IDEAL FERMI SYSTEMS

An ideal fermi gas is characterised by particles with half integral spin. They are indistinguishable, non interacting and having anti-symmetric wave function under the exchange of particles. They obey Pauli's exclusion principle.

Large no. of particles occupying the single energy state does not even arise in this case. Hence there is no B.E condensation.

Eg :: nucleons.

q-potential of the s/m is given by,

$$q = \frac{PV}{kT} = \ln Z$$

$$= \sum_{\epsilon} \ln (1 + z e^{-\beta \epsilon})$$

z - fugacity

$e^{\mu/kT}$

$\beta = 1/kT$

The value of z ranges from zero to infinity.

And the average occupational no is given by,

$$N = \langle n_\epsilon \rangle = \sum \frac{1}{z^{-1} e^{\beta \epsilon} + 1}$$

$$= \sum \frac{1}{e^{-\mu/kT} e^{\epsilon/kT} + 1}$$

$$= \sum \frac{1}{e^{\frac{(\epsilon - \mu)}{kT}} + 1}$$

Now to find the equation of state we have to start with the q-potential:

$$\frac{PV}{kT} = \sum_{\epsilon} \ln(1 + ze^{-\beta\epsilon})$$

For large volume, $\frac{PV}{kT} = \int_0^\infty \ln(1+ze^{-\beta\epsilon}) \frac{4\pi p^2 dp V}{h^3}$

The energy of fermions, $\epsilon = \frac{p^2}{2m}$.

$$\therefore \frac{PV}{kT} = \int_0^\infty \ln\left(1+ze^{-\frac{\beta p^2}{2m}}\right) \frac{4\pi p^2 dp V}{h^3}$$

Put, $\frac{\beta p^2}{2m} = x^2$

$$p^2 = \frac{2m}{\beta} x^2 \qquad \therefore \cancel{2}p\,dp = \frac{\cancel{2}m}{\beta} 2x\,dx$$

$$p = \left(\frac{2m}{\beta}\right)^{1/2} x.$$

$$\frac{PV}{kT} = \int_0^\infty \ln(1+ze^{-x^2}) \frac{4\pi V}{h^3} \left(\frac{2m}{\beta}\right)^{3/2} x^2\,dx.$$

$$= \frac{4\pi V}{h^3} (2mkT)^{3/2} \int_0^\infty \ln(1+ze^{-x^2})\, x^2\,dx.$$

Multiply and divide by $\sqrt{\pi}$:

$$= \frac{4V}{\sqrt{\pi}\, h^3} (2\pi mkT)^{3/2} \int_0^\infty \ln(1+ze^{-x^2})\, x^2\,dx.$$

$$= \frac{4V}{\sqrt{\pi}\,\lambda^3} \int_0^\infty \ln(1+ze^{-x^2})\, x^2\,dx.$$

$$\frac{P}{KT} = \frac{1}{\lambda^3} \underbrace{\frac{4}{\sqrt{\pi}} \int_0^\infty \ln(1+ze^{-x^2})\, x^2\, dx}_{f_{5/2}(z)}.$$

$f_{5/2}(z)$ - Fermi-Dirac function about 5/2.

$$\boxed{\frac{P}{KT} = \frac{1}{\lambda^3} f_{5/2}(z)} \quad — (1)$$

$$f_{5/2}(z) = z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} \dots$$

By introducing the statistical weight factor arising from the internal structure of particle,

$$(1) \longrightarrow \boxed{\frac{P}{KT} = \frac{g}{\lambda^3} f_{5/2}(z)}$$

<u>Average no: of particles:</u>

$$N = z \frac{\partial}{\partial z} \ln Z$$

$$= z \frac{\partial}{\partial z} \sum_\epsilon \ln(1+ze^{-\beta\epsilon}).$$

$$= z \sum_\epsilon \frac{1}{(1+ze^{-\beta\epsilon})} e^{-\beta\epsilon} = \sum_\epsilon \frac{ze^{-\beta\epsilon}}{(1+ze^{-\beta\epsilon})}$$

