576 LECTURE NOTES IN ECONOMICS AND MATHEMATICAL SYSTEMS

Jaroslav Zajac

# Economic Dynamics and Information

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ISBN-10 3-540-32694-4 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-32694-6 Springer Berlin Heidelberg New York



Springer is a part of Springer Science+Business Media
springeronline.com



Typesetting: Camera ready by author
Cover: Erich Kirchner, Heidelberg
Production: LE-TEX, Jelonek, Schmidt & Vöckler GbR, Leipzig

SPIN 11681069 Printed on acid-free paper – 88/3100 – 5 4 3 2 1 0

# Preface

In this book, I show how agents can learn private information about partners from asset prices by analyzing the arbitrage opportunities of financial markets. I show that we can define an equilibrium that embeds the way agents infer information from asset prices. Its properties will are presented under asymmetric information conditions. Such unobserved information improves output distribution by improving agents' ability to perceive relative prices. This can help to reduce the social loss usually encountered through the variance of local output that arises from perception errors at the local level regarding the general price level. Some settings of prospective and contemporaneous feedback rules increase such price-perception errors, while other settings enable very low price-perception errors, and also produce arbitrarily high inflation forecasts.
The benefits of hierarchy flow from the fact that it attenuates opportunism, limits the problem stemming from bounded rationality, and reduces bargaining cost deriving from asset-specificity; this explains why free agents have chosen to renounce part of their freedom of action. We model how cooperation and trust emerge and shift adaptively as relations evolve in a context of a system of buyer-supplier relations. The methodology of adaptive agents seems to deal with this interrelated structure of processes of interaction in which future decisions are adapted to past experience. A model is developed in which interactions between agents—in the making and breaking of relations on the basis of a bounded rational, adaptive and mutual evaluation of transaction partners—takes into account trust, costs, and utility.

We analyze the existence of such equilibria in economies having a measured space of agents and a continuum of agents and commodities. Excessive homogeneity with respect to agent productivity leads to instability and non-uniqueness of a given stationary state and the indeterminacy of the corresponding stationary state equilibrium. Sufficient heterogeneity leads to global saddle-path stability, uniqueness of a given stationary state and the global uniqueness of the corresponding equilibrium. The variety and variance of agent capabilities is reflected in such constructions as culture, social relations, ideology, politics, and preferences that change continuously with historical development. Information variety is an efficacious companion of systemic variety, and can support the performance of an economy, and can be an economically efficient cradle for adaptation to environmental and individual change and systemic evolution.

This book is founded on the research supported by grant VEGA of Slovak Republic No. 1/223/04.

Jaroslav Zajac, Bratislava, Slovakia, May 2006

# Contents

# 1 Rational Expectations of Efficiency and Incentive Schemes

It is in general necessary to use contemporaneous and prospective feedback policy to get on the efficient frontier as well as to move along it. Agents may operate limited financial transfers across periods and states via finitely many nominal assets. Each consumer receives a private information signal about which state will prevail at the next period. This is the main reason why agents may need to refine their information up to an information structure precluding arbitrage. We present the framework and recall the concepts of arbitrage-free information structures, refinements, and no-arbitrage prices. We also define the notion of equilibrium in an asymmetric setting, which explicitly presents consumers' behavior and the need for a refinement of information when it is not arbitrage-free at the outset. We show that every information structure $(S_i)$ has a unique coarsest arbitrage-free refinement, denoted by $(\bar{S}_i)$, which does not contain any wrong signals since $\bigcap_i S_i = \bigcap_i \bar{S}_i$, such that $\bar{S}_i$ coincides with agents' pooled information, market completeness and symmetric information.

When the initial information structure $(S_i)$ is arbitrage-free, consumers may keep their initial information sets. Hence, agents may always update their beliefs through prices, and neither the presence of another agent, nor any knowledge of other agents' characteristics is required. Constant risk aversion and a stationary state of technology imply that a market maker can observe outcome paths of the optimal incentive scheme in profits by imposing the additional assumption that the market maker observes only the time path of total profits. By subtracting total profits at time $t-1$ from total profits at time *t*, he can easily compute the increment in profits. The market maker is thus able to recover the time paths and implement the same solution he would if he could observe these time paths directly. Based on the profit values that are associated with different outcomes in a given period, this may even be possible if he observes only the cumulative final value of total profits. Moreover, by the time he observes this aggregate, the agent may have destroyed some profits by actually realizing them. The stationary of solutions to the time problem plays no role, and the agent controls the drift rate vector of a multi-dimensional Brownian motion with a diffusion process. To relate the different processes, we consider the deviations of the processes of counting variables in the time model from some suitably chosen norms and show that they converge to a

Brownian motion as period length goes to zero. If the market maker fully observes the time paths of outcomes, an optimal incentive scheme can be present if the market maker wants to implement action in the relative interior of the admissible set and if the agent's effort costs depend only on the drift in cumulative total profits that is induced by his actions. A compact set of admissible controls for the implementation of a boundary action, which is subject to weaker incentive compatibility requirements, provides scope for reducing the agent's risk. The incentive scheme is approximately optimal if the market maker only observes cumulative total profits (possibly after some profits have been destroyed by the agent).

## Rational Expectations and Efficiency

A simple way to represent the systematic components of the supply of $m_t$, is

$$m_t = \mu_1 v_{t-1} + \mu_2 \varepsilon_{t-1} + \phi i_t ,$$

where the $\mu's$ represent responses to $t-1$ shocks which were unobserved during period $t-1$, but will be assumed to be known during period *t,* following Bali and Thurston (2002). The parameter $\phi$ is interest rate information based on interest rate policy; the coefficient on $i_t$ will take the form $c_2' = c_2 - \phi$. The model is:

$$p_t = \pi_{00} + \pi_{01} v_t + \pi_{02} \varepsilon_t + \pi_{03} v_{t-1} + \pi_{04} \varepsilon_{t-1} ,$$

$$i_t = \pi_{10} + \pi_{11} v_t + \pi_{12} \varepsilon_t + \pi_{13} v_{t-1} + \pi_{14} \varepsilon_{t-1} ,$$

$$y_t = \pi_{20} + \pi_{21} v_t + \pi_{22} \varepsilon_t + \pi_{23} v_{t-1} + \pi_{24} \varepsilon_{t-1} .$$

The $\pi$ 's must follow:

$$\pi_{00} = \frac{a_0 c_2' - \gamma_0 (a_1 c_1 + c_2')}{a_1}, \quad \pi_{01} = \frac{c_2' - \gamma_1 c_1 \alpha_1}{d}, \quad \pi_{02} = \frac{-a_1(1-\beta_1) - \gamma_1 \alpha_1}{d} ,$$

$$\pi_{03} = \frac{\mu_1}{1-c'}, \quad \pi_{04} = \frac{\mu_2}{1-c_2'}, \quad \pi_{10} = \frac{\gamma_0 - a_0}{a_1}, \quad \pi_{11} = \frac{-1-\gamma_1 c_1 (1-\alpha_0)}{d} ,$$

$$\pi_{12} = \frac{-\gamma_1(1-\alpha_0) - a_1 \beta_0}{d}, \quad \pi_{13} = \frac{-\mu_1}{1-c_2'}, \quad \pi_{14} = \frac{-\mu_2}{1-c_2'}, \quad \pi_{20} = \gamma_0 ,$$

$$\pi_{21} = \frac{\gamma_1[\alpha_1 + c_2'(1-\alpha_0)]}{d}, \quad \pi_{22} = \frac{\gamma_1 a_1 [\beta_0 \alpha_1 - (1-\beta_1)(1-\alpha_0)]}{d} ,$$

$$\pi_{23} = 0, \quad \pi_{24} = 0,$$

where

$$d = a_1(1-\beta_1)[1+\gamma_1 c_1(1-\alpha_0)] + \gamma_1[\alpha_1 + c_2'(1-\alpha_0)] + a_1\beta_0(c_2' - \gamma_1 c_1 \alpha_1)$$

and $c_2' = c_2 - \phi$.

We consider that agents' forecasts of future prices are determined on the basis of period $t$ information, $E_t(p_{t+1})$, the interest rate is a signal in the output equation, and the authorities have no information advantage over the public. The policy objective has been to minimize the variance of the $z$th-island price-perception error $p_t - E_{z,t}(p_t)$, or equivalently to minimize the variance of $z$th-island output deviation from the local full-information level. An obvious source of inefficiency in bond markets is forecast error in real interest rate projections, which is entirely due in this context to the uncertainty and heterogeneity of inflation forecasts. These errors create uncertainty about returns from lending, and costs of borrowing over time. They create a wedge between expected and perceived returns across islands. We take notion of the $z$th-island forecast error in aggregate inflation forecasting and apply it to:

$$(p_{t+1} - p_t) - E_{z,t}(p_{t+1} - p_t) = \pi_{01}v_{t+1} + \pi_{02}\varepsilon_{t+1} + (\pi_{03} - \pi_{01})v_t + (\pi_{04} - \pi_{02})\varepsilon_t$$
$$- \beta_0\left[p_t + \frac{z_t}{1+\gamma_1(1-\alpha_0)} - E_{t-1}(p_t)\right] - \beta_1[i_t - E_{t-1}(i_t)],$$

whose variance is conditional on information available at $t$, which induces the following interest rate observation:

$$\psi_e = \left[\pi_{01}^2 + (\pi_{03} - \pi_{01} - \beta_0\pi_{01} - \beta_1\pi_{11})^2\right]\sigma_v^2$$
$$+ \left[\pi_{02}^2 + (\pi_{04} - \pi_{02} - \beta_0\pi_{02} - \beta_1\pi_{12})^2\right]\sigma_\varepsilon^2 + \frac{\beta_0^2\sigma_z^2}{[1+\gamma_1(1-\alpha_0)]^2}.$$

The last term on the right side of this equation is the variance of inflationary expectations as they differ across markets, while the first two terms combined represent the errors in inflation common to all islands. The setting is in effect just that, which makes the interest rate shock irrelevant as a signal about relative price. Rules of this type make output and price outcomes independent of policy settings, and are in general socially inferior to interest rate policies that are accompanied by specified money processes.

## Coalition Structure

Majumdar and Rotar (2000) consider the case of *n* random agents. A subset $S_i$ containing an agent *i* is a dependency neighborhood of *i* if *i* is independent of all agents not in $S_i$. If the agents are stochastically independent, for each agent *i*, one has $S_i = \{i\}$. Consider a sequence of economies $E_n$ where the number of elements in the dependency neighborhoods of any agent remains the same as *n* gets larger, and let us take a sequence of prices $p_n$ such that we expect excess demands in $E_n$ at prices $p_n$. If *Z* is the total amount of control produced by all agents, then the payoff to some region that produces *z* of it is $Z - c(z)$, where $c(z)$ gives the cost its generation involves. If all agents agree to this arrangement, this results in a coalition, which is then bound to jointly decide on extent of its control activity. Ray and Vohra (2001) assumed the free-rider problem across the agents in a coalition to be solved by the act of signing such a binding agreement. The aggregate payoff to coalition *S* with cardinality *s* when each of its members produces *z* is $s[sz - c(z) + Z_{-i}]$, where $Z_{-i}$, is the aggregate level of agents not included in *S*. The problem facing a coalition with cardinality *s* is to produce control of *z* per member, where *z* solves $\max sz - c(z)$. All the interest centers on a description of which coalition structure will actually form, because a coalition agreement may specify not just production levels but also transfers across coalition members. In such a structure, a coalition of size *s* will produce $z = s$ per member or $s^2$ in all, and will incur a cost per agent of $\frac{1}{2}s^2$. It follows, that if there are *m* coalitions with sizes $n = \{s_1, s_2, \ldots, s_n\}$, then a coalition of size *s* will enjoy a payoff per agent of

$$\sum_{j=1}^{m} s_j^2 - \frac{1}{2}s_i^2 .$$

The reason is that the asymmetric payoff per agent to the coalition would always dominate the payoffs for such a structure. So while there may be some inefficiency as the number of agents grows, and while enlarging the ability of coalitions to divide their surplus unequally makes no difference, the exercise has value *ex post* when some agent has already committed to standing alone but is open to re-negotiation. Let *z*(s) be its output per agent:

$$f(s) = sz(s),\ h(s) = c(z(s)),\ g(s) = f(s) - h(s) .$$

If *f*(s) denotes the aggregate output of coalition *s*, *h*(s) the corresponding cost per member, and *g*(s) the payoff per member from the activity of the coalition structure $\pi = \{S_1,\ldots\ldots,S_m\}$, then the average worth of the agent in *S* is

$$\alpha(S,\pi) = \sum_{j=1}^{m} f(s_j) - h(s)$$

.

Because all worth depends only on the sizes of the coalitions involved, we can note that

$$\alpha(s,n) = \sum_{j=1}^{m} f(s_j) - h(s)$$

,

where $n$ is some numerical coalition structure and $s \in n.$ Let $T = \{m_1, m_2,\ldots\ldots\}$ be a collection of increasing positive integers, where $m_1 = 1$. For any integer $n \geq 2$, we define the *T* decomposition of *a* as a collection $s(n) = (t_1,\ldots\ldots,t_k)$, and the smallest integer *n* with the property that *n*>*m*, and

$$g(n) \geq \alpha(t_1, s(n)) = \sum_{i=1}^{k} f(t_i) - h(t_1),$$

where $(t_1,\ldots\ldots,t_k)$ is the $T^*$ decomposition of *n*. If $d(n) = (n_1,\ldots.,n_k),\ where\ k \geq 2,\ then\ n_i \neq n_j$ for all $i,j \in \{1,\ldots..,k\},\ i \neq j.\ If\ T^* = \{m_1,\ldots.,m_i,m_{i+1},\ldots\ldots.\}$, then $m_1 = 1$, $m_2 = 2$, and $m_{i+1} < 2m_i,\ for\ all\ i \geq 2.$ This means that $f(m) + g(m) > g(2m).$ Suppose that the number of agents is *a* and the grand coalition does not form. The equilibrium coalition structure is $d(n) = (n_1,\ldots\ldots,n_k),\ where\ k \geq 2$, then $n_i \neq n_j$ for all $i,j \in \{1,\ldots..,k\}\ and\ n_k > n/2$, and a coalition *S* that forms will do so with the intention of maximizing its average worth, which we have denoted by $\alpha(S,\pi)$. This prediction is assisted by the understanding that other coalitions, when they form, will have the same motivation as *S* does. Full efficiency also obtains if the number of agents does not exceed some upper bound. Then the initial range of populations for which full efficiency obtains is just all the values of *n* for which a single agent does not want to be on its own under the assumption that all the other agents stay together. This boils down to the condition that $f(n-1) + g(1) \leq g(n).$ The cost function takes the constant elasticity form $c(z) = (1/\alpha)z^{\alpha}$ for some $\alpha > 1$. This is equivalent to the requirement that

$\lambda(n-1)^{\lambda} - n^{\lambda} + 1 \le 0,$ where $\lambda = \alpha/(\alpha - 1)$. For each $n$, let $e(n)$ denote the ratio of equilibrium to potential surplus, then $k < \log_2 n + 1$. This shows that the coalition structure of the algorithm conforms to the equilibrium and thus the algorithm follows. Formally, to each integer $n$ we assign a choice of integer $T(n) \in \{1,\ldots,n\}$; applying $T$ to $n$ and then repeatedly to $n - T(n)$ will allow us to break up any integer $n$ into a numerical structure. Let $c(n,T)$ denote this numerical structure.

Set $T(1) = 1$.

Recursively, for any integer $n > 1$, suppose that we have defined $T(m)\ for\ all\ m = \{1,\ldots,n-1\}$. Choose $T(n)$ to be the largest integer $t$ in $\{1,\ldots,n\}$ that maximizes $\alpha\{t, tc(n-t,T)\}$. Let $\alpha'(n)$ be this maximum value.

Complete this recursive definition so that $T$ is now defined on all the positive integers. Define a numerical coalition structure for a situation with $n$ agents as $c(n,T)$.

Imagine a game involving agents $i$ and $j$. Goering, *et al.* (1999) assume $i, j = 1,2,\ but\ i \ne j$, and describe a second stage profit function:

$$\pi_i = f\left(x_i, x_j \middle| v_i, v_j\right),$$

where the conjecture $v$ symbolizes the perception the agent-manager of the $i$th agent has concerning the way the agent-manager of the $j$th agent will react to a change in the $i$th agent's second stage choice variable, $x$. Assume strict concavity in $x_i$ and assume that for the rival's choice of $x_j$, we have

$$f_{s_j} \ne 0,\ f_{x_i x_j} < 0.$$

In the second stage, the conjectures $v_i\ and\ v_j$ enter as parameters in the agent's maximization, and $\pi_i = f\left(x_i(v_i,v_j), x_j(v_i,v_j)\right)$.

If agents lack information on how rivals choose managers, the following describes the first-order conditions facing agents maximizing first stage profits:

$$\frac{d\pi_i(v_i,v_j)}{dv_i} = f_{x_i}\frac{\partial x_i}{\partial v_i} + f_{x_j}\frac{\partial x_j}{\partial v_i} = 0,$$

and this simplifies to the first equality:

$$v_i = \frac{\partial x_j / \partial v_i}{\partial x_i / \partial v_i} = \frac{\partial x_j}{\partial x_i}.$$

We identify the agent with respect to $x_i$, producing

$$\frac{\partial x_j}{\partial x_i} = -\frac{\pi^j_{x_i x_i}}{\pi^j_{x_j x_j}}.$$

Implicitly differentiating these first-order conditions, we get

$$\frac{\partial x_j}{\partial v_i} = \chi \pi^j_{x_j x_i} \pi^j_{x_i v_j}$$

$$\frac{\partial x_i}{\partial v_i} = -\chi \pi^j_{x_j x_j} \pi^j_{x_i v_i}$$

, where

$$\chi = \frac{1}{\pi^i_{x_i x_j} \pi^j_{x_j x_j} - \pi^i_{x_i x_j} \pi^j_{x_j x_j}}$$

. This illustration of the fruitfulness of examining joint equilibrium results in first-order conditions isomorphic to the consistent conjectures condition as an exogenous constraint on such an equilibrium. Monetary payments, $\pi_{it} = \pi_i(a_{1t}, a_{2t})$ with action $a_{it}$, are assumed to be common knowledge. Utility payoffs, $U_i(\pi_{1t}, \pi_{2t})$, are regarded as private information, and Mason and Philips (2001) assume utility functions are combinations of two monetary payments:

$$U_i(\pi_{1t}, \pi_{2t}) = \pi_{it} + \gamma_i \pi_{jt} \quad j \neq i = 1,2.$$

We allow for $\gamma_i$ to be drawn from a range of values between –1 and 1. Letting $\delta_i$ represent the weight agent *i* places on payoffs during one period, the discounted flow of expected utility for the repeated game is:

$$V_i(s_i) = \sum_t EU_i(\pi_{1t}, \pi_{2t})\delta_i^t,$$

where *E* is the expectation operator, and the agent's goal is to maximize $V_i$:

$$\max_{a_{it}} V_{it} = \pi_{it} + \gamma_i \pi_{jt} + \delta_i E_t V_{it+1}$$

,

where $V_{it}$ represents the agent's value function for period *t* and $E_t[.]$ represents expectations taken at time *t* concerning effects realized in *t* + 1. Suppose that agent *i* predicts agent *j* will choose $a^e_{jt}$ in period *t* and that *j*'s period *t* + 1 choice is tied to period *t* behavior according to the strategy $a_{jt+1} = s_j(a_{it}, a_{jt})$, then agent ''s optimal period *t* choice, $a^*_{it}$, is:

$$\partial \pi_{it}(a^*_{it}, a^*_{jt}) / \partial a_{it} + \gamma_i \partial \pi_{jt}(a^*_{it}, a^*_{jt}) / \partial a_{it} + \delta E_t [\partial \pi_{it+1} / a_{jt+1} + \gamma_i \partial \pi_{jt+1} / \partial a_{jt+1}]$$
$$(\partial a_{jt+0} / \partial a_{it}) + \delta E_t [\partial \pi_{it+1} / \partial a_{it+1} + \gamma_i \partial \pi_{jt+1} / \partial a_{it+1}](\partial a_{it+1} / \partial_{it}) = 0$$

.

The solution to this difference equation determines agent *i*'s optimal strategy for period *t* action and period *t* – 1 action. In the event that each agent knows the other agent's strategy, these Markov strategies lead to a Nash equilibrium for the repeated game. At long-run equilibrium, agent *i* takes action $a_i^*$, and agent *i* correctly predicts that agent *j* will choose $a_j^*$, and agent *i* correctly identifies the impact of *i*'s current action upon *j*'s next period action. If agents do not know their rivals' preferences, then each agent's optimal action depends upon the strategy they predict their rival will play. For these reasons each agent has an incentive to devote time and energy to predicting each rival's behavior; upon acquisition of new information an agent will update her beliefs about all rivals. In this framework an agent may be regarded as learning about either a rival's preferences or beliefs, but the lack of common priors renders the successful construction of a Nash equilibrium unlikely. The scope of possible functions from the set of histories to the action set is so large as to render the analysis intractable. Each agent selects his or her output based on the immediate financial reward and the potential information that may accrue by obtaining more precise information on each rival's behavior. Describing the optimal strategy requires a formal description of belief formation. We assume that agents use Markov strategies:

$$a_{it} = \alpha_i + \beta_i a_{jt-1} + \eta_i a_{it-1},$$

and in the experimental design they take the form:

$\pi_i = \left(A - Ba_j\right)a_i - Ba_i^2$ for all *i*, and with these parameterizations, we may describe the long-run equilibrium actions by:

$$\left(1+\delta_1\eta_1\right)\left[A - B\left(1+\gamma_1\right)a_2^* - 2Ba_1^*\right] + \beta_2\delta_1\left[\gamma_1 A - B\left(1+\gamma_1\right)a_1^* - 2B\gamma_1 a_2^*\right] = 0,$$
$$\left(1+\delta_2\eta_2\right)\left[A - B\left(1+\gamma_2\right)a_1^* - 2Ba_2^*\right] + \beta_1\delta_2\left[\gamma_2 A - B\left(1+\gamma_2\right)a_2^* - 2B\gamma_2 a_1^*\right] = 0.$$

Let $x_{kt}$ be agent *k*'s choice in period *t* (*k* =*i* or *j*) and let $y_{it}$ be agent *i*'s prediction of agent *j*'s period *t* choice, and $\varepsilon_{jt}$ is the disturbance associated with agent *j*'s choice; we denote agent *j*'s strategy as:

$$x_{jt} = \alpha_j + \beta_j x_{it-1} + \eta_j x_{jt-1} + \varepsilon_{jt},$$

and we obtain the following prediction rule with $E_{t-1}$ (where *E* is expectation operator) at the end of period *t* – 1:

$$y_{it} = E_{t-1}x_{jt} = \alpha_j + \beta_j x_{it-1} + \eta_j x_{jt-1}.$$

In essence, agent *i* is trying to identify the slopes and intercept of a plane in $x_{it-1} - x_{jt-1} - x_{jt}$ space; at the end of period *t*, agent *i* learns rival *j*'s period *t* choice. Combined with the choices the agents made in period *t* – 1, this determines a point in $x_{it-1} - x_{jt-1} - x_{jt}$ space, and agent *i* then uses the information contained in this combination to revise prior beliefs regarding potential values of slopes $(\beta_j \ and \ \eta_i)$ and intercept $(\alpha_j)$. Based on the new information, the prediction of $\mu_j$ at time $t$ is:

$$y_{it} = \alpha_{jt} + \beta_{jt}x_{it-1} + \eta_{jt}x_{jt-1},$$

and the parameter $\mu_i$ and the precision matrix $R_t$ are:

$$\mu_t = R_t^{-1}(R_{t-1}\mu_{t-1} + X_{t-1}x_{jt}),$$

$$R_t = R_{t-1} + rX_{t-1}X'_{t-1}.$$

The evolution of these recursion relations depends on the data and the initial conditions. Following Bala and Sorger (1998), we assume that each agent naively predicts the future level of spillover to be the same as its current value, and then chooses his educational investment to maximize discounted-temporal utility. Let $L = \{1,....,N\}$ denote the set of agents in the society; for an agent $i \in L$, the capital stock of the agent at time $t \geq 1$ is represented by a number $h_{it} \geq 0$, where a higher value of $h_{it}$ represents a greater stock of effective knowledge than a lower value. The effective stock of knowledge in the next period is given by:

$$h_{i,t+1} = (1-\delta)h_{it} + q_i(h_i)a_{it},$$

where $\delta \in (0,1)$ is the rate at which his capital will depreciate in the absence of learning effort, and $h_t = (h_{i,t},....,h_{N,t})$ gives the human capital stocks of all agents at time $t$, and $q$ captures the spillover effects provided by other agents of the society upon the marginal productivity of the agent's learning effort $a_{it}$. The agents to the left and right of agent $i$ plus agent $j$ constitute the peer group for$i$. Let $\bar{h}_{it}$ denote the local average of capital for agent $i$ at time $t$, i. e.

$$\bar{h}_{ij} = \frac{h_{i-1,t} + h_{it} + h_{i+1,t}}{3}.$$

We assume:

$q_i(h_t) = q_i(h_{i-1,t}, h_{i,t}, h_{i+1,t}) = q_i(h_{it} - \bar{h}_{it})$. The agent's wages are assumed to be competitively determined at $\omega$ per unit of effective knowledge, with consumption given by:

$$c_{i,t+1} = \omega h_{i,t+1} = \omega\{(1-\delta)h_{i,t} + q_i(h_{i,t} - \bar{h}_{i,t})a_{it}\}.$$

In the course of his lifetime, the agent faces the problem:

$$\max_{\{a_{i1},a_{i,2},\ldots\ldots\}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_{i,t}, 1-a_{i,t})$$
$$c_{i,t} = \omega h_{i,t},\ t \ge 1$$
$$h_{i,t+1} = (1-\delta)h_{i,t} + q_i(h_{i,t} - \bar{h}_{i,t})a_{i,t},\ t \ge 1$$
$$a_{i,t} \in [0,1],\ t \ge 1$$
.

Since every agent is simultaneously solving this optimization problem, *i*'s learning behavior indirectly influences the capital stocks of the remaining agents over time and vice versa. We suppose that each agent's single period utility function $u$ has a form given by:

$$u(c, 1-a) = c + \theta \ln(1-a),$$

where $\theta > 0$ is a parameter . Then each chooses an educational effort $a_{i,s}$ to maximize his predicted discounted utility, given by:

$$\max_{\{a_{i,s}\}}, \sum_{s=1}^{\infty} \beta^{s-1} \{c_{i,s} + \theta \ln(1-a_{i,s})\}$$
$$c_{i,t} = \omega h_{i,s},\ s \ge 1$$
$$h_{i,s+1} = (1-\delta)h_{i,s} + \bar{q}_{i,s} a_{i,s},\ s \ge t$$
$$a_{i,s} \in [0,1],\ s \ge t$$
.

The optimal strategy for the agents is to choose a learning effort $a^*_{i,s} \in (0,1)$ satisfying

$$a^*_{i,s} = 1 - \frac{\theta(1-\beta(1-\delta))}{\beta \bar{q}_{i,s}} = 1 - \frac{\theta(1-\beta(1-\delta))}{\beta q_i(h_{i,s} - \bar{h}_{i,t})} \quad for\ all\ s \ge t$$
.

If agents have to determine in advance their supply and demand in reaction to different markets, we assume that there are $k$ suppliers having marginal costs which are normalized to zero, and that the suppliers do not have capacity constraints. Following Bolle (2001), we assume:

$$N(p,n) = n - mp,\ m \ge 0,\ n \in [\underline{n}, \bar{n}],$$

where $N$ is consumer, which may depend on the spot price $p$, and which has a stochastic component $n$. Let us define

$$S(p) = \sum_{j=1}^{k} s_j(p),\ s_{-i}(p) = \sum_{j \ne i} s_j(p)$$
$$D(p) = \sum_{j=1}^{r} d_j(p),\ d_{-i}(p) = \sum_{j \ne i} d_j(p)$$
.

We get profits of suppliers and users according to:

$$GS_i = \int_{\underline{n}}^{\bar{n}} p^*(n) s_i(p^*(n)) f(n) dn$$

$$GD_j = \int_{\underline{n}}^{\bar{n}} (w - p^*(n)) d_j(p^*(n)) f(n) dn.$$

So we can pose $i$'s maximization problem as one of choosing that $p^*(n)$, which is maximizes

$$GS_i = \int_{\underline{n}}^{\bar{n}} p^*(n) [D(p^*(n)) + n - mp^*(n) - s_{-i}(p^*(n))] f(n) dn.$$

For every $n, p^*$, we get the necessary conditions:

$D(p^*) + n - mp^* - s_{-i}(p^*) + p^*[D'(p^*) - m - s'_{-i}(p^*)] = 0$, or

$s_i(p^*) + p^*[D'(p^*) - m - s'_{-i}(p^*)] = 0.$

Sufficient conditions for reply demand functions $d_j(p)$ are given by:

$- d_j(p) + (w - p)(S'(p) + m - d'_{-j}) = 0$

$- 2[S'(p) + m - d'_{-j}(p)] + (w - p)[S'' - d''_{-j}] < 0$.

Summing for all $j$ we get:

$$\frac{D}{w - p} = rm + rS' - (r - 1)D'.$$

The general solution for $k > 0$, $r > 0$, and $k + r > 2$ is:

$$D = D^0(p) + D^h(p;a,b) = \frac{mr}{k+r-2}(w-p) + a(w-p)^{a-1} D_1(p) - \frac{b}{1-\gamma}(w-p) p^{-\gamma} S_1(w-p)$$

$$S = S^0(p) + S^h(p;a,b) = -\frac{mk}{k+r-2} p - \frac{a}{\alpha} p (w-p)^{a-1} D_1'(p) + b p^{-\gamma} S_1(w-p)$$

,

with $\alpha = \frac{r}{k + r - 1}$, $\gamma = \frac{r - 1}{k + r - 1}$,

where $D_1(p)$ and $S_1(w - p)$ are given by the series:

$$D_1(p) = \sum_{i=0}^{\infty} a_i p^i$$

$$S_1(w - p) = \sum_{i=0}^{\infty} b_i (w - p)^i$$

,

with $a_0 = 1,\ a_i \le 0\ and\ b_0 = 1,\ b_i \le 0;\ i \ge 1.$ We get:

$$D_1(0) = S_1(w) = 1,\ 0 \le D_1(p),\ S_1(p) \le 1\ \ 0 \le p \le w,$$

$$D_1'(p) < 0,\ D_1''(p) < 0;\ S_1'(w-p) = \frac{\partial S_1}{\partial(w-p)} < 0,\ S_1''(w-p) < 0,$$

$$D_1'(p) \to -\infty\ for\ p \to w,$$

$$S_1'(w-p) \to -\infty\ for\ p \to 0.$$

The functional form of the equilibrium is determined, and the equilibrium exists for every finite spread of autonomous demand, in contrast with the case of competition with supply functions alone; however, there is no tendency for market prices to converge to 0 if the spread of autonomous demand increases infinitely.

## Asymmetric Information and Incomplete Markets

Cornet and De Boisdeffre (2002) consider an economy with private information, and nominal assets in which there are finite sets *I, S,* and *J*, respectively, of agents, states of nature, and nominal assets. Each agent $i \in I$ has private information during a given period about the possible states of nature of the next period; that is, she knows that the true state will be in a subset $S_i\ of\ S$, and that the true state will not belong to the complementary set of $S_i$. Agents may operate financial transfers across states in $S'$, by exchanging a finite number of nominal assets $j \in J$, contingent on the realization of the state of nature. In the first period, each agent $i \in I$ has some private information $S_i \subset S$ about which states of the world may occur in the next period, and agents are assumed to receive no wrong information signals. A collection $\left(\sum_i\right)_{i\in I}$ such that $\bigcap_{i\in I} \sum_i \neq \otimes$ is called an information structure and $\left(\sum_i\right)$ is said to be a refinement of $(S_i)$. The order relation $\left(\sum_i^1\right) \le \left(\sum_i^2\right)$ on the set of information structures is defined by $\sum_i^1 \subset \sum_i^2$ for every *i*, and we say the non-empty subset $\bigcap_{j\in I} \sum_j$ is called the pooled information of the information structure, and it is obtained by agents when they decide to share their private information. Consider an economy where the random state of nature $s = \left(s_0, \left(s_i\right)_{i \in I}\right)$ is the product of a component $s_0 \in \sum_0$, whose probability distribution is known and common to all agents, and of components $s_i \in \sum_i$. The individual risk of agent *i*, whose realization $\overline{s}_i$ is known by each agent *I*, is revealed to the other agents at the next period. Each consumer *i* receives a private information set $S_i \subset S$, and this information may be refined to a set $\sum_i \subset S_i$. The consumption sets, endowment and preferences are defined conditional to the information set $\sum_i \subset S$ that each agent may infer. The conditional utility is defined as follows:

$$u_i\left(x \middle| \sum\right) = \sum_{s \in \sum} p_i\left(s \middle| \sum\right) v_i\left(x(0), x(s)\right),$$

and the economy can be summarized by the collection:

$$\mathrm{E} = \left[\left(I, HS, J\right), V, \left(S_i, X_i, u_i, e_i\right)_{i \in I}\right].$$

The agent *i* will then maximize her utility in her budget set $B_i\left(p, q, V, \sum_i\right)$ defined as follows:

$$B_i\left(p, q, V, \sum_i\right) := \left\{ \begin{array}{l} \left(x_i, z_i\right) \in \left(\Re_+^H\right)^{\sum_i} \times \Re^J \middle| p(0)\left[x_i(0) - e_i(0)\right] \\ \leq -q z_i, \forall s \in \sum_i, p(s)\left[x_i(s) - e_i\left(\sum_i\right)(s)\right] \leq V[s] z_i \end{array} \right\}$$

.

A no-arbitrage financial equilibrium of the economy $\mathrm{E}$ is a collection $\left(\left(S_i^*\right), \left(x_i^*\right), \left(z_i^*\right), p^*, q^*\right) \; in \; S \times \prod_{i \in I} \left(\Re_+^H\right)^{S_i^{*'}} \times \left(\Re^J\right)^I \times \left(\Re_{+1H}\right)^{S'} \times \Re^J$,

such that:

$q^* \in Q_c\left[V, \left(S_i^*\right)\right]$ and $\left(S_i^*\right)$ is self-attainable, that is, $\bigcap_{i \in I} S_i = \bigcap_{i \in I} S_i^*$;

for every $i \in I$, $\left(x_i^*, z_i^*\right)$ maximizes the utility $u_i\left(\cdot \middle| S_i^*\right)$ in the budget set $B_i\left(p, q, V, S_i^*\right)$;

$\sum_{i \in I} x_i^*(s) = \sum_{i \in I} e_i\left(S_i^*\right)(s)$ for every $s \in \bigcap_{i \in I} S_i^{*'}$;

$\sum_{i \in I} z_i^* = 0$.

Agents have no private information at time $t = \varepsilon$, and we propose refinements $\left(S_i^*\right)$ that agents can implement at equilibrium in a decentralized way through prices. By observing the given asset price $\overline{q}$, each agent will infer the information set $\overline{S}_i$ as the outcome of rational behavior, consisting in inferring the largest $q$-arbitrage-free subset of $S_i$ or any particular knowledge of characteristics of the endowments and preferences of other agents. The equilibrium condition requires that the revealed information structure $\left(S_i^*\right)$ be self-attainable, that is, $\bigcap_{i\in I} S_i = \bigcap_{i\in I} S_i = \bigcap_{i\in I} S_i^*$. The assumption that the initial information sets $S_i(i\in I)$ convey no wrong information ensures that agents will make no wrong inferences at equilibrium. For every agent $i\in I$, the strategy $\left(x_i^*, z_i^*\right)$ maximizes the utility $u_i\left(.\middle|S_i^*\right)$ in the budget set $B_i\left(p^*, q^*, V, S_i^*\right)$, with $q^*\in Q_c\left[V,\left(S_i^*\right)\right]$.

The structure $\left[V,\left(S_i\right)\right]$ is arbitrage-free if satisfies the following condition: there is no $\left(z_i\right)\in\left(\Re^J\right)^I$ such that $\sum_{i\in I} z_i = 0 \ and \ V\left[s_i\right]z_i \geq 0$ for all $i\in I$ and all $s_i\in S_i$, with at least one strict inequality. We suppose that the condition of absence of bilateral future arbitrage opportunities holds, and define the mapping $W:\left(\Re^J\right)^I \rightarrow \Re^J \times \Re^J \times \prod_{i\in I} \Re^{S_i}$ by:

$$W_z = \left(\sum_{i\in I} z_i - \sum_{i\in I} z_i, \left[\left(V\left[s_i\right]z_i\right)_{s_i\in S_i}\right]_{i\in I}\right) \ for \ z = \left(z_i\right)_{i\in I} \in \left(\Re^J\right)^I$$

Given the structure $\left[V,\left(S_i\right)\right]$, there is a unique coarsest element in $S$, denoted by $\left(\overline{S}_i\left[V,\left(S_i\right)\right]\right)_i$ such that: $\left(\overline{S}_i\right)\in S$, and $\left(\sum_i\right)\leq\left(\overline{S}_i\right)$, for every $i$. The upper bound of a finite family of information structures $\left(\sum_i^1\right)\ldots\left(\sum_i^k\right)$, denoted by $\left(\sum_i\right) := \vee_{h=1}^k\left(\sum_i^h\right)$, is defined by the relations $\sum_i := \bigcup_{h=1}^k \sum_i^h$ for every $i\in I$.

If the upper bound of a finite family of arbitrage-free information structures is also an arbitrage-free information structure, then $\left(\sum_i\right) := \left(\sum_i^1\right)\vee\left(\sum_i^2\right)$ is also an information structure. Given a structure $\left[V,\left(S_i\right)\right]$, we can define, in the set $S$, the interval $\left[\left(\underline{S}_i\right),\left(\overline{S}_i\right)\right]$, that is, the set of arbitrage-free refinements of $\left(S_i\right)$, which is then said to be fully-revealing when this interval is reduced to a unique element.

The structure $[V,(S_i)]$ satisfies one of the following equivalent assertions:

the pooled refinement and the coarsest arbitrage-free refinement coincide, $(\underline{S}_i)_i = (\overline{S}_i)_i$;

the coarsest arbitrage-free refinement $(\overline{S}_i[V,(S_i)])_i$ is symmetric;

every self-attainable arbitrage-free refinement of $(S_i)$ is symmetric.

The following conditions are equivalent:

the financial markets are complete, that is rank $V = \#S$;

for every information structure $(S_i)$, the structure $[V,(S_i)]$ is fully-revealing;

every arbitrage-free information structure $(S_i)$ is symmetric.

Let $(S_i)$ be an information structure, then its coarsest arbitrage-free refinement $(\overline{S}_i)$ is symmetric by assertion. Then the family $(S_i)$ defines an information structure that is arbitrage-free, since one can choose for a common no-arbitrage price the vector:

$$q := V[\bar{s}] + \sum_{s \in S_+} \alpha(s) V[s] = V[\bar{s}] + \sum_{s \in S_-} \alpha(s) V[s]$$

.

Let the information structure $(S_i)$ of $S$ be given. Then the following are equivalent:

the information structure $(S_i)$ is symmetric;

for every finite set $J$ and every $S \times J$ -financial matrix $V$, $[V,(S_i)]$ is fully-revealing;

for every $S \times \{1\}$ -financial return matrix $V$, $[V,(S_i)]$ is fully-revealing.

Assuming the information structure $(S_i)$ is symmetric, hence is arbitrage-free, and given the structure $[V,(S_i)]$, every no-arbitrage price $q \in \Re^J$ presents a defined information structure, denoted by $(S_i(q))$, which is the coarsest $q$-arbitrage-free refinement of $(S_i)$. This is a refinement process in the sense that the price $q$ conveys enough information for each agent to update her beliefs up to the refinement $(S_i(q))$, without any information from the other agents. Given the structure

$[V,(S_i)]$, for every agent $i$ we define the information set revealed by the price $q \in \Re^J$ as the set:

$$S_i(V,(S_i)_i,q) := S(V,S_i,q),$$

and the revealed information set $S\,(q)$ may be empty.

We now define the set $S(V,\sum\ ,q)$ as the union of all the sets in the non-empty finite set that is q-arbitrage free, where the agents need to refine their information and reach an arbitrage-free structure. The price $q \in \Re^J$ is said to be a no-arbitrage price of the structure $[V,(S_i)]$ if $q$ is a common no-arbitrage price of some refinement $(\sum_i)$ *of* $(S_i)$, that is, if there exists a refinement $(\sum_i)$ and $(S_i)$ such that $q \in Q_c[V,(\sum_i)]$.

## Optimal Incentive Schemes

At the beginning of the period the agent chooses an action that gives rise to a random profit $\overline{\pi} = \{\pi_0,.....,\pi_N\},\ \pi_0 < ..... < \pi_N$. With Hellwig and Schmidt (2002), we assume that the agent chooses the probability distribution $p$ over possible profit levels $\pi_i \in \Re$ directly at personal cost $c(p) \geq 0$. The agent's action is $p = (p_0,.....,p_N) \in P$ where $P$ is the $N$-dimensional simplex, and the agent is assumed to have c coefficient of absolute risk aversion $r > 0$. Given an incentive scheme associating the payment $s_i$ with the outcome $\pi_i$, he chooses action $p \in P$ so as to maximize his expected utility:

$$-\sum_{i=0}^{N} p_i e^{-r(s_i - c(p))}.$$

The market maker is assumed to be risk neutral, his payoff from implementing an action $p$ by an incentive scheme $s = \{s_0,.....,s_N\}$ is given by:

$$\sum_{i=0}^{N} p_i(\pi_i - s_i).$$

In an $m$-period model, the agent chooses a new action $(p_0,....,p_N) \in P$ in each period. This action determines outcome probabilities for that period, as sums of realized incentive payments, effort costs, and profits. We assume that in a period of length $\Delta$ the profit levels $\pi_i^\Delta$ and effort cost function $c^\Delta\left(p^\Delta\right)$ are given by:

$$\pi_i^\Delta = \pi_i \Delta^{\frac{1}{2}} \quad \forall i \in \{0,\ldots\ldots,N\},$$

and that the choice $\mu^\Delta$ entails expected profits are equal to:

$$\sum_{i=0}^{N} p_i^\Delta \pi_i^\Delta = \sum_{i=0}^{N} \left(p_i^\Delta - \hat{p}_i\right)\left(\pi_i - \pi_o\right)\Delta^{\frac{1}{2}} = \Delta \sum_{i=0}^{N} \left(\pi_i - \pi_0\right)\frac{p_i^\Delta - \hat{p}_i}{\Delta^{\frac{1}{2}}} = \Delta \sum_{i=1}^{N} \mu_i^\Delta,$$

and that effort costs are equal to:

$$c^\Delta\left(p^\Delta\right) = \Delta c\left(\hat{p} + \frac{p^\Delta\left(\mu^\Delta\right) - \hat{p}}{\Delta^{\frac{1}{2}}}\right) = \Delta c\left(\hat{p}_0 - \sum_{i=1}^{N}\frac{\mu_i^\Delta}{k_i}, \hat{p}_i + \frac{\mu_1^\Delta}{k_1}, \ldots, \hat{p}_N + \frac{\mu_N^\Delta}{k_N}\right) = \Delta\hat{c}\left(\mu^\Delta\right).$$

The tradeoff between expected profits and effort costs is thus not affected by the choice of *m*, and with period-length $\Delta = 1/m$. The agent chooses $\mu^{\Delta,\tau} = \left(\mu_1^{\Delta,\tau},\ldots\ldots,\mu_N^{\Delta,\tau}\right)$ in period $\tau = 1,\ldots\ldots,m$, and expected gross profits are equal to $\Delta\sum_{\tau=1}^{m}\sum_{i=1}^{N}\mu_i^{\Delta,\tau}$, and effort costs are equal to $\Delta\sum_{\tau=1}^{m}\hat{c}\left(\mu^{\Delta,\tau}\right)$. The probability vector $p^\Delta(\mu)$ that is associated with a given $\mu$ depends on $\Delta$. In the *m*-period with length $\Delta$, the market maker's problem is to choose a time path of actions $\left\{\mu^{\Delta,\tau}\right\}_{\tau=1,2,\ldots,m}$ and an incentive scheme so as to maximize:

$$\sum_{\tau=1}^{m}\sum_{i=0}^{N} p_i^\Delta\left(\mu^{\Delta,\tau}\right)\left(\pi_i^\Delta - s_i^\Delta\right) = \Delta\sum_{\tau=1}^{m}\sum_{i=1}^{N}\mu_i^{\Delta,\tau} - \sum_{\tau=1}^{m}\sum_{i=1}^{N} p_i^\Delta\left(\mu^{\Delta,\tau}\right)s_i^\Delta,$$

subject to incentive compatibility and individual rationality.

# 2 Dynamic Adjustment Processes and the Effect of the Economic System

If agents' features are different, they produce different economic systems that may still be equivalent in their economic efficiency. When environmental and agent features change, economic systems also change along their own characteristic paths. Indeed, the most important feature of an economic system is the coordination it provides among different institutions, and following from this is the need for selection from amongst alternative institutions. Coordination is, in fact, the necessary condition to govern interactions among agents in the many fields of economic activity (and to mutual advantage). Investments in institutions have externalities and produce lock-ins and irreversibility, and these features have an appropriate economic value and give rise to multiple systemic equilibria and path dependence. Their efficacy is determined by cost minimization of decision-making and production, effectiveness of incentives, and the allocation of agent features to their best possible use.

It follows that differently efficacious economic systems produce permanently different structures, and each attempt at transformation or convergence may be better described as a deep restructuring of that system necessary to adapt to environmental change and to pursue new goals. In this case, institutions may nor converge because the variant forms may affect efficiency only weakly and individuals and interest groups can resist institutional change, and neither may have an edge over the other. One might be better, but the inferior one may persist if the costs of transformation exceed the benefits. One form may more efficient in one economy than in another because of the different features of that economic system or the lack of relevant capabilities compared to the other. This makes replacing the variants economically inefficient if the value lost to externalities is greater than the efficiency gains from the new variants.

The set of agents who own a positive portion of the aggregate endowments are the set of non-negligible agents, and economists have proved the existence of equilibria with incomplete and intransitive preferences in many cases. In finite dimensional economies, various conditions have been given for the existence of an equilibrium:

- the assumption of the existence of a no-arbitrage price;
- the assumption of an absence of unbounded and utility-increasing trades;
- the assumption that the individually rational utility set is compact.

Since we assume that success on each agent task is uncertain and dependent on the amount of time allocated to the task, the probability of success on all tasks is relatively small. The problem is that of an agent's allocation of effort across multiple tasks, and in an environment in which this choice is necessary. The agent chooses how to allocate his time, and his allocation, together with a random shock, determines the selection of the structure of the agent's compensation beforehand (and later makes the payment promised for the realized production).

We prove the existence and efficiency of equilibria in infinite economies with many agents who own a positive portion of the aggregate endowment.

The Pareto optimality of a competitive equilibrium and the existence of individually rational Pareto optimal allocations suggest that it might be necessary to work with the assumption that only agents with rational Pareto optimal allocations are involved. Along the optimal decision path, convergence to the optimal steady state is from above in the high endowment treatment and from below in the low endowment treatment. In the basic design we impose an exogenous probability of terminating the economy at each time $t$, which is equivalent, from the point of view of the agent, to an infinite horizon with discounting.

## Comparative Efficiency of Economic Systems

Different economies adopt diverse formal institutions; often they rapidly end up with the same structures. Dallago (2002) supposes that institutional liberalization produces institutional convergence and that countries now share the same economic institutions (for example, prohibition of universal banking, accounting rules, regulation of insider trading, labor mobility, and competitive labor markets). There can also be different efficacious institutions and organizational structures, depending on the economic system, and including the environmental and individual features that influence the capabilities they have, the overall costs that agents have to pay, and the returns they obtain. This explains why the existence of diversity can be an efficacious adaptation to variety, variance, and variability of different environments and agent features.

Acquisitions of firms are part of a strategy of investors, so that these firms lessen their integration with other agents and markets. The development of group agreements is sometimes implemented through the initiative of firms; the process is aimed at creating greater dimensions through reintegration within productive stages that were previously externalized. This is usually made possible by technological developments and is rendered convenient by the high costs of quality and time controls over third parties or increased prices of externalized functions. We live with differences: different unit costs, different market sizes, different tastes and preferences, different resource endowment, different levels of development, and different institutions and economic systems. From this interaction between agents and conditions results a permanent renewal of differences between costs, market sizes, and preferences in various economies and economic environments, and thus also between different economic systems.

This is particularly so if the environment changes in opposite directions for different systems: if a period of adaptation follows accelerated technological innovations, if the consolidation of new markets ensues after the abrupt opening up of those markets, if stability follows a period of price shocks, and if stagnation follows the rapid expansion of capital markets, and so on. Environmental variability in the presence of different economic systems is revealed by lack of synchronization of economic performance, even if there are good returns in financial and capital markets, and gives rise to evolution of institutions and a coordination framework.

In any given economic system, agents have to invest in system-specific assets; investment in system-specific assets gives rise to individual systemic capital, which in turn influences the agents' choices and in the long run their individual features. The asymmetric distribution of systemic capital creates asymmetries in bargaining power, knowledge, and information. All this diminishes the possibility of capturing potential social gains from systemic change, and individual gain depends largely on re-distributive actions.

Systemic capital, asymmetries, and transformation costs cause path dependence of systemic change as a rational behavior due to transformation costs. All these changes are motivations for the evolution of economic systems along their particular path, e.g. the size structure of firms and the existence of configurations of firms and other agents that are characterized by cooperation and interaction and so on. The same holds for the utilization of resources and the ability to adapt to various factors and react to random events; such capabilities depend upon the features of the economic system, the agents it is based on, the interaction and coordination between them, the incentives, and the costs, risks, and uncertainties. In fact, each particular economic system generates, due to its coordinating and selecting effects, characteristic and consistent patterns of transactions and compliance costs and requirements that make stable organizational forms and configurations convenient. However, agents may be inefficient and ineffective in dealing with all the implications of variance and variability through time, under conditions of imperfect, imbalanced, or missing knowledge and information.

To avoid all this and keep variance and variability of individual features within limits that are manageable to agents, the coordinating, knowledge standardizing, and disciplining effect of the economic system is necessary. In a world of uncertainty, limits to individual capabilities, opportunism, various kinds of asymmetry, missing and imperfect knowledge, divergent goals of agents, and other forms of variety, variance, and variability, distinct agents must sustain different transaction and compliance costs and difficulties in order to coordinate their decisions and activities. The allocation and costs of defining, implementing, and controlling effective contracts, incentives, agency relations, trust, and reputation, and coordination costs to streamline divergent interests and goals and to manage externalities, and the incompatibilities and lock-ins that these create must be managed. The costs of and barriers to information collection, elaboration, and transmission and knowledge production and diffusion, and individual features in the long run take form under the coordinating effect of the system including a typical way of producing and diffusing information and knowledge, and consistent arrangements of resource allocation.

The economic system creates connections between agents and actions that can further reduce costs and disadvantages and strengthen incentives by coordinating and selecting institutions that manage the benefits and gains and the disadvantages and costs of economic activity.

This uncertainty and cost of coordination stimulates and supports the specialization of agents and the production and diffusion of knowledge and information. Specialization produces results at the cost of foregone opportunities in other directions and the cost of investment in system-specific assets. As for the implications for the interaction of agents, interaction takes place following rights and duties within specialized structures. Gains offset the disadvantages, costs, and limitations that agents sustain in order to operate (such as learning costs, the costs borne to internalize and conform to rules and to set up appropriate structures).

## Simultaneous Equilibrium

We derive this from the characterization of the simultaneous equilibrium of the market process, and we assume that agents generate beliefs with reference to their immediate history of interaction. We follow Herrendorf, *et al.* (2000), denoting the agent's discount rate by $p \in (0,\infty)$, and the relevant information for the agent's decision at time *t* is the future path of $\{n(s)\}_{s=t}^{\infty}$, so the formal condition that agent of type *i* born at time *t* chooses to work is:

$$E_t\left(\int_t^{\infty} \exp(-p(s-t))ds\right) \le E_t\left(\int_t^{\infty} a(n(s))e(i)\exp(-p(s-t))ds\right),$$

where $E_t()$ denotes the agent's expectation at *t*, and agent output is assumed to be $a(n)e(i)$. Given that the death rate is *p*, the probability that an agent born at time *t* lives at least until time $s > t$ equals $\exp(-p(s-t))$, so we have:

$$\frac{1}{e(i)} \le V(t) = (\rho + p)\int_t^{\infty} a(n(s))\exp(-(\rho+p)(s-t))ds,$$

where $V(t)$ represents the annuity value of the output stream that one unit of labor produces. A competitive equilibrium is an initial $\overline{n}_0$, paths $\{\overline{n}(t),\overline{V}(t)\}_{t=0}^{\infty}$ with $\overline{n}(0)=\overline{n}_0$, and agent choices such that at any point in time *t*, given $\overline{V}(t)$ each newborn agent's choice maximizes its expected lifetime outcome, and the paths $\{\overline{n}(t),\overline{V}(t)\}_{t=0}^{\infty}$ are consistent with the newborn's decision, and it satisfies:

$$\bar{n}(t) = \bar{n}(0) + p\int_0^t \left[F\left(\bar{V}(s)\right) - \bar{n}(s)\right]ds,$$

where agents' choices are taken into account. The equilibrium dynamics behavior is characterized by the laws of motion for $V$ and $n$, and we get the following equations:

$$\dot{V} = (\rho + p)\left[V - a(n)\right],$$
$$\dot{n} = p\left[F(V) - n\right].$$

A stationary state of the economy is a pair $(n^*, V^*)$ such that all variables are constant if the world's interest rate $r$, equals the effective discount rate: $r = \rho + p$. To ensure that an interior state exists, we make the following assumption: either $[\underline{a}, \bar{a}] \subset [1/\bar{e}, 1/\underline{e}]$ *or* $[1/\bar{e}, 1/\underline{e}] \subset [\underline{a}, \bar{a}]$.

Let $\{\bar{n}(t), \bar{V}(t)\}_{t=0}^{\infty}$ be a competitive equilibrium with $\bar{n}(0) = \bar{n}_0$, $\{\bar{n}(t), \bar{V}(t)\}_{t=0}^{\infty}$ is unique at $\bar{n}(t)$ if $\bar{V}(t)$ is the only $V \in [\underline{a}, \bar{a}]$ for which $(\bar{n}(t), V)$ is on an equilibrium path, and $\{\bar{n}(t), \bar{V}(t)\}_{t=0}^{\infty}$ is determinate at $\bar{n}(t)$ if it is locally unique; that is, there exists an $\varepsilon > 0$ such that $\bar{V}(t)$ is the only $V \in \left(\bar{V}(t) - \varepsilon, \bar{V}(t) + \varepsilon\right)$ for which $(\bar{n}(t), V)$ is on an equilibrium path. Noussair and Matheny (2000) assumed that each agent maximizes the present discounted value of current and future utility, to a sequence of source constraints and given positive capital stock $k_0$:

$$\max \sum_{t=0}^{\infty} (1 + p)^{-t} u(c_t)$$
$$c_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t, \ \forall t \geq 0,$$

where depreciation occurs at the rate $\delta \in (0,1)$, utility and production functions $u$ and $f$ are increasing, concave, and differentiable. The agent's rate of time preference $p$ is positive. Necessary and sufficient conditions for optimal choices of consumption and capital stock include the Euler equation

$u'(c_t) = (1 + p)^{-1}\left[1 - \delta + f'(k_{t+1})\right]u'(c_{t+1}), \ \forall t \geq 0,$ and the condition

$\lim_{t \to \infty} (1 + p)^{-t} u'(c_t) k_{t+1} = 0$, where

$$k_{t+1} = f(k_t) + (1 - \delta)k_t - c_t, \ \forall t \geq 0.$$

The steady-state solution is a time-invariant one where $c_t = \bar{c}$ *and* $k_{t+1} = \bar{k},\ \forall t \geq 0$:

$$\bar{c} = f(\bar{k}) - \delta\bar{k}$$
$$f'(k) = p + \delta.$$

The functions $G(k_t)$ *and* $H(k_t)$ are for capital and consumption:

$$k_{t+1} \geq k_t \Leftrightarrow c_t \leq f(k_t) - \delta k = G(k_t),$$
$$c_{t+1} \geq c_t \Leftrightarrow c_t \geq f(k_t) - \delta k_t + (k_t - \bar{k}) = H(k_t).$$

In finite dimensional economies, a no-arbitrage price for the economy is a price such that $pw_i > 0,\ \forall w_i \in W_i \setminus (0)$ where $W_i$ denotes the asymptotic cone of the agent's preferred set. The existence of a no-arbitrage price is known to be equivalent to $\operatorname{int} \sum_i W_i^0 \neq \phi.$ Similarly there exists a $\lambda$ such that $\lambda z < 0,\ \forall_z \in U_\infty \setminus (0)$ where $U_\infty$ denotes the asymptotic code of the agent's utility set, and there are utility weights for which the agent problem has a Pareto-optimal solution. We may then similarly define in the space of utility weights, the excess utility correspondence and use it to prove the existence of the equilibrium. Given a subset $C$ *of* $\Re^n$, $\operatorname{int} C$, $\partial C$, *and* $\bar{C}$ Dana and Van (2000) denote its interior, its boundary, and its closure. For a subset $C \subseteq \Re^n$, $\operatorname{int} C$ is its relative interior, when *C* is regarded as a subset of its hull. Space *F* is assumed to be a locally convex, topological space with dual $F'$, and there are *m* agents. The agent *i* is described by a consumption set $X_i \subseteq F$, the initial endowment $w_i \in X_i$, and the preference of agent *i*. They are represented by a utility function $u_i : X_i \to \Re$, and the pair $(\bar{x}, \bar{p}) \in \Pi_i X_i \times F' \setminus (0)$ is a quasi-equilibrium if $\forall i,\ u_i(x_i) > u_i(\bar{x}_i)$, which implies $\bar{p}x_i \geq \bar{p}w_i$, and $\sum_i \bar{x}_i = w.$ A pair $(\bar{x}, \bar{p}) \in \Pi_i X_i \times F' \setminus (0)$ is an equilibrium if $\forall_i,\ u_i(\bar{x}) \geq u_i(x_i)$ for every $x_i$ such that $\bar{p}x_i \leq \bar{p}w_i$, and $\sum_i \bar{x}_i = w.$ Assume $p \in \partial_n h(\lambda, 0)$ and let $(x_i^n)$ be a sequence such that $u_i(w_i + x_i^n) \to \infty.$ Then $\lambda_i[u_i(x_i(\lambda)) - u_i(w_i + x_i^n)] \geq p(\bar{x}_i - (w_i + x_i^n)),$

and $px_i^n \to \infty$, which implies that $p$ is an arbitrage-free price for the economy. We shall maintain the following assumptions: $X_i \subseteq \Re^i$ is close and convex, $\forall i$, $u_i : X_i \to \Re$ is concave and continuous and does not have a point and $u_i(w_i) = 0,$ $w$ is not weakly Pareto-optimal. However, without an assumption such as the existence of finite number of non-negligible agents, each agent's income in the quasi-equilibrium may be zero. Barut (2000) assumes that the agent set $A$ is countable, and that the commodity space $E$ is endowed with a convex topology $\tau$, so the price space $E'$ is the dual space of $E$. The consumption set of each agent $i$ is $E$, and the initial endowment of each agent $i$ $\omega_i \in E_+$, $\omega = \sum_{i\in A} \omega_i$ is the aggregate endowment for the economy. Let $P_i(x)$ be the set of commodities in $E$, that are strictly preferred to $x \in E$, and let $Q_i(x)$ be the set of commodities in $E$, that $x$ is preferred to:

$$P_i(x) = \{y \in E_+ : y \succ_i x\} \; and \; Q_i(x) = \{y \in E_+ : x \succ_i y\}.$$

An economy with a commodity-price dual space $(E, E')$ and an agent set $A$ whose preferences are $\succ_i$ for every $i \in A$, and whose endowments are $\omega_i$, will be denoted by $((E,E'), A, (\succ_i)_i, (\omega_i)_i)$. An allocation $(x_i)_{i\in A}$ and a price $p \in E'$, $p \neq 0$ are said to be a competitive equilibrium for the economy if $p.x = p.\omega_i \; and \; p.\omega_i > 0$ for some $i$, and if $y \in P(x_i)$ implies $p.y \geq p.x_i$. If the budget equality is not satisfied, $p.x_i \geq p\omega_i$, $((x_i)_{i\in A}, p)$ is called a transfer payment equilibrium. An allocation $(x_i)_{i\in A}$ is said to be a weakly Pareto optimal allocation if there is no other allocation $(y_i)_{i\in A}$ such that $y_i \succ x_i$ for all $i \in A$. An allocation $(x_i)_{i\in A}$ is said to be Pareto optimal if there is no other allocation $(y_i)_{i\in A}$ such that $y_i \succeq x_i$ for each $i \in A$ and $y_i \succ x_i$ for at least one agent $i$, and it is said to be individually rational $\omega_i \notin P_i(x_i)$ for any $i$. There exists a finite set of agents $B \subset A$, and a scalar $\varepsilon > 0$ such that $\sum_{i\in B} \omega_i > \sum_{i\in A} \omega_i$, and we obtain a competitive equilibrium from a transfer payment equilibrium in infinite economies, where $B$ is the set of non-negligible agents. Let $((x_i)_{i\in A}, p)$

be a competitive equilibrium for the economy $((E,E'),A,(\succ_i)_i,(\omega_i)_i)$, Muller (2000) supposes that if the aggregate value of the economy is $p.\omega < \infty$ and agent preference is complete, transitive and convex, then $(x_i)_{i\in A}$ is Pareto optimal.

## Allocation and Control

In the time interval [0,1] the agent controls a publicly observable output process *X* with boundary condition $X_0 = 0$ under a stochastic differential equation:

$$dX_t = f(u_t)dt + \sigma dB_t$$

where $f(u_t)$ is the instantaneous mean, $u_t = u_t(t,X)$ is the agent's control at time *t*, $\sigma$ is the diffusion rate, and *B* represents standard Brownian motion. The agent weakly prefers the random allocation (*S*(X),u) to the certain income $W_A$, and given the sharing rule *S*(X), the agent's optimal control is *u*:

$$\max_{S(X),u} E[X_1 - S(X)]$$

s.t. $dX_t = f(u_t)dt + \sigma dB_t$,

$$E\left[-\exp\left\{-r\left(S(X)-\sum_{t=0}^{(T-1)/T} c(u_t)\Delta t\right)\right\}\right] \geq -\exp\{-rW_A\}$$

, and

$$u \in \arg\max_{\bar{u}\in U} E\left[-\exp\left\{-r\left(S(X)-\sum_{t=0}^{(T-1)/T} c(\bar{u}_t)\Delta t\right)\right\}\right].$$

The sharing rule is $S_{FB} = W_A + c(u_{FB})$,

where the control $u_{tFB} = u_{FB}$ is constant over time. In a discrete-time model in which the agent repeatedly chooses the mean $f(\mu_t)\Delta t$ of a distributed random variable $\Delta X_t$, the agent will choose a low control, if $X_{1/2}$ is high, and if $X_{1/2}$ is low he will choose a high control. At time 1, the agent randomly selects one of the *T* output increments $\Delta X_t = X_{t+\Delta t} - X_t$, and the agent is compensated with a function based on $\Delta X_t$, and knows the entire history of the output process *X*. Accordingly the agent's overall control problem can be expressed as a simple multivariate optimization problem. The agent's overall control problem is:

$$\max_{u_0, u_{1/2}} -\frac{1}{2}F\left(\Delta\overline{X}\middle|u_0\right)\exp\{-r(\underline{s}-c(u_0)\Delta t-c(u_{1/2})\Delta t)\}$$
$$-\frac{1}{2}\left(1-F\left(\Delta\overline{X}\middle|u_0\right)\right)\exp\{-r(\overline{s}-c(u_0)\Delta t-c(u_{1/2})\Delta t)\}$$
$$-\frac{1}{2}F\left(\Delta\overline{X}\middle|u_{1/2}\right)\exp\{-r(\underline{s}-c(u_0)\Delta t-c(u_{1/2})\Delta t)\}$$
$$-\frac{1}{2}\left(1-F\left(\Delta\overline{X}\middle|u_{1/2}\right)\right)\exp\{-r(\overline{s}-c(u_0)\Delta t-c(u_{1/2})\Delta t)\},$$

where $\Delta\overline{X}$ denotes the cutoff, $\underline{s}$ is the payment if $\Delta X_t \le \Delta\overline{X}$, $\overline{s}$ is the payment if $\Delta X_t > \Delta\overline{X}$, *and* $F\left(\Delta\overline{X}\middle|u_t\right)$ is the probability that $\Delta X_t \le \Delta\overline{X}$, conditional upon the fact that $u$ is chosen. The agent's expected utility if the selected increment is $\Delta X_t = \Delta X_0$, weighted with a probability of 1/2 that the subinterval is selected, and random spot checks provide the agent with constant incentives over time. Given an arbitrary constant control $\overline{u}$, there exists a random spot check with payments

$$\underline{s} = W_A + c(\overline{u}) - \frac{1}{r}\ln\left[1 - rc'(\overline{u})\left(1-F\left(\Delta\overline{X}\middle|\overline{u}\right)\right)/F_u\left(\Delta\overline{X}\middle|\overline{u}\right)\right]$$
and
$$\overline{s} = W_A + c(\overline{u}) - \frac{1}{2}\ln\left[1 + rc'(\overline{u})F\left(\Delta\overline{X}\middle|\overline{u}\right)F_u\left(\Delta\overline{X}\middle|\overline{u}\right)\right],$$

such that the agent chooses $u$ in each subinterval, then the agent's participation constraint holds with equality. The agent's control problem is

$$\max_{u_0,\dots,u(T-1)/T} -\frac{1}{T}\sum_{t=0}^{(T-1)/T} F\left(\Delta\overline{X}\middle|u_t\right)\exp\left\{-r\left(\underline{s}-\sum_{t=0}^{(T-1)/T} c(u_t)\Delta t\right)\right\}$$
$$-\frac{1}{T}\sum_{t=0}^{(T-1)/T}\left(1-F\left(\Delta\overline{X}\middle|u_t\right)\right)\exp\left\{-r\left(\overline{s}-\sum_{t=0}^{(T-1)/T} c(u_t)\Delta t\right)\right\}.$$

The agent's participation constraint is then

$$-\exp\{-r(\underline{s}-c(\overline{u}))\}F\left(\Delta\overline{X}\middle|\overline{u}\right)-\exp\{-r(\overline{s}-c(\overline{u}))\}\left(1-F\left(\Delta\overline{X}\middle|u\right)\right)\ge -\exp\{-rW_A\}.$$

The agent's expected utility is then

$$E\left[X_1\middle|u_{FB}\right]-\underline{s}F\left(\Delta\overline{X}\middle|u_{FB}\right)-\overline{s}\left(1-F\left(\Delta\overline{X}\middle|u_{FB}\right)\right).$$

MacDonald and Marx (2001) assumed that payment to the agent is bounded from above by $\overline{s}$, which implies

$$E\left[X_1|u_{FB}\right]-W_A-c\left(u_{FB}\right)+\frac{1}{r}\ln\left[1+rc'\left(u_{FB}\right)\frac{F\left(\Delta\overline{X}|u_{FB}\right)}{F_u\left(\Delta\overline{X}|u_{FB}\right)}\right].$$

Because the tasks involve competing uses of time, the agent views time spent on them as part of the relative cost she attaches to the various tasks. Thus, given an arbitrary contract $c=(n,p,f)$ and effort allocation $(t,t')$ an agent earns expected utility

$$V(t,t',c)=\ell t'u(f)+\left[t(1-t')+(1-t\ell t')\right]u(p)+(1-t)(1-t')u(n)-(\xi t+t')$$

where she achieves full success and utility $u$(f) with probability $\ell t'$, achieves partial success and utility $u$(p) with probability $t(1-t')+(1-t)t'$, and achieves no success and utility $u$(n) with probability $(1-t)(1-t')$, incurring effort cost $\xi t$ from time spent on the low-cost activity and $t'$ from time spent on the high-cost activity. Schonhofer (2001) assumes that an adaptive learning rule $LR_t$ is a map that projects past realizations $\{x_i\}_{i=0}^{t}$ into the set of predictors $P$: $\psi_t=LR_t\left((x_i)_{i=1}^{t}\right)$. An adaptive learning process is a sequence of predictors $(\psi_t)_{t=0}^{\infty}$ induced by a sequence $\{LR_t\}_{t=0}^{\infty}$, and following it results in a non-autonomous dynamical system on $X$:

$$x_{t+1}=F_{\psi_t}(x_t):=F(x_t,\psi_t(x_t)),x_t\in X \quad t=0,1.....$$

At each period, $\beta_t$ is determined by the adaptive learning rule

$$\beta_t=\arg\min_{\beta}\sum_{i=1}^{t}\left(y_i-\beta^T x_{i-1}\right)^2,$$

which yields a sequence of predictors with

$$\psi_t(x_t):=\beta_t^T x_t \quad x_t\in \mathrm{X},$$

and its transformation leads to:

$$\beta_t=\left[\sum_{i=1}^{t}x_{k-1}x_{k-1}^T\right]^{-1}\sum_{k=1}^{t}x_{k-1}y_k.$$

Define

$$R_t:=\sum_{k=1}^{t}x_{k-1}x_{k-1}^T$$

. Furthermore,

$$R_{t-1}=R_t-x_{t-1}x_{t-1}^T,$$

$$\beta_t = R_t^{-1}\left[\sum_{k=1}^{t-1} x_{k-1}y_k + x_{t-1}y_t\right] = R_t^{-1}\left[R_{t-1}\beta_{t-1} + x_{t-1}y_t\right] = R_t^{-1}\left[\left(R_t - x_{t-1}x_{t-1}^T\right)\beta_{t-1} + x_{t-1}y_t\right].$$

It follows that:

$x_{t+1} = F(x_t, \psi_t(x_t)) = F(x_t, \beta_t^T x_t)$, with forecast errors given by:

$\varepsilon_t = f(x_t, \psi_t(x_t)) - \psi_t(x_t)$, where agents think $\{\varepsilon\}_{t=1}^T$ are realizations of a stochastic process in which:

- Agents believe that $y_{t+1} = \psi_t(x_t) + \varepsilon_t, \quad where\ \{\varepsilon_t\}_{t=1}^{\infty}$ is a sequence of random variables with mean 0.
- Agents make the point estimate $y_{t+1}^e = \psi_t(x_t)$, and apply a certainty equivalence principle and replace the realization of $y_{t+1}$ by its expected value given their belief.
- In period $T + 1$, agents observe their past forecast errors $\{\varepsilon_t\}_{t=1}^T$ and they use these observations to test their hypotheses.

We assume that agents use the adaptive learning rule with prices:

$$\beta_t = \arg\min_{\beta} \sum_{i=1}^{t-1} (p_i - \beta p_{t-1})^2.$$

This yields

$\beta_t = \left[\sum_{s=1}^{t-1} p_{s-1}^2\right]^{-1}\left[\sum_{s=1}^{t-1} p_{s-1}p_s\right]$, and the prediction $p_{t+1}^e = \beta_t p_t$, which means in inflation terms

$$\theta_{t+1}^e = \frac{p_{t+1}^e}{p_t} = \beta_t.$$

We show that $\theta_{t+1}^e$ can be written as a sequence of predictors $(\psi_t)$ with:

$\theta_{t+1}^e = \psi_{t-2}(\theta_{t-1}),$

$\psi$ is indexed by $t - 2$, because:

$$\sum_{s=1}^{t-1} p_{s-1}p_s = \sum_{s=1}^{t-2} p_{s-1}p_s + p_{t-2}\theta_{t-1}.$$

Then, we have the trajectories of a dynamical system:

$$\theta_{t+1} = f(\psi_{t-2}(\theta_{t-1})), \quad \psi_{t-1}(\theta_t) = \bar{f}_{t-1}(\theta_t, \theta_{t-1}).$$ Consider

$$\beta_t = \left[\sum_{s=1}^{t-1} p_{s-1}^2\right]^{-1} \left[\sum_{s=1}^{t-1} p_{s-1} p_s\right].$$

Defining $$g_{t-1} := p_{t-2}^2 \left[\sum_{s=1}^{t-1} p_{s-1}^2\right]^{-1}$$ , yields

$$\beta_t = \beta_{t-1} + g_{t-1}\left[\frac{p_{t-1}}{p_{t-2}} - \beta_{t-1}\right] = \beta_{t-1} + g_{t-1}\left[\theta_{t-1} - \beta_{t-1}\right].$$ For g we have:

$$g_t = p_{t-1}^2 \left[\sum_{s=1}^{t} p_{s-1}^2\right]^{-1} = p_{t-1}^2 \left[\sum_{s=1}^{t-1} p_{s-1}^2 + p_{t-1}^2\right]^{-1} = \left[\theta_{t-1}^{-2} g_{t-1}^{-1} + 1\right].$$

We have seen that agents cannot reject forecast errors, and any adaptive learning process with these properties is called consistent, and in this case agents do not have any incentive to switch to another learning rule. The dependence of the current value of the steady state-variable $x_t$, assumed to be a real number, on its point expectation for the next period, $x_{t+1}^e$ is denoted by:

$$x_t = F(x_{t+1}^e).$$

## Temporary Equilibrium

Chatterji and Chattopadhyay (2000) assume the steady state value of the state variable is 0, which is the fixed point of the map $F(.)$. The temporary equilibrium map $F : R \to R$ has 0 as a fixed point, $F(0)=0$, and satisfies the condition $|F(x)| \le a|x|$ for all $x$, for some $a > 0$, and the agent's predictions are given by :

$x_{t+1}^e := \beta_{t-1}^2 x_{t-1}$, and to fully specify the system with learning:

$$\beta_t = m(\omega_{t-1} x_{t-1})\beta_{t-1} + [1 - m(\omega_{t-1} x_{t-1})]\frac{F(\beta_{t-1}^2 x_{t-1})}{x_{t-1}}$$

$$\omega_t^2 = m(\omega_{t-1} x_{t-1})\omega_{t-1}^2,$$

with $m(z) = 1/(1+z^2)$. Subject to the initial conditions $\omega_0^2 = 1/\sum_{-L}^{-1} x_j^2$ and $\beta_0 = \left(\sum_{-L}^{-1} x_j x_{j-1}\right) / \left(\sum_{-L}^{-1} x_j^2\right)$, we obtain the following representation:

$$\beta_t = \frac{\sum_{-L}^{t-1} x_j x_{j-1}}{\sum_{-L}^{t-1} x_j^2}.$$

To see why the existence of a global stability interval is useful, consider the dynamics:

$$\left|\frac{x_t}{x_{t-1}}\right| = \left|\frac{F(\beta_{t-1}^2 x_{t-1})}{\beta_{t-1}^2 x_{t-1}}\right| \beta_{t-1}^2 \le a\beta_{t-1}^2,$$

and suppose that for some $n$, $|\beta_n| < 1/a \; and \; |\beta_n|^2 < 1/a$. Then for $t \ge n$, the sequence $|\beta_t|$ is decreasing and $x_t \to 0, \beta_t \to \bar{\beta}, \omega_t \to \bar{\omega}$ .Moreover, $|\bar{\beta}| \le 1.$ This comes from the property $|\beta_t| \le 1\frac{\omega_t^2 x_t^2}{2}$ and the fact that $\omega_t x_t \to 0.$ By summation over $j$ of $2|x_j x_{j-1}| \le x_j^2 + x_{j+1}^2$, one gets $\left|\sum_{-L}^{t-1} x_j x_{j+1}\right| \le \sum_{-L}^{t-1} |x_j x_{j+1}| \le \sum_{-L}^{t-1} x_j^2 + \left(\frac{x_t^2}{2}\right).$

The temporary equilibrium map is globally a contraction, $|F(x)| \le a|x|$ for some $0 < a < 1$ and for all $x$, and $F(x) \ge -K$ for some $K>0$ and all $x$, and with a positive upper bound, for some $F(x) \le Q$ for some $Q > 0$ and every x>0. Agent predictions are given by

$$x_{t+1}^e := \max\{\beta_{t-1}^2 x_{t-1}, -K\}.$$

We can say that every agent $i$ is trying to learn the decision function $\Delta_i : w \to a_i$, which maps the state $w$ of the world into the action $a_i$ that the agent should take in that state. This function will not be fixed throughout the lifetime of the agent because other agents are also engaged in some kind of learning themselves. Vidal and Durfee (1998) refer to those agents who try to directly learn $\Delta_i(w)$ as *0*-level agents, because they have no-explicit models of other agents.

