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DYNAMIC MODELING AND ECONOMETRICS IN ECONOMICS AND FINANCE 10

Topics in Applied Macrodynamic Theory

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Preface

This book is about the study of topics in macro dynamics from an applied, empirical perspective. The modeling philosophy behind most of the chapters of this book is of Keynesian nature, representing an attempt to revive this theoretical perspective on the working of the macroeconomy. The macroeconomic research pursued here is somewhat different from the mainstream literature using the Dynamic Stochastic General Equilibrium (DSGE) approach as the basic modeling device. The main features of the latter are the assumptions of intertemporally optimizing agents, rational expectations, competitive markets and price mediated market clearing through sufficiently flexible prices and wages. The New Keynesian approach to macroeconomics has, in the last decade or so, to a large extent, also adopted the DSGE framework, building on intertemporally optimizing agents and market clearing, but favoring more the concept of monopolistic competition, sticky wages and prices and nominal as well as real rigidities. An path breaking work of this type is the recent book by Woodford (2003).

However, it is well known that the intertemporal approach of smoothly optimizing agents and fast adjustments in order to establish temporal or intertemporal marginal conditions in the product market, labor and capital markets, has not been very successful to match certain stylized facts on those markets. A further deficiency of those intertemporal decision models is that macroeconomic feedback effects—and their stabilizing or destabilizing impact on the macroeconomy—have rarely been considered in those models. Yet, those feedback mechanisms, relevant for the interaction of all three markets, have been theoretically and empirically explored since the 1930s. The emphasis of the topics in our book lies on the study of the relative strength and interaction of these feedback mechanism as well as transmission channels with respect to all three markets from a Keynesian perspective. We are, in particular interested in their impact on the stability once their working is considered in the context of a fully developed dynamic system approach.

While we do not deny that forward-looking behavior and (the attempt of) intertemporal optimization by the economic agents might be relevant for the dynamics of the economy, in our view the exclusive focusing on this issues in the present academic literature leaves too many interesting, important and relevant issues aside. In particular, in the interaction of all three markets there may be nonlinear feedback mechanism at work which do not necessarily give rise to market clearing, nor necessarily to convergence of a (unique) steady state growth path. Also, as recent research has shown, there is heterogeneity of agents and beliefs present in modern economies, as well as a large variety of informational and structural frictions present in the real world. We believe that this leaves many questions open so that the true understanding of the economy might better be pursued by a variety of frameworks. Often it is said with respect to the DSGE models: One needs to use an intertemporal optimizing and rational expectations' framework, otherwise one would leave "too much money on the side walk". But one might also add, by doing so, there is a danger that one might also leave too many problems in macroeconomics on the side walk.

One central point in our book on topics of macro economics are the mechanisms generating non-cleared markets and the phenomenon of disequilibrium recurrently present in certain markets such as the labor markets. In contrast to the tradition which stresses the clearing of all markets at each instance of time, in our modeling approach, as it will be stressed at several occasions throughout this book, these disequilibrium situations are the main driving forces of the wage and price inflation dynamics. These, in turn, might act either in a stabilizing or destabilizing manner through a variety of different macroeconomic channels such as the real wage feedback channel, product market or financial market channels. As the reader will notice, the many estimations discussed throughout this book confirm the empirical plausibility of our modeling approach, showing that there are indeed different (and also valid) possibilities to specify and analyze the dynamics of the macro economy in a different way than in the DSGE framework.

Due to the fact that in our modeling approach the stability of the analyzed dynamical system is not imposed ad initio by the rational expectations assumption, which requires that the economy always "jumps" to the stable path and therefore always converges to the steady state after any type of shocks, its stability properties (and its analysis) are based on the relative strength of the interacting macroeconomic feedback channels. Such type of stability analysis, despite of its importance for the understanding of the dynamics of an economy, seems not to be relevant for the literature based on the rational expectations market clearing tradition. The existence of possibly divergent paths does not appear to be a relevant issue either. However, the ongoing occurrence of "bubbles" and "herding" in the financial markets across the world, as well as the large macroeconomic imbalances present nowadays in the global economy show that such divergent paths can indeed take place.

Our book commences with a chapter, Chap. 1, where we concentrate on the issues surrounding the problem of continuous time versus period modeling choices. In particular we study the relevance of the assumed uniform time unit for the dynamics of a model formulated in discrete time with respect to an analogous model formulated in continuous time. This preliminary analysis has two main purposes: first, to motivate the continuous time (and also the disequilibrium) modeling approach pursued throughout this book, and second, to highlight the importance of continuous and discrete time model equivalence for applied macrodynamic analysis (without or with the addition of significant and relevant time delays). Indeed, even though economic decisions might be revised in a quarterly or annual frequency at the individual level, given the large number of agents and the highly probable de-synchronization between them, a continuous- or quasi-continuous time theoretical formulation of the resulting dynamics seems more adequate at the macroeconomic level.

Part I of the book focuses on advanced topics of the dynamics of closed economies. Here the baseline AS-AD framework of advanced type is outlined and, in a variety of alternative formulations and extensions, investigated and estimated. Empirical evidence not only on the wage-price dynamics of this framework but also for the remaining equations of the model, concerning Keynesian quantity adjustment processes and a monetary policy rule, is provided. The role of wage- and price-flexibility, of income distribution as well as of monetary policy for stabilizing the dynamical economic system is analyzed in detail. Furthermore, in Chap. 5, a closer look is taken at the link between the goods and the labor markets by means of a thorough analysis of Okun's Law. As recent work has shown there might be some significant de-linking of the product and labor market dynamics. Altogether, this part provides a detailed introduction into a Keynesian AD-AS framework that, in view of the state of traditional Keynesian modeling, may be called "matured" instead of "new". It represents more of an advancement of the traditional approach in contrast to the complete overhaul of its theoretical foundations as it is characteristic for the "New Keynesian" DSGE approach.

Part II analyzes advanced topics on the dynamics and interaction of open economies. It starts from the description of an IS-LM-PC model of Dornbusch myopic foresight type in which the stock market dynamics showing the same type of foresight are incorporated, and where both sluggish price and quantity adjustment occurs in an otherwise perfect world. The questions there is to what extend does the open economy dynamics impact the domestic asset and goods market and what type of Taylor policy rule is appropriate in such a context. After this discussion of a model of rather classical type, in the following chapters somewhat alternative models are investigated. In Chap. 7 a Mundell–Fleming–Tobin model is set up in order to investigate the interaction of fiscal and current account imbalances and the dynamics of inflation. In Chap. 8, in a similar framework, an attempt is made to model and understand the dynamics of currency and financial crises—in recent times seen to unfold in many countries, in particular in emerging markets. The macroeconomic interaction of two large economies through a variety of international transmission channels is investigated in Chap. 9 by means of a semi-structural two country model and estimated for the case of the US economy and the Eurozone. In an outlook, in Chap. 10, we highlight the importance of having supply constraints in the dynamics of demand-driven macroeconomy, but show at the same time that they rarely become binding ones if active inventory policies of firms are added to the demand driven core dynamics.

This book builds to some degree on research papers written jointly also with further co-authors pursuing this line of research. We here have to thank here in particular Toichiro Asada from Chuo University, Pu Chen from Bielefeld University, Carl Chiarella from the UTS Sydney, Reiner Franke from Kiel University, Gang Gong from Tsinghua University, Florian Hartmann from Bielefeld University and Hans-Martin Krolzig from the University of Kent for their contributions and the stimulating discussions we have had with them on many occasions in the recent and more distant past. Of course, the usual disclaimer here applies.

Bielefeld and New York,
February 2008

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Notation

Steady state or trend values are indicated by a sub- or superscript "o" and foreign country variables are indicated by a superscript $*$ or f. When no confusion arises, letters F, G, H may also define certain functional expressions in a specific context. A dot over a variable $x = x(t)$ denotes the time derivative, a caret its growth rate; $\dot{x} = dx/dt$, $\hat{x} = \dot{x}/x$. In the numerical simulations, flow variables are measured at annual rates.

As far as possible, the notation tries to follow the logic of using capital letters for level variables and lower case letters for variables in intensive form, or for constant (steady state) ratios. Greek letters are most often constant coefficients in behavioral equations (with, however, the notable exceptions being π and ω, the inflation rate and the real wage).

B	outstanding government fixed-price bonds (priced at $p_b = 1$)
C	real private consumption (demand is generally realized)
E	number of equities
F	neoclassical production function or foreign bonds in Part II of the book
G	real government expenditure (demand is always realized)
I	real net investment of fixed capital (demand is always realized)
J	Jacobian matrix in the mathematical analysis
K	stock of fixed capital
L^d	employment, i.e., total working hours per year (labor demand is always realized)
L	labor supply, i.e., supply of total working hours per year
M	stock of money supply
S	total real saving; $S = S_f + S_g + S_h$

s	nominal exchange rate (η the real exchange rate)
S_p	real saving of private households
T	total real tax collections
T^c	real taxes of asset holders
W	real wealth of private households
N	actual inventories
N^d	desired inventories
$U^n = N^d/N$	inventory utilization rate
Y	real output
Y^d	real aggregate demand
Y^e	expected sales
$\bar{Y}$	output at normal use of capacity
e, V^l	employment rate
u^w, V^w	utilization rate of the employed
f_x	partial derivative
$n_z, \hat{z}$	growth rate of trend labor productivity
μ	growth rate of money supply
i	nominal rate of interest on government bonds;
ℓ	labor intensity (in efficiency units)
m	real balances relative to the capital stock; $m = M/pK$
p	price level
p_e	price of equities
r	rate of return on fixed capital, specified as $r = (pY - wL - \delta pK)/pK$, or the real interest rate $r = i - \hat{p}$
s_c	propensity to save out of capital income on the part of asset owners
s_h	households' propensity to save out of total income
u, V^c	rate of capacity utilization; $u = Y/Y^p = y/y^p$
V_c^e	expected capacity utilization
v	wage share (in gross product); $v = wL/pY$
w	nominal wage rate per hour
ω	real wage rate
y	output-capital ratio; $y = Y/K$;
y^d	ratio of aggregate demand to capital stock; $y^d = Y^d/K$
y^p	potential output-capital ratio (a constant)
z	labor productivity, i.e., output per working hour; $z = Y/L^d$
α_{ii}	interest rate smoothing coefficient in the Taylor rule

ϕ_{ip}	coefficient on inflation gap in the Taylor rule
ϕ_{iy}, ϕ_{iu}	coefficient on output gap (capacity utilization) in the Taylor rule
α_{yr}, α_{ur}	real interest rate sensitivity of the output gap (capacity utilization rate)
β_x	generically, reaction coefficient in an equation determining x, $\dot{x}$ or $\hat{x}$
β_π	general adjustment speed in revisions of the inflation climate
β_{xy}	generically, reaction coefficient related to the determination of variable x, $\dot{x}$ or $\hat{x}$ with respect to changes in the exogenous variable y
β_{pu}	reaction coefficient of u in price Phillips curve
β_{pv}	reaction coefficient of $(1+\mu)v - 1$ in price Phillips curve
β_{we}	reaction coefficient of e in wage Phillips curve
β_{wv}	reaction coefficient of $(v - v^o)/v^o$ in wage Phillips curve
δ	rate of depreciation of fixed capital (a constant)
$\eta_{m,i}$	interest elasticity of money demand (expressed as a positive number)
κ_p	parameter weighting $\hat{w}$ vs. π in price Phillips curve
κ_w	parameter weighting $\hat{p}$ vs. π in wage Phillips curve
κ	coefficient in reduced-form wage-price equations $\kappa = (1 - \kappa_p \kappa_w)^{-1}$
π^c, π_c	general inflation climate
θ	tax parameter (net of interest)
τ_w	tax rate on wages

1

Period Models, Continuous Time and Applied Macrodynamics

1.1 Introduction

In this chapter,[1] and we reconsider the issue of the (non-)equivalence of period and continuous time analysis. We stress here that period models—the now dominant model type in the macrodynamic literature—assume a single (uniformly applied) lag length for all markets, which therefore act in a completely synchronized manner. In view of this, we start in Sect. 1.2 from the methodological precept that period and continuous time representations of the same macrostructure should give rise to the same qualitative outcome, i.e., that the qualitative results of period analysis should not depend on the length of the period, see Foley (1975) for an earlier statement of this precept, as well as Medio (1991a) and Sims (1998) for related observations. A simple example where this is fulfilled is given by the conventional Solow growth model, considered in Sect. 1.3, while all chaotic period dynamics of dimension less than 3 are in conflict with this precept, see however Medio (1991a) for routes to chaos in such an environment.

A basic empirical fact moreover is that the actual data generating process in macroeconomics is by and large a daily one (and the data collection frequency now also much less than a year). This suggests that empirically oriented macromodels should be iterated with a short period length as far as actual processes are concerned and will then in general provide the same answer as their continuous-time analogues. Concerning expectations, the data

[1] This chapter is based on Asada et al. (2007), "Continuous Time, Period Analysis and Chaos from an Empirical Perspective", CEM Working Paper 144, Bielefeld University.

collection process is however of importance and may give rise to certain (smaller) delays in the revision of expectations, which however may be overcome by the formulation of extrapolating expectation mechanisms and other ways by which agents smooth their expectation formation process. We do not expect here that this implies a major difference between period and continuous time analysis if appropriately modeled, a situation which may however radically change if proper delays as for example considered in Invernizzi and Medio (1991) are taken into account.

We discuss in Sect. 1.4 a typical example from the literature (by far not the only one), where chaos results from a "too" stable continuous time approach when reformulated as a "long-period" macro-model, then exhibiting a sufficient degree of locally destabilizing overshooting. Shortening the period lengths in such chaotic macro models, i.e., iterating them with a finer step size, removes on the one hand "chaos" from such model types, while it on the other hand (and at the same time) brings the model into closer contact with what happens in the data generating process of the real world.[2]

By contrast, the chapter shows in Sect. 1.5 that baseline macromodels can give rise to complex dynamics in (quasi-)continuous time if they are sufficiently rich in their dynamical structure and dimension. We conclude from this result that the investigation of complex dynamics is of a more fundamental type when restricted to higher dimensional continuous time macrodynamics, since such approaches avoid the mixture of locally destabilizing, strongly overshooting adjustment processes (which would converge in quasi-continuous time) with the dynamics that are typical for the larger models (with interacting real and financial markets) of advanced macrodynamic literature.

1.2 The J2-Status of Macrodynamic Period Analysis

We reconsider here the issue of the (non-)equivalence of period analysis (or for brevity discrete time) and continuous time macro modeling. Period analysis is now the dominant form for models in the macrodynamic literature and thus of interest in its own right, independently of the consideration of the existence of more complicated lags in more advanced macrosystems. Discrete time macro

[2] Note in this respect again, that we focus in this chapter on standard period models and therefore do not yet consider, as in Invernizzi and Medio (1991), Medio (1991a) the role of significant delays and exponential lags in economic activity.

modeling is of course not restricted to the assumption of a single uniform and synchronized period length between all economic activities, on which this chapter is focused. For a detailed consideration of the role of significant lags in macrodynamics the reader is referred to Invernizzi and Medio (1991).

We in this respect focus on the empirical fact that the actual *data generating process* in macroeconomics is of much finer step size than the corresponding *data collection frequency* available nowadays, at least in the real markets of the economy, and that the latter is nowadays also considerably finer than one year in general.[3] This implies that empirically applicable period macromodels (using annualized data) should be iterated with a much finer frequency (approximately with step size between "1/365 year" and "1/52 year" with respect to the actual performance of economy) in order for them to generate results that may then in general equivalent to the ones of their continuous time analogue (at least in dimensions one and two). Furthermore, models that contain expectational variables may be referring to the data collection process, yet are subject to expectational smoothing and thus also updated in shorter time intervals than the actually observed data.

These empirically applicable period models—which take account of the fact that macroeconomic (annualized) data are generally updated each day—will then not be able to give rise to chaotic dynamics in dimensions one and two, suggesting that the literature on such chaotic dynamics is of questionable empirical relevance (though mathematically often demanding and of interest from this point of view). To exemplify this we consider in this chapter a 2D nonlinear monetarist baseline model that is known to be globally asymptotically stable in continuous time and that has been used in a period framework to generate from its parameters a period doubling route to chaos.

[3] This discrepancy concerning the frequency between the data generating- and the data collection processes is ignored in the majority of empirical mainstream macroeconomic models, which, focusing on aggregate macroeconomic variables available in general at a quarterly basis (such as consumption or prices), simply assume for the time intervals of the theoretical framework the same periodicity as the data collection process. This strategy which is conditioned through the data collection technology available nowadays, can be misleading when the resulting dynamic properties of the calibrated theoretical model depend not on its intrinsic characteristics, but mainly on the length of the iteration intervals. This issue becomes particularly clear in discrete-time dynamic models of dimensions one or two which exhibit chaotic properties, whereas in the continuous time analogue the occurrence of such chaotic dynamics is simply impossible.

Before doing so we however consider a simple case, the Solow growth model, where period and continuous analysis give qualitatively the same answer for any length of the period between zero and infinity. The clustering of production and investment activities at possibly very distant points in time thus does not raise in this case the question of which period length is the most appropriate one, though it may still be asked whether the assumed type of clustering of economic activities really makes sense from an applied macroeconomic point of view if periods longer than one week are considered.

In concluding, this chapter therefore proposes that continuous time modeling (or period modeling with a short period length) is the better choice to approach macrodynamical issues compared to a period model where the length of the period remains unspecified, since it avoids the empirically uninterpretable situation of a uniform period length (with a length of one quarter, year or more) with an artificial synchronization of economic decision making. If discrete time formulations (not period analysis) are considered for macroeconomic model building they should represent averages over the day as the relevant time unit for *complete* models of the real-financial interaction on the macroeconomic level (interactions which in fact should be the relevant perspective for all *partial* macroeconomic model building). The stated dominance of continuous time modeling (or quasi-continuous modeling with a period length of one day) not only simplifies the stability analysis for macrodynamic model building, but also questions the relevance of period model attractors that differ radically from their continuous time analogue.

Chiarella and Flaschel (2000) argue that a fully specified Keynesian model of monetary growth exhibits at least the six state variables, namely wage share and labor intensity (the growth component), inflation and expected inflation (the medium-run component) and expected sales and actual inventories per unit of capital (the short-run dynamics), i.e., these models easily meet the 3D requirement for the existence of strange attractors in continuous time. Also the New Keynesian baseline model with both staggered wage and price setting is at least of dimension 4, so that even still simple models of a monetary economy (with only an interest rate rule of the central bank) can be used for routes to chaos analysis without running into the danger of synthesizing basically continuous-time ideas with radically synchronized (overshooting) discrete time adjustment processes (which when appropriately bounded produce chaos also in dimensions one or two). This suggests that all techniques developed for analyzing nonlinear dynamical systems represent unquestionably a useful stock of knowledge, to be applied now (in macroeconomics) to investigate strange attractors as they may come about in continuous or quasi-continuous time of high-order macrodynamics.

As a future research agenda we therefore propose to use existing higher dimensional macromodels with small (quasi-continuous, but still of period model type) iteration step size and basic parameters broadly in line with empirical magnitudes in order to investigate by the help of behavioral nonlinearities complex attractors that allow us to apply mathematical contributions like Guckenheimer and Holmes (1983) and Wiggins (1990) to macrodynamic model building. In mathematics, the step from 2D to 3D dynamical system is a truly significant and very interesting one that should now become the focus of interest, since complex 1D and 2D macrodynamics are fairly well exploited by now. Nevertheless such low dimensional models may be useful for testing the role of certain bounding mechanisms in 2D models in particular, but should then give rise to limit cycle behavior or at most attractors of the type shown in Fig. 1.3 in the conclusions of this chapter (which by and large exhaust the possibilities of attracting sets for ordinary continuous time models in dimension 2).

Continuous vs. discrete time modeling, in macroeconomics, was discussed extensively in the 1970s and 1980s, sometimes in very confusing ways and often by means of highly sophisticated, but—as we shall show in this chapter—also by an unnecessarily complicated mathematical apparatus. There are however some statements in the literature, old and new, which suggest that period analysis in macroeconomics, i.e. discrete-time analysis where all economic agents are forced to act in a synchronized manner (with a time unit that is usually left unspecified) can be misleading from the formal as well as from the economic point of view. Foley (1975, p. 310) in particular states:

> The arguments of this section are based on a methodological precept concerning macroeconomic period models: *No substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period.*

Such a statement has however been completely ignored in the numerous analytical and numerical investigations of complex or chaotic macro-dynamics. Furthermore, from the view point of economic modeling, Sims (1998, p. 318) states:

> The next several sections examine the behavior of a variety of models that differ mainly in how they model real and nominal stickiness . . . They are formulated in continuous time to avoid the need to use the uninterpretable "one period" delays that plague the discrete time models in this literature.

Tobin (1982, p. 189), by contrast, states:

> Representation of economies as systems of simultaneous equations always strains credibility. But it takes extraordinary suspension of disbelief to imagine that the economy solves and re-solves such systems every microsecond. Even with modern computers the task of the Walrasian Auctioneer, and of the market participants who provide demand and supply schedules, would be impossible. Economic interdependence is *the* feature of economic life and we as professional economists seek to understand and explain. Simultaneous equations systems are a convenient representation of interdependence, but it is more persuasive to think of the economic processes that solve them as taking time than as working instantaneously.

In our view, a macro-dynamic analysis that is intended to consider eventually real and financial markets simultaneously must consider period analysis with a very short time-unit ("one day"), if a uniform and synchronized period length is assumed (with averaging of what happened during the day). But then, following Tobin (1982), real markets cannot be considered in equilibrium all of the time. Instead gradual adjustment of wages, prices and quantities occurs in view of labor and goods markets imbalances for which moreover convergence to real market equilibria cannot automatically be assumed, in particular if the economic fundamentals are changing in time. Real market behavior is therefore to be based on gradual adjustment processes, as suggested in Chiarella and Flaschel (2000) and extended in Chiarella et al. (2005), and it can then be discussed whether, on this basis, financial markets should be modeled by equilibrium conditions (as Tobin 1982 proposes) or also by somewhat delayed responses as well, both in short period analysis as well as in continuous time.

Such implications may be the outcome of a reconsideration of discrete vs. continuous time dynamics. The present chapter however focuses on a narrower point, namely, following Foley (1975), that discrete and continuous-time modeling should provide qualitatively the same results. We provide in this respect a positive example (the Solow model) and a negative one (the monetarist baseline model), but conclude with respect to both of them that an artificial clustering of macroeconomic activities with long intermediate intervals of inactivity should be avoided in empirically oriented macrodynamics.

In the linear case this can be motivated further by the following type of argument. We consider the economically equivalent discrete and continuous-time models[4]

$$x_{t+1} = Ax_t \quad \text{and} \quad \dot{x} = (A - I)x = Jx$$

which follow the literature by assuming an unspecified time unit 1.

Our above arguments suggest that we should generalize such an comparison and rewrite the discrete time model with a variable period length to compare it with the continuous version as follows:

$$x_{t+h} - x_t = hJx_t \quad \text{and} \quad \dot{x} = Jx.$$

This gives for their system matrices (and eigenvalues) the relationships

$$A = hJ + I, \qquad \lambda_i(A) = h\lambda_i(J) + I, i = 1, \ldots, n.$$

According to Foley's postulate both J and A should be (at least) stable matrices,[5] i.e., all eigenvalues of J should have negative real parts, while the eigenvalues of A should all be within the unit circle, if discrete and continuous time macromodels are supposed to have the same dynamic properties (as should be the case). Graphically this implies the situation shown in Fig. 1.1 (which shows that, if J's eigenvalues do not yet lie inside the unit circle shown, that they have to be moved into it by a proper choice of the time unit and thus the matrix hJ).

If the eigenvalues of the matrix J of the continuous time case are such that they lie outside the solid circle shown, but for example within a circle of radius 2, the discrete time matrix $J + I$ would—in contrast to the continuous time case—have unstable roots (on the basis of a period length $h = 1$ that generally is left implicit in such approaches). The system $x_{t+1} = Ax_t, A = J + I$ then has eigenvalues outside the unit circle (which is obtained by shifting the shown solid unit circle by 1 to the right (into the dotted one). Choosing $h = 1/2$ would however then already be sufficient to move all eigenvalues $\lambda(A) = h\lambda(J) + 1$

[4] I the identity matrix.

[5] And (even stricter) have the same number of real roots. We thank Laura Gardini, Anna Agliari, Gian-Italo Bischi, Roberto Dieci of making us aware of this additional restriction.

of $A = hJ + I$ into this unit circle, since all eigenvalues of hJ are moved by this change in period length into the solid unit circle shown in Fig. 1.1, since J's eigenvalues have all been assumed to have negative real parts and are thus moved towards the origin of the space of complex numbers when the period length h is reduced.

In view of this, we claim that sensible macro-dynamic discrete time models $x_{t+h} = (hJ+I)x_t = Ax_t$ have all to be based on a choice of the period length "h" such that at least $\|\lambda(A)\| < 1$ can be achieved (if the matrix J is stable). Since models of the real financial interaction suggest very small periods length and since the macroeconomy is updated at the least on a daily basis in reality, such a choice should always be available for the model builder. In this way it is guaranteed that linear period and continuous-time models give qualitatively the same answer, also when they are simulated numerically.[6]

As a generalizing statement and conclusion, related to Foley's (1975) observation, we would conclude that the empirical relevance of macroeconomic models specified with a uniform period length across all sectors and activities

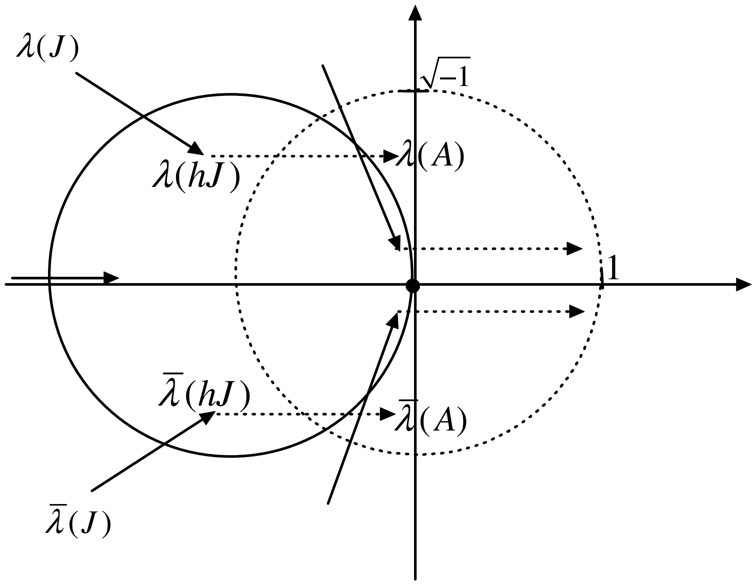

Fig. 1.1. A choice of the period length that guarantees equivalence of continuous and discrete time analysis

[6] Note again that we may even be forced to demand that the number of real roots is the same in both types of analyzes.

and with attractors whose dynamic properties differ substantially from their continuous-time analogue should be questioned.

This implies that a lot of mathematical simulations of typical macromodels should be evaluated as interesting and surely skilled mathematical exercises, but as questionable from the point of view of their empirical relevance. Period models thus in general depend on their continuous-time analogues for their results, if empirically meaningful, and thus exhibit, in terms of US migration policies, only a "J2 status" (dependent on a J1 visitor with work permission) in their macroeconomic implications.

Discrete time models (also empirically estimated ones, using annualized quarterly data in general) should be iterated approximately with step size of "1/100 year" at least as far as basic macroeconomic time series, like for example factual output levels and factual price inflation rates, are concerned and then will generally give rise to results that are equivalent to the ones of their continuous time limit. Thus, pure period models, as the one to be considered in Sect. 1.4, iterated in this manner, in general will not give rise to chaotic dynamics in dimensions 1 and 2, suggesting that the literature on such chaotic dynamics is of no empirical relevance.

Section 1.3 will reconsider the Solow growth model from the perspective of the arguments of the present section. We will find that a baseline version of the 1D Solow growth model is not subject to a discrepancy between period and continuous time analysis, however large the period length is chosen. Section 1.4 will then however provide a 2D example, in fact the monetarist baseline model of inflation and unemployment dynamics, generally interpreted to provide global convergence, that has been extended into a synchronized uniform period setup by Soliman (1996). Here, such a discrepancy comes into being in a striking way, since the continuous-time version is globally asymptotically stable for all parameter choices, while the period version exhibits routes into chaotic dynamics if certain parameters values (representing longer period lengths) are sufficiently increased. Section 1.6 concludes and provides as an outlook an example of complex dynamics derived from an applicable macromodel of dimension 4.

1.3 1D Equivalence: The Solow Model

Solow's (1956) one-good model of economic growth is based on full employment throughout, with a natural rate of labor force growth that is exogenously given. The dynamics of Solovian growth are nonlinear due to its use of a neoclassical production function. In the usual continuous time formulation it implies a monotonic one-dimensional transition towards its steady state solution for all initial values of capital-intensity. It can be varied in many ways, including differentiated saving habits, endogenous saving rates, endogenous technological change.

The Solow model of neoclassical economic growth is usually based on the following set of assumptions on the supply side of a closed macroeconomy. In the form that is presented below we still ignore capital stock depreciation and technical change for expositional reasons, see Flaschel (1993) for this and further modifications of the Solow growth model.

$$Y = F(K, L^d) \quad \text{the neoclassical production function} \tag{1.1}$$

$$S = sY, \; s = const. \quad \text{Harrod type savings function} \tag{1.2}$$

$$\dot{K} = S \text{ capital stock growth driven by household' savings decisions} \tag{1.3}$$

$$\dot{L} = nL, \; n = const. \quad \text{labor force growth} \tag{1.4}$$

$$L^d = L \quad \text{the full employment assumption} \tag{1.5}$$

$$\omega = F_L(K, L) \text{ the marginal productivity theory of employment} \tag{1.6}$$

The notation in these equations is fairly standard. We here use L^d to denote labor demand and $\omega = w/p$ to denote the real wage. Technology is described by means of a so-called neoclassical production function that exhibits constant returns to scale. There is only direct investment of savings in real capital formation in this model type, i.e., Say's Law is assumed to hold true in its most simple form:

$$I \equiv S = sY$$

with savings being strictly proportional to output and income Y. Labor is growing at a given natural rate n and is fully employed, i.e., this model simply bases economic growth on actual factor growth without any demand side restriction on the market for goods. The last of the above equations is only added to justify the full employment assumption and it does not play a role in the quantity dynamics to be considered below. These dynamics are obtained from the following reduced form representation of the above model:

$$\dot{K} = sF(K, L) \tag{1.7}$$

$$\dot{L} = nL \tag{1.8}$$

Since the state variables of these dynamics exhibit an exponential trend the model is generally only analyzed in intensive form, i.e., in terms of the variable k.[7] In intensive form the above Solow model reads:

$$\dot{k} = sF(k,1) - nk = sf(k) - nk \tag{1.9}$$

and thus gives rise to a single differential equation in the state variable k which is nonlinear due to the strict concavity of the function f.

In the form of a period model with period length h this form of the Solow model can be represented by

$$Y_{t+h} = hF(K_t, L_t) \tag{1.10}$$

$$S_{t+h} = sY_{t+h} \tag{1.11}$$

$$K_{t+h} = K_t + S_{t+h} \tag{1.12}$$

$$L_{t+h} = (1 + nh)L_t \tag{1.13}$$

We note here that the literature generally sets h equal to 1 and considers instead

$$\begin{aligned} Y_t &= F(K_t, L_t) \\ S_t &= sY_t \\ K_{t+1} &= K_t + S_t \\ L_{t+1} &= (1+n)L_t \end{aligned}$$

i.e., it assumes that output and savings occur instantaneously and that there is a uniform gestation lag in investment only (that is synchronized over the whole set of firms). This however is misleading, since production Y (a flow) grows the longer the stocks capital K (the number of machines)and L (the number of workers) are employed, i.e., output Y must vary with h. Our discrete model can be reduced to the two equations

$$K_{t+h} = K_t + shF(K_t, L_t) \tag{1.14}$$

$$L_{t+h} = (1 + nh)L_t \tag{1.15}$$

which for $h = 1$ are identical to the ones implied by the case where the role of the period h length is neglected. Using the identity $K_{t+h}/L_{t+h} =$

[7] See however Pampel (2005) for a recent discussion of problems that characterize its actual growth path $(K(t), L(t))$.

$(K_{t+h}/L_t)(L_t/L_{t+h})$, this model can again be reduced to the state variable k now given by $k_t = K_t/L_t$ and gives then rise to:

$$k_{t+h} = (k_t + shf(k_t))/(1 + nh) \tag{1.16}$$

At first sight, this law of motion of the period version of the Solow model looks quite different compared to the one in continuous time

$$\dot{k} = sF(k, 1) - nk = sf(k) - nk$$

and its discretization by way of difference quotients

$$k_{t+h} = k_t + h(sf(k_t) - nk_t) = k_t + shf(k_t) - nhk_t$$

Yet, since this last difference equation is (for small period lengths h) but an approximation to the continuous time case we have to check here whether this can also be stated with respect to $k_{t+h} = (k_t + shf(k_t))/(1 + nh)$, the law of motion of the period model. Indeed, this law of motion can be reformulated as

$$\begin{aligned}\frac{k_{t+h} - k_t}{h} &= \frac{(k_t + shf(k_t))/(1 + nh) - k_t}{h} \\ &= \frac{(k_t - (1 + nh)k_t + shf(k_t))}{(1 + nh)h} = \frac{sf(k_t) - nk_t}{1 + nh}\end{aligned}$$

For small period lengths h this expression is close to the period analogue of the intensive form continuous time case, i.e., the original extensive form period model and the original extensive form continuous time model provide nearly the same dynamics on the intensive form level for small periods h. Yet, with respect to large period lengths, we have to compare the outcome of the continuous time case with the properties of the period case directly and not via the latter approximations, which indeed depart from their original forms when the parameter h is increased.[8]

Moreover, all versions of the Solow model of this section share the same qualitative property of global monotonic convergence to the unique interior steady state of the model. This is exemplified by means of Fig. 1.2 where the mapping H of the period version of the Solow model is always strictly increasing and strictly concave and thus must cut the 45 degree line as shown in this figure if the Inada conditions are assumed to hold.[9] Note that the steady

[8] An analysis of the accuracy of solutions of nonlinear models with respect to higher order approximations can be found in Becker et al. (2007).

[9] We have to thank T. Pampel of making us aware of the fact that this indeed holds for all positive period lengths h.

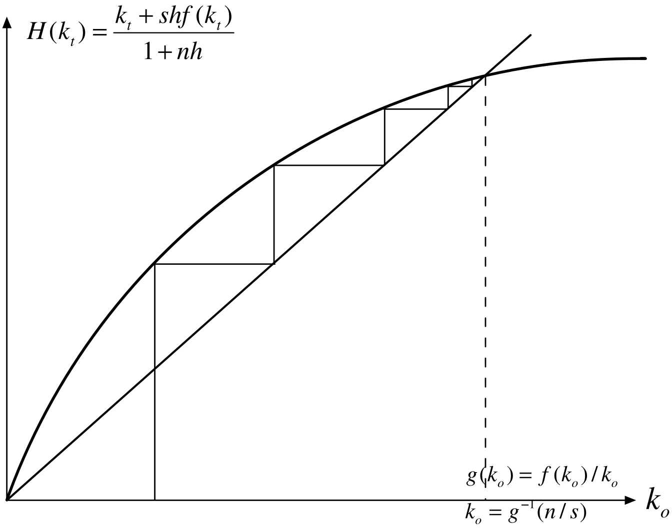

Fig. 1.2. Monotonic convergence in the h-period Solow growth model

state, to be calculated from $k_o = (k_o + shf(k_o))/(1+nh)$, is independent from the period length h.

We thus have two Solow growth model versions, using continuous time and period analysis respectively, that not only give rise to closely related reduced form dynamics, but that always share the same qualitative feature of not only convergence, but even monotonic convergence, independently of the period length that is assumed to underlie the period model. The period model may therefore assume as radical a clustering or bunching of economic activities, with huge amounts of idle time in between, but does give us the same qualitative results in a stricter sense than we demanded it to be the case in Sect. 1.2. The Solow growth model is therefore an ideal example for the fulfillment of Foley's (1975) quotation that we have given in Sect. 1.2. Nevertheless we would argue that iteration step size for this model type (with annualized capital-output ratios) should be chosen as small as one day, since in reality annualized output, investment etc. is changing every day due to the huge number of firm activities that are here aggregated.

One might however argue here that there are significant gestation lags in investment behavior in reality, between investment orders and actual production increases, and this is indeed a relevant observation. This idea was indeed already put forward in Kalecki (1935) in an important "post-Keynesian" approach that even preceded the General Theory of Keynes (1936). But this does not question our above empirical observation on nearly continuous output and investment changes, but only extends the (quasi-)continuous formulation of an empirically oriented Solow growth model towards its restatement as a delayed differential equation, a situation from which interesting results may be expected, but that is here beyond what can be and has to be treated in this chapter.[10] Such delays are of empirical relevance, but not the clustering of activities that period analysis with large period lengths is suggesting.

In closing, we briefly observe that the ideal convergence properties of the Solow growth model get lost in discrete time when there is real wage rigidity as in the Goodwin (1967) growth cycle model, to be formalized by a real wage Phillips curve $\hat{\omega} = \beta_\omega(e-1)$. We then get a synthesis of the Solow and the Goodwin model, see e.g. Flaschel (1993) for its investigation, and can recover the original Solow model as the limit case $\beta_\omega = \infty$ (if appropriately interpreted). In the continuous-time formulation we get faster and faster convergence, at first cyclical and then monotonic, as the adjustment speed β_ω of real wages is increased, i.e., the Solow model is a meaningful limit case of the Goodwin–Solow model. Yet, for period modeling—where the increase in β_ω can be related to an increase in the period length h—we get, since one eigenvalue in the continuous version is approaching $-\infty$, sooner or later instability and thus a model that must be excluded from those that are of empirical relevance.

From an empirical perspective we finally must even conclude that not all period versions of the law of motion of the Solow growth model should be used from an applied perspective, since it makes no sense to assume in this growth model that aggregate investment behavior exhibits a significant degree of clustering in time, as is assumed by period analysis with period lengths higher than a day or week. Such a statement must however be carefully distinguished from the assumption of significant gestation periods in single investment projects which—though not clustered—imply the need for using difference or differential equations with a pronounced time delay. Apart from this however, continuous or quasi-continuous time (small periods) is all that is needed from the perspective of applicable macro modeling.

Since the 2D Solow–Goodwin model (just sketched) has not yet been investigated in period form with respect to the complex dynamics it may generate under appropriate assumptions, we turn in the next section to a typical example of the literature on macroeconomic chaos where the empirically motivated

[10] See Chen (1988) for a delayed feedback model of economic growth.

postulate of the equivalence of period and continuous time analysis of Sect. 1.2 finds useful application.

1.4 2D Monetarist Baseline Analysis. Chaotic Attractors?

In a 2D period model, Soliman (1996) considers a small model of inflation dynamics of textbook type with two laws of motion, one for the expected inflation rate π_t^e and one for the rate of unemployment U_t. The model is formulated in discrete time (in a form that is directly analogous to the familiar continuous-time versions) and makes use of a uniform period length (of one year, see Soliman (1996, p. 143)) in describing the adjustment of expected inflation and unemployment based on labor market disequilibrium and a vertical LM curve (the monetarist case of IS-LM equilibrium). This latter relationship implies that real growth g_t is given (approximately) by the discrepancy between nominal money supply growth μ and the actual inflation rate π_t, to be inserted in to Okun's law as shown below. The model is a nonlinear one due to its use of a nonlinear Phillips curve in the determination of actual inflation.

The Phillips curve of this approach to inflation dynamics is indeed given by

$$\pi_t = f(U_t) + \alpha\pi_t^e, \quad 0 < \alpha \leq 1, f' < 0$$

and inflationary expectations π_t^e are adjusted adaptively according to

$$\pi_{t+1}^e = \pi_t^e + c(\pi_t - \pi_t^e), \quad 0 < c < \infty$$

The final equation of the model is given by Okun's Law in the form

$$U_{t+1} = U_t - bg_t = U_t - b(\mu - \pi_t), \quad b > 0$$

which due to its specific form assumes that the steady state value of g_t is zero.

Soliman (1996) uses this monetarist model of inflation dynamics (where the long-run Phillips curve need not be vertical) in order to explore numerically transitions from stable equilibrium points to finally chaotic attractors. Since such a result is simply impossible in continuous time, see Hirsch and Smale (1974, p. 240) for a classification of the limit sets for two-dimensional differential equation systems, the model is formulated in discrete time, using a uniform length for the period of the model, i.e., by using period analysis. In Fig. 2 of Soliman it is then shown for example that the dynamics give rise to a period doubling sequence that finally leads to chaos when the parameters μ and b, the growth rate of money supply and the Okun parameter, in the parameter space (b, μ) (jointly or separately) cross a certain critical line.

In continuous time the above model gives rise to the system of 2 differential equations:[11]

$$\dot{U} = b(\pi - \mu) = b(f(U) + \alpha\pi^e) - \mu), \quad \mu \text{ given} \tag{1.17}$$

$$\dot{\pi}^e = c(\pi - \pi^e) = c(f(U) - (1 - \alpha)\pi^e) \tag{1.18}$$

in the state variables U, π^e. The steady state of this system is given by

$$\pi^o = \mu, \qquad \pi^{eo} = \mu, \qquad U^o = f^{-1}((1 - \alpha)\pi^{eo}) \tag{1.19}$$

and is thus uniquely determined. For the Jacobian of these dynamics we get in $\Re^2$

$$J = \begin{pmatrix} bf' & b\alpha \\ cf' & -(1-\alpha)c \end{pmatrix} = \begin{pmatrix} - & + \\ - & - \text{ or } 0 \end{pmatrix}$$

if $\alpha \leq 1$ holds. According to Olech's theorem, see Flaschel (1993, Chap. 4), one obtains from this sign structure in the Jacobian J that the dynamics are globally asymptotically stable in $\Re^2$. Note however that the above system must be modified if the rate of unemployment U assumes negative values or values larger than 1.[12] We conclude that this monetarist baseline model is always asymptotically stable in the large, but may need some extra qualifications if the unemployment rate approaches its boundary conditions. Be that as it may, the eigenvalues of the considered Jacobian at the steady state always have negative real parts in the continuous time case.

[11] Note that both systems are assumed to use annualized data for reasons of comparability, i.e., yearly rates of growth which are updated in the period case the stronger, the longer the assumed period h where the discrepancy that is driving them is working.

[12] This can be done by either directly imposing the side conditions $1 \geq U \geq 0$ or by assuming an appropriate slope of the function f at $U = 0, 1$. A third possibility is to take note of a more appropriate form of Okun's law, derived from its level formulation $e = au^b$ with e, u the employment rate of labor and capital, respectively. This form is then given by

$$\hat{e} = \frac{-\dot{U}}{1 - U} = b\hat{u} \quad \text{or} \quad \dot{U} = -bg(1 - U) = -b(\mu - \pi)(1 - U)$$

This formulation avoids the situation that U can become 1, but still needs the side condition $e \leq 1$, i.e. $U \geq 0$.

As already indicated above, one should use in continuous time—as in discrete time—annualized data and variables from an empirical point of view (and for better comparison) and thus for example annual inflation rates that are in principle updated "every second" and thus can lead only to smoothly changing annualized inflation rates as time moves on. Inflation rates thus mirror a period over which they are calculated (quarters or years), but are in principle available at any "second" if the data updating process is that fast (a "day" and averages generated over the day would already normally sufficient in this respect). The variables π, π^e are therefore of the same order of magnitude, independently of the assumed period length h of the discrete-time model that is used, and they are moving nearly continuously with time under normal conditions (no hyperinflation).

In discrete time, the dynamics (1.17), (1.18) therefore read with respect to a period of length h:

$$\begin{aligned} U_{t+h} &= U_t + bh(\pi_t - \mu) \\ \pi^e_{t+h} &= \pi^e_t + ch(f(U_t) - (1-\alpha)\pi^e_t) \end{aligned}$$

These expression give rise to the formal relationship between the period model and its continuous time representation:

$$A(z_t) = hJ(z_t) + z_t, \quad z_t = (U_t, \pi^e_t)'$$

where the expressions $A(\cdot), J(\cdot)$ are viewed as nonlinear functions here. Such a translation from continuous time is necessary (and here of particularly simple type) from the applied perspective, since it is necessary to know in applied macromodels how the parameter values (and which ones) are changing if one uses for example monthly data in place of quarterly ones and wants to check the comparability of the obtained parameter estimates. In the present case, this is of the simplest type, since only the parameters b, c are changing (proportional to h) with the lengths of the considered period of the data collection process, implying larger reactions of the state variables the longer the time period that passes by until their annualized values are measured again.

Turning to local comparisons around the steady state now, we obtain from the above that the system matrix A (of partial derivatives) of the period model is again related to the Jacobian of the continuous time case as follows:

$$A = hJ + I$$

which gives for the eigenvalues $\lambda(A), \lambda(J)$ of the matrices J, A the relationship

$$\lambda(A) = h\lambda(J) + I$$

For asymptotic stability we need in the discrete time case $|\lambda(A)| < 1$, i.e., $h < \bar{h}$ where $\bar{h}$ is given by $\lambda(J) = a \pm bi, i = \sqrt{-1}$:

$$|\bar{h}\lambda(J) + 1|^2 = 1 = \bar{h}^2(a^2 + b^2) + 2a\bar{h} + 1$$

This gives (note that $a < 0$ holds in continuous time):

$$\bar{h} = \frac{2|a|}{a^2 + b^2}$$

We thus get that the discrete time case is asymptotically stable (in line with the continuous time case) if the period length of the period model satisfies $h < \bar{h}$. In an appendix we consider the bifurcation value $\bar{h}$, that separates stability from instability, in more detail and obtain in particular the result that the speed of adjustment of inflationary expectations is the most important contributor to such bifurcations (see also the numerical example below).

We provide an example that this local result should be the case for all empirically relevant parameter sizes of the model, so that there is no period doubling route and the like to chaotic attractors in this parameter range. The parameter b in Okun's Law is approximately 1/3 (Okun's rule of thumb), while the slope of the Phillips curve at the NAIRU has often been estimated as being not too far away from "one" (for annualized data). For the parameter α one generally assumes "1" (the monetarist accelerator case). We thus are left here with the adjustment speed c of inflationary expectations for which no easy estimates can be provided. Loss of local asymptotic stability occurs here solely due an adjustment of inflationary expectations that is chosen sufficiently fast. Such a loss of stability cannot occur in the continuous time case, since there is not yet a Mundell or real interest rate effect present in the Monetarist baseline model (see Sect. 1.5 for its presence in a Keynesian baseline model). Loss of stability therefore only occurs in the period version in particular due to the fact that the eigenvalues of the continuous time case become too negative in their real parts, i.e., the continuous time case becomes "too stable".

The eigenvalues of the matrix J are in this case given by ($\alpha = 1$):

$$\lambda_{1,2} = \frac{bf'}{2} \pm \sqrt{\left(\frac{bf'}{2}\right)^2 + bcf'} = 1/6 \pm \sqrt{(1/6)^2 - c/3} = a \pm bi$$

For $\bar{h}$ we then get by the above derivations ($c > 1/12$):

$$\bar{h} = \frac{\frac{2}{6}}{\frac{1}{36} + \left(\frac{c}{3} - \frac{1}{36}\right)} = \frac{1}{c}$$

For $c < 1$ we thus would get that the model could be iterated with a year as period length without a change in its stability properties (i.e., the one of its continuous time limit). But the actual macroeconomy is factually updated at least every day, i.e., the iteration step size (which is independent of the currently used timing of the data collection process) should be close to 1/365. Thus the parameter c can assume any value in the interval (1/12,365) in such a situation without loss of asymptotic stability of the daily iterated or updated rates of unemployment and inflation.

The basic problem with the numerical simulations shown in Soliman (1996) thus is that the parameter ranges that are used in the various figures are simply much too high from an empirical point of view. A macroeconomy is updated (in terms of annualized data) every day and thus moving according to ($\alpha = 1$):[13]

$$\begin{aligned} U_{t+h} &= U_t + bh\left(\mu - f(U_t) - \pi_t^e\right) \\ \pi_{t+h}^e &= \pi_t^e + chf(U_t) \end{aligned}$$

with $h = 1/100$ approximately. Soliman uses $h = 1$ and indicates by the choice of μ, etc. that this period length has to be interpreted as "1 year". Parameter values $c \in (0, 36.5), b \in (1/5, 1)$ added to her empirical Phillips curve (which has approximately -0.2 as slope at the steady state) give for the iteration parameters bh the maximum 1/365 and for ch the maximum value 1/10. This suggests with respect to her Fig. 2 for example that we are so close to the vertical axes that only the white = stable domain is relevant from the empirical point of view. The period doubling transition to chaos is thus of a purely hypothetical nature. Similar observations apply to the other simulation studies shown in Soliman (1996), as for example to her Fig. 5, where b, c are chosen much too high in order to provide an empirically relevant study of basins of attraction (which would be totally white if constrained to empirically meaningful parameter values).

We conclude again that the data generating process is to be considered to be of a much finer step size than the data collection and data processing process (to be performed such that annualized data are established on a quarterly or maybe even monthly basis) and that this leads us to a dynamical system in discrete time (even if period synchronization is assumed) that

[13] Note here that all rates in Soliman (1996) are calculated in %.

in general (for most empirically relevant macrodynamic models) is not qualitatively distinguishable in its dynamic properties from its continuous time analogue.

Such an assertion, of course, needs further investigation by means of other applied models from the macrodynamic literature. In particular with respect to expectations formation one may then argue against our conclusions, since they have to rely on the data collection process, because the data generating process becomes only visible through such an activity. Yet, even then we would expect not much difference from such a perspective, since the data collection process has meanwhile been improved very much in many areas and since there may exist means by which individuals smooth their observations. Nevertheless, it may be sensible to investigate this further, for example in differential equation systems which exhibit expectational delays.[14]

As a future research agenda we consequently propose to go beyond what has been investigated in this section and to use existing higher dimensional macromodels with small (quasi-continuous) iteration step size, still of a period model type, however updated each "day", with basic parameters broadly in line with empirical magnitudes, in order to investigate by the help of appropriate behavioral nonlinearities their possibly complex attractors, by applying mathematical studies as Guckenheimer and Holmes (1983), Wiggins (1990) and more recent work to such high order macrodynamic model building. In mathematics, the step from 2D to 3D dynamical system is a truly significant and very interesting one that should now become the focus of interest, since complex 1D and 2D macrodynamics are fairly well exploited. Nevertheless such low dimensional models may be useful for testing the role of certain bounding mechanisms in lower dimensions, but should then give rise to limit cycle behavior or at most of attractors of the type shown in Fig. 1.3 (which by and large exhaust the possibilities of attracting sets for ordinary continuous time models in dimension 2). In Sect. 1.5 we shall briefly discuss as an example a basically linear 4D continuous time model which allows for complex dynamics if a typical nonlinearity of the macrodynamic literature is added to it. Such models should become the focus of interest in the future investigation of complex dynamics, i.e., models with parameter dependent strange

[14] See Invernizzi and Medio (1991) for a detailed discussion of lags and chaos in economic dynamic models and Medio (1991b) for a discussion of continuous-time models of chaos in economics.

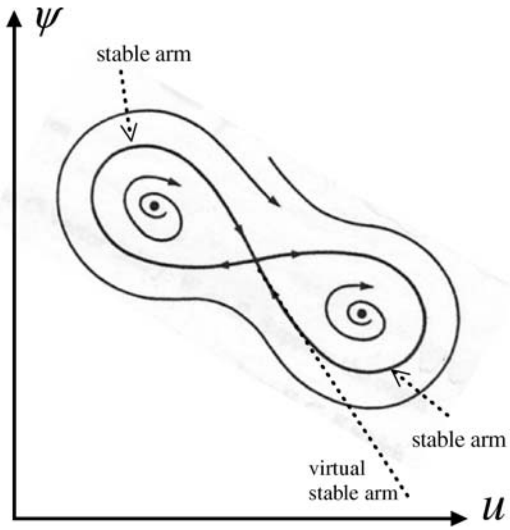

Fig. 1.3. Limit sets in continuous time planar systems that are not closed orbits

attractors that can, with this parameter range and small period lengths h, be applied to actual economies.[15]

1.5 4D Complex Keynesian Macrodynamics

In this section, we consider as an example for the emergence of complex dynamics in (quasi-)continuous time, the 4D Keynesian macro model formulated, estimated and simulated in Asada et al. (2007). The presentation of the model is slightly simplified here, in that Okun's law is assumed to be a 1:1 relationship between the employment gap on the labor market and the capacity utilization gap on the goods market, both measured as deviation from the corresponding steady state values, in place of an Okun coefficient of 1/3. It can be further simplified to a 3D system if a static interest rate policy rule is assumed (no interest rate smoothing) in place of the dynamic one shown below (or even an interest rate peg by the central bank). The four laws of

[15] In linearized rational expectations models, as they are the subject of a companion chapter Asada et al. (2007) to the present one, the virtual stable arm shown in Fig. 1.3 is in fact used in place of the true one, in order to solve these models, a fact that may lead to forward-looking reactions of economic agents that differ from those that would happen in the true model, see also Asada et al. (2003, p. 214) for details.

motion of the model are given as follows:

$$\begin{aligned}
\hat{u} &= -\alpha_{uu}(u-u_o) - \alpha_{ur}(i-\hat{p}-(i_o-\bar{\pi})) + \alpha_{u\omega}\ln(\omega/\omega_o) \\
\hat{\omega} &= \kappa\left[(1-\kappa_p)(\beta_{we}(u-u_o) - \beta_{w\omega}\ln(\omega/\omega_o))\right. \\
&\quad \left. -(1-\kappa_w)(\beta_{pu}(u-u_o) + \beta_{p\omega}\ln(\omega/\omega_o))\right] \\
\dot{\pi}^c &= \beta_{\pi^c}(\hat{p}-\pi^c) \\
\dot{i} &= -\gamma_{ii}(i-i_o) + \gamma_{ip}(\hat{p}-\bar{\pi}) + \gamma_{iu}(u-u_o)
\end{aligned}$$

with

$$\hat{p} = \kappa[\beta_{pu}(u-u_o) + \beta_{p\omega}\ln(\omega/\omega_o) + \kappa_p(\beta_{we}(u-u_o) - \beta_{w\omega}\ln(\omega/\omega_o))] + \pi^c$$

Note that the $\hat{p}$-equation has to be inserted in some of the other equations in order to arrive at an autonomous system of differential equations in the four state variables u, the rate of capacity utilization of firms, ω, the real wage, π^c, the inflationary climate and i, the nominal rate of interest. Note also that the system is close to being linear, since $\mathrm{d}\ln\omega/dt = \hat{\omega} = \dot{\omega}/\omega$ (the use of logs of real wages is derived in Blanchard and Katz (1999) for the money wage Phillips curve and here also applied to the price Philips curve and the goods market dynamics $\hat{u}$, the growth rate of capacity utilization).

Here we only briefly explain the contents of the above dynamical system and refer the reader to Asada et al. (2007) for more detailed explanations and to Asada et al. (2006) for an alternative dynamic macro approach which allows for complex dynamics in continuous time through the inclusion of typical behavioral nonlinearities far off the steady state.[16] The meaning of the above laws of motion is in fact an intuitively simple one, despite some lengthy derivation procedures concerning underlying wage and price Phillips curve in their structural form. The $\hat{u}$ equation states that the growth rate of capacity utilization depends negatively on its level (the dynamic multiplier assumption), as usual negatively on the real rate of interest and positively on the real wage (in a wage-led economy). We have reduced form price (the one in brackets) and wage Phillips curves where demand pressure in the labor and the goods market act positively (directly or indirectly) on these inflation rates, and also the inflationary climate π^c into which wage and price inflation are embedded.

[16] Note that these models also provide baseline Keynesian alternatives to the oversimplistic monetarist model we considered in Sect. 1.4 (with its trivial quantity-theory driven explanation of upper and lower turning points in economic activity and inflation rates in the continuous time case).

The difference between reduced form wage inflation and price inflation then provides the law of motion for real wages $\omega = w/p$ which then depends positively on demand pressure in the labor and negatively on demand pressure in the goods market (here both measured by the rate of capacity utilization u). The inflationary climate π^c is revised adaptively and thus follows price inflation with a certain delay. Finally, the law of motion for the nominal rate of interest is of conventional Taylor rule type. The above dynamics mirror the building blocks of New Keynesian models with staggered wages and prices, but is only built on Neoclassical model consistent expectations formation (in place of their forward looking expectations) which moreover are supplied with sufficient inertia due to their combination with the inflationary climate into which these expectations are embedded.

We know from the literature that the real rate of interest channel is destabilizing if inflationary expectations are formed sufficiently fast. Moreover, the real wage channel of the above dynamical system is unstable in a wage-led economy if the growth rate of real wages depends positively on utilization (by and large: if real wages are moving procyclically), since real wages then stimulate economic activity which in turn leads to further real wage increases and so on. The above nearly linear system is therefore likely to be governed by destabilizing forces around the steady state and thus in general purely explosive. This is the case for the base parameter set underlying the simulations shown below which is the following:

$$\beta_{pu} = 1, \quad \beta_{p\omega} = 0.4, \quad \kappa_p = 0.3,$$
$$\beta_{we} = 0.8, \quad \beta_{w\omega} = 0.4, \quad \kappa_w = 0.7, \quad \beta_{\pi^c} = 0.5,$$
$$\alpha_{uu} = 0.22, \quad \alpha_{u\omega} = 0.1, \quad \alpha_{ui} = 0.25, \quad \gamma_{ii} = 0.1, \quad \gamma_{ip} = 0.5, \quad \gamma_{iu} = 1$$

This parameter set is broadly in line with empirical observations, see Asada et al. (2007) and is here used for illustrative purposes solely. The model, when based on these parameters, is not a viable one, not even in the medium run. It is fairly well known however, see Keynes (1936) for the initial statement of this fact, that nominal wages are downwardly rigid to a certain degree. Again for illustrative purposes we assume in the following simulations that they can rise, but will not fall to a significant degree. This simple modification has significant consequences since it implies that the explosive dynamics are tamed thereby and becomes bounded or viable, since in a wage led regime we then get in depressions that real wages will start rising (due to falling prices) and thus will stimulate aggregate demand and economic activity. Yet, due to the explosive nature in the boom this will happen in an irregular way in the case of strong centrifugal forces around the steady state. We thus get the outcome that a single and very basic nonlinearity in a fully fledged Keynesian dynamical system can generate complex dynamics in 4D continuous time, a situation where bounding mechanisms are not as easy to design as in dimensions 1 or 2.

Figure 1.4 presents two bifurcation diagrams around the given set of parameter values which show the plot of local maxima and minima (in the vertical direction) plotted against one typical parameter on the horizontal axis. The first plot shows the implications of an increase in the adjustment speed of wages with respect to demand pressure on the labor market for the real wage (after a certain transient period has been passed). We see that the fluctuations in real wages become complex from 1 onwards, a value that in our empirical studies (for the USA) is however higher than the actually observed one. In theory however even infinite adjustment speeds are allowed for and indicate that not too narrowly chosen values may be of interest here. Be that as it

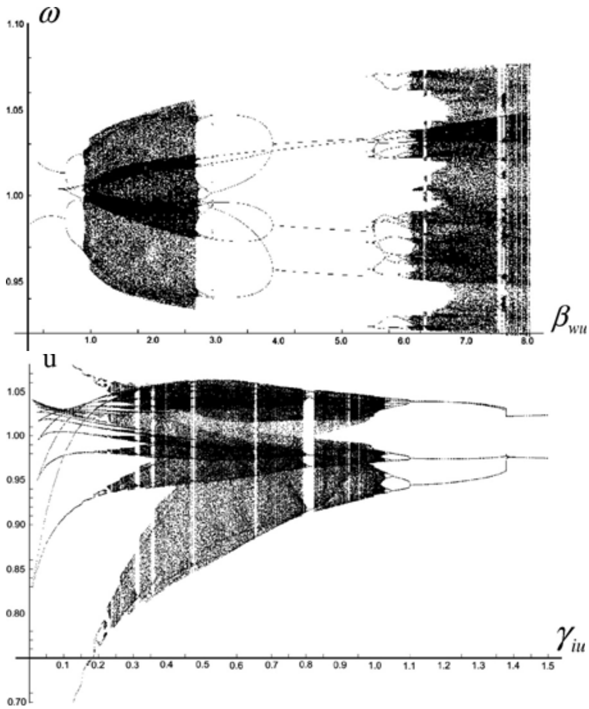

Fig. 1.4. Bifurcation diagrams for selected adjustment speeds

may, the floor in the money wage Phillips curve is indeed already sufficient to make the economy viable even up to $\beta_{we} = 8$!

In the second plot we consider the role of monetary policy and clearly see its stabilizing role as the reaction of the central bank to the capacity utilization gap is increased. It may be out of reach however for the central bank to indeed establish convergence to the steady state in the considered situation. Complex mathematical dynamics (though maybe not really complex from an economic point of view where irregularity is part of the observations that are actually made) may thus be an issue with which the central banks in the world have to deal.

1.6 Concluding Remarks

In this chapter, we have reconsidered the issue of the (non-)equivalence of discrete and continuous time macro modeling or more exact period vs. continuous time analysis. We started from the empirical fact that the actual data generating process in macroeconomics is (in the real markets) of much finer step size (daily) than the corresponding data collection process (a month/a quarter), which in turn is often much finer than the periods implicit in macromodels with chaotic attractors. This implies that empirically applicable period macromodels should be iterated at least with step size of "1/52 year" (if not even 1/365) as far as actual processes (concerning production, investment, inflation etc.) are concerned and will then in general (at least in dimensions 1 and 2) generate results that are equivalent to the ones of their continuous time analogue (assumed to exist). In our view, therefore, such empirically constrained applicable macromodels will in general not be able to give rise to chaotic dynamics in dimensions one and two, if their parameter ranges are broadly in line with empirical observations, suggesting that the literature on such chaotic dynamics is of questionable empirical relevance.

Such a statement however needs further qualification if models with dynamic expectation formation, discretionary economic policy making or gestation lags in investment are considered. In the first example, the data collection process (which is generally much cruder than the data generating process) may need extrapolating behavior of economic agents in order to allow for a smooth representation of expected magnitudes. In the second case, there is generally a smoothing process involved that transforms for example government expenditure programs from discretionary policy decisions to fairly smooth factual policy performance, due to implementation lags. The third example may be the most significant one, as already the early article by Kalecki (1935) exemplifies. This case however leads to the consideration of continuous or period models with behaviorally specific and significant realization delays (time to build), to be justified by economic reasoning, that cannot be uniformly applied to all evolving updating processes in a period model and that will also occur in an unsynchronized, non-clustered fashion across the micro-structures of the economy. Theory may assume representative firms, etc., but it must take account of the empirical fact that such agents do not act in a synchronized manner, but that unsynchronized staggered and thereby smoothed decision making must be added when going from the micro to the macro level. Such a procedure will however make macroeconomic discrete time analysis in general quasi-continuous and thus by and large equivalent to continuous time modeling if available.

However, we have to leave such considerations for future research. We have considered in this chapter only a first step towards such a discussion, by way of a mapping from ordinary differential equation systems into systems of difference equations that replaces differential quotients simply through difference quotients with a given period length or time-interval h, as in the model of Soliman (1996). Not all systems of difference equations, however, will be captured in this way (but have to be investigated then as to why they are not well-suited for small iteration steps). The conclusions of the chapter are thus for the moment only applicable to a subset of period models of the considered dimensionality. But with respect to this subset we have argued, that—from the empirical perspective—we should only allow for period lengths which give the generated period model a "quasi continuous-time" outlook. Of course, there may be exceptions to this rule for special choices of the difference and differential equation system, but we expect that macrodynamic models of conventional type will not be of such an exceptional nature.

We also conclude that applied macrodynamic period models are unlikely to give rise to chaotic dynamics in dimensions one and two and thus suggest that the extensive literature on such chaotic dynamics is of no empirical relevance, as exemplified in Sect. 1.4 by means of a monetarist baseline model. This implies as strategy for future research that one should concentrate investigations on macrodynamic models, which when represented by ordinary differential equation systems, have at least 3 state variables in order to find economically well motivated reasons for the occurrence of complex macrodynamics. But here too one needs to keep in mind that the finding of complex macrodynamics in period representations that is not present in their continuous time analogue may question such a finding from the empirical perspective.

Chiarella and Flaschel (2000) have for example argued that a fully specified balanced model of Keynesian monetary growth exhibits at the least the state variables wage share and labor intensity (the growth component), inflation and expected inflation (the medium-run component) and expected sales and actual inventories per unit of capital, i.e., meets the dimensionality condition easily. And also the New Keynesian baseline model with both staggered wage and price setting is at least of dimension 4, i.e., even still simple models of a monetary economy can be used for a bifurcation analysis as considered in Sect. 1.4 without running into the danger of synthesizing basically continuous-time ideas with radically synchronized (overshooting) discrete time adjustment processes, which when appropriately bounded can produce chaos also in dimensions one and two. This implies that all developed techniques for analyzing nonlinear dynamical systems and the experience related with them represent a useful stock of knowledge by which to investigate the strange attractors that may come about in continuous or quasi-continuous time of high order macro-systems.

As a future research agenda we therefore propose to use existing higher dimensional macromodels with small (quasi-continuous, but still of period model type) iteration step size and basic parameters broadly in line with empirical magnitudes, in order to investigate by the help of appropriate behavioral nonlinearities their possibly complex attractors, applying mathematical approaches such as in Guckenheimer and Holmes (1983), Wiggins (1990) and more recent work to such higher order macrodynamic model building. In Sect. 1.5 of this chapter we have briefly considered as an outlook an example for such a research strategy, intended to show the relevance of the occurrence of complex dynamics in the realm of applicable macromodels.

Appendix A

Indeterminacy for Large Periods h

We consider the linear approximation of the discrete time dynamics investigated in Sect. 1.3 (with $\alpha \leq 1$), which is given by:

$$A = hJ + I = \begin{pmatrix} 1 + bf'(\cdot)h & b\alpha h \\ cf'(\cdot)h & 1 - (1-\alpha)ch \end{pmatrix} \tag{A.1}$$

and want to determine the size (in its the dependence on the model's parameters) of the period length h where the considered system loses its local stability property (implying global instability unless the Phillips curve $f(\cdot)$ is made nonlinear as in Soliman 1996).

Note that $f' < 0$ in (A.1) is evaluated at the equilibrium point. The characteristic equation of this system becomes

$$\Delta(\lambda) \equiv \lambda^2 + a_1\lambda + a_2 = 0 \tag{A.2}$$

where

$$a_1 = -trace\ A = -2 + \{(1-\alpha)c - b \underset{(-)}{f'}\}h, \tag{A.3}$$

$$\begin{aligned} a_2 &= \det A = (1 + bf'h)\{1 - (1-\alpha)ch\} - bc\alpha f'h^2 \\ &= -bc \underset{(-)}{f'} h^2 + \{b \underset{(-)}{f'} - (1-\alpha)c\}h + 1 \end{aligned} \tag{A.4}$$

Then, we have

$$A_1 \equiv 1 + a_1 + a_2 = -bc \underset{(-)}{f'} h^2 > 0 \quad \text{for all } h > 0 \tag{A.5}$$

$$A_2 \equiv 1 - a_2 = b\, c\underset{(-)}{f'} h^2 + \{-b \underset{(-)}{f'} + (1-\alpha)c\}h = A_2(h) \tag{A.6}$$

$$A_3 \equiv 1 - a_1 + a_2 = -bc \underset{(-)}{f'} h^2 + 2\{b \underset{(-)}{f'} - (1-\alpha)c\}h + 4 = A_3(h) \tag{A.7}$$

The characteristic roots of (A.2) are given by

$$\lambda_1 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2}}{2}, \quad \lambda_2 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2}, \tag{A.8}$$

and the discriminant of these roots is given by

$$D \equiv a_1^2 - 4a_2 = [(1-\alpha)c\{(1-\alpha)c - 2b \underset{(-)}{f'}\} + b \underset{(-)}{f'} (b \underset{(-)}{f'} + 4c)]h^2. \tag{A.9}$$

The equilibrium point of this system of difference equations is locally stable if and only if the inequalities $|\lambda_j| < 1$ $(j = 1, 2)$ are satisfied, and it is well known that this local stability condition is equivalent to the following set of inequalities, which is called the "Cohn–Schur condition" in case of the two dimensional discrete time dynamic system (cf. Gandolfo 1996, p. 58).

$$A_j > 0 \quad (j = 1, 2, 3) \tag{A.10}$$

Proposition A.1. *The equilibrium point of this system is* (i) *locally asymptotically stable for all sufficient small values of* $h > 0$, *and* (ii) *it is unstable for all sufficiently large values of* $h > 0$ *irrespective of the values of the parameters* $b > 0,\ c > 0,\ f' < 0,$ *and* $\alpha \in (0, 1]$.

Proof.

1. We have $A_2(0) = 0$, $\frac{dA_2}{dh}\big|_{h=0} = -b \underset{(-)}{f'} + (1 - \alpha)c > 0$, and $A_3(0) = 4 > 0$ from (A.6) and (A.7), which means by continuity that we have $A_2 > 0$ and $A_3 > 0$ for all sufficiently small values of $h > 0$. On the other hand, it follows from (A.5) that the inequality $A_1 > 0$ is always satisfied. In this case, all of the local stability conditions (A.10) are satisfied.
2. It is easy to see from (A.6) that we have $A_2 < 0$ for all sufficiently large values of $h > 0$. In this case, one of the local stability conditions (A.10) is violated.

Proposition A.2.

