

**THIRD SEMESTER B.Sc. DEGREE (SUPPLEMENTARY)
EXAMINATION, NOVEMBER 2016**

(UG-CCSS)

Mathematics

MM 3B 03—CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all twelve* questions. Each question carries $\frac{1}{4}$ weightage :

- 1 Evaluate $\lim_{t \rightarrow 1} \frac{t^2 + t - 2}{t^2 - 1}$.
- 2 Define the continuity of a function f at a right end point $x = b$ of its domain.
- 3 At what points are the function $y = \frac{1}{x-2} - 3x$ is continuous.
- 4 State Rolle's theorem.
- 5 What are the critical points of f given $f'(x) = (x-1)^2(x+2)$.
- 6 Find the intervals in which the function f is increasing given $f'(x) = (x-1)(x+2)(x-3)$.
- 7 Evaluate $\lim_{x \rightarrow -\infty} \frac{2x^2 - 3}{7x + 4}$.
- 8 Write the sum without sigma notation and then evaluate the sum $\sum_{k=1}^3 (-1)^{k+1} \sin \frac{\pi}{k}$.
- 9 Evaluate $\int_0^{\pi/3} 2 \sec^2 x \, dx$.
- 10 Suppose that $\int_1^2 f(x) \, dx = 5$. Find $\int_1^2 -f(x) \, dx$.
- 11 Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.
- 12 Evaluate $\int_0^1 (x^2 + \sqrt{x}) \, dx$.

(12 \times $\frac{1}{4}$ = 3 weightage)

Turn over

II. Short Answer Type Questions. Answer *all* nine questions :

- 13 A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m. How far will a 45 N force stretch the spring.
- 14 Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = 0$, $x = 2$.
- 15 Evaluate $\int_{-\pi/4}^0 \tan x \sec^2 x \, dx$.
- 16 Evaluate $\frac{d}{dt} \int_0^{t^4} \sqrt{u} \, du$.
- 17 Find the average value of $f(x) = -3x^2 - 1$ on $[0, 1]$.
- 18 Evaluate $\sum_{k=1}^6 (3 - k^2)$.
- 19 Find the linearization of $f(x) = x^3 - x$ at $x = 1$.
- 20 Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.
- 21 Find the absolute maximum and minimum value of $g(t) = 8t - t^4$ on $[-2, 1]$.

(9 × 1 = 9 weightage)

III. Short essay or paragraph questions. Answer any *five* questions :

- 22 The line segment $x = 1 - y$, $0 \leq y \leq 1$ is revolved about the y -axis to generate a cone. Find its lateral surface area.
- 23 Find the length of the curve $x = \sin y$, $0 \leq y \leq \pi$.
- 24 Find the area of the region between the x -axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \leq x \leq 2$.
- 25 Find the smallest perimeter possible for a rectangle whose area is 16 square inches.
- 26 Using the Sandwich theorem find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.
- 27 Find the local maxima and local minima of $g(x) = -x^3 + 12x + 5$, $-3 \leq x \leq 3$.
- 28 Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.

(5 × 2 = 10 weightage)

IV. Essay Questions. Answer any *two* questions :

- 29 The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x -axis to generate a solid. Find the volume of the solid.
- 30 Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$ and the x -axis.
- 31 Prove that $\lim_{x \rightarrow 2} f(x) = 4$ if: $f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2. \end{cases}$

(2 × 4 = 8 weightage)

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Name.....

Reg. No.....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(U.G.—CCSS)

Core Course—Mathematics

MM 3B 03—CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer all questions :—

1 Let $F(t) = 2(t - 1) + 3$. Evaluate F at the input value $x + 2$.

2 If $f(x) = \sqrt{x}$ and $g(x) = x + 1$ find $(f \circ g)x$.

3 Find the domain and range of $f(x) = 1 + x^2$.

4 Evaluate $\lim_{x \rightarrow -2} \frac{2x - 4}{x^3 + 2x^2}$.

5 At what points are the function $y = \frac{1}{x - 2} - 3x$ is continuous.

6 State Rolle's theorem.

7 What are the critical points of f given

$$f'(x) = (x - 1)(x + 2)(x - 3).$$

8 Evaluate $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

9 Find dy if $y = \frac{2x}{1 + x^2}$.

10 Find the intervals in which the function f is increasing. Given $f'(x) = x(x - 1)$.

Turn over

11 The length of the longest sub interval of a partition is called its _____.

12 Evaluate $\int_0^{\pi/3} 2 \sec^2 x \, dx$.

($12 \times \frac{1}{4} = 3$ weightage)

II. Answer all *nine* questions.

13 Find the volume of the solid generated by revolving the region bounded by the lines $y = 0$, $x = 2$ and the curve $y = x^3$.

14 Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

15 Find the average value of $f(x) = -3x^2 - 1$ on $[0, 1]$.

16 Evaluate $\sum_{k=1}^6 (3 - k^2)$.

17 Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.