## Knowledge and Preferences

Any agent $i$ will learn a decision function $\delta_i : w \rightarrow a_i$ where $w$ is what agent $i$ knows about the external world and $a$ is its rational action in that state. Agents at $l$-level with other agents have two kinds of knowledge – a set of functions $\delta_{ij} : w \rightarrow a_j$ that capture agent $i$'s knowledge of each of the other agents $j$, and $\delta_i : (w,, a_{-i}) \rightarrow a_i$ which captures $i$' s knowledge of what action to take given $w$ and the collective actions $a_{-i}$ the others will take. We define $a_{-i} = \{a_1 .... a_{-1} a_{i+1} .... a_n\}$, where $n$ is the number of agents. Agents at 2 -level are assumed to have deeper knowledge about other agents; they have knowledge of the form $\delta_{ij} : (w, a_{-j}) \rightarrow a_j$. We decided that $0$-level agents would learn all their knowledge by tracking solely their own actions and the rewards thereby obtained; they receive no extra domain knowledge and learn everything from experience. The 2-level agents learn their $\delta_{ij}(w)$ knowledge from observations of others' actions, under the already stated assumption that there is common knowledge of everyone's actions; all agents see the actions taken by all others. The rest of knowledge $\delta_{ij}(w, a_{-j})$ and $\delta_i(w, a_{-i})$ is built into the 2-level agents a priori, and these agents get some input, take some action, then receive some reward. The value of $\varepsilon$ is initially 1, but decreases with time to some chosen, fixed minimum value $\varepsilon_{\min}$. That is,

$$\varepsilon_{i+1} = \begin{cases} \gamma\varepsilon_i & \text{if } \gamma\varepsilon_i > \varepsilon_{\min}, \\ \varepsilon_{\min} & \text{otherwise,} \end{cases}$$

where $0 < \gamma < 1$ is some annealing factor.

Kaplan and Wettstein (2000) consider a surplus-sharing problem where there are three agents that consume two goods, $x$ and $y$; agents are endowed with $w_i > 0$ of good $x$ and none of good $y$, and agent $i$'s preferences, can be represented by a utility function $u^i$ that is $\left[\lim_{x_i \to \theta} u_1^i(x_i, y_i) = \lim_{x_i \to \theta} u_2^i(x_i, y_i) = \infty, \lim_{x_i \to \infty} u_1^i(x_i, y_i) = \lim_{x_i \to \infty} u_2^i(x_i, y_i) = 0\right]$

An allocation consisting of the agents' consumption levels $(x\ , y_i)$ and input-output levels $x_p, y_p$ is feasible if

$$x_p + \sum_{i=1}^{3} x_i \leq \sum_{i=1}^{3} w_i$$
,

and if this condition characterizes a Pareto-optimal allocation; it can be derived in a straightforward manner given by $u_1^1/u_2^1 = u_1^2/u_2^2 = u_1^3/u_2^3 = f'$,

$$\sum_{i=1}^{3} y_i \leq y_p$$
,

$$y_p \leq f(x_p)$$
.

Each agent is fully informed about the environment, and follows a mechanism where the other agents send messages to the agent in question, who in turn decides on production and consumption plans, and the equilibrium of the mechanism gives rise to efficient outcomes. In the first stage all agents simultaneously announce prices, which determine how much agents must pay for the consumption of good *x*. Next, the agents choose suggested consumption levels of good *x*, and if the sum of suggested consumption is less than the total aggregate endowment, the difference is used as the production input. In stage 1, the prices announced by the agents are $(p_{jk}^i, p_{kj}^i)$, where $p_{jk}^i$ refers to the price chosen by agent *i* that agent *k* must pay agent *j* for each unit of good *x*. In stage 2, agents simultaneously announce *x*'s; if the sum of the *x*'s is greater than the sum of endowments *W*, the mechanism finds agents that could have prevented the sum of *x*'s from being greater than *W* and assigns them $(w_i, 0)$. If no such agents are found, it assigns $(w_i, 0)$ to all agents submitting *x*'s larger than *w*. The mechanism then determines the set of agents. In the case where $x_1 + x_2 + x_3 \geq W,\ let\ S_1 = \left\{ i \,\middle|\, \sum_{j \neq i} x_j < W \right\}$ and $S_2 = \{ i | x_i \geq w_i \}$.

The mechanism produces outputs using an input level $x_p$ that is equal to $\sum_{i=1}^{3} (w_i - x_i)$ and sets up the following system

$$y_1 = a_1 f(x_p) - (p_{31}^2 + p_{21}^3)x_1 + p_{12}^3 x_2 + p_{13}^2 x_3$$
$$y_2 = a_2 f(x_p) - (p_{32}^1 + p_{12}^3)x_2 + p_{21}^3 x_1 + p_{23}^1 x_3$$
$$y_3 = a_3 f(x_p) - (p_{23}^1 + p_{13}^2)x_3 + p_{31}^2 x_1 + p_{32}^1 x_2$$

The *a*'s can be interpreted as interim output shares $a_i f(x_p)$, and the outcome function is individually feasible by construction but is only weakly balanced for some cases of punishment, and could make it impossible for another agent to respond optimally in the fist stage.

## Contract Choice

We argue if the market makers and agents they choose to contract with are matched with each other according to economic variables, and if risk effects are an important determinant, then very risky crops will be more likely to be associated with share contracts. For social welfare it would be best for risk-neutral agents to work on the most risky crops; if equilibrium were to be the desired the outcome, the more risky crops would be associated with fixed-rent contracts, whereas the less risky crops, cultivated by risk-averse agents, would be associated with share contracts. Market makers with more ability to measure output might end up matching with agents with more risk-aversion, more credit constraints, or higher effort costs. Following Ackerberg and Botticini (2002), we assume that contract choice is given by the probity:

$$y_i = I\left(\beta_0^k + \beta_1 c_i + \beta_2 w_i + \varepsilon_i^1 > 0\right).$$

The matching equation is given by the ordered probity:

$$c_i\left(w_i, k, \varepsilon_i^2\right) = \begin{cases} 1 & if \quad \gamma_1^k w_i + \varepsilon_i^2 > \overline{C}^k \\ 0.5 & if \quad \underline{C}^k < \gamma_1^k w_i + \varepsilon_i^2 < \overline{C}^k \\ 0 & if \quad \underline{C}^k > \gamma_1^k w_i + \varepsilon_i^2, \end{cases}$$

where $y_i$ is the contract choice dummy, $c_i$ is the crop type variable, $w_i$ is the agent's wealth, and $k$ indexes the different towns.

The slopes $\gamma_1^k$ and cutoffs $\left(\underline{C}^k \ and \ \overline{C}^k\right)$ in the ordered probity matching equation are allowed to depend on town $k$, and on the variances of the assumed estimate biases $\varepsilon_i^1, \varepsilon_i^2$, which include the unobservable component of agent's risk.

# 3 Optimal Incentive Schemes with a Measure Space and Decision

The market maker is able to recover the time paths and implement the same solution he would if he could observe these time paths directly. The solution to the time problem plays no role, and the agent controls the drift rate vector of a multidimensional Brownian motion with a diffusion process. A compact set of admissible controls, via the implementation of a boundary action subject to weak incentive compatibility requirements, provides scope for reducing the agent's risk. The incentive scheme is approximately optimal if the market maker only observes cumulative total profits, possibly after some profits have been destroyed by the agent. Agents anticipate outcomes, and they factor their expectations into pricing decisions. An agent can revise his control only at the beginning of each subinterval by selecting one of the $T$ subintervals and the agent is compensated with a step function based on the output produced in this subinterval, and he optimally chooses as a constant control the mean of a normally distributed random variable. Local instability is attributable to the fact that agents extrapolate all trends in deviations from the steady state in past data. Agents' strategies consist of learning nested models of the other agents. Agents might try to manipulate the interactions for their individual benefit at the cost of global efficiency. Agents may have incentive to acquire technology, as with free-rider problem in markets, average-cost pricing, marginal cost, and serial-cost sharing. The externalities present in the environment stem from the fact that the input made by one agent affects the productivity of the input contributed by another. The agent may secure rents from his privileged knowledge of his wealth endowment, and as his wealth increases, an agent can make a larger up-front payment to the agent-owner, retail a large share of realized profit, and supply more effort.

## Distribution of Wealth

The distribution of wealth among agents affects the market maker's welfare, which is distributed widely among agents rather than concentrated in the hands of a single agent. When there is only one agent qualified to operate the market maker's project, the latter can capture all the agent's wealth only by reducing his profit share. Multiple agents are qualified to operate the project; however, in this case the optimal mechanism differs from an all-pay. Potential competition among agents can help the market maker to collect information about all the agents' wealth conditions in the absence of asymmetric information. The convergence of the market to equilibrium lags the convergence of behaviors to equilibrium, but the markets then settle into the equilibrium that would exist given rational expectations, assuming agents are able to predict perfectly. In the first stage of each round agents buy or sell rights to participate in a second stage of strategic interactions. Plott and Wiliamson (2002) assume that market equilibrium shares a common structure, given unit payoffs (*b, s*) to buyers and sellers in the second stage:

$$P \in [D(Q) + b] \cup [S(Q) - s],$$

where $D$(Q) is inverse demand correspondence and $S$(Q) is inverse supply correspondence,

$b$ is unit payoff to buyers, under some solution concept, $s$ is unit payoff to sellers.

## Risky Activities

At the beginning of the period, the agent chooses an action that gives rise to a random profit $\bar{\pi} = \{\pi_0, \ldots\ldots, \pi_N\},\ \pi_0 < \ldots\ldots < \pi_N$. Hellwig and Schmidt (2002) assume that the agent chooses the probability distribution $p$ over possible profit levels $\pi_i \in \Re$ directly at personal cost $c(p) \geq 0$. The agent's action is $p = (p_0, \ldots\ldots, p_N) \in P$ where $P$ is the $N$-dimensional simplex, and the agent is assumed to have c coefficient of absolute risk aversion $r > 0$. Given an incentive scheme associating the payment $s_i$ with the outcome $\pi_i$, he chooses action $p \in P$ so as to maximize his expected utility:

$$-\sum_{i=0}^{N} p_i e^{-r(s_i - c(p))}.$$

The market maker is assumed to be risk neutral, and his payoff from implementing an action $p$ by an incentive scheme $s = \{s_0, \ldots\ldots, s_N\}$ is given by:

$$\sum_{i=0}^{N} p_i(\pi_i - s_i).$$

In an $m$-period model, the agent chooses a new action $(p_0, \ldots., p_N) \in P$ in each period, and this action determines outcome probabilities for that period, as sums of realized incentive payments, effort costs, and profits. We assume that in a period of length $\Delta$ the profit levels $\pi_i^{\Delta}$ and effort cost function $c^{\Delta}(p^{\Delta})$ are given by:

$$\pi_i^\Delta = \pi_i \Delta^{\frac{1}{2}} \quad \forall i \in \{0,\ldots\ldots,N\}$$
,

and the choice $\mu^\Delta$ entails expected profits equal to:

$$\sum_{i=0}^{N} p_i^\Delta \pi_i^\Delta = \sum_{i=0}^{N} \left(p_i^\Delta - \hat{p}_i\right)\left(\pi_i - \pi_o\right)\Delta^{\frac{1}{2}} = \Delta \sum_{i=0}^{N} \left(\pi_i - \pi_0\right) \frac{p_i^\Delta - \hat{p}_i}{\Delta^{\frac{1}{2}}} = \Delta \sum_{i=1}^{N} \mu_i^\Delta$$
,

and the effort cost is equal to:

$$c^\Delta\left(p^\Delta\right) = \Delta c\left(\hat{p} + \frac{p^\Delta\left(\mu^\Delta\right) - \hat{p}}{\Delta^{\frac{1}{2}}}\right) = \Delta c\left(\hat{p}_0 - \sum_{i=1}^{N} \frac{\mu_i^\Delta}{k_i}, \hat{p}_i + \frac{\mu_1^\Delta}{k_1}, \ldots, \hat{p}_N + \frac{\mu_N^\Delta}{k_N}\right) = \Delta \hat{c}\left(\mu^\Delta\right)$$
.

The tradeoff between expected profits and effort costs is thus not affected by the choice of $m$ and with period length $\Delta = 1/m$; if the agent chooses $\mu^{\Delta,\tau} = \left(\mu_1^{\Delta,\tau},\ldots\ldots,\mu_N^{\Delta,\tau}\right)$ in period $\tau = 1,\ldots\ldots,m$, then expected gross profits are equal to $\Delta \sum_{\tau=1}^{m} \sum_{i=1}^{N} \mu_i^{\Delta,\tau}$, and effort costs are equal to $\Delta \sum_{\tau=1}^{m} \hat{c}\left(\mu^{\Delta,\tau}\right)$. The probability vector $p^\Delta(\mu)$ that is associated with a given $\mu$ depends on $\Delta$. In the $m$-period with length $\Delta$, the market maker's problem is to choose a time path of actions $\left\{\mu^{\Delta,\tau}\right\}_{\tau=1,2,\ldots m}$ and an incentive scheme so as to maximize:

$$\sum_{\tau=1}^{m} \sum_{i=0}^{N} p_i^\Delta\left(\mu^{\Delta,\tau}\right)\left(\pi_i^\Delta - s_i^\Delta\right) = \Delta \sum_{\tau=1}^{m} \sum_{i=1}^{N} \mu_i^{\Delta,\tau} - \sum_{\tau=1}^{m} \sum_{i=1}^{N} p_i^\Delta\left(\mu^{\Delta,\tau}\right) s_i^\Delta$$
,

subject to incentive compatibility and individual rationality.

Conditional on the agent's choices of actions, the random variables $\tilde{A}_i^{\Delta,\tau}$ *and* $\tilde{A}_i^{\Delta,\tau'}$, $\tau \neq \tau'$ are stochastically independent. We consider the incentive payments $s_i^\Delta$, $i = 0,1,\ldots,N$ that the market maker needs to implement a given action $p^\Delta$ of the agent if the period length is $\Delta$, and for the agent's maximization problem this implies that an incentive scheme $s^\Delta = \left(s_0^\Delta,\ldots\ldots,s_N^\Delta\right)$ with period length $\Delta$ must satisfy:

$$s_i^\Delta = c^\Delta\left(p^\Delta\right) - \frac{1}{r}\ln\left(1 - rc_i^\Delta + r\sum p_j^\Delta c_i^\Delta\right)$$
,

for $i = 0,1,....,N$, where $c_i^\Delta$ refers of the effort cost function $c^\Delta$ with respect to $p_i^\Delta$, and

$$s_i^\Delta = \Delta\bar{c}(\mu^\Delta) - \frac{1}{r}\ln\left(1 - r\bar{c}_i k_i \Delta^{\frac{1}{2}} + r\sum_{j=0}^{N} p_j^\Delta \bar{c}_j k_j \Delta^{\frac{1}{2}}\right).$$

If the market maker wants to implement the time path of actions $\{\mu^{\Delta,\tau}\}_{\tau=1,....,m}$, the total remuneration that has to be offered is given by:

$$\bar{s}^\Delta = \Delta\sum_{\tau=1}^{m}\bar{c}(\mu^{\Delta,\tau}) + \sum_{\tau=1}^{m}\sum_{i=0}^{N}\bar{c}(\mu^{\Delta,\tau})(\tilde{A}^{\Delta,\tau} - p_i^\Delta(\mu^{\Delta,\tau}))k_i\Delta^{\frac{1}{21}}$$

$$+\frac{r}{2}\sum_{\tau=1}^{m}\sum_{i=0}^{N}\tilde{A}_i^{\Delta,\tau}\left[\bar{c}_i(\mu^{\Delta,\tau})k_i - \sum_{j=0}^{N}\hat{p}_j\hat{c}_j(\mu^{\Delta,\tau})k_i\right]^2 \Delta + O\left(\Delta^{\frac{1}{2}}\right).$$

If outcome $i$ is realized in period $\tau$, this raises the agent's overall incentive payment by an amount $\bar{c}_i(\mu^{\Delta,\tau})k_i\Delta^{1/2}$, reflecting the marginal cost of shifting the probability mass towards outcome $i$. The Brownian motion with probability can be written in the form:

$$\bar{s} = s(Z(.), \mu(.)),$$

where $Z(.)$ is he stochastic process satisfying:

$$Z(t) = \int_0^t \mu(t')dt' + X(t).$$

Thus for all $t$ and for any $N$-dimensional process of the form $\hat{Z}(.)$, and any adapted process $\mu(.)$ we get:

$$s(\hat{Z}(.),\mu(.)) = \int_0^1 \bar{c}(\mu(t))dt + \int_0^1 \bar{c}'(\mu(t))d\hat{Z} - \int_0^1 \bar{c}(\mu(t))\mu(t)dt + \frac{r}{2}\int_0^1 \bar{c}'(\mu(t))\sum\{\bar{c}'(\mu(t))\}^T dt$$

.

The agent controls the drift rate process $\hat{\mu}(.)$ of the $N$-dimensional Brownian motion $\hat{Z}(.)$, the market maker observes the realizations of the process $\hat{Z}(.)$, but not the control process $\hat{\mu}(.)$ or the disturbance process $X(.)$. In the agents control lies only the drift rate, while the higher moments of the cumulative output process are determined by the vector $\hat{p}$, and these are $\pi_i^{\Delta} = \Delta^{\frac{1}{2}}\left(k_i - \sum_{j=1}^{N} \hat{p}_j k_j\right)$,

and the cost to the agent in each period is given by $c^{\Delta}\left(p^{\Delta}\left(\mu^{\Delta}\right)\right) = \Delta\bar{c}\left(\mu^{\Delta}\right)$.

The dimension $N$ of the Brownian motion refers to different activities of the agent, and the market maker's problem has a solution in which he induces the agent to take action in each period. To ensure that the sequence $\{\mu^{\Delta/}(.)\}$ of optimal control paths have a limit path $\mu^*(.)$, we assume that controls are restricted to a compact set $\hat{K} \subset \Re^N$. For $\Delta = 1, \frac{1}{2}, \ldots\ldots\ldots$ and $m = 1/\Delta,$ *let* $\mu^{\Delta^*} \in \Re^N$ be a control vector such that the control path $\{\mu^{\Delta^*}(.)\}$ with the value $\mu^{\Delta^*}$ solves the market maker's problem in the $m$-period problem with period length $\Delta$ when controls are restricted to lie in $\hat{K}$. The optimal incentive schemes are:

$$\bar{s}^{\Delta^*} = \sum_{\tau=1}^{m} \sum_{i=0}^{N} \widetilde{A}_i^{\Delta,\tau} s_i^{\Delta^*},$$

which implement the optimal control paths $\mu^{\Delta^*}(.)$.

For the limiting incentive scheme:

$$s(Z(.)) = \bar{c}\left(\mu^*\right) + \sum_{i=1}^{N} \bar{c}_i\left(\mu^*\right) Z_i(l) - \sum_{i=1}^{N} \bar{c}_i\left(\mu^*\right)\mu_i^* + \frac{r}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{c}_i \alpha_{ij} \bar{c}_j,$$

with $\alpha_{ij}$, to implement the control path $\mu^*(.)$, as a function of the market maker's cumulative total gross profits $\sum_{i=1}^{N} Z_i(l)$, we should require that:

$$\bar{c}_1\left(\mu^*\right) = \ldots\ldots = \bar{c}_N\left(\mu^*\right).$$

The market maker cannot observe the time paths of the individual accounts: $Z_i(t),\ i = 1, \ldots., N$ contributing to total profits, but only the time path of the aggregate $z(t) = \sum_{i=1}^{N} Z_i(t)$, and hence of total profits. The disturbance terms are independent of the agent's actions, and any change of the control process $\mu(.)$ that leaves the drift rate process $\sum_{i=1}^{N} \mu_i(.)$ of the accounting aggregate $z(.)$ unchanged will remain undetected by the market maker and will leave the distribution of payments to the agent unchanged. The agent will choose the control process $\mu(.)$ so as to achieve a given target $E(.)$ for the aggregate drift rate process $\sum_{i=1}^{N} \mu_i(.)$ at minimal cost to himself. Suppose also that $E^*$ maximizes the expression:

$$E - \gamma(E) - \frac{r}{2}\gamma'(E)^2 \sigma^2$$

over $[\underline{E}, \overline{E}]$. Then $\mu(E^*) \in K$ and, for all $i$, $\overline{c}_i(\mu(E^*)) = \gamma'(E^*) \geq 0$, and the control path $\mu^{**}(.)$ with the value $\mu^{**}(t) = \mu(E^*)$ solves the market maker's problem and observes the aggregate process $z(.) = \sum_{i=1}^{N} Z_i(.)$.

In terms of the cumulative profits process $Z(.)$, the optimal incentive scheme $s^{**}$ has the representation:

$$s^{**} = \gamma(E^*) + \gamma(E^*)\left(\sum_{i=1}^{N} Z_i(l) - E^*\right) + \frac{r}{2}\gamma'(E^*)^2 \sigma^2$$

We assume that the market maker does not observe the time path of profits, and that the agent is able to destroy profits before he reports them to the market maker. The market maker could ask for a message $w$ from some general message space and make the payment $S^{\Delta}(w, \overline{z})$, which makes the payment to the agent. Then there exists an incentive scheme $s^{\Delta}(.)$ such that, when the agent is willing to choose the control path $\mu^{\Delta}(.)$ and truthfully report total profits as $\hat{z} = \sum_{i=0}^{N} Z_i^{\Delta}(l)$, the resulting payoff for the agent is same and for the market maker is no less than under the implementation of $\mu^{\Delta}(.)$ through the scheme $S^{\Delta}(.,.)$.

If an incentive scheme implements a control path $\mu^{\Delta}(.)$, then note that the incentive scheme is feasible for the market maker because it depends only on total profits. If the incentive scheme has to be seen to make a difference, the market maker would like to penalize outcomes $\pi^{\Delta} = (1/e)\Delta^{1/2}$ and to reward outcomes $\pi^{\Delta} = \Delta^{1/2}$ in any period. Suppose that, in a model with period length $\Delta$, the market maker can implement a control process $\mu^{\Delta}(.)$.

The control processes chosen by the agent converge to the control process that he chooses when he is faced with the limit scheme. Instead of processes $\mu^{\Delta}(.),\ \mu(.)$, which refer to the time paths of actions taken by the agent, we look at the associated cumulative processes $M^{\Delta}(.)\ and\ M(.)$ where, for any $t \in [0,1]$:

$$M^{\Delta}(t) = \int_0^t \mu^{\Delta}(t')dt' \quad \text{and} \quad M(t) = \int_0^t \mu(t')dt'.$$

Suppose also that the processes $\mu^{\Delta}(.)$ take values in the interior of the set $\hat{K}$, and are satisfied for some subsequence $\{s^{\Delta'}(.)\}$ of the sequence $\{s^{\Delta}(.)\}$. The market maker can implement incentive schemes that take the form:

$$s^{\Delta^{**}}(\bar{z},\eta) = \gamma(E^*) + \gamma'(E^*)[\bar{z} - E^*] + \frac{r}{2}\gamma'(E^*)^2\sigma^2 + \eta,$$

where $E^*$ is the optimal value of the aggregate drift, to avoid running afoul of the agent's participation constraint.

What if the market maker is restricted to using incentive schemes that can be represented as functions of cumulative total gross profits? Because profits are aggregated over time and across accounts and because the agent can destroy profits unnoticed, the market maker has to use an incentive scheme that is a function of total profits. As the number of periods increases, this severely restricts his ability to infer actual profit realizations by exploiting the structure of game. The market maker cannot distinguish between profits generated by different accounts. He cannot prevent the agent from spreading his effort across accounts, and the choice of an optimal incentive scheme is eventually driven by the preferences and technologies that push the market maker to provide the agent with stationary incentives that are independent of time and histories. Note that the market maker's inability to obtain the relevant information about individual accounts stems not only from the fact that he observes only an aggregate across accounts, but the aggregation over time also plays a role in the continuous-time analysis. When the market maker is said to observe the time path of a Brownian motion $Z(.) = (Z_1(.),\ldots\ldots,Z_N(.))\ or\ z(.) = \sum_{i=1}^{N} Z_i(.)$, he is not actually observing any incremental changes that could be interpreted as rates of growth of cumulative profits at any given instant. The stochastic differential equation $dz = \sum_{i=1}^{N} dZ_i$ is meaningful only in terms of the integral version:

$$z(t_2) = z(t_1) = \sum_{i=1}^{N} \int_{t_1}^{t_2} dZ_i$$
,

involving aggregation over time as well as across accounts.

## Strategy Games and Equilibrium

The strategy of an agent can be described as a choice of prices. For each possible set of prices, the choices of $x$'s are a Nash equilibrium and, given the contingent future choices of $x$'s, the choice of prices is a Nash equilibrium, and the equilibria are Pareto optimal. The externality agent $i$ imposes on agent $j$ by consuming more input goods is the reduction of the share of output $a_j$. This externality is internalized, because in equilibrium the price he pays to each agent for consuming input goods is equal to the marginal externality he imposes on them, and these prices are chosen because each agent only chooses prices that other agents must pay each other. Any sub-game-perfect equilibrium allocation is interior, and all sub-game-perfect equilibria satisfy the following equations:

$$u_1^1 = u_2^1\left[a_1 f'(x_p) + p_{31}^2 + p_{21}^3\right]$$
$$u_1^2 = u_2^2\left[a_2 f'(x_p) + p_{32}^1 + p_{12}^3\right]$$
$$u_1^3 = u_2^3\left[a_3 f'(x_p) + p_{23}^1 + p_{13}^2\right]$$
$$a_1 f'(x_p) = p_{12}^3 = p_{13}^2$$
$$a_2 f'(x_p) = p_{21}^3 = p_{23}^1$$
$$a_3 f'(x_p) = p_{31}^2 = p_{32}^1$$
.

Denote by $x_i(p_{23}^1, p_{32}^1, p_{13}^2, p_{31}^2, p_{12}^3, p_{21}^3)$ the quantities demanded by agent $i$ at the second stage of the game. The problem that faces agent *1* in the first stage is to choose an optimal pair of prices $p_{23}^1, p_{32}^1$ :

$$\max_{p_{23}^1, p_{32}^1} u^1 \begin{bmatrix} x_1(p_{23}^1, p_{32}^1, \ldots), a_1 f\left[\sum w_i - x_i(p_{23}^1, p_{32}^1, \ldots)\right] \\ -(p_{31}^2 + p_{21}^3)x_1(p_{23}^1, p_{32}^1, \ldots) + p_{12}^3 x_2(p_{23}^1, p_{32}^1, \ldots) + p_{13}^2 x_3(p_{23}^1, p_{32}^1, \ldots) \end{bmatrix}$$
.

Differentiating with respect to $p_{23}^1$ *and* $p_{32}^1$ yields the following:

$$\left[u_1^1 - u_2^1\left(a_1 f' + p_{31}^2 + p_{21}^3\right)\right]\frac{\partial x_1}{\partial p_{23}^1} + \left(p_{12}^3 - a_1 f'\right)\frac{\partial x_2}{\partial p_{23}^1} + \left(p_{13}^2 - a_1 f'\right)\frac{\partial x_3}{\partial p_{23}^1} = 0$$

$$\left[u_1^1 - u_2^1\left(a_1 f' + p_{31}^2 + p_{21}^3\right)\right]\frac{\partial x_1}{\partial p_{32}^1} + \left(p_{12}^3 - a_1 f'\right)\frac{\partial x_2}{\partial p_{32}^1} + \left(p_{13}^2 - a_1 f'\right)\frac{\partial x_3}{\partial p_{32}^1} = 0.$$

All sub-game-perfect equilibrium allocations are Pareto optimal, and we take a competitive economy with such preferences and technology as our environment, so that the agents own their firms with shares given by $a$'s and initial endowments by $\overline{w_i} = a_i \sum_{i=1}^{N} w_i$; the budget constraint of agent $i$ is

$$x_i + qy_i \le \overline{w_i} + a_i\left[qf\left(x_p\right) - x_p\right],$$

where $q$ is the price of good $y$ in terms of good $x$, and $x_p$ is the production input. Since agents are maximizing their utility, the budget constraints are satisfied with equality:

$$x_i' + qy_i' = \overline{w} + a_i\left[qf\left(x_p'\right) - x_p'\right].$$

We want to show that for any set of prices announced in stage *1*, the second-stage game will always have a Nash equilibrium. In the new game, the $x$ outcome coincides with the announced $x_i$, and except for contingencies, and the following determine the $y_i$ outcomes:

$$y_1 = a_1 f\left(x_p\right) - \left(p_{31}^2 + p_{21}^3\right)x_1 + p_{12}^3 x_2 + p_{13}^2 x_3$$, and so on.

In this new game, the agents can guarantee a strictly positive consumption of goods, and we must show that no agent has incentive to deviate, and we must show that agent *1* is choosing a strategy that would solve the following:

$$\max_{x_1^1} u_1\left[x_1, a_1 f\left(x_p'\right) - \left(\overline{p}_{31}^2 + \overline{p}_{21}^3\right)x_1 + \overline{p}_{12}^3 x_2' + \overline{p}_{13}^2 x_3'\right]$$

$$u_{11} = u_{12}\left(a_1 f'\left(x_p'\right) + \overline{p}_{31}^2 + \overline{p}_{21}^3\right).$$

Similarly, it can be shown that announcing $x_2'$ *and* $x_3'$ solves the problems faced by agents *2* and *3*. Hence the strategies $\left(x_1', x_2', x_3'\right)$ constitute a Nash equilibrium for the second stage of the game. We have solved the mechanism whose sub-game-perfect equilibria exist and give rise to efficient outcomes, which are independent of the initial distribution of endowments. All agents have unit costs of effort, which is normalized to unity; each chooses his effort $e$ to maximize $p(e)T - e$. Each agent's effort of supply is determined by:

$p'(e)T - 1 = 0$; an agent's expected rent $\Pi$ when he supplies effort $e$, receives $T$ for success, post bond $B$, and is sure to operate is:

$$\Pi = p(e)T - e - B.$$

We will employ $\pi$ to denote the agent's expected profit from production, so

$\pi = p(e)T - e = \Pi + B$, and

$\pi = \dfrac{p(e(\pi))}{p'(e(\pi))} - e(\pi)$ defines $e(\pi)$, the level of effort the agent will deliver when he is promised. Levis and Sappington (2000) suppose the rate at which the agent's effort increases as his expected profit from production increases, $h(e) = de/d\pi = (de/dT)(dT/d\pi)$, is non-increasing in effort, and :

$h'(e) \le 0$ for all $e \ge 0$, where $h(e) = -[p'(e)]^2/[p''(e)p(e)]$, and each agent $i$ alone knows his wealth endowment $L_i \in [\underline{L}, \overline{L}]$. So the agent-owner's problem is following:

$$\underset{\{B(L_i),\pi(L_i)\}}{\max imize} \int_{\underline{L}}^{\overline{L}} \{p(e_i)V - e_i - [\pi(L_i) - B(L_i)]\} dG(L_i)$$

, subject to, all $L_i \in [\underline{L}, \overline{L}]$,

where the wealth of each agent is an independent draw of a random variable with the distribution $G(L_i)$ and corresponding density $g(L_i)$, reflecting the agent-owner's goal of maximizing her expected return, which is the difference between total expected surplus and the agent's rent. The next equation ensures the agent's participation by guaranteeing him nonnegative rent:

$$\Pi(L_i) = \pi(L_i) - B(L_i) \ge 0.$$

$$\Pi(L_i) = \max_{L_i' \in (\overline{L}_i | B(\overline{L}_i) \le L_i)} \Pi(L_i').$$

This identifies a profit-sharing option that the agent with wealth $L$ will select:

$$B(L_i) \le L_i.$$

$$e_i = e(\pi(L_i)),$$

which: defines the agent's self-interested effort choice, given the selected profit-sharing.

If $L$ denotes the wealth of agent $i$ $(i = 1,....,N)$ and $(L_i, L_{-i})$ denotes the wealth realization for all $N$ agents, $L_{-i} = (L_1,....,L_{i-1}, L_{i+1},....,L_N)$ will denote the wealth realizations for all agents other than agent $i$. $E_L$ will denote expectations over $L$, and $E_{L_{-i}}$ will denote expectations over $L_{-i}$. Each agent knows his own wealth endowment and determines agent 's effort supply:

$$\Pi_i(\bar{L}) = \lambda_i(\bar{L})[p(e)T(e) - e] - B_i(\bar{L}) = \lambda_i(\bar{L})[p(e)/p'(e) - e] - B_i(\bar{L}).$$

The market maker's problem is following:

$$\max_{\{\lambda_i(L), B_i(L), \Pi_i(L)\}} E_L\left(\sum_{i-1}^{N} \lambda_i(L)\{p(e_i(L))V - e_i(L)\} - \Pi_i(L)\right),$$

which reflects the agent's desire to maximize her expected return, i.e. the difference between the expected surplus and the rent afforded the agents.

$$B(L_i', L_{-i}) + \Pi_i(L_i', L_{-i}) = \lambda_i(L_i', L_{-i})\{p(e(L_i', L_{-i}))/p'(e(L_i', L_{-i})) - e(L_i', L_{-i})\}$$

identifies the self-interest effort choice of the agent who is selected to operate, and following ensures nonnegative rent for each agent:

$$E_{L_{-i}} \Pi_i(L_i, L_{-i}) \geq 0.$$

$$E_{L_{-i}} \Pi_i(L_i, L_{-i}) \geq E_{L_{-i}} \Pi_i(L_i', L_{-i}) \text{ for all } L_i' < L_i,$$

This ensures that each agent will truthfully reveal his wealth realization, provided all other agents do the same, by providing rents to agents who reveal higher wealth realizations.

The following give each agent's bond at the level of his wealth, and ensure that operating probabilities are defined:

$$B_i(L) \leq L_i$$

$$\lambda_i(L) \geq 0$$

$$\sum_{i=1}^{N} \lambda_i(L) \leq 1,$$

Consider a sequence of economies $\mathrm{E}_n$ where the number of elements in the dependency neighborhoods of any agent remains uniformly bounded as *n* gets larger. Let us take a sequence of prices $p_n$ such that we expect excess demand, and construct a sequence of random market-clearing prices $p_n(\omega)$, where $\omega$ refers to a particular state of the environment.

Majumdar and Rotar (2000) consider an exchange economy with $t + 1$ commodities, and an agent participating in the exchange of commodities is described by its demand function *f* defined on $S \times R_{++}$ with values in $R_+^{f+1}$, and its satisfied continuity *f* is continuous on $S \times R_{++}$, and its budget constraint $\langle p, f(p,\omega)\rangle = \omega \; for \; p \in S, \; \omega \in R_{++}$, and desirability $p_n \in S, \; \omega_n \in R_{++}$ are such that $p_n \to p \in \bar{S} - S, \; \omega_n \to \omega > 0 \; then \; |f(p_n, \omega_n)| \to \infty$.

## Effect of the Economic System

An economic system is a coordinated set of formal and informal institutions, including among the formal aspects:

- economically important laws and codified rights and duties and their enforcement;
- specific features allowed to exist in a given context, and their internal structure;
- normal relations among the suppliers of resources;
- codified practices and provisions;
- the nature, role, instruments, and goals of government.

Dallago (2002) assumes economically informal institutions include the role of the household as provider of capital and entrepreneurs, work and payment habits, consumption habits, tax morale and ethical standards, the role of ideology, and religion and belief. The system is the network of rules of the game that structures the interaction with the environment. This may develop spontaneously, by design, or by imposition, following the action of a political authority, or a powerful social class, a dictator, an ideology, a religion, or a foreign occupant. Compared with institutions, the economic system adds coordination that strengthens the impact of institutions upon activities, gives stability to social interaction, and reduces costs and problems that could derive from contradictory institutions. Therefore, the economic system supports the agents of a society by coordinating their knowledge, choices, production, and transactions. Economic activity without the economic system is certainly possible but not necessarily economically advantageous. The system reduces their costs of economic interaction and allows them to capture externalities and take advantage of coordination, and we must be able to afford the learning, adaptation, and compliance costs of operating within the system. Beyond a certain level of systemic complexity a general solution must be found to avoid rapidly increasing interaction costs and prevent the failure of interaction. Coordination and interaction among agents requires a certain degree of coherence within the system. If agents internalize norms of conduct and rules of the game, and have institutions that are mutually compatible and possibly coherent, the costs of interaction will be lower and interactions will be more effective in promoting adaptation to environmental variability and change. Although this never produces monolithic constructions, though time these processes give the economic system a well-defined structure that makes the system easily distinguishable from other systems. This means that the features and role of the government must be in line with the features of the economic system, and in a competitive market economy based on private enterprises the possibility for the government to intervene directly in the economic domain is limited. Reform is always a complex and delicate undertaking; the host of unforeseen and unwanted consequences of reform processes is testimony to the ignorance of reforms and resistance of interest groups. The economic system, through the action of agents, tries to keep the compatibility and coherence that make economic interaction easier and more effective. At any given point in time, technology is part of the environment. Technology as part of the environment means that technological change promotes systemic evolution, and existing features of the economic system contribute to select the technology that is most proper in that particular system. Stability derives from the fact that within an economic system institutions do not emerge randomly but are bundled in an orderly interaction. Since the latter features externalities, complementation, and lock-ins, the path dependence of each institution is and must related to other institutions of a particular brand. A full understanding of the way in which different economic systems and different economies adapt to such environmental transformations requires a dynamic analysis. In a sense, this is an infra-systemic comparison that may help in devising reforms to fill that gap. Different economic systems are more or less efficient depending on their proximity to this organizational structure, the types and sizes of firms and configurations of economic agents. The economic system defines relative advantages and disadvantages, which include costs, incentives, and access to markets, and network externalities, complementarities, and lock-ins, and hence ultimately the efficacy of structures.

The economic subject is exclusively the individual; by isolating the individual from his or her environment and other individuals, individualism consecrates the economic subject as sovereign over a world of objects. This world of objects is the world of alienable commodities, a world built on property relations and contracts. To postulate a sovereign, purposeful, and pre-constituted individual is to imply that an ontology of contracts is the normal state of the economy. The informal substitutes for modern insurance arrangements can only de sustained in communities where members interact frequently, so that the reputation effects will have an impact. We must note that the risks covered under such arrangements must be limited: as the scale of the damage increases and/ or as the frequency of interacting among community members decreases, such arrangements will dissolve. The evolutionary construct of taking the formal insurance markets, and the universalizing construct of taking the formal institutions for the egoistic nature of human subject, fails to demonstrate the superior rationality of modern market institutions. Posed instead as a mutual gain game, cooperative interactions appear to be efficiency enhancing alternatives to the anonymity of markets with their imperfect natures. The cooperative strategy dominates the non-cooperative one: through the institutions of reputation and punishment, the selfish economic subjects are able to enforce their contracts. The reciprocal interactions are inter-temporal exchanges within the domain of the family, showing that private intergenerational transfers of income, wealth, and in-kind services are motivated by exchange consideration. Current transfers from parents to children may be made with the expectation of future reciprocation, the most crucial among the mechanisms of enforcement that sustain such reciprocal interactions. Mutual-gain games can be treated as a supplement to the market mechanism, as a variant of exchange, thereby collapsing the former into the later through utility-maximizing selfish agents who are usually assumed to consume private and public goods. Each individual's utility then depends on his or her consumption of private goods and the sum of everyone's voluntary contribution. This use of selfish behavioral assumptions takes it for granted that each individual holds a zero conjecture (Nash) regarding the effect of his or her contribution on the contributions of others. It is particularly interesting that the economic subject remains intact in its exogenous nature and that institutions are seen as exterior to the subject, figuring is only as constraints. Observations that agents do make voluntary contributions have motivated those who want to explain these institutions by conceiving the very act of giving (the behavioral essence of the economic subject) as a commodity. Introducing ad hoc institutional framework as constraints renders the institutions of morality and ethics as constraints. The economic process of voluntary contribution does not shape the economic subject but simply constraints him or her. The household becomes a site of economic reason: reaping benefits of division of labor, extending credit for investment activities, sharing of collective goods, and risk pooling: none of these activities necessarily occurs, and there is no need to form to enjoy increasing returns or to pool risks.

The altruist's utility then depends on private and public goods, and the level of satisfaction of those who consume the public goods. This is the consideration even when the motivations of the economic subject from kind responses in repeated interactions are entirely motivated by future gains. While the cost-inefficiency of acting under competitive markets is acknowledged, in an economy with market imperfections the efficiency of market transactions is applied to the sphere of private goods/services and the agency problem between the market maker and agent, with the condition that the game provides direct punishment opportunities. Players are ready to punish, even at their own cost, those selfish types who opt for free-riding, and can induce the selfish to contribute to the provision of public goods. Enforcement becomes a problem for all economic processes only if an ontology of contracts is rendered as the horizon of the economy and the contractual fiction of a sovereign, purposeful, agent is naturalized. With reciprocity as the alternative mechanism for organizing inter-temporal exchanges, it is claimed that whether or not reciprocity is enforceable depends on the market size and agent's preferences. When agents require many different goods, a reciprocal-exchange arrangement has fewer benefits, and the market is an attractive alternative while reciprocity cannot be enforced. Reciprocal institutions that still operate in these cases can be found in providing the enforcement mechanisms that guarantee the fulfillment of contractual relations. In the case of market failures, when it is impossible to write fully enforceable contracts, reciprocities reemerge as the point of departure, also appealed to as economic comparisons need to be made between alternatives. The relative costs of making transactions in market versus in non-market institutions (family) vary, and when contracts cannot be written easily/fully, reciprocity surfaces as a usable supplementary institution. With competitive markets with complete contracts as the alternative, reciprocal activities will be represented as endowed with an inferior rationality compared to markets.

The position of markets as the gravitational center, or the implicit postulation of the ontology of contracts as the normal state of economy is assumed, although each economic agent is unified, heterogeneous, and centered around a particular essence. In order to secure the plurality of economic discourses and conceptions, we need to acknowledge the heterogeneity of the behavioral orientations with which an economic subject can identify. The distribution of different behavioral orientations in a given population is an effect of a structural mechanism. The distribution of different behavioral patterns among a population becomes an outcome of a set of structural mechanisms, and the plurality of behavioral traits in this manner entails leaving the terrain of individualism and entering that of structuralism.

Assuming for a moment these behavioral orientations, social phenomena must be explained at the level of the individual. Once the formation of the subject is explored, the boundaries between subject and object are broken, and the ways in which each shapes the other will need to be accounted for; only then can true heterogeneity be considered. We assume that our proposed taxonomy comprises all possible configurations in comparison to competitive markets with complete contracts (or it acts as a supplement to markets with imperfections). This market-centric view, structured as it is around the question of enforcement remains to be structured by an ontology of contracts, and subordinates the multiple logic and rationality of the diverse economic activities that comprise the representation of the economy once all existing alternatives to the market exchange and calculated behavior couplet are deemed either supplements or variants of market exchange. Therefore the need for a heterogeneous economy persists, not only in order to make sense of the past and the present but also to be able to propose alternatives for the present and the future. To understand the peculiarities of such socioeconomic forms requires us to recognize that the subject is shaped and constructed by the diverse set of economic and non-economic processes he or she participates in, even as he shapes and constructs them.

Organizational structures vary according to firm size and type and configurations of economic agents. Organization structures features basic coherence and permanence in time, together with evolution. Different agents rarely come as isolated entities, and they are usually in stable relations with other agents; complementary functions are the subcontracting and outsourcing relations that many firms have with other firms, or financial relations they have with financial institutions. The necessary conditions for this are that they provide agents with proper incentives, keep coordination costs low, and evolve over time following environmental changes. If only one of these elements shows variability, variety, or variance multiple equilibrium follow that make use of largely the same or similar technological devices that exchange production factors, information, knowledge, and even skills. This may produce patterns of convergence and divergence of economic performance without necessarily causing systemic convergence. The environment does not influence the economic system directly and its efficiency and the features of economic agents are as unique as the set of optimal institutions. The environment includes many variables that may differ, even in their development levels and technology. These variables are resources, external factors and the impact of random events on each of these; the variety and variance of capabilities and personalities is reflected in such constructions as culture, social relations, ideology, politics, and preferences.

When different environments and individual features change, economic systems also change along their own characteristic paths with specific externalities; through this process, the most important feature of an economic system is the coordination it provides among different institutions. Coordination is in fact the necessary condition for governing interactions among agents, and between them and the environment, for mutual advantage. Investment in institutions, and the externalities generated by such investments produce lock-ins and irreversibility. These features have an appropriate economic value and give rise to multiple systemic equilibria and path dependences. Since economic agents invest resources to operate in the given system and internalize the coordination among different institutions, economic agents in different economic systems develop different capabilities and personalities and have different cost structures. Hence efficacious organizational structures are different, even if one follows the convergence in adopting the restrictive supposition that systems are purely efficiency driven. Their efficacy is determined by cost minimization of decision making and production, incentive effectiveness, and the allocation of individual features to their best possible use.

# 4 Dynamics and Information

To achieve the goal of systemic change, access to agents' preferences is needed. But the information may not be publicly known and agents may behave strategically. A mechanism that consists of a set of strategies for each agent and a function that assigns each strategy profile an alternative characterization is needed. When agents choose whether to learn or not, avoiding costless information can be their optimal strategy, and strategic ignorance predicts a systematic bias in the agent's perceived payoff. When information flows as an investment under uncertainty, agents may undertake irreversible investments anticipating expected losses. Such decisions are taken as commitment devices against the acquisition of future information undesirable from the current perspective. Several equilibria coexist, and the agent will succeed in avoiding investments with losses (or not) depending on the degree of trustworthiness of his future behavior, and under learning abstention can be part of an equilibrium strategy. When agents have the same convex capacity, the set of Pareto-optima is independent of them and identical to the set of optima, and we focus on a pure-exchange economy in which agents are uncertain about future endowments and consume after uncertainty is resolved. We construct a stationary Markov equilibrium for an economy with fiat money, non-durable commodities, countably many time periods, and a continuum of agents. In order to hedge against random fluctuations, agents find it useful to hold fiat money, which they can borrow or deposit at appropriate rates of interest. Equilibrium analysis yields the stationary distribution of wealth across agents as a real possibility in an individual agent's optimization problem. Consider an agent facing a risky distribution of losses who can change this distribution by exerting some effort. Effort is shown to increase with risk-aversion if conditional on the occurrence of a loss; this condition in fact seems to be the main information difference between self-insurance and self-protection. Agents' specificity of information, assets, economies of scale and learning by doing develops through ongoing relationship, and agents adapt their trust in a partner as a function of that partner's loyalty, exhibited by continuation of their relationship. We let the distribution of economic activity across different organizational forms emerge from processes of interaction between these agents as they adapt future decisions to past experiences. What the agents subsequently do in that interaction is their own – possibly suboptimal – decision, which they make on the basis of their locally available, incomplete information and as a result of their own processing of that information.

## Preferences and Matching

Agents are assumed to have differential preferences for different potential partners, and when a buyer is assigned to an agent this means that he makes rather than buys. A matching algorithm produces a set of matches on the basis of individual agents' preference ranking over other agents. Klos and Nooteboom (2001) suppose that each agent assigns a score to all agents he can possibly be matched with, and the product of potential profitability and trust (interpreted as a probability of realization) would constitute expected profit, and allow agents to attach varying weights to profitability versus trust. Both economies of scale and experience effects are modeled with the following function:

$$y = \max\left[0,1 - \frac{1}{fx + 1 - f}\right],$$

and the way profits are made, then, is that suppliers may reduce costs by generating efficiencies for buyers, while buyers may increase returns by selling more varied products. On top of that basic level of trust one can develop partner-specific trust on the basis of experience in dealing with individual partners, and this yields the following specification, to reflect the increase of trust with the duration of an ongoing relation:

$$y = b + (1 - b)\left(1 - \frac{1}{fx + 1 - f}\right),$$

where $b$ is the base-level of trust and $x$ is the number of consecutive matches the agents have been involved in. After the calculation of scores the matching algorithm is applied, and the outcome is determined by the agents' preference ranking over acceptable alternatives. The agents decide how much of risky information to purchase, and the asset is sold at a price, $p$, and issues a random dividend next period paying, $d$.

## Random Dividends and Equilibrium

Assume, that each agent compares two sets ($A$, $B$) by comparing the locations in the two sets that he prefers, all $X, Y \in A^M$. We write $XR_iY$, if for some $l \in M$ and all $k \in M$, $x_l R_i y_k$. For each feasible interval and each chosen alternative $Y$. There does not exist another alternative $X$ consisting of locations taken from this interval such that all agents weakly prefer $X$ to $Y$ and some agent strictly prefers $X$ to $Y$ for all $B$, and this called Pareto optimality. If $m$ = 1, then Pareto-optimality is equivalent to the requirement that the solution chooses from all $R \in \Re^N$ *and all B.*: For all $R \in \Re^N$ and all $B$, there exists no $X \in A^M$

such that $X \subseteq B$ for all $i \in N,\ XR_i\varphi(R,B)$, and for some $j \in N,\ XP_j\varphi(R,B)$ are under Pareto-optimality. We assume that $m \leq n-1$: if the feasible interval shrinks in such a way that the previous choice remains feasible, then both choices are the same. For all $R \in \Re^N$ and all $B,B'$ such that $B \subseteq B'$, if $\varphi(R,B') \subseteq B,$ then $\varphi(R,B) = \varphi(R,B').$ are under Nash's independence of irrelevant alternatives. For each feasible interval and for each pair of preference profiles that coincide on the set of alternatives that are subsets of that interval, the social choice is the same. For all $R,R' \in \Re^N$ and all *B*, if $R|B = R'|B$, then $\varphi(R,B) = \varphi(R',B)$ are under Arrow's independence of irrelevant alternatives. A small change in the feasible interval does not affect the choice too much, and for all $R \in \Re^N,\ \varphi(R,B)$ is continuous with respect to *B*, and under interval continuity. Let $\varphi$ be a solution satisfying Nash's condition and interval continuity, and $R \in \Re^N$, and $B = [a,b]$. For $x \in [a,b],\ \varphi_1(R,[x,b])\ and\ \varphi_m(R,[a,x])$. Let *m* = 2; the extreme peaks solution is the only solution satisfying Pareto-optimality, Nash's, Arrow's and continuity. Let $\Phi$

be a decision rule and $\varphi$ a solution, the pair $(\Phi,\varphi)$ is dual, then for all $R \in \Re^N,\ \Phi(R) = \varphi(R,[0,1]).$ The agents can never gain by misreporting their preferences, and one requires that small changes in preferences do not affect the option set too much.. For all $i \in N$, all $R \in \Re^N$, and all $R_i' \in \Re,\ \Phi(R)R_i\Phi(R_i',R_{-i})$ are under thestrategy. For all $R \in \Re^N,\ all\ i \in N,\ and\ (R_i')_{i \in N} \subset \Re$, if $R_i' \to R_i$, then $\Phi(R_i',R_{-1}) \to \Phi(R)$ are under preference continuity. Let $m \in \{1,\ldots,n-1\}$ and $(\Phi,\varphi)$ be a dual pair, and under Pareto-optimality; the solution $\varphi$ satisfies Nash's condition, Arrow's independence of irrelevant alternatives, and interval continuity if the decision rule $\Phi$ satisfies strategy and preference continuity. Each agent *i* has preferences *R*, a binary relation on *A* which is complete, transitive and reflexive, and let $P_i$ be the preference relation and $I_i$ be the indifference relation. A mechanism $\Gamma$ is a pair (S,g) of a list of strategy sets $S = S_1 \times \ldots\ldots \times S_n$, where $S_i$ is the strategy set for agent *i* , and a function $g: S \to A$ which associates with each strategy an alternative in $A$. Suh (2001) supposes for a mechanism $\Gamma = (S, g)$, the outcome of the strategy profile $s = (s_1, \ldots, s_n) \in S$ is an alternative $g(s)$. Given a preference profile $R \in \Re$ and a mechanism $\Gamma = (S, g)$, a strategy profile $s \in S$ is a Nash equilibrium of the game $(\Gamma, R)$, if there is no $i \in N$ such that for some $\bar{s}_i \in S_i$, $g(\bar{s}_i, s_{-1}) P_i g(s)$. A strategy profile $s \in S$ is a strong Nash equilibrium of the game $(\Gamma, R)$ if there is no coalition $T \subseteq N$ and $T \neq \phi$ such that for all $i \in T$ and for some $\bar{s}_T \in S_T$, $g(\bar{s}_T, s_{N/T}) P_i g(s)$. For all $(a, R) \in A \times \Re$ and for all $i \in N$, let $L(i, a, R) = \{b \in A | a R_i b\}$ be the set of all alternatives which are less preferred to $a$ by agent $i$ under profile $R$. For all $\theta = ((a^1, R^1), \ldots, (a^n, R^n)) \in \Theta^F$, for all $T \in \mathrm{N}, T \neq N$ and for all $i \in N/T$, let

$$\tau(\theta, T, i) = \{j \in N \setminus T | (a^i, R^i) = (a^j, R^j)\}$$

For all $T \in \mathrm{N}$ and for all $\theta = (a^i, R^i)_{i \in N} \in \Theta^F$, let

$$B_T(\theta) = \begin{cases} A & if T = N; \\ a^* & if T = \phi\, and (a^i, R^i) = (a^*, R^*)\ for all\ i \in N; \\ \bigcap_{i \in N/T} L(N \setminus \tau(\theta, T, i), a^i, R^i) & otherwise \end{cases}$$

For all $a \in B_T(\theta)$ and for all $i \in N \setminus T$, at least one agent $j \in N \setminus \tau(\theta, T, i)$ does not prefer alternative $a$ to alternative $a^i$ under profile $R^i$, and if $(a^i, R^i) = (a^*, R^*)$ for all $i \in N \setminus T$, then for all $a \in B_T(\theta)$ at least one agent in $T$ does not prefer alternative $a$ to alternative $a^*$ under profile $R^*$. We study here an infinite-horizon strategic market game with a continuum of agents. Uncertainty is captured by a probability space $(\Omega, F, \mathrm{P})$, and there is a continuum of agents $a \in I = [0,1]$, distributed according to a non-atomic probability measure $\varphi$ on the collection $\mathrm{B}(I)$. On each time period, each agent $a \in I$ receives a random endowment $Y_n^a(w) = Y_n(a, w)$ in units of a commodity, the endowments $Y_1^a, Y_2^a, \ldots$ for a given agent $a$ are assumed to be nonnegative, integral, and independent, with common distribution $\lambda^a$. Geanakoplos, *et al.*

(2000) suppose the variables $Y_n(a,w)$ are jointly measurable in $(a, w)$, so that the total endowment is given by

$$Q_n(w) = \int Y_n(a,w)\varphi(da) > 0,$$

which is a well-defined, positive and finite random variable for every $n$. The bank sets interest rates, $r_{1n}(w) = 1 + p_{1n}(w)$ to be paid by borrowers and $r_{2n}(w) = 1 + p_{2n}(w)$ to be paid to depositors. Agents bid money for consumption of the commodity, thereby determining its price $p$, and the interest rates are assumed to satisfy:

$$1 \le r_{2n}(w) \le r_{1n}(w) \text{ and } r_{2n}(w) < \frac{1}{\beta},$$

for all $n \in \mathrm{N}$, $w \in \Omega$, where $\beta \in (0,1)$ is a fixed discount factor. Each agent has a utility function $u^a : \Re \to \Re$, for $x < 0$, $u^a(x)$ is negative and measures the disutility for agent $a$ of going bankrupt by an amount $x$, for $x > 0$, $u^a(x)$ is positive and measures the utility derived from the consumption of $x$ units of the commodity. At the beginning of time $t = a$, the price of the commodity is $p_{n-1}(w)$ and the total amount of money held in the bank is $M_{n-1}(w)$. If an agent $a \in I$ enters with wealth $S_{n-1}^a(w)$, if $S_{n-1}^a(w) < 0$, then agent $a$ has an unpaid debt from the previous time. If $S_{n-1}^a(w) \ge 0$, then the agent $a$ has fiat money on hand and plays from position $S$; an agent $a$ will play from the wealth position $\left(S_{n-1}^a(w)\right)^+ = \max\left\{S_{n-1}^a(w), 0\right\}$. Agent $a$ also begins time $n$ with information $F_{n-1}^a \subset F$, a $\sigma$-algebra of events that measures past prices $p_k$, past total endowments $Q_k$, and interest rates $r_{1k}, r_{2k}$, as well as past individual-levels, endowments, and actions $S_0^a, S_k^a, Y_k^a, b_k^a$ for $k$ –1,…..,$n$-1. Based on this information, agent $a$ bids an amount

$b_n^a(w) \in \left[0, \left(S_{n-1}^a(w)\right)^+ + k^a\right]$ of fiat money for the commodity at time $n$. The total utility that agent $a$ receives during the period is:

$$\zeta_n^a(w) = \begin{cases} u^a\left(x_n^a(w)\right), & \text{if } S_{n-1}^a(w) \ge 0 \\ u^a\left(x_n^a(w)\right) + u^a\left(S_{n-1}^a(w)/p_{n-1}(w)\right), & \text{if } S_{n-1}a < 0. \end{cases}$$

The total payoff for agent $a$ during the entire duration is the discounted $\sum_{n=1}^{\infty} \beta^{a-1}\zeta_n^a(w)$, and a strategy $\pi^a$ for an agent $a$ specifies the bids $b_\pi^a$ as random variables, which are $F_{n-1}$ measurable. A collection $\Pi = \{\pi_a, a \in I\}$ of strategies for all the agents and the collection of strategies played by the agents are admissible. There are three possible situations for agent $a$ in a given period:

(1) Agent $a$ is a depositor, this means that $n$'s bid $b_n^a(w)$ is strictly less than his wealth $\left(S_{n-1}^a(w)\right)^+ = S_{n-1}^a(w)$ and he deposits or lends the difference:

$$I_n^a(w) = S_{n-1}^a(w) - b_n^a(w) = \left(S_{n-1}^a(w)\right)^+ - b_n^a(w).$$

At the end of the period, $a$ gets back his deposit with interest, as well as his endowment's worth in fiat money and, moves to the new wealth level:

$$S_n^a(w) = r_{2n}(w)I_n^a(w) + p_n(w)Y_n^a(w) > 0.$$

(2) Agent $a$ is a borrower: this means that $a$' s bid $b_n^a(w)$ exceeds his wealth $\left(S_{n1}^a(w)\right)^+$, so he must borrow the difference:

$$d_n^a(w) = b_n^a(w) - \left(S_{n-1}^a(w)\right)^+.$$

At the end of time, $a$ owes the bank $r_{1n}(w)d_n^a(w)$, and his new wealth position is:

$$S_n^a(w) = p_n(w)Y_n^a(w) - r_{1n}(w)d_n^a(w),$$

a quantity which may be negative. Agent $a$ is then required to pay back, from his endowment $p_n(w)Y_n^a(w)$, as much of his debt $r_{1n}(w)d_n^a(w)$ as he can, and agent $a$ pays back the amount:

$$h_n^a(w) = \min\{r_{1n}(w)d_n^a(w), p_n(w)Y_n^a(w)\},$$

and his cash holdings at the end of the period are $\left(S_n^a(w)\right)^+ = p_n(w)Y_n^a(w) - h_n^a(w).$

(3) Agent $a$ neither borrows nor lends, and agent bids his entire cash-holdings $b_n^a(w) = \left(S_{n-1}^a(w)\right)^+$ and ends the time with his endowment's worth in fiat money:

$$S_n^a(w) = p_n(w)Y_n^a(w) \geq 0,$$

and agent's wealth position at the end of the period is:

$$S_n^a(w) = p_n(w)Y_n^a(w) + r_{2n}(w)I_n^a(w) - r_{1n}(w)d_n^a(w),$$

and another formula for agent's cash-holdings is:

$$\left(S_n^a(w)\right)^+ = p_n(w)Y_n^a(w) + r_{2n}(w)I_n^a(w) - h_n^a(w).$$

Let $\{r_{1n}, r_{2n}, p_n\}_{n-1}^{\infty}$ be a system of interest rates and prices. The total expected utility to an agent $a$ from a strategy $\pi^a$ when $S_0^a = s$, is given by:

$$I^a(\pi^a)(s) = \mathrm{E}\sum_{n-1}^{\infty} \beta^{n-1}\zeta_n^a(w)$$

An admissible collection of strategies $\{\pi^a, a \in I\}$ together with an initial distribution for $\{S_n^a, a \in I\}$ determines the random measures $v_n(A, w) = \int 1_A\left(S_n^a(w)\right)\varphi(da),\ A \in \mathrm{B}(\Re)$ that describe the distribution of wealth across agents for $n = 0,1,\dots\dots$ A stationary Markov equilibrium is an equilibrium $\{r_{1n}, r_{2n}, p_n\}_{n'=1}^{\infty}$, $\{\pi^a, a \in I\}$ such that, in addition to previous conditions the following are also satisfied:

- the interest rates $r$ and prices $p$ have constant values $r_1,\ r_2,\ and\ p$ ;
- the wealth distributions $v_n(.,w)$ are equal to a constant measure $\mu$;
- the quantities $M_n(w)\ and\ \vec{M}_n(w)$ have constant values $M$ and $\vec{M}$ , and
- each agent $a \in I$ follows a stationary Markov strategy $\pi^a$ , which means that the bids $b_n^a$ specified by $\pi^a$ can be written in the form:

$b_n^a(w) = c^a\left(\left(S_{n-1}^a(w)\right)^+\right)$ for every $w \in \Omega,\ n \in \mathrm{N}$ . In a stationary Markov equilibrium, then an individual agent faces a sequential optimization problem with fixed price and fixed interest rates.

## Heterogeneous Expectations

The economy is populated by utility-maximizing, infinitely-lived, forward-looking agents, whose solution are the perfect-foresight equilibrium laws of motion of all choice variables. Bomfin (2001) expresses the evolution of these variables as a function of past, current, and future states of the economy, and the assumption of heterogeneous expectations amounts to saying that agents use the same mechanism to solve their dynamic optimization problem:

$$\max \sum_{t=0}^{\infty} \beta^t \{u(c_{i,t}, 1-n_{i,t}) + \lambda_t [\exp(A_t) F(.) Y_t^{\phi} - c_{i,t} - k_{i,t} + (1-\delta) k_{i,t-1}]\},$$

where $\phi$ determines the extent to which individual output $y$ depends on aggregate output $Y$, subject to $k_{i,-1}$, the condition $\lim_{t\to\infty} \beta^t \lambda_t k_{i,t} = 0$, and given $\{A_t, Y_t\}_{t=0}^{\infty}$, $\lambda_t$ is the discounted Lagrange multiplier relevant for time $t$. After imposing a symmetry condition that says that agents who rely on the same forecasting mechanism make identical decisions, the equilibrium laws of motion take the form

$$x_{i,t} = \Pi_{i,k} \vec{K}_{i,t-1} + \Pi_{i,\lambda} \vec{\lambda}_{i,t} + \Pi_{i,e} e_{i,t},$$

where

$$x_{i,t} = [\vec{N}_{i,t}, \vec{K}_{i,t}, \vec{C}_{i,t}]', \; e_{i,t} = [A_t, Y_{j,t}]', \; and \; \vec{\lambda}_{i,t} = \sum_{h=0}^{\infty} \mu_i^{-h} (F_{i,1} e_{i,t+h+1} + F_{i,2} e_{i,t+h}).$$

For each period $t$, we assume that all agents follow a decision process where agents make their labor supply and capital accumulation decisions before being able to observe the current value of productivity shifter $A$, or the current output decision of the other agents in the economy. The allocation decision rules take the form:

$$z_{i,t} = \pi_{i,k} \vec{K}_{i,t-1} + \pi_{i,\lambda} E^{(i)} [\overline{\lambda}_{i,t} | \Omega_{t-1}] + \pi_{i,e} E^{(i)} [e_{i,t} | \Omega_{t-1}],$$

where $z_{i,t} = [\overline{N}_{i,t}, \overline{K}_{i,t}]'$, and the $\pi_i$ parameters correspond to the appropriate elements of the $\Pi_i$ matrices; $\Omega_{t-1}$ is the information set available at the beginning of period $t$, $E^{(i)}[.|\Omega_{t-1}]$ denotes the expectation of a type $i$ agent conditioned on $\Omega_{t-1}$. Therefore, the consumption decision is based on a larger information set $\Omega_{0,t} = \{\Omega_{t-1}, A_t, Y_{S,t}, Y_{R,t}\}$. Agents of different types are information-ally linked, and generate their own decision rules they must forecast the behavior of the other agents in the economy, and $\overline{Y}_{j,t}$ is an element of $e_{i,t}$, and the channel through which agents' expectations affect their behavior. The different expectations rules embedded in $E^{(S)} \; and \; E^{(R)}$ can lead to potentially different responses to the same fundamental shocks. Consider an agent with an increasing utility function $U$, this agent faces a risk of loss or accident and can engage efforts, chosen from an interval $[0, \bar{e}]$. Jullien *et al.* (1999) assume for a level $e$ of effort, his wealth is $w = W - c(e)$ with probability $(1 - p(e))$, $w = W - d(e)$ with probability $p(e)$, where the function $c(e)$ can be thought of as the cost of effort. The difference $l(e) = d(e) - c(e)$ is the loss, and $W$ is the initial wealth, and the expected utility of the agent is

$p(e)U(W - d(e)) + (1 - p(e))U(W - c(e))$. For a level of effort $e$, the end wealth is 0 if $w < W - d(e)$, $p(e)$ *if* $W - d(e) < w < W - c(e)$, and 1 if $w > W - c(e)$. Let us add risk to final wealth, so that the expected utility of the agent is:

$$pE[U(w_1)|e] + (1 - p)E[U(w_0)|e],$$

where $w_0$ *and* $w_1$ are the stochastic variables whose distribution may depend on *e*, and then a more risk-averse agent chooses a higher level of effort. Let $p(e)$, $c(e)$, and $d(e)$ be continuously differentiable, and assume $U$ is increasing and continuously differentiable and that the agent characterized by $U$ strictly prefers effort $e_0 \in [0, \bar{e}]$ to any other effort, with $0 < p(e_0) < 1$. In this case, the final wealth $w$ is a random variable with compact support, and the single-crossing condition ensures that more risk averse agent chooses a higher level of information effort.

## Dynamics of Prices

We consider groups of traders with different trading strategies, who are risk-neutral have a reasonable knowledge of value of the stock $p^*(t)$, and we assume that buying or selling order is given by:

$$x^F(t) = am(\ln p^*(t) - \ln p(t)),$$

where $m$ is the number of agents, and $a$ characterizes the strength of the reaction of the discrepancy between agents price and the market price. Kaizoji *et al.* (2002) suppose the investment attitude of agent $i$ is represented by the random variable $s_i$ of agent is updated with a heat-bath dynamics according to:

$$s_i(t+1) = +1 \ \textit{with} \ p = \frac{1}{1 + \exp(-2\beta h_i(t))},$$

$$s_i(t+1) = -1 \ \textit{with} \ 1 - p,$$

where $h_i(t)$ is the local field governing the strategic choice of the agent.

The local strategy changes of an agent, that is the decision that an agent makes, is influenced by local information as well as global information. Global information includes the information about whether the agent belongs to the majority group or the minority group of sellers or buyers at a time period. The asymmetry in size of majority versus minority groups can be captured by the value:

$$M(t) = \frac{1}{n}\sum_{i=1}^{n} s_i(t)$$
.

The goal of the interacting agents is to obtain capital gains through trading; the majority group has to expand over the next trading period. To sum up, the probability with which increasing agents in the group, withdraw from their coalition,

and the local field
$$h_i(t) = \sum_{j=1}^{m} J_{ij}S_j(t) - \alpha S_j(t)|M(t)|$$
,

with a coupling constant $\alpha > 0$, with nearest neighbor interactions $J_{ij} = J$ and $J_{ij} = 0$ for all other pairs. We assume that the agent excess demand for the stock is approximated as:

$$x^I(t) = bnM(t)$$
,

and in the system a market maker mediates the trading and adjusts the market price to the market clearing values. The balance of demand and supply is written as:

$$x^F(t) + x^I(t) = am[\ln p^*(t) - \ln p(t)] + bnM(t) = 0$$
.

Hence the market price and the trading volume are calculated as:

$$\ln p(t) = \ln p^*(t) + \lambda M(t), \quad \lambda = \frac{bn}{am}$$
,

and

$$V(t) = bn\frac{1 + |M(t)|}{2}$$
.

If $M(t) > 0$, the market price $p(t)$ exceeds the optimal price $p^*(t)$ (bull market regime), and the opposite scenario is a bear market regime. The relative change of price, the so-called log-return, is defined as:

$$\ln p(t) - \ln p(t-1) = (\ln p^*(t-1)) + \lambda(M(t) - M(t-1))$$
.

The meta-stable phases are the analog of speculative bubbles; for example, the bull market is defined as a large deviation of the market price from the optimal price. That is, there is a higher probability for extreme values to occur as compared to the case of a distribution for absolute returns $|r(t)|$, which report asymptotic behavior:

$$P(|r(t)| > x) \sim \frac{1}{x^{\mu}},$$

with an exponent $\mu$ between about 2 and 4 for stock returns, and the volatility $|r(t)|$ is called clustered volatility. Let us consider a timescale $\tau$ at which we observe price fluctuations. The log-return for duration $\tau$ is then defined as $r_\tau(t) = \ln(P(t)/P(t-\tau))$, and volatility clustering as described above is this observable defined for any interval $\tau$ ranging from minutes to more than a month or even longer. The model reproduces observations of markets as distributed returns, clustered volatility, positive correlation between volatility and trading volume, as well as self-similarity in volatility at different time-scales.

## Bounded Rationality

What we learn from experiments is that subjects often fail to play the equilibrium strategy, especially if the equilibrium notion is fairly refined, even when agents can acquire some experience through repeated play. However, for some games, players may still fail to play an equilibrium, even with experience. Among the relevant variables are those that specify the environment in which the mechanism is supposed to operate, as well as initial conditions including the learning protocols agents may use, and a social choice rule will be said to be dynamically implemented by a mechanism. For all possible environments, preferences, adjustment processes, initial conditions, and the limiting set of outcomes coincide with the first-best and is also asymptotically stable, that is, robust when subjected to arbitrarily small perturbations. Then many equilibria in the inefficient Nash equilibrium component can be limit points of the adjustment process. The sustainability of incredible threats under evolutionary dynamics for any way of simulation was first pointed put,by Ponti (2000), who shows that mechanisms pass the stability test for initial conditions sufficiently close to the desired stability properties.

For a given formal game $\wp = \{\Im, S_i, u_i\}$, denote by $x_i = \{x_i^k\},\ k \in S_i$ a generic mixed strategy for player *i*. We formalize player's behavior in terms of the mixed strategy profile, $x(t) = (x_A(t), x_B(t)) \in \Theta$, played at each point in time *t*, where $\Theta$ denotes the set of mixed strategy profiles of $\wp$.

The evolution of $x(t)$ is given by the following system of continuous-time equations:

$$\dot{x}_i^k = f_i^k((x(t)));\ k \in S_i,\ i = A, B,$$

with $f_i^k : \Theta \rightarrow \Re$ satisfying standard regularity conditions.

A function $f = \left(f_i^k\right)$ is said to yield a regular monotonic dynamic if $\gamma_i^h(x) \geq \gamma_i^k(x) \Leftrightarrow u_i(h, x_{-i}(t)) \geq u_i(k, x_{-i})$,

where $\gamma_i^k(x) = \dfrac{f_i^k(x)}{x_i^k}$ denotes the growth rate of strategy $k \in S_i$.

This condition is commonly used to capture the essence of a selective process, and the strategy played at each point in time, strategies with higher expected pay-off grow faster than poorly performing ones. The implicit assumption is that agents from each population are randomly present, so which action is to be used depends on the order of statements, which is randomly determined by the matching technology. By asymptotic stability, every trajectory starting arbitrarily close stays sufficiently close and eventually converges to the solution, and every trajectory converges to the first-best. If repeated play evolves according to such dynamics, then the first-best can (not) be dynamically implemented by $M_2(M_1)$ The failure of global convergence in the case of $M_1$ can be shown as follows: if $N\wp$ denotes the inefficient Nash equilibrium component of the game induced by $M_1$,

$$N\wp = \left\{(x_A, x_B) \in \Theta : \left(\left\{\lambda s_A^{(B,A)} + (1-\lambda) s_A^{(B,A)}\right\}\right), \left\{\mu s_B^{(A,A)} + (1-\mu) s_B^{(B,A)}\right\}$$

,

with $\lambda \in \left[\dfrac{v_B}{1+v_B}, 1\right]$ and $\mu \in [0,1]$.

All strategy profiles in $N\wp$ are outcome equivalent to the Nash equilibrium in pure strategies $\left(s_A^{(B,A)}, s_B^{(B,A)}\right)$ by which the first-best is attained in sub-game $\wp_1$ and the inefficient equilibrium is attained in sub-game $\wp_2$. $N\wp$ is reachable from a non-zero measure set of initial conditions, as follows:

$$\Omega = \left\{(x_A, x_B) \in \Theta^0 : x_A^{(B,A)} > 1 - \varepsilon_A, x_B^{(B,A)} > 1 - \varepsilon_B; 0 < \varepsilon_i \leq \eta,\ i = A, B\right\}$$

,

with $\eta$ sufficiently small. To show that all trajectories starting from $\Omega$ converge to $N\wp$, notice that $\forall x \in \Theta^0$ such that $x_A^{(B,A)} > \dfrac{v + v_B}{v + v_B + \delta}$, it must be true that $\gamma_B^{(B,A)}(x) > 0$, since $s_B^{(B,A)}$ is the unique best response to $x_A$, and implies that $x_B^{(B,A)}(t)$ is increasing, provided $x_A^{(B,A)}(t)$ is arbitrarily high.

For all $x \in \Omega$, it must be true that $\gamma_A^{(B,A)}(x) > 0$, with the differences $\left(\gamma^{(B,\alpha)}(x) - \gamma_A^{(A,A)}(x)\right)$ and $\left(\gamma_A^{(B,\alpha)}(x) - \gamma_A^{(A,B)}(x)\right)$, $\alpha = A,B$.

When $x_B^{(B,A)}$ is sufficiently high, all results are positive and bounded away from zero. This, in turn, implies $\gamma_A^{(B,A)}(x) > 0$, and $x_A^{(B,A)}(t)$ is also increasing, provided $x_B^{(B,A)}(t)$ is arbitrarily high.

