CHAPTER

7

Discount

INTRODUCTION

It is a common practice in business that money lenders require the borrowers to make a note bearing no interest until after maturity and discount this note immediately, giving the borrower only the net proceeds. In other words, amount borrowed is given to the borrower after deducting the discount at a certain rate of discount. Discount plays a major role in financial transactions. In this chapter, we discuss the method of calculating discount, compare nominal and effective rate of discounts, and establish the relation between rate of interest and rate of discount.

DISCOUNT

A discount is a deduction allowed on a financial obligation. When the value of an obligation is known at some future data, the proeess of finding its value at some earlier date is known as discounting. The rate of discount is the per cent of the maturity value charged as bank discount for a discount period of unit length of time. One year is generally taken as the unit length of time. A special case of discounting an obligation is that of finding its present value when its maturity value is known. The present value (the money received by the borrower) is commonly referred to as the net proceeds.

Consider the following example. Suppose a man requests a loan of Rs. 300 from a bank, promising to repay the loan in one year. If the bank charges 10% annual discount, the bank deducts Rs. 30 (10% of Rs. 300) from Rs. 300 and pays only Rs. 270 to the borrower, although the borrower is said to have obtained a loan of Rs. 300. The amount Rs. 270 is the present value of Rs. 300 or the net proceeds and Rs. 30 is the bank discount. At the end of the year, the borrower has to repay Rs. 300. Here the borrower is said to pay the interest in advance.

Thus, the above transaction is described as follows : The borrower receives Rs. 300 but immediately return Rs. 30 to the bank as 10% interest (in advance) on Rs. 30 and repays the loan amount Rs. 300 at the end of the year.

SIMPLE DISCOUNT

Simple discount is computed in much the same way as simple interest, with the exception that it is based on the amount rather than the principal.

Let P = proceeds/present value (amount received by the borrower).

d = discount rate per year.

t = time in years that proceeds will be held.

A = the value of obligation (amount to be paid at the end of t years).

By Definition, simple discount (D is computed as follows :

Simple discount, D = Amount × Discount rate × Time

$\therefore$ Simple discount, $D = Adt$

The proceeds received by the borrower is given by :

$$P = \text{Amount} - \text{Simple discount}$$

$$\therefore \quad P = A - Adt$$

$$\boxed{P = A(1 - dt)}$$

Example 1 : *Find the simple discount and the present value of Rs. 2,000 loan for six months at 8%.*

***Solution* :** Given A = Rs. 2,000, t = 6 months = 1/2 years, and d = 8% = 0.8

Simple Discount is given by

$$D = Adt$$

$$\therefore \quad D = 2{,}000 \times 0.8 \times 1/2$$

$$\therefore \quad D = \text{Rs. } 80.$$

Present Value is given by

$$P = A(1 - dt)$$

$$\therefore \quad P = \text{Rs. } 2{,}000\,(1 - 0.08 \times 1/2)$$

$$\Rightarrow \quad P = 2{,}000\,(1 - 0.04) = 2{,}000\,(0.96) = \text{Rs. } 1{,}920$$

$$\therefore \quad P = \text{Rs. } 1{,}920$$

Alternatively, $P = A - D$

$$\therefore \quad P = 2{,}000 - 80 = \text{Rs. } 1{,}920$$

PRESENT VALUE AT DISCOUNT RATE

The present value of an obligation discounted as bank discount is given by

$$\boxed{P = A(1 - d)^n}$$

where A = The value of obligation at the end of 'n' period.

P = Present Value of obligation.

d = Discount rate per period.

n = Number of periods.

Clearly, the discount is given by

$$\boxed{D = A - P}$$

Example 2 : *Find the present value and discount on Rs. 3,000 due in 4 years at 8% discount rate, discounted annually.*

Solution **:** The present vale is given by

$$P = A(1-d)^n$$

Here A = Rs. 3,000

The sum is discounted annually

$\therefore$ $n = 4$ and $d = 8\% = 0.08$

$\therefore$ $P = 3{,}000\,(1-0.8)^4$

$\Rightarrow$ $P = 3{,}000\,(0.92)^4 = 3{,}000 \times 0.71639$

$P = 2{,}149.18$

The preset value is Rs. 2,149.18.

