621 LECTURE NOTES IN ECONOMICS AND MATHEMATICAL SYSTEMS

Sebastian Rausch

Macroeconomic Consequences of Demographic Change

Modeling Issues and Applications

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To My Parents

Preface

This book represents a culmination of my Ph.D. research conducted at the Ruhr Graduate School in Economics and at the University of Duisburg-Essen from October 2005 to April 2008. Many people have generously contributed their time, experience and resources towards the success of this dissertation.

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Cambridge, Massachusetts, USA
February 2009 *Sebastian Rausch*

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Chapter 1
Introduction

The world is in the midst of a major demographic transition. Not only is the population growth slowing, but the age structure of the population is changing, with the share of the young falling and that of the elderly rising. The sources of population aging lie in two demographic phenomena: rising life expectancy and declining fertility. Population aging is most advanced in the industrialized countries but, with a lag, demographic trends in many developing economies will follow (United Nations, 2006). Because many developing countries are experiencing faster fertility transitions, in the future they will experience even faster population aging than the currently developed countries (Bloom and Williamson (1998) and Weil (2006)).

In the coming decades, this demographic transition will prove to be a key factor in the development of societies. From a macroeconomic perspective, the consequences of population aging are substantial and manifold. For example, a lower growth in the working age population causes aggregate labor supply to decline and reduces the amount of investment required to supply new workers with capital. As a result, capital becomes abundant and labor becomes scarce with ensuing changes in factor prices and interest rates (Krueger and Ludwig, 2007). Börsch-Supan (2003) argues it is unlikely that the decline in the labor force will be offset by higher capital intensity to prevent adverse effects on domestic production. This will have significant impacts on labor markets (Börsch-Supan, 2003) as well as capital markets (Kotlikoff, Smetters and Walliser, 2001) influencing savings and investment behavior and ultimately the growth rate of the economy (Batini, Callen and McKibbin, 2006). Demographic change also interacts with public policies through various channels. Most obviously, rising old age dependency ratios jeopardize the sustainability of current social security systems (Abel, 2003). Burtless (2006) calculates that the effect of population aging would significantly raise tax rates required to finance increased government spending. With respect to goods markets, populating aging is bound to change the composition of aggregate demand thereby changing relative prices (Lührmann, 2005).

While this list of consequences is not by any means exhaustive, an abundant body of literature has scrutinized many of the possible effects of demographic change; see, e.g., Börsch-Supan (2004) for an overview. In order to quantify these effects

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computable overlapping generations (OLG) models have emerged as the dominant analytical tool.[1] This book is related to the literature that uses OLG simulation models to study the economic consequences of demographic change. Its contribution is twofold. Firstly, it puts forward a new computational method that can be used to effectively solve large-scale OLG models. Unlike existing solution methods, the presented algorithm allows to solve high dimensional models that can incorporate a large number of heterogeneous households, many sectors and many regions. This significantly increases the range of economic questions that can be addressed with such models, hence providing a valuable addition to the toolbox of applied general equilibrium modelers. Secondly, the new computational method is applied to investigate hitherto unexplored aspects of the macroeconomic consequences of demographic change. The first of two applications investigates the interrelation between globally unsynchronized aging patterns and international trade and, moreover, looks at the distribution of gains from trade under such demographic circumstances. The second application takes a more detailed view at the economic adjustments within a country that are brought about by the ongoing and projected aging processes. To this end, a quantitative model is calibrated for the German economy to evaluate the effects on the sectoral composition of output. One integrated feature of the models presented in this book is—by virtue of the new computational method—to also look in-depth at the distributional implications of demographic change for income and welfare, both on intra- and intergenerational household levels.

Starting with the seminal work by Auerbach, Kotlikoff and Skinner (1983) and Auerbach and Kotlikoff (1987), computable OLG models have become an important tool for policy analysis, fruitfully applied in fields such as macroeconomics and public finance.[2] Apart from generic advantages of rigorously micro-founded general equilibrium models, of which OLG models are a special case, these models are appealing for at least three reasons. Firstly, they depart from the uncomfortable assumption of infinitely lived agents in the standard neoclassical growth model (Ramsey (1928), Cass (1965), and Koopmans (1965)) by assuming that economic agents have a finite lifetime. This implies a turnover of the population as at any point in time a new generation is born and an old one exits the model. Most importantly, this feature enables the study of intergenerational distributive effects of economic policies, making the OLG model a natural framework within which to investigate demographic issues. Secondly, by capturing the essence of the life-cycle theory of consumption and saving introduced by Modigliani and Brumberg (1954) and Ando and Modigliani (1963), OLG models allow to investigate the aggregate implications of life-cycle saving by individuals.[3] Thirdly, within the class of dynamic general

[1] For surveys of econometric studies that deal with demographic change see, e.g., Bryant and McKibbin (1998, 2003).

[2] See Kotlikoff (2000) for an overview. The basic model in which households live for two periods goes back to Allais (1947), Samuelson (1958), and Diamond (1965).

[3] OLG models provide an example of economies in which the competitive equilibrium may not be Pareto-optimal: life-cycle savers may overaccumulate capital, leading to equilibria in which everyone can be made better off by consuming part of the capital stock (see, e.g., Phelps (1961) and Diamond (1965)). This possible deficiency contrasts sharply with the intertemporal efficiency

equilibrium models, the OLG framework is a natural candidate for introducing heterogeneous agents into the economic analysis. By construction, OLG models feature heterogeneity of age among agents, and adding intra-cohort heterogeneity to these models involves a—more or less—trivial indexing of agents within each generation. Advancing numerical OLG models in this direction is clearly desirable because it strengthens the micro-foundation of this type of analysis, and allows to analyze in more detail the distributional implications of economic policy. Not surprisingly, this is an active area of research. In a considerable number of papers, e.g., Conesa and Krueger (1999), Kotlikoff, Smetters and Walliser (1999), Krusell and Smith (1998) Huggett and Ventura (1999), Jensen and Rutherford (2002), Krueger and Ludwig (2007) and Storesletten, Telmer and Yaron (2007).

1.1 Methodology

Large-scale OLG models have been fruitfully used to analyze a wealth of economic issues, as for example tax reform, investment incentives, government spending and debt, social security reform, monetary policy, endogenous growth, human capital accumulation, and demographic change. However, the scope in many applications and the required degree of complexity are often limited by the computational power of available numerical solution techniques. Quantitative simulation models with overlapping generations naturally involve a large number of variables and equations that describe the equilibrium behavior of economic agents and the aggregate economy. In particular, models that exhibit a rich household side including a variety of household-specific effects, a large number of heterogeneous households, and realistic lifetimes of agents, typically require "customized" solution methods that may both be costly to implement and difficult to validate. Under such circumstances, the decision to add specific features to the model may not be driven by economic reasoning alone but also by the need to take into account the computational feasibility of the numerical problem.

Chapter 2 of this research monograph, written with Thomas F. Rutherford, aims at remedying this problem by developing a new decomposition algorithm that can be used to efficiently solve large-scale OLG models. The algorithm can be implemented using "off the shelf numerical tools" which are routinely available through *GAMS*.[4] The presented approach is primarily appropriate for computing equilibria in models in which the number of agents is so large that simultaneous solution methods operating directly on the equilibrium system of equations are infeasible due to

of the Ramsey model. The analyses in this research monograph, however, focus on OLG economies that are dynamically efficient.

[4] *GAMS* (General Algebraic Modelling System) is a computer language originally developed to assist economists at the World Bank in the quantitative analysis of economic policy questions (Meeraus (1983) and Brooke, Kendrick and Meeraus (1988)). Currently, two large-scale solvers are available through *GAMS/MCP*: *MILES* employs a modified Newton algorithm and *PATH* is based on a path-following procedure. For details and more references see Rutherford (1995b).

the high dimensionality related to income and household-specific effects. The Sequential Recalibration (SR) algorithm is based on the solution of a sequence of nonlinear complementarity problems. A characteristic of many economic models is that they can be cast as a complementary problem. Rutherford (1995b) and Mathiesen (1985) have shown that a complementary-based approach is convenient, robust, and efficient.[5]

The main idea of the proposed algorithm is to solve a market economy with many households through the computation of equilibria for a sequence of representative agent economies. The dimensionality of the numerical problem is significantly reduced by an appropriate decomposition of the system of equations that defines the dynamic general equilibrium of the OLG economy. To achieve this, the OLG economy is transformed into a lower-dimensional Ramsey growth problem by replacing the entire system of overlapping generations with a single infinitely lived representative agent. Apart from this modification, both models are identical. The first step in each iteration is to compute a vector of general equilibrium prices given the economic policy (or shock) under consideration. In a second step, the candidate prices are used to evaluate the demand functions of the actual overlapping generations households. This amounts to solving a partial equilibrium relaxation of the original OLG economy that suppresses all general equilibrium interactions. The final step in each iteration uses households' quantity choices to recalibrate, i.e. to choose a set of preferences for the artificial Ramsey agent such that, given candidate prices, aggregated quantity choices by OLG households are consistent with the equilibrium behavior of the representative agent. The SR algorithm finds the equilibrium allocation of the underlying OLG economy if the sequence of prices and quantities obtained from the Ramsey growth model and the partial equilibrium relaxation converge to the true equilibrium prices and quantities.

The decomposition algorithm provides improvements in both efficiency and robustness as compared with simultaneous solution methods. To demonstrate this point, a full-blown Auerbach-Kotlikoff OLG model with production activities, capital accumulation, endogenous labor supply, and a government sector is augmented by a large number of heterogeneous households. Evaluating the algorithm's performance compared with an "integrated" complementarity-based approach as put forward by Rasmussen and Rutherford (2004), shows that the decomposition algorithm is able to solve high-dimensional OLG models that are infeasible for simultaneous solution methods. Overall, the quality of approximation is found to be excellent and decreases marginally in the degree of intra-cohort heterogeneity.

To characterize limitations of the SR algorithm, local convergence theory for a simple exchange economy due to Scarf (1960) is developed, and global conditions for convergence failure are determined by means of numerical analyses. The conditions for local stability of the implied adjustment process reduce to those of a Walrasian price tâtonnement process. Thus, the proposed algorithm belongs to a large

[5] The complementarity format embodies weak inequalities and complementary slackness, relevant features for models that contain bounds on specific variables, e.g. activity levels which cannot a priori be assumed to operate at positive intensity. Such features are not easily handled with alternative solution methods.

class of algorithms used in computational economics that are robust, efficient, and yet fail to provide global convergence. In general, however, the appropriateness of the proposed algorithm depends on the characteristics of the underlying economic model. A central result is that the algorithm may be inappropriate for applications in which there are significant income effects. This deficiency is a consequence of the simplifying nature of the adjustment process which omits both income constraints and global characteristics of the individual utility functions.

Notwithstanding these limitations, the decomposition algorithm can be beneficial for a wide range of economic applications. Due to its decomposition strategy, which operates on the household side of the economy, it is particularly tailored to models with many heterogenous households. As such, the SR algorithm can be instrumental in developing (and solving) large-scale OLG models to analyze in considerable detail both the inter- and intragenerational distributional implications of economic policy. In general, however, the decomposition method can also be advantageous for modeling tasks that need to economize on the dimensionality of the numerical problem. For instance, multi-sectoral and multi-regional OLG models typically involve a large number of equations and variables that often render simultaneous solution methods infeasible. By transforming the OLG model into a lower-dimensional Ramsey growth problem, and applying the decomposition approach, the SR algorithm can create sufficient degrees of freedom in the model architecture to accommodate such complex research designs.

The proposed algorithm also contributes to the growing literature on the integration of "micro-macro" models that combine the strengths of both computable general equilibrium and microsimulation models. The SR algorithm does not only overcome a principal weakness that is characteristic of many approaches in this strand of literature (see, e.g., Bourguignon, Robilliard and Robinson (2005) and Bourguignon and Spadaro (2006)), namely to neglect feedback effects from the micro to the macro level. Applied to the OLG context, it is also the first computational device to integrate both approaches in a dynamic framework.

Chapter 3 and Chapter 4 are based on the methodology developed in Chapter 2. The OLG models developed in both chapters exploit the advantages of the decomposition algorithm to study specific aspects of the macroeconomic implications of population aging that have received little attention in the literature. Hence, although each chapter focuses on economic analysis rather than on computational issues, they both demonstrate the flexibility, scope, and power of the SR algorithm.

1.2 Macroeconomic Consequences of Demographic Change: Focus and New Aspects

In many industrialized countries populations are aging, and the demographic transition will prove to be one of the key factors in shaping the development of societies in the coming decades. This process is driven by falling mortality rates and a decline in birth rates, which reduces population growth rates and increases the share of older people in the economy. While an extensive economic literature has scrutinized many aspects of the possible economic effects of population aging, see Börsch-Supan (2004) for an overview, this book relates to the strand of literature that uses the models and techniques pioneered by Auerbach and Kotlikoff (1987).

From a macroeconomic perspective, the most striking consequence is the change in the relative abundance of factors of production. As the demographic transition reduces population growth and decreases the share of young people in the economy, the aggregate labor force is projected to decline. At the same time, a larger share of older people—who tend to hold a larger amount of financial wealth—increases the amount of capital in the economy. Both effects imply that capital in an aging society becomes abundant relative to labor. Another way of thinking about this is to imagine that in the light of a shrinking labor force the accumulated capital stock does not depreciate fast enough to maintain a constant equipment of physical capital per worker. In other words, demographic change is bound to increase the capital-labor ratio—often referred to as the process of "capital deepening" (Kotlikoff et al., 2001). Everything else equal, factor prices will change to reflect the varying scarcities of capital and labor: the capital rental rate is projected to decline, while wage rates are bound to increase.

While this adjustment mechanism to demographic change is extensively documented in the literature, and is at the heart of most analyses, the majority of studies pays special attention to the impact of aging on the viability of social security systems in closed economies. Important examples in a closed economy are Huang, Imrohoroglu and Sargent (1997), De Nardi, Imrohoglu and Sargent (1999), Abel (2003), and Fehr, Halder and Jokisch (2004a), and in an open economy setting Attanasio, Kitao and Violante (2006) and Börsch-Supan, Ludwig and Winter (2006). Although issues related to social security are probably among the most obvious consequences of demographic change, these are by no means the only consequences. Population aging affects virtually all markets, i.e. goods markets, labor markets and capital markets.

This book focuses on an analysis of the economic consequences of demographic change per se and not just the consequences of alternative social security reform scenarios. More specifically, it tries to provide both qualitative and quantitative answers to questions such as the following: What are the aggregate economic consequences of population aging? How is individual economic behavior affected? How do returns to capital and labor evolve during the demographic transition? How is the sectoral composition of output affected? What are the consequences for the intra- and intergenerational distribution of income, wealth, and welfare? Does demographic change make households worse-off? As ongoing and projected demographic processes are not synchronized across countries or world regions, how is international trade affected? Is trade liberalization under such demographic circumstances always beneficial and how are gains from trade are distributed across generations over time and between different types of households?

1.2.1 Global Demographic Change and International Trade

A salient feature of global demographic change is the fact that the aging of populations will occur at differing paces and with a differing degree of intensity in the industrialized countries of the world. Significant aging is already under way in some economies, for example in Germany, Italy, Japan, while major demographic changes in the U.S. and China will begin in the second decade of the 21st century, and with a still longer lag, demographic trends in the developing economies will follow.

While the existing literature consistently identifies the presence of globally unsynchronized aging patterns as an important driving force of cross-border flows of capital and labor (see, e.g., Attanasio and Violante (2005), Börsch-Supan et al. (2006), INGENUE (2001), and Fehr, Jokisch and Kotlikoff (2004b)), it surprisingly overlooks its implications for international trade in goods. To fill this gap, Chapter 3 develops an augmented version of the canonical Heckscher-Ohlin model to analyze the economic consequences of trade liberalization in a world that is characterized by globally unsynchronized aging patterns. Adopting a stylized dynamic two-country two-sector two-factor model with overlapping generations it shows that unsynchronized demographic patterns emerge as a potential determinant of international trade flows. The mechanism behind this is simple: since relative factor endowments are endogenously determined by life cycle saving and labor supply behavior of overlapping generations, different extents and timing of demographic processes across countries imply international differences in relative factor endowments, and hence induces Heckscher-Ohlin trade patterns.

If demographic differences emerge as a potential determinant of international trade flows, a natural question arises whether a country or a region can benefit from diffusing part of its demographic shock by means of liberalizing its trade policy. Since trade tends to reduce the dispersion in international factor prices, opening up an economy for trade might mitigate the pressure on factor prices exerted by population aging. However, as real wages and interest rates move in opposite directions, the overall effect on income and welfare of households is ambiguous. Chapter 3 therefore provides a quantitative assessment of the welfare effects from trade liberalization in the presence of globally unsynchronized aging patterns. Given the absence of any distortion, and given the perfect neoclassical setup of the model that abstracts from any form of market imperfection, the general result is surprising, standing in contrast to what would be expected from standard neoclassical trade theory: that trade liberalization in the presence of globally unsynchronized aging patterns does not necessarily lead to welfare gains.

In the fast aging country, trade liberalization is only beneficial for older generations born before and at the beginning of the demographic transition, whereas future generations stand to incur substantial utility losses. The reason for this is the evolution of autarky-trade factor price differentials during the demographic transition. As trade is bound to increase the real return to capital and to decrease the real wage rate (relative to autarky), older asset-rich households gain while future generations suffer from losses in labor income. The model also analyzes the distribution of welfare gains and losses with respect to an intragenerational dimension. In general, low-skilled and asset-poor households in the fast aging region tend to be relatively worse off.

Employing Monte-Carlo simulation methods, the results are found to be robust for a broad range of empirically plausible parameter configurations. It is estimated that there is a very high probability for households in the fast aging region not to gain from liberalizing trade. Moreover, trading off gains from trade for older generations against welfare losses for younger households, it is found that an aggregate welfare improvement would only materialize in the presence of unrealistically high social discount rates.

1.2.2 Quantifying the Sectoral and Distributional Effects of Population Aging in Germany

Chapter 4 takes a more detailed view of the economic adjustment mechanisms to demographic change within a country. To this end, it develops a large-scale multi-sectoral OLG model with intra-cohort heterogeneity to quantify the impact of population aging on the sectoral composition of output and the distribution of income, wealth, and welfare. The model is calibrated for the German economy and features 17 sectors that represent sectoral aggregates of all 71 industries of the German Input-Output table. To allow for a meaningful analysis of the distributional implications of demographic change, the household side of the model distinguishes between eight household types that correspond to eight income classes of the German Income and Expenditure Survey ("Einkommens- und Verbrauchsstichprobe"). Demographic projections from the Federal Statistical Office of Germany are taken as the major exogenous driving force of the model.

The model focuses on two issues that have received little attention in the literature. The first objective is to quantify the impact of demographic change on the sectoral composition of output. In the model, sectoral adjustment rests on two key mechanisms. First, as technological differences across sectors generally imply that firms operate at different factor intensities, the increase in the aggregate capital-labor ratio due to population aging is absorbed quite differently across sectors. Sectors that most efficiently employ abundant capital and attract scarce labor will grow faster than the rest of the economy, while other sectors will contract due to the demographic transition. Second, as the composition of household consumption varies with age (see Federal Statistical Office of Germany (2003) for empirical evidence for Germany, and Börsch-Supan (2003) for a discussion) changes in the population structure induce a demand-side driven effect that may give rise to sectoral adjustments, too. As the analysis departs from the standard macroeconomic single-commodity world commonly employed in the literature on demographic change, it is able to incorporate sectoral shifts and relative price changes as important adjustment mechanisms to demographic change.

The second objective of the model is to quantify the distributional and welfare consequences of the demographic transition in Germany. As labor is expected to be scarce relative to capital, real wages are bound to increase while real returns to capital decrease. A priori, it is not clear how this opposite movement of factor prices is going to affect households' labor supply and life cycle savings behavior, and welfare. The quantitative model is used to estimate the welfare effects of the demographic transition along inter- and intragenerational dimensions, and to assess how the distribution of labor income and wealth evolves over time.

Over the main projection period from 2003-2050, the demographic transition in Germany will bring about a 7% decrease in output per capita. This is largely driven by a significant decrease in aggregate labor supply, about 17% over the same horizon (as the model features a labor-leisure trade-off this number already incorporates general equilibrium reactions by households), and a substantial decline in investment rates (by 26% in 2050). Population aging is shown to imply a "capital deepening" process that is reflected by an increase in the capital-labor ratio of around 2%. Real factor prices reflect the varying scarcities of factors of production, and the real capital rental rate (real wage rate) falls (increases) by 0.5% (0.56%) in 2010, and by about 1.3% (1.29%) in 2050.

The demographic transition is found to induce substantial changes in the sectoral composition of output. Sectoral change—as measured by a change in the share of sectoral output in total domestic output—ranges between -8.5% and $+6.5\%$ in year 2010 and between -23.5% and $+20\%$ in 2050. Expanding sectors are characterized by higher growth rates of sectoral employment of labor and capital relative to the economy-wide growth rates of labor and the capital stock. Accounting for structural changes in life-cycle consumption that are due to age-specific preferences does not affect the qualitative results of the benchmark model. At the aggregate level, the quantitative effects are of minor significance. This suggests that the nature of the economic transition is predominantly shaped by the negative labor supply shock. At the sectoral level, however, demand-side induced effects stemming from age-dependent consumer spending are found to be quantitatively important.

In order to evaluate the welfare consequences of the demographic transition the following thought experiment is conducted: suppose households of different age and type alive in 2003 live through the economic transition with changing factor prices induced by the demographic change, how would their welfare change relative to a situation without a demographic transition ? The answer is that young households experience substantial welfare gains because they benefit from a future path of increasing wage rates and do not suffer from too large losses of capital income on already accumulated financial wealth. In contrast, older asset-rich households tend to gain less or even lose because of declining interest rates. From an intra-generational perspective, those members of society for whom labor income constitutes a smaller part of (future) resources, i.e. , households with relatively low labor productivity, benefit less from the demographic transition.

In Germany, the demographic transition is projected to increase income inequality. The growing dispersion in household income is driven by a rise in capital income inequality, and overcompensates the projected decrease in labor income inequality. Labor income inequality decreases because a future path of increasing wages implies a tendency towards a flatter labor supply profile over the life cycle. It is also found that intergenerational factors, rather than a redistribution between different types of households within a particular age group account for the observed evolution of income inequality.

Finally, Chapter 5 concludes with a discussion of a number of simplifying assumptions that underly the economic analyses in Chapter 3 and Chapter 4. This also serves to point out possible extensions of the work presented here.

Chapter 2
Computation of Equilibria in OLG Models with Many Heterogeneous Households

2.1 Introduction

Over the past twenty years infinite horizon general equilibrium models with overlapping generations (OLG) have become an important tool for policy analysis, and have been fruitfully applied in fields such as macroeconomics or public finance (see, e.g., Auerbach and Kotlikoff (1987), and Kotlikoff (2000) for an overview). OLG models naturally involve a large number of variables and equations that describe the equilibrium behavior of economic agents. As a consequence, the development of large-scale OLG models is often limited by the computational capacity of available numerical solution methods. In particular, models that exhibit a rich household side including a variety of household-specific effects, a large number of heterogeneous households, and realistic agent lifetimes typically require "customized solution methods" which may be both costly to implement and difficult to validate.

This chapter[1] develops a decomposition algorithm based on "off the shelf numerical tools" for solving general equilibrium models with many households, of which OLG models are a special case. The presented approach is primarily appropriate for computing equilibria for models in which the number of agents is so large that simultaneous solution methods operating directly on the equilibrium system of equations are infeasible due to the high dimensionality related to income and household-specific effects. The "sequential recalibration" (SR) algorithm presented here is based on the solution of a sequence of nonlinear complementarity problems[2] although in special cases the same procedure may be implemented by solving a

[1] This chapter has been jointly written with Thomas F. Rutherford. GAMS code for the presented applications is available at http://www.mpsge.org.

[2] Rutherford (1995b) and Mathiesen (1985) have shown that a complementary-based approach is convenient, robust, and efficient. A characteristic of many economic models is that they can be cast as a complementary problem, i.e. given a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$ The complementarity format embodies weak inequalities and complementary slackness, relevant features for models that contain bounds on specific variables, e.g. activity levels which cannot a priori be assumed to operate at positive intensity. Such features are not easily handled with alternative solution methods.

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sequence of convex nonlinear programming problems. The main idea of the presented decomposition approach is to solve a market economy with many households through the computation of equilibria for a sequence of representative agent economies. Typically, the sequence of prices and quantities converges to the true equilibrium allocation.

The close connection between the allocation of a competitive market economy and the optimal solution to a representative agent's planning problem is well known and widely cited in the economic literature. Negishi (1960) was the first to use an optimization problem to characterize equilibrium allocations in a general equilibrium framework. Negishi's original paper was primarily concerned with optimization as a means of proving existence. Dixon (1975) developed the theory and computational effectiveness of "joint maximization algorithms" for multi-country trade models. Rutherford (1999b) presented the "sequential joint maximization algorithm" (SJM) which provides a simple recursive version of Negishi's method.

There are similarities between the SR and SJM algorithms. Both approaches solve subproblems representing relaxations of the equilibrium conditions. The SJM algorithm ignores consumer budget constraints but retains details of consumer demand systems. The SR algorithm employs a yet looser representation of individual consumer's demand systems by omitting both income constraints and global properties of the individual utility functions. The omission of global characteristics of preferences simplifies the model but can hinder convergence. The appropriateness of the proposed solution method therefore depends on the characteristics of the underlying model.

The decomposition approach can be useful for the computation of equilibria in large-scale general equilibrium models with many households. There are many economic questions for which heterogeneous agent models have to be used to provide answers, and an increasing amount of research employs frameworks that allow for intra-cohort heterogeneity in an OLG setup.[3] We believe that the presented approach can be beneficial for a wide range of economic applications, in particular within the class of OLG models, for the following reasons. First, by significantly reducing the computational overhead of the numerical problem at hand this method facilitates the development of OLG models which feature a complex and rich household side. This strengthens the microfoundation of the models in general and allows to analyze in detail intra- and intergenerational distributive consequences of economic policy. Second, the presented approach enables to solve OLG models that include a "realistic" number of households within each age group since the number of households in the model more closely corresponds to the number of observational units available from household survey data. This approach avoids relying on some ad-

[3] For instance, Conesa and Krueger (1999), Kotlikoff et al. (1999), and Huggett and Ventura (1999) investigate the intra-cohort distributive and welfare consequences of social security reform. Fehr (2000) looks at pension reform during the demographic transition in the case of Germany. Ventura (1999) explores the general equilibrium impact and associated distributional consequences of a revenue neutral tax reform, and Jensen and Rutherford (2002) analyze the intra- and intergenerational welfare effects of fiscal consolidation via debt reduction. This chapter concentrates on applications within the Auerbach-Kotlikoff OLG framework. For a general discussion of economies with heterogeneous agents, see, e.g., Rios-Rull (1995).

hoc aggregation of household groups, and thereby helps to enhance the empirical basis of the model. Third, and more generally, the method can also be effectively applied to OLG models which display a high dimensionality deriving from sources other than the household side. Potential applications may include multi-sectoral and multi-country models, or models which incorporate a detailed government sector.

In addition, the present contribution adds to the recent and growing body of research that deals with the integration of macro and microsimulation models, the "micro-macro" approach to modeling (Bourguignon and Spadaro, 2006). This strand of literature aims to combine the strengths of both the computable general equilibrium (CGE) paradigm and microsimulation models. While CGE models have become standard tools of quantitative policy assessment in the last twenty years, two major criticisms are their reliance on the concept of the "representative agent" and their usage of unclear aggregation procedures. The virtue of the microsimulation approach, on the other hand, is to replace representative agents with "real households" as observed in standard household surveys. This, however, is typically achieved at the cost of ignoring general equilibrium effects that are essential for policy analysis.

The simplest link between economy-wide modeling and the microsimulation approach proceeds top down, i.e. simulated policy changes obtained from an aggregate representation of the economy are passed down to a microsimulation module, as, e.g., in Bourguignon et al. (2005) and Bourguignon and Spadaro (2006). The principal weakness of the top-down approach is the absence of feedback effects from the micro to the macro level. Relatively few studies have attempted to fully integrate both approaches, most of them by means of employing an iterative strategy between the microsimulation and the CGE model (Cockburn and Cororatona (2007), Savard (2003, 2005), Arntz et al. (2006)). Also belonging to this class of models, Rutherford and Tarr (2008) applied the SR algorithm to a large-scale, static general equilibrium model with 25 sectors and 53,000 households to assess the poverty effects of Russia's WTO accession. All of these studies, however, are concerned with applications in a static framework. Clearly, covering complex behavioral responses and potential general equilibrium and macroeconomic effects in a dynamic setup is essential for many policy issues.[4]

The present chapter develops a computational technique that allows to fully integrate a comprehensive system of OLG households, that exhibits a substantial degree of intra-cohort heterogeneity, into a generic Auerbach-Kotlikoff model. We show that the positive experience of the SR algorithm for large-scale static models carries over to dynamic applications and demonstrate its effectiveness for solving OLG models with a large number of heterogeneous households. To find the equilibrium transition path of the OLG economy, the presented algorithm solves a sequence of "related" Ramsey optimal growth problems where the system of overlapping generations is replaced by an infinitely-lived representative agent. An iterative procedure

[4] Available models tend to concentrate on some specific behavior, abstracting from other important components of the demo-economic life cycle. For instance, Townsend (2002), Townsend and Ueda (2003) concentrate on saving/investment behavior under uncertainty and in different financial market environments.

between the macro and micro model is employed based on the successive recalibration of preferences of this artificial representative agent.

In order to characterize limitations of the algorithm, local convergence theory for a simple exchange economy due to Scarf (1960) is developed and a number numerical analyses are carried out to examine global conditions under which the SR algorithm may fail to converge. We show that conditions for local stability of the adjustment process reduce to those of a Walrasian price tâtonnement process, thus SR belongs to a large class of algorithms commonly used in computational economics which are robust and efficient, yet fail to provide global convergence. The presented counterexample illustrates that the SR algorithm may be ill-suited for applications in which there are significant income effects.

After having characterized limitations of the technique, this chapter explores the algorithm's performance when applied to large-scale OLG models. To this end, a prototype Auerbach-Kotlikoff model is considered which includes up to 2000 heterogeneous households within each generation differing with respect to labor productivity over the life cycle and other behavioral parameters. The performance of the SR algorithm is compared to a simultaneous solution method as suggested by Rasmussen and Rutherford (2004). We find that the proposed algorithm can provide improvements in both efficiency and robustness. It is furthermore demonstrated that the decomposition algorithm can routinely solve high-dimensional OLG models which are infeasible for conventional solution methods.

Lastly, it is important to emphasize that the decomposition method is inadequate for approximating equilibria in OLG economies that are generically *Pareto-inferior*, i.e. models in which the economy's growth rate exceeds the real interest rate (see, e.g., Diamond (1965) and Phelps (1961)). For the given model, this corresponds to a situation where population growth dominates discounting. In such circumstances there is no social planner's problem which corresponds to the OLG demand system. Whether this significantly limits the relevance and scope of the presented approach ultimately is an empirical question. Empirical evidence suggests that the incapacity of the method to deal with dynamically inefficient equilibria is of minor practical significance.[5]

The rest of the chapter is organized as follows. Section 2.3 introduces the SR algorithm for the case of a static economy and illustrates its basic logic by means of graphical analysis. Section 2.4 investigates a model where convergence of the SR algorithm fails if income effects are relatively strong. Section 2.5 demonstrates that the algorithm can be effectively applied to solve Auerbach-Kotlikoff OLG models. Furthermore, the performance of the algorithm is compared to computational experience from an integrated simultaneous solution method. Section 2.6 concludes.

[5] Abel et al. (1989) find that for the US economy the condition for dynamic efficiency seems to be satisfied in practice. Similarly, under the weak assumption that rates of return are ergodic, Barbie, Hagedorn and Kaul (2004) reach the conclusion that the US economy does not overaccumulate capital. By means of numerical analysis, Larch (1993) suggests that in the Auerbach-Kotlikoff framework rather implausible values of the pure rate of time preference, the intertemporal elasticity of substitution or the population growth rate are required to obtain non Pareto-optimal market solutions.

2.2 Calibration in CGE models

In order to facilitate the subsequent description of the algorithm and to clarify the algebraic forms used in the associated computer programs, this section reviews some fundamental aspects of calibration which underly most computable general equilibrium (CGE) models.

2.2.1 CES Preferences

Calibration refers to the process of selecting values of model parameters which ensure that the model's reference equilibrium is consistent with given data. Such data are typically obtained in the form of a social accounting matrix for a given base year. CGE models are based on parametric forms which describe technology and preferences. The most common functional form used in empirical applications is the constant-elasticity-of-substitution (CES) function. A CES utility function can be written as:

$$U(\mathbf{C}) = \left(\sum_{i=1}^{n} \alpha_i C_i^{\rho} \right)^{1/\rho} \tag{2.1}$$

where $\mathbf{C}$ denotes the vector of consumption goods C_i, $i = 1, \ldots, n$. There are $n+1$ parameters in this function, with n share parameters $\alpha_i > 0$ and a curvature parameter ρ. The latter is related to the Allen-Uzawa elasticity of substitution σ as: $\rho = 1 - 1/\sigma$ with $\rho < 1$ and $\sigma > 0$.

Consumers in CGE models are typically modeled as budget constrained utility-maximizers, so a model would incorporate the following behavioral subproblem:

$$\begin{aligned} &\max\nolimits_{c_1,\ldots,c_n} \; U(\mathbf{C}) \\ \text{s.t.} \quad &\textstyle\sum_{i=1}^{n} p_i C_i = M \end{aligned} \tag{2.2}$$

where p_i is the price of consumption good i, and M is consumer income. The consumer problem in closed-form can be solved, obtaining demand functions:

$$C_i(\mathbf{p}, M) = \frac{\alpha_i^{\sigma} M p_i^{-\sigma}}{\sum_{i'} \alpha_{i'}^{\sigma} p_{i'}^{1-\sigma}} \tag{2.3}$$

where $\mathbf{p}$ denotes the price vector, and $i \neq i'$. The calibration of preferences involves inverting this demand function to express the function parameters in terms of an observed set of prices and demands. If a consumer chooses to consume quantities $\overline{C}_i$ when commodity prices are $\overline{p}_i$, it may be concluded that the share parameters have to be given by:

$$\alpha_i = \lambda \, \overline{p}_i \overline{C}_i^{1-\rho} \tag{2.4}$$

where $\lambda > 0$ is an arbitrary scale factor[6], and the elasticity parameter, ρ, is exogenously specified. It is helpful to think of the share and scale parameters as calibrated values, determined by an agent's observed choices in a reference equilibrium, whereas the elasticity parameters are "free parameters" which are typically drawn from econometric estimates of the responsiveness of demand or supply to changes in relative prices. In traditional applied general equilibrium models, the reference quantities $\overline{C}_i$ and prices $\overline{p}_i$ are based on a benchmark equilibrium data set.

2.2.2 The Calibrated Share Form

In applied work it may be convenient to work with a different yet equivalent form of the CES utility function (Rutherford, 1995a). The *calibrated share form* is based on the observed quantities, prices and budget shares. In computational applications the calibrated form is preferable because it provides a simple parameter and functional check that is independent from second-order curvature. Normalizing the benchmark utility index to unity, the utility function can be written as:

$$U(\mathbf{C}) = \left[\sum_i \theta_i \left(\frac{C_i}{\overline{C}_i} \right)^{\rho} \right]^{1/\rho} \tag{2.5}$$

in which:

$$\theta_i = \frac{\overline{p}_i \overline{C}_i}{\sum_{i'} \overline{p}_{i'} \overline{C}_{i'}} \tag{2.6}$$

is defined as the benchmark value share of good i. Similarly, one can express the unit expenditure function as:

$$e(\mathbf{p}) = \left[\sum_i \theta_i \left(\frac{p_i}{\overline{p}_i} \right)^{1-\sigma} \right]^{1/1-\sigma}, \tag{2.7}$$

the indirect utility function as:

$$V(\mathbf{p}, M) = \frac{M}{e(\mathbf{p}) \overline{M}}, \tag{2.8}$$

and by Roy's identity the demand function as:

$$C_i(\mathbf{p}, M) = \overline{C}_i \frac{M}{e(\mathbf{p}) \overline{M}} \left(\frac{e(\mathbf{p}) \overline{p}_i}{p_i} \right)^{\sigma} \tag{2.9}$$

where $\overline{M} = \sum_i \overline{p}_i \overline{C}_i$.

[6] The consumer maximization problem is invariant with respect to positive scaling of U, hence the share parameters may only be determined up to a scale factor.

One can think of the demand function given here as a second-order Taylor approximation to the "true" demand function based on an observation of the true function at the reference point. At that point, $\overline{C}_i$ corresponds to a "zero*th* order approximation" to the utility function, $\overline{p}_i$ corresponds to the "first order approximation", and the "free parameter" σ controls the second (and higher) order properties of preferences. The benchmark prices correspond to the marginal rate of substitution—the slope of the indifference curve—at $\overline{C}$. As long as one remains in the neighborhood of $\overline{\mathbf{p}}$, the elasticity parameter σ only plays a minor role, and calibrated demand is determined largely by $\overline{\mathbf{C}}$ and $\overline{\mathbf{p}}$.

2.3 CGE with Many Households: A Decomposition Approach

This section presents the decomposition method by which a market economy with many heterogeneous households may be solved through the computation for equilibria for a sequence of representative agent economies. While the primary interest is in dynamic models, it is advantageous to introduce the algorithm for the case of a two-sector static economy in which one can provide a graphical description that serves to illustrate its basic logic.

2.3.1 A Static Economy

Consider the following static economy which is populated by a large number of heterogeneous households $h = 1, \ldots, H$ each of whom is endowed with K^h and L^h units of capital and labor, respectively. Households earn income $M^h = rK^h + wL^h$ from supplying their factor endowments inelastically at respective market prices r and w. Household $h = 1, \ldots, H$ solve:

$$\max_{c_1^h, c_2^h} U^h(\mathbf{c}^h) = \left[\sum_{i=1}^{2} \theta_i^h \left(\frac{c_i^h}{\overline{c}_i^h} \right)^{\rho^h} \right]^{1/\rho^h}$$

$$s.t. \qquad \sum_{i=1}^{2} p_i c_i^h = M^h. \tag{2.10}$$

where the utility function is written in calibrated share form. θ_i^h and $\overline{c}_i^h$ denote the benchmark value share and the benchmark consumption of good i for household h. Households are heterogeneous with respect to θ_i^h, ρ^h, K^h, and L^h.

Furthermore, there is a single representative firm which uses capital and labor services to produce two consumption goods X_i, $i, j = 1, 2$, according to a constant returns to scale production function $X_i = f_i(K, L)$, where $K = \sum_h K^h$ and $L = \sum_h L^h$. All goods and factor markets are perfectly competitive.

2.3.2 A Decomposition Algorithm

The main challenge for computing equilibria in a setup where H is very large is dimensionality. Typically, conventional simultaneous solution methods that operate directly on the equilibrium system of equations are infeasible. The proposed algorithm decomposes the corresponding numerical problem into two parts and thereby effectively manages to reduce its dimensionality. The general equilibrium of the underlying economic model is approximated by computing equilibria for a sequence of representative agent economies. In each iteration, first a general equilibrium representation of the underlying economic model is solved in which the household side is replaced by a single representative agent (RA). The second subproblem then consists of solving a partial equilibrium relaxation of the original model that retains the full structure of the household demand system. Given equilibrium prices from the previous solution of the RA economy, it is possible to compute optimal choices for each of the "real" households. In a next step, and to create a basis for successive iterations, the preferences of the "artificial" RA are recalibrated such that given candidate equilibrium prices the RA choices replicate aggregate household choices.

