CHAPTER

4

Compound Interest

INTRODUCTION

The second method of calculating interest is the compound interest method where the interest earned by an invested amount of money (principal) is reinvested so that it too earns interest. In this concept, the interest earned on the initial principle becomes a part of principal for the next period. At the beginning of every period, the principal will be the sum of the principal of the previous period and the interest earned on that principal. For instance, suppose that interest is converted annually. Then the principal for third year will be the sum of the principal for the second year and the interest for second year. Thus, in case of compound interest, interest is converted into principal and hence there is *interest* on interests. In this chapter, we study the method of computation of compound interest, present value and future value of investments, and etc.

COMPOUND INTEREST

An understanding of compound interest is very important for every common man, since every one has to think of investing money. Let us see an example to know how the concept of compound interest works out. Assume that Mr. A deposits Rs. 5,000 in a savings bank account for 3 years at 10% per annum compounded annually.

Principal at the beginning of first year = Rs. 5,000

Interest for first year = 5,000 × 0.1 = Rs. 500

∴ Amount at the end of first year = (Rs. 5,000 + 500) = Rs. 5,500

This is the principal for the second year.

Interest for second year = 5,500 × 0.1 = Rs. 550

∴ Amount at the end of second year = (Rs. 5,500 + 550) = Rs. 6,050.

This is the principal for third year.

Interest for third year = 6,050 × 0.01 = Rs. 605

∴ Amount at the end of third year = (Rs. 6,050 + 605) = Rs. 6,655

This is the amount Mr. A will receive at the end of third year against the investment of Rs. 5,000.

The total amount due at the end of the last period is called the *compound amount.* The difference between the compound amount and the original principal is called *compound interest.*

Compound Interest = Compound Amount – Initial principal

Interest may be compounded annually, half-yearly, quarterly, monthly or at any other regular periods of time. This time between two successive computation of interest is called the *conversion period.*

The number of times over which the interest is compounded in one year is known as *frequency of conversion.* For example, if the interest is compounded quarterly, then the conversion period will be 3 months and the frequency of conversion is 4, *i.e.,* 4 times in a year.

It should be noted that regardless of the frequency of conversion, interest rates are generally quoted as an annual nominal rate. So if the interest is compounded more than once in a year, then the stated annual interest rate should be converted into the rate per compounding (or conversion) period. As we have discussed earlier in the preceeding chapter, the rate per conversion period is found by dividing the annual nominal rate by the number of conversion periods in a year.

To understand conversion period and rate per conversion period more precisely, go through the following table, in which the rate per conversion period equivalent to a nominal rate of 12% compounded is given.

Conversion period	*Frequencies of conversion (m)*	*Rate per conversion period* $i = (r/m)$ %
Annually	1	$12\% = 0.12$
Semi-annually	2	$\frac{12\%}{2} = \frac{0.12}{2} = 0.06$
Quarterly	4	$\frac{12\%}{4} = \frac{0.12}{4} = 0.03$
Monthly	12	$\frac{12\%}{12} = \frac{0.12}{12} = 0.01$
Weekly	52	$\frac{12\%}{52} = \frac{0.12}{52} = 0.0023$
Daily	365	$\frac{12\%}{365} = \frac{0.12}{365} = 0.00333$

FORMULA FOR COMPOUND INTEREST

Let P be the principal earning interest compounded m times a year at a rate of i per period. Let 'n' be the number of conversion periods.

Then,

The amount at the end of first conversion period

$$A_1 = P + Pi = P(1 + i)$$

The amount at the end of second conversion period

$$A_2 = P(1 + i) + P(1 + i)\, i = P(1 + i)(1 + i) = P(1 + i)^2$$

The amount at the end of third conversion period

$$= P(1 + i)^2 + P(1 + i)^2\, i = P(1 + i)^2(1 + i) = P(1 + i)^3$$

.. . .

.....

The amount at the end of the n^{th} period $= P(1 + i)^n$

Hence, the compound amount A of a principal P at the end of n conversion periods at the rate of interest of i per conversion period is given as follows :

$$\boxed{A = P(1 + i)^n}, \; n = m \times t, \; i = \frac{r}{m}$$

Where

P → Principal amount invested or borrowed

A → Compound Amount accumulated

r → Rate of interest per annum

i → Rate of interest per conversion period

m → Number of conversion periods per year

n → Total umber of conversion periods

t → Number of year.

It should be noted that the principal amount P is invested for t years at annual rate of r and interest is compounded m times in a year. So the above formula for compound amount can also be given by

$$\boxed{A = P\left(1 + \frac{r}{m}\right)^{mt}}$$

Compound interest may be obtained by using the following formula :

Compound Interest $I = A - P$

Example 1 : *Find the compound interest earned from Rs. 16,000 for 3 year at $12\frac{1}{2}\%$ per annum ?*

Solution **:** Here, P = Rs. 16,000

rate of interest, $r = 12\frac{1}{2}\% = 0.125$

no. of periods, $n = 3$

Interest compounded annually $\therefore \; i = r$

Amount $A = P(1 + i)^n$

$\therefore$ $A = 16{,}000\,(1 + 0.125)^3 = 16{,}000\,(1{\cdot}423828)$

$= \text{Rs. } 22{,}781.25$

Now Compound Interest $I = A - P$

$= 22{,}781.25 - 16{,}000 = \text{Rs. } 6{,}781.25$

$\therefore$ Interest paid is Rs. 6,781.25

Example 2 : *If Rs. 1,750 is invested at 9% per annum interest for 10 years and interest is compounded half-year, find the amount and interest.*

Solution : $P = \text{Rs. } 1{,}750$ and $r = 9\% = 0.09$

Interest is calculated half-yearly.