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Replacing summation by integration,

$$N = \int_0^\infty \frac{ze^{-\beta p^2/2m}}{(1+ze^{-\beta p^2/2m})} \times \frac{4\pi p^2 dp V}{h^3}$$

$$N = \frac{4\pi V}{h^3} \int_0^\infty \frac{ze^{-x^2}}{(1+ze^{-x^2})} \left(\frac{2m}{\beta}\right)^{3/2} x^2 dx$$

Put $\frac{\beta p^2}{2m} = x^2$

$p^2 = \frac{2m}{\beta} x^2$

$\cancel{2} p dp = \frac{2m}{\beta} \cancel{2} x dx$

$$N = \frac{4\pi V}{h^3} (2mkT)^{3/2} \int_0^\infty \frac{ze^{-x^2}}{(1+ze^{-x^2})} x^2 dx$$

$\times \frac{\sqrt{\pi}}{\sqrt{\pi}}$

$$N = \frac{4V}{\sqrt{\pi}\lambda^3} \int_0^\infty \frac{ze^{-x^2}}{(1+ze^{-x^2})} x^2 dx .$$

$$\boxed{\frac{N}{V} = \frac{1}{\lambda^3} f_{3/2}(z)} \quad \text{— (2)}$$

$f_{3/2}(z)$ – fermi – dirac funⁿ about 3/2.

$$\frac{N}{V} = \frac{g}{\lambda^3} f_{3/2}(z).$$

where $f_{3/2}(z) = z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \dots$

Internal energy :

$$U = -\frac{\partial}{\partial \beta} \ln Z .$$

$$U = kT^2 \frac{\partial}{\partial T} \ln Z$$

ℓn =

$$U = kT^2 \frac{\partial}{\partial T}\left(\frac{pV}{kT}\right)$$

We know that $\frac{pV}{kT} = \frac{V}{\lambda^3} f_{5/2}(z)$

$$U = kT^2 \frac{\partial}{\partial T}\left(\frac{V}{h^3}(2\pi m k T)^{3/2} f_{5/2}(z)\right)$$

$$U = kT^2 \frac{V}{h^3}(2\pi m k)^{3/2} f_{5/2}(z) \frac{3}{2} T^{1/2}$$

$$= kT \frac{V}{h^3}(2\pi m k T)^{3/2} \frac{3}{2} f_{5/2}(z)$$

$$\boxed{U = \frac{3}{2} kTV \frac{f_{5/2}(z)}{\lambda^3}} \quad \text{— (3)}$$

We also have,

$$\frac{N}{V} = \frac{1}{\lambda^3} f_{3/2}(z)$$

$$V = \frac{N\lambda^3}{f_{3/2}(z)} \quad \text{— (4)}$$

We have $U = \frac{3}{2} kTV \frac{f_{5/2}(z)}{\lambda^3}$ — (5)

(4) in (5), $\Rightarrow$ $U = \frac{3}{2} kT \frac{N\cancel{\lambda^3}}{f_{3/2}(z)} \frac{f_{5/2}(z)}{\cancel{\lambda^3}}$

$$\boxed{U = \frac{3}{2} NkT \frac{f_{5/2}(z)}{f_{3/2}(z)}} \quad — (6)$$

Pressure

$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{5/2}(z)$$

$$P = \frac{kT}{\lambda^3} f_{5/2}(z) \quad — (7)$$

We have $U = \frac{3}{2} kTV \frac{f_{5/2}(z)}{\lambda^3}$

$$\frac{2}{3} \frac{U}{V} = kT \frac{f_{5/2}(z)}{\lambda^3} \quad — (8)$$

Substitute (7) in (8), $\boxed{P = \frac{2}{3} \frac{U}{V}}$

Case-1

When the density of the gas is very low or the temperature is high. (ie when $n\lambda^3 << 1$). Then the gas is said to be highly ~~degenerate~~ non-degenerate.

$$\therefore f_{5/2}(z) \approx f_{3/2}(z) \approx z$$

Then from (6), $U = \frac{3}{2} NkT$

$C_V = \frac{3}{2} Nk$ which means that the s/m is classical.

