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Mixed Cournot Duopoly with Two Production Periods: The Labour-Managed Firm and the Profit-Maximizing Firm

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ABSTRACT

This paper considers mixed Cournot duopoly competition with two production periods in which labour-managed and profit-maximizing firms compete against each other. The paper demonstrates that there exists a subgame perfect Nash equilibrium that coincides with the Stackelberg outcome in which the profit-maximizing firm is the leader and the labour-managed firm is the follower.

Keywords: Mixed Cournot duopoly; Two production periods; Labour-managed firm; Profit-maximizing firm

JEL Classification: C72; D21; L20

1. INTRODUCTION

The behaviour of profit-maximizing firms is most frequently encountered in the literature on economic theory. Furthermore, the behaviour of labour-managed firms has received increasing attention in recent years.[2] The pioneering work on the theoretical model of a labour-managed firm was conducted by Ward (1958). Since then, many economists have studied the behaviours of labour-managed firms such as Mai and Hwang (1989), Horowitz (1991), Okuguchi (1991), Stewart (1991, 1992), Cremer and Crémer (1992), Delbono and Rossini (1992), Chiarella (1993), Futagami and Okamura (1996), Lambertini (1997), Neary and Ulph (1997), Lambertini and Rossini (1998), Ireland (2003), Ohnishi (2008) and Cuccia and Cellini (2009).

We extend Saloner's (1987) pure Cournot duopoly model and consider a mixed Cournot market model in which a profit-maximizing firm competes against a labour-managed income-per-worker-maximizing firm. Saloner examines a pure Cournot duopoly model with two production periods in which profit-maximizing firms compete against each other and shows that any outcome on the outer envelope of the best response

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2 See Ireland and Law (1982), Stephan (1982), Bonin and Putterman (1987) and Putterman (2008) for excellent surveys of labour-managed firms.

functions between and including the firms' smallest Stackelberg outcomes is sustainable as a subgame perfect Nash equilibrium.[3]

We study the behaviours of a labour-managed firm and a profit-maximizing firm in a mixed Cournot model with two production periods. In the first production period, labour-managed and profit-maximizing firms simultaneously and non-cooperatively choose outputs. The chosen outputs become common knowledge and then, in the second production period, the firms simultaneously and non-cooperatively choose outputs. After the second period outputs have been chosen, the market opens.

The purpose of this study is to present the subgame perfect Nash equilibrium of mixed Cournot duopoly competition with two production periods in which a profit-maximizing firm competes against a labour-managed income-per-worker-maximizing firm.

The remainder of this paper is organized as follows. In Section 2, we formulate the model. Section 3 presents the equilibrium outcome of the model. Finally, Section 4 concludes the paper.

2. THE MODEL

We consider a mixed duopoly model with two production periods, where one labour-managed income-per-worker-maximizing firm (firm L) and one profit-maximizing firm (firm P) produce perfectly substitutable goods. In the remainder of this paper, superscripts L and P refer to firms L and P, respectively, and subscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when i and j are used to refer to firms in an expression, they should be understood to refer to L and P with $i \neq j$. The market price is determined by the inverse demand function $P(Q)$, where $Q = q^{\mathrm{L}} + q^{\mathrm{P}}$. We assume that $P' < 0$ and $P'' \leq 0$. The timing is as follows. In the first production period, firms L and P simultaneously and non-cooperatively choose outputs $q_1^{\mathrm{L}} \geq 0$ and $q_1^{\mathrm{P}} \geq 0$, respectively. The chosen outputs become common knowledge and then, in the second production period, the firms simultaneously and non-cooperatively choose outputs $q_2^{\mathrm{L}} \geq 0$ and $q_2^{\mathrm{P}} \geq 0$. After the second period outputs have been chosen, price is determined from the inverse demand function $P(q_1^{\mathrm{L}} + q_2^{\mathrm{L}} + q_1^{\mathrm{P}} + q_2^{\mathrm{P}})$, and the firms sell cumulative quantities $q^{\mathrm{L}} \equiv q_1^{\mathrm{L}} + q_2^{\mathrm{L}}$ and $q^{\mathrm{P}} \equiv q_1^{\mathrm{P}} + q_2^{\mathrm{P}}$.