$\therefore$ $i = \frac{r}{2} = 0.045$ and $n = 10 \times 2 = 20$

Amount $A = P\,(1 + i)^n$

$\therefore$ $A = 1{,}750\,(1.045)^{20}$

Let $x = (1.045)^{20}$

log $x = 20 \log 1.045 = 20 \times 0.0191 = 0.3820$

$x = \text{antilog } (0.03820) = 2.41$

Now, $A = 1{,}750 \times 2.41 = 4{,}217.50$

Interest, $I = S - P$

$= 4{,}217.50 - 1{,}750 = \text{Rs. } 2{,}467.50$

$\therefore$ The amount is Rs. 4,217.50, and interest is Rs. 2,467.50

Example 3 : *To what amount Rs. 10,000 accumulate in 6 year, if invested at 8% compounded quarterly ? [1.02)^{24} = 1.6084].*

Solution : $P = \text{Rs. } 10{,}000$, and $r = 8\% = 0.08$

Rate of interest as per conversion period

$i = \frac{r}{4} = \frac{0.08}{4} = 0.02$ and $n = 6 \times 4.24$

Amount $A = P\,(1 + i)^n$

$\therefore$ $A = 10{,}000\,(1 + 0.02)^{24} = 10{,}000\,(1.02)^{24}$

$= 10{,}000 \times 1.6084 = \text{Rs. } 16{,}084$

$\therefore$ The amount is Rs. 16,084.

Example 4 : *Find the rate of interest, if the sum of money will double itself in 10 years by investing at compound interest.*

Solution : Let P be the principal.

Let the rate of interest be r.

Interest is compounded annually, $r = i$ and $n = 10$

Given that the amount will be doubled in 10 years.

i.e. $A = 2P$

But $A = P(1 + i)^n$

$\therefore$ $2P = P(1 + i)^{10} \Rightarrow 2 = (1 + i)^{10}$

Taking log on both sides

$$\log 2 = 10 \log (1 + i)$$

$$\log (1 + i) = \frac{\log 2}{10} = \frac{0.0310}{10} = 0.0301$$

Now, $1 + i = \text{antilog } (0.0301)$

$$1 + i = 1.072$$

$$i = 0.072 = 7.2\%$$

$\therefore$ The required rate of interest is 7.2%

Example 5 : *At what rate per annum compound interest will Rs. 5,000 amount to Rs. 9,035 in 5 years if the interest is calculated quarterly ?*

Solution : $P =$ Rs. 5,000, and $A =$ Rs. 9,035

Let r be the rate of interest per annum.

Interest is calculated quarterly.

$\therefore$ $i = \frac{r}{4}$ and $n = 4 \times 5 = 20$

$$A = P(1 + i)^n$$

$$9{,}035 = 5{,}000 (1 + i)^{20}$$

$$(1 + i)^{20} = \frac{9{,}035}{5{,}000}$$

$$(1 + i)^{20} = 1.807$$

Taking log on both sides

$$20 \log (1 + i) = \log 1.807$$

$$\log (1 + i) = \frac{\log 1.807}{20} = \frac{0.2570}{20} = 0.0129$$

$$1 + i = \text{antilog } (0.0129) = 1.030$$

$$i = 1.030 - 1 = 0.030$$

$$i = \frac{r}{4} \Rightarrow r = 4 \times i$$

$$r = 4 \times 0.030 = 0.012$$

$$r = 12\%$$

$\therefore$ The required rate is 12%.

Example 6 : *In how many years an amount will triple itself at 11% compound half yearly ?*

Solution : Let P be the principal.

$$r = 11\% = 0.11$$

Interest is compounded half yearly.

$$i = \frac{r}{2} = 0.055$$

Given, $A = 3P$

But, $A = P(1+i)^n$

$\therefore$ $3P = P(1+i)^n$

$$3P = P(1+0.055)^n$$

$$3 = (1.055)^n$$

Taking log on both sides

$$n \log (1.055) = \log 3$$

$$n = \frac{\log 3}{\log 1.055}$$

$$n = \frac{0.4771}{0.0233} = 20.555$$

Now, No. of years $t = n/2$

$$t = 10.28 \text{ approx.}$$

$\therefore$ Number of years is approximately 10.28 years.

Example 7 : *The difference between compound interest and simple interest on a certain sum of money invested for 3 years at 6% per annum is Rs. 110.16. What is that amount invested ?*

Solution : Let P be the principal.

Interest is compounded annually.

$\therefore$ $t = n = 3$ and $i = r = 6\% = 0.06$

Simple interest $I_1 = Prt$

$\therefore$ $I_1 = P \times 0.06 \times 3 = 0.18\,P$

Compound interest $I_2 = A - P$

$$I_2 = P(1+i)^n - P$$

$$I_2 = P(1.06)^3 - P$$

Given that $I_2 - I_1 = 110.16$

$\Rightarrow$ $P(1.06)^3 - P - 0.18\,P = 110.16$

$\Rightarrow$ $1.191016\,P - P - 0.18\,P = 110.16$

$\Rightarrow$ $0.011016\,P = 110.16$

P = Rs. 10,000

$\therefore$ The amount deposited is Rs. 10,000.