Case - II

When the temperature is very low, (ie $T \to 0$)

which means $n\lambda^3 \to \infty$

$\therefore \frac{1}{T} = \frac{1}{0} = \infty$

$$\langle n_\epsilon \rangle = \sum \frac{1}{z^{-1} e^{\beta\epsilon} + 1} = \sum \frac{1}{e^{\frac{(\epsilon - \mu)}{kT}} + 1}$$

At very low temperature $(T=0)$

$$\langle n_\epsilon \rangle = \sum_\epsilon \frac{1}{e^{\frac{(\epsilon - \epsilon_f)}{kT}} + 1}$$

All particles state upto $\epsilon = \epsilon_f$ are completely filled with one particle per state, according to pauli's exclusion principle. And all the particle states with $\epsilon > \epsilon_f$ are empty. ϵ_f is the limiting energy and is called fermi energy.

OR Fermi energy can be defined as the energy of top most filled orbit when $T = 0$.

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i) $\epsilon > \epsilon_f$

$\Rightarrow \epsilon - \epsilon_f = +ve$

$$\langle n_\epsilon \rangle = \frac{1}{e^{1/0}+1} = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

$$\langle n_\epsilon \rangle = 0$$

$\Rightarrow$ no particles exists above the energy level ϵ_f

ii) $\epsilon < \epsilon_f$

$\epsilon - \epsilon_f = -ve$

$$\langle n_\epsilon \rangle = \frac{1}{e^{-1/0}+1} = \frac{1}{e^{-\infty}+1} = \frac{1}{\frac{1}{e^{\infty}}+1} = \frac{1}{0+1} = 1$$

$$\underline{\underline{\langle n_\epsilon \rangle = 1}}$$

<u>Calculation of fermi energy</u> :

For the calculation of fermi energy we have the condition.

no. of energy states = no. of particles.

$$\int_0^{\epsilon_f} g(\epsilon)\, d\epsilon = N$$

$$\int_0^{P_F} \frac{4\pi P^2 dP V}{h^3} \times g = N$$

$$\frac{4\pi V g}{h^3} \int_0^{P_F} P^2 dP = N$$

$$\frac{4\pi g V}{h^3} \cdot \frac{P_F^3}{3} = N$$

$$P_F^3 = \left(\frac{3Nh^3}{4\pi g V}\right)$$

$$P_F = \left(\frac{3Nh^3}{4\pi g V}\right)^{1/3}.$$

$$\therefore\ E_f = \frac{P_F^2}{2m} = \left(\frac{3N}{4\pi g V}\right)^{2/3} \frac{h^2}{2m}$$

$$\boxed{E_f = \left(\frac{3N}{4\pi g V}\right)^{2/3} \frac{h^2}{2m}}$$

Zero - point energy :

$$E_0 = \int_0^{E_f} E\, g(E)\, dE$$

$$= \int_0^{P_f} \frac{4\pi P^2 dP V}{h^3} \frac{P^2}{2m}.$$

$$= \int_0^{P_f} \frac{4\pi V}{2m h^3} P^4\, dP$$

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$$E_0 = \frac{4\pi g V}{2mh^3} \cdot \frac{P_f^5}{5}$$

$$= \frac{4\pi g V}{5h^3} \frac{P_f^2}{2m} P_f^3$$

$$= \frac{\cancel{4\pi g V}}{2m\cancel{h^3}} \frac{P_f^2}{5} \frac{3N\cancel{h^3}}{\cancel{4\pi g V}}$$

$$E_0 = \frac{3N}{5} \frac{P_f^2}{2m} = \frac{3}{5} N E_f$$

↳ Ground state energy.

Zero point pressure:

$$P = \frac{2}{3} \frac{U}{V}$$

$$= \frac{2}{3} \frac{E_0}{V}$$

$$= \frac{2}{\cancel{3}} \frac{\cancel{3}}{5} \frac{N E_f}{V} \qquad \frac{N}{V} = n.$$

$$\boxed{P = \frac{2}{5} n E_f}$$

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MAGNETIC BEHAVIOUR OF IDEAL FERMI GAS :

Pauli's Paramagnetism :

Consider 'n' electrons in volume 'V' and it is placed in external magnetic field H. Let 'μ_0' be the spin magnetic moment of single electron. Then '$\mu_0 H$' be the potential energy, where electrons are alligned in the direction opposite to the external field $\vec{H}$. And '$-\mu_0 H$' is the potential energy when it is aligned in the direction of external magnetic field.