The discount is given by

$D = A - P$

$\therefore$ $D = 3{,}000 - 2{,}149.18$

$D = 850.82$

$\therefore$ Discount is Rs. 850.82

Example 3 : *Find the present value and discount on Rs. 2,000 due in 4 years at 8% discount rate, convertible half-yearly.*

Solution **:** The present value is given by

$$P = A(1-d)^n$$

Here A = Rs. 2,000

Discount Rate = 8% = 0.08

t = 4 years

Sum is discounted half-yearly

$\therefore$ $n = 4 \times 2 = 8$ and $d = \dfrac{0.08}{2} = 0.04$

$\therefore$ $P = 2{,}000\,(1-0.04)^8$

$P = 2{,}000\,(0.96)^8 = 2{,}000\,(0.72139)$

$P = 1{,}442.78$

Thus, the present value is Rs. 1,442.78.

The discount is given by

$$D = A - P$$

Here $D = 2{,}000 - 1{,}442.78$

$$D = 557.22$$

The discount is Rs. 557.22

Example 4 : *If the present value of Rs. 6,000 due in 2 years at a certain nominal rate of discount, convertible quarterly is Rs. 5,536.47. Find the rate of discount.*

Solution : Given A = Rs. 6,000

Let d be the rate of discount.

Present value P = Rs. 5,536.47

No. of years = 2

The sum is discounted quarterly

$$\therefore \quad n = 2 \times 4 = 8$$

Rate of discount per quarter $= \frac{d}{4}$

The present value is given by

$$P = A(1-d)^n$$

Here

$$5{,}536.47 = 6{,}000\left(1-\frac{d}{4}\right)^8$$

$$\Rightarrow \quad \left(1-\frac{d}{4}\right)^8 = 0.922745$$

Taking log on both sides we get,

$$8\log\left(1-\frac{d}{4}\right) = \log(0.922745)$$

$$\Rightarrow 8\log\left(1-\frac{d}{4}\right) = -1 + 0.9650 = -0.0350$$

$$\Rightarrow \quad \log\left(1-\frac{d}{4}\right) = \frac{-0.0350}{8} = -0.004375$$

$$\Rightarrow \quad \log\left(1-\frac{d}{4}\right) = -1 + 1 - 0.004375 = -1 + 0.995625$$

$$1 - \frac{d}{4} = \text{antilog}(-1 + 0.9956)$$

$$1 - \frac{d}{4} = 0.9900$$

$$\frac{d}{4} = 0.01$$

$$\Rightarrow \quad d = 0.4 = 4\%$$

The rate of discount is 4%.

DISCOUNT CONVERTIBLE CONTINUOUSLY

We now derive the formula for the present value of an obligation discountable at a rate of discount d continuously.

Suppose that a loan of Rs. A is repayable after t years and the discount is convertible m times a year.

Let d be the nominal rate of discount. Then the present value of Rs. A due in t years at the nominal rate of discount d, convertible 'm' times a year is given by

$$P = A\left(1 - \frac{d}{m}\right)^{m \times t}$$

In case of continuous discounting, the present value P is given by

$$P = \lim_{m \to \infty}\left[A\left(1 - \frac{d}{m}\right)^{mt}\right]$$

$$\Rightarrow \quad P = A\left[\lim_{m \to \infty}\left(1 - \frac{d}{m}\right)^{mt}\right]$$

$$\Rightarrow \quad P = A\left[\lim_{m \to \infty}\left(1 - \frac{d}{m}\right)^{-mt \times \left(-\frac{d}{d}\right)}\right]$$

$$\Rightarrow \quad P = A\left[\lim_{m \to \infty}\left(1 - \frac{d}{m}\right)^{-\frac{m}{d} \times -dt}\right]$$

$$\Rightarrow \quad P = A\left[\left[\lim_{m \to \infty}\left(1 + \left(\frac{-d}{m}\right)\right)^{\frac{1}{(-d/m)}}\right]^{-dt}\right]$$