Example 8 : *The amounts for a certain sum with compound interest at a certain rate in two years and in three years are Rs. 8,820 and Rs. 9,261 respectively. Find the rate and sum.*

Solution : Let P be the principal amount and i be the rate of interest.

Amount $A = P(1 + i)^n$

At the end of two years, A = Rs. 8,820

$\therefore$ $8{,}820 = P(1 + i)^2$ ...(1)

At the end of three years, A = Rs. 9,261

$\therefore$ $9{,}261 = P(1 + i)^3$ ...(2)

Dividing (2) by (1), we get

$$\frac{9{,}261}{8{,}820} = \frac{P(1+i)^3}{P(1+i)^2}$$

$\Rightarrow$ $1.05 = 1 + i$

$\Rightarrow$ $i = 0.05 = 5\%$

Now, substituting $i = 0.05$ in equation (1) we get,

$8{,}820 = P(1.05)^2$

$\Rightarrow$ $8{,}820 = P \times 1.1025 \Rightarrow P$ = Rs. 8,000

$\therefore$ Required rate is 5% and principal is Rs. 8,000.

Example 9 : *A certain sum of money is invested at 4% p.a. compound annually. The interest for 2nd year is Rs. 25. Find the interest for 3rd year.*

Solution : Let P be the principal.

$A = P(1 + i)^n$

Amount at the end of 1st year = $P(1.04)$

Amount at the end of 2nd year = $P(1.04)^2$

Interest for 2nd year = Amount the at the end of 2nd year – Amount at the end of 1st.

Given interest for second year is Rs. 25.

$25 = P(1.04)^2 - P(1.04)$

$\Rightarrow$ $P(1.04)(1.04 - 1) = 25$

$\Rightarrow$ $P(1.04)(0.04) = 25$ ...(1)

Amount at the end of 3rd year = $P(1.04)^3$

Interest for 3rd year = Amount at the end of 3rd year – Amount at the end of 2nd year

$\therefore$ Interest for 3rd year $= P(1.04)^3 - P(1.04)^2$

$= P\ (1.04)^2[1.04 - 1]$

$= P(1.04)^2(0.04)$

$= P(1.04)(1.04)(0.04) = 25 \times 1.04$ [using (1)]

$= \text{Rs. } 26.$

$\therefore$ Interest for third year is Rs. 26.

Example 10 : *Mr. X wants to make an investment of Rs. 20,000 in one of the two banks that fetch the return after 6 years. Bank A offers 8% interest compounded annually and Bank B offers 7.8% compounded semi-annually. Which Bank should be chosen, so that Mr. X will get maximum return ?*

Solution : Amount is given by

$$A = P(1 + i)^n$$

In case of Bank A :

$P = \text{Rs. } 20{,}000$, and $r = 8\% = 0.08$

Interest is compounded annually.

$\therefore \quad i = r = 0.08$ and $n = 6$

Amount $A_1 = 20{,}000\,(1.08)^6$

$A_1 = 20{,}000 \times 1.586874$

$A_1 = \text{Rs. } 31{,}737.48$

In case of Bank B :

$P = \text{Rs. } 20{,}000$, and $r = 7.8\% = 0.078$

Interest is compounded semi-annualy.

$$\therefore \quad i = \frac{r}{2} = \frac{0.078}{2} = 0.039 \text{ and } n = 6 \times 2 = 12$$

Amount $A_2 = 20{,}000\,(1.039)^{12}$

$A_2 = 20{,}000 \times 1.582656$

$A_2 = \text{Rs. } 31{,}653.12$

Since $A_1 > A_2$, Mr. X should invest in Bank A to get maximum return.

Example 11 : *Lal deposited an amount of Rs. 50,000 in two different Banks A and B, dividing the amount into two investments. Bank A calculates interest at a rate of 7% per annum and Bank B calculates interest at the rate of 6% per annum convertible semi-annually. At the end of 3 years, he received Rs. 10,632.35 as the return on his investment. What amount he has deposited in both Banks ?*

Solution : Let Rs. X and Rs. Y be the amounts deposited in Bank A and Bank B respectively.

Total investment is Rs. 50,000

$\therefore \quad X + Y = 50{,}000$ ...(1)

Compound interest is given by

$$I = A - P \text{ where } A = P(1 + i)^n$$

Interest earned from Bank A :

$$I_1 = X(1.07)^3 - X$$

$$\Rightarrow \quad I_1 = 1.225043\,X - X$$

$$\Rightarrow \quad I_1 = 0.225043\,X$$

Interest earned from Bank B :

$$I_2 = (1.03)^6\,Y - Y$$

$$I_2 = 1.194052\,Y - Y$$

$$I_2 = 0.194052\,Y$$

Total interest earned $I = I_1 + I_2$

$\therefore \quad I = 0.225043\,X + 0.194052\,Y$

It is given that total interest received is Rs. 10,632.35

$\therefore \quad 0.225043\,X + 0.194052\,Y = 10632.35$ ...(2)

Solving (1) and (2) we get,

X = Rs. 30,000 and Y = Rs. 20,000.

