CHAPTER

5

Nominal and Effective Rates of Interest

INTRODUCTION

The value of money invested at an annual rate of r per annum compounded more than once in a year will be more than the value of money at r per annum compounded annually. This is because interest compounded more than one period will itself earn interest during subsequent period. For example, value of Rs. 100 at 6% per annum compounded semiannually is 100 $(1.03)^2$, *i.e.* Rs. 106.09, while the value of the same principal at 6% compounded annually is 100 (1.6) *i.e.* Rs. 106. So the effective return is Rs. 6.09. In this chapter, we discuss the concept of effective rate and how effective rate is applied in comparing different investment alternatives.

NOMINAL AND EFFECTIVE RATE

In transactions involving compound interest, the stated annual rate of interest is called nominal rate of interest. The actual percentage by which money grows during a year is called effective rate of interest. In other words, the effective rate is the simple interest rate that is equivalent to the nominal compound interest rate. Effective rates are also called annual yields or true interest rates. It should be noted that in case of effective rate of interest, interest is compounded only once in a year.

Let us understand the concept of nominal and effective rate of interest with a suitable example.

Let Rs. 100 be invested for one year at 10% compounded quarterly.

Here 10% is the nominal rate of interest.

The amount after one year is

$$A = 100\left(1 + \frac{0.10}{4}\right)^4 = 100\,(1.025)^4 = \text{Rs. } 110.38$$

$\therefore$ The actual interest earned on Rs. 100 is Rs. 10.38 in one year.

We say that the effective rate in this case is 10.38%.

Now at 10.38% per annum rate of interest, compounded annually,

$$A = 100\,(1 + 0.1038) = 100\,(1.1038) = \text{Rs. } 110.38.$$

Thus it is obvious that for a principal of Rs. 100, the amount at a rate of 10.38% compounded annually is equivalent to the amount at a rate of 10% compounded quarterly.

We therefore say that the effective rate is 10.38% and the corresponding nominal rate is 10% (compounded quarterly).

RELATION BETWEEN EFFECTIVE RATE AND NOMINAL RATE

Let r_e be the effective rate of interest and r be the corresponding nominal rate compounded 'm' times in a year.

Let P be the principal invested.

Let i be the rate of interest per conversion period.

Therefore $\quad i = \dfrac{r}{m}$

At r_e effect rate of interest, the amount after one year is given by

$$A_1 = P\,(1 + r_e) \qquad \ldots(1)$$

At r compounded m time in a year, the amount after one is given by

$$A_2 = P\left(1 + \frac{r}{m}\right)^m \qquad \ldots(2)$$

The amount at r_e effective is equivalent to the amount at r compounded m times a year.

$$\therefore \quad A_1 = A_2$$

$$\Rightarrow \quad P(1 + r_e) = P\left(1 + \frac{r}{m}\right)^m$$

$$\Rightarrow \quad 1 + r_e = \left(1 + \frac{r}{m}\right)^m$$

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Thus, the relation between nominal rate and effective rate is given by

$$\boxed{r_e = \left(1 + \frac{r}{m}\right)^m - 1}$$

Using this relation, we can find the effective rate equivalent to the nominal rate r compounded m times a year, *i.e.* equivalent to rate i per conversion period.

In the above relation, interest is compounded 'm' times a year at r nominal rate of interest. Now we derive the relation between the nominal rate and effective rate, if the nominal rate is r compounded continuously.

If the interest is compounded continuously at r per annum nominal rate, the effective rate of interest is given by

$$r_e = \lim_{m \to \infty} \left[\left(1 + \frac{r}{m}\right)^m - 1 \right]$$

$$\Rightarrow \quad r_e = \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^m - 1$$

$$\Rightarrow \quad r_e = \lim_{m \to \infty} \left[\left(1 + \frac{r}{m}\right)^{\frac{m}{r}} \right]^r - 1$$

$$\Rightarrow \quad r_e = \left[\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{\frac{m}{r}} \right]^r - 1$$

$$\Rightarrow \quad r_e = \left[\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{\frac{1}{\left(\frac{r}{m}\right)}} \right]^r - 1$$

$$\Rightarrow \quad r_e = \left[\lim_{x \to 0} (1 + x)^{1/x} \right]^r - 1 \qquad \because \text{ Let } x = \frac{r}{m} \text{ as } m \to \infty, x \to 0$$

$$\Rightarrow \quad r_e = e^r - 1$$

Thus the relation between the effective rate and nominal rate in case of nominal rate r compounded continuously is given by

$$\boxed{r_e = e^r - 1}$$

The nominal rate r compounded continuously and equivalent to a given effective rate, r_e, is called the *force of interest*.