Let $\quad x = -\frac{d}{m}$ as $m \to \infty \quad x \to 0$

$$\therefore \quad P = A\left[\lim_{m \to 0}(1 + x)^{\frac{1}{x}}\right]^{-dt}$$

$$\therefore \quad P = A\,e^{-dt}$$

Thus, the present value of an obligation of Re. A due in t years at d nominal rate of discount convertible continuously is given by

$$\boxed{P = A\,e^{-dt}}$$

Example 5 : *What is the present value of Rs. 2,000 due after 5 years from now if the discount is convertible continuously at a discount rate of 8%.*

Solution **:** The present value of Rs. A due in t years at d rate of discount, convertible continuously is given by

$$P = A\,e^{-dt}$$

Here A = Rs. 2,000, $d = 8\% = 0.08$ and $t = 5$

$\therefore$ $P = 2{,}000\, e^{-0.08 \times 5}$

$\Rightarrow$ $P = 2{,}000\, e^{-0.40}$

$\Rightarrow$ $P = 2{,}000 \times 0.67032 = 1{,}340.64$

$\therefore$ The present value is Rs. 1,340.64.

Example 6 : *At what rate of discount convertible continuously the present value Rs. 3,000 due in 7 years will be Rs. 2,123 ?*

Solution : Let d be the rate of discount, convertible continuously. The present value of Rs. A due in t years at d discount convertible continuously is given by

$$P = A\, e^{-dt}$$

Here P = Rs. 2,123, A = Rs. 3,000 and $t = 7$ years

$\therefore$ $2{,}123 = 3{,}000\, e^{-7d}$

$$\Rightarrow \quad e^{-7d} = \frac{2{,}123}{3{,}000} = 0.7077$$

Taking log on both sides we get

$$-7\, d \log e = \log 0.7077$$

$$\Rightarrow \quad d = -\frac{\log (0.7077)}{7 \log e}$$

$$d = \frac{-(-0.1502)}{7 \times 0.4343}$$

$$d = \frac{0.152}{3.0401} = 0.0494 = 4.94\%$$

The rate of discount is 4.94%.

NOMINAL AND EFFECTIVE RATE OF DISCOUNT

In situations where discount is converted more frequently than once each year, the stated rate of discount is called a nominal rate of discount and the rate of discount actually obtained during the year is called the effective rate of discount. In the later case, the rate of discount is always convertible annually.

Consider the following example : The discount on a maturity value of Rs. 100 due in one year at a rate of 8% per annum is Rs. 8 and the proceeds are Rs. 92 (*i.e.* Rs. 100 – Rs. 8).

Suppose that the same rate of discount is convertible semi-annually.

Then the present value of Rs. 100 due in one year would be $100\,(1 - 0.04)^2 = 92.16$.

$\therefore$ The actual discount obtained on this obligation of Rs. 100 at the end of one year is Rs. (100 – 92.16) = Rs. 7.84.

We say that the effective rate in this case is 7.84%.

Now at 7.84% rate of discount, convertible per annum, the present value we get is

$$P = 100\,(1 - 0.0784) = 100\,(0.9216) = \text{Rs. } 92.16.$$

Thus for a maturity value of Rs. 100 after one year, the net proceeds at a 7.84% rate of discount, convertible annually is equivalent to the net proceeds discounted at a rate of 8%, convertible half-yearly.

Therefore, we say that the effective rate of discount is 7.84% and the corresponding nominal rate of discount is 8% convertible semi-annually.

Let d_e denote the effective rate of discount corresponding to the nominal rate of discount d, convertible m times a year.

Let A be the maturity amount at the end of 1 year.