$\therefore$ Rs. 30,000 and Rs. 20,000 are the sums deposited in Bank A and Bank B respectively.

INTEREST COMPOUNDED CONTINUOUSLY

If a principal of P is invested for t years at an annual rate of r and interest is compounded m times a year, then the interest rate per conversion period i is $\frac{r}{m}$ and there are mt conversion periods. The compound amount A at the end of t years becomes

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

It is well known that if the interest is compounded more frequently, the compound amount will become larger for a fixed principal, time period and annual interest rate. One may think that the compound amount will be increased by indefinitely increasing the frequency of compounding, *i.e.*, $m \to \infty$. But there is an upper limit on the compound interest that can be achieved in this way.

When interest is so computed that the interest per year gets larger and larger as the number of compounding periods increases continuously, *i.e.*, $m \to \infty$, we say that interest is *compounded continuously*. Thus, in case of continuous compounding of interest, the amount A is given by

$$A = \lim_{m \to \infty}\left[P\left(1 + \frac{r}{m}\right)^{mt}\right]$$

$$\Rightarrow \quad A = P \lim_{m \to \infty}\left(1 + \frac{r}{m}\right)^{mt}$$

$$\Rightarrow \quad A = P \lim_{m \to \infty}\left(1 + \frac{r}{m}\right)^{\frac{m}{r} \times rt}$$

$$\Rightarrow \quad A = P \lim_{m \to \infty} \left[\left(1 + \frac{r}{m}\right)^{\frac{m}{r}} \right]^{rt}$$

$$\Rightarrow \quad A = P \left[\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{\frac{m}{r}} \right]^{rt}$$

$$\Rightarrow \quad A = P \left[\lim_{m \to \infty} \left(1 + \frac{1}{m/r}\right)^{\frac{m}{r}} \right]^{rt}$$

Let $x = \frac{m}{r}$

as $m \to \infty$, $x \to \infty$.

$$\therefore \quad A = P \left[\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} \right]^{rt}$$

$$\Rightarrow \quad A = Pe^{rt} \qquad \because \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

Thus, the compound amount A for a principal amount P after t years at an annual interest rate of r compounded continuously is given by

$$\boxed{A = P\,e^{rt}}$$

Remark. Value of $e = 2.718$; $\log e = 0.4343$

Example 12 : *A person deposits Rs. 5,000 in a bank which pays interest at 11% p.a. compounded continuously. How much amount will be in his account after 10 years ? ($e^{1.1} = 3.0042$)*

Solution : P = Rs. 5,000, $r = 11\% = 0.11$, $t = 10$ years

Amount $S = P \,.\, e^{rt}$

$= 5{,}000 \,.\, e^{0.11 \times 10}$

$= 5{,}000 \,.\, e^{1.1} = 5{,}000 \times 3.0042 = 15{,}021$

$\therefore$ The amount is Rs. 15,021.

Example 13 : *What is the amount after 5 years, if Rs. 10,000 is invested at 8% compounded*

(a) annually ? *(b) semi-annually ?* *(c) quarterly ?*

(d) monthly ? and *(e) continuously ?*

Solution : P = Rs. 10,000, $r = 8\% = 0.08$ and $t = 5$ years

(a) *If interest is compounded annually* :

Amount $A = P(1 + i)^n$

$i = r = 0.08$ and $n = t = 5$

Amount $A = 10{,}000\,(1 + 0.8)^5 = 10{,}000 \times 1.469328 = \text{Rs. } 14{,}693.28$

$\therefore$ The amount is Rs. 14,693.28

(*b*) *If interest is compounded semi-annually* :

$$i = \frac{r}{2} = \frac{0.08}{2} = 0.04$$

$$n = 2 \times 5 = 10$$

Amount $A = 10{,}000\,(1.04)^{10}$

$= 10{,}000 \times 1.480244 = \text{Rs. } 14{,}802.44$

$\therefore$ The amount is Rs. 14,802.44.

(*c*) *If interest is compounded quarterly* :

$$i = \frac{r}{4} = \frac{0.08}{4} = 0.02$$

$$n = 4 \times 5 = 20$$

Amount $A = 10{,}000\,(1.02)^{20}$

$= 10{,}000 \times 1.485947 = \text{Rs. } 14{,}859.47$

$\therefore$ The amount is Rs. 14,859.47

(*d*) *If interest is compounded monthly* :

$$i = \frac{r}{12} = \frac{0.08}{12} = 0.00667$$

$$n = 12 \times 5 = 60$$

Amount $A = 10{,}000\,(1.00667)^{60}$

$= 10{,}000 \times 1.490142 = \text{Rs. } 14901.42$

$\therefore$ The amount is Rs. 14,901.42

(*e*) *If interest is compounded continuously* :

Amount $A = Pe^{rt}$

$A = 10{,}000\, e^{0.08 \times 5}$

$A = 10{,}000 \times e^{0.4}$

$A = 10{,}000 \times 1.4918 = \text{Rs. } 14{,}918$

$\therefore$ The amount is Rs. 14,918.