18 Find the absolute maximum and minimum values of $f(x) = -x - 4$, $-4 \leq x \leq 1$.

19 Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.

20 Find the work done by a force of $F(x) = \frac{1}{x^2}$ N along the x -axis from $x = 1$ m. to $x = 10$ m.

21 Evaluate $\int_0^{\pi/4} \tan x \sec^2 x \, dx$.

($9 \times 1 = 9$ weightage)

III. Answer any *five* questions :—

- 22 Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \leq x \leq 4$, about the y -axis.
- 23 Find the length of the curve $y = \tan x$, $\frac{-\pi}{3} \leq x \leq 0$.
- 24 Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.
- 25 Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
- 26 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.
- 27 Find the intervals on which the function $g(t) = -t^2 - 3t + 3$ is increasing and decreasing.
- 28 About how accurately should we measure the radius r of a sphere to calculate the surface area $S = 4\pi r^2$ within 1 % of its true value.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :—

- 29 Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$, $0 \leq x \leq 1$.
- 30 Show that the centre of mass of a straight, thin strip or rod of constant density has halfway between its two ends.
- 31 State and prove the fundamental theorem of calculus.

(2 × 4 = 8 weightage)

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Name.....

Reg. No.....

**THIRD SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION NOVEMBER 2015
(UG—CCSS)**

Core Course—Mathematics

MM 3B 03—CALCULUS

Time : Three Hours

Maximum : 30 Weightage

Answer all questions.

1. If $f(x) = \sqrt{x}$ find $(f \circ f)(x)$.
2. Find the domain and range of $f(x) = \sqrt{4 - x^2}$.
3. Evaluate $\lim_{x \rightarrow -2} \frac{x^2 - 7x + 10}{x - 2}$.
4. At what points are the function $y = \frac{x^4 + 20}{5x(x - 2)}$ is continuous.
5. State the Mean Value Theorem.
6. If f is smooth in $[a, b]$ then the length of the curve $y = f(x)$ from a to b is $L =$ _____.
7. What are the critical points of f given $f'(x) = (x - 1)^2(x + 2)$.
8. Find the intervals in which the functions f is increasing given $f'(x) = (x - 1)^2(x + 2)$.
9. Evaluate $\lim_{x \rightarrow -\infty} \frac{11x + 2}{2x^3 - 1}$.
10. Write the sum without sigma notation and then evaluate the sum $\sum_{h=1}^4 \cos k\pi$.

Turn over

11. If $\int_0^2 f(x) dx = 3$ find $\int_0^2 \sqrt{3} f(x) dx$.

12. Evaluate $\int_0^4 \left(3x - \frac{x^3}{4} \right) dx$.

(12 \times $\frac{1}{4}$ = 3 weightage)

Answer all nine questions.

13. Find the volume of the solid generated by revolving the region bounded by the line $y = 0$ and the curve $y = x - x^2$.

14. Suppose that f is continuous and that $\int_0^3 f(x) dx = 3$ and $\int_0^4 f(x) dx = 7$. Find $\int_4^3 f(x) dx$.

15. Evaluate $\sum_{k=1}^7 (-2k)$.

16. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x) dx = 0$ then $f(x) = 0$ at least once in $[a, b]$.

17. Find the average value of $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$.

18. Find the linearization of $f(x) = \sqrt{x}$ at $x = 4$.

19. Evaluate $\frac{d}{dt} \int_0^{t^4} \sqrt{u} du$.

20. Evaluate $\int_0^{2\pi} \frac{\cos z}{\sqrt{4 + 3 \sin z}} dz$.

21. Find the absolute maximum and minimum values of $f(x) = \frac{-1}{x}$, $-2 \leq x \leq -1$.

(9 \times 1 = 9 weightage)

Answer any **five** questions.

22. Find the length of the curve $y = \tan x$, $-\frac{\pi}{3} \leq x \leq 0$.
23. Find the lateral surface area generated by revolving $xy=1$, $1 \leq y \leq 2$ about the y -axis.
24. Find the intervals on which the function $f(x) = 3x^2 - 4x^3$ is increasing and decreasing.
25. Using the sandwich theorem find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.
26. Express the solution of the following initial value problem as an integral

$$\text{Differential Equation} : \frac{dy}{dx} = \tan x$$

$$\text{Initial condition} : y(1) = 5$$

27. Find the area of the region enclosed by the curve $y = 2x - x^2$ and the line $y = -3$.
28. Find the volume of the solid generated by revolving the region between the y -axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$ about the y -axis.

(5 × 2 = 10 weightage)

Answer any **two** questions.

29. Find the area of the region between the x -axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \leq x \leq 2$.
30. Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.
31. Show that the centre of mass of a straight, thin strip or rod of constant density has halfway between its two ends.

(2 × 4 = 8 weightage)