Remark 1 : If the conversion period is one year, at r nominal rate of interest, that is, if $m = 1$, then the effective rate equals nominal rate.

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

$$r_e = \left(1 + \frac{r}{1}\right)^1 - 1 = 1 + r - 1 = r \qquad \because m = 1$$

Thus, $\quad r_e = r.$

Remark 2 : The effective rate of interest depends only on the nominal rate r and the number of conversion periods in a year 'm' but is independent of the principal P.

Remark 3 : The effective rate can be a useful guide for comparing alternative investment opportunities available to an individual.

Example 1 : *Find the effective rate of interest corresponding to 8% nominal rate compounded quarterly.*

Solution : Nominal rate of interest, $r = 8\% = 0.08$

Interest is compounded quarterly.

$\therefore$ No. of conversions per year $m = 4$ and $\frac{r}{m} = \frac{0.08}{4} = 0.02 = 4$

Effective rate of interest $r_e = \left(1 + \frac{r}{m}\right)^m - 1$

$$\therefore \quad r_e = (1 + 0.02)^4 - 1 = (1.02)^4 - 1$$
$$= 1.0824 - 1 = 0.824 = 8.24\%$$

$\therefore$ The required effective rate is 8.24%.

Example 2 : *Find the effective rate equivalent to the normal rate of interest 6% compounded continuously.*

Solution : In case of continuous compound interest, the effective rate is given by

$$r_e = e^r - 1$$

Here $\quad r = 6\% = 0.06$

$$\therefore \quad r_e = e^{0.6} - 1$$
$$\Rightarrow \quad r_e = 1.0618 - 1 = 0.0618 = 6.18\%$$

$\therefore$ The effective rate of interest is 6.1%.

Example 3 : *Find the effective rate of interest equivalent to the nominal rate 9% converted (a) semi-annually, (b) quarterly, (c) monthly and (d) continuously.*

Solution :

(*a*) *In case of Semi-annual conversion* :

The effective rate r_e equivalent to the nominate r converted m time a year is given by

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Given that $\quad r = 9\% = 0.09$

$\therefore$ Here, interest is compounded semi-annually,

$$\therefore \quad m = 2$$
$$\therefore \quad r_e = \left(1 + \frac{0.09}{2}\right)^2 - 1$$
$$\therefore \quad r_e = (1.045)^2 - 1$$
$$\therefore \quad r_e = 1.0920 - 1 = 0.920 = 9.2\%$$

$\therefore$ The effective rate is 9.2%.

(*b*) *In case of quarterly conversion :*

$$m = 4$$

$$\therefore \qquad r_e = \left(1 + \frac{0.09}{4}\right)^4 - 1$$

$$r_e = (1.0225)^4 - 1$$

$$r_e = 1.0931 - 1 = 0.0931 = 9.31\%$$

$\therefore$ The effective rate is 9.31%

(*c*) *In case of monthly conversion :*

$$m = 12$$

$$\therefore \qquad r_e = \left(1 + \frac{0.09}{12}\right)^{12} - 1$$

$$r_e = (1.0075)^{12} - 1$$

$$r_e = 1.0938 - 1 = 0.0938 = 9.38\%$$

The effective rate is 9.38%.

(*d*) *In case of interest converted continuously :*

The effective rate of interest r_e equivalent to the nominal rate r compounded continuously is given by

$$r_e = e^r - 1$$

Here $\qquad r_e = e^{0.09} - 1$

$$r_e = 1.0942 - 1 = 0.0942 = 9.42\%$$

The effective rate is 9.42%.

Example 4 : *Find the force of interest corresponding to the effective rate 6%.*

***Solution* :** The force of interest r corresponding to the effective rate r_e is given y

$$r_e = e^r - 1$$

Here $\qquad r_e = 6\% = 0.06$

$$\therefore \qquad 0.06 = e^r - 1$$

$$\Rightarrow \qquad e^r = 1.06$$

Taking log on both sides we get

$$r \log e = \log 1.06$$

$$r\,(0.4343) = .0253$$

$$\Rightarrow \qquad r = \frac{0.0253}{0.4343} = 0.583 = 5.83\%$$

The force of interest is 5.83%.