At d_e effective rate of discount, the present value is given by

$$P_1 = A(1-d_e) \quad ...(1)$$

At d nominal rate of discount, convertible m time a year, the present value is given by

$$P_2 = A\left(1-\frac{d}{m}\right)^m \quad ...(2)$$

Equating the equations (1) and (2), we get

$$1-d_e = \left(1-\frac{d}{m}\right)^m$$

$$d_e = 1-\left(1-\frac{d}{m}\right)^m$$

Thus, the relation between the nominal rate of discount and effective rate of discount is given by

$$\boxed{d_e = 1-\left(1-\frac{d}{m}\right)^m}$$

Now, we derive the relation between the nominal rate of discount, convertible continuously and the effective rate of discount.

In case of nominal rate d convertible continuously, the effective rate d_e is given by

$$d_e = \lim_{m\to\infty}\left[1-\left(1-\frac{d}{m}\right)^m\right]$$

$$\Rightarrow \quad d_e = 1-\lim_{m\to\infty}\left(1-\frac{d}{m}\right)^m$$

$$\Rightarrow \quad d_e = 1-\lim_{m\to\infty}\left(1-\frac{d}{m}\right)^{m\times\frac{-d}{-d}}$$

$$\Rightarrow \quad d_e = 1-\lim_{m\to\infty}\left[\left(1-\frac{d}{m}\right)^{-\frac{m}{d}}\right]^{-d}$$

$$\Rightarrow \quad d_e = 1-\lim_{m\to\infty}\left[\left(1-\frac{d}{m}\right)^{\frac{-1}{d/m}}\right]^{-d}$$

$$\Rightarrow \quad d_e = 1-e^{-d}$$

Thus, the effective rate of discount equivalent to the nominal rate of discount, convertible continuously is given by

$$\boxed{d_e = 1-e^{-d}}$$

Remark 1 : The nominal rate of discount d compounded continuously and equivalent to a given effective rate of discount r_e is called the force of discount.

Remark 2 : If the conversion period is one year at d nominal rate of discount, then the effective rate of discount will be equal to the nominal rate of discount.

If the conversion period is one year, then $m = 1$

$$\therefore \quad d_e = 1 - \left(1 - \frac{d}{m}\right)^m$$

$$\Rightarrow \quad d_e = 1 - \left(1 - \frac{d}{1}\right)^1 = 1 - (1 - d) = d$$

$$\therefore \quad d_e = d.$$

Remark 3 : The effective rate of discount (d_e) depends only on the nominal rate of discount (d_e) and the number of conversion periods in a year (m). It is independent of the maturity value A.

Example 7 : *Find the effective rate of discount equivalent to the nominal rate of discount 9% converted monthly.*

***Solution* :**

Here nominal rate of discount $d = 9\% = 0.09$

Discount is converted monthly

$$\therefore \quad m = 12$$

The effective rate equivalent to the nominal rate of discount d is given by

$$d_e = 1 - \left(1 - \frac{d}{m}\right)^m$$

Here $\quad d_e = 1 - \left(1 - \frac{0.09}{12}\right)^{12}$

$$\therefore \quad d_e = 1 - (0.9925)^{12} = 1 - 0.9136 = 0.0864$$

$$\therefore \quad d_e = 8.64\%$$

The effective rate is 8.64%.

Example 8 : *Find the effective rate of discount equivalent to the nominal rate 5% convertible continuously.*

***Solution* :** The effective rate of discount equivalent to a nominal rate of discount convertible continuously is given by

$$d_e = 1 - e^{-d}$$

Here $\quad d = 5\% = 0.05$

$$\therefore \quad d_e = 1 - e^{-0.05} = 1 - 0.95123 = 0.04877$$

$$\therefore \quad d_e = 4.88\%$$

The effective rate is 4.88%

Example 9 : *Find the nominal rate of discount, convertible semi-annually equivalent to the effective rate of discount 6%.*

