

CHAPTER

6

Equation of Value

INTRODUCTION

We have so far discussed the kinds of interest as well as the method of converting nominal value into effective rate. By using effective rate, we generally compare different investment alternatives. We have also seen that money has time value. A sum of money today will not be the same in value tomorrow. The question now is how to find the value of a certain financial at a future date which is equivalent to some other obligation(s). Answer is the equation of value. In this chapter, we study the concept of equation of value in a detailed manner.

EQUATION OF VALUE

Money has different values at different times. In business transactions, different sums are flowing at different points of time. Such flows can be equated at a particular point of time. An equation of value is an equation which states that the sum of the values, on a given date, of one set of obligations is the same as the sum of values, on this date, of another set of obligations. The date chosen for comparing the values of two sets of obligations is called the focal date or comparison date.

The equation of value is given by :

Sum of the values of one set of obligations (old obligations) on focal date } = { Sum of the values of another set of obligations (new obligations) on focal date

Let Rs. x be a non-interest bearing debt which is due at some specified time. The value of this debt at the due date is of course Rs. x . Suppose that money is worth i per period. If the debt is not repaid on due date and paid ' n ' periods after due date, then the value of the money is of course more than Rs. x and is found by multiplying x by $(1 + i)^n$. But if the debt is

repaid 'n' periods before due date, then the value of the money is less than Rs. x and is found by multiplying x by $(1 + i)^{-n}$.

Thus,

- (a) Value of Rs. x, n periods after due date at the rate i per period is $x(1 + i)^n$.
- (b) Value of Rs. x, n periods before due date at at the rate i per period is $x(1 + x)^{-n}$.

Remark :

- (a) Focal date is decided on by the lender and the borrower. In case of only one new obligation, the due date is the focal date. If there are more than one new obligations, then focal date is generally the due date of the last obligation.
- (b) The flows of money involved may be involving simple interest or compound interest. Simple interest is used only if it is mentioned in the problem, otherwise it is treated as compound interest.
- (c) It should be noted that if two set of obligations have equal values on one date, then they will have equal values on any other date, provided rate of interest is same for each obligations.
- (d) If the interest is calculated on the basis of simple interest, the conversion should be based on the methods of conversion which we have discussed in the chapter : Simple Interest.
- (e) If the problem is to find the focal date, or the number of periods of conversion (months/years), we assume that the focal date is today.

Example 1 : A man borrowed Rs. 10,000 from a money lender at 9% simple interest and agreed to make two equal payments, one due in 6 months and the other in 12 months. Find the payment at the end of 12 months.

Solution : Let Rs. x be the amount of equal payment.

Focal date is 12 months.

$$r = 9\% = 0.09$$

The old obligations and new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 10,000 now	10,000 (1.09)	Rs. x at the end of 6 months	$x(1 + (0.09) \times 1/2)$
		Rs. x at the end of 12 months	x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

∴ The equation is :

$$\begin{aligned} 10,000 (1.09) &= x (1.045) + x \\ \Rightarrow 10,900 &= 2.045 x \\ \Rightarrow x &= \text{Rs. } 5,330.07 \end{aligned}$$

Payment at the end of 12 years is Rs. 5,330.07.

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Example 2 : A man owes Rs. 2,000 due in 2 months, Rs. 1,000 due in 5 months and Rs. 1,800 due in 9 months. He wishes to discharge his obligations by two equal payments due in 6 and 12 months respectively. Find the equal payments if money is worth 6% simple interest and at the end of 1 year is the agreed focal date.

Solution : Let Rs. x be the equal payments.

The focal date is 1 year.

Rate of interest is 6% simple interest.

The old obligations and new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 2,000 due in 2 months	$2,000 \left[1 + \left(0.06 \times \frac{10}{12} \right) \right]$	Rs. x due in 6 months	$x \left[1 + \left(0.06 \times \frac{6}{12} \right) \right]$
Rs. 1,000 due in 5 months	$1,000 \left[1 + \left(0.06 \times \frac{7}{12} \right) \right]$	Rs. x in 1 year	x
Rs. 1,800 due in 9 months	$1,800 \left[1 + \left(0.06 \times \frac{3}{12} \right) \right]$		