## Imperfect Commitment

This involves the range of outcomes that give no incentive to the agent to misrepresent his type, and the problem is that of finding an optimal mechanism within the set of all conceivable mechanisms to a straightforward problem of incentive compatibility constraints. In a long-term relationship the mechanism designer has to specify a contract that covers the entire time horizon of the relationship. The revelation principle for environments in the form of the mechanism designer cannot fully commit to the outcome induced by the mechanism, and the revelation principle, is the guiding principle for the theory of implementation and mechanism design under imperfect information. The designer must be able to resist renegotiation away inefficiencies, and any action that he may take must be verifiable so that it can be specified as part of the mechanism. Since the agent anticipates this, he may realize that truthfully reporting his private information is required, and if he fails due to imperfect commitment, the result is mechanisms whose outcome cannot be replicated by a direct mechanism. We are able to prove that the payoffs on the Pareto frontier of an arbitrary mechanism may also be obtained by a direct mechanism, in which the agent's message space is the set of his types. Under this mechanism it is an optimal strategy for the agent to reveal his type truthfully and he will use this strategy with high probability; also in the presence of imperfect commitment an optimal mechanism can still be found in the set of incentive compatible direct mechanisms. The agent has to be kept indifferent between truthfully revealing his information and cheating, which may occur with nonzero probability. Bester and Strausz (2001) show this for a message set of the same dimensionality as the set of the agent's types. The mechanism designer can get the required payoff just as from a contracting problem with an arbitrary message set, and the optimality of a direct mechanism under which the agent has a weak incentive to reveal his type truthfully can be shown. All allocation consists of types of decision $X$, by which we denote all those decisions to which the market maker can contractually commit himself, and $Y$, which describes all those decisions that are not contractible and are chosen at the market maker" discretion. The market maker has to select $y \in F(x)$ when he is committed to the decision

$x \in X$. The agent is privately informed about his type $t \in T = \{t_1, \ldots, t_i, \ldots t_{|T|}\}$, and we assume that $2 \leq |T|$.

The market maker knows the probability distribution $\gamma = (\gamma_1, \ldots, \gamma_i, \ldots, \gamma_{|T|})$ of the agent's type.

The payoffs of players depend on the allocation $(x, y)$ and the agent's type. When the agent is of the type $t_i$, the market maker's payoff from $(x, y)$ *is* $V_i(x, y)$, and the agent's payoff in this situation is $U_i(x, y)$. The market maker will require the agent to provide some kind of information, and therefore chooses a message set *M* so that the agent has to select some message $m \in M$. The market maker can commit himself to a measurable decision function $x : M \to X$, and the agent can enforce the decision $x(m)$ by sending the message *m*, and a mechanism $\Gamma = (M, x)$ specifies a message set *M* in combination with a decision function $x(.)$.

The agent selects some message $m \in M$; this determines the contractually specified decision $x(m) \in X$, and the market maker uses the agent's message to update his beliefs about the agent's type and chooses some decision $y \in F(x(m))$. For given $\Gamma$, the market maker is constrained to the allocations that can be obtained through the perfect Bayesian equilibria of this game, and the expected payoffs for the market maker and the $t_i$-type agent are defined as:

$$V^*(q, y, x|M) = \sum_i \gamma_i \int_M V_i(x(m), y(m)) dq_i(m),$$

$$U_i^*(q, y, x|M) = \int_M U_i(x(m), y(m)) dq_i(m).$$

The market maker's strategy has to be optimal given his beliefs about the agent's type, and for all every $m \in M$:

$$\sum_i p_i(m) V_i(x(m), y(m)) \geq \sum_i p_i(m) V_i(x(m), y')$$

for all $y' \in F(x(m))$.

The agent anticipates the market maker's behavior *y* and chooses *q* to maximize his payoff, and for each $t_i \in T$, $q_i$ has to satisfy:

$$\int_M U_i(x(m), y(m))dq_i(m) \geq \int_M U_i(x(m), y(m))dq_i'(m) \quad for \quad all \quad q_i' \in Q$$

The market maker's belief has to be consistent with Bayes' rule on the support of the agent's message strategy, for all $t_i \in T$ and all $H \in \mathrm{M}$ with $\overline{q}(H) > 0$ it is required that:

$$\int_H p_i(m)d\overline{q}(m) = \gamma_i q_i(H)$$

.

The left-side represents the market maker's belief that he confronts agent $t_i$ upon receiving a message from the set *H*, and the right-hand side expresses the conditional probability that the agent is actually of type $t_i$ given that, under the reporting strategy *q*, a message in the set *H* is realized. For a given message set $M, (q, p, y, x|M)$ is said be incentive efficient if it is incentive feasible and there is no incentive feasible $(q', p', y', x'|M)$ such that:

$$V^*(q^*, y^*, x^*|M) > V^*(q, y, x|M)$$

and

$$U_i^*(q^*, y^*, x'|M) = U_i^*(q, y, x|M)$$

for all $t_i \in T$. The agent has the option to refuse to contract with the market maker; if we let $\overline{U}_i$ denote the payoff that the type must also satisfy individual-rationality constraints:

$$U_i^*(q, y, x|M) \geq \overline{U}_i \quad \text{for all} \quad t_i \in T,$$

for each type $t_i$ there is a message $m \in M$ such that $U_i(x(m), y(m)) \geq \overline{U}_i$.

The market maker's overall problem includes the choice of an appropriate message set *M*. For unobservable actions, the market maker's decision *y* may not be contractible because it is not publicly observable. He can perform an audit to verify the agent's type, but commitment to a specific auditing strategy is problem. If imperfect commitment arises in contracting parties for the current period after the contract *x* expires, we will suppose that the market maker offers a new contract *y*, i.e. that the contract $x^*(m)$ is inefficient, and a new contract *y* is offered, which the agent can either accept or reject. When $Z = Y$, the market maker faces no commitment and so the agent's message *m* has no direct impact on the allocation; it does not affect the market maker's decision via his beliefs about the agent's type.

With a direct mechanism $\Gamma = (M, x)$ the message set is the agent's type set, and in the game induced by $\Gamma$, when the agent announces some type, the market maker may be able to commit himself to all relevant decisions. The market maker's problem allows him to select as his message set of the agent's type, and he can restrict his choice to an allocation function $z : T \rightarrow X$ that gives the agent the constraint denoted by the following incentive compatibility conditions:

$U_i(z(t_i)) \geq U_i(z(t_j))$ for all $t_j \in T$ .

This incorporates the restrictions that the market maker faces because he is uninformed about the agent's type. The design of efficient mechanisms or optimal contracts is supported by the fact that all outcomes that are implement-able through some other type of mechanism; any incentive efficient $(q, p, y, x|M)$ can be replaced by some payoff-equivalent $(q', p, y, x|M)$ in such a way that the support of the agent's reporting strategy $q'$ contains at most $|T|$ elements.

The idea for the reporting strategy $q'$ can be explained for the case where $M = (m_1, \ldots\ldots, m_h, \ldots\ldots, m_{|M|})$ is a finite set if we assume asserts that:

$$\sum_h \alpha_h p(m_h) = \gamma ,$$

as this allows us to define the reporting strategy $q'$ by setting $q_i'(m_h) = \alpha_h q_i(m_h) / \overline{q}(m_h)$.

The support of $\overline{q}'$ contains at most $|T|$ messages and $p$ and $q'$ are consistent with Bayesian updating, since replacing $q$ by $q'$ does not alter the market maker's belief, his choice of $y$ remains optimal. The reason is that applying above procedure to the reporting behavior of one of the agents may reduce the expected reporting of another agent so that individual rationality be violated. These observations allow us to formulate the market maker's contracting problem:

$$\max_{q,p,y,x} \sum_i \sum_j \gamma_i q_i(t_j) V_i(z(t_j)) ,$$

subject to

$$U_i(z(t_i)) \geq U_i(z(t_j)) ,$$

$$U_i(z(t_i)) \geq \overline{U}_i ,$$

$$[U_i(z(t_i)) - U_i(z(t_j))] q_i(t_j) = 0 ,$$

$$y(t_i) \in \arg\max_{y \in F(x(t_i))} \sum_j p_j(t_i) V_j(x(t_i), y)$$
,

$$p_i(t_j) \sum_k \gamma_k q_k(t_j) = \gamma_i q_i(t_j)$$
,

for all $t_i, t_j \in T$. The first constraints represent the usual incentive compatibility and individual rationality restrictions, and the market maker faces additional constraints.

The lack of commitment any incentive constraint could turn out to be binding at the optimum, and one may have to result to the straightforward but tedious procedure of solving the problem by examining all possible constellations. We denote the decisions that have been implemented in the periods up to date $\tau$ by $(\bar{x}_{\tau-1}, p_{\tau-1})$; the market maker uses the agent's message *m* to update his belief according to some function $p_\tau : M_\tau \to P$.

# 5 Life Cycle and Risk-Sharing of Information Under Restriction

Competition is considered to be motive power of evolution in economical systems and in the pooled life cycles of agents, resources, their interactions and the whole system, and out-coming flows vary with the agent's potential ability to process resources. Human beings and their organizations are purposeful agents whose actions help transform the society in which they live, and society is made up of social relationships that structure the interactions between these agents. States are mutually reflective and are embedded in, or are co-constitutive of, each other, and the adoption of a collective-logic approach and agents as doubled-edged suggests itself. The realm of constraints that defines parameters to which states must adapt and a resource pool in which states-as-agents must participate in order to enhance their power or the interests of autonomous agents. States have the power of reflexive agents, such that they can enhance their power by working with social forces at various levels, not least by entering into cooperative relations with other states and non-state agents. States employ exit strategies when they seek to overcome or mitigate structural constraints by playing off the different realms against each other, and states can dip into these realms in order to counter challenges.

Thus states often dip into these realms to conform to structural imperatives: interest rate policies are indeed concerned to appease financial interests by trimming their monetary policies and keeping fiscal deficits in check. The crucial point to note is that, in pursuing these various exit and adaptive strategies linkages between realms are formed, thereby enabling the development of an increasingly integrated architecture.

## Learning and Information

The agents must choose between risky information and a risk-free assets paying zero interest. Since the actual behavior of evolutionary agents is not well understood, agents are assumed to have myopic constant absolute risk aversion preferences. They maximize $U(w) = E\left(-e^{-\gamma w}\right), \quad w = s(d - p),$ where $s$ is the number of shares held of the risky asset, the agent's only choice variable. For a distribution of $d$, the optimal solution for $s$ will be a function of the price, $p$, and the mean dividend $\bar{d}$ : $s^* = \alpha^*(\bar{d} - p)$. LeBaron (2000) assumes that learning and evolution takes place in a space that is somewhat disconnected from a real one. A candidate rule, $i$, is evaluated by how well it performs over a set of $S$ experiments, $V_i = \sum_{j=1}^{S} U_i(w_{i,j})$, where $w_{i,j}$ is the wealth obtained by rule $i$ in the $j$th experiment. That is, agents are choosing

$$\alpha^{**} = \arg\max_{\alpha i} \sum_{j=1}^{S} U_{\alpha i}(w_j), \quad and \quad E(\alpha^{**}) \neq \alpha^*.$$

Pushing $S$ to infinity exposes agents to more trials, and the chances of performing well go to zero, and we are forced to make some decision about how far back agents should look, and how much data they should look at. The agent-traders' behavior is basically random, issuing random bids and offers distributed over a predefined range, and beyond a budget restriction they continue to bid and offer randomly. The structure of the economy is based on a generations model where two period agents solve the following problem:

$$\max_{c_{t,t}c_{t,t+1}} \log c_{t,t} + \log c_{t,t+1}$$

s. t. $$c_{t,t} \leq w_1 - \frac{m_{1,t}}{p_{1,t}} - \frac{m_{2,t}}{p_{2,t}}, \qquad c_{t,t+1} \leq w_2 + \frac{m_{1,t}}{p_{1,t+1}} + \frac{m_{2,t}}{p_{2,t+1}}.$$

The amounts $m_{1,t}$ *and* $m_{2,t}$ the agents are holding in the two information periods 1,2 can both be used to purchase the one consumption good, $p_{i,t}$ is the price level of information $i$ in period $t$. Consumption, $c_{m,n}$ is for generation $m$ at time $n$. The uncertainty on the two information periods, must be equal:

$R_t = p_{1,t}/p_{1,t+1} = p_{2,t}/p_{2,t+1}$. The savings demand of the agents at $t$ is

$$s_t = \frac{m_{1,t}}{p_{1,t}} + \frac{m_{2,t}}{p_{2,t}} = \frac{1}{2}\left(w_1 - w_2\frac{1}{R_t}\right).$$

The equilibrium occurs in a general equilibrium setting with price formation, and it compares the learning dynamics to results from actual markets. Uncertainty and information in markets depends on a portfolio decision problem with a costly information signal that agents can decide to purchase. The dividend pay-out is given by $d = \beta_0 + \beta_1 y + \varepsilon$, where $y$ is the signal that can be purchased for a given cost, $c$. Agents are interested in maximizing expected one-period utility given by $E(-e^{-\gamma w_1}|\Omega)$

s. t. $$w_1 = w_0 - \theta c + x(d - p),$$

with $x$ being the number of information signals about the risky asset. There is a risk-free asset in zero net supply with zero interest, $\theta$ is 1 for informed agents and 0 for uninformed agents. The expectation is conditioned on available information, and for the informed agents, these are price and the signal $y$. For the uninformed, it is based on price alone, and with multivariate normality this leads to a demand for the risky asset of

$$x = \frac{E(d|\Omega) - p}{\gamma V(d|\Omega)}.$$

Learning takes place as the agents try to convert their information into forecasts. The informed build forecasts using the signal alone:

$E^n(d|y) = \beta_0^{i,,n} + \beta_1^{i,n}$. The uninformed base their predictions on their only piece of information, the price: $E^n(d|y) = \beta_0^{u,n} + \beta_1^{u,n} p,$ where $I$ and $U$ are the set of informed and uninformed agents. The conditional variances for each informed and uninformed agent are assumed to be, $v^{i,n}$ *and* $v^{u,n}$. Each instance of an agent carries with it a vector of parameters which describes its learning state $(\theta_n \beta_0^{i,n}, \beta_1^{i,n}, v^{i,n}, \beta_0^{u,n}, \beta_1^{u,n}, v^{u,n})$

For a given configuration of agents with a fraction $\lambda$ purchasing the signal, the equilibrium price can be determined. This is done setting aggregate demand for risky information equal to aggregate supply:

$$\sum_{n \in I} \frac{\beta_0^{i,n} + \beta_1^{i,n} y - P}{\gamma v^{i,n}} + \sum_{n \in U} \frac{\beta_0^{u,n} + \beta_1^{u,n} P - P}{\gamma v^{u,n}} = Ne.$$

Now define

$$T^I = \frac{1}{\lambda N} \sum_{n \in I} \frac{1}{\gamma v^{i,n}}, \quad \text{and} \quad T^U = \frac{1}{(1-\lambda)N} \sum_{n \in U} \frac{1}{\gamma v^{u,n}}.$$

There are the average effective risk tolerances for informed and uninformed agents, and we define aggregate $\beta's$ for $j = 0,1$:

$$\beta_j^1 = \frac{1}{T^I \lambda N} \sum_{n \in I} \frac{\beta_j^{i,n}}{\gamma v^{i,n}}.$$

$$\beta_j^U = \frac{1}{T^U (1-\lambda) N} \sum_{n \in U} \frac{\beta_j^{u,n}}{\gamma v^{u,n}}, \quad \text{and}$$

$$\lambda T^I (\beta_0^I + \beta_1^I y - P) + (1-\lambda) T^U (\beta_0^U + \beta_1^U P - P) = e,$$ which can be easily solved for $P$:

$$P = \alpha_0 + \alpha_1 y + \alpha_2 (e - \bar{e}),$$

$$\alpha_0 = \frac{\lambda\beta_0^I T^I + (1-\lambda)\beta_0^U T^U - \bar{e}}{\lambda T^I + (1-\lambda)T^U\left(1-\beta_1^U\right)},$$

$$\alpha_1 = \frac{\lambda\beta_1^I T^I}{\lambda T^I + (1-\lambda)T^U\left(1-\beta_1^U\right)},$$

$$\alpha_2 = \frac{-1}{\lambda T^I + (1-\lambda)T^U\left(1-\beta_1^U\right)}.$$

A rational expectations equilibrium is a pricing function $P(y)$, and learning parameters, such that the above forecast parameters are the correct ones for all agents, and the expected utilities of all types of agents are equal. The dividend follows a defined stochastic process:

$$d_t = \bar{d} + p\left(d_{t-1} - \bar{d}\right) + \varepsilon_t,$$

where $\varepsilon_t$ is Gaussian, independent, and identically distributed, and $p = 0.95$ for all experiments.

## Risky Expectations

The demand for risky information by agent $i$, is given by

$$s_{t,i} = \frac{E_{t,i}\left(p_{t+1} + d_{t+1}\right) - p_t(1+r)}{\gamma\sigma^2_{t,i,p+d}},$$

where $p$ is the price of the risky information at $t$, $\sigma^2_{t,i,p+d}$ is the conditional variance of $p + d$ at time $t$ for agent $i, \gamma$ is the coefficient of absolute risk aversion, and $E_{t,i}$ is the expectation for agent $i$ at time $t$. This would depend on bringing to bear all appropriate conditioning information in the market, which would include the beliefs and holdings of all other agents.

Ait-Sahalia and Lo (2000) assume that the stock information price $S$ follows the stochastic differential equation

$$\frac{dS_t}{S_t} = \mu(S_t, t)dt + \sigma(S_t, t)dZ_t,$$

where $Z$ denotes a standard Brownian motion. Let $f_t(S_T)$ denote the conditional density of $S_T$ *given* $S_t$. The agent adjusts the amount, $q$, invested in the stock at each intermediary date $\tau$ to solve optimization problem:

$$\max_{\{q_s | t \le s \le T\}} E[U(W_T)]$$

$$dW_s = \{rW_s + q_s(\mu(S_s,s) - r)\}ds + q_s\sigma(S_s,s)dZ_s,\ W_s \geq 0,\ t \leq s \leq T,$$

where $W_s$ denotes his wealth at date $s$. In equilibrium, the agent optimally invests all his wealth using the risky information in stock at every instant prior to $T(W_s = S_s \ \textit{for all}\ t \leq s \leq T)$ and then consumes the terminal informational value of the stock at $T(C_T = W_T = S_T)$. Let $J(W,S,t)$ denote the agent's indirect utility function, and condition for the agent's problem takes the form

$$\frac{\partial J(W_s,S_s,s)}{\partial W} = e^{-r(s-1)}\frac{\partial J(W_s,S_s,t)}{\partial W}\zeta_s$$

, with terminal condition at date $s = T$ given by

$$U'(W_T) = e^{-r(T-t)}U'(W_t)\zeta_T$$

, where

$$\zeta_s = \exp\left\{\int_t^s\left(\frac{\mu(S_u,u) - r}{\sigma(S_u,u)}\right)dZ_u - \frac{1}{2}\int_t^s\left(\frac{\mu(S_u,u) - r}{\sigma(S_u,u)}\right)^2 du\right\}.$$

The price at date $t$ of information about security with a single date $T$ liquidating payoff of $\psi(W_T)$ is then given by

$$E_T\left[\psi(W_T)M_{t,T}\right],$$

where $M_{t,T} = U'(W_T)\ /U'(W_t)$ is the stochastic discount factor or marginal rate of substitution between information dates $t$ and $T$. If we define the information state-price density to be

$$f_t^*(S_T) = f_t(S_T) \times E\left[\zeta_T | S_t, S_T\right] = f_t(S_T) \times \zeta_T,$$

then we can write:

$$E_t\left[\psi(W_T)M_{t,T}\right] = \int_0^\infty \psi(W_T)\frac{U'(W_T)}{U'(W_t)}f_t(S_T)dW_T$$

$$= e^{-r(T-t)}\int_0^\infty \psi(W_T)f_t^*(S_T)dW_T$$

$$= e^{-r(T-t)}E^*\left[\psi(W_T)\right].$$

The information price of any asset can be expressed as a discounted expected payoff, discounted at the risk-less rate of interest. The expectation must be taken with respect to $f^*$, a marginal rate of substitution weighted probability density function, and this density is called the state-price density, and the ratio of $f^*$ to $f$ is proportional to the agent:

$$\zeta_T = \frac{f_t^*(S_T)}{f_t(S_T)} = e^{r(T-t)}\frac{U_T'(C_T)}{U_t'(C_t)} = e^{r(T-t)}M_{t,T}.$$

In equilibrium, the agent's information preferences are denoted by $U$, and the information asset price dynamics by $f^*$. We implement this relationship using information about market prices of options and index values the information on $f^*$ and $f$, respectively, that are needed to characterize the agent preferences. The condition for the agent's optimization problem is:

$$\zeta_t(S_T) = \frac{f_t^*(S_T)}{f_t(S_T)} = \lambda e^{-r_{t,s,T}}\frac{U'(S_T)}{U'(S_t)},$$

where $\lambda$ is a constant independent of the index level, and measure of relative risk aversion $p_t(S_T)$ by observing that :

$$\zeta_t'(S_T) = \lambda e^{-r_{t,s,T}}\frac{U''(S_T)}{U'(S_t)} \Rightarrow p_t(S_T) = -\frac{S_T U''(S_T)}{U'(S_T)} = -\frac{S_T\zeta_t'(S_T)}{\zeta_t(S_T)}.$$

This suggest a estimation strategy for the measure of risk aversion $p(.)$ ; we estimate the first derivatives of $f^*() \; and \; f()$ and we can calculate:

$$\ddot{p}_t(S_T) = -\frac{S_T\zeta_t'(S_T)}{\zeta_t(S_T)} = \frac{S_T\hat{f}_t'(S_T)}{\hat{f}_t(S_T)} - \frac{S_T\hat{f}^{*\prime}(S_T)}{\hat{f}^*(S_T)}.$$

This concludes the formalization of time inconsistent preferences and the different scenarios involving uncertainty.

## Uncertain Environments

Denote by $V^t$ the inter-temporal utility from the perspective of the agent at date $t$, and denote also by $u_t$ the instantaneous utility. The relationship from the agent's perspective is:

$$V^t = u_t + \beta\sum_{i=0}^{\infty}\delta^i u_{t+1},$$

where $\beta(<1)$ is the extra weight of current rewards relative to future streams of payoffs. Brocas and Carrillo (2000) are interested in analyzing the role of information acquisition by rational agents, who are aware of their conflicts and cannot make binding commitments on future decisions. We will analyze different environments involving uncertainty, and suppose that at time $t$ = 1 the agent has to decide whether to undertake any activity or not. This activity yields either a cost at

$t = 1$ or a benefit at $t = 2$, and there is some uncertainty either about the cost, or the benefit, or both. At time $t = 0$, that is the period before undertaking the activity, the agent has the opportunity to freely learn about the realization of the uncertainty, and in an infinite horizon, activity is possible at every period. An agent with time inconsistent preferences may optimally decide to avoid collecting free information, and in the absence of learning, we have conflicting goals. On the one hand, the agent is willing to obtain information so as to improve his knowledge before undertaking any action, and the decision will be taken in a future context, whose goals do not coincide with the current ones. The agents are linked to each other by their dynamically inconsistent preferences, and the decision of an agent to privately learn his own independent probability of success will depend on the sharing of risk information. The competitive interest rate will be set conditionally on the learning decision of all the agents, and levels of entrepreneurial boldness are jointly determined in equilibrium. When the risk-free rate is low, competitive agents do not need to impose a large interest rate to satisfy their break-even constraints. At each time $t$, the agent can undertake an irreversible activity yielding an uncertain payoff.

## The Flow of Information

When the flow of information transmitted between periods is small, the information value of waiting is negative and increasing over time. Chateauneuf, Dana and Tallon (2000) assume the existence of a utility $U_i : \Re_+ \to \Re$ that is cardinal, and defined up to a transformation. The core of a capacity $v$ is defined as follows:

$$core(v) = \left\{ \pi \in \Re_+^j \sum_j \pi^j = 1 \ and \ \pi(A) \geq v(A), \forall A \in \Im \right\},$$

where $\pi(A) = \sum_{j \in A} \pi^j$, and a capacity is a set function $v : \Im \to [0,1]$ such that $v(0) = 0,\ v(S) = 1$, and for all $A, B \in \Im,\ A \subset B \Rightarrow v(A) \leq v(B)$. Core $(v)$ is a compact, convex set which may be empty, and since $1 > v(A) > 0 \forall A \in \Im, A \neq S, A \neq 0,\ any\ \pi \in core(v)$ is such that $\pi >> 0$. We now turn the definition of the Choquet integral of $f \in \Re^S$:

$$\int f dv = E_v(f) = \int_{-\infty}^{0} (v(f \geq t) - 1) dt + \int_{0}^{\infty} v(f \geq t) dt,$$

if $f^j = f(j)$ is such that $f^1 \leq f^2 \leq \ldots \leq f^k$ :

$$\int f dv = \sum_{j=1}^{k-1} [v(\{j,\dots\dots,k\}) - v(\{j+1,\dots\dots,k\})] f^j + v(\{k\}) f^k.$$

If we assume that an agent consumes $C^j$ in state $j$, and that $C^1 \leq \dots \leq C^k$, then the preferences are represented by:

$$v(C) = [1 - v(\{2,\dots\dots,k\})]U(C^j) + ..[v(\{j,\dots\dots,k\}) - v(\{j+1,\dots\dots,k\})]U(C^j) + ..v(\{k\})U(C^k).$$

In optimal risk-sharing, agents have the probability $\pi = (\pi^1, \dots\dots, \pi^k)$, $\pi^j > 0$ for all $j$, over the states of the world and a utility function defined by

$$v_i(C_i) = \sum_{j=1}^{k} \pi^j U_i(C_i^j),\ i = 1,\dots,n.$$

If agents have different probabilities $\pi_i^j,\ j = 1,\dots,k,\ i = 1,\dots,n$, over the states of the world, it is easily seen that Pareto optimality now depends on these probabilities and on aggregate risk. Optimal information risk-sharing and equilibrium analysis can be carried out when agents have identical capacities, and then the analysis can move on to different capacities. Define $D_v(w)$ as follows:

$$D_v(w) = \{\pi \in core(v) | E_\pi w = E_v w\},$$

where $D_v(w)$ is constituted of the probabilities that minimize the expected value of the aggregate endowment if $w^1 < w^2 \dots < w^k$, $D_v(w)$ contains only $\pi = (\pi^1, \dots\dots, \pi^k)$ *with* $\pi^j = v(\{j, j+1, \dots\dots, k\}) - v(\{j+1, \dots\dots, k\})$ for all $j < k$ and $\pi^k = v(\{k\})$.

If $w^1 = \dots = w^k$, the set $D_v(w)$ is equal to core ($v$). Since the Pareto optima of an economy containing agents with the same probability are independent of the probabilities, there would exist an allocation $(C_1', C_2', \dots\dots, C_n')$, such that:

$$v_i(C_i') = E_v[U_i(C_i')] \geq v_i(C_i) = E_\pi[U_i(C_i)]$$

for all $i$, and with at least one inequality. Let $(p^*, C^*)$ be an equilibrium in which all agents have utility index $U_i$ and beliefs given by $\pi \in D_v(w)$, then $(p^*, C^*)$ is an equilibrium, in which agents have the same probabilities over states $\pi^j = v(j, j+1, \dots\dots, k) - v(j+1, \dots\dots, k),\ j < k$ *and* $\pi^k = v(\{k\})$.

Optimal informational risk-sharing and equilibrium where agents have different capacities can now be considered. Let $(C_i)_{i=1}^n \in \Re_+^{kn}$, and let $\pi_i \in core(v_i)$ be such that $E_{v_i}[U_i(C_i)] = E_{\pi_i}[U_i(C_i)]$ for all $i$. Then $(C_i)_{i=1}^n$ is a Pareto optimal allocation of an economy in which agents have utility index $U_i$ and probabilities $U_i$ and probabilities $\pi_i,\ i = 1,\dots,n.$ Also $(C_i)_{i=1}^n$ is a Pareto optimal allocation of the economy in which agents are maximizing returns with utility index $U_i$ and probability

$$\pi_i^j = v_i(\{j,\dots,k\}) - v_i(\{j+1,\dots k\})\ for\ j < k\ and\ \pi_i^k = v_i(\{k\}).$$

Assume $w^1 \le w^2 \le \dots \le w^k$, and let $(p^*.C^*)$. If $0 < C_i^{*1} < \dots < C_i^{*k}$ for all $i$, then $(p^*, C^*)$ is an equilibrium of the economy in which agents are maximizing returns with utility index $U_i$ and probability $\pi_i^j = v_i(\{j,\dots,k\}) - v_i(\{j+1,\dots,k\})$, and since $v_i$ is differentiable at $C_i^*$ for every $i$, there exists a multiplier $\lambda_i$ such that $\lambda_i[U_i'(C_i^{*1})\pi_i^1,\dots,U_i'(C_i^{*k})\pi_i^k]$. Hence $(p^*, C^*)$ is an equilibrium of the economy in which agents are maximizing returns with probability $(\pi_i^j)$ for all $i$, $j$. We have

$$p^* C_i' \le p^* w_i \Rightarrow E_{\pi_i}[U_i(C_i')] \le E_{\pi_i}[U_i(C_i^*)].$$

Optimal informational risk-sharing and equilibrium without aggregate risk, when each agent's endowment is independent of the state of the world, is necessary in such economies to decentralize an optimal allocation. We thus move to an economy with $m$ goods, indexed by $\ell$, $C_{i\ell}^j$ as the consumption of good $\ell$ by agent $i$ in state $j$, and we have $C_i^j = (C_{i1}^j,\dots,C_{i,m}^j)$ and $C_i = (C_i^1,\dots,C_i^k)$. Let $p_\ell^j$ be the price of good $\ell$ available in state $j$, $p = (p_1^j,\dots,p_m^j)$, and $p = (p^1,\dots,p^k)$, which ensures that there is a solution to the agent's maximization problem, and

$$\forall i,\ \{x' \in \Re_+^m | U_i(x') \ge U_i(x)\} \subset \Re_{++}^m,\ \forall x \in \Re_{++}^m.$$

Assume all agents have identical beliefs, $\pi = (\pi^1,\dots,\pi^k)$, then at a Pareto optimum $C_i^j = C_i^{j'}$ for all $i, j, j'$, and $(p^*, C^*)$ is an equilibrium of the economy if there exists $q^* \in \Re_+^m$

such that $p^* = (q^*\pi^1, q^*\pi^2, ...., q^*\pi^k)$, and $(q^*, C^*)$ is an equilibrium of the economy $(U_i, w_i = E_\pi(w_i)),\ i = 1, ...., n.$ Thus, even with different beliefs, agents might still find it optimal to fully insure themselves: differences in beliefs do not necessarily lead agents to optimally bear some informational risk.

## Preference Relation

Each agent has a preference relation over the interval and is allowed to select which information facility to use. Such a list constitutes an option set and each agent compares option sets by comparing their elements according to his preference relation over locations. If we additionally require Pareto-optimality and interval continuity, we show that this combination only allows for the extreme peak solution. Ehlers (2001) shows that if a requirement that each single location satisfies the Nash independence requirement is imposed, then already Pareto-optimality and interval continuity are only satisfied by one solution (extreme peak solution).

Let [0,1] be the set of possible informational locations, and let $n \in N \ and \ N = \{1, ...., n\}$ be the set of agents. Each agent $i$ is equipped with complete and transitive binary relation $\Re_i$ over [0,1] as his preference relation. This relation is such that there exists a point $p(R_i) \in [0,1]$, the peak of $\Re_i$ over [0,1]. Let $m \in N \ and \ M = \{1, ...., m\}$ be the set of information (an alternative is an ordered list $X = \{x_1, ..., x_m\}$ such that for all $k \in M,\ x_i \in [0,1],\ and$ $x_1 \leq x_2 \leq .... \leq x_m$) and let $A^M$ denote the set of alternatives in a closed interval $[a,b] \subseteq [0,1]$. Let $B = \{[a,b]\}\ 0 \leq a \leq b \leq 1$ denote the set of all such intervals. Then a solution is a mapping $\varphi$ associated with every profile $\Re^N$ and every interval $B$: an alternative $\varphi(\Re, B) \in A^M$ such that $\varphi(\Re, B) \subseteq B.$ We think of each alternative as an option set from which agents can freely select.

## Equilibrium and Competition

When the individual properties of all the agents with resources and relations with each other go on by the rules of competition, we can represent it as taking place in a square lattice of size $K \times K$, and for each agent

$S^n(t) = \{S_{ij}^n(t)\}/k_{ij} \neq 0$, where $k$ is the coordination number of the cell, and agent's assets are $A_n = \{S^n\}$. Berg and Popkov (2002) suppose that the interaction between agents and resources are determined by an assimilation rule that covers the agent's growth and variable costs:

$X_{ij}(t) \in L \rightarrow X_{ij}(t+1) \in S^n$ with the probability $\rho_k^n(t)$,

if $\{M_{ij}(t) \cap S^n(t)\} \neq 0$.

Here $X_{ij}(t)$ and $X_{ij}(t+1)$ are states of the *i, j*-cell at the moments $t$ and $t+1$; $M_{ij}(t) = \{X_{t+1,j}(t); X_{t-1,j}(t); X_{i,j+1}(t); X_{i,j-1}(t)\}$ represent the *i j*-cell vicinity. Initial and boundary conditions are: initial uniform distribution of *L, Z, S* –cells, and Brownian motion of *Z* and *L* elements, and agents are individuals, $P_k^n(t) \neq P_k^r(t) \quad \forall n, r / n, r \in \{N(t)\}$, plus the following modifications of open system: $\sum A_i(t) + L(t) \neq const.$, innovations appear at the $\tau_{N+1}$ moment for the $(N+1)$ agent with less constant cost value, $P_k^{N+1}(t + \tau_{N+1}) < P_k^n(t + \tau_{N+1})$, $n = 1, \ldots, N$. Resource exchange among agents in this environment brings the evolution properties to a trajectory where one stage corresponds to a decrease in assets. This stage is obtained for separate agents if more effective agents exist with lower $P_k$ and higher $\rho_k$ values: for all agents if the system is open and the total number of resources decreases $d(A_n + L)/dt < 0$.

## Equilibrium Strategies

Each agent's private information consists of a signal $X_i$ that is affiliated with $U_i$. Let $F_X(.)$ denote the joint distribution of $X = (X_1, \ldots, X_n)$. Athey and Haile (2002) assume the joint distribution of $(U, X)$, the marginal distributions $F_{U_i}(.)$ and $F_{X_i}(.)$, allowing asymmetric agents. Using generic random variables $S = (S_1, \ldots, S_n)$ drawn from the distribution $F_S(.)$, we denote by $S^{(j:n)}$ the *j*th order statistic, with $S^{(n:n)}$ denoting the maximum, and $F_S^{(j:n)}(.)$ denotes the marginal distribution of $S^{(j:n)}$. For private values $U_i = X_i \quad \forall i$, no agent has private information relevant to another's expected utility, and in a common values model for all $i$ and $j$, where $U_i$ and $X_j$ are strictly affiliated conditional on any $\chi \subset \{X_k\}_{k \neq j}$ agent $i$ would update her beliefs about her utility, $U_i$, if she observed $X_j$ in addition to her own signal $X_i$.

Rational agents anticipate this information when forming expectations of the utility they would receive, and the value and signal distributions are assumed to be common knowledge among the agents. We restrict attention to perfect Bayesian Nash equilibria in weakly un-dominated strategies, denoted by $\beta_i(.)$ for each $i$, and further to symmetric equilibria, and then for each agent $i$ :

$$b_i = \beta_i(x_i) = E\left[U_i \middle| X_i = x_i,\ \max_{j \neq i} \beta_j(X_j) = b_i\right].$$

As the auction proceeds, agents condition on the signals of opponents who have already dropped out, and there are multiple symmetric equilibria in weakly un-dominated strategies.

In any such equilibrium, the agent's exit price $b_i$ solves:

$$b_i = \beta_i(x_i) = E\left[U_i \middle| X_i = x_i, \beta_j(X_j) = b_i \ \ \forall j \notin \{i \cup L_i\}, X_k = x_k \ \ \forall k \in L_i\right]$$

where $L_i$ denotes the set of agents who exit before $i$. When equilibrium strategies are increasing, the equilibrium bids $(B_1, \ldots\ldots, B_n)$ have the same information content as the signals $(X_1, \ldots\ldots\ldots, X_n)$, with each auction representing an independent draw from this distribution.

Agent identities and agent-specific factors such as firm size, location, or inventories are often observed, particularly for government auctions, and resale prices can provide measures of realized values. With no reserve price, all agents who learn $X_i$ would place yielding variation in $n$; such variation can also arise from participation restrictions by design in field experiments, where more potential agents may become aware of longer auctions.

Given the implications of equilibrium behavior in an auction game, the joint distribution of agents' utilities and signals is determined by the joint distribution of observables as a pair $(\Im, \Gamma)$, where $\Im$ is a set of joint distributions over the vector of latent random variables, $\Gamma$ is a collection of mappings $\gamma : \Im \to \mathrm{H}$ *and* $\mathrm{H}$ is the set of all joint distribution over the vector of observable random variables.

The equilibrium bid function is just the identity function, so the identification reduces to that of whether the joint distribution of valuations can be determined when only certain order statistics are observed. With independent private values:

$X_i = U_i \quad \forall i$, with $(X_1,......X_n)$ mutually independent, the underlying distribution of valuations is identified even when only one bid per auction is observed. The model is testable if either more than one bid per auction is observed, or transaction prices are observed from auctions; non-parametric identification relies on distribution assumptions. Parametric approaches will not be necessary and the assumptions of bidder demand, if the transaction price is observed, still holds when this is dropped as long as the identity of the winning agent is also observed. The identification argument holds even if agents are asymmetric as long as agent identities are observed, but fails when not all bids are observed. Observed bids correspond to realizations of order statistics $U^{(j:n)} = A^{(j:n)} + V$, which are dependent even when the underlying random variables are independent and when the measurement errors $\{A^{(j:n)}\}$ underlying the observed bids are dependent. If the conditional variable $V$ is determined by observable covariates $V = g_0(W_0)$ for some unknown function $g_0$, while the idiosyncratic components of agents' valuations are independent. Let private values be given by $U_i = g_i(A_i, W_0)$. Assume $W_0$ and that the transaction prices are observed if agents are asymmetric, and the identity of winner is also observed. If $A_1,.......,A_n$ are independent conditional on $W_0$, then each $F_0(.|w_0)$ is identified. Let $g_i(A_i, W_0) = A_i + g_0(W_0)\ \forall i,\ with\ g_0()$ an unknown function. If $A_1,......,A_n$ are mutually independent and independent of $W_0,\ then\ F_A()\ and\ g_0()$ are identified up to location if more than one bid is observed in each auction or the transaction price is observed in auctions with varying numbers of agents.

For $w_0$, $U_1,......,U_n$ are independent, so we can obtain for each $w_0$ in equilibrium:

$$H_B^{(n-1:n)}(b|w_0) = \Pr\left(A^{(n-1:n)} \leq b - g_0(w_0)\right) = F_A^{(n-1:n)}(b - g_0(w_0)).$$

Suppose $B^{(i:n)}\ and\ B^{(j:m)}$ are observed, let $F_U(.|w_0;i,n)\ and\ F_U(.|w_0;j,m)\ [F_A(.;i,n)\ and\ F_A(.;j,m)]$ denote the marginal distributions implied by the bid distributions $H_B^{(i,n)}(.|w_0)\ and\ H_B^{(j"m)}(.|w_0)$.

Let the distribution of values equal the distribution of bids, but un-testable without further information, and we derive positive and negative identification results for the unrestricted private values for cases in which some bids are unobserved. In the case of symmetric private values, $F_U(.)$ is not identified from the vector of bids an ascending auction, and suppose that some $k \in \{1,......,n\}$ *a subset of* $\{U^{(j:n)} : j \neq k\}$ is observed but $U^{(k:n)}$ is unobserved.

Define a set of partitions of agent indices:

$$\mathfrak{I}^k = \{(S_1, S_{k-1}, S_{n-k}) : S_1 \cup S_{k-1} \cup S_{n-k} = \{1,....n\}, |S_1| = 1, |S_{k-1}| = k-1, |S_{n-k}| = n-k\}.$$

With $k = n - 1$ this change in the underlying joint distribution preserves exchange-ability and does not change the joint distribution of the observable order statistics. Observing an extreme yields a relative identification problem since the distribution provides information about the underlying joint distribution, and:

$$\Pr(B^{(n:n)} \leq b | w) = F_A(b - g_1(w_1),.....,b - g_n(w_n)).$$

Inference from order statistics is more difficult, and has not been considered if the transaction price is observed. We can uncover the underlying joint distribution of values when agent-specific covariates with sufficient variation are available. In the case of asymmetric private values, assume $U_i = g_i(W_i) + A_i \ \forall i$, $F_A(.)$ has support equal to $\Re^n$ and a differentiable density, $(A_i, W_j)$ are independent for all *i, j*; $\sup(g_1(W_1),....,g_n(W_n)) = \Re^n$.

Then: $F_A(.)$ and each $g_i(.),\ i = 1,.....,n$ are identified up to location from observation of the transaction price and *W*, if more than one bid per auction is observed.

Then for $0 \leq m \leq n-1$:

$$\Pr(B^{n-m:n} \leq b|w) = \sum_{T \subseteq (1,...,n) s.t. |T| = m} \sum_{i \in T} \int_{-\infty}^{b} \bar{F}^T_{A,A_i}(\bar{b} - g_1(w_1),.......\bar{b} - g_n(w_n))d\bar{b},$$

and is identified from observation of $B^{(n-m:n)}$, and is equal to:

$$1 + \frac{1}{(-1)^m \binom{n-1}{m}} \int_{-\infty}^{w_n} .......\int_{-\infty}^{w_1} \frac{\partial^n}{\partial w_1....\partial w_n} \Pr(B^{n-m:n} \leq b|w) dw_1....dw_n.$$

For each *i*, then, variation in *b* and $w_i$ at this limit identifies $g_i(.)$, with this knowledge we can then determine $F_A(.)$ at any point $(a_1,....,a_n)$ through appropriate choices of *b* and *w*.

Since $\Pr\left(\max_{j\neq i} B_j \leq z \middle| B_i = b\right)$ is observable when all bids are observed, it can be used to identify $F_U(.)$ from the transaction price $B^{(n:n)}$.

When agents are asymmetric, assume that the identity of the winner is also observed if there is variation in the number of agents, and we observe the joint distribution of $\left(B^{(n:n)}, I^{(n:n)}\right)$. These marginal distributions, for each *i* and *b*, identify each $F_{U_i}(.)$ and valuations are independent, both $H_{B_i}\left(b|w_0\right)$ and $\beta_i^{-1}\left(b; w_0\right)$, since:

$$H_{B_i}\left(b|w_0\right) = \Pr\left(\beta_i\left(A_i + g_0\left(w_0\right); w_0\right) \leq b|w_0\right) = F_{A_i}\left(\beta_i^{-1}\left(b - g_0\left(w_0\right); w_0\right)\right).$$

Because valuations are conditional on $w_0$ and each agent uses the increasing bid function, this describes the relation between $H_B\left(b|w_0\right)$ and any $H_B^{(j:n)}\left(b|w_0\right)$, and if observed for at least values of *j* (or *a*), this relation is testable. Each $F_{U_i}\left(.|w_0\right)$ is identified when $\left(B^{(n-1:n)}, I^{(n:n)}\right)$ are observed, and this provides testable restrictions that fail under affiliated private or common values. Although many auction data sets either contain only the transaction price or else all bids, in procurement auctions, the agent may default or be disqualified, in which case the second-highest agent will often receive the contract. If agents are asymmetric, assume the identity of the winner $\left(I^{(n:n)}\right)$ is also observed, then the equilibrium bid functions $\beta_i(.),\ i = 1,\ldots\ldots,n$, are identified. Then the joint distribution of $\left(U^{(n:n)}, U^{(n-1:n)}\right)$ is sufficient for some policy simulations, including evaluation of reserve prices or simulation of outcomes under many alternative selling mechanisms. We consider whether this distribution is identified from incomplete sets of bids, where knowledge of the bid functions $\beta_i(.)$, which incorporate strategic responses to the distribution of opponents' bids, can provide no information about the distribution of lower bids.

Identification of a common value requires determination of the joint distribution $F_{X,U}(.)$ from parameters; joint distribution of signals and values is required for policy questions such as determination of a reserve price. It contains information about the extent of agents' residual uncertainty about their values after observing their signals, as well as the extent to which opponents have information about such valuations $U_i$. Because agent behavior depends on the information content of the signals, which is preserved by transformations, the scaling of *X* is arbitrary, and normalization of signals satisfies:

$$E\left[U_i \middle| X_i = \max_{j \neq i} X_j = x, n\right] = x,$$

and the equilibrium bidding strategy is $b(x_i) = x_i$.

If all bids are observed in a price auction, the bid distribution identifies $F_X(.)$ in a second-price sealed-bid auction unless all bids provide no further information about $F_{X,U}(.)$. It is identified from observed bids in a price sealed-bid auction, and the signals have an additive separable structure and provide a set of conditions. We assume that for each $a$ there exist known constants $(C,D) \in \Re \times \Re_+$, and random variables $(A_1, \ldots\ldots, A_n)$ with joint distribution $F_A(.)$ such that, with the normalization

$$E\left[V \middle| X_i = \max_{j \neq i} X_j = x, n\right] = x,\ X_i = C + D(V + A_i)\ \forall i.$$

Identification is problematic when some bids are unobserved; then if $U_i = V + A_i$, and $A$ are independent conditional on $V$, the range of applications in which an accurate measure is available may be limited.

The exact forms of modifications depend on the joint distribution of signals and values; there is a multiplicity of symmetric equilibria in weakly un-dominated strategies, implying that there is no unique interpretation of bids below the transaction price. The distinction between private and common values makes the problem even more difficult in ascending auctions and common values admit a continuum of equilibria. When strategic responses to changes in the level of competition are accounted for, bids can increase or decrease for a number of agents, although another problem arises in these and any other contracts in which not all bids are observed. In a second price sealed-bid, the private value model is testable against the common value alternative if we observe the transaction price $B^{(m-1:m)}$ from contracts with $m \geq 2$ agents and bids $B^{(m-1:n)}, \ldots\ldots, B^{(n-1:n)}$ from auctions with $n > m$ agents; it is also sufficient to observe $B^{(m:m)}$ at auctions with $m$ agents and bids $B^{(m,n)}, \ldots\ldots, B^{(n:n)}$ from the $n$-bidder auctions. Each agent $i$ bids

$$b_i = E\left[U_i \middle| X_i = \max_{j \neq i} X_j = x_i\right] = b(x_i : n),$$

and we must than have under common value:

$$b(x_1;n) = E\left[U_1 \middle| X_1 = X_2 = x_1,\ X_j \leq x_1,\ j = 3, \ldots, n\right]$$
$$< E\left[U_1 \middle| X_1 = X_2 = x_1,\ X_j \leq x_1,\ j = 3, \ldots\ldots, n-1\right] = b(x_1; n-1),$$

with the strict inequality following from the fact that $E\left[U_1 \middle| X_1,\ldots\ldots,X_n\right]$ increases in each $X_i$, due to strict affiliation of $(U_1,X_i)$.

Hence, $b(x_1;n)$ decreases in $a$, implying:

$$\frac{2}{n}\Pr\left(B^{(n-2:n)} \le b\right)+\frac{n-2}{n}\Pr\left(B^{n-1:n} \le b\right) > \Pr\left(B^{(n-2:n-1)} \le b\right),$$

so the first-order stochastic dominance relation then provides a test of the private values against the common values alternative. In a private values model, equilibrium bidding is exactly as in the second-price auction, and this directly implies a recurrence relation between means:

$$E\left[B^{(n-2:n-1)}\right] = \frac{2}{n}E\left[B^{(n-2:n)}\right]+\frac{n-2}{n}E\left[B^{(n-1:n)}\right].$$

In an auction with $n-1$ agents the transaction price is:

$$B^{(n-2:n-1)} = \widetilde{B}^{n-1}_{I(n-2:n-1)},$$

where $\widetilde{B}^{n}_{I(J:k)}$ denotes the random variable equal to the equilibrium bid made in an $n$-agent auction by the agent whose signal is $j$th lowest in a sample of $k$ agents. The final inequality that holds in such an equilibrium then gives:

$$E\left[\widetilde{B}_{I(n-2:n-1)}\right] > E\left[\widetilde{B}^{n}_{I(n-2:n-1)}\right],$$

and the expected bid made by the top losing agent in an $(n-1)$-agent auction is larger than the expected bid the same agent would make in an $n$-agent auction.

This implies:

$$E\left[\widetilde{B}^{n-1}_{I(n-2:n-1)}\right] > \frac{2}{n}E\left[\widetilde{B}^{n}_{I(n-2:n)}\right]+\frac{n-2}{n}E\left[\widetilde{B}^{n}_{I(n-1:n)}\right],$$

i.e.,

$$E\left[B^{(n-2:n-1)}\right] > \frac{n}{2}E\left[B^{(n-2:n)}\right]+\frac{n-2}{2}E\left[B^{(n-1:n)}\right],$$

and we observe the top two bids from auctions with $n$ and $n-1$ agents without restriction on which equilibrium describes actual behavior in ascending auctions.

Let $U_1,\ldots\ldots,U_n$ be the random variables corresponding to the valuations of the agents and let $Y_1,\ldots\ldots\ldots,Y_m$ be a sample of size $m<n$ drawn without replacement using a distribution where for $r<n$ we have:

$$\frac{n-r}{n}\overline{F}_U^{(r:n)}(y)+\frac{r}{n}\overline{F}_U^{(r+1:n)}(y) = \overline{F}_U^{(r:n-1)}(y),$$

where $\overline{F}_U^{(r:m)}(.)$ is the distribution of the $r$th order statistic, for $l \leq n$, this simplifies to:

$$\frac{n-r}{n} F_U^{(r:n)}(u) + \frac{r}{n} F_U^{(r+1:n)}(u) = \overline{F}_U^{(r:n-1)}(u)$$.

If $m = n - 1$ can be tested against the common values alternative, the top two bids in a first-price auction give us enough information to test.

In many auctions the seller announces a reserve price $r_0$, which is in the interior of the support of agents' valuations, with the probability some potential agents will be unwilling to bid, and no auction can reveal information about $F_U(u)$ for $u < r_0$, we identify the truncated distribution:

$$F_U(.|r_0) = \frac{F_U(.) - F_U(r_0)}{1 - F_U(r_0)}$$

in auctions with different numbers of participating agents.

# 6 Risk Choices, Heterogeneous Beliefs and Prediction Strategies in Allocation

It is necessary for optimizing agents to account for the asset return predictability that derives from investors' hedging demands at equilibrium, and significantly lessens the difficulty of achieving the optimal portfolio strategy. This suggests an economy where real exchange rates, real interest rates and real stock price changes follow stochastic processes whose drifts and diffusion parameters are driven by an arbitrary number of state variables, and investors thus face a stochastic opportunity set. An investor trades in stocks, bonds and bills issued in order to maximize the expected utility of his or her terminal wealth. It is challenging to find the set of state variables that are suitably priced when the state variables are identified, and the implementation of the investor strategy is difficult if not impossible. The investor's optimal strategy is shown to be associated with interest rate risk, and with the risk brought about by co-movements of interest rates and market prices of risk. For stocks the average premium for risk is a fraction of the average total premium, and we can substitute an indirect utility function that depends on consumption and a random price index, or inflation rate for the direct utility function that depends on real consumption. Since inflation rates differing across economies play the role of state variables, risk premiums are obtained; in the investor's optimal strategy, risk it is indirectly hedged against the random fluctuations of the market prices of risk for the various traded assets. We investigate why exactly returns constitute the link that return predictability to the hedging demands of investors, and different state variables that generate time-varying market returns cause risk premium on the assets to differ. The utility costs of behaving myopically, and ignoring predictability can be explored, and all the risks have consistently been shown to vary over time and consequently the length of the investor's horizon; portfolio strategies that take into account the predictability of asset returns significantly outperform others. Since ignoring the partial predictability of asset returns in designing portfolio strategies may lead to substantial losses, the investor's time horizon is known explicitly to play a crucial role in the optimal strategy design and in the fluctuations of the state variables. Accordingly, each and every risk is evolves in a stochastic manner and is driven by an arbitrary number of state variables.

## Prediction Strategies

For a large class of non-linear demand and supply scenarios, derived from utility and profit maximization, in a market that is unstable under expectations, the choice is to switch prediction strategies. Supply $S(p_p^e)$ is a function of the price expected by the producers $p_t^e$, derived from:

$$S(p_t^e) = \arg\underset{q_t t}{meax}\{p_t^e q_t - c(q_{tt})\} = (c^t)^{-1}(p_t^e).$$

Goeree and Hommes (2000) assume the cost function $c$ to be strictly convex, so that the marginal cost function can be inverted, and supply is then strictly increasing in expected price. Agent demand $D$ depends upon the current market price $p$, and market equilibrium price dynamics are given by:

$$D(p_t) = S(H(P_{t-1})),$$

and depend upon the demand curve $D$, the supply curve $S$ as well as the predictor $H$ of expected price, where $P$ denotes past prices. Market equilibrium with rational versus naive expectations is determined by

$$D(p_t) = f_{t-1}^R S(p_t) + f_{t-1}^N S(p_{t-1}),$$

where $f_{t-1}^R$ and $f_{t-1}^N$ denote the fractions of agents using the rational and naive predictors respectively, at the beginning of period $t$. For the rational expectations predictor, realized profit is given by

$$\pi_t^R = p_t S(p_t) - c(S(p_t)).$$

The profit for rational expectations is given by $\pi_t^R - C$, where $C$ is the information cost that has to be paid for obtaining forecast. For naive predictors the realized profit is given by

$$\pi_t^N = p_t S(p_{t-1}) - c(S(p_{t-1})).$$

The fraction of agents using the rational expectations predictor in period $t$ equals

$$f_t^R = \frac{\exp(\beta(\pi_t^R - C))}{\exp(\beta(\pi_t^R - C)) + \exp(\beta\pi_t^N)},$$

and the fraction of agents choosing the naïve one is

$$f_t^N = 1 - f_t^{R.}.$$

Parameter $\beta$ is called the intensity of choice; it measures how fast agents-producers switch between the two prediction strategies, the higher the intensity of choice the more rational, in the sense of evolutionary fitness, agents are in choosing their prediction strategies. The difference $m$ of the two fractions will be defined:

$$m_t = f_t^R - f_t^N,$$

so $m$ = -1 corresponds to all agents being naive, whereas $m$ = 1 means that all agents prefer the rational expectations predictor. The steady state is locally stable for all $\beta$, and when $C > 0$, there are exists a critical value $\beta_1$ such that the steady stable is stable for $0 \leq \beta < \beta_1$ and unstable for $\beta > \beta_1$.

## Preferences and Coalitions

Consider an economy with agent set $N$ and in which there are $Q$ types of goods. Konishi, Quint and Wako (2001) assume that each agent desires to consume one unit of each type. All of the goods in the market are distributed among agents, and an allocation $x$ is said to be blocked by coalition $S$ if there is another allocation $y$ with $y_1(S) = \ldots = y_Q(S) = S \text{ and } y(i) \succ_i x(i) \text{ for all } i \in S.$ We assume that each agent $i$ has preferences for which there exist real valued functions $u_i^1 : N \to \Re, \ldots u_i^Q : N \to \Re$ such that for any $(j_1, \ldots, j_q), (k_1, \ldots, k_Q) \in N^Q \text{ with } (j_1, \ldots, j_Q) \neq (k_1, \ldots, k_Q),$

$$(j_1, \ldots, j_Q) \succ_i (k_1, \ldots, k_Q) \Leftrightarrow \sum_{q=1}^{Q} u_i^q(j_q) > \sum_{q=1}^{Q} u_i^q(k_q).$$

Competitive allocation $x$ is weakly blocked by a coalition $S$ via an allocation $y$, and if agent $i \in S$ is not strictly better off by joining $S$, since each agent has a strict preference ordering.

## Preferences and Contracts

For contracts, agent's preferences are given a utility function:

$$V(\theta, x, t) = h(\theta, x) + t,$$

where $\theta$ is an individual characteristic of the agent belonging to $\Omega$ , the set of types; $x \in A$ is the agent's transfer, if an agent has his welfare level $h(\theta, x) + t.$ An incentive-compatible contract is a pair of mappings $(x, t) \text{ from } \Omega \text{ to } A \times R$ , and if for every type $\theta$ the following holds:

$$h(\theta, x(\theta)) + t(\theta) = \max_{\theta' \in \Omega} h(\theta, x(\theta')) + t(\theta').$$

For a pair of functions (x ,t), Carlier (2001) suppose that contracts are incentive-compatible if and only if:

for all $(\theta,\theta')\in\Omega^2,\ h(\theta,x(\theta))+t(\theta)\geq h(\theta,x(\theta'))+t(\theta')$.

Let (x , t) be some contract, the potential associated in the function is denoted:

$$U(x,t)(\theta)=h(\theta,x(\theta))+t(\theta),\ for\ all\ \theta\in\Omega.$$

If the agent implements an optimal incentive-compatible contract then he will make a non-negative profit with almost every type of agent.

## Markets and Choice

Gottardi and Mas-Collel (2000) consider an economy with $N$ commodities and $H$ consumers, and the choice problem of agent $h$, for $L\in\wp_+^{N,K}$, is the following:

$$\max U^h(x)$$

s. t. $x-\omega\ \in L-\Re_+^N$.

Let K < N and $\wp^{N,K}$ denote the manifold of $K$-planes in $\Re^N$, and let $x^h(L)$ denote the solution set of the above problem, describing the agent's demand at $L$. Similarly the agent's excess demand set is $z^h(L)=x^h(L)-\omega^h$. Specification of the budget constraints in $(P^h)$ allows us to capture various kinds of market structures. When $K=N-1$, we obtain the case of complete markets, and $L$ is the budget defined by the price; when $K<N-1$, the budget constraints correspond to the situation when the markets are incomplete, in which agents face a set of restrictions on the level of their net trades. The set $L$ reflects the level of prices and the specification of trading constraints. Adding individual agents' excess demands we obtain the expression of the aggregate excess demand

$$z(L)=\sum_h z^h(L)$$

which satisfies $z(L)\subset L-\Re_+^N$ for all $L\in\wp_+^{N,K}$.

A single decision-maker analyzes past experiences, and Matsui (2000) suppose the expected utility is given by

$$<(\Omega,F),A,R,f,u,\mu>,$$

where $\Omega$ is a state space, $F$ is a $\sigma$-algebra on $\Omega$, $A$ is the set of available actions, $R$ is a countable set of possible results or pieces of information, $u$ is a utility function, and $\mu$ is a probability measure on $(\Omega,F)$. The function

$f : \cup_{T=1}^{\infty} A^T \times \Omega \rightarrow \Re$ is an outcome function where $A^T$ is the cross product of $A$. The conditional measure is on $(\Omega, F)$ and satisfies

$$\mu_h(E) = \frac{\mu(E \cap \Omega_h)}{\mu(\Omega_h)}, \ \forall E \in F$$

In each period, the agent chooses an action to maximize the expected utility in that period. The agent is affected by the current action, and in making a decision he cares about the future payoff as well. The agent chooses a maximum of

$$U(a|p,h_T) = \sum_{t=1}^{T-1} s((p,a),(p^t,a^t);r^t)\mu(r^t)$$

given a problem $p$ and a history $h_T = ((p^1,a^1,r^1),\ldots,(p^{T-1},a^{T-1},r^{T-1})) \in H_T (T = 1,2,\ldots\ldots)$.

## Distribution of Assets

Consider a problem involving the distribution of a given amount of money among a number of agents, each characterized by a monetary entitlement. The vector of entitlements represents the agents' individual rights, needs, claims, benefits or loans, shares, inheritance wills and others. Herrero, Maschler and Villar (1999) suppose an allocation problem is a triple $[N,E,c]$, such that $N$ is a set of agents, $N$ describes a vector of entitlements and $E \in R$ is given budget. Let us call $\Omega$ the family of all allocation problems, and for any $\omega = [N,E,c] \in \Omega$, denote $C(\omega) = \sum_{i \in N} c_i$, and let $H(\omega)$ stand for the hyper-plane

$$H(\omega) = \left\{ z \in R^N \middle| \sum_{i \in N} z_i = E \right\}.$$

An allocation rule is a function $F : \Omega \rightarrow \cup_{i \in N} R^N$, such that for any $\omega[N,E,c] \in \Omega$, $F(\omega) \in H(\omega)$.

All the agents require that claims should be paid in full, and for any $\omega = [N,E,c]$ in $\Omega$, and each $i \in N$, $F([N,E,c]) = (0,\ldots,0,c_1,0,\ldots,0) + F([N,E - c_i, c_{-i},0])$,

so that every agent $i$ achieves the same outcome if she is given her claim and then proceeds to distribute the rest among the agents when agent $i$ has no more claims. Symmetry is a condition that says that if all agents have identical entitlements, then the rule should divide the budget equally among them. For any $\omega=[N,E,c]\in\Omega,\ if\ c_i=c_j\ \ for\,all\,i,j\in N,\ then F_i(\omega)=F_j(\omega),\ for\,all\,i,j\in N.$

If $\omega=[N,E,c]\in\Omega$, and $S\subset N,\ S\neq N$, we shall call $c_S=(c_i)_{i\in S}$.

Consistency has to do with the possibility of negotiation among the group of agents about the total amount assigned to them. If some agents leave, taking with them their shares, they cannot change their outcomes; this rule can be considered a stability feature of the solution. Compatibility is an obvious restriction on any allocation, and for any $\omega=[N,E,c]\in\Omega,\ C(\omega)=E\ implies\ F(\omega)=c.$ Let $\omega=[N.E.c]\in\Omega$ and let $E_1,E_2$ be such that $E_1+E_2=E,$ it follows that $F(\omega)=x_1+x_2,\ where\ x_1=F([N,E_1,c])\ and\ x_2=F([N,E_2,c-x_1]),$ and this is a property that prevents the manipulation of the outcome by conveniently framing the sequential process composition. Claims may also be seen as dealing with the case in which the agents are uncertain about the rights point $c$, and let $F([N,E,\lambda c+(1-\lambda)c'])=\lambda F(\omega)+(1-\lambda)F(\omega').$

This happens when the claims correspond to the value of a partnership in which agents' contributions consist of real assets and they agree to sign a contingent contract rather than wait until all uncertainties are resolved. Reflection requires that problems of image and name have identical solutions, say $C(\omega>E)$, and the agent views it in aggressive terms, when the firm is bankrupt, and it is clear that if others do the same that $c$ cannot be achieved since the firm is already bankrupt. The principle of reflection requires that it does not matter whether the agents' rights are the original claims or they are the undisputed amounts, the solution function will yield each of them the same outcome:

Let $\omega=[N,E,c]$, and let $\omega'=[N,E,r(\omega)],\ then\ F(\omega)=F(\omega').$

Suppose that new agents enter while the budget remains unaltered, and all agents initially present are equally affected by the incorporation of new claimants. For any problems $\omega=[N,E,c]$, and $\omega'=[N\cup M,E,(c,c')]\in\Omega,\ F_i(\omega)-F_i(\omega')=F_j(\omega)-F_j(\omega')$ for all $i,j\in N.$

## Decisions About Payoff

Consider a decision maker who takes on three actions $a$ , $b$ and $c$ and receives a payoff. For simplicity let us assume that there are two stages $H$ and $T$. An agent facing a random walk on the integers has to decide at each stage on an action, $a\in A.$. Lehrer and Smorodinsky (2000) assume that at each stage the random walk either goes one step right or left, and that the agent's stage payoff depends only on the random walk's location. The payoff function is $u : A \times N \to R,$ and the payoff function depends on the entire history of outcomes. Let $t$ denote time and let $\Omega$ be the finite set of nature's possible outcomes at time $t$. Let $\Omega^N$ be the set of all infinite strings of outcomes, and endow $\Omega^N$ with the natural $\sigma$ - algebra, $F$, generated by all finite cylinders. Given a sequence of stage payoffs, $\{u_t\}_{t=1}^{\infty}$, let $U_T = 1/T \sum_{t=1}^{T} u_t$ be the payoff of the given horizon decision problem, of length $T$. For an infinite horizon decision problem let $U_t = (1-r)\sum_{t=1}^{\infty} r^t u_t$ be the discounted sum of payoffs. Since the agent's belief does not coincide with the truth, an agent's strategy might be initially sub-optimal with respect to the true distribution $\mu$.

## Rational Beliefs

The major implication of aggregate risk sharing is that changes in household consumption are independent of changes in idiosyncratic income and aggregate consumption. The agent is interested in maximizing the probability of meeting her target. At time $t$ the agent faces a fixed set $L$ of random variables, where each random variable is stochastically independent of the random variable in previous periods. At each time $t$, the agent has a target $V^t$, which represents her current (and possibly uncertain) aspiration level. DellaVigna and LiCalzi (2001) denote by $X^t$ the random value chosen in $t$ and by $x^t$ the corresponding outcome of $X^t$, which becomes known to the agent right after she selects $X^t$. We assume that the agent's aspiration level is closely related to the average income that the agent has obtained in the past. Let $X^0$ be the agent's random initial aspiration level at $t = 0$, and then the outcome $x^\tau$ received for random variables $X^\tau$ is known for any $\tau = 0,1,\ldots,t-1$. The agent sets her current reference point to

$$v^t = \bar{x}^t = \left(\frac{1}{t}\right)\sum_{\tau=0}^{t-1} x^\tau .$$

The target $V^t$ is used to choose a distributed random variable with expected value $v^t = \bar{x}^t$, the reference point decreases and the agent lowers the expected value of her target. Chance is what makes this simple rule for setting the reference point interesting, and for next period is the random variable is given by:

$$\overline{X}^{+1} = \left(\frac{t}{t+1}\right)\overline{x}' + \left(\frac{1}{t+1}\right)X'.$$

The agent's reference point converges to a level that induces her to pick a random one that maximizes expected value, so that she learns to make risk neutral choices. At the reference point $v = \mu$, the optimal choice has expected value $\mu_e$, and if $v > \mu_e$ then any optimal choice has an expected value strictly lower than $v$. When the reference point $v$ is lower than $\mu_e$, the agent is ambitious in the sense that she goes after random variables with a an expected value higher than $v$. Following Smorodinsky (2000), we suppose $D$ to be the set of all bounded random variables on $R$, for any $X \in D,\ let\ F_X(.)$ denote its cumulative distribution function. For $c, d \in R,\ c > 0,$ let $Y = cX + d$ be the element of $D$ satisfying $F_Y(t) = F_X((t - d/c))$.

For any $X, Y \in R\ \ and\ \ \alpha \in [0,1]\ \ let\ \ Z = \alpha X \oplus (1-\alpha)Y$ be the random variable satisfying $F_Z(t) = \alpha F_X(t) + (1-\alpha)F_Y(t)$.

Let $\succeq$ be a complete transitive relation on $D$ and let $\sim$ be the indifference relation it induces.

Let $f : D \to R$, defined implicitly by the equation $X \sim \delta_{f(X)},\ \ where\ \ \delta_a$ denotes the random variable $a$, be the certainty equivalent functional induced by $\succeq$. The map defines a certainty equivalent functional that satisfies certain properties. The oddness axiom is motivated by the reflection effect: $f$ is an odd function, for all $X \in D,\ f(-X) = -f(X)$.

Many other properties of solutions can be described:- (a)Constant risk aversion: for all $X \in D\ \ c > 0\ \ and\ \ d \in R,\ f(cX + d) = cf(X) + d.$ (b) Betweenness: for all $X, Y \in D$, such that $f(X) < f(Y)$, and for any $\alpha \in (0,1),\ f(X) < F(\alpha X \oplus (1-\alpha)Y) < f(Y).$ (c) Continuity: $f$ is continuous the weak topology on $D$, and $f : D \to R$ is a fair solution if and only if there exists $c \in R$, such that $f = \phi_c$.

For $c = 1$, $\phi_c$ is the expectation operator, and we shall be using the following properties of solutions: (a) indifference: for any $X, Y \in D,\ f(X) = f(Y)$ implies $f(X) = f(\alpha X \oplus (1-\alpha)Y) = f(Y)$, for any $\alpha \in (0,1)$, (b) consistency: if $X \in D$ satisfies

$prob(X = a) = 1$ for some $a \in R$, then $f(X) = a.$ We show that the underlying function of $\phi_c$ is $f_{\phi_c}(p) = p^{1/c} / (p^{1/c} + (1-p)^{1/c})$. For convenience, we use the notation

$$f^c(p) := \left( \frac{p^{1/c}}{p^{1/c} + (1-p)^{1/c}} \right).$$

For any $c > 0$ and any $X \in D$, $\phi_c(X)$ is uniquely determined, and the underlying function of $\phi_c$ is $f_{\phi_c} = f^c$. Then the following holds for its underlying function:

$f_\phi(0) = 0,\ f_\phi(1/2) = (1/2),\ f_\phi(1) = 1,\ f_\phi[0,1] \subset [0,1],\ f_\phi$ is monotonically increasing, $f_\phi(p) + f_\phi(1-p) = 1,\ \forall p \in [0,1],\ f_\phi(.)$ is continuous.

If $\alpha$ and $\phi$ are two fair solutions satisfying $f_\alpha \equiv f_\phi$ then $\alpha \equiv \phi.$ The assumption about the differentiability of certainty equivalents plays an important role in expected utility and non-expected utility: if the two are close, the probability that agent 1's house burns is not independent of the individual state of agent 2. We impose the condition that this additional collective component of the risk, which appears at the individual level through endowments, vanishes at the aggregate level, and asymmetric equilibria are Pareto optimal. Moreover, the subject of symmetric Pareto optima is a smooth sub-manifold of strictly smaller dimension, and Pareto optima are budget-feasible, in which case there are no asymmetric equilibria.

## Coordinates of Endowment

Let $\omega_1 \in R^1_{++}$, $\omega_2 \in R^2_{++}$ be the endowments of agents in the polar aggregate states 1, 2; therefore in the non-polar aggregate state 3, the agent in individual state $\alpha$ has a vector of endowments:

$\omega_{3,1} = (\overline{\omega}_1, \omega^1_{3,1}) \in R^{l-1}_{++} \times R,$ where $\overline{\omega}_1$ denotes the $l-1$ coordinates of $\omega_1$.

The agent in individual state $\beta$ has a vector of endowments $\omega_{3,2} = (\overline{\omega}_2, \omega^l_{3,2})$, and the endowments in good $l$ are assumed to be aggregate state invariant: $\omega^l_{3,1} + \omega^l_{3,2} = \omega^l_1 + \omega^l_2.$ The endowment of agents $x$ and $y$ are thus:

$\omega_x = ({}_1\omega_1, \omega_2, \omega_{3,1}, \omega_{3,2})$, and $\omega_y = (\omega_1, \omega_2, \omega_{3,2}, \omega_{3,1})$, Cre`s and Rossi (2000) denote by $\omega = (\omega_1, \omega_2, \omega_{3,2}^l, \omega_{3,1}^l)$ the distribution of endowments characterizing an economy. There are $m$ agents and $l$ physical goods. A state of nature $v$ is a complete specification of the individual state faced by each agent. To each state of nature $v$ is associated an aggregate state $p$(v) describing the distribution of agents among the individual states. There are $S$ polar aggregate states in $R$, i.e. those in which all the agents are in the same individual state.

The probability of a state of nature does not depend on the identity of the agents facing the different individual risks. The probability of a state of nature $v$ is thus given by:

$$\pi(v) = \frac{\pi(p(v))}{n(p(v))}.$$

The utility function of agent $i$ will then be defined from $R_{++}^{IN}$ into $R$ by the formula: $\forall X_i \in R_{++}^{IN}, U_i(X_i) = \sum_{v=1}^{N} u_{v(i)}(x_i(v))$. The endowment of an agent who is in individual state $s$ when aggregate state $p$ occurs is $\omega_{p,s} = (\overline{\omega}_s, \omega_{p,s}^l) \in R_{++}^{l-1} \times R$. An allocation is said to be symmetric if, given that aggregate state $p$ occurs, for any state of nature belonging to $p$, the allocation of each agent only depends on his individual state. When utility functions are not state-dependent $(u_s \equiv u, \forall_s)$ and prices are symmetric, wealth is the same. Agents are described by symmetric characteristics but can still come out of exchange process with asymmetric allocations, and the generic existence of asymmetric equilibria is thereby shown.

## Monetary Exchange

We present a model with spatially differentiated agents who exchange only with their neighbors, and use the local information directly generated by these exchanges to form their decisions. The use of fiat money assumes an absence of double coincidences of wants by making assumption about which goods the individual agents want to buy and sell respectively. Manolova, Tong and Deissenberg (2003) consider an economy with $n$ agents $i$, $i = 1,\ldots,n$, and $a$ goods $j$, $j = 1,\ldots,n$, where $n > 2$, and with time $t = 0,1,2,\ldots$. At any given time $t$, the goods must be either consumed by the agent that holds them or handled over to another agent, and each agent $i$ produces in each period $t = 0,1,2,\ldots$, without costs, quantity $x_i$ of good $i$. The instantaneous utility function $U_i^t(.)$ is assumed to be time-independent and the same for all agents, i.e. $U_i^t(.) = U(.)\ \forall i$ and $t$. This condition guarantees that in any period an agent will not concentrate his or her consumption, but will spread it over time and over all available goods, and the sales of a good by an agent will be less than his or her current endowment. The agents are located on a circle, agent $i$ on the right of agent $i-1$ and on the left of agent $i+1$, and agents interact one with the other sequentially. The number of interactions between agents is unlimited – the circle is followed clockwise an infinite number of times. And there are an unlimited number of exchange cycles of length $n$. Each agent $i$ interacts with two other agents, first with left neighbor $i-1$, then with right neighbor $i+1$, and agents act as price-takers. The price-taking can be justified by assuming that the agents are not single individuals, the interaction structure is as follows, when agent $i$ interacts with agent $i-1$:

Agent $i$ initially has a quantity $m_i^{t-1}$ of money, and purchases from agent $i-1$ a vector of goods $Y_i^t$ at price $P_i^t$, with the intention to resell part $Y_{i+1}^t$ of the vector $Y_i^t$ to agent $i+1$, at the expected price $\tilde{P}_i^{t+1}$. Agent $i$ realizes an expected monetary income $\tilde{m}_i^t$, and consumes the goods $\tilde{C}_i^t$, and expects the monetary balance $\tilde{m}_i^t$ to buy goods from agent $i-1$at the expected price $\tilde{P}_{i1}^{t+1}$. The demand of agent $i$ for the goods of agent $i-1$, is the solution of the problem:

$$V_i^t = U_i^t\left(\tilde{C}_i^t\right) + \tilde{U}_i^{t+1}\left(\tilde{P}_{i1}^{t+1}, \tilde{m}_i^t\right) \rightarrow \max_{Y_i^t}.$$

Doing so, agent $i$ initially has a basket of goods $Y_i^t + X_i$, sells to agent $i+1$ a vector of goods $Y_{i+1}^t$ at price $P_{i+1}^t$, and consumes immediately the goods remaining in his or her possession, $C_i^t = Y_i^t + X_i - Y_{i+1}^t$.

The supply function of agent $i$ when selling to agent $i+1$, $Y_{i+1}^{tS} = Y_{i+1}^{tS}\left(Y_i^t + X_i, P_{i+1}^t, .\right)$ is the solution of the problem:

$$V_i^t = U_i^t\left(Y_i^t + X_i - Y_{i+1}^t\right) + \tilde{U}_i^{t+1}\left(\tilde{P}_{i2}^{t+1}, P_{i+1}^t Y_{i+1}^t\right) \rightarrow \max_{Y_{i+1}^t}$$

s. t. $Y_i^t + X_i - Y_{i+1}^t \geq 0, \quad Y_{i+1}^t \geq 0$.