1. *Suppose that* $4c > b\,|f'|$ *and* α *is sufficiently close to* 1 *(including the case of* $\alpha = 1$*). Then, the characteristic equation* (A.2) *has a set of complex roots irrespective of the value of* $h > 0$.
2. *Suppose that the characteristic equation* (A.2) *has a set of complex roots. Then, we have* $|\lambda| = 1$ *for* $h = \bar{h}$, $|\lambda| < 1$ *for all* $h \in (0, \bar{h})$, $|\lambda| > 1$ *for all* $h \in (\bar{h}, +\infty)$, *and* $\frac{d|\lambda|}{dh}\big|_{h=\bar{h}} > 0$, *where* $\bar{h}$ *is defined as* $\bar{h} \equiv \frac{1}{c} + \frac{1-\alpha}{b|f'|} > 0$, *and* $|\lambda|$ *is the modulus of the characteristic roots.*

Remark A.1. In the case where $\alpha = 1$ holds (the "Friedman" limit case) the bifurcation value $\bar{h}$ depends only on the parameter c that determines the strength of the adaptively revised expectations mechanism (in reciprocal form). Loss of local asymptotic stability occurs therefore the earlier, the faster inflationary expectations are adjusted. Such a loss of stability—we repeat—cannot occur in the continuous time case, since there is not yet a Mundell or real interest rate effect present in the Monetarist baseline model (see Sect. 1.4 for its presence in a Keynesian baseline model). Loss of stability therefore only occurs in the period version in particular due to the fact that the eigenvalues of the continuous case become too negative in their real parts, i.e., the continuous time case becomes "too stable".

Proof.

1. Suppose that $4c > b\,|f'|$ and $\alpha = 1$. Then, it follows from (A.9) that $D = b \underset{(-)}{f'} (-b\,|f'| + 4c)h^2 < 0$ for all $h > 0$. It is obvious from continuity that we have $D < 0$ even if $\alpha < 1$, as long as α is sufficiently close to 1 and $4c > b\,|f'|$.
2. Suppose that $D < 0$. Then, we have $a_2 > 0$ and

$$|\lambda| = \sqrt{\left(\frac{a_1}{2}\right)^2 + \left(\frac{\sqrt{-a_1^2 + 4a_2}}{2}\right)^2} = \sqrt{a_2} = \sqrt{a_2(h)}. \tag{A.11}$$

   It is easy to see from (A.5) that $a_2(\bar{h}) = 1$, $a_2(h) < 1$ for all x$h \in (0, \bar{h})$, $a_2(h) > 1$ for all $h \in (\bar{h}, +\infty)$, and $\frac{da_2}{dh}\big|_{h=\bar{h}} > 0$.

Remark A.2. Proposition A.2 (i) means that the large values of $c > 0$ and $\alpha \in (0, 1]$ are conductive to cyclical fluctuations.

Remark A.3. The critical value $\bar{h}$ in Proposition A.2 (ii) is decreasing function of the parameters c, b, $|f'|$, and α. This means that the increases of these parameter values have destabilizing effects.

Remark A.4. The point $h = \bar{h}$ in Proposition A.2 (ii) is in fact the Hopf bifurcation point of the two dimensional discrete time system if the additional technical conditions $\lambda_j^n(\bar{h}) \neq \pm 1$ $(n = 1, 2, 3, 4)$ are satisfied, where $\lambda_j(h)$ $(j = 1, 2)$ are the characteristic roots (cf. Gandolfo 1996, p. 492). In this case, there exist some non-constant closed orbits at some parameter values h that are sufficiently close to $\bar{h}$.

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Part I

The Closed Economy

2

The AS–AD Framework: Origins, Problems and Progress

2.1 Introduction

In this chapter we reformulate and extend the traditional AS–AD growth dynamics of the Neoclassical Synthesis, stage I with its traditional microfoundations, as it is for example treated in detail in Sargent (1987, Chap. 5).[1] Our extension in the first instance does not replace the LM curve with a now standard Taylor rule, as is done in the New Keynesian approaches (however this is treated in a later section of the chapter). The model exhibits sticky wages as well as sticky prices, underutilized labor as well as capital, myopic perfect foresight of current wage and price inflation rates and adaptively formed medium-run expectations concerning the investment and inflation climate in which the economy is operating. The resulting nonlinear 5D dynamics of labor and goods market disequilibrium (at first—in comparison with the old neoclassical synthesis—with a conventional LM treatment of the financial part of the economy) avoids striking anomalies of the conventional model of the Neoclassical synthesis, stage I. Instead it exhibits Keynesian feedback dynamics proper with in particular asymptotic stability of its unique interior steady state solution for low adjustment speeds of wages, prices, and expectations. The loss of stability occurs cyclically, by way of Hopf bifurcations, when these adjustment speeds are made sufficiently large, leading eventually to purely explosive dynamics.

[1] This chapter is based on Asada et al. (2006): "Keynesian Dynamics and the Wage-Price Spiral: A Baseline Disequilibrium Model". Journal of Macroeconomics, **28**, 90–130.

Locally we thus obtain and prove in detail (in the case of an interest rate policy rule in the place of the LM curve)—for a certain range of parameter values—the existence of damped or persistent fluctuations in the rates of capacity utilization of both labor and capital, and of wage and price inflation rates accompanied by interest rate fluctuations that (due to the conventional working of the Keynes-effect or later also in the case of an interest rate policy rule) move in line with the goods price level (or the inflation gap). Our modification and extension of traditional AS–AD growth dynamics, as investigated from the orthodox point of view in detail in Sargent (1987), see also Chiarella et al. (2005, Chap. 2), thus provides us with a Keynesian theory of the business cycle.[2] This is so even in the case of myopic perfect foresight, where the structure of the traditional approach dichotomizes into independent supply-side real dynamics—that cannot be influenced by monetary policy at all—and subsequently determined inflation dynamics that are purely explosive if the price level is taken as a predetermined variable. These dynamics are turned into a convergent process by an application of the jump variable technique of the rational expectations school (with unmotivated jumps in the money wage level however). In our new type of Keynesian labor and goods market dynamics we can, by contrast, treat myopic perfect foresight of both firms and wage earners without any need for the methodology of the rational expectations approach to unstable saddlepoint dynamics.

If the model loses asymptotic stability for higher adjustment speeds, it does so in a cyclical fashion, by way of so-called Hopf-bifurcations, which may give rise to persistent fluctuations around the steady state. However, this loss of stability (generated if some of the speed of adjustment parameters become sufficiently large) is only of a local nature (with respect to parameter changes), since eventually purely explosive behavior is the generally observed outcome, as is verified by means of numerical simulations. The model developed thus far cannot therefore be considered as being complete in such circumstances, since some additional mechanism is required to bound the fluctuations to economically viable regions. Downward money wage rigidity is the mechanism we use for this purpose. Extended in this way, we therefore obtain and study a

[2] Yet one, as must be stressed with respect to the results obtained in this chapter, with generally a long phase length for the implied cycles, due to the central role that is played by income distribution in the generation of the cycle and due to the lack of any fluctuations in the marginal propensity to consume, in investment efficiency and in the parameters characterizing the state of liquidity preference.

baseline model of the DAS–AD variety with a rich set of stability implications and a variety of patterns of the fluctuations that it can generate.

The dynamic outcomes of this baseline DAS–AD model can be usefully contrasted with those of the currently fashionable baseline or extended New Keynesian alternative (the Neoclassical synthesis, stage II) that in our view is more limited in scope, at least as far as interacting Keynesian feedback mechanisms and thereby implied dynamic possibilities are concerned. This comparison reveals in particular that one does not always end up with the typical (and in our view strange) dynamics of rational expectation models, due to certain types of forward looking behavior, if myopic perfect foresight is of cross-over type in the considered wage-price spiral, is based on Neoclassical dating of expectations, and is coupled with plausible backward looking behavior for the medium-run evolution of the economy. Furthermore, our dual Phillips-Curves approach to the wage-price spiral indeed also performs quite well from the empirical point of view,[3] and in particular does not give rise to the situation observed for the New (Keynesian) Phillips curve(s), found in the literature to be completely at odds with the facts.[4] In our approach, standard Keynesian feedback mechanisms are coupled with a wage-price spiral having a considerable degree of inertia, with the result that these feedback mechanisms work by and large (as is known from partial analysis) in their interaction with the added wage and price level dynamics.

In the next Sect. 2.2 we briefly reconsider the fully integrated Keynesian AS–AD model of the Neoclassical Synthesis, stage I, and show that it gives rise to an inconsistent real/nominal dichotomy under myopic perfect foresight—with appended explosive nominal dynamics, subsequently tamed by means of the jump variable technique of the "rational expectations approach". Money wage levels must then however be allowed to jump just as the price level, despite the presence of a conventional money wage Phillips curve, in order to overcome the observed nominal instability by means of the assumption of rational expectations (which indeed makes this solution procedure an inconsistent one in the chosen framework). We conclude that this model type—though still heavily used at the intermediate textbook level—is not suitable for a Keynesian approach to economic dynamics which (at least as a limit case of fast

[3] See Flaschel and Krolzig (2006), Flaschel et al. (2007) and Chen and Flaschel (2006).

[4] In this connection, see for example Mankiw (2001) and with much more emphasis Eller and Gordon (2003), whereas Galí et al. (2005) argue in favor of a hybrid form of the Phillips Curve in order to defend the New Phillips curve.

adaptive expectations) should allow for myopic perfect foresight on inflation rates without much change in its implications under normal circumstances.[5]

In Sect. 2.3 we briefly discuss the New Keynesian approach to economic dynamics on an extended level, with staggered wage and price adjustment. We find there too that it raises more questions than it helps to answer from the theoretical as well as from the empirical point of view, though it can be considered as a radical departure from the structural model of the old Neoclassical synthesis. Section 2.4 then proposes our new and nevertheless traditional (matured) approach to Keynesian dynamics, by taking note of the empirical facts that both labor and capital can be under- or overutilized, that both wages and prices adjust only gradually to such disequilibria and that there are certain climate expressions surrounding the current state of the economy which add sufficient inertia to the dynamics. This organic structural reformulation of the model of the old Neoclassical synthesis completely avoids its anomalies without representing a break with respect to the Keynesian part of the model, though the AS-curve in the narrow sense (of the old Neoclassical synthesis) is still present in the steady state of the model, but only of secondary importance in the adjustment processes surrounding this steady state.

The resulting 5D dynamical model is briefly analyzed with respect to its stability features in Sect. 2.5 and shown to give rise to local asymptotic stability when certain Keynesian feedback chains—to some extent well-known to be destabilizing from a partial perspective—are made sufficiently weak, including a real wage adjustment mechanism that is not so well established in the literature. The informal stability analysis presented there is made rigorous (for the case of an interest rate policy rule) in an appendix, where the calculation of the Routh–Hurwitz conditions for the relevant Jacobians is considered in great detail and where the occurrence of Hopf bifurcations (i.e. cyclical loss of stability) is also shown. Preparing the grounds for this appendix, Sect. 2.6 of the chapter replaces the LM curve view of financial markets in conventional AS–AD by a classic Taylor interest rate policy rule and also extends the wage and price Phillips curves of our baseline model such that they can be compared in a nearly one to one fashion with the New Keynesian approach towards staggered price as well as wage setting.

Section 2.7 then provides some numerical explorations of the model, which in particular illustrate the role of wage and price flexibility with respect to

[5] See Chiarella et al. (2005, Chap. 2) for the case of adaptive expectations formation.

their corresponding measures of demand pressure. This analysis does not always support the economic arguments based on the partial feedback structures considered in Sects. 2.4 and 2.5. In particular, although aggregate demand always depends negatively on the real wage, under certain conditions increasing wage flexibility may not lead to more stability. In such situations, downward money wage rigidity can indeed assist in stabilizing the economy and this in a way that creates economically still simple, but mathematically complex dynamics due to the "squeezed" working of the economy during the low inflation regime. Section 2.8 concludes.

2.2 Traditional AS–AD with Myopic Perfect Foresight. Classical Solutions in a Keynesian Setup?

In this section we briefly discuss the traditional AS–AD growth dynamics with prices set equal to marginal wage costs, and nominal wage inflation driven by an expectations augmented Phillips curve. Introducing myopic perfect foresight (i.e., the assumption of no errors with respect to the short-run rate of price inflation) into such a Phillips curve alters the dynamics implied by the model in a radical way, in fact towards a globally stable (neo-)classical real growth dynamics with real wage rigidity and thus fluctuating rates of under- or over-employment. Furthermore, price level dynamics no longer feed back into these real dynamics and are now unstable in the large. The mainstream approach in the literature is then to go on from myopic perfect foresight to "rational expectations" and to construct a purely forward looking solution (which incorporates the whole future of the economy) by way of the so-called jump-variable technique of Sargent and Wallace (1973). However in our view this does not represent a consistent solution to the dynamic results obtained in this model type under myopic perfect foresight, as we shall argue in this chapter.

The case of myopic perfect foresight in a dynamic AD–AS model of business fluctuations and growth has been considered in very detailed form in Sargent (1987, Chap. 5). The model of Sargent's (1987, Chap. 5) so-called Keynesian dynamics is given by a standard combination of AD based on IS-LM, and AS based on the condition that prices always equal marginal wage costs, plus finally an expectations augmented money wage Phillips Curve or WPC. The specific features that characterize this textbook treatment of AS–AD–WPC are that investment includes profitability considerations besides the real rate of interest, that a reduced form PC is not immediately employed in this dynamic analysis, and most importantly that expectations are rational (i.e., of the myopic perfect foresight variety in the deterministic context). Consumption is based on current disposable income in the traditional way, the LM curve is of standard type and there is neoclassical smooth factor substitution along with the assumption that prices are set according to the marginal productivity principle—and thus optimal from the viewpoint of the firm. These more or less standard ingredients give rise to the following set of equations that determine the statically endogenous variables: consumption (C), investment (I), government expenditure (G), output (Y), interest (i), prices (p), taxes (T), the profit rate (ρ), employment (L^d) and the rate of employment (e). These statically endogenous variables feed into the dynamically endogenous variables: the capital stock (K), labor supply (L) and the nominal wage level (w), for which laws of motion are also provided in the equations shown below. The equations are

$$C = c(Y + iB/p - \delta K - T), \tag{2.1}$$

$$I/K = i_1(\rho - (r - \pi)) + n, \quad \rho = \frac{Y - \delta K - \omega L^d}{K}, \ \omega = \frac{w}{p}, \tag{2.2}$$

$$G = gK, \quad g = \text{const.}, \tag{2.3}$$

$$Y \overset{IS}{=} C + I + \delta K + G, \tag{2.4}$$

$$M \overset{LM}{=} p\,(h_1 Y + h_2(i_o - i)W), \tag{2.5}$$

$$Y = F(K, L^d), \tag{2.6}$$

$$p \overset{AS}{=} w/F_L(K, L^d), \tag{2.7}$$

$$\hat{w} \overset{PC}{=} \beta_w(e - e_o) + \pi, \quad e = L^d/L, \tag{2.8}$$

$$\pi \overset{MPF}{=} \hat{p}, \tag{2.9}$$

$$\hat{K} = I/K, \tag{2.10}$$

$$\hat{L} = n \quad (= \hat{M} \quad \text{for analytical simplicity}). \tag{2.11}$$

We make the simplifying assumptions that all behavior is based on linear relationships in order to concentrate on the intrinsic nonlinearities of this type of AS–AD–WPC growth model. Furthermore, following Sargent (1987, Chap. 5), we assume that $t = (T - iB/p)/K$ is a given magnitude and thus, like real government expenditure per unit of capital, g, a parameter of the model. This excludes feedbacks from government bond accumulation and thus from the government budget equation on real economic activity. We thus concentrate on the working of the private sector with minimal interference from the side of fiscal policy, which is not an issue in this chapter. The model is fully backed-up by budget equations as in Sargent (1987): pure equity financing of firms, money and bond financing of the government budget deficit and money, bond and equity accumulation in the sector of private households. There is flow consistency, since the new inflow of money and bonds is always accepted by private households. Finally, Walras' Law of Stocks and the perfect substitute assumption for government bonds and equities ensure that equity price dynamics remain implicit. The LM-curve is thus the main representation of the financial part of the model, which is therefore still of a very simple type at this stage of its development.

The treatment of the resulting dynamics turns out to be not very difficult. In fact, (2.8) and (2.9) imply a real-wage dynamics of the type:

$$\hat{\omega} = \beta_w(l^d/l - e_o), \quad l^d = L^d/K, l = L/K.$$

From $\dot{K} = I = S = Y - \delta K - C - G$ and $\dot{L} = nL$ we furthermore get

$$\hat{l} = n - (y - \delta - c(y - \delta - t) - g) = n - (1-c)y - (1-c)\delta + ct - g,$$

with $y = Y/K = F(1, l^d) = f(l^d)$.

Finally, by (2.7) we obtain

$$\omega = f'(l^d), \text{ i.e.,} \quad l^d = (f')^{-1}(\omega) = h(\omega), \quad h' < 0.$$

Hence, the real dynamics of the model can be represented by the following autonomous 2D dynamical system:

$$\begin{aligned}\hat{\omega} &= \beta_w(h(\omega)/l - e_o),\\ \hat{l} &= n - (1-c)\delta - g + ct - (1-c)f(h(\omega)).\end{aligned}$$

It is easy to show, see e.g. Flaschel (1993), that this system is well-defined in the positive orthant of the phase space, has a unique interior steady–state, which moreover is globally asymptotically stable in the considered domain. In fact, this is just a Solow (1956) growth dynamics with a real-wage Phillips curve (real wage rigidity) and thus classical under- or over-employment dynamics if $e_o < 1$!. There may be a full-employment ceiling in this model type, but this is an issue of secondary importance here.

The unique interior steady state is given by

$$\begin{aligned}y_o &= \frac{1}{1-c}[(1-c)\delta + n + g - ct] = \frac{1}{1-c}[n + g - t] + \delta + t,\\ l_o^d &= f^{-1}(y_o), \quad \omega_o = f'(l_o^d), \quad l_o = l_o^d/e_o,\\ m_o &= h_1 y_o, \quad \hat{p}_o = 0, \quad r_o = \rho_o = f(l_o^d) - \delta - \omega_o l_o^d.\end{aligned}$$

Keynes' (1936) approach is almost entirely absent in this type of analysis, which seems to be Keynesian in nature (AS–AD), but which—due to the neglect of short-run errors in inflation forecasting—has become in fact of very (neo-)classical type. The marginal propensity of consume, the stabilizing element in Keynesian theory, is still present, but neither investment nor money demand plays a role in the real dynamics we have obtained from (2.1)–(2.11). Volatile investment decisions and financial markets are thus simply irrelevant for the real dynamics of this AS–AD growth model when *myopic* perfect foresight on the current rate of price inflation is assumed. What, then, remains for the role of traditional Keynesian "troublemakers", the marginal efficiency of investment and liquidity preference schedule? The answer again is, in technical terms, a very simple one:

We have for given $\omega = \omega(t) = (w/p)(t)$ as implied by the real dynamics (due to the $I = S$ assumption):

$$(1-c)f(h(\omega)) - (1-c)\delta + ct - g = i_1(f(l) - \delta - \omega h(\omega) - i + \hat{p}) + n, \quad \text{i.e.}$$

$$\begin{aligned}\hat{p} &= \frac{1}{i_1}[(1-c)f(h(\omega)) - (1-c)\delta + ct - g - n] - (f(l) - \delta - \omega h(\omega)) + i \\ &= g(\omega, l) + i,\end{aligned}$$

with an added reduced-form LM-equation of the type

$$i = (h_1 f(h(\omega)) - m)/h_2 + i_o, \quad m = \frac{M}{pK}.$$

The foregoing equations imply

$$\hat{m} = \hat{l}(\omega) - g(\omega, l) - i_o + \frac{m - h_1 f(h(\omega))}{h_2},$$

as the non-autonomous[6] differential equation for the evolution of real money balances, as the reduced form representation of the nominal dynamics.[7] Due to this feedback chain, $\hat{m}$ depends positively on the level of m and it seems as if the jump-variable technique needs to be implemented in order to tame such explosive nominal processes; see Flaschel (1993), Turnovsky (1997) and Flaschel et al. (1997) for details on this technique. Advocates of the jump-variable technique, therefore are led to conclude that investment efficiency and liquidity preference only play a role in appended purely nominal processes and

[6] Since the independent (ω, l) block will feed into the RHS as a time function.

[7] Note that we have $g(\omega, l) = -\rho_o$ in the steady state.

this solely in a stabilizing way, though with initially accelerating phases in the case of anticipated monetary and other shocks. A truly neoclassical synthesis.

By contrast, we believe that Keynesian IS-LM growth dynamics proper (demand driven growth and business fluctuations) must remain intact if (generally minor) errors in inflationary expectations are excluded from consideration in order to reduce the dimension and to simplify the analysis of the dynamical system to be considered. A correctly formulated Keynesian approach to economic dynamics and fluctuating growth should not give rise to such a strange dichotomized system with classical real and purely nominal IS-LM inflation dynamics, here in fact of the most basic jump variable type, namely

$$\hat{m} = \frac{m - h_1 y_o}{h_2} \quad \left[\hat{p} = -\frac{(M/K)_o \frac{1}{p} - h_1 y_o}{h_2}\right],$$

if it is assumed for simplicity that the real part is already at its steady state. This dynamic equation is of the same kind as the one for the Cagan monetary model and can be treated with respect to its forward-looking solution in the same way, as it is discussed in detail for example in Turnovsky (1997, Sect. 3.3/4), i.e., the nominal dynamics assumed to hold under the jump-variable hypothesis in AS–AD–WPC is then of a very well-known type.

However, the basic fact that the AS–AD–WPC model under myopic perfect foresight is not a consistently formulated one and also not consistently solved arises from its ad hoc assumption that nominal wages must here jump with the price level p ($w = \omega p$), since the real wage ω is now moving continuously in time according to the derived real wage dynamics. The level of money wages is thus now capable of adjusting instantaneously, which is in contradiction to the assumption of only sluggishly adjusting nominal wages according to the assumed money wage PC.[8] Furthermore, a properly formulated Keynesian growth dynamics should—besides allowing for un- or over-employed labor—also allow for un- or over-employment of the capital stock, at least in certain episodes. Thus the price level, like the wage level, should better and alternatively be assumed to adjust somewhat sluggishly; see also Barro (1994) in this regard. We will come back to this observation after the next section which is devoted to new developments in the area of Keynesian dynamics, the so-called New Keynesian approach of the macrodynamic literature.

[8] See Flaschel (1993) and Flaschel et al. (1997) for further investigations along these lines.

The conclusion of this section is that the Neoclassical synthesis, stage I, must be considered a failure on logical grounds and not a valid attempt "to formalize for students the relationships among the various hypotheses advanced in Milton Friedman's AEA presidential address (1968)", see Sargent (1987, p. 117).

2.3 New Keynesian AS–AD Dynamics with Staggered Wage and Price Setting

In this section we consider briefly the modern analog to the old neoclassical synthesis (with gradually adjusting money wages), the New Keynesian approach to macrodynamics, and this already in its advanced form, where both staggered price setting and staggered wage setting are assumed.[9] We here follow Woodford (2003, p. 225) in his formulation of staggered wages and prices, where their joint evolution is coupled with the usual forward-looking output dynamics and now in addition augmented by a derived law of motion for real wages. Here we shall only briefly look at this extended approach and leave to a later sections of the chapter a consideration of the similarities and differences between these New Keynesian dynamics and our own approach.

Woodford (2003, p. 225) makes use of the following two loglinear equations for describing the joint evolution of wages and prices (d the backward-oriented difference operator).[10]

$$\mathrm{d}\ln w_t \overset{NWPC}{=} \beta E_t(\mathrm{d}\ln w_{t+1}) + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln\omega_t,$$
$$\mathrm{d}\ln p_t \overset{NPPC}{=} \beta E_t(\mathrm{d}\ln p_{t+1}) + \beta_{py}\ln Y_t + \beta_{p\omega}\ln\omega_t,$$

where all parameters are assumed to be positive. Based on theories of staggered wage and price setting, output gaps act positively on current rates of wage and price inflation, while the wage gap is influencing negatively the current wage inflation rate and positively the current price inflation rate. Our first aim here is to derive the continuous time analog to these two equations (and the other equations of the full model) and to show on this basis how this extended model is solved by the methods of the rational expectations school.

[9] The baseline case of only staggered price setting is considered in Asada et al. (2006), while the extended case considered here is further discussed in Chiarella et al. (2005, Chap. 1).

[10] We make use of this convention throughout this chapter and thus have to write $r_{t-1} - dp_t$ to denote for example the real rate of interest.

In a deterministic setting we obtain from the above

$$\mathrm{d}\ln w_{t+1} \overset{WPC}{=} \frac{1}{\beta}[\mathrm{d}\ln w_t - \beta_{wy}\ln Y_t + \beta_{w\omega}\ln\omega_t],$$
$$\mathrm{d}\ln p_{t+1} \overset{PPC}{=} \frac{1}{\beta}[\mathrm{d}\ln p_t - \beta_{py}\ln Y_t - \beta_{p\omega}\ln\omega_t].$$

If we assume (as we do in all of the following and without much loss in generality) that the parameter β is not only close to one, but in fact set equal to one, then the last two equations can be expressed as

$$\mathrm{d}\ln w_{t+1} - \mathrm{d}\ln w_t \overset{WPC}{=} -\beta_{wy}\ln Y_t + \beta_{w\omega}\ln\omega_t,$$
$$\mathrm{d}\ln p_{t+1} - \mathrm{d}\ln p_t \overset{PPC}{=} -\beta_{py}\ln Y_t - \beta_{p\omega}\ln\omega_t.$$

Denoting by π^w the rate of wage inflation and by π^p the rate of price inflation (both indexed by the end of the corresponding period) we therefrom obtain the continuous time dynamics, (with $\ln Y = y$ and $\theta = \ln\omega$):

$$\dot{\pi}^w \overset{WPC}{=} -\beta_{wy}y + \beta_{w\omega}\theta,$$
$$\dot{\pi}^p \overset{PPC}{=} -\beta_{py}y - \beta_{p\omega}\theta.$$

From the output dynamics of the New Keynesian approach, namely

$$y_t = y_{t+1} - \alpha_{yr}(i_t - \pi^p_{t+1} - i_0), \quad \text{i.e.,} \quad y_{t+1} - y_t = \alpha_{yr}(i_t - \pi^p_{t+1} - i_0),$$

we moreover obtain the continuous time reduced form law of motion

$$\dot{y} \overset{IS}{=} \alpha_{yr}[(\phi_{ip} - 1)\pi^p + \phi_{iy}y]$$

where we have already inserted the interest rate policy rule shown below in order to (hopefully) obtain dynamic determinacy as in the New Keynesian baseline model, which is known to be indeterminate for the case of an interest rate peg, but determinate in the case of a suitably chosen active interest rate policy rule. For the following we choose the simple textbook Taylor interest rate policy rule:

$$i = i_{\mathrm{T}} = i_o + \phi_{ip}\,\pi + \phi_{iy}\,y,$$

see Walsh (2003, p. 247), which is of a classical Taylor rule type (though without interest rate smoothing yet).

There remains finally the law of motion for real wages to be determined, which setting $\theta = \ln\omega$ simply reads

$$\dot{\theta} = \pi^w - \pi^p.$$

We thus get from this extended New Keynesian model an autonomous linear dynamical system, in the variables π^w, π^p, y and θ. The uniquely determined steady state of the dynamics is given by $(0,0,0,0)$. From the definition of θ we see that the model exhibits four forward-looking variables, in direct generalization of the baseline New Keynesian model with only staggered price setting. Searching for a zone of determinacy of the dynamics (appropriate parameter values that make the steady state the only bounded solution to which the economy then immediately returns after isolated shocks of any type) thus requires establishing conditions under which all roots of the Jacobian have positive real parts.

The Jacobian of the 4D dynamical system under consideration reads:

$$J = \begin{pmatrix} 0 & 0 & -\beta_{wy} & \beta_{w\omega} \\ 0 & 0 & -\beta_{py} & -\beta_{p\omega} \\ 0 & \alpha_{yr}(\phi_{ip}-1) & \alpha_{yr}\phi_{iy} & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

For the determinant of this Jacobian we calculate

$$-|J| = (\beta_{wy}\beta_{p\omega} + \beta_{py}\beta_{w\omega})\alpha_{yr}(\phi_{ip}-1) \gtreqless 0 \quad \text{iff} \quad \beta_{i\pi} \gtreqless 1.$$

We thus get that an active monetary policy of the conventional type (with $\phi_{ip} > 1$) is—compared to the baseline New Keynesian model—no longer appropriate to ensure determinacy (for which a positive determinant of J is a necessary condition). One can show in addition, see Chen et al. (2005), via the minors of order 3 of the Jacobian J, that the same holds true for a passive monetary policy rule, i.e., the model in this form must be blocked out from consideration, at least from a continuous time perspective. There consequently arises the necessity to specify an extended or modified active Taylor interest rate policy rule from which one can then obtain determinacy for the resulting dynamics, where the steady state as again the only bounded solution and therefore, according to the logic of the rational expectations approach, the only realized situation in this deterministic set-up. This would then generalize the New Keynesian baseline model with only staggered prices, which is known to be indeterminate in the case of an interest rate peg or a passive monetary policy rule, but which exhibits determinacy for the above conventional Taylor rule with $\phi_{ip} > 1$.

The situations of unanticipated (and anticipated) shocks and their implications under the assumptions made by the jump variable method now have a long tradition in macrodynamics, so long in fact that economic, and not only mathematical justifications for this type of approach are no longer given, see Turnovsky (1997, part II) for an exception. Yet, authors working in the tradition of the present chapter have expressed doubts on the economic meaningfulness of the jump variable procedure on various occasions. These authors point to a variety of weaknesses in this contrived procedure to overcome the explosive forces of saddlepoint dynamics, or even purely explosive dynamics, and this in particular if such explosiveness is used to traverse smoothly to convergent solutions in the case of anticipated events, see Flaschel et al. (1997), Chiarella and Flaschel (2000) and Asada et al. (2003) in particular. We believe that in a fully specified, then necessarily nonlinear model of economic dynamics, an analysis along the lines of the jump variable technique represents not only an exceptional case with hyper-perfect foresight on the whole set of future possibilities of the economy (that in particular in the case of anticipated events cannot be learned), but that it is generally intractable from the mathematical point of view, and that in a nonlinear world is not unambiguously motivated through certain boundedness conditions.

We acknowledge that the jump variable technique of the *rational expectations* approach is a rigorous approach to forward looking behavior, too often however restricted to loglinear approximations, with potentially very demanding calculational capabilities even in nonlinear baseline situations. Our aim in this chapter is to demonstrate that acceptable situations of myopic perfect foresight can be handled in general without employing non-predetermined variables in order to place the economy on some stable manifold in a unique fashion such that (temporary) processes of accelerating instability can only occur until anticipated shocks are assumed to occur. Instead, local instability will be an integral part in the adjustment processes we consider, here however tamed not by imposing jumps on some non-predetermined variables that bypass instability, but by making use of certain nonlinearities in the behavior of economic agents when the economy departs too much from its steady state position. In sum, we therefore find with respect to the dynamic situation we have sketched above, that it may provide a rigorous way out of certain instability scenarios, one that does not fail on logical grounds as the one considered in the preceding section, but nevertheless one with a variety of questionable features of theoretical (as well as empirical) content that demand other solution procedures with respect to the local instability features of models with forward looking components.

We are fairly skeptical as to whether the New PC's really represent an improvement over conventional structural approaches with separate equations for wage and price inflation (as often used in macroeconometric model building). Further skepticism is expressed in Mankiw (2001) where the New (price) Phillips curve is characterized as being completely at odds with the facts. Eller and Gordon (2003) go even further and state that "the NKPC approach is an empirical failure by every measure". Galí et al. (2005) by contrast defend this NKPC by now basing it on real marginal costs in the place of an output gap and what they call a simple hybrid variant of the NKPC as derived from Calvo's staggered price setting framework. They find in such a framework "that forward-looking behavior is highly important; the coefficient on expected future inflation is large and highly significant." The criticism just quoted also applies to the extended wage and price dynamics considered above.

In order to overcome the questionable features of the New Keynesian approach to price and wage formation we now propose some modifications to the above presentation of the wage-price dynamics which will completely remove from it the problematic feature of a sign reversal (in their reduced-form representation) in front of output as well as wage gaps. This sign reversal is caused by the fact that future values of the considered state values are used on the right hand side of their determining equations, which implies that the time rates of change of these variables depend on output and wage gaps with a reversed sign in front on them. These sign reversals are at the root of the problem when the empirical relevance of such NPC's is investigated. We instead will make use of the following expectations augmented wage and price Phillips curves in the remainder of this chapter:

$$\begin{aligned}
\mathrm{d}\ln w_{t+1} &\overset{WPC}{=} \kappa_w \mathrm{d}\ln p_{t+1} + (1-\kappa_w)\pi_t^c + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln \omega_t,\\
\mathrm{d}\ln p_{t+1} &\overset{PPC}{=} \kappa_p \mathrm{d}\ln w_{t+1} + (1-\kappa_p)\pi_t^c + \beta_{py}\ln Y_t + \beta_{p\omega}\ln \omega_t.
\end{aligned}$$

We have modified the New Keynesian approach to wage and price dynamics here only with respect to the terms that concern expectations, in order to generate the potential for a wage-price spiral mechanism. We first assume that expectations formation is of a crossover type, with perfectly foreseen price inflation in the wage PC of workers and perfectly foreseen wage inflation in the price PC of firms. Furthermore, we make use in this regard of a neoclassical dating of expectations in the considered PC's, which means that we have the same dating for expectations and actual wage and price formation on both sides of the PC's. Finally, following Chiarella and Flaschel (1996) and later work, we assume expectations formation to be of a hybrid type, where a certain weight is given to current (perfectly foreseen) inflation rates and the counterweight attached to a concept that we have dubbed the inflationary climate π^c that is surrounding the currently evolving wage-price spiral. We thus assume that workers as well as firms to a certain degree pay attention to whether the current situation is embedded in a high inflation regime or in a low inflation one.

These relatively straightforward modifications of the New Keynesian approach to expectations formation will imply for the dynamics of what we call a matured Keynesian approach—to be started in the next section and completed in Sect. 2.7—radically different orbits and stability features, with in particular no need to single out the steady state as the only relevant situation for economic analysis in the deterministic set-up. Concerning microfoundations for the assumed wage-price spiral we here only note that the postulated wage PC can be microfounded as in Blanchard and Katz (1999), using standard labor market theories, giving rise to nearly exactly the form shown above (with the unemployment gap in the place of the logarithm of the output gap) if hybrid expectations formation is in addition embedded into their approach. Concerning the price PC a similar procedure may be applied based on desired markups of firms, see Flaschel and Krolzig (2006). Along these lines one in particular gets an economic motivation for the inclusion of—indeed the logarithm of—the real wage (or wage share) with negative sign into the wage PC and with positive sign into the price PC, without any need for loglinear approximations. We furthermore will use the (un-)employment gap and the capacity utilization gap in these two PC's, respectively, in the place of a single measure (the log of the output gap). We conclude that the above wage-price spiral is an interesting alternative to the—theoretically rarely discussed and empirically questionable—New Keynesian form of wage-price dynamics. This wage-price spiral will at first be embedded in a somewhat simplified form into a complete Keynesian approach in the next section, exhibiting a dynamic IS-equation as in Rudebusch and Svensson (1999), but now also including real wage effects and thus a role for income distribution in aggregate demand, exhibiting furthermore Okun's law as the link between goods and labor markets, and exhibiting of course (later on, see Sect. 2.6) the classical type of Taylor interest rate policy rule in the place of LM-curve employed for the time being.

2.4 Matured Keynesian AD–AS Analysis: A Baseline Model

We have already remarked that a Keynesian model of aggregate demand fluctuations should (independently of whether justification can be found in Keynes' General Theory) allow for under- (or over-)utilized labor as well as capital in order to be general enough from the descriptive point of view. As Barro (1994) for example observes IS-LM is (or should be) based on imperfectly flexible wages *and* prices and thus on the consideration of wage as well as price Phillips Curves. This is precisely what we will do in the following, augmented by the observation that medium-run aspects count both in wage and price adjustment as well as in investment behavior, here still expressed in simple terms by the introduction of the concept of an inflation as well as an investment climate. These economic climate terms are based on past observation, while we have model-consistent expectations with respect to short-run wage and price inflation. The modification of the traditional AS–AD model of Sect. 2.2 that we shall introduce now thus treats expectations in a hybrid way, myopic perfect foresight on the current rates of wage and price inflation on the one hand and an adaptive updating of economic climate expressions, with an exponential weighting scheme, on the other hand.

In light of the foregoing discussion, we assume here two Phillips Curves or PC's in the place of only one. In this way we provide wage and price dynamics separately, both based on measures of demand pressure $e - e_o, u - u_o$, in the market for labor and for goods, respectively. We denote by e the rate of employment on the labor market and by e_o the NAIRU-level of this rate, and similarly by u the rate of capacity utilization of the capital stock and u_o the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation, $\hat{w}, \hat{p}$, are both augmented by a weighted average of cost-pressure terms based on forward-looking perfectly foreseen price and wage inflation rates, respectively, and a backward looking measure of the prevailing inflationary climate, symbolized by π^c. Cost pressure perceived by workers is thus a weighted average of the currently evolving price inflation rate $\hat{p}$ and some longer-run concept of price inflation, π^c, based on past observations. Similarly, cost pressure perceived by firms is given by a weighted average of the currently evolving (perfectly foreseen) wage inflation rate $\hat{w}$ and again the measure of the inflationary climate in which the economy is operating. We thus arrive at the following two Phillips Curves for wage and price inflation, here formulated in a fairly symmetric way.

Structural form of the wage-price dynamics:

$$\hat{w} = \beta_w(e - e_o) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c,$$
$$\hat{p} = \beta_p(u - u_o) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^c.$$

Inflationary expectations over the medium run, π^c, i.e., the *inflationary climate* in which current wage and price inflation is operating, may be adaptively following the actual rate of inflation (by use of some exponential weighting scheme), may be based on a rolling sample (with hump-shaped weighting schemes), or on other possibilities for updating expectations. For simplicity of exposition we shall here make use of the conventional adaptive expectations mechanism. Besides demand pressure we thus use (as cost pressure expressions) in the two PC's weighted averages of this economic climate and the (foreseen) relevant cost pressure term for wage setting and price setting. In this way we get two PC's with very analogous building blocks, which despite their traditional outlook turn out to have interesting and novel implications. These two Phillips curves have been estimated for the US-economy in various ways in Flaschel and Krolzig (2006), Flaschel et al. (2007) and Chen and Flaschel (2006) and found to represent a significant improvement over single reduced-form price Phillips curves, with wage flexibility being greater than price flexibility with respect to demand pressure in the market for goods and for labor, respectively. Such a finding is not possible in the conventional framework of a single reduced-form Phillips curve.

Note that for our current version, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$, the law of motion of which is given by:

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_w(e - e_o) - (1 - \kappa_w)\beta_p(u - u_o)], \quad \kappa = 1/(1 - \kappa_w \kappa_p).$$

This follows easily from the obviously equivalent representation of the above two PC's:

$$\hat{w} - \pi^c = \beta_w(e - e_o) + \kappa_w(\hat{p} - \pi^c),$$
$$\hat{p} - \pi^c = \beta_p(u - u_o) + \kappa_p(\hat{w} - \pi^c),$$

by solving for the variables $\hat{w} - \pi^c$ and $\hat{p} - \pi^c$. It also implies the two cross-markets or *reduced form PC's* are given by:

$$\hat{p} = \kappa[\beta_p(u - u_o) + \kappa_p \beta_w(e - e_o)] + \pi^c, \quad (2.12)$$

$$\hat{w} = \kappa[\beta_w(e - e_o) + \kappa_w \beta_p(u - u_o)] + \pi^c, \quad (2.13)$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market. This traditional expectations-augmented PC formally resembles the above reduced form $\hat{p}$-equation if Okun's Law holds in the sense of a strict positive correlation between $u - u_o,\ u = Y/Y^p$ and $e - e_o,\ e = L^d/L$, our measures of demand pressures on the market for goods and for labor. Yet, the coefficient in front of the traditional PC would even in this situation be a mixture of all of the β's and κ's of the two originally given PC's and thus represent a synthesis of goods and labor market characteristics.

With respect to the investment climate we proceed similarly and assume that this climate is adaptively following the current risk premium $\epsilon(= \rho - (i - \hat{p}))$, the excess of the actual profit rate over the actual real rate of interest (which is perfectly foreseen). This gives[11]

$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\epsilon - \epsilon^m), \quad \epsilon = \rho + \hat{p} - i,$$

which is directly comparable to

$$\dot{\pi}^c = \beta_{\pi^c}(\pi - \pi^c), \quad \pi = \hat{p}.$$

We believe that it is very natural to assume that economic climate expressions evolve sluggishly towards their observed short-run counter-parts. It is however easily possible to introduce also forward looking components into the updating of the climate expressions, for example based on the p^* concept of central banks and related potential output calculations. The investment function of the model of this section is now given simply by $i_1(\epsilon^m)$ in the place of $i_1(\epsilon)$.

We have now covered all modifications needed to overcome the extreme conclusions of the traditional AS–AD approach under myopic perfect foresight as they were sketched in Sect. 2.2. The model simply incorporates sluggish price adjustment besides sluggish wage adjustment and makes use of certain

[11] Chiarella et al. (2003) in response to Velupillai (2003), have used a slightly different expression for the updating of the investment climate, in this regard see the introductory observation in Sect. 2.6.

delays in the cost pressure terms of its wage and price PC and in its investment function. In the Sargent (1987) approach to Keynesian dynamics the $\beta_{\epsilon^m}, \beta_{\pi^c}, \beta_p$ are all set equal to infinity and u_o set equal to one, which implies that only current inflation rate and excess profitabilities matter for the evolution of the economy and that prices are perfectly flexible, so that full capacity utilization, not only normal capacity utilization, is always achieved. This limit case has however little in common with the properties of the model of this section.

This brings us to one point that still needs definition and explanation, namely the concept of the rate of capacity utilization that we will be using in the presence of neoclassical smooth factor substitution, but Keynesian over- or under-employment of the capital stock. Actual use of productive capacity is of course defined in reference to actual output Y. As measure of potential output Y^p we associate with actual output Y the profit-maximizing output with respect to currently given wages and prices. Capacity utilization u is therefore measured relative to the profit maximizing output level and thus given by[12]

$$u = Y/Y^p \quad \text{with} \quad Y^p = F(K, L^p),\ \omega = F_L(K, L^p),$$

where Y is determined from the IS-LM equilibrium block in the usual way. We have assumed in the price PC as normal rate of capacity utilization a rate that is less than one and thus assume in general that demand pressure leads to price inflation, before potential output has been reached, in line with what is assumed in the wage PC and demand pressure on the labor market. The idea behind this assumption is that there is imperfect competition on the market for goods so that firms raise prices before profits become zero at the margin.

Sargent (1987, Chap. 5) not only assumes myopic perfect foresight ($\beta_{\pi^c} = \infty$), but also always the perfect—but empirically questionable—establishment of the condition that the price level is given by marginal wage costs ($\beta_p = \infty, u_o = 1$). This "limit case" of the dynamic AS–AD model of this section does not represent a meaningful model, in particular since its dynamic properties are not at all closely related to situations of very fast adjustment of prices and climate expressions to currently correctly observed inflation rates and excess profitability.

[12] In intensive form expressions the following gives rise to $u = y/y^p$ with $y^p = f((f')^{-1}(\omega))$ in terms of the notation we introduced in Sect. 2.2.

There is still another motivation available for the imperfect price level adjustment we are assuming. For reasons of simplicity, we here consider the case of a Cobb-Douglas production function, given by $Y = K^\alpha L^{1-\alpha}$. According to the above we have

$$p = w/F_L(K, L^p) = w/[(1-\alpha)K^\alpha (L^p)^{-\alpha}]$$

which for given wages and prices defines potential employment. Similarly, we define competitive prices as the level of prices p_c such that

$$p_c = w/F_L(K, L^d) = w/[(1-\alpha)K^\alpha (L^d)^{-\alpha}].$$

From these definitions we get the relationship

$$\frac{p}{p_c} = \frac{(1-\alpha)K^\alpha (L^d)^{-\alpha}}{(1-\alpha)K^\alpha (L^p)^{-\alpha}} = (L^p/L^d)^\alpha.$$

Due to this we obtain from the definitions of L^d, L^p and their implication $Y/Y^p = (L^d/L^p)^{1-\alpha}$ an expression that relates the above price ratio to the rate of capacity utilization as defined in this section:

$$\frac{p}{p_c} = \left(\frac{Y}{Y^p}\right)^{\frac{-\alpha}{1-\alpha}} \quad \text{or} \quad \frac{p_c}{p} = \left(\frac{Y}{Y^p}\right)^{\frac{\alpha}{1-\alpha}} = u^{\frac{\alpha}{1-\alpha}}.$$

We thus get that (for $u_o = 1$) upward adjustment of the rate of capacity utilization to full capacity utilization is positively correlated with downward adjustment of actual prices to their competitive value and vice versa. In particular in the special case $\alpha = 0.5$ we would get as reformulated price dynamics (see (2.12) with $\bar{u}$ being replaced by $(p_c/p)_o$):

$$\hat{p} = \beta_p(p_c/p - (p_c/p)_o) + \kappa_p \hat{w} + (1-\kappa_p)\pi^c,$$

which resembles the New Phillips curve of the New Keynesian approach as far as the reflection of demand pressure forces by means of real marginal wage costs are concerned. Price inflation is thus increasing when competitive prices (and thus nominal marginal wage costs) are above the actual ones and decreasing otherwise (neglecting the cost-push terms for the moment). This shows that our understanding of the rate of capacity utilization in the framework of neoclassical smooth factor substitution is related to demand pressure terms as used in New Keynesian approaches[13] and thus further motivating

[13] See also Powell and Murphy (1997) for a closely related approach, there applied to an empirical study of the Australian economy. We would like to stress here

its adoption. Actual prices will fall if they are above marginal wage costs to a sufficient degree. However, our approach suggests that actual prices start rising before marginal wage costs are in fact established, i.e. in particular, we have that actual prices are always higher than the competitive ones in the steady state.

We note that the steady state of the now considered Keynesian dynamics is the same as the one of the dynamics of Sect. 2.2 (with $\epsilon_o^m = 0$, $y_o^p = y_o/u_o$, $l_o^p = f^{-1}(y_o^p)$ in addition). Furthermore, the dynamical equations considered above have of course to be augmented by the ones that have remained unchanged by the modifications just considered. The intensive form of all resulting static and dynamic equations is presented below, from which we then start the stability analysis of the baseline model of the next section. The modifications of the AS–AD model of Sect. 2.2 proposed in the present section imply that it no longer dichotomizes and there is no need here to apply the poorly motivated jump-variable technique. Instead, the steady state of the dynamics is locally asymptotically stable under conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability by way of cycles (by way of so-called Hopf-bifurcations) and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high.

We no longer have state variables in the model that can be considered as being not predetermined, but in fact can reduce the dynamics to an autonomous system in the five predetermined state variables: the real wage, real balances per unit of capital, full employment labor intensity, and the expressions for the inflation and the investment climate. When the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior, assumed to come into affect far off the steady state, that bound the dynamics to an economically meaningful domain in the 5D state space. Chen et al. (2005) provide details of such an approach and its numerical investigation.

Summing up we can state that we have arrived at a model type that is much more complex, but also much more convincing, that the labor market dynamics of the traditional AS–AD dynamics of the Neoclassical synthesis, stage I. We now have five in the place of only three laws of motion, which incorporate myopic perfect foresight without any significant impact on the resulting Keynesian dynamics. We can handle factor utilization problems both for labor

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that this property of our model represents an important further similarity with the New Keynesian approach, yet here in a form that gives substitution (with moderate elasticity of substitution) no major role to play in the overall dynamics.

and capital without necessarily assuming a fixed proportions technology, i.e., we can treat AS–AD growth with neoclassical smooth factor substitution. We have sluggish wage as well as price adjustment processes with cost pressure terms that are both forward and backward looking, and that allow for the distinction between temporary and permanent inflationary shocks. We have a unique interior steady state solution of (one must stress) supply side type, generally surrounded by business fluctuations of Keynesian short-run as well as medium-run type. Our DAS–AD growth dynamics therefore exhibits a variety of features that are much more in line with a Keynesian understanding of the features of the trade cycle than is the case for the conventional modelling of AS–AD growth dynamics.

Taken together the model of this section consists of the following five laws of motion for real wages, real balances, the investment climate, labor intensity and the inflationary climate:

$$\hat{\omega} = \kappa[(1-\kappa_p)\beta_w(l^d/l - e_o) - (1-\kappa_w)\beta_p(y/y^p - u_o)], \tag{2.14}$$

$$\hat{m} = -\hat{p} - i_1\epsilon^m, \tag{2.15}$$

$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\rho + \hat{p} - i - \epsilon^m), \tag{2.16}$$

$$\hat{l} = -i_1\epsilon^m, \tag{2.17}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c), \tag{2.18}$$

with $\hat{p} = \kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)] + \pi^c$.

We here already employ reduced-form expressions throughout and consider the dynamics of the real wage, ω, real balances per unit of capital, m, the investment climate ϵ^m, labor intensity, l, and the inflationary climate, π^c on the basis of the simplifying assumptions that natural growth n determines also the trend growth term in the investment function as well as money supply growth. The above dynamical system is to be supplemented by the following static relationships for output, potential output and employment (all per unit of capital) and the rate of interest and the rate of profit:

$$y = \frac{1}{1-c}[i_1\epsilon^m + n + g - t] + \delta + t, \tag{2.19}$$

$$y^p = f((f')^{-1}(\omega)), \tag{2.20}$$

$$F(1, L^p/K) = f(l^p) = y^p, F_L(1, L^p/K) = f'(l^p) = \omega,$$

$$l^d = f^{-1}(y), \tag{2.21}$$

$$i = i_o + (h_1 y - m)/h_2, \tag{2.22}$$

$$\rho = y - \delta - \omega l^d, \tag{2.23}$$

which have to be inserted into the right-hand sides in order to obtain an autonomous system of 5 differential equations that is nonlinear in a natural or intrinsic way. We note however that there are many items that reappear in various equations, or are similar to each other, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. This dynamical system will be investigated in the next section in somewhat informal terms and, with slight modifications, in a rigorous way in the Appendix A to this chapter.

As the model is now formulated it exhibits—besides the well-known real rate of interest channel (giving rise to destabilizing Mundell-effects that are traditionally tamed by the application of the jump variable technique—another real feedback channel, see Fig. 2.1, which we have called the Rose real wage effect (based on the work of Rose (1967)) in Chiarella and Flaschel

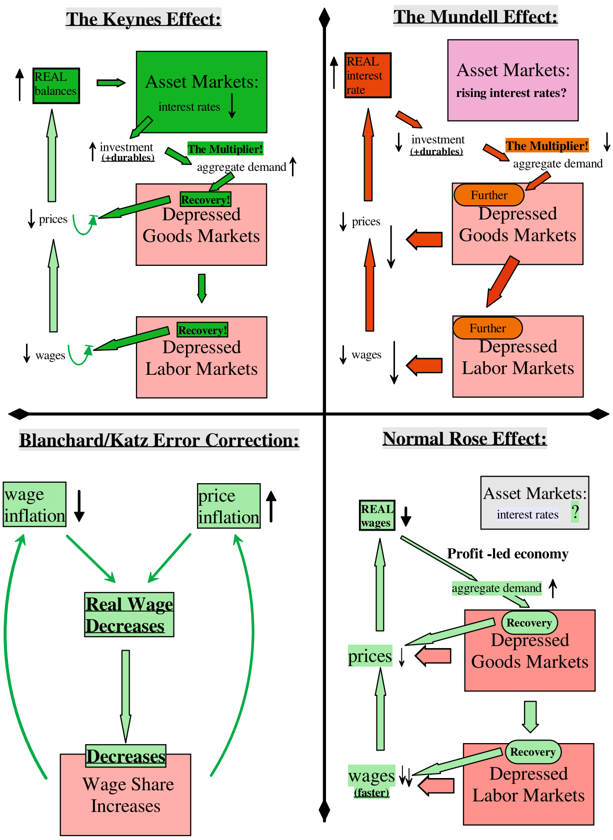

Fig. 2.1. The feedback channels of matured Keynesian macrodynamics

(2000). This channel is completely absent from the considered New Keynesian approach and it is in a weak form present in the model of the Neoclassical synthesis, stage I, due to the inclusion of the rate of profit into the considered investment function. The Rose effect only gives rise to a clearly distinguishable and significant feedback channel, however, if wage and price flexibilities are both finite and if aggregate demand depends on the income distribution between wages and profits. In the traditional AS–AD model of Sect. 2.2 it only gives rise to a directly stabilizing dependence of the growth rate of real wages on their level, while in our mature form of this AS–AD analysis it works through the interaction of the law of motion (2.14) for real wages, the investment climate and the IS-curve we have derived on this basis. The real marginal costs effect of the New Keynesian approach is here present in addition, in the denominator of the expression we are using for rate of capacity utilization, $(u = y/y^p)$ and contributes to some extent to stability should the Rose effect by itself be destabilizing.

We thus have now two feedback channels interacting in our extended DAS–AD dynamics which in specific ways exhibit stabilizing as well as destabilizing features (Keynes vs. Mundell effects and normal vs. adverse Rose effects). A variety of further feedback channels of Keynesian macrodynamics are investigated in Chiarella et al. (2000). The careful analysis of these channels and the partial insights that can be related with them form the basis of the 5D stability analysis of the next section and the appendix to this chapter. Such an analysis differs radically from the always convergent jump-variable analysis of the rational expectations school in models of the Neoclassical synthesis, stage I and stage II and many other approaches to macrodynamics.

In Fig. 2.1 we summarize the basic feedback channels of our approach to DAS–AD dynamics. We have the textbook Keynes-effect or stabilizing nominal rate of interest rate channel top-left and the therewith interacting destabilizing Mundell- or inflationary expectations effect which together with the Keynes-effect works through the (expected) real rate of interest channel. In addition we have Rose (1967)-effects working though the real wage channel. Figure 2.1 indicates that the real wage channel will be stabilizing when investment reacts more strongly than consumption to real wage changes (which is the case in our model type, since here consumption does not depend at all on the real wage) if this is coupled with wages being more flexible than prices, in the sense that (2.14) then establishes a positive link between economic activity and induced real wage changes. However if this latter relationship becomes a negative one, due to a sufficient degree of price level flexibility, this will destabilize the economy, since shrinking economic activity due to real wage increases will then indeed induce further real wage increases, due to a price level that is falling faster than the wage level in this state of depressed markets for goods and for labor (representing an adverse type of Rose-effect). We stress here that the degree of forward looking behavior in both the wage and the price level dynamics is also important, since these weights also enter the crucial equation (2.14) describing the dynamics of real wages for any changing states of economic activity. Figure 2.1 finally also shows the Blanchard and Katz wage share correction mechanism (bottom left) which will be added to the considered dynamics in Sect. 2.6.

2.5 Feedback-Guided Local Stability Investigation

In this section, we illustrate an important method used to prove local asymptotic stability of the interior steady state of the considered dynamical system, through partial motivations from the feedback chains that characterize our baseline model of Keynesian macrodynamics. Since the model is an extension of the traditional AS–AD growth model we know that there is a real rate of interest effect involved, first analyzed by formal methods in Tobin (1975), see also Groth (1992). There is therefore the stabilizing Keynes-effect based on activity-reducing nominal interest rate increases following price level increases, which provides a check to further price increases. Secondly, if the expected real rate of interest is driving investment and consumption decisions (increases leading to decreased aggregate demand), there is the stimulating (partial) effect of increases in the expected rate of inflation that may lead to further inflation and further increases in expected inflation under appropriate conditions. This is the Mundell-effect that works opposite to the Keynes-effect, but also through the real rate of interest channel as just seen; we refer the reader again to Fig. 2.1.

The Keynes-effect is the stronger the smaller the parameter h_2 characterizing the interest rate sensitivity of money demand becomes, since the reduced-form LM equation reads:

$$i = i_o + (h_1 y - m)/h_2, \quad y = Y/K, \ m = M/(pK).$$

The Mundell-effect is the stronger the faster the inflationary climate adjusts to the present level of price inflation, since we have

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c) = \beta_{\pi^c}\kappa[\beta_p(u - u_o) + \kappa_p\beta_w(e - e_o)],$$

and since both rates of capacity utilization depend positively on the investment climate ϵ^m which in turn (see (2.16)) is driven by excess profitability $\epsilon = \rho + \hat{p} - i$. Excess profitability in turn depends positively on the inflation rate and thus on the inflationary climate as the reduced-form price Phillips curve shows.

There is—as we already know—a further potentially (at least partially) destabilizing feedback mechanism as the model is formulated. Excess profitability depends positively on the rate of return on capital ρ and thus negatively on the real wage ω. We thus get—since consumption does not yet depend on the real wage—that real wage increases depress economic activity (though with the delay that is caused by our concept of an investment climate transmitting excess profitability to investment behavior). From our reduced-form real wage dynamics

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_w(e - e_o) - (1 - \kappa_w)\beta_p(u - u_o)],$$

we thus obtain that price flexibility should be bad for economic stability due to the minus sign in front of the parameter β_p while the opposite should hold true for the parameter that characterizes wage flexibility. This is a situation already investigated in Rose (1967). It gives the reason for our statement that wage flexibility gives rise to normal, and price flexibility to adverse, Rose effects as far as real wage adjustments are concerned. Besides real rate of interest effect, establishing opposing Keynes- and Mundell-effects, we thus have also another real adjustment process in the considered model where now wage and price flexibility are in opposition to each other, see Chiarella and Flaschel (2000) and Chiarella et al. (2000) for further discussion of these as well as other feedback mechanisms in Keynesian growth dynamics.

There is still another adjustment speed parameter in the model, the one (β_{ϵ^m}) that determines how fast the investment climate is updated in the light of current excess profitability. This parameter will play no decisive role in the stability investigations that follow, but will become important in the more detailed and rigorous stability analysis to be considered in the appendix to the chapter. In the present stability analysis we will however focus on the role played by h_2, β_w, β_p, β_{π^c} in order to provide one example of asymptotic stability of the interior steady state position by appropriate choices of these parameter values, basically in line with the above feedback channels of partial Keynesian macrodynamics.

The above adds to the understanding of the dynamical system (2.14)–(2.18) whose stability properties are now briefly investigated by means of varying adjustment speed parameters. With the feedback scenarios considered above in mind, we first observe that the inflationary climate can be frozen at its steady state value, here $\pi_o^c = \hat{M} - n = 0$, if $\beta_{\pi^c} = 0$ is assumed. The system thereby becomes 4D and it can indeed be further reduced to 3D if in addition $\beta_w = 0$ is assumed, since this decouples the l-dynamics from the remaining dynamical system with state variables ω, m, ϵ^m.

We intentionally will consider the stability of these 3D subdynamics—and its subsequent extensions—in very informal terms here, leaving rigorous calculations of stability criteria to the appendix (there however for the case of an interest rate policy rule in the place of our standard LM-curve). In this way we hope to demonstrate to the reader how one can proceed in a systematic way from low to high dimensional analysis in such stability investigations. This method has been already applied to various other often much more complicated dynamical systems, see Asada et al. (2003) for a variety of typical examples.

Proposition 2.1. *Assume that $\beta_{\pi^c} = 0$, $\beta_w = 0$ holds. Assume furthermore that the parameters h_2, β_p are chosen sufficiently small and that the κ_w, κ_p parameters do not equal* 1*. Then: The interior steady state of the reduced* 3D *dynamical system*

$$\begin{aligned}
\hat{\omega} &= -\kappa(1-\kappa_w)\beta_p(y/y^p(\omega) - u_o), \\
\hat{m} &= -i_1\epsilon^m - \kappa\beta_p(y/y^p(\omega) - u_o), \\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\rho + \kappa\beta_p(y/y^p(\omega) - u_o) - i - \epsilon^m),
\end{aligned}$$

is locally asymptotically stable.

Sketch of proof. The assumptions made imply that the Mundell-effect is absent from the reduced dynamics, since inflationary expectations are kept constant, and that the destabilizing component of the Rose-effect is weak. Due to the further assumption of a strong Keynes-effect, the steady state of the system is thus surrounded by centripetal forces,

Proposition 2.2. *Assume in addition that the parameter β_w is now positive and chosen sufficiently small. Then: The interior steady state of the implied* 4D *dynamical system (where the law of motion for l has now been incorporated)*

$$\hat{\omega} = \kappa[(1-\kappa_p)\beta_w(l^d/l - e_o) - (1-\kappa_w)\beta_p(y/y^p - u_o)],$$
$$\hat{m} = -i_1\epsilon^m - \kappa[\beta_p(y/y^p - u_o) + \kappa_p\beta_w(l^d/l - e_o)],$$
$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\rho + \kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)], -i - \epsilon^m),$$
$$\hat{l} = -i_1\epsilon^m,$$

is locally asymptotically stable.

Sketch of proof. In the considered situation we do no apply the Routh–Hurwitz conditions to 4D dynamical systems, as in the appendix to this chapter, but instead proceed by simple continuity arguments. Eigenvalues are continuous functions of the parameters of the model. It therefore suffices to show that the determinant of the Jacobian matrix of the 4D dynamics that is generated when the parameter β_w is made positive is positive in sign. The zero eigenvalue of the case $\beta_w = 0$ must then become positive and the three other eigenvalues continue to exhibit negative real parts if the parameter β_w is changing by a small amount solely. We conjecture—in view what is shown in the appendix in the case of an interest rate policy rule—that this proposition holds for all changes of the parameter β_w.

Proposition 2.3. *Assume in addition that the parameters β_{π^c} is now positive and chosen sufficiently small. Then: The interior steady state of the full* 5D *dynamical system (where the differential equation for π^c is now included)*

$$\hat{\omega} = \kappa[(1-\kappa_p)\beta_w(l^d/l - e_o) - (1-\kappa_w)\beta_p(y/y^p - u_o)],$$
$$\hat{m} = -\pi^c - i_1\epsilon^m - \kappa[\beta_p(y/y^p - u_o) + \kappa_p\beta_w(l^d/l - e_o)],$$
$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\rho + \kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)] + \pi^c - i - \epsilon^m),$$
$$\hat{l} = -i_1\epsilon^m,$$
$$\dot{\pi}^c = \beta_{\pi^c}(\kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)]),$$

is locally asymptotically stable.

Sketch of proof. As for Proposition 2.2, now simply making use of the rows corresponding to the laws of motion for l and m in order to reduce the row corresponding to the law of motion for π^c to the form $(0,0,0,0,-)$, again without change in the sign of the determinants of the accompanying Jacobians, allows to show here that the determinant of the full 5D dynamics is always negative. The fifth eigenvalue must therefore change from zero to a negative value if the parameter β_π is made slightly positive (but not too large), while the remaining real parts of eigenvalues do not experience a change in sign.

A weak Mundell-effect does consequently not disturb the proven asymptotic stability.

We stress again that the parameters β_p and β_{π^c} have been chosen such that adverse Rose and destabilizing Mundell-effects are both weak and accompanied by a strongly stabilizing Keynes-effect.

We formulate as a corollary to Proposition 2.3 that, due to the always negative sign of the just considered 5D determinant, loss of stability can only occur by way of Hopf-bifurcations, i.e., through the generation of cycles in the real-nominal interactions of the model.

Corollary. *Assume an asymptotically stable steady state on the basis of Proposition 2.3. Then: The interior steady state of the full 5D dynamical system will lose its stability (generally) by way of a sub- or supercritical Hopf-bifurcation if the parameters β_p or β_{π^c} are chosen sufficiently large.*

Since the model is in a natural way a non-linear one, we know from the Hopf-bifurcation theorem[14] that usually loss of stability will occur through the death of an unstable limit cycle (the subcritical case) or the birth of a stable one (the supercritical case), when destabilizing parameters pass through their bifurcation values. Such loss of stability is here possible if prices become sufficiently flexible compared to wage flexibility, leading to an adverse type of real wage adjustment, or if the inflationary climate expression is updated sufficiently fast, i.e., if the system loses the inflationary inertia—we have built into it—to a sufficient degree. These are typical feedback structures of a properly formulated Keynesian dynamics that may give rise to global instability, directly in the case of subcritical Hopf-bifurcations and sooner or later in the case of supercritical bifurcations, and thus give rise to the need to add further extrinsic behavioral nonlinearities to the model in order to bound the generated business fluctuations. Such issues will be briefly explored in the following section. They are further investigated in the next chapter of the book.

We conclude from this section that a properly specified Keynesian disequilibrium dynamics—with labor and capital both over- or underutilized in the course of the generated business fluctuations—integrates important feedback channels based on partial perspectives into a consistent whole, where all behavioral and budget restrictions fully specified. We can have damped oscillations, persistent fluctuations or even explosive oscillations in such a framework. The latter necessitate the introduction of certain behavioral non-

[14] See the mathematical appendix in Asada et al. (2003) for details.

linearities in order to allow for viable business fluctuations. However, a variety of well-known stabilizing or destabilizing feedback channels of Keynesian macrodynamics are still excluded from the present stage of the modelling of Keynesian macrodynamics, such as wealth effects in consumption or Fisher debt effects in investment behavior, all of which define the agenda for future extensions of this model type.[15]

2.6 Wage Share Error Corrections and Interest Rate Policy Rules

We have considered in Sect. 2.3 the New Keynesian approach to wage and price dynamics and have compared this approach already briefly with the two Phillips curve wage-price spiral of this chapter there (without use of real wage gaps in this baseline DAS–AD model). We recapitulate this extended wage-price spiral here briefly and include thereby Blanchard and Katz (1999) type error correction terms into our baseline DAS–AD dynamics, together with a Taylor interest rate policy rule now in the place of the LM-curve representation of the financial markets of Sect. 2.4, in order to fully show how our matured Keynesian AS–AD dynamics is differentiated from the New Keynesian approach when both approaches make use of two Phillips curves and an interest rate policy rule. In the New Keynesian model of wage and price dynamics we had:

$$\mathrm{d}\ln w_t \overset{NWPC}{=} E_t(\mathrm{d}\ln w_{t+1}) + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln\omega_t,$$
$$\mathrm{d}\ln p_t \overset{NPPC}{=} E_t(\mathrm{d}\ln p_{t+1}) + \beta_{py}\ln Y_t + \beta_{p\omega}\ln\omega_t.$$

Current wage and price inflation depend on expected future wage and price inflation, respectively, and in the usual way on output gaps, augmented by a negative (positive) dependence on the real wage gap in the case of the wage (price) Phillips curve. Assuming again a deterministic framework and myopic perfect foresight allows to suppress the expectations operator.

In order to get from these two laws of motion the corresponding Phillips curves of our matured, but conventional DAS–AD dynamics, we use Neoclassical dating of expectations in a crossover fashion, i.e., perfectly foreseen wage inflation in the price Phillips curve and perfectly foreseen price inflation in the wage Phillips curve, now coupled with hybrid expectations formation as

[15] See Chiarella et al. (2000) for a survey on such feedback channels.

in the DAS–AD model of the preceding sections. We furthermore replace the output gap in the NWPC by the employment rate gap and by the capacity utilization gap in the NPPC as in the matured Keynesian macrodynamics introduced in Sect. 2.4. Finally, we now also use real wage gaps in the MWPC and the MPPC, here based on microfoundations of Blanchard and Katz type, as in the paper by Flaschel and Krolzig (2006). In this way we arrive at the following general form of our M(atured)WPC and M(atured)PPC, formally discriminated from the New Keynesian case of both staggered wage and price setting solely by a different treatment of wage and price inflation expectations.

$$\begin{aligned} \mathrm{d}\ln w_{t+1} &\overset{\text{MWPC}}{=} \kappa_w \mathrm{d}\ln p_{t+1} + (1-\kappa_w)\pi_t^c + \beta_{we}(e_t - e_o) - \beta_{w\omega}\ln(\omega_t/\omega_o), \\ \mathrm{d}\ln p_{t+1} &\overset{\text{MPPC}}{=} \kappa_p \mathrm{d}\ln w_{t+1} + (1-\kappa_p)\pi_t^c + \beta_{pu}(u_t - u_o) + \beta_{p\omega}\ln(\omega_t/\omega_o). \end{aligned}$$

In continuous time these wage and price dynamics read

$$\begin{aligned} \hat{w} &= \kappa_w \hat{p} + (1-\kappa_w)\pi^c + \beta_{we}(e - e_o) - \beta_{w\omega}\ln(\omega/\omega_o), \\ \hat{p} &= \kappa_p \hat{w} + (1-\kappa_p)\pi^c + \beta_{pu}(u - u_o) + \beta_{p\omega}\ln(\omega/\omega_o). \end{aligned}$$

Reformulated as reduced-form expressions, these equations give rise to the following linear system of differential equations ($\theta = \ln\omega$):

$$\begin{aligned} \hat{w} &= \kappa[\beta_{we}(e-e_o) - \beta_{w\omega}(\theta-\theta_o) + \kappa_w(\beta_{pu}(u-u_o) + \beta_{p\omega}(\theta-\theta_o))] + \pi^c, \\ \hat{p} &= \kappa[\beta_{pu}(u-u_o) + \beta_{p\omega}(\theta-\theta_o) + \kappa_p(\beta_{we}(e-e_o) - \beta_{w\omega}(\theta-\theta_o))] + \pi^c, \\ \dot{\theta} &= \kappa[(1-\kappa_p)(\beta_{we}(e-e_o) - \beta_{w\omega}(\theta-\theta_o)), \\ &\quad - (1-\kappa_w)(\beta_{pu}(u-u_o) + \beta_{w\omega}(\theta-\theta_o))]. \end{aligned}$$

As monetary policy we now in addition employ a Taylor interest rate rule, in the place of an LM-curve, given by:

$$i_{\mathrm{T}} = (i_o - \bar{\pi}) + \hat{p} + \phi_{ip}(\hat{p} - \bar{\pi}) + \phi_{iu}(u - u_o), \tag{2.24}$$

$$\dot{i} = \alpha_{ii}(i_{\mathrm{T}} - i). \tag{2.25}$$

These equation describe the interest rate target i_{T} and the interest rate smoothing dynamics chosen by the central bank. The target rate of the central bank i_{T} is here made dependent on the steady state real rate of interest, augmented by actual inflation towards to a specific nominal rate of interest, and is as usually dependent on the inflation gap with respect to the target inflation rate $\bar{\pi}$ and the capacity utilization gap (our measure of the output gap). With respect to this interest rate target, there is then interest rate smoothing with strength α_{ii}. Inserting i_T and rearranging terms we get from this latter expression the following form of a Taylor rule

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o),$$

where we have $\gamma_{ii} = \alpha_{ii}$, $\gamma_{ip} = \alpha_{ii}(1 + \phi_{ip})$, i.e., $\phi_{ip} = \gamma_{ip}/\alpha_{ii} - 1$ and $\gamma_{iu} = \alpha_{ii}\phi_{iu}$.