Example 5 : *Find the nominal rate compounded quarterly which is equivalent to the effective rate 6%.*

Solution : The relation between the nominal rate and effective rate is given by

$$r_e = \left(1+\frac{r}{m}\right)^m - 1$$

Here $\quad r_e = 6\% = 0.06 \quad$ and $\quad m = 4$

$$\therefore \quad 0.06 = \left(1+\frac{r}{4}\right)^4 - 1$$

$$\Rightarrow \quad \left(1+\frac{r}{4}\right)^4 = 1.06$$

Taking log on both sides, we get

$$4\log\left(1+\frac{r}{4}\right) = \log 1.6$$

$$\Rightarrow \quad 4\log\left(1+\frac{r}{4}\right) = 0.0253$$

$$\Rightarrow \quad \log\left(1+\frac{r}{4}\right) = .0063$$

$$\Rightarrow \quad 1+\frac{r}{4} = \text{antilog}\,(0.0063)$$

$$\Rightarrow \quad 1+\frac{r}{4} = 1.0146$$

$$\Rightarrow \quad \frac{r}{4} = 0.146$$

$$\Rightarrow \quad r = 0.0584 = 5.84\%$$

$\therefore$ The nominal rate is 5.84%.

Example 6 : *Which is the better investment from the stand point of the investor : 6.2% compounded semi-annually or 6% compounded monthly.*

Solution : The effective rate (r_e) corresponding to the nominal rate r is given by

$$r_e = \left(1+\frac{r}{m}\right)^m - 1$$

In case of 6.2% compounded semi-annually :

$$m = 2, \quad \text{amd } r = 6.2\% = 0.062$$

Here

$$r_e = \left(1+\frac{0.062}{2}\right)^2 - 1$$

$$= (1+0.031)^2 - 1 = (1.031)^2 - 1$$

$$= 1.06296 - 1 = 0.6296 = 6.3\%$$

In case of interest compounded monthly :

$$r = 6\% = 0.06 \quad \text{and} \quad m = 12$$

$$r_e = \left(1 + \frac{0.06}{12}\right)^{12} - 1$$
$$= (1.005)^{12} - 1 = 1.06168 - 1$$
$$= 0.06168 = 6.17\%.$$

The effective rate of interest corresponding to the nominal rate 6.2% compounded semi-annually is greater.

$\therefore$ The first investment (6.2%) is better investment.

Example 7 : *When is a better investment : 7.8% compounded semi-annually or 8% compounded annually.*

Solution : *In case of 7.8% compounded semi-annually :*

The effective rate 'r_e' corresponding to the nominal rate r is given by

$$r_e = \left(1 + \frac{r}{m}\right)^{m} - 1$$

Here $r = 7.8\% = 0.078$ and $m = 2$

$$\therefore \quad r_e = \left(1 + \frac{.078}{2}\right)^{2} - 1$$

$$\Rightarrow \quad r_e = (1.039)^2 - 1$$

$$\Rightarrow \quad r_e = 1.0795 - 1 = 0.0795 = 7.95\%.$$

In case of 8% compounded annually :

Here $r = 8\%$

If the nominal rate r is compounded annually, then the effective rate $r_e = r$

$\therefore \quad r_e = r = 8\%$

Here, the effective rate of interest corresponding to the nominal rate of interest 8% compounded annually is greater than the effective rate corresponding to the nominal rate 7.8% compounded semi-annually.

$\therefore$ 8% compounded annually is better investment.

Example 8 : *Find the nominal rate compounded monthly equivalent to 5% compounded semi-annually.*

Solution : Let r be the nominal rate compounded monthly.

The effective rate 'r_e', corresponding to the nominal rate converted m times a year is given by

$$r_e = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$\therefore$ In case of r compounded monthly

$$m = 12 \quad \text{and} \quad r_e = \left(1 + \frac{r}{12}\right)^{12} - 1 \qquad \ldots(1)$$

In case 5% compounded semi-annually,

$$r = 5\% = 0.05 \quad \text{and} \quad m = 2$$

$$\therefore \qquad r_e = \left(1 + \frac{0.05}{2}\right)^2 - 1$$

$$\Rightarrow \qquad r_e = (1.025)^2 - 1 \qquad \ldots(2)$$

Given that (1) is equivalent to (2)

$$\therefore \quad \left(1 + \frac{r}{12}\right)^{12} - 1 = (1.025)^2 - 1$$

$$\Rightarrow \quad \left(1 + \frac{r}{12}\right)^{12} = (1.025)^2$$

Taking log on both sides, we get

$$12 \log \left(1 + \frac{r}{12}\right) = 2 \log 1.025$$

$$\Rightarrow 12 \log \left(1 + \frac{r}{12}\right) = 2 \times 0.0107$$

$$\Rightarrow \quad \log \left(1 + \frac{r}{12}\right) = \frac{2 \times 0.107}{12}$$

$$\Rightarrow \quad 1 + \frac{r}{12} = \text{antilog}\,(0.0018)$$

$$\Rightarrow \quad 1 + \frac{r}{12} = 1.004$$

$$\Rightarrow \quad \frac{r}{12} = 0.004 \Rightarrow r = 0.048 = 4.8\%$$

$\therefore$ The required nominal rate is 4.8%.