Let n_p^+ is the no. of electrons having momentum 'p' parallel to $\vec{H}$ and 'n_p^-' is the no. of electron having momentum 'p' and antiparallel to $\vec{H}$. Then the net magnetic moment

$$M = \sum_{p=0}^{\infty} (n_p^+ - n_p^-)\mu_0 \quad \text{—①}$$

∴ we have the average no. of particles,

$$n = \frac{1}{e^{\beta(\epsilon-\mu)}+1}$$

for $n_p^+ = \dfrac{1}{e^{\beta(\epsilon_p^+ - \mu)}+1}$

for $n_p^- = \dfrac{1}{e^{\beta(\epsilon_p^- - \mu)} + 1}$

where $\epsilon_p^+ = \dfrac{p^2}{2m} - \mu_0 \vec{H}$

$\epsilon_p^- = \dfrac{p^2}{2m} + \mu_0 \vec{H}$

Then $M = \sum_{p=0}^{\infty} \left[\dfrac{1}{e^{\beta((\frac{p^2}{2m} - \mu_0 H) - \mu)} + 1} - \dfrac{1}{e^{\beta((\frac{p^2}{2m} + \mu_0 H) - \mu)} + 1} \right] \mu_0$

For large volume summation over states can be replaced by integration,

$$M = \int_0^{\infty} \left[\frac{1}{e^{\beta(p^2/2m - \mu_0 H - \mu)} + 1} - \frac{1}{e^{\beta(p^2/2m + \mu_0 H - \mu)} + 1} \right] \mu_0 \frac{4\pi p^2 dp V}{h^3}$$

$$M = \frac{\mu_0 4\pi V}{h^3} \left[\underbrace{\int_0^{\infty} \frac{p^2 dp}{e^{\beta(\frac{p^2}{2m} - \mu_0 H - \mu)} + 1}}_{a} - \underbrace{\int_0^{\infty} \frac{p^2 dp}{e^{\beta(\frac{p^2}{2m} + \mu_0 H - \mu)} + 1}}_{b} \right]$$

If $a = \displaystyle\int_0^{\infty} \frac{p^2 dp}{e^{\beta(\frac{p^2}{2m} - \mu_0 H - \mu)} + 1}$

Put $\dfrac{p^2}{2m} - \mu_0 H = \varepsilon$

$p^2 = 2m(\varepsilon + \mu_0 H)$

$$P = \sqrt{2m(\varepsilon + \mu_0 H)}$$

$$P^2 = 2m(\varepsilon + \mu_0 H)$$

$$\cancel{2} P\,dP = \cancel{2} m\,d\epsilon$$

$$\therefore\ a = \int_0^\infty \frac{\sqrt{2m(\varepsilon + \mu_0 H)}\ m\,d\epsilon}{e^{\beta(\epsilon - \mu)} + 1}$$

IF $\quad b = \displaystyle\int_0^\infty \frac{p^2\,dP}{e^{\beta\left(\frac{p^2}{2m} + \mu_0 H - \mu\right)} + 1}$

$$\frac{p^2}{2m} + \mu_0 H = \varepsilon$$

Then $\quad b = \displaystyle\int_0^\infty \frac{\sqrt{2m(\varepsilon - \mu_0 H)}\ m\,d\epsilon}{e^{\beta(\epsilon - \mu)} + 1}$

Then $\quad M = \dfrac{\mu_0 4\pi V}{h^3}\left[\displaystyle\int_0^\infty \frac{\sqrt{2m(\varepsilon + \mu_0 H)}\ m\,d\epsilon}{e^{\beta(\epsilon - \mu)} + 1} - \int_0^\infty \frac{\sqrt{2m(\varepsilon - \mu_0 H)}\ m\,d\epsilon}{e^{\beta(\epsilon - \mu)} + 1}\right]$

$$M = \frac{\mu_0 4\pi V}{h^3} 2^{1/2} m^{3/2} \left[\int_0^\infty \frac{\sqrt{\varepsilon + \mu_0 H} - \sqrt{\varepsilon - \mu_0 H}}{e^{\beta(\epsilon - \mu)} + 1} d\epsilon\right]$$

$$M = \frac{\mu_0 4\pi V}{h^3} 2^{1/2} m^{3/2} \int_0^\infty \frac{\epsilon^{1/2}\left(1 + \frac{\mu_0 H}{\epsilon}\right)^{1/2} - \epsilon^{1/2}\left(1 - \frac{\mu_0 H}{\epsilon}\right)^{1/2}}{e^{\beta(\epsilon - \mu)} + 1} d\epsilon$$