The key departure from the routine use of calibration in the decomposition algorithm is the idea that the calibration of preferences occurs more than once. The first iteration of the algorithm is based on observable benchmark data, but in subsequent iterations the preferences of the RA are sequentially recalibrated to values determined in the iterative process. For the case of the static economy as described above, the SR algorithm involves the following steps:

Step 0: Initialize the Representative Agent Economy

In the computation of equilibria we will portray the choices of H households using a single representative agent. To construct an RA economy of the underlying economic model, replace (2.10) by:

$$\max_{C_1,C_2} U^k(\mathbf{C}) = \left[\sum_{i=1}^{2} \Theta_i^k \left(\frac{C_i}{\overline{C}_i^k} \right)^{\rho} \right]^{1/\rho}$$

$$s.t. \qquad \sum_{i=1}^{2} p_i C_i = w \sum_{h=1}^{H} L^h + r \sum_{h=1}^{H} K^h \tag{2.11}$$

where k is an iteration index. Factor endowments of the RA equal the sum of respective factor endowments across all households. To initialize the RA economy at a consistent data point, the data set has be constructed such that the RA model and the household model share the same optimal consumption quantities in the initial benchmark. This is achieved by setting:

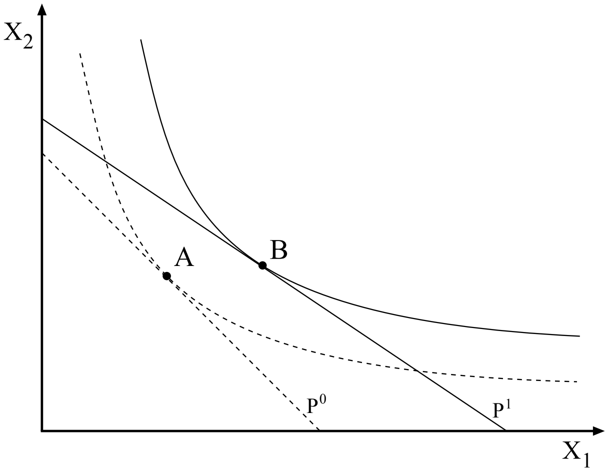

Fig. 2.1 Solution to the initial representative agent model (Step 0 and Step 1)

$$\overline{C}_i^0 = \sum_{h=1}^{H} \overline{c}_i^h \tag{2.12}$$

$$\Theta_i^0 = \frac{\overline{p}_i \overline{C}_i^0}{\sum_{i'} \overline{p}_{i'} \overline{C}_{i'}^0}, \quad i \neq i' \tag{2.13}$$

where $\overline{C}_i^0$ and Θ_i^0 denote initial consumption by the RA and the aggregate value share for good i in iteration $k = 0$, respectively.

This initial consumption point of the RA in the benchmark equilibrium is represented by point A in Figure 2.1 where initial goods prices are denoted by P^0. Benchmark prices and an arbitrary elasticity are used to extrapolate preferences in the neighborhood of the benchmark point to the global preferences of the RA, as indicated by the indifference curve which is tangent to the benchmark budget constraint at point A. The key limitation of the RA model on the demand side is that the "community indifference curve" represented by this indifference curve does not truthfully portray the response of household demand to a comprehensive change in both goods and factor prices.

Step 1: Solve for a General Equilibrium of the RA Economy

The solution to the RA model in the first iteration of the algorithm is illustrated in Figure 2.1. As depicted here, the assumed exogenous policy shock has led to an increase in factor earnings, and a reduction in the relative price of X_1 as compared with

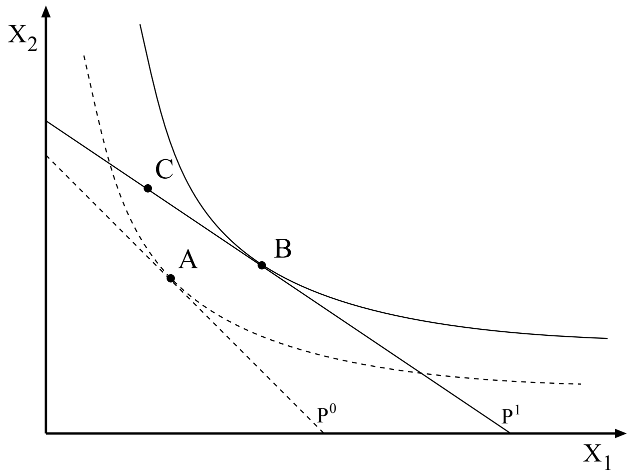

Fig. 2.2 Evaluating household demands at new prices (Step 2)

X_2. This new price situation is denoted by P^1. The RA model, based on the assumed community indifference curve and the associated change in factor and commodity prices returns point B as the optimal consumption point.

Step 2: Evaluate Household Demand Functions

In the solution program equilibrium prices are read from the RA model and evaluate the household demand vector. This produces a different point on the same budget constraint (see Figure 2.2). The household demand model is based on compensated demand functions so the aggregate budget constraint for the household demand system is equivalent to the budget constraint which applies to the RA. Point C corresponds to the aggregate demand which results from solving the individual household optimization problems. The extent to which C differs from B depends on both the difference in implicit substitution elasticities and differences in income effects.

Step 3: Recalibrate Preferences of the Representative Agent

The next step in the algorithm consists of specifying a new set of preferences for the RA model. The algorithm is termed “sequential recalibration” on the basis of this idea. After having solved one RA model, a new RA model is constructed based on a set of preferences which are locally calibrated to the aggregate consumption quantities at point C and the associated relative prices. This ensures that given prices P^1 the optimal consumption point of the new RA, point C in Figure 2.3, is consistent with the aggregated choices by households. Preferences of the RA in iteration k,

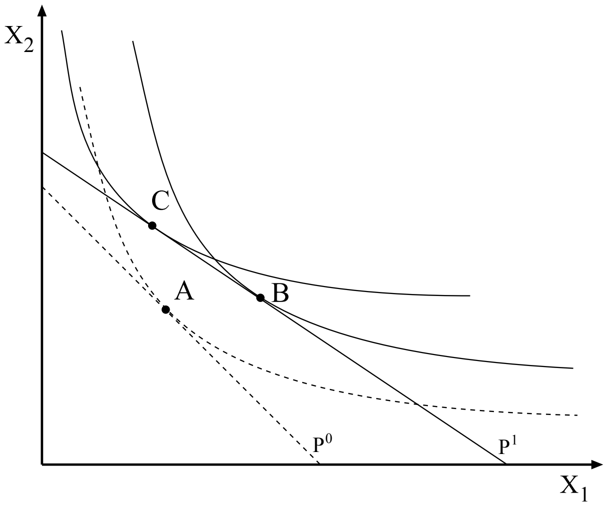

Fig. 2.3 Recalibration of preferences (Step 3)

$U^k(\mathbf{C})$ in (2.11), are based on household demands at the prices returned in iteration $k-1$:

$$\overline{C}_i^k = \sum_h c_i^h(\mathbf{p}^{k-1}, M_h^{k-1}) \tag{2.14}$$

in which $c_i^h(\mathbf{p}^{k-1})$ is the demand for good i by household h evaluated at the candidate price vector from the previous iteration $k-1$, and where factor income of household h in iteration k, $M_h^k(\mathbf{p}^k)$, is a function of prices in iteration k. Likewise, value shares in $U^k(\mathbf{C})$ are updated to:

$$\Theta_i^k = \frac{p_i^{k-1}\sum_h c_i^h(\mathbf{p}^{k-1}, M_h^{k-1})}{\sum_j p_j^{k-1}\sum_h c_j^h(\mathbf{p}^{k-1}, M_h^{k-1})} \tag{2.15}$$

where θ_i^k is the aggregate value share of good i at iteration $k-1$. The indifference curves tangent at A and B are based on identical preferences, but the indifference curve tangent at point C is based on a new set of community indifference curves, hence it may intersect the indifference curves based on RA utility in the previous iteration of the algorithm. Note that the preferences of the "real" households remain unchanged throughout the entire iteration process.

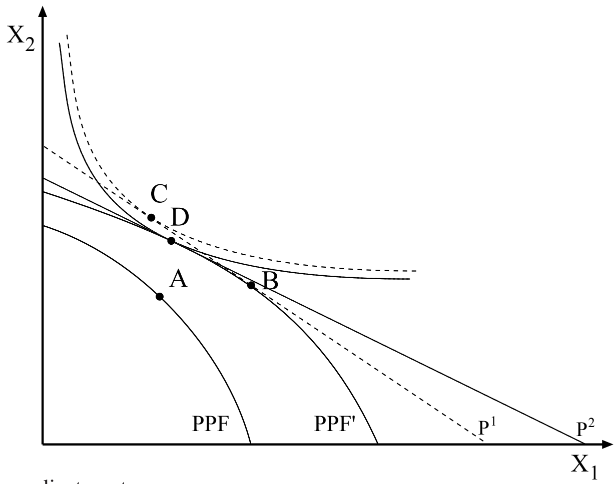

Fig. 2.4 Iterative adjustment

Iterative Adjustment

When the RA model is recalibrated at point C, both the representative agent and all households are in equilibrium at C with prices P^1, but at these prices firms will only supply quantities given by point B. Hence, due to inconsistency with the supply side of the model there is a general disequilibrium. To illustrate this idea, it is convenient to portray the supply side of the economy by a production possibility frontier (*PPF*). Assume that the policy shock produces an expansion in the *PPF* (to PPF') and a substantial change in relative prices from point A to B. The next step in the solution program is to resolve for a general equilibrium of the new RA model with recalibrated preferences at point C. Point C in Figure 2.4 becomes therefore interpreted as point A in the next iteration. The solution of this RA model is then characterized by a new optimal consumption point, here depicted by point D, and prices P^2.

Subsequent iterations involve carrying out Steps 1 to Steps 3 (Step 0 initializes the solution procedure). The process is stopped if some convergence metric, e.g., the 1-norm of the difference between the price vectors from one iteration to the next, is satisfied. Note that subsequent iterations of the algorithm only involve refinements of the demand system and result in much smaller changes in relative prices, as indicated here by the change from C to D as compared with A to B.

2.4 Convergence Theory

This section evaluates the performance of the algorithm for an economy in which the exact equilibrium is known and where the computed allocation can be compared to the true equilibrium allocation. Local convergence theory for the proposed algorithm is developed and conditions under which the adjustment process may fail to converge are identified.

The example is due to Scarf (1960) who considers a pure exchange economy with an equal number of n consumers and goods. Consumer h is endowed with one unit of good h and demands only goods h and $h+1$. Let $d_{i,h}$ denote demand for good i by consumer h. Preferences are represented by CES utility functions with the following structure:

$$U_h(d) = \left(\theta\, d_{h,h}^{\frac{\sigma}{\sigma-1}} + (1-\theta)\, d_{h+1,h}^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}} . \tag{2.16}$$

Scarf (1960) demonstrates that this economy has a unique equilibrium in which all prices are equal to unity.[7]

2.4.1 Local Convergence

As explained in Section 2.3.2, the sequential recalibration algorithm iteratively adjusts the baseline level parameter $\overline{C}_i$ (and $\Theta_i(\overline{C}_i)$) in the utility function of the representative agent. These may be normalized so that $\sum_i \overline{C}_i = n$. Market clearing commodity prices for the RA economy are determined given the baseline level parameters. Let $p_i(\overline{C})$ denote the price of good i consistent with $\overline{C} = (\overline{C}_1, \ldots, \overline{C}_n)$. In this exchange economy, let $\zeta_i\left(p(\overline{C})\right)$ denote the market excess demand function for good i that is obtained from evaluating household demand functions. Given the special structure of preferences and endowments, this function has the form:

$$\zeta_i\left(p(\overline{C})\right) = d_{i,i} + d_{i,i-1} - 1 . \tag{2.17}$$

Furthermore, let $\xi_i\left(\overline{C}\right)$ denote the value of market excess demand for good i at prices $p_i(\overline{C})$:

$$\xi_i\left(\overline{C}\right) = p_i(\overline{C})\, \zeta_i\left(p(\overline{C})\right) . \tag{2.18}$$

Of course, in equilibrium it must be true that $\xi_i(\overline{C}^*) = 0,\ \forall i$. Let the initial estimate $\overline{C}^0$ be selected on the n-simplex, i.e. $\sum_i \overline{C}_i^0 = n$. Walras' law ensures that the adjustment process $\frac{d\overline{C}_i}{dt} = \xi_i\left(\overline{C}\right)$ remains on the n-simplex: $\frac{d\sum_i \overline{C}_i(t)}{dt} = \sum_i p_i(\overline{C}(t))\, \zeta_i\left(p(\overline{C}(t))\right) = 0$.

[7] See Lemma 1 and Lemma 2 (Scarf, 1960, p.164). The parameters of this utility function correspond to Scarf's parameters a and b (Scarf, 1960, p.168) as: $\sigma = \frac{1}{1+a}$ and $\theta = \frac{b}{1+b}$.

Local convergence concerns properties of the Jacobian matrix evaluated at the equilibrium point, $\nabla\xi(\overline{C}^*) = [\xi_{ij}]$. This Jacobian has entries defined as follows:

$$\xi_{ij} \equiv \frac{\partial \xi_i}{\partial \overline{C}_j} = \begin{cases} \frac{\partial \zeta_i(p(\overline{C}))}{\partial \overline{C}_i} & i = j \\ \frac{\partial \zeta_i(p(\overline{C}))}{\partial \overline{C}_j} & i \neq j \end{cases} . \tag{2.19}$$

If all principal minors of $\nabla\xi(\overline{C}^*) = [\xi_{ij}]$ are negative, the adjustment process is locally convergent. If, however, $\frac{\partial \zeta_i(p(\overline{C}))}{\partial \overline{C}_i} > 0$, the process is locally unstable. When an equilibrium is unique and the process is uninterrupted, then local stability implies global instability.

For this model, the tâtonnement price adjustment process is unstable (in the case $n = 3$) when $\frac{\theta}{1-\theta} > \frac{1}{1-2\sigma}$ (Scarf, 1960). In the following, it is shown that the same condition implies instability for the $\overline{C}$-adjustment process of the SR algorithm. Furthermore, it is shown (numerically) that while the tâtonnement and SR price adjustment processes are locally identical, they may be quite different at points in the price space that are sufficiently far away from the equilibrium.

Given the special structure of $\zeta_i\left(p(\overline{C})\right)$, one can write:

$$\frac{\partial \zeta_i}{\partial p_i} = \frac{\partial d_{i,i}}{\partial p_i} + \frac{\partial d_{i,i-1}}{\partial p_i}, \quad \frac{\partial \zeta_i}{\partial p_{i-1}} = \frac{\partial d_{i,i-1}}{\partial p_{i-1}}, \quad \frac{\partial \zeta_i}{\partial p_{i+1}} = \frac{\partial d_{i,i}}{\partial p_{i+1}} . \tag{2.20}$$

Defining the "unit-utility" expenditure function for consumer i as:

$$e_i(p) = \left(\theta\, p_i^{1-\sigma} + (1-\theta)\, p_{i+1}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{2.21}$$

demand functions are given by:

$$d_{i,i} = \frac{\theta\, p_i}{e_i^{1-\sigma} p_i^{\sigma}}, \quad d_{i+1,i} = \frac{(1-\theta)\, p_i}{e_i^{1-\sigma} p_{i+1}^{\sigma}} . \tag{2.22}$$

Evaluating $\frac{\partial \zeta_i(p(\overline{C}))}{\partial \overline{C}_i}$ at $p^* = 1$, yields

$$\begin{aligned} \frac{\partial \zeta_i\left(p(\overline{C})\right)}{\partial \overline{C}_i} = \frac{\partial p_i}{\partial \overline{C}_i} & \left((2\sigma-2)\theta^2 + (3-2\sigma)\theta - 1\right) \\ & + \frac{\partial p_{i+1}}{\partial \overline{C}_i} \left(-\theta(1-\theta)(1-\sigma)\right) \\ & + \frac{\partial p_{i-1}}{\partial \overline{C}_i} \left(-\theta(1-\theta)(1-\sigma) + 1 - \theta\right) . \end{aligned} \tag{2.23}$$

The function $p(\overline{C})$ is defined implicitly by the equation:

$$\widetilde{\zeta}\left(p, \overline{C}\right) = 0 \tag{2.24}$$

where $\widetilde{\zeta}\left(p,\overline{C}\right)$ denotes the vector of market excess demand functions from the representative agent economy. Its *i-th* element is given by:

$$\widetilde{\zeta}_i\left(p,\overline{C}\right) = \frac{\overline{C}_i}{\left(\sum_{i'} \alpha_{i'} p_{i'}^{1-\widetilde{\sigma}}\right) p_i^{\widetilde{\sigma}}} - 1 \tag{2.25}$$

with $\alpha_{i'} = \frac{\overline{C}_i}{\sum_{i'} \overline{C}_{i'}}$ and where $\widetilde{\sigma}$ denotes the elasticity of substitution for the representative agent. In order to evaluate $\nabla_p \widetilde{\zeta}$ at $\overline{C}^*$, we make a first-order Taylor series expansion:

$$\nabla_p \widetilde{\zeta}\left(p,\overline{C}\right) d\,p + \nabla_{\overline{C}} \widetilde{\zeta}\left(p,\overline{C}\right) d\,\overline{C} = 0 \tag{2.26}$$

which gives:

$$\frac{d\,p}{d\,\overline{C}} = -\nabla_p^{-1} \widetilde{\zeta}\, \nabla_{\overline{C}} \widetilde{\zeta}\,. \tag{2.27}$$

Evaluating gradients at $p^* = \overline{C}^* = 1$ yields:

$$\nabla_p \widetilde{\zeta} = \begin{pmatrix} \frac{-(2\widetilde{\sigma}+1)}{3} & \frac{-(1-\widetilde{\sigma})}{3} & \frac{-(1-\widetilde{\sigma})}{3} \\ \frac{-(1-\widetilde{\sigma})}{3} & \frac{-(2\widetilde{\sigma}+1)}{3} & \frac{-(1-\widetilde{\sigma})}{3} \\ \frac{-(1-\widetilde{\sigma})}{3} & \frac{-(1-\widetilde{\sigma})}{3} & \frac{-(2\widetilde{\sigma}+1)}{3} \end{pmatrix} \tag{2.28}$$

$$-\nabla_p^{-1} \widetilde{\zeta} = \begin{pmatrix} \frac{\widetilde{\sigma}+2}{3\widetilde{\sigma}} & \frac{\widetilde{\sigma}-1}{3\widetilde{\sigma}} & \frac{\widetilde{\sigma}-1}{3\widetilde{\sigma}} \\ \frac{\widetilde{\sigma}-1}{3\widetilde{\sigma}} & \frac{\widetilde{\sigma}+2}{3\widetilde{\sigma}} & \frac{\widetilde{\sigma}-1}{3\widetilde{\sigma}} \\ \frac{\widetilde{\sigma}-1}{3\widetilde{\sigma}} & \frac{\widetilde{\sigma}-1}{3\widetilde{\sigma}} & \frac{\widetilde{\sigma}+2}{3\widetilde{\sigma}} \end{pmatrix} \tag{2.29}$$

$$\nabla_{\overline{C}} \widetilde{\zeta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.30}$$

Hence, in the neighborhood of the equilibrium:

$$\frac{\partial\, p_i(\overline{C})}{\partial\, \overline{C}_i} = \frac{\widetilde{\sigma}+2}{3\widetilde{\sigma}} > 0, \quad \frac{\partial\, p_i(\overline{C})}{\partial\, \overline{C}_j} = 0\,. \tag{2.31}$$

From (2.23) it therefore follows that the adjustment process in $\overline{C}$ is locally unstable if:

$$(2\sigma - 2)\theta^2 + (3 - 2\sigma)\theta - 1 > 0 \tag{2.32}$$

which is equivalent to the condition for instability of a simple price tâtonnement adjustment process as demonstrated by Scarf (1960, p.169).

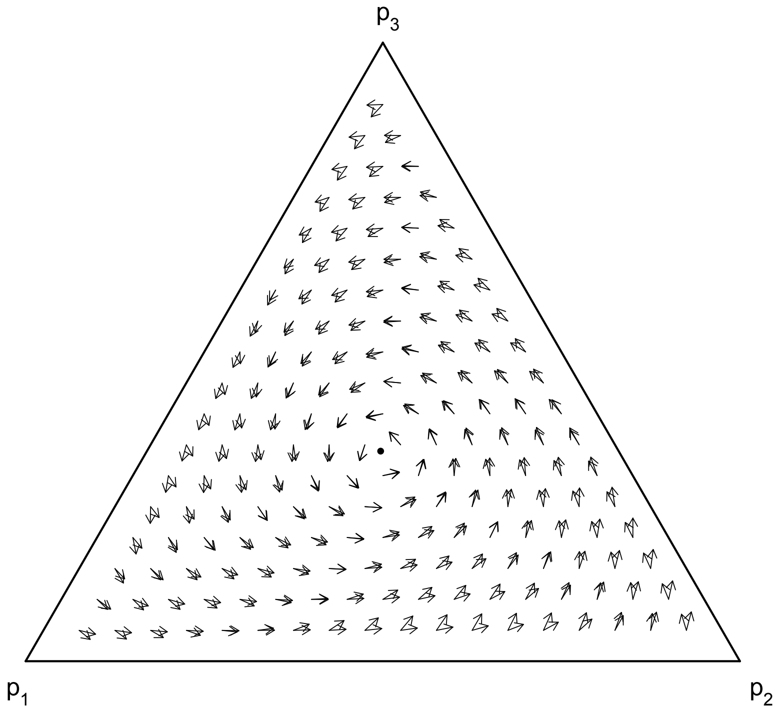

Fig. 2.5 Comparison of SR and tâtonnement fields

2.4.2 Global Convergence

Although the local behavior of the price tâtonnement and the SR algorithm adjustment processes are identical, they produce different search directions away from a neighborhood of the equilibrium. This is apparent in Figure 2.5 where the two vector fields are superimposed. Only local to the equilibrium where price effects dominate income effects, do the two fields coincide exactly, as indicated by (2.32). As one moves further away from the center of the simplex, the vector fields become more divergent. It is found that there are cases in which the SR algorithm does not converge even though the price tâtonnement is globally stable. This convergence failure is a manifestation of the simplifying nature of the adjustment process. By solving a sequence of representative agent economies the SR algorithm omits both income constraints and global properties of the individual utility functions. While the omission of global characteristics of preferences reduces the dimensionality of the model significantly, this may at the same time hinder convergence.

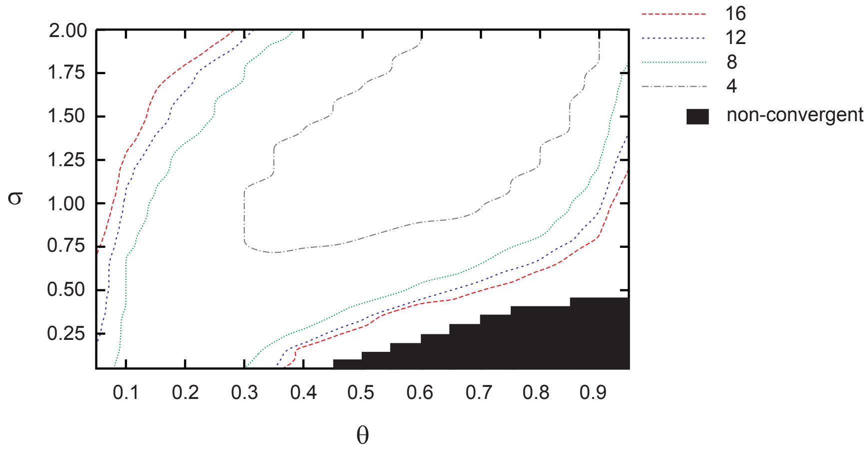

Fig. 2.6 Global convergence behavior for different configurations of θ and σ

To assess the global convergence properties of the SR algorithm, a grid search over the behavioral parameters σ and θ is performed. We let the algorithm start from a disequilibrium point $p = (0.2, 0.2, 0.6)$ where local equilibrium dynamics are absent. Figure 2.6 reveals that convergence of the SR algorithm fails for combinations of small values for σ and high values for θ.[8] For these parameter configurations, income effects are relatively strong vis-à-vis substitution effects. In cases where convergence is achieved, the presence of significant income effects means that more iterations are required to find the true equilibrium. If, however, income effects are relatively weak, the SR algorithm only requires a modest number of iterations. The appropriateness of the presented solution method therefore depends on the characteristics of the underlying model.

One last remark is in order. To guarantee convergence of the SR algorithm, it is necessary to select a sufficiently large value for $\widetilde{\sigma}$, the elasticity of substitution of the representative agent. If $\widetilde{\sigma}$ is too low, convergence may fail even if income effects are relatively weak. Non-convergent behavior, however, that occurs in the bottom right corner of Figure 2.6 is robust with respect to $\widetilde{\sigma}$. The choice of $\widetilde{\sigma}$ is entirely innocuous since this parameter bears no economic significance for the behavior of "real" households in the underlying economic model. Computational experience suggests to use values of order $\widetilde{\sigma} \geq 1$.

[8] For both parameters, a grid resolution of 0.05 is chosen, $\widetilde{\sigma}$ is set to one, and a maximum of 1000 iterations is allowed for. The adjustment process is said to converge if the 1-norm of differences between a computed price vector and the equilibrium point drops below some metric δ, i.e. $\|p_i - p_i^*\|_1 < \delta$, where p_i^* denotes the analytical equilibrium solution. We set $\delta = 0.01$.

2.5 OLG Models with Many Households

This section presents a decomposition algorithm for solving overlapping generations models with many heterogeneous households. The proposed algorithm is an application of the SR approach with a few elaborations specific to the OLG context. As in the static setting, an equilibrium allocation is approximated by computing equilibria for a sequence of representative agent economies. In the case of OLG, the representative agent economies are Ramsey optimal growth problems where the system of overlapping generations is replaced by a single infinitely-lived agent.

The algorithm is demonstrated for a simple prototype Auerbach-Kotlikoff OLG economy with production activities, intra-cohort heterogeneity, a labor-leisure choice, and a government sector.[9] We solve for the effects of a tax reform that is introduced unexpectedly in year zero, and then evaluate the performance of the algorithm against numerical solutions that are available from simultaneous solution methods.

2.5.1 A Prototype Auerbach-Kotlikoff OLG Model

2.5.1.1 Households

Time is discrete and extends from $t = 0,\ldots,\infty$. There is no aggregate or household-specific uncertainty. The economy is populated by overlapping generations of heterogeneous agents. A household of generation g and type $h = 1,\ldots,H$ is born at the beginning of year $t = g$, lives for $N+1$ years, and is endowed with $\omega_{g,h,t} = \omega\,(1+\gamma)^g$ units of time in each period $g \leq t \leq g+N$, and $\pi_{g,h,t}$ is an index of labor productivity over the life cycle.[10] γ denotes the exogenous steady-state growth rate of the economy. Leisure time, $\ell_{g,h,t}$, enters in a CES function with consumption, $c_{g,h,t}$, to create full consumption, $z_{g,h,t}$. Expressed with present value prices, the optimization problem is:

[9] The example is an adapted version of the production model presented in Rasmussen and Rutherford (2004). A closed economy version of their model with intra-cohort heterogeneity is considered. While a single-sector model is investigated here, the logic can be readily extended to a multisectoral framework.

[10] ω is a constant income scaling factor which is determined in the initial calibration procedure to reconcile household behavior with the aggregate benchmark data. For more details see Rasmussen and Rutherford (2004).

$$
\begin{aligned}
\max_{c_{g,h,t},\,\ell_{g,h,t}} \quad u_{g,h}\left(z_{g,h,t}\right) &= \sum_{t=g}^{g+N} \left(\frac{1}{1+\rho}\right)^{t-g} \frac{z_{g,h,t}^{1-1/\sigma_h}}{1-1/\sigma_h} \\
s.t. \qquad\qquad z_{g,h,t} &= \left(\alpha c_{g,h,t}^{\nu} + (1-\alpha)\,\ell_{g,h,t}^{\nu}\right)^{\frac{1}{\nu}} \\
p_{c,t}\, c_{g,h,t} + p_{y,t}\, i_{g,h,t} &\leq p_{l,t}\, \pi_{g,h,t}\left(\omega_{g,h,t} - \ell_{g,h,t}\right) + p_{r,t}\, k_{g,h,t} + p_{y,t}\, \zeta_{g,h,t} \\
k_{g,h,t+1} &\leq (1-\delta)\, k_{g,h,t} + i_{g,h,t} \\
\ell_{g,h,t} &\leq \omega_{g,h,t} \\
c_{g,h,t},\, \ell_{g,h,t} &\geq 0 \\
k_{g,h,g} &\leq \bar{k}_{g,h,g}\,, \quad i_{g,h,t+N} + (1-\delta)\, k_{g,h,t+N} \geq 0\,,
\end{aligned}
\tag{2.33}
$$

where ρ is the utility discount factor, σ_h is the intertemporal elasticity of substitution for household h, $\sigma_\nu = 1/(1-\nu)$ is the uniform elasticity of substitution between consumption and leisure, α is a share parameter, and $p_{x,t}$, $x = \{y,c,l,r\}$, denotes the price for the single output good, the price for consumption, the wage rate, and the capital rental rate, respectively. All prices refer to after-tax prices. Heterogeneity relates to intra-cohort differences in labor productivity and the intertemporal elasticity of substitution. Households have access to a storage technology: they can use one unit of the output good to obtain one unit of the capital good next period. We denote the investment into this technology by $i_{g,h,t}$. We do not restrict $i_{g,h,t}$, because we want to permit households to borrow against future labor income. Private capital $k_{g,h,t}$ depreciates at an annual rate of δ. $\bar{k}_{g,h,g}$ denotes the capital holdings of generation g at the beginning of life $t = g$. Initial old generations, i.e. generations born prior to period zero, are endowed with a non-zero amount of assets: $\bar{k}_{g,h,0} \neq 0$, $\forall g = -N, \ldots, -1, \forall h$. The initial asset distribution for these generations is selected such that the economy is on a balanced growth path. We assume that newborn households enter with zero assets: $\bar{k}_{g,h,g} = 0$, $\forall g \geq 0, \forall h$. We furthermore rule out that households die in debt. In each period of the life cycle households receive $\zeta_{g,h,t}$ units of the output good as a lump-sum transfer from the government. For simplicity, we assume that these transfers are allocated to each generation and type according to its share in the total population. We moreover assume that the government has no outstanding debt in the initial steady state and hence households' total assets in period zero are equal to the value of the capital stock, i.e. $\sum_{g=-N}^{0} \sum_{h=1}^{H} \bar{k}_{g,h,t} = (1+\bar{r})\, K_0$, where $\bar{r}$ is the steady state interest rate and K_0 is the initial aggregate capital stock.[11]

2.5.1.2 Firms

There is a single representative firm which in each period t uses capital and labor services to produce a single output good Y_t according to a linearly homogeneous

[11] We therefore implicitly assume that the government has no outstanding debt at period zero. A situation with non-zero initial government debt slightly complicates the calibration procedure but is conceptually straightforward (see Rasmussen and Rutherford (2004)).

production function $Y_t = F(K_t, L_t)$. All goods and factor markets are perfectly competitive.

2.5.1.3 Government

The government agent collects revenue from levying taxes on consumption, and on capital and labor income. Tax revenue is spent on government expenditure (G_t) and on total transfers to households ($T_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \zeta_{g,h,t}$). We assume that the consumption tax rate (τ_t^c) adjusts such that the government budget is balanced on a period-by-period basis:

$$\tau_t^r \, p_{r,t} \, R_t + \tau_t^l \, p_{l,t} \, L_t + \tau_t^c \, p_{y,t} \, C_t = p_{y,t} \, G_t + p_{y,t} \, T_t \tag{2.34}$$

where τ_t^r and τ_r^l are the net tax rates on capital and labor income, respectively.

2.5.1.4 Aggregate Economy Restrictions

Given inter- and intragenerational heterogeneity, the following feasibility conditions must be satisfied:

$$L_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \omega_{g,h,t} - \ell_{g,h,t} \tag{2.35}$$

$$I_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} i_{g,h,t} \tag{2.36}$$

$$K_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} k_{g,h,t} \tag{2.37}$$

$$C_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} c_{g,h,t} \, . \tag{2.38}$$

The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = (1-\delta) K_t + I_t \, . \tag{2.39}$$

Finally, the single output good may be used for household consumption, investment, or government consumption implying the following condition for balance between aggregate supply and demand:

$$F(K_t, L_t) = C_t + I_t + G_t \, . \tag{2.40}$$

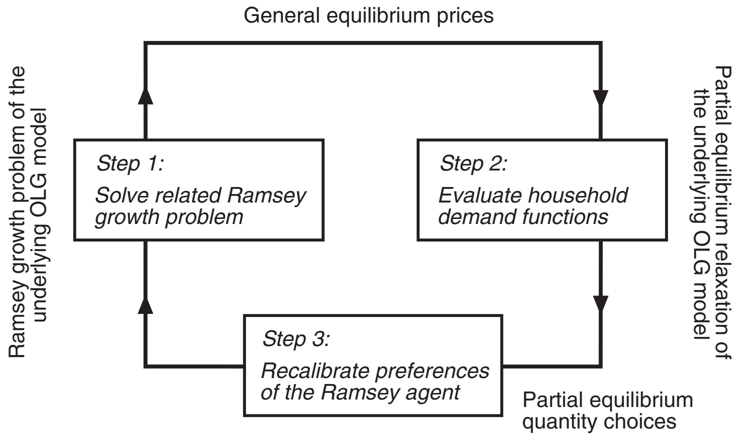

Fig. 2.7 Solving OLG by Ramsey: steps in the decomposition algorithm

2.5.2 *A Decomposition Algorithm for OLG Models*

The unknown equilibrium allocation of the OLG economy described by (2.33)-(2.40) is approximated by computing equilibria for a sequence of "related" Ramsey (optimal) growth problems. Figure 2.7 provides a schematic exposition of the steps involved in the decomposition procedure. Each iteration comprises the following three steps. In the first step, we solve the general equilibrium of the "related" Ramsey growth problem (Section 2.5.2.1) which retains the full structure of the production side of the model but replaces the system of OLG households by a representative infinitely-lived consumer agent. The second step computes optimal household behavior given the equilibrium prices from the Ramsey economy (Section 2.5.2.3). This step can be viewed as solving a partial equilibrium relaxation of the underlying economy that ignores general equilibrium interactions with the production side of the model but retains the full structure of the OLG demand system. In the third step, we construct a new Ramsey optimal growth problem by recalibrating the preferences of the "artificial" Ramsey agent based on households' choices from Step 2 (Section 2.5.2.4). Subsequent iterations proceed with analogous steps. Typically, the sequence of prices and quantities computed by the algorithm converges to the true equilibrium allocation. In what follows, we provide more details on each of the involved steps.

2.5.2.1 The "Related" Ramsey Growth Problem

As the "related" Ramsey optimal growth problem, we define a model of the underlying OLG economy in which the system of overlapping generations is replaced by a single infinitely-lived representative agent, henceforth called the Ramsey agent. Apart from this modification, the entire economic structure of the OLG model including the behavior of other agents, market structure, the number of sectors etc. is unchanged. The Ramsey agent solves the following optimization problem:

$$
\begin{aligned}
\max_{C_t,\mathscr{L}_t} U(Z_t) &= \left[\sum_{t=0}^{T} \Theta_t^k \left(\frac{Z_t}{\overline{Z}_t^k}\right)^{1-1/\widehat{\sigma}}\right]^{\frac{\widehat{\sigma}}{\widehat{\sigma}-1}} \\
s.t. \qquad Z_t &= \left(\Delta_t^k \left(\frac{C_t}{\overline{C}_t^k}\right)^{\nu} + \left(1-\Delta_t^k\right)\left(\frac{\mathscr{L}_t}{\overline{\mathscr{L}}_t^k}\right)^{\nu}\right)^{\frac{1}{\nu}} \\
C_t + I_t + G_t &= F(K_t, \Omega_t^k - \mathscr{L}_t) \\
K_{t+1} &\leq (1-\delta) K_t + I_t \\
\mathscr{L}_t &\leq \Omega_t^k \\
C_t, \mathscr{L}_t &\geq 0 \\
K_0 &\leq \Psi \\
K_{T+1} &= \hat{K}_{T+1}
\end{aligned}
\tag{2.41}
$$

where C_t, $\mathscr{L}_t$, Z_t, K_t, I_t, and Ω_t^k now denote consumption, leisure time, full consumption, the capital stock, investment, and the time endowment by the Ramsey agent, respectively, and where $\widehat{\sigma}$ is the intertemporal elasticity of substitution. k denotes an iteration index.[12]

The initial capital stock in the Ramsey economy is given by the aggregate capital stock of the OLG economy in year zero:

$$
\Psi = \sum_{g=-N}^{0} \sum_{h=1}^{H} \overline{k}_{g,h,0} \,. \tag{2.42}
$$

To approximate the infinite-horizon Ramsey economy by a finite-dimensional complementarity problem, we use the *state-variable targetting* method suggested by Lau, Pahlke and Rutherford (2002) in which the target post-terminal capital stock ($\hat{K}_{T+1}$) is chosen at a level such that investments grow at the steady-state rate in the last period:

$$
\frac{I_T}{I_{T-1}} = 1 + \gamma. \tag{2.43}
$$

[12] Note that the nested lifetime utility function $U(Z_t)$ is written in calibrated share form. We monotonically transform preferences in (2.33) to obtain a linear homogenous CES representation. This does not alter the underlying preference orderings and hence optimization yields the same demand functions.

2.5.2.2 Initialization

In order to initialize the "related" Ramsey growth problem, it is first necessary to characterize a baseline reference path of the OLG economy. For simplicity, we assume that the economy is initially on a balanced growth path and employ a steady-state calibration procedure proposed by Rasmussen and Rutherford (2004) which proceeds in two steps. In the first step, the optimal profile of decision variables for a reference generation of type h is computed subject to given aggregate benchmark data in year zero. The second step involves extrapolating the results from the household calibration model, together with remaining elements in the aggregate dataset, to set up a baseline reference path.

Given this reference path, we choose an initial set of preferences for the representative agent such that the Ramsey growth problem endogenously reproduces the baseline reference path of the underlying OLG economy. This is accomplished by selecting appropriate reference levels and value share parameters for $U(Z_t)$ in (2.41):

$$\overline{C}_t^0 = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \overline{c}_{g,h,t} \tag{2.44}$$

$$\overline{\mathscr{L}}_t^0 = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \overline{\ell}_{g,h,t} \tag{2.45}$$

$$\overline{Z}_t^0 = \overline{C}_t^0 + \overline{\mathscr{L}}_t^0 \tag{2.46}$$

$$\Delta_t^0 = \frac{\overline{p}_t \overline{C}_t^0}{\overline{p}_t \overline{C}_t^0 + \overline{p}_t \overline{\mathscr{L}}_t^0} \tag{2.47}$$

$$\Theta_t^0 = \frac{\overline{p}_t \overline{Z}_t^0}{\sum_{t'} \overline{p}_{t'} \overline{Z}_{t'}^0} \tag{2.48}$$

where $\overline{p}_t, t = 0, \ldots, T$ and $\overline{x}_{g,h,t}, x = \{c, \ell\}$, denote benchmark prices and household quantity choices, respectively. The superscript "0" indicates starting values.

The Ramsey agent is endowed with units of "productive" time, Ω_t, that reflect households' labor productivity in a given period:

$$\Omega_t^0 = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \pi_{g,h,t} \left(\omega_{g,h,t} - \overline{\ell}_{g,h,t}\right) + \overline{\ell}_{g,h,t} \tag{2.49}$$

where $\overline{\ell}_{g,h,t}$ is benchmark leisure time by households.

The algorithm is started off by solving the initial Ramsey growth problem as defined by (2.34), and (2.41)-(2.49). If no policy change is implemented, the given specification of the Ramsey economy ensures that it can reproduce the initial steady state path of the underlying OLG economy as an equilibrium solution.

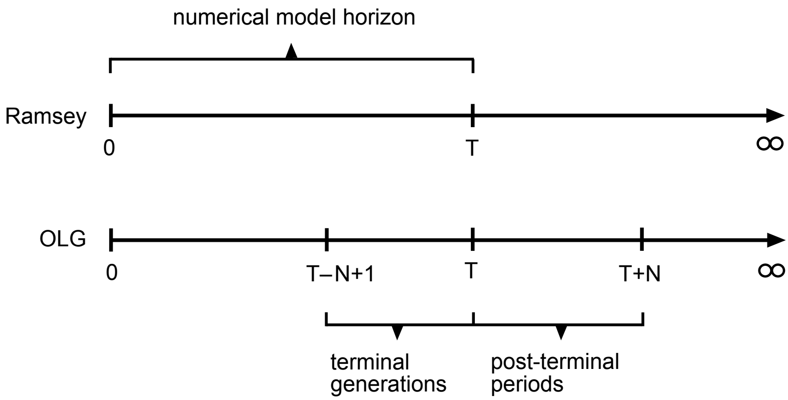

Fig. 2.8 Approximating OLG by Ramsey: the issue of terminal generations

2.5.2.3 The Partial Equilibrium Relaxation

The second step of the algorithm solves a partial equilibrium relaxation of the underlying OLG economy which retains full details of the household demand system but ignores general equilibrium effects. Hence, any interactions via commodity and factor markets and with the production side of the economy are suppressed. Given equilibrium prices from the previous solution of the "related" Ramsey growth problem, we evaluate demand functions for each generation and type that originate from the set of household problems in (2.33).