Example 9 : *What nominal rate compounded quarterly will be equivalent to 10% compounded continuously ?*

Solution : Let r be the nominal rate compounded quarterly.

The effective rate r_e, corresponding to the nominal rate r converted m times a year is given by

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

In case of r compounded quarterly

$$m = 4$$

$$\therefore \qquad r_e = \left(1 + \frac{r}{4}\right)^4 - 1 \qquad \ldots(1)$$

The effective rate r_e, corresponding the nominal rate r converted continuously is given by

$$r_e = e^r - 1$$

In case of 10% compounded continuously

$$r = 10\% = 0.10$$

$$\therefore \quad r_e = e^{0.10} - 1 \qquad \text{...(2)}$$

Given that (1) is equivalent to (2)

$$\therefore \quad \left(1 + \frac{r}{4}\right)^4 - 1 = e^{0.10} - 1$$

$$\Rightarrow \quad \left(1 + \frac{r}{4}\right)^4 = e^{0.10}$$

Taking log on both sides, we get

$$4 \log \left(1 + \frac{r}{4}\right) = 0.10 \times \log e$$

$$\Rightarrow \quad 4 \log \left(1 + \frac{r}{4}\right) = 0.10 \times 0.4343$$

$$\Rightarrow \quad \log \left(1 + \frac{r}{4}\right) = \frac{0.04343}{4} = 0.0109$$

$$\Rightarrow \quad 1 + \frac{r}{4} = \text{antilog}\,(0.0109)$$

$$\Rightarrow \quad 1 + \frac{r}{4} = 1.0254 \Rightarrow \frac{r}{4} = 0.0254$$

$$\Rightarrow \quad r = 0.1016 = 10.16\%$$

The nominal rate is 10.16%.

Example 10 : *A money-lender charges interest at the rate of 10 paise per rupee per month, payable in advance. What effective rate of interest does he charge per annum ?*

Solution **:** The money lender charges interest at the rate of 10 paise per rupee per month, payable in advance.

Therefore, 10 paise may be treated as interest on 90 paise for one month.

$\therefore$ The interest rate per month $i = \frac{10}{90} = \frac{1}{9}$.

$$i.e. \quad \frac{r}{12} = \frac{1}{9}$$

The effective rate r_e corresponding to the nominal rate r is given by

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Here $\quad m = 12$

$$\therefore \quad r_e = \left(1 + \frac{1}{9}\right)^{12} - 1$$

$$\Rightarrow \quad r_e + 1 = \left(\frac{10}{9}\right)^{12}$$

Taking log on both sides we get

$$\log(1 + r_e) = 12 \log\left(\frac{10}{9}\right)$$

$$\Rightarrow \quad \log(1 + r_e) = 12\,[\log 10 - \log 9]$$

$$\Rightarrow \quad \log(1 + r_e) = 12\,(1 - 0.9542)$$

$$\Rightarrow \quad \log(1 + r_e) = 12\,(0.0458)$$

$$\Rightarrow \quad \log(1 + r_e) = 0.5496$$

$$\Rightarrow \quad 1 + r_e = \text{antilog}\,(0.5496)$$

$$\Rightarrow \quad 1 + r_e = 3.545$$

$$r_e = 2.545 = 254.5\%$$

The effective rate is 254.5%

Example 11 : *A deposited an amount in a bank for 10 years at the effective rates of interest 3% per annum for 4 years, 4% per annum for next 4 year and 5% per annum for the last two years. B deposited the same amount in an another bank at a constant rate of interest compounded semi-annually. After 10 years if both A and B get same accumulate amount, at what rate of interest B deposited his money ?*

***Solution* :** Let Re. 1 be the principal.

The amount is given by $A = P(1 + i)^n$

For A, the Amount after 10 years is

$$A_1 = 1\,(1.03)^4\,(1.04)^4\,(1.05)^2 \qquad \ldots(1)$$

Let r be the rate of interest for B.

Interest compounded semi-annually, $\therefore \; i = \frac{r}{2}$.