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using binomial expansion,

$$M = \frac{\mu_0 4\pi V 2^{1/2} m^{3/2}}{h^3} \int_0^\infty \frac{\epsilon^{1/2}\left(1+\frac{\mu_0 H}{2\epsilon}\right) - \epsilon^{1/2}\left(1-\frac{\mu_0 H}{2\epsilon}\right) d\epsilon}{e^{\beta(\epsilon-\mu)}+1}$$

$$M = \frac{\mu_0 4\pi V 2^{1/2} m^{3/2}}{h^3} \int_0^\infty \frac{\epsilon^{1/2}\left(1+\frac{\mu_0 H}{2\epsilon} - 1 + \frac{\mu_0 H}{2\epsilon}\right) d\epsilon}{e^{\beta(\epsilon-\mu)}+1}$$

$$M = \frac{\mu_0 4\pi V 2^{1/2} m^{3/2}}{h^3} \int_0^\infty \frac{\epsilon^{1/2} \frac{2\mu_0 H}{2\epsilon} d\epsilon}{e^{\beta(\epsilon-\mu)}+1}$$

$$M = \frac{\mu_0^2 H \, 4\pi V 2^{1/2} m^{3/2}}{h^3} \int_0^\infty \frac{\epsilon^{1/2} d\epsilon}{e^{\beta(\epsilon-\mu)}+1}$$

$$\boxed{M = \mu_0^2 H \frac{4\pi V 2^{1/2} m^{3/2}}{h^3} \left[\mu^{1/2}\right]}$$

$$\text{where } \mu^{1/2} = \int_0^\infty \frac{\epsilon^{1/2} d\epsilon}{e^{\beta(\epsilon-\mu)}+1}$$

This is the net magnetic moment. Then the magnetisation is, (intensity of magnetisation)

$$I = \frac{M}{V}$$

$$I = \frac{4\pi \mu_0^2 H 2^{1/2} m^{3/2}}{h^3} \cdot \mu^{1/2}$$

Then Susceptibility, $\chi = \frac{\partial I}{\partial H}$.

$$\chi = \frac{4\pi \mu_0^2 \, 2^{1/2} \, m^{3/2}}{h^3} \mu^{1/2}$$

At high temp. χ is inversely proportional to T and at low temperature χ is independent of T.

we have $\quad E_f = \left(\frac{3N}{4\pi V}\right)^{2/3} \frac{h^2}{2m}$

$$m = \left(\frac{3N}{4\pi V}\right)^{2/3} \frac{h^2}{2E_f}$$

E_f is replaced by 'μ'

$$\therefore m = \left(\frac{3N}{4\pi V}\right)^{2/3} \frac{h^2}{2\mu}$$

$$m^{3/2} = \frac{3N}{4\pi V} \cdot \frac{h^3}{(2\mu)^{3/2}}$$

$$\therefore \chi = \frac{4\pi \mu_0^2 \, 2^{1/2} \, 3N \, h^3 \, \mu^{1/2}}{h^3 \cdot 4\pi V \, (2\mu)^{3/2}}$$

$$\chi = \frac{3N}{V} \frac{\mu_0^2}{2\mu}$$

$$\boxed{\chi = \frac{3}{2\mu} \frac{\mu_0^2}{v}}$$

$v = \frac{V}{N}$

Replace 'μ' by ϵ_F,

$$\chi = \frac{3\mu_0^2}{2\epsilon_f v}$$

We know that $\epsilon_f = kT_F$

$$\therefore \chi = \frac{3\mu_0^2}{2kT_f v}$$

ie, χ for a paramagnetic substance is a +ve quantity.

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(ii) Diamagnetic Property :- Landu

The magnetism arising from the quantization of orbital motion of a charged particle in the presence of an external magnetic field. This is called diamagnetism. Diamagnetic substance repel in a magnetic field and it have -ve magnetic susceptibility.