Since the interest rate is temporarily fixed by the central bank, we must have an endogenous money supply now and get that the law of motion of the original model

$$\hat{m} = -\hat{p} - i_1\epsilon^m,$$

does now no longer feed back into the rest of the dynamics.

Taken together the revised AS–AD model of this section consists of the following five laws of motion for the log of real wages, the nominal rate of interest, the investment climate, labor intensity and the inflationary climate:

$$\begin{aligned}
\dot{\theta} &= \frac{1}{1 - \kappa_w\kappa_p}[(1 - \kappa_p)(\beta_{we}(e - e_o) - \beta_{w\omega}(\theta - \theta_o)), \\
&\quad -(1 - \kappa_w)(\beta_{pu}(u - u_o) + \beta_{w\omega}(\theta - \theta_o))], \\
\dot{i} &= -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o), \\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\epsilon - \epsilon^m), \\
\hat{l} &= -i_1\epsilon^m, \\
\dot{\pi}^c &= \beta_{\pi^c}(\hat{p} - \pi^c),
\end{aligned}$$

with $\hat{p} = \kappa[\beta_{pu}(u - u_o) + \beta_{p\omega}(\theta - \theta_o) + \kappa_p(\beta_{we}(e - e_o) - \beta_{w\omega}(\theta - \theta_o))]$.

This dynamical system is to be supplemented by the following static relationships for output, potential output and employment (all per unit of capital), the rate of interest and the rate of profit:

$$\begin{aligned}
y &= \frac{1}{1 - c}[i_1\epsilon^m + n + g - t] + \delta + t, \\
y^p &= f((f')^{-1}(\exp\theta)), \\
&\quad F(1, L^p/K) = f(l^p) = y^p, F_L(1, L^p/K)) = f'(l^p) = \omega, \\
l^d &= f^{-1}(y), \\
u &= y/y^p, \quad e = l^d/l, \\
\rho &= y - \delta - \omega l^d, \quad \epsilon = \rho - (i - \hat{p}), \\
i_o &= \rho_o + \bar{\pi},
\end{aligned}$$

which have to be inserted into the right-hand sides of the dynamics in order to obtain an autonomous system of five differential equations that is nonlinear in a natural or intrinsic way.

The interior steady state solution of the above dynamics is given by:

$$y_o = \frac{1}{1-c}[n+g-t]+\delta+t,\ l_o^d = f^{-1}(y_o),\ l_o = l_o^d/e_o,\ y_o^p = y_o/u_o,$$
$$l_o^p = f^{-1}(y_o^p),\ \omega_o = f'(l_o^p),\ \hat{p}_o = \pi_o^c = \bar{\pi},$$
$$\rho_o = f(l_o^d) - \delta - \omega_o l_o^d,\ i_o = \rho_o + \hat{p}_o,\ \epsilon_o = \epsilon_o^m = 0.$$

Note that income distribution in the steady state is still determined by marginal productivity theory, since it does not yet play a role in aggregate demand in the steady state.

Despite formal similarities in the building blocks of the New Keynesian AS–AD dynamics and the above matured Keynesian DAS–AD dynamics, the resulting reduced form laws of motion, see Sect. 2.3, have not much in common in their structure and nothing in common in the applied solution strategies. The New Keynesian model has four forward-looking variables and thus demands for its determinacy four unstable roots, while our approach only exhibits myopic perfect foresight of a crossover type and thus allows again, with respect to its all variables, for predeterminacy and for stability results as in the preceding section and as shown in the mathematical appendix of this chapter.

We note in this regard that there are many items that reappear in various equations, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. Using such linear dependencies and the knowledge we have about the feedback structure of the dynamics we can then show the following proposition:

Proposition 2.4. *Assume that the parameters β_{pu}, $\beta_{p\omega}$ in the price PC are not chosen too large and that the parameters κ_p, β_{p^m} and i_1, γ_{ii} are chosen sufficiently small. Then: The interior steady state of the above 5D dynamical system is locally asymptotically stable.*

Proof. See the mathematical Appendix A of this chapter.

We thus see that the assumption about the price PC, the Mundell effect, the degree of interest rate smoothing and the speed with respect to which investment is adjusted to profitability changes can be decisive for the stability of the dynamics. However, this is only one set of sufficient stability conditions for the considered dynamics, which and not all a necessary selection yet. Further combination of the working of destabilizing Mundell-effects, Rose real-wage effects and the strength the inflation targeting process may be found that ensure stability, yet relevant parameter choices can also be found where the dynamics are not viable without the addition of extra behavioral nonlinearities, a topic that is considered in the next section by means of numerical simulations of the dynamics (in the case of an LM-curve as well of a Taylor interest rate rule).

2.7 Downward Nominal Wage Rigidities

Let us now turn to some numerical simulations of our matured Keynesian analysis of the working of the wage-price spiral. In Fig. 2.2 we show the maximum real parts of eigenvalues as functions of the crucial adjustment speeds of

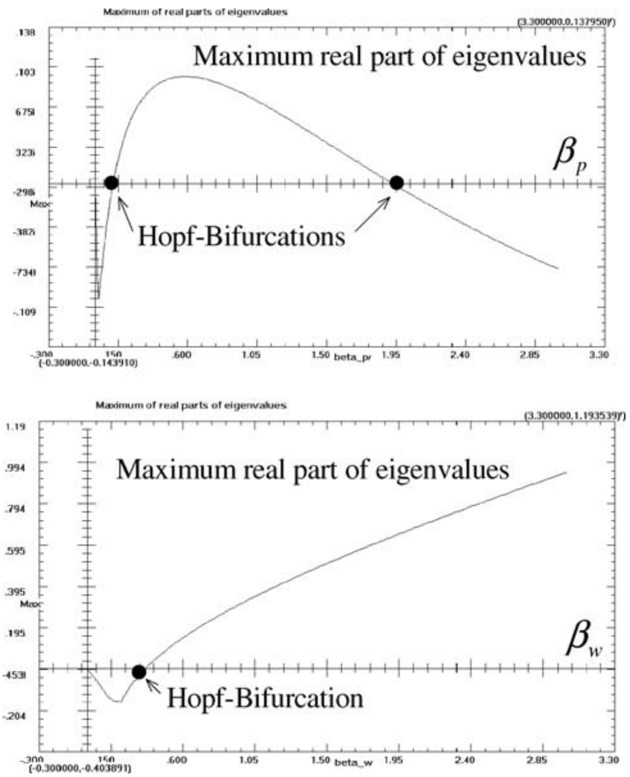

Fig. 2.2. Loss of stability and reestablishment of stability by way of Hopf-bifurcations

prices and wages with respect to the demand pressure on their corresponding markets. We see from these graphs that increasing wage flexibility is initially stabilizing and increasing price flexibility destabilizing (as expected from our partial consideration of the real wage channel). But fairly soon the role played by these parameters becomes reversed, approaching thereafter in fact a second Hopf-bifurcation point in each case. Thus, sooner or later, the partial insights gained from our consideration of Rose-effects are overturned and further wage flexibility and price flexibility then start to do just the opposite of what these partial arguments would suggest. This shows that a 5D dynamical system (and the numerous local asymptotic stability conditions it exhibits) can be much more complicated than is suggested by partial formal or even verbal economic reasoning.

Starting from this observation we now consider situations where the loss of stability has become a total one, giving rise to economic fluctuations, the amplitude of which increases without bound. From the perspective of previous work of ours[16]—and the reversal in the stability features just observed—we expect that complete or partial downward rigidity of money wages may be the cure in such a situation, in line with what has been suggested already by Keynes (1936), since wage adjustment is then destabilizing, while price adjustment is not. We thus now consider situations where money wages can fall at most with the rate $f \leq 0$, which alters our WPC in an obvious way, leading to a kink in it if the floor f is reached. Figure 2.3 provides a typical outcome of the dynamics if downwardly rigid money wages are added to an explosive situation where the economy is not at all a viable one and in fact subject to immediate breakdown without such rigidity.

If the money wage Phillips curve is augmented by the assumption that money wages can rise as described, but cannot fall at all, we get a situation of a continuum of steady states (for money supply growth equal to natural growth $\hat{M} = n$ and thus no steady state inflation). This is due to the zero root hysteresis that then occurs, and the thereby implied strong result that the economy will then converge rapidly to the situation of a stable depression where wages become stationary. This stable depression depends in its depth on the initial shock the economy was subject to and is indeed a persistent one, since money wages do not fall (whereby on the one hand economic breakdown is avoided, at the cost of more or less massive underemployment on the other hand). If, by contrast, money wages can fall, but will do so at most at the rate

[16] See e.g. Chiarella and Flaschel (2000).

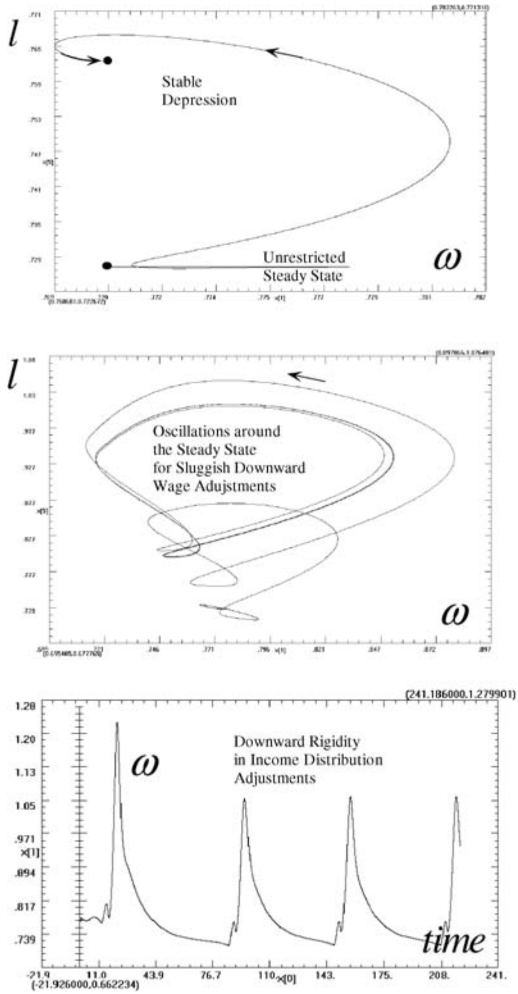

Fig. 2.3. Stable depressions or persistent fluctuations through downwardly rigid money wages (phase length approximately sixty years). *Note:* The parameter set used was: $\omega(0) = 0.770$, $m(0) = 9.088$, $\epsilon^m(0) = 0$, $\ell(0) = 0.727$, $\pi^c(0) = 0$, $\beta_\omega = 0.5$, $\beta_p = 0.5$, $\beta_{\pi^c} = 0.32$, $\beta_{\epsilon^m} = 0.3$, $\alpha = 0.3$, $\kappa_w = 0.5$, $\kappa_p = 0.5$, $s_c = 0.2$, $t^n = 0.25$, $\delta = 0.05$, $n = 0.05$, $g = 0.3$, $\bar{u} = 1.0$, $\bar{e} = 1.0$, $h_1 = 0.1$, $h_2 = 0.1$, $i = 0.25$, $wage\text{-}floor = 0.0$, $w_{shock} = 1.01$

of for example −0.01, the steady state instead remains uniquely determined (as shown in this chapter) and—though surrounded by strongly explosive forces—it is not totally unstable, due to the limit cycle situation that is then generated by the operation of the floor to the speed of money wage declines.

This type of floor makes depressions much longer than recoveries, but avoids the situation where the economy can be trapped in a stable depression as in the case of complete downward rigidity of money wages. The two situations just discussed are illustrated by the Fig. 2.3 in the real wage and labor intensity phase subspace of the full 5D dynamics. In this figure, the latter situation is also augmented by a time series plot for the real wage with its characteristic asymmetry between booms and depressions, with a total phase length of around sixty years of the generated income distribution dynamics. This is in broad agreement with observed empirical phase plots of this sort for example for the U.S. economy, see Chen et al. (2005). Money supply policy rules can dampen the fluctuations shown, but are in general too weak to allow for a disappearance of such endogenously generated and very persistent business cycles in the private sector.

Note that the employment rate of an economy is inversely related to the fluctuations in the full employment labor intensity ratio l that is shown in Fig. 2.3. A high value of l therefore signifies a low employment rate and thus the situation of a long-lasting depression from where the economy is slowly recovering. Normal employment, by contrast, is given when the state variable l exhibits a low value and is accompanied by the instability the economy is subject to if the kink in the money wage PC is not in operation. The economy is then in a very volatile state, which is however moving into a new depression the more the kink in the money wage PC comes into operation again. The working of the kink is clearly shown in the bottom Fig. 2.3 where we have only sluggishly falling real wages until a new recovery phase sets in.

It is one important implication of such a downward floor to the speed of money wage declines that it can easily generate complex dynamics from the mathematical point of view. This is due to the fact that the economy is hitting the kink often in slightly distinct situations after each unstable recovery and thus works each time through the depression phase in a different way, leading to a clearly distinguishable upswing thereafter. Such a situation is exemplified in Fig. 2.4 where the irregularity of the fluctuations in the real wage ω is shown over a time horizon of four thousand years in the top figure. In the bottom figure we in addition show the projection of the cycle into the $\omega - l$ phase plane. One can see there the small corridor through which the dynamics are squeezed on the left hand side for low real wages and the in principle explosive fluctuations that are generated thereafter, but kept under control again and in an increasing manner through the kink in the money wage PC.

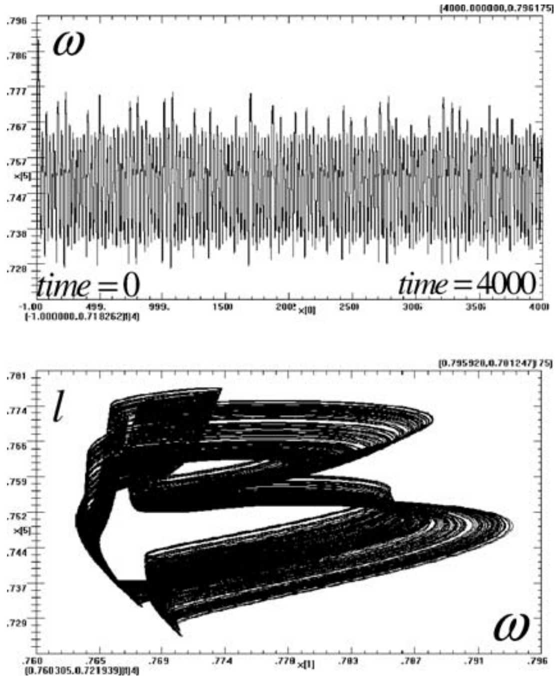

Fig. 2.4. Mathematically complex dynamics with basically economically similar long-term fluctuations in growth and income distribution. *Note:* The parameter set used was: $\omega(0) = 0.770$, $m(0) = 9.088$, $\epsilon^m(0) = 0$, $\ell(0) = 0.727$, $\pi^c(0) = 0$, $\beta_\omega = 0.2$, $\beta_p = 0.5$, $\beta_{\pi^c} = 1.1$, $\beta_{\epsilon^m} = 0.3$, $\alpha = 0.3$, $\kappa_w = 0.7$, $\kappa_p = 0.3$, $s_c = 0.2$, $t^n = 0.25$, $\delta = 0.05$, $n = 0.05$, $g = 0.3$, $\bar{u} = 1.0$, $\bar{e} = 1.0$, $h_1 = 0.1$, $h_2 = 0.1$, $i = 1$, $wage\text{-}floor = -0.0049$, $w_{shock} = 1.01$

An indication of the range of complex behavior can be obtained by considering bifurcation diagrams (showing the local maxima and minima of a state variable as one parameter of the model is increased along the horizontal axis). In Fig. 2.5 we show such a diagram for ω with respect to the savings rate $s(= 1 - c)$. As s increases the bifurcation diagram indicates that two-cycles for ω give way to periods of complex behavior interspersed with period of high order cycles. Of course, average savings ratios above 25-percent are not too likely from the economic point of view, so that the economic range for the savings parameter is significantly smaller than the one shown in Fig. 2.5. Visible is however that higher savings ratios increase the tension in our model economy. This also holds true for the case of an interest rate policy, as shown

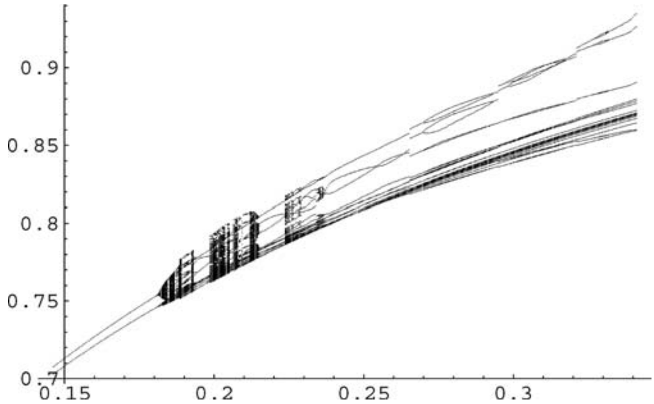

Fig. 2.5. Mathematically complex dynamics: Bifurcation diagram 2.4

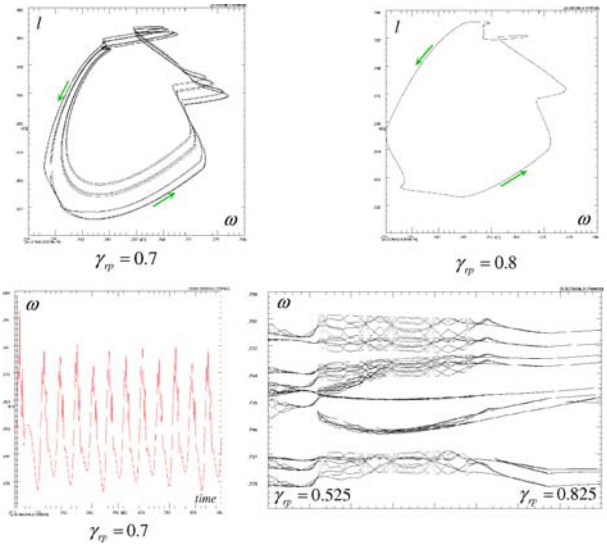

Fig. 2.6. Complexity reducing interest rate policy?

in Fig. 2.6, where we in fact would get convergence for saving rates below $s = 0.03$-percent solely.

In Fig. 2.6,[17] we instead show for a higher savings rate period doubling situations (top-left and bottom-left) that can be reduced to simple limit cycles (top-right) by increasing the strength of the reaction of the Central Bank with

[17] The parameter set here is: $\beta_w = 0.2, \beta_p = 0.1, \beta_{pi^m} = 0.72, \beta_{\epsilon^m} = 0.8, \gamma_{ii} = 0.1, \gamma_{ip} = 0.7, \gamma_{iu} = 0.1, \alpha = 0.3, \kappa_w = 0.5, \kappa_p = 0.5, s = 0.1, t^n = 0.3, \delta = 0.05, n = 0.05, g = 0.3, u_o = 1.0, e_o = 1.0, \bar{\pi} = 0, i = 0.2.$

respect to the inflation gap. Yet, due to the fast adjustment of the inflationary climate with respect to current inflation rates that is here assumed, not much more can be achieved by monetary policy in the considered case. Figure 2.6, bottom-right shows in this respect again a bifurcation diagram which indicates complex types of limit cycle behavior for parameter values γ_{ip} below 0.7, but thereafter the establishment of simpler limit cycles which cannot made simpler however or even turned into convergent dynamics as the parameter γ_{ip} is further increased, even when increased much beyond 0.825 (not shown in this figure). Monetary policy may reduce dynamic complexity to a certain degree, but may be incapable to turn persistent business fluctuations generated in the private sector of the economy into damped oscillations.

Underlying Fig. 2.6 is a floor parameter $f = 0.02$, i.e., wage inflation cannot even be reduced below 2 percent. In addition we, however, here assume that wages regain their assumed flexibility if the rate of employment falls below 80 percent. Without this latter assumption cycles would be much larger than shown in Fig. 2.6, i.e., we here have a case where a return to wage flexibility in deep depressions improves the stability of the dynamics, though the floor to money wage inflation in between is indeed of help, since its removal would lead to explosive business fluctuations. A wage Phillips curve with three regimes (two regime changes) as investigated empirically in Filardo (1998) may therefore be better than one with only two in a situation where partial Rose effects indicate that wage flexibility is stabilizing while price flexibility is not.

2.8 Conclusion

Summing up the main results of this chapter, we have been able to generate damped business fluctuations, persistent oscillations or even complex dynamics from a matured, but conventional synthesis of Keynesian AS–AD dynamics with an advanced description of its wage-price module as a wage-price spiral, when in addition simple (plausible) regime changes in the money wage Phillips curve are taken into account. There are thus no fancy nonlinearities necessary in a by and large conventional type of AS–AD disequilibrium dynamics in order to obtain interesting dynamic outcomes. Some further stability may be achieved through monetary policy to a certain degree. However the cycle generating mechanisms in the private sector are often too strong to be overcome completely by the mechanisms analyzed in this chapter. This is so since in a situation of possibly fairly explosive dynamics the downward money wage rigidity provides a stabilizing influence on the dangers for economic breakdown arising from inflationary or deflationary spirals and their implications, but not on other broader destabilizing tendencies.

We stress that we have achieved viable or bounded dynamics through behavioral assumptions that concern the private sector and not—as in the New Keynesian approach of Sect. 2.3—only by way of an interest rate policy of the Central Bank that is sufficiently advanced and active such that all roots of the Jacobian of the dynamics become unstable. In the latter case, boundedness comes about by assumption in a totally unstable linear(ized) environment and not by changes in agents' behavior when the economy departs too much from the steady state.

Appendix A

Rigorous Stability Analysis (Interest Rate Policy Case)

In this appendix we provide the proof for Proposition 2.4 of the chapter. We refer the reader to Sect. 2.6 for the original formulation of the laws of motion to be considered here and their interior steady state solution. The static relationships supplementing the laws of motion introduced in Sect. 2.6 and their partial derivatives are reformulated for the subsequent proof as follows:

$$y = \frac{1}{1-c}[i_1\varepsilon^m + n + g - t] + \delta + t = y(\varepsilon^m), \tag{A.1}$$

$$y_\varepsilon = dy/\varepsilon^m = \frac{1}{1-c} i_1 > 0,$$

$$y^p = f((f')^{-1}(\exp\theta)) = y^p(\theta); \tag{A.2}$$

$$y^p_\theta = dy^p/d\theta = (f'(l^p)/f''(l^p))(\exp\theta) < 0,$$

$$l^d = f^{-1}(y(\varepsilon^m)) = l^d(\varepsilon^m); \quad l^d_\varepsilon = dl^d/d\varepsilon^m = y_\varepsilon/f' > 0, \tag{A.3}$$

$$u = y(\varepsilon^m)/y^p(\theta) = u(\varepsilon^m, \theta); \quad u_\varepsilon = \partial u/\partial\varepsilon^m = y_\varepsilon/y^p > 0,$$

$$u_\theta = \partial u/\partial\theta = -y y^p_\theta/(y^p)^2 > 0, \tag{A.4}$$

$$e = l^d(\varepsilon^m)/l = e(\varepsilon^m, l); \quad e_\varepsilon = \partial e/\partial\varepsilon^m = l^d_\varepsilon/l > 0,$$

$$e_l = \partial e/\partial l = -l^d/l^2 < 0, \tag{A.5}$$

$$\rho = y - \delta - \omega l^d = y(\varepsilon^m) - \delta - (\exp\theta)l^d(\varepsilon^m) = \rho(\varepsilon^m, \theta);$$

$$\rho_\varepsilon = \partial\rho/\partial\varepsilon^m = \{1 - (\exp\theta)/f'(l^d)\}y_\varepsilon$$

$$= \{1 - f'(l^p)/f'(l^d)\}y_\varepsilon > 0 \ \text{ if } \ l^d < l^p,$$

$$\rho_\theta = \partial\rho/\partial\theta = -(\exp\theta)l^d < 0, \tag{A.6}$$

$$i_o = \rho_o + \bar{\pi}, \tag{A.7}$$

$$\hat{p} = \frac{1}{1-\kappa_w\kappa_p}[\beta_{pu}\{u(\varepsilon^m,\theta) - u_o\} + \beta_{p\omega}(\theta - \theta_o)$$

$$+\kappa_p\{\beta_{we}(e(\varepsilon^m, l) - e_o) - \beta_{w\omega}(\theta - \theta_o)\}] + \pi^m = f(\theta, \varepsilon^m, l) + \pi^m,$$

$$f_\theta = \partial f/\partial\theta = \frac{1}{1-\kappa_w\kappa_p}(\beta_{pu}u_\theta + \beta_{p\omega} - \kappa_p\beta_{w\omega}),$$
$$f_\varepsilon = \partial f/\partial\varepsilon^m = \frac{1}{1-\kappa_w\kappa_p}(\beta_{pu}u_\varepsilon + \kappa_p\beta_{we}e_\varepsilon) > 0,$$
$$f_l = \partial f/\partial l = \frac{1}{1-\kappa_w\kappa_p}\kappa_p\beta_{we}e_l < 0, \tag{A.8}$$

because of the inequalities $0 < \kappa_w < 1$ and $0 < \kappa_p < 1$. In this case we have

$$\varepsilon = \rho - (i - \hat{p}) = \rho(\varepsilon^m, \theta) - i + f(\theta, \varepsilon^m, l) + \pi^m = \varepsilon(\theta, r, \varepsilon^m, l, \pi^m);$$
$$\varepsilon_\theta = \partial\varepsilon/\theta = \rho_\theta \underset{(-)}{\lim} + f_\theta \underset{(?)}{\lim},\ \varepsilon_i = \partial\varepsilon/\partial i = -1 < 0,$$
$$\varepsilon_\varepsilon = \partial\varepsilon/\partial\varepsilon^m = \rho_\varepsilon + f_\varepsilon > 0,$$
$$\varepsilon_l = \partial\varepsilon/\partial l = f_l < 0,\ \varepsilon_\pi = \partial\varepsilon/\partial\pi^m = 1 > 0. \tag{A.9}$$

We will make the following two assumptions in the derivation of the propositions of this appendix:

Assumption A.1. The parameters β_{pu} and $\beta_{p\omega}$ are not extremely large so that we can have $\varepsilon_\theta = \rho_\theta + f_\theta < 0$. Substituting the above static relationships into the dynamic equations of Sect. 2.6, we have the following five dimensional system of nonlinear differential equations

$$\left.\begin{aligned}
&\text{(i) } \dot{\theta} &&= \tfrac{1}{1-\kappa_w\kappa_p}\left[(1-\kappa_p)\{\beta_{we}(e(\varepsilon^m, l) - e_o) - \beta_{w\omega}(\theta - \theta_o)\}\right.\\
&&&\quad \left.-(1-\kappa_w)\{\beta_{pu}(u(\varepsilon^m, \theta) - u_o) + \beta_{w\omega}(\theta - \theta_o)\}\right]\\
&&&\quad = F_1(\theta, \varepsilon^m, l),\\
&\text{(ii) } \dot{i} &&= -\gamma_{ii}(i - i_0) + \gamma_{ip}\{f(\theta, \varepsilon^m, l) + \pi^m - \bar{\pi}\}\\
&&&\quad +\gamma_{iu}\{u(\varepsilon^m, \theta) - u_o\} = F_2(\theta, i, \varepsilon^m, l, \pi^m),\\
&\text{(iii) } \dot{\varepsilon}^m &&= \beta_{\varepsilon^m}\{\varepsilon(\theta, i, \varepsilon^m, l, \pi^m) - \varepsilon^m\} = F_3(\theta, i, \varepsilon^m, l, \pi^m),\\
&\text{(iv) } \dot{l} &&= -i_1\varepsilon^m l = F_4(\varepsilon^m, l),\\
&\text{(v) } \dot{\pi}^m &&= \beta_{\pi^m} f(\theta, \varepsilon^m, l) = F_5(\theta, \varepsilon^m, l).
\end{aligned}\right\} \tag{A.10}$$

The equilibrium solution of this system is given in Sect. 2.6. We assume that

Assumption A.2. At the equilibrium point we have $l^d < l^p$ so that $\rho_\varepsilon > 0$ holds true.

Now, let us investigate the local stability/instability of the equilibrium point of the system given by (A.10). We can write the Jacobian matrix of this system *at the equilibrium point* as follows

$$J = \begin{bmatrix} F_{11} & 0 & F_{13} & F_{14} & 0 \\ F_{21} & -\gamma_{ii} & F_{23} & F_{24} & \gamma_{ip} \\ \beta_{\varepsilon^m}(\rho_\theta + f_\theta) & -\beta_{\varepsilon^m} & -\beta_{\varepsilon^m}(1-\rho_\varepsilon - f_\varepsilon) & \beta_{\varepsilon^m} f_l & \beta_{\varepsilon^m} \\ 0 & 0 & -i_1 l_0 & 0 & 0 \\ \beta_{\pi^m} f_\theta & 0 & \beta_{\pi^m} f_\varepsilon & \beta_{\pi^m} f_l & 0 \end{bmatrix}, \tag{A.11}$$

where

$$\begin{aligned} F_{11} &= \frac{-1}{1-\kappa_w\kappa_p}[(2-\kappa_p-\kappa_w)\beta_{w\omega} + (1-\kappa_w)\beta_{pu} \underset{(+)}{u_\theta}] < 0, \\ F_{13} &= \frac{1}{1-\kappa_w\kappa_p}[(1-\kappa_p)\beta_{we} \underset{(+)}{u_\varepsilon} - (1-\kappa_w)\beta_{pu} \underset{(+)}{u_\varepsilon}], \\ F_{14} &= \frac{1}{1-\kappa_w\kappa_p}[(1-\kappa_p)\beta_{we} \underset{(-)}{e_l}] < 0, \\ F_{21} &= \gamma_p \underset{(?)}{f_\theta} + \gamma_u \underset{(+)}{u_\theta}, \\ F_{23} &= \gamma_p \underset{(+)}{f_\varepsilon} + \gamma_u \underset{(+)}{u_\varepsilon} > 0, \\ F_{24} &= \gamma_p \underset{(-)}{f_l} < 0. \end{aligned}$$

The sign pattern of the matrix J becomes as follows

$$sign\, J = \begin{bmatrix} - & 0 & ? & - & 0 \\ ? & - & + & - & + \\ - & - & ? & - & + \\ 0 & 0 & - & 0 & 0 \\ ? & 0 & + & - & 0 \end{bmatrix}. \tag{A.12}$$

The characteristic equation of this system can be written as

$$\Gamma(\lambda) = \lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0, \tag{A.13}$$

where coefficients $a_i, i = 1, \ldots, 5$ are given as follows.

$$a_1 = -trace\, J = -\underset{(-)}{F_{11}} + \gamma_{ii} + \beta_{\varepsilon^m}(1 - \underset{(+)}{\rho_\varepsilon} - \underset{(+)}{f_\varepsilon}) = a_1(\beta_{\varepsilon^m}), \tag{A.14}$$

$$\begin{aligned}
a_2 &= \text{sum of all principal second-order minors of } J \\
&= \begin{vmatrix} F_{11} & 0 \\ F_{21} & -\gamma_r \end{vmatrix} + \beta_{\varepsilon^m} \begin{vmatrix} F_{11} & F_{13} \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) \end{vmatrix} + \begin{vmatrix} F_{11} & F_{14} \\ 0 & 0 \end{vmatrix} \\
&\quad + \begin{vmatrix} F_{11} & 0 \\ \beta_{\pi^m} f_\theta & 0 \end{vmatrix} + \beta_{\varepsilon^m} \begin{vmatrix} -\gamma_{ii} & F_{23} \\ -1 & \rho_\varepsilon + f_\varepsilon - 1 \end{vmatrix} + \begin{vmatrix} -\gamma_{ii} & F_{24} \\ 0 & 0 \end{vmatrix} \\
&\quad + \begin{vmatrix} -\gamma_{ii} & \gamma_{ip} \\ 0 & 0 \end{vmatrix} + \beta_{\varepsilon^m} \begin{vmatrix} -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ -il_0 & 0 \end{vmatrix} \\
&\quad + \beta_{\varepsilon^m}\beta_{\pi^m} \begin{vmatrix} -(1-\rho_\varepsilon - f_\varepsilon) & 1 \\ f_\varepsilon & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ \beta_{\pi^m} f_l & 0 \end{vmatrix}, \\
&= -\underset{(-)}{F_{11}}\gamma_r + \beta_{\varepsilon^m}\{-\underset{(-)}{F_{11}}(1-\rho_\varepsilon - f_\varepsilon) - \underset{(?)}{F_{13}}(\rho_\theta + \underset{(-)}{f_\theta}) \\
&\quad + \gamma_r(1-\rho_\varepsilon - f_\varepsilon) + \underset{(+)}{F_{23}} + il_0 \underset{(-)}{f_l} - \beta_{\pi^m} \underset{(+)}{f_\varepsilon}\} \\
&= a_2(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
a_3 &= -(\text{sum of all principal third-order minors of } J), \\
&= -\beta_{\varepsilon^m} \begin{vmatrix} F_{11} & 0 & F_{13} \\ F_{21} & -\gamma_{ii} & F_{23} \\ \rho_\theta + f_\theta & -1 & -(1-\rho_\varepsilon - f_\varepsilon) \end{vmatrix} - \begin{vmatrix} F_{11} & 0 & F_{14} \\ F_{21} & -\gamma_r & F_{24} \\ 0 & 0 & 0 \end{vmatrix} \\
&\quad - \begin{vmatrix} F_{11} & 0 & 0 \\ F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \beta_{\pi^m} f_\theta & 0 & 0 \end{vmatrix} - \beta_{\varepsilon^m} \begin{vmatrix} F_{11} & F_{13} & F_{14} \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ 0 & -i_1 l_0 & 0 \end{vmatrix} \\
&\quad - \beta_{\varepsilon^m}\beta_{\pi^m} \begin{vmatrix} F_{11} & F_{13} & 0 \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) & 1 \\ f_\theta & f_\varepsilon & 0 \end{vmatrix} - \begin{vmatrix} F_{11} & F_{14} & 0 \\ 0 & 0 & 0 \\ \beta_{\pi^m} f_\theta & \beta_{\pi^m} f_l & 0 \end{vmatrix} \\
&\quad - \beta_{\varepsilon^m} \begin{vmatrix} -\gamma_{ii} & F_{23} & F_{24} \\ -1 & -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ 0 & -i_1 l_0 & 0 \end{vmatrix}
\end{aligned}$$

$$-\beta_{\varepsilon^m}\beta_{\pi^m}\begin{vmatrix} -\gamma_{ii} & F_{23} & \gamma_{ip} \\ -1 & -(1-\rho_\varepsilon - f_\varepsilon) & 1 \\ 0 & f_\varepsilon & 0 \end{vmatrix} - \begin{vmatrix} -\gamma_{ii} & F_{24} & \gamma_{ip} \\ 0 & 0 & 0 \\ 0 & \beta_{\pi^m} f_l & 0 \end{vmatrix}$$

$$-\beta_{\varepsilon^m}\beta_{\pi^m}\begin{vmatrix} -(1-\rho_\varepsilon - f_\varepsilon) & f_l & 1 \\ -i_1 l_0 & 0 & 0 \\ f_\varepsilon & f_l & 0 \end{vmatrix},$$

$$= \beta_{\varepsilon^m}[-\underset{(-)}{F_{11}}\gamma_{ii}(1-\rho_\varepsilon - f_\varepsilon) + \underset{(?)}{F_{13}}\,\underset{(?)}{F_{21}} - \underset{(?)}{F_{13}}\gamma_{ii}\underset{(-)}{(\rho_\theta + f_\theta)} - \underset{(-)}{F_{11}}\,\underset{(+)}{F_{23}}$$

$$+\underset{(-)}{F_{14}}\, i_1 l_0 \underset{(-)}{(\rho_\theta + f_\theta)} - \underset{(-)}{F_{11}}\, i_1 l_0 \underset{(-)}{f_l} - \underset{(-)}{F_{24}}\, i_1 l_0 + \gamma_{ii} i_1 l_0 \underset{(-)}{f_l}$$

$$+\beta_{\pi^m}\{-\underset{(?)}{F_{13}}\,\underset{(+)}{f_\theta} + \underset{(-)}{F_{11}}\,\underset{(+)}{f_\varepsilon} + \gamma_{ip}\underset{(+)}{f_\varepsilon} - \gamma_{ii}\underset{(+)}{f_\varepsilon} + \underset{(-)}{f_l}\, i_1 l_0\}],$$

$$= a_3(\beta_{\varepsilon^m}, \beta_{\pi^m}). \tag{A.16}$$

$a_4 =$ sum of all fourth-order minors of J,

$$= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \left\{ \begin{vmatrix} -\gamma_{ii} & F_{23} & F_{24} & \gamma_{ip} \\ -1 & -(1-\rho_\varepsilon - f_\varepsilon) & f_l & 1 \\ 0 & -1 & 0 & 0 \\ 0 & f_\varepsilon & f_l & 0 \end{vmatrix} \right.$$

$$+ \begin{vmatrix} F_{11} & F_{13} & F_{14} & 0 \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) & f_l & 1 \\ 0 & -1 & 0 & 0 \\ f_\theta & f_\varepsilon & f_l & 0 \end{vmatrix}$$

$$+(1/i_1 l_0)\begin{vmatrix} F_{11} & 0 & F_{13} & 0 \\ F_{21} & -\gamma_{ii} & F_{23} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & -(1-\rho_\varepsilon - f_\varepsilon) & 1 \\ f_\theta & 0 & f_\varepsilon & 0 \end{vmatrix}$$

$$\left. + \begin{vmatrix} F_{11} & 0 & F_{13} & F_{14} \\ F_{21} & -\gamma_{ii} & F_{23} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ 0 & 0 & -1 & 0 \end{vmatrix} \right\},$$

$$= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \left\{ \begin{vmatrix} -\gamma_{ii} & F_{24} & \gamma_{ip} \\ -1 & f_l & 1 \\ 0 & f_l & 0 \end{vmatrix} + \begin{vmatrix} F_{11} & F_{13} & F_{14} \\ 0 & -1 & 0 \\ f_\theta & f_\varepsilon & f_l \end{vmatrix} \right.$$

$$+(1/i_1 l_0)F_{11} \begin{vmatrix} -\gamma_{ii} & F_{23} & \gamma_{ip} \\ -1 & 1-\rho_\varepsilon - f_\varepsilon & 1 \\ 0 & f_\varepsilon & 0 \end{vmatrix}$$

$$\left. +(1/i_1 l_0)F_{13} \begin{vmatrix} F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & 1 \\ f_\theta & 0 & 0 \end{vmatrix} + \begin{vmatrix} F_{11} & 0 & F_{14} \\ F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & f_l \end{vmatrix} \right\},$$

$$= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \{ -f_l(\gamma_{ip}-\gamma_{ii}) - F_{11}f_l + F_{14}f_\theta - (1/i_1 l_0)F_{11} f_\varepsilon(\gamma_{ip}-\gamma_{ii})$$

$$+(1/i_1 l_0)F_{13} f_\theta(\gamma_{ip}-\gamma_{ii}) - F_{11}\gamma_{ii} f_l - F_{14}F_{21} + F_{14}\gamma_{ii}(\rho_\theta + f_\theta)$$

$$+F_{11}\gamma_{ip}\},$$

$$= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 [- \underset{(-)}{f_l} \{(\gamma_{ip}-\gamma_{ii}) + \underset{(-)}{F_{11}}(1+\gamma_{ii})\}$$

$$+\underset{(-)}{F_{14}}\{- \underset{(?)}{F_{21}} + \underset{(-)}{\rho_\theta}\ \gamma_{ii} + (1+\gamma_{ii})\ \underset{(?)}{f_\theta}\}$$

$$+\{-\underset{(-)}{F_{11}}\ \underset{(+)}{(f_\varepsilon/i_1 l_0)} + \underset{(?)}{(F_{13}/i_1 l_0)}\ \underset{(?)}{f_\theta}\}(\gamma_{ip}-\gamma_{ii}) + \underset{(-)}{F_{11}}\ \gamma_{ip}]$$

$$= a_4(\beta_{\varepsilon^m}, \beta_{\pi^m}). \tag{A.17}$$

$$a_5 = -\det J = -\beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \begin{vmatrix} F_{11} & 0 & F_{13} & F_{14} & 0 \\ F_{21} & -\gamma_{ii} & F_{23} & F_{24} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & -(1-\rho_\varepsilon - f_\varepsilon) & f_l & 1 \\ 0 & 0 & -1 & 0 & 0 \\ f_\theta & 0 & f_\varepsilon & f_l & 0 \end{vmatrix},$$

$$= -\beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \begin{vmatrix} F_{11} & 0 & F_{14} & 0 \\ F_{21} & -\gamma_{ii} & F_{24} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & f_l & 1 \\ f_\theta & 0 & f_l & 0 \end{vmatrix},$$

$$= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \left\{ -F_{11} \begin{vmatrix} -\gamma_{ii} & F_{24} & \gamma_{ip} \\ -1 & f_l & 1 \\ 0 & f_l & 0 \end{vmatrix} - F_{14} \begin{vmatrix} F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & 1 \\ f_\theta & 0 & 0 \end{vmatrix} \right\},$$

$$= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 (\underset{(-)}{F_{11}}\ \underset{(-)}{f_l} - \underset{(-)}{F_{14}}\ \underset{(?)}{f_\theta})(\gamma_{ip}-\gamma_{ii}) = a_5(\beta_{\varepsilon^m}, \beta_{\pi^m}). \tag{A.18}$$

We can define the Routh–Hurwitz terms Δ_j $(j = 1, 2, \ldots, 5)$ as follows

$$\left.\begin{aligned}
&\text{(i)} \quad \Delta_1 = a_1 = a_1(\beta_{\varepsilon^m}),\\
&\text{(ii)} \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix} = a_1 a_2 - a_3 = \Delta_2(\beta_{\varepsilon^m}, \beta_{\pi^m}),\\
&\text{(iii)} \quad \Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = a_3 \Delta_2 + a_1(a_5 - a_1 a_4),\\
&\qquad = a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 + a_1 a_5 = \Delta_3(\beta_{\varepsilon^m}, \beta_{\pi^m}),\\
&\text{(iv)} \quad \Delta_4 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ 1 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \end{vmatrix} = a_4 \Delta_3 - a_5 \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & 1 & a_2 \end{vmatrix},\\
&\qquad = a_4 \Delta_3 + a_5(-a_1 a_2^2 - a_5 + a_2 a_3 + a_1 a_4),\\
&\qquad = a_4 \Delta_3 + a_5(a_1 a_4 - a_5 - a_2 \Delta_2) = \Delta_4(\beta_{\varepsilon^m}, \beta_{\pi^m}),\\
&\text{(v)} \quad \Delta_5 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ 1 & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 1 & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix} = a_5 \Delta_4 = \Delta_5(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned}\right\} \tag{A.19}$$

It is well known that the equilibrium point of the five dimensional dynamical system (A.10) is locally stable if and only if the following Routh–Hurwitz conditions for stable roots are satisfied

$$\Delta_j > 0 \text{ for all } j \in \{1, 2, \ldots, 5\}. \tag{RH}$$

It is also well known that the set of conditions RH can be expressed in any of the four following alternative forms, which are called Lienard–Chipart conditions (cf. Gandolfo 1996, p. 223)

$$\left.\begin{aligned}
&\text{(a)}\ a_5 > 0,\ a_3 > 0,\ a_1 > 0,\ \Delta_3 > 0,\ \Delta_5 > 0,\\
&\text{(b)}\ a_5 > 0,\ a_3 > 0,\ a_1 > 0,\ \Delta_2 > 0,\ \Delta_4 > 0,\\
&\text{(c)}\ a_5 > 0,\ a_4 > 0,\ a_2 > 0,\ \Delta_1 > 0,\ \Delta_3 > 0,\ \Delta_5 > 0,\\
&\text{(d)}\ a_5 > 0,\ a_4 > 0,\ a_2 > 0,\ \Delta_2 > 0,\ \Delta_4 > 0.
\end{aligned}\right\} \tag{LC}$$

It follows from Lienard–Chipart conditions that the following conditions are *necessary* (but not sufficient) conditions for local asymptotic stability.

$$a_j > 0 \quad \text{for all} \quad j \in \{1, 2, \ldots, 5\}. \tag{A.20}$$

From the relationships $\gamma_{ii} = \alpha_{ii}$ and $\gamma_{ip} = \alpha_{ii}(1 + \phi_{ip})$ we always have

$$\gamma_{ip} - \gamma_{ii} = \alpha_{ii}\phi_{ip} > 0, \tag{A.21}$$

which means that we have the following expressions

$$\begin{aligned} a_4 = \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 [& -\underset{(-)}{f_l}\{\alpha_{ii}\phi_{ip} + \underset{(-)}{F_{11}}(1+\alpha_{ii})\} \\ & + \underset{(-)}{F_{14}}\{-\underset{(?)}{F_{21}} + \underset{(-)}{\rho_\theta}\,\alpha_{ii} + (1+\alpha_{ii})\underset{(?)}{f_\theta}\} \\ & + \{-\underset{(-)}{F_{11}}\underset{(+)}{(f_\varepsilon/i_1 l_0)} + \underset{(?)}{(F_{13}/i_1 l_0)}\underset{(?)}{f_\theta}\}\alpha_{ii}\phi_{ip} + \underset{(-)}{F_{11}}\,\alpha_{ii}(1+\phi_{ip})], \end{aligned} \tag{A.22}$$

$$a_5 = \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 (\underset{(-)}{F_{11}}\,\underset{(-)}{f_l} - \underset{(-)}{F_{14}}\,\underset{(?)}{f_\theta})\alpha_{ii}\phi_{ip}, \tag{A.23}$$

where $(f_\varepsilon/i_1 l_0)$ and $(F_{13}/i_1 l_0)$ are *independent* of the parameter $i_1 > 0$.

We can easily see that the following relationships are satisfied

$$\left.\begin{array}{l} \text{(i)}\ \ f_l = 0 \text{ if } \kappa_p = 0 \\ \text{(ii)}\ f_\theta = 0 \text{ and } F_{21} > 0 \text{ if } \kappa_p = \beta_{pu} = \beta_{p\omega} = 0. \end{array}\right\} \tag{A.24}$$

Therefore, we can obtain the following results

$$\begin{aligned} a_4 = {} & \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 [\underset{(-)}{F_{14}}(-\underset{(+)}{F_{21}} + \underset{(-)}{\rho_\theta}\,\alpha_{ii}) + \alpha_{ii}\,\underset{(-)}{F_{11}}\{-\underset{(+)}{(f_r/i_1 l_0)} + 1 + \phi_{ip}\}] \\ & \text{if } \kappa_p = \beta_{pu} = \beta_{p\omega} = 0, \end{aligned} \tag{A.25}$$

$$\begin{aligned} a_5 = {} & -\beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0\,\underset{(-)}{F_{14}}\,\beta_{p\omega}\alpha_{ii}\phi_{ip} > 0 \\ & \text{for all } (\beta_{\varepsilon^m}, \beta_{\pi^m}, i_1, \beta_{p\omega}) > (0,0,0,0) \text{ if } \kappa_p = 0. \end{aligned} \tag{A.26}$$

Now, let us assume as follows

Assumption A.3. The parameters $\kappa_p > 0$, $\beta_{pu} > 0$, $\beta_{p\omega} > 0$, and $\gamma_{ii} = \alpha_{ii} > 0$ are sufficiently small.

Lemma A.1. *Under Assumptions A.1–A.3, we have $a_4 > 0$ and $a_5 > 0$ for all $(\beta_{\varepsilon^m}, \beta_{\pi^m}, i) > (0,0,0)$.*

Proof. This result follows directly from (A.25), (A.26) and Assumption A.3 by continuity. ■

Lemma A.2. *Under Assumptions A.1–A.3, we have $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $\Delta_2 > 0$, $\Delta_3 > 0$, and $\Delta_4 > 0$ for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i_1 > 0$ are sufficiently small.*

Proof. We can easily see that the following equalities are satisfied

$$\begin{aligned}\lim_{i_1\to 0} y_\varepsilon &= \lim_{i_1\to 0} l^d_\varepsilon = \lim_{i_1\to 0} u_\varepsilon = \lim_{i_1\to 0} e_\varepsilon = \lim_{i_1\to 0} \rho_\varepsilon = \lim_{i_1\to 0} f_\varepsilon = \lim_{i_1\to 0} F_{13} \\ &= \lim_{i_1\to 0} F_{23} = 0. \end{aligned} \tag{A.27}$$

Therefore, we have the following inequalities from (A.14)–(A.19)

$$\lim_{i_1\to 0} \ a_1(\beta_{\varepsilon^m}) = -\underset{(-)}{F_{11}} + \gamma_{ii} + \beta_{\varepsilon^m} > 0 \quad \text{for all } \beta_{\varepsilon^m} \geq 0, \tag{A.28}$$

$$\lim_{i_1\to 0} \ a_2(\beta_{\varepsilon^m},0) = -\underset{(-)}{F_{11}}\gamma_{ii} + \beta_{\varepsilon^m}(-\underset{(-)}{F_{11}}+\gamma_{ii}) > 0 \quad \text{for all } \beta_{\varepsilon^m} \geq 0, \tag{A.29}$$

$$\lim_{i_1\to 0} \ a_3(\beta_{\varepsilon^m},0) = -\beta_{\varepsilon^m}\underset{(-)}{F_{11}}\gamma_{ii} > 0 \quad \text{for all } \beta_{\varepsilon^m} > 0, \tag{A.30}$$

$$\begin{aligned}\lim_{i_1\to 0} \ \Delta_2(\beta_{\varepsilon^m},0) &= (\gamma_{ii} - \underset{(-)}{F_{11}})\{\beta^2_{\varepsilon^m} + (\gamma_{ii} - \underset{(-)}{F_{11}})\beta_{\varepsilon^m} - \underset{(-)}{F_{11}}\gamma_{ii}\} > 0 \\ &\text{for all } \beta_{\varepsilon^m} \geq 0, \end{aligned} \tag{A.31}$$

$$\begin{aligned}\lim_{i_1\to 0} \ \Delta_3(\beta_{\varepsilon^m},0) &= \lim_{i_1\to 0}\{a_3(\beta_{\varepsilon^m},0)\Delta_2(\beta_{\varepsilon^m},0)\} > 0 \\ &\text{for all } \beta_{\varepsilon^m} > 0. \end{aligned} \tag{A.32}$$

These inequalities imply that we have $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $\Delta_2 > 0$, and $\Delta_3 > 0$ for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i > 0$ are sufficiently small (by continuity).

Next, let us turn to the analysis of the term Δ_4. Substituting (A.19)(iii) into (A.19)(iv), we obtain

$$\begin{aligned}\Delta_4(\beta_{\varepsilon^m},\beta_{\pi^m}) &= a_4(a_3\Delta_2 + a_1a_5 - a_1^2a_4) + a_1a_4a_5 - a_5^2 - a_2a_5\Delta_2, \\ &= (a_3a_4 - a_2a_5)\Delta_2 + a_1a_4(2a_5 - a_1a_4) - a_5^2, \\ &= \beta_{\varepsilon^m}\beta_{\pi^m}i_1l_0[(a_3\tilde{a}_4 - a_2\tilde{a}_5)\Delta_2(\beta_{\varepsilon^m},\beta_{\pi^m}) \\ &\quad + a_1\tilde{a}_4(2a_5 - a_1a_4) - \tilde{a}_5a_5], \\ &= \beta_{\varepsilon^m}\beta_{\pi^m}i_1l_0\phi(\beta_{\varepsilon^m},\beta_{\pi^m}), \end{aligned} \tag{A.33}$$

where $\tilde{a}_j = a_j/\beta_{\varepsilon^m}\beta_{\pi^m}i_1l_0$ $(j = 4,5)$. We can easily see that

$$\lim_{i_1\to 0}\phi(\beta_{\varepsilon^m},0) = [\lim_{i_1\to 0}\{a_3(\beta_{\varepsilon^m},0)\tilde{a}_4 - a_2(\beta_{\pi^m},0)\tilde{a}_5\}]\{\lim_{i_1\to 0}\Delta_2(\beta_{\varepsilon^m},0)\}, \tag{A.34}$$

where $\lim_{i_1\to 0}\Delta_2(\beta_{\varepsilon^m},0) > 0$ is satisfied for all $\beta_{\varepsilon^m} \geq 0$ because of (A.31).

From (A.24), (A.25), (A.26), and (A.30) we can obtain the following result if $\kappa_p = \beta_{pu} = \beta_{p\omega} = 0$:

$$\lim_{i_1 \to 0}\{a_3(\beta_{\varepsilon^m},0)\tilde{a}_4 - a_2(\beta_{\varepsilon^m},0)\tilde{a}_5\} = -\beta_{\varepsilon^m} \underset{(-)}{F_{11}}\, \alpha_{ii}[\underset{(-)}{F_{14}}(-\underset{(+)}{F_{21}} + \underset{(-)}{\rho_\theta}\, \alpha_{ii}) + \alpha_{ii} \underset{(-)}{F_{11}}\{-\underset{(+)}{(f_i/i_1 l_0)} + 1 + \phi_{ip}\}], \tag{A.35}$$

which will be positive for all $\beta_{\varepsilon^m} > 0$ if $\gamma_{ii} = \alpha_{ii} > 0$ is sufficiently small. From (A.33), (A.34), and (A.35) we have $\Delta_4 > 0$ for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i_1 > 0$ are sufficiently small under Assumptions A.1–A.3 by continuity reasons. This completes the proof of Lemma A.2. ∎

The following two propositions are our main results.

Proposition A.1. (i) *Under Assumptions A.1–A.3, the equilibrium point of the system (A.10) is locally asymptotically stable for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i > 0$ are sufficiently small.*

(ii) *Suppose that $\beta_{\varepsilon^m} > 0$. Then, the equilibrium point of the system (A.10) is locally unstable for all sufficiently large values of $\beta_{\pi^m} > 0$.*

Proof. (i) Lemma A.1 and Lemma A.2 imply that all of the conditions (LC)(b) (or alternatively, all of the conditions (LC)(d)) are satisfied for all $\beta_{\varepsilon^m} > 0$ under Assumptions A.1–A.3 if $\beta_{\pi^m} > 0$ and $i_1 > 0$ are sufficiently small.

(ii) Suppose that $\beta_{\varepsilon^m} > 0$. Then, we have $a_2 < 0$ for all sufficiently large values of $\beta_{\pi^m} > 0$. In this case, one of the necessary conditions for local stability (20) is violated. ∎

Proposition A.2. *We posit Assumptions A.1–A.3 and assume that $i_1 > 0$ is sufficiently small. Furthermore, β_{ε^m} is fixed at an arbitrary positive value, and we select $\beta_{\pi^m} > 0$ as a bifurcation parameter. Then, there exists at least one bifurcation point $\beta^0_{\pi^m}$ at which the local stability of the equilibrium point of the system (A.10) is lost as the parameter β_{π^m} is increased. At the bifurcation point, the characteristic equation (A.13) has at least one pair of pure imaginary roots, and there is no real root $\lambda = 0$.*

Proof. Existence of the bifurcation point $\beta^0_{\pi^m}$, at which the local stability of the system is lost, is obvious from Proposition A.1 by continuity. By the very nature of the bifurcation point, the characteristic equation (A.13) must have at least one root with zero real part at $\beta_{\pi^m} = \beta^0_{\pi^m}$. But, we can exclude a real root $\lambda = 0$, because we have $\Gamma(0) = a_5 > 0$. ∎

Remark. In general, the following two cases are possible.

(A.1) At the bifurcation point, the characteristic equation (A.13) has a pair of purely imaginary roots and three roots with negative real parts.

(A.2) At the bifurcation point the characteristic equation (A.13) has two pairs of purely imaginary roots and one negative real root.

The case (A.1) corresponds to the so called "Hopf bifurcation", and in this case we can establish the existence of the closed orbits at some parameter values β_{π^m} which are sufficiently close to the bifurcation value (cf. Gandolfo 1996, Chap. 25 and in Asada et al. 2003, the mathematical appendix). On the other hand, in the case (A.2) one of the conditions for Hopf bifurcations is not satisfied. The case (A.1) will be more likely to occur than the case (A.2), and the case (A.2) will occur only by accident. Even in the case (A.2), however, the existence of the cyclical fluctuations is ensured at some range of the parameter values β_{π^m} which are sufficiently close to the bifurcation value, because of the existence of two pairs of the complex roots.

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3

Wage–Price Dynamics: Basic Structural Form, Estimation and Analysis

3.1 Introduction

3.1.1 The Phillips Curve(s)

Following the seminal work in Phillips (1958) on the relation between unemployment and the rate of change of money wage rates in the UK, the "Phillips curve" was to play an important role in macroeconomics during the 1960s and 1970s, and modified so as to incorporate inflation expectations, survived for much longer.[1] The discussion on the proper type and the functional shape of the Phillips curve has never come to a real end and is indeed now at least as lively as it has been at any other time after the appearance of Phillips (1958) seminal paper. Recent examples for this observation are provided by the paper of Galí et al. (2001), where again a new type of Phillips curve is investigated, and the paper by Laxton et al. (1999) on the typical shape of the expectations augmented price inflation Phillips curve. Blanchard and Katz (1999) investigate the role of an error-correction wage share influence theoretically as well as empirically and Plasmans et al. (1999) investigate on this basis the impact of the generosity of the unemployment benefit system on the adjustment speed of money wages with regard to demand pressure in the market for labor.

[1] This chapter is based on the chapter: "Wage–Price Phillips Curves and Macroeconomic Stability: Basic Structural Form, Estimation and Analysis." (by P. Flaschel and H. Krolzig). In: C. Chiarella, P. Flaschel, R. Franke and W. Semmler (eds.): *Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels.* Contributions to Economic Analysis (Series Editors: B. Baltagi, E. Sadka and D. Wildasin), Amsterdam: Elsevier, 2006, 7–47.

Much of the literature has converged on the so-called *"New Keynesian Phillips curve,"* based on Taylor (1980) and Calvo (1983). Indeed, McCallum (1997) has called it the *"closest thing there is to a standard formulation"*. Clarida et al. (1999) have used a version of it as the basis for deriving some general principles about monetary policy. However, as has been recently pointed out by Mankiw (2001): *"Although the new Keynesian Phillips curves has many virtues, it also has one striking vice: It is completely at odd with the facts"*. The problems arise from the fact that although the price level is sticky in this model, the inflation rate can change quickly. By contrast, empirical analyzes of the inflation process (see, *inter alia*, Gordon (1997)) typically give a large role to "inflation inertia".

Rarely, however, at least on the theoretical level, is note taken of the fact that there are in principle two relationships of the Phillips curve type involved in the interaction of unemployment and inflation, namely one on the labor market, the Phillips (1958) curve, and one on the market for goods, normally not considered a separate Phillips curve, but merged with the other one by assuming that prices are a constant mark-up on wages or the like, an extreme case of the price Phillips curve that we shall consider in this chapter.

For researchers with a background in structural macroeconometric model building it is, however, not at all astonishing to use two Phillips curves in the place of only one in order to model the interacting dynamics of labor and goods market adjustment processes or the wage-price spiral for simplicity. Thus, for example, Fair (2000) has recently reconsidered the debate on the NAIRU from this perspective, though he still uses demand pressure on the market for labor as proxy for that on the market for goods (see Chiarella and Flaschel (2000) for a discussion of his approach).

In this chapter we, by contrast, start from a traditional approach to the discussion of the wage-price spiral which uses different measures for demand and cost pressure on the market for labor and on the market for goods and which distinguishes between temporary and permanent cost pressure changes. Despite its traditional background—not unrelated however to modern theories of wage and price setting, see Appendices A.2 and A.3—we are able to show that an important macrodynamic feedback mechanism can be detected in this type of wage-price spiral that has rarely been investigated in the theoretical as well as in the applied macroeconomic literature with respect to its implications for macroeconomic stability. For the U.S. economy we then show by detailed estimation, using the software package PcGets of Hendry and Krolzig (2001), that this feedback mechanism tends to be a destabilizing one. We finally demonstrate on this basis that a certain error correction term in the money-wage Phillips curve or a Taylor interest rate policy rules that is augmented by a wage gap term can dominate such instabilities when operated with sufficient strength.

3.1.2 Basic Macro Feedback Chains. A Reconsideration

The Mundell Effect

The investigation of destabilizing macrodynamic feedback chains has indeed never been at the center of interest of mainstream macroeconomic analysis, though knowledge about these feedback chains dates back to the beginning of dynamic Keynesian analysis. Tobin has presented summaries and modeling of such feedback chains on various occasions (see in particular Tobin (1975, 1980 and 1993). The well-known Keynes effect as well as Pigou effect are however often present in macrodynamic analysis, since they have the generally appreciated property of being stabilizing with respect to wage inflation as well as wage deflation. Also well-known, but rarely taken serious, is the so-called Mundell effect based the impact of inflationary expectations on investment as well as consumption demand. Tobin (1975) was the first who modeled this effect in a 3D dynamic framework (see Scarth (1996) for a textbook treatment of Tobin's approach). Yet, though an integral part of traditional Keynesian IS-LM-PC analysis, the role of the Mundell is generally played down as for example in Romer (1996, p. 237) where it only appears in the list of problems, but not as part of his presentation of traditional Keynesian theories of fluctuations in his Chap. 5.

Figure 3.1 provides a brief characterization of the destabilizing feedback chain underlying the Mundell effect. We consider here the case of wage and price inflation (though deflation may be the more problematic case, since there is an obvious downward floor to the evolution of the nominal rate of interest (and the working of the well-known Keynes effect) which, however, in the partial reasoning that follows is kept constant by assumption).

For a given nominal rate of interest, increasing inflation (caused by an increasing activity level of the economy) by definition leads to a decrease of the real rate of interest. This stimulates demand for investment and consumer durables even further and thus leads, via the multiplier process to further increasing economic activity in both the goods and the labor markets, adding

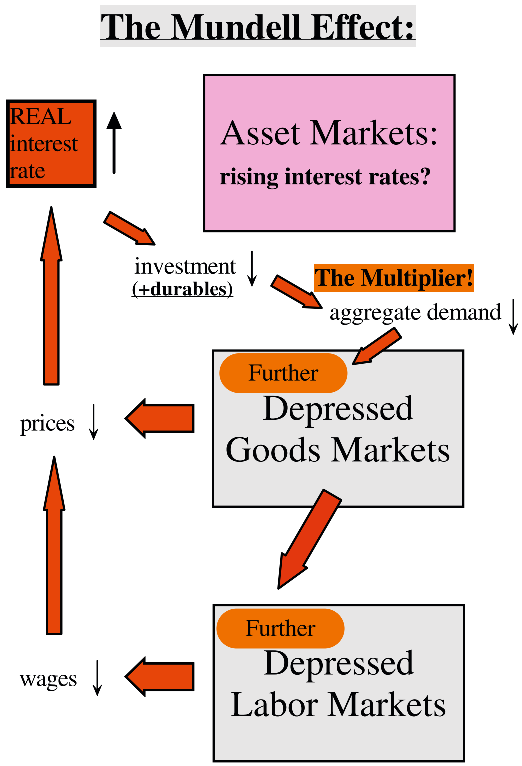

Fig. 3.1. Destabilizing Mundell effects

further momentum to the ongoing inflationary process. In the absence of ceilings to such an inflationary spiral, economic activity will increase to its limits and generate an ever accelerating inflationary spiral eventually. This standard feedback chain of traditional Keynesian IS-LM-PC analysis is however generally neglected and has thus not really been considered in its interaction with the stabilizing Keynes- and Pigou effect, with works based on the seminal paper of Tobin (1975) being the exception (see Groth 1993, for a brief survey on this type of literature).

Far more neglected is however an—in principle—fairly obvious real wage adjustment mechanism that was first investigated analytically in Rose (1967) with respect to its local and global stability implications (see also Rose 1990). Due to this heritage, this type of effect has been called Rose effect in Chiarella and Flaschel (2000), there investigated in its interaction with the Keynes- and the Mundell effect, and the Metzler inventory accelerator, in a 6D Keynesian model of goods and labor market disequilibrium. In the present chapter we intend to present and analyze the working of this effect in a very simple IS growth model—without the LM curve as in Romer (2000)—and thus with a direct interest rate policy in the place of indirect money supply targeting and its use of the Keynes effect (based on stabilizing shifts of the conventional LM-curve). We classify theoretically and estimate empirically the types of Rose effects that are at work, the latter for the case of the U.S. economy.

Stabilizing or Destabilizing Rose effects?

Rose effects are present if the income distribution is allowed to enter the formation of Keynesian effective demand and if wage dynamics is distinguished from price dynamics, both aspects of macrodynamics that are generally neglected at least in the theoretical macroeconomic literature. This may explain why Rose effects are rarely present in the models used for policy analysis and policy discussions.

Rose effects are however of great interest and have been present since long—though unnoticed and not in full generality—in macroeconometric model building, where wage and price inflation on the one hand and consumption and investment behavior on the other hand are generally distinguished from each other. Rose effects allow for at least four different cases depending on whether consumption demand responds stronger than investment demand to real wage changes (or vice versa) and whether—broadly speaking—wages are more flexible than prices with respect to the demand pressures on the market for labor and for goods, respectively. Figures 3.2 and 3.3 present two out of the four possible cases, all based on the assumption that consumption demand depends positively and investment demand negatively on the real wage (or the wage share if technological change is present).

In Fig. 3.2 we consider first the case where the real wage dynamics taken by itself is stabilizing. Here we present the case where wages are more flexible with respect to demand pressure (in the market for labor) than prices (with respect to demand pressure in the market for goods) and where investment responds stronger than consumption to changes in the real wage. We consider again the case of inflation. The case of deflation is of course of the same type with all shown arrows simply being reversed. Nominal wages rising faster than prices means that real wages are increasing when activity levels are high. Therefore, investment is depressed more than consumption is increased, giving rise to a decrease in aggregate as well as effective demand. The situation on the market for goods—and on this basis also on the market for labor—is

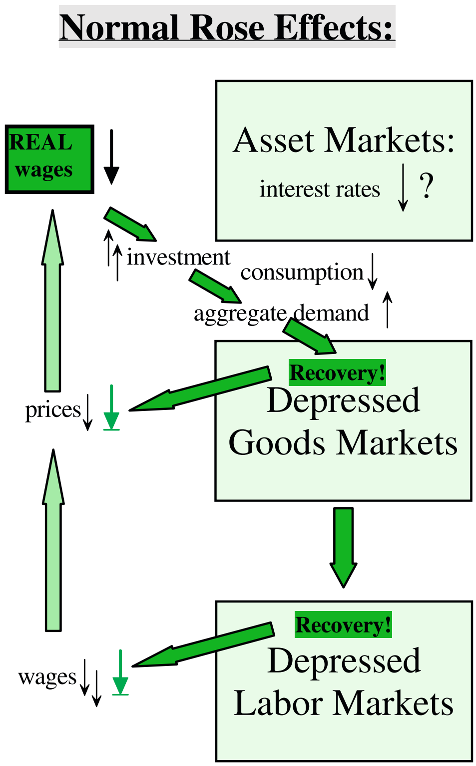

Fig. 3.2. Normal Rose effects

therefore deteriorating, implying that forces come into being that stop the rise in wages and prices eventually and that may—if investigated formally—lead the economy back to the position of normal employment and stable wages and prices.

The stabilizing forces just discussed however become destabilizing if price adjustment speeds are reversed and thus prices rising faster than nominal wages, see Fig. 3.3. In this case, we get falling real wages and thus—on the basis of the considered propensities to consume and invest with respect to real wage changes—further increasing aggregate and effective demand on the goods market which is transmitted into further rising employment on the market for labor and thus into even faster rising prices and (in weaker form) rising wages. This adverse type of real wage adjustment or simply adverse Rose effect can go on for ever if there is no nonlinearity present that modifies either investment or consumption behavior or wage and price adjustment speeds

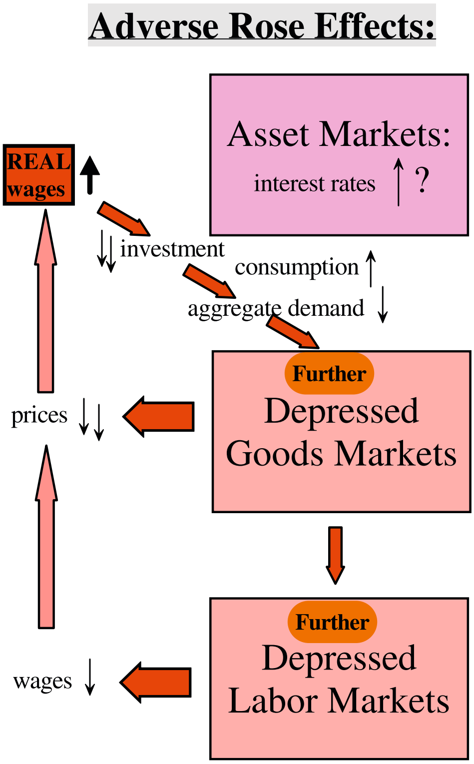

Fig. 3.3. Adverse Rose effects

such that normal Rose effects are established again, though of course supply bottlenecks may modify this simple positive feedback chain considerably.[2]

Since the type of Rose effect depends on the relative size of marginal propensities to consume and to invest and on the flexibility of wages vs. that of prices we are confronted with a question that demands for empirical estimation. Furthermore, Phillips curves for wages and prices have to be specified in more detail than discussed so far, in particular due to the fact that also cost pressure and expected cost pressure do matter in them, not only demand

[2] The type of Rose effect shown in Fig. 3.3 may be considered as the one that characterizes practical macro-wisdom which generally presumes that prices are more flexible than wages and that IS goods market equilibrium—if at all—depends negatively on real wages. Our empirical findings show that both assumptions are not confirmed, but indeed both reversed by data of the US economy, which taken together however continues to imply that empirical Rose effects are adverse in nature.

pressure on the market for goods and for labor. These specifications will lead to the result that also the degree of short-sightedness of wage earners and of firms will matter in the following discussion of Rose effects. Our empirical findings in this regard will be that wages are considerably more flexible than prices with respect to demand pressure, and workers roughly equally short-sighted as firms with respect to cost pressure. On the basis of the assumption that consumption is more responsive than investment to temporary real wage changes, we then get that all arrows and hierarchies shown in Fig. 3.3 will be reversed. We thus get by this twofold change in the Fig. 3.3 again *an adverse Rose effect* in the interaction of income distribution dependent changes in goods demand with wage and price adjustment speeds on the market for labor and for goods.