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

\therefore The equation is

$$2,000 \left[1 + \left(0.06 \times \frac{10}{12} \right) \right] + 1,000 \left[1 + \left(0.06 \times \frac{7}{12} \right) \right] + 1,800 \left[1 + \left(0.06 \times \frac{3}{12} \right) \right] = x \left[1 + \left(0.06 \times \frac{6}{12} \right) \right] + x$$

$$\Rightarrow 2,000 (1 + 0.05) + 1,000 (1 + 0.035) + 1,800 (1 + 0.015) = x (1 + 0.03) + x$$

$$\Rightarrow (2,000 \times 1.05) + (1,000 \times 1.035) + 1,800 (1.015) = x (1.03) + x$$

$$\Rightarrow 2,100 + 1,035 + 1,827 = 2.03 x$$

$$\Rightarrow 2.03 x = 4,962$$

$$\Rightarrow x = 2,444.33$$

\therefore The installment amount is Rs. 2,444.33.

Example 3 : What single payment 5 years hence will discharge the debt of Rs. 800 and Rs. 500 in 3 years and 9 years respectively, if the money is worth 6% compounded quarterly ?

Solution : Let Rs. x be the amount due in 5 years

The focal date = 5 years

$$r = 6\% \quad \therefore i = \frac{r}{4} = \frac{0.06}{4} = 0.015$$

The old obligations and new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
800 due in 3 years	$800 (1.015)^{2 \times 4}$	x at the end of 5 years	
500 due in 9 years	$500 (1.015)^{-4 \times 4}$		x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

$$\text{The equation is : } 800 (1.015)^8 + 500 (1.015)^{-16} = x$$

$$\Rightarrow x = 900.8 + 394.94$$

$$\Rightarrow x = \text{Rs. } 1,295.74$$

\therefore The required payment is Rs. 1,295.74.

Example 4 : A debt of Rs. 5,000 due in five years is to be repaid by a payment of Rs. 2,000 now and a second payment at the end of 6 years. How much should the second payment be if the rate of interest is 6% compounded quarterly ?

Solution : Let Rs. x be the second payment.

The focal date is 6 years.

Rate of interest $r = 6\% = 0.06$.

Interest is compounded quarterly.

$$\therefore i = \frac{r}{4} = \frac{0.06}{4} = 0.015$$

The old obligations and new obligations are shown in the following table:

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 5,000 due in 5 years	$5,000 (1.015)^4$	Rs. 2,000 now	$2,000 (1.015)^{6 \times 4}$
		Rs. x at the end of 6 years	x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

$$5,000 (1.015)^4 = 2,000 (1.015)^{6 \times 4} + x$$

$$\Rightarrow 5,306.82 = 2,859.01 + x$$

$$\Rightarrow x = 5,306.82 - 2,859.01$$

$$\Rightarrow x = 2,447.81$$

\therefore The second payment is Rs. 2,447.81

Example 5 : Mr. X owes Rs. 1,000 due in 1 year and Rs. 3,000 due in 4 years. He agrees to pay Rs. 2,000 today and the remainder in 2 years. How much he pay at the end of 2 years if the money is worth 5% compounded semi-annually ?

Solution : Let Rs. x be the final payment due at the end of 2 years.

The focal date is 2 years.

$$r = 5\% = 0.05$$

Interest is calculated half-yearly

$$\therefore i = \frac{0.05}{2} = 0.025$$

The old obligations and the new obligations are shown in the following table.

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 1,000 due in 1 year	1,000 $(1.025)^{1 \times 2}$	Rs. 2,000 today	2,000 $(1.025)^{2 \times 2}$
Rs. 3,000 due in 4 years	1,000 $(1.025)^{-2 \times 2}$	Rs. x due in 2 years.	x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

The required equation

$$\therefore 1,000 (1.025)^2 + 3,000 (1.025)^{-4} = 2,000 (1.025)^4 + x$$

$$\Rightarrow 1,050.625 + 2,717.852 = 2,207.626 + x$$

$$\Rightarrow 3,768.477 = 2,207.626 + x$$

$$\therefore x = \text{Rs. } 1,560.85$$

\therefore The amount to be paid is Rs. 1,560.85.

Example 6 : A debt of Rs. 2,000 due in 2 years and Rs. 3,000 due in 7 years is to be repaid by a single payment of Rs. 1,000 now and 2 equal payments which are due 1 year from now and 4 years from now. If the interest rate is 6% compounded annually, how much will be the equal payments ?

Solution : Let Rs. x be the equal payment.