Landu showed that diamagnetism arises from the quantization of orbits of charged particle in MF.

In a uniform magnetic field the intensity 'B' directed along z-axis, a charged particle would follow a helical path whose axis is parallel to the z-axis and proj^n on the x-y plane is a circle. Motion along z-dir^n has a constant linear velocity 'v' while with xy plane it has a constant ang. velocity 'ω', it arises from the Lorentz force $\frac{Bev}{c}$ and centripetal force balances this force. ie $\frac{mv^2}{r}$

$$\frac{Be}{c} = \frac{mv}{r}$$

$$\omega = \frac{Be}{cm} \quad -(1) \qquad \frac{v}{r} = \omega$$

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Landu solved this problem quantum mechanically,

Schrodinger equation for a free particle,

$$\hat{E}\psi = \hat{H}\psi$$

$$\frac{p^2}{2m}\psi = E\psi \quad \text{—②}$$

In electromagnetic field vector potential A will replace p by,

$$p - \frac{e}{c}A$$

∴ ② becomes,

$$\frac{(p - \frac{e}{c}A)^2}{2m}\psi = E\psi$$

$$\frac{(-i\hbar\nabla - \frac{eA}{c})^2\psi}{2m} = E\psi \quad \text{—③}$$

Landu solved this equation and find the energy eigenvalues

$$E_j = \frac{P_z^2}{2m} + (j + 1/2)\hbar\omega \qquad j = 0, 1, 2 \ldots$$

$$E_j = \frac{(\hbar k_z)^2}{2m} + (j + 1/2)\hbar\omega \quad \text{—④} \qquad P_z = \hbar k_z$$

These are called Landau levels the degeneracy of Landau level is given by,

$$g = \frac{BeL^2}{Ch} \quad \text{—⑤}$$

energy levels are split ~~been~~ having same energy

L^2 - area of orbit

5

The equ^n of state in GC ensemble is given by,

$$\frac{PV}{kT} = \ln Z \quad \text{—} (6)$$

Let M be the magnetic moment and B is applied MF. In the case of gas work done is pdv, similarly in case of mag. field work done is MdB. So we find that $P \to M$ and $B \to V$,

$\therefore$ equ^n of state becomes, $\frac{MB}{kT} = \ln Z(z,V,T)$ — (7)

differentiate w.r.t B, $\frac{M}{kT} = \frac{\partial}{\partial B} \ln Z(z,V,T)$

$\div V$ $\quad \frac{M}{kTV} = \frac{1}{V}\frac{\partial}{\partial B} \ln Z(z,V,T)$ — (8)

$\frac{M}{V}$ - magnetic moment / unit volume. I = Intensity of magnetization.

$$\frac{I}{kT} = \frac{1}{V}\frac{\partial}{\partial B} \ln Z(z,V,T)$$

$$I = kT \frac{\partial}{\partial B} \underline{\frac{\ln Z.(z,V,T)}{V}} \quad \text{—} (9)$$

For ideal Fermi gas, $Z = \prod_p \ln\left(1+ze^{-\beta \epsilon_p}\right)$

ϵ_p are Landau levels of $\bar{e}$ in mag. field,

$$\epsilon_p = \frac{P_z^2}{2m} + (j+1/2)\hbar\omega$$

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Substitute ϵ_p in the expression for Z,

$$Z = \prod_p \left[1 + z e^{-\beta\left(\frac{p_z^2}{2m} + (j+1/2)\hbar\omega\right)}\right]$$

$$\ln Z = \Sigma \ln \left(1 + z e^{-\beta\left(\frac{p_z^2}{2m} + (j+1/2)\hbar\omega\right)}\right) \quad \text{--- } 10$$

Expand the equ^n $\ln(1+x) = x + \frac{x^2}{2} \dots$ and retaining the 1st term

$$\ln Z = \Sigma \, z e^{-\beta\left(\frac{p_z^2}{2m} + (j+1/2)\hbar\omega\right)}$$

$$= \Sigma \, z \, e^{-\frac{\beta p_z^2}{2m}} e^{-\beta(j+\frac{1}{2})\hbar\omega} \quad \text{--- } (11)$$