***Solution* :** The effective rate of discount d_e equivalent to a nominal rate of discount is given by

$$d_e = 1 - \left(1 - \frac{d}{m}\right)^m$$

Here $\quad m = 2$ and $d_e = 6\% = 0.06$

$$0.06 = 1 - \left(1 - \frac{d}{2}\right)^2$$

$$\Rightarrow \quad \left(1 - \frac{d}{2}\right)^2 = 1 - 0.06 = 0.94$$

Taking log on both sides we get

$$2 \log \left(1 - \frac{d}{2}\right) = \log 0.94 = \log (9.4 \times 10^{-10})$$

$$\Rightarrow 2 \log \left(1 - \frac{d}{2}\right) = -1 + 0.9731 = -0.0269$$

$$\Rightarrow \log \left(1 - \frac{d}{2}\right) = -0.01345 = -1 + 0.98655$$

$$\Rightarrow 1 - \frac{d}{2} = \text{antilog} (-1 + 0.98655)$$

$$\Rightarrow 1 - \frac{d}{2} = 9.6939 \times 10^{-1} = 0.96939$$

$$\frac{d}{2} = 0.03061$$

$$d = 0.03061 \times 2 = 0.06121 = 6.12\%$$

∴ The nominal rate is 6.12%.

Example 10 : *What nominal rate of discount convertible continuously, will be equivalent to the effective rate of discount 7% ?*

***Solution* :** The effective rate of interest, equivalent to a nominal rate of discount d, convertible continuously is given by

$$d_e = 1 - e^{-d}$$

Here $\quad d_e = 7\% = 0.07$

$$\therefore \quad 0.07 = 1 - e^{-d}$$

$$\Rightarrow \quad e^{-d} = 0.93$$

$$\Rightarrow \quad e^{d} = \frac{1}{0.93} = 1.0752$$

Taking log on both sides we get

$$d \log e = \log 1.0752$$

$$\Rightarrow \quad d = \frac{\log 1.0752}{\log e} = \frac{0.03140}{0.4343} = 0.0723$$

$$\therefore \quad d = 7.23\%$$

The effective rate is 7.23%

Example 11 : *Find the nominal rate of discount convertible half-yearly which is equivalent to the nominal rate of discount 8% convertible quarterly.*

Solution : The effective rate of discount equivalent to a nominal rate of discount, convertible m times a year is given by

$$d_e = 1 - \left(1 - \frac{d}{m}\right)^m$$

Let d_1 be the required nominal rate of discount.

At 8% nominal rate of discount, convertible half-yearly, the effective rate of discount is

$$d_e = 1 - \left(1 - \frac{d_1}{2}\right)^2 \quad \ldots(1)$$

At 8% nominal rate of discount convertible quarterly, the effective rate of discount is

$$d_e = 1 - \left(1 - \frac{0.08}{4}\right)^2 \quad \ldots(2)$$

Given that the effective rates of discount (1) and (2) are equivalent.

$$\therefore \quad 1 - \left(1 - \frac{d_1}{2}\right)^2 = 1 - \left(1 - \frac{0.08}{4}\right)^4$$

$$\Rightarrow \quad \left(1 - \frac{d_1}{2}\right)^2 = \left(1 - \frac{0.08}{4}\right)^4$$

$$\Rightarrow \quad \left(1 - \frac{d_1}{2}\right)^2 = (0.98)^4 = 0.9224$$

Taking log on both sides, we get

$$2 \log \left(1 - \frac{d_1}{2}\right) = \log (0.9224)$$

$$\Rightarrow 2 \log \left(1 - \frac{d_1}{2}\right) = -0.0351$$

$$\Rightarrow \quad \log \left(1 - \frac{d_1}{2}\right) = \frac{-0.0351}{2} = -0.0176$$

$$\Rightarrow \quad 1 - \frac{d_1}{2} = \text{antilog}(-0.0176)$$

$$\Rightarrow \quad 1 - \frac{d_1}{2} = 0.9603$$

$$\Rightarrow \quad \frac{d_1}{2} = 0.0397$$

$$\Rightarrow \quad d_1 = 2 \times 0.0397 = 0.0794 = 7.94\%$$

The nominal rate of discount is 7.94%.