3.1.3 Outline of the Chapter

In view of the above hypothesis, the chapter is organized as follows. Section 3.2 presents a simple Keynesian macrodynamic model where advanced wage and price adjustment rules are introduced and in the center of the considered model and where—in addition—the income distribution and the real rate of interest matter in the formation of effective goods demand. We then investigate the stability implications of this macrodynamic model for the case of a stabilizing Rose effect resulting from the dominance of investment behavior in effective demand and the sluggishness of price dynamics and inflationary expectations. Since the steady growth path is found to be unstable even under the joint occurrence of stable Rose and weak Mundell effects, a standard type of interest rate policy rule[3] is therefore subsequently introduced to enforce convergence to the steady state, indeed also for fast revisions of inflationary expectations and thus stronger destabilizing Mundell effects. Section 3.3 investigates empirically whether the type of Rose effect assumed in Sect. 3.2 is really the typical one. We find evidence (in the case of the U.S. economy) that wages are indeed more flexible than prices. Increasing wage flexibility is thus bad for economic stability (while price flexibility is not) when coupled with the observation that consumption demand responds stronger than investment demand to temporary real wage changes.

[3] The discussion of such interest rate or Taylor policy rules originates from Taylor (1993), see Taylor (1999a), for a recent debate of such monetary policy rules and Clarida et al. (1998) for an empirical study of Taylor feedback rules in selected OECD countries.

In Sect. 3.4, this type of destabilizing Rose effect is then incorporated into our small macrodynamic model and the question of whether and which type of interest rate policy can stabilize the economy in such a situation is reconsidered. We find that a standard Taylor interest rate rule is not sufficient due to its specific tailoring that only allows to combat the Mundell type feedback chain—which it indeed can fight successfully. In case of a destabilizing Rose or real wage effect the tailoring of such a Taylor rule must be reflected again in order to find out what type of rule can fight such Rose effects. We here first reintroduce wage share effects considered by Blanchard and Katz (1999) into the money-wage Phillips curve which—when sufficiently strong—will stabilize a system operating under standard Taylor rule. Alternatively, however, the Taylor rule can be modified to include an income distribution term, which enforces convergence in the case where the wage share effect in the money wage Phillips curve is too weak to guarantee this.

We conclude that the role of income distribution in properly formulated wage-price spirals represents an important topic that is very much neglected in the modern discussion of inflation, disinflation and deflation.

3.2 A Model of the Wage–Price Spiral

This section briefly presents an elaborate form of the wage-price dynamics or the wage-price spiral and a simple theory of effective goods demand, which however gives income distribution a role in the growth dynamics derived from these building blocks. The presentation of this model is completed with respect to the budget equations for the four sectors of the model in the Appendix A.1 to this chapter. The wage-price spiral will be estimated, using U.S. data, in Sect. 3.3 of the chapter.

3.2.1 The Wage–Price Spiral

At the core of the dynamics to be modeled, estimated and analyzed in this and the following sections is the description of the money wage and price adjustment processes. They are provided by (3.1) and (3.2):

$$\hat{w} = \beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o) + \kappa_w(\hat{p} + n_x) + (1 - \kappa_w)(\pi + n_x), \tag{3.1}$$

$$\hat{p} = \beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(v - v_o) + \kappa_p(\hat{w} - n_x) + (1 - \kappa_p)\pi. \tag{3.2}$$

In these equations for wage inflation $\hat{w} = \dot{w}/w$ and price inflation $\hat{p} = \dot{p}/p$ we denote by U^l and U^c the rate of unemployment of labor and capital, respectively, and by n_x the rate of Harrod–neutral technological change. v is the wage share, $v = wL^d/pY$.

Demand pressure in the market for labor is characterized by deviations of the rate of unemployment U^l from its NAIRU level $\bar{U}^l$. Similarly demand pressure in the market for goods is represented by deviations of the rate of underemployment U^c of the capital stock K from its normal underemployment level $\bar{U}^c$, assumed to be fixed by firms. Wage and price inflation are therefore first of all driven by their corresponding demand pressure terms.

With respect to the role of the wage share u, which augments the Phillips curves by the terms $\beta_{w_2}(v - v_o)$ and $\beta_{p_2}(v - v_o)$, we assume that increasing shares will dampen the evolution of wage inflation and give further momentum to price inflation (see Franke 2006, for details of the effects of a changing income distribution on demand driven wage and price inflation). As far as the money-wage Phillips curve is concerned, this corresponds to the error-correction mechanism described in Blanchard and Katz (1999). In Appendix A.2, we motivate this assumption within a wage-bargaining model. A similar, though less strong formulation has been proposed by Ball and Mofitt (2001), who—based on fairness considerations—integrate the difference between productivity growth and an average of past real-wage growth in a wage-inflation Phillips curve.

In addition to demand pressure we have also cost–pressure terms in the laws of motions for nominal wages and prices, of crossover type and augmented by productivity change in the case of wages and diminished by productivity change in the case of prices. As the wage-price dynamics are formulated we assume that myopic perfect foresight prevails, of workers with respect to their measure of cost pressure, $\hat{p}$, and of firms with respect to wage pressure, $\hat{w}$. In this respect we follow the rational expectations school and disregard model–inconsistent expectations with respect to short-run inflation rates. Yet, in the present framework, current inflation rates are not the only measuring root for cost pressure, so they enter wage and price inflation only with weight $\kappa_w \in [0, 1]$ and $\kappa_p \in [0, 1]$, respectively, and $\kappa_w \kappa_p < 1$. In addition, both workers and firms (or at least one of them) look at the inflationary climate surrounding the perfectly foreseen current inflation rates.

A novel element in such cost-pressure terms is here given by the term π, representing the inflationary climate in which current inflation is embedded. Since the inflationary climate envisaged by economic agents changes sluggishly, information about macroeconomic conditions diffuses slowly through the economy (see Mankiw and Reis 2001), wage and price are set staggered (see Taylor 1999a), it is not unnatural to assume that agents, in the light of past inflationary experience, update π by an adaptive rule. In the theoretical model,[4] we assume that the medium-run inflation beliefs are updated adaptively in the standard way:

$$\dot{\pi} = \beta_{\pi}(\hat{p} - \pi). \tag{3.3}$$

In two Appendices A.2, A.3 we provide some further justifications for the two Phillips curves here assumed to characterize the dynamics of the wage and the price level. Note that the inflationary climate expression has often been employed in applied work by including lagged inflation rates in price Phillips curves, see Fair (2000) for example. Here however it is justified from the theoretical perspective, separating temporary from permanent effects, where temporary changes in both price and wage inflation are even perfectly foreseen. We show in this respect in Sect. 3.4 that the interdependent wage and price Phillips curves can however be solved for wage and price inflation explicitly, giving rise to two reduced form expressions where the assumed perfect foresight expressions do not demand for forward induction.

For the theoretical investigation, the dynamical equations (3.1)–(3.3) representing the laws of motion of w, p and π are part of a complete growth model to be supplemented by simple expressions for production, consumption and investment demand and—due to the latter—also by a law of motion for the capital stock. These equations will allow the discussion of so-called Mundell and Rose effects in the simplest way possible and are thus very helpful in isolating these effects from other important macrodynamic feedback chains which are not the subject of this chapter. The econometric analysis to be presented in the following section will focus on the empirical counterparts of the Phillips curves (3.1) and (3.2) while conditioning on the other macroeconomic variables which enter these equations.

3.2.2 Technology

In this and the next subsection we complete our model of the wage price spiral in the simples way possible to allow for the joint occurrence of Mundell and Rose effects in the considered economy.

[4] In the empirical part of this chapter we will simplify these calculations further by measuring the inflationary climate variable π as a 12 quarter moving average of $\hat{p}$.

For the sake of simplicity we employ in this chapter a fixed proportions technology:[5]

$$y^p = Y^p/K = \text{ const.}, \qquad x = Y/L^d, \qquad \hat{x} = \dot{x}/x = n_x = \text{ const.}$$

On the basis of this, the rates of unemployment of labor and capital can be defined as follows:

$$U^l = \frac{L - L^d}{L} = 1 - \frac{Y}{xL} = 1 - yk,$$
$$U^c = \frac{Y^p - Y}{Y^p} = 1 - \frac{Y}{Y^p} = 1 - y/y^p,$$

where y denotes the output-capital ratio Y/K and $k = K/(xL)$ a specific measure of capital-intensity or the full employment capital-output ratio. We assume Harrod–neutral technological change: $\hat{y}^p = 0, \hat{x} = n_x =$ const., with a given potential output-capital ratio y^p and labor productivity $x = Y/L^d$ growing at a constant rate. We have to use k in the place of K/L, the actual full employment capital intensity, in order to obtain state variables that allow for a steady state later on.

3.2.3 Aggregate Goods Demand

As far as consumption is concerned we assume Kaldorian differentiated saving habits of the classical type ($s_w = 1 - c_w = 1 - c \geq 0,\ s_c = 1$), i.e., real consumption is given by:

$$C = cvY = c\omega L^d, \quad v = \omega/x,\ \omega = w/p \text{ the real wage} \tag{3.4}$$

and thus solely dependent on the wage share v and economic activity Y. For the investment behavior of firms we assume

$$\frac{I}{K} = i_1\left((1-v)y - (i-\pi)\right) + n, \tag{3.5}$$

$$y = \frac{Y}{K}, \quad n = \hat{L} + \hat{x} = n + n_x \text{ trend growth.} \tag{3.6}$$

The rate of investment is therefore basically driven by the return differential between $\rho = (1-u)y$, the rate of profit of firms and $i - \pi$, the real rate of interest on long-term bonds (consols), only considered in its relation to the

[5] We neglect capital stock depreciation in this chapter.

budget restrictions of the four sectors of the model (workers, asset-holders, firms and the government) in Appendix A.1 to this chapter.[6]

This financial asset is needed for the generation of Mundell (or real rate of interest) effects in the model, which as we will show later can be neutralized by a Taylor-rule.

Besides consumption and investment demand we also consider the goods demand G of the government where we however for simplicity assume $g = G/K = \text{const.}$, since fiscal policy is not a topic of the present chapter.

3.2.4 The Laws of Motion

Due to the assumed demand behavior of households, firms and the government we have as representation of goods-market equilibrium in per unit of capital form ($y = Y/K$):

$$cvy + i_1((1-v)y - (i-\pi)) + n + g = y, \tag{3.7}$$

and as law of motion for the full-employment capital-output ratio $k = K/(xL)$:

$$\hat{k} = i_1((1-v)y - (i-\pi)). \tag{3.8}$$

Equations (3.1), (3.2) furthermore give in reduced form the two laws of motion (3.9), (3.10), with $\kappa = (1-\kappa_w\kappa_p)^{-1}$:

$$\begin{aligned}\hat{v} = \kappa\Big[&(1-\kappa_p)\Big\{\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o)\Big\}\\ &-(1-\kappa_w)\left\{\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(v - v_o)\right\}\Big],\end{aligned} \tag{3.9}$$

$$\hat{p} = \pi + \kappa\left[\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(v - v_o) + \kappa_p\Big\{\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o)\Big\}\right]. \tag{3.10}$$

The first equation describes the law of motion for the wage share u which depends positively on the demand pressure items on the market for labor

[6] We consider the long-term rate r as determinant of investment behavior in this chapter, but neglect here the short-term rate and its interaction with the long-term rate—as it is for example considered in Blanchard and Fischer (Sect. 10.4)—in order to keep the model concentrated on the discussion of Mundell and Rose effects. We thus abstract from dynamical complexities caused by the term structure of interest rates. Furthermore, we do not consider a climate expression for the evolution of nominal interest, in contrast to our treatment of inflation, in order to restrict the dynamics to dimension 3.

(for $\kappa_p < 1$) and negatively on those of the market for goods (for $\kappa_w < 1$).[7] The second equation is a reduced form price Phillips curve which combines all demand pressure related items on labor and goods market in a positive fashion (for $\kappa_p > 0$). This equation is far more advanced than the usual price Phillips curve of the literature.[8] Inserted into the adaptive revision rule for the inflationary climate variable it provides as further law of motion the dynamic equation

$$\dot{\pi} = \beta_\pi \kappa \left[\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(v - v_o)\right. \tag{3.11}$$

$$\left. + \kappa_p \left\{\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o)\right\}\right]. \tag{3.12}$$

We assume for the time being that the interest rate i on long-term bonds is kept fixed at its steady-state value i^o and then get that (3.8), (3.9) and (3.12), supplemented by the static goods market equilibrium equation (3.5), provide an autonomous system of differential equations in the state variables v, k and π.

It is obvious from (3.9) that the error correction terms β_{w_2}, β_{p_2} exercise a stabilizing influence on the adjustment of the wage share (when this dynamic is considered in isolation). The other two β-terms (the demand pressure terms), however, do not give rise to a clear-cut result for the wage share subdynamic. In fact, they can be reduced to the following expression as far as the influence of economic activity, as measured by y, is concerned (neglecting irrelevant constants):

$$\kappa \left[(1 - \kappa_p)\beta_{w_1} k - (1 - \kappa_w)\beta_{p_1}/y^p\right] \cdot y.$$

In the case where output y depends negatively on the wage share v we thus get partial stability for the wage share adjustment (as in the case of the error correction terms) if and only if the term in square brackets is negative (which is the case for β_{w_1} sufficiently large). We have called this a normal Rose effect in Sect. 3.1, which in the present case derives—broadly speaking—from investment sensitivity being sufficiently high and wage flexibility dominance.

[7] The law of motion (3.9) for the wage share u is obtained by making use in addition of the following reduced form equation for $\hat{w}$ which is obtained simultaneously with the one for $\hat{p}$ and of a very similar type:
$\hat{w} = \pi + \kappa \left[\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o) + \kappa_w \left\{\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(v - v_o)\right\}\right].$

[8] Note however that this reduced form Phillips curve becomes formally identical to the one normally investigated empirically, see Fair (2000) for example, if $\beta_{w_2}, \beta_{p_2} = 0$ holds and if Okun's law is assumed to hold (i.e. the utilization rates of labor and capital are perfectly correlated). However, even then the estimated coefficients are far away from representing labor market characteristics solely.

In the case where output y depends positively on v, where therefore consumption is dominating investment with respect to the influence of real wage changes, we need a large β_{p_1}, and thus a sufficient degree of price flexibility relative to the degree of wage flexibility, to guarantee stability from the partial perspective of real wage adjustments. For these reasons we will therefore call the condition[9]

$$\alpha = (1-\kappa_p)\beta_{w_1}k_o - (1-\kappa_w)\beta_{p_1}/y^p \begin{Bmatrix} < \\ > \end{Bmatrix} 0 \iff \begin{Bmatrix} \text{normal} \\ \text{adverse} \end{Bmatrix} \text{Rose effects}$$

the critical or α condition for the occurrence of *normal (adverse) Rose effects*, in the case where the flexibility of wages (of prices) with respect to demand pressure is dominating the wage-price spiral (including the weights concerning the relevance of myopic perfect foresight). In the next section we will provide estimates for this critical condition in order to see which type of Rose effect might have been the one involved in the business fluctuations of the U.S. economy in the post-war period.

Note finally with respect to (3.10) and (3.12) that $\dot{\pi}$ always depends positively on y and thus on π, since y always depends positively on π. This latter dependence of accelerator type as well as the role of wage share adjustments will be further clarified in the next subsection.

3.2.5 The Effective Demand Function

The goods-market equilibrium condition (3.7) can be solved for y and gives

$$y = \frac{n+g-i_1(i_o-\pi)}{(1-v)(1-i_1)+(1-c)v} = \frac{n+g+i_1(\pi-i_o)}{1-i_1+(i_1-c)v}. \tag{3.13}$$

We assume $i \in (0,1), c \in (0,1]$ and consider only cases where $v < 1$ is fulfilled which, in particular, is true close to the steady state. This implies that the output-capital ratio y depends positively on π. However, whether y is increasing or decreasing in the labor share v depends on the relative size of c and i_1. In the case of $c = 1$, we get the following dependencies:

$$y_v = \frac{(n+g-i_1(i_o-\pi))(i_1-1)}{[(1-v)(1-i_1)]^2} = \frac{y}{1-v},$$
$$\rho_v = -y-(1-v)y_v = 0.$$

[9] Note here that $1/k = xL/K$ and $y^p = Y^p/K$ are approximately of the same size, since full employment output and full capacity output generally do not depart too much from each other.

As long as y is positive and v smaller than one, we get a positive dependence of y on v. The rate of profit ρ is independent of the wage share v due to a balance between the negative cost and the positive demand effect of the wage share v.[10]

Otherwise, i.e. if the consumption propensity out of wage income is strictly less than one, $c < 1$, we have that

$$y_v = \frac{(c - i_1)y}{(1 - i_1)(1 - v) + (1 - c)v} \geq 0 \quad \text{iff} \quad c \geq i_1, \tag{3.14}$$

$$\rho_u = -y + (1 - v)y_v < 0, \tag{3.15}$$

where the result for the rate of profit $\rho = (1 - v)y$ of firms follows from the fact that y_v clearly is smaller than $y/(1 - v)$.

Therefore, if a negative relationship between the rate of return and the wage share is desirable (given the investment function defined in (3.8), then for the workers consumption function, the assumption $c < 1$ is required: $C/K = cvy, c \in (0, 1)$.

3.2.6 Stability Issues

We consider in this subsection the fully interacting, but somewhat simplified 3D growth dynamics of the model which consist the following three laws of motion (3.16)–(3.18) for the wage share v, the full employment capital-output ratio k and the inflationary climate π:[11]

$$\hat{u} = \kappa[(1 - \kappa_p)(\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o)) - (1 - \kappa_w)\beta_{p_1}(\bar{U}^c - U^c)], \tag{3.16}$$

$$\hat{k} = i_1((1 - v)y - (i - \pi)), \tag{3.17}$$

$$\dot{\pi} = \beta_\pi \kappa[\beta_{p_1}(\bar{U}^c - U^c) + \kappa_p(\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o))], \tag{3.18}$$

[10] We note that the investment function can be modified in various ways, for example by inserting the normal-capacity-utilization rate of profit $\rho^n = (1 - v)(1 - \bar{U}^c)y^p$ into it in the place of the actual rate ρ, which then always gives rise to a negative effect of u on this rate ρ^n and also makes subsequent calculations simpler. Note here also that we only pursue local stability analysis in this chapter and thus work for reasons of simplicity with linear functions throughout.

[11] We therefore now assume—for reasons of simplicity—that $\beta_{p_2} = 0$ holds throughout, a not very restrictive assumption in the light of what is shown in the remainder of this chapter. Note here that two of the three laws of motion (for the wage share and the inflationary climate) are originating from the wage-price spiral considered in this chapter, while the third one (for the capital output ratio) represents by and large the simplest addition possible to arrive at a model on the macro level that can be considered complete.

where $U^l = 1 - yk$ and $U^c = 1 - y/y^p$.

During this section, we will impose the following set of assumptions:[12]

- A.1 The marginal propensity to consume is strictly less than the one to invest: $0 < c < i_1$ (the case of a profit-led aggregate demand situation or briefly of a profit-led economy).
- A.2 The money-wage Phillips curve is not error-correcting w.r.t. the wage share: $\beta_{w_2} = 0$.
- A.3 The parameters satisfy that $v_o \in (0, 1)$ and $\pi_o \geq 0$ hold in the steady state.
- A.4a The nominal interest rate i is constant: $i = i_o$ (the case of an interest rate peg).
- A.4b There is an interest rate policy rule in operation which is of an active type:[13] $i = \rho_o + \pi + \phi_{ip}(\pi - \bar{\pi})$ with $\phi_{ip} > 0$, ρ_o the steady-state real rate of interest, and $\bar{\pi}$ the inflation target.

Assumption (A.1) implies that (i) $y_v < 0$ as in (3.14), (ii) $U_v^l > 0$ and $U_v^c > 0$ since the negative effect of real wage increases on investment outweighs the positive effect on consumption, and (iii) $\rho_v < 0$ with $\rho = (1 - v)y$ (the alternative scenario with $c > i_1$ is considered in Sect. 3.4). (A.2) excludes the potentially stabilizing effects of the Blanchard–Katz-type error-correction mechanism (will be discussed in Sect. 3.4.2 for the money-wage Phillips curve). (A.3) ensures the existence of an interior steady state. Assumptions (A.4a) and (A.4b) stand for different monetary regimes and determine the nominal interest rate in (3.17) and the algebraic equation for the effective demand which supplements the 3D dynamics.

For the neutral monetary policy defined in (A.4a), we have that output y is an increasing function of the inflationary climate π:

$$y = \frac{n + g + i_1(\pi - i_o)}{(1 - v)(1 - i_1) + (1 - c)v}. \tag{3.19}$$

By contrast, assumption (A.4b), the adoption of a Taylor interest rate policy rule, implies that the static equilibrium condition is given by

$$y = \frac{n + g - i_1(\rho_0 + \phi_{ip}(\pi - \bar{\pi}))}{(1 - i_1)(1 - v) + (1 - c)v}, \tag{3.20}$$

[12] In Sect. 3.4 we will relax these assumptions in various ways.

[13] I.e., the coefficient in front of the inflation climate expression is in sum larger than 1.

which implies a negative dependence of output y on the inflationary climate π.[14]

Proposition 3.1 (The Unique Interior Steady State Position). *Under assumptions* (A.1)–(A.4a)*, the interior steady state of the dynamics* (3.16)–(3.18) *is uniquely determined and given by*

$$y_0 = (1-\bar{U}^c)y^p, \quad k_0 = (1-\bar{U}^l)/y_o,$$
$$v_0 = 1/c + (n+g)/y_0, \quad \rho_o = (1-v_o)y_o.$$

Steady-state inflation in the constant nominal interest regime (A.4a) *is given by:*

$$\pi_o = i_o - (1-v_o)y_o,$$

and under the interest rule (A.4b) *we have that:*

$$\pi_o = \bar{\pi}, \quad i_o = \rho_o + \bar{\pi}$$

holds true.

The proof of Proposition 3.1 is straightforward. The proofs of the following propositions are in the mathematical Appendix A.5. ■

The steady state solution with constant nominal interest rate (A.4a) shows that the demand side has no influence on the long-run output-capital ratio, but influences the income distribution and the long-run rate of inflation. In the case of an adjusting nominal rate of interest (A.4b), the steady state rate of inflation is determined by the monetary authority and its steering of the nominal rate of interest, while the steady-state rate of interest is obtained from the steady rate of return of firms and the inflationary target of the central bank.

Proposition 3.2 (Private Sector Instability). *Under assumptions* (A.1)–(A.4a)*, the interior steady state of the dynamics* (3.16)–(3.18) *is essentially repelling (exhibits at least one positive root), even for small parameters* β_{p_1}, β_π*.*

[14] Note that our formulation of a Taylor rule ignores the influence of a variable representing the output gap. Including the capacity utilization gap of firms would however only add a positive constant to the denominator of the fraction just considered and would therefore not alter our results in a significant way. Allowing for the output gap in addition to the inflation gap may also be considered as some sort of double counting.

A normal Rose effect (stability by wage flexibility and instability by price flexibility in the considered case $c < i_1$) and a weak Mundell effect (sluggish adjustment of prices and of the inflationary climate variable) are thus not sufficient to generate convergence to the steady state.[15]

Proposition 3.3 (Interest Rate Policy and Stability). *Under assumptions* (A.1)–(A.3)*, the interest rule in* (A.4b) *implies asymptotic stability of the steady state for any given adjustment speeds* $\beta_\pi > 0$ *if the price flexibility parameter* β_{p_1} *is sufficiently small.*

As long as price flexibility does not give rise to an adverse Rose effect (dominating the trace of the Jacobian of the dynamics at the steady state), we get convergence to the steady state by monetary policy and the implied adjustments of the long-term real rate of interest $i - \pi$ which increase i beyond its steady state value whenever the inflationary climate exceeds the target value $\bar{\pi}$ and *vice versa*. The present stage of the investigation therefore suggests that wage flexibility (relative to price flexibility), coupled with the assumption $c > i_1$ and an active interest rate policy rule is supporting macroeconomic stability. The question however is whether this is the situation that characterizes factual macroeconomic behavior.

An adverse Rose effect (due to price flexibility and $c < i_1$) would dominate the stability implications of the considered dynamics: the system would then lose its stability by way of a Hopf-bifurcation when the reaction parameter ϕ_{ip} of the interest rate rule is made sufficiently small. However, we will find in the next section that wages are more flexible than prices with respect to demand pressure on their respective markets. We thus have in the here considered case $c < i_1$ that the Rose effect can be neglected (as not endangering economic stability), while the destabilizing Mundell effect can indeed be tamed by an appropriate monetary policy rule.

3.3 Estimating the U.S. Wage–Price Spiral

In this section we analyze U.S. post-war data to provide an estimate of the two Phillips curves that form the core of the dynamical model introduced in Sect. 3.2. Using *PcGets* (see Hendry and Krolzig 2001), we start with a general,

[15] In the mathematical Appendix A.5, it is shown that the carrier of the Mundell effect, $\dot{\pi}_\pi$, will always give the wrong sign to the determinant of the Jacobian of the dynamics at the steady state.

Table 3.1. Data

Variable	Transformation	Mnemonic	Description of the untransformed series
U^l	UNRATE/100	UNRATE	Unemployment Rate
U^c	1−CUMFG/100	CUMFG	Capacity Utilization: Manufacturing.Percent of Capacity
w	log(COMPNFB)	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, 1992=100
p	log(GDPDEF)	GDPDEF	Gross National Product: Implicit Price Deflator, 1992=100
$y - l^d$	log(OPHNFB)	OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons, 1992=100
v	$\log\left(\frac{\text{COMPRNFB}}{\text{OPHNFB}}\right)$	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour, 1992=100

Note that w, p, l^d, y, v now denote the logs of wages, prices, employed labor, output and the wage share (1992=1) so that first differences can be used to denote their rates of growth. Similar results are obtained when measuring the wage share as unit labor costs (nonfarm business sector) adjusted by the GNP deflator

dynamic, unrestricted, linear model of $\hat{w} - \pi_t$ and $\hat{p} - \pi_t$ which is conditioned on the explanatory variables predicted by the theory and use the general-to-specific approach to find an undominated parsimonious representation of the structure of the data. From these estimates, the long-run Phillips curves can be obtained which describe the total effects of variables and allow a comparison to the reduced form of the wage-price spiral in (3.1) and (3.2).

3.3.1 Data

The data are taken from the Federal Reserve Bank of St. Louis (see http://www.stls.frb.org/fred). The data are quarterly, seasonally adjusted and are all available from 1948:1 to 2001:2. Except for the unemployment rates of the factors labor, U^l, and capital, U^c, the log of the series are used (see Table 3.1).

For reasons of simplicity as well as empirical reasons, we measure the inflationary climate surrounding the current working of the wage-price spiral by an unweighted 12-month moving average:

$$\pi_t = \frac{1}{12} \sum_{j=1}^{12} \Delta p_{t-j}.$$

This moving average provides a simple approximation of the adaptive expectations mechanism (3.3) considered in Sect. 3.2, which defines the inflation climate as an infinite, weighted moving average of past inflation rates with declining weights. The assumption here is that people apply a certain window (twelve quarters) to past observations, here thus of size 12, without significantly discounting their observations.

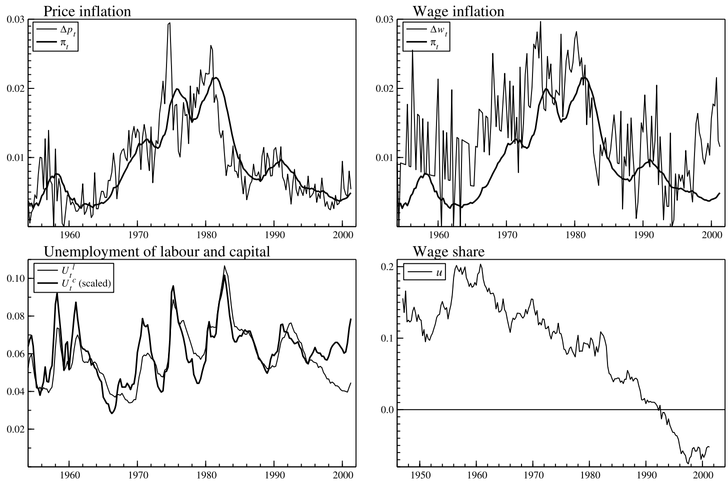

Fig. 3.4. Price and wage inflation, unemployment and the wage share

The data to be modeled are plotted in Fig. 3.4. The estimation sample is 1955:1–2001:2 which excludes the Korean war. The number of observations used for the estimation is 186.

3.3.2 The Money-Wage Phillips Curve

Let us first provide an estimate of the wage Phillips curve (3.1) of this chapter: We model wage inflation in deviation from the inflation climate, $\Delta w - \pi$, conditional on its own past, the history of price inflation, $\Delta p - \pi$, measured by the same type of deviations, overall labor productivity growth, $\Delta y - \Delta l^d$, the unemployment rate, U^l, and the log of the labor share, $v = w + l^d - p - y$, by means of (3.21):

$$\Delta w_t - \pi_t = \nu_w + \sum_{j=1}^{5} \gamma_{wwj} \left(\Delta w_{t-j} - \pi_{t-j}\right) + \sum_{j=1}^{5} \gamma_{wpj} \left(\Delta p_{t-j} - \pi_{t-j}\right) + \sum_{j=1}^{5} \gamma_{wxj} \left(\Delta y_{t-j} - \Delta l^d_{t-j}\right) + \sum_{j=1}^{5} \gamma_{wuj} U^l_{t-j} + \alpha_w v_{t-1} + \varepsilon_{wt}, \tag{3.21}$$

where ε_{wt} is a white noise process. The general model explains 43.7% of the variation of $\Delta w_t - \pi_t$ reducing the standard error in the prediction of quarterly changes of the wage level to 0.467%:

$$RSS = 0.0036, \quad \hat{\sigma} = 0.0047, \quad R^2 = 0.4373, \quad \bar{R}^2 = 0.3653,$$
$$\ln L = 1011, \quad AIC = -10.6298, \quad HQ = -10.4752, \quad SC = -10.2482.$$

Almost all of the estimated coefficients of (3.21) are statistically insignificant and therefore not reported here. This highlights the idea of the general-to-specific (*Gets*) approach (see Hendry 1995, for an overview of the underlying methodology) of selecting a more compact model, which is nested in the general but provides an improved statistical description of the economic reality by reducing the complexity of the model and checking the contained information. The *PcGets* reduction process is designed to ensure that the reduced model will convey all the information embodied in the unrestricted model (which is here provided by (3.21)). This is achieved by a joint selection and diagnostic testing process: starting from the unrestricted, congruent general model, standard testing procedures are used to eliminate statistically-insignificant variables, with diagnostic tests checking the validity of reductions, ensuring a congruent final selection.

In the case of the general wage Phillips curve in (3.21), *PcGets* reduces the number of coefficients from 22 to only 3, resulting in a parsimonious money-wage Phillips curve, which just consists of the demand pressure U^l_{t-1}, the cost pressure $\Delta p_{t-1} - \pi_{t-1}$ and a constant (representing the integrated effect of labor productivity and the NAIRU on the deviation of nominal wage growth from the inflationary climate),[16]

$$\Delta w_t - \pi_t = \underset{(0.00163)}{0.0158} + \underset{(0.101)}{0.266}\,(\Delta p_{t-1} - \pi_{t-1}) - \underset{(0.0271)}{0.193}\,U^l_{t-1}, \tag{3.22}$$

without losing any relevant information:

$$RSS = 0.0039, \quad \hat{\sigma} = 0.0046, \quad R^2 = 0.3755, \quad \bar{R}^2 = 0.3686,$$
$$\ln L = 1001, \quad AIC = -10.7297, \quad HQ = -10.7087, \quad SC = -10.6777.$$

An F test of the specific against the general rejects only at a marginal rejection probability of 0.5238. The properties of the estimated model (3.22) are illustrated in Fig. 3.5. The first graph (upper LHS) shows the fit of the model over time; the second graph (upper RHS) plots the fit against the actual values of $\Delta w_t - \pi_t$; the second graph (lower LHS) plots the residuals and the last graph (lower RHS) the squared residuals. The diagnostic test results shown in Table 3.2 confirm that (3.22) is a valid congruent reduction of the general model in (3.21).

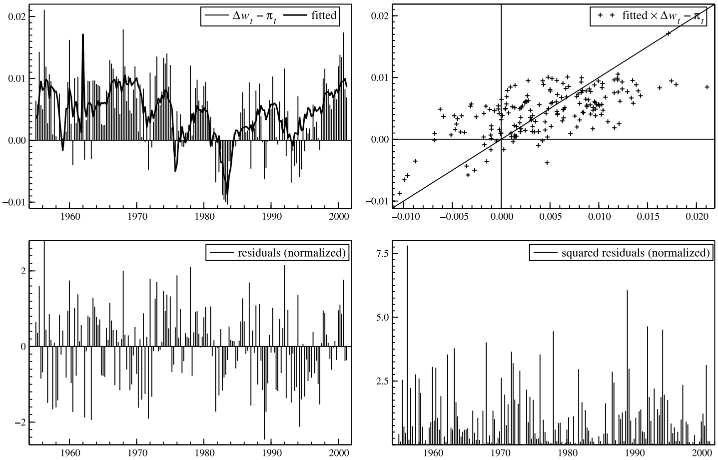

Fig. 3.5. Money wage Phillips curve

Table 3.2. Diagnostics

Diagnostic test	Wage Phillips curve				Price Phillips curve			
	(3.21)		(3.22)		(3.23)		(3.24)	
$F_{Chow(1978:2)}$	0.993	[0.5161]	0.866	[0.7529]	0.431	[0.9999]	0.421	[1.0000]
$F_{Chow(1996:4)}$	0.983	[0.4829]	0.771	[0.7315]	0.635	[0.8672]	0.551	[0.9288]
$\chi^2_{normality}$	0.710	[0.7012]	0.361	[0.8347]	0.141	[0.9322]	0.483	[0.7856]
$F_{AR(1-4)}$	1.915	[0.1105]	1.276	[0.2810]	2.426	[0.0503]	1.561	[0.1869]
$F_{ARCH(1-4)}$	1.506	[0.2030]	0.940	[0.4421]	1.472	[0.2133]	3.391	[0.0107]
F_{hetero}	0.615	[0.9634]	1.136	[0.3411]	0.928	[0.6072]	1.829	[0.0346]

Reported are the test statistic and the marginal rejection probability

With respect to the theoretical wage Phillips curve (3.1)

$$\hat{w} = \beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(v - v_o) + \kappa_w(\hat{p} + n_x) + (1 - \kappa_w)(\pi + n_x),$$

we therefore obtain the quantitative expression

$$\hat{w} = 0.0158 - 0.193U^l + 0.266\hat{p} + 0.734\pi.$$

We notice that the wage share and labor productivity do play no role in this specification of the money-wage Phillips curve. The result on the influence of the wage share is in line with the result obtained by Blanchard and Katz (1999) for the U.S. economy.

[16] We have $E(\hat{p} - \pi) = 0, E(\hat{w} - \pi) = 0.0045$ and $\bar{U}^l = E(U^l) = 0.058$.

3.3.3 The Price Phillips Curve

Let us next provide an estimate of the price Phillips curve (3.2) for the U.S. economy. We now model price inflation in deviation from the inflation climate, $\Delta p - \pi$, conditional on its own past, the history of wage inflation, $\Delta w - \pi$, overall labor productivity growth, $\Delta y - \Delta l^d$, the degree of capital under-utilization, U^c by means of (3.23), and the error correction term, u:

$$\Delta p_t - \pi_t = \nu_p + \sum_{j=1}^{5} \gamma_{ppj} \left(\Delta p_{t-j} - \pi_{t-j}\right) + \sum_{j=1}^{5} \gamma_{pwj} \left(\Delta w_{t-j} - \pi_{t-j}\right)$$
$$+ \sum_{j=1}^{5} \gamma_{pyj} \left(\Delta y_{t-j} - \Delta l^d_{t-j}\right) + \sum_{j=1}^{5} \gamma_{puj} U^c_t + \alpha_p v_{t-1} + \varepsilon_{pt}, \tag{3.23}$$

where ε_{pt} is a white noise process. The general unrestricted model shows no indication of misspecification (see Table 3.2) and explains a substantial fraction (63.8%) of inflation variability. Also note that the standard error of the price Phillips curve is just half the standard error in the prediction of changes in the wage level, namely 0.259%:

$$RSS = 0.0012, \quad \hat{\sigma} = 0.0026, \quad R^2 = 0.6376, \quad \bar{R}^2 = 0.5810,$$
$$\ln L = 1122, \quad AIC = -11.7843, \quad HQ = -11.6015, \quad SC = -11.3334.$$

There is however a huge outlier ($\hat{\varepsilon}_{pt} > 3\hat{\sigma}$) associated with the oil price shock in 1974 (3) so a centered impulse dummy, I(1974:3), was included.

Here, the model reduction process undertaken by *PcGets* limits the number of coefficients to 9 (while starting again with 22) and results in the following price Phillips curve:

$$\Delta p_t - \pi_t = \underset{(0.0011)}{0.0046} + \underset{(0.0413)}{0.12} \left(\Delta w_{t-1} - \pi_{t-1}\right) + \underset{(0.0397)}{0.0896} \left(\Delta w_{t-3} - \pi_{t-3}\right)$$
$$+ \underset{(0.0691)}{0.254} \left(\Delta p_{t-1} - \pi_{t-1}\right) + \underset{(0.0653)}{0.196} \left(\Delta p_{t-4} - \pi_{t-4}\right)$$
$$- \underset{(0.0634)}{0.18} \left(\Delta p_{t-5} - \pi_{t-5}\right) - \underset{(0.0232)}{0.0467} \left(\Delta y_{t-1} - \Delta l^d_{t-1}\right)$$
$$- \underset{(0.0055)}{0.0287} U^c_{t-1} + \underset{(0.0026)}{0.0099} \mathrm{I}(1974{:}3)_t, \tag{3.24}$$

$$RSS = 0.0012, \quad \hat{\sigma} = 0.0026, \quad R^2 = 0.6074, \quad \bar{R}^2 = 0.5897,$$
$$\ln L = 1114, \quad AIC = -11.8870, \quad HQ = -11.8238, \quad SC = -11.7309.$$

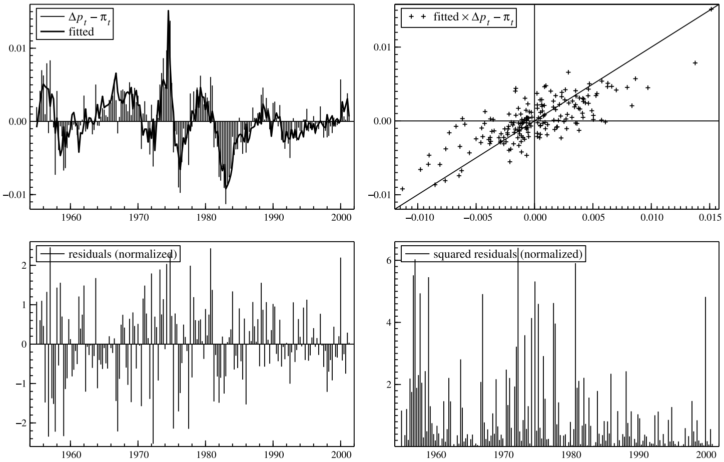

Fig. 3.6. Price Phillips curve

The reduction is accepted at a marginal rejection probability of 0.7093. The fit of the model and the plot of the estimation errors are displayed in Fig. 3.6.

The long-run price Phillips curve implied by (3.24) is given by:

$$\begin{aligned}\Delta p - \pi = \underset{(0.0016)}{0.0063} + \underset{(0.0608)}{0.286}\,(\Delta w - \pi) - \underset{(0.0305)}{0.064}\,\left(\Delta y - \Delta l^d\right)\\ -\underset{(0.0079)}{0.0393}U^c + \underset{(0.00366)}{0.0135}\,\mathrm{I}(1974{:}3). \qquad (3.25)\end{aligned}$$

With respect to the theoretical price Phillips curve

$$\hat{p} = \beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(v - v_o) + \kappa_p(\hat{w} - n_x) + (1 - \kappa_p)\pi,$$

we therefore obtain the quantitative expression

$$\hat{p} = 0.006 - 0.039U^c + 0.286\hat{w} + 0.714\pi,$$

where we ignore the dummy and the productivity term in the long-run Phillips curve.[17] We notice that the wage share and labor productivity do again play no role in this specification of the money-wage Phillips curve. The result that demand pressure matters more in the labor market than in the goods market is in line with what is observed in Carlin and Soskice (1990, Sect. 18.3.1), and the result that firms are (slightly) more short-sighted than workers may be due to the smaller importance firms attach to past observations of wage inflation.

[17] From the perspective of the theoretical equation just shown this gives by calculating the mean of U^c the values $\bar{U}^c = 0.18, n_x = 0.004$.

3.3.4 System Results

So far we have modeled the wage and price dynamics of the system by analyzing one equation at a time. In the following we check for the simultaneity of the innovations to the price and wage inflation equations. The efficiency of a single-equation model reduction approach as applied in the previous subsection depends on the absence of instantaneous causality between $\Delta p_t - \pi_t$ and $\Delta w_t - \pi_t$ (see Krolzig 2003). This requires the diagonality of the variance-covariance matrix Σ when the two Phillips curves are collected to the system

$$z_t = \sum_{j=1}^{5} A_j z_{t-j} + Bq_t + \epsilon_t, \tag{3.26}$$

which represents $z_t = (\Delta p_t - \pi_t, \Delta w_t - \pi_t)'$ as a fifth-order vector autoregressive (VAR) process with the vector of the exogenous variables $q_t = (1, U^c_{t-1}, U^l_{t-1}, \Delta y_{t-1} - \Delta l^d_{t-1}, \mathrm{I}(1974:3))'$ and the null-restrictions found by *PcGets* being imposed. Also, ϵ_t is a vector white noise process with $\mathsf{E}[\epsilon_t \epsilon_t'] = \Sigma$.

Estimating the system by FIML using PcGive10 (see Hendry and Doornik 2001) gives almost identical parameter estimates (not reported here) and a log-likelihood of the system of 1589.34. The correlation of structural residuals in the $\Delta w - \pi$ and $\Delta p - \pi$ equation is just 0.00467, which is clearly insignificant.[18] Further support for the empirical Phillips curves (3.22) and (3.24) comes from a likelihood ratio (LR) test of the over-identifying restrictions imposed by *PcGets*. With $\chi^2(44) = 46.793[0.3585]$, we can accept the reduction. The presence of instantaneous non-causality justifies the model reduction procedure employed here, which was based on applying *PcGets* to each single equation in a turn.

The infinite-order vector moving average representation of the system corresponding to the system in (3.26) is given by

$$z_t = \sum_{j=0}^{\infty} \Psi_j B q_{t-j} + \sum_{j=0}^{\infty} \Psi_j \epsilon_{t-j}, \tag{3.27}$$

where $\Psi(L) = A(L)^{-1}$ and L is the lag operator. By accumulating all effects, $z = A(1)^{-1}Bq$, we get the results in Table 3.3.

[18] Note that under the null hypothesis, the FIML estimator of the system is given by OLS. So we can easily construct an LR test of the hypothesis $\Sigma_{12} = \Sigma_{21} = 0$. As the log-likelihood of the system under the restriction is 1587.51. Thus the LR test of the restriction can be accepted with $\chi^2(1) = 3.6554[0.0559]$.

Table 3.3. Static long run solution

	Constant	U^c	U^l	$\Delta y - \Delta l^d$	I(1974:3)
$\Delta w - \pi$	0.0189	−0.0113	−0.2093	−0.0184	+0.0039
	(0.1109)	(0.0090)	(0.0300)	(0.0028)	(0.0011)
$\Delta p - \pi$	0.0118	−0.0426	−0.0600	−0.0693	+0.0146
	(0.0680)	(0.0340)	(0.0228)	(0.0105)	(0.0040)
$\Delta w - \Delta p$	0.0071	+0.0313	−0.1493	+0.0509	−0.0107

Derived from the FIML estimates of the system in (3.26)

Note here that all signs are again as expected, but that the estimated parameters are now certain compositions of the β, κ terms and are in line with the values of these parameters reported earlier. Taking into account all dynamic effects of U^l and U^c on wage and price inflation, real wage growth reacts stronger on the under-utilization of the factor labor U^l than of the factor capital U^c.

3.3.5 Are There Adverse Rose Effects?

The wage Phillips curve in (3.22) and the price Phillips curve in (3.24) can be solved for the two endogenous variables $\hat{w}$ and $\hat{p}$. The resulting reduced form representation of these equations is similar to (3.9) and (3.10), but for wages and prices and simplified due to the eliminated Blanchard–Katz-type error correction terms (*i.e.*, $\beta_{w_2} = \beta_{p_2} = 0$):

$$\hat{w} - \pi = \kappa \left[\beta_{w_1}(\bar{U}^l - U^l) + \kappa_w \beta_{p_1}(\bar{U}^c - U^c)\right], \tag{3.28}$$

$$\hat{p} - \pi = \kappa \left[\beta_{p_1}(\bar{U}^c - U^c) + \kappa_p \beta_{w_1}(\bar{U}^l - U^l)\right], \tag{3.29}$$

with $\kappa = (1 - \kappa_{w_1}\kappa_{p_1})^{-1}$.

For the US economy, we found that wages reacted stronger to demand pressure than prices ($\beta_{w_1} > \beta_{p_1}$), that β_{w_2}, β_{p_2} and wage share influences as demand pressure corrections could be ignored (as assumed in Sect. 3.2) and that wage-earners are roughly equally short-sighted as firms ($\kappa_w \approx \kappa_p$). Furthermore, using the FIML estimates of the static long run solution of system $\hat{w} - \pi$, $\hat{p} - \pi$ reported in Table 3.3, we have the following empirical equivalents of (3.28) and (3.29):

$$\hat{w} - \pi \approx 0.019 - 0.209U^l - 0.011U^c, \tag{3.30}$$

$$\hat{p} - \pi \approx 0.012 - 0.060U^l - 0.043U^c, \tag{3.31}$$

where we abstract from the dummy and productivity term.

These calculations imply with respect to the critical condition (α) derived in Sect. 3.2,

$$\alpha = (1-\kappa_p)\beta_{w_1} k_o - (1-\kappa_w)\beta_{p_1}/y^p \approx 0.714 \cdot 0.209 - 0.734 \cdot 0.043 \approx 0.118 > 0, \tag{3.32}$$

if we assume that $k = K/(xL)$ and $1/y^p = K/Y^p$ are ratios of roughly similar size, which is likely since full-employment output should be not too different from full-capacity output at the steady state.

Hence, the Rose effect will be of adverse nature if the side-condition $i < c$ is met. For the U.S., this condition has been investigated in Flaschel et al. (2001) in a somewhat different framework (see Flaschel et al. 2002, for the European evidence). Their estimated investment parameter i_1 is 0.136, which should be definitely lower than the marginal propensity to consume out of wages.[19] Thus the real wage or Rose effect is likely to be adverse. In addition to what is known for the real rate of interest rate channel and the Mundell effect, increasing wage flexibility might add further instability to the economy. Advocating more wage flexibility may thus not be as unproblematic as it is generally believed.

Given the indication that the U.S. wage-price spiral is characterized by adverse Rose effects, the question arises which mechanisms stabilized the U.S. economy over the post-war period by taming this adverse real wage feedback mechanism. Some aspects of this issue will be theoretically investigated in the remainder of the chapter. But a thorough analysis from a global point of view must be left for future theoretical and empirical research on core nonlinearities possibly characterizing the evolution of market economies.

The results obtained show that (as long as goods demand depends positively on the wage share) the wage-price spiral in its estimated form is unstable as the critical condition (α) creates a positive feedback of the wage share on its rate of change. We stress again that the innovations for obtaining such a result

[19] In the context of our model, one might want to estimate the effective demand function $y = [(n+g-i_1(i_o-\pi)]/[(1-v)(1-i_1)+(1-c)v]$. In view of the local approach chosen, it would in fact suffice to estimate a linear approximation of the form $y = a_0 + a_1 v + a_2(i-\pi)$, where $\text{sign}(a_1) = \text{sign}(c-i_1)$ and $a_2 < 0$ holds. However, in preliminary econometric investigations, we found a_1 being statistically insignificant so that no conclusions could be drawn regarding the sign of $c-i$.

are the use of two measures of demand pressure and the distinction between temporary and permanent cost pressure changes (in a cross-over fashion) for the wage and price Phillips curves employed in this chapter.

3.4 Wage Flexibility, Instability and an Extended Interest Rate Rule

In Sect. 3.2, we found that a sufficient wage flexibility supports economic stability. The imposed assumption $c < i$ ensured that the effective demand and thus output are decreased by a rising wage share; thus deviations from the steady-state equilibrium, are corrected by the normal reaction of the real wage to activity changes. In contrast, sufficiently flexible price levels (for given wage flexibility) result in an adverse reaction of the wage share, since a rising wage share stimulate further increases via output contraction and deflation.

Motivated by the estimation results presented in the preceding section, we now consider the situation where $c > i_1$ and $\alpha > 0$ holds true with respect to the critical Rose condition (α). The violation of the critical condition implies that $\hat{v}$ depends positively on y. In connection with $c > i_1$, i.e., $y_v > 0$ it generates a positive feedback from the wage share v onto its rate of change $\hat{v}$. Thus sufficiently strong wage flexibility (relative to price flexibility) is now destabilizing. This is the adverse type of Rose effect.

3.4.1 Instability Due to an Unmatched Rose Effect

Here we consider the simplified wage-price dynamics (3.16)–(3.18) under the assumption $i_1 < c$ instead of (A.1). If, in the now considered situation, monetary policy is still inactive (A.4a), the Rose effect and the Mundell effect are both destabilizing the private sector of the economy:

Proposition 3.4 (Private Sector Instability). *Assume* $i_1 < c$, *i.e.,* $y_v > 0$, *i.e., an economy that is now wage-led,* $\alpha > 0$ *and* $\kappa_p < 1$. *Then, under the assumptions* (A.2)–(A.4a) *introduced earlier, the interior steady-state solution of the dynamics* (3.16)–(3.18) *is essentially repelling (exhibits at least one positive root).*

Let us consider again to what extent the interest rate policy (A.4b) can stabilize the economy and in particular enforce the inflationary target $\bar{\pi}$. We state here without proof that rule (A.4b) can stabilize the previously considered situation if the adjustment speed of wages with respect to demand pressure in the labor market is sufficiently low. However, this stability gets lost if wage flexibility is made sufficiently large as is asserted by the following proposition, where we assume $\kappa_p = 0$ for the sake of simplicity.

Proposition 3.5 (Instability by an Adverse Rose Effect). *We assume (in the case $i_1 < c$) an attracting steady-state situation due to the working of the monetary policy rule* (A.4b). *Then: Increasing the parameter β_{w_1} that characterizes wage adjustment speed will eventually lead to instability of the steady state by way of a Hopf bifurcation (if the parameters κ_p, i_1, ϕ_{ip} are jointly chosen sufficiently small). There is no reswitching to stability possible, once stability has been lost in this way.*

Note that the proposition does not claim that there is a wage adjustment speed which implies instability for any parameter value ϕ_{ip} in the interest rate policy rule. It is also worth noting that the instability result is less clear-cut when for example $\kappa_p > 0$ is considered. Furthermore, increasing the adjustment speed ϕ_{ip} may reduce the dynamic instability in the case $\kappa_p = 0$ (as the trace of the Jacobian is made less positive thereby). In the next subsection we will however make use of another stabilizing feature which we so far neglected in the considered dynamics due to assumption (A.2): the Blanchard Katz error correction term $\beta_{w_2}(v - v_o)$ in the money-wage Phillips curve.

3.4.2 Stability from Blanchard–Katz Type "Error Correction"

We now analyze dynamics under the assumption $\beta_{w_2} > 0$. Thus money wages react to deviations of the wage share from its steady-state value. In this situation the following proposition holds true:

Proposition 3.6 (Blanchard–Katz Wage Share Correction). *Assume $i_1 < c$, i.e., $y_v > 0$, $\alpha > 0$ and $\kappa_p < 1$. Then, under the interest rate policy rule* (A.4b), *a sufficiently large error correction parameter β_{w_2} implies an attracting steady state for any given adjustment speed $\beta_\pi > 0$ and all price flexibility parameters $\beta_{p_1} > 0$. This stability is established by way of a Hopf bifurcation which in a unique way separates unstable from stable steady-state solutions.*

We thus have the result that the Blanchard–Katz error correction term if sufficiently strong overcomes the destabilizing forces of the adverse Rose effect in Proposition 3.5.

Blanchard and Katz (1999) find that the error correction term is higher in European countries than in the U.S., where it is also in our estimates insignificant. So the empirical size of the parameter β_{w_2} may be too small to achieve the stability result of Proposition 3.6. Therefore, we will again disregard the error correction term in the money-wage Phillips curve (A.2) in the following, and instead focus on the role of monetary policy in stabilizing the wage-price spiral.

3.4.3 Stability from an Augmented Taylor Rule

The question arises whether monetary policy can be of help to avoid the problematic features of the adverse Rose effect. Assume now that there interest rates are determined by an augmented Taylor rule of the form,

$$i = \rho_o + \pi + \beta_{r_1}(\pi - \bar{\pi}) + \beta_{r_2}(v - v_o), \quad \beta_{r_1}, \beta_{r_2} > 0, \tag{3.33}$$

where the monetary authority responds to rising wage shares by interest rate increases in order to cool down the economy, counter-balancing the initial increase in the wage share.

The static equilibrium condition is now given the

$$y = \frac{n + g - i_1(\rho_0 + \beta_{r_1}(\pi - \bar{\pi}) + \beta_{r_2}(v - v_o))}{(1 - i_1)(1 - v) + (1 - c)v}.$$

Thus the augmented Taylor rule (3.33) gives rise to a negative dependence of output y on the inflationary climate π as well as the wage share v.

We now consider the implications for the stability of the steady state:

Proposition 3.7 (Wage-Gap Augmented Taylor Rule). *Assume $i_1 < c$, $\alpha > 0$ and $\kappa_p < 1$. Then: A sufficiently large wage-share correction parameter β_{r_2} in the augmented Taylor rule (3.33) implies an attracting steady state for any given adjustment speed $\beta_\pi > 0$ and all price flexibility parameters $\beta_{p_1} > 0$. This stability is established by way of a Hopf bifurcation which in a unique way separates unstable from stable steady-state solutions.*

Thus, convergence to the balanced growth path of private sector of the considered economy is generated by a modified Taylor rule that is augmented by a term that transmits increases in the wage share to increases in the nominal rate of interest. To our knowledge such an interest rate policy rule that gives income policy a role to play in the adjustment of interest rates by the central bank has not yet been considered in the literature. This is due to the general neglect of adverse real wage or Rose effects which induce an inflationary spiral independently from the one generated by the real rate of interest or Mundell effect, though both of these mechanisms derive from the fact that real magnitudes always allow for two interacting channels by their very definition, wages versus prices in the case of Rose effects and nominal interest versus expected inflation in the case of Mundell effects.

3.5 Conclusions

In context of the "Goldilocks economy" of the late 1990s, Gordon (1998) stressed the need for explaining the contrast between decelerating prices and accelerating wages as well as the much stronger fall of the rate of unemployment than the rise of the rate of capital utilization. The coincidence of the two events is exactly what our approach to the wage-price spiral would predict: wage inflation is driven by demand and cost pressures on the labor market and price inflation is formed by the corresponding pressures on the goods markets.

Based on the two Phillips curves, we investigated two important macrodynamic feedback chains in a simple growth framework: (i) the conventional destabilizing Mundell effect and (ii) the less conventional Rose effect, which has been fairly neglected in the literature on demand and supply driven macrodynamics. We showed that the Mundell effect can be tamed by a standard Taylor rule. In contrast, the Rose effect can assume four different types depending on wage and price flexibilities, short-sightedness of workers and firms with respect to their cost-pressure measures and marginal propensities to consume c and invest i_1 in particular (where we argued for the inequality $i_1 < c$, i.e., a wage-led situation). The following table summarizes these four cases in a compact way:

Table 3.4. Four scenarios for the real wage channel

Critical α-condition	Profit-led regime	Wage-led regime
$\alpha < 0$	Unstable	Stable
$\alpha > 0$	Stable	Unstable

Empirical estimates for the U.S.-economy then suggested the presence of adverse Rose effects: the wage level is more flexible than the price level with respect to demand pressure (and workers roughly equally short-sighted as firms with respect to cost pressure). We showed that this particular Rose effect can cause macroeconomic instabilities which can not be tamed by a conventional Taylor rule. But this chapter also demonstrated means by which adverse real interest rate and real wage rate effects may be modified or dominated in such a way that convergence back to the interior steady state is again achieved. We proved that stability can be re-established by (i) an error-correction term in the money-wage Phillips curve (as in Blanchard and Katz 1999),[20] working with sufficient strength, or (ii) a modified Taylor rule with monetary policy monitoring the labor share (or real unit labor costs) and reacting in response to changes in the income distribution.

In this chapter, we showed that adverse Rose effects are of empirical importance, and indicated ways of how to deal with them by wage or interest rate policies. In future research, we intend to discuss the role of Rose effects for high and low growth phases separately, taken account of the observation that money wages may be more rigid in the latter phases than in the former ones (see Hoogenveen and Kuipers 2000, for a recent empirical confirmation of such differences and Flaschel et al. 2003, for its application to a 6D Keynesian macrodynamics). The existence of a "kink" in the money-wage Phillips curve should in fact increase the estimated) wage flexibility parameter further (in the case where the kink is not in operation). Furthermore, the robustness of the empirical results should be investigated (say, by analyzing the wage-price spiral in other OECD countries). Finally, more elaborate models have to be considered to understand the feedback mechanisms from a broader perspective (see Flaschel et al. 2001, 2002 for first attempts of the dynamic AS–AD variety).

[20] The related error correction in the price Phillips curve should allow for the same conclusion, but has been left aside here due to space limitations.

Appendix A

A.1 The Sectoral Budget Equations of the Model

For reasons of completeness, we here briefly present the budget equations of our four types of economic agents (see Sargent 1987, Chap. 1, for a closely related presentation of such budget equations, there for the sectors of the conventional AS–AD growth model). Consider the following scenario for the allocation of labor, goods and assets:

$$cvpY + \dot{B}^d = vpY + \bar{i}B \text{ (workers: consumption out of} \tag{A.1}$$

$$\text{wage income and saving deposits)} \tag{A.2}$$

$$p_b\dot{B}^d + p_e\dot{E}^d = B + (1-v)pY \text{ (asset–holders: bond and} \tag{A.3}$$

$$\text{equity holdings)} \tag{A.4}$$

$$pI = p_e\dot{E} \text{ (firms: equity financed investment)} \tag{A.5}$$

$$\bar{i}B + B + pG = \dot{B} + p_b\dot{B} \text{ (government: debt financed} \tag{A.6}$$

$$\text{consumption)} \tag{A.7}$$

where $g = G/K = \text{const}$. In these budget equations we use a fixed interest rate $\bar{r}$ for the saving deposits of workers and use—besides equities—perpetuities (with price $p_b = 1/i$) for the characterization of the financial assets held by asset-holders. Due to this choice, and due to the fact that investment was assumed to depend on the long-term expected real rate of interest, we had to specify the Taylor rule in terms of i in the body of the chapter. These assumptions allow to avoid the treatment of the term structure of interest rate which would make the model considerably more difficult and thus the analysis of Mundell or Rose effects more advanced, but also less transparent. For our purposes the above scenario is however fully adequate and very simple to implement.

Furthermore, we denote in these equations the amount of saving deposits of workers by B (and assume a fixed interest rate $\bar{i}$ on these saving deposits). Outstanding bonds (consols or perpetuities) are denoted by B and have as their price the usual expression $p_b = 1/i$. We finally use p_e for the price of shares or equities E. These equations are only presented for consistency reasons here and they immediately imply

$$p(Y - C - I - G) = (\dot{B}^d - \dot{B}) + p_b(\dot{B}^d - \dot{B}) + p_e(\dot{E}^d - \dot{E}) = 0.$$

We have assumed goods–market equilibrium in this chapter and assume in addition that all saving deposits of workers are channeled into the government sector ($\dot{B}^d = \dot{B}$). We thus can also assume equilibrium in asset market flows via a perfect substitute assumption (which determines p_e, while p_b is determined by an appropriate interest rate policy rule in this chapter). Note that firms are purely equity financed and pay out all profits as dividends to the sector of asset holders. Note also that long-term bonds per unit of capital $b = B/(pK)$ will follow the law of motion

$$\dot{b} = r(b + g - s_w vy) - (\hat{p} + \hat{K})b$$

which—when considered in isolation (all other variables kept at their steady-state values)—implies a stable evolution of such government debt b towards a steady-state value for this ratio if $i^o - \hat{p}_o = \rho_o < n$ holds true. Since fiscal policy is not our concern in this chapter we only briefly remark that this is the case for government expenditure per unit of capital that is chosen sufficiently small:

$$g < \frac{nv_o(1-c)}{1-v_o}.$$

Similarly, we have for the evolution of savings per unit of capital $b = B/(pK)$ the law of motion

$$\dot{b} = s_w vy + (\bar{i} - (\hat{p} + \hat{K}))b$$

which—when considered in isolation—implies convergence to some finite steady-state value if $\bar{r} < \hat{p}_o + n$ holds true. Again, since the Government Budget Restraint is not our concern in this chapter, we have ignored this aspect of our model of wage-price and growth dynamics.

A.2 Wage Dynamics: Theoretical Foundation

This subsection builds on the paper by Blanchard and Katz (1999) and briefly summarizes their theoretical motivation of a money-wage Phillips curve which is closely related to our dynamic equation (3.1).[21] Blanchard and Katz assume—following the suggestions of standard models of wage setting—that real wage expectations of workers, $\omega^e = w_t - p_t^e$, are basically determined by the reservation wage, $\bar{\omega}_t$, current labor productivity, $y_t - l_t^d$, and the rate of unemployment, U_t^l:

[21] In this section, lower case letters (including w and p) indicate logarithms.

$$\omega_t^e = \theta\bar{\omega}_t + (1-\theta)(y_t - l_t^d) - \beta_w U_t^l.$$

Expected real wages are thus a Cobb-Douglas average of the reservation wage and output per worker, but are departing from this normal level of expectations by the state of the demand pressure on the labor market. The reservation wage in turn is determined as a Cobb-Douglas average of past real wages, $\omega_{t-1} = w_{t-1} - p_{t-1}$, and current labor productivity, augmented by a factor $a < 0$:

$$\bar{\omega}_t = a + \lambda\omega_{t-1} + (1-\lambda)(y_t - l_t^d).$$

Inserting the second into the first equation results in

$$\omega_t^e = \theta a + \theta\lambda\omega_{t-1} + (1-\theta\lambda)(y_t - l_t^d) - \beta_w U_t^l,$$

which gives after some rearrangements

$$\begin{aligned}\Delta w_t &= p_t^e - p_{t-1} + \theta a - (1-\theta\lambda)[(w_{t-1} - p_{t-1}) - (y_t - l_t^d)] - \beta_w U_t^l \\ &= \Delta p_t^e + \theta a - (1-\theta\lambda)v_{t-1} + (1-\theta\lambda)(\Delta y_t - \Delta l_t^d) - \beta_w U_t^l,\end{aligned}$$

where Δp_t^e denotes the expected rate of inflation, v_{t-1} the past (log) wage share and $\Delta y_t - \Delta l_t^d$ the current growth rate of labor productivity. This is the growth law for nominal wages that flows from the theoretical models referred to in Blanchard and Katz (1999, p. 70).

In this chapter, we proposed to operationalize this theoretical approach to money-wage inflation by replacing the short-run cost push term Δp_t^e by the weighted average $\kappa_w \Delta p_t^e + (1-\kappa_w)\pi_t$, where Δp_t^e is determined by myopic perfect foresight. Thus, temporary changes in the correctly anticipated rate of inflation do not have full impact on temporary wage inflation, which is also driven by lagged inflation rates via the inflationary climate variable π_t. Adding inertia to the theory of wage inflation introduced a distinction between the temporary and persistent cost effects to this equation. Furthermore we have that $\Delta y_t - \Delta l_t^d = n_x$ due to the assumed fixed proportions technology. Altogether, we end up with am equation for wage inflation of the type presented in Sect. 3.2.1, though now with a specific interpretation of the model's parameters from the perspective of efficiency wage or bargaining models.[22]

[22] Note that the parameter in front of v_{t-1} can now not be interpreted as a speed of adjustment coefficient. Note furthermore that Blanchard and Katz (1999) assume that, in the steady state, the wage share is determined by the

A.3 Price Dynamics: Theoretical Foundation

We here follow again Blanchard and Katz (1999, Sect. IV) see also Carlin and Soskice (1990, Chap. 18), and start from the assumption of normal cost pricing, here under the additional assumption of our chapter of fixed proportions in production and Harrod neutral technological change. We therefore consider as rule for normal prices

$$p_t = \mu_t + w_t + l_t^d - y_t, \quad \text{i.e.,} \quad \Delta p_t = \Delta\mu_t + \Delta w_t - n_x,$$

where μ_t represents a markup on the unit wage costs of firms and where again myopic perfect foresight, here with respect to wage setting is assumed. We assume furthermore that the markup is variable and responding to the demand pressure in the market for goods $\bar{U}^c - U_t^c$, depending in addition negatively on the current level of the markup μ_t in its deviation from the normal level $\bar{\mu}$. Firms therefore depart from their normal cost pricing rule according to the state of demand on the market for goods, and this the stronger the lower the level of the currently prevailing markup has been (markup smoothing). For sake of concreteness let us here assume that the following behavioral relationship holds:

$$\Delta\mu_t = \beta_p(\bar{U}^c - U_{t-1}^c) + \gamma(\bar{\mu} - \mu_{t-1}),$$

where $\gamma > 0$. Inserted into the formula for price inflation this in sum gives:

$$\Delta p_t = \beta_p(\bar{U}^c - U_{t-1}^c) + \gamma(\bar{\mu} - \mu_{t-1}) + (\Delta w_t - n_x).$$

In terms of the logged wage share $v_t = -\mu_t$ we get

$$\Delta p_t = \beta_p(\bar{U}^c - U_{t-1}^c) + \gamma(v_{t-1} - \bar{v}) + (\Delta w_t - n_x).$$

As in the preceding subsection of the chapter, we again add persistence the cost pressure term $\Delta w_t - n_x$ now in the price Phillips curve in the form of the inflationary climate expression π and thereby obtain in sum (3.2) of Sect. 3.2.1.

firms' markup $u = -\mu$ (both in logs) to be discussed in the next subsection. Therefore the NAIRU can be determined endogenously on the labor market by $\bar{U}^l = \beta_w^{-1}\left[\theta a - (1-\theta\lambda)\bar{\mu} - \theta\lambda(\Delta y_t - \Delta l_t^d)\right]$. The NAIRU of their model therefore depends on both labor and goods market characteristics in contrast to the NAIRU levels for labor and capital employed in our approach.

A.4 Routh–Hurwitz Stability Conditions and Hopf Bifurcations

We consider the matrix of partial derivatives at the steady state of the 3D dynamical systems of this chapter in (v, k, π), the so-called Jacobian J, in detail represented by:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}.$$

We define the principal minors of order 2 of this matrix by the following three determinants:

$$J_1 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}, \quad J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}, \quad J_3 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}.$$

We furthermore denote by a_1 the negative of the trace of the Jacobian $-\text{trace}\, J$, by a_2 the sum of the above three principal minors, and by a_3 the negative of the determinant $|J|$ of the Jacobian J. We note that the coefficients $a_i, i = 1, 2, 3$ are the coefficients of the characteristic polynomial of the matrix J.

The Routh Hurwitz conditions (see Lorenz 1993) then state that the eigenvalues of the matrix J all have negative real parts if and only if

$$a_i > 0, i = 1, 2, 3 \quad \text{and} \quad a_1 a_2 - a_3 > 0.$$

These conditions therefore exactly characterize the case where local asymptotic stability of the considered steady state is given.

Supercritical Hopf bifurcations (the birth of a stable limit cycle) or subcritical Hopf bifurcations (the death of an unstable limit cycle) occur (if asymptotic stability prevailed below this parameter value) when the following conditions hold simultaneously for an increase of a parameter β of the model (see Wiggins 1990, Chap. 3):

$$a_3(\beta) > 0, (a_1 a_2 - a_3)(\beta) = 0, (a_1 a_2 - a_3)'(\beta) > 0.$$

We note here that the dynamics considered below indeed generally fulfill the condition $a_3 > 0$ and also $J_2 = 0$, the latter up to Proposition 3.6 and due to the proportionality that exists between the laws of motion (3.16), (3.18) with respect to the state variables u, π.

A.5 Proofs of Propositions 3.2–3.7

In the following we present the mathematical proofs of the Propositions 3.2–3.7 of the chapter. The proofs involve the stability analysis of the 3D dynamics in (3.16) to (3.18) under certain parametric assumptions and different monetary regimes and are based on the Routh–Hurwitz conditions just considered.

Proof of Proposition 3.2. Choosing β_{p_1} or β_π sufficiently large will make the trace of J, the Jacobian of the dynamics (3.16)–(3.18) at the steady state, unambiguously positive and thus definitely lead to local instability.