The summation is replaced by integration for the 1st term,

no. of states $\frac{dp\,dz}{h}$ for 1-D,

$$\Sigma \, z e^{-\beta p_z^2/2m} = \int_0^\infty z e^{-\beta p_z^2/2m} \frac{dp\,dz}{h}$$

$$= \frac{Lz}{h} \int_0^\infty e^{-\beta p_z^2/2m} \, dp_z$$

$$= \frac{Lz}{h} \int_0^\infty e^{-u} \frac{m\,du}{\beta\left(\frac{2mu}{\beta}\right)^{1/2}}$$

$$= \frac{Lz}{h} \int_0^\infty e^{-u} \left(\frac{m}{2m\beta u}\right)^{1/2} du$$

$$= \frac{Lz}{h} \left(\frac{m}{2\beta}\right)^{1/2} \int_0^\infty e^{-u} u^{-1/2} \, du$$

$$= \frac{Lz}{h} \left(\frac{m}{2\beta}\right)^{1/2} \sqrt{\pi}$$

$$\frac{\beta p_z^2}{2m} = u$$

$$2p_z \, dp_z = \frac{2m \, du}{\beta}$$

$$dp_z = \frac{\cancel{2}m \, du}{\beta p_z}$$

$$dp_z = \frac{\cancel{2}m \, du}{\beta\left(\frac{2mu}{\beta}\right)^{1/2}}$$

$$= \frac{L_z}{h} \left(\frac{\pi m}{2\beta}\right)^{1/2} \quad —(12)$$

Now eqn (11) becomes,

$$\ln Z = \left(\frac{\pi m}{2\beta}\right)^{1/2} \frac{L_z}{h} \sum e^{-\beta(j+1/2)\hbar\omega} \quad —(13)$$

Now we introduce the degeneracy of Landau level, and the fact that each level can accomodate two $\bar{e}$, ($\uparrow$ & $\downarrow$)

$$\ln Z = \frac{2gL_z}{h} \left(\frac{\pi m}{2\beta}\right)^{1/2} \sum e^{-\beta(j+1/2)\hbar\omega} \quad —(14)$$

$$= \frac{gL_z}{h} \sqrt{2\pi mkT} \sum e^{-\beta(j+1/2)\hbar\omega}$$

$$= \frac{gL_z}{\lambda} \left[e^{-\frac{\beta\hbar\omega}{2}} + e^{-\frac{3}{2}\beta\hbar\omega} + \dots\right]$$

$$\ln Z = \frac{gL_z}{\lambda} \left(\frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}\right) \quad —(15)$$

put $\frac{\beta\hbar\omega}{2} = x$

$$\frac{\hbar\omega}{2kT} = x \quad ; \quad \frac{h\omega}{2\pi\, 2kT} = x$$

$$\ln Z = \frac{gL_z}{\lambda} \left(\frac{e^{-x}}{1-e^{-2x}}\right)$$

$$= \frac{gL_z}{\lambda} \left(\frac{1}{e^{x}-e^{-x}}\right) = \frac{gL_z}{\lambda} \frac{1}{\left(1+x+\frac{x^2}{2!}\dots\right) - \left(1-x+\dots\right.}$$

$$= \frac{gLz}{\lambda} \frac{1}{\left(2x + \frac{2x^3}{3!} + \cdots\right)} = \frac{gLz}{\lambda} \frac{1}{2x\left(1+\frac{x^2}{6}\right)}$$

$$= \frac{gLz}{2\lambda x}\left(1+\frac{x^2}{6}\right)^{-1}$$

$$\ln z = \frac{gLz}{2\lambda x}\left(1-\frac{x^2}{6}\right)$$

$\frac{gLz}{2\lambda x}$ can be evaluated as follows,

$$= \frac{gLz}{2\lambda x} \Rightarrow \quad g = \frac{BeL^2}{ch}$$

$$= \frac{BeL^3 z}{2\lambda x ch} \qquad x = \frac{\hbar\omega}{4\pi kT}$$

$$= \frac{BeL^3 z \, 4\pi kT}{2\lambda \hbar\omega ch} = \frac{BeL^3 \, 2\pi kTz}{ch^2\omega\lambda}$$

$$\times \frac{m}{m} = \frac{BeL^3 \, 2\pi mkTz}{ch^2\omega m\lambda} \qquad L^3 = V$$