One recognizes that agent $i+1$spends his income on goods from agent $i$, that $P_{i+1}^t Y_{i+1}^t = m_{i+1}^t$, and from the equilibrium condition $Y_i^{tS}\left(Y_{i-1}^t + X_{i-1}, P_i^t, .\right) = Y_i^{tD}\left(m_i^{t-1}, P_i^t, .\right)$, we see that the structure of the interaction between agents $i-1$ and $i$ $i$ and $i+1$ respectively does not depend upon $t$. Money circulates counterclockwise, goods clockwise, there is full conservation of money, and equal loss of money is balanced by the buying agent. At any point of time, some agents always hold a larger monetary balance than others, and goods are fully consumed within the period when they are produced. We assume that the price anticipations are forming according to:

$$\tilde{P}_i^{t+1} = \tilde{P}_{i1}^{t+1} = \tilde{P}_{i2}^{t+1} = P_i^t, \quad \forall i,t$$
,

and this implies that the agents believe that they will be able to sell to the right neighbor at a price equal to the price at which they buy from the left neighbor. We start with a situation where all agents initially have the same monetary balance $m$ but no goods, that they have no money but an arbitrary weakly positive basket of goods, and that all agents produce the same quantity of their respective goods. The economy has a unique, stable point equilibrium such that for $k = 0,1,\ldots\ldots.,n-1$:

$$p_i(j) = 2^{-k/2} p_i(i),\ y_i(j) = y_i(i)2^k$$
,

$$c_i(j) = \begin{cases} y_i(i) + x_i - y_{i+1}(i) \\ y_i(j) - y_{i+1}(j),\ j \neq i \end{cases}$$
,

with $k = j - i$ mod $n$. At the point equilibrium, all agents realize the same instantaneous utility $\overline{U}$ , and this shows that at the equilibrium, when buying from agent $i-1$, agent $i$ pays the lowest price for the good produced by $i-1$.

The highest price is paid for ''s own good; agents buy more of the goods that are produced at home than goods that are produced further away. Since all production is consumed within each exchange cycle, and since every agent spends of his monetary balance on goods, we have:

$$\sum_{i=1}^{n} y_i(j) = x,\ \sum_{j=0}^{n} p_i(j) y_i(j) = m$$
.

The utility of all agents is higher at the efficient symmetric competitive equilibrium, where:

$$p_i(j) = \frac{m}{x},\ C_i = \left(\frac{x_1}{n}, \frac{x_2}{n}, \ldots\ldots, \frac{x_n}{n}\right) \quad \forall i,j$$
,

hence agents consume too much of neighboring goods, and too little of distant ones. Assume that agents initially have the same monetary balance $m$ and the economy is at its point equilibrium, as a counterpart, its monetary balance increases to $m + \Delta m$ instead of $m$ at the point equilibrium. Over time, the economy converges towards a new equilibrium, driven by the circulation of $\Delta m$ around the circle; this equilibrium is now periodic, with period $n$ - *1*. At the equilibrium, all agents realize their utilities and sell and consume the quantities over $n$ - *1* exchange cycles, lower than the ones realized at the initial point equilibrium, reflecting the fact that there are now prediction errors due to the fluctuating demand.

The agent receives, in every period immediately before she interact with her neighbor, an additional real amount of money $\Delta m^{t}$ corresponding to the real amount of money in the economy given its growth at a positive rate. Utility starts decreasing again if the growth rate of money is too high, suggesting that there is an optimal level of monetary creation, and that at the point of equilibrium there is locally excessive trade in some goods, and locally insufficient trade in others. A permanent injection of money tends to correct these excessive and insufficient levels of trade.

## Asset Allocation

There are $N$ sources of risk across the $M$ economies, represented by $N$ independent Brownian motions $\{Z_i(t); t \in [0, \tau_E]; i = 1,\ldots\ldots,N\}$ defined on a probability space $(\Omega, F, P)$ where $\Omega$ is the state space, $F$ is the $\sigma$-field representing measurable events and $P$ is the probability measure. In each economy, the consumption ogf goods is such that every variable is expressed in real terms from the investor's viewpoint, and these sources of risk could summarize the various shocks affecting the real exchange rates, and all asset prices must be converted using the real exchange rates $e_j(t)$. Lioui and Poncet (2003) assume that each such structure is driven by an arbitrary number of factors, and all the portfolio strategies followed by investors are admissible. These strategies consist in determining at each time $t$ the number of units of all available assets. The investor's horizon is denoted $\tau$, with $\tau < \min(\tau_{j\ell})_{j\ell \in [1, M \times L]}$, which ensures that all assets are long-lived from her viewpoint, and as the investment opportunity set fluctuates randomly due to the presence of state variables, her utility function is assumed to exhibit relative risk aversion to ensure explicit solutions.

The relative risk aversion utility function is the iso-elastic utility such that:

$$u(V(\tau,\omega)) = \frac{1}{\alpha} V(\tau,\omega)^{\alpha}, \quad \omega \in \Omega, \quad 0 < \alpha < 1,$$

where $(1-\alpha)$ is the constant of relative risk aversion. The utility characterizes a Bernoulli investor and uniquely possesses the myopic property:

$$u(V(\tau,\omega)) = \ln(cV(\tau,\omega)), \quad \omega \in \Omega.$$

The investor's portfolio problem can then be written as:

$$\max E^{P}\left[\frac{V(\tau)^{\alpha}}{\alpha}\right] \quad s.t. \quad E^{P}\left[\frac{V(\tau)}{h(\tau)}\right] = V(0),$$

where $0 < \alpha < 1$ and $h(\tau)$ is the value at date $\tau$ of the optimal growth portfolio, which makes the $h$-denominated value process of any portfolio a martin-gale under probability measure $P$. Formally, $h(t)$ is defined as

$$h(t) = B_1(t)\frac{dP}{dQ}\Big|_{F_t},$$

and the portfolio is the Bernoulli investor's optimal portfolio, and the preference parameter is $\alpha$, which is investor specific, and depends on the investor's horizon $\tau$.

The definition of the market prices of risk $\phi(t)$ is associated with the risky assets, and this speculative part shows up in the numerator instead of the drifts of the price processes, and we can write the investor's value function as a function of the state variables and derive the optimal demands. This would have produced $K$ hedges against the instantaneous risks associated with the $K$ state variables. The interest rate risk relates to the random evolution of money market account value $B_1(t)$ accruing at the stochastic rate $r_1(t)$, and the risk associated with the random fluctuations. An information-based component hedges against unfavorable shifts in the investment opportunity set, and the rational investor wants to protect herself against situations in which wealth is reduced because of such shocks.

The asset that the investor implicitly uses is not the money market account, which is common to all investors; thus the role of the investor's horizon has emerged in a natural way in the optimal portfolio strategies. Thus $\hat{\sigma}_J(\alpha;t,\tau)$, the diffusion vector of the stochastic process $d\hat{J}(.)/\hat{J}(.)$, is a measure of the risk associated with the random volatility. Accordingly, it is also investor specific as it depends on both the investor's risk aversion coefficient and his horizon, and $\hat{\theta}(t,\tau)$ plays the role of a state variable that encompasses the random fluctuations. In a complete market, all the risks brought about by the economic factors must be embedded in the stochastic discount factor, so that the market price of risk sums up all the relevant information available on the market. This is because the ultimate concern of all investors is real consumption, and the latter variable encompasses all the sources of risk that affect the economy. Shifts in the investment opportunity set are due solely to random changes in the parameters of the deviation risk, or currency risk, and interest rates are stochastic but the drifts and diffusion parameters are due to the relevant stochastic processes. The risk associated with the fluctuations is given by:

$$\hat{J}(\alpha,t,\tau) = E_t^P\left[\hat{\theta}(t,\tau)^{\frac{\alpha}{\alpha-1}}\right] = l(\alpha,\tau,r_1(t),\hat{r}(t) - r_1(t)1_{M-1}),$$

and investors will hedge against real interest rate differentials.

## Occupational Choice

Suppose age-discrimination is not observed by the employer, so old workers cannot be paid less than young, this being ruled out by law or social norm. If $S > 0$, then young workers work harder than older ones at the same wage, and are strictly more attractive to employers. The greater incentives come from the entrepreneurial rents, not from a higher wages. Forcing the wages to be equal therefore yields ambiguous welfare results. Each firm would prefer to hire young workers. The expected fraction of old workers in the firm is:

$$\hat{\alpha} = \frac{n-1}{2n},$$

and the entrepreneur's expected profit can be expressed as:

$$S = \delta A - k - \frac{1}{2}\delta c\left(e^0\right)^2 = \delta\left(ne^0 + \frac{n+1}{2}\frac{S}{c}\right)\left(1 - ce^0\right) - k - \delta\frac{1}{2}c\left(e^0\right)^0.$$

This equation determines $e^0 = e^0(S)$ as a function of *S*. We note that the effort levels and wages are different, as follows:

$$S\beta\left(e^0\right) = \delta\left(ne^0\left(1 - ce^0\right) - \frac{1}{2}c\left(e^0\right)^2\right) - k,$$

where

$$\beta\left(e^0\right) = 1 - \delta(n+1)\left(1 - ce^0\right)/2c < 1.$$

Given that $\beta'\left(e^0\right) > 0$, and since the lowest value $e^0$ can take is $1/2c$, *if* $\beta\left(e^0\right) < 0$

for some feasible value of $e^0$, it must be that $\beta(1/2c) > 1$, *or* $4c/(n+1) < \delta$.

But that implies, for any $S > 0$, the effort level $e^0$ of old workers must now be greater than if $h^y > h^0$ were allowed, and hence $e^y$ will be higher as well, although the equilibrium value of *S* will now be naturally different.

# 7 Design, Knowledge and Stabilization of Economy

An agent or an institution (government, firm, social planner, etc.) aims to contract with the population of agents, and give them incentives to perform actions by means of monetary transfers. When markets are incomplete, the budget constraints each agent faces can always be reduced to the condition that his net trades are in commodities below $K$-dimensional planes. Agents can choose between different prediction strategies; those with higher fitness in the recent past are selected more than those with lower fitness, and a rational route to randomness, that is a bifurcation route to chaos, is observed. Stated differently, when agents become more sensitive to differences in evolutionary fitness, equilibrium prices fluctuate, and the only non-linearity comes from the heterogeneity in expectations of beliefs. Decisions under uncertainty are made by analogies to previously encountered problems. An ambitious agent, whose aspiration level is adjusted over time, tends to optimize if the same problem is repeated sufficiently many times. The problem is one of distributing a given amount of a divisible good among a set of agents who may have individual entitlements. We refer to the distribution of an estate among a group of agents, when these have claims but also are held collectively responsible for the discrepancies between rights and worth. We assume that instead of many identical agents, the economy is populated by a number of agents who differ in their endowments and non-acquired skills. Market mechanisms are analyzed under the strong assumption that all actions are made simultaneously at known equilibrium prices. Norms of cooperation are increasingly recognized to be crucial for typical market transactions. Rational agents uphold cooperative social norms in order to preserve reputations for trustworthiness, which are valuable in their market and non-market interactions. If all agents were in fact opportunistic maximizers, this fact would presumably become common knowledge to the agents and there would no room for uncertainty on the part of one agent about his counterpart's type. We assume that a group of agents having a common belief about some proposition is a strong condition, because it requires that each agent believes the proposition, and it is therefore important to analyze situations that can bring common beliefs into existence. Heterogeneity usually arises because an agent's income is affected not only by aggregate but also by idiosyncratic income shocks. Only idiosyncratic fluctuations in preferences or measurement error should account for the dispersion of consumption changes. Households face numerous forms of risk – e.g. region specific, industry specific, idiosyncratic and aggregate. The utility function of an agent is based on a reference point, and most of agents tend to the reflection effect: they are risk averse in gains and risk seeking in losses with respect to their reference point. However, at each period the agent maintains a dual risk attitude which never withers away; therefore, learning to make risk neutral choices takes place without the agent learning to be risk neutral, and it would be interesting to study the risk attitudes of experienced versus inexperienced agents in a symmetric world. The reflection effect in the context of a preference, such as being constantly risk averse, states that if we adding the same constant to two distributions, or multiplying them by the same positive constant, this will not change the preference relation between agents. With reference to asymmetric equilibria the coordination failures they might induce, and the question of when they are optimal, suggest that an asymmetric equilibrium might not be as desirable as a symmetric equilibrium. But if one considers a perturbation of a symmetric model where agents, if not identical, are quite similar, those asymmetric equilibria probably do not vanish, even though they become more difficult to identify. This clearly adds a collective component to the risks faced by each agent.

## Competitive Equilibrium

However, as the agent accumulates information when more observations become available, he may learn to optimize. Analyzing a competitive equilibrium in an economy populated by a set of utility-maximizing heterogeneous agents and profit-maximizing agent, Maliar and Maliar (2001) construct a planner's problem of generating the optimal allocation which is identical to the competitive equilibrium in a decentralized economy. The consumer side of economy consists of a set of agents $S$, and the measure of agent $s$ is denoted by $d\omega^s$, where $\int_S d\omega^s = 1$. The agents are heterogeneous in skills and initial endowments, and the skills are intrinsic, permanent characteristics of the agents. We denote the skills of an agent $s \in S$ *by* $e^s$. An infinitely-lived agent $s \in S$ seeks to maximize the expected sum of momentary utilities $u(c_{tt}^{ss}, l\ )$, discounted at the rate $\delta \in (0,1)$, by choosing a path for consumption $c$, and leisure. The utility function $u$ is continuously differentiable. In period $t$ the agent owns capital stock $k_t^s$ and rents it to the firm at rental price $r_t$, he supplies to the firm $n_t^s$ units of labor in exchange for income $n_t^s e^s w_t$, where $w_t$ is the wage paid for one unit of efficient labor. The total time endowment of the agent is $n_t^s + l_t^s = 1$, and capital depreciates at the rate $d \in (0,1)$. The agent faces uncertainty about the future returns of capital, and the agent can insure himself against uncertainty by trading state contingent claims

$\{m_t^s(\theta)\}_{\theta\in\Theta}$, where $\Theta$ denotes the set of all possible realizations of productivity shocks. The problem solved by agent $s$ is:

$$\max_{\{c_t^s, n_t^s, k_{kt+1}^s, m_t^s, (\theta)\}_{\theta\in\Theta, t\in T}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t^s, 1-n_t^s)$$

s. t.

$$c_t^s + k_{t+1}^s + \int_{\Theta} p_t(\theta) m_{t+1}^s(\theta) d\theta = (1-d+r_t)k_t^s + n_t^s e^s w_t + m_t^s(\theta_t).$$

Therefore, the problem of the firm is the following:

$$\max_{k_t, h_t} \pi_t = \theta_t f(k_t, h_t) - r_t k_t - w_t h_t.$$

The agents' preferences subject to the economy's resource constraint are given by:

$$\max_{\{c_t^s, n_t^s, k_{t+1}, \}_{t\in T}^{s\in S}} E_0 \sum_{t=0}^{\infty} \delta^t \int_S \lambda^s u(c_t^s, 1-n_t^s) d\omega^s$$

s. t.

$$c_t + k_{t+1} = (1-d)k_t + \theta_t f(k_t, h_t),$$

where $c$ and $h$ are aggregate consumption and aggregate efficiency hours and $\{\lambda^s\}^{s\in S}$ is a set of welfare weights. Specifically, it restricts the value of commodities consumed by the agent over his lifetime to be equal to his endowment of wealth and value and the value of his lifetime labor income. It turns out that with heterogeneity in both wealth and skills, the aggregation demand for physical hours worked, $a$, is not linear in wealth any longer, however, demand for efficiency hours worked $n_t^s, e_s^s$ is. Assume the agent's momentary utility is

$$u(c_t^s, n_t^s) = \frac{\left((c_t^s)^{\mu}(1-n_t^s)^{1-\mu}\right)^{1-\eta} - 1}{1-\eta}, \quad 1 > \mu > 0, \; \eta > 0.$$

Then, the equilibrium can be described as an agent utility maximization problem:

$$\max_{\{c_t, h_t, k_{t+1}, \}_{t\in T}} E_0 \sum_{t=0}^{\infty} \delta^t \frac{\left(c_t^{\mu}(1-h_t)^{1-\mu}\right)^{1-\eta} - 1}{1-\eta},$$

giving a set of equations which relates individual and aggregate allocations

$$c_t^s = c_t f^s, \; n_t^s = 1 - (1-h_t)\frac{f^s}{e^s},$$

and a set of equations which determines the agent-specific parameters $\{f^s\}^{s\in S}$

$$f^s = \frac{k_0^s + e^s E_0 \sum_{t=0}^{\infty} \delta^t \frac{u_1(c_t,h_t)}{u_1(c_0,h_0)} w_t}{E_0 \sum_{t=0}^{\infty} \delta^t \frac{u_1(c_t,h_t)}{u_1(c_0,h_0)} (c_t + w_t(1-h_t))}$$

where $f$ is the share of consumption of agent $s \in S$ in aggregate consumption. Assume that the agents have momentary utility of the type:

$$u(c_t^s, n_t^s) = \frac{(c_t^s)^{1-\gamma} - 1}{1-\gamma} + B\frac{(1-n_t^s)^{1-\sigma} - 1}{1-\sigma}, \ \gamma, \sigma, B > 0.$$

Then, the equilibrium can be characterized by a utility maximization of a single-agent:

$$\max_{\{c_t, h_t, k_{t+1}\}_{t \in T}} E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + XB\frac{(1-h_t)^{1-\sigma} - 1}{1-\sigma} \right\},$$

a condition which identifies the parameter $X$

$$X = \left( \int_S (e^s)^{1-1/\sigma} (f^s)^{\gamma/\sigma} d\omega^s \right)^{\sigma},$$

and a set of individual and aggregate allocations

$$c_t^s = c_t f^s, \quad n_t^s = 1 - (1-h_t) X^{-1/\sigma} (e^s)^{-1/\sigma} (f^s)^{\gamma/\sigma},$$

and conditions which determine the agent's shares of consumption $\{f^s\}^{s \in S}$ :

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{u_1(c_t,h_t)}{u_1(c_0,h_0)} \left[ c_t f^s - w_t e^s \left(1 - (1-h_t) X^{-1/\sigma} (e^s)^{-1/\sigma} (f^s)^{\gamma/\sigma}\right) \right] = k_0^s,$$

where $f^s = (\lambda^s)^{1/\gamma} \Big/ \int_S (\lambda^s)^{1/\gamma} d\omega^s.$

The utility of the agent depends on the joint distributions of wealth and skills through the parameter $X$ independently of the number of heterogeneous agents. The effect of the parameter $\sigma$ on the group's consumption is determined by the value of $\gamma$. The variability of hours worked decreases with $\sigma$ under any value of $\gamma$, and this implies that if $\gamma > 1$, an increment in $\sigma$ induces the low skilled agents to work less and high skilled agents to work more, while if $\gamma < 1$, the opposite is true.

## Knowledge and Decision Strategy

Guttman (2001) assumes respect for choice of optimal investment in knowledge $e$ as the agent's decision problem, which can be written as:

$$\max_{e} \sum_{T=1}^{T} [\pi_T - (\delta_T \bar{x} \ /T)] - e + \frac{\bar{x}[p(e) + [1 - p(e)]\theta]}{1 + r},$$

where the maturity period is divided into $T$ stages, with probability $p$ that current knowledge expenditures $e$ presently being raised will be of trustworthy types. Differentiating with respect to $e$ and equating to zero to the obtain condition for optimal knowledge expenditure $e$, we obtain:

$$\frac{p'(e)(1-\theta)\bar{x}}{1+r} - 1 = 0$$

, and

$$\frac{(1-\theta)\bar{x}}{1+r} p''(e) d\hat{e} - \frac{p'(e)\bar{x}}{1+r} d\theta = 0,$$

which implies

$$\frac{d\ddot{e}}{d\theta} = \frac{p'(e)}{(1-\theta)p''(e)} < o.$$

The individual agent takes $\theta$ as given when deciding her optimal $e$, which depends on their knowledge expenditures. Since all variables are determined in a Nash equilibrium, each household takes the other households' knowledge expenditures as given in determining its own $e$. Given a minimum proportion of trustworthy types $p_{\min}$, if $p < p_{\min}$ all agents will defect in their market and nonmarket transactions throughout their careers. In equilibrium, an opportunistic agent will maintain his transactions if he upholds the social norms of upholding his commitments. There are three possible equilibrium strategy options consistent with a perfect Bayesian equilibrium. If agent $i$ expects his opponent is of an opportunistic type and will defect at some stage $t$, then he will defect simultaneously at that stage; if $i$ expects preempt his opponent to defect at some stage $t$, he will then defect one stage earlier; if $i$ expects his opponent to wait, $i$ cooperates nevertheless in that stage. In order to preserve $i$'s reputation for being trustworthy, $i$ then continues to cooperate with different trading partners until stage $T$, at which point $i$ defects.

The expected payoff of a simultaneous switch (SS) strategy is

$$E\pi_{ss} = (t-1)\alpha + (1+b)p, \ where \ \alpha = 1 - \frac{\bar{x}}{T}, \ and \ \beta = -b - \frac{\bar{x}}{T},$$

where the agent expects $t - 1$ stages of cooperation, in which he receives a payoff of $\alpha$ per stage.

The expected payoff of the preempt strategy, where one preempts at stage $t - 1$, is

$$E\pi_{preempt} = (t-2)\alpha + 1 + b.$$

Here the calculation follows that in the previous strategy, except that the defection occurs one stage earlier, and the payoff of 1 + *b* at the defection stage is certain, rather than having a probability of *p*. The expected payoff of the wait strategy is

$$E\pi_{wait} = [(T-2)\alpha + \beta + 1 + b](1-p) + [(T-1)\alpha + 1 + b]p.$$

If the trading partner at stage *t* is an opportunistic type, with a probability of (1 – *p*), in stage *t* the agent in question is preempted, giving a payoff of $\beta$ In stage *T*, the agent plays against trustworthy type, giving a payoff of 1 + *b*. In the remaining *T* – 2 stages, the agent cooperates with his opponent, giving a payoff of $\alpha$ per stage. If the trading partner at stage *t* is a trustworthy type, which has a probability of *p*, then the agent in question cooperates jointly with his partner for *T* – 1 stages, giving a payoff of 1 + *b*.

## Communication

Let *A* be a finite set of *a* agents, and let us suppose that an agent *a*, communicates to the other *n* – 1 agents of *A*. Colombetti (1999) designates *a* as the speaker, and the other *n* – 1 agents as the audience. For a given proposition $\varphi$, *a* successfully communicates that $\varphi$ if:

- $\varphi$ is a common belief of *A*;
- that *a* intends to communicate that $\varphi$ is also a common belief of *A*;
- $B_\varphi\varphi$, meaning that all believe that $\varphi$ is a common belief of *A*;
- $I_a\varphi$, meaning that agent *a* intends to perform $\varphi$;
- $C_a\varphi$, meaning that agent *a* communicates that $\varphi$ to the audience $A - \{a\}$.

Communication should be defined so that it is:

$(F_C)$ $\quad C_a\varphi = B_\varphi(\varphi \wedge I_a C_a\varphi)$, $F_C$ is the fixed-point axiom of mutual belief, or communication. We have the fixed-point axiom: $C_a^{'}\varphi = B_\varphi\left(I_a B_\varphi\varphi \wedge I_a C_a^{'}\varphi\right)$.

That $C_a^{'}$ is weaker than $C_a$ is reflected in the fact that $C_a\varphi$ entails $C_a^{'}$, and establishing which of the two notions of communications are mutually interdefinable can be carried out through the following schemes:

$$C^{'}\varphi = B_\varphi I_a C_a\varphi$$

$$C_a\varphi = B_\varphi \varphi \wedge C_a{}' \varphi.$$

$$C_a{}'' \varphi = B_\varphi\left(B_a\varphi \wedge I_a C_a{}'' \varphi\right),$$

When it is successful, communication that $\varphi$, besides implying the common belief of the intention to communicate that $\varphi$, also implies the common belief that the speaker believes that $\varphi$.

The endowment of agent $i$ relative to the per capita endowment $y_t^i$, can take on a low value, $y^L$, and a high value, $y^H$. The growth rate $a_t = A_t/A_{t-1}$, can also take on recession value, $a^R$, and a boom value, $a^B$, and both processes are assumed to be first-order Markov processes.

## Consumption and Shock

Agent $i$'s problem is as follows:

$$\max_{\{C_t^i, B_t^i\}_{i=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

s. t. $\quad C_t^i + q_t B_t^i = y_t^i A_t + B_{t-1}^i,$

where $C$ is the amount of consumption of agent $i$ in period $t$, $B$ is the demand for one unit of the consumption commodity in the next period, $q$ is the price, and the interest rate is $r_t = (1-q_t)/q_t$. Den Haan (2001) shows that the first-order conditions for the maximization problem of the agent are the two-part Kuhn-Tucker conditions:

$$q_t\left[C_t^i\right]^{1-\gamma} \geq \beta E_t\left[C_{t+1}^i\right]-\gamma \ \ and \ \left(B_t^i + \bar{b}A_t\right)\left(q_t\left[C_t^i\right]^{-\gamma} - \beta E_t\left[C_{t+1}^i\right]^{-\gamma}\right) = 0.$$

Let $F_t^L \ and \ F_t^H$ be the cumulative distribution function of the agents who receive a low-income shock and the high-income shock, respectively. Consequently, $dF^L \ and \ dF^H$ have mass at only one level of $b_{t-1}$ in an economy with two agents, and in an economy with a continuum of types, there will be a wide variety of holdings an each point in time. The state variables of agent $i$ are $b_{t-1}^i, y_t^i$, and the aggregate state variables $\bar{a}_t$. The equilibrium condition is given by

$$\int_{-b/a_t}^{\infty} b\left(b_{t-1}, y^L, \bar{a}_t\right) dF_t^L\left(b_{t-1}\right) + \int_{-b/a_t}^{\infty} b\left(b_{t-1}, y^H, \bar{a}_t\right) dF_t^H\left(b_{t-1}\right) = 0.$$

A solution to the model consists of a consumption function, $c\left(b_{t-1}, y_t, a_t, F_t^L, F_t^H\right)$, an investment function, $b\left(b_{t-1}, y_t, a_t, F_t^L, F_t^H\right)$, a price function, $q\left(a_t, F_t^L, F_t^H\right)$, and the functionals $F^L\left(a_t, a_{t-1}, F_{t-1}^L, F_{t-1}^H\right)$ *and* $F^H\left(a_t, a_{t-1}, F_{t-1}^L, F_{t-1}^H\right)$ that describe the laws of motion of $F_t^L$ *and* $F_{t-1}^H$, respectively.

## Risk Sharing and Design

Each worker seeks a job, and each firm $f_i$ seeks $q_i$ workers, a matching is a subset of $F \times W$, identified with a correspondence $\mu : F \cup W \to F \cup W$ such that $\mu(w) = f$ and $w \in \mu(f)$ if and only $(f, w)$ is a matched pair. If no matched pair contains *w*, then $\mu(w) = w$. Roth (2002) supposes an agent has complete and transitive preferences rather than remaining unmatched and waiting for the less desirable post-match market. The preferences of firms and workers will be given as lists of acceptable agents on the other side of the market, and firm $f_i$'s preference list $P(f_i) = w_i, w_j, \ldots\ldots, w_k$ of acceptable workers does not define its preferences over groups. That is, if a firm prefers worker *w* to $w'$, this means that the firm would prefer to add *w* instead of $w'$ to any group of other workers. A matching $\mu$ is blocked by an individual *k* if $\mu(k)$ is unacceptable to *k*, and it is blocked by a pair of agents $(f, w)$ if

$f >_w \mu(w)$.

When preferences are responsible, the set of stable matching equals by weak domination a game whose rules are any workers and firm may be matched. Suppose worker *w* prefers firm *f* to her outcome $\mu_w(w)$, but not when the firm has a dominant strategy for each worker to state her preferences.

If the market outcome is unstable, there is an agent who has the incentive to circumvent the match, because the market can do a lot of parallel processing. Consider a worker who has received an offer from her choice firm, which indicates whether they produce a matching that is stable with respect to the submitted preferences. The set of markets that come closest to providing a crisp comparison are the markets offering a different matching; in a controlled environment, we can examine the effect of a different matching, and define a priority for each firm-worker pair as a function of their mutual ranking.

That is, the first priority is to match firms and workers who each list one another as their mutual first choice, the second priority is to match a firm and worker such that the firm gets its second choice while the worker gets his first choice. This can produce unstable matching if a desirable firm and worker rank subjects who would initially gain within a decentralized matching market with sufficient competition. Once subjects had time to adapt to this market, the matching mechanism would be made available for those subjects who did not make early matches. In each matching market, firms could hire workers, and likewise each worker could accept a job, firms being restricted to one offer in each period. Contracts would be binding a firm whose offer was accepted, and a worker who accepted an offer could not make other matches in a later period. The reason this produces a stable matching is that no firm ever regrets any rejections it issues. This creates a potential instability involving firm *g* which now regrets having rejected worker *c*, like the deferred acceptance procedure will no longer be able to operate in one pass through worker *c*'s preferences, since to do so would miss this kind of instability. Instabilities could be sequenced in such a way that the process would converge to a stable matching, creating a new matching in which *f* and *w* are matched with one another, perhaps leaving a worker previously matched to *f* unmatched, and a position previously occupied by *w* vacant. Knowing that other bidders have comparable estimates would reduce the chance that the bidder's own estimate was mistaken; this means that each bidder has to treat his own estimate with caution. However not all potential complementarities can be clearly defined by the technology, and so a design question is how to structure the auction to allow bidders to take into account any important complementarities in their valuations. To determine the winning bids when bidders bid on different packages, the auction has to determine the revenue-maximizing set of packages, and those that may later become part of the revenue-maximizing set as higher bids for other packages come in. The difference in rules has a dramatic effect on the distribution of bids over time: in terms of price discovery early bids are less informative about final selling prices, and differences in rules for ending the auction have the same effect. This difference in ending rules is the only difference between the auctions studied, and bidders are randomly assigned to each kind of action, rather than self-selecting themselves between auctions. These complications come in the strategic environment, and consequently in the possible outcomes, the strategies available to the agents, and the behavior of agents. In the long term, success will be not merely be determined by how we understand the principles that economic interactions.

## Incomplete Markets

We consider inside information when the dynamics of the prices also features jumps, and consider a market in which there is a risk-less asset $S^0$ and a risky asset, $S$. The dynamics of the risky asset are given by the equation:

$$dS_t = S_{t-}(\mu dt + \sigma dW_t + \phi dM_t).$$

Here $W = (W_t, t \geq 0)$ is a Brownian motion and $(M_t, t \geq 0)$ is the martingale of a Poisson process, and the coefficients $\mu, \sigma, \phi$ are constants. Because the times of the jumps are observed and the sizes of the jumps are observed and the sizes of the jumps are known, an equivalent martingale measure is a probability $Q$, equivalent to $P$ such that the process $S$ can be described. Elliott and Jeanblanc (1999) consider an informed agent who, from time 0, knows any one of the following pieces of information: the number of jumps over the interval $[0,T]$, $i.e.\ N_T$ ; the times when the jumps occur, i. e., $(N_s, s \leq T)$; the path trajectory of the Brownian motion, i. e., $(W_s, s \leq T)$; the terminal value of the Brownian motion $W$, i. e. $W_T$ ; and the terminal value of the underlying asset, i. e., $S_T$.

In an incomplete market, the informed agent knows any hedge-able $\Im_T$ random variable. There does not exist a probability measure for the informed agent up to time $T$. The informed agent knows the price of the $\zeta$ contingent claim and he obtains an arbitrage opportunity. This arbitrage opportunity is revealed to the informed agent only at maturity, the optimization problem of terminal wealth for such an informed agent has no solution on the interval $[0,t]\ for\ t < T$ because the informed agent may obtain arbitrary wealth at terminal time. In the market driven by Brownian noises, the informed agent knows the terminal value of Brownian motion, and the dynamics of the price are given by:

$$dS_t = S_t(\mu_t dt + \sigma_1(t)dW_1(t) + \sigma_2(t)dW_2(t)),$$

where the informed agent knows $W$. The non-informed agent and the informed one assign prices to contingent claims of the form $h(S_T)$ which are hedge-able for any $h$, as we show that for the uninformed agent:

$$dS_t = S_t[\mu_t dt + \sigma_1(t)dW_1(t) + \sigma_2(t)dW_2(t)] = S_t[\mu_t dt + \sigma_3(t)dW_3(t)].$$

The filtration of the prices is equal to the filtration of $W_3$ and the incompleteness of the market is due to a specification of the filtration, and the market is complete if we consider contingent claims to be measurable with respect to

$\sigma(S_t, t \le T)$. The agent will compute the price of a contingent claim $h(S_T)$ as the expectation under the probability $P$ for the uninformed agent in $dS_t = S_t\sigma_3(t)d\overline{W}_t$ and for the informed agent, under the appropriate risk probability $P^*$, which is proved to exist in $dS_t = S_t\sigma_3(t)d\overline{W}_t^*$.

The dynamics of risky asset are given by:

$$dS_t = S_{t-}(\mu dt + \phi dM_t),$$

the arbitrage opportunity is the case $\frac{\mu}{\lambda\phi} \ge 1$; the risky asset has a return rate $\mu - \lambda\phi$ which is greater, and the jumps increase the value of this asset.

For the informed agent, the value of the underlying risky asset is affected the return of the risky asset, and the arbitrage opportunity comes from the knowledge of the times of the jumps, and from the fact that the agent knows if the jump is positive or negative. If there existed a martingale measure for the informed agent, the dynamics of the price would be of the form $dS_t = S_{t-}\phi d\overline{M}^*(t)$, where $\overline{M}^*(t)$ is a compensated martingale. The informed agent knows that $S_t \ge m(1+\phi)^{N_t}$ and he can buy at the last lowest price before $T$ and sell at a higher price.

We suppose the informed agent knows $N_T$ from time 0, and denote by $\Im_t$ the filtration generated by the price of the risky asset:

$$\Im_t \underline{\underline{def}} \sigma(S_s, s \le t) = \sigma(W_s, M_s, s \le t).$$

The uninformed agent can use portfolios that are measured with respect to $\Im_t$, and the informed agent will use the enlarged filtration:

$$\Im_t^* \underline{\underline{def}} \Im_t \vee \sigma(N_T).$$

However, the contrary to the case of the Brownian bridge, any $\sigma(N_s, s \le t)$-local martingale has finite variation, if the agent knows that $N_T = 1$, the single jump occurs with a uniform law on $[0,T]$ if the agent knows that $N_T = n$, as soon as he observes the occurrence of the $m$th jump.

For the informed agent, the dynamics of the price are given by:

$$dS_t = S_{t-}\left[\mu dt + \phi(\Lambda_t - \lambda)dt + \sigma dW_t + \phi dM_t^*\right].$$

Under $P^T$,

$$W^T(t) \underline{\underline{def}} W(t) - \int_0^t \psi_s dt$$,

is a Brownian motion and

$$M^\gamma(t) \underline{\underline{def}} M(t) - \int_0^t \lambda \gamma_s ds = N_t - \int_0^t \lambda(1+\gamma_s)ds$$,

yields a martingale.

This market is incomplete and, the uninformed agent must consider a range of viable prices for any the set of expectations $E_Q(\zeta)$ of the terminal payoff under any martingale measure $Q$, and this set is an interval and it is quite large. For the informed agent:

$$dS_t = S_{t-}\left[\mu dt + \phi(\Lambda_t - \lambda)dt + \sigma dW_t + \phi dM_t^*\right]$$,

and the pair $(W, M^*)$ has the property of predictable representation.

Let us note that after time $\tau^*$, the informed agent has to choose the risk premium $\psi^*$ given by:

$$\mu + \sigma\psi^* - \lambda\phi = 0$$.

For the informed agent, the price of the underlying asset is:

$$dS_t = S_{t-}\left(\sigma dW_t^{*\gamma} + \phi dM_t^{*\gamma}\right)$$,

and

$$S_t = S_0 \wp\left(\sigma W^{*\gamma}\right)_t \wp\left(\phi M^{*\gamma}\right)_t$$.

The uninformed agent $U$, and an informed one $I$, are dealing in the same market with an appreciation of the continuous risk premium, their jumps are linked by:

$$\lambda(1+\gamma) = \Lambda_t\left(1+\gamma_t^*\right)$$.

The relation

$$\left(1+\gamma_t^*\right) = \frac{T-s}{N_T - N_s}\lambda(1+\gamma)$$,

shows how the informed agent will be affected by the jump risk.

The non-arbitrage condition is equivalent to the existence of $\gamma$ such that $\gamma > -1$ and $\mu + \lambda\phi\gamma = 0$, and this implies that

$$\frac{\lambda\phi - \mu}{\lambda\phi} > 0$$.

Suppose $\alpha_t \; and \; \beta_t$ are predictable processes representing the portfolio of the uninformed agent, that $\alpha_t$ are the amounts of risk-less assets owned at time *t*, and the $\beta_t$ are the number of units of *S* held at time *t*. The wealth of the investor is then $X_t = \alpha_t + \beta_t S_t$, and if $c_t \geq 0$ is an adapting process representing the consumption, *X* satisfies the self-financing condition:

$$dX_t = \beta_t dS_t - c_t dt = \pi_t X_{t-}(\mu dt + \phi dM_t) - c_t dt = \pi_t X_{t-}\phi d\overline{M}_t - c_t dt,$$

where $\pi_t = \beta_t S_t / X_t$ is the proportion of the wealth invested in *S* at time and $\overline{M}_t = \mu t + \phi M_t$, is a *Q* martingale. Then the Lagrangian is $E\left(\int_0^T u(c_s)ds + g(X_T) - v\left(L_T X_T + \int_0^T L_s c_s ds - x\right)\right)$, where *v* is the Lagrange multiplier.

The optimal pair is given by:

$$\overline{c}_t = -\overline{u}'(\overline{v}L_t),\; X_T^* = -\overline{g}'(\overline{v}L_T),$$

with $\overline{v}$ such that:

$$\overline{v} = \frac{1+T}{x},\; \overline{X}_T = \frac{x}{(1+T)L_T},\; \overline{c}_t = \frac{x}{(1+T)L_t}.$$

The optimal portfolio is easily obtained from the representation $dX_t = \beta_t dS_t - c_t dt$, and can be proved to be $\overline{\pi}_t = \dfrac{\mu}{\phi(\lambda\phi - \mu)}$.

The current optimal wealth is such that $X_t L_t = E(X_T L_T | \wp_t)$ leads to:

$$\overline{X}_t = \frac{x\Phi(T-t)}{\Phi(T)} L_t^{\beta},$$

where $\Phi(\tau) = \Gamma(\beta+1)\exp[\Gamma(\beta+1)\tau] + \exp[\Gamma(\beta+1)\tau] - 1.$

If the informed agent knows $N_T$, he has an arbitrage opportunity and so his expected potential wealth $X_t,\; t \geq 0$ exceeds that of an uninformed agent:

$$dX_t = \pi_t X_{t-}(\mu dt + \sigma dW_t + \phi dM_t) - c_t dt.$$

The informed agent knows $N_T$ from time 0, and his wealth evolves according to the dynamics:

$$dX_t^* = \pi_t X_{t-}^*\left[(\mu + \phi(\Lambda_t - \lambda))dt + \sigma dW_t + \phi dM_t^*\right].$$

The optimal portfolio of the informed agent is time-varying and jumps as soon as a jump occurs in the prices. The agent must maximize at each $(s,\omega)$ the quantity:

$$\pi\mu + \Lambda_s(\omega)\ln(1+\pi\phi) - \lambda\pi\phi - \frac{1}{2}\pi^2\sigma^2 .$$

Therefore, the maximum expected wealth for the informed agent is greater than that of the uninformed agent, because the informed agent can use any strategy available to the uninformed agent. The optimal portfolio $\pi^*$ of the informed agent is now given by $\mu + \phi\left[\Lambda(1+\pi^*\phi)^{\alpha-1} - \lambda\right] + (\alpha-1)\pi^*\sigma^2 = 0$,

and the optimal wealth is $X_t^* = x\Phi^*(t)\left[L_t^{*\gamma}\right]^\beta$, with $\beta = 1/(\alpha-1)$.

The uninformed agent might detect information from the actions and investment strategy of the informed agent, although the price process *S* and the number of shares $\beta\ of\ S$, the total wealth and the proportion $\pi$ of the wealth invested in *S* are not known. For the informed agent, the optimal $\pi^*$ is continuous between jump times, at a jump time *t*, the uninformed agent will observe the quantities $\beta_{t-}^* S_{t-}\ and\ \beta_t^* S_t$ of the informed agent:

$$\frac{\beta_t^* S_t}{\beta_{t-}^*} = (1+\pi_{t-}^*\phi)\frac{\pi_t^*}{\pi_{t-}^*},$$

which can be written $(1+\theta_{t-}\phi)$ for some $\theta_{t-}$.

After a jump the uninformed agent might conclude the informed agent is using the strategy $\theta$, and after observing the actions of the informed agent at more jump times more values of $\theta$ will be detected. This may signal to the uninformed agent that the informed agent does have some knowledge. A strategy to avoid disclosing inside knowledge might be for him to use $\overline{\pi}$ until the time of the last jump and then to use an optimal $\pi^*$ over the remaining period to *T*.

## Collateral Structure

In incomplete markets, economies have equilibria if some mechanism has been established under debt constraints for trees with finitely many branches at each node. These debt constraints are added to the budget set, but do not follow from budgetary considerations. The state prices that are used as a present value process are chosen jointly with the other equilibrium variables, without having an objective criterion among the continuum of possibilities compatible with the absence of arbitrage. The collateral obligation coefficients are for all agents in the economy.

Araujo, Páscoa and Torres-Martínez (2002) show the existence of equilibrium for economies with and infinite horizon, where assets are countable through many periods and the number of branches at each node. The consumer would have an interest in running into debt, and using credit to pay debt interests, with debt constraints putting a uniform limit on paying the debt on time. The equivalent martingale measure suggests a solution that consists of a condition on the budget restriction. In this way, the arbitrary imposition of the transversal condition is strengthened by arbitrary choices whose budget sets include an implicit debt constraint, and assumes that the agent's choices are restricted to portfolios. The assets traded at each node are real and, and initial endowments at each node are bounded and preferences described by utilities are additive in time and in states of nature. Agents have the possibility of defaulting on their promises, and the omission of payments gives rise to seizure by the lenders. Each agent has the minimum from the values of real return and this is established at the moment of the negotiation. Let $S$ denote the set of states of nature, and the available information in $\Im$ at period $t$ is given by the partition $\Im_t$ for every agent, so the information in the economy is given by a family: $\Im = (\Im_0, \Im_1, \ldots\ldots, \Im_{\tau-1})$.

There exists a finite set of commodities, $\aleph$, at each node of the event-tree $\wp^{\tau}$, and we characterize each agent $i$ in $F$ by an endowment process $\omega^i = (\omega^i(\zeta,l)(\zeta,l) \in L^{\tau} \times L)$. Each agent chooses a consumption plan $x^i = (x^i(\zeta,l):(\zeta,l) \in L^{\tau} \times L) \in X^i$ in the event-tree and a plan of collateral $y^i \ in \ \tilde{X}^i = \{y \in X^i : y(\zeta) = 0 \forall \zeta \in L^{\tau}, b_{\zeta}^{\tau} = 0\}$.

The function $A : L^{\tau} \times J \to \Re_{-}^{i}$ characterizes the promises of the asset structure, so $A(\zeta, j)$ describes the bundles yielded by asset $j$ at the immediate successor nodes of $\zeta^-$. At each node $\zeta$ of the event tree, denote by $\theta^i(\zeta, j)$ the number of units of the asset $j \in J(\zeta)$ bought by agent $i$ at the node.

Each lender expects to receive the minimum between the claim and the market value of the collateral, the utility functions that represent the agents' preferences $\hbar = (U^i)_{i \in J}$, the agent endowment processes $W = (\omega^i)_{i \in J}$, the asset structure $A$, and depreciation $\Upsilon$.

An equilibrium for the economy $\wp_{ex}^{\tau}$ is a vector $[(\bar{x}, \bar{y}, \bar{\theta}, \bar{\varphi}); (\bar{p}, \bar{q})]$, and the pair $(\bar{\theta}, \bar{\varphi})$ satisfies:

$$\sum_{i \in J} \overline{\theta}^i = \sum_{i \in J} \overline{\varphi}^i .$$

If the structure of depreciation were the same for commodities, then for the agents in the equilibrium at each node in the economy, the total demand of commodities is equal to the endowment accumulated until this date. That is:

$$\sum_{i \in J} \left[ \overline{x}^i(\zeta) + C^{\zeta} \overline{\varphi}^i(\zeta) \right] = \sum_{i \in J} \sum_{k=0}^{\bar{t}(\zeta)} Y\left(\zeta, \zeta^{-k}\right) \omega^i\left(\zeta^{-k}\right),$$

where $\zeta^{-k}$ denotes the $k$-times predecessor of the node $\zeta$, the consumption allocations being uniformly bounded in equilibrium.

Therefore agents are indifferent to the amount of asset $j$ traded. Let $(x, y, \theta, \varphi)$ be an allocation the feasibility conditions:

$$\sum_{(l,i) \in J \times \wp} \left[ x^i(\zeta_0, l) y^i(\zeta_0, l) + \sum_{j \in J(\zeta_0)} C_{j,l}^{\zeta_0} \varphi^i(\zeta_0, j) \right] = \sum_{(l,j) \in J \times T} \omega^i(\zeta_0, l) .$$

When the asset's market feasibility conditions hold, we achieve with a finite set of utility maximizing consumers and auctioneers at each node the maximization of the value of the excess demand in the markets.

# 8 Decision Process, Assets and Information

The exchange economy involves *a* agents, *l* consumption goods and money, and agent ''s private information is represented by type $t^i \in T^i (i = 1,.....,n)$, where $T^i$ is a finite set. Forges, Mertens and Vohra (2002) assume that $q(t^i) > 0,\ \forall t^i \in T^i$, and agent ''s initial endowment is $e^i \in \Re_{+1l}$.

The set of actions of agent *i* is a copy $A^i\ of T^i$, and a coalition *S* is a subset of *N*. Let us define those actions $\forall a \in A_S$:

$$X_S(a) = \left\{ x = (x^i)_{i \in S} \in (\Re^l_+)^S \middle| \sum_{i \in S} x^i \le \sum_{i \in S} e^i(a^i) \right\}.$$

Let $\Delta(Y)$ be the set of probability distributions over *Y*, a mechanism for coalition *S* is a pair $(\xi_S, \mu_S)$ of maps such that:

$$\xi_S(.|a) = \xi_S(a) \in \Delta(X_S(a)),\ \forall a \in A_S,$$

$$\mu_S : A_S \to \Re^S : a \to (\mu^i_S(a))_{i \in S}.$$

Random money transfers would generate the characteristic function, and the mechanism $(\xi_S, \mu_S)$ is incentive-compatible if the strategies $a^i = t^i$ for every agent $i \in S$ form a Nash equilibrium. The characteristic function of the exchange economy is:

$$V^*(S) = \{v \in \Re^n(\xi_S, \mu_S),\ that\ v^i \le U^i_{\xi_S, \mu_S}\ \ \forall i \in S\}.$$

Thus $V^*$ can be described by the transfer utility function:

$$v^*(S) = \max\left\{ \sum_{i \in S} U^i_{\xi_S, \mu_S} |(\xi_S, \mu_S) \right\},$$

The presence of monetary transfers makes it possible that the characteristic function $v(S)$ can be expressed in terms of the utility from consumption goods:

$$v(S) = \max_{\xi_S}\left\{\sum_{i\in S}\sum_{t\in T} q(t)w^i\left(t,\xi_S^i(t_S)\right)\right\}.$$

Let, for $t,a \in T^i$, $u_{ta}^i$ denote $i$'s expected utility from $\xi$ when of type $t$ and claiming to be of type $a$, with transfers $\mu_t^i$ to agent $i$ claiming to be of type $t$ being incentive compatible. Money transfers are likely to facilitate the fulfillment of the incentive compatibility constraints.

For every incentive mechanism $(\xi_N, \mu_N)$, there exists a money scheme $v_N$ such that there is incentive for budget balance:

$$\sum_{i\in N} v_N^i(t) = 0 \quad \forall t \in T,$$

and the agents' beliefs are independent, the mechanism $(\xi_N, \overline{\mu}_N)$, where:

$$\overline{\mu}_N^i(t) = \overline{\mu}_N^i(t^i) = \sum_{t^{-i}} q(t^{-i})\mu_N^i(t^i, t^{-i}) \quad \forall i \in N,\ t \in T.$$

Under balance, we specify the definition:

$$v^*(N) = \max_{\xi_N}\left\{\sum_{i\in N}\sum_{t\in T} q(t)\int_{\Re_-^l} w^i(t,x^i)\xi_N(dx^i|t)\right\},$$

where the maximum is over all mechanisms $\xi_N$ for which there exists a money transfer scheme $\mu_N$. For every mapping $f: T \to \Re$, there exists a money transfer scheme $\mu: T \to \Re^n$ and:

$$\sum_{t^{-i}} q(t^{-i}|t^i)\mu^i(t^i,t^{-i}) \geq \sum_{t^{-i}} q(t^{-i}|t^i)\mu^i(a^i,t^{-i}) \quad \forall i \in N,\ \forall t^i, a^i \in T^i.$$

It can be made efficient by specifying:

$$\xi_N(t) \in \arg\max_{x\in X_N(t)}\left\{\sum_{i\in N} w^i(t,x^i)\right\},$$

such that $(\xi_N, \mu_N)$ is incentive compatible. For an analysis of conditions under Bayesian and dominant strategy incentive compatibility we must involve independent private values.

The beliefs of agents being independent, take the form:

$$v^*(S) = \max_{\xi_s}\left\{\sum_{i\in S}\left[\int_{\Re_+^l} w^i(s,x^i)\xi_S(dx^i|s) + \int_{\Re_+^l} w^i(t,x^i)\xi_S(dx^i|t)\right]\right\},$$

where the maximum is over mechanisms $\xi_S$, and monetary transfers play a role as they do under complete information, but here they also facilitate the fulfillment of the incentive constraints.

Let preferences, endowments, and the probability over states of nature be given; some sub-coalition can get more with an incentive compatible mechanism that Pareto dominates this expected utility. The incentive constraint for the first-best mechanism must still be satisfied in the neighborhood contradictions. For each sub-coalition *S*, and every agent of it, an incentive compatible mechanism exists giving this agent the whole surplus above the individually rational levels. Each of many mechanisms can be involved, *T* being finite; make a finite number of possible money transfers away from each individual and give this to each agent as his initial endowment of money.

We assume that there is an agent producing a kind of perishable consumption good, which sells at price *p*(t) units at time *t*. The agent uses labor, which, at time *t*, costs $w(t)$ units of currency per unit of time. It can produce *R*(t, L) units of the consumption goods per unit of time when it employs *L* units of labor. Chiarolla and Haussmann (2001) assume that profits are distributed as dividends to the shareholders at a rate given by:

$\delta'(t) = p(t)R(t,L(t)) - w(t)L(t).$ There are *J* agents in the economy and they provide the labor, so $0 \le L(t) \le J$. Then:

$$L(t) \in \arg\max_{0\le L\le L_{\max}} \{p(t)R(t,L) - w(t)L\}.$$

The price per share of the asset is:

$$dA(t) = \beta(t)dt + \alpha(t)dW(t),\ t\in[0,T],$$

*A*(t) represents the value of the productive asset at time *t*. The real-valued process $\beta$ and the row diffusion process will be determined by:

$$\int_0^T \left[|\beta(t)| + |\alpha(t)|^2\right]dt < \infty.$$

The *J* agents in the economy provide the labor, consume the goods and own the productive assets. To allow the agents to hedge all the risk and to finance their consumption , tradable at the prices *f*, *n* =0,….,N with:

$$\begin{cases} df_0(t) = r(t)f_0(t)dt & t\in[0,T],\ f_0(0)=1 \\ df_n(t) = f_n(t)[b_n(t)dt + a_n(t)dW(t)] & t\in[0,T],\ n = 1,2,\dots,N \end{cases}$$

we make the assumption

$a(t)a(t)^T \geq \varepsilon I$ and hence define the market price of risk $\theta$ as the solution of

$$a(t)\theta(t) = b(t) - r(t)V_N$$

We now clarify the problem facing the *j*-th agent; he has initial endowment of $\varepsilon_j$ shares of the productive asset with

$$\sum_{j=1}^{J} \varepsilon_j = 1$$

The agent chooses leisure process $\lfloor I_j(t) : t \in [0,T] \rfloor$ with

$$I_j(t) \in [0,1]$$

The agent chooses consumption process $\{c_j(t) : t \in [0,T]\}$ measured in units of the commodity with

$$\inf_{t \in [0,T]} c_j(t) \geq 0, \quad \sup_{t \in [0,T]} < \infty$$

The agent chooses productive asset share process $\pi_j(t) : t \in [0,T]$ *with* $\pi_j(0) = \varepsilon_j$.

The agent chooses financial asset portfolio process $\{\phi_{j0}, \phi_j\}\{t\} : t \in [0,T]$ where $\tilde{\phi}_j(t) = (\phi_{j,1}(t), \ldots\ldots\ldots, \phi_{j,N}(t))$ *and* $\phi_{j,k}(0) = 0,\ k = 0, \ldots, N.$

The agent's earnings process is a measurable, bounded process

$$e_j(t) = w(t)(1 - I_j(t)),$$

measured in units of currency. We shall see that another interpretation is possible: the agent works at maximum intensity to produce a wage endowment stream, $w(t)$, and then leisure at rate $I_j(t)$, so the agent buys out the fraction *I* of her work effort. If agent *j* does not modify her initial productive asset portfolio but sticks with her initial endowment, then her income process will simply be given by

$$\tilde{e}_j(t) := e_j(t) + \varepsilon_j \delta(t) = w(t)(1 - I_j(t)) + \varepsilon_j \delta(t).$$

The aggregate income process of the *J* agents is:

$$\tilde{e}(t) = \sum_{j=1}^{J} \tilde{e}_j(t) = \sum_{j=1}^{J} e_j(t) + \delta(t),\ t \in [0,T].$$

The wealth of agent $j$ at time $t$ is given by $X_j(t) := \pi_j(t)A(t) + \phi_j(t)f(t)$, so the wealth lies in the holdings of productive assets and financial assets. We require that $X$ satisfies the budget equation

$$X_j(t) = \varepsilon_j A(0) - \int_0^1 p(c)c_j(s)ds + \int_0^1 e_j(s)ds + \int_0^1 \pi_j(s)\delta(s)ds + \int_0^1 \pi_j(s)dA(s) + \int_0^1 \phi_j(s)df(s),\ t \in [0,T]$$

where an interest rate $r$, a spot price $p$, a productive asset price $A$, a dividend rate $\delta$, and a wage rate $w$, a quadruple $(c_j, I_j, \pi_j, \phi_j)$ of consumption, leisure, productive asset share, and the financial asset portfolio are all feasible for agent $j$ and maximize the agent's expected total utility from consumption and leisure:

$$E\left\{\int_0^T U^j(t, c_j(t), I_j(t))dt\right\}.$$

The market is in equilibrium if there exist

$$A(t) = \tilde{E}\left\{\int_t^T \exp\left[-\int_t^s r(u)du\right]\delta(s)ds \middle| F_t\right\} = \frac{1}{\xi}E\left\{\int_t^T \xi(s)\delta(s)ds \middle| F_t\right\},\ t > 0,$$

where $A$ is the expected value under the risk-neutral probability of the discounted future dividend stream. Each agent $j$ =1,….,J takes

$$E\left\{\int_0^T \xi(t)[p(t)c_j(t) + w(t)I_j(t)]dt\right\} \le E\left\{\int_0^T \xi(t)(\varepsilon_j\delta(t) + w(t))dt\right\}.$$

The agent has some private information $\theta$ about his constant marginal cost of production.

## Private Information

The risk neutral agent's utility is $u = t - \theta q$, where $t$ denotes, respectively, the transfer the agent receives from the intermediary and the output produced. Faure-Grimaud and Martimort (2001) assume that the intermediary receives a monetary transfer from the agent. The agent gets private information about his productivity parameter= $\theta$, with probability $\varepsilon$. Agents select their actions, $\mu_1$ and $\mu_2$, which are unobservable to the shareholders. Agents are risk-averse, and their preferences can be represented by utility functions with the absolute risk aversion coefficient, $r$. Choi (2001) assumes that the payoff from division $i$ depends on the agent's actions and a random shock as follows:

$$x_i = \mu_i + \gamma\mu_j + \varepsilon_i,\ i = 1,2 \text{ and } i \ne j,$$

where $\gamma > 0$ is a synergy parameter. In an organization, agents are in charge of their own operations, and the payoffs are given by

$$x_1 = \mu_1 + \gamma\mu_2 + \varepsilon_1 \quad and \quad x_2 = \mu_2 + \gamma\mu_1 + \varepsilon_2.$$

The compensation for each agent is:

$$s_i(x_1, x_2) = \alpha_{1i}x_1 + \alpha_{2i}x_2 + \beta_i, \quad i = 1,2.$$

The incentive compatibility constraints are:

$$\alpha_{11} = \mu_1 - \gamma\alpha_{21} \quad and \quad \alpha_{22} = \mu_2 - \gamma\alpha_{12}.$$

The optimal incentive contracts are as follows:

$$\alpha_{21}^* = \frac{(\gamma\sigma_1^2 - \sigma_{12})\mu_1^*}{\gamma^2\sigma_1^2 + \sigma_2^2 - 2\gamma\sigma_{12}},$$

$$\alpha_{12}^* = \frac{(\gamma\sigma_2^2 - \sigma_{12})\mu_2^*}{\gamma^2\sigma_2^2 + \sigma_1^2 - 2\gamma\sigma_{12}},$$

$$\mu_1^* = \left[1 + r\sigma_1^2\left(1 - \frac{(p\sigma_2 - \gamma\sigma_1)^2}{\gamma^2\sigma_1^2 + \sigma_2^2 - 2\gamma p\sigma_1\sigma_2}\right)\right]^{-1}(1+\gamma),$$

$$\mu_2^* = \left[1 + r\sigma_2^2\left(1 - \frac{(p\sigma_1 - \gamma\sigma_1)^2}{\gamma^2\sigma_2^2 + \sigma_1^2 - 2\gamma p\sigma_1\sigma_2}\right)\right]^{-1}(1+\gamma),$$

where the choice of organizational form is affected by the magnitude of synergy and the correlation parameter, assuming equal risk $\sigma$, equal risk aversion $r$, and equal opportunity wages in two divisions $p$. Therefore, the stronger the synergy gain is, the less probable the decentralized structure, since the agents can utilize the synergetic gains more efficiently.

## Portfolio Process

For each agent $i$, $p$ is the probability that the agent succeeds. Bardsley (2001) assumes the probability that $K$ is included in the portfolio of a successful process is:

$$p_K = \prod_{k \in K} p_k,$$

while the probability of a successful process is

$$p_K^N = \prod_{k \in K \notin K}\prod p_k(1 - p_t).$$

The agent chooses success probabilities $p$ which requires effort $e$(p). The agent's utility is separable in income and effort, so his expected utility is:

$$U = \sum_K p_N^K u_K - e(p)$$
,

where $u$ is the utility associated with the transfer $x$. The agent chooses $p$ to maximize $U$ subject to a reservation utility. The participation constraint is:

$$e(p) = \sum_K p_N^K u_K$$
.

The Kuhn Tucker compatibility conditions are:

$$\sum_{\substack{K \\ j\in K}} P_N^{K-j} u_K - \sum_{\substack{K \\ j\notin K}} P_{N-j}^K u_K = e_j(p), \quad if \quad p_j > 0,$$

$$\sum_{\substack{K \\ j\in K}} P_N^{K-j} u_K - \sum_{\substack{K \\ j\notin K}} P_{N-j}^K u_K \le e_j(p), \quad if \quad p_j = 0.$$

Then the Lagrangean is:

$$L(p) = \sum_J P_N^J x_j + \gamma\left(e(p) - \sum_J P_N^J u_j\right) + \sum_J \lambda_J \left(e_J(p) \sum_{\substack{J \\ j\in K}} P_{N-j}^{J-j} u_j + \sum_{\substack{J \\ j\notin J}} P_{N-j}^J u_J\right).$$

The first–order conditions may be written in the following form:

$$(x_K' - \gamma) P_N^K - \sum_{j\in K} \lambda_j P_{N-j}^{K-j} + \sum_{j\notin K} \lambda_j P_{N-j}^K = 0$$
,

where $x_K'$ is an abbreviation for $x'(u_K)$. If $p$ is a risky portfolio, then $\lambda_j \ge 0$. Thus, unless risk aversion declines with wealth, the agent is rewarded to a decreasing degree for multiple successes. The payment made to the agent if $K$ is the set of actions that succeeds is:

$$x_K = x_0 + \sum_{i\in K} \frac{\lambda_j}{p_j(1-p_j)}.$$

## Consumption

We assume that at any time $t$ the agent can choose a rate $c$(t) of consumption, taken from her bank account. Framstad, Oksendal and Sulem (2001) let $X$(t), $Y$(t) denote the amount of money invested in, and then the evolution follows:

$$dX(t) = dX^{cAM}(t) = (rX(t) - c(t))dt - (1+\lambda)dA(t) + (1-\mu)dM(t)$$
$$X(0'') = x \in \Re$$

$$dY(t) = dY^{AM}(t) = Y(t'')\left(\alpha dt + \sigma dW(t) + \int_{-1}^{\infty} \eta \tilde{N}(dt, d\eta)\right) + d\mathrm{A}(t) - d\mathrm{M}(t)$$

$$Y(0'') = Y \in \Re$$

Here $\mathrm{A}(t), \mathrm{M}(t)$ represent the cumulative purchase and sale, respectively, of stocks up to time *t*. The coefficients $\lambda \geq 0, \mu \in [0,1]$ represent the constants of proportionality of the transaction costs.

A voting scheme is strategy-proof if no agent can gain by voting strategically, no matter what the other agents do. Let $N = \{1,...,n\} \quad n \geq 1$ denote the set of agents. These agents are assumed to have preferences on space $\Re^2$, there is a point $x \in \Re^2$ and a metric $\delta \ on \ \Re^2$ such that *y* is weakly preferred to *z* if and only if $\delta(y,x) \leq \delta(z,x)$ for all $y,z \in \Re^2$.

## Vote Strategy

A voting scheme is a map $\varphi : \left(\Re^2\right)^N \rightarrow \Re^2$. Elements of $\left(\Re^2\right)^N$ are called profiles. Stel (2000) assumes strategy-proof-ness means that it is always optimal to vote for one's bliss point, no matter what the other agents do. Pareto optimality requires that the compromise is always optimal in the sense that there is no other point which is for all agents at least as good the compromise point and better than the compromise point for at least one agent.

What is meant by weak conditional strategy-proof-ness, anonymity, and non-dictatorship? Weak coalition strategy-proof-ness means that all agents belonging to some subset of *N* have the same bliss point; they cannot gain by simultaneously voting points for all $M \subset N, x \in \Re^2 \ \ and \ \ p,q \in \left(\Re^2\right)^N$. Anonymity means that interchanging agents does not affect the compromise point $\varphi\left(p^{\sigma}\gamma\right) = \varphi(p)$ for every $p \in \left(\Re^2\right)^N$. Non-dictatorship means that there is no agent who completely determines the compromise point, and there is no $i \in N$ with $\varphi(p) = p(i) \quad for \quad all \ p \in \left(\Re^2\right)^N$. Non-dictatorship means that there is no agent who completely determines the compromise point, in the sense that point always coincides with his reported point. Thus we are left with the class of coalition-dependent median schemes. A weighted median scheme is easy to compute, and the assignment of weights by which it is defined can be achieved quite naturally.

## Stochastic Volatility

Any reasonable strategy for an agent has to depend in some way on the unobservable latent state process of an incomplete market situation, but also on partial information. We proceed to determine a risk minimizing strategy assuming full knowledge also of the latent process. We obtain the risk minimizing hedging strategy under partial information. The full information strategy on the sub-filtration describing the available partial information comes from observing the prices at discrete random times. This leads to a filtering problem with market point process observations.

Frey and Runggaldier (1999) consider some underlying filtered probability space $(\Omega, F, (F_t)_{0 \le t \le T}, P)$ and some terminal date *T*, and the dynamics of the latent process *X* that is assumed to influence the asset volatility. *X* is a finite state Markov chain over a time grid with step $\Delta > 0$, the state space is $E_M := \{x_1, \ldots\ldots, x_M\}$ and the transition probability is $\Pi = \{p_{ij}\}_{i,j=1,\ldots\ldots,M}$ .

*Then X* is a diffusion, and we have:

$$dX_t = a(t, X_t)dt + \eta(t, X_t)dw_t^1 ,$$

with coefficients such that there exists a unique weak sense solution.

The asset price *S* follows a stochastic volatility of the form:

$$dS_t = S_t \sqrt{v(t, S_t, X_t)} dw_t^0$$

for a second Brownian motion $w^0$ , which is independent of the filtration generated by *X*. Besides *S*, there is a risk-free asset *B* traded in the market from the viewpoint of an agent who observes the precise value of the stock price $S_t$ , at random points in time $T_1 < T_2 < \ldots.$ The random times $T_n$ are modeled as jump times of some point process $N = (N_t)$ , assuming that $\lambda$ being non-decreasing makes sense from the economic viewpoint. The agent is likely to monitor the market more frequently in periods when the market is very active or when a lot of new economic information emerges during high volatility periods. Since observing $S_t$ , we assume that the information available to agents comes from $L_t$ at the random times $T_n$ . The information available to agent can be an information process *Y* defined via:

$Y_t := L_{T_{N_t}}$ ,

where the jump times of *Y* and *N* coincide. The information available to the agent can be modeled by the sub-filtration $\{F_t^Y\}$, generated by the process *Y*, which can be expressed as:

$F_t^Y = \{N_s, s \le t; L_{T_i}, T_i \le t\}$ ,

and

$F_T^Y = \{N_s, s \le T; L_{T_i}, T_i \le T; L_T\}$ ,

gives the value of $L_T$ .at the date $T$

We shall consider the problem of hedging a contingent claim $H \in F_T^S$ when the information of agent is restricted to the filtration $\{F_t^Y\}$, and we shall use the criterion of risk minimization, and the criteria of local and remaining risk. In a dynamic $\{F_t\}$- trading strategy $(\xi_t, \eta_t) = \{(\xi_t, \eta_t), 0 \le t \le T\}$ is a rule to hold $\xi_t$ units of the risky asset *S* and $\eta_t$ units of *B* at time *t*. The value process of this strategy is given by:

$V_t := V_t(\xi, \eta) := \xi_t X_t + \eta_t$ ,

and the strategy is said to hedge against *H* if $V = H$ . We define the cost process of a trading strategy via:

$$C_t := C_t(\xi, \eta) := V_t - \int_0^t \xi_t dS_t$$ .

In computing his strategy, the agent has the information contained in $\{F_t^Y\}$ at his disposal, where $\xi \; is \; \{F_t^Y\}$-predictable and $\eta$ is $\{F_t^Y\}$-adapted. The $\{F_t^Y\}$-risk process $R^Y(\xi, \eta)$ of an $\{F_t^Y\}$-admissible strategy is defined as:

$R_t^Y(\xi, \eta) := E\{(C_T(\xi, \eta) - C_t(\xi, \eta))^2 | F_t^Y\}$ .

Compute $\{F_t^Y\}$-risk-minimizing strategies, determined under full-information, for an agent who is able to use $\{F_t\}$-trading strategies, and the set of $\{F_t^Y\}$-admissible strategies. We determine a risk minimizing hedging strategy and define a *P*-martingale:

$g(t, S_t, X_t) := E\{H(S_T) | F_t\}$ ,

which is justified by the Markov property of $(S_t, X_t)$.
This leads to the representation:

$$H(S_T) = g(0, S_0, X_0) + \int_0^T g_S(t, S_t, X_{t-}) dS_t + M_T^{(1)},$$

which is the required decomposition of $H(S_T)$, as the finite variation martingale $M^{(1)}$ is orthogonal to the $P$-martingale $S$, and we have the representation:

$$H(S_T) = g(0, S_0, X_0) + \int_0^T g_S(t, S_t, X_{t-}) dS_t + M_T^{(2)},$$

which proves the claim, as $S$ and $M^{(2)}$ are again orthogonal.

The agent does not receive significant new information between the jump times of $N$, and if the time between the jumps is small. The purpose is present a method to compute conditional expectations of the form $E\{F(X_{T_i}) | F_{T_i}^Y\}$, where $F(.)$ is continuous and bounded, $X_t$, is the unobserved process.

The $T_i$'s are the jump times of the process $N$, at which the prices of the risky asset are given by the $S_t$'s, and this a stochastic filtering problem, which is related to Bayes' formula:

$$E^P\{F(X_t) | F_t^Y\} = \frac{E^Q\{F(X_t)\Lambda_t^{-1} | F_t^Y\}}{E^Q\{\Lambda_t^{-1} | F_t^Y\}}.$$

The independence of $(X_t)$ *and* $(Y_t)$ under $Q$ are expectations of a functional of the process $(X_t)$, in which $Y$ is fixed at the observed values. We now explain how this measure transformation leads for formulas for the distribution of $X_t$ given observations up to time $t$.

## Liquidity Constraint

The investigation now turns to the characterization of optimal policies when the investor has no extra wealth other than labor income, and examines how the agent evaluates his human capital. Koo (1999) has shown that when an agent has some risk aversion but is not liquidity constrained in income, risk lowers the implicit future labor income and thereby lowers consumption. He applied the martingale approach to a consumption and portfolio selection problem with labor income and liquidity constraints. The income risk is spanned by risks in assets, i. e. liquidity constraints are a unique source of market imperfection.

There is an economic agent whose preference over consumption profiles is given by time separable utility function:

$$u = E\left\{\sum_{t=0}^{T}(1+\delta)^{-t}v(c_t)\right\},$$

where $\delta$ is the subjective rate of time preference, and $v(c)$ is given by the risk aversion function:

$$v(c)\begin{cases}\dfrac{c^{1-\gamma}}{1-\gamma} & if \quad \gamma \neq 1 \\ \log c & if \quad \gamma = 1.\end{cases}$$

The agent receives labor income $y_t$ at the beginning of time period $[t, t+1]$, with risk-less bonds and risky assets, with a random real return. Given a consumption profile $(c_t)$ and a portfolio profile $(\pi_t)$, assets, $A$, evolve according to:

$$A_{t+1} = R(A_t + y_t - c_t) + \pi_t(\tilde{R}_{t+1} - R).$$

The constraint will be satisfied if the agent faces a margin requirement and the restriction of agent value function $V$ is defined to be the maximized value of his expected utility. Under suitable regularity conditions, there exists a unique consumption and investment policy where income shocks are permanent. The agent's consumption and investment in the risky asset are homogeneous functions of degree 1 given the current wealth $x_t$ and labor income $y_t$. Then the consumption and portfolio policy is given by:

$$c_t = c(z_t)y_t \quad \pi_t = \pi(z_t)y_t,$$

where

$$z_t = \frac{x_t}{y_t}.$$

The value function $V$ is homogeneous of degree $1-\gamma$ in wealth, $x_t$, and labor income, $y_t$, where $\gamma$ is the coefficient of risk aversion of the period utility function. Let $\phi(z)$ be a function where:

$$V(x_t, y_t) = \begin{cases}\phi(z_t)y_t^{1-\gamma} & if \quad \gamma \neq 1 \\ \phi(z_t) + \dfrac{1+\delta-1/(1+\delta)^{T-t}}{\delta} & if \quad \gamma = 1.\end{cases}$$

Let the marginal rate of substitution $p$ between labor income and wealth be defined by:

$$p = \frac{V_y(x,y)}{V_x(x,y)},$$

and let $q$ be defined by:

$$q = \begin{cases} (1-\gamma)\dfrac{V(x,y)}{(x+p(z)y)^{1-\gamma}} & if \quad \gamma \neq 1 \\ \dfrac{e^{\delta/\left(1+\delta-1/(1+\delta)^{T-t}\right)}V(x,y)}{(x+py)} & if \quad \gamma = 1. \end{cases}$$

Let $c_t$ be the consumption at time $t$, which takes the form:

$$c_t = \begin{cases} q(z_t)^{-(1/\gamma)}(x_t + p(z_t)y_t) & if \quad \gamma \neq 1 \\ \dfrac{\delta}{1+\delta-1/(1+\delta)^{T-t}}(x_t + p(z_t)y_t) & if \quad \gamma = 1. \end{cases}$$

If there exists income risk and liquidity constraints, then the marginal rate of substitution between wealth and current labor income is denoted by $p(z)$, for $a < z < b, \quad and \quad p'(z) \geq 0$.

This is intuitive in the sense that as the ratio of wealth to income increases, the effects of risk and liquidity constraints become smaller, and human capital is more valuable to the agent. The following proposition applies when the agent does not have any assets carried over from previous period, i.e. when $A_t = 0$ for the consumption and portfolio selection problem. It shows that consumption is more likely to be lower than wealth, where the agent's investment opportunity sets the risk-less and risky assets. Then the consumption at $z_t = x_t / y_t$ is equal to current wealth if

$$\frac{1}{1+\delta} E\overline{g}^{-\gamma}\left(R + w^*\left(\overline{R} - R\right)\right) \leq 1.$$

If the above inequality is valid, there exist $\overline{z} > 1$ such that the ratio of the risky asset holdings to total investment is equal to $w^*$ for all $1 < z_t \leq \overline{z}$.

Consumption is less than current wealth for all $z_t = x_t / y_t \geq 1$ if

$$\frac{1}{1+\delta} E\overline{g}^{-\gamma}\left(R + w^*\left(\overline{R} - R\right)\right) > 1,$$

is equal to the discounted expected value of the future marginal utility of money, and if it is smaller than the current marginal utility of money. Then the agent would want to consume more than current wealth if there were no liquidity constraints. If we make the approximation $(1+\delta)e^{a} = 1+\delta+a$, then

$$\frac{r-\delta+\overline{w}\mu}{\gamma}+\frac{\gamma}{2}\sigma_g^2 > g$$,

where $r = R-1$ is the risk-free rate and $\mu = E\overline{R} - R$ is the return premium on the risky asset.

Inequality shows that consumption is likely to be lower than wealth when the agent's investment opportunity set has both the risk-less and risky assets. The existence of a broader opportunity to invest increases the agent's marginal utility of money, and this tends to reduce consumption below wealth.

These conditions can be expressed as:

$$E\overline{R} - R > \gamma\sigma_{R,g}$$,

where the right-hand side can be regarded as the risk-premium on human capital. When wealth is small, the agent buys the risky asset up to the limit if the return premium on risky asset is greater than the risk-premium on human capital, and invests up the maximum limit, where:

$$w^* = \left[\frac{E\overline{R}-R}{\sigma_R^2}-\frac{\gamma\sigma_{R,g}}{\sigma_R^2}\right]\left(\frac{1}{\gamma+p'(z)}\right)\left(1+\frac{p(z)}{z}\right)+\frac{\sigma_{R,g}}{\sigma_R^2}$$.

The agent does not consider only wealth when making investment decisions, but also the value of human capital, and wants to invest part of the value in the risky asset and thereby to hedge risk in the implicit value, so the coefficient of relative risk aversion is adjusted upward by $p'(z)$.

When wealth is small and the return premium on the risky asset is greater than the risk-premium on human capital, it is invested in the risky asset, because the value of human capital is much greater than the currently saved money. The agent wants to sell short the risky asset as a hedge against the risk in human capital.

If income shocks have both permanent and transitory components, then the functions $p$ and $q$ can be defined for labor income as follows:

$$y_{t+1} = \overline{y}_{t+1}s_{t+1}$$.