Yet, even if β_{p_1} and β_π are sufficiently small, we get by appropriate row operations in the considered determinant the following sequence of result for the sign of $\det J$:

$$
\begin{aligned}
|J| &\hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & + \\ - & 0 & + \end{vmatrix} \hat{=} -(+)\begin{vmatrix} - & + \\ - & + \end{vmatrix} \hat{=} \begin{vmatrix} -y_0 & +1 \\ y_v & y_\pi \end{vmatrix} \\
&= y_0 \begin{vmatrix} -1 & +1 \\ \frac{c-i_1}{(1-v)(1-i_1)+(1-c)v} & \frac{i_1}{(1-v)(1-i_1)+(1-c)v} \end{vmatrix} \\
&= \frac{y_0}{(1-v)(1-i_1)+(1-c)v} \begin{vmatrix} -1 & +1 \\ c-i_1 & i_1 \end{vmatrix} \\
&= \frac{cy_0}{(1-v)(1-i_1)+(1-c)v} > 0.
\end{aligned}
$$

One of the necessary and sufficient Routh–Hurwitz conditions for local asymptotic stability is therefore always violated, independently of the sizes of the considered speeds of adjustment. ∎

Proof of Proposition 3.3. Inserting the interest rule in (A.4b) into the $y(c,\pi)$ and $i_1(\rho-(i-\pi))$ functions gives rise to the functional dependencies

$$
\begin{aligned}
y &= y(v,\pi) = \frac{n+g-i_1[\rho_0+\phi_{ip}(\pi-\bar{\pi})]}{(1-v)(1-i_1)+(1-c)v}, \quad y_v<0, y_\pi<0, \\
i_1 &= i_1(\rho-(i-\pi)) = i_1(v,\pi), \quad i_{1,u}<0, i_{1,\pi}<0.
\end{aligned}
$$

The signs in the considered Jacobian are therefore here given by

$$
J = \begin{pmatrix} - & + & - \\ - & 0 & - \\ - & + & - \end{pmatrix}
$$

if β_{p_1} is chosen sufficiently small (and thus dominated by wage flexibility β_{w_1}). We thus then have trace $J < 0$ ($a_1 = -$ trace $J > 0$) and

$$J_3 = \begin{vmatrix} - & + \\ - & 0 \end{vmatrix} > 0, \quad J_1 = \begin{vmatrix} 0 & - \\ + & - \end{vmatrix} > 0, \text{ i.e.,}$$

$a_2 = J_1 + J_2 + J_3 > 0$ for β_{p_1} sufficiently small. Next, we get for $|J|$ with respect to signs:

$$|J| \hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & - \\ - & 0 & - \end{vmatrix} \hat{=} -(+) \begin{vmatrix} - & - \\ - & - \end{vmatrix} \hat{=} \begin{vmatrix} -y_0 & -\phi_{ip} \\ y_u & y_\pi \end{vmatrix} = - \begin{vmatrix} -y_0 & -\phi_{ip} \\ \frac{c-i_1}{N} y_0 & \frac{-i_1 \phi_{ip}}{N} \end{vmatrix}$$

$$= -\phi_{ip}(y_o/N) \begin{vmatrix} -1 & -1 \\ c - i_1 & -i \end{vmatrix} = -\phi_{ip} \frac{y_o}{N} c < 0$$

since $N = (1-v)(1-i_1) + (1-c)v > 0$ at the steady state. Therefore: $a_1, a_2, a_3 = -|J|$ are all positive.

It remains to be shown that also $a_1 a_2 - a_3 > 0$ can be fulfilled. Here it suffices to observe that a_1, a_2 stay positive when $\beta_{p_1} = 0$ is assumed, while a_3 becomes zero then. Therefore $a_1 a_2 - a_3 > 0$ for all adjustment parameters β_{p_1} chosen sufficiently small. These qualitative results hold independently of the size of β_π and ϕ_{ip} (with an adjusting size of β_{p_1} however). ■

Note in addition that the trace of J is given by

$$\kappa\beta_{p_1}/y^p[(1-\kappa_w)(i_1 - c)y - \beta_\pi \phi_{ip} i_1]/((1-i_1)(1-v) + (1-c)v)$$

as far as its dependence on the parameter β_{p_1} is concerned. Choosing β_π or ϕ_{ip}, for given β_{p_1}, sufficiently small will make the trace of J positive and thus make the steady state of the considered dynamics locally unstable.

Proof of Proposition 3.4. With $i \equiv i_o$, we have for the Jacobian J of the dynamics at the steady state:

$$J = \begin{pmatrix} + & + & + \\ - & 0 & + \\ + & + & + \end{pmatrix}$$

and thus in particular trace $J > 0$ and

$$|J| \hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & + \\ + & 0 & + \end{vmatrix} = -(+) \begin{vmatrix} - & + \\ + & + \end{vmatrix} > 0.$$

Thus there is at least one positive real root, which establishes the local instability of the investigated interior steady state solution. ■

Proof of Proposition 3.5. For the considered parameter constellations, the Jacobian J is given by

$$J = \begin{pmatrix} + & + & - \\ - & 0 & - \\ + & + & - \end{pmatrix}.$$

This Jacobian first of all implies

$$|J| \hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & - \\ + & 0 & - \end{vmatrix} = -(+) \begin{vmatrix} - & - \\ + & - \end{vmatrix} < 0$$

and thus for the Routh–Hurwitz condition $a_3 = -|J| > 0$ as necessary condition for local asymptotic stability. We assert here without detailed proof that local stability will indeed prevail if β_{w_1} is chosen sufficiently close to zero, since $|J|$ will be close to zero then too and since the Routh–Hurwitz coefficients a_1, a_2 are both positive and bounded away from zero. Wages that react sluggishly with respect to demand pressure therefore produce local stability in the case $c > i_1$.

This is indeed achieved for example by the assumption $\kappa_p = 0$: Obviously, trace of J is then an increasing linear function of the speed parameter β_{w_1} in the considered situation, since this parameter is then only present in J_{11} and not in J_{33}. This proves the first part of the assertion, if note is taken of the fact that $|J|$ does not change its sign. Eigenvalues therefore cannot pass through zero (and the speed condition for them is also easily verified). The second part follows from the fact that $a_1a_2 - a_3$ becomes zero before trace $J = -a_1$ passes through zero, but cannot become positive again before this trace has become zero (since $a_1a_2 - a_3$ is a quadratic function of the parameter β_{w_1} with a positive parameter before the quadratic term and since this function is negative at the value β_{w_1} where trace J has become zero). ■

Proof of Proposition 3.6. The signs in the Jacobian of the dynamics at the steady state are given by

$$J = \begin{pmatrix} - & + & - \\ - & 0 & - \\ - & + & - \end{pmatrix}$$

if β_{w_2} is chosen sufficiently large (and thus dominating the wage flexibility β_{w_1} term). We thus have trace $J < 0$ ($a_1 = -\text{trace}J > 0$) and

$$J_3 = \begin{vmatrix} - & + \\ - & 0 \end{vmatrix} > 0, \quad J_1 = \begin{vmatrix} 0 & - \\ + & - \end{vmatrix} > 0, \quad \text{sign} J_2 = \text{sign} \begin{vmatrix} - & - \\ + & - \end{vmatrix} > 0, \text{ i.e.,}$$

$a_2 = J_1 + J_2 + J_3 > 0$, in particular due to the fact that the $\beta_{w_i}, i = 1, 2$-expressions can be removed from the second row of J_2 without altering the size of this determinant.

Next, we get for $|J|$ with respect to signs:

$$|J| \hat{=} \begin{vmatrix} - & + & - \\ - & 0 & - \\ + & 0 & - \end{vmatrix} \hat{=} -(+) \begin{vmatrix} - & - \\ + & - \end{vmatrix} < 0$$

since the $\beta_{w_i}, i = 1, 2$-expressions can again be removed now from the third row of $|J|$ without altering the size of this determinant.

Therefore: a_1, a_2, and $a_3 = -|J|$ are all positive as demanded by the Routh–Hurwitz conditions for local asymptotic stability. There remains to be shown that also $a_1 a_2 - a_3 > 0$ can be fulfilled. In the present situation this however is an easy task, since—as just shown—$|J|$ does not depend on the parameter β_{w_2}, while $a_1 a_2$ depends positively on it (in the usual quadratic way). Finally, the statement on the Hopf bifurcation can be proved in a similar way as the one in Proposition 3.5. ■

Proof of Proposition 3.7. Inserting the Taylor rule

$$i = \rho_o + \pi + \beta_{r_1}(\pi - \bar{\pi}) + \beta_{r_2}(v - v_o), \quad \beta_{r_1}, \beta_{r_2} > 0$$

into the effective demand equation

$$y = \frac{n + g - i_1(i_o - \pi)}{(1 - v)(1 - i_1) + (1 - c)v}$$

adds the term

$$\tilde{y} = -\frac{i_1 \beta_{r_2}(v - v_o)}{(1 - v)(1 - i_1) + (1 - c)v}$$

to our former calculations—in the place of the β_{w_2} term now. This term gives rise to the following additional partial derivative

$$\tilde{y}_v = -\frac{i\beta_{r_2}}{(1 - v_o)(1 - i_1) + (1 - c)v_o}$$

at the steady state of the economy. This addition can be exploited as the β_{w_2} expression in the previous subsection used there to prove Proposition 3.7. ■

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4

Estimation and Analysis of an Extended AD–AS Model

4.1 Introduction

The present chapter intends to provide empirical evidence for a crossover type of interaction of wage and price inflation rates, or more briefly for the wage-price spiral, and additionally formulates and estimates within a Keynesian Disequilibrium framework a macroeconomic model for the U.S. economy.[1] It presents the feedback structures of this (semi-) reduced form of a macromodel and its stability implications, first on a general level and then on the level of the sign and size restrictions obtained from empirical estimates of the five laws of motion of the dynamics. These estimates, undertaken from the U.S. economy for quarterly data 1965.1–2001.1 also allow us to discuss asymptotic stability for the estimated parameter sizes and to determine stability boundaries.

Our approach builds as the nowadays popular New Keynesian approach on gradual wage and price adjustments, employing two Phillips-curves to relate wage and price dynamics to factor utilization rates. We use the same formal structure for the variables that drive wage and price inflation rates

[1] This chapter is based on the article "Measuring the interaction of wage and price Phillips curves for the U.S. economy" (P. Chen and P. Flaschel). *Studies in Nonlinear Dynamics and Econometrics*, 2006, **10**, 1–35, and the chapter "Keynesian Macrodynamics and the Phillips Curve. An estimated baseline macro-model for the U.S. economy." (P. Chen, C. Chiarella, P. Flaschel, W. Semmler). In: C. Chiarella, P. Flaschel, R. Franke and W. Semmler (eds.): *Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels.* Contributions to Economic Analysis (Series Editors: B. Baltagi, E. Sadka and D. Wildasin), Amsterdam: Elsevier, 2006, 229–284.

(utilizations rates and real wages), but with microfoundations consistent with the Blanchard and Katz (1999) reconciliation of wage Phillips curve and current labor market theories. The basic difference in the wage-price module is that we augment this structure by hybrid expectations formation where the forward-looking part is based on a neoclassical type of dating and where expectations are of cross-over type—we have price inflation expectations in the wage Phillips curve (wage PC) and wage inflation expectations in the price Phillips curve (price PC). Our formulation of the wage-price dynamics permits therefore an interesting comparison to New Keynesian work that allows for both staggered price and wage setting. Concerning the IS-curve we make use of a law of motion for the rate of capacity utilization of firms that depends on the level of capacity utilization (the dynamic multiplier), the real rate of interest and finally on the real wage and thus on income distribution. New Keynesian authors often use a purely forward-looking IS-curve (with only the real rate of interest effect) and a Phillips curve which has been criticized from the empirical point of view; see in particular Fuhrer and Rudebusch (2004) and Eller and Gordon (2003). Since we distinguish between the rate of employment of the labor force and that of the capital stock, namely the rate of capacity utilization, we employ some form of Okun's law to relate capacity utilization to employment.

Up to this link between the working of the real markets, the chapter starts from the same theoretical framework and attempts to demonstrate empirically (with a new and improved data set) that the measurement of two structural wage and price Phillips curves, one for the labor market and one for the goods market, produces theoretically and empirically much more elaborate results than the reduced-form estimate of a single Philips curve that directly relates price inflation to demand pressure in the labor market without much justification. This improvement in theoretical and empirical content is obtained, since we take into account (in addition to market-specific measures of demand pressure) that cost pressure measures for workers and firms should be based on backward-looking (medium-run) averages as well as forward-looking (perfectly foreseen) price and wage inflation rates (for wage earners and firms, respectively). This crossover use of such hybrid measures for the accelerator terms in the wage and price inflation dynamics is based on work by Chiarella and Flaschel (1996, 2000), and it now also characterizes (without use of a crossover relationships) the New Keynesian approach to staggered wage and price inflation; see for example Woodford (2003), though the New Keynesian wage and price Phillips curves differ considerably in spirit from the ones proposed in this chapter. We also provide evidence for the presence of Blanchard and Katz (1999) error correction terms in both the wage and the price Phillips curve for the U.S. economy after World War II and also add to this situation an estimated link between goods and labor markets performance in the form of an extended Okun's law, where in addition insider-outsider aspects are distinguished and taken into account. In this way, the critical α-condition separating normal from an adverse real wage adjustment, as implied by our interacting Phillips curves approach, is now estimated in an integrated way and not just based on the assumption of fixed proportions in production, as it was the case in Flaschel and Krolzig (2006).

The remainder of the chapter is organized as follows. In Sect. 4.2 we briefly reconsider the wage and price level based structural equations estimated by Fair (2000) and show that they may easily be turned into ordinary wage and price inflation Phillips curves when account is taken of the parameter sizes estimated by Fair (2000), arguing that such separate wage and price inflation Phillips curves, when reformulated in sufficiently general terms with respect to demand as well as cost pressures items, can give rise to various real wage adjustment patters, two normal or stabilizing ones and two adverse or destabilizing ones. In Sect. 4.3 we compare our approach to gradual wage and price adjustments with a deterministic and continuous time representation of the New Keynesian macromodel with staggered wage and price setting as discussed for example in Woodford (2003), highlighting significant formal similarities but also important differences especially in the treatment of inflationary expectations that give rise to radically different results for the implied wage-price dynamics. Reduced-form expressions and the resulting critical α-condition for an explosive behavior of our wage-price spiral are briefly discussed in Sect. 4.4. Section 4.5 provides 3SLS estimate of our structural wage-price Phillips curves, including estimates of Okun's Law as the link between goods and labor market pressure, distinguishing in addition inside employment rates from the employment rate on the external labor market. In Sect. 4.6 we then present a complete formulation of an estimable baseline Keynesian DAS–DAD growth dynamics model, however with a more simplified labor market module. Section 4.7 considers the feedback chains of the reformulated model and derives cases of local asymptotic stability and of loss of stability by way of Hopf-bifurcations. In Sect. 4.8 we then estimate the model to find out sign and size restrictions for its behavioral equations and we study which type of feedback mechanisms may have applied to the U.S. economy after World War II. Section 4.9 investigates in detail the stability properties of the estimated model. Section 4.7 analyzes on the one hand the stability problems that occur when there is a floor to money wage deflation and the role of monetary policy in such a case. Section 4.10 draws some concluding remarks from this study.

4.2 Structural Models of the Wage–Price Spiral

> In the early 1980s, there began a movement away from the estimation of structural price and wage equations to the estimation of reduced-form price equations ... The current results (see below, P.F.) call into question this practice in that considerable predictive accuracy seems to be lost when this is done.
>
> R. Fair (2000, p. 69)

Fair's observation holds especially for applied work where it appears to be quite natural to express labor market and goods market dynamics by a single Phillips curve with demand pressure based on the external labor market (the rate of employment on this market, not hours worked within the firms) and with cost pressure in the two markets represented by a single expected inflation rate. Rigid markup pricing is one possible justification for such reduced form inflation dynamics. Yet, if in fact such reduced form PC's are explicitly derived from separate wage and price equations, the very special situation underlying this reduced form approach to wage-price dynamics becomes obvious.

In order to motivate our own formulation of such wage-price dynamics, a wage-price spiral in fact, we start briefly from the two structural wage and price equations estimated in Fair (2000). His structural equations for wage and price formation are of the form

$$\ln p_t = \beta_0 + \beta_1 \ln p_{t-1} + \beta_2 \ln w_t + \beta_3 \pi^m_{t-1} + \beta_4 U_{t-1},$$
$$\ln w_t = \gamma_0 + \gamma_1 \ln w_{t-1} + \gamma_2 \ln p_t + \gamma_3 \ln p_{t-1} + \gamma_4 U_{t-1},$$

where w and p represent wage and price levels, π^m denotes import price inflation and where U denotes the unemployment rate in these two structural equations. The estimation of these two equations by two-stage least-squares (with time trend and a specific constraint in addition) gives in Fair's (2000) paper the result shown in Table 4.1.

Table 4.1. Fair's (2000) estimated price and wage

Estimation period: 1954:1–1998:1, Estimations method: 2SLS					
$\ln p_t = \beta_0 + \beta_1 \ln p_{t-1} + \beta_2 \ln w_t + \beta_3 \pi^m_{t-1} + \beta_4 U_{t-1} + \beta_5 t + \epsilon_t$					
$\ln w_t = \gamma_0 + \gamma_1 \ln w_{t-1} + \gamma_2 \ln p_t + \gamma_3 \ln p_{t-1} + \gamma_4 U_{t-1} + \gamma_5 t + \mu_t$					
	Estimate	t-Stat.		Estimate	t-Stat.
β_0	0.0778	1.65	γ_0	−0.0709	−1.60
β_1	0.9225	284.47	γ_1	0.9887	109.53
β_2	0.0200	2.51	γ_2	0.7513	8.86
β_3	0.0403	13.61	γ_3	−0.7564	−0.28
β_4	−0.1795	−8.51	γ_4	0.000181	2.61
β_5	0.00088	1.01	γ_5	−0.0104	constrained coefficient
SE	0.00294		SE	0.00817	

The result of his estimation provides us approximately with the following two inflation relationships for the U.S. economy, when note is taken of the fact that the obtained parameter values suggest a reformulation of Fair's wage and price level curves towards rates of wage and price inflation. In terms of growth rates $\mathrm{d}\ln x = \hat{x}$, $x = w, p$ they can indeed be simplified and approximated by:

$$\hat{p}_t = 0.08 - 0.18 U_{t-1},$$
$$\hat{w}_t = -0.07 + 0.75 \hat{p}_t.$$

We do not think that the structure represented by these two equations is developed enough from the theoretical perspective to really represent a structural approach to the wage-price spiral in the U.S. economy. Fair's recommendation to use two structural wage and price curves in the place of a single reduced form Phillips curve for price inflation is an appropriate one, but one should employ for each market his own measure of demand pressure and not a single one for both. Furthermore, inflationary expectations should enter the wage-price spiral in an explicit, from today's perspective necessarily hybrid way. We shall fulfill this latter demand by a mixture (a weighted average) of short-run perfectly foreseen inflation rates (with Neoclassical, not New Keynesian dating of expectations) and an expression for the medium-term inflationary climate into which these short-run expectations are embedded. This adds persistence to an approach which is known to be problematic when only myopic perfect foresight expectations are considered. We thus reconsider the issue of interacting wage and price dynamics from a considerably more general structural point of view, with an emphasis on measuring the parameters involved in such a wage-price spiral and not yet on predictive accuracy as in the quotation from Fair's paper we started from.

In Chiarella et al. (2000, Chap. 2), Fair's wage-price dynamics has been reformulated as a wage price spiral as follows:

$$\hat{w} = \beta_{we}(e - e_o) + \beta_{weh}(u^w - 1) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c, \tag{4.1}$$

$$\hat{p} = \beta_{pu}(u - u_o) + \beta_{pn}(n - 1) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^c. \tag{4.2}$$

These authors use two separate measures of demand pressure for wages and prices,[2] determined in the labor and the goods market, respectively. In the above wage-price dynamics, $e - e_o = U_o - U$, $u^w - 1$ are denoting (if positive) excess labor demand on the external labor market (in terms of labor market utilization) and excess labor demand (in terms of overtime worked) within firms, and $u - u_o$, $n - 1$ (if positive) are denoting excess demand on the market for goods in terms of utilized capacity u and in terms of a desired/actual inventory ratio n. In the following investigation of this wage-price spiral we will set β_{pn} equal to zero however and will thus only pay attention to capacity utilization rates e, u on the labor and the goods market in their deviation from their NAIRU(tilization) rates e_o, u_o. We will then compare the outcomes on the labor market with the results that are obtained when the rate u^w is used in the place of the rate e, i.e., when an insider or workforce utilization view is replacing the measurement of the employment rate in terms of heads.

This formulation of wage and price Phillips curves represents in our view the minimum structure one should start from in a non-reduced-form investigation of wage and price dynamics, which should only be simplified further—for example with respect to the reduced form equations it implies—if there are definite and empirically motivated reasons to do so. Generally however all parameters of the structural wage and price Phillips curves will show up in their reduced form representations which therefore cannot be interpreted in terms of labor market phenomena or goods market characteristics alone, as in the mainstream literature and in Fair's (2000) approach.

[2] $\hat{w}$, $\hat{p}$ wage and price inflation and π^c, our measure of an inflation climate, here simply a weighted average of past price inflation rates, and κ_w, $\kappa_p \in (0, 1)$ the weights of current price and wage inflation in the employed cost-pressure terms, see Chiarella and Flaschel (1996) for the original formulation and Flaschel and Krolzig (2006) for a first estimation of this wage-price spiral.

Up to work of Rose (1967, 1990), it remained fairly unnoticed that having specific formulations of measures of demand and cost pressure on both the labor market and the goods market must, when taken together, imply that either increasing wage or price flexibility with respect to these demand pressures must then always be destabilizing, depending on marginal propensities to consume and to invest with respect to changes in the real wage. Figure 4.1 attempts to illustrate this assertion with respect to rising wages and prices if aggregate demand is pushed into an upward direction, through increasing consumption demand caused by real wage increases (with investment demand kept constant), and falling prices and wages, caused by falling investment demand due to rising real wages (with consumption demand kept constant). In both cases we consider situations where wages are more flexible than prices and vice versa. We have—broadly speaking—normal real wage reaction patterns (leading to converging real wage adjustment and thus economic stability from this partial point of view), if investment is more responsive to real wage changes than consumption and if wages are more flexible with respect to demand pressure on their market than prices with respect to their measure of demand pressure (with additional assumptions concerning the forward looking component in the cost pressure items as will be see later on).

In this case, aggregate demand depends negatively on the real wage and real wages tend to fall in the depression (thereby reviving economic activity via corresponding aggregate demand changes), since the numerator in real wages is reacting stronger than their denominator. The opposite occurs, of course, if it holds—in the considered aggregate demand situation—that wages are less flexible than prices with respect to demand pressure, which is not unlikely in cases of a severe depression. In such cases it would therefore be desirable to have that consumption responds stronger than investment to real wage changes, since the implied real wage increases would then revive the economy. There is a fourth case when—in the latter demand situation—wages are more flexible than prices, where again an adverse real wage adjustment would take place leading the economy via falling real wages into deeper and deeper depressions as long as this situation remains in existence.

Figure 4.1 provides a graphical illustration of these possibilities of a real wage feedback channel within the wage-price spiral. It considers only the limit cases discussed above where only one demand component is changing and only one price is flexible. It can however easily be reinterpreted in stressing the components that are more flexible than the other ones (that are kept

$$w/p \uparrow \begin{cases} \mapsto & C \uparrow \Rightarrow Y^d \uparrow \Rightarrow Y \uparrow \Rightarrow e, u^w \uparrow \Rightarrow w \uparrow \Rightarrow w/p \uparrow \\ \mapsto & C \uparrow \Rightarrow Y^d \uparrow \Rightarrow Y \uparrow \Rightarrow u \uparrow \quad\quad \Rightarrow p \uparrow \Rightarrow w/p \downarrow \\ \mapsto & I \downarrow \Rightarrow Y^d \downarrow \Rightarrow Y \downarrow \Rightarrow e, u^w \downarrow \Rightarrow w \downarrow \Rightarrow w/p \downarrow \\ \mapsto & I \downarrow \Rightarrow Y^d \downarrow \Rightarrow Y \downarrow \Rightarrow u \downarrow \quad\quad \Rightarrow p \downarrow \Rightarrow w/p \uparrow \end{cases}$$

Fig. 4.1. Normal vs. adverse real wage adjustments (Y^d aggregate demand and Y the output level)

constant in the four possible scenarios considered in this figure. Figure 4.1, reinterpreted in this way, immediately suggest that the exact type of real wage adjustment occurring within the considered wage-price spiral can only be determined by empirical investigation and—as will be shown—will depend moreover on the shortsightedness of workers and firms with respect to the current rate of price and wage inflation, respectively.

We conclude that wage and price Phillips curves which pay sufficient attention to demand as well as cost pressure items on the market for labor as well as on the market for goods may give rise to interesting dynamic phenomena with respect to the type of real wage adjustment they imply. This definitely deserves closer inspection than was the case so far in the macrodynamic literature. The present chapter wants to discuss in this respect possible theoretical and (for the U.S. economy after World War II) empirical outcomes, in continuation and extension of the results achieved in Flaschel and Krolzig (2006), and thus wants to provide a definite answer for a specific country over a specific time interval. In the following section we will moreover follow Chiarella et al. (2005, Chap. 5) and take Blanchard and Katz (1999) error correcting real wage influences (in addition to demand pressure terms) into account in both the wage and the price Phillips curve.

The hope is that interest in further investigation of the questions raised in this chapter will be stimulated by its results on the type and form of the wage-price spiral obtained for the U.S. economy, for other countries, for high versus low inflation regimes, for more refined measures of demand pressure, for integral and derivative besides proportional demand pressure influences and more.

4.3 New Keynesian Phillips Curves and the Wage–Price Spiral: A Brief Comparison

In this section we consider briefly the New Keynesian approach to macrodynamics, here already in its advanced form, where both staggered price and wage setting are assumed. We here follow Woodford (2003, p. 225) in his formulation of staggered wages and prices, which there too implies a derived law of motion for real wages, but do not yet include New Keynesian IS-dynamics and the Taylor interest rate policy rule here. As in this New Keynesian formulation of the wage-price dynamics, we ignore technical change here, but will introduce labor productivity growth in our empirical investigations below. We shall only briefly look at this extended New Keynesian approach in order to compare its formulation of wage-price dynamics with ours below. It will turn out—somewhat surprisingly, but from a formal perspective solely—that their approach differs from ours only in their handling of inflationary expectations, where we use hybrid expectations formation, neoclassical dating of expectations, cross-over cost-push linkages (and two measures of demand pressure, a labor market stock and a goods market flow measure in the place of a single output gap right from the start).

Woodford (2003, p. 225) provides the following two loglinear equations as representation of the joint evolution of staggered wages and prices, the wage and price Phillips Curves of the New Keynesian approach. In these equations we denote by $\ln w$, $\ln p$ the logs of wage and the price level, by y the log of output (with normal output set equal to one) and by ω the real wage w/p (with steady state wages also set equal to one):

$$\hat{w}_t \stackrel{NWPC}{=} \beta E_t(\hat{w}_{t+1}) + \beta_{wy} y_t - \beta_{w\omega}\omega_t,$$
$$\hat{p}_t \stackrel{NPPC}{=} \beta E_t(\hat{p}_{t+1}) + \beta_{py} y_t + \beta_{p\omega}\omega_t.$$

All parameters shown are assumed to be positive. Our first objective is to derive the continuous time analog of these two equations, describing the New Wage Phillips Curve and the New Price Phillips Curve, and to show how this extended model is to be solved from the New Keynesian perspective and the rational expectations methodology.

In a deterministic setting, the above translates into

$$\hat{w}_{t+1} = \frac{1}{\beta}[\hat{w}_t - \beta_{wy} y_t + \beta_{w\omega}\omega_t],$$
$$\hat{p}_{t+1} = \frac{1}{\beta}[\hat{p}_t - \beta_{py} y_t - \beta_{p\omega}\omega_t].$$

If we assume that the parameter β is not only close to one, but equal to one, this yields (with a reversal of all parameter signs):

$$\hat{w}_{t+1} - \hat{w}_t = -\beta_{wy} y_t + \beta_{w\omega}\omega_t,$$
$$\hat{p}_{t+1} - \hat{p}_t = -\beta_{py} y_t - \beta_{p\omega}\omega_t.$$

Denoting by π^w the rate of wage inflation and by π^p the rate of price inflation, these equations can be recasted into continuous time:

$$\dot{\pi}^w \stackrel{NWPC}{=} -\beta_{wy} y + \beta_{w\omega}\omega, \tag{4.3}$$
$$\dot{\pi}^p \stackrel{NPPC}{=} -\beta_{py} y - \beta_{p\omega}\omega, \tag{4.4}$$
$$\dot{\omega} \stackrel{RWPC}{=} \pi^w - \pi^p = (\beta_{py} - \beta_{wy})y + (\beta_{p\omega} + \beta_{w\omega})\omega. \tag{4.5}$$

This reformulation of the originally given New Keynesian wage and price PC's shows that there has occurred a complete sign reversal on the right hand side of the NWPC and the NPPC as compared to the initially given situation. This occurs in combination with the use of rates of changes of inflation rates on the left hand sides of the NWPC and the NPPC. The continuous-time equations for the NWPC and the NPPC also imply—as shown in (4.5)—a law of motion for the log of real wages, and thus a three-dimensional system (which is coupled with a forward-looking law of motion for (the log of) output and a Taylor interest rate policy rule in the New Keynesian approach).

From the output dynamics of the New Keynesian approach, namely

$$y_t = y_{t+1} - \alpha_{yi}(i_t - \pi^p_{t+1} - i_0), \quad \text{i.e.,} \quad y_{t+1} - y_t = \alpha_{yr}(i_t - \pi^p_{t+1} - i_0),$$

we obtain the continuous time reduced form law of motion

$$\dot{y} \stackrel{IS}{=} \alpha_{yr}[(\phi_{ip} - 1)\pi^p + \phi_{iy} y]$$

where we have already inserted an interest rate policy rule in order to (hopefully) obtain determinacy as in the New Keynesian baseline model, which is known to be indeterminate for the case of an interest rate peg. Here we have chosen the simple Taylor interest rate policy rule

$$i = i_{\mathrm{T}} = i_o + \phi_{ip}\pi + \phi_{iy} y,$$

see Walsh (2003, p. 247), which is of a classical Taylor rule type (though without interest rate smoothing yet).

There remains finally the law of motion for real wages to be determined, which setting $\theta = \ln \omega$ simply reads

$$\dot{\theta} = \pi^w - \pi^p.$$

We thus get from this extended New Keynesian model an autonomous linear dynamical system, in the variables π^w, π^p, y and θ. The, in general, uniquely determined steady state of the dynamics is given by $(0, 0, 0, i_o)$. From the definition of θ we see that the model exhibits four forward-looking variables, in direct generalization of the baseline New Keynesian model with only staggered price setting. Searching for a zone of determinacy of the dynamics (appropriate parameter values that make the steady state the only bounded solution of the dynamics to which the economy then immediately returns after isolated shocks of any type) thus requires establishing conditions under which all roots of the Jacobian have positive real parts.

The Jacobian of the 4D dynamical system under consideration reads:

$$J = \begin{pmatrix} 0 & 0 & -\beta_{wy} & \beta_{w\omega} \\ 0 & 0 & -\beta_{py} & -\beta_{p\omega} \\ 0 & \alpha_{yr}(\phi_{ip} - 1) & \alpha_{yr}\phi_{iy} & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

For the determinant of this Jacobian we calculate

$$-|J| = (\beta_{wy}\beta_{p\omega} + \beta_{py}\beta_{w\omega})\alpha_{yr}(\phi_{ip} - 1) \gtreqless 0 \quad \text{iff} \quad \phi_{ip} \gtreqless 1.$$

We thus get that an active monetary policy of the conventional type (with $\phi_{ip} > 1$) is—compared to the baseline New Keynesian model—no longer appropriate to ensure determinacy (for which a positive determinant of J is a necessary condition).[3] One can show in addition, see Chen et al. (2004), via the minors of order 3 of the Jacobian J, that the same holds true for a passive monetary policy rule, i.e., the model in this form must be blocked out from consideration. There consequently arises the necessity to specify an extended or modified active Taylor interest rate policy rule from which one can then obtain determinacy for the resulting dynamics, i.e., the steady state as the only bounded solution and therefore, according to the logic of the rational

[3] Gali (2008, Chap. 6) claims—and illustrates this claim numerically—that the consider model type will imply determinacy if $\beta_{ip} > 1$ holds. We prove this claim to be correct in Flaschel et al. (2008), but argue there that his suggested reformulation of the model by means of backward looking elements is not the appropriate solution compared to our purely forward looking treatment of the considered dynamics.

expectations approach, the only realized situation in this deterministic set-up. This would then generalize the New Keynesian baseline model with only staggered prices, which is known to be indeterminate in the case of an interest rate peg or a passive monetary policy rule, but which exhibits determinacy for a conventional Taylor rule with $\phi_{ip} > 1$.

We briefly observe here that when the discrete time dynamics makes use of a system matrix J the system matrix of the continuous time analog is given by $J - I$, I the identity matrix. The eigenvalues of the discrete time case are thus all shifted to the left by 1 in the continuous time analog. In the case of the considered dynamics this means that determinacy in the continuous time case implies determinacy in the discrete time case, but the same does not at all hold for indeterminacy in the place of determinacy. The discrete time case therefore can be determinate, though the continuous time case has been shown to be indeterminate, for example simply because the stable roots of the continuous time case are situated to the left of -1. In this example, a very stable root in the continuous system may cause strong overshooting divergence in the discrete situation and thus turn stable roots into unstable ones. We would consider the occurrence of such a situation as resulting from over-synchronization in the considered market structure, since theoretical discrete time systems are then allowed only to react in the discrete point in time $t, t+1, \ldots$. Depending on the period length that is underlying the model this can mean (in the case of one quarter) that shopping can only be done every three months which—if implemented by law on an actual economy—would make it probably a very unstable one. Discrete time modeling is important in empirical analysis due to data availability, but should not be implemented as a theoretical model, unless it can be checked that it is not in stark contradiction compared to the case where all difference quotients are replaced by differential quotients. There are processes in agriculture and biology where discrete time analysis is reasonable by itself, but this statement does not carry over to the macrolevel of industrialized economies, where staggered price and wage setting is not restricted to four points in time within a year, and where therefore an assumption of this type can give rise to instability results simply due to over-synchronization and a lack of smoothness, aspects that are very questionable from an macroeconomic point of view. We conclude that the lack of determinacy in continuous time is also a problem for the discrete time analogue that should not be overcome by making the period length so large that stable processes are in fact turned into unstable ones.

There are a variety of critical arguments raised in the literature against the New Phillips Curves of the (baseline) model of New Keynesian macrodynamics, see in particular Mankiw (2001) and recently Eller and Gordon (2003) for particular strong statements on the empirical irrelevance of such PC's.[4] These and other criticisms also apply to the above extended wage and price dynamics. In view of these and other critiques, as well as in view of the approach established in Chiarella and Flaschel (2000) and by further work along these lines, see in particular Chiarella et al. (2005), we propose the following modifications to the above New Keynesian wage-price dynamics, which will remove the questionable feature of a sign reversal in the role of output and wage gaps, caused by the fact that future values of the considered state variables are used on the right hand side of their determining equations, implying that the time rates of change of these variables depend on output and wage gaps with a reversed sign in front on them. These sign reversals are at the root of the problem when the empirical relevance of such NPC's is investigated.

We tackle this issue by using the following expectations augmented wage and price Phillips curves, which provide a wage-price spiral in the sense of the preceding section that (from a formal perspective) is in close correspondence to the New Keynesian approach. The letter "M = Matured" in front of these wage PC and price PC denotes their traditional orientation, however certainly in a matured form from the perspective of macroeconomic theorizing.[5]

$$\mathrm{d}\ln w_{t+1} \overset{MWPC}{=} \kappa_w \mathrm{d}\ln p_{t+1} + (1-\kappa_w)\pi_t^c + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln \omega_t], \quad (4.6)$$

$$\mathrm{d}\ln p_{t+1} \overset{MPPC}{=} \kappa_p \mathrm{d}\ln w_{t+1} + (1-\kappa_p)\pi_t^c + \beta_{py}\ln Y_t + \beta_{p\omega}\ln \omega_t]. \quad (4.7)$$

We have modified the New Keynesian approach to wage and price dynamics here only with respect to the terms that concern expectations, in order to generate the potential for a wage-price spiral. We first assume that expectations formation is of a crossover type, with perfectly foreseen price inflation in the wage PC of workers and perfectly foreseen wage inflation in the price PC of firms. Furthermore, we make use in this regard of a neoclassical dating in the considered PC's, which means that—as is usually the case in the reduced form PC—we have the same dating for expectations and actual wage and price

[4] With respect to the New Phillips curve it is stated in Mankiw (2001): "Although the new Keynesian Phillips curves has many virtues, it also has one striking vice: It is completely at odd with the facts."

[5] To simplify the presentation we have assumed here again that the steady state value of the real wage has the value 1.

formation on both sides of the PC's. Finally, following Chiarella and Flaschel (1996), we assume expectations formation to be of a hybrid type, where a certain weight is given to current (perfectly foreseen) inflation rates (κ_w, κ_p) and the counterweight attached to a concept that we have dubbed the inflationary climate π^c that is surrounding the currently evolving wage-price spiral. We thus assume that workers as well as firms to a certain degree pay attention to whether the current situation is embedded in a high inflation regime or in a low inflation one.

These relatively straightforward modifications of the New Keynesian approach to expectations formation will imply for this what we call matured Keynesian approach to wage and price dynamics—to be completed in the next section—radically different solutions and stability features, with in particular no need to single out the steady state as the only relevant situation for economic analysis in the deterministic set-up here considered (when goods market dynamics and interest rates rules are added to the model and when note is taken of the fact that all variables are forward-looking in the considered New Keynesian framework). Concerning microfoundations for the assumed wage-price spiral we note here that the wage PC can be microfounded as in Blanchard and Katz (1999), using wage curves from standard labor market theories, if hybrid expectations formation is added to the Blanchard and Katz approach. We thus obtain from Blanchard and Katz (1999) in particular a foundation for the fact that it is indeed the log of the real wage or the wage share that should appear on the right hand side of the wage PC (due to their theoretical starting point, given by an expected real wage curve). We will call the ω expressions in the MWPC (and the MPPC) Blanchard and Katz error corrections terms in the following. Concerning the price PC a similar procedure can be applied, based on desired markups of firms and implied expected real wages (now with the rate of capacity utilization gap $u-u_o$ in the place of the employment rate gap).[6] Along these lines, we obtain an economic motivation for including the log of real wages with a negative sign into the MWPC and with a positive sign into the MPPC, without any need for loglinear approximations. We furthermore use the employment gap $e - e_o$ (a stock measure) and the capacity utilization gap $u - u_o$ (a flow measure) in these two PC's, respectively, in the place of a single measure (the log of the output gap, y), in order to distinguish between the demand forces that drive wages and those

[6] See Chiarella et al. (2005) and Flaschel and Krolzig (2006) for an alternative motivation of the MWPC and the MPPC.

that drive prices. This wage-price spiral will be embedded into a complete Keynesian approach in the next section, exhibiting a dynamic IS-equation as in Rudebusch and Svensson (1999), but now also including real wage effects and thus a role for income distribution, exhibiting furthermore Okun's law as the link from goods to labor markets, and exhibiting of course the classical type of a Taylor interest rate policy rule in the place of an LM-curve.

In continuous time the above two Phillips curves (4.6), (4.7) read (with $\hat{\omega} = \hat{w} - \hat{p}$!):

$$\hat{w} \overset{MWPC}{=} \kappa_w \hat{p} + (1-\kappa_w)\pi^c + \beta_{we}(e - e_o) - \beta_{w\omega}\omega, \tag{4.8}$$

$$\hat{p} \overset{MPPC}{=} \kappa_p \hat{w} + (1-\kappa_p)\pi^c + \beta_{pu}(u - u_o) + \beta_{p\omega}\omega. \tag{4.9}$$

This is the model of the wage-price spiral that we will investigate from the analytical perspective in the next section and from the empirical perspective in the section thereafter.

We conclude that this model of a wage-price spiral is an interesting alternative to the—theoretically rarely investigated and empirically questionable—New Keynesian form of wage-price dynamics. This wage-price spiral, when implanted into a somewhat conventional Keynesian macrodynamical model, will produce stability results as they are expected from a Keynesian theory of the business cycle, with much closer resemblance to what is stated in Keynes (1936) "Notes on the trade cycle" than is the case for the New Keynesian theory of business fluctuations (which—when there are cycles at all—is entirely based on the Frisch paradigm, see Chen et al. (2006) for details).

4.4 Real-Wage Dynamics: The Critical Stability Condition

We now derive reduced form expressions from the wage and price PC's of the Sect. 4.2, one for the real part of the overall dynamics (for the real wage) and one for the nominal part of the dynamics (for price inflation), where both of these reduced form dynamics are now driven by mixtures of excess demand expressions on the market for goods and for labor (and within firms) plus real wage error correction, and—in the case of the price inflation rate—in addition by the inflationary climate (accelerator) term with a unity coefficient in front of it.

Note in this respect first that the wage and price Phillips curves of the preceding sections are of the general form

$$\hat{w} = \beta_{w's}(\,\cdot\,) + \kappa_w \hat{p} + (1-\kappa_w)\pi^c,$$
$$\hat{p} = \beta_{p's}(\,\cdot\,) + \kappa_p \hat{w} + (1-\kappa_p)\pi^c,$$

where demand pressure and error correction expressions $\beta_{w's}, \beta_{p's}$ for the labor and the goods market may be formulated as advanced or numerous as possible and sensible. Appropriately reordered, these equations are just two linear equations in the two unknowns $\hat{w}-\pi^c$, $\hat{p}-\pi^c$, the deviations of wage and price inflation from the inflationary climate currently prevailing. They can be uniquely solved for $\hat{w}-\pi^c$, $\hat{p}-\pi^c$, when the weights applied to current inflation rates, $\kappa_w, \kappa_p \in [0,1]$, fulfill $\kappa_w\kappa_p < 1$, then giving rise to the following reduced-form expressions for wage and price inflation rates, detrended by our concept of the inflationary climate into which current inflation is embedded:

$$\hat{w} - \pi^c = \frac{1}{1-\kappa_w\kappa_p}[\beta_{w's}(\,\cdot\,) + \kappa_w \beta_{p's}(\,\cdot\,)], \tag{4.10}$$

$$\hat{p} - \pi^c = \frac{1}{1-\kappa_w\kappa_p}[\beta_{p's}(\,\cdot\,) + \kappa_p \beta_{w's}(\,\cdot\,)]. \tag{4.11}$$

Note that all demand pressure variables are acting positively on the deviation of nominal wage and price inflation rates from the inflationary climate variable π^c. Integrating across markets for example the two PC's approach (4.1), (4.2) thus implies that two qualitatively different measures for demand pressure in the markets for labor as well as for goods have to be used both for money wage and price inflation for describing their deviation from the prevailing inflation climate, formally seen equivalent to a standard expectations augmented PC of the literature, see Laxton et al. (2000) for a typical example (with only one measure of demand pressure, the one on the labor market). Furthermore two different types of NAIRU's (one on the labor and one on the goods market) are here present in the integrated nominal wage and price PC which in general cannot be identified with each other (without knowledge of their link, i.e., Okun's law).

As a special case of the general reduced form (4.8) and (4.9) we obtain in the light of the preceding section and its representation of a wage-price spiral (4.8), (4.9) the following detailed equations for real wage growth and price inflation dynamics. Note that these two equations for the growth rate of real wages $\omega = w/p$ and price inflation $\hat{p}$ are equivalent to the two structural equations from a mathematical perspective.

$$\hat{\omega} = \kappa[(1-\kappa_p)(\beta_{we}(e-e_o) - \beta_{w\omega}\ln\omega) - (1-\kappa_w)(\beta_{pu}(u-u_o) + \beta_{p\omega}\ln\omega)],$$
$$\hat{p} = \kappa[\beta_{pu}(u-u_o) + \beta_{p\omega}\ln\omega + \kappa_p(\beta_{we}(e-e_o) - \beta_{w\omega}\ln\omega)] + \pi^c.$$

On the basis of the law of motion for the real wage $\omega = w/p$ we get as critical condition for the establishment of a positive dependence of the growth rate of real wages on economic activity the following term:

$$\alpha = (1-\kappa_p)\beta_{we} - (1-\kappa_w)\beta_{pu} \begin{Bmatrix} < \\ > \end{Bmatrix} 0 \iff \begin{Bmatrix} \text{normal} \\ \text{adverse} \end{Bmatrix} \text{RE},$$

the critical α condition for the occurrence of *normal (respectively: adverse) real wage effects*, if we assume that the rate of employment e and the rate of capacity utilization u are related to each other by an elasticity coefficient of unity, (which they are not in reality). Following Okun (1970) one might however argue that the relationship between these two rates is of the kind:

$$\frac{e}{e_o} = \frac{u}{u_o}^{b} \quad \text{or} \quad \ln e = b \ln u + const, \quad \text{i.e.,} \quad d\ln e = bd\ln u$$

with $b = 1/3$ according to Okun's own estimates. In this case we have to use

$$\alpha = (1-\kappa_p)\beta_{we} b u_o/e_o - (1-\kappa_w)\beta_{pu}$$

as term in the above critical α-condition in order to distinguish normal from adverse real wage adjustment patterns, see our estimates in the next section.

In the next section we shall reformulate this one step Okun link between goods and labor markets as a two stage procedure, leading from changes in the capacity utilization rate u of firms to the utilization rate of their labor force u^w and from there to the employment rate e on the labor market (i.e., from overtime work to new employees in the place of further increases in overtime work).[7] We shall also allow there for the possibility that insiders (the workforce of firms and their utilization rate u^w) determine the measure of demand pressure that drives wage inflation and not so much the outside employment rate, which there provides a second model of the working of the wage-price spiral (where of course only the first stage of okun's law is needed in order to close the model as far as supply side aspects are concerned).

4.5 Estimating the Wage–Price Spiral for the U.S. Economy

So far we have argued from the theoretical perspective that the PC approach to describe labor and goods market behavior is better modelled as a 2D dynamic system instead of a single labor market oriented PC (or goods market

[7] And similarly from decreases in utilization rate to reductions in the workforce employed by firms.

PC as in the baseline New Keynesian approach). In this section we are now going to provide empirical answers to the issues raised in the last two sections, i.e.:

- Do the two PC's as described in (4.1) and (4.2) provide a suitable model structure to capture the dynamics of the wage-price spiral implied by the empirical data?
- What is an appropriate empirical specification of the demand pressure terms in the two PC's including the quantity link between goods and labor markets (Okun's law)?
- How can we evaluate diverse specifications of the PC's and the resulting types of a wage-price spiral (outsider vs. insider formulations)?

In the following section we will give empirical answers to the above questions, while an econometric analysis of a more general model is provided and compared in the appendix to this chapter. Note with respect to the following that all variables in the displayed formula are expressed in logarithms now (through a convenient loglinear approximation of the utilization gaps of the model), up to Fig. 4.1 which provides the economic time series underlying the theoretical model in their original form.

4.5.1 Data Description

The empirical data for the relevant variables discussed above are taken from Economic Data—FRED® at http://research.stlouisfed.org/fred2/. The data shown below are quarterly, seasonally adjusted, annualized where necessary and are all available from 1947:1 to 2004:4. Up to the rate of unemployment they represent the business sector of the U.S. economy. We will make use in our estimations below of the range 1961:1 to 2004:4 solely, i.e., roughly speaking of the last five business cycles that characterized the evolution of the U.S. economy. We thus neglect the evolution following World War II to a larger degree (starting with the time when John F. Kennedy came into office and with the subsequent adoption of Keynesian economic policies).

Note that the time series of the variables employed in our model can be and have all been constructed from these basic time series.[8] We now use as inflationary climate expression π^c a moving average of price inflation over the past 12 quarters with linearly declining weights (as an especially simple

[8] This data set now employs a homogeneous sectoral measure of the wage share in the place of the hybrid one used in Flaschel and Krolzig (2006).

Table 4.2. Data Set I

Variable	Description of the untransformed series
e	Unemployment Rate (%), in logs
u^w	Hours of All Persons, Business Sector, Index 1992 = 100, log. e_h: ratio to the long run trend calculated by HP-filtering, in logs
u	Log Deviation of Real Gross Domestic Product to Real Potential Gross Domestic Product, in Billions of Chained 2000 Dollars, Alternatively: Capacity Utilization: Business Sector (%)
w	Business Sector: Compensation Per Hour, Index 1992 = 100, in logs
p	Implicit Business Sector Price Deflator, Index 1992 = 100, in logs
z	Output Per Hour of All Persons, Business Sector, Index 1992 = 100, in logs
$\ln v$	Log of Wage Share centered at 60%

measure of this inflationary climate expression). The graphs of the time series of our model's variables are shown in Fig. 4.1. Note that we are making use of the variable z, the labor productivity, its rate of growth $\hat{z}$ and the variable $\ln v = \ln(w/(pz))$, the log wage share or log real unit wage costs in addition to the variables employed in the preceding sections, since this is needed from the empirical perspective, but was ignored in the theoretical comparison with the New Keynesian wage and price dynamics considered in Sect. 4.3. The Blanchard and Katz (1999) error correction terms are thus now represented as in their paper by the log of the wage share and the growth rate of labor productivity is now added with a positive (negative) parameter value to the structural wage (price) Phillips curve. We note that the approach of Blanchard and Katz suggests that the parameters in front of $\ln v$ and $\hat{z}$ are of the same size, but of opposite sign in the wage Phillips curve and considerably less than one (if not zero as they claim it to hold for the U.S. economy). A similar observation holds for the price PC with opposite signs, see Flaschel and Krolzig (2006) in this regard.[9]

Before we start with our empirical investigation, we examine the stationarity of the relevant time series. The shown graphs of the series for wage and

[9] Note that the vertical lines shown in the graphs indicate the starting period of our empirical estimates.

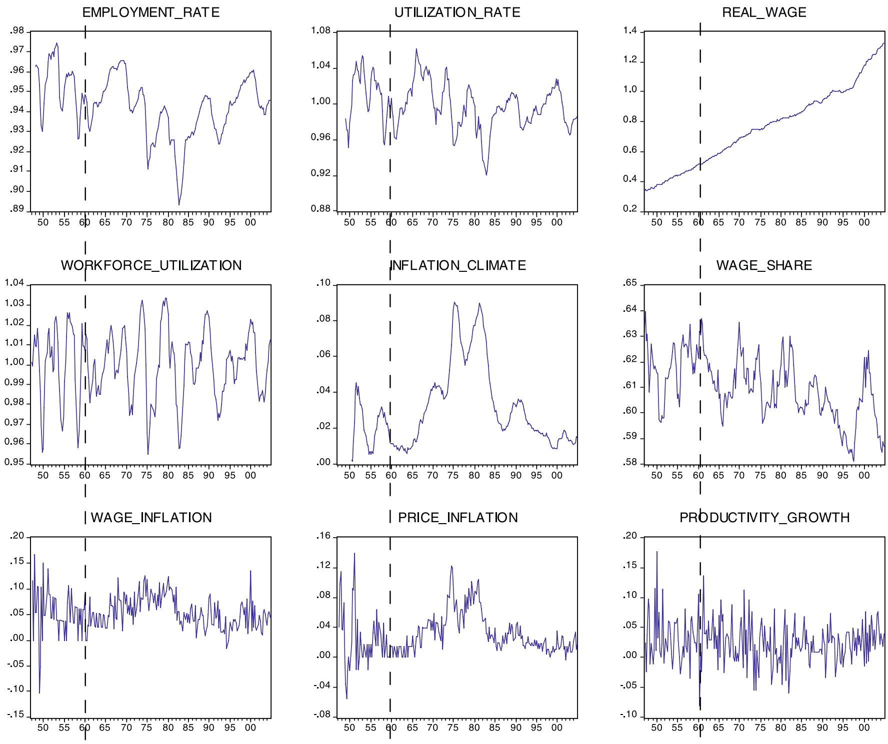

Fig. 4.2. Graphs of the time series of the variables of the model

Table 4.3. Summary of Dickey Fuller test results

Variable	Sample	Critical value	Test Statistic
$\hat{w}$	1947:02 TO 2004:04	−3.41	−4.75
$\hat{p}$	1947:02 TO 2004:04	−3.41	−4.14
e	1947:02 TO 2004:04	−3.41	−3.69
u^w	1947:02 TO 2004:04	−3.41	−6.80
u	1947:02 TO 2000:04	−3.41	−4.90
$\hat{z}$	1947:02 TO 2004:04	−3.41	−9.58

price inflation, capacity utilization rates and labor productivity growth suggest the stationarity of the time series (as expected). In addition we carry out the augmented Dickey Fuller unit root test for each series. The test results are reported in Table 4.3. The unit root tests confirms our expectation.

4.5.2 Estimation Results

When one estimates the model of this chapter in its most general form, with both rates of employment, e, u^w in the money wage Phillips curve, and with an Okun's Law for employment inside the firm sector, u^w, as a function of u, primarily representing the production technology of the economy, supplemented by an Okun's Law for e as a function of u^w, representing the employment policy of firms, and when one finally allows that the weights concerning actual inflation and the inflationary climate need not sum to one, one gets in the case of a three-stage least-square estimate all parameter signs as suggested by theory, though not always with a convincing t-statistics in particular. Removing the insignificant variables from the right hand side of our structural equations then provides us with the following two alternative approaches, one with the insiders' employment rate solely and one with the employment rate on the labor market solely, which as we shall see will both perform quite well as competing approaches to labor market phenomena. We thus shall test below the following two specific wage-price spiral models of our general approach to supply side macrodynamics:

Model I (Outsider Approach)

$$\begin{aligned}
\hat{w}_t &= a_1\hat{p}_t + a_2\pi_t^c + a_3 e_{t-1} - a_4 \ln v_{t-1} + a_5\hat{z}_t + a_6 + \epsilon_{1t}\\
\hat{p}_t &= b_1\hat{w}_t + b_2\pi_t^c + b_3 u_t + b_4 \ln v_{t-1} - b_5\hat{z}_t + b_6 + b_7 d74 + \epsilon_{2t}\\
\widehat{e}_t^h &= c_1\hat{u}_t + c_2\hat{u}_{t-1} + c_3\hat{u}_{t-2} + \epsilon_{3t}\\
\hat{e}_t &= d_1\widehat{e}_t^h + d_2\widehat{e}_{t-1}^h + \epsilon_{4t}
\end{aligned}$$

Model I makes use of the outside employment rate solely in the money wage PC and summarizes the variables that are then involved in a complete measurement of the resulting wage price spiral (with all parameters of the model collected in a single constant parameter in the equations to be estimated). It now exhibits the influence of the growth rate of labor productivity in addition to what was formulated in Sects. 4.2 and 4.3. Note that Blanchard and Katz (1999) show for their microfounded money wage PC that the parameters in front of $\ln v$, $\hat{z}$ should be equal in size, but opposite in sign (which they approximately are in our subsequent estimates). Note also that we make use the current utilization rate in the place of the lagged one in the price inflation equation. Okun's law is formulated in two steps here, leading from capacity utilization to workforce utilization and from there to the outside employment rate. Here, the lag structure shown above performed best in the estimates that were considered. This shows that a distributed lags of past growth rates of utilization rates explains growth in workforce participation and labor-force utilization better than just a single term on the right hand side of these equations. The symbol $d74$ denotes a dummy variable to take account of the influence of the first oil crisis in 1974 on the price level.

Model II (Insider Approach)

$$\hat{w}_t = a_1\hat{p}_t + a_2\pi_t^c + a_3 e_{t-1}^h - a_4 \ln v_{t-1} + a_5\hat{z}_t + a_6 + \epsilon_{1t}$$
$$\hat{p}_t = b_1\hat{w}_t + b_2\pi_t^c + b_3 u_t + b_4 \ln v_{t-1} - b_5\hat{z}_t + b_6 + b_7 d74 + \epsilon_{2t}$$
$$\hat{e}_t^h = c_1\hat{u}_t + c_2\hat{u}_{t-1} + c_3\hat{u}_{t-2} + \epsilon_{3t}$$

In the second formulation of the model to be estimated, where insiders (the utilization of the workforce employed by firms) are the ones that represent demand pressure in the money wage PC, we just replace the variable e by u^w and can of course then suppress the second stage in the formulation of Okun's Law. The three-stage least-square estimates of these two possible theoretical approaches to the labor market and to the wage-price spiral are shown in the tables below.

Unrestricted estimation shows that the estimated coefficients a_1 and a_2, and b_1 and b_2 sum approximately to unit respectively, which confirms our general formulation of the price and wage Phillips curves in the theoretical part of this chapter. In Table 4.4 we report the results of the constrained estimates where these sums are restricted to unity. Note that wage earners are much more short-sighted than firms with respect to the weight they give current rates of inflation. We have moreover in the shown estimates that demand pressure matters more in the labor market than in the goods market. More importantly, and in contrast to the arguments put forth in Blanchard and Katz (1999), their error correction terms now indeed matter for the U.S. economy and this holds true for both the wage and the price PC. Blanchard and Katz (1999) furthermore have with respect to their augmented wage PC, see their (6), that the coefficient in front of labor productivity should be equal in size (but opposite in sign) to the one in front of the wage share, which is approximately true in our estimated wage PC, and thus fairly different from unity (as it may be suggested by standard steady state calculations in standard macrodynamic models. A similar argument applies to the parameter in front of labor productivity growth in the price PC).

In sum, the estimated parameter values suggest for the critical α-condition derived in Sect. 4.4 the approximate value $\alpha = -.005$ if only the unlagged

Table 4.4. Three-stage least-square estimates with outsider employment rate e solely and two-step formulation of Okun's law

Wage PC			Price PC			Okun's Law		
Dependent Variable $\hat{w}$			Dependent Variable $\hat{p}$			Dependent Variables u^w, e		
Variable	Estimate	t-values	Variable	Estimate	t-values	Variable	Estimate	t-values
$\hat{p}$	0.51	2.8	$\hat{w}$	0.27	6.9	$u(0)$	0.50	11.2
π^c	0.49	–	π^c	0.73	–	$u(-1)$	0.23	5.2
e	0.63	4.8	u	0.21	3.2	$u(-2)$	0.13	3.0
$\ln v$	−0.12	−6.7	$\ln v$	0.16	3.2	$u^w(0)$	0.39	16.1
$\hat{z}$	0.14	2.88	$\hat{z}$	−0.12	-3.5	$u^w(-1)$	0.09	4.0
			$d74$	−0.04	4.9			
			$const.$	0.07	3.0			
R^2	0.49		R^2	0.78		R^2	0.91, 0.98	
$\bar{R}^2$	0.48		$\bar{R}^2$	0.77		$\bar{R}^2$	0.91, 0.98	
RSS	0.02		RSS	0.01		RSS	0.01, 0.008	
DW	1.97		DW	1.58		DW	1.76, 1.86	

terms of our formulation of Okun's law are taken into account. Yet, this result indicates that real wage adjustments to activity changes may be uncertain in sign, in particular if the lagged terms in Okun's law are also taken into account. The transmission of business fluctuations into real wage changes may therefore here be characterized as being weak and uncertain in sign, and may therefore change sign in particular if certain subperiods of the here considered time interval are to be investigated. If even $\kappa_w = 1$ is assumed as restriction, we must of course have a positive value for α in the critical condition that translates changes in economic activity into real-wage growth. Real wages are then definitely moving procyclically, though lagging behind economic activity with a quarter phase displacement.

Table 4.5 presents the estimation results of Model II. They are the same as the ones shown in Table 4.4 as far as parameter signs are concerned, also with respect to the Blanchard and Katz error correction coefficients. Parameter sizes are however somewhat different, in particular as far as the parameter κ_w in front of $\hat{p}$ in the WPC is concerned which is now fairly close to unity, indicating that the inflation climate is of not much importance in the formation of wage inflation. Demand pressures in the two PC's are now less important, while error correction is now working with more strength. However, wages are still more flexible than prices with respect to their measures of

Table 4.5. Three-stage least-square estimates with insider employment rate u^w solely and one-step formulation of Okun's law

Wage PC			Price PC			Okun's Law		
Dependent Variable $\hat{w}$			Dependent Variable $\hat{p}$			Dependent Variables u^w		
Variable	Estimate	t-values	Variable	Estimate	t-values	Variable	Estimate	t-values
$\hat{p}$	0.86	4.1	$\hat{w}$	0.35	–	$u(0)$	0.48	10.5
π^c	0.14	–	π^c	0.65	6.4	$u(-1)$	0.23	5.1
u^w	0.29	2.0	u	0.18	2.9	$u(-2)$	0.10	2.3
$\ln v$	−0.23	−2.6	$\ln v$	0.17	3.4			
$\hat{z}$	0.24	4.6	$\hat{z}$	−0.13	−4.1			
const.	−0.1	−2.2	$d74$	0.04	4.4			
			$const.$	0.08	3.2			
R^2	0.44		R^2	0.77		R^2	0.91	
$\bar{R}^2$	0.43		$\bar{R}^2$	0.76		$\bar{R}^2$	0.91	
RSS	0.02		RSS	0.01		RSS	0.01	
DW	1.84		DW	1.62		DW	1.74	

demand pressure and workers remain more short-sighted than firms concerning medium-run inflation dynamics. Again the parameters for error correction and labor productivity are by and large equal in size, but opposite in sign. And with respect to the critical parameter α we now approximately get the value 0.14, again by only employing the unlagged term in the estimated Okun law and thus now a positive value that can be considered a lower bound for the implied adverse working of the wage price spiral.

Summing up, Table 4.4 claims that conditions on the external labor market are the important ones for the working of the U.S. wage-price spiral, while Table 4.4 states the same for the inside employment of the workforce of firms, though the parameter in front of inside demand pressure in the wage PC is smaller than the corresponding one for the outside demand pressure term. When one uses the parameter restrictions shown above, but integrates again both measures of demand pressure into the considered wage PC, it is however again suggested that outside demand pressure is the significant (dominant) one. We however admit here that this point deserves closer inspection in future research, also from the theoretical point of view. Be that as it may, only estimate II shows that real wage dynamics are labor-market led and will thus imply instability of real wage adjustment in situations where aggregate goods demand is wage-led and thus increasing with economic activity. The other estimate (model I) suggests in addition that real wage growth is not strongly driven by economic activity levels on an average and thus indicates that unstable real wage adjustment may be possible, but will be small in degree.

4.6 Keynesian Macrodynamics: Empirical Reformulation of a Baseline Model

In this section we reformulate the theoretical disequilibrium model of AS–AD growth of Asada et al. (2006) in order to obtain a somewhat simplified version that is more suitable for empirical estimation and for the study of the role of contemporary interest rate policy rules. We dispense with the LM curve of the original approach and replace it here by a Taylor type policy rule. In addition we use dynamic IS as well as employment equations in the place of the originally static ones, where with respect to the former the dependence of consumption and investment on income distribution now only appears in an aggregated format. We use Blanchard and Katz (1999) error correction terms both in the wage and the price Phillips curve and thus give income distribution a role to play in wage as well as in price dynamics. Finally, we will again have inflationary inertia in a world of myopic perfect foresight through the inclusion of a medium-run variable, the inflationary climate in which the economy is operating, and its role for the wage–price dynamics of the considered economy.

We start from the observation that a Keynesian model of aggregate demand fluctuations should (independently of whether justification can be found for this in Keynes' General Theory) allow for under- (or over-) utilized labor *as well as* capital in order to be general enough from the descriptive point of view. As Barro (1994) for example observes, IS-LM is (or should be) based on imperfectly flexible wages and prices and thus on the consideration of wage as well as price Phillips Curves. This is precisely what we will do in the following, augmented by the observation that also medium-run aspects count both in wage and price adjustments, here formulated in simple terms by the introduction of the concept of an inflation climate. We have moreover model-consistent expectations with respect to short-run wage and price inflation. The modification of the traditional AS–AD model that we shall consider thus treats—as already described in the preceding section—expectations in a hybrid way, with crossover myopic perfect foresight of the currently evolving rates of wage and price inflation on the one hand and an adaptive updating of an inflation climate expression with exponential or any other weighting schemes on the other hand.

We consequently assume, see also the preceding section, two Phillips Curves in the place of only one. In this way, we can discuss wage and price dynamics separately from each other, in their structural forms, now indeed both based on their own measure of demand pressure, namely $e - e_o, u - u_o$, in the market for labor and for goods, respectively. We here denote by e the rate of employment on the labor market and by e_o the NAIRU-level of this rate, and similarly by u the rate of capacity utilization of the capital stock and u_o the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation rates, $\hat{w}$, $\hat{p}$, are both augmented by a weighted average of corresponding cost-pressure terms, based on forward looking myopic perfect foresight $\hat{p}, \hat{w}$, respectively, and a backward looking measure of the prevailing inflationary climate, symbolized by π^c.

We thereby arrive at the following two Phillips Curves for wage and price inflation, which in this core version of Keynesian AS–AD dynamics are—from a qualitative perspective—formulated in a fairly symmetric way.[10] We stress that we include forward-looking behavior here, without the need for an application of the jump variable technique of the rational expectations school in general and the New Keynesian approach in particular as will be shown in the next section.[11]

The structural form of the wage-price dynamics:

$$\hat{w} = \beta_{w_1}(e - e_o) - \beta_{w_2} \ln(\omega/\omega_o) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c, \tag{4.12}$$

$$\hat{p} = \beta_{p_1}(u - u_o) + \beta_{p_2} \ln(\omega/\omega_o) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^c. \tag{4.13}$$

Somewhat simplified versions of these two Phillips curves have been estimated for the US-economy in various ways in Flaschel and Krolzig (2006), Chen and Flaschel (2006) and Flaschel et al. (2007) and have been found to represent

[10] With respect to empirical estimation one could also add the role of labor productivity growth. But this will not be done here in order to concentrate on the cycle component of the model, caused by changing income distribution in a world of stable goods market and interest rate dynamics. With respect to the distinction between real wages and unit wage costs we shall therefore detrend the corresponding time series such that the following types of PC's can still be applied.

[11] For a detailed comparison with the New Keynesian alternative to our model type see Chiarella et al. (2005).

a significant improvement over the conventional single reduced-form Phillips curve. A particular finding was that wage flexibility was greater than price flexibility with respect to their demand pressure measure in the market for goods and for labor,[12] respectively, and workers were more short-sighted than firms with respect to their cost pressure terms. Note that such a finding is not possible in the conventional framework of a single reduced-form Phillips curve. Inflationary expectations over the medium run, π^c, i.e., the inflationary climate in which current inflation is operating, may be adaptively following the actual rate of inflation (by use of some linear or exponential weighting scheme), may be based on a rolling sample (with hump-shaped weighting schemes), or on other possibilities for updating expectations. For simplicity of the exposition we shall make use of the conventional adaptive expectations mechanism in the theoretical part of this chapter, namely

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c). \tag{4.14}$$

Note that for our current version of the wage-price spiral, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$, the law of motion of which is given by (with $\kappa = 1/(1-\kappa_w\kappa_p)$):

$$\begin{aligned}\hat{\omega} = \kappa\,[&(1-\kappa_p)(\beta_{w_1}(e-e_o) - \beta_{w_2}\ln(\omega/\omega_o))\\ &-(1-\kappa_w)(\beta_{p_1}(u-u_o) + \beta_{p_2}\ln(\omega/\omega_o))].\end{aligned}$$

This follows easily from the following obviously equivalent representation of the above two PC's,

$$\begin{aligned}\hat{w} - \pi^c &= \beta_{w_1}(e-e_o) - \beta_{w_2}\ln(\omega/\omega_o) + \kappa_w(\hat{p} - \pi^c),\\ \hat{p} - \pi^c &= \beta_{p_1}(u-u_o) + \beta_{p_2}\ln(\omega/\omega_o) + \kappa_p(\hat{w} - \pi^c),\end{aligned}$$

by solving for the variables $\hat{w} - \pi^c$ and $\hat{p} - \pi^c$. It also implies the following two across-markets or *reduced form Phillips Curves*:

$$\begin{aligned}\hat{p} =\ & \kappa[\beta_{p_1}(u-u_o) + \beta_{p_2}\ln(\omega/\omega_o) + \kappa_p(\beta_{w_1}(e-e_o) - \beta_{w_2}\ln(\omega/\omega_o))] + \pi^c,\\ \hat{w} =\ & \kappa[\beta_{w_1}(e-e_o) - \beta_{w_2}\ln(\omega/\omega_o) + \kappa_w(\beta_{p_1}(u-u_o) + \beta_{p_2}\ln(\omega/\omega_o))] + \pi^c,\end{aligned}$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market.

[12] For lack of better terms we associate the degree of wage and price flexibility with the size of the parameters β_{w_1}, β_{p_1}, though of course the extent of these flexibilities will also depend on the size of the fluctuations of the excess demands in the market for labor and for goods, respectively.

The remaining laws of motion of the private sector of the model are as follows:

$$\hat{u} = -\alpha_u(u - u_o) \pm \alpha_\omega \ln(\omega/\omega_o) - \alpha_r(i - \hat{p} - (i_o - \bar{\pi})), \tag{4.15}$$

$$\hat{e} = \alpha_{e_1}(u - u_o) + \alpha_{e_2}\hat{u}. \tag{4.16}$$

The first law of motion is of the type of a dynamic IS-equation, see also Rudebusch and Svensson (1999) in this regard, here represented by the growth rate of the capacity utilization rate of firms. It has three important characteristics; (i) it reflects the dependence of output changes on aggregate income and thus on the rate of capacity utilization by assuming a negative, i.e., stable dynamic multiplier relationship in this respect, (ii) it shows the joint dependence of consumption and investment on the real wage (which in the aggregate may in principle allows for positive or negative signs before the parameter α_ω, depending on whether consumption or investment is more responsive to real wage changes), and (iii) it shows finally the negative influence of the real rate of interest on the evolution of economic activity. Note here that we have generalized this law of motion in comparison to the one in the original baseline model of Asada et al. (2006), since we now allow for the possibility that also consumption, not only investment, depends on income distribution as measured by the real wage. We note that we also use $\ln \omega$ in the dynamic multiplier equation, since this variable will be used later on to estimate this equation.