In order to obtain a good approximation of the underlying OLG economy, it is necessary to compute optimal household demand for all households and types in each period of the numerical model that runs from $t = 0, \ldots, T$. This information then forms the basis for the recalibration of preferences of the Ramsey agent in the subsequent step of the algorithm. A complication arises for periods $T - N + 1 \leq t \leq T$ in which generations are born that live beyond T (henceforth called terminal generations). Figure2.8 illustrates this issue. To compute the optimal decision profiles of these agents, it is essential to account for their behavior over the full life cycle. With the last cohort of households being born in period T, this means that there are N post-terminal periods that have to be included in the analysis, which we denote by $\hat{t} = T, \ldots, T + N$. We resolve this issue by employing a steady-state closure rule which postulates that the economy has reached a steady state by period $T - N + 1$. This additional restriction is not binding if T is chosen sufficiently large.[13] Exploiting this fact, prices for post-terminal periods can be inferred from the following steady-state projection:

[13] The specific choice of T depends on the nature of the policy shock that is considered. In the numerical examples below we set $T = 150$.

$$x^k_{\hat{t}} = \frac{x^k_T}{(1+r_\infty)^{\hat{t}-T}} \tag{2.50}$$

where $x^k_{\hat{t}} = \{pt^k_{g,h,\hat{t}}, pt^k_{y,\hat{t}}, pt^k_{l,\hat{t}}\}$ denote the post-terminal price for full consumption, the output good, and the market wage all obtained in iteration k, respectively. Correspondingly, x^k_T refer to respective prices in the terminal period. The price for full consumption can be obtained as:

$$p^k_{g,h,t} = e_{g,h,t}\left(p^k_{c,t}\,,\, p^k_{l,t}\right) \tag{2.51}$$

where $e_{g,h,t}(\cdot)$ denotes the unit expenditure function for $z_{g,h,t}$. For future reference, let $\overline{p}_{g,h,t}$ denote respective benchmark prices. In (2.50), $r_\infty = p^c_{T-1}/p^c_T - 1$ defines the endogenous (steady-state) interest rate in the terminal period.

Finally, the lifetime income of generation g and type h, evaluated at candidate prices from iteration k, is given by:

$$M^k_{g,h} = \sum_{t=g}^{g+N} \pi_{g,h,t}\, p^k_{l,t}\, \omega_{g,h,t} + p^k_{y,t}\, \zeta_{g,h,t} + p^k_{r,0}\, \overline{k}_{g,h,g}\,. \tag{2.52}$$

A similar formula applies to the lifetime income of terminal generations where for post-terminal periods projected prices according to (2.50) are used. For future reference, let $\overline{M}_{g,h}$ denote the lifetime income at benchmark prices.

We are now in a position to compute household demand. In order to reduce computational complexity, we solve the dual problem making use of formulas (2.7)-(2.9) (see the Appendix). Let $e_{g,h}(\mathbf{p}^k_{g,h})$ denote the expenditure function for a unit of $u_{g,h}$ which—given the specific structure of preferences[14]—can be constructed using the vector of prices for the full consumption good, $\mathbf{p}^k_{g,h}$ (including projected prices). Indirect utility can then be written as:

$$V_{g,h}(\mathbf{p}^k_{g,h}\,,\, M^k_{g,h}) = \frac{M^k_{g,h}}{e_{g,h}(\mathbf{p}^k_{g,h})\,\overline{M}_{g,h}} \quad \forall g\,, \forall h\,. \tag{2.53}$$

Applying Roy's identity, optimal household demand (in the context of the partial equilibrium relaxation) for full consumption, goods consumption and leisure, respectively, evaluated at the *candidate price vector* $\mathbf{p}^k_{g,h}$, are updated in each iteration according to:

[14] Note that in the given case of homothetic preferences, the unit expenditure function conveys all information concerning the underlying preferences.

$$z_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) = \bar{z}_{g,h,t}\, V_{g,h}(\mathbf{p}^k_{g,h}, M^k_{g,h}) \left(\frac{e_{g,h}(\mathbf{p}^k_{g,h})\, \overline{p}_{g,h,t}}{p^k_{g,h,t}} \right)^{\sigma_h} \tag{2.54}$$

$$c_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) = \bar{c}_{g,h,t}\, z_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) \left(\frac{p^k_{g,h,t}\, \overline{p}_t}{p^k_{c,t}\, \overline{p}_{g,h,t}} \right)^{\sigma_v} \tag{2.55}$$

$$\ell_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) = \bar{\ell}_{g,h,t}\, z_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) \left(\frac{p^k_{g,h,t}\, \overline{p}_t}{p^k_{l,t}\, \pi_{g,h,t}\, \overline{p}_{g,h,t}} \right)^{\sigma_v} . \tag{2.56}$$

2.5.2.4 Recalibration of the Ramsey Agent's Preferences

The last step in each iteration is to construct a new Ramsey optimal growth problem by recalibrating the Ramsey agent's preferences based on optimal household choices from the previous step. This is accomplished by updating level parameters in (2.41) according to:

$$\overline{C}^{k+1}_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} c_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) \tag{2.57}$$

$$\overline{\mathscr{L}}^{k+1}_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \ell_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) \tag{2.58}$$

$$\overline{Z}^k_t = \overline{C}^k_t + \overline{\mathscr{L}}^k_t \tag{2.59}$$

and value share parameters in (2.41) according to:

$$\Delta^{k+1}_t = \frac{p^k_{c,t}\, \overline{C}^k_t}{p^k_{c,t}\, \overline{C}^k_t + p^k_{l,t}\, \overline{\mathscr{L}}^k_t} \tag{2.60}$$

$$\Theta^{k+1}_t = \frac{p^k_{c,t}\, \overline{C}^k_t + p^k_{l,t}\, \overline{\mathscr{L}}^k_t}{\sum_{t'} \left(p^k_{c,t'}\, \overline{C}^k_{t'} + p^k_{l,t'}\, \overline{\mathscr{L}}^k_{t'} \right)} . \tag{2.61}$$

With varying prices, households adjust their labor supply, and hence the composition of the labor force with respect to age and household type is altered. But since labor productivity depends on these two socio-economic characteristics, aggregate labor productivity in the underlying OLG economy also changes. We therefore adjust the time endowment of the Ramsey agent in each iteration according to:

$$\Omega^{k+1}_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \pi_{g,h,t} \left(\omega_{g,h,t} - \ell_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) \right) + \ell_{g,h,t}(\mathbf{p}^k_{g,h}, M^k_{g,h}) . \tag{2.62}$$

Thus, the newly constructed Ramsey optimal growth problem in iteration $k+1$ consists of solving (2.41) (subject to (2.42) and (2.43) with updated preference parameters and time endowment as defined by (2.57)–(2.62). This completes the description of the algorithm.

2.5.3 Algorithmic Performance

The OLG economy presented above has no analytical solution. In order to evaluate the algorithm, we therefore compare its performance to those of conventional simultaneous/direct solution methods. As a benchmark, we take a complementarity-based approach as suggested by Rasmussen and Rutherford (2004).

The base case parametrization of the economy is as follows. Households live for 51 years or $N = 50$. We set $\bar{r} = 0.05$, $\gamma = 0.01$, $\delta = 0.07$, $\nu_h = 0.8$, $\beta = 0.32$, and $\alpha = 0.8$. In the numerical analysis, we test the performance of the algorithm for a different number of household types H and also allow for various degrees of intra-cohort heterogeneity. For simplicity, we assume that σ_h, $h = 1,\ldots,H$, are generated by random draws from a uniform distribution defined over $[\underline{\sigma}, \overline{\sigma}]$. Likewise, differences in labor productivity are generated by randomly drawing $a_{g,h}$ from a uniform distribution with support $\underline{a} \leq a_{g,h} \leq \bar{a}$, where the parameter $a_{g,h}$ enters the labor productivity profile over the life cycle as: $\pi_{g,h,t} = \exp\left[4.47 + a_{g,h}(t-g) - 0.00067(t-g)^2\right]$. Furthermore, it is assumed that each type has equal size in the total population. The values for the aggregate data including tax payments in the initial benchmark are based on Input-Output tables for the U.S. economy in 1996 and are presented at the top of the corresponding computer programs. We solve the model for $T = 150$ years.

2.5.3.1 Solving For a Policy Shock: A Fundamental Tax Reform

We now present an illustrative application of the decomposition algorithm by solving for the effects of a policy change that in year zero unexpectedly and permanently reduces the capital income tax and introduces a consumption tax to endogenously balance the government budget. The capital income tax is reduced from a benchmark value of 28.4% to 22.9%.

We start out by considering a case where $H = 1$, $\sigma_h = 1.2$, and $a_{g,h} = 0.04$. Figure 2.9 shows the sequence of time paths for investment that emerges from the iterative procedure of the algorithm. The true transition path to a new steady state of the OLG economy as computed by the benchmark simultaneous solution method is labeled “OLG”. “Iteration 1” plots the impact of the tax reform scenario after the first iteration of the solution method. This is equivalent to what would be obtained from solving a Ramsey optimal growth model. Each subsequent iteration of the algorithm produces a new time path for investment that eventually converges to the true solution. We terminate the search process if $\|p^k_{c,t} - p^*_{c,t}\|_1 < 10^{-6}$. For the current model, this is achieved after 34 iterations. Figure 3.7 shows a similar picture for the welfare

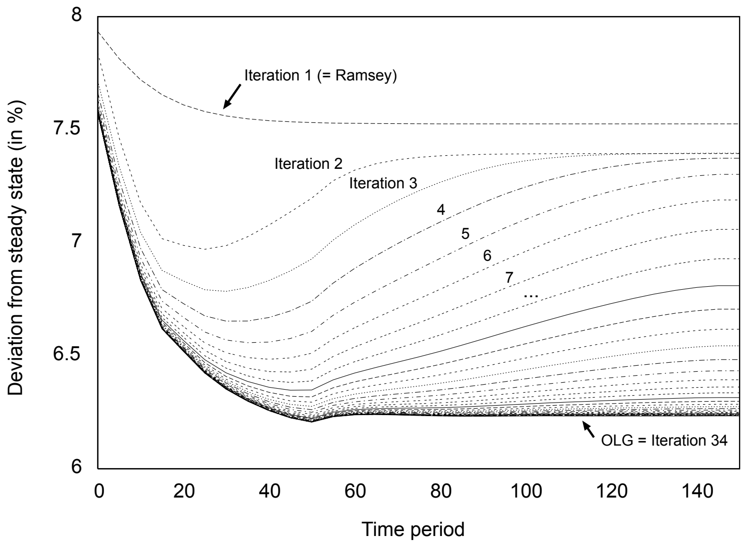

Fig. 2.9 Solving OLG by Ramsey: sequence of investment time paths

change experienced by each generation.[15] Note that in terms of welfare changes, stopping after the first iteration corresponds to a situation which would results from a pure top-down approach that fails to take into account general equilibrium feedback effects from the micro to the macro level.

To assess the quality of the approximation, we use the following two measures. First, the approximation error e^k reports the 1-norm of differences between computed consumption prices and true equilibrium prices as calculated by the benchmark method: $e^k = \|p^k_{c,t} - p^*_{c,t}\|_1$. As e^k constitutes a summary statistic which is defined over the entire model horizon, it says little about whether price deviations of the computed from the true price path lie within a tolerable bandwidth. As a second measure, we therefore report the maximum distance error τ^k which is defined as: $\tau^k = \max\{|p^k_{c,t} - p^*_{c,t}|\}$.

Figure 2.11 plots e^k as a function of the number of iterations. The approximation error quickly decreases and then converges to zero. After the first few iterations the decomposition technique only involves refinements of the demand system, and consequently, subsequent changes in relative prices are small.

[15] Kehoe and Levine (1985) have shown that the OLG framework may permit multiple equilibria for certain parameter values. In such cases, indeterminacy would manifest itself as sensitivity to the truncation date. None of the models presented here are sensitive to T, provided that it is sufficiently large. This and the general robustness of the models provide evidence that the equilibria are unique. Kotlikoff (2000) reaches the same conclusion regarding the uniqueness of equilibria in the OLG models he has been working with.

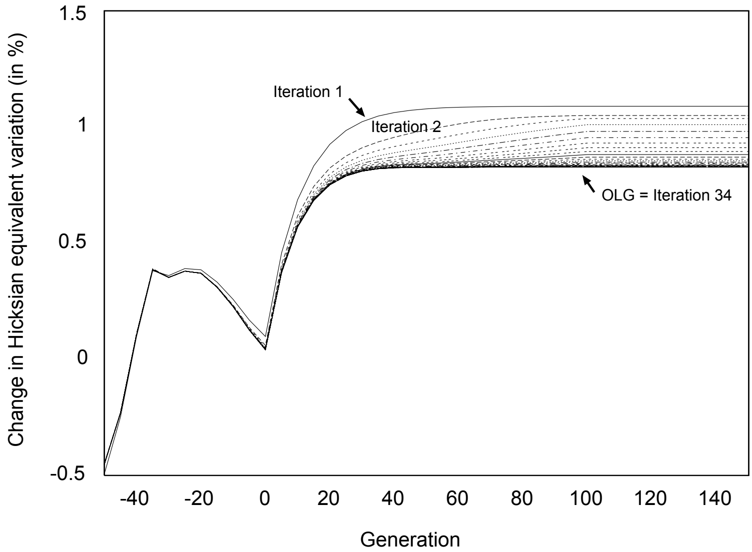

Fig. 2.10 Solving OLG by Ramsey: sequence of welfare changes by generation

2.5.3.2 Robustness and Accuracy

In order to explore the capacity of the algorithm to solve large-scale OLG models, we examine its performance for a different number of households and various degrees of intra-cohort heterogeneity. We look again at the effects of the tax reform scenario as described above. Given the simple specification for the source of intra-cohort differences, we vary the extent of heterogeneity, denoted by Γ, by changing the support for the distributions from which σ_h and $a_{g,h}$ are drawn.[16]

Table 2.1 reports results from a series of runs where the number of households within each generation is increased while holding fixed the degree of intra-cohort heterogeneity. The quality of approximation is excellent (τ_k is around 10^{-4}). As the number of household types increases, the proposed decomposition procedure become advantageous.[17] Most importantly, it is shown that the algorithm can provide improvements in robustness as compared to the benchmark simultaneous solution method which quickly becomes infeasible for models in which $H \geq 100$.

To examine the performance of the algorithm in the presence of a substantial degree of heterogeneity among households, we report results for different configura-

[16] We consider the following sets of choices for $\{(\underline{\sigma},\overline{\sigma}),(\underline{a},\overline{a})\}$ ordered by their implied degree of heterogeneity: $\Gamma_1 = \{(1.00,1.50),(0.2,0.3)\}$, $\Gamma_2 = \{(1.00,1.50),(0.2,0.4)\}$, $\Gamma_3 = \{(0.25,0.75),(0.2,0.3)\}$, $\Gamma_4 = \{(0.25,0.75),(0.2,0.4)\}$, $\Gamma_5 = \{(0.25,1.25),(0.2,0.3)\}$, $\Gamma_6 = \{(0.25,1.25),(0.2,0.4)\}$, $\Gamma_7 = \{(0.25,2.00),(0.2,0.3)\}$, $\Gamma_8 = \{(0.25,2.00),(0.2,0.4)\}$.

[17] All reported running times refer to an implementation on a Dual Core 2 GHz processor machine.

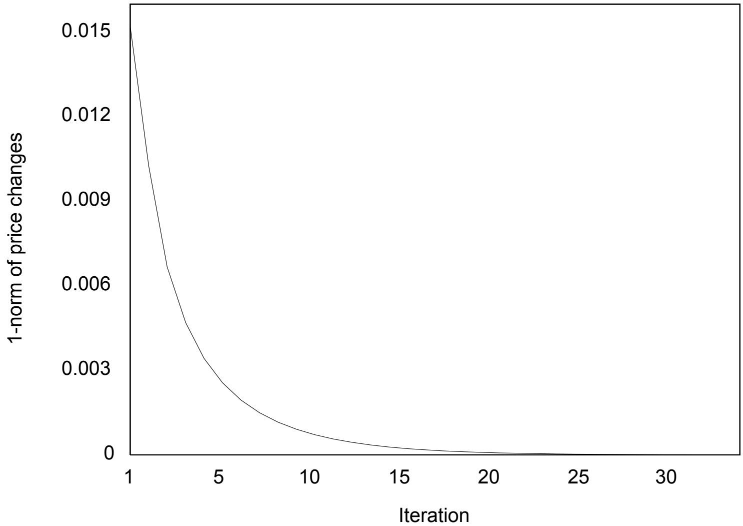

Fig. 2.11 Solving OLG by Ramsey: approximation error e^k

tions of Γ. We set $H = 50$ so that the benchmark solution method is feasible and the calculation of approximation errors is available. Not surprisingly, the approximation quality of the method is decreasing with the degree of heterogeneity. Overall, the quality of approximation is still very good: computed prices fall within a reasonably small interval around the true equilibrium price path (τ^k is around $10^{-5} - 10^{-3}$).

Motivated by the discussion of the potential convergence failure of the SR algorithm in the presence of significant income effects (Section 2.4), we conduct a number of sensitivity analyses for behavioral parameters governing intra-period and intertemporal substitution and income effects (see Table 2.3). Looking first at the intra-period dimension, we find that combinations of too small ν and α can pose serious problems for the decomposition approach. Although the search process is terminated within a modest number of iterations, both approximation measures indicate a rather poor quality of approximation for $\alpha \leq 0.5$. If α is too small, the equilibrium behavior over the life cycle of a household displays periods with zero labor supply in old ages, i.e. there is endogenous retirement. This happens because the shadow price of time exceeds the market wage rate. In the presence of such corner solutions, it is harder to portray the choices of OLG households by using a representative agent which in turn explains why the approximation error increases.

As for the role of intertemporal income effects, we do not experience problems of convergence or a poor quality of approximation (results not shown). However, the speed of convergence (in terms of the number of iterations required for convergence) is the slower, the larger is σ_h. This finding indicates that income effects

Table 2.1 Convergence performance and approximation error

H ($\Gamma = \Gamma_1$)	Number of iterations	Approx. error e^k (last iteration)	Max. distance τ^k (last iteration)	CPU computing time
1	37	10^{-6}	10^{-7}	0 min 13 s (3.78)
10	36	0.002	10^{-4}	0 min 21 s (1.17)
50	35	0.005	10^{-4}	1 min 10 s (0.18)
100	35	—	—	2 min 06 s (×)
500	34	—	—	6 min 14 s (×)
1000	34	—	—	10 min 48 s (×)
2000	29	—	—	30 min 31 s (×)

Note: Figures in parentheses denote running time of the decomposition algorithm expressed as a fraction of the running time as required by the benchmark simultaneous solution method. A "×" indicates infeasibility of the simultaneous solution method.

Table 2.2 Approximation errors for different Γ

Γ ($H = 50$)	Number of iterations	Approx. error e^k (last iteration)	Max. distance τ^k (last iteration)	CPU computing time
Γ_1	35	0.005	10^{-4}	1 min 10 s (0.18)
Γ_2	35	0.005	10^{-4}	0 min 56 s (1.14)
Γ_3	79	0.001	10^{-5}	2 min 14 s (0.39)
Γ_4	74	0.004	10^{-4}	2 min 23 s (0.42)
Γ_5	58	0.005	10^{-4}	6 min 14 s (0.29)
Γ_6	55	0.011	10^{-3}	1 min 44 s (0.26)
Γ_7	41	0.011	10^{-3}	1 min 10 s (0.18)
Γ_8	37	0.021	10^{-3}	1 min 15 s (0.22)

Note: Figures in parentheses denote running time of the decomposition algorithm expressed as a fraction of the running time as required by the benchmark simultaneous solution method.

stemming from an intertemporal reallocation of resources do not cause problems of convergence.

2.6 Concluding Remarks

This chapter develops a decomposition approach which can be applied to solve high-dimensional static and dynamic general equilibrium models with many households. We demonstrate its effectiveness for computing equilibria in large-scale OLG models which are infeasible for conventional simultaneous/direct methods. We find that the proposed algorithm provides an efficient and robust way to approximate general equilibrium in models with a large number of heterogeneous agents if income effects remain sufficiently weak. The appropriateness of the solution method there-

Table 2.3 Convergence behavior in the presence of strong income effects

ν , α ($H = 50$)	Number of iterations	Approx. error e^k (last iteration)	Max. distance τ^k (last iteration)	CPU computing time
0.50 , 0.80	33	0.015	10^{-3}	1 min 06 s (0.17)
0.25 , 0.80	30	0.035	10^{-3}	1 min 05 s (0.18)
0.80 , 0.50	42	0.014	0.013	1 min 27 s (0.19)
0.80 , 0.25	53	0.042	0.037	1 min 33 s (0.25)
0.50 , 0.50	42	0.117	0.011	1 min 15 s (0.23)
0.25 , 0.25	47	0.171	0.015	1 min 42 s (0.25)

Note: Figures in parentheses denote running time of the decomposition algorithm expressed as a fraction of the running time as required by the benchmark simultaneous solution method.

fore depends on the characteristics of the underlying model and the nature of the implemented policy shock.

We believe that the approach can be beneficial for a wide range of economic applications. In particular, it is advantageous for modeling tasks which necessitate to economize on the dimensionality of the corresponding numerical problem. Potential applications may include multi-country and multi-sectoral OLG models, and analyses of relevant policy issues—such as, e.g., population aging, trade policy, and poverty—which require detailed account of the distributional effects on a household level while at the same time taking into account general equilibrium effects. Moreover, the decomposition approach may prove useful for the further development of fully-integrated static and dynamic microsimulation models that incorporate the essential macroeconomic linkages required for a comprehensive policy analysis.

Chapter 3
Trade Liberalization and Global Demographic Change: A Quantitative Assessment

3.1 Introduction

The developed world stands at the fore of a historically unprecedented demographic transition. Over the next several decades the number of elderly in the U.S., the EU, and Japan will more than double. A salient feature of this global demographic change is the fact that population aging will occur at differing paces and with differing intensities in the industrialized countries of the world. Significant aging is already underway in some economies, for example Germany, Italy, and Japan. Major demographic changes in the U.S. and China begin in the second decade of the 21st century (Figure 3.1). With a still longer lag, the demographic trends will be manifest in developing economies as well. The presence of globally unsynchronized aging patterns and the ongoing globalization of the world economy make it necessary to enhance our understanding of the interaction of demographic factors and international capital, labor, and commodity flows.

While the existing literature consistently identifies the presence of globally unsynchronized aging patterns as an important driving force of cross-border flows of capital (see, e.g., Attanasio and Violante (2005), Börsch-Supan et al. (2006), INGENUE (2001)) and labor (see, e.g., Fehr et al. (2004b)), it—quite surprisingly—overlooks its implications for international trade in goods. This chapter aims to fill this void by developing a computable dynamic Heckscher-Ohlin model with overlapping generations (OLG) to evaluate the welfare effects of trade liberalization in the presence of global demographic change. The purpose of our analysis is twofold. First, we demonstrate that unsynchronized regional demographic patterns emerge as a potential determinant of international trade flows. Then, we ask whether trade liberalization under these circumstances can be beneficial. Based on standard neoclassical trade theory, international trade economists typically argue that a country is better off under an open trade regime relative to autarky.[1] Exploiting the substantial amount of household heterogeneity present in the model, it is possible to derive

[1] There is a vast literature which tries to assess whether trade liberalization is beneficial. Among those which tend to find a positive effect, are, e.g., Edwards (1993), Frankel and Romer (1999),

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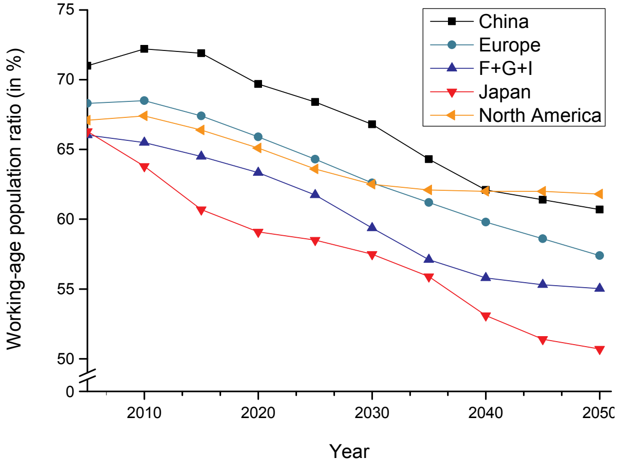

Fig. 3.1 Unsynchronized global aging patterns

Note: The working-age population ratio is defined as the population aged between 15-64 as a fraction of the total population. 'F+G+I' denotes the (unweighted) country average for France, Germany, and Italy. Source: Own calculations based on United Nations, World Population Prospects: The 2004 Revision. Medium projection variant.

welfare statements with respect to the inter- and intragenerational distribution of welfare gains and losses across regions. In contradiction to the "standard view" that international trade is Pareto-superior, we find that trade liberalization in the presence of globally unsynchronized demographic patterns does not necessarily imply welfare gains, i.e. openess may be immiserizing. This holds both for the household and social level, where in the latter case we use a range of different social welfare functionals to check for the sensitivity of utility aggregation across time and household types.

There are several innovations in this chapter which set it apart from the existing literature. First, we develop an augmented version of the canonical Heckscher-Ohlin model which combines key elements of the factor proportions theory of international trade with a computable OLG model in the tradition of Auerbach and Kotlikoff (1987). Such a framework enables to study the interaction of demographic change and international trade, and furthermore, introduces a role for relative prices and the sectoral allocation of resources to shape the economic response to demographic change. By adopting a standard macroeconomic single-commodity view of the world, existing studies (Attanasio and Violante (2005), Börsch-Supan et al. (2006), and Fehr et al. (2004b)) are neither capable of taking into account the impacts of relative price changes nor can they shed light on the patterns of demographically-induced trade in goods that are produced with different factor intensities.

and Glenn Harrison and Tarr (1997). Of course, this view has been subject to criticism. See, e.g., criticisms by Rodrik (1992) and Harrison and Hansen (1999).

The second innovation of this chapter is to analyze the distribution of gains from trade, both across time and generations of households and within each age group. We are not aware of any similar attempt in the literature. Besides the heterogeneity of age among overlapping generations, the model also features a substantial amount of intra-cohort heterogeneity: household types differ with respect to age-specific labor productivity and other key parameters governing the relative magnitudes of intra- and intertemporal income and substitution effects. Third, and in terms of computational methodology, we employ the decomposition algorithm proposed in Chapter 2 that approximates the equilibrium allocation of the OLG economy by solving a sequence of appropriately chosen two-country Ramsey growth models. Despite of the large dimensionality of the numerical problem at hand—resulting from the system of overlapping generations and the two-country setup—this decomposition method makes it possible to compute the transitional and steady state equilibrium effects of all endogenous model variables in response to a demographic shock.

The results of this chapter are as follows. Regionally unsynchronized demographics patterns lead to the emergence of international trade flows. This result is based on a simple yet convincing model mechanism. In a nutshell, an economy with a growing fraction of older people is bound to increase its accumulation of capital and bound to decrease its aggregate labor supply. Capital becomes abundant relative to labor because life-cycle savings and labor supply behavior of overlapping generations entails that older households tend to hold more assets and work less compared to younger ones. Factor prices change accordingly to reflect the varying scarcity of factors of production, i.e. wages go down and real interest rates increase. This in turn creates differences in production costs of commodities that are produced with different factor intensities. The induced sectoral change is characterized by a relative contraction (expansion) of the labor-intensive (capital-intensive) sector. Under autarky, differences in demographic patterns across regions translate into international differences in the relative abundance of factors of production: a relatively fast aging country becomes capital-abundant and labor-scarce vis-á-vis a slow aging country. Under such circumstances, opening up a country for trade leads to the emergence of demographically-induced Heckscher-Ohlin trade flows.

If demographic differences emerge as a potential determinant of international trade flows, the question arises whether a country or a region can benefit from diffusing part of its demographic shock abroad by means of liberalizing its trade policy. Since trade creates a tendency to reduce the dispersion in international factor and commodity prices, opening up an economy for trade might “cushion” the adverse consequences of population aging. However, as real wages and interest rates move in different directions the overall effect on income and welfare is not unambiguous. Moreover, the dynamic paths of equilibrium factor prices hinge on the extent and timing of the demographic change in both regions. Thus, the qualitative and quantitative effects from factor price changes will be distinct for each generation. Lastly, households are not only heterogeneous with respect to their age but also exhibit fundamental differences in preferences and ability. This point is taken into account by allowing for 10 different household types within each generation.

In contrast to what one would expect from standard neoclassical trade theory, gains from trade in the presence of unsynchronized global aging patterns are not guaranteed, i.e. openess may be immiserizing. This is a result of the evolution of autarky-trade factor price differentials during the demographic transition. We find that, for the fast aging region, trade liberalization is only beneficial for current old generations that are born before and in the beginning of the demographic transition. Future generations of all household types stand to incur substantial utility losses. The intra-generational distribution of welfare gains and losses depends on the level of labor productivity and the propensity to save throughout the life cycle. "Poor" households, i.e. low-skilled and asset-poor households, tend to be relatively worse off. We perform a systematic sensitivity analysis over a broad range of empirically plausible parameters and find that there is a $80,7\%$ probability for households in the fast aging region not to gain from trade liberalization. Moreover, looking at the benefits of trade from a country/region perspective, we find that aggregate social welfare gains require unrealistically high discount rates of about 8% and more. Despite of the lack of social desirability for an open trade regime, the political assessment of trade liberalization by means of an "hypothetical democratic referendum" undertaken at the beginning of the demographic transition would support a liberal stance of trade policy at the expense of future generations.

A substantial and growing literature has been calling attention to the economic implications of unsynchronized global aging patterns in open economies. Besides empirical studies[2] computable overlapping generations (OLG) models have emerged as the dominant analytical tool. For instance, Attanasio and Violante (2005), Feroli (2003), Brooks (2003), and Domeij and Floden (2004) overwhelmingly find that unequal demographic trends across countries can potentially induce global capital flows if capital markets are integrated.[3] A second strand of the literature analyzes the role of pension reform and associated savings patterns for international capital flows in the light of population aging (Attanasio and Violante (2005), Börsch-Supan et al. (2006), INGENUE (2001)) and finds that capital flows from fast aging countries to the rest of the world will initially be substantial but that trends are reversed when aging progresses and households decumulate savings. Moreover, the status quo of public pension systems is shown to crucially determine the magnitudes of interna-

[2] See Bryant and McKibbin (1998, 2003) for good surveys of multi-country macroeconometric simulation models that incorporate demographic change.

[3] For example, Feroli (2003) uses a multi-region OLG model that is calibrated to match demographic differences among the major industrialized countries over the past 50 years and finds that demographic differences can explain not all but some of the observed long-term capital movements; Brooks (2003) finds that retirement saving by aging baby boomers will raise the supply of capital substantially above investment in both the European Union and North America, causing both regions to export large amounts of capital to Latin America and other emerging markets in the years ahead. Beyond 2010, however, baby boomers in the European Union and North America will dissave in retirement, causing both regions to become capital importers. This shift will be financed by capital flows from Latin America and other emerging markets, while Africa will remain dependent on foreign capital for the foreseeable future because of continued high population growth. Domeij and Floden (2004) empirically test to what extent international capital flows predicted by the model match historically observed current account positions and find that a small but significant fraction of international capital movements can be explained by the simulation model.

tional capital movements. A series of papers by Fehr, Jokisch and Kotlikoff (2003), Fehr et al. (2004a), and (n.d.) develop closed- and open economy OLG models that allow, among many other features, for joint mobility of capital and labor across regions. For instance, Fehr et al. (2004b) find that even a significant expansion of immigration, whether across all skill groups or among particular skill groups, will do remarkably little to alter the major capital shortage, tax hikes, and reductions in real wages that can be expected along the demographic transition.

The rest of this chapter is organized as follows. Section 3.2 sets out the analytical framework. Section 3.3 explains our calibration strategy and specifies the region-specific demographic dynamics. Section 3.4 presents the main model results looking at the economic implications of unsynchronized global aging patterns under autarky and trade, and also discusses the inter- and intragenerational distribution of gains from trade. Section 3.4.5 performs a number of sensitivity analyses to provide a comprehensive quantitative assessment of trade liberalization under global demographic change. Section 3.5 concludes.

3.2 Model Formulation

To provide a quantitative assessment of trade liberalization under global demographic change, we develop a numerical $2 \times 2 \times 2$ (two regions–two factors–two commodities) dynamic general equilibrium model that combines elements of the factor-proportions theory of international trade with an OLG framework in the tradition of Auerbach and Kotlikoff (1987).[4] To isolate the effects of globally unsynchronized aging patterns, we adopt the simplest possible model structure and use a neoclassical constant-returns-to-scale growth model where all markets are perfectly competitive. There is no aggregate or household-specific uncertainty. Time is discrete and extends to infinity, $t = 0, \ldots, \infty$. The two regions, indexed by $r, s = a, b$, are assumed to be identical with respect to production technologies (and structure) and household preferences. Within regions, overlapping households differ with respect to labor productivity and a number of behavioral parameters that govern the intra-period and life-cycle behavior. The two regions differ only with respect to the exogenously imposed population dynamics.

International borrowing and lending is not permitted which implies in this model that trade is balanced in each period. In doing so, we focus on the international exchange of goods as the sole link between regions and therefore rule out international capital flows. Moreover, we do not consider any form of government activity and in particular abstract from a public sector pension system. It can be argued, however,

[4] In the international trade literature, OLG models are still used very rarely. Exceptions are models that have been put forward by Mountford (1998), Sayan (2005), and Bajona and Kehoe (2006)—all extending the seminal contribution by Galor (1992) to a two-country context. To maintain analytical tractability, however, they abstract from a number of structural model components, e.g., multi-period lived agents and endogenous labor supply, that are pivotal for the type of analysis we carry out here.

that the model describes a world where a fully-funded pension system is in place that implicitly operates through the (private) life-cycle savings behavior of households.

3.2.1 Heterogeneous Households

Households live over a deterministic lifespan of $a = 0, \ldots, N$ periods. A generation $g = 0, \ldots, \infty$ is born at the beginning of each year $t = g$. Generations born prior to year zero and that are alive in the first model period are indexed by $-N, \ldots, -1$. Within each generation, a household of type h, $h = 1, \ldots, H$, is characterized by the following set of socio-economic characteristics:

$$\Omega_h = \{\Pi_h, \sigma_h, \nu_h, \alpha_h\} \tag{3.1}$$

where Π_h denotes a labor productivity profile over the life cycle, σ_h is the intertemporal elasticity of substitution, $\sigma_{cl,h} = 1/(1-\nu_h)$ is the elasticity of substitution between consumption and leisure, and α_h determines the relative importance of material consumption vis-à-vis leisure consumption. The presence of low and high-skilled workers as well as different preferences with regard to the substitutability of intra- and intertemporal consumption implies different savings and consumption behavior over the life cycle that in turn give rise to qualitatively and quantitatively different welfare implications of trade liberalization for households (for more details on the empirical specification of household parameters see section 3.3.1). Note that this setup implies that at each point in time there are $H \times (N+1)$ different household types that coexist. In the numerical examples below, we set $N = 14$ and $H = 10$, so that $H \times (N+1) = 150$.

In each period over the life cycle households are endowed with units of time that they allocate between labor and leisure. Households are assumed to be for-ward-looking individuals that form rational point expectations (perfect foresight) over the infinite horizon. Lifetime utility of generation g and type h in region r, $u_{g,h,r}$, is additively separable over time and is of the constant-intertemporal-elasticity-of-substitution form (CIES). The representative agent of each generation and type chooses optimal consumption and leisure paths over his life cycle subject to lifetime budget and time endowment constraints. The optimization problem for generation g and household type h in region r is given by:

$$\begin{aligned} \max_{c_{r,g,h,t},\,\ell_{r,g,h,t}} \quad & u_{r,g,h}\left(z_{r,g,h,t}\right) = \sum_{t=g}^{g+N} \left(\frac{1}{1+\rho}\right)^{t-g} \frac{z_{r,g,h,t}^{1-\sigma_h^{-1}}}{1-\sigma_h^{-1}} \\ s.t. \quad & z_{r,g,h,t} = \left(\alpha_h c_{r,g,h,t}^{\nu_h} + (1-\alpha_h)\, \ell_{r,g,h,t}^{\nu_h}\right)^{\frac{1}{\nu_h}} \end{aligned}$$

$$\sum_{t=g}^{g+N} p_{r,a,t}\, c_{r,g,h,t} \leq p_{r,k,t}\, \overline{k}_{r,g,h,g} + \sum_{t=g}^{g+N} p_{r,l,t}\, \pi_{g,h,t}\, (\omega_{r,g} - \ell_{r,g,h,t})$$
$$\ell_{r,g,h,t} \leq \omega_{r,g}$$
$$c_{r,g,h,t} \geq 0 \quad , \quad \ell_{r,g,h,t} \geq 0\,. \tag{3.2}$$

Here, material consumption c and leisure consumption ℓ are combined to form a composite consumption good z.[5] ρ is the utility discount factor, and $p_{r,x,t}$, $x = \{a,k,l\}$, denote the price for the output good, the purchase price of capital, and the wage rate, respectively. $\pi_{g,h,t}$ is an index of labor productivity over the life cycle and $\omega_{r,g}$ denotes the periodic time endowment of generation g. All household types within a generation are of equal population size. The present value of total consumption expenditure over the lifetime cannot exceed the present value of labor income. This rules out that households die in debt. In each period of the life cycle, time allocated to leisure consumption cannot exceed the total time endowment. Choices for material and leisure consumption are restricted to be nonnegative.[6]

$\overline{k}_{r,g,h,g}$ denotes the capital holdings of generation g at the beginning of life $t = g$. Initial old generations, i.e. generations born prior to period zero, are endowed with a non-zero amount of capital. The initial distribution of capital across these generations is selected such that the economy is on a balanced growth path (for details on the calibration procedure see Appendix 3.5.1). We assume that newborn households enter with zero capital: $\overline{k}_{r,g,h,g} = 0$, $\forall\, g \geq 0$, and $\forall\, h$.

The time endowment of a generation ($\omega_{r,g}$) evolves according to:

$$\omega_{r,g+1} = (1 + \gamma_{r,g})\, \omega_{r,g} \tag{3.3}$$

where $\gamma_{r,g}$ is the region-specific and time-dependent fertility rate. The size of the generation born at the beginning of year zero is normalized to unity. Note that there is no growth in time endowments over the life cycle. Thus, while the number of households across generations increases over time, the size of a cohort over its life cycle remains constant. Total population $N_{r,t}$ in region r at time t is given by $N_{r,t} = H \sum_{g=t-N}^{t} \omega_{r,g}$. The population growth rate $\widetilde{\gamma}_{r,t}$ can thus be defined as:

$$\widetilde{\gamma}_{r,t} = \frac{N_{r,t+1}}{N_{r,t}} - 1\,. \tag{3.4}$$

To maintain a balanced growth path, the age structure of the population has to be stationary which requires that $\gamma = \widetilde{\gamma} = const.$[7]

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[5] Households first decide how to allocate their lifetime income over time. Given the expenditure for z, households decide in a second stage how much to spend on consumption and leisure. The assumption of multi-stage budgeting is innocuous if and only if the utility function u is weakly separable and the sub-utility functions z are homothetic. Both conditions are satisfied in this model.

[6] Note that due to the convex structure of CES-preferences the nonnegativity constraints on c and l are never binding in the optimum.

[7] Furthermore, it is straightforward to show that the lower γ, the lower is $\widetilde{\gamma}$ and the lower is also the working-age population ratio, denoted by $W_{r,t}$, which we define here as the number of people

3.2.2 Firms and Production Structure

Production occurs within a period and technologies are stationary over time.[8] Competitive firms in both regions have access to two linearly homogenous production technologies of the constant-elasticity-of-substitution (CES) form, index-ed by $i, j \in \{1,2\}$, that combine services of capital, $K_{t,r,i}$, and labor, $L_{t,r,i}$ to produce a homogenous good:

$$Y_{i,r,t} = \left(\alpha_i K_{i,r,t}^{\varepsilon_i} + (1-\alpha_i) L_{i,r,t}^{\varepsilon_i}\right)^{1/\varepsilon_i} \tag{3.6}$$

where α_i is a sector-specific share parameter and where $\sigma_{Y,i} = 1/(1-\varepsilon_i)$ is the elasticity of substitution. For future reference, let $p_{i,t}$ denote the price of the traded good i. Factors of production can move freely across sectors within a region but are assumed to be immobile internationally. We make the assumption that production of $Y_{1,r,t}$ is capital-intensive relative to $Y_{2,r,t}$:

$$\alpha_1 > \alpha_2 \,. \tag{3.7}$$

Capital intensity reversals at all factor and commodity prices are ruled out by the functional form of (3.6) and the additional assumption that $\varepsilon_1 = \varepsilon_2$.[9] This assumption is central to the results and determines along with changes in the relative abundance of factors the structure of trade patterns.