Example 14 : *An amount of Rs. 5,000 is invested at an annual rate of 9% compounded continuously. How long it will take to this principal to amount to Rs. 8,580 ?*

Solution : Compound Amount $A = P\,e^{rt}$

Given that $P = \text{Rs. } 5{,}000,\ r = 9\% = 0.09, A = \text{Rs. } 8{,}580$

Substituting these values,

$$8{,}850 = 5{,}000\, e^{0.09\,t}$$

$$\Rightarrow \quad e^{0.09t} = \frac{8{,}580}{5{,}000} \Rightarrow e^{0.09t} = 1.716$$

Taking log on both sides,

$$\log e^{0.09t} = \log 1.716$$
$$0.09 \times t \times \log e = \log 1.716$$
$$0.09 \times t \times 0.4343 = 0.2345$$
$$t = \frac{0.2345}{0.09 \times 0.4343} = 6 \text{ years}$$

$\therefore$ 6 years are required.

Example 15 : *A sum of Rs. 4,000 is deposited in a savings bank account that fetches Rs. 885.60 as compound interest at a rate of r% per annum, compounded continuously after 4 years. Find r.*

Solution : Principal P = Rs. 4,000

Compound Interest $A - P$ = Rs. 885.60

$\therefore$ $A = 4{,}000 + 885.60$ = Rs. 4,885.60

Given that $t = 4$

It is to find the rate of interest r.

Compound Amount $A = Pe^{rt}$

Substituting the values, we get

$$4{,}885.60 = 4{,}000\, e^{4r}$$
$$\Rightarrow \quad e^{4r} = \frac{4{,}885.60}{4{,}000} \Rightarrow e^{4r} = 1.2214$$

Taking log on both sides,

$$\log e^{4r} = \log 1.2214$$
$$4r \log e = \log 1.2214$$
$$4 \times r \times 0.4343 = 0.0867$$
$$r = 0.05 = 5\%$$

$\therefore$ The required rate of interest is 5%.

PRESENT VALUE

Every one plans for future. One may think of his son's education, or daughter's marriage for the future of his children. When making such kinds of future plans, we must know well in advance what amount should be invested to receive the certain designed sum in the future. This original principal to be invested is called present value or the capital value of the desined amount.

Thus, if money is worth 'i' per conversion period, the present value of the compounded amount A due in 'n' conversion periods is the principal 'P' which is invested now at the rate of 'i' per period. We derive the formula from the compound interest formula.

We know that $A = P\,(1 + i)^n$

$$\Rightarrow \quad P = A\,(1 + i)^{-n}$$

Thus the present value of amount A due n conversion periods hence at the rate of 'i' per period is given by

$$\boxed{P = A(1+i)^{-n}}$$

The quantity $(1+i)^{-n}$, denoted by 'v', is called the *discount factor*. v represents the present value of Re. 1 due n periods hence at the rate of i per period.

It should be noted that P and A represent the value of the same obligation at different dates. P is the present value of a given obligation while A is the future value of the same obligation. The difference between the two amounts is known as *compound interest* or *compound discount*.

i.e., compound discount $= A - P$

To derive the formula for the present value in case of interest compounded continuously, we solve the formula for the compound amount.

Compound amount is given by

$$A = P e^{rt} \Rightarrow P = Ae^{rt}$$

Thus, the present value of compound amount A due at the end of t years at the annual rate of r compounded continuously is given by

$$\boxed{P = Ae^{-rt}}$$

Example 16 : *Find the present value of Rs. 300 at rate of interest of 6% per annum payable 5 years hence.*

Solution : Hence P = Rs. 300

$i = r = 6\% = 0.06$

$n = 5$

Present value is given by

$$P = A(1+i)^{-n}$$

$$\therefore \quad P = 300\,(1.06)^{-5} = 300\,(0.74726)$$

$$= \text{Rs. } 224.18$$

The required present value is Rs. 224.18.

Example 17 : *How much should be invested at 5% per annum so that after 3 years the amount will be Rs. 8,000, when the interest is compounded (a) annually ? (b) half-yearly ? (c) quarterly ? and (d) continuously ?*

Solution : Here P = Rs. 8,000, $r = 5\% = 0.05$, $t = 3$ years.

(*a*) *In case of interest compounded annually* :

$i = r = 0.05$ and $n = t = 3$

Present value $P = A(1+i)^{-n}$

$$P = 8{,}000\,(1.05)^{-3}$$

$$P = 8{,}000\,(0.86384) = \text{Rs. } 6{,}910.70$$

$\therefore$ Amount to be invested is Rs. 6,910.70

(*b*) *In case of interest compounded semi-annually* :

$$i = \frac{r}{2} = \frac{0.05}{2} = 0.025$$

$$n = 2 \times t = 2 \times 3 = 6$$

Present value $P = 8{,}000\,(1.025)^{-6}$

$\therefore$ $P = 8{,}000\,(0.8623) = \text{Rs. } 6{,}898.40$

$\therefore$ Amount to be invested is Rs. 6,896.40

(*c*) *In case of interest compounded quarterly* :

$$i = \frac{r}{4} = \frac{0.05}{4} = 0.0125$$

$$n = 4 \times t = 4 \times 3 = 12$$

Present value $P = 8{,}000\,(1.0125)^{-12}$

$\therefore$ $P = 8{,}000\,(0.86151) = \text{Rs. } 6{,}892.10$

$\therefore$ Amount to be invested is Rs. 6,892.10

(*d*) *In case of interest compounded continuously* :

Present value $P = A\,e^{-rt}$

$\therefore$ $P = 8{,}000\,e^{-0.05 \times 3} = 8{,}000 \times e^{-0.15}$

$P = 8{,}000 \times 0.86071 = \text{Rs. } 6{,}885.68$

$\therefore$ Amount to be invested is Rs. 6,885.68.