$$\frac{gLz}{2\lambda x} = \frac{BeL^3 z}{c\lambda\omega m\lambda^2} = \frac{BeVz}{c\lambda\omega m\lambda^2}$$

$$\times \frac{N}{N} \Rightarrow \frac{BeVz}{c\lambda\omega m\lambda^2} \times \frac{N}{N} = \boxed{\frac{Bez\,N}{c\omega m\lambda^3}} \qquad \frac{Be}{cm} = \omega$$

$$= \underline{\underline{N}} \qquad \frac{\lambda^3}{v} = z$$

$\ln z = N(1 - x^2/6)$

$$I = kT \frac{\partial}{\partial B} \frac{\ln Z(z,V,T)}{V}$$

$$= kT \frac{\partial}{\partial B} \frac{N\left(1 - \frac{x^2}{6}\right)}{V} \qquad \frac{N}{V} = \frac{1}{v}$$

$$= \frac{kT}{v} \frac{\partial}{\partial B} \left(1 - \frac{x^2}{6}\right)$$

$$x = \frac{\hbar\omega}{2kT}$$

$$I = +\frac{kT}{v} \frac{\partial}{\partial B} \left[1 - \frac{\left(\frac{\hbar B e}{2cmkT}\right)^2}{6}\right] \qquad \omega = \frac{Be}{cm}$$

$$x = \frac{\hbar Be}{cm2kT}$$

$$I = -\frac{kT}{v} \cancel{\frac{\partial}{\partial B}} \left(\frac{\left(\frac{e\hbar}{2kT}\right)^2 2B}{6m^2c^2}\right)$$

$$\chi = \frac{I}{B} = \frac{-kT}{v} \left[\frac{\left(\frac{e\hbar}{2kT}\right)^2 2}{6m^2c^2}\right]$$

$\frac{e\hbar}{2mc} = \mu_0$ – Bohr magneton (5.788×10^{-5} eV/T)

9.274×10^{-24} J/T

$$\chi = \frac{-1}{3kTv} \left(\frac{e\hbar}{2mc}\right)^2$$

$$\frac{1}{v} = \bar{n}$$

$$= \frac{-1}{3kTv} \mu_0^2$$

$$\boxed{\chi = \frac{-\bar{n}}{3kT} \mu_0^2}$$

Electron Gas in metals :-

The theory of conduction $\bar{e}$s in metals is developed by SOMMERFIELD using FD statistics, free electron model of metal was developed treating $\bar{e}$ gas as a MB system by Drude and Lorentz, but this theory has many discrepancy with exptl results. 1) The important one is about the specific heat of metals. According to their theory the contribution towards specific heat is only from lattice vibration and no contribution from $\bar{e}$-gas.

Fermi temperature can be calculated and it is found to be in the range of $10^4 - 10^5$. In all metals, $\bar{e}$ gas in metal is highly degenerate, Fermi s/m at room temp is the ratio $\frac{kT}{E_F} = \frac{T}{T_F} \simeq 10^{-2}$. This gives only contribution less than 1% to the specific heat and contribution from $\bar{e}$ gas is negligible, at ordinary temperature.

The specific heat of metal is almost completely determine by the vibrational modes of lattice and very little contribution from the conduction $\bar{e}$s.

As temp. ↓ses specific heat due to lattice vibration also decreases. Finally become considerably smaller than classical value. A stage comes when the two contribution become comparable, contribution from lattice vibration being proportional to T^3 (debye theory) and become even smaller than the electronic specific heat which is proportional to 'T'

In general we may write at low temp,

$$C_v = AT + BT^3$$

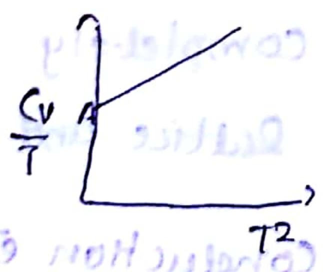
A and B are temperature independent constants for metals.

As temp falls lattice contribution falls faster than electronic contribution.

So we can write

$$\frac{C_v}{T} = A + BT^2$$

A plot b/w $\frac{C_v}{T}$ and T^2 gives a straight line



y intercept → A

Slope → B