The component $\overline{y}_t$ represents part of income, which has a permanent effect on future income, and the component $s_t$ represents a temporary shock. The value function can be written as $V(x_t\overline{y}_t, s_t)$, so that the marginal rate of substitution $p$ can be defined by:

$$p = \frac{V_{\overline{y}}(x_t, \overline{y}_t, s_t)}{V_x(x_t, \overline{y}_t, s_t)}$$.

The implicit value of human capital increases for each level of transitory income shocks as wealth increases.

## System Dynamics

Shocks and bubbles are becoming more frequent events in economy. The broad-based move to increased capital market liberalization has managed to make contagion more likely. The need to understand the logic of financial panics is greater than ever. Harvey (2002) presents a comprehensive, useful, and unique explanation of economic fluctuations, and illustrates its usefulness by modeling it using system dynamics. As the stock of capital rises with net increases in investment, the expectation of profit from future additions to the stock of capital will decline. Marginal efficiency of capital is a function of how much the existing stock of capital differs from some objectively determined target-level that would presumably satisfy current demand. Though it creates an underlying logic for the appearance of a turning point, by itself it may actually yield a stable equilibrium. To begin with, making the interest rate a function of demand for cash in this sort of system implies that it will be simultaneously a function and a determinant of investment. This is an economy in time, where there must exist a sequence of events. Today's investment is a result of yesterday's marginal efficiency of capital and rate of interest, which implies that firms make profit projections and secure funding one period ahead of actual spending. This leads to forecasts in an environment of uncertainty, and held with confidence, which are based largely on the agent's impressions of what others believe the market will do. Agents' optimism, combining with that of investors, lead to them to continue to expect positive returns well beyond the point that the increasing size of the stock of capital and rising costs of production would suggest. Modeling of expectations necessitates the certain adjustments.

Investment is affected by speculative marginal efficiency, a bandwagon effect occurs with consecutive positive values for objectives that can lead to shocks. A shock in expectations occurs whenever there are consecutive periods during which the speculative capital exceeds the objective, and at that point there is a collapse in the speculative capital.

Specification of a trade cycle consists of:

$$Investment = 100^{*}(M\,\mathrm{arg}inal\ efficiency\ of\ capital - \mathrm{int}erest\ rate)$$

;

$$Net\ investment = Investment - Depreciation$$

;

Stock of capital = Previous Stock of Capital + Net Investment;

Marginal Efficiency of Capital = (Target Stock of Capital – Stock of Capital)/Stock of Capital)*100;

Marginal Efficiency of Capital = (Target Stock of Capital – Stock of Capital).

With a system dynamics framework, it can be demonstrated that Harvey offers a viable explanation of shocks and bubbles. It also becomes clear that key the to panic and collapse lies in the way in which agents form expectations, with other factors adding vital elements.

# 9 Endowments, Money, Debt and Risky Choice of Decision

Agents mistakenly interpret data as the result of a stochastic process even though they are actually generated. Schonhofer (2001) supposes that agent-based decision models influence use for their estimation. Let $X \in \Re^n$ be the space of the economic variables $x_t \in X$ and $y_t \in \mathrm{Y} \subset \Re^q$, where we have

$$x_{t+1} = F_\phi(x_t).$$

An adaptive learning rule *LR* is a map, which maps past realizations $\{x_i\}_{i=1}^{\infty}$ into a set of predictors *P,* where

$$\psi_t = LR_t\left(\{x_i\}_{i=0}^{\infty}\right)$$

This results in:

$$x_{t+1} = F_\psi(x_t) = F(x_t, \psi_t(x_t)),\ x_t \in X \quad t = 0,1,\ldots..$$

Forecast errors in a dynamic system with adaptive learning are given by

$$\varepsilon_t = f(x_t, \psi_t(x_t)) - \psi_t(x_t).$$

Agents believe that $y_{t+1} = \psi_t(x_t) + \varepsilon_t$, where $\{\varepsilon_t\}_{t=1}^{\infty}$ is a sequence of random variables with mean 0. The agents make the point estimate $y_{t+1}^e = \psi_t(x_t)$, and they apply a certainty equivalence principle and replace the unknown *y* by its expected value given their belief. In period $T + 1$ they observe their past forecasts errors $\{\varepsilon_t\}_{t=1}^{T}$ and they use these observations to test the hypothesis.

## Endowment of Sources

Each agent is initially endowed with several units of each type of commodity and a positive amount of money. Yang (2000) supposes that each agent initially owns at most one unit of each type of commodity and a positive amount of money, and that no agent demands more than one unit of each type of commodities, and considers an exchange economy. Commodities of the same type are subject to quality differentiation, but have all the same function for the agents. Each agent owns several units of each type of commodity, and a certain amount of money.

We consider an exchange economy in which there are $n$ agents, denoted by $I$, and $k$ different types of commodities, and $\prod_{j=1}^{k} R^n$ denotes the space of all commodities. Each agent $i$ owns one unit of each type of commodity, denoted by $\omega^i$, and an amount of money denoted by $m$. In a real-life economy some agents – sellers, initially, own many products and other agents – buyers, initially, own no product but wish to buy products by using money. Assume that each agent has a utility function $u$. The agent's portfolio decisions determine the rates of return and affect the performance of the rules and determine their survival in this environment. An agent of generation $t$ solves the following maximization problem at time $t$:

$$\max \ln c_t(t) + \ln c_t(t+1)$$

$$c_t(t) \leq w^1 - \frac{m_1(t)}{p_1(t)} - \frac{m_2(t)}{p_2(t)},$$

$$c_t(t+1) \leq w^2 + \frac{m_1(t)}{p_1(t+1)} + \frac{m_2(t)}{p_2(t+1)},$$

where $m$ represents the agent's nominal holdings of currency 1 and currency 2 acquired at time $t$. Agents' savings, denoted by

$s$(t), in the first period of life , are equal to the sum of real holdings of currency 1, $m_1(t)/p_1(t)$

and real holdings of currency 2 , $m_2(t)/p_2(t)$, and $p$(t) is the nominal price of the good in terms of currency 1 and currency 2 at time $t$. The rates of return on currency 1 and currency 2 are equal to

$$R(t) = \frac{p_1(t)}{p_2(t)},$$

where $R$(t) is gross real rate of return between $t$ and $t$+1. The agent's savings $s$(t) derived from the agent's maximization problem are given by

$$s(t) = \frac{m_1(t)}{p_1(t)} + \frac{m_2(t)}{p_2(t)} = \frac{1}{2}\left[w - w^2 \frac{1}{R(t)}\right],$$

and then the equilibrium condition requires that aggregate savings equal real world money supply, so that

$$S(t) = N\left[w^1 - w^2 \frac{p_1(t+1)}{p_1(t)}\right] = \frac{H_1(t)}{p_1(t)} + \frac{H_2(t)e}{p_1(t)},$$

where $H_1(t)$ is the nominal supply of currency 1 at time $t$ and $H_2(t)$ is the nominal supply of currency 2 at time $t$ , and exchange rate is $e$. The agent-government of country $i$ finances the purchases of $G$ units of consumption good by issuing currency $i$. The agent-government $i$'s policy at time $t$ is given by

$$G_i = \frac{H_i(t) - H_i(t-1)}{p_i(t)},$$

and the condition for the monetary equilibrium is:

$$G_1 + G_2 = S(t) - S(t-1)R(t-1).$$

Consider the condition for equilibrium at time $t = 1$:

$$G_1 + G_2 = S(1) - \frac{H_1(0) + eH_2(0)}{p_1(1)}.$$

The paths of the equilibrium inflation rates are

$$\pi_w(t+1) = \frac{w_1}{w_2} + 1 - \frac{G_w}{2Nw^2} - \frac{w^1}{w^2}\frac{1}{\pi_w(t)}.$$

Each agent $i$, $i \in [1, N]$, in generation $t$ decides to consume some amount at time $t$ and of the balance, one part, represented by the fraction $\lambda_i(t) \in [0,1]$ is that agent $i$' s savings placed in currency 1 and the fraction $1 - \lambda_i(t)$ is placed into currency 2. The prices of the consumption in terms of currency $p$ are:

$$p_1(t) = H_1(t-1) \; / \; \left(\sum_i^N \lambda_i(t) s_i(t) - G_1\right),$$

$$H_1(t-1) = \sum_i^N \lambda_i(t-1) s_i(t-1) p_1(t-1) \quad and$$

$$p_2(t) = H_2(t-1) \Bigg/ \left(\sum_i^N (1 - \lambda_i(t)) s_i(t) - G_2\right), \text{ and}$$

$$H_2(t-1) = \sum_i^N (1 - \lambda_i(t-1)) s_i(t-1) p_2(t-1).$$

Given the prices $p$, the fraction $\lambda_i(t)$ will determine the holdings of currency 1 of agent $i$ of generation $t$

$$m_{i,1}(t) = \lambda_i(t) s_i(t) p_1(t),$$

and the holdings of currency 2 will be give by

$$m_{i,2}(t) = (1 - \lambda_i(t)) s_i(t) p_2(t).$$

The fitness $\mu_{i,t-1}$ is given by the ex-post value of the utility function of agent $i$ of generation $t - 1$:

$$\mu_{i,t-1} = \ln c_{i,t-1}(t-1) + \ln c_{i,t-1}(t).$$

## Asset Exchange

Agents may change their stock purchase patterns and switch their strategies. Aoki (2000) supposes that there are a number $K$ of types of $n$ agents who participate in selling, buying or producing goods. An example of a transition rate is:

$$w(n, n - e_j + e_i) = \lambda_{ji} d_j n_j c_i (n_i + h_i).$$

The transition rate specifies the entry rate of type $i$ agents, and that of the exit or departure rate of type $j$ agents, and the latter specifies the transition intensity of changing types by agents from type $j$ to type $i$. In the entry transition rate $c\ n$ stands for attractiveness of a group of agents.. The constant term $c\ h$ stands for the innovation niche effects, and $e$ is in the $i$ th position which is the only non-zero element, and $\lambda_{ji} = \lambda_{ij}$ for all $j, k$ pairs, and where $i, j = 1,2, \ldots,$K, and assume that $d_j \geq c_j > 0, \ and \ h_j > 0$. We introduce $a_i$ as the number of types with exactly $i$ agents, and we have an inequality

$$\sum_i a_i = K_n \leq K,$$

where $K_n$ is the number of groups formed by the $n$ agents , and

$$\sum_i i a_i = n,$$

which is an accounting identity. Parameter $\theta$ influences the number of groups formed by the agents, and controls the correlation between agent's types or classifications. The expected value of $a$ is given by

$$E(a_j) = \sum_{w(n)} a_j \pi_n(a) = \frac{\theta}{j} \frac{n!}{(n-j)!} \frac{\theta^{(n-j)}}{\theta^{(n)}}.$$

The probability of the $(j + 1)$st draw of the agents from the population is given by

$$E\left[\sum_i (1 - p_i)^j p_j\right] = \int_0^1 (1 - x)^j x \theta x^{-1} (1 - x)^{\theta - 1} dx.$$

Suppose that fraction $x$ describes the population by types of exchangeable agents. Then

$$w(n, n + 1) = \alpha,$$

where $\alpha$ is the transition rate if $n < n^*$, and

$$w(n, n + 1) = \beta,$$

where β is the transition rate if $n > n^*$. By the balance condition we obtain

$$\pi_n = c_1 \left(\frac{\alpha}{\beta}\right)^n,$$

for $n < n^*$, and for $n > n^*$

$$\pi_n = c_2\left(\frac{\beta}{\alpha}\right)^{n-n^*}.$$

The transition rate $w(a, a + e_i) = \alpha$ represents the process in which agents join a group of size $i$ at rate $\alpha$, $i = 1,2,\ldots$. The transition rate $w(a, a - e_i) = \beta$ refers to the rate at which agents leave that group. Suppose that the transition rate on one r-group and one s-group form

one u-group, this can be written

$w(a, a - e_r - e_s + e_u) = \lambda_{rsu} a_r a_s$.

The balance condition for the equilibrium distribution is of the form

$$\pi(a) = B\prod_r \frac{c_r^a}{a_r!},$$

provided there are positive numbers $c$, such that

$c_r c_s c \lambda_{rsu} = c_u \mu_{rsu}$.

## Rationing and Allocation

We now consider the problem of distributing goods in the face of a variable set of agents' claims. A rationing problem is defined as a pair $(\tau, g)$, where Kaminski (2000) supposes $\tau \in \sum$ as restriction and $g$ as the amount of a good to be divided. A $\gamma = (g_1, g_2, \ldots\ldots, g_n)$ is an allocation for $(\tau, g)$ if

$$|\tau| = n, \ \sum_{i=1}^{n}(g_i) = g$$

and $g_i \in dom(\tau_i)$ *for* $1 \le i \le n$. An allocation rule is a mapping $F : \prod \rightarrow U_{n \in N}\Re_+^n$, the value of $F$ for a problem $(\tau, g)$ is a $(F(\tau, g)_k)_{k-1,\ldots,|\tau|}$. We say that a type $\eta$ is a restriction of $\tau$ *if* $\eta$ results from deleting some coordinates in $\tau$. A rule $F$ is consistent if, for all problems $(\tau, g) \in \prod$, and for types $\eta \in \sum$, $\eta$ is a restriction of $\tau$. Consistency requires that the agents from any subset divide this amount in the same way the good is divided in the larger set. A rule $F$ is monotonic if for all pairs of agents $(\tau, g^*), (\tau, g^{**}) \in \prod$, *if* $g^* > g^{**}$ *then* $F(\tau, g^*) > F(\tau, g^{**})$ Whenever continuity appears, it is assumed that $T$ is a separable topological space, which include $\Re^n$ or certain spaces of real-valued functions. A rule $F$ is anonymous if all agents $(\tau, g) \in \prod$, and for all permutations of coordinates in $\tau, \sigma, F(\tau^a \sigma, g) = F(\tau, g)^a \sigma$. A rule $F$ is $R$-equitable if for all agents $(\tau, g) \in \prod$, no transfer is $R$-justified for $F(\tau, g)$.

The following agent correspondences are frequently considered in choice theory.

Utilitarian min: $L^2(u_i, u_j, g) = Arg \max \min\{u_i(x), u_j(g - x)\}$, for $0 \leq x \leq g$.

Nash: product: $N^2(u_i, u_j; g) = Arg \max[u_i(x) - u_i(0)][u_j(g - x) - u_j(0)]$, for $0 \leq x \leq g$.

Utilitarian: summation: $U^2(u_i, u_j; g) = Arg \max\{u_i(x) + u_j(g - x)\}$, *for* $0 \leq x \leq g$.

$L^2$ is single-valued and can be consistently extended to agent rules.

## Strategic Information

To achieve the goal of selecting alternatives based on strategic information, information about agents' preferences is needed. Suh (2001) supposes that the information may not be publicly known and agents may behave strategically. A mechanism consists of a set of strategies for each agent and a function that assigns each strategy profile to an alternative. The set $Z$ of equilibrium outcomes of the game given by the mechanism coincides with the set of alternatives selected by the correspondence for all possible preference profiles. The Nash equilibrium applies to environments where agents cannot form coalitions; hence agents can make unilateral deviations. Let $A$ be an arbitrary set of alternatives. Let $N = \{1, \ldots, n\}$ be a finite set of agents. Each agent $i \in N$ has preferences $R_i$, defining a binary relation on $A$ which is complete, transitive and reflexive. Let $P$ be the strict preference relation, and let $I$ be the indifference relation. Let $\Re_i$ be the set of agent $i$'s admissible preferences so a preference profile is a list $R = (R_1, \ldots, R_n) \in \Re$. A correspondence is a mapping $F$ which associates with each preference $R \in \Re$ a non-empty subset of $A$. A mechanism $\Gamma$ is a pair $(S, g)$ of a list of strategy sets $S$, where $S_i$ is the strategy set for agent $i$, and a function $g : S \rightarrow A$ which associates with each strategy outcome $s$ as alternative $g(s)$., and is evaluated by the profile $R$, and defines a game $(\Gamma, R)$. Given a preference profile $R$, and a mechanism $\Gamma = (S, g)$, a strategy profile $s \in S$ is a Nash equilibrium of the game $(\Gamma, R)$ if there is no $i \in N$ such that for some $\bar{s}_i \in S_i$, $g(\bar{s}_i, s_{-1}) P_i g(s)$. Hence a mechanism $\Gamma$ implements the correspondence $F$ in Nash equilibrium if $NA(\Gamma, R) = F(R)$ for all $R \in \Re$.

## Selection and Identification Preferences

We consider a model of selection of different agents' transmissions mechanisms, assuming the economy is populated by a continuum of agents. Each agent is identified by a cultural or preference trait, a parameter of his utility function, and agents are in fact able to influence these traits, with different psychological abilities to evaluate preferences. Bisin and Verdier (2001) suppose that the environment can be of two types, $e \in \{a, b\}$, yielding a Markov chain on state space $\{a, b\}$, with transition matrix:

$$\begin{bmatrix} 1-p & p & 0 & 0 \\ 0 & 0 & p & 1-p \\ 1-p & p & 0 & 0 \\ 0 & 0 & p & 1-p \end{bmatrix}.$$

These notions suggest various equilibria, which yield Pareto efficient allocations even in the presence of some types of increasing returns from production technologies , and they do not need to take profit shares as given, and the core equivalence holds for share equilibria under regularity.

## Behavior Allocation

Equilibrium allocation using a Nash equilibrium is a solution used to describe any agent's self-interested behavior. Tian (2000) supposes that a Nash equilibrium is a strictly non-cooperative notion and is concerned with specifying an agent's deviations that cannot be improved by any unilateral deviation from a prescribed strategy profile. As a result, although a Nash equilibrium may be easy to reach, it may not be stable in the sense that there may exist a group of agents who can improve it by forming a coalition. Although Nash equilibrium may result in a more stable equilibrium outcome, it requires more information about the communication network in operation and other agents' characteristics. Agents in some coalitions will cooperate and in some other coalitions they will not, so it allows for the possibility of agents manipulating coalition patterns. A ratio equilibrium allows each agent to produce a single good, while the generalized ratio equilibrium allows for the production of multiple goods. For a convex and differentiable production technology, this can be regarded as Lindahl equilibrium allocations, with shares proportional to the consumption of goods and services. Each agent's characteristic is denoted by $e_i = (w_i^0, R_i)$, where $w$ is the true initial endowment of the good and $R$ is the preference ordering. An allocation (x, y) is Pareto-optimal with respect to the preference profile $R = (R_1, ..., R_n)$, if it cannot be improved upon by $N$. It is individually rational with respect to the preference profile if it cannot be improved upon by any agent $i$.

Given a profit share $\theta \in \Delta^{n-1}$, an allocation $(x^*, y^*)$ is a $\theta$-Lindahl equilibrium allocated for economy $e$ if it is feasible and if prices $q$ exist. Let $M$ denote the $i$-th agent message domain, and let $M \prod_{i=1}^{n} M_i$ denote the message space. The outcome function is $h_i(m) = (X_i(m), Y(m))$, and the mechanism consists of (M, h). A message $m^* = (m_1^*, ...., m_n^*) \in M$ is said to be a Nash equilibrium of the mechanism (M, h) for an economy $e$ if, for each $i \in N$ *and* $m_i \in M_i$,

$$h_i(m^*) R_i h_i(m_i, m_{-1}^*),$$

where $(m, m_{-1}^*) = (m_1^*, ..., m_{i-1}^*, m, m_{i+1}^*, ..., m_n^*) h(m^*)$ is called a Nash equilibrium allocation of the mechanism for the economy $e$. A message $m^* = (m_1^*, ......, m_n^*) \in M$ is said to be a Nash equilibrium of the mechanism (M , h) for an economy $e$ if there does not exist any coalition $S$ where $m_S \in \prod_{i \in S} M_i$ such that for all $i \in S$,

$$h_i(m_S m_{-S}^*) P_i h_i(m^*),$$

$h(m^*)$ is called a Nash allocation of the mechanism for the economy $e$. The component $w$ denotes agent $i$'s endowment , which is necessary to guarantee the feasibility even at dis-equilibrium points. The component $y$ is the goods consumption proposed by agent $i$, the component $t$ is the tax contribution proposed by agent $i$. The component $\gamma_i$ is a shrinking index of agent $i$ used to shrink the goods consumptions of other agents. The component $\eta_i$ is the penalty index of agent $i$ when the announced price vectors and proposed allocation for goods by agent $i$ deviates from agent $i$ + 1. Define agent $i$'s share function $\theta_i : M \to \Re_{++}$ by

$$\theta_i(m) = \frac{q_i(m) t_{i+1}}{p_i(m) t_{i+1}}.$$

If $m^*$ is a Nash equilibrium , then $w_i^* = w_i^0$ for all $i \in N$. The first agent uses his private information, but the second agent chooses to contradict the first one.

## Probability Assignment

The agent's payoffs depend on the ex-post probability assignment of the market that an agent is smart or dumb. Each type is in charge of a firm. Small changes of speculator's beliefs may drive agents to equilibrium with a self-fulfilling attack. Sbracia and Zaghini (2001) study the role played by agents' beliefs about the fundamentals in a market crisis. The mean of the distribution can result in a shift from a model with multiple equilibria to a model with an attack equilibrium. Since $\theta$ is not known when agents choose their actions, there cannot be any multiple equilibrium for a given $\eta$ over $[0,1]$. The agent-state, who knows the state of agent-fundamentals $\theta$, takes his decision, and he will use a decision rule $\psi(\alpha,\theta)$ such that:

$$\psi(\alpha,\theta)=\begin{cases} leave, & if\ v-c(\alpha,\theta)\leq 0\\ defend, & otherwise.\end{cases}$$

Let us determine the expected payoff of a generic speculator, assuming:

if a speculator $i$ refrains from attacking , she gets 0 whatever to others do;

if a speculator $i$ attacks while all other agents attack, her expected payoff is given by:

$$\int_0^1 \left(e^* - f(\theta)-t\right)\eta(\theta)d\theta = e^* - E\left[f\left(\overline{\Theta}\right)\right]-t,$$

since, for any level of $\theta$, an attack by all agent-speculators induces the government to leave the previous policy. If a speculator $i$ attacks while all agents refrain from attacking, her expected payoff is:

$$\int_0^{\underline{\theta}} \left(e^* - f(\theta)-t\right)\eta(\theta)d\theta - \int_{\underline{\theta}}^1 t\eta(\theta)d\theta,$$

where in the first interval the government leaves the defense, and in second interval it is maintained. The strategy profile in which all agents attack the market is an equilibrium if $u(a_i,a_{-i})\geq 0$, and the strategy profile in which all agents refrain from attacking is an equilibrium if $u(a_i,n_{-i})\leq 0$. Thus we have :

$$u(a_i,a_{-i}) = e^* - E\left[f\left(\overline{\Theta}\right)\right]-t, \text{ and}$$

$$u(a_i,n_{-i}) = e^* p - E\left[f\left(\overline{\Theta}\right) \middle| \overline{\Theta}\leq\overline{\theta}\right]p-t.$$

We can show that there are multiple equilibria if:

$$e^*\in\left[E\left[f\left(\overline{\Theta}\right)\right]+t, E\left[f\left(\overline{\Theta}\right)\middle|\overline{\Theta}\leq\underline{\theta}\right]+ t/p\right].$$

Let us denote the above interval with $E \equiv [e_1, e_2]$, and we can verify that the condition $e^* \in E$ is a reasonable requirement for multiple equilibria. If $E$ is not empty, we can find that $E \neq \theta$, if

$$p \leq \frac{t}{t + E\left[f(\overline{\Theta})\right] - E\left[f(\overline{\Theta}) \middle| \overline{\Theta} \leq \underline{\theta}\right]} = s,$$

where $s \in (0,1)$. For a given strategy of agents-speculators, let $\pi(x)$ be their belief about the share of agents-attackers when the message is $x$. This belief is to be determined in equilibrium, and must be consistent with speculators' equilibrium strategies. Messages are distributed on:

$$a(\theta, \pi) = \frac{1}{2\delta} \int_{\theta-\delta}^{\theta+\delta} \pi(x) dx.$$

Hence, the payoff of an agent who attacks when the state of fundamentals is "do not know $\theta$" and their belief is $\pi$ when they receive message $x$ can be written as:

$$u_a(x, \pi) = \frac{1}{2\delta} \int_{x-\delta}^{x+\delta} h_a(\theta, \pi) d\theta = \frac{1}{2\delta} \int_{C_x} \left(e^* - f(\theta)\right) d\theta - t.$$

Let us consider a symmetric equilibrium; we have $\pi(x) = 0 \Leftrightarrow u_a(x, \pi) \leq 0 \ and \ \pi(x) = 1 \Leftrightarrow u_a(x, \pi) > 0.$

The function $u_a(k, I_k)$ is assumed to be strictly decreasing and continuous in $k$, and $I_k$ represents the expected share of attackers and agent-speculators beliefs, so we have:

$$a(\theta, I_k) = \begin{cases} 1 & if \quad \theta \leq k - \delta \\ \frac{1}{2} - \frac{1}{2\delta}(\theta - k) & if \quad k - \delta < \theta \leq k + \delta \\ 0 & if \quad \theta < k + \delta \end{cases}.$$

Therefore, agents optimal rule is: attack; global markets show the importance of common knowledge of agents' actions for the result of multiple equilibrium to be obtained, and also give insight about the likelihood of the equilibrium of compete and information games being obtained.

Note that such an economy would be considered vulnerable to an attack, or fragile because for some unbiased agent's expectations there can be an equilibrium with attack.

Assume that there is a $[0,1]$ continuum of infinitely lived agents and $N$ types of agents. Each type is specialized in consumption and production. When agent $i$ consumes $q$ units of his consumption good he enjoys utility $u(q)$, when he produces $q$ units of his production good he suffers dis-utility $c(q)$. The agent enters a trading process characterized by bilateral random matching, and trading partners arrive to an agent according a Poisson process with a rate $\beta > 0$. The meeting technology presents the number of agents with whom a given agent is matched based on the arrival rate times the number of agents involved in exchange. Agents' trading histories are treated as information. All assets are storable, and the economy begins with a proportion of $M$ agents as money holders, and $1 - M$ agents with production opportunities. Li (2001) assumes specialization in production and consumption, so that agent $i$ produces what agent $i+1$consumes while agent $i+1$produces good $i+2$, which is single-coincidence meeting. If both agree to trade, agent $i$ becomes a creditor and agent $i+1$ a debtor. There is a technology that allows agents to consume only if they posses a particular object, and as long as future consumption yields a positive value to the debtor, he will repay the debt and regain his collateral as soon as possible. In a credit trade an asset is issued in exchange for $q_c$ units of good. In a monetary trade between a money holder and producer, the former gives unit of money in exchange for $q_m$ units of good. In a trade between a money holder and debtor, an amount of good, $q_d$ is predicted in exchange for money. The debtor then repays his debt, regains his collateral, and becomes a producer; the creditor gets repayment and becomes a money holder. When a creditor meets a producer of his consumption good, the former can sell his asset for $q_{sc}$ amount of production. When a creditor or money holder can acquire a producer's consumption good, there is no trade to the upper bound of unity on asset holdings. Circulating debt overcomes trading frictions to such an extent that money is driven out of circulation. In each meeting, agents bargain over the quantity of goods surrendered for unit of some asset. If bargaining involves the provision of goods, consumption occurs and an asset is created or assets change hands, by assuming that consumers make take-it-or-leave-it offers to their trading agents who produce goods. Let $V_i$, $i = p,m,c,d$ , be the expected discounted utility from beginning a period with no assets, money, credit, or debt. The implications of the bargaining rule that agents who produce do not gain in a trade give us the following terms of trade:

$$q_m = V_m - V_p$$

$$q_c = V_c - V_p$$

$$q_{mc} = V_m - V_{sc}$$

$$q_{sc} = V_{sc} - V_p.$$

In a trade between a money holder and producer, the trade surplus is $V_m - V_p - q_m$ for the producer and $u(q_m) + V_p - V_m$ for the money holder. The trade is acceptable to the money holder if

$$u(q_m)+V_p-V_m=u(q_m)-q_m\geq 0.$$

The credit trade yields trade surplus if

$$u(q_c)+V_d-V_p\geq 0.$$

Since agents who buy second-hand assets at the meeting acquire the expected value of getting repayment, the trade surplus for the money holder is $u(q_{mc})+V_{sc}-V_m$. Money holders are willing to buy second-hand assets if:

$$u(q_{mc})+V_{sc}-V_m\geq 0.$$

This trade nets the producer an expected value of $V_{sc}$. Creditors are willing to trade if

$$u(q_{sc})+V_p-V_{sc}\geq 0,$$

of trade surplus. We need to check whether it is the best response for money holders to propose $q_m$ to those who produce their consumption goods:

$$(P_p+P_d)[u(q_m)-q_m]\geq P_d\lfloor u(V_p-V_d)-q_m\rfloor,$$

is satisfied in equilibrium. There may exist other equilibria in which money holders use other strategies.

Given that money holders propose $q_m$ we need to check whether the debtor is willing to engage in trade:

$$V_p-V_d-q_m\geq 0.$$

The payoff of repaying the debt is the continuing ability to participate in the economy; the penalty on failure to repay the debt is being forced to pay the cost of acquiring money.

A monetary trade is possible between a money holder and producer. If the money holder issues an asset, he consumes $q_c$ units of good, and after consumption he will immediately make repayment. Since money is a more valuable asset than credit, agents have no incentive to conduct trade with credit when they have money in hand. Agents choose trading strategies to maximize their expected lifetime utility, taking into account others' strategies and steady-state conditions. An equilibrium is $(V_p,V_m,V_d,V_c)$, and $q_m>0$, $V_p-V_d-q_m<0$ where:

$$rV_p=M(V_m-V_p-q_m)$$

$$rV_m=(1-M)\lfloor u(q_m)+V_p-V_m\rfloor.$$

One can show that for all $M\in(0,1)$ there exists a unique equilibrium.

Potentially there are four types of agents in this economy: money holder, producer, creditor, and debtor, is the measures of each being denoted by $M, P_p, P_c,\ and\ P_d$, respectively, and

$$P_p+P_c+P_d=1-M.$$

In a stationary equilibrium, the steady-state condition for the distribution of types is described by:

$$MP_d = (P_p)^2,$$

where agents are willing to engage in credit trade, and debtors are willing to trade with money holders and make repayments, which equates the outflow and inflow into the fraction who are debtors. The outflow equals the fraction of such agents who meet and trade with money holders and repay the debt. The inflow equals the proportion of producers who have meetings in which they issue an asset and becomes a debtor.

We have following value functions with respect to flow returns:

$$rV_p = P_p \max\lfloor u(q_c) + V_d - V_p, 0 \rfloor,$$

which sets the flow value to a producer equal to the probability of a meeting multiplied by the gain of issuing assets to consume $q_c$ units of good and become a debtor;

$$rV_m = (P_p + P_d)\max\lfloor u(q_m) + V_p - V_m, 0 \rfloor + P_c \max[u(q_m) + V_{sc} - V_m, 0],$$

which sets the flow return of holding money equal to the probability of meeting a producer or a debtor, multiplied by the gain of trading , plus the probability of meeting a creditor, multiplied by the expected utility of changing asset positions;

$$rV_d = M(-q_m + V_p - V_d),$$

which describes the flow return to a debtor, which is the expected value of meeting a money holder, producing to gain money and repaying the debt ;

$$rV_c = P_p \max[u(q_{sc}) + V_p - V_c, V_{sc} - V_c] + M \max(-q_{mc} + V_m - V_c, V_{sc} - V_c) + (1 - P_p M)(V_{sc} - V_c),$$

which sets the flow return to a creditor equal to the gain of selling his asset to a producer and money holder, plus the gain of moving to the repayment.

The creditor's expected value in the repayment is:

$$V_{sc} = MV_m + (1 - M)V_c,$$

which describes the value to a creditor at the beginning, which is the probability of the debtor getting money to repay the debt multiplied by the gain of acquiring repayment, plus the continuation value.

We get:

$$q_{mc} = (1 - M)(q_m - q_c),$$

$$q_{sc} = q_c + M(q_m - q_c),$$

which implies $q_{sc} > q_c$ in equilibrium, so second-hand assets are sold at a higher price than the newly issued assets.

An equilibrium with active credit trade is a vector of value functions $V = (V_p, V_m, V_d, V_c, V_{sc})$, quantities of trade $Q = (q_m, q_c, q_{mc}, q_{sc})$, and distribution of agents $P = (P_p, P_c, P_d)$ such that $q_m > 0,\ q_c > 0,\ q_m \geq q_c$.

The algorithm to find such an equilibrium is as follows:

$$f(q_c,q_m)=(P_p+P_d)u(q_m)-[1+r-M(P_c+P_p)]q_m+P_cu[(1-M)(q_m-q_c)]-P_pu[q_m-q_c]$$
$$+[1+r-P_d-M(P_c+P_p)]q_c,$$
$$g(q_c,q_m)=[M(r+M+2P_p)-P_pM(r+M+P_p)]q_m+P_p(r+M+P_p)u[q_c+M(q_m-q_c)]$$
$$-P_p(r+M)u(q_c)-(r+M+P_p)[r+M+P_p(1-M)]q_c.$$

for the remaining work for finding an equilibrium consists in showing that the solutions $q_m, q_c \le \hat{q}$ satisfy:

$$q_m \ge q_c > 0,$$
$$u(q_c) > q_m(r+P_p)/P_p.$$

This result implies liquidity for agents who are not too impatient for debt to exist and circulate. The debt is circulated and is primarily used as a medium of exchange, making it a close substitute for money, and credit dominates money in rate of return. After a random duration of time, $t_d$, and the interest rate here by

$$p=\frac{1}{t_d}\ln\left(\frac{q_m}{q_c}\right).$$

The notion of expected interest rate, $E_p = M\ln(q_m/q_c)$, can be used to check how interest rate is affected by parameters, through the relative price of credit and money. Let $W$ denote the welfare criterion where:

$$W = P_pV_p + MV_m + P_dV_d + P_cV_c,$$

which can be interpreted as long-run expected utility of a agent, not conditional on her current status. When there are multiple equilibria welfare is compared according to the welfare criterion $W$. Thus even if agents were allowed to accumulate assets, they would conduct credit trade when they do not have enough purchasing power in a meeting.

That is, agents continue to use the individually maximizing trading strategies, which is called an open market operation for the exchange of assets in this random-matching economy.

# 10 Market Mechanisms, Portfolios and Uncertainty of Decisions

The agent's portfolio $(\alpha,\theta)$ is constrained to take values in $A$, and for $v=(v_0,v_-)$

$$\delta(v)=\sup_{(\alpha,\theta)\in A}-\left(\alpha v_0+\theta^T v_-\right)$$

Portfolio-mix constraints are given by:

$$A=\left\{(\alpha,\theta)\in R^{n-1};\alpha+\sum_{k=1}^{n}\theta_k\geq 0,\theta\in M\left(\alpha+\sum_{k=1}^{n}\theta_k\right)\right\},$$

where $M$ is subset of $R$. The minimum capital required is $K\geq 0$:

$$A=\left\{(\alpha,\theta)\in R^{n+1};\alpha+\sum_{k=1}^{n}\theta_k\geq K\right\}.$$

For the value of the agent's portfolio at time $t$, we have:

$$W(t)=d_0+\int_0^t\left(\alpha(\tau)+r(\tau)+\theta(\tau)^T\mu(\tau)\right)d\tau+\int_0^t\theta(\tau)^T\sigma(\tau)dw(\tau)-\int_0^t\left(c(\tau)-d(\tau)\right)d\tau-C(t),$$

$$W(t)\geq -K,\quad W(T)\geq 0.$$

The agent is allowed not to reinvest some of his wealth if he chooses to do so, and it states that the wealth at any time $t$ equals the initial wealth, plus trading gains, minus the cumulative net withdrawals. While the agent is allowed to borrow against future income and thus to have short-term deficits, the final wealth must be sufficient to cover any amount borrowed.

The agent's consumption problem can be stated as

$$\max_{c\in C_+^*}U(c)$$

$$E^{Q_v}\left[\int_0^T\beta_v(t)\left(c(t)-d(t)-\delta v(t)\right)dt\right]\leq d_0\quad\forall\quad v\in N^*.$$

Let $K$ denote the set of consumption processes:

$$K = \left\{ c \in C_+^* : E^{Q_v}\left[\int_0^T \beta_v(t)(c(t) - d(t) - \delta v(t))dt\right] \leq d_0 \quad \forall v \in N^* \right\}.$$

Let $W = (W_1, W_2)$ be a standard probability space, consider an agent who wants to maximize her expected life-utility from consumption $E\left[\int_0^\infty e^{-\beta t} u(c(t))dt\right]$. Munk (2000) supposes that for this purpose, the agent can invest in a risky asset with a price process $S(.)$ given by

$$dS(t) = S(t)(bdt + \sigma dW_1(t)) \quad S(0) > 0$$
,

and in a risk-less asset with a constant rate of return $r$. The agent has an initial wealth endowment $x \geq 0$ and receives income from non-traded assets at a rate given by the process $Y(.)$, where

$$dY(t) = Y(t)\left(\mu dt + \delta\rho dW_1(t) + \delta\sqrt{1-\rho^2}\, dW_2(t)\right), \quad Y(0) = y > 0$$
.

Here $b, \sigma, \mu, \ and \ \delta$ are positive constants, and $\rho \in (-1,1)$ is the correlation between changes in the risky asset price $S$ and changes in the income rate $Y$. Define $X$(t) as the time $t$ value of the agent's portfolio of assets, and hence $X$(t) is the liquid wealth of the agent, which given a consumption process $c$ and a portfolio process $\pi$ evolves as

$$dX(t) = (rX(t) + \pi(t)(b - r) - c(t) + Y(t))dt + \pi(t)\sigma dW_1(t), \quad X(0) = x$$
.

Now assume that the agent has a CRRA (constant relative risk aversion) utility function

$$u(c) = c^\gamma, \quad 0 < \gamma < 1,$$

where $1 - \gamma$ is the coefficient of relative risk aversion. Indeed, for $x > 0$ and $y > 0$, the homogeneity property of the value function implies that the mappings $B(.)$ and $A(.)$ are :

$$B(z) = \gamma \frac{F(z)}{F'(z)} - z,$$

$$A(z) = \left(\frac{\gamma}{F'(z)}\right)^{\gamma/(1-\gamma)} / F(z).$$

The value function can be written as

$$v(x,y) = A(z)^{\gamma-1}(x + B(z)y)^\gamma, \quad x > 0, y > 0,$$

and hence

$$F(z) = A(z)^{\gamma-1}(z + B(z))^{\gamma}.$$

## Opening Markets

In the complete market where there are no liquidity constraints and the income rate is spanned, the certainty wealth equivalent of lifetime income is:

$$p(t) = \exp\left\{-rt - \frac{1}{2}\left(\frac{b-r}{\sigma}\right)^2 t - \frac{b-r}{\sigma}W_1(t)\right\},$$

and we define the constraint $\lambda = r - \mu + \delta(b-r)/\sigma$ .

Because all agents are rational, it is possible to obtain explicit welfare measures of the benefits and costs of markets more often. Smith (2001) supposes that opening markets more often increases unconditional welfare for all reasonable parameter values. Generation $t$ traders have preferences

$$\ln(C_t) + \beta \ln(C_{t+1}),$$

where $C_{t+i}$ denotes consumption of the good during time period $t + i$ and $\beta > 0$ is a subjective discount factor, and each agent receives $y$ units of consumption goods. Long-term investment is accomplished by purchasing shares of an asset in a competitive market, and this asset could be interpreted as stocks, bonds, pollution permits, or any other asset that is traded. The supply of this asset is $K$ shares, and ownership of a unit of the asset has dividend $D$ per period. The number of new agents entering the market at a given point in time is represented by a random variable $\tilde{x}$, and $Q_t \equiv x_{t1} + x_{t2}$ is the total number of agents. With expanded market access (EMA), agents are free to trade at any instant during a time period, and with restricted market access (RMA) agents may trade only at certain moments within a time period, and the index $k \in \{EMA, RMA\}$ will subsequently be used to denote the trading regime. In any time period, there are many generations of agents alive, and let $V_{t_2-1}^{j,k}$ denote a type $j$ old agent's shareholdings at the start of time period $t$, because they were determined in the last trading period. Let $\mu_{t_1}^{j,k}$ denote the fraction of $V_{t_2-1}^{j,k}$ sold at $t_1$, and if $P_{it}^{k}$ denotes the asset price, then the decision problem of agents at $t_1$ is

$$\max_{\mu_{t_1}^{j,k}} E_{t_1}^{k} \ln\left(C_t^{j,k}\right),$$

subject to

$C_t^{j,k} = V_{t_2-1}^{j,k}\left[D + \mu_{t_1}^{j,k} P_{t_1}^{k} + \left(1-\mu_{t_1}^{j,k}\right)P_{t_2}^{k}\right]$, where $E$ denotes expectation, conditional on information at $t_1$. The solution is:

$$V_{t_2}^{2,k} = \frac{\beta y}{P_{t_2}^{k}(1+\beta)}.$$

In comparison, decision problem at $t_2$ reflects the possibility that they may have traded at: $\max_{V_{t_2}^{1,k}} \ln\left(C_t^{1,k}\right) + \beta E_{t_2}^{k} \ln\left(C_{t+1}^{1,k}\right)$

$$C_t^{1,k} = y - P_{t_2}^{1,k} V_{t_2 1,k} + V_{t_1}^{1,k}\left(P_{t_2}^{k} - P_{t_1}^{k}\right),$$

$$C_{t+1}^{1,k} = V_{t_2}^{1,k}\left[D + \mu_{t_1+1}^{k} P_{t_1+1}^{k} + \left(1-\mu_{t_1+1}^{k}\right)P_{t_2+1}^{k}\right].$$

Given the stochastic process $x$, a competitive equilibrium in the *EMA* regime is a pair of price processes *P* and a set of asset-trading strategies *V* is:

$$x_{t_1} V_{t_1}^{1,k} = \mu_{t_1}^{k} K,$$

$$x_{t_1} V_{t_1}^{1,k} + x_{t_2} V_{t_2}^{2,k} = K,$$

and the equilibrium price of the *RMA* is:

$$P_{t_2}^{R} = \frac{\beta Q_t y}{(1+\beta)K}.$$

If *S* denotes the agent's information set at time *t* in the *RMA* regime, then it can be shown that

$$E_{t_1}^{R} P_{t_2}^{R} > E_{t_1}^{E}\left[\mu_{t_1}^{E} P_{t_1}^{E} + \left(1-\mu_{t_1}^{E}\right)P_{t_2}^{E}\right], \text{ if}$$

$$E\left(Q_t \middle| S_{t_1}\right) > E\left(Q_t \middle| x_{t_1}\right) + \left(\frac{x_{t_1}}{\beta}\right)\left(1 - \frac{E\left[G(Q_t)\middle| x_{t_1}\right]}{G\left[E\left(Q_t \middle| x_{t_1}\right)\right]}\right).$$

The conditional mean of the weighted average asset price in the *EMA* regime is less than the conditional mean of the asset price in the *RMA* regime.

## Market Information Flow

The amount that markets are open effects the information flow from markets and this can be beneficial to conditional welfare depending on the quantitative information that agents receive. The consumer's goal is to achieve:

$$\max E_t\left[\sum_{s=1}^{T} \beta^{s-1} u(c_s, h_s)\right],$$

where $\beta$ is the constant time preference, $u$ is utility, $c$ is current consumption , $h$ is habit stock. Carroll (2000) assumes that the utility function is given by:

$$u(c,h) = \frac{\left(c/h^{\gamma}\right)^{1-\rho}}{1-\rho}$$

If consumption is positive then $h$ will always be positive and the constant relative risk aversion utility function can be used for $v$ without danger of introducing any negative utility.

The existence and regularity of rational expectations equilibrium for any informational structure is derived from prices. Citanna and Villanacci (2000) suppose there are $H$ agents, each receiving $1 \le N_h < \infty$ signals about the future realization of states of the world , with $N_h > 1$ for at least one $h$. The economy signal space is N, and individual information can be represented through a partition of the space N of signals, and each agent partition is common knowledge.

It is denoted by $\Im_h$ , its cardinality is $J_h$, and its generic element is $N_h^j$. When only one agent is informed, $h$ = 1 has a partition with each element $a$ as an equivalence class, all other agents have the whole set $N$, and they are uninformed, and the informed agent may not be perfectly informed, in the sense of knowing with certainty what state of the world will occur after observing the signal $n$. Each agent has a prior distribution $\pi_h$ over the state and signal spaces, and acts upon receiving the signal and any other information that may refine the partition $\Im_h$. Agents can exchange $I$ assets, with $S > I > 1$, making plans at time $t = 0$ contingent upon the signal received; the assets are identified with a payoff $Y$. We assume that $Y > 0$ and that $Y$ is in a general position. Then $b_h(n) \in \Re^I$ represents portfolio holdings if signal $a$ occurs. Let $q(n) \in \Re^I$ be the price of these assets. Also set $\Im(q)$ to be the partition on $N$ induced by a signal-by-signal normalized $q$ , with cardinality $J(q)$ and with

$$N^j = \left\{ n \in N \middle| q(n)/\|q(n)\| = q(j)/\|q(j)\| \right\} \quad for \quad j = 1,2,\dots J(q).$$

The agents use this information to make decisions, and agents' utility is denoted by $v_h(x_h, \pi_h, \Im_h(q))$ to stress the fact that conditional probabilities depend on $\Im_h(q)$.

The rational expectations equilibrium agent's goal is to solve:

$$\max v_h(x_h; \pi_h, \Im_h(q))$$

$$q(n)b_h(n) = 0 \qquad \text{for all } n,$$

$$\Psi(n)z_h(n) = Yb_h(n) \quad \text{for all } n \text{, and}$$
$$\sum z_h(n) = 0$$
$$\sum b_h(n) = 0,$$

where $\Psi(n)$ is a matrix of prices. For given $p, q, e, Y$:

$$\max v_h(x_h; \pi_h, \Im(_h q)).$$

In essence, asymmetric information economies and allow control of asset prices even if all agents are restricted, that is only partially informed.

The agent's attitudes toward risk are critical in determining the relation between the relative risk and the relative dispersion of its price. If individual preferences exhibit risk substitutability, and if the degree of future period relative risk aversion is a function of consumption, then the relative risk of an asset corresponds to greater asset price volatility.

## Volatility of Assets

If preferences are additive time-separable, the volatility of asset prices does not depend on their relative risk. Drees and Eckwert (2000) study economies populated by an agent who owns a share of each asset at the beginning of period 1. After observing the state $y_1$, the agent receives all due dividends and uses them to buy consumption goods and additional shares to be carried over into next period. Thus the agent's choices are constrained by

$$c + p^{\pi}(z-1) + p^{q}(s-1) = h(y) + g(y)$$
$$\tilde{c} = zh(\tilde{y}) + sg(\tilde{y}),$$

where $c$ denotes the agent's consumption, $z$ and $s$ are the holdings of assets $\pi$ and q , and the assets' payoffs $h$(y), $g$(y). The agent's preference over random lifetime allocations can be described by the expected value of utility function: $E\{U(c,\tilde{c})\}$, where $E$ is the expectations operator. If the agent knows the distribution of payments, then the necessary and sufficient conditions for his decision problem are:

$$p^{\pi}(y)E\{U_1(c,\tilde{c})\} = E\{U_2(c,\tilde{c})h(\tilde{y})\},$$
$$p^{q}(y)E\{U_1(c,\tilde{c})\} = E\{U_2(c.\tilde{c})g(\tilde{y})\}.$$

Using the agent conditions:

$$p^{\pi}(y)\int U_1(c(y),\tilde{c}(\tilde{y}))dF(\tilde{y}) = \int U_2(c(y),\tilde{c}(\tilde{y}))h(\tilde{y})dF(\tilde{y}),$$

where

$$c(y) = h(y) + g(y), \tilde{c}(\tilde{y}) = h(\tilde{y}) + g(\tilde{y}).$$

The relative price volatility of assets with payoff patterns can be ranked according to their systematic risks. Consider the expected real rates of return,

$$r_\pi(y) = \frac{E\{h(\tilde{y})\}}{p^\pi(y)}, \quad r_q(y) = \frac{E\{g(\tilde{y})\}}{p^q(y)},$$

where the difference between the real rates of return $r$ equals the difference between the conditional risk premium of assets, and is a convenient measure of relative risk.

Asset $q$ is less risky than asset $\pi$, if

$$S(\tilde{y};F) := \int_{\underline{y}}^{\bar{y}} s(y')dF(y') \overset{(\geq)}{\leq} 0 \quad \forall \tilde{y} \in [\underline{y}, \bar{y}] \quad \forall F \in \Phi,$$

where $s$ is the difference between the normalized returns on assets $q$ and $\pi$.

## Market Mechanisms

$G$ and $F$ are $C^1$ functions supported over $[o,1]$, and $E$ contains the environment $(G^u, F^u)$.

Satterthwaite and Williams (2002) discuss a market game $\phi_m$ of size $m$ over $E$ that consists of:

a strategy set $A_i$ for each of the $2m$ traders;

an outcome mapping $\zeta_m : \left(\prod_{i=1}^{2m} A_i\right) \times E \rightarrow ([0,1] \times \Re)^{2m}$ that specifies for each agent his probability of receiving a unit along with a transfer as functions of the profile of strategies and the environment;

the selection of a Bayesian-Nash equilibrium in the game previously defined for each environment $(G,F) \in E$. This market game takes the approach that the outcome mapping $\zeta_m$ can depend upon the environment and an equilibrium is specified for each environment, which is a theme in mechanism design.

A market mechanism over $E$ is a sequence $\Phi = (\phi_m)_{m \in N}$ in which $\phi_m$ is a market game of size $m$ over $E$. In the efficient allocation, buyers whose values are among the $m$ values/costs purchase units from sellers whose costs are among the $2m$ agents. The value $\phi_m(G,F)$ denotes the expected gains from trade achieved by the $2m$ agents in the selected equilibrium of the market game. The measure of error in a market game is the relative inefficiency $e(\phi_m, G, F)$, which is the fraction of the expected potential gains from trade in the environment $(G, F)$, that is:

$$e(\phi_m, G, F) = \frac{\Gamma_m(G, F) - \phi_m(G, F)}{\Gamma_m(G, F)}.$$

An individually rational and budget balanced mechanism $\Phi$ is necessarily inefficient, regardless of the size of market *m*, and has the following properties:

symmetry, meaning each buyer uses the function $B_m(.)$ and each seller uses the function $S_m(.)$ to select his bid/ask as a function of his value/cost; non-dominated strategies, i.e. at every $v_i, c_j \in [0,1]$, $B_m(v_i) \le v_i$ and $S_m(c_j) \ge c_j$; non-triviality, meaning the sets $\{v_i | B_m(v_i) > 0\}$ and $\{c_j | S_m(c_j) < 1\}$ have positive measure, which implies that trade occurs with positive probability. An equilibrium satisfying these properties is denoted $(B_m, S_m)$, and each rule for selecting such an equilibrium defines a different approach. The requirement that strategies be non-dominated insures that each equilibrium satisfies individual rationality, and the rule that all trades are consummated at a market-clearing price ensures that every equilibrium satisfies budget balance. The existence of an equilibrium in distributional strategies satisfies non-dominated strategies and non-triviality, which are required for the existence of a symmetric equilibrium in symmetric Bayesian game.

Given a set *E* of environments and a set *M* of mechanisms, a mechanism $\Phi$ is worst-case asymptotic optimal over *E* among mechanisms if there exists a $\eta \in \Re^+$ such that:

$$e^{wr}(\phi_m, E) \le \eta e^{wor}(\phi_m^*, E),$$

for all $m \in \mathrm{N}$.

The mechanism operates over time, communicates trades at a number of different prices, and runs surpluses and deficits. Let $\Phi^{ce} = (\phi_m^{ce})_{m \in \mathrm{N}}$ denote a mechanism with such a property; it can be described as a market game that solves the constrained problem:

$$\min_{\phi_m} e(\Phi_m, G, F),$$

subject to the individual rationality and budget balance conditions. Any individually rational and budget-balanced mechanism satisfies:

$e^{wor}(\Phi_m, E) \ge e(\phi_m, G^u, F^u) \ge e(\phi_m^{cs}, G^u, F^u)$ for each *m*, where if the first inequality is true then $(G^u, F^u) \in E$.

There exists a positive number $\gamma$ such that:

$e(\Phi_m^{ce}, G^u, F^u) \geq \frac{\gamma}{m^2}$ is the relative inefficiency of the constrained efficient mechanism in the environment. An agent's $\alpha$-virtual utility is defined as a function of his value/cost, the environment $(G,F)$, and a parameter $\alpha \in [0,1]$.

Buyer $i$' s $\alpha$-virtual utility function is:

$$\Psi_\alpha^b(v_i) = v_i + \alpha \frac{G(v_i) - 1}{g(v_i)},$$

and seller $j$' s function is:

$$\Psi_\alpha^s(c_j) = c_j + \alpha \frac{F(c_j)}{f(c_j)}.$$

The equilibrium in each market game $\phi_m$ for each environment $(G,F)$ exists if $\phi_m$ allocates the $m$ units to buyers and sellers whose values are such that $t_{(m+1)} \leq t_{(m+2)} \leq .... \leq t_{(2m)}$.

Let $\sigma_m = (v_1, ......., v_m, c_1, ....., c_m)$ *and* $\alpha \in [0,1]$, and for buyer $i$ define $p_\alpha^i(\sigma_m)$ as:

$$p_\alpha^i(\sigma_m) = \begin{cases} 1 & if \quad \Psi_\alpha^b(v_i) \geq t_{(m+1)} \\ 0 & if \quad \Psi_\alpha^b(v_i) < t_{(m+1)} \end{cases},$$

and for seller $j$ define $q_\alpha^j(\sigma_m)$ as:

$$q_\alpha^j(\sigma_m) = \begin{cases} 1 & if \quad \Psi_\alpha^s(c_j) \leq t_{(m)} \\ 0 & if \quad \Psi_\alpha^s(c_j) > t_{(m)} \end{cases}.$$

These are indicator functions that equal one if only if the agent trades in the sample $\sigma_m$ when items are allocated by an $\alpha$-market game.

Let $U_i(V_i)$ *and* $V_i(c_j)$ be the expected utilities of buyer $i$ with value $v_i$ and seller $j$ with cost $c_j$ respectively. The function

$$Sur(\alpha, mG, F) = \wp\left[\left(\sum_{i=1}^{m} \Psi_1^b(v_i) p_\alpha^i(\sigma_m)\right) - \left(\sum_{j=1}^{m} \Psi_1^s(c_j) q_\alpha^i(\sigma_m)\right)\right],$$

is a $2m$-dimensional equation, and it characterizes the efficient market game in this environment, and then expected surplus is:

$$Sur(\alpha,m,G,F) = \wp_{\sigma_m}\left(\sum_{i=1}^{m} r_\alpha^i(\sigma_m) - \sum_{j=1}^{m} s_\alpha^j(\sigma_m)\right).$$

Consider an $A$-mechanism, and for sequence $A = (\alpha_m)_{m\in\mathrm{N}}$ define the market game $\Phi_m^{2,A}$ with a price mechanism $\Phi^{2,A} = (\phi_m^{2,A})_{m\in\mathrm{N}}$. Allocate the $m$ units to the agents with $\alpha_m$-virtual utilities, first by assigning items to those agents whose utilities are above $t_{(m+1)}$, second to buyers whose utilities equal $t_{(m+1)}$, and last to sellers using a fair decision whenever necessary. Each buyer who purchases a unit pays $(\Psi_{\alpha_m}^b)^{-1}(t_{(m)})$ as his price and each seller receives $(\Psi_{\alpha_m})^{-1}(t_{(m-1)})$, and an agent who successfully trades cannot rig prices in his favor by changing his reported value/cost for the dominant strategy of each agent. The $m$ items are allocated to agents in the dominant strategy equilibrium.

Let $H(t_{(m)},t_{(m+1)})$ denote the expected number of trades conditional on the values of the utilities; then the expected surplus is:

$$H(t_{(m)},t_{(m+1)})\left((\Psi_{\alpha_m}^b)^{-1}(t_{(m)}) - (\Psi_{\alpha_m}^s)^{-1}(t_{(m+1)})\right),$$

so buyers pay and $(\Psi_{\alpha_m}^s)^{-1}(t_{(m+1)})$ is the price that sellers receive.

If the environment is regular, then:

$$Sur(\alpha,m,G,F) = \wp\left[H(t_{(m)},t_{(m+1)})\left((\Psi_\alpha^b)^{-1}(t_{(m)}) - (\Psi_\alpha^s)^{-1}(t_{(m+1)})\right)\right],$$

which gives the expression using the $A$-mechanism.

There exists a constant $\tau \in \Re^+$, such that the values $\alpha_m^*(G^u,F^u)$ characterize the efficient market game $\phi_m^{ce}$ in the environment. The expected value of the unrealized gains from trade will be a portion of the losses in the event $D$. This condition implies that the buyer with value $s_{(m+1)}$, and the seller with cost $s_{(m)}$ should trade for the sake of efficiency. The unrealized gains from trade are at least $s_{(m+1)} - s_{(m)}$ in event $D$. Define $w = s_{(m+1)} - s_{(m)}$ and let $p(w;m)$ denote its density function giving, for any value of $w$, the probability that $s_{(m+1)}$ is a buyer's value and $s_{(m)}$ is a seller's cost.

To show that $\gamma > 0$ we regard $\tau$ as a variable and turning to $e(\phi_m^{ce},G^u,F^u)$, we have:

$$e\left(\phi_m^{ce}, G^u, F^u\right) = \frac{\Gamma_m\left(G^u, F^u\right) - \phi_m^{ce}\left(G^u, F^u\right)}{\Gamma_m\left(G^u, F^u\right)} > \frac{\gamma}{m\Gamma_m\left(G^u, F^u\right)}.$$

The expected gains from trade $\Gamma_m\left(G^u, F^u\right)$ are limited by the fact that at most *m* trades can be made, each value one or less. The rate at which its worst case error converges to zero is as fast as possible, and quantifies a sense in which trade should move to using other mechanisms. The mechanism reduces performance over all environments, and it summarizes performance over all sizes of markets with a rate as the sole statistic.

## Public Information

The value obtained by the firm, denoted by $V_0$ is the expected liquidation value obtained from the following density function with success probability *r*:

$$V_0 = \sum_{s=0}^{N} \binom{N}{s} (ru)^s \left((1-r)d\right)^{N-s} = \left(ru + (1-r)d\right)^N.$$

Shin (2003) supposes that at the initial date nothing is known about the value of the firm before some time passes. At an interim time, not enough time has elapsed for the agent to know the outcomes of all projects, but the outcomes of some of the projects will have been realized. There is a probability $\theta$ for a given outcome of a project, and whether the outcome is revealed/known is independent across projects. By the date 2, uncertainty is resolved, and outcomes of all the projects become common knowledge. The firm is liquidated, and consumption takes place, without disclosing how much private information the agent has at the time of disclosure if such information is deemed to be unfavorable.

The information available to the agent at the interim date can be summarized by the pair $(s, f)$, where *s* is the number of successes observed and *f* is the number of failures observed, and the agent's disclosure strategy *m* maps his information to the pair $(s', f')$.

Since the initial and final price of the firm is based on information, this gives rise to a game of incomplete information. The market is modeled as an agent in the game who sets the price of the firm to its actuarially fair value based on all the available evidence. The market's strategy is the pricing function:

$$(s', f') \mapsto V_1,$$

where the market aims to set the price of the firm to its fair value in the game in order to minimize the loss function:

$$(V_1 - V_2)^2,$$

where $V_2$ is the liquidation value of the firm.

The market then sets $V_1$ equal to the expected value of $V_2$, as generated by its disclosure strategy, and the agent anticipates the optimal response of the market, and chooses the disclosure that maximizes $V_1$. The expected value of the firm is:

$$u^s d^f \sum_{i=0}^{N-s-f} \binom{N-s-f}{i} (ru)^i ((1-r)d)^{N-s-f-i} = u^s d^f (ru + (1-r)d)^{N-s-f}$$

We will show that this strategy can be supported in equilibrium, and will derive the implications for the agent in the sense that:

$$V_1(s,f) \geq V_1(s',f').$$

We will confine our attention in what follows to an equilibrium in which the agent uses a strategy such that the interim prices that occur with positive probability are of the form $V_1(s,0)$. At the interim date, the agent's disclosure at date 1 of successes up to that point cannot exceed realized successes, as denoted by:

$$h(k|s),$$

the probability of $k$ realized successes on disclosure of $s$ successes.

Let $q = (r - \theta r)/(1 - \theta r)$ when the agent follows the strategy

$$h(k|s) = \begin{cases} \binom{N-s}{k-s} q^{k-s}(1-q)^{N-k} & if \quad s \leq k \\ 0 & othervise. \end{cases}$$

The residual uncertainty can also characterized by a density function in which the probability of success of an undisclosed project is given by $q = (r - \theta r)/(1 - \theta r)$. To state the closure property it is better to use the comparative information, the $q$-matrix is an upper bound with the property: $B(p)B(q) = B(p + q - pq)$. The $(i,s)$-th entry of $B(p)B(q)$ is given by:

$$\sum_{j=i}^{s} \binom{N-i}{j-i} p^{j-i}(1-p)^{N-j} \binom{N-j}{s-j} q^{s-j}(1-q)^{N-s} = \binom{N-i}{s-i} (p+q-pq)^{s-i}[(1-p)]^{N-s}.$$

In particular, for $p = r\theta \ and \ q = (r - r\theta)/(1 - r\theta)$, we have:

$$B(r) = B(p)B(q),$$

where distribution over successes must be equal to the average of the distributions weighted by the probability of each disclosure. When $\theta$ is large, the agent is well informed about the true number of successes, and the agent is fully informed in the limit. The market's response is to draw all the weight, so the agent is not always fully informed, and the market must make allowances for some pooling between genuinely uninformed types of the agent. Any favorable shift in the disclosure of successes is matched by a corresponding opposite shift in the posterior probability of success, and the trade off between $p$ and $q$ is given by:

$$\frac{dq}{dp} = -\frac{1-q}{1-p}.$$

There is a sequential equilibrium in which the agent uses the strategy:

$$V_1(s) = u^s(qu + (1-q)d)^{N-s},$$

where $q = (r - \theta r)/(1 - \theta r)$.

The equilibrium pricing rule must specify a price for all feasible reports $(s, f)$, and the beliefs given these out-of-equilibrium disclosures must receive positive probability. The agent follows this strategy, while the market's pricing rule is given by:

$$V_1(s, f) = \begin{cases} u^s(qu + (1+q)d)^{N-s} & if \quad f = 0 \\ u^s d^{N-s} & if \quad f > 0. \end{cases}$$

Then define the sequence $\{q_t\}_{t=0}^{T}$ as follows:

$$q_0 = r,$$

$$q_t = \frac{q_{t-1} - q_{t-1}\theta_t}{1 - q_{t-1}\theta_t},$$

where $\theta_t$ is increasing over time, $q_t$ is decreasing over time. At date $t$, the probability that the agent announces the success of a project is given by $r\theta_t$. The uncertainty over the disclosed projects is governed by the density function with success probability $q_t$, and is greatest when it is near the edge of the spiral. Residual uncertainty remains when the agent is not able to reveal any successes or failures; the mapping is from the interim value of the firm at a given date to the size of the residual uncertainty. When information arrives through the disclosures of interested agents, the residual uncertainty is at its greatest when the news is bad and corresponds to the greatest asymmetry of information. The first period return can be denoted as $R_1 = \frac{V_1}{V_0}$, and is a random variable that takes value:

$$\frac{u^s(qu + (1-q)d)^{N-s}}{(ru + (1-r)d)^N},$$

with probability:

$$\binom{N}{s}(r\theta)^s(1-r\theta)^{N-s}.$$

The second period return, denoted as $R_2 = \frac{V_2}{V_1}$ is the random variable taking value:

$$\frac{u^{s+j}d^{N-s-j}}{u^s(qu+(1-q)d)^{N-s}},$$

with probability:

$$\binom{N-s}{j}q^j(1-q)^{N-s-j}.$$

In such an environment, more information is disclosed voluntarily by the firms leading to differences in value, and agents must be compensated for the increased uncertainty of returns. The price of the firm at the interim date is obtained from the prices across the realized numbers of successes at that date. The price of the firm at the interim date as a function of the disclosed number of successes $s$ is:

$$\frac{u^s\sum_{i=0}^{N-s}\binom{N-s}{i}q^i(1-q)^{N-s-i}\left(u^{s+i}d^{N-s-i}\right)^{1-\alpha}}{\sum_{i=0}^{N-s}\binom{N-s}{i}q^i(1-q)^{N-s-i}\left(u^{s+i}d^{N-s-i}\right)^{-\alpha}} = u^s\left[\frac{qu^{1-\alpha}+(1-q)d^{1-\alpha}}{qu^{-\alpha}+(1-q)d^{-\alpha}}\right]^{N-s}.$$

The expected payoff at the final date is $u^s(qu+(1-q)d)^{N-s}$, so that the expected second period return is:

$$E(R_2|s) = \left(\frac{qu+(1-q)d}{\pi u+(1-\pi)d}\right)^{N-s},$$

where $\pi = \frac{qu^{-\alpha}}{qu^{-\alpha}+(1-q)d^{-\alpha}}$.

Risk averse investors expect a return that is higher than the actuarially fair rate, and the more risk averse the investors are, the greater must be the price discount. The greater the residual uncertainty, the lower must be the price relative to its expected payoff in order to compensate a risk averse investor, and the greater is the information asymmetry between the agent and the market. For any $k > s$, we have, by Bayes' rule:

$$\frac{h(k|s)}{h(k-1|s)} > \frac{h(k+1|s+1)}{h(k|s+1)} \Leftrightarrow \frac{kh(k)}{h(k-1)} > \frac{(k+1)h(k+1)}{h(k)},$$

where the density $h(.)$ over the number of successful projects is such that:

$\frac{kh(k)}{h(k-1)}$, which is a decreasing function of $k$.

The theory has been developed in the context of corporate disclosure, and sovereign risk has been difficult to capture in an asset pricing setting since the notion of default in opportunistic behavior.

# 11 Stability of Markets, Uncertainty, and Strategic Behavior

The expected discounted present values of the investment options facing any particular agent at any point in time depend upon the current and future investment strategies adopted by all agents. The trade-offs that agents face in this environment appear to be that agents might be expected to learn the strategies, and the Nash equilibrium predictions have be tested using populations of rational agents. Duffy (2001) supposes the agents are constrained to playing strategies consistent with the Nash equilibrium where the best response of the agents is to speculate.

There are agents of types $i = 1,2,3$ and type $i$ desires to consume good $i$, but produces good $i + 1$ modulo 3. The per period cost of storing one unit of good $j$ is $c_j$ and it is assumed that $0 < c_1 < c_2 < c_3$, and all agents are assumed to receive the same utility from consumption $u > c_3$, and have a discount factor $\beta \in (0,1)$. In every period, all $N$ agents are randomly paired with one another. Each pair engage in bilateral exchange, and when an agent successfully trades for his consumption good $i$, he consumes that good receiving utility $u$. When an agent is unsuccessful in trading for his consumption, his net payoff for the round is negative, corresponding to the storage cost of the good he holds after any trading has occurred. In situations where an agent $i$ with production good $i + 1$ in storage, is randomly paired with an agent storing good $i + 2$, consider agent $i$'s strategy $s_i(t)$. Let $s_i(t) = 0$ if type $i$ refuses to trade good $i + 1$ for good $i + 2$ and let $s_i(t) = 1$ if type $i$ offers to trade good $i + 1$ for good $i + 2$. Each agent solves problem in which they maximize the expected present value of their discounted utility from consumption net of storage costs over an infinite horizon by choosing optimal trading strategies. A Nash equilibrium calls for type 2 agents to always offer to trade their high storage cost good 3 for the less -costly-to store good 1 $(s_2 = 1)$, and for type 3 agents to refuse to offer to trade their low storage cost good 1 for the more costly-to-store good 2 $(s_3 = 0)$, and these trading strategies, referred to as storage cost considerations, govern the trading decision. Let the net payoff in period $t$ to individual agent $j$, of type $i$, from storing good $i + 1$ be denoted by

$$v_{i+1}^{j}(t) = \sum_{\tau=1}^{t-1} I^{s}(\tau)\gamma_{i+1} - \sum_{\tau=1}^{t-1} I^{f}(\tau)\gamma_{t+2},$$

where $I^{s}(\tau)$ is an function that is equal to 1 if agent $j$ was storing good $i + 1$ in period $\tau$ and was successful in trading it for good $i$, and is equal to zero otherwise, and $I^{f}(\tau)$ is an function that is equal to 1 if agent $j$ was storing good $i + 1$ in period $\tau$ and failed to trade it for good $i$, and is equal to zero otherwise. Accordingly, the opportunity cost to agent type $i$ of storing good $i + 1$ and failing to trade for good $i$ in the following round is $\gamma_{i+2}$. Let the type $i$ agent's utility gain from storing good $i + 1$ at time $t$ be denoted by $\gamma_{i+1} = -c_{i+1} + \beta u.$

When the agent fails to trade either good $i + 1$ or good $i + 2$ for good $i$, the payoff of storing either good is negative and equal to the foregone net utility gain the agent would have received had he held the other good in storage. We define the probability that agent $j$ of type $i$ plays strategy $s_i = 0$ by:

$$\Pr\left[s_i^j(t) = 0\right] = \frac{e^{x_i^j(t)}}{1 + e^{x_i^j(t)}}.$$

Recall that the strategy $s_i = 0$ is the strategy of refusing to trade the type $i$ agent's produced good $i + 1$ for good $i + 2$. The probability that agent $j$ of type $i$ plays strategy $s_i = 1$ is then given by $1 - \Pr\left[s_i^j(t) = 0\right]$.

In our discussion of the equilibrium selection problem, we can note that the information tends to be perfectly transmitted, as preferences tend to coincide. There are agents, a sender $S$ and a receiver $R$, and $S$ observes the value $s$, and then sends a message to $R$; $R$ then chooses some action $x$ in $\left[\underline{x}, \overline{x}\right]$ which affects his utility as well as $S$'s.

If the equilibrium exists with $p$ intervals then for each $q \leq p$ there exists one with $q$ intervals as well where no information is transmitted at all. Spector (2000) supposes that for any $p$ there exists $\varepsilon_0 > 0$ such that if $\varepsilon < \varepsilon_0$ there exists an equilibrium in which the message set has $p$ elements $m_1, \ldots\ldots, m_p$, and $S$ sends the message $m$. In a general theoretical structure the problem can be converted into one about solving a differential inequality, but the results are heuristic.

## Consumption and Portfolios

We study the problem of optimal consumption and portfolio rules for an agent whose preferences over consumption are given as:

$$E\left[\int_0^T e^{-\delta t}u(z(t),y(t))dt\right],$$

where

$$z(t) = z_0 e^{-\beta t} + \beta\int_0^t e^{-\beta(t-s)}c(s)ds$$

$$y(t) = y_0 e^{-\phi t} + \phi\int_0^t e^{-\phi(t-1)}c(s)ds$$

The consumption rate process $c(t)$ denotes the consumption at time $t$ . The processes $z(t)$ and $y(t)$ are derived from consumption using the weighting factors $\beta$ *and* $\phi$, with $\beta > \phi$ ,and $z_0 \geq 0$ *and* $y_0 \geq 0$ are given constants and $\delta > 0$ captures the impatience of the agent. Yang (2000) supposes agents with such preferences treat consumption and the felicity function as depending only on weighted averages of past consumption. The model represents preferences over service flows from irreversible purchases of a durable good that decays over time. The good provides a flow of services proportional to the stock $z(t)$ of a good at time *t*. The standard of living of the agent is given by $y(t)$ and reflects past consumption experience, and is at a much slower rate than the stock of the durable $(\beta > \phi)$. This represents the preferences for consumption of a dual-purpose commodity. We provide the agent with sources of utility that have different half-lives. Labor income follows an arbitrarily bounded stochastic process, short-sale and borrowing constraints, as well as incomplete markets. The uncertainty is represented by a probability space and the investment opportunities are represented by $n + 1$ long-lived assets. Its price process, denoted by $B$, is given by

$$B(t) = \exp\left(\int_0^t r(\tau)d\tau\right),$$

for some interest rate process $r$. The remaining $n$ assets are risky, and letting $S$ denote their price process and $D$ their cumulative dividend process, we assume that :

$$S(t) + D(t) = S(0) + \int_0^t I_S(\tau)\mu(\tau)d\tau + \int_0^t I_S(\tau)\sigma(\tau)dw(\tau)$$

where $I_S(t)$ denotes the matrix with elements $S(t)$. Trading takes place continuously, where $\alpha(t)$ and $\theta_k(t)$ denote the money amount invested at time $t$ in bonds and the $k$th risky asset, with the set of admissible trading strategies being denoted by $\Theta$. Preferences for an agent over consumption patterns are given by

$$U(c) = E\left[\int_0^T e^{-\delta t} u(z(t), y(t)) dt\right].$$

The agent is endowed with some initial wealth and non-negative stochastic income process $d$ with

$$\int_0^T B(t)^{-1} d(t) dt \le K_d \quad \text{for some } K_d > 0.$$

## Relative Risk

Information about the risk characteristics of the asset is revealed by the behavior of equilibrium prices and depends on the agent's attitudes toward risk. The volatility of equilibrium asset prices is related to the risk of the assets if the measure of future period relative risk aversion is strictly decreasing in current period of consumption. It is evident that the aggregate consumption and the prices of both assets are positively related to the random variable $y$. Suppose that $y$ takes on a high value in period 1. Then the agent is more risk averse in period 2, and if $y$ in period 1 is low, then current consumption is low and the agent discounts the risk associated with random future asset returns at a lower rate.

Thus portfolios of assets with relatively stable dividends can exhibit considerable price volatility, which would be compared to the dividend process.

We consider the problem of a regulator who desires to contract with an agent for the delivery of $q$ units of a good. The agent's productivity parameter $\theta$ is known by the agent, but the regulator only assesses it with a probability distribution $F(.)$ over $\Theta \equiv \lfloor \theta, \bar{\theta} \rfloor$. Bontems and Bourgeon (2000) suppose that production $q$ depends on both the agent's productivity parameter $\theta$ and the agent's effort $a$, according the relation $q = Q(a, \theta)$. The type-$\theta$ agent's objective is to maximize the following utility:

$$t - V(a, \theta),$$

where $t$ is the transfer given by the regulator. High productivity agents are efficient but expensive, whereas low productivity agents are less efficient but cheaper. The market maker may base contracts upon the production level $q$ or the effort level $a$ at the same time. With a production-based contract, the type-$\theta$ agent who announces $\tilde{\theta}$ has to exert an effort $a = h(q(\tilde{\theta}),\theta)$ that satisfies

$$q(\tilde{\theta}) = Q(h(q(\tilde{\theta}),\theta),\theta)$$

to obtain the corresponding transfer $t_q(\tilde{\theta})$. His ex-ante utility is given by

$$u_q(\theta,\tilde{\theta}) \equiv t_q(\tilde{\theta}) - V(h(q(\tilde{\theta}),\theta),\theta).$$

Rearranging term yields

$$\dot{U}_q(\theta) = \Delta(h(q(\theta),\theta),\theta)\partial_a V(h(q(\theta),\theta),\theta)$$

where $\Delta$ is the function given by

$$\Delta(a,\theta) = \frac{\partial_\theta Q(a,\theta)}{\partial_a Q(a,\theta)} - \frac{\partial_\theta V(a,\theta)}{\partial_a V(a,\theta)}.$$

$\Delta$ is the difference between marginal rates of substitution of effort for the agent's type, corresponding respectively to the production side and the disutility side. With an effort-based contract, the utility of a type-$\theta$ agent who announces $\tilde{\theta}$ is given by

$$u_e(\theta,\tilde{\theta}) \equiv t(\tilde{\theta}) - V(e(\tilde{\theta}),\theta).$$

The rate of growth of informational rents for an incentive contract is given by

$$\dot{U}(\theta) = \dot{U}_q(\theta) - p(\theta)(\dot{U}_e(\theta) - \dot{U}_q(\theta)).$$

With probability $p$, the agent is thus able to exploit the difference between $\dot{U}_e$ and $\dot{U}_q$ to modify $\dot{U}$. The agent, chooses $c \in C$ in order to maximize her welfare $W$, which is restricted to the sum of the consumer and taxpayer surplus

$$W(c) \equiv p(\theta)(Q(e(\theta),\theta) - t_e(\theta)) + (1 - p(\theta))(q(\theta) - t_q(\theta)).$$

The optimal contract is consequently the same as in a situation where she is constrained to use only output schemes, and when agents share the same effort disutility function, input-based contracts allow the agent to reach the first best allocation. Indeed, production based contracts involve an additional cost as the agent has to compensate the low type-agents; the rent gained overstating their type is higher the lower their disutility of effort. Under an output-based contract, the agents' incentive to overstate their type is reduced since it implies a higher production goal to be met with a low productivity. So $\dot{U}_q$, the rate of growth of informational rents associated with an output-based contract, is positive.