Example 12 : *The amount of Re. 1 in 2 years at a certain nominal rate of interest plus the present value of Re. 1 due in 2 years, at the same nominal rate of discount, both rates convertible half-yearly, is 2.01080162. Find the rate of discount.*

Solution : Let r be the nominal rate of interest convertible half yearly.

$$\therefore \quad i = r/2$$

Given that this is also the nominal rate of discount.

$\therefore$ The amount of Re. 1 in 2 years = $(1 + i)^4$ and the present value of Re. 1 due in 2 years = $(1 - i)^4$

Given that

$$(1 + i)^4 + (1 - i)^4 = 2.01080162$$

$$\Rightarrow \quad 2i^4 + 12i^2 + 2 = 2.01080162$$

$$\Rightarrow \quad 2i^4 + 12i^2 - 0.01080162 = 0$$

$$\Rightarrow \quad i^4 + 6i^2 - 0.00540081 = 0$$

$$\Rightarrow \quad i^2 = \frac{-6 \pm \sqrt{36 + 0.02160324}}{2}$$

$$\Rightarrow \quad i^2 = \frac{-6 \pm \sqrt{36.02160324}}{2}$$

$$\Rightarrow \quad i^2 = \frac{-6 \pm 6.0018}{2}$$

Rejecting the negative values, we get

$$i^2 = \frac{-6 + 6.0018}{2} = \frac{0.0018}{2} = 0.009$$

$$\therefore \quad i = 0.03$$

$$r = 2 \times i = 2 \times 0.03 = 0.06 = 6\%$$

The rate of discount is 6%.

DISCOUNT TO COMPOUND INTEREST

The present value of a sum of money payable at a future date is called the discounted value of the amount due at the future date. The difference between the amount due at the future date and its present value is called the discount over this period.

The present value or capital value of an amount A discounted for n period at a rate of interest i per period is determined using the compound interest formula. The present value of Rs. A due in n periods discounted at i rate of interest per period is that principal which is invested now at an interest rate of i per period.

$\therefore$ The present value of an amount A discounted for n periods at a rate of interest i per period is given by

$$\boxed{P = A\ (1 + i)^{-n}}$$

$(1 + i)^{-n}$ is called the discount factor.

The discount is given by

$$D = A - P$$

$$\Rightarrow \quad D = A - A(1+i)^{-n}$$

$$\Rightarrow \quad D = A(1-(1+i)^{-n})$$

Discount obtaining using this relation is referred to as compound discount.

It should be noticed that instead of rate of discount, if the rate of interest is given to find the discount, we use the above formula, derived from compound interest formula.

Example 13 : *Determine the present value of Rs. 5,000 due in 5 years from now invested at 8% compounded annually. What is the compound discount of this investment ?*

***Solution* :** The present value is given by

$$P = A(1+i)^{-n}$$

Here A = Rs. 5,000

Interest is compounded annually.

$$\therefore \quad i = r = 8\% = 0.08 \text{ and } n = t = 5$$

$$\therefore \quad P = 5{,}000\,(1+0.08)^{-5}$$

$$P = 5{,}000\,(1.08)^{-5} = 5{,}000\,(0.68058) = 3402.90$$

$\therefore$ The present value is Rs. 3,402.90.

The compound discount is given by

$$D = A - P$$

$$\therefore \quad D = 5{,}000 - 3{,}402.90 = 1{,}597.10$$

$\therefore$ The compound discount is Rs. 1,597.10.

RELATIONSHIP BETWEEN RATE OF DISCOUNT AND RATE OF INTEREST

From the discussion made so far, it is obvious that discount and interest are two different ways of looking at the same problem of discounting. For example, the discount on an obligation of Rs. 100 due in one year at the rate of discount of 6% per annum is Rs. 6 and the net proceeds are Rs. 94.