3 Matsumura (2003) adopts Saloner's duopoly model with two production periods and investigates endogenous roles in a mixed duopoly where a welfare-maximizing public firm competes with a profit-maximizing private firm. He finds that in equilibrium the public firm cannot play the role of the Stackelberg leader, while the private firm can.

Firm L's income per worker is given by

$$\omega^{L} = \frac{P(q^{L}+q^{P})q^{L} - rq^{L} - f}{l(q^{L})} \qquad ...(1)$$

where $r > 0$ is the unit cost of capacity, $f > 0$ is the fixed cost, and l is the labour input function. We assume that $l' > 0$ and $l'' > 0$. This assumption means that the marginal quantity of labour used is increasing.

Furthermore, firm P's profit is given by

$$\pi^{P} = P(q^{L} + q^{P})q^{P} - rq^{P} - wl(q^{P}) - f, \qquad ...(2)$$

where $w > 0$ denotes the wage rate.

Firm L's best response function is defined by

$$R^{L}(q^{P}) = \arg\max_{q^{L}\geq 0}\left[\frac{P(q^{L}+q^{P})q^{L} - rq^{L} - f}{l(q^{L})}\right]. \qquad ...(3)$$

Since $l''> 0$, $R^{L}(q^{P})$ is upward sloping.[4] Furthermore, firm P's best response function is defined by

$$R^{P}(q^{L}) = \arg\max_{q^{P}\geq 0}[P(q^{L}+q^{P})q^{P} - rq^{P} - wl(q^{P}) - f]. \qquad ...(4)$$

$R^{P}(q^{L})$ is downward sloping. These reaction functions ensure that there exists a unique Cournot-Nash equilibrium, which is denoted by (N^{L},N^{P}) .

We illustrate both firms' reaction curves, which are drawn in Figure 1. R^{i} denotes firm i's reaction curve. For intuitive explanations, this figure is drawn very simply. R^{L} is upward sloping, whereas R^{P} is downward sloping. That is, firm L treats quantities as strategic complements, while in the case of profit-maximizing behaviour, quantities are strategic substitutes.[5] The reaction curves intersect at point N.

We use subgame perfection as our equilibrium concept. Since inverse demand is defined only for non-negative outputs, it is ensured that all outputs obtained in equilibrium are non-negative.

4 For the reaction functions of labour-manage firms, see Stewart (1991), Delbono and Rossini (1992), Futagami and Okamura (1996), Lambertini and Rossini (1998) and Ohnishi (2008).