## Technology Adjustment

Moretto (2000) constructs a technology-adoption continuous-time workable model that incorporates the property that it is more expensive to go first than when others have already done so. The strategic nature of the waiting time effect leads to delayed adoption, which reinforces the option value of waiting due to irreversibility.

Agents belonging to the same industry are confronted with the opportunity to abandon their present production process, and to adopt a new one by paying a sunk switching cost *C*. However, agents face uncertainty about the benefits per unit of time $x_t$, which are described by a Brownian motion:

$$dx_t = \alpha x_t dt + \sigma x_t dW_t \text{ , with } \alpha, \sigma > 0 \text{ ,}$$

where $dW_t$ is the increment of a standard Wiener process. When the evolutionary pattern of the technological change's operating benefits is independent of the number of agents who have abandoned the old technology, we assume that the agent's investment cost depends on the number of agents who have already adopted the new technology:

$$C_i(\theta, n) = \theta_i k(n), \quad i = 1, \ldots t \quad n = 1, \ldots .t \text{ ,}$$

where *k*(a) stands for the pure investment costs, net of transition costs and the old technology's value, and $\theta_i$ s player *i* s private valuation parameter, reflecting agent *i*'s private perception of forgone potentially more valuable investment opportunities in the future.

Whilst *k*(2) and *k*(1) are common knowledge, $\theta_i$ is private information , and takes on values in $\Theta \subseteq R_+$ with cumulative distribution $G(\theta_i)$ and density $g(\theta_i)$, which are public knowledge and common to both agents. Agents' types are independent, so they do not convey information about the other agents' valuation parameters. If agent *i* takes the option value, evaluated at time zero, this can be expressed as:

$$V_i(x;1) = \max\left[0, E_0\left\{\int_{T_i}^{\infty} x_t e^{-rt} dt - \theta_i k(1) e^{-rT_i} \middle| x_0 = x\right\}\right] \text{ ,}$$

while for agent *j* :

$$V_j(x;2) = \max\left[0, E_0\left\{\int_{T_j}^{\infty} x_t e^{-rt} dt - \theta_j k(2) e^{-rT_j} \middle| x_0 = x\right\}\right] \quad for\ j \neq i \text{ ,}$$

where $r > \alpha$ is the discount rate , and $T_j \geq T_i$ stands for the stochastic stopping times at which agents will find it convenient to change the technology. We can interpret $\theta k(2)e^{-rT}$ and $\theta k(1)e^{-rT}$ as the opportunity costs of switching technologies with and without network benefits respectively. Since we are assuming that the investment expenditure cannot be recovered, adoption of a new technology is slowed down by the uncertainty about the technological change's net operating benefits. The technological change is decelerated by each agent's hope to get the network benefit $\theta[k(1) - k(2)]$, and the uncertainty about the other agent's investment opportunity cost makes it valuable to wait to see how things go for the others before switching.

Specifically, each agent *i* will optimally select the change time according to his current information about the state *x* and the distribution parameter *G*, as the opportunity cost of switching depends on $\theta$, and *x* is assumed to determine the agent's actions, and the investment ability per unit of time does not depend on the number of agents. In continuous time, we represent the agent's countervailing interests by the following strategy. Let us begin by determining the optimal investment rule, i.e. when each agent chooses his trigger value and continues to use it regardless of how the game evolves. If threshold levels $\tilde{x}_i^+$ and $\tilde{x}_i^{++} \in X$ exist such that $0 < \tilde{x}_i^{++} < \tilde{x}_i^+$, then an equilibrium involves each agent playing the following strategy:

$$V_i(x;\tilde{x}_i^{++}) = A_i(\tilde{x}_i^{++})x^{\beta_i} \equiv \left(\frac{\tilde{x}_i^{++}}{r-\alpha} - \theta_i k(2)\right)\left(\frac{x}{\tilde{x}_i^{++}}\right)^{\beta_i} \quad \text{for } i = 1,\ldots,t$$

and

$$V_i(x;\tilde{x}_i^{+}) = A_i(\tilde{x}_i^{+})x^{\beta_i} \equiv \left(\frac{\tilde{x}_i^{+}}{r-\alpha} - \theta_i k(1)\right)\left(\frac{x}{\tilde{x}_i^{+}}\right)^{\beta_i} \quad \text{for } i = 1,\ldots,t.$$

We will identify three operating actions: if *x* is less than $\tilde{x}_i^{++}$, the agent never switches. If *x* falls into the action $[\tilde{x}_i^{++}, \tilde{x}_i^{+}]$, the agent waits until the other agent has switched before switching himself, and if *x* rises to $\tilde{x}_i^{+}$ without the other agent changing, he switches unilaterally. Agents should wait to obtain more information about the difference in revenues between the old technology and the new one before switching. The use of information regarding a rival's position allows one agent to wait longer, in hope of inducing the other to give up, and then to gain the resulting network benefits. Agent *i*' s option value at time zero of investing at time *T*, if agent *j* is still using the old technology and agent *j*' s strategy is *T*, is given by:

$$V_i\left(x;\tilde{x}_i^*\right) = V_i^S\left(x;\tilde{x}_i^*\right) + V_i^F\left(x;\tilde{x}_i^*\right).$$

The value to invest is given by the option value. The agent observes the realization of the state variable $x$, moreover, as time goes by. If $x$ hits new upper levels without the other agent switching, he learns that the probability of this happening in future may decrease. Using Bayes' rule the relationship is given by:

$$F_t\left(\tilde{x}_j^*;u_t\right) = \frac{F\left(\tilde{x}_j^*;x\right) - F\left(u_t;x\right)}{1 - F\left(u_t;x\right)}, \text{ where } u_t = \sup_{0<s<1}\left(x_s\right).$$

The agent invests and gets the benefit measured by the discounted value of the state variable $x$ from $T$ downward. He gives up the opportunity to invest, valued at $V_i(x)$. The information about the state is in the variable $x$ and is revised as soon as new information becomes available. The social optimum is always obtainable when there is no longer any lack of information. If the investment cost benefit $\theta$ is public information, the Nash equilibrium in pure strategies involves coordination. There is a common value $\tilde{x}^{**}$ above which agents coordinate their switch to the new technology. When $\theta_1, \theta_2$ are known to agents and if $x \geq \tilde{x}^{**} = \sup\left(\tilde{x}_1^{**}, \tilde{x}_2^{**}\right)$, then the perfect equilibrium involves all agents switching.

For each period, $n$ =1,2,.... , the agent $\alpha \in I$ receives a random endowment $Y^\alpha(w) = Y_n(\alpha, w)$ in units of a single commodity.

## Money and Consumption

In the commodity market, agents bid money for consumption of a commodity, thereby determining its price $p_n(w)$. In Geanakoplos, *et al.* (2000) interest rates are assumed to satisfy:

$$1 \leq r_{2n}(w) \leq r_{1n}(w) \quad r_{2n}(w) < \frac{1}{\beta},$$

where $\beta \in (0,1)$ is a fixed discount factor. At the beginning of day $t$ = n, the price of the commodity is $p_{n-1}(w)$ and the total amount of money held in the bank is $M_{n-1}(w)$. An agent $\alpha \in I$ enters the day with wealth $S_{n-1}^\alpha(w)$. If $S_{n-1}^\alpha(w) < 0$, then the agent has an unpaid debt from the previous day, which is assessed a punishment of $u^{\alpha}\left(S_{n-1}^{\alpha}(w)\ /p_{n-1}(w)\right)$, and the debt is then forgiven. If $S_{n-1}^{\alpha}(w) \geq 0$, then the agent has fiat money on hand and plays from position *S*. The agent also begins day *n* with information $F_{n-1}^{\alpha} \subset F$, a $\sigma$-algebra of events that measures past prices $p_k$, past total endowments $Q_k$ and interest-rates $r_{1,k}, r_{2,k}$, as well as past wealth-levels, endowments, and actions $S_0^{\alpha}, S_k^{\alpha}, Y_k^{\alpha}, b_k^{\alpha}$ for $k$ =1,….,n-1. Based on this information, the agent bids an amount:

$$b_n^{\alpha}(w) \in \left[0, \left(S_{n-1}^{\alpha}(w)\right)^+ + k^{\alpha}\right],$$

of fiat money for the commodity on day *n*. The constant $k^{\alpha} \geq 0$ is an upper bound on allowable loans. Consequently, the total bid is given by:

$$B_n(w) = \int b_n^{\alpha}(w)\varphi(d\alpha) > 0.$$

The total utility that agent receives during the period is:

$$\xi_n^{\alpha}(w) = \begin{cases} u^{\alpha}\left(x_n^{\alpha}(w)\right), & if\ S_{n-1}^{\alpha}(w) \geq 0 \\ u^{\alpha}\left(x_n^{\alpha}(w)\right) + u^{\alpha}\left(S_{n-1}^{\alpha}(w)\ /p_{n-1}(w)\right), & if\ S_{n-1}^{\alpha}(w) < 0 \end{cases}.$$

The total payoff for agent during the entire duration is the discounted sum $\sum_{n-1}^{\infty} \beta^{n-1}\xi_n^{\alpha}(w)$. A strategy $\pi^{\alpha}$ for an agent specifies the bids *b* as random variables that are *F* measurable for every $n \in N$. A collection $\Pi = \{\pi_{\alpha}, \alpha \in I\}$ of strategies for all agents is admissible. Now there are three possible situations for agent on day *n:*

(a) The agent is a depositor: this means that bid $b_n^{\alpha}(w)$ is strictly less than his wealth, and he deposits the difference:

$$I_n^{\alpha}(w) = S_{n-1}^{\alpha}(w) - b_n^{\alpha}(w) = \left(S_{n-1}^{\alpha}(w)\right)^+ - b_n^{\alpha}(w).$$

At the end of day, agent gets back his deposit with interest, as well as his endowment's worth in fiat money and thus, moves to the new wealth level:

$$S_n^{\alpha}(w) = r_{2n}(w)I_n^{\alpha}(w) + p_n(w)Y_n^{\alpha}(w) > 0.$$

(b) The agent is a borrower; he must borrow the difference:

$$d_n^{\alpha}(w) = b_n^{\alpha}(w) - \left(S_{n-1}^{\alpha}(w)\right)^+.$$

Thus, the agent pays back the amount:

$$h_n^{\alpha}(w) = \min\left\{r_{1n}(w)d_n^{\alpha}(w), p_n(w)Y_n^{\alpha}(w)\right\}$$

and his cash holdings at the end of the period are $\left(S_n^\alpha(w)\right)^+ = p_n(w)Y_n^\alpha(w) - h_n^\alpha(w)$.

(c) The agent neither borrows and not lends, he bids his entire cash-holdings $b_n^\alpha(w) = \left(S_{n-1}^\alpha(w)\right)^+$ and ends the day with exactly his endowment's worth in fiat money:

$$S_n^\alpha(w) = p_n(w)Y_n^\alpha(w) \geq 0.$$

We can write a formula for the agent's wealth position at the end of period:

$$S_n^\alpha(w) = p_n(w)Y_n^\alpha(w) + r_{2n}(w)I_n^\alpha(w) - r_{1n}(w)d_n^\alpha(w),$$

and another formula for agent's cash-holdings:

$$\left(S_n^\alpha(w)\right)^+ = p_n(w)Y_n^\alpha(w) + r_{2n}(w)I_n^\alpha(w) - h_n^\alpha(w).$$

A real possibility in an agent's optimization problem involves properties of the invariant measures for associated optimally controlled Markov chains.

## Dynamics of Money

The selling mode presents an agent $N$ who sells goods which are traded via $N-1$ intermediary agents to consumers at level $n = 0$. The goods are returned by a second chain where agent $N$

buys goods, which are traded via $N-1$ different intermediaries following the buying mode. The circular geometry now allows the scenario that in selling (or buying) mode each agent can sell to (or buy from) $z-1$ agents. Bornholdt and Wagner (2002) suppose the agents to constitute the sites or nodes, while goods and money flow along the links of the tree. For the amount of goods flowing between agents we use the variable:

$$\overline{q}_{n,i} = \frac{1}{(z-1)^{n-1}} q_{n,i}.$$

If an agent $n$ sells $q$ at the price $p$ he gains the utility:

$$u_n^{(S)} = I_n \overline{q} p - \overline{c}(\overline{q}), \text{ where}$$

$I_n$ denotes the value of money and $\overline{c}(\overline{q})$ the decrease of $u$ by losing $q$. If the agent buys $q$ at price $p$ the utility reads:

$$u_n^{(B)} = d(\overline{q}) - I_n \overline{q} p.$$

In equilibrium, all the money values $I_n$, the goods $q_n$ increase with $(z-1)^{n-1}$, and each agent $n$ can choose his own money value, amount of bought goods $q_{n+1}$, and price $p_{n,i}$ for sold goods $q_{n,i}$. For the utility functions $d$ and $\bar{c}$, power laws have been used:

$$d(x) = \frac{1}{\beta} x^{\beta} \quad \beta < 1$$
,

and for $\bar{c}$ a power law is:

$$\bar{c}(x) = \frac{1}{\alpha} x^{\alpha} \quad with \quad \alpha > 1$$
.

And we get:

$$c(r) = \frac{\alpha - \beta}{\alpha\beta} (\beta r)^{\alpha/(\alpha-\beta)}$$
.

We have the optimization for each agent $n$:

$$u_n^{(B)} = \frac{1}{\beta} \bar{q}_{n+1}^{\beta} - I_n \bar{q}_{n+1} p_{n+1} \quad n = 0, \ldots\ldots, N-1$$
,

$$u_n^{(S)} = I_n \sum_i \left[ \bar{q}_{ni} p_{ni} - \bar{c}(\bar{q}_{ni}) \right] \quad n = 1, \ldots\ldots, N.$$

The resulting values of traded goods and money flow from $g_{n,i}$, $n-1,\ i$, to $n$:

$$g_{n,i} = q_{n,i} p_{n,i}$$
,

where g and $q$ are given by:

$$q_{n,i} = (z-1)^{n-1} \left[ c'\left( \frac{I_n}{I_{n-1,i}} \right) \right]^{1/\beta}$$
,

$$g_{n,i} = (z-1)^{n-1} \frac{1}{I_{n-1,i}} c'\left( \frac{I_n}{I_{n-1,i}} \right)$$
.

The value of utilities at the maximum are given by:

$$u_n^{(B)} = \frac{1-\beta}{\beta} c'\left( \frac{I_{n+1}}{I_n} \right)$$
,

and by

$$u_n^{(S)} = \sum_{i=1}^{z-1} c\left( \frac{I_n}{I_{n-1,i}} \right)$$
.

This leads to:

$$q_{n,i} = (z-1)^{n-1}\left[c'\left(\frac{I_{n-1,i}}{I_n}\right)\right]^{1/\beta},$$

$$g_{n,i} = (z-1)^{n-1}\frac{1}{I_n}c'\left(\frac{I_{n-1,i}}{I_n}\right),$$

and the utility functions at maximum are:

$$u_n^{(B)} = \frac{1-\beta}{\beta}\sum_{i=1}^{z-1} c'\left(\frac{I_{n-1,i}}{I_n}\right)$$

$$u_n^{(S)} = c\left(\frac{I_n}{I_{n+1}}\right).$$

The agents are on a circle and consider the money ratio at site *n*, and in the selling mode we have:

$$\Delta g(n+1,n,i) = (z-1)\frac{g_{n-1,i}}{g_n} = \frac{I_n}{I_{n-1,i}}\left[c'\left(\frac{I_n}{I_{n-1,i}}\right)/c'\left(\frac{I_{n+1}}{I_n}\right)\right].$$

Therefore in the selling mode $r < 1$ implies inflation, while values $r > 1$ imply deflation. In the buying mode *r* is given by:

$$\Delta g(n+1,n,i) = \frac{I_{n+1}}{I_n}\left[c'\left(\frac{I_{n-1,i}}{I_n}\right)/c'\left(\frac{I_n}{I_{n+1}}\right)\right].$$

The change of $I_n$ results in new *q*, *p* values and for $z > 2$ money conversation involves a sum of $\Delta g$ over *i*. Suppose agent *n* sells the amount $q = \sum_i q_{n,i}$, which is bought by agents $n-1,i$; if they cooperate, they optimize their common utility:

$$u^{(B)} = \sum_i \frac{1-\beta}{\beta}\overline{q}_{n,i}^{\beta},$$

which as a function of $I_{n-1,i}$, subject to $\beta < 1$ $u^{(B)}$, has a maximum for $q_{n,i}$, which in turn implies $I_{n-1,i}$. In terms of the ratios *r* it reads:

$$c'(r_n) = r_{n-1}c'(r_{n-1}).$$

For power laws the recursion depends on the ratio:

$$\gamma = \frac{\beta}{\alpha},$$

which can be called elasticity, and is given by:

$$\ln r_n = \gamma^{N-1-n} \ln r_{N-1}.$$

Now $r_0'$ can be chosen arbitrarily due to the replacement $r_n \to 1/r_n$:

$$\ln r_n' = \gamma^n \ln r_0'.$$

The money values $I_N$ of agent $N$ and $I_0$ of the other agents are random, and their choice depends on the relative weight the agents place on their utilities in the buying or selling mode. A seller dominated scenario leads to $I_N > I_0$, and the ratio:

$$\frac{\bar{q}_N}{\bar{q}_0} = \left(\frac{I_N}{I_0}\right)^{1/(\alpha-\beta)},$$

which may find values. Any possible utility function for the dynamics would be rather complicated, since $\Delta g$ on a tree connects agent $n+1$ with agents $n-1,i$ corresponding to a next-to-next neighbor interaction. The dynamics of the money values will be based on a utility function $H$, where $H_M$ contains the effect of the market maker, and $H_A$ is due to the agents. The of utilities can be solved in the selling mode by:

$$u_A(z) = c(z) + \frac{1-\beta}{\beta} c'(z).$$

The market maker's part must favor $\Delta g(n+1,n,i) = 1$, and this establishes money conversation and a certain cooperation between the agents $n$, $i$ to prefer money values $I_{n,i}$.

We have at the following utility function for the dynamics of $r$:

$$H(r) = \sum_x u_A(r_x) + \sum_{x>y} u_M(\Delta g(x,y)),$$

and a possible equilibrium corresponds to the maximum. The change $I_n'$ is accepted with probability:

$$p = e^{\beta r \min(0, \Delta H)},$$

where $\Delta H = H(r') - H(r)$ denotes the change in the utility function.

In the same way we obtain:

$$G^{(0)}(\sigma_x) = e^{(z-1)L\delta_{\sigma_x,1}},$$

with the self-interest constant:

$$L = \frac{\beta_T}{z-1}\left[u_A(r_0) - u_A(1)\right].$$

The equilibrium for the dynamical variables can be written as:

$$w(\sigma) = \frac{1}{Z}\prod_x G^{(0)}(\sigma_x)\prod_{y<x} G^{(1)}(\sigma_x,\sigma_y).$$

We introduce the tree distribution $T_n(\sigma_x)$ of length $|x| = n$ corresponding to the product of all factors $G^{(0)}$ *and* $G^{(1)}$, which are summed over all spins $\sigma_y$ *with* $|y| \geq |x|$:

$$T_n(\sigma_x) = \frac{1}{Z_T}\sum_{\{\sigma_y, y>x\}}\prod_{|y|\geq|x|}\left(G^{(0)}(\sigma_y)\prod_{|y'|\geq|y|} G^{(1)}(\sigma_y,\sigma_{y'})\right).$$

For agent $N$ this yields the equilibrium distribution $w_1(\sigma_N)$, for other agents a tree can be expressed by trees of length $n-1$ in the following way:

$$T_n(\sigma) = G^{(0)}(\sigma)\sum_{\sigma_1,\ldots\ldots,\sigma_{z-1}}\prod_{i=1}^{z-1} G^{(1)}(\sigma,\sigma_i)T_{n-1}(\sigma_i).$$

For the latter this reads:

$$w_{n+1} = f(w_n),$$

$$f(w) = \left[e^{-Ky^2+L}\frac{1+e^{(2y-1)K}w}{1+e^{-K}w}\right]^{z-1},$$

which allows the calculation of $w_n$, and the value of $r_x$ for the agent $x = N$ is related to $w_N$ by the inflation parameter:

$$M = \left\langle\frac{\ln r_x}{\ln r_0}\right\rangle = \left(\delta_{\sigma_x,1}\right)_T = \frac{w_N}{1+w_N}.$$

They correspond to a homogeneous value of the inflation parameter on the system, which can exhibit different phases. The form of $f(w)$ shows that the point, and $K$ has to be larger than a value given by:

$$K_c = \frac{1}{\gamma} \ln \frac{z}{z-2},$$

and we need L in the form of :

$$L_{\pm} = K\left(\gamma^2 + \frac{1}{z-1}\right) - 2\gamma K \begin{cases} \dfrac{1}{z-1} \\ 1 \end{cases}.$$

The money value regulating market makers can achieve a stable economy with an inflation parameter $M$ for given agent parameters $L$ and $\gamma$. A mechanism should exist in order to recycle the money flow, and an inflationary value $r_0 < 1$ is preferred for this purpose.

# 12 Market Making, Information, Beliefs, and Chaos

There are $N$ strategic agents each of whom receive a signal $s$. Based on strategy, an agent submits a market order $x_i$, the market makers observe some information about the order flow and base their prices on this information. Bagnoli, Viswanathan and Holden (2001) suppose this is a game of incomplete information because market makers do not know the signals strategic agents receive and so seek a Bayes-Nash equilibrium. We mean market makers choose price functions that they observe about the submitted orders, that is, if $(x_1,\ldots\ldots,x_N,u_1,\ldots\ldots,u_L)$ is the vector of submitted orders, then market makers observe $z = \sum_{i=1}^{N} x_i + u$.

With a game, one starts at the stage and works backward to find sequentially rational Bayes-Nash equilibrium. A market maker's expected profit is $E\left\{(p-v)\frac{z}{k}\{z\right\}$, if one of the $k \le K$ market makers quotes a favorable price for agents. The Nash equilibrium is for market makers to offer $p(z) = E\{v|z\}$ for equilibrium values of $z$, and this identification of the price charged equaling the expected value of a share embeds a strategy function for each strategic agents. In equilibrium the strategic agents' strategies, we seek conditions such that each market maker's if support of $X$ and the support of $u$ permits aggregate net order flow to be any real number. Strategic agents infer that $p(z) = \mu_N + \lambda_N z$. The $i$th strategic agent's problem is:

$$\max_{x_i} E\{(v - p(z))x_i|s\} = \max_{x_i}(s - \mu_N - \lambda_N(x_i + X_{\sim i} + E[u]))x_i,$$

where $x_i$ is the order she places, $u$ is agents' net order, and $X_{\sim i}$ is the sum of the orders placed by other strategic agents. There is unique Nash equilibrium in which strategic agents choose the strategy:

$$x(s) = \frac{s - \mu_N - \lambda_N E\{u\}}{(N+1)\lambda_N} = \beta_N(s - \mu_N - \lambda_N E[u]),$$

where each strategic agent's order is a function of her information.

A random variable $y_1$ is an $N$-fold average of another random variable $y_2$ if $y_1$ has the same distribution as the average of $N$ random variables each with the distribution as $y_2$.

For $N$, there is equilibrium, and its intercept is $E[s]-\lambda_N E[u]$ and its slope is

$$\lambda_N = \frac{1}{(N+1)\beta_N},$$

which is characterized by:

$$\phi_s(N\beta_N t) = e^{iN(\beta_N E[s]-E[u])}\phi_u(t)^N \quad \forall t.$$

If the distributions of the signal and agents have affect the equilibrium price function. As a result, we will be able to find combinations of distributions that lead to the price functions. If there are $N$ strategic agents, then there is a unique equilibrium if $\hat{x}$ and $\hat{u}$ have finite expectations. Potentially the most important application is that it allows us to readily illustrate that many combinations of distributions produce the same equilibrium. They indicate that one parametric restriction is that there be exactly $N$ strategic agents. They restrict market makers to choose a price that is the best estimate of the terminal value of the asset.

## Private Information

For the information acquisition problem, we assume that agents make some decision prior to private information. Such analyses provide insight into how much and how quickly information is absorbed in prices and issues surrounding the regulation of insider trading, and one must be able find an equilibrium for all $N$. Remaining decisions may be made after obtaining the information. Expected profits of acquiring information for $N-1$:

$$E\left[E\{(v-\mu_N-\lambda(X+u))x|s\}\right] = \frac{1}{(N+1)\lambda_N}E\left[(s-\mu_N-\lambda_N E[u])^2\right].$$

If agents make some decisions prior to learning the information, we will determine conditions for an equilibrium for all strategic agents and then add the condition that the random variables are the class of stable distributions. If informed agents make all decisions after learning the information, then there is an equilibrium for all $N$ if $s$ and $u$ are stable random variables with finite expectations. The characteristic function $\phi$ of any stable distribution can be written as:

$$\ln\phi(t) = \delta\left(it\gamma - |t|^\alpha + it\omega_A(t,\alpha,\eta)\right),$$

where

$$\omega_A(t,\alpha,\eta) = \begin{cases} |t|^{\alpha-1}\eta\tan(\pi\alpha/2) & if \quad \alpha \neq 1 \\ -\eta(2/\pi)\ln|t| & if \quad \alpha = 1 \end{cases}$$

for $0 < \alpha \le 2,\ -1 \le \eta \le 1,\ \delta > 0$ and $\gamma$ is a finite, real number.

Any stable random variable can be written as a combination of random variables. If they are stable, then their sum is stable and we will have an equilibrium for all $N$ if $s$ has that same distribution. If strategic agents make decisions before learning the information, an additional condition applies to the game, namely that there is a equilibrium for all $N$ if $s$ and $u$ are random variables and, for each $N$, there is only one equilibrium. Conditions for an equilibrium of all $N$ can be expressed as:

$$\ln\phi_{\hat{s}}\left(\frac{Nt}{(N+1)\lambda_N}\right) = N\ln\phi_{\hat{u}}(t) \quad \forall t$$

If $\hat{s}\ and\ \hat{u}$ are stable, then we obtain:

$$\delta\left(\frac{N}{(N+1)\lambda_N}\right)^{\alpha}|t|^{\alpha} = N\delta'|t|^{\alpha'} \qquad \forall t,$$

where $\lambda_N$ is:

$$\lambda_N = \left(\frac{\delta}{N\delta'}\right)^{1/\alpha}\left(\frac{N}{N+1}\right).$$

The $i$th strategic agent chooses an order, $x_i$ to maximize her expected wealth given her signal, $s_i$. Since $z = x_i + X_{\sim i} + u$, the price function solves:

$$\max_{x_i} E\left[x_i(v - \mu_N - \lambda_N(x_i + X_{\sim i} + u))|s_i\right].$$

Given that market makers choose a price function, a Bayes-Nash equilibrium has strategic agents choosing:

$$x(\hat{s}_i) = \left(\frac{\xi}{\lambda_N(2+\xi(N-1))}\right)\hat{s}_i = \beta_N\hat{s}_i.$$

There is an equilibrium if $(\hat{s},u)$ have expectations and:

$$\phi_{\hat{v}}(N\beta_N t)^{\frac{z-\xi}{N\xi}} = \phi_{\hat{v}}(\beta_N t)^{N(1-\xi)/\xi}\phi_{\hat{u}}(t) \quad \forall t.$$

If differentially informed strategic agents use strategies and $v$, $u$ which have finite moments, then there is an equilibrium for all $N$ if $v$ and $u$ are normal. The aggressiveness of the strategic agents $\beta$, and the result for distributed random variables is that each strategic agent trades less aggressively. Assume that a strategic agent's order is executed with probability $p$ and, conditional on a strategic agent's order being selected, each of their orders is equally likely to be selected. We will consider that strategic agents are differentially informed and then specialize when they are informed. Market makers choose prices after observing the randomly chosen order that will be executed, *z*, and although all market makers submit prices, only one will execute the order. Each strategic agent solves:

$$\max_{x_i} E\{v - \mu - \lambda z | s_i\} x_i$$
,

and the first-order condition is:

$$x_i(\hat{s}_i) = \frac{\xi}{2\lambda}\hat{s}_i = \beta\hat{s}_i$$
,

and the second-order condition is satisfied because $\lambda > 0$.

The information can be obtained when every strategic agent observes *s* and $E\{v|s\} = s$, independent of the number of agents there are for an equilibrium to exist, regardless of the number of strategic agents. This can be done by noting that we can express these conditions by:

$$\phi_s(\beta t) = e^{it(\beta E[s] - E[u^*])}\phi_{u^*}(t) \quad \forall t$$
,

where using the appropriate characteristic functions yields the solutions. Since market makers are risks, $E[v|Z] = p[Z]$ in equilibrium. Any distributions that support a equilibrium, and the market makers observe each order. Since informed strategic agents submit the same order, the conditional probability that any *N* elements of *Z* are this common order is a weighted average of $E\{v|\xi\} = \mu_N + 2\lambda_N \xi \text{ for } \xi$, equal to the common value of the *N* elements and the weights are the probabilities submitted by strategic agents given *Z*.

Market makers can commit to not executing an order vector that contains more than one order for each agent. There are circumstances where such a condition is sensible. We expect strategic agents, and we restrict our attention to environments with exactly one strategic agent. No matter what the strategic agent's total order, $x_i$, it will affect market makers' inferences. If there are $L \geq 1$ agents and one strategic agent who may split her order as she chooses, then there is an equilibrium under the same distribution assumptions.

## Equilibrium and Heterogeneity

The lifetime utility of an unemployed agent of type $\varepsilon$ is denoted by $V_0(\varepsilon)$ and that of an agent when employed at a firm of type *p* and paid wage

$w$ is $V(\varepsilon, w, p)$. Whenever that condition is met, any type-$p$ firm will want to hire any type-$\varepsilon$ unemployed agent upon finding him on the search market, and will offer the wage $\phi_0(\varepsilon, p)$. That compensates the agent for his opportunity cost of employment, which is defined by: $V(\varepsilon, \phi_0(\varepsilon, p), p) = V_0(\varepsilon)$.

The agent's future employment prospects depend on both the type of firm at which he works and his personal ability, and the minimum wage at which a type-$\varepsilon$ unemployed agent is willing to work at a given type-$p$ firm depends on $p$ and $\varepsilon$. Postel-Vinay and Robin (2002) note that an agent accepts a move to a potentially better match with a firm of type $p'$ if the latter offers at least the wage $\phi(\varepsilon, p, p')$ defined by:

$$V(\varepsilon, \phi(\varepsilon, p, p'), p') = V(\varepsilon, \varepsilon p, p).$$

The agent would rather stay at his current firm if he is promoted to the wage $\phi(\varepsilon, p', p)$, which makes him indifferent between staying and joining the type-$p'$ firm, where:

$$U(\phi(\varepsilon, p, p')) = U(\varepsilon p) - \frac{\lambda_1}{p + \delta + \mu} \int_p^{p'} \overline{F}(x) U'(\varepsilon x) dx.$$

The wage paid by firm $p'$ is less than the maximal wage firm $p$ can afford to pay, and the integral term represents the option value of turning down the type-$p$ firm to work at the type-$p'$ firm. The option value further positively depends on the frequency of outside offers $(\lambda_1)$ and the likelihood of high-$p$ draws, and it negatively depends on the overall job termination rate $\delta + \mu$, which tends to reduce the probability that an outside wage offer arrives before the match breaks up. The amount of inter-temporal transfer depends on the discount rate and the coefficient of relative risk. Firms always offer their reservation wages to workers.

Let us define the threshold firm type $q(\varepsilon, w, p)$ by the equality:

$$\phi(\varepsilon, q(\varepsilon, w, p), p) = w,$$

where the value $q(\varepsilon, w, p)$ is such that the competition between firm $p$ and firm $p'$ for agent $\varepsilon$ raises the agent's wage above $w$. Consider an agent of type $\varepsilon$ currently employed by a firm of type $p$ at wage $w$ and let this agent be contracted by a firm of type $p'$. If $p' \le q(\varepsilon, w, p)$, nothing changes. If $p \ge p' > q(\varepsilon, w, p)$, the agent obtains a wage raise $\phi(\varepsilon, p', p) - w > 0$ from her current employer. If $p' > p$, and the agent moves to firm $p'$ for a wage $\phi(\varepsilon, p, p')$ that may be greater or smaller than *w*, and firm *p* loses its agent. Agents with long tenures have on average received more offers and consequently earn higher wages, and can generate firm-to-firm agent movements with wage cuts when the tenure profile in the new firm is expected to be increasing over a very long time span. It is then optimal for the firms to offer upward sloping wage-tenure contracts in order to prevent excessive turnover.

Over an agent's lifetime, the sequence of *p* values marks the effect of experience as reflected by job-to-job mobility, and agents can gradually find better technological support for their ability. The distribution of wages can be estimated from individual wages, and presumes that an agent of type $\varepsilon$ of a firm of type *p* is currently paid a wage *w* that is either equal to $\phi(\varepsilon, b, p)$ if *w* is the first salary after unemployment. Finally, wage mobility is the outcome of a price competition between the incumbent employer and another firm of type *q*. Some terms can now be defined.

Unemployment rate:

$$u = \frac{\delta + \mu}{\delta + \mu + \lambda_0}.$$

Distribution of firm types across employed agents, the fraction of agents employed at a firm:

$$L(p) = \frac{F(p)}{1 + \kappa_1 \overline{F}(p)},$$

and the density of agents in firms of type *p* follows from:

$$\ell(p) = \frac{1 + \kappa_1}{\left[1 + \kappa_1 \overline{F}(p)\right]^2} f(p),$$

with $\kappa_1 = \lambda_1 / (\delta + \mu)$.

Within-firm distribution of agents' types, and the density of matches $(\varepsilon, p)$ is:

$$\ell(\varepsilon, p) = h(\varepsilon)\ell(p).$$

Within-firm distribution of wages, and the fraction of ability is:

$$G(w|\varepsilon, p) = \left(\frac{1 + \kappa_1 \overline{F}(p)}{1 + \kappa_1 \overline{F}[q(\varepsilon, w, p)]}\right)^2 = \left(\frac{1 + \kappa_1 L(\varepsilon, w, p)}{1 + \kappa_1 L(p)}\right)^2.$$

Steady-state equilibrium conditions that allow average size of a firm of productivity *p* can be written as

$$M\ell(p)/\gamma(p),$$

and it is:

$$\frac{M\ell(p)}{\gamma(p)}=\frac{M(1+\kappa_1)}{\left[1+\kappa_1\overline{F}(p)\right]^2}\times\frac{f(p)}{\gamma(p)},$$

where the first term on the right-hand-side says that high-productivity firms have more market power and should be proportionately bigger. The way hiring effort varies with $p$ is as it would be with random matching, and results from the assumptions of heterogeneity and undirected search. We also have information on the time in days of the individual's employment spell as well as the number of hours worked during a homogeneous period of the business cycle.

The set $\{w_i, i=1,\ldots\ldots,N\}$ is a set of $N$ draws from the steady-state wage distribution. There are two obvious choices: firm size and firm mean earning utility, because the value of the steady-state size of a firm with productivity $p$ is a function of the weights $f(p)/\gamma(p)$.

We derive the steady-state equilibrium conditional expectation of any function $T(w)$ of the wage paid by a firm of type $p$ to any its agents taken at random, as:

$$E\{U(w)|p\}=U(p)-\left(1+\kappa_1\overline{F}(p)\right)^2\int_0^p\frac{1+(1-\sigma)\kappa_1\overline{F}(q)}{\left[1+\kappa_1\overline{F}(q)\right]^2}U'(q)dq,$$

where $\sigma=p/(p+\delta+\mu)$.

A better capacity for predicting wage dynamics would have been suspect, as a large part of wage mobility is likely to reflect idiosyncratic labor productivity shocks. These can be explained by moral hazard considerations but they are also likely to reflect fluctuations in market conditions or firm productivity.

## Dynamic Equilibrium and Ambiguity

We have the probability space $(\Omega,F,P)$. Let $W=(W_t)$ be a $d$-dimensional Brownian motion defined on $(\Omega,F,P)$ *and* $F=\{F_t\}_{0\le t\le T}$ be the filtration that it generates, representing the information available to the individual. Consumption processes lie in $\wp$, a subset of the set of measurable processes, and they define a utility process $(V_t(c))$ for each consumption process $c$ in $\wp$ as follows:

$$V_t(c)=\min_{Q\in \mathrm{P}}E_Q\left[\int_t^T e^{-\beta(s-t)}u(c_s)ds\,|F_t\right],$$

where $\mathrm{P}$ is a set of priors on the state space $(\Omega,F_T)$. Abbreviate $V_0(.)$ *by* $V(.)$ and refer to it as the multiple-priors utility.

Epstein and Miao (2003) suppose a density generator $\theta$ is a process that delivers the continuous-time counterpart and generates a *P*-martingale $\left(z_t^\theta\right)$ via this equation:

$$dz_t^\theta = -z_t^\theta \theta_t dW_t, \quad z_0^\theta = 1$$

or equivalently:

$$z_t^\theta = \exp\left\{-\frac{1}{2}\int_0^t \|\theta_s\| ds - \int_0^t \theta_s dW_s\right\}, \quad 0 \le t \le T$$

Then the set of priors is:

$$\mathrm{P} = \left\{Q^\theta : \theta \in \Theta \textit{ and } Q^\theta \textit{ is defined by } \frac{dQ^\theta}{dP} = z_T^\theta\right\}.$$

An important feature of this specification of $\mathrm{P}$ is that it delivers dynamic consistency in the sense that:

$$V_t = \min_{Q \in \mathrm{P}} E_Q\left[\int_t^\tau e^{-\beta(s-t)} u(c_s) ds + e^{-\beta(\tau - t)} V_\tau \middle| F_t\right], \; 0 \le t \le \tau \le T$$

A super-gradient for *V* at the consumption process *c* is a process $(\pi_t)$ satisfying:

$$V(c') - V(c) \le E_P\left[\int_0^T \pi(c_t' - c_t) dt\right], \textit{ for all } c' \textit{ in } \wp$$

Then:

$$V_t(c) = A_t\left(c_t^\alpha - 1\right)/\alpha - \frac{1}{\beta p}\frac{(p-\beta)}{\alpha} + e^{\beta(t-T)}\frac{1}{\beta p}\frac{\left[p - \beta e^{(p-\beta)(t-T)}\right]}{\alpha},$$

where

$A_t = p^{-1}\left[1 - \exp(p(t-T))\right]$ and the associated volatility is:

$$\sigma_t = A_t c_t^\alpha s^c.$$

This describes the information structure and preferences of agents, and they have the common information structure represented by the Brownian filtration. The differing belief of the individuals described below as asymmetric information reflect differing prior views prior about the environment. Each individual have a multiple-priors utility functions on $\wp$ and for $i = 1,2,\ldots,c^i$ *and* $\left(V_t^i(.)\right)$ denote *i*'s consumption and utility processes. Preferences differ as individuals have different sets of priors, that is, different ambiguity parameters $\kappa^i$.

Investment opportunities are represented by risky assets earning the instantaneous interest rate $r$ with dividend streams $Y$. Thus dividends are described by:

$$D_{tt}^{T} = \begin{cases} \left(0, \int_0^t Y_s^1 ds, \int_0^t Y_s^2 ds\right) & if \quad 0 \le t \le T, \\ \left(1, \int_0^T Y_s^T ds, \int_0^T Y_s^2 ds\right) & if \quad t = T. \end{cases}$$

In equilibrium, the gain process $S + D$ is shown by:

$$d(S_t + D_t) = \mu_t^G dt + s_t^G dW_t \,,$$

where $\int_0^T \left|\gamma_t \mu_t^G\right| dt + \int_0^T \gamma_t^T s_t^G \left(s_t^G\right)^T \gamma_t dt < \infty$ ,

where $\gamma_t = \left(\gamma_{0,t}, \gamma_{1,t}, \gamma_{2,t}\right)^T$ and $\gamma_{n,t}$ represents the number of assets.

The aggregate endowment or output process is $(Y_t)$ and we assume that:

$$Y_t = Y_t^1 + Y_t^2 + \Phi_t \,,$$

where $(\Phi_t)$ is the part of output that is not traded. The following defines the economy:

$$\mathrm{E} = \left((\Omega, F, \Im, P), (W_t), \left(u, \beta, \kappa^i, \gamma_0^i\right)_t, (D_t), (Y_t)\right).$$

An equilibrium for the economy $\mathrm{E}$ is a tuple $\left(\left(c^i\right)_t, p\right)$ where $p$ is a valued state price process, and $c^i$ solves:

$$\sup_{e^i \in \wp} V^i\left(e^i\right) \qquad \text{subject to}$$

$$E\left[\int_0^T p_s \left(e_s^i - \frac{1}{2}\Phi_s\right) ds\right] \le \gamma_{i,0}^i E\left[\int_0^T p_s Y_s^i ds\right] + \gamma_{j,0}^i E\left[\int_0^T p_s Y_s^J ds\right].$$

The risk rate and the asset price are related by:

$$r_t dt = dS_t^0 / S_t^0 .$$

Let the returns process for the risky assets be:

$$dR_t^n = \frac{dS_t^n + Y_t^n dt}{S_t^n} .$$

The equilibrium has complete markets, and the state price process satisfies:

$$-dp_t / p_t = r_t dt + \eta_t dW_t, \quad p_0 = 1 \,,$$

where $\eta_t = s_t^{-1}(b_t - r_t 1)$.

We show that the density generators that support equilibrium consumption processes are:

$$\theta_t^{*1} = (0, \kappa_1)^1 \quad and \quad \theta_t^{*2} = (\kappa_2, 0)$$ for all $t$.

There exists an equilibrium $((c^i)_i p)$ where:

$$c_t^1 = +\frac{1}{1+\lambda\varsigma_t} Y_t, \quad c_t^2 = \frac{\lambda\varsigma_t}{1+\lambda\varsigma_t} Y_t$$

and

$$p_t = \frac{e^{-\beta t} z_t^{\theta^{*i}}}{(c_t^i / c_0^i)}.$$

The interest rate $r$ satisfies:

$$r_t = \beta + \mu^Y - s^Y s^Y - \left[\kappa_2 s_1^Y - \frac{c_t^1}{Y_t}\left(\kappa_2 s_1^Y - \kappa_1 s_2^Y\right)\right],$$

the market price of uncertainty $\eta_t$ is:

$$\eta_t = s^Y + \begin{bmatrix} \kappa_2 c_t^2 / Y_t \\ \kappa_1 c_t^1 / Y_t \end{bmatrix}.$$

Excess returns for the risky assets are:

$$b_t^1 - r_1 = s_t^1 s^Y + \left(\kappa_2 \frac{c_t^2}{Y_t} s_t^{11} + \kappa_1 \frac{c_t^1}{Y_t} s_t^{12}\right),$$

$$b_t^2 - r_t = s_t^2 s^Y + \left(\kappa_1 \frac{c_t^1}{Y_t} s_t^{22} + \kappa_2 \frac{c_t^2}{Y_t} s_t^{21}\right).$$

Wealth processes satisfy:

$$\overline{X}_t^i = \beta^{-1}\left(1 - e^{-\beta(T-t)}\right) c_t^i,$$

and trading strategies for the risky assets are given by a volatility:

$$\overline{s}_t = \frac{1 - e^{-\beta(T-t)}}{\beta} Y_t s^Y - \left(s_t^1\right)^T S_t^1 - \left(s_t^2\right)^T S_t^2.$$

Here $\kappa_j^i$ measures $i$'s ambiguity about the shock in location $j$, where:

$$0 < \kappa_1^2 - \kappa_1^1 < s_1^Y, \quad 0 < \kappa_2^1 - \kappa_2^2 < s_2^Y,$$

so there are differences in ambiguity parameters, and an equilibrium exists. The new process takes the form:

$$\varsigma_t = \exp\left\{\frac{1}{2}\left(\left(\kappa_1^2\right)^2 + \left(\kappa_2^2\right)^2 - \left(\kappa_1^1\right)^2 - \left(\kappa_2^1\right)^2\right)t - \left(\kappa_1^2 - \kappa_1^1\right)W_t^1 - \left(\kappa_2^2 - \kappa_2^1\right)W_1^2\right\},$$

corresponding to the equilibrium density generators: $\theta_t^{*1} = \left(\kappa_1^1, \kappa_2^1\right)^1,\ \theta_t^{*2} = \left(\kappa_1^2, \kappa_2^2\right)^1$. Each location's portfolio of risky assets consists of the mean-efficient portfolio $\overline{X}_t^i\left(s_t s_t^T\right)^{-1}\left(b_t - r_t 1\right)$ and a component $-\frac{1}{2}\left(s_t^T\right)^{-1}\overline{s}_t$ that hedges the risk due to the non-traded endowment.

We use the aggregate output process $(Y_t)$ to measure ambiguity.

Individual consumption levels depend not only on the aggregate endowment but also on allocation specific shocks $W_t^n$, and we noted above that $\left(W_t^1, \ldots W_t^n + \kappa_1 t\right)$ is a Brownian motion under $Q^{\theta^{*1}}$. As a multiple-priors decision-maker, allocation evaluates prospects through the worst-case scenario. The ambiguity-adjusted probabilities affect consumption because, *i*'s marginal rate of substitution between time 0 and time *t* consumption is:

$$MRS_{0,t}^i = \frac{e^{-\beta t}u'\left(c_t^i\right)}{u'\left(c_0^i\right)}\frac{dQ^{\theta^{*i}}}{dP}\bigg|_{F_t}.$$

We define

$$\varsigma_t = \frac{dQ^{\theta^{*2}}/dP}{dQ^{\theta^{*1}}/dP}\bigg|_{F_t}$$, which under the latter effect takes the precise form:

$$\varsigma_t = \frac{c_t^2/c_0^2}{c_t^1/c_0^1},$$

giving equal relative consumption growth rates of the allocations. The presence of disagreement leads to individual consumption paths in some of these shares and the following relation between output growth and the disagreement process can be specified:

$$\mathrm{cov}_t\left(\frac{d\varsigma_t}{\varsigma_t}, \frac{dY_t}{Y_t}\right) = -\kappa_2 s_1^Y + \kappa_1 s_2^Y < 0.$$

The volatilities are given by:

$$s_t^{c,1} = s^Y + \frac{c_t^2}{Y_t}\begin{bmatrix}\kappa_2 \\ -\kappa_1\end{bmatrix},$$

$$s_t^{c,2} = s^Y + \frac{c_t^1}{Y_t}\begin{bmatrix} -\kappa_2 \\ \kappa_1 \end{bmatrix},$$

which implies that:

$$s_t^{c,n} s_t^{c,n} = s^Y s^Y + 2(\kappa_n)^{n+1}\left(\frac{c_t^{n+1}}{Y_t}\right)^{n+1} + 2\kappa_n\left(\frac{c_t^{n+1}}{Y_t}\right)\left(s_n^Y - s_{n+1}^Y\right),$$

$$s_t^{c,n+1} s_t^{c,n+11} = s^Y s^Y + 2(\kappa_n)^2\left(\frac{c_t^n}{Y_t}\right)^2 - 2\kappa_n\left(\frac{c_t^n}{Y_t}\right)\left(s_n^Y - s_{n+1}^Y\right),$$

and the difference in variables is:

$$s_t^{c,n} s_t^{c,n} - s_t^{c,n+1} s_t^{c,n+1} = 2(\kappa_n)^2\left(\frac{c_t^{n+1} - c_t^n}{Y_t}\right) + 2\kappa_n\left(s_n^Y - s_{n+1}^Y\right).$$

We show that like risk, ambiguity drives down the risk-less rate, and their effects are captured in the variance of output growth. The risk rate is stochastic and varies over time between the extremes $\beta + \mu^Y - s^Y s^Y - s_{n+1}^Y \kappa_n$ *and* $\beta + \mu^Y - s^Y s^Y - s_n^Y \kappa_{n+1}$ depending on the distribution of aggregate consumption. Thus as the distribution of wealth shifts in favor of $a$, the ambiguity in the economy falls, and ambiguity act to increases the market price of uncertainty with qualitative features of its effects. The time variation of $\eta_t$ is of particular interest, and the risk premium term is the covariance of asset returns with growth rate of aggregate consumption. The properties of the equilibrium depend on how that output is distributed between the dividend streams $(Y_t^i)$ of the traded assets and the non-traded endowment, and we assume that:

$$Y_t^i / Y_t = v(W_t^i),$$

and

$$dY_t^n / Y_t^n = a_t^n dt + \left[s_n^Y + \frac{v'(W_t^n)}{v(W_t^n)}\right] dW_t^n + s_{n+1}^Y dW_t^{n+1},$$

$$dY_t^{n+1} / Y_t^{n+1} = a_t^{n+1} dt + s_n^Y dW_t^n + \left[s_{n+1}^Y + \frac{v'(W_t^{n+1})}{v(W_t^{n+1})}\right] dW_t^{n+1},$$

for suitable drifts $a_t^n$ *and* $a_t^{n+1}$.

The returns process in terms of the Brownian driving process that is appropriate for each agent, leads to:

$$dR_t^n = \left(b_t^n - \kappa_n s_t^{n,n+1}\right)dt + s_t^{n,n} dW_t^n + s_t^{n,n+1} d\left(W_t^{n+1} + \kappa_n t\right)$$

$$dR_t^n = \left(b_t^n - \kappa_{n+1} s_t^{n,n}\right)dt + s_t^{n,n} d\left(W_t^n + \kappa_2 t\right) + s_t^{n,n+1} dW_t^2 .$$

From the explicit expressions for the returns' volatility we have to interpret elicited probability measures as including an adjustment for ambiguity. This extension moves predictions in the direction of helping to resolve the puzzles concerning bias in consumption and equity.

We now consider forward-looking, expected-utility-maximizing consumers who face liquidity constraints and earning uncertainty. One of the implications of time limits is that consumers' current welfare choices affect their future opportunity sets. Grogger and Michalopoulos (2003) represents a substantial departure from the entitlement regime, and it gives consumers an incentive to use their benefits in the future. The reason is that welfare acts as insurance in the consumer's utility-maximization problem, and lengthen the horizon over which it could be used to smooth consumption. We show that the less likely households are to use welfare, the greater the difference between their remaining eligibility horizon and their available stock of benefits. Since the available stock of benefits is equal to their timeless welfare use, the remaining eligibility horizon is largely determined by age dependence. The reason is that such households will become age-ineligible for welfare before they can reach the time limit. The estimated effects of time limits exhibit age dependence that is largely consistent with the prediction, and appears well before the households could have exhausted their benefits.

This can possibly be interpreted in terms of team participation and productivity. The forming of teams is economically desirable when they make possible gains from complementarities in production among agents, facilitate gains from specialization by allowing each agent to accumulate task-specific human capital, or encourage gains from knowledge transfer of idiosyncratic information that may be valuable to other team members. Hamilton, Nickerson and Owan (2003) note obvious concerns about moral hazard, and many firms do in fact introduce teams even when individual task assignment is feasible and provide team-based incentives in the hope of improving productivity. This suggests that we need to analyze other behavioral responses to team structure and free-riding predicted by market maker approach. Agent heterogeneity could shape team productivity by facilitating mutual learning, thereby enhancing team productivity. The effects of agent heterogeneity on productivity depend on agent participation in teams, which suggests that low-ability agents could expect that teaming up with higher-ability agents would raise their pay.

# 13 Volatility, Information, and Return Distribution

## Realized Volatility

At time zero, the agents start with a functional relationship, which they know up to some finite-dimensional vector of unknown parameters as an arbitrary estimate $\hat{\theta}_0$. When the next period arrives, agents observe new information, update their beliefs, and then make current period decisions accordingly. Chen and White (1998) suppose the process repeats itself each period with agents having estimates $\hat{\theta}_n$, n=1,2......, and agents learn non-parametrically the set of asymptotically stable solutions to a population moment condition *M*. Modeling the consumption, hours of work and savings behavior of an agent in a life cycle setting under uncertainty requires specifications for preferences, asset accumulation constraints, and forms of uncertainty. In each period *t* the agent-household is assumed to choose its current and planned consumption, leisure, and savings in order to maximize the expected value of the time-preference-discounted stock of total utility over the remaining lifetime, given by

$$E_t\left\{\sum_{k=t}^{T}\frac{1}{(1+p)^{k-t}}U(k)\right\},$$

where *p* is the rate of time preference, *E* indicates that the agent accounts for all information available in period *t* when calculating expected value, and utility at age *t* is given by the function *U*. The accumulation is determined by the function

$$H(t+1)=H(h(t),H(t),\beta(t)),$$

where *H* denotes the stock of human capital, *h* is the account of human capital produced in period $t$, and $\beta(t)$ is a vector of factors (including depreciation rate), which translate current production and human capital endowments into future stocks. The value of *h* depends on the magnitude

$$h(t)=h(X(t),Z(t),\eta(t)),$$

where $X$ is the account of time devoted to human capital investment, $Z$ is the input of market goods, and $\eta(t)$ is a vector of exogenous variables affecting production. Hours of work must satisfy the time constraint

$$\tau = N(t) + L(t) + X(t),$$

where $\tau$ is the total number of hours in a period, $N$ are hours of work , and $L$ is time spent in non -market activities (e.g. leisure).

The following analysis assumes that property, income, prices, wages, and tastes in future periods are all uncertain. Define the value function corresponding to period $t+1$ by

$$V(t+1) = \max\left[U(t+1) + \frac{1}{1+p}E_{t+1}\left\{\sum_{k=t+2}^{T}\frac{1}{(1+p)^{k-t-1}}U(k)\right\}\right],$$

where the maximization is carried out under the constraints, and $V$ shows the expected lifetime utility. In order to measure the risk we define the constants $K$ and $b>0$ by the expression

$$E_t(V(\Omega(t+1) - K, H(t+1), t+1)) = E_t\{V(\Omega(t+1) + b\varepsilon, t+1)\},$$

where $\varepsilon$ is an error term , $\Omega$ is the endowment of non-human wealth, and $K$ is as the risk premium that an agent assigns to the wealth stream $b\varepsilon$ . Treating $K$ as a function of $b$ and differentiating the above equation, we obtain

$$\frac{dK}{db} = \frac{1}{E_t\lambda(t+1)}E_t\{\lambda(t+1)\varepsilon\},$$

where $\lambda$ is the Lagrange multiplier associated with the budget constraint given by marginal utilities of consumption and leisure, and the specification of preferences determines the relation of $\lambda(t+1)$ to current variables, including wealth, wages, and prices, and to factors needed to calculate the future distributions of these variables.

The tax analysis considers schemes in which payment of taxes depends on a function of the agent's current and past incomes. The payment of taxes in period $t$, denoted by $Q$, is determined according to the function

$$Q(t) = Q\{I(t), I(t-1), t\},$$

where

$$I(k) = W(k)h(k) + Y(k)$$

specifies total income received in period $k$, and I is the accumulated income. Various forms of income, which are taxed at different rates, are replaced by earnings, $W$, and the different types of property income $Y$.

The agent's objective function is

$$U\{C(t), L(t), \pi(t)\} + \frac{1}{1+p}E_t\{V(\Omega(t), Y(t+1), I(t), H(t+1), t+1)\},$$

where $\pi$ may include such factors affecting preferences as family size or marital status, and such variables as health status, and the function *V*(.) is maximum expected lifetime utility attainable in period *t+1* given endowments $\Omega(t), Y(t+1), ,I(t) \; and \; H(t+1)$.

The agent's objective function in period *t* then takes the form

$$U\{C(t),D(t)+d(t),L(t),\pi(t)\}+\frac{1}{1+p}E_t\{V(\Omega(t),Y(t+1)I(t),H(t+1),D(t+1),t+1)\},$$

where the variables $D \; and \; d$ specify the endowment of consumer durables owned at the start of period $t$ and the net purchase of durables respectively

The value function *V*(t+1) is computed by maximizing expected utility subject to the new budget constraint and the accumulation constraints for durables.

Each agent *h* has preferences represented by the utility function

$$U_h(c) = E\left\{\sum_{t=0}^{\infty}\beta_h^t u_h(c_t)\right\}.$$

A competitive equilibrium for an economy is a collection of $\Im$ measurable portfolio holdings $\{(\theta_t^1,\theta_t^2)\}$ and asset prices $\{q_t\}$, such that $\theta^1+\theta^2=1$ for each agent *i*, and $\theta^i \in \arg\max U_i(c)$ s. t. $c_t = e_t + \theta_{t-1}(q_t+d_t)-\theta_t q_t$.

For each economy $\Im$, agent *i*'s equilibrium portfolio holding has to lie in the bounded set I. That is for each equilibrium and for all *t*, we have $\theta_t^1 \in I$.

Andersen et al. (2003), for example, consider an *a*-dimensional price process defined on a probability space, $(\Omega, F.P)$, and assume that the asset prices through time *t*, including the relevant state variables, are included in the information set $F_t$.

We have the following characterization of the asset price vector process,: $p=(p(t))_{t\in[0,T]}$, which may be written as the sum of a finite variation and predictable mean component, $A=(A_1,\ldots\ldots,A_n)$, and a local martingale, $M=(M_1,\ldots\ldots,M_n)$. These may each be decomposed into:

$$p(t)=p(0)+A(t)+M(t)=p(0)+A^c(t)+\Delta A(t)+M^c(t)+\Delta M(t),$$

where the finite-variation predictable components, $A^c \; and \; \Delta A$ are continuous and pure jump processes, while the local martingales, $M^c \; and \; \Delta M$ are continuous sample-path and compensated jump processes respectively. If we assume predictable jumps are associated with genuine jump risk, then:

$$P[\operatorname{sgn}(\Delta A(t)) = -\operatorname{sgn}(\Delta A(t) + \Delta M(t))] > 0,$$

where $\operatorname{sgn}(x) = 1 \; for \; x \geq 0 \; and \; \operatorname{sgn}(x) = -1 \; for \; x < 0$.

Whenever $\Delta A(t) \neq 0$, there is a predictable jump in the price, and hence it enables arbitrage unless there is a simultaneous jump in the martingale component, $\Delta M(t) \neq 0$.

We denote the return over $[t-h, t] \; by \; r(t,h) = p(t) - p(t-h)$,

where $r(t)$ inherits all the main properties of $p(t)$ and may likewise be decomposed into the predictable mean component, $A$, and the local martingale, $M$. The continuous component (such as a Brownian motion and any jump in the mean) must be accompanied by a corresponding predictable jump of unknown magnitude in the compensated jump martingale, $\Delta M$.

The latter jump event will typically occur when unanticipated information hits the market, and the former type of predictable jump may be associated with the release of information according to plan. It is worth noting that any slight uncertainty about the precise timing of the information of predictability and removes the jump in the mean process. Because the return process is a semi-martingale it plays a critical role in development. For any arbitrage-free process with finite mean, $[r,r] = \{[r,r]\}_{t \in [0,T]}$, and for an increasing sequence of random partitions we have:

$$[r_i, r_j]_t = [M_i, M_j]_t = [M_i^c, M_j^c]_t + \sum_{0 \leq s \leq t} \Delta M_i(s) \Delta M_j(s).$$

Jump components only contribute to the co-variation if there are simultaneous jumps in the price path for the $i$th and $j$th asset. Under the weak auxiliary condition ensuring the above property, this variation is induced by the innovations to the return process.

We refer to such measures, obtained from actual high-frequency data, as realized volatility. The variation induced by the genuine return innovations is represented by the martingale component. The conditional return covariance matrix at time $t$ over $[t, t+h]$, where $0 \leq t \leq t+h \leq T$, is then given by:

$$\operatorname{cov}(r(t+h,h) | F_t) = E([r,r]_{t+h} - [r,r]_t | F_t) + \Gamma_A(t+h,h) + \Gamma_{AM}(t+h,h) + \Gamma'_{AM}(t+h,h).$$

We proceed by verifying the equality for an arbitrary element of the covariance matrix. We have:

$$\operatorname{var}(M_i(t+h) - M_i(t) | F_t) = E([M_i, M_i]_{t+h} [M_i, M_i]_t F_t) = E([r_i, r_i]_{t+h} - [r_i, r_i]_t | F_t),$$

where the result holds for the elements of the covariance matrix.

It follows from the above that:

$$E\left([M_i,M_j]_{t+h}-[M_i,M_j]_t|F_t\right)=1/2\begin{bmatrix}\mathrm{var}\left([M_i(t+h)+M_j(t+h)]-[M_i(t)+M_j(t)]|F_t\right)\\ -\mathrm{var}\left(M_i(t+h)-M_i(t)|F_t\right)-\mathrm{var}\left(M_j(t+h)-M_j(t)|F_t\right)\end{bmatrix}$$
$$=\mathrm{cov}\left([M_i(t+h)-M_i(t)],[M_j(t+h)-M_j(t)]|F_t\right).$$

If the mean process $\{A(s)-A(T)\}_{s\in[t,t+h]}$, conditional on information at time $t$, is independent of the return innovation process, $\{M(u)\}_{u\in[t,t+h]}$, then the conditional return covariance matrix for $0\le t\le t+h\le T$ is given by:

$$\mathrm{cov}\left(r(t+h,h)|F_t\right)=E\left([r,r]_{t+h}-[r,r]_t|F_t\right)+\Gamma_A(t+h,h).$$

If the mean process $\{A(s)-A(t)\}_{s\in[t,t+h]}$, conditional on information at time $t$, is a function over $[t,t+h]$, then the conditional return covariance matrix equals the conditional expectation of the return variation process:

$$\mathrm{cov}\left(r(t+h,h)|F_t\right)=E\left([r,r]_{t+h}-[r,r]_t|F_t\right).$$

This follows as the variation represents the actual variability of the return on innovations, and the conditional covariance matrix is the expectation of this quantity.

Although such feedback effects may be present in high-frequency returns, they are likely in some magnitude over all frequencies; it is also worth stressing that variation process is compatible with the existence of an asymmetric return-volatility relation.

The expression involving $\Gamma_A$ accommodates evolving random variation in the conditional mean process, and hence the former remains the critical determinant of the return volatility over shorter horizons. This observation follows from the fact that over horizons of length $h$, the variance of the mean return is of order $h$. We would expect the within-day variance of the expected daily return to be much smaller than the expected daily return fluctuations induced. This occurs when there is a leverage effect, or asymmetry, by which the volatility impacts the mean drift. Some assets will be associated with an increase in return volatility, which in turn raises the risk premium and the return drift.

Because the above results carry implications for the measurement of return volatility, we may obtain a useful benchmark under somewhat more restrictive conditions, including a continuous price process. For any $a$-dimensional arbitrage -free price process $p$, with continuous path and a full rank of the associated variation process $[r,r]_t$, we have:

$$r(t+h,h)=p(t+h)-p(t)=\int_0^h\mu_{t+s}ds+\int_0^h\sigma_{t+s}dW(s),$$

where $\mu_t$ denotes an predictable vector, $\sigma_s = \left(\sigma_{(i,j),s}\right)_{i,j=1,\dots,n}$ is a matrix, $W(s)$ is a Brownian motion. Integration with respect to $s$ gives:

$$\int_0^h \mu_{t+s} ds = \left( \int_0^h \mu_{1,n+s} ds, \dots, \int_0^h \mu_{n,t+s} ds \right),$$

and integration of the associated vector gives:

$$\int_0^h \sigma_{t+s} dW(s) = \left( \int_0^h \sum_{j=1,\dots,n} \sigma_{(i,j),t-s} dW_j(s), \dots\dots\dots, \int_0^h \sum_{j=1,\dots,n} \sigma_{(n,j),t+s} dW_j(s) \right)$$

We have:

$$P\left[ \int_0^h \left(\sigma_{(i,j),t+sd}\right)^2 ds \right] = 1, \quad 1 \le i, j \le n,$$

and letting $\Omega_s = \sigma_s \sigma'_s$, the variation process takes the form:

$$[r,r]_{t+h} - [r,r]_t = \int_0^h \Omega_{t+s} ds.$$

This implies that no individual asset returns can be spanned by a portfolio created by the remaining assets, and a parallel representation may be achieved on an extended probability space. For any arbitrage-free price process with conditional mean and volatility processes $\mu_s$ *and* $\sigma_s$ independent of the innovation process $W(s)$, we have:

$$r(t+h,h) \Big| \sigma\left(\mu_{t+s}, \sigma_{t+s}\right)_{s \in [0,h]} \sim N\left( \int_0^h \mu_{t-s}, ds, \int_0^h \Omega_{t+s} ds \right),$$

where $\sigma\left(\mu_{t+s}, \sigma_{t+s}\right)_{s \in [0,n]}$ denotes the $\sigma$-field generated by $\left(\mu_{t+s}, \sigma_{t+s}\right)_{s \in [0,h]}$.

The volatility part is:

$$\int_0^h \sigma_{t+s} dW(s) = \left( \int_0^h \left(\sigma_{(1),t+s}\right)' dW(s), \dots\dots, \int_0^h \left(\sigma_{(n),t+s}\right)' dW(s) \right)',$$

where $\sigma_{(i),s} = \left(\sigma_{(i,1),s}, \dots\dots, \sigma_{(i,n),s}\right)'$, so that $\int_0^h \left(\sigma_{(i),t+s}\right)' dW(s)$.

A typical element representing the *j*th volatility factor loading of asset *i* over $[t, t+h]$ takes the form: $I_{i,j}(t+,h) = \int_0^h \sigma_{(i,j),t+s} dW(s), \quad for \quad 1 \le i,j \le n$.

Then $I_{i,j}(t) = B\{\lfloor I_{i,j}, I_{i,j} \rfloor_t\}$,

where $I_{i,j}(t)$ is a continuous local martingale, and $B(t)$ denotes a standard Brownian motion.

We have:

$$I_{i,j}(t+h,h) = I_{i,j}(t+h) - I_{i,j}(t)$$
$$= B(\lfloor I_{i,j}, I_{i,j} \rfloor_{t+h}) - B(\lfloor I_{i,j}, I_{i,j} \rfloor_t),$$

where the Brownian motion is distributed: $N(0, \lfloor I_{i,j}, I_{i,j} \rfloor_{t+h} - \lfloor I_{i,j}, I_{i,j} \rfloor_t)$.

Conditional on the realization of the volatility path the variation is measurable, and the conditional of $I_{i,j}(t+h,h)$ follows. The distribution characterization is conditional on the path realization of $(\mu_s, \sigma_s)_{s \in [t,t+h]}$, and the conditional mean variation is negligible relative to the return volatility.

This implies a conditional return distribution, and some recent evidence suggests the possibility of jumps in asset prices, rendering paths discontinuous.

For some asset classes there is evidence of leverage effects that may indicate a connection between concurrent returns and volatility innovations.

## Debt Constraints

This implies that agent's portfolio holding will violate any explicit debt constraint with positive probability. Therefore, it also violates the implicit debt constraint condition. Judd, Kubler, and Schmedders (2000) show that for an economy $\Im_t$, adapted processes $q$ and $\{(\theta_t^1, \theta_t^2)\}$ form an equilibrium if and only if for all $t$ the following equations and market clearing conditions are satisfied: $c_t^i = e_t^i + \theta_{t-1}^i(q_t + d_t) - \theta_t^i q_t$. With respect to agent heterogeneity, the current exogenous state does not constitute a sufficient characteristic of the current environment.

The latter will also include endogenous variables, because the distribution of wealth or the agents' portfolio holdings will clearly influence equilibrium prices.

We consider learning algorithms that evolve according to the formulae

$$\hat{\theta}_{n+1} = \hat{\theta}_n + a_n M_n(\xi_n, \theta_n),$$

$$\xi_{n+1} = R_n\left(\hat{\xi}^n, \hat{\theta}^{n+1}, Z_{n+1}\right), \quad n = 0,1,2......,$$

where

$$\hat{\xi}^n \equiv \left(\hat{\xi}_0, ...., \hat{\xi}_n\right), \quad \hat{\theta}^{n+1} = \left(\hat{\theta}_0, ...., \hat{\theta}_{n+1}\right).$$

Since $R$ embodies agents' actions, the agents choose their actions $\hat{\xi}_n$ from a bounded set $\Xi$. This accommodates the possibility that $M$ may increase with n and $\hat{\theta}_n$.

Each firm makes its production decision at $t$ before the realization of a stochastic demand $d_t = a_1 - a_2 p_t + v_t$, where $v$ represents unobservable demand shocks.

Each agent $i$ has a cost function and observes production shocks $X$ and $w$ before making his supply decision. $X$ is a sequence of shocks observable by all agents, while $w$ is a sequence of shocks observable by firm $i$. Hence, agent $i$'s optimal output at $t$ is $q_t = p^e_{i,t} + b'_t X_t + w_{i,t}$, where $p^e_{i,t}$ is the firm's guess about $p$. The market supply is $s_t = \int_0^1 p^e_{i,t} di + b' X_t + w_t$, with $b = \int_0^1 b_i di,, \quad w_t = \int_0^1 w_{i,t} di$.

The market clearing condition implies that

$$p_t = (a_2)^{-1}\left[a_1 - b' X_t - \int_0^1 p^e_{pi,t} di\right] + u_t, \quad u_t = \left(v_t - w_t\right) / a_2.$$

Since we want to express learning, we have

$$\hat{\beta}_t = \hat{\beta}_{t-1} + t^{-1}(Q_t)^{-1}\tilde{X}_t\left(\hat{p}_t - \tilde{X}'_t \hat{\beta}_{t-1}\right)$$
$$Q_t = Q_{t-1} + t^{-1}\left(\tilde{X}_t, \tilde{X}'_t - Q_{t-1}\right),$$

where

$$\hat{p}_t = (a_2)^{-1}\left\lfloor a_1 - \tilde{X}'_t \hat{\beta}_{t-1} - X'_t - b\right\rfloor + \mu_t.$$

Firm $i$'s output at $t$ is

$q = p^e_{i,t} + f_i(X_t) + w_{i,t}$, where $p^e_{i,t}$ is its guess about $p$. The market average supply is

$$s_t = \int_0^1 p^e_{i,t} di + f(X_t) + w_t$$, where $$\tilde{f}(X_t) \equiv \int_0^1 f_j(X_j) dj, \quad w_t \equiv \int_0^1 w_{i,t} di.$$

The market clearing condition implies:

$$p_t = (a_2)^{-1}\left[a_1 - \int_0^1 p_{i,t}^e di - \tilde{f}(X_t)\right] + u_t$$
.

In stochastic nonlinear models a variable of interest $x$ are generated according to

$x_t = H\left(G(x_{t+1}, v_{t+1})^e, v_t\right)$ and $v$ is a shock. At time $t$ agents observe $G(x_t, v_t)$ and $v$ and form a guess $G(x_{t+1}, v_{t+1})^e$ in order to generate $x_t = H(G(.))$, provided that agents guess that the economy is in an equilibrium steady state with unknown parameter $G(x_{t+1}, v_{t+1})^e = b$ and they use a learning process to estimate $b$ by $\hat{b}_t$ at time $t$. The actual economy generated by $x_t = H(\hat{b}_t, v_t)$ will converge to the rational steady state solution $x_t = \tilde{x}(v_t)$ with

$$\tilde{x}(v) = H(\tilde{b}, v) = H\left[E\left(G(\tilde{x}(v_2), v_2)\middle| v_2\right), v\right]$$
.

Instead, agents would more plausibly guess $G(x_{t+1}, v_{t+1})^e = \theta(v_t)$. Given that agents know the functional forms of $H$ and $G$, we consider the situation in which, at the beginning of period $t$, agents observe $v$ and guess $G(x_{t+1}, v_{t+1})^e = \hat{\theta}_{t-1}(v_t)$.

At the end of period $t$, agent $i$ observes both agents' actions $\xi_{1,t}$, *and* $\xi_{2,t}$ and forms a belief $\theta_{ij,t+1}$ about agent $j$'s action $\xi_{j,t+1}$ in the next period, where the belief $\theta_{ij,t+1}$ is a probability density over agent $j$'s possible actions.

Agent $i$ then chooses his action for the next period, $\xi_{i,t+1}$, according to his response function $R_{i,t}$, which is derived from maximizing his current period's expected payoff given his beliefs. Agent $i$ will choose $\xi_{i,t+1} \in [0,1]$ in order to maximize

$$\int_0^1 Q_{i,t+1}\left(\xi_{i,t+1}, \xi_{i,t}, Z_{i,t+1}, \xi_j\right)\theta_{ij,t+1}\left(\xi_j\right)d\xi_j$$
,

where $Z$ is a payoff shock to agent $i$ known at time (t+1), which is unknown to the other agent for all time periods. It is a stationary dependent random process.

Given his belief $\hat{\theta}_{ij,t+1}$, agent $i$'s action at time $t+1$, $\hat{\xi}_{i,t+1} = R_{i,t}\left(\xi_{i,t}, \hat{\theta}_{ij,t+1}, Z_{i,t+1}\right)$ is a solution to

$$\int_0^1 (d/d\xi_i)\left[Q_{i,t+1}\left(\hat{\xi}_{i,t+1}, \xi_{i,t}, Z_{i,t+1}, \xi_j\right)\right]\hat{\rho}_{ij,t+1}, \left(\xi_j\right)d\xi_j = 0$$

Here we obtain almost-sure convergence in a Hilbert space, with feedback and Markovian shocks $Z$, in which agents learn about value function over time. Important questions include the speed of learning and the asymptotic distribution of the estimator $\hat{\theta}_n$, with feedback rule:

$$\hat{\xi}_n = p\left(\hat{\xi}_{n-1}, \hat{\theta}_n, Z_n\right).$$

## Transformation of Information

The transformations taking place form the probabilistic occurrence of quantum items of information, called 'tokens', which are delivered to the agent by the medium at random times. Falmagne and Doignon (1997) suppose tokens are formalized as ordered pairs (x, y) of alternatives and include agents. An agent is rational from the start and remains rational through all the transformations induced by the occurrences of the tokens, which will transform this empty agent into another agent. If $R$ is an agent on $X$, we can create another agent on the same set by adding or removing some pair.

For any agent $R$, we can manufacture another agent on the same set either by removing a pair from $R^L$, or by adding to $R$ a pair from $R^O$. ($R$ may be empty.) Let $S$ be the group of all agents on a particular finite set $X$.