However the same Rs. 6 may be treated as the interest on Rs. 94 for one year. Thus, the rate of interest per annum corresponding to 6% per annum rate of discount will be 6.38% $\left(\frac{6}{94} \times 100\right)$.

Present value of Rs. 100 at 6.38% rate of interest convertible annually is Rs. 94.

Present value of Rs. 100 at 6% rate of discount convertible annually is Rs. 94.

Thus, 6% per annum rate of discount is equivalent to 6.38% per annum rate of interest.

Now we derive the relation between the rate of interest and the rate of discount.

Let Re. 1 is payable after one year.

Let d and r be the rate of discount and the rate of interest convertible annually respectively.

The present value of Re. 1 due in 1 year at d annual rate of discount is given by

$$P_1 = 1\,(1-d)^1 = 1-d \qquad \ldots(1)$$

The present value of Re. 1 due in one year at r annual rate of interest is given by

$$P_2 = 1\,(1+r)^{-1} = \frac{1}{1+r} \qquad \ldots(2)$$

The present value P_1 is equivalent to the present value P_2.

$$\therefore \qquad 1-d = \frac{1}{1+r}$$

$$\Rightarrow \qquad d = 1-\frac{1}{1+r} = \frac{1+r-1}{1+r} = \frac{r}{1+r}$$

Thus, we get

$$\boxed{d = \frac{r}{1+r}}$$

This relation is used to find the rate of discount correspondig to the rate of interest 'r'.

Again, $$1-d = \frac{1}{1+r}$$

$$\Rightarrow \qquad 1+r = \frac{1}{1-d}$$

$$\Rightarrow \qquad r = \frac{1}{1-d}-1 = \frac{1-(1-d)}{1-d} = \frac{d}{1-d}$$

Thus, we get

$$\boxed{r = \frac{d}{1-d}}$$

This relation is used to find the rate of interest corresponding to the rate of discount d.

Example 14 : *Find the rate of discount corresponding to a rate of interest 6%.*

***Solution* :** The rate of discount d corresponding to a rate of interest r is given by

$$d = \frac{r}{1+r}$$

Here $$r = 6\% = 0.06$$

$$\therefore \qquad d = \frac{0.06}{1+0.06} = \frac{0.06}{1.06} = 0.586 = 5.66\%$$

The rate of discount is 5.66%.

Example 15 : *Find the rate of interest corresponding to a rate of discount of 8%.*

***Solution* :** The rate of interest r, corresponding to a rate of discount d is given by

$$r = \frac{d}{1-d}$$

Here $$d = 8\% = 0.08$$

$$\therefore \qquad r = \frac{0.08}{1 - 0.08} = \frac{0.08}{.92} = 0.8696 = 8.7\%$$