The sectoral output good $Y_{i,r,t}$ can be traded on international markets. There are no transportation costs or any other type of trade barriers. Domestically produced and imported quantities of $Y_{i,r,t}$ are combined by a CES-aggregator to form a final good $A_{t,r}$:

$$A_{r,t} = \left(\alpha_a a_{1,r,t}^{\varepsilon_a} + (1-\alpha_a) a_{2,r,t}^{\varepsilon_a}\right)^{1/\varepsilon_a} \tag{3.8}$$

where α_a is a share parameter and $\sigma_a = 1/(1-\varepsilon_a)$ denotes the elasticity of substitution. $a_{i,r,t}$ denotes the amount of the tradable good i used in the production of $A_{r,t}$. Let $p_{r,a,t}$ denote the price for the final good in region r. The final good can be used for domestic consumption and investment purposes. For future reference, let $p_{r,re,t}$ denote the capital rental rate.

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aged between $0-0.6N$ as a fraction of the total population, i.e. :

$$W_{r,t} = \frac{H \sum_{g=t-N+0.6N}^{t} \omega_{r,g}}{N_{r,t}} \,. \tag{3.5}$$

[8] We solely focus on the role of population aging in determining growth dynamics and do not consider technological change. Adding another exogenous engine of growth does not change the results with respect to the implications of differential population dynamics.

[9] See Wong (1990).

3.2.3 Feasibility Conditions and Model Closure

In the following, let upper case letters denote the respective aggregate variable. Given intergenerational heterogeneity, we impose the following feasibility conditions to ensure that individual household behavior is consistent with the aggregate economy:

$$L_{r,t} = \sum_{g=t-N}^{t} \sum_{h=1}^{H} \omega_{r,g,t} - \ell_{r,g,h,t} \tag{3.9}$$

$$I_{r,t} = \sum_{g=t-N}^{t} \sum_{h=1}^{H} i_{r,g,h,t} \tag{3.10}$$

$$K_{r,t} = \sum_{g=t-N}^{t} \sum_{h=1}^{H} k_{r,g,h,t} \tag{3.11}$$

$$C_{r,t} = \sum_{g=t-N}^{t} \sum_{h=1}^{H} c_{r,g,h,t} \,. \tag{3.12}$$

The feasibility conditions for factors of production are:

$$K_{1,r,t} + K_{2,r,t} \leq K_{r,t} \tag{3.13}$$

$$L_{1,r,t} + L_{2,r,t} \leq L_{r,t} \,. \tag{3.14}$$

World markets for traded good i, $i = 1,2$, in period t, clear if:

$$\sum_{r} Y_{i,r,t} = \sum_{r} a_{i,r,t} \,. \tag{3.15}$$

The law of motion for the aggregate capital stock for each region is given by:

$$K_{r,t+1} = (1-\delta)\,K_{r,t} + I_{r,t} \,. \tag{3.16}$$

In each period t, the composite consumption good can be used for consumption and investment:

$$A_{r,t} = C_{r,t} + I_{r,t} \,. \tag{3.17}$$

Finally, and to close the model, we impose the restriction that international borrowing and lending is not permitted.[10] In addition, we assume that neither of the regions has fiat money, i.e. $\sum_{g=-N}^{0} \overline{k}_{r,g,h,g} = (1+\bar{r})\,\overline{K}_r$. Here, $\bar{r}$ and $\overline{K}_r$ denote the steady state interest rate and the base year capital stock in region r, respectively. The assumption that international trade is balanced in each period can then be written as:

$$p_{r,a,t}\,(A_{r,t} - C_{r,t} - I_{r,t}) = 0 \,. \tag{3.18}$$

[10] Bajona and Kehoe (2006) show in a similar setting that if international borrowing and lending takes place, i.e. a country can run a temporary trade imbalance, the equilibrium patterns of capital, international borrowing and lending, and trade are indeterminate.

This condition can be derived by rewriting the lifetime budget constraint in (3.2) as a series of periodic budget constraints, aggregating the periodic constraints over all generations and household types that are alive at a given point in time, and finally making use of Euler' theorem for homogeneous functions.

3.2.4 Equilibrium

Definition 1: Given initial endowments of of capital, $\overline{k}_{r,g,h,g}$, for generations $g = -N, \ldots, -1$, for all h, i.e. those born prior to year zero, a *competitive equilibrium* is a collection of choices for households $\{c_{r,g,h,t}, \ell_{r,g,h,t}\}_{t=0}^{\infty}$, $t = g, \ldots, g+N$, for all $g = -N, \ldots, \infty$, for all h, and for the representative firm in each traded industry $\{Y_{i,r,t}, K_{i,r,t}, L_{i,r,t}\}_{t=0}^{\infty}$, $i = 1,2$, and for the representative firm in the final goods sector, $\{A_{r,t}, Y_{1,r,t}, Y_{2,r,t}\}_{t=0}^{\infty}$, and prices $\{p_{r,a,t}, p_{r,l,t}, p_{r,re,t}, p_{r,k,t}, \ p_{i,t}\}_{t=0}^{\infty}$, $i = 1,2$, and for all regions, $r = a,b$, such that:

1. Given prices $\{p_{r,a,t}, p_{r,l,t}, p_{r,re,t}, p_{r,k,t}, p_{i,t}\}_{t=0}^{\infty}$, the choices $\{c_{r,g,h,t}, \ell_{r,g,h,t}\}$ solve the respective optimization problem in (3.2).
2. Given prices $\{p_{r,a,t}, p_{r,l,t}, p_{r,re,t}, p_{r,k,t}, p_{i,t}\}_{t=0}^{\infty}$, the representative firm in each traded industry, $i = 1,2$, and in the final goods sector maximizes profits, i.e. $\{K_{i,r,t}, L_{i,r,t}\} \in \arg\ \max_{K_{i,r,t}, L_{i,r,t} \geq 0} \ p_{i,t} Y_{r,i,t}(K_{i,r,t}, L_{i,r,t}) - p_{r,re,t} K_{i,r,t} - p_{r,l,t} L_{i,r,t}$ and $\{a_{1,r,t}, a_{2,r,t}\} \in \arg\ \max_{a_{1,r,t}, a_{2,r,t} \geq 0} \ p_{r,a,t} A_{r,a,t}(a_{1,r,t}, a_{2,r,t}) - p_{1,t} a_{1,r,t} - p_{2,t} a_{2,r,t}$, respectively.
3. The sequences of household choices $\{c_{r,g,h,t}, \ell_{r,g,h,t}\}_{t=0}^{\infty}$, and production plans, $\{Y_{i,r,t}, \ K_{i,r,t}, L_{i,r,t}\}_{t=0}^{\infty}$, and $\{A_{r,t}, Y_{1,r,t}, Y_{2,r,t}\}_{t=0}^{\infty}$, satisfy the feasibility conditions (4.33)–(3.18).

Definition 2: A *steady state* is a collection of choices for households $\left\{\widehat{c}_{r,g,h,t}, \widehat{\ell}_{r,g,h,t}\right\}_{t=0}^{\infty}$, and for the representative firm in each traded industry $\{\widehat{Y}_{i,r,t}, \widehat{K}_{i,r,t}, \widehat{L}_{i,r,t}\}_{t=0}^{\infty}$, and for the representative firm in the final goods sector, $\{\widehat{A}_{r,t}, \widehat{Y}_{1,r,t}, \widehat{Y}_{2,r,t}\}_{t=0}^{\infty}$, and prices $\{p_{r,a,t}, p_{r,l,t}, p_{r,re,t}, p_{r,k,t}, \ p_{i,t}\}_{t=0}^{\infty}$, that satisfy the conditions of a competitive equilibrium for appropriate initial endowments of capital $\overline{k}_{r,g,h,g} = \widehat{k}_{r,g,h,g}$, and where the age structure of the population is stationary, i.e. $\gamma = \widetilde{\gamma} = const$. Here, we set $v_t = \widehat{v}$ for all t, where v represents a generic variable.

Table 3.1 Base case parametrization of intracohort household heterogeneity

Household type	σ_h	$\sigma_{cl,h}$	α_h	$\Pi_h = \{\zeta_{\text{scale},h}, \zeta_{\text{age},h}, \zeta_{\text{age2},h}\}$
$h = 1$ (poorest household)	0.50	0.50	0.45	$\{0.50, 0.0450, 0.00400\}$
$h = 2$	0.60	0.60	0.50	$\{0.55, 0.0475, 0.00400\}$
$h = 3$	0.70	0.70	0.55	$\{0.60, 0.0500, 0.00375\}$
$h = 4$	0.80	0.80	0.60	$\{0.65, 0.0525, 0.00375\}$
$h = 5$	0.90	0.90	0.65	$\{0.70, 0.0550, 0.00355\}$
$h = 6$	1.00	1.00	0.70	$\{0.75, 0.0575, 0.00355\}$
$h = 7$	1.20	1.10	0.75	$\{0.80, 0.0600, 0.00325\}$
$h = 8$	1.30	1.20	0.80	$\{0.85, 0.0625, 0.00325\}$
$h = 9$	1.40	1.30	0.85	$\{0.90, 0.0650, 0.00300\}$
$h = 10$ (richest household)	1.50	1.40	0.90	$\{1.00, 0.0675, 0.00300\}$

3.3 Calibration and the Nature of Demographic Shocks

The model outlined above does not possess an analytical solution and therefore has to be solved numerically. Appendix 3.5 formulates the dynamic general equilibrium of the model as a mixed-complementary problem (MCP) and provides more details on the steady state calibration of the model and the solution algorithm that is used to compute the transitional dynamics and steady state effects of the demographic simulation experiments. For solving the model, we use the decomposition algorithm presented in Chapter 2. In the present context, the equilibrium of the OLG economy is approximated by solving a sequence of appropriately chosen two-country, two-sector Ramsey growth models.

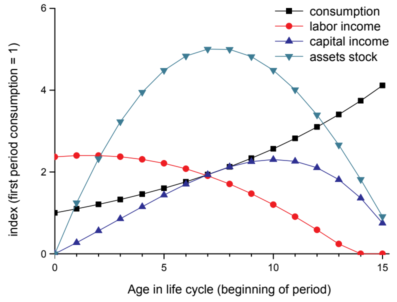
3.3.1 Calibration Strategy

Given the oversimplified and illustrative nature of the model, we adopt a naive calibration approach that builds on standard values from the literature.[11] Section 3.4.5 performs a number of sensitivity analyses to check for the robustness of the results.

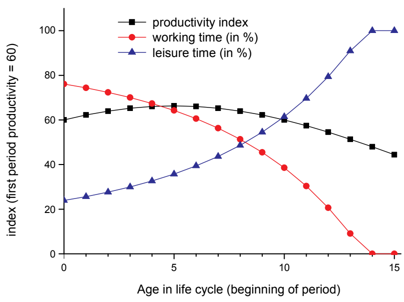
For the technology parameters, we set $\alpha_1 = 0.7$, $\alpha_2 = 0.3$, $1/(1-\varepsilon_i) = 1$, $i = 1,2$, $\alpha_a = 0.5$, $1/(1-\varepsilon_a) = 1$, and $\delta = 0.1$.[12] The model is calibrated to a steady state equilibrium where we assume that along the baseline path quantities grow with $1+\bar{\gamma} = 1.02$ and present value prices decline with $1+\bar{r} = 1.1$. We reconcile individual decisions by the OLG households with aggregate data in the base year by

[11] Numerical values for the exogenous model parameters are taken from Backus, Kehoe and Kydland (1992) and Ambler, Cardia and Zimmermann (2002).

[12] For the base case, we assume Cobb-Douglas technologies for $Y_{i,r,t}$ and $A_{r,t}$, i.e. $\varepsilon_i = \varepsilon_a = 0$. For these values the CES functions in (3.6) and (3.8) are not defined. However, as ε_i and ε_A approach zero, the isoquants of the CES production functions look very much like the isoquants of the Cobb-Douglas production function.



(a) Baseline profiles and asset stocks



(b) Productivity index and allocation of time

Fig. 3.2 Calibrated life cycle behavior for $h = 3$ (initial steady state)

Note: Optimal life cycle behavior for all generations and household types along the baseline path is derived in the first calibration step using a steady state assumption. For details see Appendix 3.5.1. The household calibration model is parameterized as shown in Table 3.1.

calibrating the discount factor ρ such that the sum of the value of individual consumption demands equals aggregate consumption. Likewise, to match individual asset holdings with the value of the aggregate capital stock in the base year, we introduce a scaling factor on the time endowment of households and select its value accordingly.[13]

Naturally, intra-period and intertemporal household behavior crucially hinges on assumptions regarding various elasticities parameters. The way of parameterizing household behavior takes the extreme standpoint that the uncertainty of empirical parameter estimates is to a large extent a result of differences in underlying household preferences. Hence, we incorporate "uncertainty" in a direct fashion by allowing for different preferences within each age group. In 3.4.5 we use Monte-Carlo simulation methods to check for the general robustness of results over a broad range of empirically plausible parameter configurations.

Following Auerbach and Kotlikoff (1987), we assume that labor productivity follows a hump-shaped profile over the life cycle:

$$\pi_{a,h} = \zeta_{\text{scale},h}\, e^{\left(4.47 + \zeta_{\text{age},h}\, a - \zeta_{\text{age2},h}\, a^2\right)} / e^{4.47}\,. \tag{3.19}$$

For each household type, we slightly perturb their numerical specification thereby allowing for differences in labor productivity. Labor productivity across generations of a given household type is assumed to be identical. Table 3.1 shows the parameter specification which is used in the numerical simulation later on. The model is solved in one-year intervals and $N = 14$ which implies that households live for 15 years.

[13] Since income is linear in this scaling factor and demand is linear in income, such a scaling parameter has no economic significance. For more details of the steady state calibration procedure see Appendix 3.5.1.

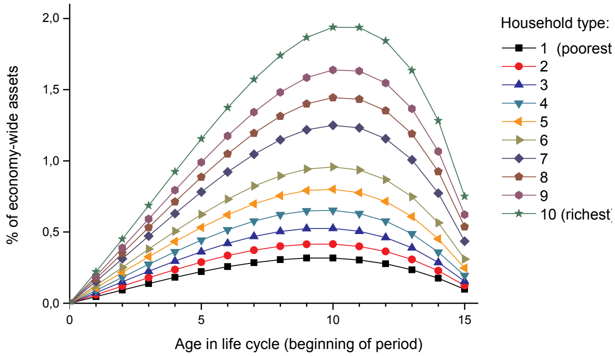


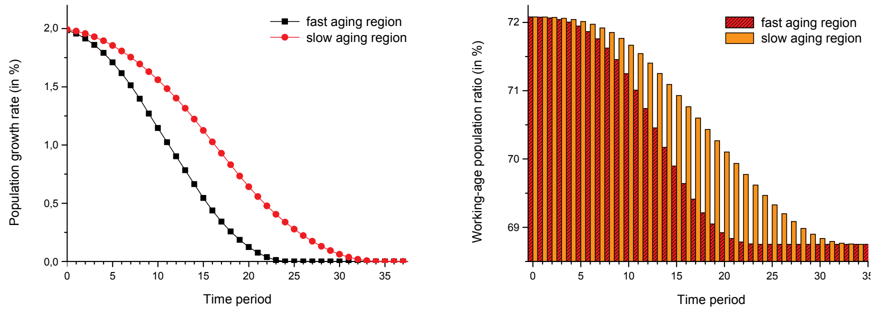
Fig. 3.3 Baseline equity shares by household type and age

Note: The plot shows the amount of assets held by different household types through the life cycle as a percentage fraction of economy-wide assets (along the steady state baseline path).

This is found to be sufficient for generating a meaningful life cycle behavior on the part of the households and for studying the implications of population aging.

Figure 3.2, Panel (a), shows the calibrated steady state income and consumption profiles and asset stocks over the life cycle for a household of type $h = 3$. For the interpretation of the results later on, it is important to understand the nature of the life cycle behavior which builds on the foundation developed by Modigliani and Brumberg (1954). Households arrange savings so as to smooth consumption over their lifetime—as it is implied by the intertemporal Euler equation. Due to the hump-shaped productivity profile, time devoted to labor and consequently labor income is increasing in the first years and then decreases—as it is implied by the intratemporal Euler equation for leisure. Households accumulate assets in younger to middle ages and run down their asset positions when being old; capital income evolves accordingly. In the last period of the life cycle, consumption is entirely financed out of savings since households optimally chose not to earn labor income. Figure 3.2, Panel (b), shows the allocation of leisure and working time over the life cycle. With the given parametrization, labor supply drops to zero in the two last periods of the life cycle with the reservation wage exceeding the market wage. An attractive feature of this analysis is the use of complementarity programming to accommodate endogenous regime shifts—as here, for instance, optimal retirement decisions by households.

As a consequence of the differences in intra- and intertemporal behavior of households, there are large dissimilarities with respect to the optimal wealth accumulation path. Figure 3.3 illustrates this point and shows the amount of assets that is being held by each household type throughout the lifetime as a fraction of economy-wide assets.



(a) Population growth rate (b) Working-age population ratio

Fig. 3.4 Demographic shocks and population aging

Note: Panel (a) shows the changes in the population growth rate $\widetilde{\gamma}_{r,t}$ as implied by the fertility rate shocks. Panel (b) shows the impact on the working-age population ratio $W_{r,t}$.

3.3.2 Region-specific Population Dynamics

Population aging is modeled by assuming that fertility rates decrease over time.[14] Shocks to the fertility rate rate alter the age composition of the population. Globally unsynchronized aging patterns are modeled by assuming that the speed of decline in fertility rates varies across regions. Region a is taken to be the fast aging region vis-à-vis a slow aging region b. In the initial steady state, both regions are characterized by an identical fertility rate $\bar{\gamma}$. From year zero onwards population dynamics begin to diverge and both regions experience a linear decline in fertility rates. Eventually, fertility rates in both region converge to a new and common long-run level $\gamma^{SS} < \bar{\gamma}$. More specifically, the time profile for the fertility rates in region r is given by:

$$\gamma_{r,g} = \begin{cases} \bar{\gamma} & \text{if} \quad g \leq 0 \\ \bar{\gamma} - \Delta_r t & \text{if} \quad 0 < g \leq B_r \\ \gamma^{SS} & \text{if} \quad g > B_r\,, \end{cases} \tag{3.20}$$

where $\Delta_a = 0.02$, $\Delta_b = 0.01$, $B_a = 10$, $B_b = 20$. Without loss of generality, we set $\gamma^{SS} = 0$. Figure 3.4 shows the implied changes in population growth rates and the working-age population ratio. Note that $\gamma^{SS} < \bar{\gamma}$ implies that the fraction of old people in the new steady steady is lower than in the initial equilibrium. Also, it is important to bear in mind that the demographic transition extends beyond the point in time where fertility rates have converged to a new long-run level: it nearly takes a lifetime of a generation until the age structure becomes stationary again.

[14] The number of periods over the life cycle is kept constant. Hence, we do not consider changes in longevity.

3.4 Model Results and Discussion

We now present simulation results obtained from imposing differential population dynamics under alternative assumptions about the integration of international goods markets. As a starting point and to understand why differences in regional population dynamics prepare the ground for Heckscher-Ohlin trade patterns, it is helpful to first look at each of the two regions under autarky. Such a situation might for example reflect prohibitively high transportation costs or the presence of protectionist trade policies. Under autarky, we compare the consequences of demographic change to the initial steady state situation. Second, we look at the full two-country model where international goods markets are perfectly integrated. Here, the autarky case serves as a benchmark.

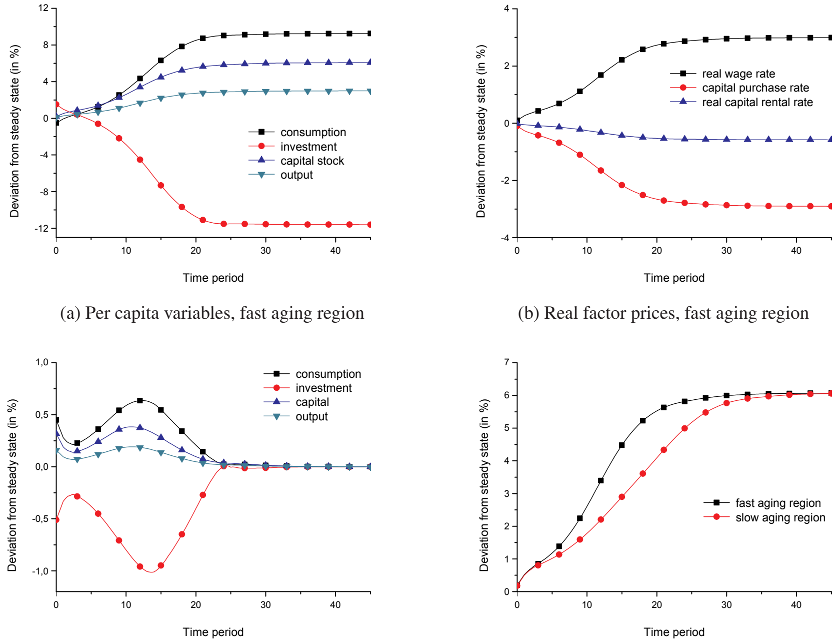
3.4.1 Demographic Change in a Two-Sector Economy Under Autarky

Figure 3.5, Panels (a) – (c), show the evolution of key macroeconomic variables for the fast aging region under autarky.[15] A decline in fertility rates γ increases the capital stock, consumption, and output, and decreases investment (all variables refer to per capita terms). Moreover, the real wage rate increases whereas the real interest rate declines.

There are a number of direct and indirect effects that explain these equilibrium outcomes (the relative strength of each effect depends on the specific parametrization of the model—an issue which we will explore further in Section 3.4.5). As the most direct effect, the decrease in γ leads to a decrease of the overall population which means that existing resources have to be shared by less people. The implied decline in the working-age population ratio means that the number of young relative to the old has decreased. Since the propensity to save is higher for younger people, and because older people tend to hold more assets than younger ones, investment decreases and the capital stock increases (both in per capita terms).[16] These shifts in relative factor endowments affect real factor prices which in turn alter economic incentives to accumulate capital and to supply labor. Both, the capital deepening and the decline in the (potential) labor force makes labor scarce relative to capital, and hence, the real wage rate increases while the real capital rental rate (as well as the real interest rate which moves in parallel) decreases. Despite the decline in the labor force, per capita output is shown to increase due to the increase in the per capita

[15] Although this model features elastic labor supply on the part of the households, we do not distinguish between the "per capita" and "per worker" concept. From a qualitative viewpoint both concepts lead to identical results while quantitative differences are negligible.

[16] The result of a higher steady state level of per capita capital and output in response to a permanently lower population growth rate is also obtained in a Solow-Swan growth framework. In the Ramsey model, however, changes in the population growth rate have only transitional effects on these per capita variables since there is by construction no "leverage" for the compositional shift in the age structure of a population. For more details see, e.g., Barro and Sala-i-Martin (2004).



(a) Per capita variables, fast aging region

(b) Real factor prices, fast aging region

(c) Per capita growth rates, fast aging region

(d) Per capita capital stock, fast aging region

Fig. 3.5 Per capita macroeconomic variables under autarky

Note: The plots show the transition to a new long-run equilibrium in response to demographic shocks as specified in eq. (3.20). Panel (a): Consumption is defined as total private sector consumption. Investment is gross investment. Output is defined as the quantity of the composite good A produced at a given point in time. Panel (b): Factor prices are nominal factor prices deflated by the consumer price index.

capital stock and the fact that output has to be shared among less people. Finally, since in this model population growth is the only engine of long-run growth, growth rates of per capita variables converge back to zero in the new steady state equilibrium (Panel (c)). Section 3.4.5 demonstrates that these outcomes are qualitatively robust over a broad range of parameter configurations that is supported by empirical literature.

Factor price changes imply different costs of production for each sector and therefore bring about a sectoral change in the aging economy. Figure 3.6 shows that the output of the capital-intensive sector expands relative to the labor-intensive sector (Panel (a)) and that the relative price of the capital-intensive commodity declines (Panel (b)). The movement in factor price changes is an instance of what is known as the "Rybczynski effect": the fall in the relative price of commodity 1 lowers the real return to the factor used intensively in the production of that good, here capital, and hence the capital rental rate falls, and increases the real return to the other factor, here labor, and hence the wage rate rises.[17] Since in each period changes in commodity prices are a weighted average of changes in factor prices, the wage rate (capital rental rate) increases (decreases) in percentage terms by more than the relative price of commodity 1.

We can summarize the key results of this section as follows:

1. Population aging leads to a higher per capita capital stock during the transition and in the new long-run equilibrium.
2. In a two-sector economy in which sectoral output is produced with different factor intensities, population aging leads to a relative contraction (expansion) of the labor-intensive (capital-intensive) sector. The real wage rate (capital rental rate) increases (decreases) during the transition and in the new long-run equilibrium.

3.4.2 Globally Unsynchronized Aging Patterns and International Trade

Differences in the extent and timing of aging patterns across regions lead to quantitatively distinct responses in both economies. Figure 3.5, Panel (d), shows that the capital deepening process is less pronounced in the slow aging region as compared to the fast aging region. Since the working-age population ratio in the fast aging region is higher throughout the demographic (and economic) transition, the per capita capital stock in the slow aging region is lower than in the fast aging region, i.e. the slow (fast) aging region becomes relatively labor (capital) abundant. As a result, the "dampened Rybczynski effect is weaker for the slow aging region implying a weaker relative expansion of the capital-intensive sector (see Figure 3.6, Panel (a)).

Under autarky, differences in relative factor endowments cause equilibrium factor prices to diverge (Figure 3.6). The wage rate (capital rental rate) in the capital-abundant/labor-scarce region is higher (lower) than in the capital-scarce, labor-abundant region. Despite identical production technologies in both regions, differentials in factor prices imply different costs of production which in turn translate into international relative price differences of sectoral output under autarky (Figure 3.6, Panel (b)). The fast aging region therefore has a cost advantage in producing the capital-intensive commodity whereas the slow aging region has a cost advantage in the production of the labor-intensive commodity. Hence, the relative price of commodity 1 in the fast aging region stays below the one in the slow aging region as long as differences in relative factor endowments across regions are sustained.

Divergence of relative commodity prices under autarky create incentives for trade. If international trade is liberalized, each region will export the good where it has a cost advantage. Exploiting the higher relative price of commodity 1 in the slow aging region, the fast aging, capital-abundant region consequently exports the

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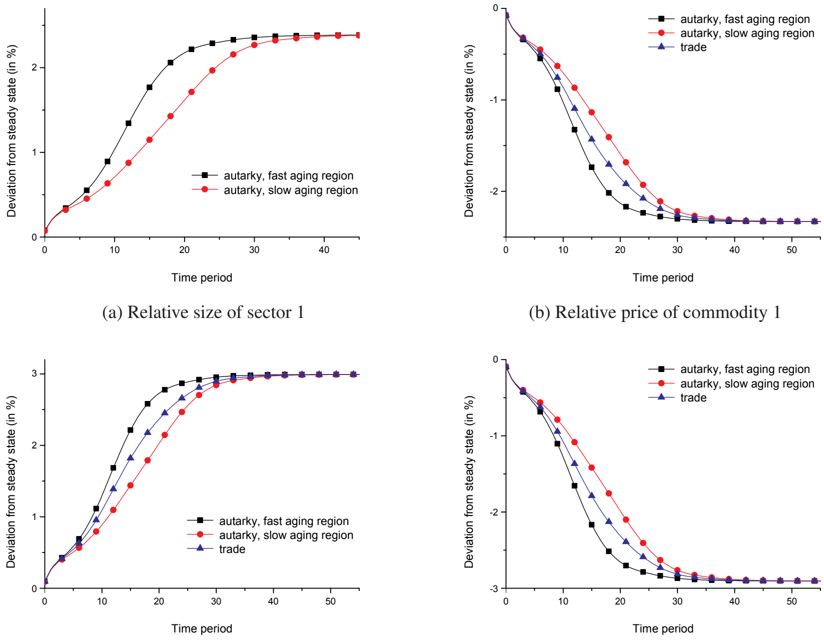
[17] Note that we only observe a "dampened" Rybczynski effect due to feedback effects of induced commodity price changes which cushion—but not reverse—the "pure" Rybczynski effect that would be obtained if prices were fixed. See, e.g., Jones (1965, pp.562).



(a) Relative size of sector 1 (b) Relative price of commodity 1

(c) Real wage rate (d) Real rental rate of capital

Fig. 3.6 Sectoral change and impact on equilibrium commodity and factor prices

Note: The plots show the transition to a new long-run equilibrium in response to demographic shocks as specified in eq. (3.20). The relative size of the capital-intensive sector 1 is measured as the ratio of sectoral outputs. The relative price of commodity 1 is defined as the ratio of the price for commodity 1 and 2. Real factor prices are nominal factor prices deflated by the consumer price index.

capital-intensive commodity 1, whereas the slow aging, labor-abundant region exports the labor-intensive commodity 2. Each region specializes in the production of the good which it can produce more efficiently thereby even more increasing productivity of the factor used intensively in this sector. Comparing trade with autarky, capital is more productive in the capital-intensive sector in the fast aging region and is less productive in the labor-intensive sector in the slow aging region—vice versa for the productivity of labor (not shown). Free trade establishes a common relative price across regions where the autarky prices provide lower and upper bounds (Figure 3.6, Panel (b)). Trade in goods is a perfect substitute for trade in factors and therefore leads under the assumptions of this model to the equalization of factor prices (Figure 3.6).

Although immobile internationally, capital is shifted between regions to the extent that it is embedded in traded goods. By exporting the capital-intensive commodity during the transition, the capital abundant region is thus characterized by a lower capital stock per capita under trade as compared to autarky, whereas per capita capital in the labor abundant region, which exports the labor-intensive good, is higher under trade as compared to autarky (not shown).

We can summarize the key results of this section as follows:

1. Globally unsynchronized aging patterns affect the relative abundance of factors of production across regions and lead to differentials in commodity and factor prices under autarky. The per capita capital stock in the fast aging region is higher than in the slow aging region.
2. If international goods markets are integrated (and if goods are produced with different factor intensities), the presence of globally unsynchronized aging patterns leads to demographically-induced Heckscher-Ohlin trade. The fast (slow) aging region specializes in the production and export of the capital-intensive (labor-intensive) commodity during the transition.

3.4.3 Welfare Effects of Trade Liberalization Under Global Demographic Change

In contrast to what one would expect from standard neoclassical trade theory, moving from an autarky situation to a world where goods can move freely across borders does not unambiguously result in welfare gains on a household level. Depending on the particular set of socio-economic characteristics of a household type and generation—including age, labor productivity and other principal characteristics that determine intra- and intertemporal behavior—gains from trade are not guaranteed. Figure 3.7 shows the percentage change in Hicksian equivalent variation (EV) by generation and household type for the fast aging region.[18]

On an *intergenerational* basis, trade liberalization in the presence of globally unsynchronized aging pattern has very distinct quantitative and qualitative welfare consequences for current and future generations. Depending on the specific household type, generations born in the run up to and the beginning of the demographic transition gain (lifetime utility of generations $g = -15, \ldots, 8$ increases) whereas generations born in the midst of the transition and thereafter (generations $g > 9$ for all household types) stand to incur substantial utility losses. As the age structure of populations in both regions converges, the adverse welfare effects for newly born generations diminish.

On an *intragenerational* basis, whether welfare gains materialize for current old generations depends on the particular household characteristics. Figure 3.8, Panel (a), takes a cross-sectional view and shows the distribution of welfare changes across household types for generation $g = 5$. It is evident that "poor" households, i.e. low-skilled workers and those households that hold relatively few assets throughout their life cycle, lose from trade liberalization whereas richer households gain overall.

[18] Due to the artificial symmetry of this two-country model, welfare changes for the slow aging region are diametrically opposed to what is observed in the fast aging region. In the following discussion, we focus on the fast aging region.

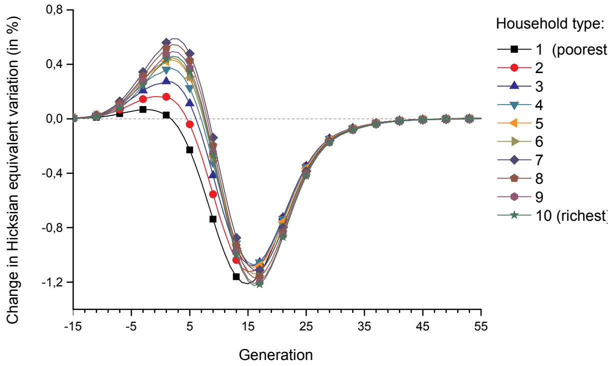


Fig. 3.7 Intergenerational distribution of welfare effects by household types

Note: Changes in Hicksian equivalent variation resulting from a trade liberalization in the presence of globally unsynchronized aging patterns. The graph shows welfare changes by generation and household type for the fast aging region.

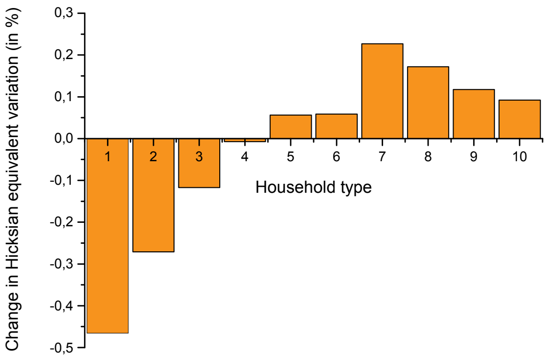


Fig. 3.8 Intragenerational distribution of welfare effects (for $g = 5$)

Note: Changes in Hicksian equivalent variation resulting from a trade liberalization in the presence of globally unsynchronized aging patterns across household types for generation $g = 5$, fast aging region.

Why are there winners and losers from trade liberalization in the presence of globally unsynchronized aging patterns ? What explains that future generations will experience welfare losses whereas current old generations gain ? Why are low-skilled workers and those households that exhibit relatively low levels of savings throughout their lifetime most adversely affected by global demographic change ? The answers lie in the evolution of autarky-trade factor price differentials. The pressure on real factor prices that stems from the region-specific demographic change is mitigated by the specialization patterns in production and the international exchange of goods.

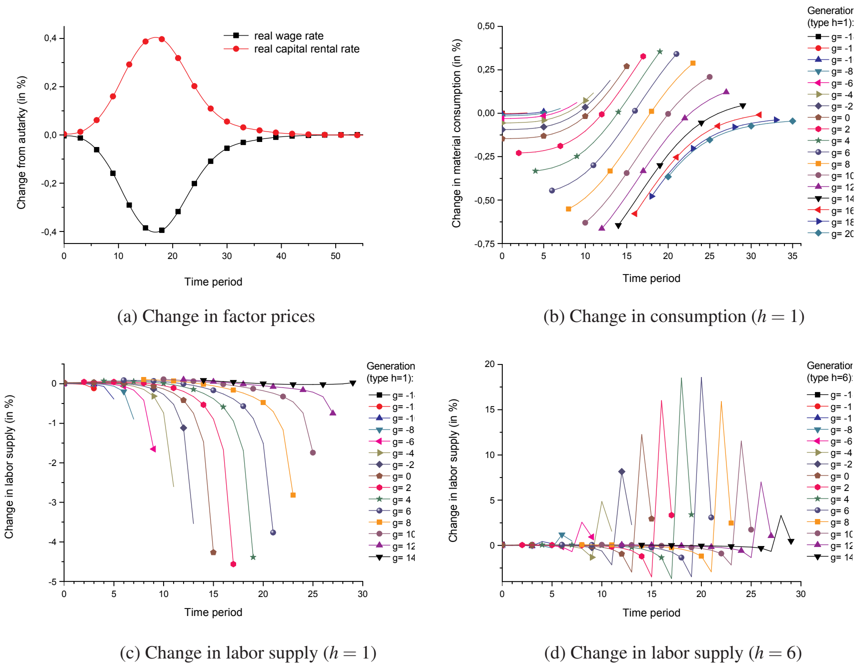


Fig. 3.9 Factor price changes and impact on household decisions (trade vs. autarky)

Note: The plots show compare trade with autarky under population dynamics as specified in (3.20). All plots refer to the fast aging region. Panel (a): Change in real factor prices. Panel (b): Change in material consumption, $c_{g,h,t}$, for household type $h = 1$. Panel (c) and (d): Change in labor supply, $\omega_{g,t} - \ell_{g,h,t}$, for household type $h = 1$ and $h = 6$, respectively.

International trade equalizes the real factor prices and therefore increases the real capital rental rate and decreases the real wage rate in the fast aging region relative to autarky (Figure 3.9, Panel (a)).

International trade can thus be viewed as a mechanism which arbitrages away differences in global demographic patterns as the integrated world economy as a whole now faces an "average demographic scenario" with factor and commodity prices being averages of autarky equilibrium prices. Changes in real factors prices alter the consumption and labor supply/leisure choices of households directly affecting utility as in (3.2) and therefore explain why some households gain and others lose.

3.4.3.1 Household Consumption and Labor Supply

To better understand the distribution of intra- and intergenerational welfare changes from trade liberalization, this section studies in more detail how the global demographic transition affects household decisions on a micro level.

Consumption and labor supply decisions are equilibrium outcomes that depend on a number of direct and indirect effects—some of them work in different directions—and that are determined by key household parameters and by the magnitude of prices changes that households experience over their lifetime.[19] Consumption choices over the life cycle of a generation are directly affected by changes in the real interest rate since, firstly, for a given level of savings higher interest rates make households wealthier (*wealth effect*) and, secondly, changes in the interest rate affect the relative price of intertemporal consumption. Specifically, higher interest rates imply that consumption today becomes more expensive vis-à-vis consumption tomorrow, and hence households allocate consumption towards later periods of their life (*substitution effect*). Second, there is an indirect effect on consumption which stems from changes in the wage rate: for a given level of labor supply, lower wage rates depress lifetime income and consumption therefore decreases (*wage effect*). Labor supply decisions respond to changes in the real wage rate (*price effect*). The intuitive case is that labor supply is increasing in the wage rate. Intertemporally optimizing households may, however, increase labor supply in response to a decline in the wage rate so as to prevent large fluctuations in periodic income which would be inconsistent with the objective of consumption smoothing. In such a case, leisure consumption is a Giffen good. Interest rate changes also affect labor supply decisions since for a given level of savings, higher interest rates imply that households have become more wealthy, and because there is disutility from working this creates a tendency for labor supply to fall (*interest rate effect*).

Figure 3.9, Panel (b), shows the change in material consumption, $c_{g,h,t}$, over the respective lifetime for each generation $g = -14,\ldots,20$ of household type $h = 1$ in the fast aging region. Recall from Panel (a) that the gap between autarky and trade factor prices increases during the demographic transition and diminishes eventually as the working-age population ratios in both region converge. For generations born prior to year zero, we see that old-age consumption is slightly increased due to a positive wealth and substitution effect. Consumption in earlier periods is almost unchanged because the drop in wage rates and hence negative wage effects are relatively small. For subsequent generations, i.e. moving along the x-axis from the left to the right in Panel (b), losses in wage income become increasingly substantial and lead to larger and larger reductions of consumption in earlier periods of the life cycle. At the same time, the interest rate gap widens which strengthens the positive income and intertemporal substitution effect causing increases in old-age consumption. However, as the extent of real factor price changes over the lifetime of a generation becomes less favorable, losses in wage income cannot be compensated for by the positive income effect and hence there is a downward adjustment in consumption for all periods of the life cycle ($g \geq 16$). From year 17 onwards, the trade-autarky differentials in factor prices begin to diminish and households born during this period experience smaller and smaller decreases in wage rates over their lifetime. Hence, moving across generations, the size of reductions in young-age con-

[19] For the interpretation of results, we concentrate on the link from price changes to household quantity decision, and neglect the fact that in general equilibrium prices and quantities are determined simultaneously.

sumption diminishes. Consumption changes across generations for different household types are very similar (not shown).

Figure 3.9, Panel (c), shows the impact on labor supply, $\omega_{g,t} - \ell_{g,h,t}$, over the respective lifetime for each generation $g = -14, \ldots, 14$ of household type $h = 1$ in the fast aging region. Generations born prior to the demographic shocks, leave labor supply largely unchanged as real wages in early stages of the life cycle when labor productivity is high are almost unchanged. At the same time, they benefit in older ages from higher interest rates on their savings and therefore adjust labor supply downwards in later periods of their life cycle. As the wage and interest rate gaps between trade and autarky situations broaden, decreases in labor supply become increasingly substantial for generations when moving, both, over the life cycle of a given generation as well as across generations. There are relatively large drops in labor supply for generations $g = 0$ and $g = 2$ which belong—looking at the respective welfare changes in Figure 3.7—to the biggest winners of trade liberalization. Albeit decreases in real wages during younger ages, the positive wealth effect is strong enough to make these households significantly better off. For subsequent generations $g \geq 4$, factor price changes over their respective lifetime become increasingly less favorable implying—for a given age—a decrease in periodic labor supply that is smaller as compared to preceding generations. This is necessary to prevent an otherwise too substantial decline in lifetime income.