A distance $d$ on the group of all subsets of a set $E$ is obtained by $d(R,S) = |(R \setminus S) \cup (S \setminus R)|$ for any two subsets $R$ and $S$. Groups of sets have also been investigated in the context of knowledge spaces, which are structures playing a role in the design of efficient algorithms for the assessment of knowledge. The preference relation at time $t = 0$ is the empty set, and we assume that at random times $t_1, t_2, \ldots\ldots, t_n, \ldots\ldots$ quantum tokens of information on the alternatives and their relationships are delivered by the environment. A positive token conveys a message such as '*x is preferred to y*', while a negative token may carry the meaning that '*x is not preferred to y*'. These two types of tokens are denoted by $x\,y$ and $\widetilde{xy}$, respectively. In terms of random variables, for any $t > 0$, $S$ specifies the state of the agent at time $t$, $N$ indicates the number of tokens arising in the half open interval (0,1] of time $t$, and $T$ means the last token presented before or at time $t$. We shall also use the random variable

$N_{t,t+\delta} = N_{t+\delta} - N_t$ to specify the number of tokens arising in the half-open interval (t, $t+\delta$ ]. If $R$ is the state at time $t$ and a single token $\tau$ occurs between times $t$ and $t+\delta$, then $R\Diamond\tau$ is defined, regardless of past events before time $t$, by the formula

$$P\left(S_{t+\delta} = S \middle| T_{t+\delta} = \tau, N_{t,t+\delta} = 1, S_t = R, \varepsilon_t\right)$$

$$= P\left(S_{t+\delta} = S \middle| T_{t+\delta} = \tau, N_{t,t+\delta} = 1, S_t = R\right).$$

The agent is endowed with rationality in the form of some group of relations, of which the agents, and the feasible changes are represented by the addition of a pair to the state.

## Prices and Equilibrium

If the market price were not the competitive one, but lower, some of the unsatisfied consumers would realize they could attract a seller by offering an $\varepsilon$ over and above the market price. Serrano and Volij (2000) suppose a similar conclusion would hold for sellers if the market price were higher than the equilibrium one. Each agent $i$ counts on his own endowment $Z_i^{*x}(0)$. Once the allocation $x$ is realized, agent $i$ may make a proposal to agent $j$. All trade under these contracts gives rise to the set of bundles $Z_i^{*x}(1)$ that agent $i$ considers to be achievable. Agent $j$ will think that the bundles in the set $Z_j^{*x}(1)$ are achievable for him. If agents look further, the same thought process continues. Generally agent $i$ believes that the bundles in the set $Z_i^{*x}(t)$ are achievable for him. On the basis of this belief, he constructs the set $Z_i^{*x}(t+1)$, and, as $t \to \infty$, this interactive algorithm yields the limit $Z_i^{*x}(t)$ over which agents will make their final consumption decisions.

The processes are distinct when one takes a finite number of interactions, but they coverage to the same set of bundles in the limit. While in convergence, an agent seeks cooperation with an increasing number of agents, and arbitrageurs envision more and more bilateral contracts among the same finite set of agents.

Each agent must prefer his net trade to any combination of the individual net trades in the economy, where each agent faces the same set of net trades.

For every $i \in N$ and $x_i \in X_i$, define the preferred and weakly preferred sets:

$$P_i(x_i) = \left\{z_i \in X_i \middle| z_i \succ_i x_i\right\}$$

$$W_i(x_i) = \{z_i \in X_i | z_i \succsim x_i\}.$$

Denote agent 1's initial endowment $\{\omega_1\}$ by $Z_i^{*x}(0)$ and agent 2's endowment by $Z_2^{*x}(0)$. Suppose that agent 1 proposes a trade in which agent 2 would give agent 1 his endowment in exchange for a bundle that leaves agent 2 no worse than at *x*. In taking arbitrage to its ultimate consequences, agents could envision more complicated contracts that would be still acceptable and lead to achievable bundles. Thus, in order to see what is the set of bundles from which he can consume, an agent thinks about a process of mutually beneficial trades. Fix an allocation *x* in $\Im$. We have

$$Z_i^{*x}(0) = \{\omega_i\} \qquad \forall i \in N.$$

For t>0 define

$$Z_i^{*x}(t) = Z_i^{*x}(t-1) \cup \bigcup_{k \in N \setminus \{i\}} \left[Z_i^{*x}(t-1) + Z_k^{*x}(t-1) - W_k(x_k)\right], \quad i \in N,$$

and

$$Z_i^{*x} = \bigcup_{t \in N} Z_i^{*x}(t).$$

Fix an allocation *x* in $\Im$ and an agent *i*. Then *Z* is defined as

$Z_i^x(0) = \{\omega_i\} \qquad \forall i \in N$, and for t > 0 define

$$Z_i^x(t) = \bigcup_{\substack{S \in N \\ i \in S}} \left[\sum_{k \in S} Z_k^*(t-1) - \sum_{k \in S \setminus \{i\}} W_k(x_k)\right], \quad i \in N$$

and

$Z_i^x = \bigcup_{t \in N} Z_i^x(t)$. In this process, an agent is allowed to imagine trades with any subset of the *n* agents in the economy.

## Risk and Life-Cycle

Executive compensation increases with stock market returns on equity and with respect to measures of net income, and in competitive equilibrium agents are paid the value of their marginal product, which is partly determined by agents' prices. If agents are risk-friendly, such portfolios seem sub-optimal; there are reasons why risk-averse agents voluntarily hold stock in their own firms, and the rationale for agents holding financial claims to their own firms seems somewhat contrived in economies lacking private information. Margiotta and Miller (2000) suppose that agents are risk-averse, the only reason for imposing additional risk on them is to induce an action that they would not otherwise undertake. They are advocates of the performance to compensation ratio and see it as a benchmark for measuring moral hazard from estimated structural parameters of a model in which agents preferences are implicitly laid out.

We study also an agent's inter-temporal consumption problem, given labor decisions and an implied distribution of future compensation, when an economy with aggregate shocks is supported by complete competitive markets complemented by a public disclosure condition. Agents observe the economic conditions, but they cannot monitor managerial effort perfectly. The firms are infinitely lived, and agents are hired by the market makers from a population. The choices of an agent are taken over the consumption and leisure allocation he receives in each period in his life. Let $c_{nt}$ denote consumption by agent *a* in period *t*. There are levels of labor activity, and we express the *n*th agent choice in period *t* by a vector $I_{nt} = (I_{0nt}, I_{1nt}, I_{2nt})$, where $I_{jnt} \in (0,1)$ is an indicator function for choice $j \in (0,1,2)$ and

$$\sum_{j=0}^{j=2} I_{jnt} = 1$$

.

If $I_{0nt} = 1$, it means that the agent has retired. This activity is publicly observed. Thus lifetime utility can be expressed as:

$$-\sum_{t=\underline{n}}^{t=\bar{n}} \beta^t (a_0 I_{0nt} + a_1 I_{1nt} + a_2 I_{2nt}) \exp(-pc_{nt})$$

,

where $\beta \in (0,1)$ is the common objective discount factor, *a*, is a utility parameter associated with choosing $I$, and *p* is the coefficient of absolute risk aversion. Let $w_{nt}$ denote the compensation received by the *n*th agent at the beginning the period *t*, and let $a_{n,t-1}$ denote the shareholder's equity at the end of period *t – 1*. Define $z_{nt} \equiv w_{nt} / a_{n,t-1}$ as the accounting effect of the agent on the firm's returns realized at the beginning of next period *t*, and denote by $\pi_{nt}$ the return to the *n*th agent at time *t*. Further, $\pi_t$ denotes the return on the market portfolio. Let

$$x_{nt} \equiv \pi_{nt} - \pi_t + z_{nt}$$

.

Here *x* is a random choice in the previous period *t*. We derive the valuation function, or indirect utility, by choosing a consumption sequence that maximizes

$$-E_t\left[a_0 \sum_{s=t}^{\bar{n}} \beta^s \exp(-p_{ns})\right]$$

subject to the budget constraint

$$\acute{E}\left(\sum_{s=t}^{\bar{n}} \lambda_s c_{ns}\right) \leq \lambda\, e_{nt}$$

. All the contingent claims prices $\lambda_s$ in periods $s \in \left(t,....,\bar{n}\right)$ help to determine the optimal consumption streams. Let $p$ denote the price of an asset that, contingent on the history through date $t$, pays a unit of consumption in periods $t$ through $u$. Then

$$p_{tu} \equiv E_t\left(\sum_{s=t}^{u} \lambda_s\right).$$

Let $q$ denote the price of an asset that pays the random quantity consumption units in period $t$ through $u$:

$$q_{tu} \equiv E_t\left[\sum_{s=t}^{u} \lambda_s \left(\ln \lambda_s - s \ln \beta\right)\right].$$

In the next step of consumption, the functional form of the two-period expected utility is

$$-a_j \beta^t \exp\left(-pc_{nt}\right) - a_0 E_t\left[p_{t+1,\bar{n}} \exp\left(-\frac{q_{t+1,\bar{n}} + p\lambda_{t+1} + e_{n,t+1} + p\lambda_{t+1} w_{n,t+1}}{p_{t+1,\bar{n}}}\right)\right],$$

while the budget constraint is

$$\lambda_t c_{nt} + E_t\left(\lambda_{t+1} e_{n,t+1}\right) \leq \lambda_t e_{nt}.$$

There is extensive evidence for the existence of such preferences, given the assumption that agents have preferences for reciprocity.

## The Game and Information

While agents may play short games many times over the course of their lifetimes, they are assumed to be randomly re-matched after each play of the game with a new agent-partner. Guttman (2000) supposes that agents have no information about the past moves of their agent-partners, implying that the population is sufficiently large so that the likelihood of an encounter with an agent with whom they have previously been matched is negligible.

We assume a population of agents, of whom the proportion $r \in [0,1]$ of (row) players are reciprocator *(R)* types and the remaining (column) players are opportunist *(O)* types. We assume that the set of *R* type players changes from a given generation $g_1$ to the following generation $g_2$ according to the following evolutionary rule:

$$r(g_2) \gtreqless r(g_1) \quad as \quad \overline{\pi}_i(g_1) \gtreqless \overline{\pi}_0(g_1)$$

,

where $\overline{\pi_i}\ (i = r,0)$ is the mean realized payoff for agent of type $i$. For a finite sized population, a finite number of transactions over the agent's lifetime is a subset of all possible matchings, so $\overline{\pi}$ will be distributed around expected payoff $E\pi_{ti}$.

The agent chooses his optimal strategy given his preferences from one to the next generation of life times, and we have $E\pi_t = 1 \quad and \quad E\pi_0 = 0$. Thus, for all $r > 0$, $E\pi_t : E\pi_0$, so that for any proportion of reciprocators, a reciprocator monitors the opportunist's move before choosing his own move. If $c < 1$, the expected payoff for the $R$-type is

$E\pi_t = [r \cdot 1] + [(1-r)(1-c)] = 1 - c(r-1)$ and a similar calculation for expected payoff for opportunist type gives $E\pi_0' = r$. In equilibrium, we have $1 + cr - c = r$.

If $c > 1$, the reciprocators do not monitor opportunists' moves and the expected payoffs are $E\pi_t = r$ and $E\pi_0 = 0$. When agents cannot identify their opponent's type, and $c \geq 1$, there is an all-opportunists type population. This possibility arises because the agents will change their strategies as the proportion of opportunists rises in the population. As $r$ falls, a point will be reached at which the reciprocators will start monitoring their opponents' moves, or alternatively will switch to defection. The condition for cooperation to be optimal is then

$r + (1-r)b > ra_t$, which implies

$$r > \frac{-b}{1-b-a_t}$$

. It was suggested above that as $r$ declines, the reciprocators will eventually switch to monitoring, and their opponents will be induced to cooperate, leading to the joint-cooperate outcome with a payoff of unity.

## Consumption and Income

This motivates a life-cycle framework, where we have argued that finite lives and a life-cycle pattern in risk are important for understanding the mapping between income and consumption as per Storesletten, Telmer and Yaron, (2001). Agents are indexed by their age, $h$, where $h \in \mathrm{H} = \{1,2,\ldots\ldots,H\}$. Each of the $H$

age cohorts consists of a number of agents who face uncertain lifetimes with a maximum length of $H$ years. Each year, a new cohort of agents is born and some positive fraction of each existing cohort dies.

We use $\phi_h$ to denote the unconditional probability of surviving up to age $h$. We also use $\xi_h = \phi_h / \phi_{h-1}$ to denote the probability of surviving up to age $h$ conditional on being alive at age $h - 1$. Each agent is characterized by a preference ordering over consumption distributions, an endowment process, and an asset market position. Preferences for an unborn agent are represented by

$$E \sum_{h=1}^{H} \beta^h \phi_h u(c_h)$$

,

where $\beta$ denotes the utility discount factor, $c_h$ denotes the consumption of an $h$ year old agent , $u$ is utility function, and the expectation is assumed to be conditional on the state of the economy prior to birth. An agent begins to work at age $h = 1$ and receives an endowment $n_h$. Denoting the value function of an agent of age $h$ as $V_h$, the agent's choice problem can be represented in the following way:

$$V_h(z_h, a_h, Z, \mu) = \max_{a'_{h+1}} \left\{ u(c_h) + \beta \frac{\phi_{h+1}}{\phi_h} E\left[ V'_{h+1}(z'_{h+1}, a'_{h+1}, Z', G(\mu, Z)) \middle| z_h, Z \right] \right\}$$

,

where $\mu$ is the distribution of agents with respect to age, $a_h$ denotes asset holdings at the beginning of the period, and $a'_{h+1}$ denotes asset holdings at the end of the period. $Z$ is a productivity shock.

Welfare comparisons can be made across alternative economies, and the welfare gain associated with moving from economy $A$ to economy $B$ is measured as the proportional compensation required to increase consumption in a way that will make the agent indifferent between the two economies. The welfare gain for age-cohort $h$ can now be expressed as the number $\psi_h$ that solves the problem

$$V_h^A(\cdot, Z, \mu; \psi_h) = \max_{a'_{h+1}} \left\{ u[c(1 + \psi_h)] + \beta \frac{\phi_{h+1}}{\phi_h} E V_{h+1}^A(\cdot, Z, \mu, \psi_h) \right\}$$

subject to the constraint that the proportional change $\psi_h$ in consumption be held equal across agents of age $h$ . The agents go through fundamental structural changes and use risk neutral capital that is sufficient to ensure a run-free optimal outcome.

# 14 Shocks, Uncertainty, and Preferences Over Information

We assume that the preferences are complete, transitive and monotone, which means if partition *A* is finer than partition *B*, the decision maker strictly prefers *A*. Then decision maker is indifferent between total ignorance and receiving a signal that tells her if the state 1/2 has occurred or not. Ex-post knowledge of the state 1/2 may be valuable, but since it is a probability event the signal is worthless to decision maker. Dubra and Echenique (2001) believe that monotonicity as an assumption for it is dubious and that many people would be indifferent between ignorance and the 1/2-signal above. The question is, what is the behavioral assumption for the analysis of information? This describes a choice problem where the decision maker must first choose the information structure that he finds more useful for a second choice problem involving bets.

Suppose that partition $\tau$ is finer than $\tau'$, so there is an element $k'$ *of* $\tau'$ that gives a collection of sets $\{k_a\}$ and $k^c$ is the set of the rest of the elements of the set. The agent must choose between $\tau$ *and* $\tau'$. After he is informed in what element of the chosen partition the true state lies, he must choose the robustness of a utility representation, in which case we need to analyze arbitrary preferences over the information, and the representation breaks down. The representation rests on a large number of indifferences, and we show how preferences arise naturally if the agent is an optimizer, which suggests that agents dislike uncertainty. That is, agents value information not only to make plans, but also for its own sake. If agents have priors over the state space, preferences for information exist in partitions of the state space. We model information by partitions of a set of possible states of nature, $\Omega$. A partition $\tau$ of $\Omega$ is a collection of subsets, and for each state of nature $\omega$ there is a corresponding element of $\tau$. A decision maker whose information is represented by $\tau$ is informed if contains the true state of nature that has occurred. Let $P(\Omega)$ be the set of all partitions of $\Omega$, if $\tau, \tau' \in P(\Omega)$, say that $\tau'$ is finer than $\tau \neq \tau'$, if for every $A \in \tau'$, there is *B* in $\tau$ such that $A \subseteq B$.

If a decision maker has information represented by $\tau'$, she is informed of the event $B \in \tau'$. If her information had been represented by $\tau$, she would have known that a certain event $A \supseteq B$ occurred. The utility theory is not useful tool in the analysis of the value of information. Suppose, that there is a function $u : P(\Omega) \to \Re$. Foe each $x \in (0,1)$ let:

$$\tau_x = \{\{y\} : 0 \leq y < x\} \cup [x,1],$$

$$\tau'_x = \{\{y\} : 0 \leq y \leq x\} \cup (x,1].$$

But there is a rational number $r(x)$ such that $u(\tau_x) < r(x) < u(\tau'_x)$, thus:

$$u(\tau_x) < r(x) < u(\tau'_x) < u(\tau_{\bar{x}}) < r(\bar{x}) < u(\tau'_{\bar{x}}).$$

Let $\Omega = [0,1]$ and let *P* be a set of probability measures on $\Omega$. A decision maker must choose an action in $A = [0,1]$ after observing a signal about the state of nature. The utility in event *B* when action *a* is chosen is:

$$U(B,a) = \inf_{p(B)>0} \int_B \frac{u(\overline{\omega},a)}{p(B)} dp(\overline{\omega}).$$

If for all $\omega \in \Omega$,

$$U(k_\tau(\omega) \cap k_{\tau'}(\omega), a[k_\tau(\omega)]) \geq U(k_\tau(\omega) \cap k_{\tau'}(\omega), a[k_{\tau'}(\omega)]),$$

and there exist $\overline{\omega} \text{ and } p \in P \text{ with } p(\overline{\omega}) > 0$ such that $\tau \succ \tau'$. The decision maker believes that a state will occur in which choosing the optimal action under $\tau$ will make her better off than choosing the optimal action under $\tau'$, using Bayesian updating on all priors in *P*.

Any $\tau \in T(\Omega)$ has a countable number of elements, say $\tau = \{[A_k : k \in \mathrm{N}]\}$. Let:

$$u(\tau) = \sum_{k \in \mathrm{N} : A_k \neq \otimes} \inf A_k,$$

then *u* represents a preference relation over information on $\Omega$. For any $A \subseteq \Omega \text{ with } \omega \in A$ and at least elements

$$\{A, A^c\} \preceq \tau \preceq \{\{\omega\}, A \setminus \{\omega\}, A^c\}.$$

We consider state of nature in which the decision maker gains relatively little from being perfectly informed about this state, where a utility representation for partition arises. A decision maker must choose an action, an element in a compact set *A*, after observing a signal about the state of nature, and knowledge is represented by the probability measure $\mu$ over $\Omega$, given a probability space $(\Omega, F, \mu)$. Let $u : \Omega \times A \to \Re$ be the decision maker's measurable state utility function, and let $k_\tau(\omega)$ be the element of $\tau$ that contains $\omega$. When $\omega$ is realized, the decision maker is informed, and let:

$$a^*(\omega) \in \arg\max_{a \in A} \int_{k_\tau(\omega)} u(\overline{\omega}, a) d\mu(\overline{\omega})$$
,

so that for each $\omega$, $a^*(\omega)$ is the decision maker's optimal choice, given her signal $k_\tau(\omega)$.

The preference relation is represented by a utility function $U : P(\Omega) \to \Re$ such that:

$$U(\tau) = \int_\Omega \left\{ \int_{k_\tau(\omega)} u(\overline{\omega}, a^*(\omega)) d\mu(\overline{\omega}) \right\} d\mu(\omega)$$
,

for some action space $A$, $u : \Omega \times A \to \Re$ and beliefs $\mu$.

The value of more information is that decision maker has fewer restrictions on her choice of action. If the decision maker faces a limited number of alternative actions without more information, she must choose an action in *A* after observing a signal about the state of nature. Suppose state $\omega$ occurs and the decision maker is informed of the element of the partition that has occurred, say $B = k_\tau(\omega)$. Each partition $\tau$ generates a function $a_\tau : \Omega \to A$. Let $f : P(\Omega) \to A^\Omega$ be the map that takes partitions into functions from $\Omega$ *to* $A : f(\tau) = a_\tau$.

## Choice and Endowment

The choice set must contain the endowments and satisfy an arbitrage-freeness condition, which says that each agent's set of choice must contain all bundles that can be achieved through acceptable connection with other agents, where for each agent *i*, the set *C* is the budget set

$$H_i^p = \{x_i \in \Re^l : px_i \le p\omega_i\}$$
,

where *p* is an equilibrium price vector. Arbitrage free equilibria are therefore anonymous across agents in these environments, regardless of one's initial situation, the set of net trades that each agent considers available is independent. The choice set uses unlimited sales since using the upper contour sets of the agent's preferences make payments to agents for their resources. If an agent can find an arbitrage opportunity moving along a particular direction in the set of net trades, there exist net trades that are also arbitrage opportunities. An allocation $(x_i)_{i \in N}$ in $\Im$ is said to be a coalition *S* if there exists $i \in S$ with

$$P_i(x_i) \cap \left[ \sum_{k \in S} \omega_k - \sum_{k \in S \setminus \{i\}} W_k(x_k) \right] \neq \otimes$$

When dealing with fluctuations, thick market externalities may appear more relevant through learning by doing, so we should expect externalities coming from the market. Agents supply a variable quantity of labor hours and may save a fraction of their income by holding productive and nominal capital outside money. Rental return on capital and wages are paid by the agents at the end of the period when agents buy consumption goods.

## Dynamic Signaling of Information

Both good- and bad-quality types incur a cost in producing a good and can produce either unit at each time period. Each time that a seller produces a good, he will be successful with probability *g* and unsuccessful with probability 1 – *g*, whereas for a bad type the probabilities are *b* and 1 – *b*, where $1 > g > b > 0$.

Bar-Isaac (2003) supposes that sellers and buyers are share a common belief that with probability $\lambda_t$ the seller is good and with probability $1 - \lambda_t$ he is bad. After observing the seller's decision to continue trading, buyers revise their beliefs to $\mu_t$ and the price will be $\mu_t g + (1 - \mu_t)b$.

The success or failure of the good can only be determined if its realization is publicly observable, and the price of the good in any period varies with the history of the seller's previous successes and failures. The sellers begins with some initial reputation $\lambda_0$, and we can summarize:

she decides whether or not to trade in this period; on the basis of this decision beliefs are revised to the interim belief $\mu_t$; the seller then produces a unit at a cost *c* and sells it;

the buyer then consumes the good and its realization as a success or failure is publicly observed; the seller's end of period reputation is $\lambda_{t+1} = \mu_t$.

Buyers' strategies are the prices that they bid for the good, given their beginning of period belief, and all the propositions characterize a Markov equilibrium. We suppose that the seller has no private information regarding her type, that and her belief regarding her own type is identical to her public reputation. In her decision about whether to stop selling or not, the seller takes into account the profit that she expects to earn in that period and the value of the information generated about her ability. Her reputations after a success and failure are given respectively by the following:

$$\lambda^s = \frac{\lambda g}{\lambda g + (1-\lambda)b},$$

and

$$\lambda^f = \frac{\lambda(1-g)}{\lambda(1-g) + (1-\lambda)(1-b)}.$$

Information implies that the only factor that has an influence on a seller's value of trading is her current reputation, and can be defined by the following:

$$V^u(\lambda) = \lambda g + (1-\lambda)b - c + \beta\{\lambda g + (1-\lambda)b\}\max\{0, V^u(\lambda)\}$$
$$+ (1 - \lambda g - (1-\lambda)b)\max\{0, V^u(\lambda^f)\},$$

which represents the profit in the current period, and discounted future expected profit.

If there is a very low belief that the seller is good then she would have to sell at a loss in the current period, and the posterior belief derived above following Bayes' rule would not be much different.

There exists a critical reputation level $\lambda^u \in (0,1)$, such that if the reputation at the beginning of a period, $\lambda$, is greater than or equal to this critical level, then there is trade and if $\lambda < \lambda^u$, trade ceases. Since $V^u(\lambda)$ is increasing in $\lambda$, it follows that for all $\lambda \geq \lambda^u$, $V^u(\lambda) \geq 0$, so that the seller will seek to trade. Further note that $\lambda^u$ can be thought of as an absorbing barrier for reputation, if ever reputation falls below this below this level.

With the mechanism in equilibrium, a seller's self-confidence when she sells at a loss acts as a signal of her quality. The private information allows a multiplicity of equilibria in contrast to the unique outcome when the seller is uninformed. Potential buyers therefore take account of the fact that the seller is willing to trade in assessing the likelihood that she is good. The value of a sale for a good type and a bad type can be written as:

$$V^g(\lambda) = \mu g + (1-\mu)b - c + \beta\{g\max\{0, V^g(\mu)\} + (1-g)\max\{0, V^g(\mu^f)\}\},$$

and

$$V^b(\lambda) = \mu g + (1-\mu)b - c + \beta\{b\max\{0, V^b(\mu^s)\} + (1-b)\max\{0, V^b(\mu^f)\}\}.$$

What differs is the short-term profit for an uninformed seller, which is $\lambda g + (1-\lambda)b - c$, and the interim reputation is revised to $\mu$, which will be determined by equilibrium strategies and beliefs. The only difference between equations is that in the value function it occurs with probability *g* rather than *b*, so the value of a sale should lead to an increase in reputation $V(\lambda)$. Good sellers value continuation more highly than bad sellers do, at almost all levels of reputation. This is a consequence of the buyers' knowledge that a good seller is relatively more willing to incur a loss to continue trading, which induces a lower bound for the reputation and a limit to how low the sale price can fall. If a bad type stayed in the market in equilibrium regardless of her current reputation, then at some reputation level the sum of the value and the short-term profit would be negative, since buyers' beliefs could drop arbitrarily low. Thus at some reputation level, a bad seller would drop out for sure. Then buyers would assume that any seller seeking to trade must be a good type and so would be willing to pay $g$ at all periods in the future. A bad seller would then clearly prefer to stay in and earn $\frac{g-c}{(1-\beta)}$.

Thus in equilibrium bad seller employs a mixed strategy of continuing to trade, and such mixed strategies on the part of a bad seller allows some continuity in the beliefs of buyers.

Consider the value function for a bad type if buyers believed that sellers never stopped trading regardless of type:

$$W^b(\lambda) = \lambda g + (1-\lambda)b - c + \beta b \max\{0, W^b(\lambda^s)\} + \beta(1-b)\max\{0, W^b(\lambda^f)\},$$

where the price in the short run is $\lambda g + (1-\lambda)b$.

Suppose that the seller's reputation is $\lambda \in (0,1)$. Then in equilibrium, in which if $\lambda < \lambda^*$, a good seller trades for sure and bad seller continues trading with probability $d(\lambda)$ and ceases trading with a probability $(1-d(\lambda))$, where

$$d(\lambda) = \frac{\lambda(1-\lambda^*)}{(1-\lambda)\lambda^*}.$$

This occurs for high reputation levels, that is above $\lambda^*$, but if her reputation falls then a bad type would cease trading with a mixed strategy with probability $d(\lambda)$ such that she would be indifferent between her reputation and the fact that a good seller prefers to sell, which in equilibrium she does.

Thus for a good seller $\lambda^*$ acts as a reflecting barrier and for a bad seller it is further worth noting that $\lambda^* < \frac{c-b}{g-b}$, where this latter value is the reputation level that allows the seller in with respect to the frequency with which each type generates success. The profit realized depends only on buyers' beliefs, and possible off-equilibrium actions are not selling when one's reputation is high. Any other behavior is equilibrium action, and the buyers' beliefs are contingent on observing that the seller's willingness to trade should be non-decreasing in the beginning of the belief period.

An informed market maker, who knew the seller's type and wished to maximize the payoffs to the seller and the buyers, would ensure that there were taxes or some other reason in virtue of which the seller could only appropriate a fraction of the buyer's valuation. Then the value of having any reputation would be lower for good and bad sellers alike. The value for a good seller is higher than the value for a bad seller, and so a good seller would never cease trading so that is as before. Suppose that the seller keeps only a fraction $\gamma$ of the price, lowering the potential gains from trading for a seller whether good or bad. Thus increasing a per unit tax reduces $\gamma$. A good seller always sells, and there exists a $\lambda^*(\gamma)\in(0,1)$ such that a bad seller sells with certainty if the belief $\lambda \geq \lambda^*(\gamma)$ and sells with probability $d(\lambda,\gamma)$ otherwise. The lower the proportion of the consumer valuation that the seller can appropriate, the more likely that a bad seller would cease trading, and the greater the level of efficiency. The seller has some imperfect private information concerning her own type, which is not open to the information that buyers commonly hold. A seller who has private information regarding her own type will have a lower continuation value than a seller with encouraging information. The reputation based on the observation that the seller is willing to continue trading would raise the seller's assessment of her quality, and this level would be reached before a seller with private information was induced to cease trading. The seller privately receives either a high or low signal; if she receives a high signal then she believes initially that she is good with probability $\overline{\lambda}_0$ and if she receives a low signal the corresponding probability is $\underline{\lambda}_0$, where $1 > \overline{\lambda}_0 > \underline{\lambda}_0 > 0$.

We assume that this signal is private information, and that initially the buyers have some belief, $r_0$ about the seller having received a high signal and a probability $(1-r_0)$ about the seller having received a low signal. The information set of the seller is superior to the buyers' information set, and in this environment there are a number of different beliefs to bear in mind. The seller is not informed as to her own type and so revises her belief that she is good depending on her history and also on the signals that she receives. We denote the belief at time $t$ of a seller with a good signal by $\overline{\lambda}_t$ and the belief of a seller with a bad signal by $\underline{\lambda}_t$, and these depend on a commonly observed history. All agents can calculate $\overline{\lambda}_t(h_t)$ *and* $\underline{\lambda}_t(h_t)$, and we further assume that $r_0$ is common knowledge so that in equilibrium so is $r_t(h_t)$.

In this environment, the seller does not have information as to her type and revises her belief that she is good in the light of previous history. The seller's signal is private information, there is an equilibrium in which a seller with a high signal behaves in the same way that she would if that signal were public. There is an equilibrium in which the set of histories $\overline{PH}$ induce a seller with a high private signal to stop selling. A seller with a high signal prefers to continue selling longer than a seller with a low signal. The seller's decision to continue selling is informative and its influence on buyers' beliefs can be no higher than if the buyers were certain that the seller had received a high signal. There will be a range of histories for which sellers who had received low signals will employ mixed strategies and continue selling with some probability. In contrast with those equilibria, a consequence of imperfect private information is that there will be a range of histories for which sellers who had received a high signal will trade with certainty and sellers who had received a low signal will cease trading with certainty. There exists a range of reputation levels $[\lambda^u, \lambda^*(1)]$ for which there is separation of sellers in that sense.

The low-signal seller might cease trading with some positive probability, so that his public reputation would move closer to the belief of a seller who had a good signal. In each period the seller has the opportunity to cease trading with probability $\alpha$, which is essentially a measure of how strategic the seller can be. In any equilibrium there exist reputation levels $0 < l < h < 1$ such that if the reputation is denoted by $\lambda$, then if $\lambda < l$ then bad and good types would cease trading, and if $\lambda > h$ then if given the opportunity both types would continue trading. If a seller has a very low reputation, it takes a long time to recover even if things go as well as possible. Buyers' beliefs are non-decreasing and $\mu(\lambda)$ is non-decreasing in $\lambda$, and for any $\alpha \in (0,1)$ self-confidence leads to equilibrium.

In any equilibrium in which buyers' beliefs are non-decreasing in beginning of period, there exist values $0 \le k \le l \le m \le h \le 1$ such that the seller makes strategic decisions to continue as follows:

- with reputations $\lambda < k$, both good and bad types would cease trading for sure;

- for $k \le \lambda < l$, a good type would continue trading with some probability and a bad type would cease trading for sure;

- for $l \le \lambda < m$, a good type would continue trading for sure and a bad type would cease trading for sure;

- for $m \le \lambda < h$, a good type would continue trading for sure, which ensures that in equilibrium the reputation would be $h$;

- for $h \le \lambda$, both types would continue trading for sure.

## Labor Market

Labor market imperfections due to incomplete or imperfect information prevent agents from borrowing against their human capital, while productive capital is accepted as collateral to secure a loan. Cazzavillan, Lloyd-Braga and Pintus (1998) suppose that agents behave like agents living for two periods: they choose optimally their labor supply for today and, consequently, their next period consumption demand. In each period $t \in N$ , a consumption good is produced combining labor $l$ and the capital stock $k_{t-1} \geq 0$ resulting from previous period. Agents take real rental prices of capital and labor and determine their input demands by equating the private marginal productivity of each input to its real price, and the real competitive equilibrium wage is:

$$\Omega = A(k,l)\omega(a) = A(k/a)^{\upsilon}\psi(a)\omega(a),$$

where $A$(k , l) is the total factor productivity function, social returns to scale are $1+v$ , and $A(k,l) = Al^{\psi}\psi(a)$ . An agent solves the utility optimization problem

$$\max\{V_2(c_{t+1}^{w}/B) - V_1(l_t)\} \text{ subject to } p_{t+1}c_{t+1}^{w} = w_t l_t, \quad c_{t+1}^{w} \geq 0, \quad l_t \geq 0,$$

where $B$ > 0 is a scaling factor , $c_{t+1}^{w}$ is the next period consumption , $l$ is the labor supply, $p_{t+1} > 0$ is the next period price of consumption, and $w$ > 0 is the nominal wage rate , with $w_t = p_t\Omega_t$ . The utility functions $V_1(l)$ and $V_2(c)$ are continuous for $0 \leq l \leq l^{w}$ and $c \geq 0$, where $l^{w} > 0$ is the agents' endowment of labor. Since agents save wage income in the form of money, the equilibrium money market condition is:

$$\Omega(a_t, k_{t-1})l_t = M/p_t,$$

where $M$ > 0 is the constant money supply and $p$ > 0 is the current nominal price of consumption.

In decision making under uncertainty we study the situation $(S;\Omega;F;X)$, corresponding to expected utility in uncertainty aversion, in which there exists an urn of known composition which is preferred to the given urn of unknown composition, and $S$ is a set of states of nature, $\Omega$ is an algebra on $S$ , $X$ is a set on consequences and $F$ is a set of every possible act from $S$ to $X$.

## Expected Utility

Montesano and Giovannoni (1996) suppose expected utility is given by:

$$CEU(f) = u(x_n) + \sum_{i=1}^{n-1} (u(x_i) - u(x_{i+1}))v\left(\bigcup_{j=1}^{i}\right).$$

In this situation, $\succ$ reveals uncertainty aversion if $\exists p : 2^S \to [0,1]$ such that $EU(f) \geq CEU(f)$,

where $CEU = (Max \ \ u;I;v)$ $EU = (Max \ \ u;I;p)$ and $p$ is a probability, and the preference relation reveals neutrality to uncertainty if $v$ is a probability. Obviously, there is neutrality to uncertainty if and only if the capacity is a probability. If $\succ$ reveals aversion to (or appeal of) to *increasing* uncertainty, then it also reveals aversion (or appeal) generally.

If

$$UA(af \oplus (1-a)g) \leq aUA(f) + (1-a)UA(g) \quad \forall f, g \in F \quad \forall a \in [0,1],$$

where $UA(f) = EU(f) - CEU(f)$. If we evaluate the uncertainty aversion of an act on the basis of the difference between its *EU* and *CEU*, we may also see that uncertainty aversion increases (or decreases) when uncertainty increases (or decreases). We call a situation a "decision making under uncertainty" situation where probability $p$ is assigned to events is $2^S$, and situation is $(S, 2^S, F, X, p)$, where $(S, 2^S, F, X)$ is a case of risk dependent expected utility we have

$$RDEU(q) = u(x_n) - \sum_{i=1}^{n-1} (u(x_i) - u(x_{i+1}))\varphi\left(\sum_{j=1}^{i} p(s_j)\right),$$

where $\varphi$ is an increasing distortion function, and random process $q = (x_1; p(s_1), \ldots\ldots\ldots, x_n; p(s_n))$.

For a situation, the following statements are equivalent: $\succ$ reveals risk aversion of the first order, and $\varphi(z) \leq z \quad \forall z \in [0,1]$. If $u$ is a strictly increasing function, for any act $f \in F$, the uncertainty premium $UP(f) = u^{-1}(EU(f)) - u^{-1}(CEU(f))$, and the risk premium is analogous.

## Investments, Flexibility, and Productivity

The problem of investing in the setup and production process displays some features that are similar to the choice between flexibility and productivity. An investment in the setup process is appropriate when there is a high initial setup cost $F_0$, interest rates $i$ are low, initial lot sizes $Q_0$ are small, and demand rates $d$ are high. Hofmann (1998) supposes the total cost per period is:

$$K = \frac{Fd}{Q} + \frac{1}{2}lQ\left(1 - \frac{d}{p}\right) + iCF,$$

where the interest amount $iCF$ shows the economic consequences of the investment per period. The optimal capital investment is $CF_c^*$, the optimal production quantity is, $Q_c^*$ and the critical parameter is $\overline{\alpha}_c$. The solutions are:

$$Q_c^* = \frac{2i}{\alpha l(1 - (d/p))},$$

$$CF_c^* = \frac{1}{\alpha}\ln\left(\frac{\alpha^2 F_0 dl(1 - (d/p))}{2i^2}\right),$$

$$\overline{\alpha}_c = i\sqrt{2/[F_0 dl(1 - (d/p))]}.$$

The total capital investment equals $I = CF + CC$,

where $CC$ is the additional expenditure for the investment in the production process. The optimal expenditure $CC^*$ for an investment in the production process is:

$$CC^* = \frac{1}{\beta}\ln\left(\beta\frac{cp_0}{i}\frac{1 - e^{-iQ^*/p}}{1 - e^{-iQ^*/d}}\right).$$

Thus $CC^*$ depends on the potential savings associated with reducing the production cost when $\beta > \overline{\beta}$. The critical parameter is defined as:

$$\overline{\beta} = \frac{i}{cp_0}\frac{1 - e^{-iQ^*/d}}{1 - e^{-iQ^*/p}},$$

where the impact of a reduced interest rate $i$ is even larger as for the investment process.

We obtain the critical parameter $\overline{\gamma}$ for an investment to increase the production rate as:

$$\bar{\gamma} = \frac{\left(1 - e^{iQ^*/d}\right)e^{iQ^*/p_0}}{\left(cp/p_0\right)Q^*},$$

where high per unit of time production costs *cp*, interest rates *i* and production rates $p_0$ result in a critical parameter $\bar{\gamma}$.

Whereas an investment which reduces production costs decreases the optimal expenditure necessary, an investment which increases production rates may lead to as an increase in the optimal capital expenditure. If an investment in the production process enhances the technological ability to reduce costs, small production rates support increasing capital expenditures for investments. In these systems, the product variety, its heterogeneity, as well as characteristics of the production system, have an additional impact on the capital expenditure for each investment alternative.

## Conditional Desirability

Conditional attitudes are not the attitudes an agent is disposed to acquire upon learning that a condition holds. A conditional desirability measure for degrees of conditional desire is proposed, and implies that an agent's degrees of conditional belief are conditional probabilities. Bradley (1999) considers the following definition of conditional probability: if $\Pr ob(Y) \neq 0$,

$$\Pr ob(X|Y) = \Pr obXY / \Pr obY.$$

An agent's degrees of conditional belief are conditional probabilities on a pair of inconsistencies if an agent's degrees on conditional belief do not equal the ratio of her degrees of unconditional belief. There are two ways in which one might expect an agent's degree of desire for some *X* to be influenced by the supposition that *Y* is true. The first is the effect of the truth of *Y* on the subjective value for the agent of the information that *X* is true, where the conditional desirability, *Des,* of *X* given *Y* depends on the desirability of *XY*. The second is the effect of the truth of *Y* on the degree to which the agent believes *X*, so the conditional desirability of *X* given *Y* depends on the probabilistic dependence of *X* on *Y*. If $\Pr ob(X,Y) = 1$, then $Des(X,Y) = DesT$, so the various conditional desirability measures on *T* are not equated. If $\Pr ob(X,Y) \neq 0$ $Des(X|Y) = DesXY - DesY + DesT$.

Notice that if an agent is sure that some condition is false then there will be no component of her unconditional attitudes that builds on the consideration she gives to that condition's possible truth. Think of the desirability of a proposition as the amount of money that an agent is prepared to spend to make it true. A rational agent's degrees of conditional desire are conditional abilities just as her rational degrees of conditional belief are conditional probabilities, so if *Prob* is a measure of her degrees of belief, then it is such a measure for any $Y$ having non-zero probability. If $XY = F$, $XZ \neq F$, and $YZ \neq F$, then

$$Des(X \vee Y|Z) = \frac{Des(X|Z) \cdot Prob(X|Z) + Des(Y|Z) \cdot Prob(Y|Z)}{Prob(X|Z) + Prob(Y|Z)},$$

if $XYZ \neq F$ and $X \sim YZ \neq F$ then:

$$Des(X|Z) = Des(XY|Z) \cdot Prob(Y|XZ) + Des(X \sim Y|Z) \cdot Prob(\sim Y|XZ),$$

$$Des(X|Z) \cdot Prob(X|Z) + Des(\sim X|Z) \cdot Prob(\sim X|Z) = DesT,$$

$$Prob(X|Z) = \frac{DesT - Des(\sim X|Z)}{Des(X|Z) - Des(\sim X|Z)}.$$

If *Des* is a desirability measure of an agent's degrees of desire, then *Des(Y)* is a desirability measure of her degrees of conditional desire given $Y$. A rational agent's degrees of conditional belief are conditional probabilities and they can be derived from the assumption that an agent's degrees of conditional desire are conditional abilities and that conditional desirability functions are desirability measures of degrees of conditional desire. Then:

$$Prob_Y X = Prob(X|Y) = ProbXY/ProbY.$$

If an agent's degrees of conditional desire are measured by the desirability function $Des_y$, then her degrees of conditional belief are measured by the probability function $Prob_y$, and conditional desirability functions are the right desirability measures of a rational agent's conditional desires. Let $\geq$ be an unconditional preference relation for any non-contradictory proposition $A$, and let $\geq_A$ be a relation on sets of propositions consistent with $A$. An agent's conditional preferences given $A$ should relate to her unconditional preferences in accordance with: $Y \geq_A Z \quad AY \geq AZ$, i.e. she cannot consistently prefer the truth of $AY$ to that of $AZ$ and not prefer that, given $A$, $Y$ be true rather than $Z$. There exist a desirability measure $Des_A$ and a probability measure $Prob_A$ on $\Omega_A$ such that $X \geq_A Y \quad Des_A X \geq Des_A Y$. That means that the agent's unconditional preferences imply the existence of conditional desirability and probability functions, which agree with the fact that her conditional preferences given $A$ imply the existence of unconditional desirability and probability functions agreeing with her unconditional preferences. Agents are either risk averse in proportion $\theta$, or risk neutral in proportion $1-\theta$, and each risk averse agent has an endowment $K$ in period 0 and nothing in any other period. The total endowment in the economy in period 0 is $\theta+(1+\theta)K$, and for each unit of resource invested in period 0, the return is $R$ in period 1.

The investment may be liquidated in period 1, which can be recovered. Gangopadhyay and Singh (2000) suppose $c_{ii}$ denotes the consumption of a type $i$ agent in period $i$ = 1,2. The expected utility of a risk averse agent in period 0 is

$$EU'^{p}=\int_{0}^{1}\left[tu\left(c_{11}^{p}\right)+(1-t)pu\left(c_{22}^{p}\right)\right]dF(t)$$,

from which it follows that we only need to consider $c$; $p$ is the discount rate, 0 < p< 1. Similarly, for a risk neutral agent

$$EU^{p}=\int_{0}^{1}\left[tc_{11}^{p}+(1-t)pc_{22}^{p}\right]dF(t)$$,

and the assumption $\mu R>1$ guarantees that agents prefer the long-term returns.

If an agent invests on her own , then $EU^{p}=t^{p}u(1)+\left(1-t^{p}\right)pu(R)$.

# 15 Externalities, Market Information and Contracts

In a wide variety of games, agents take actions in the first stage that determine the set of constant marginal costs used to play a game in the second stage. Suppose $n(\geq 2)$ agents with constant marginal costs play a game, and in Nash equilibrium each agent produces a positive output $q > 0$, for $i = 1, \ldots, n$. Denote agent $i$ 's marginal cost as $c$ and the inverse demand as $P = P(Q)$, where $P$ is the market price and $Q = \sum_{i=1}^{n} q_i$ is industry output. Agent i 's profit is $P(Q)q_i - c_i qi$. Following Salant and Shaffer (1999), let $\Delta c_i$ denote the change in agent $i$'s marginal cost and $\Delta q_i$ denote the induced change in agent $i$'s equilibrium output. Then the change in aggregate production costs is:

$$\sum_{i=1}^{n} (c_i + \Delta c_i)(q_i + \Delta q_i) = \sum_{i=1}^{n} \Delta c_i \Delta q_i + \sum_{i=1}^{n} c_i \Delta q_i + \sum_{i=1}^{n} q_i \Delta c_i .$$

Then, aggregate production costs $\sum c_i q_i$ strictly decrease after the rearrangement if and only if the variance of the marginal costs strictly increases. Since both industry revenue $(QP(Q))$ and gross domestic surplus $\left(\int_{u=0}^{Q} P(u)du\right)$ depend only on industry output, industry profits and the social surplus both increase if aggregate production costs decline.

## Asymmetric Information

Suppose all agents have the same marginal cost initially, and aggregate production costs are then larger, while industry profits and social surplus are smaller than they would be under any other configuration, then the right-hand side of equation simplifies to:

$$q\sum_{i=1}^{n}\Delta c_i + \sum_{i=1}^{n}(c + \Delta c_i)\Delta q_i$$

.

Since the rearrangement of marginal costs affects neither industry revenue nor consumer surplus while it strictly reduces aggregate production costs, industry profits and the social surplus must increase. Frequently, each agent in an industry can produce at lower cost once experience is gained with a production process. Let $x$ denote the list of prior actions of the $n$ agents; such actions can be production, resource extraction, level of allowed pollution and R&D investment, among others. Denote the cost to agent $i$ of action $x$ as $f^i(x)$. Denote industry action costs as $F(x) = \sum_{i=1}^{n} f^i(x)$. Denote the sum of the marginal costs of production in the last stage as $h(x) = \sum_{i=1}^{n} c^i(x)$. Let $x_i = x^*$ for $i$ = 1,......,n be any action combination in which all agents invest the same amount $x^*$, and let $c(x^*)$ be the sum of the second-stage marginal costs of production. When agents invest, industry profit can be written as a function of a variable:

$$\prod(x_i;x^*) = R(x_i;x^*) - I(x_i;x^*) - C(x_i;x^*),$$

where $I(x_i;x^*) = F(x_i, g(x_i;x^*), x^*, \ldots\ldots, x^*)$ is the aggregate cost of action in the first stage,

$$C(x_i;x^*) = \sum_{i=1}^{n} c^i(x_i, g(x_i;x^*), x^*, \ldots\ldots, x^*)q^i(x_i, g(x_i;;x^*), x^*, \ldots\ldots, x^*)$$

is the aggregate cost of production in the second stage, and $R(x_i;x^*)$ is industry revenue then. Similarly social surplus can be written as:

$$W(x_i;x^*) = S(x_i;x^*) - I(x_i;x^*),$$

where $S(x_i;x^*)$ is gross consumer surplus. Evidently, $x^*$ is a local minimum whenever

$$\prod{}''(x^*;x^*) = W''(x^*;x^*) = -I''(x^*;x^*) - C''(x^*,x^*) > 0.$$

Recall that $x^*$ represents an action combination in which all agents invest $x^*$. Denote by $x^{\Pi} = (x^{\Pi}, \ldots, x^{\Pi}) \in R^N_{++}$ the action combination that maximizes industry profit subject to the constraint that all agents invest equally. Denote by $x^W = (x^W, \ldots\ldots, x^W) \in R^N_{++}$ that investment combination which maximizes social surplus subject to the same constraint, and then choose $x^* = x^{\Pi}$ and

$x^* = x^W$ respectively. Asymmetric investment by agents are required at the first stage in order to maximize joint profits and social surplus respectively:

$$-\left(2(F_{11}-F_{12})-2F_2\frac{\sum_{i=1}^{n}\left(c_{11}^i-c_{12}^i\right)}{\sum_{i=1}^{n}c_2^i}\right)-\left(\frac{4\left(c_1^1-c_1^2\right)^2}{P^1}\right)>0$$

,

where $P'$ is evaluated at $Q(x^*)=\sum_{i=1}^{n}q^i(x^*)$, and $x^*$ equals $x^{\Pi}$, *respectively* $x^W$

.

The set of actions with a marginal-cost-sum $\left(\sum_{i=1}^{n}c^i(x)\right)$ no larger than a specified level is convex, and generates the smallest investment cost at the first stage. Investment costs are minimized when the *n* agents invest equally, and overall costs can be reduced at the first stage even though doing so increases investment costs.

We study games that are played between an informed sender and an uninformed receiver. The agent-sender is privately informed about his type $t \in \Im$ . The sender sends a message $m \in \mathrm{M}$ to the receiver, who responds with an action $a \in \mathrm{A}$. Payoffs to both agents depend on the sender's private information *t*, and the receiver's action *a*. Blume, DeJong, Kim and Sprinkle (1998) assume a strategy for the sender maps types *t* into messages *m*, for the receiver a strategy maps messages *m* into action *a*. A strategy pair that is a Nash equilibrium is a mutual best reply. We are interested in an environment with repeated interactions among a large population of agents. Agents are randomly designated as either senders or receivers, and are paired using a random matching procedure. Prior to the first stage, senders are informed about their respective types, and send a message of either *A* or *B*, $\mathrm{M}=(A,B)$, to the receiver they are paired with. In the second stage, receivers take actions $a_1 \;\; or \;\; a_2$ after receiving the message *A* or *B* from the agent they are paired with. Each sender and receiver pair then learns the sender type, message sent, action taken, and payoffs received. All agents next receive information about all sender types and all messages sent by them.

Participating agents will need to deal with uncertainty about both prices and location in a multi-dimensional product space. We analyze the case where the consumers are fully informed about their individual values $\mu$, the producer knows the distribution $F(\mu)$, and both aim to maximize their expected gain. We compute the conditional distribution:

$$\tilde{g}(\mu,\omega) = \frac{g(\mu,\omega)}{1-G(\Delta)}\Theta(\omega-\Delta) = \mu e^{-\mu(\omega-\Delta)}\Theta(\omega-\Delta)$$

.

The average valuation for this conditional distribution is $\omega(\mu,\Delta) = \Delta + \mu^{-1}$.

To compute the producer's expected profit as a function of $\Delta\ and\ F$:

$$\langle\Pi[F,\Delta]\rangle = \frac{1}{(\mu_{max}-\mu_{min})}\int_{\mu_{min}}^{\mu'} d\mu\left(F+N(\Delta-\gamma)e^{-\mu\Delta}\right) = \frac{(\mu'-\mu_{min})F+N\left(1-\frac{\gamma}{\Delta}\right)\left(e^{-\Delta\mu_{min}}-e^{-\Delta\mu'}\right)}{(\mu_{max}-\mu_{min})}$$

,

where $\mu' = \min(\mu_{max}, \tilde{\mu})$. For a variety of reasons, a consumer has to rely on adaptive estimates of its valuation distribution *g*. We suppose that consumer *i* knows that *g* is an exponential distribution, but does not know its individual parameter $\mu_i$.

## Externalities

Consider a model in which trade with each agent *i* is denoted by $x_i \in \aleph_i$, and externalities among agents arise, because each agent's utility depends on other agents' trades and each agents *i*'s payoff is $u_i(x) - t_i$. Let $\Re^*$ denote the set of trade profiles maximizing the total surplus of the *N+1* parties:

$$\Re^* = \arg\max_{x\in\aleph_1\times\ldots\times\aleph_N} f(x) + \sum_i u(x)$$

,

where *f(x)* is agent net benefit. Segal (1999) supposes that due to downstream competition, each agent *i's* utility $u_i(x_i, x_{-i})$ is decreasing in other agent's purchases $x_{-i}$. We analyze that the agents commits to a get $\{(x_i,t_i)\}_{i\in N}$ of observable bilateral contract offers to agents and agents simultaneously decide whether to accept or reject their respective offers. Applications are vertical contracting, exclusive dealing for contracts to *N* agents, selling an object, common insurance, common agency, takeovers, debt workouts, acquisition for monopoly, network externalities, and pure public goods. In public offers the agent commits to a set $\{(x_i,t_i)\}_{i\in N}$ of publicly observable bilateral contract offers to agents, and agents simultaneously decide whether to accept or reject their respective offers, under the following participation constraints:

$$u_i(x) - t_i \geq u_i(0, x_{-1}) \quad for \quad all \quad i \in N$$

.

The right-hand side of the inequality represents the reservation utility of agent $i$, and the set of his profit maximizing trade profiles can be defined as $\Re = \underset{x \in \aleph_1 \times \ldots \times \aleph_N}{\arg\max} f(x) + \sum_i u_i(x) - \sum_i u_i(0, x_{-1})$.

If each agent's reservation utility does not depend on other agents' trades, and when externalities on non-traders are absent. Externalities on non-traders are positive (negative) if $u_i(0, x_{-1})$ is non-decreasing (non-decreasing) in $x_{-i} \in \aleph_{-i} \quad for \quad all \quad i$. An agent's acceptance decision depends on his beliefs about offers extended to other agents. Agent $i$ holds passive beliefs an offer $(x_i, t_i)$ if and only if $u_i(x_i, \hat{x}_{-i}) - t_i \geq u_i(0, \hat{x}_{-i})$, where $(\hat{x}_1, \ldots\ldots, \hat{x}_N) \in \wp$.

If for agent $i$ and all $x_{-i}, x'_{-i} \in \aleph_{-i}, u_i(x_i, x'_{-i}) - u_i(x_i, x_{-i})$ does not depend on $x_i \in \aleph_i$, then $\wp = \Re$. Externalities are increasing (decreasing) if for each agent $i$, $u_i(x_i, x_{-i})$ has increasing differences in $(x_i, x_{-i})[\,(-x_i, x_{-i})\,]$, for all $x_{-i}, x'_{-i} \in \aleph_{-i}$ with $x'_{-i} \geq x_{-i}, u_i(x_i, x'_{-i}) - u_i(x_i, x_{-i})$ being non-decreasing (non-increasing) in $x_i \in \aleph_i$. With increasing (decreasing) externalities, the externality imposed on agent $i$ by increasing other agents' trades is more (less) positive when he trades more.

Since agents' participation constrains the offer equilibrium $\hat{X}$, each agent $i$ with $x_i = 1$ pays $t_i = U(1, \hat{X}) - U(0, \hat{X} - 1)$. Consider a deviation from the equilibrium in which the agent offers $x = t_i = 0 \quad to \quad \hat{X} - X$ agents who previously had $x = 1$. Since the deviation must be unprofitable we have:

$$F(\hat{X}) + \hat{X}[U(1,\hat{X}) - U(0,\hat{X}-1)] \geq F(X) + X[U(1,\hat{X}) - U(0,\hat{X}-1)] \geq F(X) + X[U(1,X) - U(0,X-1)]$$

,

where the second inequality follows from the condition of increasing externalities when agents are heterogeneous or different agents trade different positive amounts in equilibrium. The firm value is given by

$$v(X) = \begin{cases} \bar{v} & when \quad X \geq 0.5, \\ \underline{v} < \bar{v} & otherwise. \end{cases}$$

Then the raider can make each tendering shareholder pivotal by ensuring that X = 0.5 in equilibrium. Suppose that the raider bids $\underline{v}$. If each shareholder holds an indivisible share and $N$ is even, there exists an equilibrium in which exactly

$N/2$ shareholders tender. Consider a sequence of environments with $N$ identical agents, $N = 1,2,\ldots,$ Let the utility function of each agent $i$ in the environment with $N$ agents be $x_i a(X) + (1/N)\beta(X)$, so that the total surplus function $W(X) = Xa(X) + \beta(X)$ is independent of $N$. Assume also that the trade domain of each agent is $\aleph_i = \aleph/N$, where $\aleph$ is a compact subset of $\Re_+$. The set of feasible aggregate trades with $N$ agents is $\sum_i \aleph_i =^N \aleph \; /N$. An agent's inability to respond may or may not be observed by the agent himself. For these strategies to constitute a Nash equilibrium, each agent must prefer to accept when he knows that others accept whenever they can, i.e., the following participation constraints must be satisfied:

for all $i \in N$. The set $M_N = \{X(N) : X \in \sum \Re_{\wp^N}\}$ of equilibrium aggregate trades in the agent's profit-maximizing mechanisms can then be described as:

$$M_N = \underset{X \in \sum \wp^N(N)}{\arg\max}\, \pi_N(X),$$

with

$$\pi_N(X) = W(X) - R_N(X) = \pi_\infty(X) + \beta(X) - R_N(X),$$

where

$$R_N(X) = \min\left\{(1/N)\sum_{i=1}^{N} \beta(X(N \setminus i)) : X \in \sum \wp^N |X\right\}$$

is the minimum sum of agents' reservation utilities consistent with equilibrium aggregate trade $X$. The sequence $\{\wp^N\}_{N=1}^{\infty}$ of mechanism families in the asymptotic setting is asymptotically adequate and continuous. However, discontinuity of the fully optimal mechanism seems unrealistic in environments in which $N$ is large and the agent cannot forecast the number of accepting agents precisely:

$$E_A\{u_i(x(A)) - t_i(A)\} \geq E_A\{u_i(0, x_i(A \setminus i))\}$$

Let $A \subset N$ denote the random set of agents who are able to respond to the agent's offer; without loss of generality we can restrict attention to direct mechanisms, in which all agents who are able to respond accept, in equilibrium. When the actions of $N$ agents stochastically affect a bounded real random variable, the agents' average expected ex ante influence on this variable is bounded by a number that goes to zero as $N \to \infty$ provided that noise does not vanish too quickly.

The agent's expected profit can be written as

$$E_A\left[f(x(A)) + \sum_{i \in A} t_i(A)\right] = E_A\left[f(x(A)) + \sum_{i \in A}(u_i(x(A)) - u_i(0, x_{-i}(A \setminus i)))\right],$$

where the values of $x$(A) and $t$(A) for $A$ are relevant for agent $i$'s acceptance decision, expected payment is $E_A[t_i(A)]$. The agent's expected profit can be written as function of the mechanism's aggregate representation $X = \sum_i x_i$:

$$\pi_N(X) = E_A\left[F(X(A)) + X(A)\alpha(X(A)) + \frac{1}{N}\sum_{i\in A}[\beta(X(A)) - \beta(X(A\setminus i))]\right].$$

The agent chooses the $N$-dimensional trade and transfer vectors $x$(A) and $t$(A) for each acceptance set $A$, and the utility of each agent $i$ depends not only on the aggregate trade $X$, but also on his own trade $x$, and payment $t$. The average agent asymptotically takes the expected reservation utility $E_A\beta(X(A))$ as given, and the total surplus would depend on the allocation of the aggregate trade among agents.

Depending on the consumer's belief about the dynamics of the environment, he might wish to place more or less weight on recent observations. Since the means of the distribution $g(\mu;\omega) = \mu\exp^{-\mu\omega}$, flexible approach to estimating $\mu$ is to start with an assumed.

The possibility of monitoring has been investigated in three-layer hierarchies, and we want analyze the problem of collusion in a hierarchical relationship. There are risk agents such as a principal P, a supervisor S, and manager A. The agents are a productive unit and produce output $x = \theta + e$, which is determined by a random productivity parameter $\theta$, and the agent's effort $e$. The agent's utility is $u_A = t - \psi(e)$ and his reservation utility is present. While the output $x$ is publicly observable, both $\theta$ and $e$ are private information of the agent.

In each period, agents may trade a single perishable good and money, which can be stored from one period to the next. Agents are endowed with amounts of the good in each period and

agent endowment is $\omega = [\omega_1, \omega_2]$. The consumption of the agent at time $t$ will be denoted $[x_{it}^t, x_{it}^{t+1}]$ for $i=1,\ldots.,n$. Utility is additive and separable $u(x_t^t, x_t^{t+1}) = U(x_t^t) + V(x_t^{t+1})$. Trade takes place at trading posts in which agents offer some amount $q$ of the good for sale, and make money bids $b$ for purchase of a share, and savings are denoted by $m$. This agent's strategy set is then $S_t = \{(q_t, b_t, m_t) \in R^s \mid \omega \geq q_t > 0\}$. Each agent is allocated a share of the amount of money bid in the proportion: $B_t^s = \sum_{i=1}^{n} b_{it}^s$ for $s=t,t+1$ and $Q_t^s = \sum_{i=1}^{n} q_{it}^s$ for $s=t,t+1$.

If the budget constraints are satisfied, the trading mechanism is:

$$x_t^t = \omega_1 - q_t^t + \frac{Q_t}{B_t} b_t^t \quad \text{and} \quad x_t^{t+1} = \omega_2 - q_t^{t+1} + \frac{Q_{t+1}}{B_{t+1}} b_t^{t+1}.$$

Now, a typical agent is given the bids and offers of all other agents, and it is determined that the solution to the problem of a market game of agents is indeterminate at the Nash equilibrium.

Define $p_n^t = \frac{\hat{B}_t^n - m}{\hat{Q}^n}$, and suppose that $\{p_t^n\}_{n=1}^{\infty}$ is a sequence of interior Nash equilibrium prices for the $n$-fold replication of the market game.

The marginal rate of substitution of the endowment is $\frac{U'(\omega_1)}{V'(\omega_2)} < 1.$

However, in the offer-constrained market game, there is a restriction on the amount of the good offered for sale, and these agents prefer thick markets, because the game is non-cooperative, so there is no way for any agent to increase the offers of other agents. It is useful to define functions for the backward dynamics as $\theta_t = \varphi(\theta_{t+1})$, where mapping $\varphi$ satisfies $v_1[\varphi(\theta)] = v_2(\theta)$. We get the condition:

$$R_2(\omega_2 + \hat{\theta}) > \frac{(\omega_2 + \hat{\theta})(\tilde{Q} - 2\hat{\theta})}{\hat{\theta}(\tilde{Q} - \hat{\theta})} + \frac{(\omega_2 + \hat{\theta})}{\hat{\theta}} \left[ \frac{2\hat{\theta} + \hat{Q}}{\hat{Q} + \hat{\theta}} + R_1(\omega_1 - \hat{\theta}) \frac{\hat{\theta}}{\omega_1 - \hat{\theta}} \right].$$

If agents may have different offers, $\hat{Q}$ can remain positive as $\tilde{Q} \to 0$ since for any agent $i$, we have $\hat{Q}_i = \tilde{Q}_i + q_{i2} - q_{i1}$. Hence, by choosing agent $i$'s offers appropriately, in the type of economy with $n>1$, we have $\hat{Q} = (n-1)q_1 + nq_2$ and $\tilde{Q} = nq_1 + (n-1)q_2$.

Suppose the economy adds agents, but simultaneously reduces individual offers such that $Q$ remains unchanged, in essence keeping the market thin. Prices will remain positive since although bids and offers are shrinking, aggregate bids and offers remain positive and finite. The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial e_h} = q - (\lambda_2 + \lambda_3)\psi'(e_h) \le 0,$$

$e_h = 0$ and $e_h \dfrac{\partial L}{\partial e_h} = 0$ $e_h \dfrac{\partial L}{\partial e_h} = 0$

$$\frac{\partial L}{\partial \tilde{t}_h} = -q + (\lambda_2 + \lambda_3) \le 0$$

$\tilde{t}_h \ge 0$ and $\tilde{t}_h \dfrac{\partial L}{\partial \tilde{t}_h} = 0$

$$\frac{\partial L}{\partial e_l} = 1 - q - \lambda_1\psi'(e_l) + \lambda_3\psi'(e_l - \Delta\theta) \le 0$$

$e_l \ge 0$ and $e_l \dfrac{\partial L}{\partial e_l} = 0$

$$\frac{\partial L}{\partial \tilde{t}_l^N} = -(1-q)p\alpha + \lambda_1 p\alpha - \lambda_3 p(1-\alpha) \le 0$$

$\tilde{t}_l^N \ge 0$ and $\tilde{t}_l^N \dfrac{\partial L}{\partial \tilde{t}_l^N} = 0$

$$\frac{\partial L}{\partial \tilde{t}_l^0} = -(1-q)(1-p) + \lambda_1(1-p) - \lambda_3(1-p) \le 0$$

$\tilde{t}_l^0 \ge 0$ and $\tilde{t}_l^0 \dfrac{\partial L}{\partial \tilde{t}_l^0} = 0$,

plus the constraints.

Let $(t_l, x_l)$ and $(t_h, x_h)$ be the contract designed for an agent who claims to be a type $\theta_l$ and $\theta_h$ respectively, denoting $e_i = x_i - \theta_i$, $i \in \{l, h\}$, and the participation constraints:

$$t_i - \psi(e_i) \ge \overline{U}, \quad i \in \{l, h\},$$

with compensation utility $\overline{U}$.

Let $t_l^r$ and $w^r$ denote the compensation for the agent and supervisor on the report $r \in \{O, N, S\}$.

The decision problem faced by the agent is

$$\max \int_0^\infty U(c,l)e^{-\delta t}dt$$

subject to the flow budget constraint

$$\dot{c} + \dot{m} + \dot{k} = f(k,l) + s - (n+\pi)m - nk - cv(m)$$

where

c – per capita consumption,
l – labor effort,
m – per capita real money balances,
k – per capita capital stock,
f(.,.) – per capita output,
s – per capita government transfers,
n – population growth rate,
$\pi$ - inflation rate.
$\upsilon(.)$ - liquidity cost per unit of consumption,
$\delta$ - rate of time preferences,
t – time.

Non-parametric recursive estimators such as kernel, orthogonal series, spline, wavelet, and neural network estimators are implemented by putting

$$H_{l(n)} = \left\{ \theta(x) = \sum_{j=1}^{k(n)} \beta_j \phi(\gamma_0 + \gamma_j' x) : |\beta| + |\gamma| \le b_n < \infty \right\},$$

with

$$b_n \to \infty, \quad k(n) \to \infty \quad \text{as} \quad n \to \infty.$$

The procedure introduced the possibility that agents do not know any fixed, bounded set. Heuristically, the feedback present in learning systems introduces a memory of the arbitrarily distant past. A belief space is a measurable space, each point of which is associated with a probability measure on the space.

For each event E and real number p

$$B^p(E) = \{\omega | t(\omega)(E) \ge p\}$$

is the event that the probability ascribed by the agent to E is at least p and we this is the event the agent is certain of. Obviously, in equilibrium the agent will choose the loans and effort to solve the following optimization problem:

$$\text{B(S,L):} \quad \max_{l,d,e} \quad U(c_1, c_2, e),$$

$$\text{subject to} \quad (l,d) \in L(S),$$

$$c_1 = w + s_1 + l,$$

$$c_2 = \max[s_2 - d, \min(s_2, \lambda)].$$

The agent then promises to repay the amount d in the second period and d<0 corresponds to withdrawal of savings and the contract takes the form of a sharing rule $S = (s_1, s_2(.))$. The agents' preferences are represented by a utility function $U(c_1, c_2, e)$, where $c_t \geq 0$ is the consumption expenditure in period t and $e \geq 0$ represents the total effort expended. Let w be the agent's initial wealth if the agent accepts the contract S. And given the bankruptcy protection level $\lambda$, an agent with contract S and aggregate debts d, the agent's incentives could be affected by buying or selling shares in the firm.

The agent's attitudes towards risk are critical in determining the relation between the relative risk and the relative dispersion of its price. If individual preferences exhibit risk substitutability, and if the degree of future period relative risk aversion is a function of consumption , then the relative risk of an asset corresponds to greater asset price volatility. If preferences are time separable, the volatility of asset prices does not depend on their relative risk. The economy is populated by agents who own shares of each asset at the beginning of period 1. After observing the state $y_1$, the agent receives all dividends due and uses them to buy consumption goods and additional shares to be carried over into next period. Thus the agent's choices are constrained by

$$c + p^{\pi}(z-1) + p^{q}(s-1) = h(y) + g(y)$$

$$\tilde{c} = zh(\tilde{y}) + sg(\tilde{y}),$$

where $c$ denotes the agent's consumption, $z$ and $s$ are the holdings of assets $\pi$ and q , and the assets' payoffs are $h$(y), $g$(y). We assume that the agent's preferences over random lifetime allocations can be described by the expected value of utility function:

$$E\{U(c, \tilde{c})\},$$

where $E$ is the expectations operator. If the agent knows the distribution of payments, then the necessary and sufficient conditions for his decision problem are

$$p^{\pi}(y)E\{U_1(c, \tilde{c})\} = E\{U_2(c, \tilde{c})h(\tilde{y})\},$$

$$p^{q}(y)E\{U_1(c, \tilde{c})\} = E\{U_2(c.\tilde{c})g(\tilde{y})\}.$$

Using the agent conditions, we get:

$$p^{\pi}(y)\int U_1(c(y), \tilde{c}(\tilde{y}))dF(\tilde{y}) = \int U_2(c(y), \tilde{c}(\tilde{y}))h(\tilde{y})dF(\tilde{y}),$$

where

$$c(y) = h(y) + g(y), \tilde{c}(\tilde{y}) = h(\tilde{y}) + g(\tilde{y}).$$

The relative price volatility of assets with payoff patterns can be ranked according to their systematic risks. Consider the expected real rates of return,

$$r_{\pi}(y) = \frac{E\{h(\tilde{y})\}}{p^{\pi}(y)}, \quad r_q(y) = \frac{E\{g(\tilde{y})\}}{p^{q}(y)}.$$

The difference between the real rates of return $r$ equals the difference between the conditional risk premium of assets and is a convenient measure of relative risk.

Asset $q$ is less risky than asset $\pi$ if

$$S(\tilde{y};F):=\int_{\underline{y}}^{\bar{y}} s(y')dF(y')\overset{(\geq)}{\underset{\leq}{}}0 \quad \forall \tilde{y}\in[\underline{y},\bar{y}] \quad \forall F\in\Phi$$

,

where $s$ is the difference between the normalized returns on assets $q$ and $\pi$.

The information about the risk characteristics of the asset revealed by the behavior of equilibrium prices depends on the agent's attitudes toward risk. The volatility of equilibrium asset prices is related to the risk of the assets if the measure of future period risk aversion is strictly decreasing (increasing) in current period consumption. It is evident that the aggregate consumption and the prices of both assets are positively related to the random variable $y$. Suppose that $y$ takes on a high value in period 1. Then current consumption is high and the agent is more risk averse in period 2. If $y$ in period 1 is low, then current consumption is low and the agent discounts the risk associated with random future asset returns at a lower rate.

Thus portfolios of assets with relatively stable dividends can exhibit considerable price volatility, which would be compared to the dividend process.

We consider the problem of a regulator who desires to contract with an agent for the delivery of $q$ units of a good. The agent's productivity parameter $\theta$ is known by the agent, but the regulator only assesses it with a probability distribution $F(.)$ over $\Theta \equiv [\underline{\theta},\bar{\theta}]$. The production $q$ depends on both the agent's productivity parameter $\theta$ and the agent's effort $a$, according the relation $q = Q(a,\theta)$. The type-$\theta$ agent's objective is to maximize the following utility:

$$t - V(a,\theta)$$

,

where $t$ is the transfer given by the regulator. High productivity agents are efficient but expensive, whereas low productivity agents are less efficient but cheaper. The regulator may base contracts upon the production level $q$ or the effort level $a$ at the same time. With a production-based contract, the type-$\theta$ agent who announces $\tilde{\theta}$ has to exert an effort $a = h(q(\tilde{\theta}),\theta)$ that satisfies

$$q(\tilde{\theta}) = Q(h(q(\tilde{\theta}),\theta),\theta)$$

to obtain the corresponding transfer $t_q(\tilde{\theta})$. His ex ante utility is given by

$$u_q(\theta,\tilde{\theta}) \equiv t_q(\tilde{\theta}) - V(h(q(\tilde{\theta}),\theta),\theta).$$

Rearranging terms yields

$$\dot{U}_q(\theta) = \Delta(h(q(\theta),\theta),\theta)\partial_a V(h(q(\theta),\theta),\theta)$$

where $\Delta$ is the function given by :

$$\Delta(a,\theta) = \frac{\partial_\theta Q(a,\theta)}{\partial_a Q(a,\theta)} - \frac{\partial_\theta V(a,\theta)}{\partial_a V(a,\theta)}$$

$\Delta$ is the difference between marginal rates of substitution of effort for agent's type corresponding respectively to the production side and the disutility side. With an effort-based contract, the utility of a type-$\theta$ agent who announces $\tilde{\theta}$ is given by

$$u_e(\theta,\tilde{\theta}) \equiv t(\tilde{\theta}) - V(e(\tilde{\theta}),\theta).$$

The rate of growth of informational rents for an incentive contract is given by

$$\dot{U}(\theta) = \dot{U}_q(\theta) - p(\theta)(\dot{U}_e(\theta) - \dot{U}_q(\theta)).$$

Using the probability *p*, the agent is thus able to exploit the difference between $\dot{U}_e$ and $\dot{U}_q$ to modify $\dot{U}$. The agent, chooses $c \in C$ in order to maximize her welfare *W*, which is restricted to the sum of the consumer and taxpayer surplus:

$$W(c) \equiv p(\theta)(Q(e(\theta),\theta) - t_e(\theta)) + (1 - p(\theta))(q(\theta) - t_q(\theta)).$$

## Output-Based Contracts

The optimal contract is consequently the same as in situation where she is constrained to use only output schemes, and when agents share the same effort disutility function, input-based contracts allow the agent to reach the first best allocation. Indeed, production based contracts involve an additional cost and the agent has to compensate the low type-agents, as the rent gained overstating their type is higher the lower their disutility of effort. Under an output-based contract, the agents' incentive to overstate their type is reduced since it implies a higher production goal to be met with a low productivity. So $\dot{U}_q$, the rate of growth of informational rents associated with an output-based contract is positive.

## Endowment and Commodities

On each period, *n* =1,2... ..., each agent $\alpha \in I$ receives a random endowment $Y^\alpha(w) = Y_n(\alpha, w)$ in units of a single commodity. In the commodity market, agents bid money for consumption of the commodity, thereby determining its price $p_n(w)$. Geanakoplos, et al. (2000) suppose the interest rates are assumed to satisfy:

$$1 \le r_{2n}(w) \le r_{1n}(w) \quad r_{2n}(w) < \frac{1}{\beta},$$

where $\beta \in (0,1)$ is a fixed discount factor. At the beginning of day $t$ = n, the price of the commodity is $p_{n-1}(w)$ and the total amount of money held in the bank is $M_{n-1}(w)$. An agent $\alpha \in I$ enters the day with wealth $S_{n-1}^{\alpha}(w)$. If $S_{n-1}^{\alpha}(w) < 0$, then the agent has an unpaid debt from the previous day, is assessed a punishment of $u^{\alpha}\left(S_{n-1}^{\alpha}(w) \; / p_{n-1}(w)\right)$. The debt is then forgiven. If $S_{n-1}^{\alpha}(w) \ge 0$, then the agent has fiat money on hand and plays from position $S$. The agent also begins day $n$ with information $F_{n-1}^{\alpha} \subset F$, a $\sigma$-algebra of events that measures past prices $p_k$, past total endowments $Q_k$, and interest-rates $r_{1,k}, r_{2,k}$, as well as past wealth-levels, endowments, and actions $S_0^{\alpha}, S_k^{\alpha}, Y_k^{\alpha}, b_k^{\alpha}$ for $k$ =1,….,n-1. Based on this information, the agent bids an amount:

$$b_n^{\alpha}(w) \in \left[0, \left(S_{n-1}^{\alpha}(w)\right)^{+} + k^{\alpha}\right],$$

of fiat money for the commodity on day $n$. The constant $k^{\alpha} \ge 0$ is an upper bound on allowable loans. Consequently, the total bid is:

$$B_n(w) = \int b_n^{\alpha}(w)\varphi(d\alpha) > 0.$$

The total utility that the agent receives during the period is:

$$\xi_n^{\alpha}(w) = \begin{cases} u^{\alpha}\left(x_n^{\alpha}(w)\right), & if\ S_{n-1}^{\alpha}(w) \ge 0 \\ u^{\alpha}\left(x_n^{\alpha}(w)\right) + u^{\alpha}\left(S_{n-1}^{\alpha}(w) \; / p_{n-1}(w)\right), & if\ S_{n-1}^{\alpha}(w) < 0 \end{cases}.$$

The total payoff for agent during the entire duration is the discounted sum $\sum_{n-1}^{\infty} \beta^{n-1}\xi_n^{\alpha}(w)$. A strategy $\pi^{\alpha}$ for an agent specifies the bids $b$ as random variables which are $F$ measurable for every $n \in N$. A collection $\Pi = \{\pi_{\alpha}, \alpha \in I\}$ of strategies for all agents is admissible. Now there are three possible situations for agent on day $n$.