The focal date is 4 years

$$i = r = 6\% = 0.06$$

The old obligations and now obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 2,000 due in 2 years	2,000 $(1.06)^2$	Rs. 1,000 now	1,000 $(1.06)^4$
Rs. 3,000 due in 7 years	3,000 $(1.06)^{-3}$	Rs. x in 1 year	$x (1.06)^3$
		Rs. x due in 4 years	x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

∴ The required equation

$$2,000 (1.06)^2 + 3,000 (1.06)^{-3} = 1,000 (1.06)^4 + x (1.06)^3 + x$$

$$\Rightarrow 2,247.2 + 2,518.86 = 1,262.48 + 1.191x + x$$

$$\Rightarrow 2.191x = 3,503.58$$

$$\therefore x = 1,599.01$$

∴ The equal payment is Rs. 1,599.01.

Example 7 : Mr. X agrees to pay Rs. 800 due in 2 years without interest and Rs. 300 due in 9 years with 6% annual effective rate of interest. He wishes to repay these debts in 2 equal installments due 4 years and 5 years respectively. If money worth 4% converted semi-annually, how much should each installment be ?

Solution : Let Rs. x be the amount of equal installments.

The focal date is 5 years.

$$r = 4\% = 0.04$$

Interest is compounded semi-annually.

$$\therefore i = \frac{r}{2} = 0.02.$$

The old obligations and new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 800 due in 2 years	$800 (1.02)^{3 \times 2}$	Rs. x due in 4 years	$x (1.02)^{1 \times 2}$
Rs. 300 due in 9 years with 6% effective rate interest [i.e. $300 (1.06)^9$]	$300 (1.06)^9 (1.02)^{-4 \times 2}$	Rs. x due in 5 years	x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

∴ The equation is

$$800 (1.02)^6 + 300 (1.06)^9 (1.02)^{-8} = x (1.02)^2 + x$$

$$\Rightarrow 800 (1.12616) + 30 (1.68948) (0.85439) = 1.0404x + x$$

$$\Rightarrow 900.93 + 432.59 = 2.0404x$$

$$\Rightarrow 1,333.52 = 2.0404x$$

$$\therefore x = \text{Rs. } 653.56$$

∴ The required amount is Rs. 653.56.

Example 8 : Mr. X secured two loans from a bank : one for Rs. 8,000 due in 3 years and another one for Rs. 15,000 due in 6 years, both at an interest rate of 10% per annum compounded semi-annually. The bank has agreed to allow the two loans to be consolidated into one loan payable in 5 years at the same interest rate. What amount will Mr. X be required to pay the bank at the end of 5 years ?

Solution : Let Rs. x be the amount to be repaid by Mr. X.

The focal date is 5 years.

$$r = 10\% = 0.10$$

Interest is compounded semi-annually.

$$\therefore i = \frac{r}{2} = \frac{0.10}{5} = 0.05$$

The old obligations and the new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 8,000 due in 3 years @ 10% p.a. compounded semi-annually i.e. $8,000 (1.05)^6$	$8,000 (1.05)^6 (1.05)^4$	Rs. x at the end of 5 years	x
Rs. 15,000 due in 6 years @ 10% p.a. compounded semi-annually i.e. $5,000 (1.05)^{12}$	$15,000 (1.05)^{12} (1.05)^{-2}$		

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

\therefore The equation is :

$$8,000 (1.05)^6 (1.05)^4 + 15,000 (1.05)^{12} (1.05)^{-2} = x$$

$$\Rightarrow 8,000 (1.05)^{10} + 15,000 (1.05)^{10} = x$$

$$\Rightarrow 8,000 (1.62889) + 15,000 (1.62889) = x$$

$$13,031.12 + 24,433.35 = x$$

$$\Rightarrow x = 37,464.47$$

\therefore Mr. X has to pay Rs. 37,464.47.

Example 9 : A debt of Rs. 1,500 due in 3 years without interest, Rs. 2,000 due in 5 years with 8% p.a. compounded half-yearly and Rs. 5,000 due in 7 years with 7% annually effective rate of interest is to be repaid by 3 equal installments due 3 years, 4 years, 5 years, respectively. If the money is worth 6% convertible quarterly, how much should be each installment ?

Solution : Let Rs. x be the amount of each installments.