5 The concepts of strategic complements and substitutes are due to Bulow, Geanakoplos and Klemperer (1985).

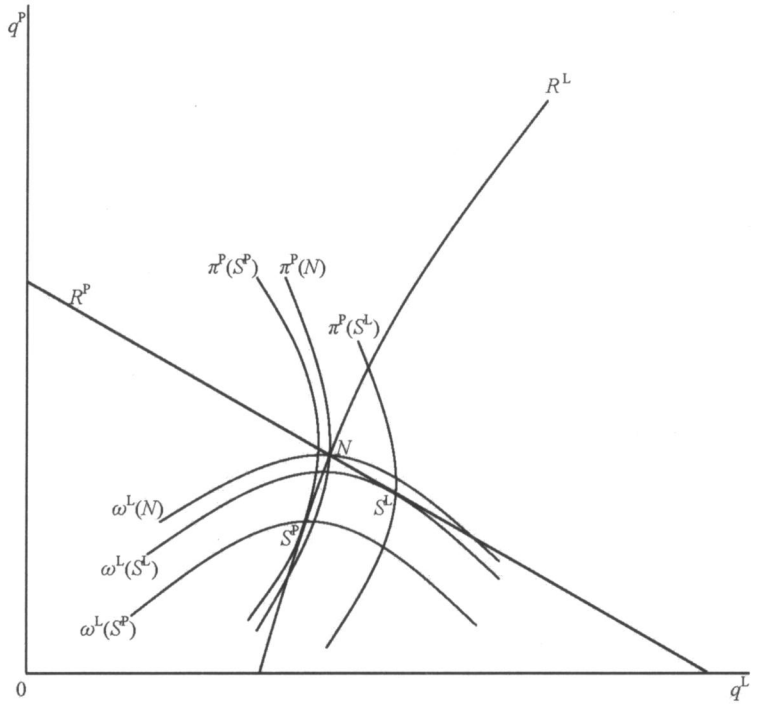

Figure 1. Reaction Curves in the Quantity Space

3. EQUILIBRIUM

In this section, we begin by defining firm i's Stackelberg leader output. Firm i selects q^i , and firm j selects q^i after observing q^i . A Stackelberg leader output for firm L (respectively firm P) is q^L (respectively q^P) such that $[P(q^L + R^P(q^L))q^L - rq^L - f]/l(q^L)$ (respectively $P(q^P + R^L(q^P))q^P - rq^P - wl(q^P) - f$) is maximized. The Stackelberg equilibrium is denoted by (L^i, F^j) , where L^i is the leader's output and F^j is the follower's. We now state the following lemma.

Lemma 1:

(i) $L^L > N^L$;

(ii) $L^P < N^P$.

Proof: First, we prove that firm L's Stackelberg leader output is higher than its Cournot output. Firm L maximizes $\bar{\omega}^L(q^L, R^P(q^L))$ with respect to q^L. Therefore, firm L's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial \omega^{L}}{\partial q^{L}} + \frac{\partial \omega^{L}}{\partial q^{P}} \frac{\partial R^{P}}{\partial q^{L}} = 0, \qquad \ldots(5)$$

where $\partial \omega^{L}/\partial q^{P} = P' q^{L}$ is negative from $P' < 0$, and $\partial R^{P}/\partial q^{L}$ is also negative. To satisfy (5), $\partial \omega^{L}/\partial q^{L}$ must be negative. From $P' < 0$, $P'' \leq 0$, $l' > 0$ and $l'' > 0$, ω^{L} and $\partial \omega^{L}/\partial q^{L}$ are concave, and at the Cournot equilibrium point, $\partial \omega^{L}/\partial q^{L}$ is zero. Thus, Lemma 1 (i) follows.

Second, we prove that firm P's Stackelberg leader output is lower than its Cournot output. Firm P maximizes $\bar{\pi}^{P}(q^{P}, R^{L}(q^{P}))$ with respect to q^{P}. Therefore, firm P's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial \pi^{P}}{\partial q^{P}} + \frac{\partial \pi^{P}}{\partial q^{L}} \frac{\partial R^{L}}{\partial q^{P}} = 0, \qquad \ldots(6)$$

where $\partial \pi^{P}/\partial q^{L}$ is negative, and $\partial R^{L}/\partial q^{P}$ is positive. To satisfy (6), $\partial \pi^{P}/\partial q^{P}$ must be positive. From $P' < 0$, $P'' \leq 0$, $l' > 0$ and $l'' > 0$, π^{P} and $\partial \pi^{P}/\partial q^{P}$ are concave, and at the Cournot equilibrium position, $\partial \pi^{P}/\partial q^{P}$ is zero. Thus, Lemma 1 (ii) follows. Q.E.D.