Figure 3.9, Panel (d), reminds us that labor supply responses are complex decisions that critically depend on the interplay of socio-economic characteristics. Unlike low-skilled workers of type $h = 1$, households with middle and high labor productivity increase old-age labor supply (here shown for $h = 6$). In response to the fall in wage rates, and due to high labor productivity even in older ages, these households find it optimally to devote a smaller fraction of time to leisure consumption as compared to autarky.

We can summarize the key results of this section as follows:

1. International trade constitutes a "mechanism" which arbitrages away regional demographic differences thereby mitigating the pressure on real factor prices that is exerted by demographic change in a closed economy. Hence, in the fast aging region, the real capital rental increases and the real wage rate decreases relative to autarky.
2. Gains from trade in the presence of globally unsynchronized aging patterns are not guaranteed. This is a result of the different evolution of real factor price changes during the demographic transition. For the fast aging region:

   - trade liberalization is only beneficial for current old generations that are born before and in the beginning of the demographic transition. Future generations of all household types stand to incur substantial utility losses.
   - the intragenerational distribution of welfare gains and losses depends on the level of labor productivity and the propensity to save throughout the life cycle. "Poor" households, i.e. low-skilled and asset-poor households, tend to be relatively worse off.

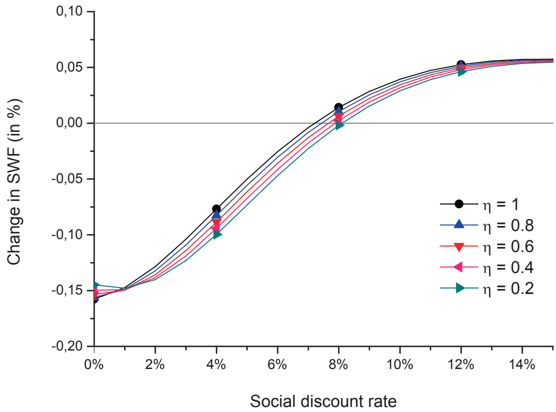


Fig. 3.10 Sensitivity of social welfare changes to inequality parameter

Note: Change in the social welfare as defined in (3.21) comparing trade with autarky for different values of $\widetilde{\rho}$ and Σ (fast aging region). A low value of $\widetilde{\rho}$ on the x-axis implies a high weight on the utility of future generations. If $\eta = 1$, a more utilitarian approach is adopted, whereas smaller values of η mean that a greater weight is placed on equity.

3.4.4 Aggregate Social Welfare

In this section, we take a first stab on whether trade liberalization in a world which is characterized by differential population dynamics is desirable from a social standpoint. We apply a very direct social welfare function approach and specifically assume that aggregate welfare can be measured as:

$$\mathrm{SWF}_r = \left(\sum_{g,h} \Theta_{r,g,h}\, u^{\eta}_{r,g,h} \right)^{1/\eta} \tag{3.21}$$

where $\Sigma = 1/(1-\eta)$ is an index of the elasticity of substitution across welfare change for different agents, and $\Theta_{r,g,h}$ is a weighting factor which accounts for discounting and population:

$$\Theta_{r,g,h} = \omega_{r,g}\,(1-\widetilde{\rho})^{g}\,. \tag{3.22}$$

In this expression, $\omega_{r,g}$ is the number of households represented by the generation and household type, and $\widetilde{\rho}$ is the social discount rate which determines the relative contribution of future generations to aggregate social welfare. When $\widetilde{\rho}$ is smaller, future generations play a larger role in defining social welfare and vice versa. Social welfare is also influenced by the interhousehold substitution elasticity Σ which captures trade-offs in welfare for different types and households born at different times.

The critical weakness of the concept of a social welfare function is the more or less arbitrary choice on how to trade off utility between current and future generations. Figure 3.10 therefore shows the sensitivity of aggregate welfare changes in the fast aging region with respect to the social discount rate and the interhousehold substitution elasticity. A clear picture emerges: trade liberalization in the presence of globally unsynchronized aging patterns is only beneficial for unrealistically high social discount rates of about 8% and greater.[20]

Another perspective on the social desirability of trade liberalization in a world of global demographic change can be provided by conducting an "hypothetical democratic referendum". As in a stylized one-person, one-vote Western democracy, we assume that each generation and household type alive at a given point in time can dispense one vote which carries equal weight. A vote for trade liberalization is taken if the change in the Hicksian equivalent variation is positive; otherwise, people vote against it. In such a referendum, trade liberalization would almost surely be voted down—except for the first periods of the demographic transition (up to year 6) where current old generations who benefit from trade are alive. fThe political outcome of this thought experiment, however, is quite stark: although trade liberalization has been shown not to be beneficial from a social perspective, prevailing political preferences at the beginning of the demographic transition would support a liberal stance of trade policy at the expense of future generations.[21]

We can summarize the key results of this section as follows:

1. From a social standpoint, trade liberalization for the fast aging region in the presence of globally unsynchronized aging patterns is only beneficial for a range of unrealistically high social discount rates.
2. Although not beneficial from a social perspective, the political assessment of trade liberalization by means of an "hypothetical democratic referendum" undertaken at the beginning of the demographic transition, would support a liberal stance of trade policy at the expense of future generations.

3.4.5 Sensitivity Analysis

3.4.5.1 Piecemeal Analysis

In order to explore the robustness of the results with respect to key model parameters, and to develop a better understanding of the impact of each single parameter in the model, we first conduct a number of "piecemeal" sensitivity analyses. Table 3.2 shows results from variations in either δ, ε_i, $i = 1, 2$, or ε_a while holding all other parameters constant. The figures in the table refer to percentage deviations of

[20] As a first approximation, it seems reasonable to use the market interest rate for discounting social investments, thereby respecting private preferences. Also, it is often argued that policy makers are more patient than private citizens, therefore suggesting a social discount rate below the interest rate (Caplin and Leahy, 2004).

[21] Of course, there is a time inconsistency problem here. Future generations want current old generation not to liberalize trade, yet if international goods markets were to remain separated, they themselves would vote for trade liberalization.

Table 3.2 Sensitivity of per capita variables under autarky

Parameter	Value	Δ*capital* (*in %*)	Δ*output* (*in %*)	Δ*investment* (*in %*)	Δ*consum.* (*in %*)
	0.1	6.07	2.99	-11.61	9.25
	0.3	1.64	0.82	-4.72	4.50
δ	0.5	0.81	0.40	-3.07	3.06
	0.7	0.50	0.25	-2.29	2.33
	1.0	0.29	0.15	-1.68	1.72
	0.5	4.89	2.39	-12.61	8.82
	0.8	5.70	2.80	-11.93	9.11
	1.5	6.69	3.31	-11.61	9.48
$\sigma_{Y,i}$, $i = 1,2$	1	6.07	2.99	-11.09	9.25
	2	7.09	3.51	-10.76	9.63
	3	7.55	3.75	-10.38	9.80
	5	7.98	3.97	-10.02	9.97
	0.5	5.90	2.91	-11.75	9.19
	0.8	6.01	2.96	-11.66	9.22
	1.5	6.22	3.07	-11.49	9.30
σ_A	1	6.07	2.99	-11.61	9.25
	2	6.35	3.13	-11.38	9.35
	3	6.58	3.25	-11.19	9.44
	5	6.92	3.43	-10.90	9.56

Note: The table shows results from "piecemeal" sensitivity analysis with respect to δ, ε_i, $i = 1,2$, and ε_a (only one parameter is changed while the others are being held constant). The figures refer to deviations of steady state values of per capita variables in response to population aging as specified in Section 3.3. All figures refer to the fast aging region. For the "piecemeal" sensitivity analysis all household parameters are held constant.

steady state values in response to population aging in the fast aging region under the autarky case. Varying key technology parameters confirms that the results arising from solving the model with the base case parametrization are qualitatively robust. In all cases, macroeconomic variables move in same directions. The intensity of economic adjustment in response to the demographic shocks is the smaller the higher is δ, and the lower are $\sigma_{Y,i}$ and σ_A. Intuitively, a higher rate of depreciation leads to less capital accumulation. A higher elasticity of substitution between capital and labor means that it is possible to increasingly substitute away from costly labor inputs to capital services which have become relatively cheap due to the capital deepening. This implies lower costs of production which in turn translate into smaller reductions in investment, a higher capital stock, and stronger (positive) reactions of output and consumption. Similarly, a higher substitutability between sectoral goods means that the price of the final good A decreases as input costs decline due to a shift towards the less expensive capital-intensive commodity. Furthermore, the piecemeal sensitivity analysis reveals that the ranking in terms of the quantitative effects as shown in Table 3.2 is also maintained during the transition phase (not shown).

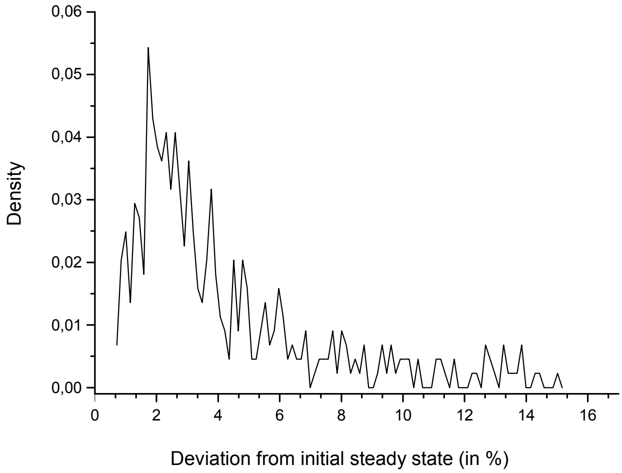


Fig. 3.11 Sample frequency distribution of per capita capital stock

Note: Sample frequency distribution of the percentage deviation of the per capita steady state capital stock from the initial steady state under autarky (fast aging region). In the systematic sensitivity analysis, we execute 1,000 simulations of the autarky model drawing key household and technology parameters from uniform probability distributions.

3.4.5.2 Systematic Sensitivity Analysis

In order to get a comprehensive picture of what drives the results, and to allow for the interaction of all parameters in the model, we conduct a systematic sensitivity analysis and execute 1,000 simulations of, both, the autarky and trade model. In each simulation, values for the key parameters are drawn from uniform probability distributions. The support for the distribution of each parameter is chosen so as to cover a range of empirically plausible parameter values that is well-supported by the literature; we mainly draw on Backus et al. (1992) and Ambler et al. (2002).[22] Figure 3.11 plots the sample frequency distribution of the percentage deviation of the per capita steady state capital stock from the initial steady state under autarky (fast aging region). The main message arising from this graph is that population aging in this economy unambiguously leads to a positive impact on the per capita capital stock. The sample mean of this distribution is an increase of 4.15%. There is a 83.5% chance that the increase in the per capital capital stock is smaller than 7%.

Similarly, we perform a systematic sensitivity analysis to check for the sensitivity of welfare gains and losses from trade liberalization in the presence of globally unsynchronized aging patterns (Figure 3.12). To compute the "overall' sample fre-

[22] The lower and upper bounds for the uniform probability distributions of each respective parameter are: $0.5 < \varepsilon_i < 5$, $0.5 < \varepsilon_a < 5$, $0.1 < \delta < 1$, $0.5 < \alpha_h < 0.9$, $0.25 < \sigma_h < 1.5$, $0.25 < \sigma_{cl,h} < 1.5$. In all simulations, labor productivity is parameterized as in the base case scenario.

quency distribution of changes in Hicksian equivalent variation, we pool welfare estimates across all different household types and age-groups that are alive during the economic transition period (up to $t = 55$). The sample comprises in total $710,000$ households which differ with respect to their age, i.e. the point in life when they are affected by demographic shocks, labor productivity, and other key behavioral parameters that govern the intra- and intertemporal behavior.[23] The resulting sample distribution is roughly symmetric and its sample mean is a decrease in EV equal to -0.24%. This compares roughly with what one could expect from loose visual inspection of Figure 3.7. The sample variance is 1.3. Based on the sample distribution, there is a 80.7% probability that welfare gains are negative. The analysis therefore unequivocally demonstrates that, unlike classical trade theory would suggest, trade liberalization in the presence of globally unsynchronized aging patterns—other things being equal—bears a high risk of negative welfare effects for households in the relatively fast aging region.

3.4.5.3 Uniqueness of Equilibria

A major concern of the sensitivity analysis was to check for the potential existence of multiple equilibria. Galor (1992) pointed out the possibility of global indeterminacy of the perfect-foresight equilibrium in the two-sector neoclassical OLG model. In principle, this contingency cannot be ruled out by the specific structure and assumptions of the model. The non-existence of a closed form solution makes it impossible to analytically characterize the dynamical system and establish conditions for uniqueness. Results from the previous sensitivity analyses, however, suggest that changes in key parameters of the model do not lead to qualitative reversals. Moreover, varying parameter values alters equilibrium outcomes of endogenous variables in a 'monotonic way'. Finally, we experimented with the choice of T. Kehoe and Levine (1985) have shown that in the OLG framework indeterminacy would manifest itself as sensitivity to the truncation date. None of the models presented here are sensitive to T, provided that it is sufficiently large. We interpret all these findings as evidence that the equilibria are unique.

3.5 Concluding Remarks

Despite the fact that the existing literature has consistently identified globally unsynchronized aging patterns to be a potentially important driving force of cross-border flows of capital and labor, the implications for the international exchange of goods have been largely overlooked. This chapter fills the gap and develops a numerical dynamic general equilibrium framework that combines elements of the factor-proportions theory of international trade with those of Auerbach and Kot-

[23] Running 1,000 simulations of the trade model drawing key parameters from uniform probability distributions takes approximately six days on a dual core 2 GHz computer.

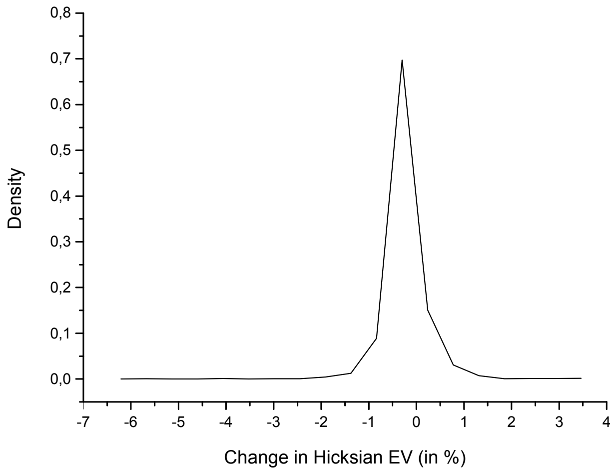


Fig. 3.12 Sample frequency distribution of welfare gains from trade liberalization

Note: Sample frequency distribution of the percentage change in Hicksian equivalent welfare comparing trade with autarky in the presence of globally unsynchronized aging patterns (fast aging region). In the systematic sensitivity analysis, we execute 1,000 simulations of the autarky and trade model drawing key household and technology parameters from uniform probability distributions. The sample comprises in total $710,000$ households that are alive during the economic transition phase and which differ with respect to their age, i.e. the point in life they are affected by demographic shocks, and labor productivity and other key behavioral parameters that govern the intra- and intertemporal behavior.

likoff (1987)-type OLG models to analyze the economic consequences of trade liberalization in the presence of global demographic change.

We demonstrate that demographics emerge as a potential determinant of international trade flows. To the extent that age-dependent savings and labor supply decisions by households have an impact on the aggregate capital stock and labor force, population aging alters the relative abundance of factors of production and thereby—in the presence of globally unsynchronized aging processes—gives rise to Heckscher-Ohlin trade patterns. International trade can be viewed as a "mechanism" which potentially arbitrages away regional demographic differences, and thereby mitigates the pressure on real wages and interest rates that is created by population aging in a closed economy.

Moreover, we provide a detailed analysis of the inter- and intragenerational distribution of gains from trade. In contrast to what one would expect from standard neoclassical trade theory, gains from trade under regional unsynchronized aging patterns are not guaranteed. Who gains from trade liberalization depends on their specific set of socio-economic characteristics. We find, for instance, that in the fast aging region current old generations born before and in the beginning of the demographic transition benefit whereas future generations stand to incur substantial utility losses. The intragenerational distribution of welfare gains and losses depend on the level of labor productivity and the propensity to save throughout the life cycle. Low-skilled workers and asset-poor households tend to be relatively worse off. On a regional level, gains from trade for the fast aging region require implausibly high social discount rates. The results are robust over a broad range of empirically plausible parameter configurations. We perform a systematic sensitivity analysis and find that there is 80.7% probability that households in the fast aging region do not gain from trade.

Of course, the results obtained are subject to several caveats which may also serve as suggestions for future research. First, by focussing on the goods market channel as the only linkage between countries, the critical assumption is made that factors of production are internationally immobile. Integrating cross-border migration and international capital flows into the model would add an additional mechanism to arbitrage away regional differences in observed population dynamics. Other factors being equal, this would most likely reduce the magnitudes of adjustment responses of economies including a lower level of interregional goods flows and less pronounced changes in households' welfare.

Second, the model abstracts from any government activity and in particular does not consider a public sector pension system. As pointed out above, recent studies emphasize the role of a public pension system in determining households' savings behavior. The model takes an extreme standpoint on this issue by assuming the presence of a fully-funded pension system which implicitly operates through private life-cycle savings decisions of households.

Third, in reality there are numerous obstacles preventing international goods markets from being fully integrated. As long as barriers to trade exert a symmetrical effect on both regions, the qualitative implications of the analysis should be preserved. However, cross-country differences in the active set of trade policy instruments are likely to result in an asymmetric distribution of gains and losses from trade.

Fourth, and probably most critical, is the fact that we carry out the analysis in an idealized neoclassical world. For instance, introducing elements of imperfect competition, increasing returns to scale and making different assumptions about labor markets are needed to enhance the policy relevance of the simulation experiments. Also, it has been shown that if the basic dichotomy of the Heckscher-Ohlin framework is extended to account for multiple skill-related categories of workers (Wood, 1994), country groups (Davis, 1996) and traded goods (Feenstra and Hanson, 1996), then the main distributive prediction of the Heckscher-Ohlin theory is theoretically undetermined. Whether the inclusion of those factors would materially alter the conclusions is a question for future research.

Appendix: Numerical Simulation of the Model

This appendix provides a sketch of the major issues involved in numerical simulation of the model. In particular, we (i) briefly explain the calibration to a steady state, (ii) provide an overview of the adopted solution algorithm, and (iii) fully characterize the general equilibrium of the numerical model in a mixed-complementarity format.

3.5.1 Steady State Calibration

We use the calibration procedure for OLG models which is described in detail in Rasmussen and Rutherford (2004). The OLG economy is calibrated to an initial steady state equilibrium in which all quantities grow with constant rate $1+\bar{\gamma}$ and present value prices decline with the interest rate $1+\bar{r}$. The virtue of this approach is that the calibration of the OLG demand system is independent from the specific model structure of the production side. The calibration procedure is carried out in two steps. Demographic shocks are imposed as a counterfactual scenario and are not incorporated in the initial steady state calibration procedure.

The first step solves for the optimal behavior of a single reference generation taking into account the existence of aggregate consistency conditions which ensure that individual choices by OLG households match the aggregate behavior of the economy. Since in a steady state we know the full path of present value prices, optimal behavior for all generations (and household types) can be inferred from the optimal behavior of a single generation. We pose the *household calibration model* as a mixed-complementarity problem (MCP) allowing for corner solutions in labor supply (we indicate the associated complementary variable to each equilibrium condition using the "perp" operator, "$\perp$"). Table A–3.3 defines all model variables and parameters. For generation $g=0$, the solution to the household utility maximization problem given by (3.2) is fully characterized by the following system of equations (3.23) – (3.35) (since both regions share the same steady state equilibrium, the region index is dropped):

Definition of full consumption:

$$z_{a,h} = \left(\alpha_h \, c_{a,h}^{\nu_h} + (1-\alpha_h)\, \ell_{a,h}^{\nu_h}\right)^{1/\nu_h} \quad \perp \; z_{a,h} \tag{3.23}$$

FOCs for material consumption:

$$\frac{\partial U(c_{a,h}, \ell_{a,h}, \rho)}{\partial c_{a,h}} = \lambda_h \, \bar{p}_a \quad \perp \; c_{a,h} \tag{3.24}$$

FOCs for leisure:

$$\frac{\partial U(c_{a,h}, \ell_{a,h}, \rho)}{\partial \ell_{a,h}} = \eta_h \quad \perp \; \ell_{a,h} \tag{3.25}$$

FOCs for price of labor time:

$$\eta_h = \lambda_h \, \bar{p}_a \, \pi_{a,h} \quad \perp \; l_{a,h} \tag{3.26}$$

FOCs for price of lifetime income:

$$\sum_a \bar{p}_a \, c_{a,h} = \sum_a \bar{p}_a \, \pi_{a,h} \, l_{a,h} \quad \perp \; \lambda_h \tag{3.27}$$

FOCs for price of time:

$$\omega_{\text{scale}} = \ell_{a,h} + l_{a,h} \quad \perp \; \eta_h \tag{3.28}$$

Price index for full consumption:

$$pz_{a,h} = \left(\alpha_h^{\sigma_{\nu_h}} \, \bar{p}_a^{\,1-\sigma_{\nu_h}} + (1-\alpha_h)^{\sigma_{\nu_h}} \, \left(\frac{\eta_{a,h}}{\lambda_h}\right)^{1-\sigma_{\nu_h}} \right)^{1/(1-\sigma_{\nu_h})} \quad \perp \; pz_{a,h} \tag{3.29}$$

Present value assets over the life cycle:

$$ca_{a,h} = \sum_{a'=0}^{a} \bar{p}_{a'} \left(\pi_{a',h} \, l_{a',h} - c_{a',h} \right) \quad \perp \; ca_{a,h} \tag{3.30}$$

Value of assets held by age:

$$cma_{a,h} = ca_{a,h} \left(\frac{\bar{p}_a}{(1+\bar{\gamma})^a} \right) \quad \perp \; cma_{a,h} \tag{3.31}$$

Aggregate consumption:

$$ccc_h = \sum_a \frac{c_{a,h}}{(1+\bar{\gamma})^a} \quad \perp \; ccc_h \tag{3.32}$$

Aggregate assets:

$$ccma_h = \sum_a cma_{a,h} \quad \perp \; ccma_h \tag{3.33}$$

To fix aggregate consumption at the benchmark value select ρ such that:

$$\sum_h ccc_h = \bar{C} \quad \perp \; \rho \tag{3.34}$$

To fix aggregate assets at the benchmark value select ω_{scale} such that:

$$\sum_h ccma_h = (1+\bar{r}) \, \bar{K} \quad \perp \; \omega_{\text{scale}} \, . \tag{3.35}$$

To calibrate the model to the benchmark values of aggregate consumption and aggregate assets, we add two additional constraints to the household problem and endogenize the discount factor and a scaling factor on the periodic time endowment.

In the second calibration step, the results from the household calibration model are used to set up the entire baseline path for all time periods and including those generations that were born prior to year zero. This involves extrapolating the optimal behavior of the reference generation (including all different household types) by making use of the steady-state assumption.

3.5.2 Solution Algorithm

To solve the OLG economy that has been formulated in section 3.2, we use a decomposition algorithm which has been described in Chapter 2. In what follows, we provide a brief description of how the algorithm can be applied in the present context of a two-country, two-sector OLG model. Each iteration of the algorithm comprises the following steps:

1. Obtain the general equilibrium prices of the "related" Ramsey growth problem. The "related" Ramsey growth problem is characterized by a model in which the demand system of OLG households has been replaced by a single representative infinitely-lived agent. In this model, all other structural elements are identical to those in the underlying OLG economy.
2. Given general equilibrium prices from the representative agent model, evaluate OLG households' demand functions. This step retains the full structure of the OLG demand system but suppresses any general equilibrium feedback effects.
3. Use the quantity choices from the partial equilibrium relaxation, to recalibrate the preferences of the representative agent.
4. Return to Step 1 and repeat the procedure until the algorithm converges.

3.5.2.1 Equilibrium in Mixed-Complementarity Format

We now formulate the equilibrium of the "related" Ramsey optimal growth model as a mixed-complementarity problem. The structure of equilibrium is exploited by the GAMS program and the solver we use to compute the transition and steady state of the infinite-horizon economy.[24] Rutherford (1995b) and Mathiesen (1985) have shown that a complementary-based approach is convenient, robust, and efficient. A characteristic of economic models is that they naturally involve a complementary problem, i.e. given a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$. A fundamental advantage of formulating an economic model as a

[24] For an overview of applied general equilibrium modeling with GAMS (General Algebraic Modeling System) and the GAMS/MPSGE subsystem see e.g. Rutherford [1998,1999a], and http://www.mpsge.org.

complementary problem is the ability to express weak inequalities and complementary slackness.

An equilibrium in complementarity format can be posed as a system of three classes of nonlinear inequalities in three sets of economic variables: commodity and factor prices, activity levels, and income levels.

1. The first class of equilibrium conditions are zero profit conditions which state that all production activities earn zero excess profit in equilibrium. Zero-profit condition exhibit complementary slackness with respect to the associated activity level.
2. The second class of equilibrium conditions are market clearance conditions. They state that supply must be greater than or equal to demand for each primary factor and produced good. Market clearance conditions exhibit complementary slackness with respect to market prices.
3. The third class of equilibrium conditions are income balance equations that state that a household's expenditure must not exceed his income.

Note that complementary slackness is a feature of the equilibrium allocation even though it is not imposed as an equilibrium condition, per se (Rutherford, 1995b). This means that in equilibrium, any production activity which is operated makes zero profit and any production activity which earns a negative net return is idle. Likewise, any commodity which commands a positive price has a balance between aggregate supply and demand, and any commodity in excess supply has an equilibrium price of zero.

Consequently, the *"related" Ramsey optimal growth problem* of the underlying OLG economy is fully characterized by the system of equations (3.36) – (3.53). Table A–3.4 defines all variables and parameters. We indicate the associated complementary variable to each equilibrium condition using the "perp" operator, "$\perp$"):

zero profit for sector i:

$$\left(\alpha_i^{1/(1-\varepsilon_i)}\, p_{r,l,t}^{\varepsilon_i/(\varepsilon_i-1)} + (1-\alpha_i)^{1/(1-\varepsilon_i)}\, p_{r,re,t}^{\varepsilon_i/(\varepsilon_i-1)}\right)^{1-1/\varepsilon_i} = p_{i,t} \quad \perp\ Y_{i,r,t} \tag{3.36}$$

zero profit for final goods sector:

$$\left(\alpha_a^{1/(1-\varepsilon_a)}\, p_{1,t}^{\varepsilon_a/(\varepsilon_a-1)} + (1-\alpha_a)^{1/(1-\varepsilon_a)}\, p_{2,t}^{\varepsilon_a/(\varepsilon_a-1)}\right)^{1-1/\varepsilon_a} = p_{r,a,t} \quad \perp\ A_{r,t} \tag{3.37}$$

zero profit for capital accumulation:

$$\text{for } t=0,\ldots,T-1: \quad p_{r,k,t} = p_{r,re,t}\,(1-\tau_{r,t}) + p_{r,k,t+1}\,(1-\delta) \quad \perp\ K_{r,t} \tag{3.38a}$$

$$\text{for } t=T: \quad p_{r,k,t} = p_{r,re,t}\,(1-\tau_{r,t}) + pkt_r\,(1-\delta) \quad \perp\ K_{r,t} \tag{3.38b}$$

zero profit for investment:

$$\text{for } t=0,\ldots,T-1: \quad p_{r,a,t} = p_{r,k,t+1}\,(1+\tau_{r,t}) \quad \perp\ I_{r,t} \tag{3.39a}$$

$$\text{for } t = T: \quad p_{r,a,t} = pkt_r \quad \perp \; I_{r,t} \tag{3.39b}$$

zero profit for material consumption:

$$p_{r,a,t} = p_{r,c,t}\,(1+\tau_{r,t}) \quad \perp \; C_{r,t} \tag{3.40}$$

zero profit for full consumption:

$$\left(\Lambda_{r,t}^{1/(1-\nu)}\, p_{r,c,t}^{\nu/(\nu-1)} + (1-\Lambda_{r,t})^{1/(1-\nu)}\, p_{r,l,t}^{\nu/(\nu-1)}\right)^{1-1/\nu} = p_{r,z,t} \quad \perp \; Z_{r,t} \tag{3.41}$$

zero profit for utility:

$$\prod_{t=0}^{T} p_{r,z,t}^{\Theta_{r,t}} = p_{r,u} \quad \perp \; U_r \tag{3.42}$$

market clearance for sector i output:

$$\sum_r Y_{i,r,t} = \sum_r A_{r,t}\,\frac{\partial p_{r,a,t}}{\partial p_{i,t}} \quad \perp \; p_{i,t} \tag{3.43}$$

market clearance for consumption:

$$\sum_r C_{r,t} = Z_{r,t}\,\frac{\partial p_{r,z,t}}{\partial p_{r,c,t}\,(1+\tau_{r,t})} \quad \perp \; p_{r,c,t} \tag{3.44}$$

market clearance for final output:

$$A_{r,t} = I_{r,t} + C_{r,t} \quad \perp \; p_{r,a,t} \tag{3.45}$$

market clearance for capital services:

$$K_{r,t}\,(1-\tau_{r,t}) = \sum_i Y_{i,r,t}\,\frac{\partial p_{i,t}}{\partial p_{r,re,t}} \quad \perp \; p_{r,re,t} \tag{3.46}$$

market clearance for labor services:

$$L_{r,t} = \sum_i Y_{i,r,t}\,\frac{\partial p_{i,t}}{\partial p_{r,l,t}} \quad \perp \; p_{r,l,t} \tag{3.47}$$

market clearance for capital stock:

$$\text{for } t = 0: \quad \sum_{g,h} \overline{k}_{r,g,h,g} = K_{r,t} \quad \perp \; p_{r,k,t} \tag{3.48a}$$

$$\text{for } t = 0,\ldots,T-1: \quad K_{r,t-1}\,(1-\delta) + I_{r,t-1} = K_{r,t} \quad \perp \; p_{r,k,t} \tag{3.48b}$$

$$\text{for } t = T: \quad K_{r,t}\,(1-\delta) + I_{r,t} = TK_r \quad \perp \; pkt_r \tag{3.48c}$$

market clearance for utility:

$$p_{r,u}\, U_r = inc_r \quad \perp \; p_{r,u} \tag{3.49}$$

market clearance for full consumption:

$$p_{r,z,t}\, Z_{r,t} = p_{r,u}\, U_r \quad \perp \; p_{r,z,t} \tag{3.50}$$

income definition:

$$inc_r = \sum_{t=0}^{T} p_{r,l,t}\, \Omega_{r,t} + p_{r,k,0} \sum_{g} \bar{k}_{r,g,g} + TK_r\, pkt_r \quad \perp \; inc_r \tag{3.51}$$

constraint for current account premium:

$$p_{r,l,t}\, \Omega_{r,t} + p_{r,re,t}\, K_{r,t} = \left(p_{r,k,t}\, I_{r,t} + p_{r,c,t}\, C_{r,t}\right)\, (1 - \tau_{r,t}) \quad \perp \; \tau_{r,t} \tag{3.52}$$

constraint for post-terminal capital stock:

$$I_{r,T}/I_{r,T-1} = Y_{r,T}/Y_{r,T-1} \quad \perp \; TK_r\,. \tag{3.53}$$

T denotes the last period of the numerical model. To approximate the underlying infinite horizon economy by a finite-dimensional complementarity problem we choose a "state variable targetting" approach as proposed by Lau et al. (2002). The infinite horizon economy can be decomposed into two distinct problems where one runs from $0, \ldots, T$ and the other one runs from $T+1, \ldots, \infty$.[25] Both subproblems are linked through the post-terminal capital stock in period $T+1$. The level of post-terminal capital is computed endogenously by requiring that investment grows at the same rate as sectoral output or any other "stable" quantity in the model (see (3.53)).

The current account premium, $\tau_{r,t}$, adjusts to satisfy the additional constraint (3.52) which ensures that trade is balanced in each period.

The share parameters $\Lambda_{r,t}$ and $\Theta_{r,t}$ are initially chosen in a way such that the "related" Ramsey optimal growth problem displays the same baseline steady state path as the underlying OLG economy. $\Omega_{r,t}$ denotes the time endowment of the representative agent which is a productivity-adjusted sum of the time endowments of OLG households.

3.5.2.2 Partial Equilibrium Relaxation of the Underlying OLG Economy

Let **p** denote the price vector of equilibrium prices that are obtained from solving the "related" Ramsey optimal growth problem from the previous step. Given the CES preferences of OLG households, it is possible to obtain from the optimization problem in (3.2) closed-form demand functions for all generations and all household types:

---

[25] Note that this method for approximating the infinite horizon relies on the assumption of time-separable utility functions.

Table 3.3 List of variables and parameters (household calibration model)

Variable	*Definition*
$c_{a,h}$	Material consumption by household type h at age a
$\ell_{a,h}$	Leisure consumption by household type h at age a
$z_{a,h}$	Full consumption (composite of leisure and material consumption)
λ_h	Shadow price of lifetime income of household type h
η_h	Shadow price of time of household type h
$l_{a,h}$	Labor supply by household type h at age a
ω_{scale}	Scaling factor on time endowment (parameter used for calibration)
$pz_{a,h}$	Price of full consumption for household type h at age a
$ca_{a,h}$	Present value of assets over the life cycle for household type h at age a
$cma_{a,h}$	Value of assets held by age and household type
ccc_h	Total consumption by households of type h
$ccma_h$	Total value of assets by households of type h
ρ	Discount factor (parameter used for calibration)
Parameter	*Definition*
$\bar{r}$	Steady state interest rate
$\bar{\gamma}$	Steady state population growth rate
$\bar{p}_a = (1+\bar{r})^{-a}$	Steady state index of present value prices
α_h	Weight on material consumption for household of type h
σ_h	Intertemporal elasticity of substitution for household of type h
$\sigma_{cl,h}$	Intratemporal elasticity of substitution for household of type h
$\pi_{a,h}$	Index of labor productivity for household type h at age a
$\bar{K}$	Benchmark value of capital stock
$\bar{C}$	Benchmark value of aggregate consumption

$$\mathbf{x} = D(\Psi, \mathbf{p}) \tag{3.54}$$

where $\mathbf{x}$ represents a generic variable representing $c_{r,g,h,t}$ and $\ell_{r,g,h,t}$, and Ψ denotes relevant household parameters. Note that evaluating $\mathbf{x}$ at $\mathbf{p}$ is equivalent to solving a partial equilibrium relaxation of the underlying OLG economy in which all general equilibrium interactions due to quantity adjustments are suppressed.

3.5.2.3 Recalibration of Preferences of the Representative Agent Model

The last step of each iteration involves updating share parameters $\Lambda_{r,t}$ and $\Theta_{r,t}$ according to:

$$\Lambda_{r,t} = \frac{p_{r,c,t} \sum_{g=t-N}^{t} \sum_{h=1}^{H} c_{r,g,h,t}}{\sum_{g=t-N}^{t} \sum_{h=1}^{H} p_{r,c,t}\, c_{r,g,h,t} + p_{r,l,t}\, \ell_{r,g,h,t}} \tag{3.55}$$

$$\Theta_{r,t} = \frac{\sum_{g=t-N}^{t} \sum_{h=1}^{H} p_{r,c,t}\, c_{r,g,h,t} + p_{r,l,t}\, \ell_{r,g,h,t}}{\sum_{t=0}^{T} \left(\sum_{g=t-N}^{t} \sum_{h=1}^{H} p_{r,c,t}\, c_{r,g,h,t} + p_{r,l,t}\, \ell_{r,g,h,t} \right)} \tag{3.56}$$

Table 3.4 List of variables and parameters ("related" Ramsey optimal growth problem)

Variable	*Definition*
U_r	Utility for representative agent in region r
inc_r	Lifetime income of representative agent in region r
$Y_{i,r,t}$	Output of sector i in region r at time t
$A_{r,t}$	Final good (CES aggregate of tradable Y's) in region r at time t
$Z_{r,t}$	Full consumption (material consumption + leisure) in region r at time t
$I_{r,t}$	Aggregate investment demand in region r at time t
$K_{r,t}$	Aggregate capital stock in region r at time t
TK_r	Aggregate post-terminal capital stock in region r
$C_{r,t}$	Aggregate consumption demand in region r at time t
$p_{r,a,t}$	Price of final good at time t in region r
$p_{r,c,t}$	Price of consumption at time t in region r
$p_{i,t}$	Price of tradable sectoral output i at time t
$p_{r,u}$	Price of a unit of utility for representative agent in region r
$p_{r,l,t}$	Market wage rate at time t in region r
$p_{r,z,t}$	Price of full consumption at time t in region r
$p_{r,re,t}$	Capital rental rate at time t in region r
$p_{r,k,t}$	Price of capital at time t in region r
pkt_r	Post-terminal price of capital in region r
$\tau_{r,t}$	Current account premium at time t in region r
Parameter	*Definition*
$\Lambda_{r,t}$	Weight on material consumption
$\Theta_{r,t}$	Weight on periodic full consumption in intertemporal utility
α_i	Capital share parameter in sector i
α_a	Share parameter in production of A
ν	Elasticity parameter (material consumption vs. leisure)
ε_i	Elasticity parameter in sector i production (capital vs. labor)
ε_a	Elasticity parameter in production of A (commodity i vs. j)
δ	Periodic capital depreciation rate
$\Omega_{r,t}$	Aggregate time endowment at time t in region r

where prices represent general equilibrium prices from Step 1 and quantities represent partial equilibrium choices obtained under Step 2. Similarly, the time endowment of the representative agent, $\Omega_{r,t}$, is updated in each iteration. Subsequently, the algorithm involves re-solving the "related" Ramsey optimal growth problem which is given by (3.36)–(3.53) using the updated values for Λ and $\Theta_{r,t}$.

Chapter 4
Quantifying the Sectoral and Distributional Effects of Demographic Change in Germany

4.1 Introduction

The accelerating pace of the demographic transition in Germany will prove to be one of the key factors in the development of society in the coming decades. This process is driven by falling mortality rates and a decline in birth rates, which reduces population growth rates and increases the share of older people in the economy. Figure 4.1, based on demographic projections by the Federal Statistical Office of Germany, illustrates the impact of population aging on the population growth rate, defined here as the growth rate of the adult population, and the ratio of the working-age population (aged 20-64) to total population (20-94) for the period 2003-2050. Population growth rates are predicted to decline and will even turn negative in 2014. As a consequence, the population starts shrinking in around 2017 (not shown). The working-age population ratio, as a crucial indicator of aging, is projected to decrease sharply by 13.8% from 0.74 in 2003 to 0.62 in 2050.

A considerable amount of the economic literature has scrutinized many aspects of how the demographic transition is going to affect future economic activity. This chapter focuses on two issues that have received surprisingly little attention. The first objective of this chapter is to quantify the impact of demographic change on the sectoral composition of output. In the model, the sectoral adjustment to demographic change rests on two key mechanisms. First, as technological differences across sectors generally imply that firms operate at different factor intensities, the increase in the aggregate capital-labor ratio that arises from population aging is absorbed differently across sectors. Sectors that employ abundant capital and attract scarce labor most efficiently will grow faster than the rest of the economy, while other sectors will contract due to the demographic transition. Differing costs of production also imply that relative goods prices change with ensuing general equilibrium consequences. Second, as the composition of household consumption varies with age (see Federal Statistical Office of Germany (2003) for empirical evidence for Germany, and Börsch-Supan (2003) for a discussion) changes in the population structure induce a demand-side driven effect that can potentially give rise to sec-

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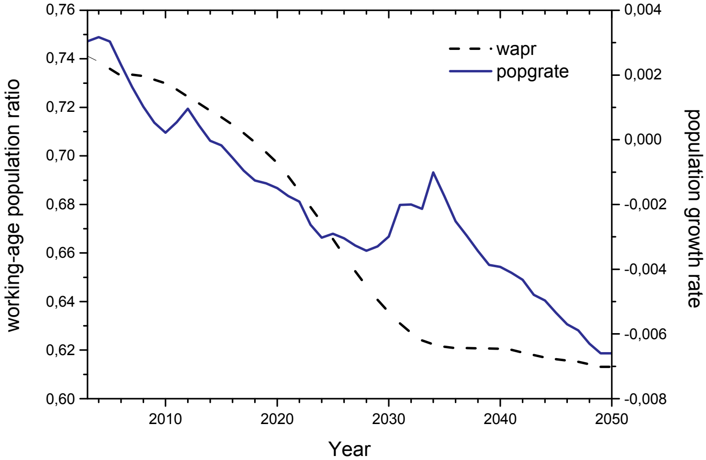

Fig. 4.1 Evolution of working-age population ratio and population growth rate

Note: Own calculations based on demographic projections by the Federal Statistical Office of Germany, *Eleventh Coordinated Population Forecast 2005, Variant 1-W2*. The working-age population ratio is here defined as the population aged 20-64 as a fraction of total adult population (aged 20-94). Population growth rates refer to the growth rate of the adult population.

toral adjustments as well. Consequently, and in contrast to the existing literature, a proper quantitative assessment of the transitional-dynamic impact of population aging requires to go beyond the standard macroeconomic single-commodity world.