The focal date is 5 years

$$r = 6\% = 0.06$$

Given that interest is compounded quarterly

$$\therefore i = \frac{r}{4} = \frac{0.6}{4} = 0.015$$

The old obligation and new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 1,500 due in 3 years without interest	$1,500 (1.015)^{2 \times 4}$	Rs. x due in 3 years	$x(1.015)^{2 \times 4}$
Rs. 2,000 due in 5 years at 8% compounded half yearly interest [i.e. $(2,000 (1.04)^{10})$]	$2,000 (1.04)^{10}$	Rs. x due in 4 years	$x (1.015)^{1 \times 4}$
Rs. 5,000 due in 7 years at 7% effective rate [i.e. $5,000 (1.07)^7$]	$5,000 (1.07)^7 (1.015)^{-2 \times 4}$	Rs. x due in 5 years	x

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

$$1,500 (1.015)^8 + 2,000 (1.04)^{10} + 5,000 (1.07)^7 (1.015)^{-8} = x (1.015)^8 + x (1.015)^4 + x$$

$$1,689.73 + 2,960.48 + 7127.35 = 1.1264x + 1.0613x + x$$

$$11,777.56 = 3.1877x$$

$$x = \frac{11,777.56}{3.1877}$$

$$x = \text{Rs. } 3,694.69$$

\therefore The required amount is Rs. 3,694.69

Example 10 : A debt of Rs. 3,000 which is due 6 years from now is instead to be paid off by 3 payments : Rs. 500 now, Rs. 1,500 in 3 years and final payment of Rs. 475 at the end of n years. The rate of interest is 6% effective. Find the value of n .

Solution : Let the focal date be today

$$i = r = 6\% = 0.06$$

The old obligations and new obligations are shown in the following table :

Old Obligations	Value of each at focal date	New Obligations	Value of each at focal date
Rs. 3,000 due in 6 years	$3,000 (1.06)^{-6}$	500 now	500
		1,500 in 3 years	$1,500 (1.06)^{-3}$
		475 in ' n ' years	$475 (1.06)^{-n}$

The equation of value is given by :

$$\left. \begin{array}{l} \text{Sum of the values of old obligations} \\ \text{at focal date} \end{array} \right\} = \left\{ \begin{array}{l} \text{Sum of the values of new obligations} \\ \text{at focal date} \end{array} \right.$$

∴ The equation is

$$3,000 (1.06)^{-6} = 500 + 1,500 (1.06)^{-3} + 475 (1.06)^{-n}$$

$$\Rightarrow 3,000 \times 0.705 = 500 + 1,500 \times 0.8396 + 475 (1.06)^{-n}$$

$$\Rightarrow 2,115 = 500 + 1,259 + 475 (1.06)^{-n}$$

$$\Rightarrow 475 (1.06)^{-n} = 356$$

$$(1.06)^{-n} = \frac{356}{475} \Rightarrow (1.06)^n = \frac{475}{356}$$

$$\Rightarrow (1.06)^n = 1.3343$$

∴ Taking log on both sides

$$n \log 1.06 = \log 1.3343$$

$$n = \frac{\log 1.3343}{\log 1.06}$$

$$n = \frac{0.1252}{0.0253}$$

$$n = 4.95 \approx 5 \text{ years}$$

∴ The value of n is 5 years

EXERCISES

1. A man borrowed Rs. 10,000 from a money lender at 8% simple interest and agreed to pay Rs. 5,000 of loan in 6 months. What payment one year from now will settle the debt ?
[Ans. Rs. 5,600]
2. Mr. X purchased a television for Rs. 6,000. He paid Rs. 500 cash down and agreed to pay the balance at 5% simple interest. If he paid Rs. 3,000 three months after purchase and Rs. 1,500 six months later, what final payment one year after the date of purchase will discharge his obligations ? Assume that the focal date is at the end of 12 months.
[Ans. Rs. 1,125]
3. Ram borrows Rs. 50,000 now and agrees to repay Rs. 10,000 in 2 months and Rs. 15,000 in 6 months. What final payment should he make at the end of 18 months to settle down his indebtedness, at 12% simple interest, assuming that the focal date is today ?
[Ans. Rs. 31,829.27]
4. At 5% simple interest, find the value of the following obligations : Rs. 2,000 due today, Rs. 5,000 due in 6 months with interest 6% per annum and Rs. 1,000 due in 1 year with interest at 8% per annum.
 - (a) use today as the focal date.
 - (b) use 1 year from today as the focal date. [Ans. (a) Rs. 17,310.10, (b) Rs. 18,178.75]
5. Mr. X owes Rs. 500 due in 2 months, Rs. 1,000 due in 5 months and Rs. 1,500 due in 8 months. He agrees to pay two equal payments, one due in 6 months and the other