Example 18 : *What is the present value of Rs. 15,000 due after 5 years from now if the interest is compounded continuously at the rate of interest of 6% ?*

Solution : Here $P = \text{Rs. } 15{,}000$, $t = 5$ years, $r = 6\% = 0.06$

Present value $P = A\,e^{-rt} = \dfrac{A}{e^{rt}}$

Substituting the values,

$$P = \frac{15{,}000}{e^{0.06 \times 5}} \Rightarrow P = \frac{15{,}000}{e^{0.3}}$$

Let $x = e^{0.3}$

$$\log x = 0.3 \log e = 0.3 \times 0.4343$$

$$\log x = 0.13029$$

$$x = \text{antilog } (0.13029)$$

$$x = 1.3496$$

$\therefore$ $P = \dfrac{15{,}000}{1.3496} = \text{Rs. } 11114.40$

The present value is Rs. 11,114.40.

VARYING RATE OF INTEREST

In the previous sections, we have assumed that the rate of interest is fixed through out the duration of the transaction. But practically the interest rates change from time to time due to the policy of the Banks or any other financial institution or the decisions taken by the Government. Thus, a bank that charges the rate of interest at 8% per annum, may increase it to 9% or reduce to 7% in the future.

The compound amount at changing rates is calculated as follows :

Suppose that a principal of Rs. P is deposited in a bank and the bank compounds interest at 'i_1' per conversion period for n_1 periods.

Then the compound Amount will be $P(1 + i_1)^{n_1}$.

We assume that suddenly the interest rate is increased to i_2 per conversion period and the deposit is withdrawn at the end of another n_2 period.

$\therefore$ The compound amount at the end of $n_1 + n_2$ periods will be

$$A = P(1 + i_1)^{n_1}(1 + i_2)^{n_2}$$

In this manner, we can calculate the compound amount, if the interest rate gets changed any number of times.

Example 19 : *A person deposited Rs. 1,000 in a bank at 5% compounded annually. After 5 years the rate of interest increased to 6% and after four more years, the rate of interest was further increased to 7%. The money was withdraw at the end of 12 years. What amount he will get when he withdraws his deposit after 12 years ?*

Solution : For first 5 years

$$i = r = 5\% = 0.05, P = \text{Rs. } 1{,}000, \; n = 5$$

Amount after 5 years

$$A = P(1 + i)^n$$

$$\therefore \quad A = 1{,}000\,(1 + .05)^5 = 1{,}000\,(1.05)^5$$

$$A = 1{,}000\,(1.27628) = \text{Rs. } 1276.28$$

After 5 years, for next 4 years

$$i = r = 6\% = 0.06, P = \text{Rs. } 1{,}276.28, \; n = 4$$

Amount after 9 years

$$A = 1276.28\,(1 + 0.06)^4$$

$$= 1276.28\,(1.06)^4$$

$$= 1276.28 \times 1.26248 = \text{Rs. } 1611.28$$

After 9 years $i = r = 7\% = 0.07$

Amount is withdrawn at the end of 12 years

$\therefore \quad n = 3, \; P = \text{Rs. } 1{,}611.28$

Amount after 12 years, $A = 1611.28\,(1 + 0.07)^3$

$$= 1611.28\,(1.07)^3$$
$$= 1611.28 \times 1.225 = \text{Rs. } 1973.83$$

$\therefore$ The amount the end of 12 years is Rs. 1973.83.

Example 20 : *Mr. X deposited Rs. 10,000 in State Bank of India. The bank calculates the interest at 8% p.a. compounded semi-annually. After 5 years, he again deposits an amount of Rs. 8,000 in his account. The bank has increased the rate of interest from 8% to 10% at the end of 8th year. If he withdraws the total amount at the end of 10 years, what amount will have been accumulated in his account ?*

Solution : *Incase of deposit of Rs. 10,000 :*

Rate of interest is 8% for first 8 years and 10% for next two years.

For first 8 years,

$$P = \text{Rs. } 10{,}000,\ r = 8\% = 0.08$$

Interest compounded semi-anually,

$$\therefore \quad i = \frac{0.08}{2} = 0.04, \text{ and } n = 8 \times 2 = 16$$

Amount is given by $A = P(1 + i)^n$

$\therefore$ Amount at the end of 8 years

$$A = 10{,}000\,(1 + 0.04)^{16}$$
$$A = 10{,}000\,(1.04)^{16}$$
$$A = 10{,}000\,(1.872792) = \text{Rs. } 18{,}727.92$$

This is the principal for 9th year

$\therefore$ For next two years

$$P = \text{Rs. } 18{,}727.92,\ r = 10\% = 0.10,\ n = 2 \times 2 = 4$$
$$i = \frac{0.10}{2} = 0.05$$

Amount at the end of 10th year

$$A = 18{,}727.92\,(1 + 0.05)^4$$
$$= 18{,}727.92\,(1.05)^4$$
$$= 18{,}727.92 \times 1.2155 = \text{Rs. } 22{,}763.90 \qquad \ldots(1)$$

In case of deposit of Rs. 8,000 for 5 years starting from 6th year :

Interest rate is 8% for first 3 years, and 10% for next two years.