We now explain the equilibrium of the mixed duopoly model by using Figure 1. Here, points N, S^{L} and S^{P} denote the Cournot equilibrium outcome, the Stackelberg equilibrium outcome where firm L is the leader, and the Stackelberg equilibrium outcome where firm P is the leader, respectively. In addition, $\omega^{L}(N)$, $\omega^{L}(S^{L})$ and $\omega^{L}(S^{P})$ are firm L's iso-income-per-worker curves through N, S^{L} and S^{P}, respectively, and $\pi^{P}(N)$, $\pi^{P}(S^{L})$ and $\pi^{P}(S^{P})$ are firm P's iso-profit curves through N, S^{L} and S^{P}, respectively. From this figure, we see that $\omega^{L}(S^{P}) > \omega^{L}(S^{L}) > \omega^{L}(N)$ and $\pi^{P}(S^{P}) > \pi^{P}(N) > \pi^{P}(S^{L})$. Each firm prefers S^{P} to S^{L}. That is, firm L prefers the role of the follower, while firm P prefers the role of the leader. Therefore, S^{L} cannot be sustained as an equilibrium. Furthermore, both firm L's income per worker and firm P's profit are highest at S^{P} on $S^{P}NS^{L}$, and hence the equilibrium is at S^{P}.

The main result of this paper is described by the following proposition, which means that firm L cannot play the role of the Stackelberg leader, while firm P can.

Proposition 1: In the mixed Cournot duopoly model with two production periods, there exists a subgame perfect Nash equilibrium that occurs at the Stackelberg solution where firm P is the leader and firm L is the follower.

Proof. We consider the optimal output of each firm. Firm L aims to maximize its income per worker and can choose its Cournot output. Lemma 1 (i) shows that firm L's Stackelberg leader output is higher than its Cournot output. Hence, firm L's Stackelberg leader profit exceeds its Cournot profit. Lemma 1 (ii) states that firm P's Stackelberg leader output is lower than its Cournot output. Since $\partial\omega^L/\partial q^P = P'q^L < 0$, decreasing q^P increases ω^L given q^L. Hence, firm L's follower profit exceeds its Cournot profit. If firm L's income per worker is higher for its Stackelberg leader output than its Stackelberg follower output, then it wants to choose its Stackelberg leader output. On the other hand, if firm L's income per worker is higher for its Stackelberg follower output than its Stackelberg leader output, then it wants to choose its Stackelberg follower output. Because cycling of choices is impossible, the outcome is either (L^L, F^P) or (F^L, L^P).

Firm P aims to maximize its own profit and can choose its Cournot output. Lemma 1 (ii) states that firm P's Stackelberg leader output is lower than its Cournot output. Hence, firm P's Stackelberg leader profit exceeds its Cournot profit. Lemma 1 (i) shows that firm L's Stackelberg leader output is higher than its Cournot output. Since $\partial\pi^P/\partial q^L = P'q^P < 0$, increasing q^L decreases π^P given q^P. Firm P's follower profit is lower than its Cournot profit, and hence firm P does not want to choose its follower output.

A strategy for firm i specifies an output for the first period, and an output for the second period that is a function of q_1^L and q_1^P. We see that the equilibrium outcome is decided by the value of q_1^i. Our equilibrium concept is subgame perfection and all information in the model is common knowledge. Firm P chooses q_1^P associated with its Stackelberg leader solution, and thus the proposition follows. Q.E.D.

It is well known that in the profit-maximizing duopoly model, each firm prefers the role of the leader. Saloner (1987) investigates a two-production-period duopoly model in which duopolists are profit-maximizing firms and shows that many outcomes, including the Cournot-Nash and the two Stackelberg points, are found to be subgame perfect Nash equilibrium outcomes. On the other hand, in our case, the Stackelberg outcome where firm L is the follower is sustainable as a subgame perfect Nash equilibrium.

4. CONCLUSION

We have examined a mixed Cournot duopoly model with two production periods in which a profit-maximizing firm competes against a labour-managed firm, and have shown that there is a subgame perfect Nash equilibrium that coincides with the Stackelberg outcome where the profit-maximizing firm is the leader and the labour-managed firm is

the follower. There are many studies dealing with mixed market models that incorporate labour-managed firms. We will pursue further research on labour-managed firms in the future.

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