- due in 10 months. Find the payment, if money is worth at 6% simple interest and at the end of 10 months is the agreed focal date. [Ans. Rs. 1,514.85]
6. X owes Y Rs. 1,000 due in 6 months without interest and Rs. 2,000 with interest for $1\frac{1}{2}$ years at 4% due in 9 months. Y agrees to accept 3 equal payments, one due today, another in 6 months and the third in 1 year. Find the equal payments using 1 year from today as focal date, if money is worth 5% to Y . [Ans. Rs. 1,031.38]
7. A debt of Rs. 4,000 due 3 years hence and another Rs. 10,000 due 8 years hence are to be repaid by a single payment 4 years hence. If the rate of interest is 6% per annum effective, how much is this payment? [Ans. Rs. 12,160.94]
8. A debt of Rs. 200 due 2 years hence and another of Rs. 500 due 7 years hence are to be paid off by a single payment 3 years hence. If the rate of interest is 5% per annum effective, how much is this payment? [Ans. Rs. 621.35]
9. A debt of Rs. 10,000 due in 4 years is to be repaid by a payment of Rs. 3,000 now and a second payment at the end of 6 years. How much should be the second payment, if the rate or interest is 8% compounded quarterly? [Ans. Rs. 6,891.28]
10. A debt of Rs. 30,000 which is due 6 years from now is to repaid by three payments : Rs. 5,000 now, Rs. 15,000 in 3 years and a final payment at the end of 5 years. If the interest rate is 6% compounded annually, how much is the final payment? [Ans. 4,756.76]
11. A man owes Rs. 10,000 due in 1 year and Rs. 30,000 due in 4 years. He agrees to pay Rs. 20,000 today and the remainder in 2 years. How much must he pay at the end of 2 years if money is worth 5% compounded semi-annually? [Ans. Rs. 15,608.4]
12. A loan of Rs. 50,000 due 5 years from now and Rs. 50,000 due 10 years from now is to be repaid by a payment of Rs. 20,000 in 2 years, a payment of Rs. 40,000 in 4 years and a final payment at the end of 7 years. If the interest rate is 5% compounded annually, how much is the final payment? [Ans. Rs. 26,486.4]
13. Mr. X agrees to pay Rs. 20,000 due in 3 years without interest and Rs. 40,000 due in 6 years with rate of interest 4% per annum compounded semi-annually. He wishes to repay these debts in 2 equal installments at the end of 1 year and 3 years respectively. If money is worth 4% effective, what equal payment Mr. X has to repay? [Ans. Rs. 31,273]
14. A owes B two sums of money : Rs. 1,000 plus interest at 7% compounded annually which is due in 5 years and Rs. 2,000 + interest at 8% compounded semi-annually which is due in 7 years. If both debts are paid off by a single payment at the end of 6 years, find the amount of payment if the money is worth 6% compounded quarterly. [Ans. Rs. 4,751.73]
15. A man owes Rs. 8,000 due in 2 years with out interest and 3,000 due in 9 years with 6% annual effective rate of interest. He wishes to repay these debts in two equal

installments due 4 and 5 years hence, respectively. If money is worth 4% converted semi-annually how much should each installment be ? [Ans. Rs. 6535.58]

16. A debt of Rs. 10,000 which is due today is to be repaid by 4 equal yearly payments. If the interest rate is 5% compounded quarterly, how much should be each installment be

- (a) if the first installment is given today ?
(b) if the first installment is given 1 year from today ?

[Ans. (a) Rs. 4,029.32, (b) Rs. 3,860.40]

17. The sum of Rs. 2,000, 3,000 and 4,000 are due at the end of 2, 4 and 8 years respectively. It is proposed to replace this series of payments by a single sum of Rs. 9,000 payable at the end of 'n' years. The rate of interest 10% p.a. effective. Find the value of n. [Ans. 5.04 years]

18. Mr. X borrows Rs. 50,000 due 5 years from now and Rs. 50,000 due 10 years from now. This obligation are to repaid by the following agreements : a payment of Rs. 20,000 in 2 years, a payment of 40,000 in 4 years and a payment of Rs. 26,500 in 'n' years. If the interest rate is 5% compounded annually, find the values of n ?

[Ans. 7 years]