The second objective of this chapter is to quantify the distributional and welfare consequences of the demographic transition in Germany. As labor is expected to be scarce relative to capital, real wages are bound to increase while real returns to capital decrease. A priori, it is not clear how the opposite movement of factor prices will affect households' labor supply, life cycle savings behavior, and welfare. We compare the welfare effects along an inter- and intragenerational dimension, and also assess how the distribution of labor income and wealth evolves over time.

To investigate these issues, we use demographic projections from the Federal Statistical Office of Germany, together with a large-scale overlapping generations (OLG) model pioneered by Auerbach and Kotlikoff (1987). We extend the model in two directions which are both necessary for the questions we want to address. First, the model features 17 sectors that represent sectoral aggregates of all 71 industries of the German economy (as distinguished by official Input-Output data). Second, apart from the heterogeneity of age of households, we enrich the model by allowing for intra-cohort heterogeneity. The model distinguishes eight household types within each generation that correspond to the eight-income classes according to the German Income and Expenditure Survey (EVS: *Einkommens- und Verbrauchsstichprobe*). Intra-cohort heterogeneity is due to differences in labor productivity and the level of transfer income. Abstracting from this heterogeneity would not allow a meaningful analysis of the distributional consequences of changes in factor prices.

The results of this chapter are as follows. Over the main projection period from 2003-2050, the demographic transition in Germany will bring about a 7% decrease in output per capita. This is largely driven by a significant decrease in aggregate labor supply, about 17% over the same horizon (as we model a labor-leisure trade-off this number already incorporates general equilibrium reactions by households), and a substantial decline in investment rates (minus 26% in 2050). Population aging is shown to imply a "capital deepening" process which is reflected by an increase in the capital-labor ratio of around 2%. As the aging of society shifts the age composition of the population towards older households that (on average) hold a larger amount of assets, capital becomes relatively abundant, while labor, also due to the slower growth in the arrival of new workers, becomes relatively scarce. The higher capital-labor ratio reduces the need for firms to invest in new physical capital, and thus investment rates fall. Real factor prices reflect the varying scarcities of factors of production, and it is projected that the real capital rental rate (real wage rate) falls (increases) by 0.5% (0.5%) in 2010 and by about 1.3% (1.3%) in 2050.

The demographic transition induces substantial changes in the sectoral composition of output. Sectoral changes—as measured by a change in the share of sectoral output in total domestic output—range between about −8.5% and +6.5% in 2010 and between −23.5% and +20.0% in 2050. Expanding sectors are characterized by higher growth rates of sectoral employment of labor and capital relative to the economy-wide growth rates of labor and the capital stock. Among the sectors that benefit most from the demographic transition endowments due to population aging are *Public Services*, *Textiles and leather products*, *Education*, and *Health*. Output shares for the latter two sectors increase by almost one fifth between 2003 and 2050. The sectors that contract most, relatively to total domestic output, are *Metals*, *Machinery* (around −10%) and *Construction, Energy, and Water* (−23.51%). *Personal Goods*, *Mining*, and *Real Estate Services* contract by less (around −2% to −4%). Output shares for the remaining sectors *Mineral and Chemical Products*, *Food Products*, *Printing and Publishing*, *Wholesale and Retail Trade*, *Transportation and Communication*, and *Banking and Insurance Services* change by a relatively small extent (around −1.7% to +1.7%).

Accounting for structural changes in life-cycle consumption that are due to age-specific preferences does not affect the qualitative results of the benchmark model, and has only minor quantitative effects for aggregate variables. This underlines that the nature of the macroeconomic transition is predominantly shaped by the negative labor supply shock. On the sectoral level, however, demand-side induced effects stemming from age-dependent consumer spending are found to be quantitatively important.

In order to evaluate the welfare consequences of the demographic transition we ask the following question: suppose households of different age and type alive in 2003 were to live through the economic transition with changing factor prices induced by the demographic change. Then how would their welfare change relative to a situation without a demographic transition? We find that young households experience substantial welfare gains because they benefit from a future path of increasing wage rates and do not suffer strongly from large losses of capital income on already accumulated financial wealth. Newborn households in 2003 obtain welfare gains in the order of 14% and more in lifetime income. In contrast, older asset-rich households tend to gain less or even lose because of declining interest rates. From an intra-generational perspective, we find that those members of society for whom labor income constitutes a smaller part of (future) resources, i.e. households with relatively low labor productivity, benefit less from the demographic transition. Welfare changes for households with the highest labor productivity are estimated to be almost twice as large in terms of equivalent variation in lifetime income as for the least productive households. This underlies the need to account for intra-cohort heterogeneity.

The demographic transition is projected to increase income inequality in Germany, indicated by an increase in the Gini coefficient by 8.4%. The growing dispersion in household income is driven by a rise in capital income inequality and overcompensates the projected decrease in labor income inequality. As capital income is proportional to wealth, and because the demographic transition induces a shift towards older asset-rich households, capital income inequality is bound to increase. Labor income inequality decreases because a future path of increasing wages implies a flatter labor supply profile over the life cycle. We also find that intergenerational factors rather than a redistribution between different types of households within a particular age group account for the observed evolution of income inequality.

Despite the economic relevance of sectoral changes as an adjustment mechanism to demographic change, we are only aware of one example in the literature that addresses this issue. Fougere, Mercenier and Merette (2007) construct a multi-sectoral OLG model for the Canadian economy which features, among other factors, segmented labor markets and age-dependent preferences. Their analysis focuses on labor market effects of population aging that are induced by shifts in the sectoral composition of output and does not investigate the consequences for welfare and the distribution of income and wealth. Compared to their model, we include endogenous labor supply, intra-cohort heterogeneity, and also allow for international borrowing and lending, whereas they require trade to be balanced in each period.

More generally, this chapter relates to the literature that uses models and techniques pioneered by Auerbach and Kotlikoff (1987) to study the economic consequences of population aging. The majority of these studies pays special attention to the impact of aging on the viability of social security systems in closed economies (see e.g. Huang et al. (1997), De Nardi et al. (1999), Abel (2003), and Fehr et al. (2004a)) or in open economy settings (Attanasio et al. (2006), Börsch-Supan et al. (2006)). Relative to the literature we see the contribution of this chapter in (i) taking into account sectoral adjustments that influence the economy's transition path (ii) in evaluating the welfare consequences of population aging per se and not just the alternative social security reform scenarios and (iii) in incorporating intra-cohort heterogeneity which allows to analyze the distributional consequences of changing factor prices due to the demographic shifts. A similar type of welfare analysis is carried out by Krueger and Ludwig (2007) in a multi-region model with household heterogeneity that, however, also lacks a disaggregated sectoral structure. Furthermore, and in contrast to the present model that is specifically calibrated to the German economy, their framework is geared towards studying the implications of population aging for international capital flows among a number of world regions.

The fact that multi-sectoral OLG models virtually do not appear in the literature on demographic change—Fougere et al. (2007) being the notable exception—is partly due to the computational burden that is associated with solving such large-scale models. Especially for the type of analysis that we pursue in this chapter, the numerical problem at hand involves a large number of dimensions resulting from the joint presence of an array of important model features such as realistic lifetime of agents, heterogeneous households, a multi-sectoral structure, and demographic dynamics. In order to compute the equilibrium transition path for the model economy, the decomposition algorithm from Chapter 2 is used. Note that with a conventional integrated solution methods, e.g., as put forward by Rasmussen and Rutherford (2004) for the case of solving OLG models in a complementarity format, this type of analysis would not be feasible.

The remainder of this chapter is organized as follows. In the next section we describe and formulate the quantitative model in a mixed-complementarity format. Section 4.3 discusses the calibration of model parameters and describes the various data used. Section 4.4 presents results for the benchmark model. In Section 4.5 we conduct a number of sensitivity analyses to check for the robustness of the results, including age-specific consumption behavior. Section 4.6 concludes, and separate appendices contain more information about details of the model formulation and the adopted computational strategy.

4.2 Model Description

To quantify the economic consequences of the demographic transition for Germany, we construct an overlapping generations model ad modum the Auerbach and Kotlikoff (1987) model. The model features 17 sectors and 8 different household types within each generation. Households have a deterministic lifespan of 74 periods, hence at each point in time there are $74 \times 8 = 592$ different household types. We adopt the small open economy assumption and model international trade according to the standard assumption that foreign and domestic goods are imperfect substitutes.[1] We do not impose any restriction on the balance of payments other than that the present value of exports must equal the present value of imports over the infinite horizon, and therefore allow for international borrowing and lending. The analysis is carried out under the hypothesis of a stylized neoclassical world without market imperfections and in which agents behave fully rationally. The model is calibrated using German Input-Output data for the year 2003 and the numerical specification

[1] Without this last assumption only the net trade balance would matter and the levels of imports and exports would be indeterminate.

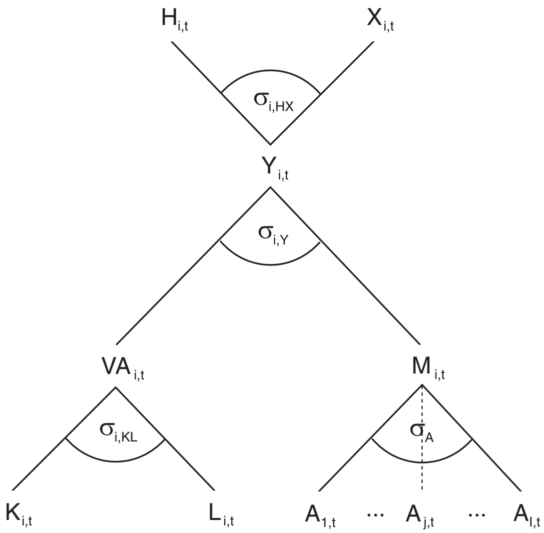

Fig. 4.2 Nesting structure of production

of the households side partly draws on data from the 2003 German Income and Expenditure Survey. Finally, demographic data is taken from the Federal Statistical Office of Germany and serves as the major exogenous driving process.

4.2.1 Firms

4.2.1.1 Production

Figure 4.2 provides a schematic overview of the nesting structure of sectoral production. In each production sector $i, j = 1, \ldots, I$, a representative firm produces a homogeneous output ($Y_{i,t}$). We use a nested constant-elasticity-of-substitution (CES) production function to reflect empirical evidence on the substitution possibilities. In the top nest, a material composite ($M_{i,t}$) is combined in fixed proportions with aggregate value added ($VA_{i,t}$), i.e. $\sigma_{i,Y} = 0, \forall i$. M consists of intermediate inputs with fixed coefficients (Leontief production structure), whereas $VA_{i,t}$ consists of capital ($K_{i,t}$) and labor ($L_{i,t}$) services, trading off at a constant elasticity of substitution. Adopting the calibrated share form (see Appendix 4.6), the unit cost function for each sector i can be written as[2]:

[2] There are no explicit production functions in the model because all necessary information is contained in the dual cost functions.

$$C_{i,t} = \Omega_{Y,i} \left(\beta_i^{VA} C_{VA,i,t} + (1-\beta_i^{VA}) C_{M,i,t}\right) \quad (4.1)$$

with

$$C_{VA,i,t} = \left[\beta_i^L \left(\frac{p_{l,t}}{\overline{p}_{l,t}}\right)^{1-\sigma_{VA,i}} + (1-\beta_i^L) \left(\frac{p_{r,t}}{\overline{p}_{r,t}}\right)^{1-\sigma_{VA,i}}\right]^{\frac{1}{1-\sigma_{VA,i}}} \quad (4.2)$$

$$C_{M,i,t} = \sum_j \frac{p_{A,j,t}}{\chi_j} \quad , \quad i \neq j \quad (4.3)$$

where $C_{VA,i,t}$ and $C_{M,i,t}$ =cost index of intermediary aggregate VA and M, respectively, $\Omega_{Y,i}$ =scale parameter, β_i^{VA} = benchmark value share of VA in total cost, β_i^L =benchmark value share of L in VA aggregate, $p_{l,t}$ =wage rate, $p_{r,t}$ =rental rate of capital, $p_{A,j,t}$ =price of Armington good j, χ_j =fixed quantity of Armington good j, $\sigma_{VA,i}$ =elasticity of substitution between capital and labor in the VA nest. Variables with a "bar" superscript denote benchmark values.

4.2.1.2 Factor Demand

Capital and labor are perfectly mobile across sectors. Each individual firm is assumed to be small in relation to its respective sector and operates on perfectly competitive markets taking goods and factor prices as given, and earning zero profits. Cost minimization yields the following demand functions for primary factors at the sectoral level (applying Shepard's Lemma):

$$K_{i,t} = Y_{i,t} \left(C_{VA,i,t} \frac{\overline{p}_{r,t}}{p_{r,t}}\right)^{\sigma_{VA,i}} \quad (4.4)$$

$$L_{i,t} = Y_{i,t} \left(C_{VA,i,t} \frac{\overline{p}_{l,t}}{p_{l,t}}\right)^{\sigma_{VA,i}} . \quad (4.5)$$

Demand for intermediate input j by sector i is given by:

$$A_{i,j,t} = Y_{i,t}\, \chi_j^{-1} \quad , \quad i \neq j . \quad (4.6)$$

4.2.1.3 Capital Accumulation and Investment

The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = (1-\delta) K_t + I_t \quad (4.7)$$

where K_t =aggregate capital stock, I_t =gross aggregate investment, and δ =periodic depreciation rate. The associated dual cost functions for producing capital services are given by:

$$p_{k,t} = p_{r,t} + p_{k,t+1} (1-\delta) \quad (4.8)$$

where $p_{k,t}$ =purchase price of capital. Investment goods are produced from sectoral outputs with fixed production coefficients. A unit of the investment good adds to next period's capital stock. The investment technology is given by the following dual cost function:

$$\Omega_I \left(\sum_{i=1}^{I} \beta_i^{IA} \frac{p_{A,i,t}}{\overline{p}_{A,i,t}} \right) \geq p_{k,t+1} \tag{4.9}$$

where β_i^{IA} =value share of intermediate good i in total investment, and Ω_I =scale parameter.

4.2.2 *Foreign Trade*

4.2.2.1 Product Differentiation for Exports

Domestically produced sectoral goods are converted through a constant elasticity of transformation (CET) function into specific goods destined for the domestic market ($H_{i,t}$) and goods destined for the export market ($X_{i,t}$). The derived pricing equation is given by:

$$\left[\beta_i^X \left(\frac{p_{f,t}}{\overline{p}_{f,t}} \right)^{1+\sigma_{HX}} + \beta_i^H \left(\frac{p_{H,i,t}}{\overline{p}_{H,i,t}} \right)^{1+\sigma_{HX}} \right]^{\frac{1}{1+\sigma_{HX}}} = \frac{p_{Y,i,t}(1+t_{Y,i})}{\overline{p}_{Y,i,t}(1+\overline{t}_{Y,i})} \tag{4.10}$$

where β_i^X =value share of exports, β_i^H =value share of domestic consumption in domestic production, $p_{f,t}$ =price for foreign exchange, $p_{H,i,t}$ =price for $H_{i,t}$, $p_{Y,i,t}$ =price for $Y_{i,t}$, $t_{Y,i}$ =output tax in sector i, σ_{HX} = elasticity of transformation. Following the small open economy assumption, export and import prices in foreign currency are not affected by the behavior of the domestic economy. In other words, the small open economy faces infinitely elastic world export demand and world import supply functions.

4.2.2.2 Armington Assumption

Similarly to the export side, we adopt the Armington assumption of product heterogeneity for the import side. A CES function characterizes the choice between imported and domestically produced varieties of the same good. The associated pricing equation is given by:

$$\frac{p_{A,i,t}(1+t_{AR,i})}{\overline{p}_{A,i,t}(1+\overline{t}_{AR,i})} = \Omega_{A,i} \left[\beta_i^{AH} \left(\frac{p_{H,i,t}}{\overline{p}_{H,i,t}} \right)^{1-\sigma_{AR,i}} + \beta_i^{AM} \left(\frac{p_{f,t}}{\overline{p}_{f,t}} \right)^{1-\sigma_{AR,i}} \right]^{\frac{1}{1-\sigma_{AR,i}}} \tag{4.11}$$

where $A_{i,t}$ =Armington good, $\Omega_{A,i}$ =scale parameter, β_i^{AH} =value share of domestic production in domestic consumption, β_i^{AM} =value share of imports, $p_{A,i,t}$ =price of Armington good, $t_{AR,i}$ = tax on Armington goods, $\sigma_{AR,i}$ = elasticity of substitution of sector i.

The representation of foreign and domestic goods as imperfect substitutes has the implication that although there is a constant interest rate on the international bond market, the domestic interest rate may deviate from the world market rate during a transition period. This happens because building up the domestic capital stock requires domestic as well as imported inputs. An increase in the capital stock thereby drives up the relative price of domestic output and the domestic interest rate until the economy settles in a new steady state with constant relative prices.

4.2.2.3 Supply of and Demand for Domestic and Foreign Goods

From (4.10), supply of sectoral production i to the domestic and export market is:

$$\frac{H_{i,t}}{\overline{H}_{i,t}} = \frac{Y_{i,t}}{\overline{Y}_{i,t}} \left(\frac{p_{Y,i,t}(1+t_{Y,i})}{\overline{p}_{Y,i,t}(1+\overline{t}_{Y,i})} \frac{\overline{p}_{H,i,t}}{p_{H,i,t}} \right)^{-\sigma_{HX}} \tag{4.12}$$

$$\frac{X_{i,t}}{\overline{X}_{i,t}} = \frac{Y_{i,t}}{\overline{Y}_{i,t}} \left(\frac{p_{Y,i,t}(1+t_{Y,i})}{\overline{p}_{Y,i,t}(1+\overline{t}_{Y,i})} \frac{\overline{p}_{f,t}}{p_{f,t}} \right)^{-\sigma_{HX}} \tag{4.13}$$

and from (4.11), Armington demand for domestic and imported variety of good i is given by:

$$\frac{H_{i,t}}{\overline{H}_{i,t}} = \frac{A_{i,t}}{\overline{A}_{i,t}} \left(\frac{p_{A,i,t}(1+t_{AR,i})}{\overline{p}_{A,i,t}(1+\overline{t}_{AR,i})} \frac{\overline{p}_{H,i,t}}{p_{H,i,t}} \right)^{\sigma_{AR,i}} \tag{4.14}$$

$$\frac{IM_{i,t}}{\overline{IM}_{i,t}} = \frac{A_{i,t}}{\overline{A}_{i,t}} \left(\frac{p_{A,i,t}(1+t_{AR,i})}{\overline{p}_{A,i,t}(1+\overline{t}_{AR,i})} \frac{\overline{p}_{f,t}}{p_{f,t}} \right)^{\sigma_{AR,i}} . \tag{4.15}$$

4.2.3 *Households*

The household side of the model is disaggregated both within and across generations. In each period, eight new households corresponding to eight income classes enter the model. Each household is characterized by overlapping generations and has a finite and known lifespan in which they engage in market activities. In each period over the life cycle, households are endowed with units of productive time that they allocate between labor and leisure. Households are assumed to be forward-looking individuals that form rational point expectations (perfect foresight). Each household maximizes lifetime utility by choosing optimal consumption and leisure paths over the life cycle subject to a lifetime budget and time endowment constraint. There is no aggregate or household-specific uncertainty.

4.2.3.1 Utility Maximization

A household of generation g and type $h = 1,\dots,H$ is born at the beginning of year $t = g$, lives for $N+1$ years, and is endowed with $\omega_{g,t}$ units of time in each period $g \le t \le g+N$. Time endowment, i.e. the size of a household, varies across generation and time and is selected to match real German demographic data. Generations born prior to year zero and that are alive in the first model period are indexed by $g = -N,\dots,-1$. Adopting again the calibrated share form, the optimization problem for a generation $g = -N,\dots,\infty$ and household type $h = 1,\dots,H$, expressed with present value prices, is given by:

$$\max_{c_{g,h,t},\,\ell_{g,h,t}} u_{g,h}\left(z_{g,h,t}\right) = \left[\sum_{t=g}^{g+N} \theta_{g,h,t} \left(\frac{z_{g,h,t}}{\bar{z}_{g,h,t}}\right)^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\frac{\sigma_h}{1-\sigma_h}} \tag{4.16}$$

subject to:

$$\frac{z_{g,h,t}}{\bar{z}_{g,h,t}} = \left[\alpha_h \left(\frac{c_{g,h,t}}{\bar{c}_{g,h,t}}\right)^{\frac{1-\nu_h}{\nu_h}} + (1-\alpha_h)\left(\frac{\ell_{g,h,t}}{\bar{\ell}_{g,h,t}}\right)^{\frac{1-\nu_h}{\nu_h}}\right]^{\frac{\nu_h}{1-\nu_h}} \tag{4.17a}$$

$$\frac{c_{g,h,t}}{\bar{c}_{g,h,t}} = \left[\theta_h^F \left(\frac{a_{1,g,h,t}}{\bar{a}_{1,g,h,t}}\right)^{\frac{1-\kappa_h}{\kappa_h}} + \theta_h^{NF}\left(\frac{c_{g,h,t}^{NF}}{\bar{c}_{g,h,t}^{NF}}\right)^{\frac{1-\kappa_h}{\kappa_h}}\right]^{\frac{\kappa_h}{1-\kappa_h}} \tag{4.17b}$$

$$\frac{c_{g,h,t}^{NF}}{\bar{c}_{g,h,t}^{NF}} = \left[\sum_{i\in NF} \varphi_{i,h,t}\left(\frac{a_{i,g,h,t}}{\bar{a}_{i,g,h,t}}\right)^{\frac{1-\sigma_{NF}}{\sigma_{NF}}}\right]^{\frac{\sigma_{NF}}{1-\sigma_{NF}}} \tag{4.17c}$$

$$\begin{aligned}\sum_{t=g}^{g+N} p_{a,g,h,t}\, c_{g,h,t} \le{} & p_{k,t}\,\bar{k}_{g,h,g} + p_{f,t}\,\bar{b}_{g,h,g} \\ & + \sum_{t=g}^{g+N} p_{l,t}\,\pi_{g,h,t}\left(\omega_{g,h,t} - \ell_{g,h,t}\right) + p_{f,t}\,\zeta_{g,h,t}\end{aligned} \tag{4.17d}$$

$$\ell_{g,h,t} \le \omega_{g,h,t} \tag{4.17e}$$

$$c_{g,h,t} \ge 0 \quad , \quad \ell_{g,h,t} \ge 0\,. \tag{4.17f}$$

Figure 4.3 visualizes the nesting structure of lifetime utility. Lifetime utility ($u_{g,h}$) is additively separable over time, and instantaneous utility ($z_{g,h,t}$) enters according to a constant-intertemporal-elasticity-of-substitution (CIES) function. $\theta_{g,h,t}$ = share parameter which accounts for discounting, and $\sigma_h =$ is the intertemporal elasticity of substitution. In the first sub-nest, a consumption aggregate ($c_{g,h,t}$) and leisure consumption ($\ell_{g,h,t}$) enter in a CES fashion, and $\nu =$elasticity of substitution between consumption aggregate and leisure, and $\alpha_h =$value share of material consumption. We distinguish between food and non-food products as empirical price elasticities typically differ substantially. Food products ($a_{1,g,h,t}$) and an aggregate of non-food products ($c^{NF}_{g,h,t}$) enter in a CES function. $\theta^F_h =$value share of food products, $\theta^{NF}_h =$value share of non-food products, and $\kappa_h =$elasticity of substitution between food and non-food products. In the bottom nest, non-food consumption of Armington goods ($a_{i,g,h,t}$) enters in a CES aggregator trading off at a constant elasticity of substitution of σ_{NF}.[3] $\varphi_{i,h,t}$ is the value share of a single non-food consumption good i in $c^{NF}_{g,h,t}$. $\varphi_{i,h,t}$ is assumed to differ across households types and time to allow for distinct preferences across households that belong to different income classes; for instance, expenditures for food products typically make up a larger fraction of disposable income for low-income than for richer households. Also, the composition of final demand is assumed to vary with age, i.e. preferences are age-dependent. Empirically, older people consume more health care services than younger households while spending on transport and communication typically declines with age.[4]

Utility maximization is subject to a number of constraints as described by (4.17d)–(4.17f). The present value of total consumption expenditure over the lifetime cannot exceed the present value of lifetime income which is derived from supplying labor ($\omega_{g,h,t} - \ell_{g,h,t}$) and transfer income where $\zeta_{g,h,t}$ is a lump sum transfer from the government which is denominated in the price of foreign exchange. This rules out that households die in debt. For later reference let lifetime income be defined as follows: $inc_{g,h} = \sum_{t=g}^{g+N} p_{l,t}\, \pi_{g,h,t}\,(\omega_{g,h,t} - \ell_{g,h,t}) + p_{f,t}\, \zeta_{g,h,t}$. We do not consider altruistic behavior thereby ruling out bequests or any other form of intergenerational transfers. $p_{x,t}$, $x = \{k,l,f\}$, denote the purchase price of capital, the wage rate, and the price of foreign exchange, respectively. $p_{a,g,h,t}$ is the price for the (household-specific) composite consumption good. $\pi_{g,h,t}$ is an index of labor productivity over the life cycle. $\overline{k}_{g,h,g}$ denotes the asset holdings of generation g at time $t = g$ that represent claims on the domestic capital stock. $\overline{b}_{g,h,g}$ represents the amount of assets being held in foreign currency. Current old generations, i.e. generations born prior to period zero, are endowed with non-zero assets of either type whereas all newborn households enter the economy with zero assets: $\overline{k}_{g,h,g} = \overline{b}_{g,h,g} = 0$, $\forall\, g \geq 0$, and $\forall\, h$. Total asset holdings of current old generations that are denominated in $p_{k,t}$ must equal the value of the domestic capital stock. We assume that the government and the trade deficit are both denominated in the price of foreign exchange. Hence, the total value of net foreign assets in period zero is equal to the present value of fu-

[3] Note that in the present setup, the assumption of multi-stage budgeting is innocuous since the utility function u is weakly separable and the sub-utility functions z and c are homothetic.

[4] In reality, preferences may not only be age- but also cohort-dependent. Empirical evidence for Germany, however, suggests that cohort effects appear to be small (Börsch-Supan, 2004). The adopted notion of household preferences thus assumes that preferences vary with age but are stable across generations. For a more detailed discussion see Section 4.5.1.

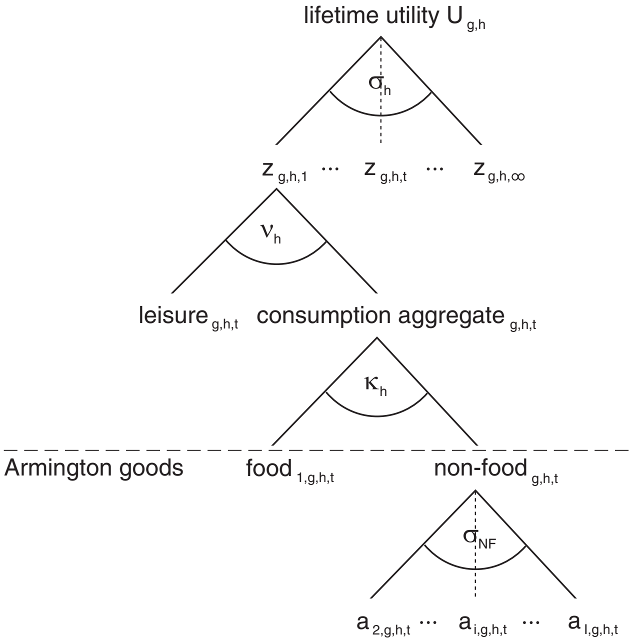

Fig. 4.3 Utility nesting

ture trade and government deficits over the infinite horizon.[5] Finally, in each period of the life cycle, time allocated to leisure consumption cannot exceed the time endowment $\omega_{g,h,t}$ (4.17e). Choices for material and leisure consumption are restricted to be nonnegative (4.17f).

4.2.3.2 Pricing Equations and Household Demand

The dual cost functions that correspond to the CES functions (4.16)–(4.17c) in the household optimization problem are given by:

$$\frac{p_{u,g,h}}{\overline{p}_{u,g,h}} = \left[\sum_{t=g}^{g+N} \theta_{g,h,t} \left(\frac{p_{z,g,h,t}}{\overline{p}_{z,g,h,t}} \right)^{1-\sigma_h} \right]^{\frac{1}{1-\sigma_h}} \tag{4.18}$$

[5] The initial distribution of domestic and foreign assets across current old generations is selected such that the economy is on a balanced growth path (for details on the calibration procedure see Section 4.3).

$$\frac{p_{z,g,h,t}}{\overline{p}_{z,g,h,t}} = \left[\alpha_h \left(\frac{p_{c,g,h,t}}{\overline{p}_{c,g,h,t}} \right)^{1-\nu_h} + (1-\alpha_h) \left(\frac{p_{\ell,g,h,t}}{\overline{p}_{\ell,g,h,t}} \right)^{1-\nu_h} \right]^{\frac{1}{1-\nu_h}} \tag{4.19}$$

$$\frac{p_{c,g,h,t}}{\overline{p}_{c,g,h,t}} = \left[\theta_h^F \left(\frac{p_{A,i,t}(1+\tau_c)}{\overline{p}_{A,i,t}(1+\overline{\tau}_c)} \right)^{1-\kappa_h} + \theta_h^{NF} \left(\frac{p^{NF}_{c,g,h,t}}{\overline{p}^{NF}_{c,g,h,t}} \right)^{1-\kappa_h} \right]^{\frac{1}{1-\kappa_h}} \tag{4.20}$$

$$\frac{p^{NF}_{c,g,h,t}}{\overline{p}^{NF}_{c,g,h,t}} = \left[\sum_{i \in NF} \varphi_{i,h,t} \left(\frac{p_{A,i,t}(1+\tau_c)}{\overline{p}_{A,i,t}(1+\overline{\tau}_c)} \right)^{1-\sigma_{NF}} \right]^{\frac{1}{1-\sigma_{NF}}} \tag{4.21}$$

where $p_{u,g,h}$ =price of intertemporal utility, $p_{z,g,h,t}$ =price for $z_{g,h,t}$, $p_{c,g,h,t}$ =price for composite consumption good, $p^{NF}_{c,g,h,t}$ =price index for non-food aggregate, τ_c =time-invariant tax on consumption, and $p_{\ell,g,h,t}$ =price for leisure consumption. Instead of dividing households into groups of workers and retirees, the model allows for optimal endogenous retirement decisions on behalf of the households. The equilibrium allocation involves a corner solution with respect to labor supply if the reservation wage exceeds the market wage rate, i.e. if:

$$p_{\ell,g,h,t} \geq p_{l,t}\, \pi_{g,h,t}\,. \tag{4.22}$$

A positive amount of labor is supplied if (4.22) holds with equality. For future reference, labor supply is denoted by $l_{g,h,t}$. Household demand functions can be written as:

$$\frac{z_{g,h,t}}{\overline{z}_{g,h,t}} = \frac{u_{g,h}}{\overline{u}_{g,h}} \left(\frac{p_{u,g,h}}{\overline{p}_{u,g,h}} \frac{\overline{p}_{z,g,h,t}}{p_{z,g,h,t}} \right)^{\sigma_h} \tag{4.23}$$

$$\frac{c_{g,h,t}}{\overline{c}_{g,h,t}} = \frac{z_{g,h,t}}{\overline{z}_{g,h,t}} \left(\frac{p_{z,g,h,t}}{\overline{p}_{z,g,h,t}} \frac{\overline{p}_{c,g,h,t}}{p_{c,g,h,t}} \right)^{\nu_h} \tag{4.24}$$

$$\frac{\ell_{g,h,t}}{\overline{\ell}_{g,h,t}} = \frac{z_{g,h,t}}{\overline{z}_{g,h,t}} \left(\frac{p_{z,g,h,t}}{\overline{p}_{z,g,h,t}} \frac{\overline{p}_{\ell,g,h,t}}{p_{\ell,g,h,t}} \right)^{\nu_h} \tag{4.25}$$

$$\frac{a_{i,g,h,t}}{\overline{a}_{i,g,h,t}} = \frac{c_{g,h,t}}{\overline{c}_{g,h,t}} \left(\frac{p_{c,g,h,t}}{\overline{p}_{c,g,h,t}} \frac{\overline{p}_{A,i,t}}{p_{A,i,t}} \right)^{\kappa_h} for\ i = 1, \tag{4.26a}$$

$$\frac{a_{i,g,h,t}}{\overline{a}_{i,g,h,t}} = \frac{c_{g,h,t}}{\overline{c}_{g,h,t}} \left(\frac{p_{c,g,h,t}}{\overline{p}_{c,g,h,t}} \frac{\overline{p}^{NF}_{c,g,h,t}}{p^{NF}_{c,g,h,t}} \right)^{\kappa_h} \left(\frac{p^{NF}_{c,g,h,t}}{\overline{p}^{NF}_{c,g,h,t}} \frac{\overline{p}_{A,i,t}}{p_{A,i,t}} \right)^{\sigma_{NF}} ,for\ i \in NF\,. \tag{4.26b}$$

4.2.4 Government

The government collects revenue from levying ad-valorem taxes ($t_{Y,i}$) on sectoral production ($X_{i,t}$ and $H_{i,t}$), and ad-valorem taxes ($t_{AR,i}$) on Armington production ($A_{i,t}$). We assume that the government budget is balanced over the infinite horizon, and that there is a constant tax rate on consumption (τ_c) that is determined endogenously such that:

$$p_{f,0}A_{G,0} + \sum_{t=0}^{\infty} \Phi_t = \sum_{t=0}^{\infty} \Gamma_t \tag{4.27}$$

where $A_{G,0}$ is the initial level of assets held by the government, which enters the model as an exogenous endowment. Total per period tax revenue is given by: $\Phi_t = \sum_{i=1}^{I} t_{Y,i}\left(p_{X,i,t}X_{i,t} + p_{H,i,t}H_{i,t}\right) + t_{AR,i}\, p_{A,i,t}A_{i,t} + \tau_c p_{C,t}C_t$, where p's are relevant prices and C_t denotes aggregate consumption. Government expenditure comprises government consumption (G_t) and total transfers to households (T_t) which are denominated in the price of foreign exchange: $\Gamma_t = p_{G,t}G_t + p_{f,t}T_t$. Government demand combines inputs from the production sectors in fixed proportions:

$$\frac{p_{G,t}}{\overline{p}_{G,t}} = \sum_{i=1}^{I} \beta_i^{GA} \frac{p_{A,i,t}}{\overline{p}_{A,i,t}} \tag{4.28}$$

where β_i^{GA} =value share of intermediate good i in total government consumption, $p_{G,t}$ =price of total government consumption.

4.2.5 *Market Clearing Conditions and Model Closure*

Armington goods can be used as intermediate inputs in production, or enter in household, government and investment demand. The market clearing condition for good i is therefore given by:

$$A_{i,t} = C_{A,i,t} + I_{i,t} + G_{i,t} + \sum_{j} A_{i,j,t} \quad , \quad i \neq j \tag{4.29}$$

where due to the presence of inter- and intragenerational household heterogeneity, total private consumption of good i is:

$$C_{A,i,t} = \sum_{g=t-N}^{t} \sum_{h=1}^{H} a_{i,g,h,t} \tag{4.30}$$

and $I_{i,t}$ and $G_{i,t}$ denote the investment and government demand for Armington good i, respectively. The market clearing conditions for primary factors of production are:

$$K_t = \sum_{i=1}^{I} K_{i,t} \tag{4.31}$$

$$L_t = \sum_{i=1}^{I} L_{i,t} \tag{4.32}$$

where aggregate labor supply (L_t) also has to be consistent with individual household decisions:

$$L_t = \sum_{g=t-N}^{t} \sum_{h=1}^{H} l_{g,h,t}\, \pi_{g,h,t}\,. \tag{4.33}$$

Using (4.12) and (4.14), the market clearing condition for sectoral output i destined for the home market is:

$$\frac{Y_{i,t}}{\overline{Y}_{i,t}}\left(\frac{p_{Y,i,t}(1+t_{Y,i})}{\overline{p}_{Y,i,t}(1+\overline{t}_{Y,i})}\,\frac{\overline{p}_{H,i,t}}{p_{H,i,t}}\right)^{-\sigma_{HX}} = \frac{A_{i,t}}{\overline{A}_{i,t}}\left(\frac{p_{A,i,t}(1+t_{AR,i})}{\overline{p}_{A,i,t}(1+\overline{t}_{AR,i})}\,\frac{\overline{p}_{H,i,t}}{p_{H,i,t}}\right)^{\sigma_{AR,i}}. \tag{4.34}$$

The shadow price of "time", or the price for leisure, adjusts to equalize available time resources and the amount of time that goes into labor supply and leisure activities:

$$\omega_{g,h,t} = l_{g,h,t} + \ell_{g,h,t} \tag{4.35}$$

where $l_{g,h,t}$ is a complementary variable determined by (4.22). The market clearing condition for government consumption is given by:

$$G_t = \overline{G}_t\,. \tag{4.36}$$

The market clearing condition for the capital stock is given by:

$$K_{t-1}(1-\delta) + I_{t-1} + \sum_{g=t-N}^{t}\sum_{h=1}^{H}\overline{k}_{g,h,g} = K_t\,. \tag{4.37}$$

This equation simply restates (4.7) and also takes into account initial asset holdings by households alive in year zero that represent claims on the domestic capital stock (third summand on the left hand side). A similar version holds for the post-terminal capital stock (see Appendix 4.6, equation (4.51)). Lifetime utility is given by:

$$u_{g,h} = \frac{inc_{g,h}}{p_{u,g,h}}\,. \tag{4.38}$$

Prices for the consumption aggregates $z_{g,h,t}$ and $c_{g,h,t}$ that enter utility are given by the following market clearing conditions:

$$z_{g,h,t} = z^D_{g,h,t} \tag{4.39}$$

$$c_{g,h,t} = c^D_{g,h,t} \tag{4.40}$$

where $z^D_{g,h,t}$ and $c^D_{g,h,t}$ denote respective demands given by (4.23) and (4.24). Finally, we fix the value of the trade deficit over the infinite horizon and it may therefore deviate from the baseline level in individual periods. Instead of imposing this relationship directly, however, we require balance in all goods and transfers denominated in foreign exchange. Because of the assumption of perfect international capital markets the current value price of exports and imports is constant and we therefore only operate with a single price for foreign exchange determining the level, denoted by p_f. The present value price for foreign currency is given by $p_{f,t} = (1+\overline{r})^{-t}p_f$ where $\overline{r}$ denotes the world interest rate. The market clearing condition for foreign currency is:

$$\sum_t p_{f,t}\left(D_t - T_t + \sum_{g=t-N}^{t}\sum_{h=1}^{H}\zeta_{g,h,t} + \overline{b}_{g,h,g}\right) = \sum_t\sum_i p_{f,t}\left(IM_{i,t} - X_{i,t}\right) \tag{4.41}$$

where D_t denotes the periodic government deficit denominated in the price of foreign exchange.

4.2.6 Equilibrium

We now define the dynamic general equilibrium of this economy in a complementarity format. Rutherford (1995b) and Mathiesen (1985) have shown that a complementary-based approach is convenient, robust, and efficient. A characteristic of economic models is that they naturally involve a complementary problem, i.e. given a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$. A fundamental advantage of formulating an economic model as a complementary problem is the ability to express weak inequalities and complementary slackness. This formulation of the equilibrium is exploited by the GAMS program and the solver we use to compute the transition and steady state of the infinite-horizon economy.[6] An equilibrium in complementarity format is represented by a vector of activity levels, a non-negative vector of prices, and a non-negative vector of incomes such that:

1. no production activity makes a positive profit (*zero profit conditions*),
2. excess supply (supply minus demand) is non-negative for all goods and factors (*market clearance conditions*), and
3. expenditure does not exceed income (*budget constraints*).