In the second law of motion, for the rate of employment, we assume that the employment policy of firms follows—in the form of a generalized Okun Law—the rate of capacity utilization (and the thereby implied rate of over- or underemployment of the employed workforce) partly with a lag (measured by $1/\beta_{e_1}$), and partly without a lag (through a positive parameter α_{e_2}). Employment is thus assumed to adjust to the level of current activity in somewhat delayed form which is a reasonable assumption from the empirical point of view. The second term, $\alpha_{e_2}\hat{u}$, is added to take account of the possibility that Okun's Law may hold in level form rather than in the form of a law of motion, since this latter dependence can be shown to be equivalent to the use of a term $(u/u_o)^{\alpha_{e_2}}$ when integrated, i.e., the form of Okun's law in which this law was originally specified by Okun (1970) himself.

The above two laws of motion therefore reformulate in a dynamic form the static IS-curve (and the employment this curve implies) that was used in Asada et al. (2006). They only reflect implicitly the there assumed dependence of the rate of capacity utilization on the real wage, due to on smooth factor substitution in production (and the measurement of the potential output this implies in Asada et al. (2006)), which constitutes another positive influence of the real wage on the rate of capacity utilization and its rate of change. This simplification helps to avoid the estimation of separate equations for consumption and investment C, I and for potential output Y^p.

Finally, we no longer to employ here a law of motion for real balances as was still the case in Asada et al. (2006). Money supply is now accommodating to the interest rate policy pursued by the central bank and thus does not feedback into the core laws of motion of the model. As interest rate policy we assume the following classical type of Taylor rule:

$$i_{\mathrm{T}} = (i_o - \bar{\pi}) + \hat{p} + \phi_{ip}(\hat{p} - \bar{\pi}) + \phi_{iu}(u - u_o), \tag{4.17}$$

$$\dot{i} = \alpha_{ii}(i_{\mathrm{T}} - i). \tag{4.18}$$

The target rate of the central bank i_{T} is here made dependent on the steady state real rate of interest augmented by actual inflation back to a nominal rate, and is as usually dependent on the inflation gap and the capacity utilization gap (as measure of the output gap). With respect to this target there is then interest rate smoothing with strength α_{ii}. Inserting i_{T} and rearranging terms we get from this expression the following from of a Taylor rule

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o), \tag{4.19}$$

where we have $\gamma_{ii} = \alpha_{ii}$, $\gamma_p = \alpha_{ii}(1 + \phi_{ip})$, i.e., $\phi_{ip} = \gamma_{ip}/\alpha_{ii} - 1$ and $\gamma_{iu} = \alpha_{ii}\phi_{iu}$.

We thus allow now for interest rate smoothing in this rule in contrast to Sect. 4.3. Furthermore, the actual (perfectly foreseen) rate of inflation $\hat{p}$ is used to measure the inflation gap with respect to the inflation target $\bar{\pi}$ of the central bank. Note finally that we could have included (but have not done this here yet) a new kind of gap into the above Taylor rule, the real wage gap, since we have in our model a dependence of aggregate demand on income distribution and the real wage. The state of income distribution matters for the dynamics of our model and thus should also play a role in the decisions of the central bank. All of the employed gaps are measured relative to the steady state of the model, in order to allow for an interest rate policy that is consistent with it.

We note that the steady state of the dynamics, due to its specific formulation, can be supplied exogenously. For reasons of notational simplicity we choose: $u_o = 1, e_o = 1, \omega_o = 1$, $\pi_o^c = \bar{\pi} = 0.005$, $i_o = 0.02$ in the later estimation of the model by means of quarterly US-data. As the model is formulated now it exhibits five gaps, to be closed in the steady state and has five laws of motion, which when set equal to zero, exactly imply this result, since the determinant of the Jacobian of the dynamics is shown to be always non-zero in the next section of the chapter. Note finally that the model becomes a linear one when utilization gaps are approximated by logs of utilization rates.

The steady state of the dynamics is locally asymptotically stable under certain sluggishness conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability cyclically (by way of so-called Hopf-bifurcations) if the system becomes too flexible, and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high, as we shall show below. If the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior—like downward money wage rigidity—to manifest themselves at least far off the steady state in order to bound the dynamics to an economically meaningful domain in the considered 5D state space. Chen et al. (2004) provide a variety of numerical studies for such an approach with extrinsically motivated nonlinearities and thus undertake its detailed numerical investigation. In sum, therefore, our dynamic AS–AD growth model here and there will exhibit a variety of features that are much more in line with a Keynesian understanding of the characteristics of the trade cycle than is the case for the conventional modeling of AS–AD growth dynamics or its radical reformulation by the New Keynesians (where—if non-determinacy can be avoided by the choice of an appropriate Taylor rule—only the steady state position is a meaningful solution in the related setup we considered in the preceding section).

Taken together the model of this section consists of the following five laws of motion (with the derived reduced form expressions as far as the wage-price spiral is concerned and with reduced form expressions by assumption concerning the goods and the labor market dynamics):[13]

[13] As the model is formulated we have no real anchor for the steady state rate of interest (via investment behavior and the rate of profit it implies in the steady state) and thus have to assume here that it is the monetary authority that enforces a certain steady state values for the nominal rate of interest.

The 5D Dynamical Model

$$\hat{u} \overset{Dyn.IS}{=} -\alpha_u(u-u_o) \pm \alpha_\omega(\omega-\omega_o) - \alpha_r(i-\hat{p}-(i_o-\bar{\pi})), \quad (4.20)$$

$$\hat{e} \overset{O.Law}{=} \beta_{e_1}(u-u_o) + \beta_{e_2}\hat{u}, \quad (4.21)$$

$$\dot{i} \overset{T.Rule}{=} -\gamma_{ii}(i-i_o) + \gamma_{ip}(\hat{p}-\bar{\pi}) + \gamma_{iu}(u-u_o), \quad (4.22)$$

$$\hat{\omega} \overset{RWPC}{=} \kappa[(1-\kappa_p)(\beta_{w_1}(e-e_o) - \beta_{w_2}\ln(\omega/\omega_o)) \\ -(1-\kappa_w)(\beta_{p_1}(u-u_o) + \beta_{p_2}\ln(\omega/\omega_o))], \quad (4.23)$$

$$\dot{\pi}^c \overset{I.Climate}{=} \beta_{\pi^c}(\hat{p}-\pi_c) \quad , \quad (4.24)$$

The above equations represent, in comparison to the baseline model of New Keynesian macroeconomics, the IS goods market dynamics (7), here augmented by Okun's Law as link between the goods and the labor market (8), and of course the Taylor Rule (9), and a law of motion (10) for the real wage $\hat{\omega} = \pi^w - \pi^p$ that makes use of the same explaining variables as the New Keynesian one considered in Sect. 4.3 (but with inflation rates in the place of their time rates of change and with no accompanying sign reversal concerning the influence of output and wage gaps), and finally the law of motion (11) that describes the updating of the inflationary climate expression.[14] We have to make use in addition of the following reduced form expression for the price inflation rate or the price PC, our law of motion for the price level p in the place of the New Keynesian law of motion for the price inflation rate π^p:

$$\hat{p} = \kappa[\beta_{p_1}(u-u_o) + \beta_{p_2}\ln(\omega/\omega_o) \\ +\kappa_p(\beta_{w_1}(e-e_o) - \beta_{w_2}\ln(\omega/\omega_o))] + \pi_c, \quad (4.25)$$

which has to be inserted into the above laws of motion in various places in order to get an autonomous nonlinear system of differential equations in the state variables: capacity utilization u, the rate of employment e, the nominal rate of interest i, the real wage rate ω, and the inflationary climate expression π_c. We stress that one can consider (4.25) as a sixth law of motion of the considered dynamics which however—when added—leads a system determinant which is zero and which therefore allows for zero-root hysteresis for certain variables of the model (in fact in the price level if the target rate of

[14] In correspondence to the Blanchard and Katz error correction terms in our wage and price PC, we here make also use of the log of the real wage in the law of motion which describes goods market dynamics, partly due also to our later estimation of the model.

inflation of the Central Bank is zero and if interest rate smoothing is present in the Taylor rule). We have written the laws of motion in an order that first presents the dynamic equations also present in the baseline New Keynesian model of inflation dynamics, and then our formulation of the dynamics of income distribution and of the inflationary climate in which the economy is operating.

With respect to the empirically motivated restructuring of the original theoretical framework, the model is as pragmatic as the approach employed by Rudebusch and Svensson (1999). By and large we believe that it represents a working alternative to the New Keynesian approach, in particular when the current critique of the latter approach is taken into account. It overcomes the weaknesses and the logical inconsistencies of the old Neoclassical synthesis, see Asada et al. (2006), and it does so in a minimal way from a mature, but still traditionally oriented Keynesian perspective (and is thus not really "New"). It preserves the problematic stability features of the real rate of interest channel, where the stabilizing Keynes effect or the interest rate policy of the central bank is interacting with the destabilizing, expectations driven Mundell effect. It preserves the real wage effect of the old Neoclassical synthesis, where—due to an unambiguously negative dependence of aggregate demand on the real wage—we had that price flexibility was destabilizing, while wage flexibility was not. This real wage channel is not really discussed in the New Keynesian approach, due to the specific form of wage-price dynamics there considered, see the preceding section, and it is summarized in Fig. 4.3 for the situation where investment dominates consumption with respect to real wage changes. In the opposite case, the situations considered in this figure will be reversed with respect to their stability implications.

The feedback channels just discussed will be the focus of interest in the now following stability analysis of our D(isequilibrium)AS–D(isequilibrium)AD dynamics. We have employed reduced-form expressions in the above system of differential equations whenever possible. We have thereby obtained a dynamical system in five state variables that is in a natural or intrinsic way nonlinear (due to its reliance on growth rate formulations). We note that there are many items that reappear in various equations, or are similar to each other, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. This dynamical system will be investigated in the next section in somewhat informal terms with respect to some stability assertions to which it gives rise. A rigorous

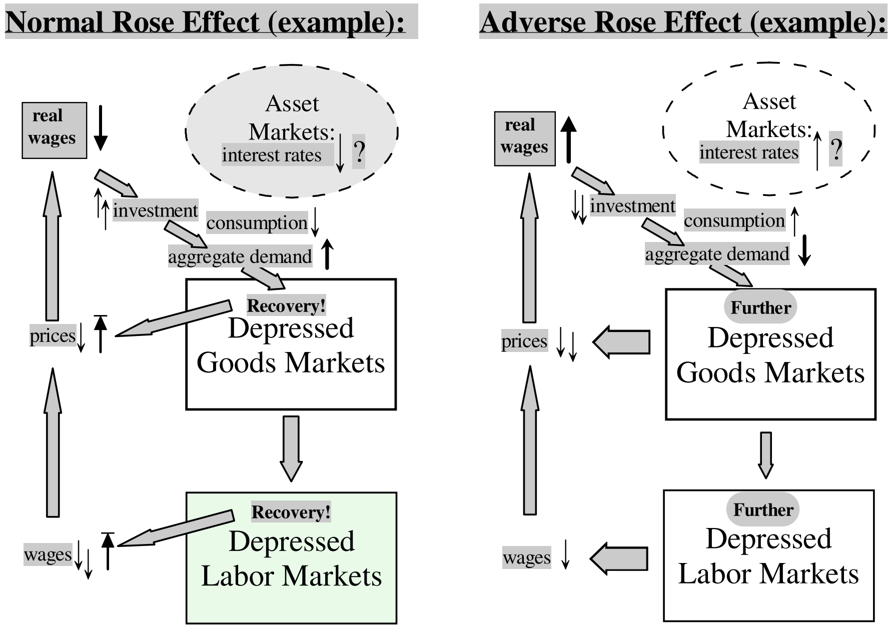

Fig. 4.3. The Rose effects: The real wage channel of Keynesian macrodynamics

proof of local asymptotic stability and its loss by way of Hopf bifurcations can be found in Asada et al. (2006), there for the original baseline model. For the present model variant we supply a more detailed stability proofs in Chen et al. (2004), where also more detailed numerical simulations of the model are provided.

4.7 5D Feedback-Guided Stability Analysis

In this section we illustrate an important method to prove local asymptotic stability of the interior steady state of the dynamical system (4.20)–(4.24) (with (4.25) inserted wherever needed) through partial considerations from the feedback chains that characterize this empirically oriented baseline model of Keynesian dynamics. Since the model is an extension of the standard AS–AD growth model, we know from the literature that there is a real rate of interest effect typically involved, first analyzed by formal methods in Tobin (1975), see also Groth (1992). Instead of the stabilizing Keynes-effect, based on activity-reducing nominal interest rate increases following price level increases, we have here however a direct steering of economic activity by the interest rate policy of the central bank. Since the (correctly anticipated) short-run real rate of interest is driving investment and consumption decisions (increases leading to decreased aggregate demand), there is furthermore the activity stimulating (partial) effect of increases in the rate of inflation (as part of the real rate of interest channel) that may lead to accelerating inflation under appropriate conditions. This is the so-called Mundell-effect that normally works in opposition to the Keynes-effect, but through the same real rate of interest channel as this latter effect. Due to our use of a Taylor rule in the place of the conventional LM curve, the Keynes-effect is here implemented in a more direct way towards a stabilization of the economy (coupling nominal interest rates directly with the rate of price inflation) and it is supposed to work more strongly the larger the choice of the parameters γ_{ip}, γ_{iu}. The Mundell-effect by contrast is stronger the faster the inflationary climate adjusts to the present level of price inflation, since we have a positive influence of this climate variable both on price as well as on wage inflation and from there on rates of employment of both capital and labor.

There is a further important potentially (at least partially) destabilizing feedback mechanism as the model is formulated. Excess profitability depends positively on the rate of return on capital and thus negatively on the real wage ω. We thus get—since consumption may also depend (positively) on the real wage—that real wage increases can depress or stimulate economic activity depending on whether investment or consumption is dominating the outcome of real wage increases (we here neglect the stabilizing role of the additional Blanchard and Katz type error correction mechanisms). In the first case, we get from the reduced-form real wage dynamics:

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_{w_1}(e - e_o) - (1 - \kappa_w)\beta_{p_1}(u - u_o)],$$

that price flexibility should be bad for economic stability, due to the minus sign in front of the parameter β_p, while the opposite should hold true for the parameter that characterizes wage flexibility. This is a situation that was already investigated in Rose (1967). It gives the reason for our statement that wage flexibility gives rise to normal, and price flexibility to adverse, Rose effects as far as real wage adjustments are concerned (if it is assumed—as in our theoretical baseline model—that only investment depends on the real wage). Besides real rate of interest effects, establishing opposing Keynes- and Mundell-effects, we thus have also another real adjustment process in the considered model where now wage and price flexibility are in opposition to each other, see Chiarella and Flaschel (2000) and Chiarella et al. (2000) for further discussion of these as well as of other feedback mechanisms of such Keynesian growth dynamics. We observe again that our theoretical DAS–AD growth dynamics in Asada et al. (2006)—due to their origin in the baseline model of the Neoclassical Synthesis, stage I—allows for negative influence of real wage changes on aggregate demand solely, and thus only for cases of destabilizing wage level flexibility, but not price level flexibility. In the empirical estimation of the model (4.20)–(4.24) we will indeed find that this case seems to be the one that characterizes our empirically and broader oriented dynamics (4.20)–(4.24).

The foregoing discussion enhances our understanding of the feedback mechanisms of the dynamical system (4.20)–(4.24) whose stability properties will now be investigated by means of varying adjustment speed parameters appropriately. With the feedback scenarios considered above in mind, we first observe that the inflationary climate can be frozen at its steady state value, $\pi_o^c = \bar{\pi}$, if $\beta_{\pi^c} = 0$ is assumed. The system thereby becomes 4D and it can indeed be further reduced to 3D if in addition $\alpha_\omega = 0$, $\beta_{w_2} = 0, \beta_{p_2} = 0$ is assumed, since this decouples the ω-dynamics from the remaining system dynamics u, e, i. We will consider the stability of these 3D subdynamics—and its subsequent extensions—in informal terms only here, reserving rigorous calculations to the alternative scenarios provided in Chen et al. (2004). We nevertheless hope to be able to demonstrate to the reader how one can indeed proceed systematically from low to high dimensional analysis in such stability investigations from the perspective of the partial feedback channels implicitly contained in the considered 5D dynamics. This method has been already applied successfully to various other, often more complicated, dynamical systems; see Asada et al. (2003) for a variety of typical examples.

Before we start with our stability investigations we establish the fact that for the dynamical system given by (4.20)–(4.24) loss of stability can in general only occur by way of Hopf-bifurcations, since the following proposition can be shown to hold true under mild—empirically plausible—parameter restrictions.

Proposition 4.1. *Assume that the parameter γ_{ii} is chosen sufficiently small and that the parameters $\beta_{w_2}, \beta_{p_2}, \kappa_p$ fulfill $\beta_{p_2} > \beta_{w_2}\kappa_p$. Then: The* 5D *determinant of the Jacobian of the dynamics at the interior steady state is always negative in sign.*

Sketch of proof. We have for the sign structure in the Jacobian under the given assumptions the following situation to start with (we here assume as limiting situation $\gamma_{ii} = 0$ and have already simplified the law of motion for e by means of the one for u through row operations that are irrelevant for the size of the determinant to be calculated):

$$J = \begin{pmatrix} \pm & + & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ + & + & 0 & + & + \\ - & + & 0 & - & 0 \\ + & + & 0 & + & 0 \end{pmatrix}.$$

We note that the ambiguous sign in the entry J_{11} in the above matrix is due to the fact that the real rate of interest is a decreasing function of the inflation rate which in turn depends positively on current rates of capacity utilization.

Using the second row and the last row in their dependence on the partial derivatives of $\hat{p}$ we can reduces this Jacobian to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}$$

without change in the sign of its determinant. In the same way we can now use the third row to get another matrix without any change in the sign of the corresponding determinants

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}.$$

The last two columns can under the observed circumstances be further reduced to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}$$

which finally gives

$$J = \begin{pmatrix} 0 & 0 & - & 0 & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}.$$

This matrix is easily shown to exhibit a negative determinant which proves the proposition, also for all values of γ_{ii} which are chosen sufficiently small. ■

Proposition 4.2. *Assume that the parameters β_{w_2}, β_{p_2}, α_ω and β_{π^c} are all set equal to zero. This decouples the dynamics of u, e, i from the rest of the system. Assume furthermore that the partial derivative of the first law of motion* (7) *depends negatively on u, i.e., $\alpha_u > \alpha_{ur}\kappa\beta_{p_1}$, so that the dynamic multiplier process, characterized by α_u, dominates this law of motion with respect to the overall impact of the rate of capacity utilization u. Then: The interior steady state of the implied* 3D *dynamical system*

$$\hat{u} = -\alpha_u(u - u_o) - \alpha_{ur}(i - \hat{p} - (i_o - \bar{\pi})), \tag{4.26}$$

$$\hat{e} = \beta_{e_1}(u - u_o), \tag{4.27}$$

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o), \tag{4.28}$$

is locally asymptotically stable if the interest rate smoothing parameter γ_{ii} and the employment adjustment parameter β_e are chosen sufficiently small in addition.

Sketch of proof. In the considered situation we have for the Jacobian of these reduced dynamics at the steady state:

$$J = \begin{pmatrix} - & + & - \\ + & 0 & 0 \\ + & + & - \end{pmatrix}.$$

The determinant of this Jacobian is obviously negative if the parameter γ_{ii} is chosen sufficiently small. Similarly, the sum of the minors of order 2: a_2, will be positive if β_e is chosen sufficiently small. The validity of the full set of Routh-Hurwitz conditions then easily follows, since trace $J = -a_1$ is obviously negative and since det J is part of the expressions that characterize the product $a_1 a_2$. ■

Proposition 4.3. *Assume now that the parameter α_ω is negative, but chosen sufficiently small, while the error correction parameters β_{w_2}, β_{p_2} are still kept* *at zero. Then: The interior steady state of the resulting* 4D *dynamical system (where the state variable* ω *is now included)*

$$\hat{u} = -\alpha_u(u - u_o) - \alpha_\omega(\ln \omega/\omega_o) - \alpha_{ur}((i - \hat{p}) - (i_o - \bar{\pi})), \tag{4.29}$$

$$\hat{e} = \beta_{e_1}(u - u_o), \tag{4.30}$$

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o), \tag{4.31}$$

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_{w_1}(e - e_o) - (1 - \kappa_w)\beta_{p_1}(u - u_o), \tag{4.32}$$

is locally asymptotically stable.

Sketch of proof. It suffices to show in the considered situation that the determinant of the resulting Jacobian at the steady state is positive, since small variations of the parameter α_ω must then move the zero eigenvalue of the case $\alpha_\omega = 0$ into the negative domain, while leaving the real parts of the other eigenvalues—shown to be negative in the preceding proposition—negative. The determinant of the Jacobian to be considered here—already slightly simplified—is characterized by

$$J = \begin{pmatrix} 0 & + & - & - \\ + & 0 & 0 & 0 \\ 0 & + & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}.$$

This can be further simplified to

$$J = \begin{pmatrix} 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}$$

without change in the sign of the corresponding determinant which proves the proposition. ■

We note that this proposition also holds where $\beta_{p_2} > \beta_{w_2}\kappa_p$ holds true as long as the thereby resulting real wage effect is weaker than the one originating from α_ω. Finally—and in sum—we can also state that the full 5D dynamics must also exhibit a locally stable steady state if β_{π^c} is made positive, but chosen sufficiently small, since we have already shown that the full 5D dynamics exhibits a negative determinant of its Jacobian at the steady state under the stated conditions. Increasing β_{π^c} from zero to a small positive value therefore must mover the corresponding zero eigenvalue into the negative domain of the plane of complex numbers.

Summing up, we can state that a weak Mundell effect, the neglect of Blanchard–Katz error correction terms, a negative dependence of aggregate demand on real wages, coupled with nominal wage and also to some extent price level inertia (in order to allow for dynamic multiplier stability), a sluggish adjustment of the rate of employment towards actual capacity utilization and a Taylor rule that stresses inflation targeting therefore are here (for example) the basic ingredients that allow for the proof of local asymptotic stability of the interior steady state of the dynamics (4.20)–(4.24). We expect however that indeed a variety of other and also more general situations of convergent dynamics can be found, but have to leave this here for future research and numerical simulations of the model. Instead we now attempt to estimate the signs and also the sizes of the parameters of the model in order to gain insight into the question to what extent for example the US economy after World War II supports one of the real wage effects considered in Fig. 4.3 and also the possibility of overall asymptotic stability for such an economy, despite a destabilizing Mundell effect in the real interest rate channel. Due to proposition 1 we know that the dynamics will generally only loose asymptotic stability in a cyclical fashion (by way of a Hopf-bifurcation) and will indeed do so if the parameter β_{π^c} is chosen sufficiently large. We thus arrive at a radically different outcome for the dynamics implied by our mature traditional Keynesian approach as compared to the New Keynesian dynamics. The question that naturally arises here is whether the economy can be assumed to be in the convergent regime of its alternative dynamical possibilities. This of course can only be decided by an empirical estimation of its various parameters which is the subject of the next section.

4.8 Estimating the Model

In Sect. 4.9 we have then considered certain special cases of the general model which allowed for the derivation of asymptotic stability of the steady state and its loss of stability by way of Hopf bifurcations if certain speed parameters become sufficiently large. In the present section we now provide empirical estimates for the laws of motion (4.20)–(4.24) of our disequilibrium AS–AD model, by means of the structural form of the wage and price Phillips curve, coupled with the dynamic multiplier equation, Okun's law and the interest rate policy rule. These estimates, on the one hand, serve the purpose of confirming the parameter signs we have specified in the initial theory-guided formulation of the model and to determine the sizes of these parameters in addition. On the other hand, we have three different situations where we cannot specify the parameter signs on purely theoretical grounds and where we therefore aim at obtaining these signs from the empirical estimates of the equations whenever this happens.

There is first of all, see (4.20), the ambiguous influence of real wages on (the dynamics of) the rate of capacity utilization, which should be a negative one if investment is more responsive than consumption to real wage changes and a positive one in the opposite case. There is secondly, with an immediate impact effect if the rates of capacity utilization for capital and labor are perfectly synchronized, the fact that real wages rise with economic activity through money wage changes on the labor market, while they fall with it through price level changes on the goods market, see (4.22). Finally, we have in the reduced form equation for price inflation a further ambiguous effect of real wage increases, which there lower $\hat{p}$ through their effect on wage inflation, while speeding up $\hat{p}$ through their effect on price inflation, effects which work into opposite directions in the reduced form price PC (4.25). Mundell-type, Rose-type and Blanchard–Katz error-correction feedback channels therefore make the dynamics indeterminate on the general level.

In all of these three cases empirical analysis will now indeed provide us with definite answers as to which ones of these opposing forces will be the dominant ones. Furthermore, we shall also see that the Blanchard and Katz (1999) error correction terms do play a role in the US-economy, in contrast to what has been found out by these authors for the money wage PC in the U.S. However, we will not attempt to estimate the parameter β_{π^c} that characterizes the evolution of the inflationary climate in our economy. Instead, we will use moving averages with linearly declining weights for its representation, which allows us to bypass the estimation of the law of motion (4.24). We consider this as the simplest approach to the treatment of our climate expression (comparable with recent New Keynesian treatments of hybrid expectation formation), which should later on be replaced by more sophisticated ones, for example one that makes use of the Livingston index for inflationary expectations as in Laxton et al. (2000) which in our view mirrors some adaptive mechanism in the adjustment of inflationary expectations.

We take an encompassing approach to conduct our estimates. The structural laws of motion of our economy, see Sect. 4.6, have been formulated in an intrinsically nonlinear way (due to certain growth rate formulations). We note that single equations estimates have suggested the use of only α_{e_2} in the equation that describes the dynamics of the employment rate. In the wage-price spiral we use—in line with Blanchard and Katz's theoretical derivation (1999)— the log of unit wage costs, removing their significant downward trend in the employed data appropriately. Note here again that we use the log of the unit wage costs in the dynamic multiplier equations as well.

We conduct our estimates in conjunction with time-invariant estimates of all the parameters of our model. This in particular implies that Keynes' (1936) explanation of the trade cycle, which employed systematic changes in the propensity to consume, the marginal efficiency of investment and liquidity preference over the course of the cycle, find no application here and that—due the use of detrended measures for income distribution changes and unit-wage costs—also the role of technical change is downplayed to a significant degree, in line with its neglect in the theoretical equations of the model presented in section 6. As a result we expect to obtain from our estimates long-phased economic fluctuations, but not yet long-waves, since important fluctuations in aggregate demand (based on time-varying parameters) are still ignored and since the dynamics is then driven primarily by slowly changing income distribution, indeed a slow process in the overall evolution of the U.S. economy after World War II.

To show that such an understanding of the model is a suitable description of (some of) the dynamics of the observed data, we first fit a corresponding 6D VAR model to the data to uncover the dynamics in the six independent variables there employed. We then identify a linear structural model that parsimoniously encompasses the employed VAR. Finally, we contrast our nonlinear structural model, i.e., the laws of motion (4.12) to (4.16) in structural form (and the Taylor rule), with the linear structural VAR model and show through a J test[15] that the nonlinear model is indeed preferred by the data. In this way we show that our (weakly) nonlinear structural model represents a proper description of the data.

The relevant variables for the following investigation are the wage inflation rate, the price inflation rate, the rates of utilization of labor and of capital,

[15] See Davidson and MacKinnon (1993) for details.

the nominal interest rate, the log of average unit wage cost,[16] to be denoted in the following by: $\mathrm{d}\ln w_t$, $\mathrm{d}\ln p_t$, e_t, u_t, i_t and uc_t, where uc_t is the cycle component of the log of the time series for the unit real wage cost, filtered by the bandpass filter.[17]

4.8.1 Data Description

The empirical data of the corresponding time series stem from the Federal Reserve Bank of St. Louis data set (see www.stls.frb.org/fred). The data are quarterly, seasonally adjusted and concern the period from 1965:1 to 2001:2. Except for the employment rates of the factors labor, e, and capital, u (and of course the interest rate and the derived inflation climate) the log of the series are used in Table 4.6 (note however that the intermediate estimation step of a linear structural VAR makes use of the logs of both utilization rates however).

We use $\ln w_t$ and $\ln p_t$, i.e., logarithms, in the place of the original level magnitudes. Their first differences $\mathrm{d}\ln w_t$, $\mathrm{d}\ln p_t$ thus give the current rate of wage and price inflation (backwardly dated). We use π_t^{12} in this section to denote specifically a moving average of price inflation rates with linearly decreasing weights over the past 12 quarters, interpreted as a particularly simple measure for the inflationary climate expression of our model, and we denote by e, u (U^l, U^c) the rates of (under-)utilization of labor and the capital stock.

There is a pronounced downward trend in part of the employment rate series (over the 1970's and part of the 1980's) and in the wage share (normalized to 0 in 1996). The latter trend is not the topic of this chapter which concentrates on the cyclical implications of changed in income distribution. Wage inflation shows three trend reversals, while the inflation climate representation clearly show two periods of low inflation regimes and in between a high inflation regime.

We expect that the six independent time series for wages, prices, capacity utilization rates, the growth rate of labor productivity and the interest rate (federal funds rate) are stationary. The graphs of the series for wage and price inflation, capacity utilization rates, $\mathrm{d}\ln w_t$, $\mathrm{d}\ln p_t$, $\ln e_t$, $\ln u_t$ seem to confirm

[16] Or alternatively the real wage which does not modify the obtained results in significant ways.

[17] For details on the bandpass filter see Baxter and King (1995a, 1995b, 1999).

Table 4.6. Data Set II

Variable	Description of the untransformed series
$U^l = 1 - e$	Unemployment Rate
$U^c = 1 - u$	Capacity Utilization: Manufacturing, Percent of Capacity
$\ln w$	Nonfarm Business Sector: Compensation Per Hour, 1992 = 100, in logs
$\ln p$	Gross Domestic Product: Implicit Price Deflator, 1996 = 100, in logs
$\ln z = \ln y - \ln l^d$	Nonfarm Business Sector; Output Per Hour of All Persons, 1992 = 100, in logs
uc	Nonfarm Business Sector: Real Compensation Per Output Unit, 1992 = 100, in logs
$\mathrm{d}\ln z$	Growth Rate of Labor Productivity
i	Federal Funds Rate

Table 4.7. Summary of Dickey Fuller-test results

Variable	Sample	Critical Value	Test Statistic
$\mathrm{d}\ln w$	1965:01 TO 2000:04	−3.41000	−3.74323
$\mathrm{d}\ln p$	1965:01 TO 2000:04	−3.41000	−3.52360
$\ln e$	1965:01 TO 2000:04	−2.86000	−2.17961
$\ln u$	1965:01 TO 2000:04	−3.41000	−3.92688
i	1965:01 TO 2000:04	−2.86000	−2.67530

our expectation. In addition we carry out the DF unit root test for each series. The test results are shown in Table 4.7.

The applied unit root test confirms our expectations with the exception of e and i. Although the test cannot reject the null of a unit root, there is no reason to expect the rate of unemployment and the federal funds rate to be unit root processes. Indeed we expect that they are constrained in certain limited ranges, say from zero to 0.3. Due to the lower power of the Dickey Fuller test, this test result should only provide hints that the rate of unemployment and the federal funds rate exhibit strong autocorrelations, respectively.

4.8.2 Estimation of the Unrestricted VAR

Given the assumption of stationarity, we can construct a VAR model for the 6 variables of the structural model to mimic their DGP (data generating process of these 6 variables) by linearizing our given structural model in a straightforward way.

$$\begin{pmatrix} \mathrm{d}\ln w_t \\ \mathrm{d}\ln p_t \\ \ln e_t \\ \ln u_t \\ i_t \\ uc_t \end{pmatrix} = \sum_{k=1}^{P} \begin{pmatrix} a_{11k} & a_{12k} & a_{13k} & a_{14k} & a_{15k} & a_{16k} \\ a_{21k} & a_{22k} & a_{23k} & a_{24k} & a_{25k} & a_{26k} \\ a_{31k} & a_{32k} & a_{33k} & a_{34k} & a_{35k} & a_{36k} \\ a_{41k} & a_{42k} & a_{43k} & a_{44k} & a_{45k} & a_{46k} \\ a_{51k} & a_{52k} & a_{53k} & a_{54k} & a_{55k} & a_{56k} \\ a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} \mathrm{d}\ln w_{t-k} \\ \mathrm{d}\ln p_{t-k} \\ \ln e_{t-k} \\ \ln u_{t-k} \\ i_{t-k} \\ uc_{t-k} \end{pmatrix}$$

$$+ \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix} d74 + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{pmatrix}. \tag{4.33}$$

To determine the lag length of the VAR we apply sequential likelihood tests. We start with a lag length of 24, at which the residuals can be taken as white noise process. The sequence likelihood ratio test procedure gives a lag length of 11. The test results are listed below.

- $H_0 : P = 20$ v.s. $H_1 : P = 24$
  Chi-Squared(144)= 147.13 with Significance Level 0.91
- $H_0 : P = 16$ v.s. $H_1 : P = 20$
  Chi-Squared(144)= 148.92 with Significance Level 0.41
- $H_0 : P = 12$ v.s. $H_1 : P = 16$
  Chi-Squared(36)= 118.13 with Significance Level 0.94
- $H_0 : P = 11$ v.s. $H_1 : P = 12$
  Chi-Squared(36)= 42.94 with Significance Level 0.19
- $H_0 : P = 10$ v.s. $H_1 : P = 11$
  Chi-Squared(36)= 51.30518 with Significance Level 0.04

According to these test results we use a VAR(12) model to represent a general model that should be a good approximation of the DGP. Because the variable uc_t is treated as exogenous in the structural form (4.12)–(4.19) of the dynamical system , we factorize the VAR(12) process into a conditional process of $\mathrm{d}\ln w_t$, $\mathrm{d}\ln p_t$, $\ln e_t$, $\ln u_t$, i_t given uc_t and the lagged variables, and the marginal process of uc_t given the lagged variables:

$$\begin{pmatrix} \mathrm{d}\ln w_t \\ \mathrm{d}\ln p_t \\ \ln e_t \\ \ln u_t \\ i_t \end{pmatrix} = \sum_{k=1}^{P} \begin{pmatrix} a^*_{11k} & a^*_{12k} & a^*_{13k} & a^*_{14k} & a^*_{15k} & a^*_{16k} \\ a^*_{21k} & a^*_{22k} & a^*_{23k} & a^*_{24k} & a^*_{25k} & a^*_{26k} \\ a^*_{31k} & a^*_{32k} & a^*_{33k} & a^*_{34k} & a^*_{35k} & a^*_{36k} \\ a^*_{41k} & a^*_{42k} & a^*_{43k} & a^*_{44k} & a^*_{45k} & a^*_{46k} \\ a^*_{51k} & a^*_{52k} & a^*_{53k} & a^*_{54k} & a^*_{55k} & a^*_{56k} \end{pmatrix} \begin{pmatrix} \mathrm{d}\ln w_{t-k} \\ \mathrm{d}\ln p_{t-k} \\ \ln e_{t-k} \\ \ln u_{t-k} \\ i_{t-k} \\ uc_{t-k} \end{pmatrix}$$

$$+ \begin{pmatrix} c^*_1 \\ c^*_2 \\ c^*_3 \\ c^*_4 \\ c^*_5 \end{pmatrix} + \begin{pmatrix} b^*_1 \\ b^*_2 \\ b^*_3 \\ b^*_4 \\ b^*_5 \end{pmatrix} d74 + \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} uc_t + \begin{pmatrix} e^*_{1t} \\ e^*_{2t} \\ e^*_{3t} \\ e^*_{4t} \\ e^*_{5t} \end{pmatrix}, \qquad (4.34)$$

$$uc_t = c_6 + \sum_{k=1}^{P} \begin{pmatrix} a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} \mathrm{d}\ln w_{t-k} \\ \mathrm{d}\ln p_{t-k} \\ \ln e_{t-k} \\ \ln u_{t-k} \\ i_{t-k} \\ uc_{t-k} \end{pmatrix} + e_{6t}. \qquad (4.35)$$

We now examine whether uc_t can be taken as an "exogenous" variable. The partial system (4.34) is exactly identified. Hence the variable uc_t is weakly exogenous for the parameters in the partial system.[18] For the strong exogeneity of uc_t, we test whether $\mathrm{d}\ln w_t$, $\mathrm{d}\ln p_t$, $\ln e_t$, $\ln u_t$, i_t Granger cause uc_t. The test is carried out by testing the hypothesis: $H_0 : a_{ijk} = 0, (i = 6; j = 1, 2, 3, 4, 5; k = 1, 2, \ldots, 12)$ in (4.35) based on the likelihood ratio

- Chi-Squared(60)=57.714092 with Significance Level 0.55972955

The result of the test is uc_t is strongly exogenous with respect to the parameters in (4.34). Hence we can investigate the partial system (4.34) taking uc_t as exogenous.

4.8.3 Estimation of the Structural Model

As discussed in Sect. 4.6, the law of motion for the real wage rate, (4.23), represents a reduced form expression of the two structural equations for $\mathrm{d}\ln w_t$ and $\mathrm{d}\ln p_t$. Noting again that the inflation climate variable is defined in the

[18] For a detailed discussion of this procedure, see Chen (2003).

estimated model as a linearly declining function of past price inflation rates, the dynamics of the system (4.12)–(4.19) can be rewritten in linearized form as shown in the following equations:[19]

$$\mathrm{d}\ln w_t = \beta_{w_1}\ln e_{t-1} - \beta_{w_2} uc_{t-1} + \kappa_w \mathrm{d}\ln p_t + (1-\kappa_w)\pi_t^{12} + c_1 + e_{1t}, \quad (4.36)$$

$$\mathrm{d}\ln p_t = \beta_{p_1}\ln u_{t-1} + \beta_{p_2} uc_{t-1} + \kappa_p \mathrm{d}\ln w_t + (1-\kappa_p)\pi_t^{12} + c_2 + e_{2t}, \quad (4.37)$$

$$\mathrm{d}\ln e_t = \alpha_{e_2}\mathrm{d}\ln e_t + e_{3t}, \quad (4.38)$$

$$\mathrm{d}\ln u_t = -\alpha_u \ln u_{t-1} - \alpha_{ur}(i_{t-1} - \mathrm{d}\ln p_t) - \alpha_\omega uc_{t-1} + c_4 + e_{4t}, \quad (4.39)$$

$$\mathrm{d}r_t = -\gamma_{ii} i_{t-1} + \gamma_p \mathrm{d}\ln p_t + \gamma_{iu}\ln u_{t-1} + c_5 + e_{5t}. \quad (4.40)$$

Obviously, the model (4.36)–(4.40) is nested in the VAR(12) of (4.34). Therefore we can use (4.34) to evaluate the empirical relevance of the model (4.36)–(4.40). First we test whether the parameter restrictions on (4.34) implied by (4.36)–(4.40) are valid.

The linearized structural model (4.36)–(4.40) puts 349 restrictions on the unconstrained VAR(12) of the system (4.34). Applying likelihood ratio methods we can test the validity of these restrictions. For the period from 1965:1 to 2001:2 we cannot reject the null of these restrictions. The test result is the following:

- Chi-Squared(349) = 361.716689 with Significance Level 0.34902017

Obviously, the specification (4.36)–(4.40) is a valid one for the data set from 1965:1 to 2001:2. This result shows strong empirical relevance for the laws of motions as described in (4.12)–(4.19) as a model for the U.S. economy from 1965:1 to 2001:2. It is worthwhile to note that altogether 349 restrictions are implied through the structural form of the system (4.12)–(4.19) on the VAR(12) model. A p-value of 0.349 thus means that (4.12)–(4.19) is a much more parsimonious presentation of the DGP than VAR(12), and henceforth a much more efficient model to describe the economic dynamics for this period.

To get a result that is easier to interpret from the economic perspective, we transform the model (4.36)–(4.40) back to its originally nonlinear form (4.12)–(4.19):[20]

[19] Note here that the difference operator d is to be interpreted as backward in orientation and that the nominal rate of interest is dated at the beginning of the relevant period. The linearly declining moving average π_t^{12} in turn concerns the past twelve price inflation rates.

[20] Note that $\mathrm{d}i_t = -\gamma_{ii} i_{t-1} + \gamma_{ip}\mathrm{d}\ln p_t + \gamma_{iu} u_{t-1} + c_5 + e_{5t}$ can also be represented by $i_t = (1-\gamma_{ii})i_{t-1} + \cdots$ in the equations to be estimated below.

$$\begin{aligned}
\mathrm{d}\ln w_t &= \beta_{w_1} e_{t-1} - \beta_{w_2} uc_{t-1} + \kappa_w \mathrm{d}\ln p_t + (1-\kappa_w)\pi_t^{12} + c_1 + e_{1t}, && (4.41)\\
\mathrm{d}\ln p_t &= \beta_{p_1} u_{t-1} + \beta_{p_2} uc_{t-1} + \kappa_p \mathrm{d}\ln w_t + (1-\kappa_p)\pi_t^{12} + c_2 + e_{2t}, && (4.42)\\
\mathrm{d}\ln e_t &= \alpha_{e_2} \mathrm{d}\ln u_t + e_{3t}, && (4.43)\\
\mathrm{d}\ln u_t &= -\alpha_u u_{t-1} - \alpha_\omega uc_{t-1} - \alpha_{ur}(i_{t-1} - d\ln p_t) + c_4 + e_{4t}, && (4.44)\\
\mathrm{d}i_t &= -\gamma_{ii} i_{t-1} + \gamma_{ip} \mathrm{d}\ln p_t + \gamma_{iu} u_{t-1} + c_5 + e_{5t}. && (4.45)
\end{aligned}$$

This model therefore differs from the model (4.36)–(4.40) by referring now again to the explanatory variables u and e instead of $\ln u$ and $\ln e$ which were necessary to construct a linear VAR(12) system. We compare on this basis the model (4.41)–(4.45) with the model (4.36)–(4.40) in a non-nested testing framework. Applying the J test to such a nonlinear estimation procedure, we get significant evidence that the model (4.41)–(4.45) is to be preferred to the model (4.36)–(4.40).

Model	J test
H_1 : Model of (4.36) – (4.40) is true	$t_\alpha = 4.611$
H_2 : Model of (4.41) – (4.45) is true	$t_\phi = -0.928$

We have already omitted in the following summaries of our model estimates the insignificant parameters in the displayed quantitative representation of the semi-structural model and also the stochastic terms. By putting furthermore the NAIRU expressions and all other expressions that are here still assumed as constant into overall constant terms, we therefore finally obtain the following (approximate) Two Stage Least Squares estimation results (with t-statistics in parenthesis):

$$\begin{aligned}
\mathrm{d}\ln w_t &= \underset{(3.95)}{0.13}\, e_{t-1} - \underset{-1.94}{0.07}\, uc_{t-1} + \underset{(2.61)}{0.49}\, \mathrm{d}\ln p_t + \underset{(2.61)}{0.51}\, \pi_t^{12} \underset{(-3.82)}{-0.12},\\
\mathrm{d}\ln p_t &= \underset{(2.32)}{0.04}\, u_{t-1} + \underset{(2.52)}{0.05}\, uc_{t-1} + \underset{(2.32)}{0.18}\, \mathrm{d}\ln w_t + \underset{(2.32)}{0.82}\, \pi_t^{12} - \underset{(-6.34)}{0.04},\\
\mathrm{d}\ln e_t &= \underset{(14.62)}{0.18}\, \mathrm{d}\ln u_t,\\
\mathrm{d}\ln u_t &= \underset{(-5.21)}{-0.14} u_{t-1} - \underset{(-4.72)}{0.94}\, (i_{t-1} - \mathrm{d}\ln p_t) - \underset{(-4.84)}{0.54}\, uc_{t-1} + \underset{(5.41)}{0.12},\\
\mathrm{d}i_t &= \underset{(24.82)}{-0.08} i_{t-1} + \underset{(1.2)}{0.06} \mathrm{d}\ln p_t + \underset{(2.46)}{0.01}\, u_{t-1} - \underset{(-2.19)}{0.01}.
\end{aligned}$$

We thus here get that Blanchard and Katz error correction terms matter in particular in the labor market, that the adjustment speed of wages is larger than the one for prices with respect to their corresponding demand pressures and that wage earners are more short-sighted than firms with respect to the influence of the inflationary climate expression. Okun's law which relates the growth rate of employment with the growth rate of capacity utilization is below a 1:5 relationship and thus in fact represents a fairly weak relationship.

There is a strong influence of the real rate of interest on the growth rate of capacity utilization in the error correcting dynamic multiplier equation and also a significant role for income distribution in this equation. Since this role is based on a negative sign we have the result that the economy is profit-led, i.e., investment behavior (which is assumed to depend negatively on real unit wage costs) dominates the outcome of a change in income distribution. With respect to the interest rate policy we finally obtain a sluggish form of interest rate smoothing, based on a passive policy rule (with a coefficient 0.06/0.08 in front of the inflation gap).

Next we compare the preceding situation with the case where the climate expression π^c is based on a 24 quarter horizon in the place of the 12 quarter horizon we have employed so far.

$$\begin{aligned}
\mathrm{d}\ln w_t &= 0.12e_{t-1} - 0.06uc_{t-1} + 0.71\mathrm{d}\ln p_t + 0.29\pi_t^{24} - 0.10,\\
\mathrm{d}\ln p_t &= 0.04u_{t-1} + 0.09uc_{t-1} + 0.38\mathrm{d}\ln w_t + 0.62\pi_t^{24} - 0.03,\\
\mathrm{d}\ln e_t &= 0.18\mathrm{d}\ln u_t,\\
\mathrm{d}\ln u_t &= -0.14u_{t-1} - 0.94(i_{t-1} - \mathrm{d}\ln p_t) - 0.54uc_{t-1} + 0.12,\\
\mathrm{d}i_t &= -0.09i_{t-1} + 0.07\mathrm{d}\ln p_t + 0.01u_{t-1} - 0.01.
\end{aligned}$$

We see that the application of a time horizon of 24 quarters for the formation of the inflationary climate variable does not alter the qualitative properties of the dynamics significantly as compared to the case of a moving average with linearly declining weights over 12 quarters only (which approximately corresponds to a value of $\beta_{\pi^c} = 0.15$ in an adaptive expectations mechanism as used for the theoretical version of the model in Sect. 4.6). Even choosing only a six quarter horizon for our linearly declining weights preserves the qualitative features of our estimated model and also by and large the stability properties of the dynamics as we shall see later on, though inflationary expectations over the medium run are then updated with a speed comparable to the ones used for the price PC in hybrid New Keynesian approaches:

$$\begin{aligned}
\mathrm{d}\ln w_t &= 0.12e_{t-1} - 0.08uc_{t-1} + 0.27\mathrm{d}\ln p_t + 0.73\pi_t^{6} - 0.11,\\
\mathrm{d}\ln p_t &= 0.03u_{t-1} + 0.02uc_{t-1} + 0.10\mathrm{d}\ln w_t + 0.90\pi_t^{6} - 0.03,\\
\mathrm{d}\ln e_t &= 0.18\mathrm{d}\ln u_t,\\
\mathrm{d}\ln u_t &= -0.14u_{t-1} - 0.94(i_{t-1} - \mathrm{d}\ln p_t) - 0.54uc_{t-1} + 0.12,\\
\mathrm{d}i_t &= -0.08i_{t-1} + 0.06\mathrm{d}\ln p_t + 0.01u_{t-1} - 0.01.
\end{aligned}$$

We thereby arrive at the general qualitative result that wages are more flexible than prices with respect to their corresponding measures of demand pressure and that wage earners are more short-sighted than firms with respect to the weight they put on their current (perfectly foreseen) measure of cost pressure as compared to the inflationary climate that surrounds this situation. Blanchard and Katz (1999) type error correction mechanisms play a role both in the wage PC and also in the price PC for the U.S. economy and have the sign that is predicted by theory, in contrast to what is found out by these two authors themselves. We have the validity of Okun's law with an elasticity coefficient of less than 20 percent and have the correct signs for the dynamic multiplier process as well as with respect to the influence of changing real rate of interests on economic activity. Finally, the impact of income distribution on the change in capacity utilization is always a negative one and thus of an orthodox type, meaning that rising average unit wage costs will decrease economic activity, and will therefore imply at least from a partial perspective that increasing wage flexibility is stabilizing, while increasing price flexibility (again with respect to its measure of demand pressure) is not.

We conclude from the above that it should be legitimate to use the system estimate with π^{12} as inflation climate term for the further evaluation of the dynamic properties of our theoretical model of Sect. 4.6, in order to see what more can be obtained as compared to the theoretical results of Sect. 4.7 when empirically supported parameter signs and sizes are (approximately) taken into account. As a further support for this parameter approximation we finally also report single equations estimates for our 5D system in order to get a feeling for the intervals in which the parameter values may sensibly assumed to lie:[21]

$$\begin{aligned}
\mathrm{d}\ln w_t &= 0.19e_{t-1} - 0.07uc_{t-1} + 0.16\mathrm{d}\ln p_t + 0.84\pi_t^{12} - 0.17,\\
\mathrm{d}\ln p_t &= 0.05u_{t-1} + 0.05uc_{t-1} + 0.09\mathrm{d}\ln w_t + 0.91\pi_t^{12} - 0.04,\\
\mathrm{d}\ln e_t &= 0.16\mathrm{d}\ln u_t,\\
\mathrm{d}\ln u_t &= -0.14u_{t-1} - 0.93(i_{t-1} - \mathrm{d}\ln p_t) - 0.54uc_{t-1} + 0.12,\\
\mathrm{d}i_t &= -0.10i_{t-1} + 0.10\mathrm{d}\ln p_t + 0.01u_{t-1} - 0.01.
\end{aligned}$$

Again parameter sizes are changed to a certain degree. We do not expect however that this changes the stability properties of the dynamics in a qualitative sense and we will check this in the following section from the theoretical as well as numerical perspective.

The above by and large similar representation of the sizes of the parameter values of our dynamics thus reveal various interesting assertions on the relative importance of demand pressure influences as well as cost pressure effects in

[21] Details on the t-statistics of the subsequently reported results can be found in the appendix of the working paper version of chapter.

the wage-price spiral of the U.S. economy. The Blanchard and Katz error correction terms have the correct signs and are of relevance in general. Okun's law holds as a level relationship between the capacity utilization rate and the rate of employment, basically of the form $e/e_o = (u/u_o)^b$ with an elasticity parameter b of about 18 percent. The dynamic IS equation shows the from the partial perspective stabilizing role of the multiplier process and a significant dependence of the rate of change of capacity utilization on the current real rate of interest. There is a significant and negative effect of real unit wage costs (we conjecture: since investment dominates consumption) on this growth rate of capacity utilization, which in this aggregated form suggests that the economy is profit-led as far as aggregate goods demand is concerned, i.e., real wage cost increases significantly decrease economy activity.

Finally, for the Taylor interest rate policy rule, we get the result that interest rate smoothing takes place around the ten percent level, and that monetary policy is to be considered as somewhat passive ($\gamma_p/(1-\gamma_{ii}) < 1$) in such an environment as far as the inflation gap is concerned, and that there is only a weak direct influence of the output gap on the rate of change of the nominal rate of interest. It may therefore be expected that instability can be an outcome of the theoretical model when simulated with these estimated parameter values. Finally, we note that it is not really possible to recover the steady state rate of interest from the constant in the above estimated Taylor rules in a statistically significant way, since the expression implied for this rate by our formulation of the Taylor rule would be:

$$i_o = (const + \gamma_{ip}\bar{\pi} + \gamma_{iu})/\gamma_{ii},$$

which does not determine this rate with any reliable statistical confidence. This also holds for the other constants that we have assumed as given in our formulation of Keynesian DAS–DAD dynamics.

In sum the system estimates of this section provide us with a result that confirms theoretical sign restrictions. They moreover provide definite answers with respect to the role of income distribution in the considered disequilibrium AS–AD or DAS–DAD dynamics, confirming in particular the orthodox point of view that economic activity is likely to depend negatively on real unit wage costs. We have also a negative real wage effect in the dynamics of income distribution, in the sense that the growth rate of real wages, see our reduced form real wage dynamics in Sect. 4.6, depends—through Blanchard and Katz error correction terms—negatively on the real wage. Its dependence on economic activity levels however is somewhat ambiguous, but in any case small. Real wages therefore only weakly increase with increases in the rate of capacity utilization which in turn however depends in an unambiguous way negatively on the real wage, implying in sum that the Rose (1967) real wage channel is present, but may not dominate the dynamic outcomes.

Finally, the estimated adjustment speed of the price level is so small that the dynamic multiplier effect dominates the overall outcome of changes in capacity utilization on the growth rate of this utilization rate, which therefore establishes a further stabilizing mechanism in the reduced form of our multiplier equation. The model and its estimates thus by and large confirm the conventional Keynesian view on the working of the economy and thus provide in sum a result very much in line with the traditional ways of reasonings from a Keynesian perspective. There is one important qualification however, as we will show in the next sections, namely that downward money wage flexibility can be good for economic stability, in line with Rose's (1967) model of the employment cycle, but in opposition to what Keynes (1936) stated on the role of downwardly rigid money wages. Yet, the role of income distribution in aggregate demand and wage vs. price flexibility was not really a topic in the General Theory, which therefore did not comment on the possibility that wage declines may lead the economy out of a depression via a channel different from the conventional Keynes-effect.

4.9 Stability Analysis of the Estimated Model

In the preceding section we have provided definite answers with respect to the type of real wage effect present in the data of the U.S. economy after World War II, concerning the dependence of aggregate demand on the real wage, the degrees of wage and price flexibilities and the degree of forward-looking behavior in the wage and price PC. The resulting combination of effects and the estimated sizes of the parameters (in particular the relative degree of wage vs. price flexibility) suggest that their particular type of interaction is favorable for stability, at least if monetary policy is sufficiently active.

We start the stability analysis of the semi-structural theoretical model with estimated parameters from the following reference situation (the system estimate where the inflationary climate is measured as by the twelve quarter moving average):

$$\begin{aligned}
\mathrm{d}\ln w_t &= 0.13e_{t-1} - 0.07uc_{t-1} + 0.49\mathrm{d}\ln p_t + 0.51\pi_t^{12} - 0.12,\\
\mathrm{d}\ln p_t &= 0.04u_{t-1} + 0.05uc_{t-1} + 0.18\mathrm{d}\ln w_t + 0.82\pi_t^{12} - 0.04,\\
\mathrm{d}\ln e_t &= 0.18\mathrm{d}\ln u_t,\\
\mathrm{d}\ln u_t &= -0.14u_{t-1} - 0.94(i_{t-1} - \mathrm{d}\ln p_t) - 0.54uc_{t-1} + 0.12,\\
\mathrm{d}i_t &= -0.08i_{t-1} + 0.06\mathrm{d}\ln p_t + 0.01u_{t-1} - 0.01.
\end{aligned}$$

We consider first the 3D core situation obtained by totally ignoring adjustments in the inflationary climate term, by setting $\pi^c = \bar{\pi}$ in the theoretical model, and by interpreting the estimated law of motion for e in level terms, i.e., by moving from the equation $\hat{e} = b\hat{u}$ to the equation $e = e_o(u/u_o)^b$, with $b = 0.18$ (and $e_o = u_o = 1$ for reasons of simplicity and without much loss of generality). On the basis of our estimated parameter values we furthermore have that the expression $\beta_{p_1} - \kappa_p\beta_{w_1}$ is approximately zero (slightly positive), i.e., the weak influence of the state variable ω in the reduced form price PC will not be of relevance in the following reduced form of the dynamics (which however is not of decisive importance for the following stability analysis). Finally, the critical condition for normal or adverse Rose effects

$$\alpha = (1-\kappa_p)\beta_{w_1}b - (1-\kappa_w)\beta_{p_1} \approx 0$$

is also—due to the measured size of the parameter b—close to zero (which is of importance for stability analysis, see the matrix J below). Rose real-wage effects are thus not very strong in the estimated form of the model, at least from this partial point of view, despite a significant negative dependence of capacity utilization on real unit wage costs (the wage share).

Under these assumptions, the laws of motion (4.20)–(4.24)—with the reduced form price PC inserted again—can be reduced to the following qualitative form (where the undetermined signs of a_1, b_1, c_1 do not matter for the following stability analysis and where are assumed to be sufficiently close to 0):

$$\hat{u} = a_1 - a_2\, u - a_3\, i - a_4\, \ln\omega, \tag{4.46}$$

$$\dot{i} = b_1 + b_2\, u - b_3\, i \pm b_4 \ln\omega, \tag{4.47}$$

$$\hat{\omega} = c_1 \pm c_2\, u - c_4\, \ln\omega, \tag{4.48}$$

since the dependence of $\hat{p}$ on u is a weak one, to be multiplied by 0.17 in the comparison with the direct impact of u on its rate of growth, and thus does not modify the sign measured for the direct influence of this variable on the growth rate of the capacity utilization rate significantly. Note with respect to this qualitative characterization of the remaining 3D dynamics, that the various influences of the same variable in the same equation have been aggregated always into a single expression, the sign of which has been obtained from the quantitative estimates shown above. We thus have to take note here in particular of the fact that the reduced form expression for the price inflation rate has been inserted into the first two laws of motion for the capacity utilization dynamics and the interest rate dynamics, which have been rearranged on this basis so that the influence of the variables u and ω appears at most only once, though both terms appear via two different channels in these laws of motion, one direct channel and one via the price inflation rate.

The result of our estimates of this equation is that the latter channel is not changing the signs of the direct effects of capacity utilization (via the dynamic multiplier) and the real wage (via the aggregate effect of consumption and investment behavior). We note again that the parameter c_2 may be uncertain in sign, but will in any case be close to zero, while the sign of b_4 does not matter in the following. A similar treatment applies to the law of motion for the nominal rate of interest, where price inflation is again broken down into its constituent parts (in its reduced form expression) and where the influence of e in this expression is again replaced by u through Okun's Law. Finally, the law of motion for real wages themselves is obtained from the two estimated structural laws of motion for wage and price inflation in the way shown in Sect. 4.6. We have the stated very weak, but possibly positive influence of capacity utilization on the growth rate of real wages, since the wage Phillips curve slightly dominates the outcome here and an unambiguously negative influence of real wages on their rate of growth due to the signs of the Blanchard and Katz error correction terms in the wage and the price dynamics.

On this basis, we arrive—if we set the considered small magnitudes equal to zero—at the following sign structure for the Jacobian at the interior steady state of the above reduced model for the state variables u, i, ω:

$$J = \begin{pmatrix} - & - & - \\ + & - & 0 \\ 0 & 0 & - \end{pmatrix}.$$

We therefrom immediately get that the steady state of these 3D dynamics is asymptotically stable, since the trace is negative, the sum a_2 of principal minors of order two is always positive, and since the determinant of the whole matrix is negative. The coefficients $k_i, i = 1, 2, 3$ of the Routh Hurwitz polynomial of this matrix are therefore all positive as demanded by the Routh

Hurwitz stability conditions. The remaining stability condition is

$$k_1k_2 - k_3 = (-\text{trace } J)\, k_2 + \det J > 0.$$

With respect to this condition we immediately see that the determinant of the Jacobian J, given by:

$$J_{33}(J_{11}J_{22} - J_{12}J_{21})$$

is dominated by the terms that appear in k_1k_2, i.e., this Routh-Hurwitz condition is also of correct sign as far as the establishment of local asymptotic stability is concerned. The weak and maybe ambiguous real wage effect or Rose effect that is included in the working of the dynamics of the private sector thus does not work against the stability of the steady state of the considered dynamics. Ignoring the Mundell effect by assuming $\beta_{\pi^c} = 0$ therefore allows for an unambiguous stability result, basically due to the stable interaction of the dynamic multiplier with the Taylor interest rate policy rule, augmented by real wage dynamics that in itself is stable due to the estimated signs (and sizes) of the Blanchard error correction terms, where the estimated negative dependence of the change in economic activity on the real wage is welcome from an orthodox point of view, but does not really matter for the stability features of the model. The neglectance of the Mundell effect therefore leaves us with a situation that is close in spirit to the standard textbook considerations of Keynesian macrodynamics. Making the β_{π^c} slightly positive does not overthrow the above stability assertion, since the determinant of the 4D case is positive in the considered situation, see also Chen et al. (2004), where it is also shown in detail, that a significant increase of this parameter must lead to local instability in a cyclical fashion via a so-called Hopf-bifurcation (if $\gamma_{ii} < \gamma_{ip}$).

Figure 4.4 shows simulations of the estimated dynamics where indeed the parameter β_{π^c} is now no longer zero, but set equal to $0.075, 0.15$ in correspondence to the measures π^{12}, π^6 of the inflationary climate used in our estimates (these values arise approximately when we estimate β_{π^c} by means of these moving averages). We use a large real wage shock (increase by ten percent) to investigate the response of the dynamics (with respect to capacity utilization) to such a shock. The resulting impulse-responses are unstable ones in the case of the estimated policy parameter $\gamma_{ip} = 0.6$ as shown in the two graphs on the left hand side of Fig. 4.4. This is due to the fact that the estimated passive Taylor rule allows for a positive real eigenvalue that leads to divergence when the dominant root of the estimated situation has run its course brought the dynamics sufficiently close to the steady state. Increasing the parameter γ_{ip} towards 0.8 (a Taylor rule right at the border towards an active one) or even to 0.12 (where ϕ_{ip} assumes a standard value of 0.5) moves the positive real eigenvalue into the negative half plane of the plane of complex numbers and thus makes the dynamics produce trajectories that converge back to the steady state as shown in Fig. 4.4 on its left hand side. On its right hand side we show in the top figure how the positive real eigenvalue (the maximum of the real parts of the eigenvalues of the dynamics) varies with the parameter γ_{ip}. We clearly see that instability is reduced and finally removed as this policy parameter is increased.[22] More active monetary policy leading to stability is a result that holds for all measures of the inflation climate as shown for the case π^6 in Fig. 4.4. Bottom right we finally show in this figure that there are complex roots involved in the considered situation (there for the case π^{12}). Adjustment to the steady state is therefore of a cyclical nature, though only weakly cyclical as shown in this figure). In sum we therefore get that active Taylor rules as estimated for the past to decades (but not for our larger estimation period) will bring stability to the dynamics, since the dominant root then enforces convergence to the steady state without any counteracting force close to the steady state. In the considered range for the parameter β_{π^c} the overall responses of the dynamics are then by and large of the shown type, i.e., the system has strong, though somewhat cyclical stability properties over this whole range, if monetary policy is made somewhat more active than estimated, independently of the particular combination of the speed of adjustment of the inflationary climate and the set of other parameter values we have estimated in the preceding section. The impulse-response situation shown in Fig. 4.4 is the expected one. The same holds true for the response of e, $\hat{w}$, $\hat{p}$, i which all decrease in the contractive phase shown in Fig. 4.4.

We again note that the system is subject to zero root hysteresis, since the laws of motion for e, u are here linearly dependent (since α_{e_1} has been estimated as being zero), i.e., it need not converge back to the initially given steady state value of the rate of capacity utilization which was assumed to be 1. Note also that the parameter estimates are based on quarterly data, i.e., the plots in Fig. 4.4 correspond to 25 years and thus show a long period of adjustment, due to the fact that all parameters have been assumed as time-

[22] Note that—due to the estimated form of Okun's law—one eigenvalue of the 5D dynamics must always be zero.

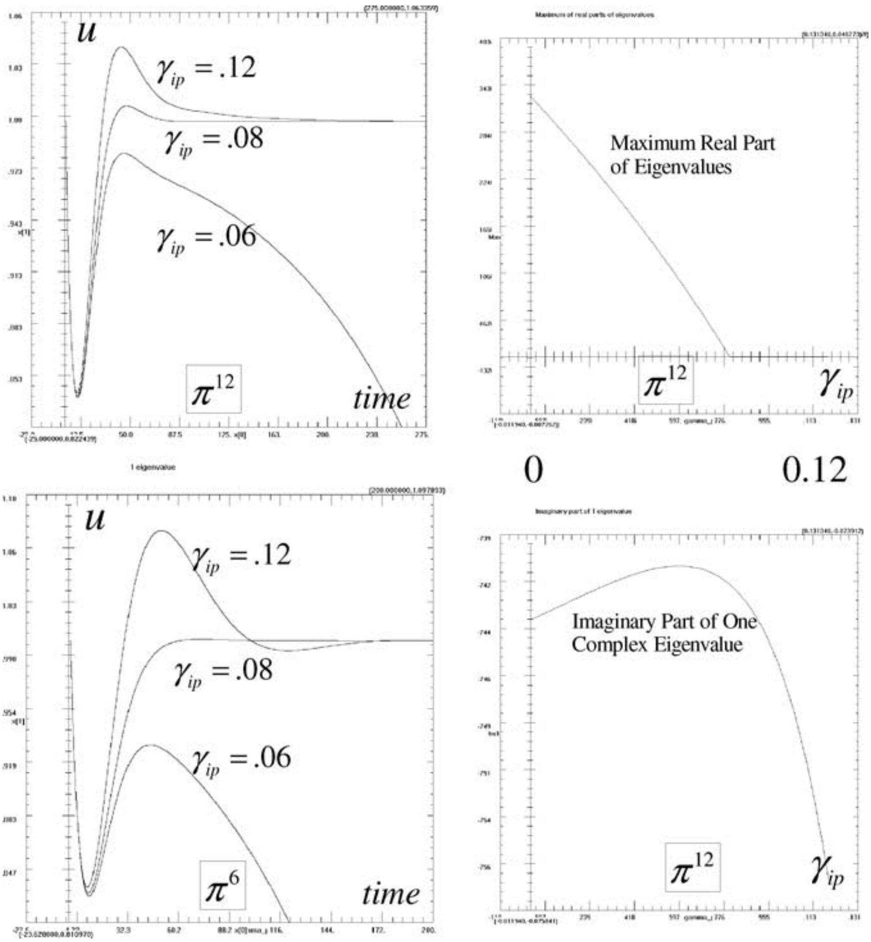

Fig. 4.4. Responses to real wage shocks in the range of estimated parameter values

invariant, so that only the slow process of changing income distribution and its implications for Keynesian aggregate demand is thus driving the economy here.

Next we test in Fig. 4.5 the stability properties of the model if one of its parameters is varied in size. We there plot the maximum value of the real parts of the eigenvalues against specific parameter changes shown on the horizontal axis in each case. We by and large find (also for parameter variations that are not shown) that all partial feedback chains (including the working of the Blanchard and Katz error correction terms) translate themselves into corresponding 'normal' eigenvalue reaction patterns for the full 5D dynamics.

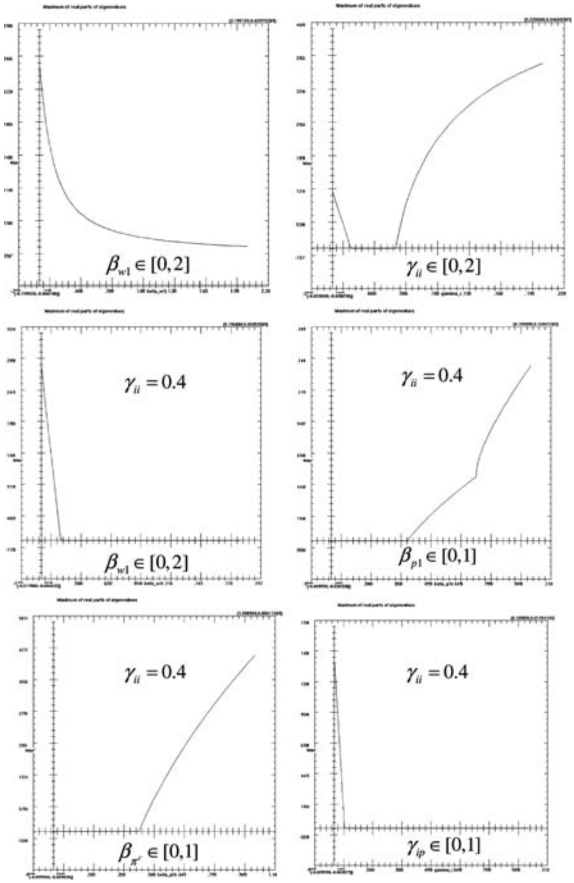

Fig. 4.5. Eigenvalue diagrams for varying parameter sizes

With respect to the wage flexibility parameter β_{w_1} we see in Fig. 4.5 top-left that its increase helps to reduce the instability of the system with respect to the estimated parameter set, but is not able by itself to enforce convergence. The same holds true for the other adjustment parameters in the wage and the price Phillips curves. Top-right we then determine the range of values for the parameter γ_{ii} where local asymptotic stability is indeed established and find that this is approximately true for a the interval $(0.1, 0.6)$, while large parameters values imply a switch back to local instability. Interest rate smoothing of a certain degree therefore can enforce convergence back to the steady state. We thus now change the estimated parameter set in this respect and assume for the parameter γ_{ii} the value 0.4 in the remainder of Fig. 4.5.

The two plots in the middle of Fig. 4.5 then show that too low wage flexibility and too high price flexibility will destabilize the dynamics again. this is what we expect from the real wage effect in a profit led economy, due o what has been said on normal and adverse Rose effects. Furthermore, concerning the Mundell effect, we indeed also find that an adjustment of the inflationary climate expression that is too fast induces local asymptotic instability and is therefore destabilizing (see Fig. 4.5, bottom-left). Finally, bottom-right we see that a Taylor interest rate policy rule that is too passive with respect to the inflation gap will also endanger the stability we have created by increasing the speed of interest rate smoothing. Such eigenvalue diagrams therefore nicely confirm what is know from partial reasoning on Keynesian macrodynamic feedback chains. Note here that increasing price flexibility is also destabilizing via the Mundell-effect, since the growth rate $\hat{u}$ of economic activity can thereby be made to depend positively on its level (via the real rate of interest channel, see (4.25)), leading to an unstable augmented dynamic multiplier process in the trace of J under such circumstances. Moreover, such increasing price flexibility will give rise to a negative dependence of the growth rate of the real wage on economic activity (whose rate of change in turn depends negatively on the real wage) and thus leads to further sign changes in the Jacobian J. Increasing price flexibility is therefore bad for the stability of the considered dynamics from at least two perspectives.