(a) The agent is a depositor: this means that bid $b_n^{\alpha}(w)$ is strictly less than his wealth, and he deposits the difference:

$$I_n^{\alpha}(w) = S_{n-1}^{\alpha}(w) - b_n^{\alpha}(w) = \left(S_{n-1}^{\alpha}(w)\right)^{+} - b_n^{\alpha}(w).$$

At the end of day, agent gets back his deposit with interest, as well as his endowment's worth in fiat money and thus, moves to the new wealth level:

$$S_n^\alpha(w) = r_{2n}(w)I_n^\alpha(w) + p_n(w)Y_n^\alpha(w) > 0.$$

(b) The agent is a borrower: he must borrow the difference:

$$d_n^\alpha(w) = b_n^\alpha(w) - \left(S_{n-1}^\alpha(w)\right)^+.$$

Thus, agent pays back the amount:

$$h_n^\alpha(w) = \min\left\{r_{1n}(w)d_n^\alpha(w), p_n(w)Y_n^\alpha(w)\right\}$$

and his cash holdings at the end of the period are $\left(S_n^\alpha(w)\right)^+ = p_n(w)Y_n^\alpha(w) - h_n^\alpha(w)$.

(c) the agent neither borrows, and not lends: the agent bids his entire cash-holdings $b_n^\alpha(w) = \left(S_{n-1}^\alpha(w)\right)^+$ and ends the day with exactly his endowment's worth in fiat money:

$$S_n^\alpha(w) = p_n(w)Y_n^\alpha(w) \geq 0.$$

We can write a formula for agent's wealth position at the end of period:

$$S_n^\alpha(w) = p_n(w)Y_n^\alpha(w) + r_{2n}(w)I_n^\alpha(w) - r_{1n}(w)d_n^\alpha(w),$$

and another formula for agent's cash-holdings:

$$\left(S_n^\alpha(w)\right)^+ = p_n(w)Y_n^\alpha(w) + r_{2n}(w)I_n^\alpha(w) - h_n^\alpha(w).$$

A real possibility in an agent's optimization problem are the invariant measures for associated optimally controlled Markov chains.

# 16 Cycles, Chaos, Stabilization and Technological Innovation

We calculate the fully optimal rule under complete information for the welfare effect of alternative stabilizations. Henderson and Kim (1999) assume that the agents are described on the unit interval $f \in [0,1]$, and the problem is to find:

$$\max_{p_{f,t+j}} \Im \ddot{\delta}_{t,t+j} \left( s_p P_{f,t+j} Y_{f,t+j} - W_{t+j} L_{f,t+j} \right)$$

.

In the period $t + j$, firm $f$ sets the price $P$, produces output $Y$ and employs the amount $L$, for which it pays the wage index $W$ per unit. All firms receive an output subsidy $s$. Each element of the infinite dimensional vector $\ddot{\delta}$ is a stochastic discount factor and we use $\Im$ to indicate an expectation taken over the states in period $t + j$ based on period $t$ information. The production function of firm $f$ is

$$Y_{f,t+j} = \frac{L_{f,t+j}^{(1-\alpha)} X_{t+j}}{1-\alpha},$$

where $X$ is a productivity shock that hits all firms. Household $h$ chooses quantities of $Y$ to minimize the cost of producing a unit of $Y$ given that $P$ is the minimum cost. The problem for household $h$ in period $t$ is to find:

$$\max_{\left(C_{h,s}, M_{h,s}, B_{h,s}, B_{h,s}^{g}, W_{h,s+j}\right)} \Im_t \sum_{s=1}^{\infty} \beta^{s-t} \left( \frac{C_{h.s}^{1-p} - 1}{1-p} - \frac{\chi_0 L_{h,s}^{1+\chi}}{Z_s (1+\chi)} \right) U_s$$

.

In period $s$ household $h$ chooses its expenditure on the output index $C_{h,s} = \min\left( C_{h,s}, \frac{M_{h,s}}{P_s V_s} \right)$ and its holdings of money $M$, which imply a consumption realization $C$. The household also chooses its wage rate in period $s + j$, $W$ and agrees to supply labor $L$ , while $B$ represents the quantity of such claims purchased by the household. The household also chooses its holding of government bonds $B_{h,s}^{g}$, which pay $I$ units of currency in every state of nature in the period

s +1 and pays lump sum taxes $T$. The household receives labor subsidy $s$, and there is a good demand $U$, a money demand $V$, and labor supply shocks $Z$.

We impose the restrictions that $0 < \beta < 1,\ p \geq 0, \quad and \quad \chi \geq 0,\ \Im_t$ indicates an expectation over the various states in period $s$ based on period $t$ information and the utility depends positively on the consumption realization and negatively on labor supply.

The government budget constraint is:

$$\frac{M - M_{-1} + B^g - I_{-1}B^g_{-1}}{P} = G + (s_p - 1)Y + (s_W - 1)\frac{W}{P}L - T,$$

where $G$ is real government spending. The government budget constraint is balanced period by period and the real government spending is zero:

$$\frac{i_{-1}B^g}{P} + (s_p - 1)Y + (s_w - 1)\frac{W}{P}L - T = 0,$$

where $i$ is nominal interest rate. With flexible wages and prices the firms have:

$$Y = \frac{L^{\bar{a}}X}{\bar{a}}, \quad P = \frac{L^aW}{X}, \quad \frac{\chi_0 L^{\bar{\chi}}}{WZ} = \frac{L}{Y^pP}, \quad \beta I\Im\left(\frac{U_{+1}}{Y^p_{+1}P_{+1}}\right) = \frac{U}{Y^pP},$$

$M = PYT$ for production, price, wage demand and money, with equilibrium condition $C = Y$ and where $\bar{a} = 1 - a \quad and \quad \bar{\chi} = 1 + \chi$.

Wage contracts and flexible prices are:

$$P = \frac{L^aW}{X}, \quad \frac{1}{W}\Im\left(\frac{\chi_0 L^{\bar{\chi}}U}{Z}\right) = \Im\left(\frac{LU}{Y^pP}\right).$$

With a wage contract, wages must be set one period in advance without knowledge of current shocks. The optimal hybrid rule makes the nominal wage invariant to demand $U$, money $V$ and productivity shocks $X$ under price level stabilization. Employment and output are more volatile and for labor supply shocks, outputs are less volatile than they would be with the optimal policy. With only wage contracts, the more important productivity disturbances, the worse are all agents from nominal income targeting, or elasticity of the disutility of labor is high with wage contracts and nominal income targeting performs very well.

## Money and Capital

Consider an economy populated with infinitely-lived agents where the total wealth of economic agents is divided between money and physical capital. Petrucci (1999) supposes that the current value of the Hamiltonian for the optimization problem can be expressed as:

$$H = \bar{U}[y(1 - I)] + \lambda\{f(k, l) + s - (n + \pi)m - nk - y(1 - I)[1 + v(m)]\},$$

where $\lambda$ is the co-state variable associated with physical capital and real money balances, and it represents the marginal utility of non-human wealth. The other variables are: $U$ = utility function,

$I$ = labor effort, $c$ = per capita consumption, $m$ = per capita real money balances, $k$ = per capita capital stock, $f(.,.)$ = per capita output , $s$ = pre capita government transfers,

$n$ = population growth rate, $\pi$ = inflation rate, $v(.)$ = liquidity costs per unit of consumption,

$\delta$ = rate of time preference, $t$ = time. Since the government follows a money supply growth rule, real money balances per capita evolve through time according to the law of motion:

$\dot{m} = m(\theta - \pi - n)$, where $\theta$ represents the nominal money growth rate. Money created is transferred by the government to the agent as a lump-sum transfer: $s = \theta m$ . The resource constraints of the economy are given by:

$$f(k,I) = \gamma(1-I)[1+v(m)] + \dot{k} + nk.$$

The macroeconomic equilibrium for the economy together with relevant accumulation equations and money market equilibrium conditions follows:

$$\gamma \bar{U}'[\gamma(1-I)] = \lambda\{f_I(k,I) + \gamma[1+v(m)]\},$$

$$f_k(k,I) + \pi = -\gamma(1-I)v'(m),$$

$$\dot{\lambda} = \lambda[\delta + n - f_k(k,I)],$$

$$\dot{m} = m(\theta - \pi - n),$$

$$\dot{k} = f(k,I) - \gamma(1-I)[1+a(m)] - nk.$$

The steady-state equilibrium is attained when $\dot{\lambda} = \dot{m} = \dot{k} = 0$ is characterized by following set of equations:

$$\gamma \bar{U}'[\gamma(1-\bar{I})] = \bar{\lambda}\{f_I[f_k^{-1}(\delta+n)] + \gamma[1+v(\bar{m})]\},$$

$$-\gamma(1-\bar{I})v'(\bar{m}) = \delta + n + \bar{\pi},$$

$$f_k(\bar{k},\bar{I}) = \delta + n,$$

$$\bar{\pi} = \theta - n,$$

$$f(\bar{k},\bar{I}) = \gamma(1-\bar{I})[1+v(\bar{m})] + n\bar{k},$$

$$\bar{c} = \gamma(1-\bar{I}).$$

Higher inflation results in a lower demand for real balances, because the opportunity cost of money holdings, $\delta + \theta$, increases. And the inflationary shock raises the unit price of consumption, *1+n,* which in turn implies lower consumption.

## Choice and Distribution

Agents in the model are distinguished by their type, and each agent may be one of a finite number of possible types $\omega \in \Omega = \{\omega_1, ......, \omega_n\}$, where $\omega_n$ by ordering represents the best type of agent. Banks and Surdaram (1998) suppose agent types are realizations of a common distribution $\pi = (\pi_1, ...., \pi_n) \in P(\Omega)$ with probability distributions *P*. Agents generate rewards $r \in R$ and the distribution depends on the action *a* the agent takes in that period. The set of actions available to any agent in any period is taken to interval $A = [a^{\min}, a^{\max}] \subset R$ and if the agent takes the action *a*, then the reward *r* is realized according to the distribution $F(.|a)$.

The utility of an agent in period of activity depends on the action taken by the agent utility $u(a, \omega)$ and creates the choice problem. Agents discount future utilities by the common factor $\delta \in [0,1]$. Let *g(x , y)* be a function from $R \times R$ into *R* and *g* is said to be super-modular in *x* and *y* and we have :

$$g(x, y) - g(x, \hat{y}) > g(\hat{x}, y) - g(\hat{x}, \hat{y}),$$

and super-modularity requires that as *y* increases, increasing *x* increases *g* as well. Then, the agent's utility in any period of activity will be $s(r) - c(a, \omega)$ and expected utility the agent obtains is:

$$u(a, \omega) = \int s(r) dF(r|a) - c(a, \omega).$$

An strategy for an agent is a first period action as a function of the agent's type, and a second period action as a function of his type and the knowledge generated in the first period. We will denote a strategy for an agent of type $\omega_k$ *by* $(\mu_k, \gamma_k)$ where $\mu_k \in P(A)$ is a mixed action for an agent of type $\omega_k$ in the first period. And $\gamma_k : S \to A$ is a function a type $\omega_k$ in the second period action using the realized first period knowledge. Let $\theta$ denote the set of all possible strategies $(\mu, \gamma)$.

The agent's first period actions imply there exist intervals $[a_k, b_k] \in P(A)$ such that $b_k \leq a_{k+1}$

For k = 1,.....,n – 1, and $\mu_k[a_k, b_k] = 1$ for all *k*. The agent's second period actions imply that for all $r \in R$, we have $\gamma_k(r) \leq \gamma_{k+1}(r)$ for k = 1,....,n – 1 will be optimal if

$$\gamma_k(r) \in \arg\ \max\{u(a, \omega_k)|a \in A\} \text{ for all } r \in S .$$

In the first period, an agent solves

$$\max\left\{u(a, \omega_k) + \delta \int_{r \geq \bar{r}} U(\omega_k) dF(r|a)|a \in A\right\}.$$

We suppose that the strategies are:

For $k = 1, \ldots, n-1, \quad \gamma_{k+1}(r) \geq \gamma_k(r) \quad for \quad all \quad r \in S$.

The expected second period utility following the strategy is:

$$v_2(r, \mu, \gamma) = \sum_{k=1}^{n} \left[\int \beta_k(r) v(\hat{r}) f(\hat{r}|\gamma_k(r)) df\right].$$

For any $m \in P(A)$, $\varphi(r|m)$ is the probability of observing the reward $r$ under the mixed action $m$ :

$$\varphi(r|m) = \int_A f(r|a) m(da).$$

All agents are going to take optimal actions and beliefs shift towards agents' types upon acquiring knowledge. Then, since any agent must be replaced after each period, the choice is of the form:

$$\max\{v_2(r.\mu, \gamma) + aV^*(\mu, \gamma), V^*(\mu, \gamma)\},$$

showing that $V^*$ is rotationally tedious . The agent's best response to the other agent strategy $(\mu, \gamma)$ is to use cut-off rules $C(\mu, \gamma, \pi)$ .

The agent's action can be any measurable selection satisfying $\gamma_k(r) \in M(\omega_k)$ for all $r \in S$, where *M(.)* is the set of optimal second-period actions for the agents.

There exists an equilibrium in strategies $(\sigma^*, \mu^*, \gamma^*)$ where $\sigma^*$ is cut-off strategy, and $(\mu^*, \gamma^*)$ is a type strategy.

Agents take higher actions in the first period than in the second period in the presence of adverse selection, and performance effects are present in the later setting, implying all types take higher actions.

All agents are dismissed after one period and take actions implying:

$$(1-a)V(\pi)=\sum_{k\in K}\pi_k E\left(a_k^m\right)$$ , where $$E\left(a_k^m\right)=\int v(\hat{r})dF\left(\hat{r}\middle|a_k^m\right)$$ .

## Technological Progress

Technological progress would increase the return to education without affecting the return to ability. Agents with a lower level of ability would find it beneficial to acquire education, and the ability dispersion would widen among educated agents and would narrow among the uneducated ones. Galor and Moav (2000) suppose the new technological level may reflect in the long run either a skill-biased or skill-saving technological change, and argue that the transition to the new technological state mostly saves in the short-run. In the transition to the steady state, the rate of technological progress increases monotonically, wage inequality within and between agents increases, the average wage of skilled agents increases despite the increase in their number, and the average wage of unskilled agents may decline with their relative supply. The output produced in time *t*, *Y*, is

$$Y_t=F\left(K_t,A_tH_t\right)\equiv A_tH_tf\left(k_t\right) \qquad k_t\equiv K_t\ /\left(A_tH_t\right),$$

where *K* and *H* are the quantities of physical capital and efficiency units of labor input , *A* is the technological level, and *f* is monotonic increasing and concave, satisfying the existence of an interior solution for the agent-producers in a competitive environment. The wage rate per efficiency unit of labor is $\omega_t$ , and the rate of returns to capital is *r* at time *t* , and agents choose the level of employment of capital *K* and labor input *H* . Agents' demand for factors of production is given by:

$$r_t=f'(k_t)\,,\quad \omega_t=A_t\left\{f(k_t)-f'(k_t)k_t\right\}\equiv A_t\omega(k_t).$$

We present the labor input *H* as a weighted sum of the number of efficiency units of skilled agents *h*, and unskilled agents *l*:

$$H_t=\beta h_t+l_t\left(1-\delta g_t\right),$$

where $\beta>1$, and $0<\delta g<1$ is the reduction in the aggregate weight given to efficiency units of unskilled agents due to rate of technological progress from period *t* –*1* to period *t* in interval $0<g_t<1$ and determines the demand for skilled agents.

In each period a generation of agents are born and live as individuals, households, firms, and state. They are represented by a utility function, and agents of generation *t* face an occupational choice about knowledge. Agents who choose to become skilled devote a fraction $0<\tau<1$ to acquire them. They supply a fraction $1-\tau$, of their potential efficiency units as skilled agent earning the competitive market wage $\omega_t^s$ or as unskilled agents earning $\omega_t^n$ per efficiency unit.

The efficiency units of labor of a member *i* of generation *t* depends positively on agent's ability $a_t^i$, and negatively on the rate of technological change *g*, with number of skilled agents *h*, and unskilled agents *l*:

$$h_t^i = (1-\tau)\{a_t^i - (1-a_t^i)g_t\} \quad and \quad l_t^i = 1-(1-a_t^i)g_t$$

.

The potential number of efficiency units of labor is diminished due to the transition from the existing technological state due to the erosion effect, and with superior level of technology – the productivity effect. The wage rate per efficiency unit, the agent's income *I* is:

$$I_t^{i,s} = I^s(a_t^i, g_t, A_t) \quad and \quad I_t^{i,u} = I^u(a_t^i, g_t, A_t)$$

.

Agents, whose would choose to become skilled, or unskilled with a level of technology are: $a_t = \dfrac{1-\delta g_t + \delta g_t^2}{1+\delta g_t^2}$,

which is function of ability $a$. The aggregate efficiency units of skilled labor $h$, and unskilled labor $l$ at time *t*, are:

$$h_t = \int_{a_t}^{1} (1-\tau)\{a_t^i - (1-a_t^i)g_t\}da_t^i \quad and \quad I_t = \int_0^{a_t} \{1-(1-a_t^i)g_t\}da_t^i$$

.

The effect of the rate of technological progress on the return to skilled and unskilled agents, the composition of the labor force and wage inequality, and the composition of the agents on the rate of technological progress, is that agents can adjust consumption and labor input in response to technology shocks.

## Shock and Choice

Hence the agents want to smooth consumption that is very unresponsive to shocks and this elasticity of substitution also has an effect on the optimal choice after a technology shock. Lettau and Uhlig (2000) suppose labor is more productive and agents want to work more now to take advantage of the higher wages, until the introduction of technology when agents are induced to work less because they earn more per unit.

The utility of an agent for a stochastic sequence of individual consumption *C* is:

$$U\{(C_t)_{t=0}^{\infty}\} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t; X_t)$$

.

The marginal rate of substitution can be written in terms of consumption and the surplus ratio:

$$M_{t+1} = \beta \frac{u'(C_{t+1};X_{t+1})}{u'(C_t;X_t)},$$

where the stochastic sequence of $X$ is regarded by agents and tied to the stochastic sequence of consumption $C$.

The agent solves:

$$\max_{C,L_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, X_t^c, L_t, X_t^l).$$

$$C_t + K_t = (d_t + (1-\delta))K_{t-1} + w\, N_t + \pi_t.$$

Agents derive utility from consumption $C$ and leisure $L$, $K$ denotes the capital stock chosen at date $t$ and owned by the agents, $d$ are dividends per unit of old capital, N=1 – L is labor, $w$ denotes wages, $\pi$ denotes agent profit, and $\delta$ is the depreciation rate. The agent maximizes profit:

$$\pi_t = \max_{(K_{t-1}^d, N_t)} Y_t - d_t K_{t-1}^d - w_t N_t^d, \text{ where}$$

$$Y_t = \tilde{Z}_t (K_{t-1}^d)^p (N_t^d)^{1-p},$$

is output and $K_{t-1}^d$, $N_t^d$ are demanded capital and labor, technology $Z$ grows at rate $g$.

$$\mathrm{Y}_t = \left[ \int_0^1 y_t(j)^{(\theta-1)/\theta} v_t(j)^{1/\theta} dj \right]^{\theta/1-\theta},$$

where $Y$ is a production technology defined over intermediate inputs units of final goods that are produced with $y$ units of each input $j$, and $v(j)$ represents a productivity shocks.

## Productivity Shocks

Fagnart, Licandro and Portier (1999) assume a maximizing profit function, where the agent faces input prices $p$, supply constraints, $q$ and productivity shocks $v$, and the problem is:

$$\max_{(y_y(j))} \mathrm{Y}_t - \int_0^1 p_t y_t(j) dj.$$

The solution can be described by the following system:

$$y_t(j) = \begin{cases} p_t^{-\theta} Y_t v_t(j) & if \ v_t(j) \le \tilde{v}_t \\ q_t & if \ v_t(j) \ge \tilde{v}_t \end{cases},$$

with $$\tilde{v}_t = \frac{q_t}{p_t^{-\theta} Y_t},$$

where $\tilde{v}$ represents the critical value of the productivity shock $v$ for which the unconstrained demand equals the supply constraints $q$.

Final output supply $Y$ can be written as

$$Y_t = \left\{ \left[p_t^{-\theta} Y_t\right]^{(\theta-1/\theta)} \int v dF(v) + (q_t)^{(\theta-1/\theta)} \int v^{1/\theta} dF(v) \right\}^{\theta/(\theta-1)}.$$

The price decision of any agent is thus the solution to the problem:

$$\max_{p_t} E_v \left[ \left( p_t - \frac{\omega_t}{A_t x_t^a} \right) y_t \right],$$

where the expectation operator $E$ represents the agent's expectation when the only remaining uncertainty bears on $v$, and the wage rate is $\omega$.

The preference of households are represented by a time separable utility function $U$:

$$U_t = E_t \left[ \sum_{s=t}^{\infty} \beta^{t-1} \left( \log(c_s) + v(1 - l_s) \right) \right],$$

where $E$ represents the expectation operator given the information available in $t$, and , and $l$ are the consumption and labor supply during $s \ge t$, $\beta$ is a constant subjective discount rate. The household enters into period $t$ with a predetermined level of financial asset $a$ and during the period it receives a wage income, firm's profit $\Pi_t$, and chooses how much to consume, to work and to save. The budget constraint is:

$$a_{t+1} + c_t \le (1 + r_t) u_t + \omega_t l_t + \Pi_t,$$

and the household problem can be written recursively as:

$$V^H(a_t) = \max_{c_t, l_t, a_{t+1}} \left\{ \log(c_t) + v(1 - l_t) + \beta E_t \left[ V^H(a_{t+1}) \right] \right\}.$$

The equilibrium of the relative price $p$ in the market share of an input is:

$$p_t = \left\{ \left[ \int v df(v) \right] + (\tilde{v}_t)^{(\theta-1)/\theta} \left[ \int v^{1/\theta} df(v) \right] \right\}^{1/\theta-1}.$$

We denote the savings of an agent in period $t$ by $m$, the bids by $b$ and offers by $q$ ; the agents strategy set $S$ is then:

$$S_t = \{(q_t, b_t, m_t) \in R^S | \omega_t \geq q_t >> 0\}.$$

## Allocation and Equilibrium

Each agent is allocated a share of the aggregate offer of the good in the proportion that his bid bears to the aggregate bid $B$. Goenka, Kelly and Spear (1998) suppose that:

$$B_t^S = \sum_{i=1}^{n} b_{it}^S \qquad \text{for } s = t, t+1.$$

Each agent is allocated a share of the aggregate amount of money bid in the proportion his offer of the good bears to the aggregate offer $Q$:

$$Q_t^S = \sum_{i=1}^{n} q_{it}^S \qquad \text{for } s = t, t+1.$$

If equilibrium prices are positive, lifetime bids must satisfy the condition:

$$b_t^t + b_t^{t+1} \geq 0.$$

The agent period budget constraint is:

$$b_t^t Q_t = B_t q_t^t - m_t Q_t.$$

Define

$$p_t^n = \frac{\hat{B}_t^n - m}{\hat{Q}^n},$$

where $\hat{B}_t = [B_t - b_t^t]$, and $\hat{Q}_t = Q_t - q_t^t$, and suppose that $\{p_t^n\}_{n=1}^{\infty}$ is a sequence of interior Nash equilibrium prices for the $a$-fold replication of the market game. Let state variable

$$\theta_t = \frac{\hat{Q}m}{(\hat{B}_t - m)}$$, which then yields

$$U'(\omega_1 - \theta_t)\frac{[\theta_t^2 + \hat{Q}\theta_t]}{\hat{Q}} + V'(\omega_2 + \theta_{t+1})\frac{[\theta_{t=1}^2 - \tilde{Q}\theta_{t+1}]}{\tilde{Q}} = 0,$$

where $\tilde{Q}_t^n = nq_1 + (n-1)q_2$.

If endowments are not Pareto optimal, then there exists a steady-state equilibrium when agents find it optimal to save or wish to borrow, but the steady-state equilibrium can be Pareto ranked. The agent's own offer is optimal in the sense that any change in the offer can be achieved by a change in the bid.

## Dynamics Environment

Wages and employment dynamics are adequately captured when norms adjust slowly to the environment and real wages may be rigid and this explains why employment fluctuates largely in response to shocks. Collard and de la Croix (2000) suppose that a household has to define a consumption-saving plan such that it maximizes discounted expected utility:

$$\max_{c_{j,t}, e_{j,t}, k_{j,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log c_{j,t} - d_{j,t} \left( e_{j,t} - \phi - \gamma \log\left(\frac{w_t}{w_t^a}\right) - \psi \log\left(\frac{w_t}{w_t^s}\right)\right)^2 \right].$$

The index $j$ identifies a variable as pertaining to individual $j$, and $c$ and $e$ denote consumption and effort, while $\phi,\ \gamma,\ and\ \ \psi$ are positive parameters $d$ is a dummy variable which takes the value 1 when the worker is employed and zero otherwise. The current wage is $w_j$ and the current alternative wage is $w_j^a$, which serves as a benchmark for the agent-workers to evaluate their salary, $k$ denotes physical capital, while $0 < \beta < 1$ is the psychological discount factor. Since all agents are assumed to be identical ex ante, the unemployed will be drawn randomly, leading to an unemployment rate:

$$u_t = 1 - l_t.$$

We assume that there exists a firm that produces a good that can either be consumed or accumulated. Its technology is described by:

$$y_t = a_t k_t^a (l_t e_t)^{1-a},$$

where $k$ denotes the firm capital stock, $l$ the level of employment, $e$ the effort level, and the technological shock $a$. The firm seeks to maximize its profit:

$$\max_{k_t, w_t, l_t} y_t - w_t l_t - r_t k_t.$$

The first-order necessary conditions are:

$$(1-a)\frac{y}{l_t} = w_t,$$

$$(1-a)\frac{y_t}{e_t}\left(\frac{\gamma + \psi}{w_t}\right) = l_t,$$

$$a\frac{y_t}{k_t} = r_t$$
.

These equations determine the optimal employment decision of the firm: it hires agents until the marginal product of labor equals the real wage, and corresponds to the wage setting behavior of the firm. It states that the firm will increase wages until the marginal cost implied equals the marginal return in terms of effort, and last equation furnishes the physical capital demand of the firm.

In equilibrium, we have the market clearing conditions:

$$k_t = \int_0^1 k_{j,t}dj \quad and \quad c_t = \int_0^1 c_{jt}dj$$
,

and
$$e_t = \phi + \gamma \log\left(\frac{w_t}{w_t^a}\right) + \psi \log\left(\frac{w_t}{w_{t-1}}\right)$$
,

$$w_t = (1-a)\frac{y_t}{l_t}$$
,

$$1 = \beta E_t\left[\frac{c_t}{c_{t-1}}\left(a\frac{y_{t+1}}{k_{t+1}} + 1 + \delta\right)\right]$$
,

$$k_{tt+1} = y_t - c_t + (1-\delta)k_t$$
,

which is the goods market condition. Households will raise their effort with increases in the ratio of the current wage to an alternative wage, and effort increases with the ratio of the current wage to past wage. The past wages are treated either as personal norms or social norms, and are useful to solve the business cycle.

Firms attempt to obtain their advantage over rivals when the discounted value of profits is highest relative to the costs of developing the innovation, and the aggregate level of profit in the economy is increasing, and is thus complementary with the innovation decisions of agents in the other sector.

## Innovation Cycles

Welfare can rise with cycle length in innovation cycles even though the innovation technology is a form of self-enforcing multiplier connections between innovating agents. Francois and Shi (1999) suppose a continuum of agents with a unit of measure and each agent has one unit of primary input, labor, which can be allocated to innovating new technologies and producing intermediate goods. Given the price $p$ for each input $i$, the profit of the final good producer is

$$\max_x \left[ \exp\left( \int_0^1 \ln x_{it} di \right) - \int_0^1 p_{it} x_{it} di \right],$$

which yields the following demand for each input $i$ and final good $y$:

$$x_{it}^d = \frac{y_t}{p_{it}}.$$

When the market for good $i$ clears, the innovator's profit is:

$$\pi_{it} = p_{it} x_{it} - w \frac{x_{it}}{\gamma^{k_i}}$$ , with productivity $\gamma^{k_i}$.

If more innovators target the same date for the innovation process, more labor will be released from the innovation to the production of the final good when the innovators implement their new technologies. The project-specific human capital $h$ is:

$$h_{t+n}^{t+1} = \left( n^{p/\theta} \sum_{s=1}^{n} l_{t+s}^{\theta/(1+\theta)} \right)^{1/\theta+1},$$

with $1+\theta$ denoting the elasticity of substitution between labor inputs at different times.

The innovator's problem is:

$$\max_{(l,n)} \left\{ \beta^{n+1} \pi_{t+n+1} \phi h(l_{t+1,\dots\dots,} l_{t+n}; n) - \sum_{s-1}^{n} \beta^t w_{t+s} l_{t+s} \right\}.$$

During the time sequence of events through such an N+*1* cycle, the new innovative effort finally ceases and project successes are realized.

With all innovators following the strategy described by $\{L_{t+1},\dots\dots,L_{t+N};N\}$ under the success probability of an innovation, which is $a_{t+N+1} = \phi H$ , we have:

$$\frac{\pi_{t+N+1}}{w_{t+N+1}} = \frac{\gamma - 1}{\gamma - (\gamma - 1)\phi H}.$$

Aggregate labor inputs into innovation rise and aggregate labor inputs into production of final good fall, which implies that the profit of successful innovation falls, and the profit / wage ratio is higher in $t+N+1$ than in period $t+1$. Given all other innovators use

(h ,N), no individual innovator has incentive to choose n<N if

$$\frac{1}{\beta\phi} > \frac{\gamma - 1}{\gamma}(1 - L_{t+2}).$$

The expected rate of return to the innovator is:

$$\phi \frac{\pi_{t+1}}{w_{t+1}} = \phi(1 - L_{t+2})\frac{\gamma - 1}{\gamma},$$

and in a stationary cycle we have

$$\frac{\pi_{t+N+2}}{w_{t+1}} = \left(\frac{\gamma - 1}{\gamma}\right)(1 - L_{t+1})\gamma^{\phi H}.$$

A firm can delaying innovation or implement it immediately. If there were some uncertainty as to the arrival date of innovations in an $N$ cycle, firms would aim to innovate after $N$ periods, but those receiving innovations before or after this time may have an incentive to delay until the main body of economy-wide innovations is implemented.

Equilibria with different lengths of cycles can be compared using the discounted present value of aggregate income.

The income levels within the job cycle in an (N – 1) period cyclical equilibrium are:

$$y_{(j+1)(N+1)} = \frac{\gamma^{1+j\phi H}}{\gamma - (\gamma - 1)\phi H}.$$

The discounted present value of income in such an equilibrium is:

$$Y(N) = \frac{\left(1 - \beta^{N+1}\right) / (1 - \beta)}{1 - \beta^{N+1}\gamma^{\phi H}}.$$

Welfare increases with the cycle length in line with human capital $H$ and productivity $\phi$:

$$H'(N) > \frac{1 - \gamma^{-\phi H}}{\phi\left(1 - \beta^{N+1}\right)}\left(-\frac{\ln \beta}{\ln \gamma}\right).$$

Its purpose is to depict the counteracting forces which contribute to the net effect of cycle length on output, and hence welfare, over time. If the input of $D$ units of goods when technology $A$ is available produces $h$ units of research, then the accumulation of research is given by:

$$H(t) = \int_{t=t_j}^{t} h_j[D(s)]ds, \quad t \in \left[t_j, t_{j+1}\right], \quad j = 0,1,2,.......$$

When the economy using the $j$-th technology accumulates $H$ units of research, the new innovation $a_{j+1}$ becomes available, increasing the level of technology to $A$

## Technology Cycle

Freeman, Hong and Peled (1999) examine the social optimum, defined to be the maximum feasible utility of the agent, as a solution of a market makers' problem. The analysis of the social optimum involves the choice of time paths for consumption $C$, research effort $D$, and capital stocks $K$, as well as the points in time at which the new technologies become available, given the initial capital stocks $K_0$, and the initial technology $A_0$ :

$$W(K_0, A_0) = \max_{C(.),D(.),K(.),[t_j \mid j=1,2,\ldots\ldots]} \int_{t=0}^{\infty} e^{-pt} u[C(t)]dt$$
.

Within each technology cycle, $[t_j, t_{j+1}]$, the allocation of resources solves an optimal control problem. Define $V(T, K_0, K_\gamma, A)$ as the maximal within-cycle discounted utility, given a cycle of total length $T$, beginning and end of cycle capital stocks $K_0$ *and* $K_\gamma$, when the available technology is $A$ :

$$V[T, K_0, K_\gamma, A] = \max_{\{C(\tau),D(\tau),K(\tau) \mid \tau \in [0,T]\}} \left\{ \int_{\tau=0}^{T} e^{-p\tau} \frac{[C(\tau)]^{1-\sigma}}{1-\sigma} d\sigma \right\}$$
.

The market –maker's problem has a recursive structure, which can be summarized by

$$W(K_0, A_0) = \max_{T,K_\gamma} \{ V(T, K_0, K_\gamma, A_0) + e^{-pT} W(K_\gamma, \gamma A_0) \}$$
.

For any $\tau \in [0,T]$ let

$$c_j(\tau) = C_j(\tau) \; / \; A_j$$
,

$$d_j(\tau) = D_j(\tau) \; / A_j$$
,

$$k_j(\tau) = K_j(\tau) \; / A_j$$
.

The resource constraint becomes

$$\dot{k}_j(\tau) = k_j(\tau)^{1-a} - \delta k_j(\tau) - c_j(\tau) - d_j(\tau)$$
,

and the within-cycle value function is:

$$v(T, .k_0, k_T) \equiv V(T, K_0, K_T, A) \; / A^{1-\sigma}$$
.

Of particular interest will be a cycle $s$ of length $T^*$ where the technology-scaled variables are cycle independent, and the optimal trajectories of these variables are how previous cycle multiplied by the technology improvement factor $\gamma$ and the recursive representation of the market maker problem implies:

$$w\left(k_0^*\right) = \frac{\int_{\tau=0}^{T^*} e^{-p\tau}\left(c^*(\tau)^{1-\sigma}\right) / (1-\sigma) d\sigma}{1 - e^{-pT^*}\gamma^{1-\sigma}}.$$

Along the optimal path, the marginal value should be obtained by any of several possible uses: consumption, production and research and we have:

$$\frac{\partial W}{\partial K}\left[K_j\left(T_j\right), \gamma A_j\right] \equiv \frac{\partial W}{\partial K}\left[K_{j+1}(0), A_{j+1}\right] = u'\left[C_{j+1}(0)\right].$$

The growth rate of consumption is the difference between the marginal product of capital $\delta$ and the constant $p$, and growth rate of consumption immediately after an innovation exceeds the rate immediately before the innovation and:

$$\mu_{j2}(\tau)^{-1/\phi} d(\tau) = e^{p\tau/\sigma} c(\tau)^{\sigma/\phi}, \text{ and}$$

$$\frac{\dot{d}(\tau)}{d(\tau)} = \frac{1}{\phi}\left[(1-a)k(\tau)^{-a} - \delta\right].$$

The differences in the cycles of growth rates of research and consumption come from differences in $\phi$ and $\sigma$, the relative curvatures of the functions representing utility and knowledge production, and differences in the rate of time discount.

Research effort is:

$$\gamma(\mu_2)^{1/\phi} c(0)^{\sigma/\phi} \geq (\mu_2)^{1/\phi} e^{pT/\phi} c(T)^{\sigma/\phi}.$$

Investment increases at the time of innovations and continuity of consumption and capital stock paths in a stationary social optimum can be written as:

$$\gamma\dot{k}(\tau) = \gamma k(\tau)^{1-a} - \gamma\delta k(\tau) - \gamma c(\tau) - \gamma d(\tau).$$

Stationary equilibrium in the optimal terminal time $T$ may be written as:

$$\frac{c(T)}{1-\sigma} + \frac{d(T)}{1-\phi} + \dot{k}(T) = p\gamma^{1-\sigma} w[k(0)]c(T)^{\sigma},$$

where $k$ is the technology stock of physical capital . Households create capital and knowledge, which is accumulated into innovation of $a_j = A_j - A_{j-1}$, and each household chooses consumption $C_j(\tau)$, research effort $D_j(\tau)$, capital per plant $K_j(\tau)$, and the time of the next innovation $t_j$ taking as given the capital rental rate $p_j(\tau)$ and the innovation rental rate $\theta_j(\tau)$ paid by each producer so as to maximize its lifetime utility. The household budget constraint at moment $\tau$ of the $j$th cycle may then be written as:

$$\dot{K}_j(\tau) = \pi_j(\tau)K_j(\tau) - \delta K_j(\tau) + \theta_j(\tau)A_j - C_j(\tau) - D_j(\tau).$$

These innovations in the production process may take the form of new knowledge or improvements in an economy's physical infrastructure. Government expenditures, prices, or the size of technological improvements, and costs or delays in adopting innovations of technologies will likewise influence the pattern of the cycle based on changes in the relative profitability of alternative forms of investment, even absent other sources of fluctuations.

# 17 Information Cascades, Expectations, and Risky Choices

Confidence has been related to liquidity preference in an intentionally generic way thus far, and is now highlighted. The points made follow given the possibility of learning, precaution and speculation. Liquidity provides the agent with flexibility to revise decisions, altering the composition of his portfolio in the future. Once agents buy capital goods, they are stuck with them, and they may simultaneously demand liquidity to protect themselves against unwanted circumstances that cannot be reliably anticipated. Lee (1998) presents the mechanism of information aggregation failure, which relies on the notion of information cascades and also agent behavior. As informational cascade is an event in which a sequence of agents takes actions as they want try to exploit the information available from knowledge of previous action choices. Each agent makes a decision based on his own private information and the memory of agents' previous decisions.

When perceived uncertainty increases and confidence decreases, the liquidity preference is reinforced, because they realize that their assessment of the likelihood of things going wrong has become even less reliable. If considerable inflation is anticipated, other liquid assets may be demanded for speculative purposes instead of money.

Faced with a sequence of risks, agents maximize the expected utility from cash and risky assets conditional on the information available at time of decision. Each agent $i$ is endowed with wealth, but a different private signal $\theta^i$ is correlated with the value of risky assets. Given this description, the agent's optimization problem is:

$$\max E_t^i \left\{ u\left( W_t^i + Yx_t^i + \sum_{t-1}^{T+1} (Y - p_t) z_t^i \left(z_t^i\right)_{t-1}^{T+1} \right) \right\},$$

where $E_t^i$ is the expectation on agent's information at market round $t$, $p$ is the risky asset price at $t$, $z$ is the agent market orders at rounds $t$ to $T+1$, $W$ is agent cash at the beginning of market round $t$, and $x$ is risky asset at beginning of market round and we denote the price history by $p = (p_0, p_1, \ldots\ldots, p_t)$ and agents behave as price takers with cash and risky assets. The agents are not allowed the strategy, which misrepresents the private signal when decision-making agents do not consider the consequence on the price path from non-market operations. When making the decision whether to place a non-zero market order, the agents do not consider the consequence on the price path from non-market operations. The decision procedure of the agent is consistent with information that can be inferred from the price sequence about the private signals.

If there are many agents whose information was not reflected in the market price, the simultaneous aggregation of those signals may bring change in the risky asset market. An informational avalanche takes place if some agents make a non-zero market order and thus the occurrence of the information is a product in which hidden information get revealed to the market. The optimal trading strategy indexes a stochastic process for the security price, since agents are allowed to market more than once to exploit the information advantage due to their private signal while the second market allows the agent to adjust a risky position.

Since the agent solves the optimization problem with a rational expectation as to the future evolution on the market, the first market order indeed fully reveals the private signal underlying the order.

The optimal market strategy may prevent the agent making an order if the gain is outweighed by the cost and the liquidation value of the asset is 1 in state $G$ and 0 in state $B$, where $\mu_0$ is the probability of the state $G$ before market round 1. There exist $\overline{\mu}(\theta)$ and $\underline{\mu}(\theta)$ such that for $\overline{\mu}(\theta) \geq \mu \geq \underline{\mu}(\theta)$ the agent newly arriving in market round $t$ with a private signal $\theta$ places a market order. The evolution of the asset price is the sequential market structure under the transaction cost, and may prevent the correct information aggregation even in the long run. The turbulence in the asset market during the informational cascade results from the price correction to reflect the information distributed in the economy at each moment. The arrival of private signals in such a sequence is characterized by: boom, euphoria, trigger and panic, if agents who arrive in the market during euphoria make market orders.

Agents are described as acting rationally based on all information available at the moment of their investment decisions, and they fail to aggregate their private information because of transaction costs. The difficulty lies in fact that the small errors each agent makes tend to be cancelled out at the equilibrium.

## Market Signals

In the sequence of signals, is necessary that agents with favorable signals move first to set an optimistic tone for the market, because many agents with low signals may have been induced to buy at high prices thus ignoring their private information.

Given a market with a non-risky asset and certain number of risky assets:

$$J(S,V) :=_{\psi} \inf E^{P}_{S,V} v\{l([H_T - V_T(\psi)]^+)\},$$

where $H$ is a liability to be hedged at some fixed future time $T$. Denote by $V_T(\psi)$ the value at $T$ of a portfolio corresponding to a self-financing investment strategy $\psi$, for the initial value $S$ of the asset in the portfolio, for a given initial capital $V$, and where $l$ is a suitable loss function with $l(0)$. The choice of the probability for the evolution of the risky asset can be denoted:

$$\inf_{\psi} \sup_{p \in P} E^{p}_{S,V} \left\{ l\left( \left[ H_T - V_T(\psi) \right]^{+} \right) \right\}$$

.

where $P$ is a real world probability measure and uncertainty is only in the stock appreciation rate. Runggaldier and Zaccaria (2000) suppose the economic agents have to assess the impact of various choices of the initial capital.

Consider a market in which a risky asset with price $P$ evolves according to

$$dS_t = a(Z_t)S_t dt + \sigma(Z_t)S_t d\omega_t,$$

where the process $Z$ is assumed to give transition probabilities and generate different model models for $S$.

Let $X_n = (S_n, Z_n, V_n^{+}, \psi_n^{1})$,

which implies that discrete random variables $\varpi_n$ then take values with probabilities:

$$p_i(Z_n) := P(\varpi_n = \varpi^{i} | Z_n) = \int_{\Omega} p(\varpi_n | Z_n) d\varpi_n.$$

If $Z$ is not observed, then the values of $p^{i}(Z_{n-1})$ have to be replaced by $E\left(p^{i}(Z_{n-1}) | \hbar^{S}_{n-1}\right)$,

where $\pi^{k}_{n-1} := P\{Z_{n-1} := k | \hbar^{S}_{n-1}\}$

The transition probabilities $p_{\pi_{n-1}, \pi_n}$ can be obtained by these means and this in effect leads to:

$$\pi^{k}_{n}(i) = P\{Z_n = k | \pi_{n-1}, \varpi_{n-1} = \varpi^{i}\}; \; i = 1, \ldots, M$$

.

Notice that by $\varpi_n = S_{n+1} / S_n$, we have $\hbar^{S}_{n} = \hbar^{\varpi \cdot}_{n-1}$. At each transition, both $S$ and $\pi_n$ have cause $M$ to grow along an only partly recombining tree and this leads to intractability for longer horizons.

We provide necessary and sufficient conditions for a dynamically consistent agent to prefer signals. If the agent is an expected utility optimizer, then she prefers more information to less regardless of the set of actions available. Grant, Kajii and Polak (2000) suppose that the reason agent sometimes chooses a less informative signal is an attempt to self-commit. The agents can perform randomization for themselves, but they are always expected to prefer more informative signals. We say that an agent is dynamically consistent if actual behavior within a decision tree conforms to that which would have chosen are able to commit. We focus on trees in which and agent chooses an action after a resolution of a signal. A typical tree, $\tau$ then consists of:

a finite set of signal realizations $(s_1,\ldots,s_n)$;

the posterior beliefs on the state space $\Omega$, $(P_1,\ldots\ldots,P_n)$ induced by these realizations,

the finite action sets , $(A_1,\ldots\ldots,A_n)$ *each* $A_i \in A$, *with* the realizations,

the probabilities $(q_1,\ldots\ldots,q_n)$ of observing these realizations,

and a consequence function $c$ in $C$.

Let $T$ denote the set of trees $\{(s_i,P_i,A_i;q_i)_{i=1}^{n};c\}$ and let $\succ_T$ be a complete and transitive preference relation over trees.

Suppose that $\{(P_i,q_i)_{i=1}^{N}\}$ and $\{(P_i',q_i')_{i=1}^{N'}\}$ are the distributions of posteriors on $\Omega$ induced by the signals $(S,\lambda)$, and $(S',\lambda')$ respectively; let $n$ denote an early-resolution randomization over the actions $A$, and let $(m_i')_{i=1}^{N'}$ denote the agent's early-resolution random behavior in tree $\tau' = \{(s_i',P_i',A,q_i')_{i\acute{e}1}^{N'},c\}$ induced by the signal $\{(s_1',\ldots\ldots,s_N'),\lambda'\}$. For any $\alpha$ in [0,1] and any pair of randomizations $m$ and $m'$, we have random behavior. Single action information loving implies that this bifurcation is preferred and that the tree is induced by the more informative signal.

Let $(a_i')_{i=1}^{N'}$ in $A^N$ denote the agent's behavior $b_{\tau'}$ and suppose that this behavior induces the stage process.

## Decision Making Under Uncertainty

Decision-making under uncertainty is based on the state of expectations, the end pursued, and the perception of constraints. Rationality of decision-making requires consistency within the state of expectations, within the ends pursued, as well as consistency between the course of agent and the state of expectations. Since uncertainty has to do with the lack or limitation of knowledge, the possibility of rationality under uncertainty depends on how the relation between rationality in this deeper sense and knowledge is dealt with. This criterion applies to the rationality of the state of expectations and by extension to the rationality of behavior based on this state of expectation. They are the aspects in whose case the lack of knowledge prevents the agent from determining what is rational.

In the formation of the state of expectations, knowledge has to be supplemented by an optimistic disposition to face uncertainty and by creativity. Situations of uncertainty are not reduced to a simple dichotomy between action and inaction, or to different types of action, depending on the quality and intensity of the optimistic disposition to face uncertainty.

Agents may be aware that the uncertainty will never be completely eliminated ex ante and still wait until it is hopefully reduced to a level that they, given their uncertainty aversion, consider tolerable enough for them to sacrifice liquidity. Moreover, since some information does not exist at the time of decision, there is no asymmetry regarding such information and, if surprises occur, there are grounds for liquidity preference other than waiting.

Creativity is as an ability to see and to do things in a novel way, and, it is an important source of uncertainty. Each agent's creativity may be strong, weak, or even absent; as a determinant of expectations, creativity is forward-looking, but it is often associated with originality in interpreting the past and the present. These aspects may be part of the agent's environment or of society at large, and spontaneous optimism or pessimism as the factor through which an optimistic disposition indirectly influences expectations and confidence, is considered. The existence of uncertainty justifies liquidity preference on the grounds that unexpected events may require sudden unforeseen expenses, or the cash flows from less liquid assets may turn out to be less than expected. The balances held for this purpose defend the agent from not being able to meet liabilities. Under risk, agents demand liquidity knowing with full confidence the chances of things going wrong, and under ambiguity they do this knowing the list of all possible events. Under uncertainty, things may go wrong in contracts in an unpredictable way, because an event may occur that is unimaginable ex ante.

The possibility of such occurrences provides a reason for liquidity demand independent of risk aversion and ambiguity aversion.

## Information Choices

We construct a particular information portion for each agent and show that it captures the notion of information in the sense that it is the finest within the class of information portions. Battigalli and Bonanno (1999) suppose that the information completion of $G$ is an n-tuple $(\mathrm{K}_1,\ldots.,\mathrm{K}_n)$ ,

where for each agent $i = 1,\ldots\ldots,a,$ $\mathrm{K}_i$ is a partition of the set of modes $T$ and coherence with information structure, that contains $t$ coincides with $h$:

$$t \in h \in \mathrm{H}_i \Rightarrow [t]_i = h$$

Agents remember what choices $c$ they made:

$$(p(t) \in h) \wedge (t \in S(h,c)) \Rightarrow [t]_i \subseteq S(h,c)$$
,

where $p$(t) denotes the previous mode of $t$ and $S(h, c)$ the set of successor of modes in $h$ following choice $c$ at $h$. If mode $x$ is a successor of mode $t$, then terminal mode that can be reached:

$$t \prec x \Rightarrow (\forall x' \in [x]_i, \exists t' \in [t]_i : t' \prec x').$$

Agents know that for mode $t$, the cell containing the number of the previous mode of $t$ $\ell(t)$, and $T^h$ is the set of stage-$k$ modes: $\ell(t) = k \Rightarrow [t]_i \subseteq T^h$.

Given form $G$ and information completion $K$, for agents $i$, we can use knowledge operator $\mathrm{K}_i$ and KE is the event that agent $i$ knows E:

$$K_i E = \{t \in T \;\|[t]_i \subseteq E\}.$$

Agents remember what choices they made.

## Coalitions of Agents

A coalition can pool agents' resources and core solutions are sought, described, and computed via programs. Sandsmark (1999) supposes that an agent faces a problem of fairly tractable nature and any individual's problem assumes the form of a concave maximization.

Let $\Im$ denote a probability space of outcomes $\xi$, each one happening with probability $p(\xi)$. This means that there is a random mapping $\xi \mapsto (c^2, A_{21}, A_{22}, (b_i^2)_{i \in I})(\xi)$ and contracts must be signed before outcomes are known. The first-stage decision $x^I \in \Re_+^{n_1}$ denotes a choice made under uncertainty, and the second-stage decision $x^2(\xi) \in \Re_+^{n_2}$ denotes the adjustment when the random outcome $\xi \in \Im$ becomes known and agents are averse.

That is, if there is an infinite sequence $\xi_1, \xi_2, \ldots\ldots$ of independent variables, all distributed according to $p(.)$, then it holds that

$$\left[b_i^2(\xi_1)\ddot{y}^2(\xi_1) + \ldots\ldots + b_i^2(\xi_r)\ddot{y}^2(\xi_r)\right] / r \to \sum_{\xi \in \Im} p(\xi)\left[b_i^2(\xi)\ddot{y}^2(\xi)\right] = E\left[b_i^2 \ddot{y}^2\right],$$

as $r \to \infty$. Suppose agent $i \in I$ has already made his first-stage decision $x_i^1$, with or without collaboration, so agents may still benefits from cooperation at the second stage and optimal solution is associated with it:

$$V_2\left(b_1^2(\xi) - A_{21}x^1, \xi\right) = \max\left\{c^2(\xi)x^2 : A_{22}(\xi)x^2 \le b_1^2(\xi) - A_{21}(\xi)x^1, x^2 \ge 0\right\},$$

where $b$ is construed as a resource bundle owned by $i$, $x$ is an activity plan worth $c\,x$, $c$ displays marginal contributions , and $A$ represents the technology.

Let $x_i$ represent an agent's production activity belonging to a space $X$ and payoff $f_i(x_i)$, and be constrained by inequalities $g_i(x_i) \succ 0$ not necessarily of the same sort:

$$x_i, \hat{x}_i \in X_i,\ \mu \in [0,1] \Rightarrow g_i(\mu x_i + (1-\mu)\hat{x}_i) \succ \mu g_i(x_i) + (1-\mu) g_i \hat{x}_i$$

and assume that any nonempty coalition $S \subseteq I$ can ensure itself a characteristic value:

$$v(S) := \max\left\{\sum_{i \in S} f_i(x_i) : \sum_{i \in S} g_i(x_i) \succ 0\right\}.$$

The problem of coalition $S$ then has the following form:

$$L_S(y) := \max_{x_i \in X_i, i \in S} L_S(x, y),$$

subject to $y \in Y$, where the Lagrangian of group of agents is:

$$L_S(x, y) := \sum_{i \in S} [f_i(x_i) + (y, g_i(x_i))],$$

and we get:

$$L_S(y) = \max_{x_i \in X_i, i \in S} L_S(x_i, y) = \max_{x_i \in X_i, i \in S} \sum_{i \in S} [(y_i, b_i) - (y, A_i x_i) - c_i(x_i)]$$

$$= (y, b_S) + \sum_{i \in S} \max_{x_i \in X_i} [(-A_i^T y, x_i) - c_i(x_i)]$$

$$= (y, b_S) + \sum_{i \in S} c_i(A_i^T y).$$

Now, for every coalition $S$ that has agreed upon and made a first-choice bundle $x_S^1 = (x_i^1)_{i \in S}$, there is a second-stage value

$$v_2(x_S^1, S) := \sup_{x_S^2}\left\{\sum_{i \in S} f_i^2(x_i^1, x_i^2) : \sum_{i \in S} g_i^2(x_i^1, x_i^2) \succ 0\right\}.$$

Ex ante, the second-stage is uncertain and has the expected value

$$f_i^2(x_i^1, x_i^2(.)) := EF_i(., x_i^1, x_i^2(.)).$$

For each coalition $S$, having made a first-choice bundle $x$, and for realization $\xi \in \Im$, there is a second-stage realized value:

$$v_2(\xi, x_S^1, S) := \sup_{x_S^2}\left\{\sum_{i \in S} F_i(\xi, x_i^1, x_i^2(\xi)) : \sum_{i \in S} G_{ij}(\xi, x_i^1, x_i^2(\xi)) \geq 0, \forall j \in J\right\}.$$

Ex ante, an agent knows that $\xi$ will come up with positive probability $p(\xi)$, and the expected stage value is :

$$Ev_2\left(x_S^1,S\right):=\sum_{\xi\in S}p(\xi)v_2\left(\xi,x_S^1,S\right).$$

Then

$$v(S):=\max_{x_S^1}\left\{\sum_{i\in S}f_i^1\left(x_i^1\right)+Ev_2\left(x_S^1,S\right):\sum_{i\in S}g_i^1\left(x_i^1\right)\succ 0\right\}$$

defines a stage nonlinear stochastic process.

## Information Scenarios

What agents know is based on the information available to them and learned through practice without necessarily being discursively based on different views on what is considered knowledge, which leads to different theories of economic reality.

Agents interact with other agents and the distribution of strategies depends on the relative size of payoff. Hoffmann (1999) considers interactions and learning between agents endowed with memories of previous processes. The separate analysis of interactions and learning may reveal what each effect individually has on agent behavior. Agents may be uncertain as to their own relative performances, and thus about the scope for improvement through learning.

We have scenarios where the agent's information is perfect, or normal, or imperfect. Tzur and Yarri (1999) suppose that the agents have incentives to misrepresent by overstating bad outcomes, which has a favorable effect on the cost of the agent's incentive, and that the more the risk of the agent, the greater this effect is. They also suppose that agents possess imperfect information.

The agents operate in the market, with the goal of maximizing price with asymmetries in information.

The equilibrium consists of the market price *P*, which would be higher for $\hat{x}_1\left(P=x_1\right)$ than for $\hat{x}_2\left(P=x_2\right)$. Thus the agent has incentives to deviate from disclosing *x* even when be observes *s*.

The minimum efficient penalty level $C^*$, must satisfy the condition that agent is indifferent between lying and telling the truth:

$P_2=\pi\left(P_2-C^*\right)+\left(1-\pi\right)P_1$ , because the agent has tried to misrepresent with probability $\pi$ .

Solving for *C*, we obtain:

$C^*=\dfrac{1-\pi}{\pi}\left(x_1-x_2\right)$. The agent reports the truth for $C\geq C^*$ and the price reflects the agent's true value. Accounting has a dual role: stewardship and information. We assume effort choice from the binary set $\left(e_H,e_L\right)$, $e_H>e_L$ and that the higher this effort, the higher the agent's expected value. That is, an agent makes a decision and he chooses an effort level after signing contracts with other agents $S \in (S_1, S_2)$, and his decision is unobservable. The risk- and work-averse agent makes his decision by maximizing the utility function, which is separable in monetary compensation and effort: $U(S_i, e_k) = EU(S_i) - e_k,\ ,i = 1,2,\ k = H, L$. At the beginning of the period agents sign a contract with the agent based on a financial report *S*, the agent chooses his effort and makes unobservable decisions. At the end of period, the economic outcome is realized and observed only by the agent and he prepares a financial report. The agent designs the contract:

$$\max_S E[P]\ ,\ E[U(S)|e_H] - e_H \geq U(R)\ ,$$

which gives a reservation utility constraint. Because the contract must guarantee the agent his expected compensation had he worked elsewhere and earned *R*:

$$E[U(S)|e_H] - e_H \geq EU(S|e_L) - e_L\ ,$$

since the agent prefers less effort , the contract must provide him with incentives to choose $e_H$.

The effort aversion of the agent has resulted from $S_1 - S_2 > 0$ and since the likelihood of a high value of $x_1$ increases with his effort, which the agent is not motivated to exert, the agent is motivated to misrepresent, stating a low income $x_2$ . The agent would misrepresent his income as $x_2$ with probability $1 - \pi$ .

We can appreciate the effect on the cost of the agent's incentives $E(S) - R$ . The agent's risk aversion has opposing effects on cost of incentives, and the more-risk averse the agent, the greater the risk-premium required given his utility constraint. Which effect dominates depends on the substitution between effort-aversion and utility over outcome. As $\pi$ increases, the ability of the agent to work less declines and when the technology is more sensitive to the agent's effort, it becomes easier to motivate the agent to exert $e_H$ .

The minimal penalty level satisfies the condition that the agent is indifferent between lying and telling the truth:

$$U(S_2) = \pi[U(S_2 - C)] + (1 - \pi)U(S_1) .$$

If the agent's information is imperfect, the agent randomizes between selling and misrepresenting, $\forall i\ ,\ 0 < \gamma_i < 1\ ,\ P_1 = P_2$ .

This price behavior gives the agent incentives to affect the price by concealing unfavorable information. Let *A* and *B* denote agents with different measures of risk aversion with utilities given by $U^A(.)\ and\ U^B(.)$, and assume that *A* is more risk than *B*.

Denote $\phi \equiv \alpha_H + (1+\alpha_H)(1-\pi)$ *and* $\delta \equiv (e_H - e_L) / \pi(\alpha_H - \alpha_L)$,

$\phi U(S_1) + (1-\phi)U(S_2) = e_H + U(R)$,

$U(S_1) = \delta + U(S_2)$. As the difference between $S_1$ *and* $S_2$ decreases, the contract becomes less risky and incentive constraints are:

$\alpha_1 U(S_1) + (1-\alpha_1)U(S_2) - e_H \geq \alpha_2 U(S_1) + (1-\alpha_2)U(S_2) - e_L$

and this yields:

$(\alpha_1 - \alpha_2)[U(S_1) - U(S_2)] \geq e_H - e_L$.

By Bayes' rule, the probability, conditional on $x_1^F$, is

$$\Pr(i = x_1 | x_1^F) = \frac{\Pr(x_1^F, \ddot{x} = x_1)}{\Pr(x_1^F)},$$

by the preliminary step:

$\Pr(x_1^F, \ddot{x} = x_1) = \alpha[p\gamma_1 + \pi p(1-\gamma_1) + (1-p)(1-\gamma_2) + \pi(1-p)\gamma_2]$,

$\Pr(x_1^F, \ddot{x} = x_2) = (1-\alpha)(1-\pi)[p(1-\gamma_2) + (1-p)\gamma_1]$,

the market price is :

$P_1 = \delta(x_1)x_1 + (1-\delta_1)x_2 = \delta(x_1)(x_1 - x_2) + x_2$,

$P_2 = \delta(x_2)x_1 + (1-\delta_2)x_2 = \delta(x_2)(x_1 - x_2) + x_2$,

with the expected price being:

$\alpha p P_1 + (1-\alpha)\pi(1-p)P_2 + (1-\alpha)(1-\pi)(1-p)P_1$,

which the agent can misrepresent :

$\alpha\pi p P_1 + \alpha(1-\pi)pP_2 + (1-\alpha)(1-p)P_2$.

## Preferences of Consumption

We wish to explore the existence of an equilibrium for agents with a measure space of agents $(T, \Pi, \mu)$, a separable Banach commodity space $\mathrm{H}$ whose positive cone admits an interior point, and an independent preference correspondence $P$: $T \times L_1(\mu, X) \to \mathrm{H}$, where $L_1(\mu, X)$ is the set of selections of the consumption correspondence $X : T \to \mathrm{H}$. The preference correspondence $P$ having the representation $P(t,x) = \{\xi \in X(t) : u(t,\xi,x) > u(t, p(x)(t), x)\}$,

where $u$ is an utility function, and $p$ is a choice function selecting a representative from each equivalence class $x \in L_1(\mu, \mathrm{H})$. Noguchi (2000) assumes the condition of every consumption set *X(t)* being a group of norm- compact valued sub-correspondences $\phi$, and we have a sub- economy $E^{\phi}$ of economy $E$, where we can utilize methods for obtaining an equilibrium $(x_{\phi}, p_{\phi}) \in L_1(\mu, \phi) \times \Delta$ with a continuum of agents and a separable Banach commodity space.

For $(t, p) \in T \times \Delta$, define:

$$A_1(t,p) = \left\{ \xi \in \phi(t) : (p, \xi) \leq \int_S \theta(t,s) \Pi(x,p) + (p, e(t)) \right\},$$

and for $(x, y, p) \in L_1(\mu, \phi|_T) \times L_1(\pi, \phi|_S) \times \Delta$ :

$$P_2(y,p) = \left\{ z \in L_1(\pi, \phi|_S) : \int_S (p, z(s)) > \int_S (p, y(s)) \right\},$$

$$P_3(x,y,p) = \left\{ q \in \Delta : \left\langle q - p, \int_T x(t) - \int_S y(s) - \int_T e(t) \right\rangle > 0 \right\}.$$

Agents exchange goods and money, which are differentiated and agents have potential use for all of them.

## Assets and Equilibrium

Agents have utilities in assets, have sufficient money endowments to afford any group of objects priced below their reservation values, and have reservation values which are satisfy the cardinality condition. Beviá, Quinzii and Silva (1999) suppose this cardinality condition requires that for agent the marginal utility of an object depends only on the number of objects to which it is added, not on their characteristics.

Consider an economy *e* with a finite set *I* of agents, and agents preferences are: the utility that agent $i \in I$ derives from consuming a set of objects *A* can be characterized by a reservation value *V(i, A)*, which represents the quantity of money that agent *i* is ready to sacrifice in order to consume the objects in *A*. The utility of agent *i* holding *m*, units of money and the set *A* objects is thus

$$u_i(A, m_i) = V(i, A) + m_i.$$

For all $i \in I$ the reservation value function *V(i,.)*, defined on the power set $P(\Omega)$, is assumed to be weakly increasing $A \subset B$ . Agents' endowments $(\overline{A}_i, \overline{m}_i)_{i \in I}$ with $\overline{m} \geq 0$ and $\bigcup_{i \in I} \overline{A}_i = \Omega$ are assumed, and when the price of a set $A$ of objects is less than the reservation value $V(i, A)$, agent $i$ can afford to buy the objects in $A$.

For an economy $e \in \varepsilon$ an assignment $\sigma$ of objects to agents is thus a partition of the objects among the agents. Let $\sum(I,\Omega)$ denote all possible assignments.

Such an assignment $\sigma$, satisfying

$$\sum_{i \in I} V(i,\sigma(i)) \geq \sum_{i \in I} V(i,\tau(i)), \quad \forall \tau \in \sum(I,\Omega)$$

is called an efficient assignment.

If agent $i$ buys the set $A$ of objects he will pay $p(A) = \sum_{\alpha \in A} p(\alpha)$, and the demand of objects $D(i,p)$ of agent $i$ on a market at prices $p$ is:

$$D(i,p) = \{A \in P(\Omega) | V(i,A) - p(A) \geq V(i,B) - p(B), \quad \forall B \in P(\Omega)\}$$

.

The demand of agent $i$ for an asset is then $m_i = \overline{m}_i + p(\overline{A}_i) - p(A)$, which ensures equilibrium on the market for invisible goods. This implies that the market for assets is also in equilibrium and an economy $e \in \varepsilon$ has a competitive equilibrium if and only if every efficient assignment $\sigma$ of $e$ can be supported by a price $p$. Under reservation value functions if it is satisfied for all $A$, $B$ in $P(\Omega)$:

$$V(i, A \cup B) \leq V(i,A) + V(i,B) - V(i, A \cap B)$$

.

Heuristically, the agent $i$ is satisfied when the price of an object $\beta$ increases while the prices of all other objects stay the same: then the objects other than $\beta$ which were demanded by agent $i$ are still demanded by this agent. The utility function $V(i,.)$ satisfies the cardinality condition if the marginal contribution of an object to agent $i$'s utility depends only on the number of objects to which it is added:

$$V(i,A) - V(i, A \setminus \alpha) = V(i,B) - V(i, B \setminus \alpha)$$

,

and the cardinality condition makes sense only for invisible objects which have the same function, since otherwise the marginal utility of an object for an agent depends on the composition and if there are objects of different natures, then each group of objects can be attributed separately.

Let $\sigma$ be an efficient assignment of the objects $\Omega$ to the $I$ agents, and these changes induce changes in social welfare analogous to the changes in social welfare accompanying a marginal change in the supply of a good:

$$U(\Omega) = \sum_{i \in I} V(i,\sigma(i))$$

,

for any efficient assignment $\sigma \ of \ \Omega$ and $p^M$ is the change in social welfare when the object $\alpha$ is taken out of the available objects:

$$p^M(\alpha) = U(\Omega) - U(\Omega \setminus \alpha), \quad \alpha \in \Omega .$$

Let $\alpha \ and \ \beta$ be two objects in $\Omega$ and let *C* and *D* be two subsets of $\Omega$ such that

$$V(i, C \cup \alpha) - V(i, C \cup \beta) = V(i, D \cup \alpha) - V(i, D \cup \beta) .$$

The social welfare is associated with

$$U(\Omega \cup \overline{\alpha}) = \max\left\{\sum_{i \in I} V(i, p(i)) \middle| p \in \sum{}'(I, \Omega, \cup \overline{\alpha})\right\}$$, if

$$p_m(\alpha) = U(\Omega \cup \overline{\alpha}) - U(\Omega), \quad \alpha \in \Omega .$$

We assume the property that the agent $\overline{i}$ who is assigned $\alpha$ under $\sigma$ also receives $\alpha$ under *p*, and that $p^M \ and \ p_m$ are equilibrium prices supporting the efficient allocation of the objects $\Omega$.

Let $\sigma$ be an efficient assignment of $\Omega$ among *I* for all $i \in I$, and if $\alpha \in \sigma(i)$ then

$$p^M(\alpha) \le V(i, \sigma(i)) - V(i, \sigma(i) \setminus \alpha) ,$$

if $\beta \notin \sigma(i), \ then \ p^M(\beta) \ge V(i, \sigma(i) \cup \beta) - V(i, \sigma(i))$.

If $\sigma$ is an efficient assignment of $\Omega$ among *I*, then $p^M$ supports $\sigma$, for all $i \in I$, if

$$\alpha \in \sigma(i), \ then \ p_m(\alpha) \le V(i, \sigma(i)) - V(i, \sigma(i) \setminus \alpha)$$, if

$\beta \notin \sigma(i),$ then

$$p_m(\beta) \ge V(i, \sigma(i) \cup \beta) - V(i, \sigma(i)) .$$

The set of prices supporting an efficient assignment $\sigma \ of \ \Omega$ is the set of solutions to the inequalities

$$V(i, \sigma(i)) - p(\sigma(i)) \ge V(i, A) - p(A), \quad \forall A \in \Omega .$$

For reservation values, the cardinality condition implies that an agent's demands satisfy the gross substitution property. Then if *A* and *B* are subsets of *D(i,p)*, and if $|B| < |A|$, then for every $\alpha$ such that $\alpha \in A, \ \alpha \notin B$, then

$B \cup \alpha$ is in $D(i, p)$, and if *p* and *p'* are prices such that $p' \geq p$, if *A* is a subset of *D(i,p)* of maximum cardinality, then for all $B \in D(i, p'), |B| \leq |A|$.

Suppose, that the reservation value *V(i,.)* of agent *i* satisfies the cardinality condition and then agent *i*'s demand satisfies the gross substitute property. It holds for the reservation value function $V(i, A) = \max\{V(i, \alpha), \alpha \in A\}$, where agents have use for only one object, where the marginal utility of every object is positive.

The liquidity premium of liquid assets reflects the agent's confidence in estimates of returns from other, less liquid assets, and it is determined by the agent's uncertainty aversion and uncertainty perception. Beyond this confidence, the agent may have different specific degrees of confidence in estimates of returns from different assets, because he may perceive some assets as involving more uncertainty than others. Dequech (2000) supposes the liquidity premium seems to be the factor through which we should accommodate the fact the expected depreciation of asset does not make all the other assets equally more interesting to the agent.

The proper way of representing this diversity of agent doubts regarding the return from many different assets is by discounting the expected flow of payments of each asset at a rate of discount specific to that asset:

$$(1 + \delta_i) = (Q_i - C_i + L_i + A_i)(1 + \alpha_{si})P_i \quad ,$$

where $\delta_i$ is the own-rate of interest *i* ; *Q, C, L* and *A* represent quasi-rents, carrying costs, liquidity premiums and appreciation in nominal values , $\alpha_{si}$ is a rate of discount reflecting the degree of uncertainty and *P* is the asset market price, for a assets, i = 1,2,….,n.

In the case of money A = 1,0 is an indicator of degree of, perceived uncertainty, $\alpha_g$ , an indicator of uncertainty aversion, $\beta$ ; an indicator of the asset's degree of liquidity, $\gamma$ ; and the appreciation of asset *i* in terms of the prices of other assets, *A*:

$$L_i = L_i(\alpha_g, \beta, \gamma_i, A_i)$$

$$\partial L_i / \partial \alpha_g > 0; \quad \partial L_i / \partial \beta > 0; \quad \partial L_i / \partial \gamma_i > 0; \quad \partial L_i / \partial A_i < 0 \quad ,$$

*A* is relevant for speculation. The liquidity premium attributed to an asset by a agent reflects in part the possibility of taking advantage of the expected depreciation of other assets, a possibility that illiquid assets such as capital goods do not allow it, but this affects that asset's rate of return directly through *A*, and not via *L*, and the confidence the agent has in these expected gains. If the agent is aware of uncertainty, it is rational to have some positive degree of liquidity preference beyond that due to the transactions motive. The choice of a specific degree of liquidity involves rationality, typically represented by the purchase of capital goods. The more the agent wants to play it safe, the larger the proportion of assets he will have in his portfolio. The knowledge on which to base this decision is incomplete and not fully reliable as a guide to action.

The results show that the likelihood of selection of the riskier choice increases as the pair becomes more similar, and these choice patterns are consistent with well-known independence violations of expected utility. A significant proportion of agents exhibit intransitive choice patterns predicted under similarity effects, but not allowed under generalized expected utility models for risky choice. Buschena and Zilberman (1999) presume the existence of risky alternatives, and agents faces a choice between dissimilar probability distributions $p^0$ *and* $q^0$ over a common outcome set, $x^0 \subseteq \Re_n^+$, and consider a number of risky choices defined by their probabilities $p$ and $q$, where:

$$p^i = \lambda^j p^0 + \left(1 - \lambda^j\right) r^j \text{, and } q^j = \lambda^j q^0 + \left(1 - \lambda^j\right) r^j,$$

where $r^j$ is another probability distribution over $x^0$, and $0 < \lambda^j < 1$. Assuming the independence of expected utility, if $p$ is preferable to $q$, then $p$ should be preferable to $q$ for all $\lambda^j$ *and* $r^j$. Perceptions of similarity are consistent with objective measures of similarity: the more similar the pair, the more likely the riskier choice, and the selection likelihood for the riskier alternative within a risky pair increases when the agent also selects the riskier alternative in a relatively more dissimilar pair.

Similarity between the alternatives $s_{ij}$ will be tested for agent $i$ and for each risky choice pair $\left(p^j, q^j\right)$ through:

$$s_{ij} = s\left(D_j, x_j^0, \alpha_i, k_{ij}, \varepsilon_{ij}^s\right),$$

where $D$ is distance the outcome vector for the pair $x$, instruments specific to the agents $\alpha_i$, and of the elicitation process $k$, and a random term drawn from an asymptotically normal distribution.

Choice models are used in an effort to relate choice to similarity and the model also includes choice over a base dissimilar pair defined by probability vectors $(p,q)$ over outcome vector as the pairs to test the strength of expected utility model for risky choice.

We consider that each agent must prefer his net trade to be any nonlinear combination of the individual net trades in the economy from the arbitrage-free condition. We denote by $\Re_l^+$ the $l$-dimensional Euclidean space and for every $i \in N$ and $x_i \in X_i$, we define the preferred and the weakly interactive choice sets:

$$P_i(x_i) = \left\{z_i \in X_i \middle| z_i \succ_i x_i\right\}$$

$$W_i(x_i) = \left\{z_i \in X_i \middle| z_i \succ_i x_i\right\}.$$

An exchange economy is a system where $N$ is a finite set that contains one or more agents, and for each agent $i \in N,\ X_i \subseteq \Re^i$ is $i$'s consumption set.

Consider an exchange economy where agents are, trading with each other and constructing an interactive choice set, a set achievable through a sequence of possible trades with other agents, and each agent chooses optimally a budget set with a different behavior.

# 18 Conclusion

Many important economic decisions involve fundamental uncertainty, in the sense that the agents do not know the list of all possible relevant events. However even such uncertainty does not imply complete ignorance, because of the existence of stabilizing social practices. And complete knowledge does not exist for agents at the time of making the most relevant decisions.

We recall the relationship between more informative signals and the distribution of posteriors that they induce. Within each given tree, the agent chooses an action at each decision mode, following each signal realization. With resolution randomization, the agent can be thought of as selecting a non-random action at the second stage, with that choice depending jointly on the realization of the randomizing device and of the signal. If the agent has access randomization devices, it seems natural to allow random behavior.

We define sets of nodes that represent the information of agent and need not correspond to agent's actual information sets. A Pareto optimal allocation is a feasible allocation, such that there does not exist any other feasible allocation weakly preferred by all agents and strictly preferred by at least one agent. They are the only Pareto optimal allocations that can possibly be obtained as competitive equilibrium allocations; they are found by maximizing the sun of agents' utilities subject to the feasibility constraints. The existence of an equilibrium is guaranteed if the agents have utility for at most one object, and where agents can consume several objects, additional restrictions must be placed on the reservation value functions. The objects typically have different values for different agents, if the agents are more sensitive to the effect of each object than to the general effect that a group of objects produce together. The characterization of the cardinality condition is as follows: a given reservation value function satisfies the cardinality condition if and only if there exists an increasing function.

Confidence is the factor behind the possibility of learning and precaution and we have shown how speculation, while involving confidence, depends primarily on expectations of asset deprecation or appreciation. The choice is considered to be rational to the extent that it is based on knowledge and consistent with the end of pecuniary gain. Liquidity preference is partly based of knowledge and therefore has partly rational grounds.

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