Zero-profit conditions exhibit complementary slackness with respect to associated activity levels, market clearing conditions exhibit complementary slackness with respect to market prices, and budget constraints define income variables. Adopting this type of equilibrium formulation, Table 4.1 shows each inequality, its associated or so-called complementary variable, and the number of unknowns. In order to approximate the infinite horizon economy by a finite dimensional complementarity problem, it is necessary to impose a number of additional constraints on the system of equations defined above. See Section 4.6 for more details.

4.3 Calibration and Data

In this section we discuss the specification of model parameters, the data that is used as model inputs, and aspects of the numerical implementation. In terms of

[6] For an overview of applied general equilibrium modeling with GAMS (General Algebraic Modeling System) and the GAMS/MPSGE subsystem see e.g. Rutherford(1998,1999a), and http://www.mpsge.org.

Table 4.1 Inequalities and associated complementary variables

Complementary variable	Inequality	Number of unknowns
Activity level	Zero profit inequality	
$Y_{i,t}$	(4.1)	I × T
K_t	(4.8)	T
I_t	(4.9)	T
$A_{i,t}$	(4.11)	T
G_t	(4.28)	T
$u_{g,h}$	(4.18)	G × H
$z_{g,h,t}$	(4.20)	G × H × (N+1)
$c_{g,h,t}$	(4.21)	G × H × (N+1)
$l_{g,h,t}$	(4.22)	G × H × (N+1)
Price	Market clearing inequality	
$p_{Y,i,t}$	(4.14)	I × T
$p_{A,i,t}$	(4.29)	I × T
$p_{r,t}$	(4.31)	T
$p_{l,t}$	(4.32)	T
$p_{k,t}$	(4.37)	T
$p_{f,t} = p_f(1+\bar{r})^{-t}$	(4.41)	1
$p_{u,g,h}$	(4.38)	G × H
$p_{\ell,g,h,t}$	(4.35)	G × H × (N+1)
$p_{G,t}$	(4.36)	T
$p_{z,g,h,t}$	(4.39)	G × H × (N+1)
$p_{c,g,h,t}$	(4.40)	G × H × (N+1)
Income variable	Income definition	
$inc_{g,h}$	(4.17d)	G × H
gov	(4.52)	1
Variable	Additional constraints[1]	
τ_c	(4.27)	1

1: See Appendix 4.6 for more details.

calibration, we need to choose parameters governing the demographic transition, the production technology, tax rates, endowments, and preferences. Table 4.12 provides an overview of the model parameters.

4.3.1 Numerical Implementation

The equilibrium path of the overlapping generations model is determined by employing the decomposition algorithm which has been described in Chapter 2. The algorithm approximates the equilibrium of the OLG economy by solving a sequence of appropriately chosen Ramsey growth problems. In each iteration, demand functions of OLG households are evaluated given a candidate price vector that has been computed solving a “related” representative agent Ramsey growth model. In the subsequent iteration, the general equilibrium of an updated Ramsey growth problem is computed. Updating involves the recalibration of the (artificial) representative agent's utility function based on households' "partial equilibrium" quantity choices from the previous iteration. The model is solved in one-year intervals.

4.3.2 Thought Experiment and Calibration of the Model

The way we use the model may be somewhat unconventional in CGE modeling because we do not perform counterfactual policy analysis but rather use the model as a projection tool that is fed with demographic data. The thought experiment which underlies the simulations is the following: we take a time-varying demographic structure as the exogenous driving process that evolves according to demographic projections by the Federal Statistical Office of Germany. This transition of the population structure induces a transition path of the economy.

We calibrate the model to the base year 2003 using Input-Output data for Germany and various other data sources.[7] For the present application calibration is a complex task because non-stationarity of the population data implies that the economy is not on a balanced growth path.[8] Our calibration strategy proceeds in two steps. First, we assume (contra-factually) an initial steady state situation for the base year 2003, and exploit this assumption to calibrate a number of household parameters (see Section 4.3.6).[9] Then, in order to find the baseline equilibrium path in the presence of non-stationary population dynamics and to ensure that the base year data is reproduced endogenously, it is necessary to impose a number of restrictions on the entire transition model. In our case, technology parameters $\Omega_{Y,i}$, $\forall i$, are calibrated such that the model replicates sectoral output data in the first model period. Similarly, $\Omega_{A,i}$, $\forall i$, and Ω_I are solved for endogenously to match base year values of Armington production and investment, respectively.

To ensure consistency between household behavior and aggregate benchmark data, it is necessary to solve endogenously for two of the parameters in the individuals' optimization problem. Rasmussen and Rutherford (2004) present a calibration

[7] Our calibration strategy which focuses exclusively on observations of a base year—as for the OLG context, e.g., in Jensen and Rutherford (2002) and Rasmussen and Rutherford (2004)—constitutes one possible way of calibrating OLG models to empirical data. This procedure differs from the approach in Auerbach and Kotlikoff (1987) or Altig, Auerbach, Kotlikoff, Smetters and Walliser (2001) where parameters are simply chosen so as to generate "reasonable" outcomes by informally matching long term averages of statistical data. A more systematic approach would be to perform a so-called "dynamic calibration" by choosing time-varying model parameters to match observed time series data. This is an ongoing area of research; see, e.g., Ludwig (2005) who suggests to estimate free model parameters by using method of moments methodology that sets to zero the average discrepancy between actual and predicted (simulated) values. Future work needs to compare how alternative approaches to model calibration may influence model results.

[8] See, e.g., Wendner (1999) for a detailed discussion.

[9] We select initial asset holdings $\overline{k}_{g,h,g}$ and $\overline{b}_{g,h,g}$ by assuming (contra-factually) that the economy is on a balanced growth path, and use the procedure as described in Rasmussen and Rutherford (2004). The time preference rate ρ is calibrated such that the value of total household consumption equals aggregate consumption in the base year.

procedure that can be used for situations where the economy is assumed to be on a balanced growth path. It is, however, straightforward to extend their procedure to non-steady state situations. In the present case, the nature of the decomposition algorithm that we used to compute the transition path of the economy, and that makes it possible to solve such a large-scale model in the first place, prevents the application of such a calibration procedure.[10] As a consequence, the value of private consumption and the value of total domestic and foreign assets deviate from the benchmark data.

4.3.3 Input-Output Data

We employ a social accounting matrix (SAM) for Germany, which is based on Input-Output (IO) data for the year 2003 issued by the Federal Statistical Office of Germany, and which distinguishes 17 production sectors.[11] Table 4.2 lists the names (and acronyms) of the industrial sector aggregates and describes the subsumed industries from the original IO data compounded in each sectoral aggregate. This level of aggregation captures most of the interesting sectors of the German economy that would be affected by demographic change. In particular, transportation and communication, banking and insurance services, real estate services, education, and health services are included as separate sectors.

For aggregation, we include all 71 industries of the German IO table. We adjust base year investments to obtain a *micro-consistent*[12] SAM which is consistent with a momentary general equilibrium in the base year. The calibration of technologies using base year data on prices and quantities is a standard exercise in CGE modeling, and hence needs no further discussion (see, e.g., Robinson (1991) and Rutherford (1999a) for a general discussion and Rasmussen and Rutherford (2004, pp.1405) for an application in an OLG context). Based on the IO data for Germany, we are able to obtain numerical values for all share parameters (β's and χ's).

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[10] Since the algorithm decomposes the OLG economy into a representative agent model and a partial equilibrium problem, and involves an iterative procedure between these two subproblems, imposing calibration restrictions on the representative agent model does only work if the parameters used for calibration do not alter demand functions of OLG households. Otherwise, the algorithm's search dynamics are profoundly altered because the endogenized parameters become part of the adjustment process. In this case, we find that the "Sequential Recalibration" algorithm does not converge, and is therefore not useful in finding the true equilibrium allocation. This also explains why we have only used technology parameters for calibration of the aggregate economy.

[11] Our SAM is based on the Input-Output table 1.1., Issue 18 (2), Federal Statistical Office of Germany.

[12] The SAM incorporates an internally consistent set of relationships. Among the most important identities in the SAM are zero profits for each production activity, market clearance for all commodities and factors, budget balance for households and the government, and the value of imports equals the value of exports, net of capital flows. In fact, an Arrow-Debreu type applied general equilibrium model, such as this one, must be based on a SAM; otherwise the data would be inconsistent with the underlying behavioral assumptions of the model (which are based on optimizing behavior of all agents subject to budget constraints and adding up conditions).

Table 4.2 Industrial sector aggregates

Sectoral aggregate in model	Single industries and CPA[1] numbers in German IO table
Agriculture and Fisheries (AGF)	01,02,05: Erzeugnisse der Land- und Forstwirtschaft, Fischerei
Mining (MIN)	10-14: Bergbauerzeugnisse, Steine und Erden
Food Products (FOP)	15,16: Nahrungs- und Futtermittel, Getrnke, Tabakerzeugnisse
Textiles and Leather Products (TXL)	17,19: Textilien, Bekleidungs- und Lederwaren
Printing and Publishing (PRP)	20-22.3: Erzeugnisse des Holz-, Papier-, Druck- und Verlagsgewerbes
Mineral and Chemical Products (MIC)	23-26.8: Mineralölerzeugnisse, chemische Erzeugnisse, Glas, Keramik, bearbeitete Steine und Erden
Metals (MET)	27.1-28: Metalle
Machinery (MAC)	29-35: Maschinen, Fahrzeuge, Datenverarbeitungsgeräte,elektrotechn. Geräte
Personal Goods (PGO)	36,37: Möbel, Schmuck, Musikinstrumente u.ä.
Construction, Energy, and Water (CEW)	40.1-41: Bauarbeiten, Energie und Wasser
Wholesale and Retail Trade (WRT)	50-52,55: Handelsleistungen und DL des Gastgew.
Transportation and Communication (TRC)	60.1-64: Verkehrs- und Nachrichtenübermittlungs-DL
Banking and Insurance Services (BIN)	65-67: DL der Kreditinstitute und Versicherungen
Real Estate Services (RES)	70-74: DL des Grundstcks- und Wohnungswesens, Unternehmens-DL
Public Services (PBS) digung,	75.1-75.3,90-95: DL der öffentl. Verwalung, Vertei- DL von Interessensvertretungen,Kirchen u.ä., Kultur-, Sport- und Unterhaltungs-DL, Abfallbeseitigungs- u. sonstige, Entsorgungsleistungen,
Sonstige DL,	DL privater Haushalte
Education (EDU)	80: Erziehungs- und Unterrichts-DL
Health (HEA)	85: DL des Gesundheits-, Veterinär- und Sozialwesens

Note: 1: For reference we give CPA numbers that identify each single industry that is used in aggregation. CPA = Classification of Products by Activity according to Eurostat. Abbreviations used: DL = Dienstleistungen.

4.3.4 Representation of the Household Sector

Within each generation we distinguish between eight different socio-economic groups representing the eight income classes according to the German Income and Expenditure Survey (EVS) for the year 2003.[13] The EVS is a representative household survey by the Federal Statistical Office of Germany. The 2003 sample comprises 53,432 households. Table 4.3 summarizes the principal characteristics of the household types. Note that transfers to households comprise pensions, unemployment benefits and various social transfers.[14] This broad definition of transfer income

[13] Source: *Einkommens- und Verbrauchsstichprobe — Einnahmen und Ausgaben privater Haushalte*, Issue 15 (4), Federal Statistical Office of Germany.

[14] This definition follows the classification in the original EVS data.

Table 4.3 Household characteristics according to EVS

Household group	Annual income (in €)	Population size[1]	Labor share	Annual transfers per capita (in €)
h=1 (poorest)	(< 10800)	3.04	0.01	6,096
h=2	(10800 – 15600)	4.67	0.03	8,496
h=3	(15600 – 18000)	2.32	0.02	8,580
h=4	(18000 – 24000)	5.30	0.07	9,960
h=5	(24000 – 31200)	5.61	0.10	12,276
h=6	(31200 – 43200)	7.32	0.22	11,712
h=7	(43200 – 60000)	5.54	0.25	12,024
h=8 (richest)	(> 60000)	4.31	0.32	14,436

Note: The different socio-economic groups represent the eight income classes according to the German Income and Expenditure Survey (EVS) for the year 2003. 1: Population size refers to the (extrapolated) number of households in million. Labor share denotes the share of the sum of labor incomes earned by all households in this income class in the economy-wide labor income.

explains the positive correlation with the level of total household income. In the simulations later on, transfer income per capita is held constant.

Following Auerbach and Kotlikoff (1987), we assume an age-related labor efficiency endowment profile which has the following form:

$$\pi_{g,h,t} = \hat{\zeta}_{scale,h}\, exp\left[4.47 + \lambda_0 (t-g) - \lambda_1 (t-g)^2\right] / exp[4.47]$$

where $\lambda_0 = 0.032$, $\lambda_1 = 0.003$. Not surprisingly, results are sensitive with respect to the numerical specification of $\pi_{g,h,t}$, and we will explore this in Section 4.5. To match the observed labor share of each income class, we augment their specification by proportionally scaling the labor productivity index by a factor $\hat{\zeta}_{scale,h}$. It is assumed that transfers within each household group between individuals of different age are distributed equally, i.e. the level of transfers is kept constant over the life cycle. We set the value share of material consumption (α_h) equal to 0.8 for all h. Expenditure shares for consumption of Armington good i ($\varphi_{i,h,t}$) are taken from the value shares of aggregate consumption as given by the IO table. Hence, in our benchmark model, households across groups *and* age are assumed to have homogenous preferences and only differ with respect to labor productivity and transfer income. We will relax this assumption later on to analyze the effect of structural changes in the final demand composition of old vis-à-vis young people. Homogeneity of preferences in the baseline considerably eases the calibration of the model and, more importantly, provides a way of disentangling the demand-side induced effects of population aging.

4.3.5 Demographic Projections

Demography is taken as the major exogenous driving force in the simulation model.[15] We use detailed demographic data by the Federal Statistical Office of Germany that covers the years from 2003 to 2050.[16] Population data is given at an annual frequency for age-groups of individual years. We assume that households enter economic life at the age of 16 and have a deterministic lifetime of 74 periods (the maximum age of households in the model is therefore 90 years). Population statistics enter the model via the time endowment parameter $\omega_{g,h,t}$ in (4.17d). In order to obtain the size of household groups we use data on population shares from Table 4.3. We assume that population aging affects all household groups equally. To extrapolate population data beyond 2050 age-group specific population growth rates are projected linearly such that the working-age population ratio remains constant between 2050 and 2100. After 2100, we assume a constant population growth rate to eventually produce a stable population which supports a final steady state equilibrium.

4.3.6 Calibration of Free Parameters

4.3.6.1 Calibration of Utility Functions

Calibration of the parameters of the utility function requires the integration of empirical estimates for labor supply and consumption demand. We set the intertemporal elasticity of substitution (σ_h) to 0.8, $\forall h$. In the literature values range from 0.5 (Jensen and Rutherford, 2002) to around 1.0 (Krueger and Ludwig, 2007). The elasticity of substitution between consumption and leisure (ν_h) is partly determined by uncompensated labor supply elasticities. Although there is information in the literature about uncompensated labor supply elasticities, we are not aware of empirical estimates that take into account variations with age. Due to this lack of information, we cannot calibrate ν_h, and assume a uniform and time-independent elasticity for all household groups. Estimates typically lie around 0.8 (Altig et al., 2001) and 1.0 (Jensen and Rutherford, 2002). We set $\nu = 0.8$. Following Börsch-Supan et al. (2006), the consumption share parameter (α_h) is set to 0.6, $\forall h$.

To calibrate the remaining elasticity parameters we use empirical estimates on uncompensated price elasticities of consumption demand. Given data on share parameters (θ^F, θ^{NF}, and φ_i), and given the nesting structure of utility (see Figure 4.3) we can solve recursively, in line with Rutherford (1995b), for the elasticities of

[15] Assuming exogenous demographic processes is of course a simplifying assumption since, in the long run, neither fertility nor mortality and of course not migration are exogenous to economic activity.

[16] For the demographic projections, we use the results of the *Eleventh Coordinated Population Forecast 2005, Variant 1-W2* issued by the Federal Statistical Office of Germany.

interest, κ and σ_{NF}:

$$\varepsilon_F = -\nu\theta^F(1-\alpha) + \theta^F\alpha - \kappa(1-\theta^F) \tag{4.42}$$

$$\overline{\varepsilon}_{NF} = -\kappa\overline{\theta}^{NF}(1-\theta^{NF}) - \nu\overline{\theta}^{NF}(1-\alpha) - \overline{\theta}^{NF}\theta^{NF}\alpha - \sigma^{NF}(1-\overline{\theta}^{NF}) \tag{4.43}$$

where ε_F =uncompensated own-price elasticity of food demand, $\overline{\varepsilon}_{NF}$ =average uncompensated own-price elasticity of demand for non-food goods, and $\overline{\theta}^{NF}$ =average share of individual non-food goods in the non-food aggregate. With respect to the specification of price elasticities of demand we draw on Boeters et al. [2008] and set $\varepsilon_F = -0.222$ and $\overline{\varepsilon}_{NF} = -0.563$.

4.3.6.2 Elasticities of Substitution in Production

The main reference for elasticities of substitution in German production is Böhringer, Boeters, and Feil (2005,p.103) who draw on estimates by Falk and Koebel (1997). Using the demand equation (4.5), the following relationship between the uncompensated labor demand elasticity and the elasticity of substitution between labor and capital in sector i applies:

$$\eta_i = -(1-\beta_i^L)\sigma_{VA,i} - \theta_i^L \tag{4.44}$$

where θ_i^L =value share of labor in total production costs of sector i.[17] Available data on uncompensated elasticities from Böhringer et al. (2005) distinguishes between unskilled and skilled labor. For the homogeneous labor in the model, we take averages of these estimates and set: $\eta_{AGF} = -0.33$, $\eta_{MIN} = -0.16$, $\eta_{TXL} = \eta_{PRP} = \eta_{MIC} = \eta_{MET} = \eta_{FOP} = \eta_{MAC} = \eta_{PGO} = -0.80$, $\eta_{CEW} = -0.48$, $\eta_{WRT} = \eta_{TRC} = -0.19$, and $\eta_{BIN} = \eta_{RES} = \eta_{PBS} = \eta_{EDU} = \eta_{HEA} = -0.44$.

The following Armington elasticities are taken from Welsch (2008)[18]: $\sigma_{AR,AGF} = 0.081$, $\sigma_{AR,MIN} = 0.915$, $\sigma_{AR,FOP} = 0.841$, $\sigma_{AR,TXL} = 0.926$, $\sigma_{AR,PRP} = 0.300$, $\sigma_{AR,MIC} = 1.600$, $\sigma_{AR,MET} = 0.687$, $\sigma_{AR,MAC} = 2.360$, $\sigma_{AR,PGO} = 0.184$, $\sigma_{AR,CEW} = \sigma_{AR,WRT} = \sigma_{AR,TRC} = \sigma_{AR,BIN} = \sigma_{AR,RES} = \sigma_{AR,PBS} = \sigma_{AR,EDU} = \sigma_{AR,HEA} = 0.871$. All sectoral aggregates are treated as tradable goods. The elasticity of transformation between domestically supplied and exported goods is uniformly set to $\sigma_{HX} = 2$.

The depreciation rate of the capital stock is derived using information from the Federal Statistical Office of Germany on the (gross) value of capital and depreciation.[19] Based on this, we calculate an annual depreciation rate of $\delta = 0.029$.

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[17] For the relation between compensated and uncompensated elasticities, see, e.g., Hamermesh (1993).

[18] Due to differences in industry classification, we use the country average of .871 for those sectoral aggregates for which there is no counterpart in Welsch (2008).

[19] *Kapitalstockrechnung in Deutschland, 2006*, Federal Statistical Office of Germany, pp. 1121. The depreciation rate is calculated as the the ratio of *Abschreibungen auf das Anlagevermögen in jeweiligen Preisen* to *Kapitalstock*.

4.3.7 Government

Across all scenarios, we assume that the government maintains fixed expenditure shares for all sectoral goods according to the baseline path. It is important to stress that this assumption neglects to take into account the growing political influence of a larger share of older generations, something which is likely to affect the preferences of the government. Reported equilibrium outcomes, e.g., with respect to sectoral changes in public services and health care therefore have to be interpreted as conservative estimates. Furthermore, to provide a reasonable basis for welfare analysis, it is critical to assume that government transfers per capita in each age and household group remain fixed. Lastly, we hold government expenditures constant in per capita terms. Without this assumption the share of government consumption in real GDP would become unrealistically large.

4.4 Results of the Benchmark Model

The model allows us to investigate a wealth of economic effects arising from the ongoing and projected population aging in Germany. The population dynamics induce a transition path of the economy both in terms of aggregate and sectoral variables as well as in terms of cross-sectional and intergenerational distributions of income, wealth and welfare. The analysis of these changes will provide us with answers as to how the change in the demographic structure of the economy affects key macroeconomic and household variables by changing relative goods prices and returns to capital and labor.

We compute the model equilibrium from 2003 to 2200 and report simulation results for the main projection period of interest, from 2003 to 2050. We do not report results after 2050 because these hinge crucially on the assumptions concerning the projection of demographic data beyond 2050. Actually, simulation results sufficiently close to 2050 are already sensitive to demographic projections made beyond 2050 because of the forward-looking nature of the model, and therefore have to be interpreted with care. The base year 2003 is taken as the main year of reference to evaluate future changes in the projected time path of variables.

The benchmark model assumes that household preferences do not vary with age. In Section 4.5, we will modify this assumption in order to assess the quantitative importance of demand-side induced effects of population aging that results from structural changes in the composition of old-age consumption.

4.4.1 Macroeconomic Effects of Population Aging

Table 4.4 reports on the dynamic impact of population aging on key aggregate statistics. The projected percentage changes are presented with respect to the base year equilibrium solution in 2003. Output per capita is projected to steadily decrease during the transition, ending up in 2050 on a 7% lower level than in the year 2003. The change in output per capita is mainly due to two effects. As the most direct effect, the decrease in the overall population means that existing resources have to be shared by less people. However, the ongoing aging of the population also directly reduces the labor force which, as a factor of production, suppresses output. Aggregate labor supply (measured in efficiency units) is shown to drop substantially by around 17%. This second negative effect largely dominates and output per capita declines. The consumption-output ratio increases by around 1.4% in 2010 and by almost 10% in 2050. This is consistent with the increasing share of older people in the economy that are dis-savers and hence consume a larger fraction out of their periodic income as compared to younger households. Consumption per capita, however, stays roughly constant. The investment-output ratio drastically drops by about 10% in 2010, and by 26% in 2050. As the population ages and the labor force declines it is optimal to reduce the capital with which these fewer workers work. The aging of the population brings about a "capital deepening" which is reflected by an increase in the capital-labor ratio of around 0.6-2% over the projection period. As the aging of society shifts the age composition of the population towards older households that (on average) hold a larger amount of assets, capital becomes relatively abundant, while labor, also due to the slower growth in the arrival of new workers, becomes scarce. The higher capital-labor ratio reduces the need for firms to invest in new physical capital. Thus investment rates fall. The movement of real factor prices is reflected by the varying relative scarcities of factors of production during the demographic transition. We observe that the real capital rental rate is projected to fall by 0.5% in 2010 and about 1.3% in 2050. Real wages follow almost exactly the inverse path of the rate of return to capital.

Table 4.4 Impact of population aging on key aggregate statistics (benchmark model)

Variable / Year	2005	2010	2015	2020	2030	2040	2050
Output per capita (%)	-0.31	-1.30	-2.02	-2.64	-3.62	-5.79	-6.60
Effective labor supply (%)	-0.37	-1.73	-2.63	-3.99	-7.63	-12.15	-16.96
Capital-labor ratio (%)	0.63	1.76	1.83	1.88	1.85	2.08	1.78
Interest rate (%)	-0.21	-0.51	-0.49	-0.54	-0.83	-1.03	-1.10
Real wage rate (%)	0.23	0.56	0.58	0.68	1.03	1.24	1.29
Investment-output ratio (%)	-3.41	-9.55	-12.97	-16.34	-19.74	-23.75	-26.28
Consumption per capita (%)	0.10	0.70	-0.26	-0.44	-0.39	-0.16	-0.01
Consumption-output ratio (%)	0.32	1.39	1.80	2.25	3.35	5.98	9.22

Note: Results refer to percentage changes with respect to the base year 2003. Output is defined as the sum of domestic sectoral production. Real factor prices are deflated by the consumption price index. Consumption comprises private and government consumption.

4.4.2 Sectoral Change During the Demographic Transition

This section analyzes the impact of population aging at the sectoral level. In the benchmark model, we do not consider age-specific preferences, and hence the effects on sectoral output induced by population aging are merely driven by supply side factors which result from the decline in the effective labor supply and the "capital deepening" process. It is evident that the demographic transition induces significant changes in the sectoral composition of output. Changes in output shares in 2010 range between -8.49% and $+6.49\%$, and are even more pronounced in 2050 ranging between -23.51% and $+19.98\%$. Among the sectors that benefit most from varying relative factor endowments due to population aging are *Public Services*, *Textiles and leather products*, *Education*, and *Health*. Output shares for the latter two sectors increase by almost 20% between 2003 and 2050. The sectors that contract most, relatively, to total domestic output are *Metals*, *Machinery* (around -10%) and *Construction, Energy, and Water* (-23.51%). *Personal Goods*, *Mining*, and *Real Estate Services* contract by less (around -2% to -4%). Output shares for the remaining sectors *Mineral and Chemical Products*, *Food products*, *Printing and Publishing*, *Wholesale and Retail Trade*, *Transportation and Communication*, and *Banking and Insurance Services* change by a relatively small extent (around -1.7% to $+1.7\%$).

Table 4.5 Sectoral change due to population aging (benchmark model)

Sector / Year	2005	2010	2015	2020	2030	2040	2050
Agriculture and Fisheries	0.61	1.92	2.04	1.88	0.81	0.93	0.74
Mining	-0.40	-1.30	-1.74	-2.12	-2.33	-3.11	-3.93
Food Products	0.82	2.55	2.84	2.81	1.73	2.04	1.96
Textiles and Leather Products	2.41	6.49	8.81	11.11	13.13	14.51	13.38
Printing and Publishing	0.03	0.10	-0.06	-0.31	-0.88	-1.25	-1.76
Mineral and Chemical Products	-0.20	-0.74	-0.92	-1.01	-0.83	-1.25	-1.78
Metals	-1.12	-3.54	-4.15	-4.68	-5.10	-7.17	-9.38
Machinery	-1.00	-3.24	-3.61	-3.90	-4.08	-6.09	-8.41
Personal Goods	-0.10	-0.21	-0.57	-1.13	-2.41	-3.22	-4.22
Construction, Energy, and Water	-3.03	-8.49	-11.57	-14.61	-17.68	-21.25	-23.51
Wholesale and Retail Trade	0.26	0.85	0.83	0.62	-0.25	-0.43	-0.81
Transportation and Communication	0.22	0.67	0.73	0.65	0.10	-0.04	-0.35
Banking and Insurance Services	0.33	1.05	1.13	1.02	0.31	0.36	0.28
Real Estate Services	-0.18	-0.41	-0.63	-1.00	-1.86	-2.38	-2.85
Public Services	-0.20	-0.07	0.05	0.55	2.86	6.20	11.84
Education	-0.24	-0.02	0.29	1.34	5.60	11.04	19.98
Health	-0.18	0.21	0.59	1.59	5.41	10.66	19.29

Note: Figures refer to percentage changes in the sectoral share of output in total domestic output relative to the base year 2003. An increase in the share of sectoral output from one period to another means that this particular sector is growing faster / decreasing less rapidly than total domestic production.

The expansion/contraction of sectors can be explained by looking at the evolution of sectoral employment of capital and labor (Table 4.6). As evident from the discussion of the macroeconomic effects, population aging brings about an increase in the economy-wide capital-labor ratio. Since the magnitude of the overall demographic shock—recall that aggregate effective labor supply falls by 16% until 2050—is significant, we consequently find that the capital-labor ratio in each single sector increases, too (results not shown). To get an intuition for why some sectors benefit and others contract, it is therefore necessary to look at how quantities of labor and capital used in production evolve over time relative to aggregate labor supply and the capital stock. Looking at these statistics indicates which sectors employ most productively abundant capital and attract scarce labor. Note that in Table 4.6 an increase in the sectoral share of labor in aggregate effective labor means that the contraction of labor that is being employed in this sector is smaller than the overall decline in aggregate labor supply that occurs due to population aging. Similarly, an increase in the sectoral share of capital implies that the quantity of capital used in this particular sector grows faster than the economy-wide capital stock.

Consequently, all sectors which benefit from population aging are characterized by an increase in both the share of sectoral employment of capital and labor; e.g., the share of employment of labor in *Health* monotonically rises from 5.17% in 2005 to 6.12% in 2050, an increase by almost 20%. Capital usage in this sector also increases over the projection horizon by 20% from 3.90% in 2005 to 4.70% in 2050. A faster decrease of sectoral labor employment relative to the decline in the aggregate labor supply in conjunction with a steadily decreasing share of sectoral capital in the economy-wide capital stock explains why sectoral output shares for, e.g., *Metals*, *Machinery*, *Personal Goods*, and *Construction, Energy, and Water* decline between 2005 and 2050. The patterns of sectoral activity for sectors that first expand and then contract can also be explained by referring to Table 4.6. In line with the evolution of the sectoral output share, e.g., the share of employment in *Wholesale and Retail Trade* first slightly increases from 16.27% in 2005 to 16.50% in 2015, but is then declining until 2050 and drops to 16.16%. A similar qualitative pattern is observed for the evolution of the capital share in this sector.

Overall, the sectoral change that is due to supply side factors induced by population aging, i.e. the decrease in labor supply and the "capital deepening" process, produces quantitatively substantial effects. We will turn back to this issue in Section 4.5.1 in which we aim to quantify the demand side effects of population aging that result from age-specific preferences.[20]

[20] As the study by Fougere et al. (2007) is the only point of reference for the type of analysis presented here, a couples of remarks are in order. It is important to bear in mind that both models differ along various dimensions. One major difference is that in their model labor supply is exogenous whereas the present model features a labor-leisure trade off. Given that the evolution of aggregate labor supply is a key driver of results, this certainly complicates a comparison. Furthermore, the quantitative nature of the demographic process, which is the major exogenous driving force of the model, the numerical specification of age-specific preferences, and the underlying Input-Output data are quite different for Germany and Canada. Notwithstanding these considerations, the macroeconomic predictions of both studies are qualitatively similar. To see this, compare Table 4.4 with Table 9, p. 705, in Fougere et al. (2007). Due to differences in the sectoral aggrega-

Table 4.6 Impact of population aging on sectoral employment and capital use

	Share of sectoral employment in aggregate labor (in %)							Share of sectoral capital in aggregate capital (in %)						
Sector / Year	2005	2010	2015	2020	2030	2040	2050	2005	2010	2015	2020	2030	2040	2050
AGF	0.664	0.678	0.680	0.680	0.673	0.674	0.670	1.663	1.688	1.694	1.696	1.682	1.685	1.679
MIN	0.612	0.611	0.609	0.608	0.606	0.602	0.594	0.185	0.184	0.184	0.184	0.184	0.183	0.181
FOP	1.884	1.925	1.933	1.934	1.907	1.910	1.898	1.268	1.302	1.309	1.314	1.312	1.323	1.322
TXL	1.078	1.129	1.160	1.191	1.216	1.231	1.210	0.474	0.500	0.514	0.530	0.550	0.562	0.555
PRP	2.212	2.225	2.225	2.222	2.203	2.192	2.170	1.802	1.817	1.819	1.821	1.825	1.826	1.816
MIC	5.296	5.294	5.295	5.296	5.292	5.262	5.206	3.301	3.316	3.319	3.332	3.374	3.379	3.361
MET	5.061	4.958	4.937	4.914	4.877	4.762	4.622	2.567	2.530	2.521	2.519	2.534	2.493	2.434
MAC	20.173	19.822	19.836	19.848	19.809	19.362	18.751	6.475	6.433	6.441	6.477	6.588	6.510	6.346
PGO	0.766	0.769	0.768	0.764	0.752	0.745	0.733	0.344	0.347	0.347	0.346	0.346	0.345	0.341
CEW	4.919	4.674	4.524	4.374	4.213	4.029	3.895	4.666	4.414	4.276	4.140	4.001	3.831	3.717
WRT	16.271	16.480	16.504	16.489	16.331	16.300	16.161	8.335	8.404	8.423	8.426	8.373	8.365	8.324
TRC	5.598	5.664	5.677	5.680	5.645	5.638	5.594	6.115	6.159	6.178	6.190	6.172	6.170	6.144
BIN	4.774	4.844	4.855	4.856	4.819	4.822	4.796	3.614	3.649	3.661	3.666	3.649	3.655	3.648
RES	10.915	10.980	10.972	10.947	10.856	10.809	10.708	48.40	48.42	48.43	48.37	48.08	47.88	47.59
PBS	9.790	9.867	9.894	9.954	10.169	10.497	11.001	6.246	6.267	6.289	6.336	6.494	6.709	7.057
EDU	4.824	4.862	4.885	4.940	5.136	5.397	5.803	0.640	0.642	0.646	0.654	0.682	0.718	0.774
HEA	5.163	5.218	5.245	5.303	5.495	5.768	6.188	3.902	3.925	3.949	3.998	4.156	4.367	4.702

Note: The relative expansion/contraction of sectors induced by population aging as indicated by the changes in sectoral output shares (see Table 4.5) can be explained by looking at the evolution of shares of sectoral employment and capital use. Whether a sector expands/contracts depends on how factors of production evolve over time *relative* to aggregate labor and capital. The statistics displayed above indicate which sectors most productively employ abundant capital and attract scarce labor. An increase in the sectoral share of labor in aggregate labor means that the contraction of labor that is being employed in this sector is smaller than the overall decline in aggregate labor supply that is due to population aging. An increase in the sectoral share of capital implies that the quantity of capital used in this sector contracts less than the economy-wide capital stock.

4.4.3 Distributional and Welfare Consequences of the Demographic Transition

In the previous sections we have documented substantial changes in factor prices induced by the aging of the population, amounting to a decline of about 1.1% in the real return to capital and an increase in the real wage rate of about 1.3% until 2050. In this section we aim to quantify the distributional and welfare effects arising from these factor price changes.

4.4.3.1 Evolution of Inequality

To characterize the evolution of income inequality over the projection period we compute Gini coefficients[21] for three different (sub-)sets of households: first, the Gini coefficient labeled "*all households*" uses income observations for households across generations and household types that are alive at time t, second, "*intergenerational*" looks at the impact on the intergenerational dispersion of income (for a given household group), and, third, *intragenerational* computes the Gini coefficient that measures the evolution of inequality within a particular household group (for a given generation).

Figure 4.4 displays the evolution of income inequality for total income, labor income and capital income which is derived from financial wealth for all households. Total income comprises labor income, transfer income, and capital income. Looking at the distribution that includes agents of different age and type, we observe a significant increase in total income inequality from 0.59 in 2005 to 0.64 in 2050, an increase in the Gini coefficient of about 8.4%. Inequality increases although labor income inequality declines over the same horizon from 0.58 in 2005 to 0.53 in 2050 because the dispersion in capital income rises by around 4% from 0.73 in 2010 to 0.76 in 2050.

Both, the evolution of inequality for capital and labor income is mainly due to the impact of the demographic transition on the respective intergenerational distribution—changes in inequality within a generation are very little. Figure 4.5 dis-

tion of industries and the aforementioned quantitative differences in input data, comparing results at the sectoral level is not instructive.

[21] The Gini coefficient is a summary statistic of the Lorenz curve, and is here calculated as the relative mean difference, i.e. the mean of the difference between every possible pair of observations $(x_{g,h,t}, x_{g',h',t})$ of a generation g and household type h at a given point in time t relative to the size of the mean:

$$G_t = \frac{n+1}{n} \frac{\sum_{\{g,g'\}} \sum_{\{h,h'\}} |x_{g,h,t} - x_{g',h',t}|}{2n^2 \mu_t}. \tag{4.45}$$

Here, n denotes the total number of households alive at t and μ_t is the arithmetic mean. The Gini coefficient ranges from a minimum value of zero, when all individuals have equal income (the numerator in (4.45) then equals zero), to a theoretical maximum of one in an infinite population in which all income is concentrated in one hand (to see this, divide (4.45) by $2n^2$, substitute the numerator by an expression which depends on the number of non-zero pairs, and apply l'Hôpital's rule as $n \to \infty$).

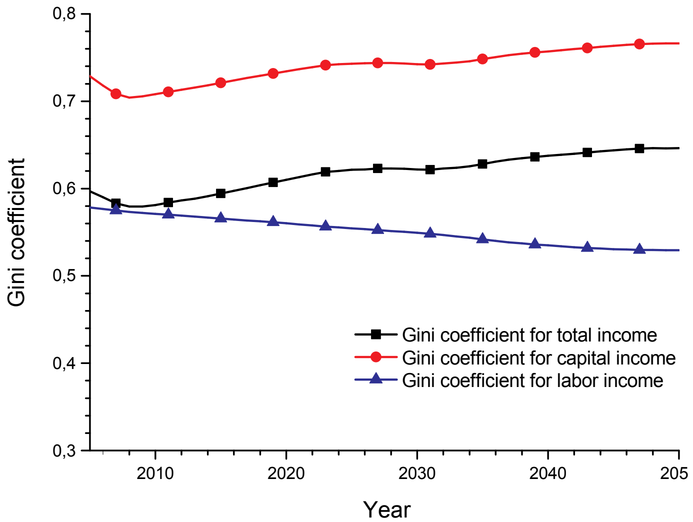

Fig. 4.4 Evolution of income inequality (all households)

Note: Gini coefficients are calculated on the basis of the entire sample which comprises households across different types *and* time.

plays the Gini coefficients that are based on samples of income observations which pool households across time for a given household type. Inequality of the intergenerational capital income distribution increases by around 18% (for $h = 1$ from 0.53 in 2005 to 0.63 in 2050) and the pattern of the increase is similar to the evolution of the corresponding Gini coefficient in Figure 4.4 (Gini coefficients for different household types move in parallel, hence only those for $h = 1$ and $h = 8$ are shown). The reason for the increase in intergenerational capital income inequality—which ultimately drives the evolution of total income inequality—is that older households hold more assets than younger households. Since capital income is proportional to wealth, and as the demographic transition induces a shift towards older, asset-rich households, capital inequality is bound to increase. Impacts on the intragenerational distribution of capital income are minor (not shown).

The decline in inequality of the intergenerational labor income distribution (again, a similar pattern is observed for different household types) stems from the significant drop in aggregate labor supply which induces a continuous rise in the real wage rate. Steadily increasing returns to labor diminish the role played by labor productivity over the life cycle in shaping labor supply decisions: households face a trade-off between supplying labor in middle ages when their productivity is high and wages are low and supplying labor in older ages when labor productivity is relatively low but wages have increased. Labor supply over the life cycle still follows a hump-shaped pattern but the increase in wage rates in the course of the

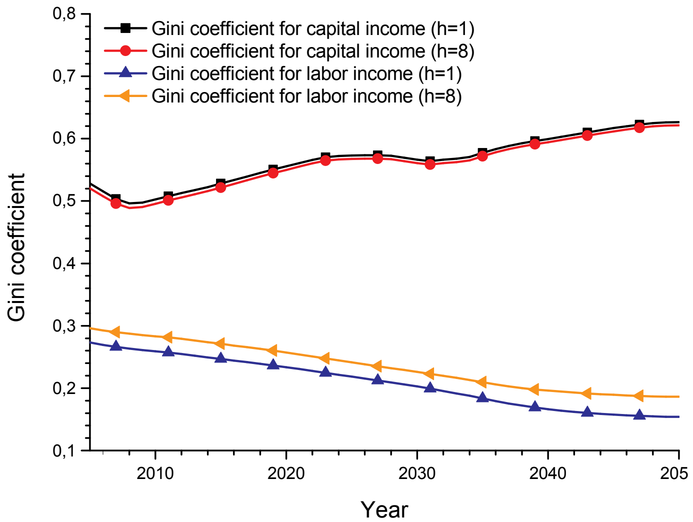

Fig. 4.5 Evolution of intergenerational income inequality

Note: Gini coefficients are calculated on the basis of a subset of households which comprises households of different generations but holds fixed a particular household type

projection period implies a tendency for a "flatter" labor income profile. This effect explains the significant drop in intergenerational inequality of labor income from 0.28 in 2005 to 0.15 in 2050.