For B, the amount after 10 years is

$$A_2 = 1\,(1 + i)^{20} = (1 + i)^{20} \qquad \ldots(2)$$

To get same accumulated amount,

$$(1) = (2)$$

$$\Rightarrow \quad (1 + i)^{20} = (1.03)^4\,(1.04)^4\,(1.05)^2$$

$$\Rightarrow \quad (1 + i)^{20} = 1.4516$$

Taking log on both sides we get

$$20 \log(1 + i) = \log(1.4516)$$

$$\log(1 + i) = \frac{0.1620}{20} = 0.0081$$

$$1 + i = \text{antilog}\,(0.0081)$$
$$1 + i = 1.019$$
$$i = 0.019 = 1.9\%$$
$$r = 2 \times i = 2 \times 1.9 = 3.8\%$$

Rate of interest for B is 3.8%.

EXERCISES

1. Find the effective rate of interest corresponding to 12% nominal rate compounded monthly. [**Ans.** 12.68%]
2. What is the effective rate which is equivalent to a nominal rate 10% per annum compounded semi-annually ? [**Ans.** 10.25%]
3. Find the effective rate equivalent to the nominal rate 5% compounded continuously. [**Ans.** 5.13%]
4. Find the effective rate equivalent to the nominal rate 8% compounded continuously. [**Ans.** 8.33%]
5. Find the effective rate equivalent to the nominal rate of 7%, converted (*a*) quarterly, (*b*) monthly and (*c*) continuously. [**Ans.** (*a*) 7.19%, (*b*) 7.22%, (*c*) 7.25%]
6. Find the effective rate equivalent to the nominal rate 6% converted (*a*) monthly, (*b*) continuously. [**Ans.** (*a*) 6.16%, (*b*) 6.18%]
7. Find the force of interest corresponding to the effective rate 8%. [**Ans.** 7.69%]
8. What annual rate compounded continuously is equivalent to an effective rate of 5% ? [**Ans.** 5.82%]
9. Find the force of interest corresponding to the effective rate 9%. [**Ans.** 8.61%]
10. Find the effective rate of interest corresponding to nominal rate of 5% per annum compounded (*i*) semi-annually, (*ii*) quarterly, and (*iii*) monthly. [**Ans.** (*i*) 5.06%, (*ii*) 5.09%, (*iii*) 5.12%]
11. A certain bank offers an interest rate of 6% per annum compounded annually. A competing bank compounds its interest continuously. What nominal rate should the competing bank offer so that the effective rates of the two banks will be equal ? [**Ans.** 5.82%]
12. Find the nominal rate compounded monthly equivalent to the effective rate 12% per annum. [**Ans.** 11.4%]
13. Which is a better investment from the stand point of the investor : 4% per annum compounded quarterly or 4.1% effective ? [**Ans.** 4.1% effective]
14. Which is better from the stand point of investor : 6.1% converted quarterly or 6% converted continuously ? [**Ans.** 6.1 quarterly]
15. Which is a better investment : 5.5% compounded semi-annually or 5% compounded monthly ? [**Ans.** 5.5 semi-anually]

16. Bank 'A' offers interest at an annual rate of 9.1% compounded semi-annually, and bank 'B' offers interest at 9% compounded monthly. Which Bank offers the better deal? [**Ans.** Bank B]

17. Find the nominal rate compounded quarterly equivalent to 6% compounded semi-annually. [**Ans.** 5.92%]

18. What nominal rate of interest compounded monthly will be equivalent to 8% compounded continuously? [**Ans.** 8.04%]

19. Find the compound interest compounded semi-annually equivalent to 9% compounded monthly. [**Ans.** 9.18%]

20. A money lender charges interest at the rate of 5 paise per one rupee per quarter, payable in advance. What effective rate does he change per annum? [**Ans.** 22.8%]

21. A money lender charges interest at the rates of 10 rupees per 100 rupees per half year, payable in advance. What effective rate of interest does he charge per annum? [**Ans.** 23.5%]

22. A deposits an amount in a bank for 16 years at the effective rates of interest 3% per annum for 10 years, 4% per annum for 4 years and 5% per annum for the last two years. B deposits the same amount in another bank at a constant force of interest. After 16 years if both A and B get same accumulated amount, at what rate of interest B deposited his money? [**Ans.** 3.43%]

23. What do you mean by nominal rate of interest? How does it differ from effective rate of interest? Establish the relationship between the nominal and effective rate of interest (a) when compounded m times a year and (b) when compounded continuously? When will nominal rate be equal to effective rate of interest?