Let us return now to our analytical stability considerations again. The destabilizing role of price flexibility is enhanced if we add to the above stability analysis for the 3D Jacobian the law of motion for the inflationary climate surrounding the current evolution of price inflation. Under this extension we go back to a 4D dynamical system, the Jacobian J of which is obtained by augmenting the previous one in its sign structure in the following way (see again (4.25)):

$$J = \begin{pmatrix} - & - & - & + \\ + & - & 0 & + \\ 0 & 0 & - & 0 \\ + & 0 & 0 & 0 \end{pmatrix}.$$

As the positive entries J_{14}, J_{41} show, there is now a new destabilizing feedback chain included, leading from increases in economic activity to increases in inflation and climate inflation and from there back to increases in economic activity, again through the real rate of interest channel (where the inflationary climate is involved due to the expression that characterizes our reduced form price PC). This destabilizing, augmented Mundell effect must become dominant sooner or later (even under the estimated simplified feedback structure) as the adjustment speed of the climate expression β_{π^c} is increased. This is obvious from the fact that the only term in the Routh-Hurwitz coefficient a_2 that depends on the parameter β_{π^c} exhibits a negative sign, which implies that a sufficiently high β_{π^c} will make the coefficient a_2 negative eventually. The Blanchard and Katz error correction terms in the fourth row of J, obtained from the reduced form price Phillips curve, that are (only as further terms) associated with the speed parameter β_{π^c}, are of no help here, since they do not appear in combination with the parameter β_{π^c} in the sum of principal minors of order 2. In this sum the parameter β_{π^c} thus only enters once and with a negative sign implying that this sum can be made negative (leading to instability) if this parameter is chosen sufficiently large. This stands in some contrast to the estimation results where the there defined inflation climate term has been varied significantly without finding a considerable degree of instability.

Assuming—as a mild additional assumption—that interest rate smoothing is sufficiently weak furthermore allows for the conclusion that the 4D determinant of the above Jacobian exhibits a positive sign throughout. We thus in sum get that the local asymptotic stability of the steady state of the 3D case extends to the 4D case for sufficiently small parameters $\beta_{\pi^c} > 0$, since the eigenvalue that was zero in the case $\beta_{\pi^c} = 0$ must become negative due to the positive sign of the 4D determinant (since the other three eigenvalues must have negative real parts for small β_{π^c}). Loss of stability can only occur through a change in the sign of the Routh-Hurwitz coefficient k_2, which can occur only once by way of a Hopf-bifurcation where the system looses its local stability through the local death of an unstable limit cycle or the local birth of a stable limit cycle. This result is due to the destabilizing Mundell-effect of a faster adjustment of the inflationary climate into which the economy is embedded, which in the present dynamical system works through the elements J_{14}, J_{41} in the Jacobian J of the dynamics and thus through the positive dependence of economic activity on the inflationary climate expression and the positive dependence of this climate expression on the level of economic activity.

To sum up we have established that the 4D dynamics will be convergent for sufficiently small speeds of adjustments β_{π^c}, and for a monetary policy that is sufficiently active, while they will be divergent for parameters β_{π^c} chosen sufficiently large. The Mundell effect thus works as expected from a partial perspective. There will be a unique Hopf bifurcation point $\beta^H_{\pi^c}$ in between (for γ_{ii} sufficiently small), where the system loses asymptotic stability in a cyclical fashion. Yet sooner or later purely explosive behavior will be indeed be established (as can be checked by numerical simulations), where there is no longer room for persistent economic fluctuations in the real and the nominal magnitudes of the economy.

In such a situation global behavioral nonlinearities must be taken into account in order to limit the dynamics to domains in the mathematical phase space that are of economic relevance. Compared to the New Keynesian approach briefly considered in Sect. 4.3 of this chapter we thus have that—despite many similarities in the wage-price block of our dynamics—we have completely different implications for the resulting dynamics which—for active interest rate policy rules—are convergent (and thus determined from the historical perspective) when estimated empirically (with structural Phillips curves that are not all at odds with the facts) and which—should loss of stability occur via a faster adjustments of the inflationary climate expression—must be bounded by appropriate changes in economic behavior far off the steady state and not just by mathematical assumption as in the New Keynesian case. Furthermore, we have employed in our model type a dynamic IS-relationship in the spirit of Rudebusch and Svensson's (1999) approach, also confirmed in its backward orientation by a recent article of Fuhrer and Rudebusch (2004). One may therefore state that the results achieved in this and the preceding section can provide an alternative of mature, but traditional Keynesian type that does not lead to the radical—and not very Keynesian—New Keynesian conclusion that the deterministic part of the model is completely trivial and the dynamics but a consequence of the addition of appropriate exogenous stochastic processes.

4.10 Instability, Global Boundedness and Monetary Policy

Next we will express some conjectures concerning other scenarios. Based on the estimated range of parameter values, and for an active monetary policy rule, the preceding section has shown that the model then exhibits strong convergence properties, with only mild fluctuations around the steady state in the case of small shocks, but with a long downturn and a long-lasting adjustment in the case of strong shocks (as in the case of Fig. 4.4, where a 10 percent increase in real wages shocks the economy). Nevertheless, the economy is reacting in a fairly stable way to such a large shock and thus seems to have the characteristics of a strong shock absorber. Figure 4.4 however is based on estimated linear Phillips curves, i.e., in particular, on wage adjustment that is as flexible in an upward as well as in a downward direction. It is however much more plausible that wages behave differently in a high and in a low inflation regime, see Chen and Flaschel (2005) for a study of the wage PC along these lines which confirms this common sense statement. Following Filardo (1998) we here go even one step further and indeed assume a three regime scenario as shown in Fig. 4.6 where we make use of his Fig. 4.5 and for illustrative purposes of the parameter sizes there shown[23] (though they there refer to output gaps on the horizontal axis, inflation surprises on the vertical axis and a standard reduced form Phillips curve relating these two magnitudes):

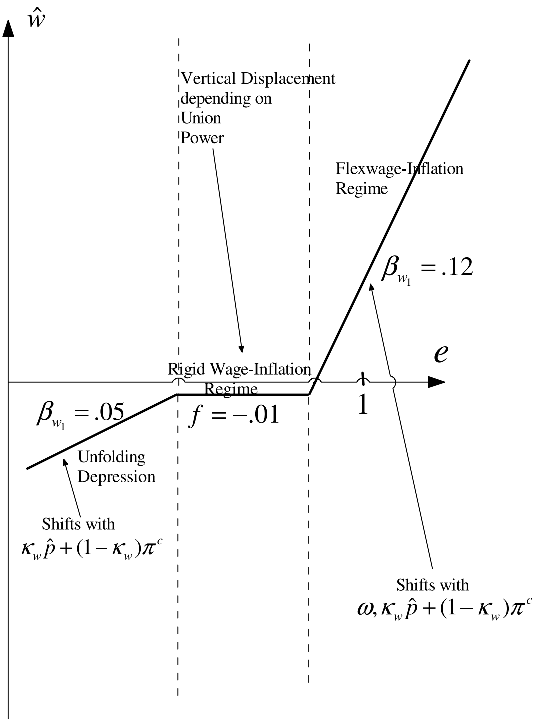

Fig. 4.6. Three possible regimes for wage inflation

[23] Here adjusted to quarterly data.

Figure 4.6 suggests that the wage PC of the present model is only in effect if there holds simultaneously that wage inflation is above a certain floor f—here (following Filardo) shown to be negative[24]—and the employment rate is still above a certain floor $\underline{e}$, where wage inflation starts to become (downwardly) flexible again. In this latter area (where wage inflation according to the original linear curve is below f and the employment rate below $\underline{e}$) we assume as form for the resulting wage-inflation curve the following simplification and modification of the original one:

$$\hat{w} = \beta_{w_1}(e - \underline{e}) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c,$$

i.e., we do not consider the Blanchard and Katz error correction term to be in operation then any more. In sum, we therefore assume a normal operation of the economy if both lower floors are not yet reached, constant wage inflation if only the floor f has been reached and further falling wage inflation or deflation rates (as far as demand pressure is concerned) if both floors have been passed. Downward wage inflation or wage deflation rigidity thus does not exist for all states of a depressed economy, but can give way to its further downward adjustment in severe states of depression in actual economies.[25]

In Fig. 4.7 we consider a situation as depicted in Fig. 4.4, i.e., the working of a wage Phillips curve as it was already formulated in Rose (1967) and again contractive real wage shocks.[26] Top-left we plot the rate of capacity utilization against the nominal rate of interest and obtain that the economy now adjusts to a fairly simple persistent fluctuation in this projection of the 5D

[24] By contrast, this floor is claimed and measured to be positive for six European countries in Hoogenveen and Kuipers (2000).

[25] An example for this situation is given by the German economy, at least since 2003.

[26] The parameters of the plot are the estimated one with the exception of $\beta_{\pi^c} = 0.4$, $\gamma_{ip} = 0.12$. The first parameter has therefore been increased in order to get local instability of the steady state and the policy parameter has been increased in order to tame the resulting instability to a certain degree. Moreover, we have assumed in this plot that the steady state value of e coincides with the value where wages become downwardly flexible again, i.e., we here only switch of the Blanchard and Katz error correction term in the downward direction, but have added a floor $f = 0.0004$ to wage inflation for employment rates above th steady state level ($u_o = 0.9$ now). This combination of wage regimes indeed tames the explosive dynamics and gives rise to a limit cycle attractor instead as is shown below.

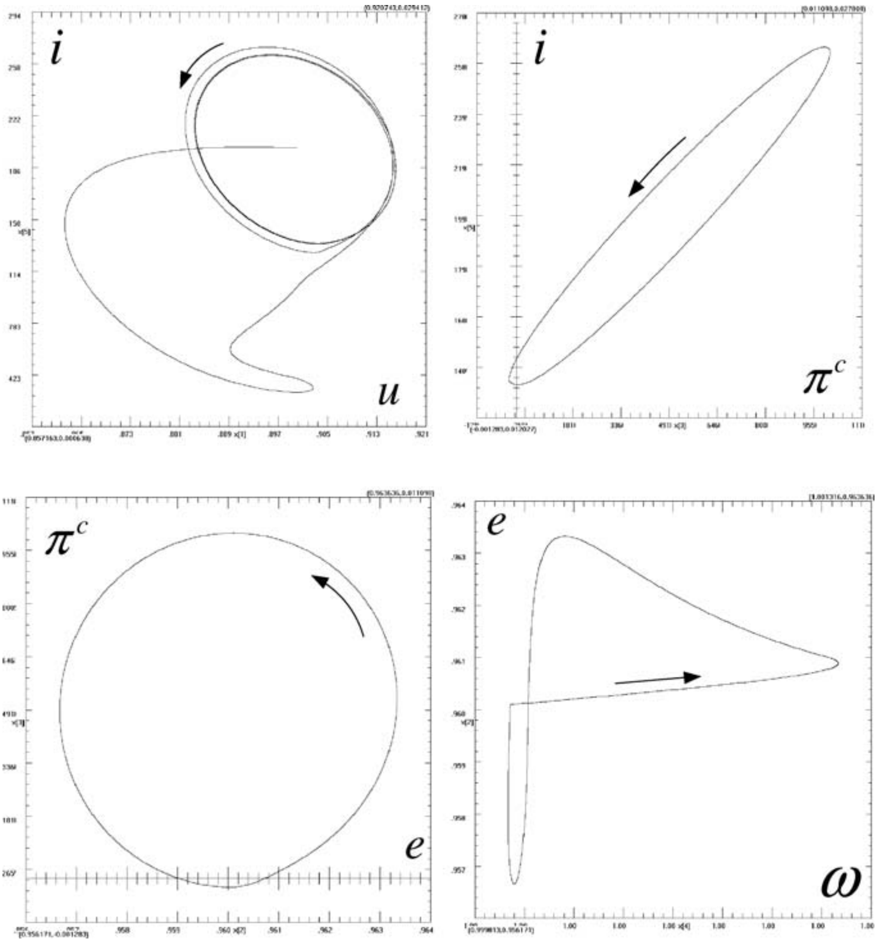

Fig. 4.7. Rose-Filardo type wage Phillips curves and the emergence of persistent business fluctuations

phase space with an overshooting interest rate adjustment, since for example the interest rate keeps on rising though economic activity has started to fall already for quite a while. There is a strict positive correlation between the rates of utilization of capital and labor, i.e., all assertions made with respect to one utilization rate also hold for the other one. The reason for this overshooting reaction of interest rate policy is that this policy closely follows the inflation gap and not the utilization gap, here represented with respect to the inflation climate term in the Fig. 4.7, top-right. This figure also shows that deflation is indeed occurring in the course of one cycle, though only weakly in a brief subperiod of it.

As already indicated there is also overshooting involved in the phase plot between the rate of employment and the inflation climate (Fig. 4.7, bottom-left), i.e., the model clearly generates periods of stagflation and also periods where disinflation is coupled with a rising employment rate. This pattern is well-known from empirical investigations. Less close to such investigations, see Flaschel and Groh (1995) for example, is the pattern that is shown bottom-right in Fig. 4.7, i.e., a phase plot between the real wage and the rate of employment which according to the Goodwin (1967) model of a growth cycle should be also an overshooting one with a clockwise orientation which in Fig. 4.7 is only partly visible in fact. Taken together, we however have the general result that a locally unstable steady state can be tamed towards the generation of a persistent fluctuation around it if wages become sufficiently flexible far off the steady state (both in an upward as well as in a downward direction).

We next show that the corridor $(\underline{e}, e_o)$ where the second regime in Fig. 4.6 applies may be of decisive importance for the resulting dynamics. Small changes in the size of this interval can have significant effects on the observed volatility of the resulting trajectories as the Fig. 4.8 exemplifies. In Fig. 4.8 we lower the value of $\underline{e}$ from 0.96 to 0.959, 0.585, 0.958, 0.9575 and see that the limit cycle is becoming larger and is approached in more and more complicated ways. In the case 0.575 we finally get a quite different limit cycle with lower i, u on an average and only a small amplitude which is shown in enlarged form in Fig. 4.8 bottom-right.

Finally, increasing the opposing forces β_{π^c} and γ_{ip} even further to 1.4 and 0.6 respectively, and assuming now $\underline{e} = 0.94 < e_o = 1$, i.e., a large range where there is a floor to money wages (and adjusting the interest rate such that it does not become negative along the trajectories that are shown) provides as with an (somewhat extreme) scenario where even complex dynamics are generated from the mathematical point of view (not directly from the economic point of view) as is shown in Fig. 4.9.

Turning now to the effects of monetary policy we first show in Fig. 4.10 the situation where the economy is strongly convergent to the steady state in the case of the active monetary policy underlying Fig. 4.4. Adding a global floor to this situation radically changes the situation and implies economic collapse once this floor is reached, since real wages are then rising due to falling prices.

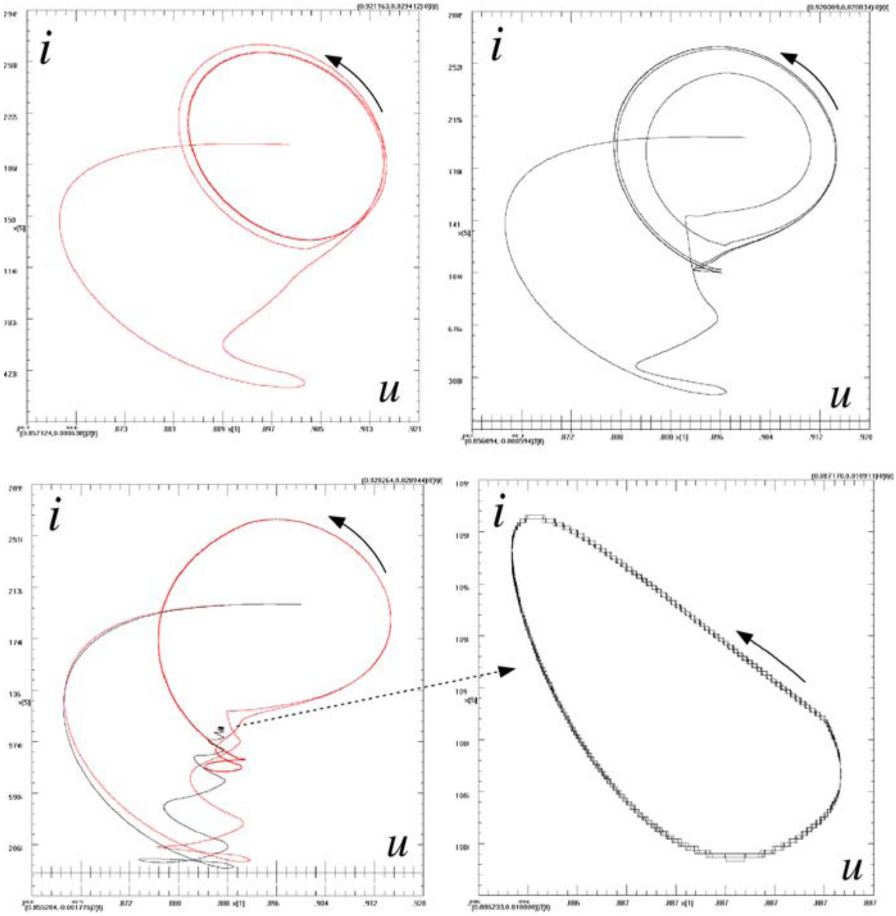

Fig. 4.8. Changes in the regime where rigid wage inflation or deflation prevails

This situation is again shown to be prevented if wages become flexible again (in a downward direction) at 92 percent of employment. Monetary policy that is then assumed more active either with respect to the utilization gap or the inflation gap can however prevent both situations from occurring, when it implies—as shown—that the floor to money wages can be avoided to come into operation thereby.

The question arises whether a monetary policy is working the better the stronger its reaction is with respect to the inflation gap, i.e., the larger the parameter γ_{ip} becomes. From an applied perspectives it is not to be expected that this is to be the case in reality, since active monetary policy is surely limited by some cautiousness bounds from above. In this 5D dynamical system

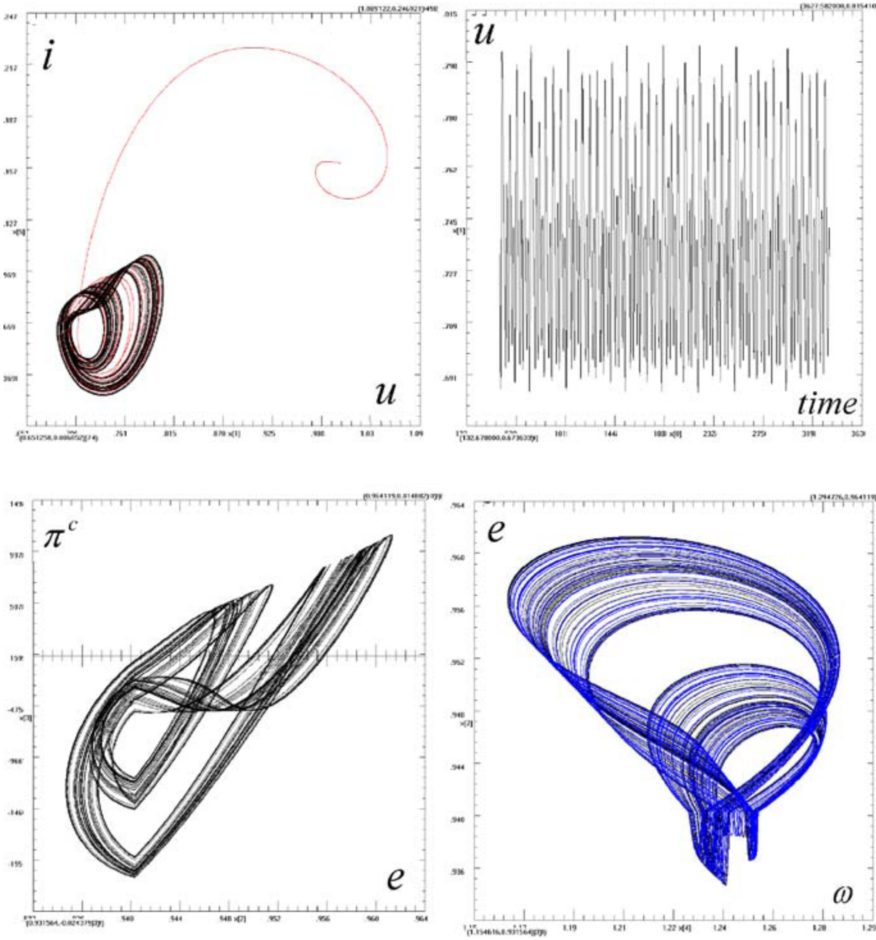

Fig. 4.9. Depressed complex dynamics with long deflationary episodes

with its still simple trajectories it may however be a theoretical possibility. Figure 4.11 now exemplifies this again by means of phase plots, i.e., of projections of the full dynamics into the u, i plane. We see in this figure top-left with respect to our estimated parameter set (but with $\beta_{\pi^c} = 0/4$ now again) that a more active monetary policy $(0.12 \rightarrow 0.15)$ enlarges the generated limit cycle (see Fig. 4.7) and thus makes the economy more volatile. By contrast, see Fig. 4.11, top-right, a lower value of $\gamma_p = 0.10$ as compared to Fig. 4.7, makes the dynamics in fact convergent with smaller cycles when the transient behavior is excluded, but with a long transient than in the case of Fig. 4.11, top-left. This longer transient behavior can be made of an extreme type—with

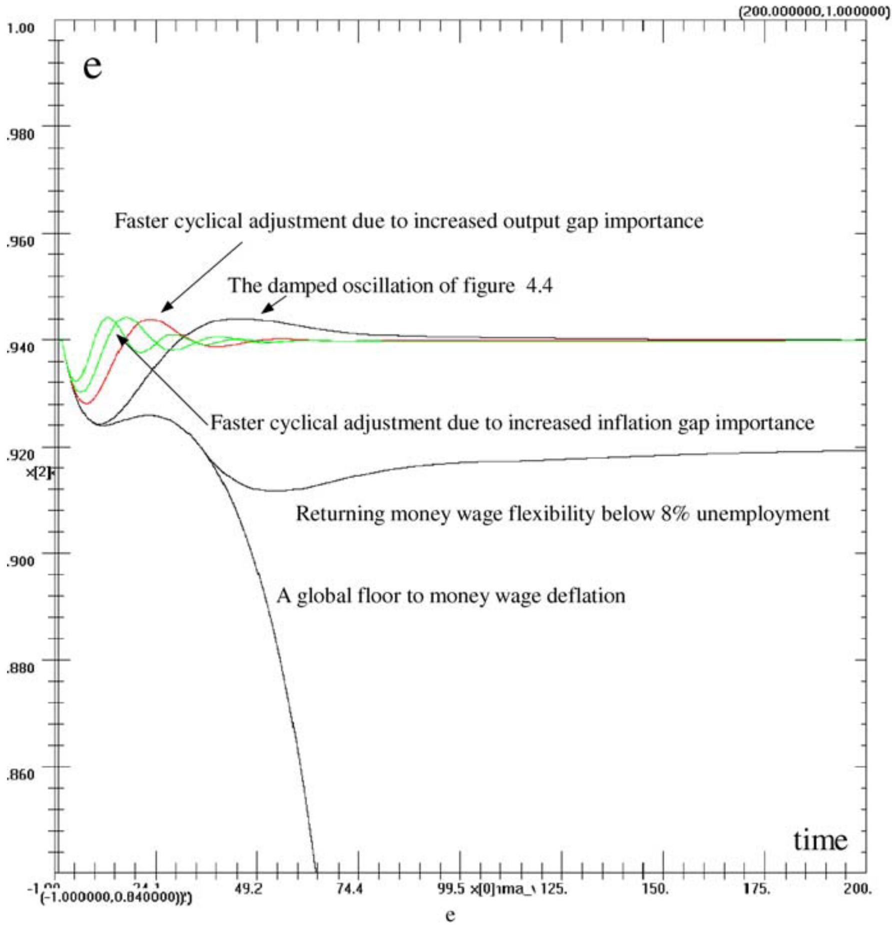

Fig. 4.10. The existence of floors and more active monetary policy rules

severely underutilized capital along the depressed transient cycles—when the policy reaction to the inflation gap is further reduced (to 0.092), see Fig. 4.11, bottom-right and -left. The degree of activeness of monetary policy matters therefore a lot for the business fluctuations that are generated and this in a way with clear benchmark for the appropriate choice of the parameter γ_{ip}.

In Fig. 4.12, finally, we vary the parameter $\bar{\pi}$ that characterizes the inflation target of the Central Bank. A first implications of such variations is shown in the plot top-left where we show the limit cycle of Fig. 4.7 once again, now together with two trajectories that are based on the assumptions $f = \bar{\pi} = 0.0004$ and $\bar{=}0.0004 > \pi = 0.0002$ concerning the temporary floor in money wages and the inflation target of the central bank. In the first case the

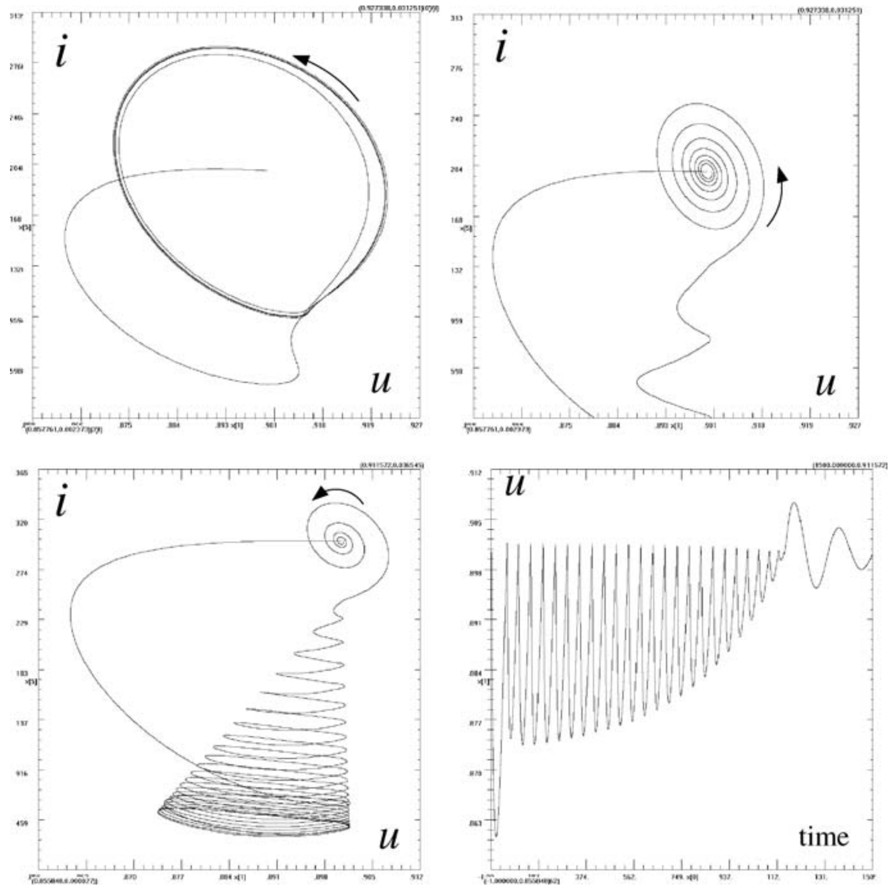

Fig. 4.11. Corridor problems for active monetary policy rules

limit cycle disappears completely and we get instead convergence to the steady state, though with a long transient again. In the second case of an even more restrictive monetary policy we no longer get complete convergence back to the steady state, but instead convergence to a small limit cycle below this steady state, shown in enlarge form in Fig. 4.12, top-right. Lowering $\bar{\pi}$ even further (to zero) gives the same result, but with a slightly more depressed limit cycle now, see Fig. 4.12, bottom-left. In Fig. 4.12, bottom-right, finally, we compare an inflation target 0f 0.05 with an in fact deflation target of the Central bank of -0.002 (both not topical themes in monetary policy). In the first case the persistent fluctuation of the initial situation gets lost and is changed into a business fluctuation with increasing amplitude, that is the economy becomes an unstable one. In the second case we now get in a pronounced way a stable,

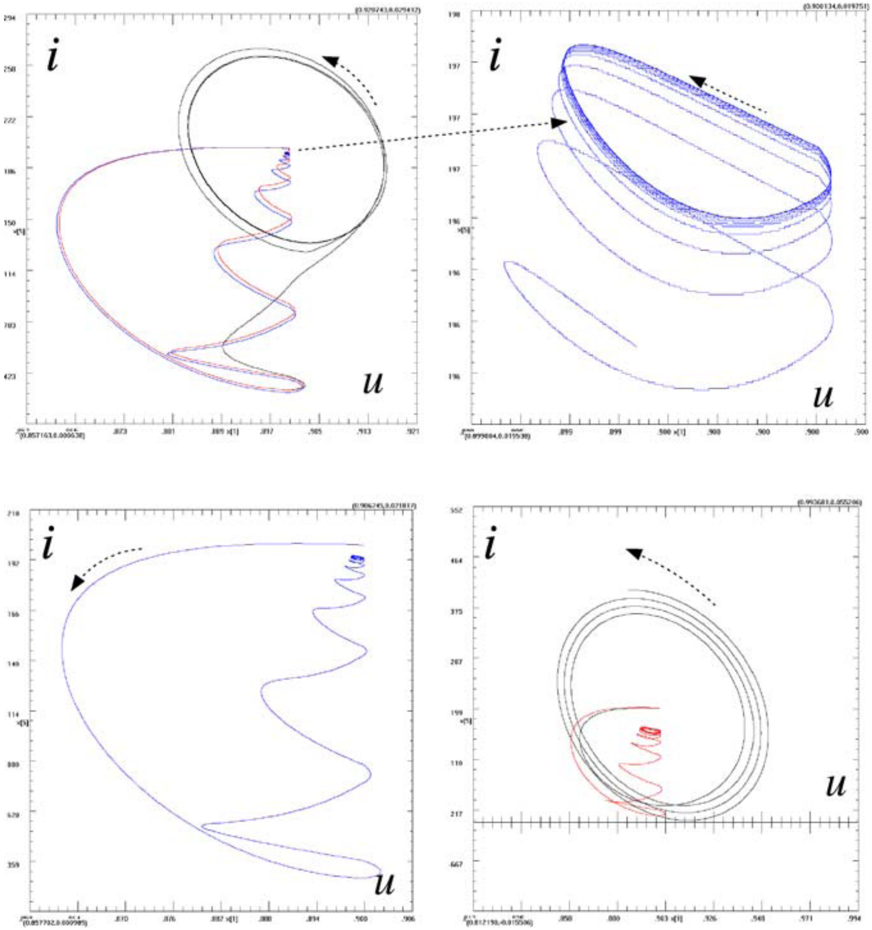

Fig. 4.12. Tight and lose inflation targets

but depressed limit cycle below the NAIRU levels for the rates of capacity utilization.

We conclude from these few simulation exercises on other scenarios in the neighborhood of our estimated Keynesian disequilibrium dynamics, that one might observe a variety of interesting scenarios when certain kinks in money wage behavior and changes in the adjustment speed of our inflationary climate expression are taken into account. These changes furthermore show that further investigations of such behavioral nonlinearities are needed, see Chen and Flaschel (2006) and Flaschel et al. (2007) for some attempts into this direction for the U.S. economy.

4.11 Conclusions and Outlook

We have considered in this chapter a significant extension and modification of the traditional approach to AS–AD growth dynamics, primarily by way of an appropriate reformulation of the wage-price block of the model, that allows us to avoid the dynamical inconsistencies of the traditional Neoclassical Synthesis. It also allows us to overcome the empirical weaknesses and theoretical indeterminacy problems of the New Keynesian approach that arise from the existence of only purely forward looking behavior in baseline models of staggered price and wage setting. Conventional wisdom, based on the rational expectations approach, however is here used to avoid the latter indeterminacy problems by appropriate extensions of the baseline model that enforce its total instability (the existence of only unstable roots), implying that the steady state represents the only bounded trajectory in the deterministic core of the model (to which the economy then immediately returns when hit by a demand, supply or policy shock).

By contrast, our alternative approach—which allows for sluggish wage as well as price adjustment and also for certain economic climate variables, representing the medium-run evolution of inflation—completely bypasses the purely formal imposition of such boundedness assumptions. Instead it allows us to demonstrate in a detailed way, guided by the intuition behind important macroeconomic feedback channels, local asymptotic stability under certain plausible assumptions (indeed very plausible from the perspective of Keynesian feedback channels), cyclical loss of stability when these assumptions are violated (if speeds of adjustment become sufficiently high), and even explosive fluctuations in the case of further increases of the crucial speeds of adjustment of the model. In the latter case further behavioral nonlinearities have to be introduced in order to tame the explosive dynamics, for example as in Chiarella and Flaschel (2000, Chaps. 6, 7) where a kinked Phillips curve (downward wage rigidity) is employed to achieve global boundedness.

The stability features of these—in our view properly reformulated—Keynesian dynamics are based on specific interactions of traditional Keynes- and Mundell-effects or real rate of interest effects with real-wage effects. In the present framework—for our estimated parameters—these effects simply imply that increasing price flexibility will be destabilizing but increasing wage flexibility might be stabilizing. On the other hand, of monetary policy responds sufficiently strong to the output or inflation gap there can be less wage flexibility to obtain boundedness of fluctuations. This is based on the estimated fact that aggregate demand here depends negatively on the real wage and on the extended types of Phillips curves we have employed in our new approach to traditional Keynesian growth dynamics. The interaction of these three effects is what explains the obtained stability results under the (in this case not very important) assumption of myopic perfect foresight, on wage as well as price inflation, and thus gives rise to a traditional type of Keynesian business cycle theory, not at all plagued by the anomalies of the textbook AS–AD dynamics.[27]

Our model provides an array of stability results, which however are narrowed down to damped oscillations when the model is estimated with data for the U.S. economy after World War II (and monetary policy is made somewhat more active in the theoretical model than estimated). Yet, also in the strongly convergent case, there can arise stability problems if the linear wage PC is modified to allow for some downward money wage rigidities. In such a case, prices may fall faster than wages in a depression, leading to real wage increases and thus to further reductions in economic activity. We have shown in this regard how the reestablishment of downward movement may be avoided leading then to a persistent business fluctuations of more or less irregular type and thus back to a further array of interesting stability scenarios. As we have shown monetary policy can avoid such downward movements and preserve damped oscillatory behavior, primarily through the adoption of a target rate of inflation that is chosen appropriately, and in case of the establishment of persistent fluctuations of the above type, reduce such fluctuations by a controlled activation of its response to output gaps or a choice of its response to inflation gaps in a certain corridor, as was shown by way of numerical examples. Therefore, not a simple answer can be given into which direction monetary policy should be changed in order to make the economy less and not indeed more volatile.[28]

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[27] See Chiarella et al. (2005) for a detailed treatment and critique of this textbook approach.

[28] The model of the current chapter is numerically further explored in Chen et al. (2004), in order to analyze in greater depth, the interaction of the various feedback channels present in the considered dynamics. There is made use of LM curves as well as Taylor interest rate policy rules, kinked Phillips curves and Blanchard and Katz error correction mechanisms in order to investigate in detail the various ways by which locally unstable dynamics can be made bounded and thus viable. The question then is which assumption on private behavior and fiscal and monetary policy—once viability is achieved—can reduce the volatility of the resulting

Taking all this together, our general conclusion here is that the here proposed framework does not only overcome the anomalies of the Neoclassical Synthesis, Stage I, but also provides a coherent alternative to its second stage, the New Keynesian theory of the business cycle, as for example sketched in Galí (2000). Our alternative to this approach to macrodynamics is based on disequilibrium in the market for goods and labor, on sluggish adjustment of prices as well as wages and on myopic perfect foresight interacting with certain economic climate expression. The rich array of dynamic outcomes of our model provide great potential for further generalizations. Some of these generalizations have already been considered in Chiarella et al. (2000) and Chiarella et al. (2005). Our overall approach, which may be called a disequilibrium approach to business cycle modelling of mature Keynesian type, thus provides a theoretical framework within which to consider the contributions of authors such as Zarnowitz (1999) who also stresses the dynamic interaction of many traditional macroeconomic building blocks.

persistent fluctuations. Our work on related models suggests that interest rate policy rule may not be sufficient to tame the explosive dynamics in all conceivable cases, or even make them convergent. But when viability is achieved—for example by downward wage rigidity—we can then investigate parameter corridors where monetary policy can indeed reduce the endogenously generated fluctuations of this approach to Keynesian business fluctuations.

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5

Linking Goods with Labor Markets: Okun's Law and Beyond

5.1 Introduction

The present chapter starts out from the widely accepted view that employment is directly affected by the growth of economic activity.[1] This is a central connection for both a characterization of empirical data and the identification of structural change, and for the conception of theoretical macroeconomic models dealing with policy issues. The basis for a discussion on the effects here involved is the statistical relationship between the variations of (un)employment and GDP which is well-known as Okun's law (the seminal reference is Okun 1962). This law states that though employment rises with output, the changes are not one-to-one. Okun found that the GDP growth rate must be equal to its potential growth just to keep the unemployment rate constant, and more specifically that a one percent increase in output above its trend line would only lead to a 0.3 percentage decrease in the rate of unemployment.

While the relationship has for along time been considered to be a fairly stable regularity in the industrialized countries, this notion is now no longer taken for granted. Regarding the U.S. economy, over the past few years the general impression has gained ground that the stronger increase in output was not accompanied by a commensurate increase in employment, such that the job creation in relation to growth has been falling in the U.S. whereas it has been rising in major European countries like France and Germany. Therefore, the present chapter also seeks to inquire into the time variability

[1] This chapter is based on Franke (2006b): "Themes on Okun's Law and Beyond", SCEPA Technical Report, New School for Social Research, New York. We would like to thank Reiner Franke for allowing us to this material.

in the relationship. However, since in this discussion a variety of topics will be addressed, our investigation focuses on U.S. data only.

Because of the central importance of the time-varying coefficients in the empirical relationships, the chapter begins with an overview of the econometric concepts that are underlying their estimation. An elaboration on these points in an extra section seems necessary since in many non-technical presentations the details of the method employed do not become exactly clear. This is especially true for the widely used Kalman filter, which often is apparently used in conjunction with additional judgement by the researchers without them making this explicit. One might furthermore conjecture that similar time paths of a regression coefficient, for example, would be obtained by less fashionable procedures that rest on more intelligible presumptions.

Section 5.2 is a methodological discussion of these often neglected niceties in the presentation of practical work. It cannot do without some mathematical notation, but it tries to keep it to a minimum and to discuss the basic properties of different procedures by means of illustrative examples. In particular, in contrast to the (stochastic, random walk) procedures using the Kalman filter to estimate time-varying coefficients, we will also propose a (deterministic) approach that is based on so-called spline functions. At the end of the section we will make clear it why we prefer the latter method over the Kalman filter variants, on what (inevitable) parameterization we settle down as a default, and what general properties of the time paths it tends to generate.

This comparison of the deterministic and stochastic method is done in Sect. 5.2.3, which we hope can be largely understood on its own. The formal presentation of the two approaches themselves in the previous tow subsections might thus be skipped.

Section 5.3 takes up an idea from the literature that exploits Okun's law and the purported relatively stable link between output and the employment rate to obtain an alternative estimation of a natural rate of unemployment. By avoiding any theorizing about wage and price inflation, this approach is more parsimonious than the usual natural rate concept of the NAIRU. Because of the central role of the output gap here and also in other parts of the chapter, the section also discusses the topic of detrending, with the conclusion that the familiar recipes in this field should be seriously reconsidered. We will, specifically, conclude that Hodrick-Prescott filtering already does a good job, where, however, it seems appropriate to increase the familiar smoothing parameter $\lambda = 1{,}600$ for quarterly data considerably.

Section 5.4 turns to an investigation of Okun's law itself, studying both a level and a first-difference version regarding output and employment. Time variations of the Okun coefficient are examined by means of regressions with a rolling sample period and, allowing more flexibility, by the spline function method introduced and discussed in Sect. 5.2. It will, in particular, be found that the main tendency of the evolution of the Okun coefficient over time is somewhat different from the time path obtained by Semmler and Zhang (2005) in a most recent contribution to Okun's law.

These investigations are carried out in Sects. 5.4.1 and 5.4.2. Subsequently, Sect. 5.4.3 goes beyond the regularities between output and the employment rate and examines the most important macroeconomic variables that contribute to this connection. To this end, the employment rate is decomposed into its constituent parts of demand and supply, i.e. employment and the labor force. It is furthermore useful to consider working hours and decompose total output into labor productivity (output per hour), hours per job, the employment rate, and the labor force. Concentrating on a business cycle perspective, we compute the trend deviations of these variables and study their comovements. It can in this way be revealed that for some variables the patterns of the business cycle fluctuations or their amplitudes have changed over the last 15 years. This, in particular, holds true for employment and hours and so, as a consequence, for labor productivity, too; but also the cyclical behavior of the labor force has changed substantially.

The next two sections are devoted to modelling the output-employment nexus and estimating the reaction coefficients that characterize the structural relationships. The two models that we put forward assume both delayed adjustments of employment in response to gaps between desired and actual magnitudes. Regarding hours and the utilization of the workforce, the first model, which is dealt with in Sect. 5.5, works with the simplification of a—within the short period—fixed relationship between hours and output (a production function for hours, so to speak). In this way the relationships between the variables can be reduced to a parsimonious building block of a recruitment policy of firms, which only incorporates the employment rate and its rate of change as endogenous variables, and the output gap as an exogenous variable. This module is characterized by just two structural parameters, which come out very satisfactorily in the estimation.

The second model in Sect. 5.6 is theoretically more ambitious and treats the determination of hours and the number of jobs at the same level, assuming the same kind of decision making of firms. Formally, the employment rate as well as the utilization of the workforce become dynamic variables then, and we also point out in which way this model augments the atheoretical short-cut formulation of Okun's law. On the whole there are now seven structural parameters to estimate, six of which turn out to be highly significant and make perfect economic sense. On the basis of this evidence we afterwards investigate possible variations of these coefficients over time and relate the results to the changes of the Okun coefficient that have been found in Sect. 5.4. Section 5.7 concludes this chapter.

5.2 Foundations for Regressions with Time-Varying Coefficients

In applied work on time-varying coefficients in the relationships of economic variables—such as, typically, estimations of a time-varying NAIRU—the focus is usually on the results and possible alternative specifications in the selection of variables or lags. The underlying approach and the estimation method is supposed to be understood. However, properly understanding the method and the assumptions associated with it is, at least to the nontechnical outsider, no straightforward issue. This section, therefore, contains a short discussion of some basic points and how this chapter takes account of them.

To begin with, let generally y_t be the series that is to be "explained" by k independent, explanatory variables.[2] Suppose time-varying coefficients are associated with the first q variables, while the coefficients on the other variables are fixed over the sample period. Denoting the vectors of these variables by the letters x and z, respectively, the regression equation reads

$$y_t = \gamma_t' x_t + \beta' z_t + u_t \;\; \mathrm{var}(u_t) = \sigma^2 \;\; (x_t, \gamma_t \in \mathbb{R}^q, z_t, \beta \in \mathbb{R}^{k-q}) \tag{5.1}$$

(all vectors are column vectors, γ_t' and β' are the corresponding row vectors). The disturbances u_t are assumed to be independently and identically distributed around zero.

[2] To avoid clumsy notation, we write y_t for the value which variable y attains in period t as well as for the entire time series from $t = 1$ until T (which would correctly be denoted as $\{y_t\}$ or even $\{y_t\}_{t=1}^T$). It will be clear from the context which of the two notions applies.

Regarding the law governing the changes in γ_t, two different approaches can be found in the literature. One is that of a stochastic random walk, and the other conceives the γ_t as deterministic functions of time.

5.2.1 The Random Walk Approach

The first approach is dominant in the literature. It is indeed hard to find any exceptions, and the stochastic framework as well as the random walk specification are hardly ever justified over possible alternatives. It is already much of a discussion if it is mentioned that estimation can be done by making use of the Kalman filter, a key advantage of which is that it can generate standard errors for nonlinear functions of the parameters (such as the ratios occurring in estimations of the NAIRU).

The random walk approach views the coefficients $\gamma_{j,t}$ in (5.1) as changing slowly and *unsystematically* over time ($j = 1, \ldots, q$). The latter means that in each period t an increase or decrease of a single coefficient is equally likely to occur, so that the expectation of $\gamma_{j,t}$ is equal to the value $\gamma_{j,t-1}$ that this coefficient has attained in the period before. Formally, the statement amounts to the hypothesis of a random walk,

$$\gamma_{j,t} = \gamma_{j,t-1} \ + \ v_{j,t}, \qquad \text{var}(v_{j,t}) = \sigma_j^2 \quad (j = 1, \ldots, q), \tag{5.2}$$

where likewise the errors $v_{j,t}$ are i.i.d. with mean zero; more specifically a normal distribution is usually assumed (especially if estimation is done by maximum likelihood).

It is important to note that the $\gamma_{j,t}$ obtained from estimations of (5.1), (5.2) do not estimate the "true" values of the coefficients. This would in fact be a meaningless statement since the latter are random variables. What is estimated are the *expected means* of the *probability distributions* of the $\gamma_{j,t}$ (with respect to, in particular, the variances $\sigma_1^2, \ldots, \sigma_q^2$).

Estimation of (5.1), (5.2) is not standardized. A first distinction one has to pay attention to concerns the role of the random walk variances $\sigma_1^2, \ldots, \sigma_q^2$; whether they are exogenously prespecified or endogenously determined within an estimation procedure.

Exogenous Random Walk Variances

In the early applications of (5.1), (5.2) to the problem of time-varying coefficients, the variances $\sigma_1^2, \ldots, \sigma_q^2$ were treated as exogenous parameters. It is clear that any specification of them prejudices the qualitative features of the time paths of the $\gamma_{j,t}$. The polar case of assuming $\sigma_j^2 = 0$ implies a completely constant coefficient, while a positive variance allows the coefficient to vary by a limited amount each time period. If no limit were placed on the ability of the coefficient to vary each period, then it would jump up and down and soak up all the residual variation in (5.1).

Hence, what assumption on such a variance should be made? With respect to a coefficient, or a nonlinear combination of coefficients like the NAIRU—which according to intuition or economic theory should shift slowly—Gordon (1997, p. 22) makes the explicit proposal of what he calls a 'smoothness' prior, which avoids overly strong volatility. As he puts it, the coefficient "can move around as much as it likes, subject to the qualification that sharp quarter-to-quarter zig-zags are ruled out".

To be exact, the degree of smoothness of a time-varying $\gamma_{j,t}$ does not depend on the variance σ_j^2 in (5.2), but on its size relative to the (estimated) variance in (5.1), the so-called signal-to-noise ratio σ_j^2/σ^2. Nevertheless, setting σ_j^2 requires judgement; judgement of a kind that Gordon (p. 22, fn 14) describes as analogous to the choice of the smoothness parameter λ for the Hodrick-Prescott filter in the detrending of growth variables.[3]

Figure 5.1 gives an example of the implications of different variances for the time paths of the parameter of interest.[4] The diagram is taken from Gordon (1997, p. 21), where on the basis of three alternative random walk variances he estimates a Phillips curve with time-varying coefficients which, then, de-

[3] It might be argued that there is one value of λ for the Hodrick-Prescott (HP) filter that is most commonly employed for quarterly data, i.e., $\lambda = 1600$; and that although there is no such convention as yet for the choice of the signal-to-noise ratio for the random walk variances, the profession might tend to agree on a suitable order of magnitude. However, in other work Gordon makes it clear that the recommended λ should not be adopted so automatically and that the appropriate degree of smoothness needs to be considered for each time series in its own context. We will have to return to the parameterization of the HP filter later in this chapter.

[4] From the references given and other empirical work by Gordon, one can assume that these estimations employ the Kalman filter. This is a one-sided (backward-looking) filter that additionally requires choosing an initial condition for the time-varying parameter (or the NAIRU itself in the present case). However, there does not seem to exist a foolproof procedure for this choice, as one can infer, for example, from the short discussion in Laubach (2001, p. 222).

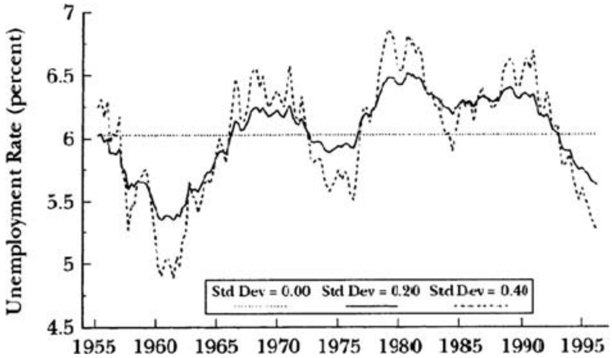

Fig. 5.1. NAIRU time paths resulting from different exogenous random walk variances. (Reproduced from Gordon (1997, p. 21, Fig. 1))

liver a time-varying NAIRU. After presenting the plot, Gordon notes that it is the (bold) solid line that meets his smoothness criterion, a "series that exhibits substantial movements but just avoids sharp quarter-to-quarter zigzags" (p. 22).[5]

The example also demonstrates that Gordon's view of what he still accepts as "smooth" may not be generally shared. In particular, there may well be other proponents in the NAIRU discussion who would find it more appropriate to have the temporary decline between 1972 and 1975 smoothed out.

Estimated Random Walk Variances

While the intention of endogenizing the random walk variances in the estimation procedure is to get rid of the judgement, or arbitrariness, just discussed, this goes at the expense of transparency since matters become even more technical (beyond the problem indicated in fn 4). In addition, there are several distinct methods to be found in applied work.

Assuming that the random walk disturbances are normally distributed, the coefficients γ_t can be jointly estimated with the signal-to-noise ratios by maximum likelihood using the Kalman filter (standard references are Harvey (1989), and Hamilton (1994, Chap. 13.8)). A grave problem here arising is the so-called "pile-up problem" (surveyed by Stock 1994, Sect. 4). From analytical results and the existing experience with this method, one would expect to

[5] From the standard errors of the regressions that Gordon reports in Table 1 (p. 25), one can infer that the solid line is generated by a signal-to-noise ratio of $0.20/0.88 \approx 0.23$.

find estimates of the signal-to-noise ratios to be zero with high probability, even if their true values are strictly larger (cf. Laubach 2001, p. 221). Some researchers even consider the problem so serious that they prefer to have recourse to the exogenously fixed variances, choosing signal-to-noise ratios that are in line with other empirical studies (Llaudes 2005, p. 15) or are checked "interactively" with estimated variances (Laubach 2001, pp. 222f).[6]

Schlicht (1989, 2003) has proposed a procedure that he simply calls the VC method (VC for "varying coefficients"), or more specifically the VC moment estimator, to distinguish it from its close cousin, the VC likelihood estimator. The general claim is that, for linear models, the VC moment estimator is mathematically and descriptively more transparent, and also statistically superior to the Kalman filter (see Schlicht and Ludsteck 2005, pp. 3f), for brief statements of these points, before they go into the details).[7] In many cases, nevertheless, the VC moment estimator, the VC likelihood estimator, and the Kalman filter produce almost identical results (Schlicht and Ludsteck 2005, p. 4, from a computational point of view, the moment estimator is sometimes, in poorly conditioned cases, superior to the other two methods).

To get an idea of the main conceptions and also to avoid as many technical details as possible, we cast the approach in terms of a consistency property that the—endogenous—variances have to satisfy. It actually amounts to a fixed-point argument. To this end, consider the following three steps. First, given variances $\sigma^2, \sigma_1^2, \dots, \sigma_q^2$ and the data y_t, x_t, z_t in (5.1) and (5.2), one can define suitably formatted matrices X, y, P, S and then, stacking the $\gamma_{j,t}$ in one $(q \cdot n)$-vector γ, compute the expected values of the $\gamma_{j,t}$ as

$$\hat{\gamma} = (X'X + \sigma^2 P'S^{-1}P)^{-1}X'y \tag{5.3}$$

(for a proof of (5.3) and also (5.6) below, see Schlicht (1989, pp. 11f)). A least squares interpretation of what is behind these "expected values" is given in a moment.

[6] Another method often referenced is Stock and Watson (1998), which promises to obtain median-unbiased estimates of the signal-to-noise ratios. Unfortunately, the article itself can only be understood by trained econometricians. Llaudes (2005, p. 16) reports to have tried the method in his NAIRU estimations but did not find the results very satisfactory (because of too little precision). So he discards this alternative approach, too (and goes on by fixing the variances exogenously).

[7] Despite these claims, the VC method is not to be considered a substitute but a complement to the Kalman filter, which is applicable to a much larger set of problems (Schlicht and Ludsteck 2005, p. 28).

In the second step, the n-vectors of the corresponding residuals $\hat{u}, \hat{v}_1, \ldots, \hat{v}_q$ are obtained from (5.1) and (5.2). In the third step, their variances $\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots,$ $\hat{\sigma}_q^2$ are computed. Taken together, we have a mapping $F\colon \mathbb{R}_+^{q+1} \to \mathbb{R}_+^{q+1}$, which can be briefly sketched as

$$F : (\sigma^2, \sigma_1^2, \ldots, \sigma_q^2) \longrightarrow \hat{\gamma} \longrightarrow (\hat{u}, \hat{v}_1, \ldots, \hat{v}_q) \longrightarrow (\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2). \quad (5.4)$$

Consistency prevails if an array $\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2$ constitutes a fixed-point of (5.4), such that

$$F(\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2) = (\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2). \quad (5.5)$$

The formulation is still somewhat loose, since a precise argument has to refer to expected values (see (50) in Schlicht 1989, p. 17), but (5.5) can nevertheless be considered to describe the core of the estimation problem and the principle that determines the signal-to-noise ratios.

Though the determination of $\hat{\gamma}$ by (5.3) is derived from a different calculus, it can be shown that (at least with respect to the fixed-point variances) this expression minimizes the weighted squares of sums of the residuals,

$$\sum_{t=1}^{n} u_t^2 \; + \; \sum_{t=2}^{n} \sum_{j=1}^{q} (\sigma^2/\sigma_j^2)\, v_{t,j}^2 . \quad (5.6)$$

It is interesting to notice that the squared random walk residuals $v_{t,j}^2$ are weighted by the reciprocal values of the signal-to-noise ratios.

For the practical purpose of solving the fixed-point problem (5.5) one has to go deeper into the details. It here suffices to mention that it can be equivalently expressed as the unconstrained minimization of a suitably defined loss function in the variances. Schlicht (1989, especially pp. 27–31) notes that the solution is not quite equal to a more common maximum likelihood estimation, but the two are asymptotically equivalent (that is, as the number of data points becomes large enough). With respect to the wide-spread use of the Kalman filter (Kalman–Bucy filtering, to be more accurate), Schlicht (2003, p. 8) also contains a short section where he argues that his procedure is statistically and conceptually superior.

Another great advantage of Schlicht's VC method is that the algorithm that minimizes the loss function is freely available in the net (Schlicht 2005) and fairly convenient to use. The algorithm itself is an iteration procedure (a gradient method). Its convergence cannot be generally proved, but Schlicht reports that under reasonable circumstances, if the estimation is not excessively ill-conditioned, convergence is no problem.[8] Thus, Schlicht's estimation procedure appears to be a reliable alternative to the approaches mentioned above. An general evaluation of the general features of the resulting time paths of the coefficients, whether there is too much or too little variation in them, is postponed until Sect. 5.2.3.

5.2.2 Deterministic Spline Functions of the Coefficients

The alternative to stochastic variations of the coefficients in (5.1) are deterministic time paths. In this framework a coefficient γ_j may be generally conceived as being determined by a vector $\tilde{z}$ of several, or perhaps many, variables, which can be generally described as $\gamma_j = \tilde{f}_j(\tilde{z})$. From all of the vectors $\tilde{z}$ in an economically meaningfully domain, only $\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_T$ have been actually realized, giving rise to $\gamma_{j,t} = \tilde{f}_j(\tilde{z}_t)$. If one does not aim at information about the general law $\tilde{f}_j(\cdot)$, which might be a futile idea anyway, but is content with referring to the realized vectors only, $\gamma_{j,t}$ can be directly expressed as a function of time, $\gamma_{j,t} = f_j(t)$.

The functions $f_j(\cdot)$ should not be too arbitrary, of course, and not too irregular, either. The analysis is thus restricted to functions that can be parameterized by S coefficients $c_{j,s}$, where all $f_j(\cdot)$ belong to the same class of functional specifications. This leads us to specify the time-varying coefficients in (5.1) as

$$\gamma_{j,t} = f(t; c_{j,1}, \ldots, c_{j,S}) \quad (j = 1, \ldots, q). \tag{5.7}$$

Given the functional form of the mapping $(t, c_1, \ldots, c_S) \mapsto f(t, c_1, \ldots, c_S)$, the estimation of (5.1), (5.7) amounts to estimating the parameters $c_{j,s}$ ($1 \le j \le q$, $1 \le s \le S$). This can be done by more ordinary methods than the ones discussed above, that is, by least squares minimization. However, since the regression equations are no longer linear in these coefficients, (5.1) and (5.7) represent a nonlinear least squares estimation.

A most straightforward specification of a time path, which pronounces possible "structural breaks", is one composed of linear segments. Here the

[8] Private communication. The gradient method here employed is comparable to the usual maximum likelihood estimation procedures. Under the quoted "reasonable circumstances", the results from this algorithm are identical to those from a global and more sophisticated algorithm. In a Mathematica package, the latter can be downloaded from http://library.wolfram.com/infocenter/MathSource/5195.

sample period is subdivided into $S-1$ equally spaced intervals $[t_s, t_{s+1}]$, and coefficient $\gamma_{j,t}$ assumes value $c_{j,s}$ if $t = t_s$, while the connection between γ_{j,t_s} and $\gamma_{j,t_{s+1}}$ is a straight line ($1 \leq s \leq S-1$). Since the outcome often does not look too "nice" and tends to overemphasize regime changes in the coefficients, where also the dating of the "breaks" may be inappropriate, this specification will not be employed for presentation purposes. It can, however, be quite informative in exploratory work.

Smoother time paths are obtained by polynomials over the sample period. An even more attractive device are so-called "splines", which combine smoothness and flexibility. A spline function is (not a linear function but) a polynomial between each pair of the knot points $(t_s, c_{j,s})$ ($s = 1, \ldots, S$), whose coefficients are determined "slightly" non-locally. The non-locality is designed to guarantee smoothness in the interpolated function, such that the left-hand and right-hand side derivatives coincide at the knot points. Most often cubic splines are employed, which means the local polynomials are of third degree and constrained to have equal first and second derivatives at the knot points.[9] Despite the high flexibility between two adjacent knot points, it is remarkable that the approach introduces no additional degrees of freedom over such a segment: regarding the time-varying coefficient $\gamma_{j,t}$ there are still no more than the S knot point parameters $c_{j,s}$ to be estimated.

Splines are therefore the method of choice in this chapter to specify the deterministic time paths of the coefficients in a regression equation. Nevertheless, another choice still remains to be made, namely, the number of segments. It is obvious that the variability in the estimated time paths can be increased (and the fit in (5.1), (5.7) "improved") by increasing the number of segments. On the other hand, one is usually interested in only the great tendencies in the evolution of a coefficient; or the theoretical background suggests narrow, though informal, bounds on its volatility. Hence, the number of segments should be rather limited. For concreteness, let us say that normally a segment should not be much shorter than ten years. Certain episodes might nevertheless be usefully checked for considerable up and downs, which would reduce such a segment to perhaps five years.

Staiger et al. (1997, pp. 36) are one example in the literature on the NAIRU where this unemployment rate is estimated by using a cubic spline. The out-

[9] In finer detail, the splines employed in this chapter are "natural splines", which have zero second derivatives on both sides of their boundaries (see Press et al. 1986, pp. 78, 86ff).

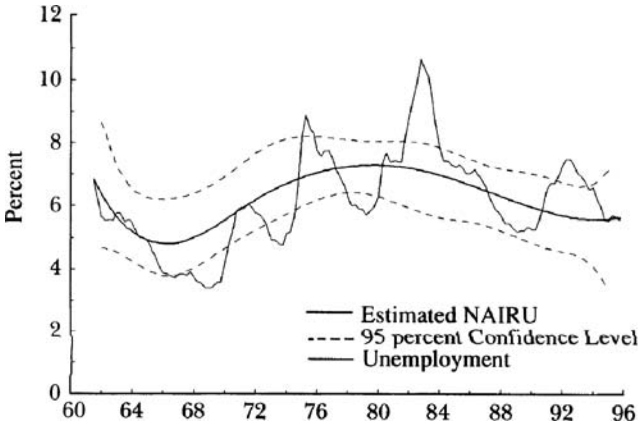

Fig. 5.2. NAIRU time path estimated from a cubic spline function (Reproduced from Staiger et al. 1997, p. 38)

come can be seen in Fig. 5.2, which reproduces their Fig. 2 (p. 38). We need not discuss the slight differences from Gordon's regression, concerning the underlying price index for inflation and the additional explanatory variables representing the supply shocks. For the present purpose it suffices to notice the general features of the paths in Figs. 5.1 and 5.2, where evidently the deterministic approach produces much less variability in the series.[10]

5.2.3 A Comparison of the Stochastic and Deterministic Approach

In modelling time-varying coefficients, the comparison of Figs. 5.1 and 5.2 already gives some insights into the implications of the random walk and the deterministic spline approach. It has, however, to be noted that Gordon's time path in Fig. 5.1 is based on his exogenous setting of the random walk variance. His "smoothness criterion" that rejects volatile short-term reversals as implausible is seen to work, but it still leaves a considerable degree of variability. The question we have to ask is if this will still be the kind of outcome when the random walk variances are endogenously determined and the Kalman filter approach is replaced with Schlicht's (1989) estimation method.

[10] Besides, Fig. 5.2 illustrates that also for the deterministically determined coefficients a confidence band can be constructed around the estimated time path, which in this case is a nonlinear function of regression coefficients; the issue is discussed in Staiger et al. (1997, p. 37).

The question is nontrivial, considering the observation made in the literature and quoted in Sect. 5.2.1 that maximum likelihood estimation with endogenous signal-to-noise ratios using the Kalman filter suffer from the "pile-up problem", which introduces a severe downward bias for the random walk variances, and considering the asymptotic equivalence of Schlicht's approach to a maximum likelihood problem.

To compare the outcome of the random walk and spline approach, consider a relationship between two highly correlated variables. For the present illustration, let us use quarterly data on e_t^{dev}, the deviation of the employment rate from some (deterministic) trend, and the output gap y_t.[11] Figure 5.3 shows the comovements of the two variables, which are both measured in percent, and the lower amplitude of the fluctuations of the employment rate. The contemporaneous correlation coefficient between e_t^{dev} and y_t over the sample period 1960:1 − 2004:4 is 0.89. It is slightly higher for the lagged output gap, $\mathrm{Corr}(e_t^{dev}, y_{t-1}) = 0.92$. A short description of the relationship

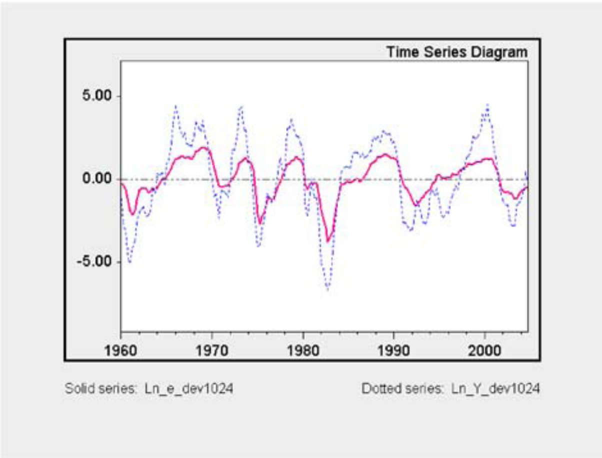

Fig. 5.3. Trend deviations of output (dotted) and the employment rate (solid series)

[11] The details of the specification of these variables, and their appropriateness, are discussed later. Here they only serve for illustration. The variables were constructed within the AELSA–FP software from the unemployment rate and output data in the Fair–Parke database, where partly the names of the new time series have been automatically assigned by the software (and we maintained them). The names can still be seen from the legend of Fig. 5.3, which is directly taken over as a software output.

between the two variables, which also accounts for possible variations in the degree of the correlation, is thus given by the regression equation

$$e_t^{dev} = \beta_t \, y_{t-1} \; + \; u_t \quad (1960{:}1-2004{:}4) \tag{5.8}$$

(u_t being the residuals). An OLS estimate of a constant coefficient yields $\beta = 0.43$, with $R^2 = 0.84$ (and, of course, considerable autocorrelation). The spline estimate (based on 5 segments) varies around this value, as shown by the bold line in Fig. 5.4. Despite the flexibility of a spline function, the fit in (5.8) is not much improved by this approach (and it does not remove the autocorrelation, either); R^2 increases no higher than 0.86.

On the other hand, the fit implied by the random walk raises R^2 to 0.95. Figure 5.4 makes it clear that this soaking up of the residual variation is achieved by suitable ups and downs, within short intervals of time, of the slope coefficient β_t (the thin line). Obviously, worries that the "pile-up problem" might carry over are not validated. Right on the contrary, the volatile series in Fig. 5.4 will probably not even match Gordon's informal "smoothness" criterion.

The short-term variation in the coefficient β_t in Fig. 5.4 is, in fact, typical for the outcome of estimations employing the random walk hypothesis. Hence, if economic theory suggests a low variability in the time-varying coefficients, or if in a brief and atheoretical description of the relationships between the variables we wish to concentrate on the main and outstanding tendencies, then

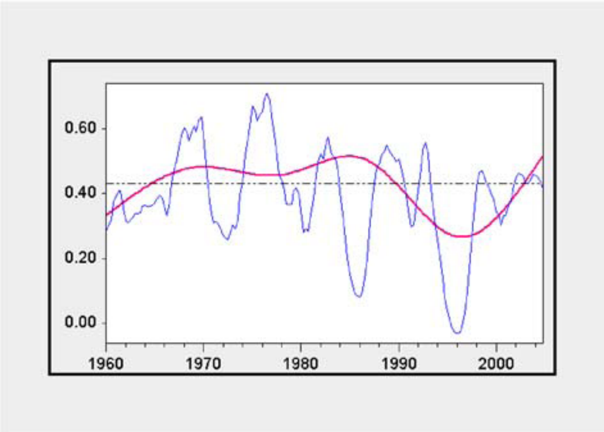

Fig. 5.4. Coefficient β_t in (5.8) from spline and random walk estimation (bold and thin line, respectively)

we should either use the deterministic spline approach or the random walk method with an exogenous prespecification of the variances of the coefficients. The latter, however, is not easily standardized since different orders of magnitude of the coefficients and different variances of the regression residuals u_t will require different "suitable" variances for the stochastic components $v_{j,t}$ of the time-varying coefficients. Moreover, our idea of a "suitable" variability in the time paths of a coefficient may not be very different from the outcome of a (likewise "suitable") spline estimation of deterministic coefficients. Therefore, we will decide in favor of the deterministic spline approach.

Having made this decision, still the number of segments has to be chosen. Figure 5.5 juxtaposes the evolution of β_t for the spline functions composed of 3, 5, 7, and 10 segments, respectively. The bold line is based on 5 segments and is reproduced from Fig. 5.4. The non-oscillating dashed line, obviously, is obtained on the basis of 3 segments, the dotted line from 7, and the thin solid line from 10 segments. The main difference between the four specifications is their ability to let β_t decrease in the mid-1990s, where the segments must be as short as $45/10 = 4.5$ years to get a more pronounced minimum than that of the bold line. Note also that Figs. 5.4 and 5.5 coincide in scale, which shows that even spline functions based on 10 segments have significantly less variability than the coefficients deriving from the random walk estimation.

An additional criterion besides a desired, or acceptable, degree of variability is the goodness-of-fit achieved thereby. It may thus be asked if the

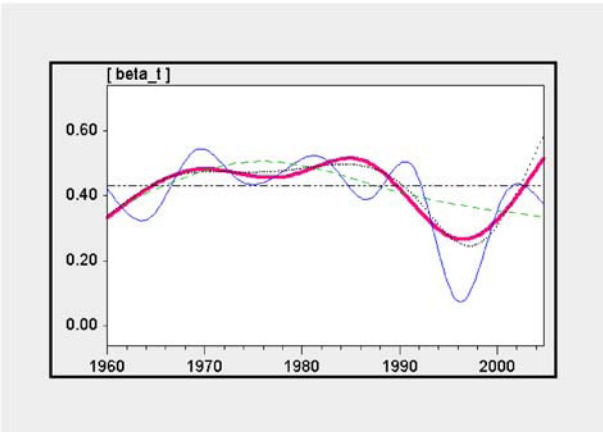

Fig. 5.5. Coefficient β_t in (5.8) from splines based on 3, 5, 7, and 10 segments, respectively

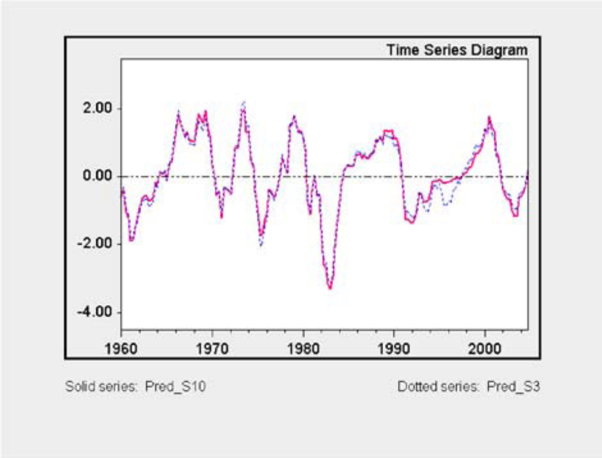

Fig. 5.6. Predictions of e_t^{dev} in (5.8) from 10-segment (solid line) and 3-segment (dotted line) spline functions

fit brought about by 10 segments is so much better than that from 3 or 5 segments. Table 5.1, which in its first two rows reports the standard error of the regression and the sum of squared residuals for the four specifications, shows that the improvement by a higher number of segments is rather limited. Comparing the 7-segment to the 5-segment specification, already the standard error indicates that the better fit is not worth the increase in the number of parameters. According to the more elaborated measure of the Bayesian information criterion (BIC, to be minimized), the parsimony of the 3-segment specification outweighs the better fit of all the other specifications, while the Akaike information criterion (AIC), which penalizes additional parameters less heavily, would prefer the other extreme of the 10-segment specification. The ambiguity of AIC and BIC is a further indicator that the four alternatives are not essentially different.