The rate of interest is 8.7%

EXERCISES

1. Find the simple discount and the present value of Rs. 5,000 loans for 4 month at 9% rate of discount. [**Ans.** Rs. 150, Rs. 4,850]
2. Find the rate of discount at which present value of a loan of Rs. 6,000 for one year will be Rs. 5,400. [**Ans.** 10%]
3. The proceeds of a Rs. 2,000 loans for one year at simple discount were Rs. 1,800. What was the simple discount rate. [**Ans.** 10%]
4. Find the present value and discount on Rs. 10,000 due in one year at discount rate 4% convertible quarterly. [**Ans.** Rs. 9,603, Rs. 397]
5. Find the present value and discount on Rs. 6,000 due in 2 years at 4% convertible quarterly. [**Ans.** Rs. 5,536.47, Rs. 463.53]
6. Find the present value and discount on Rs. 6,000 due in 3 years at 4% effective rate of discount. [**Ans.** Rs. 5,308.42, Rs. 691.58]
7. If the present value of Rs. 1,000 due in 3 years at a certain nominal rate of discount, convertible semi-annually is Rs. 832.97, find the rate of discount. [**Ans.** 6%]
8. If the present value of Rs. 3,000 due in 2 years at a certain effective rate of discount is Rs. 2,707. Find the rate of discount. [**Ans.** 5%]
9. Find the effective rate of discount equivalent to the nominal rate of discount 5% convertible monthly. [**Ans.** 4.88%]
10. Find the effective rate of discount equivalent to the nominal rate of discount 6% convertible continuously. [**Ans.** 5.82%]
11. What effective rate will be equivalent to the nominal rate of discount 8%, convertible (*a*) quarterly ?, (*b*) semi-annually ? [**Ans.** (*a*) 7.76%, (*b*) 7.84%]
12. What effective rate of discount will be equivalent to the nominal rate of discount 12% convertible (*a*) semi-annually? (*b*) quarterly? (*c*) monthly ? and (*d*) continuously ?
[**Ans.** (*a*) 11.64%, (*b*) 11.47% (*c*) 11.36%, (*d*) 11.31%]
13. The amount of Re. 1 in 2 years at a certain nominal rate of interest plus the present value of Re. 1 due in 2 years, at the same nominal rate of discount, both rates convertible half-yearly, is 2.00480032. Find the rate of discount. [**Ans.** 4%]
14. The amount of Re. 1 in one year at a certain nominal rate of interest plus the present value of Re. 1 due in one year, at the same nominal rate of discount, both rates convertible quarterly is 2.00480032. Find the rate of discount. [**Ans.** 8%]
15. Find the nominal rate of discount convertible quarterly which is equivalent to the effective rate of discount 6%. [**Ans.** 6.12%]
16. What is the nominal rate of discount convertible continuously, which is equivalent to the effective rate of discount 5% ? [**Ans.** 5.16%]

17. Find the nominal rate of discount convertible half-yearly which is equivalent to the nominal rate of discount 9% convertible quarterly. [**Ans.** 8.92%]

18. Find the nominal rate of discount convertible quarterly which is equivalent to the nominal rate of discount 12% convertible monthly. [**Ans.** 11.88%]

19. Which nominal rate of discount, convertible monthly will be equivalent to the effective rate of discount 7% ? [**Ans.** 7.2%]

20. Find the present value of Rs. 5,000 due after 4 years from now at a rate of rate of discount of Rs. 6%, convertible continuously. [**Ans.** Rs. 3,933.15]

21. What is the present value of Rs. 4,000 payable at the end of 7 years at 5% rate of discount, convertible continuously. [**Ans.** Rs. 2,818.76]

22. At what rate of discount the present value of Rs. 2,000 payable after 5 years from now will be Rs. 1,340.64, if the discount is convertible continuously. [**Ans.** 8%]

23. Find the rate of interest convertible continuously, if the present value of Rs. 8,000 due in 4 years will be Rs. 5,362.56. [**Ans.** 10%]

24. The present value of an obligation of Rs. 7,000 due in n year at a rate of discount of 7% convertible continuously is Rs. 5,290.46. Find n. [**Ans.** 4 years]

25. Find the rate of discount corresponding to a rate of interest of (*i*) 5%, (*ii*) 8%, and (*iii*) 10%. [**Ans.** (*i*) 4.76%, (*ii*) 7.41%, (*iii*) 9.09%]

26. Find the rate of interest corresponding to a rate of discount of (*i*) 4%, (*ii*) 7% and (*iii*) 9%. [**Ans.** (*i*) 4.17%, (*ii*) 7.53%, (*iii*) 9.89%]

27. Distinguish between nominal rate of discount and effective rate of discount. Establish the relation between the nominal rate of discount and effective rate of discount (a) when convertible m times a year and (b) when convertible continuously. Under what condition, nominal rate of discount will be equivalent to effective rate of discount ?

28. What do you mean by force of discount ?

29. Establish the relation between rate of discount and rate of interest.