In our model, population aging has a minor impact on the intragenerational inequality of labor income over the life cycle. Figure 4.6 plots the evolution of the Gini coefficient over the lifetime of a generation. Not surprisingly, the dispersion of labor income within a generation follows the hump-shaped profile of labor productivity over the life cycle: increasing wages magnify intra-cohort differences in labor productivity and hence increase labor income inequality.

Overall, however, intergenerational rather than intragenerational factors account for the evolution of capital and labor income inequality. Since intergenerational equity decreases the dispersion of the dispersion of the income distribution during the demographic transition increases.

4.4.3.2 Welfare Consequences of the Demographic Transition

Due to the demographic transition lifetime utility of a household changes because he faces different factor prices than without changes in the demographic structure. Specifically, households face a path of declining interest rates and increasing wages, relative to a situation without population aging. How do these opposite movements

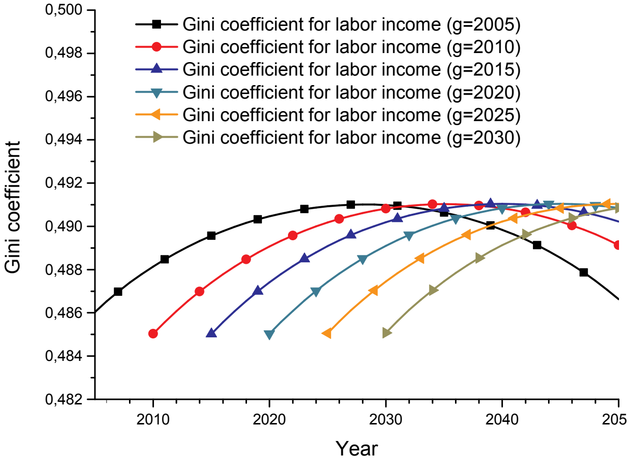

Fig. 4.6 Evolution of intragenerational income inequality

Note: Gini coefficients are calculated on the basis of a subset of households which comprises households of different types but holds fixed a particular generation.

of factor prices affect welfare? How are welfare gains and losses distributed inter- and intragenerationally?

For welfare comparison we solve each household's problem under two different scenarios. Let denote $\mathbf{p}_{aging}$ the vector of equilibrium prices as documented in the previous section. Alternatively, consider a "no aging"-scenario where prices are held constant at their 2003 value, and let $\mathbf{p}_{no\ aging}$ denote this hypothetical price situation. As the basis of comparison we use the Hicksian equivalent variation ($EV_{g,h}$) which measures the percentage change in lifetime income that would be equivalent to the price change as implied by the demographic transition in terms of the impact on lifetime utility. Using the indirect utility function we can write more formally:

$$v_{g,h}\left(inc_{no\ aging,g,h}\left(1+EV_{g,h}\right),\mathbf{p}_{no\ aging}\right)=v_{g,h}\left(inc_{aging,g,h},\mathbf{p}_{aging}\right) \tag{4.46}$$

where $inc_{no\ aging,g,h}$ and $inc_{aging,g,h}$ denote lifetime income under the respective scenario. Positive numbers of $EV_{g,h}$ indicate that households obtain welfare gains from the general equilibrium effects of the demographic changes, negative numbers imply welfare losses.

Table 4.7 documents these numbers for generations that are alive in 2003. We make several observations. First, newborn agents experience massive welfare gains from changing factor prices induced by the demographic transition equivalent of more than 14% in lifetime income. The demographic transition induces a substantial increase in real wage rates and a future path of declining interest rates. The dominating effect for newborns is the substantial increase in wage rates because these agents have not yet accumulated financial wealth and thus do not suffer from a loss of capital income on already accumulated financial wealth, in contrast to older

Table 4.7 Welfare consequences of population aging (equivalent variation in %)

generation born in	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$
2003	14.7	16.7	18.9	20.4	22.4	23.7	24.3	25.2
1995	13.8	15.4	17.1	18.4	20.1	21.1	21.6	22.4
1990	12.2	13.5	15.1	16.2	17.6	18.6	19.0	19.7
1980	5.0	5.8	6.7	7.4	8.2	8.8	9.1	9.5
1970	1.1	1.5	1.9	2.2	2.6	2.9	3.0	3.3
1960	-0.5	-0.4	-0.2	-0.1	0.0	0.1	0.2	0.3
1950	-0.5	-0.5	-0.4	-0.4	-0.3	-0.3	-0.3	-0.3

households. Of course, lower returns to capital make it more difficult to accumulate assets to finance old-age consumption. This effect, however, is largely dominated by increases in labor income. Second, positive welfare consequences for agents alive in 2003 decrease substantially with age, and even turn negative for households born before 1960. Due to relatively low labor productivity in old age, these agents do not benefit from increases in wages but, on the other hand, face lower interest rates on their already accumulated asset stock. This drives down capital income and depresses consumption, and hence welfare decreases. This pattern of intergenerational welfare impacts is stable across different types of households. Third, given that the welfare impact is largely driven by increasing wages, it is not surprising that those members of society for whom labor income constitutes a smaller part of (future) resources benefit less from the demographic transition. Recall that a major source for heterogeneity of households within a generation is labor productivity. High-skilled type $h = 8$ households consequently benefit almost twice as much from the demographic transition relative to least productive type $h = 1$ households.

4.5 Sensitivity Analysis

In this section we discuss how the results derived from the benchmark model hinge on a number of structural model assumptions as well as on the numerical specification of key model parameters. More specifically, we want to assess the quantitative importance of structural changes in old age consumption that are a result of age-specific preferences. Futhermore, by means of piecemeal sensitivity analysis, we identify key model parameters that decisively influence the nature and strength of the economic adjustment.

Table 4.8 Age-specific distribution of consumer spending across sectoral goods

Sectoral good	< 25	25-35	35-45	45-55	55-65	65-70	70-80	80+
AGF	0.021	0.021	0.022	0.022	0.021	0.022	0.022	0.021
MIN	0.015	0.015	0.015	0.015	0.015	0.015	0.017	0.019
FOP	0.177	0.171	0.182	0.181	0.172	0.173	0.174	0.169
TLX	0.064	0.062	0.062	0.060	0.056	0.054	0.052	0.046
PRP	0.027	0.028	0.030	0.029	0.030	0.032	0.032	0.029
MIC	0.069	0.075	0.072	0.077	0.079	0.078	0.071	0.067
MET	0.005	0.005	0.005	0.005	0.006	0.006	0.005	0.005
MAC	0.093	0.100	0.095	0.099	0.096	0.092	0.077	0.065
PGO	0.027	0.029	0.030	0.029	0.031	0.032	0.030	0.029
CEW	0.036	0.036	0.036	0.035	0.037	0.038	0.041	0.045
WRT	0.021	0.023	0.022	0.023	0.021	0.019	0.015	0.011
TRC	0.086	0.072	0.062	0.063	0.055	0.050	0.045	0.039
BIN	0.027	0.027	0.027	0.028	0.029	0.028	0.030	0.039
RES	0.240	0.235	0.238	0.232	0.244	0.247	0.270	0.297
PBS	0.061	0.063	0.065	0.063	0.065	0.068	0.069	0.068
EDU	0.015	0.017	0.017	0.011	0.007	0.005	0.004	0.003
HEA	0.015	0.019	0.021	0.027	0.036	0.041	0.046	0.048

Note: Own calculations based on the German Income and Expenditure survey (*Einkommens- und Verbrauchstichprobe*) 2003 and a "z-matrix (*Konsumverflechtungstabelle*), both Federal Statistical Office of Germany.

4.5.1 Age-specific Preferences: How Important Are Structural Changes in Final Consumption Demand ?

Apart from the most direct effect of population aging, the substantial decline in the supply of effective labor, a second demand-side induced effect may be important. Empirical data for Germany suggests that the composition of consumer spending varies with age.

Table 4.8 displays the age-specific distribution of spending shares across the 17 sectors in the model.[22] Most notably, spending shares for *Health*, *Construction, Energy, and Water*, and *Banking and Insurance Services* rise with age. The spending share on *Health* goods is three times larger for households aged 80 and older than for the youngest age group. Spending shares for *Education*, *Transport and Communication*, *Food Products*, *Wholesale and Retail Trade*, and *Textiles and Leather Products* decline with age. Given the variation of spending shares with age for a number of

[22] The numbers in Table 4.8 are based on household data from the German Income and Expenditure Survey 2003 (*Einkommens- und Verbrauchsstichprobe*) issued by the Federal Statistical Office of Germany. The original data lists age-specific consumer spending by consumption categories that, however, do not match with the sectoral structure of our model. The age-specific distribution of spending shares across sectoral goods can be recovered using a so-called "Z-matrix" or "bridge-matrix" that maps consumer spending from the consumption categories to the 71 industries of the German Input-Output table.

sectors, one might therefore expect additional changes in the sectoral composition of output as the population shifts towards older people.

This view is supported by Lührmann (2005) who estimates age-specific household demands for a set of eight composite goods using a quadratic almost ideal demand system based on EVS data.[23] On the contrary, a study by Schaffnit-Chatterjee (2007)—which is however based on a crude back-of-the envelope calculation—finds that the expenditure shares of consumption categories in aggregate expenditures are not significantly influenced by population aging. The present study is the first attempt in the literature to quantify these effects using a comprehensive general equilibrium model.

One remark is in order. The measurement of age-specific consumption behavior is more complicated than Table 4.8 suggests. As these age-specific spending shares are calculated on the basis of cross-sectional data, they confound age, cohort, and time effects. Unfortunately, there is no suitable panel data for Germany available to address this problem. However, looking at repeated cross-sections Börsch-Supan (2003) finds that these age-consumption profiles did not change much over the past 20-25 years—hence cohort effects appear to be small and probably do not bias Table 4.8 to a substantial extent. Assuming that the pattern of age-specific spending remains much the same in the future, we are able to compute the adjustment path for an economy where households' preferences vary with age. As in the benchmark model, we assume that households of different type have the same preferences.[24] Table 4.9 suggests that accounting for age-specific preferences has only minor effects in terms of the macroeconomic consequences of the demographic transition. A comparison with the results displayed in Table 4.4 reveals that all key aggregate statistics move in the same direction. This underlines that the nature of the economic transition is predominantly shaped by the negative labor supply shock. Also, from a quantitative viewpoint, both models produce very similar results: in the model with age-specific preferences, output per capita and effective labor supply decline slightly more, movements in real factor prices are almost identical, and the investment-output ratio decreases slightly less whereas the consumption-output ratio is marginally higher relative to the benchmark model.

On the sectoral level, a similar observation applies. Comparing results in Table 4.10 with the respective sectoral impacts shown in Table 4.5 suggests that whether a particular sector expands or contracts (relative to total domestic output) is solely determined by the negative labor supply effect: the pattern of changes in output

[23] Her results (compare Table 3, p. 29) suggest significant increases in aggregate expenditure shares for health (6.6%) and energy (5.3%) and a decline for transport (−7.6%) and leisure (−4.4%). A major shortcoming of her econometric approach, however, is to ignore possible feedback effects that stem from changes in relative prices.

[24] A remark concerns the calibration of technology parameters which has been carried out under the benchmark household preference structure. As we use the same values for technology parameters for this "counterfactual", the model does not endogenously reproduce the base year values. On the other hand, recalibrating technologies would change the adjustment of sectoral activity in response to a demographic shock, and would make a comparison between models difficult. For this reason, we keep all parameter values—except for those relating to age-sensitive preferences—constant.

Table 4.9 Impact of population aging on key aggregate statistics (model with age-specific preferences)

Variable / Year	2005	2010	2015	2020	2030	2040	2050
Output per capita (%)	-0.51	-1.85	-2.91	-3.84	-5.20	-7.44	-8.13
Effective labor supply (%)	-0.61	-2.39	-3.59	-5.13	-8.81	-13.27	-18.00
Capital-labor ratio (%)	0.72	1.98	2.13	2.20	2.08	2.26	1.96
Real capital rental rate (%)	-0.11	-0.27	-0.36	-0.49	-0.63	-0.85	-1.09
Real wage rate (%)	0.20	0.46	0.42	0.49	0.80	1.02	1.08
Investment-output ratio (%)	-3.21	-8.86	-11.87	-14.78	-17.54	-21.23	-23.88
Consumption-output ratio (%)	0.59	2.17	3.13	4.10	5.01	7.72	9.80

Note: Results refer to percentage changes with respect to the base year 2003. Output is defined as the sum of domestic sectoral production. Real factor prices are deflated by the consumption price index. Consumption comprises private and government consumption.

shares across sectors and time that is obtained from the model with age-specific preferences is roughly identical to the pattern obtained from the benchmark model.

From a quantitative viewpoint, however, demand side induced effects do matter. Most obvious, quantitative differences between both models are due to a direct effect that stems from age-related consumer demand. For instance, the age-specific increase in the spending share of *Health* as shown in Table 4.8 translates into an increase in the respective output share as compared to the benchmark model. Direct demand side effects that are in line with the movement of age-specific spending shares are also observed for a number of other sectors, e.g., *Public Services*, *Real Estate*, *Banking and Insurance Services*, *Construction, Energy, and Water*. Sectoral activity is, however, also indirectly affected by changes in intermediate demand. This explains why sectoral output shares for some sectors do not move in the direction one might expect from looking at spending shares in Table 4.8. For instance, output shares for *Wholesale and Retail Trade* increase relative to the benchmark model although consumer spending shares diminish with age. Table 4.13 displays input-output data which underlies the sectoral model structure. Looking at column "WRT" reveals which sectors use *Wholesale and retail trade* products and services as an intermediate input in production. Sectors that heavily rely on *Wholesale and Retail Trade* products, i.e. FOP, TRC, RES, PBS, are exactly those that exhibit an increase (a less strong decline) in sectoral output shares relative to the benchmark model.

Overall, we can summarize that accounting for structural changes in life-cycle consumption that are due to age-specific preferences does not affect the qualitative results of the benchmark model. Quantitative effects on the aggregate economy level are relatively small. The reason for this is that total consumption expenditures by older households (aged 60-74) only make up 24% of total private consumption in 2003, and do not exceed 30% in 2050. This suggests that the level and the growth of old age consumption are both not large enough to translate compositional effects in household consumption into significant economy-wide effects. For this to happen, either a more sizable shift towards older people in the population or a higher level of old age consumption per se would be required. In contrast, quantitative effects on the sectoral level are significant. The presence of intermediate demand linkages implies, however, that for some sectors the pattern of changes in sectoral output shares is not in line with the movement of age-dependent spending shares.

Table 4.10 Sectoral change due to population aging (model with age-specific preferences)

Sector / Year	2005	2010	2015	2020	2030	2040	2050
Agriculture and Fisheries	1.06	3.14	3.79	3.99	3.14	3.28	2.96
Mining	-0.12	-0.47	-0.81	-1.10	-1.44	-2.01	-2.83
Food Products	1.33	3.93	4.82	5.18	4.29	4.56	4.26
Textiles and Leather Products	1.84	4.93	6.23	7.33	7.67	8.41	7.74
Printing and Publishing	0.20	0.55	0.57	0.47	0.04	-0.24	-0.78
Mineral and Chemical Products	-0.05	-0.31	-0.54	-0.66	-0.64	-0.93	-1.48
Metals	-1.29	-3.94	-4.81	-5.41	-5.82	-7.64	-9.85
Machinery	-1.20	-3.75	-4.44	-4.83	-5.03	-6.79	-9.09
Personal Goods	0.05	0.18	0.12	-0.15	-1.04	-1.77	-2.81
Construction, Energy, and Water	-3.02	-8.31	-11.17	-13.95	-16.56	-20.00	-22.41
Wholesale and Retail Trade	0.36	1.10	1.35	1.37	0.86	0.72	0.34
Transportation and Communication	0.28	0.81	1.04	1.13	0.86	0.77	0.45
Banking and Insurance Services	0.14	0.46	0.59	0.62	0.47	0.53	0.59
Real Estate Services	-0.04	-0.03	-0.01	-0.14	-0.68	-1.08	-1.57
Public Services	-0.16	0.03	0.41	1.25	4.36	8.22	14.65
Education	-0.28	-0.13	0.41	1.82	7.13	13.38	23.61
Health	-0.28	0.31	0.73	2.31	7.95	14.73	25.90

Note: Figures refer to percentage changes in the sectoral share of output in total domestic output relative to the base year 2003. An increase in the share of sectoral output from one period to another means that this particular sector is growing faster (decreasing less rapidly) than total domestic production.

4.5.2 *Piecemeal Sensitivity Analysis*

Table 4.11 shows the effects of changing key model parameters from the base case parametrization of Section 4.3. For ease of reference, base case results are displayed in the first row of Table 4.11. For each parameter that is used in the sensitivity analysis, we report results for one value that is below and one value that is above the base case. Table 4.11 documents that changing parameter values can produce perceivable quantitative effects. Most importantly, however, it is found that all qualitative model results are preserved.

Since the negative labor supply shock significantly drives the overall results, parameters that govern labor supply decisions of households are found to have the greatest impact. A lower value for the intertemporal elasticity of substitution means that households are less willing to tolerate fluctuations in consumption over their life cycle. Given decreasing interest rates which make it harder to accumulate wealth (and earn capital income), a higher degree of consumption smoothing can only be achieved if labor supply is more elastic. Hence, in light of increasing wage rates along the demographic transition, lower values for σ_h imply a weaker contraction in aggregate labor supply. A lower value of the intratemporal elasticity of substitution between leisure and consumption makes labor supply less elastic, and hence the decrease in aggregate labor supply is more pronounced. Similarly, a lower weight on material consumption (a larger weight on leisure) in instantaneous utility implies that labor supply is less sensitive to changes in the wage rate. The extent of the contraction in output per capita for different values of these three household parameters can therefore be explained by movements in aggregate labor supply.

A lower value for the capital depreciation rate stimulates capital accumulation and somewhat mitigates the decline in output per capita. A lower value for δ strengthens the role of capital accumulation relative to endogenous variations in labor supply as an adjustment mechanism to the demographic transition. A larger decrease in aggregate labor supply is therefore consistent with a smaller reduction in output per capita. Uniformly lowering the elasticity of substitution between capital and labor in value-added for all sectors by 30% has only minor effects. A lower degree of substitutability means that firms are flexible in adjusting to a situation with changing factor price changes, and hence losses in output per capita are slightly higher.

In terms of the impact on factor price changes, the decrease in the real return to capital ranges from -0.29% to -0.71% in 2010 and from -0.23% to -1.22% in 2040 for the scenarios considered. Increases in real wage rates lie between 0.30% and 0.72% in 2010 and between 0.25% and 1.40% in 2050. The evolution of income inequality as measured by changes in the Gini coefficient is found not to be very sensitive: increases from 2010 to 2040 range between $7.3 - 10.1\%$.

The sensitivity of results with respect to the Armington elasticity of substitution is found to be low. This underlines the overall robustness of results. Recall that the assumption of imperfect substitutability between domestic and foreign goods implies that the domestic interest rate can deviate during the transition period from the exogenous world market interest rate; in the limit of perfect substitutability the domestic interest rate would be exogenous in all periods. As such, $\sigma_{AR,i}$ indirectly affects the capital accumulation process. A lower value of $\sigma_{AR,i}$ means that the economy cannot rely as much on imported goods to build up its capital stock. This negative effect is buffered by a smaller decline in aggregate labor supply. Both effects imply that the capital-labor ratio increases by less.

4.6 Concluding Remarks

This chapter employs a multi-sectoral OLG model with intra-cohort heterogeneity to analyze the economic consequences of the demographic transition in Germany.

Table 4.11 Piecemeal sensitivity analysis (benchmark model)

Parameter		Output per capita[a]		Eff. labor supply[a]		Cap.-labor ratio[a]		Real return to capital[a]		Real wage rate[a]		Gini coeff. income		EV[b]
		2010	2040	2010	2040	2010	2040	2010	2040	2010	2040	2010	2040	
Base case parametrization[c]		-1.30	-5.79	-1.73	-12.15	1.76	2.08	-0.51	-1.03	0.56	1.24	0.582	0.639	14.71
Intertemporal ES	$\sigma_h = 0.60$	-0.15	-1.42	-1.15	-10.97	1.46	1.56	-0.56	-1.23	0.62	1.47	0.584	0.641	14.62
	$\sigma_h = 0.99$	-2.12	-8.10	-2.69	-14.03	2.28	2.78	-0.39	-0.50	0.42	0.57	0.580	0.635	12.22
Intratemporal ES	$\nu_h = 0.60$	-2.29	-7.97	-2.84	-13.70	2.32	2.61	-0.29	-0.23	0.30	0.25	0.618	0.664	13.92
	$\nu_h = 0.99$	-0.94	-4.89	-1.50	-11.97	1.63	1.98	-0.53	-1.15	0.58	1.37	0.570	0.628	14.15
Consum. share param.	$\alpha_h = 0.40$	-2.53	-8.57	-3.47	-14.49	2.79	2.85	-0.71	-0.65	0.72	0.67	0.587	0.641	11.92
	$\alpha_h = 0.80$	-0.84	-5.40	-0.85	-11.53	1.11	1.55	-0.37	-1.15	0.44	1.40	0.576	0.633	14.37
Cap. depreciation rate	$\delta = 0.025$	-0.95	-4.72	-1.96	-12.71	1.82	1.93	-0.49	-0.76	0.53	0.93	0.583	0.641	14.69
	$\delta = 0.035$	-1.43	-6.07	-1.43	-11.34	1.62	1.90	-0.49	-1.22	0.52	1.36	0.589	0.632	11.60
ES in value-added[d]	*low*	-1.40	-6.13	-1.69	-12.12	1.58	1.62	-0.55	-1.03	0.60	1.24	0.581	0.638	13.53
	high	-1.25	-5.58	-1.80	-12.23	1.96	2.63	-0.48	-1.02	0.53	1.24	0.582	0.639	13.79
ES in Armington production[e]	*low*	-1.14	-5.12	-1.26	-11.20	1.48	1.55	-0.43	-0.93	0.50	1.25	0.585	0.640	14.37
	high	-2.29	-7.98	-2.28	-13.04	1.86	2.01	-0.28	-0.34	0.29	0.38	0.578	0.632	11.58

Note: All results are computed using the benchmark model. ES=elasticity of substitution. a : Numbers refer to percentage change relative to 2003. b : Equivalent variation in lifetime income (in %) for a newborn generation in 2003 of type $h = 1$. c : $\sigma_h = \nu_h = 0.8$, $\alpha_h = 0.6$, $\delta = 0.029$. For each parameter, the base case value lies between the two values used in the sensitivity analysis. d : *low* (*high*) refers to a situation where the elasticities of substitution in value-added for *all* sectors have been reduced (increased) by 30% relative to the base case parametrization. e : *low* (*high*) refers to a situation where the elasticities of substitution in Armington production for all sectors have been reduced (increased) by 50% relative to the base case parametrization.

While the analysis is carried out for Germany, some of the results may be applicable to other major industrialized countries facing similar demographic transitions.

The contribution of this chapter is twofold. First, we argue that shifts in the sectoral structure constitute an important channel of adjustment to population aging. We document that the demographic transition is bound to induce significant changes in the sectoral composition of output ranging—as measured by changes in output shares—from -8.49% to $+6.49\%$ in the year 2010 and from -23.51% to $+19.98\%$ in 2050. While accounting for structural changes in life-cycle consumption due to age-specific preferences does not affect the qualitative results of the benchmark model, and has only minor quantitative effects for aggregate variables, it is found that sectoral effects are quantitatively important. Second, we analyze the distributional and welfare consequences that follow from factor price changes induced by the demographic transition. Newborn households in 2003 experience substantial welfare gains in the order of 14% and more in lifetime income that arise from increases in wages and declines in returns to capital. In contrast, older households that have already accumulated assets and low-skilled households for whom labor income constitutes a smaller part of future resources benefit less and may even lose. Income inequality in Germany is projected to increase due to the demographic transition, as measured by a 8.4% increase in the Gini coefficient between 2003 and 2050.

While this chapter discloses a number of important channels of economic adjustment to an aging population, it also abstracts from several issues that can play a role in shaping the economic transition. First, it is important to bear in mind that we operate in a “frictionless” environment where all endogenous adjustments are driven by relative price changes. This can only partly be justified by the long-run character of the analysis. Second, as we do not take a stand on how social security deals with population aging we implicitly assume the presence of a fully-funded private pension system that operates through the optimal life cycle savings behavior of households. In this light, the results can be viewed as a benchmark scenario that abstracts from any public sector-induced distortions to private savings decisions. Third, by adopting the small open economy assumption we rule out the possibility that differences in the extent and timing of demographic processes across countries can shape the economic transition. It has been demonstrated that the presence of globally unsynchronized aging patterns may induce a substantial amount of international capital flows (see, e.g., Attanasio et al. (2006) and Börsch-Supan et al. (2006)) or can affect international trade (see Chapter 3). Fourth, although the assumption of imperfect substitutability between domestic and foreign goods implies that the domestic interest rate can deviate from the exogenous world interest rate, the small open economy assumption limits the impact of population aging on the interest rate. For example, employing multi-region OLG models Börsch-Supan et al. (2006) and Krueger and Ludwig (2007) find that demographic change tends to reduce the interest rate by about 0.5-1 percentage points whereas the present model projects a decline of about 0.2 percentage points over a comparable time horizon. Finally, as suggested by Ludwig, Schelkle and Vogel (2007), higher returns to labor may make it optimal for young households to obtain a better education, thereby increasing the supply of effective labor. Thus, accounting for endogenous human capital accumulation may mitigate the macroeconomic impact of demographic change.

Appendix 1: Calibrated Share Form

Numerical calculation of economic equilibrium requires the choice of concrete functional forms for production possibilities and preferences. In applied modeling, nested constant elasticity of substitution (CES) functions (including Leontief and Cobb-Douglas specifications as subcases) are most common. Such functions have certain mathematical properties (regularity) that ease the numerical analysis considerably, but are still flexible enough to allow for an appropriate representation of economic behavior. CES functions are most easily written in the so-called calibrated share form, which eases the calculation of free parameters, since it is based on quantities, prices, and value shares that are directly observable from the benchmark data. Following Rutherford (1995b), the standard CES production function in calibrated share form reads as:

$$\frac{Y}{\overline{\overline{Y}}} = \left[\sum_i \theta_i \left(\frac{X_i}{\overline{\overline{X}}_i} \right)^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}. \tag{4.47}$$

The associated dual cost function in calibrated share form is given by:

$$\frac{c_y}{\overline{c}_y} = \left[\sum_i \theta_i \left(\frac{p_{x,i}}{\overline{p}_{x,i}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{4.48}$$

and demands for the individual factors of production (applying Shepard's Lemma):

$$\frac{X_i}{\overline{\overline{X}}_i} = \frac{Y}{\overline{\overline{Y}}} \left(\frac{c_y}{\overline{c}_y} \frac{\overline{p}_{x,i}}{p_{x,i}} \right)^{\sigma} \tag{4.49}$$

where Y =output quantity, θ_i =benchmark value share of X_i in the production of Y, X_i =input factor, c_y =cost of Y, $p_{x,i}$ =input price of X_i, and σ =elasticity of substitution.

Appendix 2: Approximating the Infinite Horizon

In order to solve a finite approximation of the model with a T-period model horizon (where T is the last period of the numerical model), we use a state variable targeting approach as suggested by Lau et al. (2002). The post-terminal capital stock (KT) can be determined as part of the equilibrium calculation by targeting the associated control variable investment (I_T) in the terminal period. In the present model this is achieved by adding the following constraint to the model:

$$\frac{I_T}{I_{T-1}} = \frac{Y_{AGF,T}}{Y_{AGF,T-1}} \qquad \perp KT \tag{4.50}$$

where we use the "⊥" operator to indicate the complementarity aspect. The price for post-terminal capital (*pkt*) is determined by a last period version of (4.37):

$$K_T\,(1-\delta)+I_T = + \sum_{g=t-N}^{t}\sum_{h=1}^{H} \overline{kt}_{g,h,g}\,KT \qquad \perp pkt \tag{4.51}$$

where $\overline{kt}_{g,h,g}$ =assets holdings of terminal generations, i.e. those households that live beyond T.

Following Rasmussen and Rutherford (2004), government income over the infinite horizon is given by:

$$gov = p_{f,t}\,T_t + \sum_t \Phi_t + p_{f,0}\overline{B}\,\frac{1+\overline{r}}{\overline{r}} - p_{f,T}\,\overline{B}\,\frac{1}{\overline{r}} \tag{4.52}$$

where the second summand on the right hand side denotes the present value of future government deficits over the infinite horizon and the third summand gives the value of terminal assets in the terminal period using the steady-state assumption to sum over the infinite horizon. Thus, the government agent cannot consume all of its wealth in the last model period but is "endowed" with a negative amount of assets that is sufficient to finance future government deficits over the infinite horizon consistent with a steady state equilibrium at time T. When the government budget is balanced over the infinite horizon, the terminal level of government assets is endogenous. This is necessary to account for the fact that a policy change may cause the government's asset position in period T to deviate from the baseline level. Again, following Rasmussen and Rutherford (2004, p.1401), we therefore break (4.27) into two equations that are imposed on the model as additional constraints:

$$p_{f,0}A_{G,0} + p_{f,T}\,A_{G,T} = \sum_{t=0}^{T}(\Phi_t - \Gamma_t) \tag{4.53}$$

$$p_{f,T}\,A_{G,T} = \frac{1}{\overline{r}}\,(\Phi_T - \Gamma_T)\,. \tag{4.54}$$

Equation (4.53) states the required balance within the model horizon as a function of the endogenous level of terminal assets, $A_{G,T}$. Equation (4.54) gives the value of terminal assets as a function of income and expenditure in the terminal period using the steady-state assumption to sum over the infinite horizon.

Table 4.12 Overview of model parameters

TECHNOLOGY PARAMETERS	
β_i^{VA}	benchmark value share of value-added in total costs
β_i^{L}	benchmark value share of labor in value-added
χ_i	fixed-coefficient (Leontief) in Armington production
β_i^{IA}	benchmark value share of intermediate good in investment
β_i^{X}	benchmark value share of exports
β_i^{H}	benchmark value share of domestic consumption in domestic production
β_i^{AH}	benchmark value share of domestic production in domestic consumption
β_i^{AM}	benchmark value share of imports
$t_{AR,i}$	tax on Armington goods
$t_{Y,i}$	output tax in sector sector i
$\sigma_{VA,i}$	elasticity between capital and labor in value-added
σ_{HX}	elasticity of transformation
$\sigma_{AR,i}$	elasticity of substitution in Armington production
δ	annual capital depreciation rate
$\Omega_{Y,i}$	total factor productivity in production of $Y_{i,t}$
$\Omega_{A,i}$	total factor productivity in production of $A_{i,t}$
Ω_I	total factor productivity in composite investment good
HOUSEHOLD PARAMETERS	
$\bar{r}$	annual steady-state interest rate
θ_h^{F}	benchmark value share of food products
θ_h^{NF}	benchmark value share of non-food products
$\varphi_{i,h,t}$	benchmark value share of a single non-food product in non-food aggregate
σ_h	intertemporal elasticity of substitution
ν_h	intratemporal elasticity of substitution between consumption and leisure
κ_h	elasticity of substitution between food and non-food products
σ_{NF}	elasticity of substitution between non-food products
α_h	benchmark value share of composite consumption
ρ	time preference parameter
$\pi_{g,h,t}$	labor productivity index over the life cycle

Table 4.13 Intermediate demand structure

	AGF	MIN	FOP	TXL	PRP	MIC	MET	MAC	PGO	CEW	WRT	TRC	BIN	RES	PBS	EDU	HEA
AGF	72.16	0.42	304.26	2.75	18.03	2.63	0.00	0.00	0.17	0.00	8.08	0.65	0.06	6.50	10.96	0.58	6.77
MIN	3.89	28.36	5.85	0.64	4.57	291.94	30.51	4.22	0.44	80.68	4.35	0.35	0.65	0.87	5.89	1.10	1.88
FOP	42.71	0.00	266.83	0.96	0.00	11.71	0.00	0.00	0.00	0.08	114.09	1.47	0.00	0.12	10.97	3.48	45.40
TXL	1.14	0.17	0.47	79.35	0.31	7.46	1.07	11.79	10.72	3.42	3.73	0.55	0.17	0.28	2.98	0.11	4.96
PRP	1.81	2.38	32.65	6.03	280.75	33.11	11.81	51.42	30.15	60.72	69.57	17.45	10.64	63.84	30.08	11.55	10.30
MIC	44.53	4.66	42.56	33.11	58.88	1017.87	74.69	279.73	19.06	305.81	45.6	68.67	1.32	23.43	27.72	7.99	60.99
MET	3.62	6.92	12.48	1.54	4.46	31.32	697.59	537.76	14.97	112.38	16.52	7.50	0.30	3.19	12.91	0.98	1.78
MAC	11.22	13.82	10.79	3.0	7.06	39.52	46.94	2059.39	5.59	161.55	50.05	71.20	3.94	24.85	63.57	3.75	45.07
PGO	0.01	0.01	0.02	0.65	3.89	3.27	11.16	19.0	34.11	0.3	1.02	0.14	0.15	0.34	8.12	0.61	0.59
CEW	11.9	10.14	28.57	6.57	25.95	58.72	50.6	51.18	4.29	184.69	62.5	31.95	12.6	191.58	61.25	23.09	43.61
WRT	23.83	6.53	88.58	23.15	39.71	84.82	83.97	228.94	32.49	120.43	167.22	75.06	12.97	27.53	67.88	8.59	50.62
TRC	3.11	7.57	54.95	3.0	33.02	74.51	39.35	150.94	9.4	27.36	396.89	751.14	38.76	40.31	71.79	24.93	12.80
BIN	11.83	2.08	16.32	1.76	12.64	28.94	14.85	55.34	3.58	55.35	80.74	58.49	594.24	293.77	99.17	13.8	32.12
RES	50.17	25.51	157.74	12.4	109.67	291.02	96.79	550.94	28.9	311.84	593.57	224.52	426.51	1038.78	249.38	34.27	150.80
PBS	5.61	4.83	18.1	1.49	37.41	35.85	14.11	35.41	2.74	64.26	69.89	24.49	16.1	151.11	274.2	8.26	39.84
EDU	0.49	0.13	1.61	0.28	1.03	3.55	2.32	6.92	0.33	3.42	5.32	3.87	2.93	35.94	11.15	65.28	1.97
HEA	4.72	0.00	0.1	0.00	0.00	0.25	0.00	0.29	0.0	0.0	0.59	0.00	0.00	0.06	1.50	0.48	34.44

Note: Extract of the social accounting matrix (SAM) that shows the intermediate demand structure between sectors. The SAM is based on Input-Output table 1.1, Issue 18(2), Federal Statistical Office of Germany. Original data is aggregated using the sectoral categories as shown in Table 4.2. Figures above refer to 100 Million Euro.

Chapter 5
Concluding Remarks

This book presents a new computational method to efficiently solve large-scale Auerbach-Kotlikoff OLG models that are infeasible for simultaneous solution methods. The proposed decomposition algorithm is based on the solution of a sequence of nonlinear complementarity problems. The main idea is to approximate equilibria of an OLG economy by solving a sequence of representative agent (Ramsey optimal growth) problems. It has been demonstrated that this algorithm can be used to solve high-dimensional OLG models that comprise a large number of sectors and regions and a rich household side, including a large number of heterogeneous households and a variety of household-specific effects. Given the limitations of integrated approaches, the new method thus significantly increases the range of economic questions that can be investigated with these models, providing a valuable addition to the toolbox of applied general equilibrium modelers. One important advantage of the decomposition algorithm presented here is that it facilitates the development of large-scale OLG models designed to investigate in-depth the distributional consequences of economic policy along an intra- and intergenerational dimension.

The usefulness and flexibility of the new approach has been demonstrated by applying it to various aspects of the macroeconomic analysis of demographic change that have received little attention in the literature. Chapter 3 develops a dynamic two-country Heckscher-Ohlin model with overlapping generations to analyze the consequences of globally unsynchronized aging patterns for international trade and the distribution of gains from trade under such demographic circumstances. Chapter 4 employs a multi-sectoral OLG model with intra-cohort heterogeneity, calibrated to the German economy and projected demographic trends, to analyze the consequences of population aging for the sectoral composition of output, and the distribution of income, wealth, and welfare.

Since the main results have already been summarized in Chapter 1, the remainder of this chapter provides a discussion of a number of simplifying assumptions that underly the economic analyses in Chapter 3 and Chapter 4. The following discussion also serves to highlight possible extensions of the work presented here.

First, all models considered here abstract from government policies. While the interaction of demographic change and public policies is a wide field, social security

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is clearly a relevant dimension to the analysis. A public pension system can strongly influence private savings behavior. Consequently, a remarkable number of papers investigates the adjustments required in the pension system due to demographic shifts. Important examples include Huang et al. (1997), De Nardi et al. (1999), and Abel (2003). Krueger and Ludwig (2007) show that the impact on factor prices induced by the demographic transition is to some extent influenced by the design and reform of the pension system. Against this background, the results presented in this book ought to be viewed as a benchmark world in which there is a fully-funded pension system in place that operates through private life-cycle savings.

Second, the analyses are carried out in a "frictionless" environment in which all endogenous adjustments are driven by relative price changes. For instance, the assumption of frictionless labor markets may be particularly critical for the European economies in which unions play an important role in the wage setting process. Also, the hypothesis of perfect competition on goods markets may not reflect actual market structures for particular sectors of the economy. In general, however, accounting for market frictions is surely more of an issue for an analysis that is—unlike the models presented in this book—concerned with the short-run effects of the demographic transition.

Third, it is assumed that demographic dynamics are exogenous to economic development. There is a consensus in the literature that in the long run neither fertility nor mortality are exogenous to economic growth (see, e.g., Barro and Becker (1988), Barro and Becker (1989), and Boldrin and Jones (2002)). Furthermore, it is plausible to assume that international migration is endogenous and reacts to international differences in income.

Fourth, and maybe most critical is the assumption that individuals are fully rational and forward-looking. All models presented here assume that household behavior follows the rational expectations paradigm. While this assumption considerably reduces the complexity of the analysis and, from a practical modeling point of view, enables a relatively simple implementation of household behavior, there are certainly good reasons to question the rationality of agents. For instance, empirical evidence (for an overview see Attanasio (1999)) shows that households do not smooth consumption as much as the life cycle theory suggests. At a more general level, the economic literature on bounded rationality indicates that households sometimes violate their preferences (Kahneman, 2003). It is an open question of research how to integrate such behavioral assumptions into a general equilibrium framework. Possible routes to achieve this are, e.g., to impose additional constraints on household behavior while maintaining the assumption of full rationality (Altig et al. 2001) or to depart from this hypothesis by assuming myopic rule-of-thumb behavior (Campbell and Mankiw, 1991).

Fifth, as suggested by Ludwig et al. (2007), higher returns to labor may make it optimal for young households to obtain a better education, thereby increasing the effective supply of labor. Thus, endogenous human capital accumulation may mitigate the macroeconomic impact of demographic change.

Sixth, the analyses presented here abstract from intergenerational altruism whereby successive generations are linked through recursive altruistic preferences that give rise to intergenerational transfers or bequests. Accounting for such linkages between generations can mitigate the effects due to population aging as the economic adjustment can be spread over a longer horizon. However, as shown by Altonji, Hayashi and Kotlikoff (1992), there is strong evidence against the idea that members of extended families are altruistically linked in a way that would support the infinite horizon representation as suggested by Barro (1974).

Lastly, abstracting from any form of uncertainty in the model, and assuming rational expectations on the part of the households, implies that agents have perfect foresight with respect to future economic and demographic developments. Several recent papers go beyond a purely deterministic framework and incorporate different forms of uncertainty in the analysis. Imrohoroglu, Imrohoroglu and Joines (2003) and Conesa and Krueger (1999) add idiosyncratic uncertainties to study questions related to social security reform in an OLG context. Using an OLG model with stochastic production and incomplete markets, Krueger and Kubler (2006) find that the introduction of an unfunded social security system can lead to a Pareto improvement even when the economy is dynamically efficient. There are, however, virtually no studies which analyze the consequences of demographic change in an stochastic OLG framework. A notable exception is Krueger and Ludwig (2007) who employ a model with uninsurable labor productivity and mortality risk. They find that uninsurable idiosyncratic uncertainty makes life cycle savings profiles more realistic. Incorporating uncertainty in complementarity formulations of equilibrium models is difficult to achieve and is currently on the agenda of some researchers (Böhringer and Rutherford, 2007).

The previous discussion points out possible extensions for the work presented here. In summary, the work presented here can be extended in many ways along the lines discussed above making it possible to investigate new aspects of the economic consequences of demographic change. Moreover, relaxing a number of simplifying assumptions may improve the empirical fit of the models and enhance the policy relevance of the analyses. It is a question for future research whether the inclusion of those factors would materially alter the results presented here.

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