For first three years

$$P = \text{Rs. } 8{,}000,\ r = 8\% = 0.08,\ n = 3 \times 2 = 6$$
$$i = \frac{0.08}{2} - 0.04$$

Amount at the end of 8th year

$$A = 8{,}000\,(1 + .04)^6 = 8{,}000\,(1.04)^6$$
$$= 8{,}000\,(1.2597) = \text{Rs. } 10{,}077.69$$

This is the principal for 9th year.

$\therefore$ For next two years

$$P = \text{Rs. } 10{,}077.69,\ r = 10\% = 0.10,\ n = 2 \times 2 = 4$$

$$i = \frac{0.10}{2} = 0.05$$

Amount at the end of 10th year

$$A = 10077.69\,(1 + 0.05)^4 = 10077.69\,(1.05)^4$$

$$= 10077.69\,(1.2155) = 12249.43 \qquad \ldots(2)$$

Total Amount = (1) + (2) = Rs. 22,763.90 + Rs. 12,249.43 = Rs. 35,013.33

$\therefore$ Mr. X will receive Rs. 35,013.33 at the end of 10 years.

Example 21 : *To what sum will Rs. 2,000 amount in 8 years if invested at 6% effective rate for the first 2 years, at 6% compounded semi-annually for the next 3 years at 6% compounded continuously thereafter ?*

Solution : $P = \text{Rs. } 2{,}000$

For first 2 years :

Interest is compounded annually.

$$r = i = 6\% = 0.06, \text{ and } n = 2$$

Amount is given by $A = P\,(1 + i)^n$

$$A = 2{,}000\,(1 + 0.06)^2 = 2{,}000\,(1.06)^2$$

$$A = \text{Rs. } 2{,}247.20$$

For next 3 more years :

$$P = \text{Rs. } 2{,}247.2 \text{ and } r = 6\% = 0.06$$

Interest is calculated semi-annually.

$\therefore$ $$i = \frac{r}{2} = 0.03, \text{ and } n = 3 \times 2 = 6$$

Amount at the end of 5th year

$$A = 2{,}247.20\,(1.03)^6 = \text{Rs. } 2683.27$$

For last 3 years :

$$P = \text{Rs. } 2{,}683.27 \text{ and } r = 6\% = 0.06$$

Interest is calculated continuously.

Amount at the end of 8th year

$$A = Pe^{rt}$$

$$= 2683.27 \times e^{0.06 \times 3} = 2683.27 \times e^{0.18}$$

$$= 2683.27 \times 1.1972 = \text{Rs. } 3212.42$$

Amount at the end of 8th year is Rs. 3,212.42.

EXERCISES

1. Find the compound amount and compound interest of Rs. 1,00,000 invested for 6 years at 8% compounded quarterly. [**Ans.** Rs. 1,60,843, Rs. 60,843]
2. If Rs. 1,000 is invested at an annual rate of interest of 15%, what is the amount after 5 years if the interest is compounded monthly ? [**Ans.** Rs. 2107.18]
3. Find the compound amount of Rs. 2,000 for four years at 6% converted (*a*) annually, (*b*) semi-annually, (*c*) quarterly, and (*d*) monthly.
[**Ans.** (*a*) Rs. 2524.95, (*b*) Rs. 2,533.54, (*c*) Rs. 2,537.97, (*d*) Rs. 2,540.97]
4. An amount of Rs. 50,000 is deposited into a savings bank account that pays at the rate of 6% p.a. What amount will be there in the account, if the interest is compounded (*a*) annually, (*b*) quarterly, and (*c*) monthly ?
[**Ans.** (*a*) Rs. 1,60,300, (*b*) Rs. 1,62,000, (*c*) Rs. 1,65,500]
5. Find the compound interest for a sum of Rs. 700 invested for 15 years at 8% compounded semi-annually. [**Ans.** Rs. 1570.37]
6. In how many years an investment of Rs. 5,000 will amount to Rs. 7,000, if it is invested at 8% compounded quarterly ? [**Ans.** 4.25 years]
7. At what rate per cent per annum compound interest will Rs. 2,000 amount to Rs. 3,000 in 3 years, if the interest is reckoned half-yearly ? [**Ans.** 14%]
8. In how many years will an amount double itself at 12.2% compounded annually ?
[**Ans.** 6.02 years]
9. At what rate per cent will Rs. 40,000 yield Rs. 13,240 compound interest in 3 years ?
[**Ans.** 10%]
10. How long will it take for a principal to double if the money is worth 12% compounded monthly ? [**Ans.** 5.84 years]
11. A sum of money is put at compound interest for two years at 20% per annum. It would fetch Rs. 482 more, if the interest is payable half-yearly than if it is payable yearly. Find the sum. [**Ans.** Rs. 20,000]
12. The difference between simple interest and compound interest on a sum for 3 years at 5% per annum is Rs. 76.25. Find the sum. [**Ans.** Rs. 10,000]
13. The difference between simple interest and compound interest on a sum of money invested for 4 years at 5% per annum is Rs. 150. find the sum. [**Ans.** Rs. 9677.40]
14. A person aged 22 has contributed Rs. 25 to his provident fund during the current month. What will this particular contribution amount to by the time he retires at age 60, assuming a rate of interest 7% p.a. ? [**Ans.** Rs. 326.98]
15. The compound interest on Rs. 8,000 in two years at certain rate is Rs. 820 and in three years, it is Rs. 1,261 at the same rate. Find the rate of interest. [**Ans.** 5%]
16. Mr. *X* deposited Rs. 10,000 in a savings account for 3 years offering progressive rate of interest. Bank pays 10% per annum compounded semi-annually for the first year, 12% per annum compounded quarterly for the second year and 13% per annum compounded continuously for the third year. Find the amount at the end of 3 years.
[**Ans.** Rs. 14,131.14]

17. If Rs. 2,000 is deposited in a savings account that earns interest at an annual rate of 6% compounded continuously, what is the value of the account after 3 years ?

[**Ans.** Rs. 2,107.18]

18. At what rate will a principal amount triple itself in 12 years, if the interest is compounded continuously ? [**Ans.** 9.15%]

19. How much should be invested at 6% per annum so that after 4 years, the amount will be Rs. 25,000, when the interest is compounded continuously ? [**Ans.** Rs. 19,655]

20. How long will it take for Rs. 4,000 to account to Rs. 7,000, if it is invested at 7% compounded continuously ? [**Ans.** 8 years]

21. In how many years a principal amount P will become double, if money is worth 6% compounded continuously ? [**Ans.** 11.5 years]

22. Mr. A deposited Rs. 5,000 in a bank which calculates interest continuously. After 10 years, the amount accumulated was Rs. 15,020. At what rate bank calculated interest ? [**Ans.** 11%]

23. The population of a town is 8,00,000. During the first year, the population increased by 25%. During the second year, the population increased by 20%. During the third year, the population increased by 10%. Find the population of the town after 3 years.

[**Ans.** 13,20,000]

24. A sum of Rs. 2,000 is invested at a rate of interest of 5% per annum. After 7 years, the rate of interest was changed to 5% per annum convertible half-yearly. After a further period of 3 years, the rate was again changed to 6% per annum convertible quarterly. What is the accumulated value at the end of 15 years from commencement ? [**Ans.** Rs. 4,395.61]

25. What is the present value of Rs. 500 due at the end of 10 years, the rate of interest being 5% p.a. for the first four years from now and 6% convertible half-yearly for the next 6 years ? [**Ans.** Rs. 288.51]

26. A sum of Rs. 1,000 is due at the end of 10 years 6 months. The present interest rates are 7% per annum but it is expected that there will be a fall in the rates after 6 years bringing down the rate to 6% per annum. Find the present value of the sum of money under these assumption. [**Ans.** Rs. 512.65]

27. The difference between the accumulated values of a sum of money accumulated for 10 years at an effective rate of 4% per annum and the accumulated value of the same sum of money over the same period at 4% per annum payable quarterly is Rs. 150. Find the sum. [**Ans.** Rs. 17,401]

28. A promises to pay B a sum of Rs. 200 at the end of 3 years and another Rs. 400 at the end of 5 years from now. What immediate cash payment should B accept in lieu of the above payments, if interest is compounded at 5% per annum ? [**Ans.** Rs. 486.18]

29. Mr. Singh wants to accumulate Rs. 50,000 after 20 years for the purpose of his daughter's marriage. How much money should be set a side now, if interest is compounded continuously at an annual rate of 5% ? [**Ans.** Rs. 18,394]

30. Find the present value of Rs. 2,500 due after 4 years, if the interest rate is 6% per annum compounded (*a*) annually, (*b*) quarterly, (*c*) monthly, and (*d*) continuously.
[**Ans.** (*a*) Rs. 2,137, (*b*) Rs. 1,974.72, (*c*) Rs. 1,980.98, (*d*) Rs. 1,965.40]

31. A person invests a certain sum of money in a bank paying interest 5% effective and plans to receive Rs. 5,000 in 5 years. What is the compound discount of this investment ? [**Ans.** Rs. 1082.4]

32. What is the year zero deposit at 8% per annum compounded quarterly to obtain a compound amount of Rs. 1,00,000 after 5 years ? How much interest will be earned during this period ? [**Ans.** Rs. 67,297.15, Rs. 32,702.85]

33. Find the compound interest (compounded annually) on Rs. 10,000 for four years at 10% for the first year, 12% for the second year, 14% for the third year and 15% for the fourth year. [**Ans.** Rs. 6,151.52]

34. Which yields the higher rate of interest—a fixed deposit in a bank which gives Rs. 1,629 after 5 years for every Rs. 1,000 deposited or a National savings certificate which gives Rs. 1,901 after 6 years for Rs. 1,000 ? [**Ans.** FD : 10.24%, NSC : 11.28%]

35. Mr. *A* has a right to receive an amount of Rs. 1,000 at the end of 12 years from now. This right has been sold to Mr. *B* for a present value calculated at the rate of 8% per annum. The money thus received was invested by Mr. *A* in a deposit account at 9% per annum payable half-yearly. After 8 years the account had to be closed and then Mr. *A* invested the amount available at 6% per annum in another bank. How much has Mr. *A* gained or lost in this transaction, as at the end of 12 years ?
[**Ans.** Gain of Rs. 13.90]