Figure 5.6 depicts the predictions of e_t^{dev} in (5.8) that result from the estimations with a 10-segment (solid line) and a 3-segment (dotted line) spline function. There is only one time interval where the two visibly differ from each other, which are the years between 1994 and 1997. A look at Fig. 5.3 explains why: in this episode the employment rate returns earlier to normal than output returns to its trend. In the simple regression approach (5.8) calls for lower values of the coefficient β_t, and the 10-segment specification has greater potential for that.

Table 5.1. Fit of (5.8) under alternative number of segments for the spline function

	Segments			
	3	5	7	10
SER :	0.436	0.430	0.432	0.417
SSR :	34.000	32.224	32.132	29.409
AIC :	218.8	213.2	216.7	206.7
BIC :	231.6	232.3	242.2	241.8

There may, however, be two different interpretations of this phenomenon. First, one views the basic pattern of output as not much different from the employment rate, but its return to normal in these years was two times set back by adverse shocks, which employers considered to be temporary (and possibly not so severe) and so did not cut back on employment. This would indeed be captured by a weaker reactions β_t. On the other hand, the employment rate itself, which after all is a composed variable, was subjected to special influences in the episode. In this case the decrease in β_t may not be overinterpreted (as suggested by the 10-segment spline). As long as this question is not investigated in greater detail, the bold line in Fig. 5.5 from the 5-segment spline estimation is perhaps an acceptable compromise between these two points of views.

Also apart from the specific context, the 5-segment spline generates a motion that exhibits some variability in the coefficient but not "too much". For the remainder of this chapter, these experiments lead us to begin our investigation of time-varying coefficients with a 5-segment spline. Subsequently we will check this result with shorter and longer segments and report the modifications thus brought about if they appear important; otherwise the presentation will be confined to the 5-segment choice.

5.3 Okun's Law and the Natural Rate of Unemployment

In this section we take up an idea from the literature that exploits Okun's law and the purported relatively stable link between output and employment to obtain an alternative estimation of a natural rate of unemployment, which avoids the inflation context from which this concept, in the form of the NAIRU, is usually derived. However, because of the central role of the output gap, which it plays not only here but also in other parts of this chapter, we have first to discuss the topic of how to detrend a growing time series. We will actually find that the familiar recipes in this field should be seriously reconsidered.

5.3.1 The Problem of Detrending

Stochastic and Deterministic Trends

A fundamental issue in the analysis of systematically growing time series (or nonstationary series in general) is the notion of the trend. In this respect we are here interested in methods whose detrending procedures generate fluctuations that can be interpreted as forming part of the business cycle. Again, we are facing the alternative between deterministic and stochastic approaches.

Beginning with Nelson and Plosser (1982), it has been argued that the trends in macroeconomic time series are stochastic, so that much of the variation that used to be considered as business cycles would actually be permanent shifts in trend. While this stochastic view of the world soon became predominant, the pendulum has, in the meantime, swung back from that consensus. In a succinct summary of recent research on this issue, it can be concluded that "at the very least there is considerable uncertainty regarding the nature of the trend in many macroeconomic time series, and that, in particular, assuming a fairly stable trend growth path for real output—perhaps even a linear deterministic trend—may not be a bad approximation" (Diebold and Rudebusch 2001, p. 8).[12] Against this background, we feel generally legitimated to work with the notion of a deterministic trend.

Because of its high flexibility, a widespread deterministic concept is (still) the Hodrick-Prescott filter.[13] Having made this decision, it remains to deal with the one degree of freedom of the filter, that is, with the choice of the

[12] This short paper is a slightly revised version of the introductory chapter of their comprehensive book on business cycles (Diebold and Rudebusch 1999).

[13] In recent times, the band-pass (BP) filter developed by Baxter and King (1995) has gained in popularity. This procedure rests on spectral analysis and so is mathematically more precise about what constitutes a cyclical component. The BP(6,32) filter preserves fluctuations with periodicities between six quarters and eight years while eliminating all other fluctuations, both the low frequency fluctuations that are associated with trend growth and the high frequencies associated with, for instance, measurement error. More exactly, with finite data sets the BP(6,32) filter approximates such an ideal filter. As it turns out, for the time series with relatively low noise (little high frequency variation) the outcome of the BP(6,32) filter is almost identical to Hodrick-Prescott filtering under the usual

smoothing parameter λ. Whereas the familiar value for quarterly data is $\lambda =$ 1,600, we will cast doubt on that convention.

Before turning to this problem, it should be outlined that applications of the stochastic approach are not foolproof and have their arbitrary elements, too. As an example, consider Gordon's (2003, pp. 219ff) Kalman filter approach to detrend labor productivity. An important first observation is the limited scope of the approach: it may be useful for some time series but not for others. For example, after estimating the stochastic trend *growth rate* of productivity, Gordon (p. 223, fn 16) points out that despite considerable effort, estimations of a corresponding log-level model failed because of implausibly low variation in the implied trend growth rates. With respect to the employment rate, Gordon (p. 224) points out that "no smoothing parameter of the Kalman filter [which amounts to setting the random walk variance] was found to achieve the desired degree of stability". In this case he even resorts to a completely different detrending procedure, also distinct from the Hodrick-Prescott filter as his complementary option (see below).

Regarding the specification of the stochastic approach in the present context, denote labor productivity as z_t and let $x_t \in \mathbb{R}^m$ be a suitable set of m explanatory variables. Then Gordon estimates the trend growth rate as the time-varying intercept γ_t in the equations

$$\mathrm{d}\ln z_t = \gamma_t + \beta' x_t + u_t, \tag{5.9}$$

$$\gamma_t = \gamma_{t-1} + v_t, \tag{5.10}$$

where u_t and v_t are assumed to be normally distributed with variances σ^2 and σ_v^2. As in previous work mentioned above, Gordon treats s_v^2 as an exogenous parameter at the researcher's command. He probably requires the variance to meet his smoothness criterion for the trend deviations (see Sect. 5.2.1). In addition, he makes explicit reference to another *a priori* judgement when he discards one value of s_v^2 with an otherwise reasonable outcome because of a high terminal growth rate of $\gamma_t = 3.38\%$ in 2003:2, which is not compatible with the view that the 2002–03 surge in actual productivity was an ephemeral event (Gordon 2003, p. 222).

While the Hodrick-Prescott filter is a univariate procedure that examines a time series on its own, (5.9) also uses outside information. Gordon (p. 219) considers it an advantage of the Kalman filter that the equation may specify

smoothing parameter $\lambda = 1{,}600$. For real national U.S. output, this is exemplified in King and Rebelo (1999, p. 933, Fig. 1).

any additional number of variables (x_t) to control for determinants of actual changes of the dependent variable that do not represent fundamental causes of changes in the trend. As for the changes in productivity, the x variables "could include changes in unemployment or the output gap, or dislocations caused by short-run events such as strikes or temporary changes in oil prices" (p. 219). Certainly, the selection of the explanatory variables x_t in (5.9) is based on judgement, which may not always be unanimous. The differences caused by including or omitting a specific variable can be illustrated by Fig. 5.7, which reproduces the bottom panel of Fig. 1 in Gordon (2003, p. 220). The dashed line is a Hodrick-Prescott trend of $d \ln z_t$ (at annualized growth rates) with a larger smoothing parameter than usual, $\lambda = 6{,}400$. The dotted line results from estimating (5.9), (5.10) without any variables x_t. As the variance σ_v^2 is chosen by Gordon, this kind of trend growth rate comes fairly close to the Hodrick-Prescott outcome.[14] If the current and four leading (not lagged) values of the change in a specifically constructed output gap are included, a piece of information unavailable to the other two filters, several distinct features are obtained. The solid line of the with-gap Kalman trend has a smoother profile over the 1978 – 88 decade; it registers a slower trend in 1962–68 (when the output gap was rising); and it registers a faster trend in 1968–76 (when the output gap was declining).

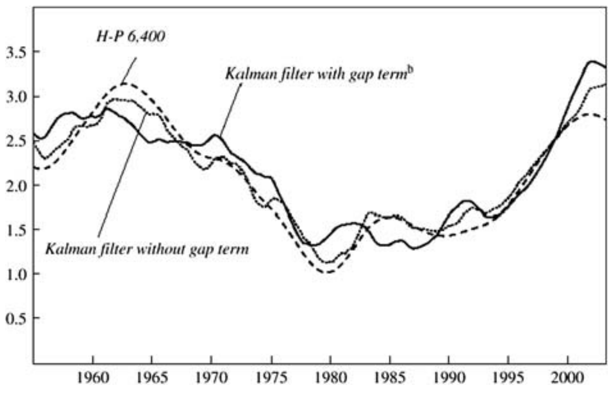

Fig. 5.7. Trend productivity growth rates obtained from Hodrick-Prescott and two specifications of the Kalman filter (Reproduced from Gordon 2003, p. 220; bottom panel of Fig. 1)

[14] Incidentally, having the variance σ_v^2 endogenously determined as described in Sect. 5.2.1 causes the series γ_t to coincide with the dependent variable.

In justifying a choice of one trend over the other, Gordon makes explicit reference to "visual inspection" (p. 221), which is used together with some basic *a priori* ideas of how the trend line should, or should not, look like. Nevertheless, accepting the general necessity of judgement and specifically the with-gap Kalman trend in Fig. 5.7, it may be asked whether similar outcomes could be achieved by a more parsimonious univariate method, simply by exploiting the degree of freedom inherent in it. This brings us back to the Hodrick-Prescott filter and a discussion of its smoothing parameter.

The Output Gap and the Smoothing Parameter in the Hodrick-Prescott Filter

The popularity of the Hodrick-Prescott (HP) filter to detrend a time series is certainly due to the fact that it is easy to understand and to use in estimation. It is, in any case, more transparent than the Kalman filter. Nevertheless, HP detrending requires the specification of one smoothing parameter λ. At one extreme, the choice of $\lambda = 0$ yields a trend that exactly tracks every value of the series being detrended. At the other extreme, a parameter of infinity yields a straight line.

As is well-known, Hodrick and Prescott themselves endorsed the benchmark $\lambda = 1{,}600$ for quarterly data. It has not only become the default value in econometric software, but it is also hard to find an empirical study of the business cycle working with the HP filter that does not follow this recommendation. As a rule, the matter is not even discussed (if $\lambda = 1{,}600$ is made explicit at all). The profound skepticism by Gordon (2003) against $\lambda = 1{,}600$ is really an exception, when he characterizes this value as implying "implausibly large accelerations and decelerations of the trend *within* each business cycle" (p. 218, emphasis added). He illustrates this property by quoting Hodrick and Prescott's (1997, p. 9) conclusion that the entire economic boom of the 1960s resulted from an acceleration of trend, rather than a deviation of actual output above trend. This evaluation, he incriminates, ignores outside information, "such as the fact that the unemployment rate was unusually low and the capacity utilization rate was unusually high" (Gordon 2003, p. 218).

This criticism again indicates that a trend, as output of an econometric procedure, is only accepted if it satisfies some (informal) criteria. As a consequence of the observation of excess sensitivity of the 1,600 parameter when it is applied to the growth rates of labor productivity, Gordon (p. 221) goes on and tries the higher values $\lambda = 6{,}400$ and $\lambda = 25{,}600$. This choice indeed reduces the flexibility in the trend series, such that in this respect these filters would become more satisfactory. But now a more detailed element of judgement comes in: "the 25,600 parameter has the disadvantage that its inability to 'bend' causes it to date the beginning of the productivity growth revival of the 1990s well before 1995, and it measures the productivity growth trend in 2002–03 at a relatively low 2.35 percent a year".

The consequence that Gordon draws from all these advantages and disadvantages is creative but unusual. He decides not to rely exclusively on one of the trends examined. Specifically, he takes both the HP 6,400 trend (6,400 since its end-of-period growth rate is somewhat higher than the low 2.35 percent just mentioned) as well as the with-gap Kalman filter and constructs as his trend the average of the two series (i.e., the average of the solid and dotted line in Fig. 5.7).[15]

This solution may be condemned as completely arbitrary (especially if one postulates that all time series in an investigation should be subjected to the same detrending procedure and the principle cannot be overall maintained). On the other hand, one may appreciate that not all is left to a technical mechanism but that, from case to case, specific outside information or *a priori* beliefs or postulates are invoked to select among a number of different options. Actually, this is what the expression "*ad-hoc*" literally means.

Back to this chapter, because of the high variability of the estimated trend we do not expect that the Kalman filter with endogenous determination of the random walk variances will be of much help, while treating the variances as exogenous and setting them ourselves would result in trend lines that similarly could also be obtained by HP filtering with a suitable smoothing parameter.[16] For this reason we concentrate on the HP filter right away, explore alternative values of the smoothing parameter λ, and choose a value that generates an "acceptable" outcome. Should no parameter value be able to achieve this, we might discuss some, admittedly *ad-hoc*, "corrections" of the trend line. As it turns out, detrending the level of output of firms, Y, does not run into

[15] If one wishes to avoid a premature rise of the trend growth rate in the 1990s, a later increase might also be enforced by a segmented linear trend line with suitable break points. In fact, the Kalman filter may not be the only reasonable alternative.

[16] In some cases the Kalman filter might produce (desirable) effects over some episodes of the sample period that cannot be obtained by HP. But then it would be poorly understood why the Kalman filter is here more "successful"—whereas in other cases it is not.

such problems.[17] Figure 5.8 shows the alternative outcomes of the annualized trend growth rates that are implied by the HP trend of $\ln Y$ (obtained by multiplying the first differences of the latter by 400). Here and in all what follows, the trend itself is computed over a longer span of time than shown in the diagram (over the period 1952:1–2005:2); so at least to the left there are no end-of-period problems. The thin solid line is based on the familiar value $\lambda = 1{,}600$. This series still exhibits such distinct within-business cycle fluctuations that it could hardly be sold as representing the growth of potential output. According to this kind of evidence, the absence of any discussion on the appropriateness of the parameter value is indeed somewhat astonishing. In any case, the thin solid line in Fig. 5.8 leads us to reject $\lambda = 1{,}600$ as a reasonable parameter.

The other three lines demonstrate how an increase in the smoothing parameter dampens the within-cycle variation. The dashed, dotted and bold lines are generated by $\lambda = 6{,}400$, 25,600 and 102,400, respectively. Even a value as high and unfamiliar as $\lambda = 25{,}600$ implies a trend growth rate that is not entirely convincing. Hence, we dare to settle down on the bold line in Fig. 5.8 and the underlying $\lambda = 102{,}400$. This concept suggests that potential output grows at 4 percent per year in the mid-1960s and that its growth rate then steadily declines to 3.10 percent in recent times, which is a feature

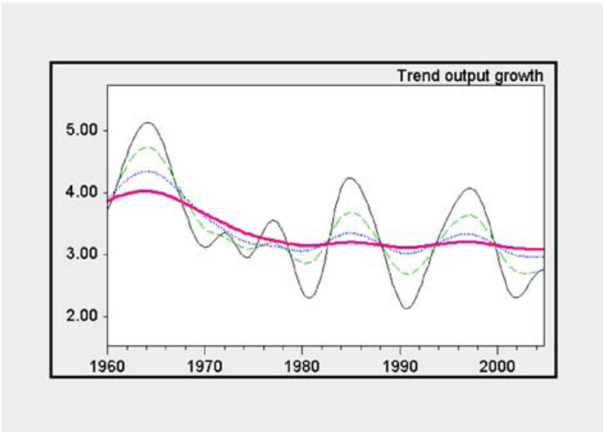

Fig. 5.8. Growth rates of trend output obtained from HP filtering of the levels under alternative smoothing parameters (see text)

[17] The firm sector comprises nonfarm nonfinancial corporate business, nonfarm noncorporate business, and farm business.

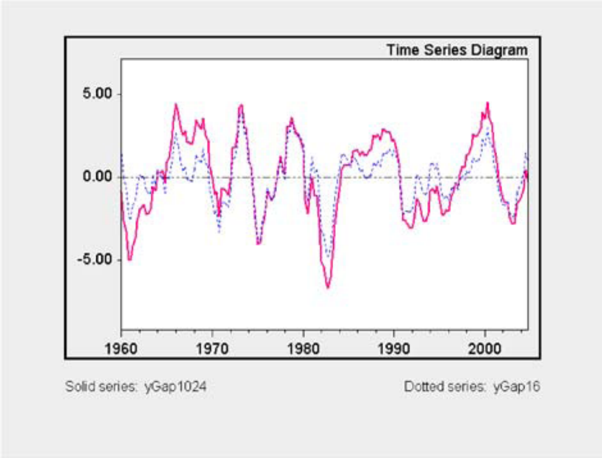

Fig. 5.9. Output gap from HP filtering under λ = 102,400 (solid line) versus λ = 1,600 (dotted line)

that seems to make good economic sense. The solid line in Fig. 5.9 displays the percentage deviations of output Y from the HP 102,400 trend. The series constitutes the output gap y_t, which will be underlying all of the following investigations ($y_t = 100 \cdot (\ln Y_t - \ln Y_t^\star)$, if $Y^\star$ denotes the trend level). It is contrasted with the well-known outcome of filtering by HP 1,600. While most of the qualitative features are not essentially altered, two differences are noteworthy: the first transitory peak in 1984 resulting for the HP 1,600 output gap disappears in our notion of y_t; and in the 1990s our y_t passes the zero level at a much later date.

Regarding the quantitative features it is obvious that the HP 1,600 gap series must yield a lower variability. Especially some (but not all) of the turning points move closer to the zero level. Although these differences do not seem too pronounced, the overall variability of the HP 1,600 gap as measured by the standard deviation is considerably smaller than that of the HP 102,400 gap; with 1.61 percentage points it indeed amounts to only two thirds of the 2.42 percentage points that we compute for y_t. Besides, the latter also exhibits stronger persistence; its first-order autocorrelation is 0.94 versus 0.87 for the HP 1,600 gap.

Having thus decided the problem of output detrending, the results in Figs. 5.8 and 5.9 may be finally compared to the outcome that Gordon (2003) obtains when he applies his method to the growth rate of GDP (instead of our levels and the output of the firm sector). Figure 5.10 reproduces his

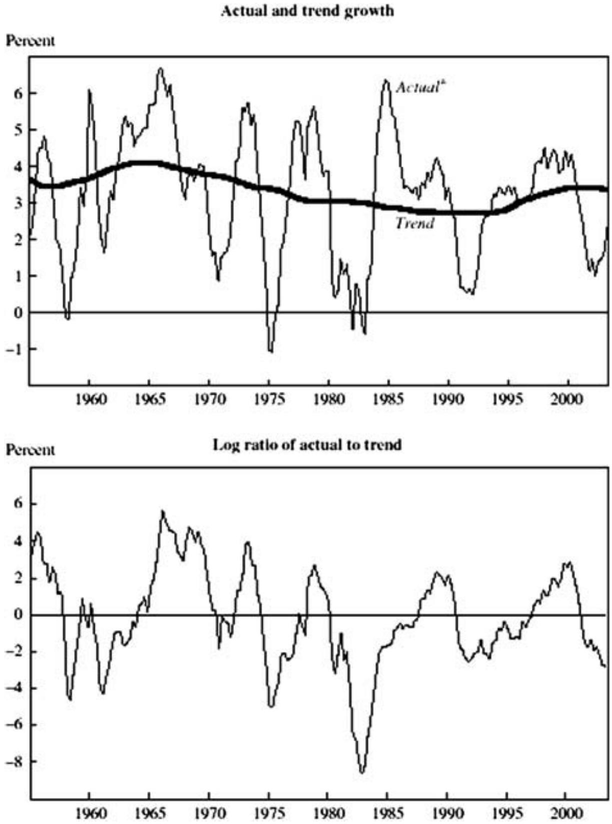

Fig. 5.10. Real GDP: trend growth and deviations of levels from trend as estimated by Gordon (2003) (Reproduced from Gordon 2003, p. 227. "Actual" in upper panel is eight-quarter change of GDP)

Fig. 3 (p. 227). The trend line in the upper panel is the average of HP 6,400 and the Kalman filter with the exogenously chosen random walk variance. It is quite similar in its variability to the bold line in Fig. 5.8. It differs, however, in its moderate upward tendency from 1993 on, which is not present in Fig. 5.8. The resulting percentage trend deviations in the level of GDP in the lower panel of Fig. 5.10 is in many features similar to the solid line in Fig. 5.9, except that the GDP gap series crosses the zero line later than our output gap, and that its peak in the mid-1960s and its trough in 1982 are more pronounced.

5.3.2 Deriving the Natural Rate of Unemployment from the Data

Although the idea of a "natural rate of unemployment" is an omnipresent reference in macroeconomic theory, there is little agreement as to what precisely the natural rate is or how it is to measured. Consulting Rogerson (1997), one finds nine different definitions in the literature. An often quoted reference is the presidential address by M. Friedman (1968) to the American Economic Association, where he characterizes the natural rate as being "ground out by the Walrasian system of general equilibrium equations . . . embedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobilities, and so on". Thus, as a minimum, the natural rate is conceived as an equilibrium rate of unemployment, which depends critically on the institutional characteristics of the economy and may vary depending on demographics and institutions. As the latter change slowly over time, the natural rate of unemployment (NRU henceforth) should be a smoothly evolving time series.

Atheoretical Trend Lines

Whatever concepts may have been put forward in economic theory to define a NRU, for measurement purposes it is a reasonable idea to proxy it by the trend component of some time series filter. For us this will most conveniently be the HP filter. Figure 5.11 displays the actual unemployment rate (the thin solid line) and lays the HP trend lines trough it that arise from three different values of the smoothing parameter λ.

The dotted line is generated by $\lambda = 1{,}600$. The standard parameter again causes considerable trend variability. A movement within 15 years from 3.9% in 1967/68 up to 8.3% in 1982/83 does not appear very "natural", quite apart from the within-cycle variation. Both features are smoothed out if $\lambda = 102{,}400$ is adopted, as shown by the bold line. Here the trend steadily rises from 5.0% at the beginning of the 1960s to a high of 7.24% in 1982:4, and then steadily declines to the original 5.0% at the end of the sample period. The dashed line with almost identical levels at the beginning and end of the period, exhibits an even lower amplitude; it only rises to 6.64% in 1983. With $\lambda = 1{,}000{,}000$, the underlying value of the smoothing parameter is, however, unprecedently high.

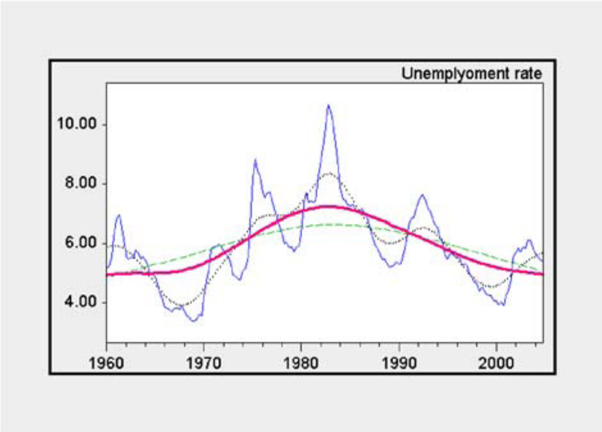

Fig. 5.11. Rate of unemployment and HP trend generated by different λ

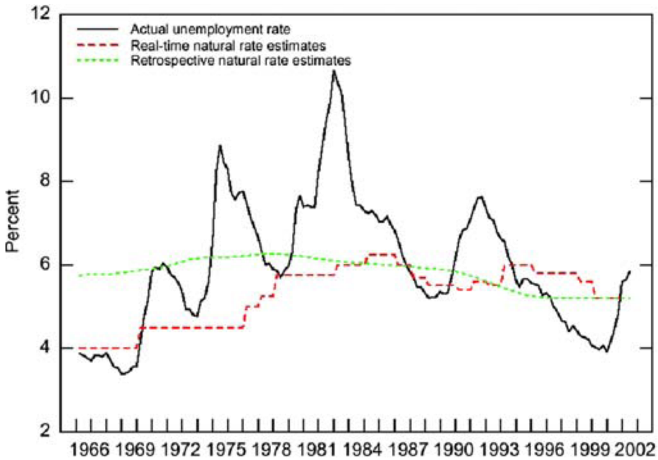

Fig. 5.12. NAIRU estimate by CBO (as of 2002; dotted line) (*Note*: Reproduced from Orphanides and Williams 2005, p. 1931, Fig. 2. The solid line is the actual unemployment rate, the dashed line is a real-time series, i.e., values of the NAIRU as it was estimated in that particular point in time, on the basis of the then available data)

Despite the unusual HP smoothing, the flatness of the bold and the dashed lines are not too implausible to proxy for a natural rate of unemployment. This is exemplified by comparing them with the current estimate (as of 2002) of the NAIRU from the Congressional Budget Office, which is the dotted line in Fig. 5.12 (the diagram has been extracted from Orphanides and Williams

2005, p. 1931, Fig. 2). The CBO concept yields an even flatter line than its counterparts in Fig. 5.11.

On the other hand, the two NRU proxies in Fig. 5.11 behave in a similar way as the NAIRU time path by Staiger et al. (1997) in Fig. 5.2 above, especially if the latter's confidence band is taken into account. Interestingly, Gordon's (1997) estimates from Fig. 5.1 move in a narrower corridor but display more within-cycle variability. Contrasting the bold and dotted lines from our HP trend in Fig. 5.11 with the alternatives in Figs. 5.1, 5.2 and 5.12, there is no reason that would disqualify the result of the atheoretical approach *a priori*. The two lines may even be conceived as sort of a compromise.

Estimation of the NRU by Okun's Law

In the discussion of the HP trends in Fig. 5.11, the natural rate of unemployment has implicitly been identified with the NAIRU. In fact, the interchangeable use of the terms is common practice in the literature. Grant (2002, pp. 96f) insists that these are two different theoretical concepts that should be more carefully distinguished. The natural rate of unemployment is a microeconomic Walrasian equilibrium outcome in which labor markets are cleared by wages and prices, whereas the NAIRU is fundamentally a disequilibrium, or Keynesian, macro outcome where "the inflationary forces of the excess demand markets balance the disinflationary forces of the excess supply markets" (p. 97).

We can here leave it open whether this is an accurate characterization of the two notions. More interesting is Grant's idea of how (in lack of suitable microeconomic data) to estimate the NRU at the macro level. Instead of referring to the inflation context, he proposes to exploit the relatively stable link between output and employment that has already been observed by Okun (1962). As Grant (2002, p. 98) writes, "Okun's conceptual framework of this employment-output link lends itself nicely both to Friedman's intuitive exposition and to econometric specification. Unemployment can be considered to be the sum of three components. Frictional unemployment and structural unemployment may exist due to microeconomic market imperfections which inhibit fluid job search and matching. Both may exist alongside unfilled labor demand. A cyclical unemployment gap exists due to deficiencies or excesses in aggregate product demand from the economy's sustainable potential."

Now, the latter are conveniently captured by the output gap y_t as we have discussed it in Sect. 5.3.1.[18] If we let UR_t designate the actual unemployment rate, Okun's link between the gap in unemployment and the output gap can then be specified econometrically as

$$\mathrm{UR}_t = \gamma_{0,t} - \gamma_{y,t} y_t + u_t . \tag{5.11}$$

Apparently, $\gamma_{0,t}$ is the econometric estimate of unemployment that would exist when the economy was running at the level of capacity given by the estimated potential output. The time path of $\gamma_{0,t}$ is Grant's output-based estimate of the NRU (Grant 2002, p. 98)). In contrast to the NAIRU relationship between the labor market and general price inflation, $\gamma_{0,t}$ in (5.11) is an estimate of the relationship between labor and product markets (p. 108).

In estimating (5.11), Grant follows the usual procedure and assumes that the coefficients $\gamma_{0,t}$ and $\gamma_{y,t}$ evolve as random walks, for which he employs the Kalman filter (p. 104). However, he makes no mentioning of the random walk variances, whether they are part of the estimation or set exogenously. The output gap he uses refers to GDP and is based on a HP trend (either HP 1,600 or HP 10,000; Grant does not make clear which one). The time path of $\gamma_{0,t}$ that Grant obtains is reproduced in Fig. 5.13 (taken from Grant 2002, p. 107, Fig. 4); it is the solid line in the diagram (HPNRU). For a better evaluation of its properties, Grant contrasts this NRU with a NAIRU estimate by Gordon (the dashed line TVNAIRU), which appears to be a slightly smoothed version of the NAIRU that was presented in Fig. 5.1 (the solid line there). The differences between the two time series are striking. First, while Gordon's NAIRU (or at least its "trend") increases over the 1960s and 1970s by about 0.5 percentage points, the NRU decreases by about 1.2 percentage points until 1970 and then again increases up to its initial level, which it reaches another 15 years later. Equally remarkable is the second difference. In contrast to the decline of the NAIRU from the mid-1980s until 1997, the NRU stays constant over these years. In these respects, the differences of Grant's NRU estimate from the NAIRU estimates by Staiger et al. (1997; see Fig. 5.2), the CBO (see Fig. 5.12), or from our HP trends in Fig. 5.11 are all qualitatively the same.

[18] To check the robustness of his results, Grant considers several detrending procedures, including HP filtering. Regarding the NRU estimates he concludes that they are of secondary importance. More precisely, the results of a subgroup of detrending procedures that contain two HP filters are fairly similar. This finding allows us to concentrate on our HP 102,400 trend right away, also when we compare our results to Grant's results.

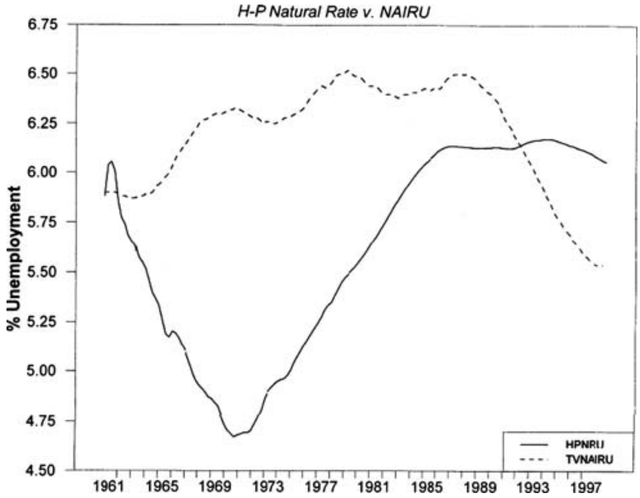

Fig. 5.13. Grant's NRU estimation from (5.11) (solid line). (*Note*: Reproduced from Grant 2002, p. 107, Fig. 4. The dashed line is a NAIRU estimate by Gordon)

Do we get similar results when we now estimate (5.11) by specifying the time-varying coefficients as discussed in Sect. 5.2.1? The output gap y_t for such a regression is obtained from detrending $\ln Y_t$ by HP 102,400, which results in the solid line in Fig. 5.9 (we have checked that the results do not change much under filtering output by HP 1,600). The sample period is again 1960:1–2004:4.

Consider first the outcome of $\gamma_{0,t}$ when we likewise adopt the random walk hypothesis for the coefficients, and the variances are endogenously determined. The resulting time path is drawn as the thin solid line in Fig. 5.14 (note that the diagram has the same scale as Fig. 5.11). The series maintains the main cyclical pattern of the actual unemployment rate, so that it already looks like a compressed image of it. This estimate of the NRU is, in particular, very different from Grant's application of the Kalman filter. The diagram points this out by the dashed line, which with its three linear segments is a stylized reproduction of his estimate in Fig. 5.13 (the solid line there). Actually, the systematic deviations of Grant's NRU from the thin solid line seem somewhat surprising. As a possible explanation, we can think of an extremely low signal-to-noise ratio that Grant may have fixed from the outside, though this issue should have certainly been discussed. In this respect one might suspect that his kind of estimation comes close to detrending the unemployment rate by a segmented linear trend. However, computing such a trend of three equally

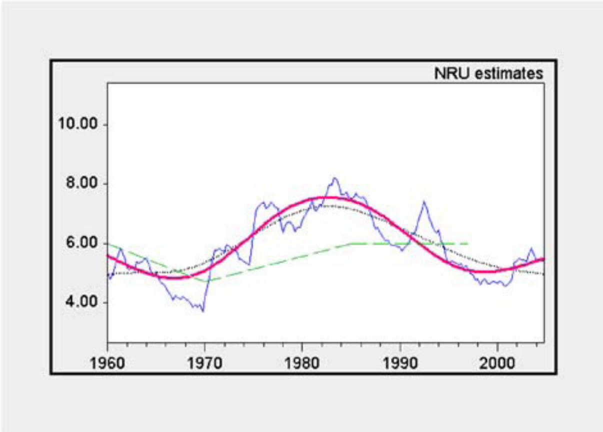

Fig. 5.14. NRU estimates of (5.11): random walk (thin solid line) and spline function (bold line) (*Note*: The spline function is based on five equally spaced segments. Dotted line is the HP 102,400 trend of the unemployment rate (see Fig. 5.11), dashed line is a stylized reproduction of Grant's estimate in Fig. 5.13)

spaced segments over the shortened sample period 1960:1–1997:4 yields a different picture from the dashed line in the diagram: in the first third of the time span this trend slightly rises from 5.0 to 5.2 percentage points, then increase up to 8 points, and in the last third returns to the initial five percentage points. So, a puzzle remains.

Alternatively to the stochastically varying coefficients $\gamma_{0,t}$ and $\gamma_{y,t}$ in (5.11), we should also adopt the deterministic approach of a spline function. As indicated above, five segments seem a reasonable compromise between too much and too little variability (we have checked that it is). The result is the bold line in Fig. 5.14. The feature that it is much smoother than the random walk estimate will have been expected and conforms to what has been discussed in Sect. 5.2.3. It is also interesting to compare it with the atheoretical HP 102,400 trend of unemployment, which is represented by the dotted line (this is just the bold line in Fig. 5.11). Although the spline function estimation of the natural rate of unemployment contains more structure, the outcome comes fairly close to the HP trend.

To sum up, four conclusions emerge from this investigation: (*i*) Grant's estimation of the NRU is (presently) not convincing. (*ii*) The NRU time path of the random walk estimation with endogenous variances is too variable. (*iii*) The spline function estimation of the NRU appears reasonable, and is also quite in line with the more involved NAIRU estimates that one finds in the literature. (*iv*) The HP 102,400 trend does a good job, too.

5.4 Okun's Law as a Time-Varying Statistical Regularity

5.4.1 Different Specifications of the Relationship

The negative relationship between unemployment and output as it is summarized by Okun's law is usually specified in two different versions, where one is based on the levels of the two variables and the other on their rates of change. If we refer to the rate of employment rather than unemployment and generally allow for time variability in the relationship, the two versions read

$$e_t - e_t^\star = \beta_t \, (Y_t - Y_t^\star)/Y_t^\star \; + \; u_t, \tag{5.12}$$

$$e_t - e_{t-1} = \beta_t \, (gY_t - gY_t^\star) \; + \; u_t. \tag{5.13}$$

The letter e denotes the employment rate, Y is an output variable, gY its rate of growth, and u_t are the residuals in a regression (as in (5.1) above). The underlying time unit is a quarter and here and in the following all rates of change are annualized percentage numbers (thus, $gY_t = 400 \cdot (Y_t - Y_{t-1})/Y_{t-1}$). A star symbol indicates trend values. The coefficient β_t will be referred to as the "Okun coefficient", where however a word of caution has to be added since occasionally this expression is used for the reciprocal $\tilde{\beta}_t = 1/\beta_t$ in the reversed equation, such as $gY_t - gY_t^\star = \tilde{\beta}_t \, (e_t - e_{t-1}) + \tilde{u}_t$, for example.

In the present research context it is particularly interesting to relate our discussion to (selected parts) of two recent papers by Semmler and Zhang (2005) and Hemraj et al. (2006). There a third specification is investigated, namely (in our notation),

$$e_t - e_{t-1} = \beta_t \, (gY_t - gY_{t-1}) \; + \; u_t \tag{5.14}$$

(see Semmler and Zhang 2005, p. 4, fn 1). Although this relationship looks a lot like (5.13), it introduces an accelerationist element that is not present in (5.13). It is thus an open question whether (5.13) and (5.14) will lead to similar results. Unfortunately, the authors present their new specification without comment, so the reader is left with no hint as to what may have motivated this choice.

Let us begin with a constant Okun coefficient for the US economy. Using quarterly real GDP from the International Statistical Yearbook over the period 1961–2000, Semmler and Zhang (2005, p. 4) estimate (5.13) with a highly significant $\beta_t = \beta =$ const. and a constant trend rate of growth as[19]

$$\Delta e_t = 0.364 \cdot (gY_t - 3.3) + u_t, \qquad R^2 = 0.734. \tag{5.15}$$

The value for R^2 is surprisingly high given that a quarterly rate of change is regressed on just one explanatory variable. So we should first check this result with our quarterly data from the Fair-Parke software package for the firm sector.[20] In fact, on this basis we get for the same sample period a considerably lower goodness-of-fit, and also the (still highly significant) Okun coefficient is much smaller:

$$\Delta e_t = 0.232 \cdot (gY_t - 3.36) + u_t, \qquad R^2 = 0.394. \tag{5.16}$$

Despite these differences it is remarkable that the trend rates of growth in (5.15) and (5.16) are practically identical.

Taking the (at least theoretical) notion into account that causality runs from output to employment and that the adjustments of the latter may take place with some delay, we should try several lags of output growth on the right-hand side of (5.13). Indeed, including three lags of gY, all of the coefficients come out as distinctly significant and they decline with increasing lags even more nicely than could have been expected. The Okun coefficient furthermore rises from 0.232 to a more familiar order of magnitude like 0.424. On the other hand, the goodness-of-fit improves but still falls short of that in (5.15). In sum, the estimation yields

$$\Delta \tilde{e}_t = 0.424 \cdot (0.42 \cdot gY_t + 0.26 \cdot gY_{t-1} + 0.17 \cdot gY_{t-2} + 0.15 \cdot gY_{t-3} - 3.46), \qquad R^2 = 0.612.$$

(Note that the coefficients on the growth rates add up to unity.)

The accelerationist version of (5.14) proves to be an unsuitable alternative to (5.13). An estimation of (5.14) actually shows no relationship between Δe_t and ΔgY_t; we get an insignificant coefficient $\beta = 0.020$ and $R^2 = 0.004$ indicates the absence of any fit. A similar finding for their data might be the reason that for a constant coefficient estimation, Semmler and Zhang (2005)

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[19] Hemraj et al. (2006, Table 1 on p. 4)) want to begin their discussion with the same result but present the t-ratios instead of the β-coefficient.

[20] To be precise, Y is real output of the firm sector, whereas for lack of more detailed data e is 1 minus the economy-wide rate of unemployment.

prefer the specification given by (5.13), too. However, one wonders why (5.14) should then be a better basis for their subsequent estimation of time-varying coefficients. At least, a discussion of this point would have been helpful.

For the complementary level specification given by (5.12) of Okun's law we first need to decide on the trend. On the basis of the discussion in Sect. 5.3 and in order to have a uniform concept of detrending, we settle down on the Hodrick-Prescott procedure with the high smoothing parameter $\lambda = 102{,}400$. Thus, we define

$$e_t^{dev} := e_t - e_t^\star, \qquad e_t^\star \text{ the HP 102,400 trend of } e_t, \tag{5.17}$$

$$y_t := \ln Y_t - \ln Y_t^\star, \qquad \ln Y_t^\star \text{ the HP 102,400 trend of } \ln Y_t, \tag{5.18}$$

y_t is the output gap, which is also used in many other applications (except for our smoothing parameter), and e_t^{dev} may correspondingly be called the employment gap.

Because of the adjustment lags in employment already mentioned above and since it gives a better fit, we regress the employment gap not on the contemporaneous output gap but on one lag of y_t. In this way we obtain for the period 1961–2000,

$$e_t^{dev} = 0.407 \cdot y_{t-1} \; + \; u_t, \qquad R^2 = 0.840. \tag{5.19}$$

(The result is, however, not essentially different from the unlagged regression, which yields an Okun coefficient 0.396 with $R^2 = 0.786$.) The coefficient $\beta = 0.407$ is quite in line with the coefficient 0.424 in (5.17) and suggests that, when dealing with a growth rate specification of Okun's law, we should better include the three lags of the output growth rate. In any case, for the firm sector an Okun coefficient

$$\beta \approx 0.40 \tag{5.20}$$

is still a good summary to describe the response of employment to changes in output.

5.4.2 Time Variations in the Okun Coefficient

The Okun coefficient depends (in part) on how firms adjust the number of jobs in response to temporary deviations in output from "normal". This adjustment depends in turn on such factors as the international organization of firms and the legal and social restrictions on hiring and firing. It is well-known that therefore the coefficient is quite distinct across different countries; and in conformity with everyday economic intuition the United States, where the labor market institutions are relatively flexible, exhibit the largest coefficient among the major industrialized countries (see, e.g., Blanchard 2003, p. 185, and Semmler and Zhang 2005, p. 4).

Blanchard (and others, of course) furthermore argues that the coefficient is also likely to change over time. Increased competition in goods markets has led firms in most countries to reconsider and reduce their commitment to job security (in exchange for the workers' loyalty), and their pressure on the government has resulted in a weakening of the legal constraints on hiring and firing. From this one expects a larger response of employment to fluctuations in output and thus a larger value of β, and indeed this holds true in many Western countries.

For the United States, however, the picture is less clear. In an estimation of the constant coefficient version of (5.13) with annual rates of change, Blanchard (2003, p. 185) subdivides the forty years from 1960 to 2000 in two equally long samples and obtains for each subperiod the same coefficient $\beta = 0.39$ (in stark contrast to Germany, UK and Japan, where the coefficient for the second half is considerably higher). As shown in Table 5.2, a very similar outcome is obtained if the same procedure is applied to our quarterly data from the firm sector. Re-estimating (5.17) and (5.19) correspondingly, the coefficient is even lower in the second period 1981–2000, although very slightly so.

Besides, the simultaneously estimated trend rate of growth in (5.17) is $gY^\star = 3.91\%$ for 1961–1980 and $gY^\star = 3.00\%$ for 1981–2000, which confirms the confidence in these estimations.

We can extend the idea of comparing different subsamples by estimating the regression over consecutive intervals of time and plotting the values of the resulting Okun coefficients as a (presumably) rather smooth time series. For simplicity, let us consider the level version of Okun's law, the regression approach of (5.19) over a shorter period of time. Choose the length of the

Table 5.2. Estimations of the Okun coefficient β over two subperiods

	1961–2000	1961–1980	1981–2000
β from Blanchard	0.390	0.390	0.390
β from (5.17)	0.424	0.444	0.413
β from (5.19)	0.407	0.411	0.402

rolling sample period shorter than the 20 years of Table 5.2 but not too short; 10 years, say. We also exploit the full sample of data that we have available from the Fair-Parke package, whose final quarter is 2005:2. The outcome of this battery of regressions is shown in Fig. 5.15, where at time t the coefficient β is plotted that results from an estimation over the *past* 10 years.

Figure 5.15 gives a more pronounced picture than Table 5.2. The most conspicuous feature in the diagram is the decline (rather than an increase) of the coefficient from 1990 on. Note, however, that the coefficient at t = 1995:1, for example, does not capture a connection prevailing in this quarter, but summarizes the relationship between output and employment from 1985:1 until 1995:1.[21] Hence, the reasons for the decline of the coefficient in the 1990s have already to be sought in the 1980s. It may also be observed that though β is consistently falling over the 1990s, the reduction is not overly dramatic (the scale on the y-axis exaggerates the phenomenon a bit).

The advantage of the method of the rolling sample period is its simplicity; its meaning as well as its limitations are immediately clear. On the other hand, each $\beta = \beta_t$ is separately estimated from the others and only uses the information of the short sample. Therefore integrated methods are considered

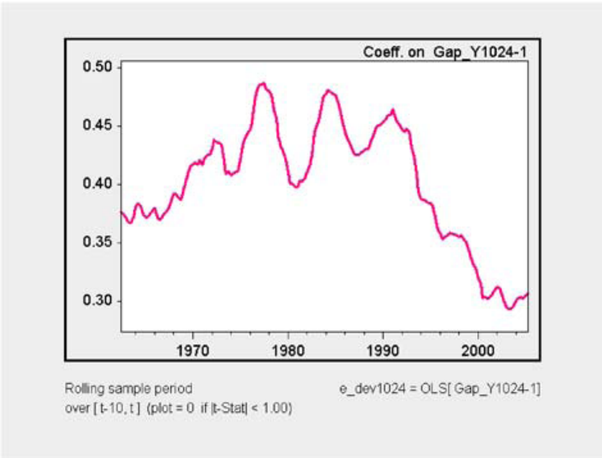

Fig. 5.15. Estimation of the level version of Okun's law with a rolling sample period of 10 years

[21] This could perhaps be emphasized by plotting the coefficient at the mid-point of the rolling sample interval, at t = 1990:1 in this case.

to be preferable, where the whole series of the β_t is estimated in a joint effort.[22] In any case, it is interesting to compare Fig. 5.15 to a readily available plot of a time-varying coefficient from the literature that is based on such an integrated approach. To this end we reproduce in Fig. 5.16 the panel for the U.S. data of Fig. 3 in Semmler and Zhang (2005, p. 7); which is one of four such panels for the U.S. and other countries.[23] This diagram plots the deviations of the time-varying β_t from the full sample estimate of a constant $\beta = \bar{\beta}$. The common feature of Figs. 5.15 and 5.16 is the decline of the Okun coefficient that sets in around 1990, and that the coefficient tends to increase in the decades before. The main difference is the relative size of the changes in the two periods before and after 1990. In Fig. 5.16 the increase of β_t in the first stage is roughly twice

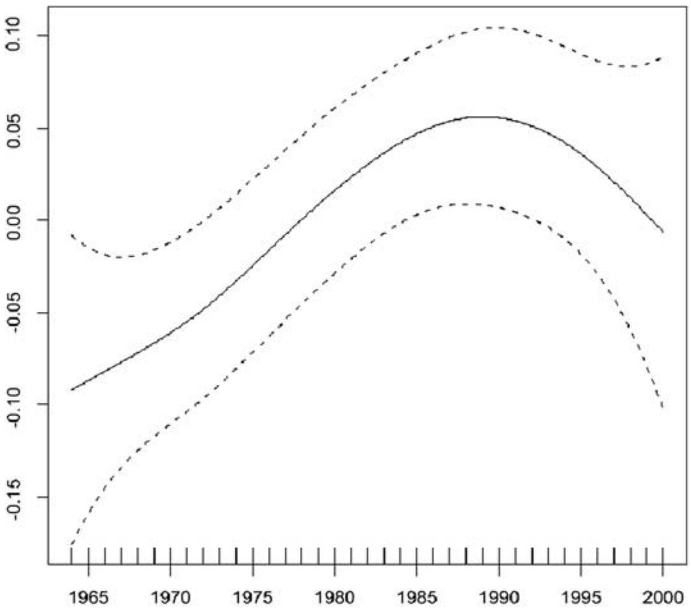

Fig. 5.16. The time-varying Okun coefficient from Semmler and Zhang (2005) (*Note*: Reproduced from Semmler and Zhang 2005, p. 7, Fig. 3. The y-axis gives the deviations of β_t from the estimate of a constant $\bar{\beta}$ over the full sample period. The dotted lines are confidence bounds)

[22] Although in our opinion this argument is not fully convincing. Part of the search for technical refinements may well be an end in itself (or convention or just an exhibition of technical skills).

[23] The (4-panel) diagram can also be found in Hemraj et al. (2006, p. 5).

as strong as the subsequent decline, whereas in Fig. 5.15 it is almost the other way round.

Apart from this asymmetry, it is remarkable that the changes of the Okun coefficient are of a similar order of magnitude. Unfortunately, Fig. 5.16 does not indicate the levels of the β_t (the original paper by Semmler and Zhang does not make them explicit, either). Moreover, the estimations of these coefficients are based on the accelerationist version (5.14) of Okun's law, where in lack of a more detailed presentation in Semmler and Zhang (2005) we have already expressed our doubts whether this specification can provide a sound basis. Nevertheless, Fig. 5.16 prompts us to estimate a time path of the Okun coefficient with the spline method described (and selected) in the methodological Sect. 5.2.2 above, using our firm sector data.[24] As before, we should try both the level and the first-difference specification of Okun's law. Beginning with the latter, which is closer to the accelerationist version underlying Fig. 5.16, we include the three lags of (5.17). Regarding the trend growth of output, the constant rate of 3.46% in (5.17) is replaced with a variable HP 102,400 trend $gY_t^\star$. Thus, we estimate

$$e_t - e_{t-1} = \sum_{k=0}^{3} \gamma_{k,t}\,(gY_{t-k} - gY_{t-k}^\star) \; + \; u_t \tag{5.21}$$

with the spline method. Our time-varying Okun coefficient β_t is then given by the sum of the coefficients on the single output growth rates,

$$\beta_t = \sum_{k=0}^{3} \gamma_{k,t} . \tag{5.22}$$

Using 3, 5, 7, and 10 segments over the sample period 1960:1–2004:4, the result is shown in Fig. 5.17. Generally the time patterns of the β_t are closer to Fig. 5.15 than to Fig. 5.16 from Semmler and Zhang. This even holds true for the least variable dashed line in Fig. 5.17 that is generated by the 3-segment spline version, where the decline of β_t begins much earlier and is proportionally much than in Fig. 5.16.

The more variable time paths in Fig. 5.17 arising from the 5-, 7-, and 10-segment splines share the property of a decline sometime in the second half of

[24] Semmler and Zhang make use of an estimation approach that is substantially different from all of the methods mentioned in Sect. 5.2. It is somewhat reminiscent of the general Hodrick-Prescott idea, where, however, the smoothing parameter is now endogenously determined; see Semmler and Zhang (2005, pp. 5f) for a rough sketch.

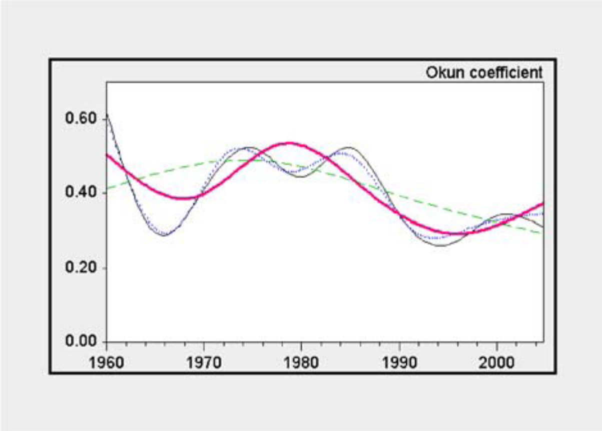

Fig. 5.17. The time-varying Okun coefficient from (5.21), (5.22) (*Note*: The estimations are based on the following number of segments for the spline functions: 3 (dashed line), 5 (bold), 7 (dotted), and 10 (thin solid line))

the sample period. Because of the different degrees of variability it begins at different dates, but it always sets in at least five years earlier than in Fig. 5.16. This finding calls into question the interpretation of Fig. 5.16 that is given in Hemraj et al. (2006, p. 5), according to which the diagram "shows that the response of unemployment to growth rates steadily moved down since the beginning of the 1990s. The U.S. case clearly shows a decline of the response of employment to economic growth—thus, a jobless recovery, as some have called it." The changes indicated by the Okun coefficient are apparently less unique than that.[25] Figure 5.18 checks the results of Fig. 5.17 by employing the level version (5.12) of Okun's law, where again β_t is determined by deterministic splines on the basis of 3, 5, 7 and 10 segments, respectively. This approach, too, yields falling values of the Okun coefficient, though the decline begins later than in Fig. 5.17. The diagram also reveals a feature that was only relatively weakly indicated in Fig. 5.17, namely, from the mid-1990s on the coefficient begins to rise again—in Fig. 5.18 to previous or even higher levels. As discussed in the methodological section, if asked for a final decision regarding the number of the underlying segments for the splines, we would settle down on the 5-segment versions that are plotted as the bold lines in

[25] We do not wish to deny that the recovery after the 1991 recession was of a jobless nature. But this has to be shown by more detailed methods—and the authors in fact do this on the subsequent pages.

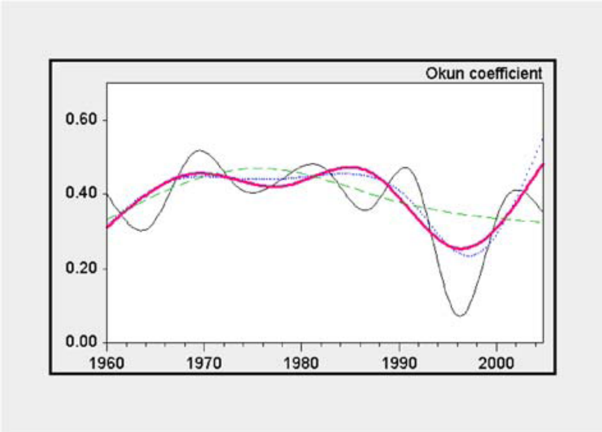

Fig. 5.18. The time-varying Okun coefficient from (5.12) (*Note*: The estimations are based on the following number of segments for the spline functions: 3 (dashed line), 5 (bold), 7 (dotted), and 10 (thin solid line))

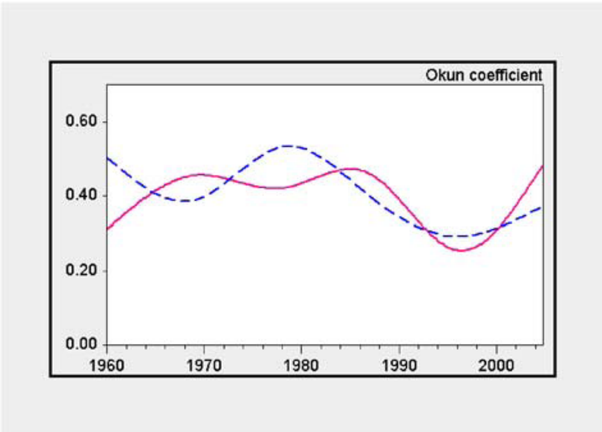

Fig. 5.19. Time-varying coefficients β_t from the level (solid line) and first-difference (dashed line) specification of Okun's law (*Note*: The diagram reproduces the bold lines of the 5-segment estimations from Figs. 5.17 and 5.18)

Figs. 5.17 and 5.18. For a better comparison, they are reproduced together in an extra time series diagram. Figure 5.19 shows that the coefficients arising from the two approaches of levels and first differences to Okun's law vary over a similar range. Also their troughs in the 1990s are not too widely apart. The earlier variations, however, are "out of phase". This indicates that these motions should not be overinterpreted, or the interpretation must take the specific version into account to which it refers. If we do not go into these details, the differences in the oscillation of β_t can be reconciled by concluding that until the early 1980s the Okun coefficient did not vary too systematically. By contrast, the subsequent decline of the coefficient with a more or less strong "recovery" appears to be a more universal phenomenon.

5.4.3 Comovements of the Components of Output and Employment

Interpretations of changes of the Okun coefficient in the literature, or at least their formulations, tend to identify employment with the employment rate, thus speaking of the "response of employment" to the variations in output or output growth. This habit neglects that the employment rate is a composite variable, or it assumes that there are no significant changes in the evolution of the other variable(s) involved. In this subsection we want to shed some light on this issue, where we go beyond the simple definition of the employment rate as a ratio of labor demand and supply. The light will perhaps not be very bright but it will still suffice to make it clear that an explanation of the changes in the Okun coefficient over the last 10 or 15 years must take more variables into account than e and Y.

To this end we decompose total output into labor productivity, hours per job, the employment rate, and the labor force. We define:[26]

E employment, i.e. number of jobs
H total hours worked per quarter
L the labor force (number of heads)

e employment rate, $e = E/L$
h hours per job, $h = H/E$
z output per hours (labor productivity), $z = Y/H$

[26] The empirical time series entering here are readily available from the Fair-Parke database. E is the series they call JF (number of jobs in the firm sector, in mill.); hours H are obtained from their series HF (number of hours paid per job in the firm sector, per quarter), thus $H = \text{HF} \times \text{JF}$; the ratio E/L for the firm sector is identified with the economy-wide employment rate, which is $1 - \text{UR}$ (UR the unemployment rate); consequently, the somewhat artificial variable labor force 'in the firm sector' is constructed by dividing the number of jobs by the employment rate, $L = \text{JF}/(1 - \text{UR})$. Incidentally, Fair-Parke's own series PROD for labor productivity in fact coincides with the series Y/H.

Similarly as in Gordon (2003, p. 212), for example, we consider the output identity,[27]

$$Y = \frac{Y}{H} \cdot \frac{H}{E} \cdot \frac{E}{L} \cdot L = z \cdot h \cdot e \cdot L. \tag{5.23}$$

In studying phenomena like the aforementioned jobless growth, the increase of trend productivity growth over the past 15 years has certainly a role to play. In this chapter, however, we limit ourselves to changes in the relationship between output and employment that abstract from this kind of structural change. That is, we are looking for possible changes in factors that are more directly related to a business cycle frequency. Accordingly, we begin by first detrending the variables. As before, the trend is uniformly given by the HP 102,400 filter, from which we define for an arbitrary dynamic variable $x = x_t$ and its trend $x_t^\star$,

$$x_t^{dev} = (x_t - x_t^\star)/x_t^\star. \tag{5.24}$$

Although somewhat clumsy, we will maintain the superscript '*dev*' to avoid any confusion on that. By logarithmic differentiation of (5.23) we then see that the output gap y_t is composed of the following sum of percentage deviations,[28]

$$y_t = z_t^{dev} + h_t^{dev} + e_t^{dev} + L_t^{dev}. \tag{5.25}$$

This output gap decomposition can also be solved for the employment gap, which gives

$$e_t^{dev} = y_t - z_t^{dev} - h_t^{dev} - L_t^{dev}. \tag{5.26}$$

Comparing this identity to (5.12), it is seen that the regularity of Okun's law, with a stable coefficient β, is dependent on labor productivity and hours per

[27] Gordon (2003) works with a more detailed decomposition and relates output, not to the labor force L, but to the working-age population N, so that L in (5.23) becomes $(L/N) \cdot L$ and he can also consider the labor force participation rate. Apart from that, his output measure is GDP, whereas quarterly productivity data are only available for the firm sector. In this way he additionally introduces (i) a so-called 'mix effect', which he defines as the ratio of output per payroll employee in the total economy to that in the firm sector; and (ii) the ratio of total employment in the payroll survey to that in the household survey (cf. Gordon 2003, p. 212). For our purposes we can neglect this detailed differentiation and soak up all these effects in the labor force variable L.

[28] To be exact, e_t^{dev} as defined by (5.24) now differs from the definition as a pure difference $e_t - e_t^\star$ in (5.17). Of course, the numerical differences are only minor.

job moving in a procyclical fashion (and a smaller amplitude than the output gap itself), while the cyclical variations of the labor force are preferably small or at least unsystematic. We are now going to check if, or how far, this view is (still) warranted.

The dotted in each of the six panels in Fig. 5.20 displays the output gap y_t as the measure of the business cycle. It is contrasted with the four components of y_t in (5.25). The two panels at the bottom add the trend deviations of jobs E_t^{dev} and hours H_t^{dev}. The latter two variables are in fact strongly procyclical, where hours move roughly one-to-one with output and perhaps somewhat surprisingly, there are longer spells of time where also the amplitude of employment is not much smaller than that of output. We can conclude from the latter that the main reason for a coefficient β_t in a relationship like $e_t^{dev} = \beta\, y_t + u_t$ to be around 0.40 derives from the fact that the labor force is not growing at a nearly constant rate but shows some variation around its trend path, too. In fact, given that E_t and Y_t move relatively closely together so that for illustrative purposes one may write $E_t^{dev} \approx \gamma\, Y_t^{dev} = \gamma\, y_t$, where γ is not much less than unity, the equation

$$e_t^{dev} = E_t^{dev} \; - \; L_t^{dev} \quad \approx \quad \gamma\, y_t - \; L_t^{dev} \tag{5.27}$$

demonstrates that $\beta_t \approx 0.40$ and the more persistent variations of this coefficient are mainly due to the behavior of the supply variable, i.e., the fluctua-

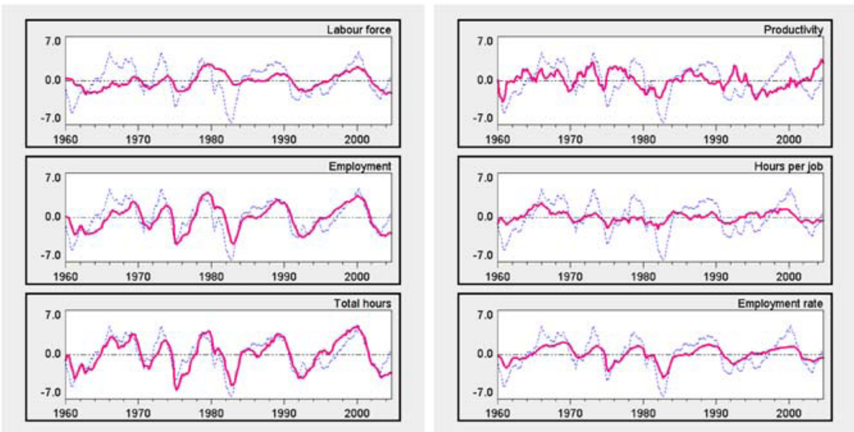

Fig. 5.20. Percentage trend deviations z_t^{dev}, h_t^{dev}, e_t^{dev}, L_t^{dev}, E_t^{dev}, H_t^{dev} (the dotted line is the output gap y_t)

tions of the labor force. However, L_t^{dev} in the fourth panel of Fig. 5.20 displays no consistent cyclical pattern.

Regarding the labor demand variable it is nonetheless remarkable that while most of the sample period employment moves synchronously with output or lags slightly behind, we observe a certain lead of employment in the mod-1990s. This not only holds true for the number of jobs but also for total hours.

The employment rate itself appears to maintain roughly the same pattern over the whole 45 years of the sample period. In this respect, hours per job (h_t^{dev}) do not seem to be a very dramatic variable, either. In contrast, the cyclical pattern of labor productivity $z_t^{dev} = Y_t^{dev} - H_t^{dev} = y_t - H_t^{dev}$ in the top panel of Fig. 5.20 undergoes a severe change. It is largely procyclical, with a certain lead, until the mid-1980s. But from the end of the 1980s on, this type of behavior has completely disappeared!

The summary statistics for the amplitudes and comovements of the variables in Table 5.3 make these qualitative observations on possible "regime shifts" more precise. The table subdivides the 45 years from 1960 to 2004 into the two periods 1960:1–1983:4 and 1984:1–2004:4; the corresponding figures are given in the first two rows for each variable e_t^{dev}, E_t^{dev}, etc. In addition, special emphasis is put on the 1990s, for which purpose the third row adds the results for the fifteen years 1990:1–2004:4. We point out the following features arising from this investigation.

1. The cyclical behavior of the employment rate does not show great changes over the entire sample period. On average, the employment rate lags one or two quarters behind the output gap, while the amplitude of its variations has slightly decreased (rather than increased). The latter information is given in the second column, which computes the standard deviation of the variables and expresses it as a fraction of the standard deviation of the output gap.
2. Total employment lags one or two quarters behind output. There is a weak indication that the delay in the employment adjustments may have somewhat shortened in the last twenty years, but the evidence with this quarterly data is not yet very conclusive. Much stronger, however, is the evidence of an increasing amplitude in the fluctuations of the number of jobs, from 3:4 in relation to output to almost exactly 1:1 (cf. the bold face figure in the second column).
3. Total hours exhibited a one-quarter lag in former times but now move synchronously with output. Even more importantly, a substantial "over-

Table 5.3. Comovements with the output gap

Series x	σ_x/σ_y	Cross correlations between y at time t and x at time						
		$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$
	0.49	0.47	0.62	0.78	0.90	0.94	0.91	0.83
e^{dev}	0.42	0.63	0.72	0.78	0.84	0.86	0.84	0.77
	0.42	0.61	0.71	0.79	0.85	0.86	0.82	0.73
	0.75	0.13	0.32	0.52	0.69	0.78	**0.82**	0.79
E^{dev}	0.95	0.62	0.71	0.78	0.84	0.86	0.86	0.81
	1.02	0.64	0.72	0.80	0.85	**0.87**	0.85	0.79
	0.88	0.35	0.53	0.70	0.84	**0.89**	0.87	0.80
H^{dev}	1.19	0.69	0.76	0.81	0.83	0.83	0.80	0.73
	1.29	0.72	0.78	0.83	**0.85**	0.84	0.79	0.71
	0.54	**0.67**	0.65	0.58	0.48	0.28	0.09	−0.07
z^{dev}	0.66	−0.03	−0.01	0.01	0.02	−0.05	−0.12	−0.16
	0.70	**−0.32**	−0.24	−0.18	−0.14	−0.21	−0.28	−0.31
	0.33	0.65	0.68	**0.70**	0.69	0.59	0.46	0.32
h^{dev}	0.33	0.68	0.69	0.66	0.61	0.53	0.44	0.32
	0.36	**0.78**	0.77	0.73	0.66	0.57	0.46	0.34
	0.45	−0.30	−0.15	0.00	0.16	0.28	0.37	**0.42**
L^{dev}	0.55	0.59	0.66	0.74	0.78	0.81	0.82	0.80
	0.62	0.63	0.70	0.78	0.81	0.84	**0.85**	0.81

Note: The first row for each variable is based on the subperiod 1960:1–1983:4, the second on 1984:1–2004:4, the third on 1990:1–2004:4. σ denotes the standard deviation

reaction" of hours has in the meantime developed. While until the 1980s hours varied somewhat weaker than output, in a proportion of roughly 9:10, this relationship has reversed and on average, from the 1990s on, a one percent change in output leads to a 1.3 percent change in hours worked.[29]

4. Obviously, the latter feature must affect the cyclical properties of labor productivity $z = Y/H$. Actually, until the 1980s z_t^{dev} was (perhaps not strongly but) distinctly procyclical, with a lead of three quarters, whereas over the last fifteen years productivity moves in a weak but rather counter-

[29] In a next step it may be interesting to study this relationship separately for booms and recessions.

cyclical fashion. In contrast to the changing features of total employment and hours, this phenomenon is more clearly visible in the top panel of Fig. 5.20.

5. The utilization of the workforce as represented by hours per worker has not changed very much. It is nevertheless notable that the previously contemporaneous movements now exhibit a slight lead of one or two quarters and that the connection with output has become somewhat tighter.
6. The labor force as the supply variable has substantially changed its cyclical characteristics. It was weakly procyclical with a one-year lag and a relative amplitude of 0.45 until the 1980s.[30] Over the last fifteen years the amplitude has increased to 0.62. Also, the labor force now follows economic activity much more closely, as indicated by the cross correlation coefficient of 0.85 at a lag of two quarters. Besides, the connection is not much weaker for the contemporaneous movements.

To return to Okun's law, we simplify the comovements of employment and the labor force with output and posit $E_t^{dev} \approx \gamma_E \, y_t$, $L_t^{dev} \approx \gamma_L \, y_t$. This allows us to relate the employment rate to the output gap as

$$e_t^{dev} \;=\; E_t^{dev} - L_t^{dev} \approx (\gamma_E - \gamma_L)\, y_t \;=:\; \beta\, y_t \tag{5.28}$$

which is of the same form as (5.12) above. The coefficient γ_E may be approximated by the ratio of the standard deviations σ_E/σ_y multiplied by the contemporaneous cross correlation coefficient between E_t^{dev} and y_t; and analogously for the coefficient γ_L on the labor force. For the first and third sample period considered in Table 5.3 we then obtain:

$$\begin{aligned} &\text{1960:1–1983:4} \;: \quad \beta = \gamma_E - \gamma_L = 0.75 \cdot 0.69 - 0.45 \cdot 0.16 = 0.45 \\ &\text{1990:1–2004:4} \;: \quad \beta = \gamma_E - \gamma_L = 1.02 \cdot 0.85 - 0.62 \cdot 0.81 = 0.36 \end{aligned}$$

Hence the proxy for the Okun coefficient from this back-of-the-envelope calculation does not only decline, the numbers are also of a similar order of magnitude to that in Fig. 5.18 for the time-varying level specification of Okun's law.

In this way we can see two different mechanisms acting on β. First the difference of the amplitudes of E_t^{dev} and L_t^{dev}. The amplitudes both increase, and since the amplitude of E_t^{dev} rises more than that of L_t^{dev}, we have a positive

[30] The cross correlation of L_{t-k}^{dev} with y_t is 0.44 for a lag $k = 4$ over this subperiod, which is slightly higher than the coefficient 0.42 for a three-quarter lag in Table 5.3.

influence on β. The second mechanism originates with the contemporaneous cross correlations of E_t^{dev} and L_t^{dev} with output, both of which increase, too. Here, however, the change for L_t^{dev} is much stronger than for E_t^{dev}, so that on the whole the negative influence from the labor force becomes dominant. This stylized explanation underlines the significance of the labor supply variable and identifies the changes in its cyclical behavior as the most important contribution to the recent variation in the Okun coefficient.

In other words, if one wants to study macroeconomic changes in the employment policy of firms over the business cycle, one should better directly refer to the volume of employment rather than to the rate of employment as a composite variable, even if Okun's law has an honorable tradition. The brief summary in point 2 above is here a first pertinent observation.

5.5 A Model of a Simple Recruitment Policy of Firms

This short section is devoted to a simplified determination of hours and employment that would allow an easy integration into a low-dimensional macrodynamic framework. Concerning hours it completely abstracts from cyclical variations of labor productivity. By contrast, as regards the number of jobs, it proposes an active recruitment policy of firms in a straightforward manner, which is based on the following points.

1. The adjustments of employment to the changes in production are not instantaneous but take place in a gradual manner.
2. Firms hire additional workers (above normal growth) if the workforce is currently overutilized, they (relatively) decrease the number of jobs if the average employee works less than normal.
3. Firms pay attention to the situation on the labor market, in the sense that they are willing to operate at higher utilization rates of the workforce, without creating additional jobs, if the labor market tightens, i.e. if the employment rate has risen.

We repeat the notation in order to be self-contained. Thus, to specify the ideas, let H denote total hours, E the number of jobs, or employment, $h = H/E$ the average hours per job, and h^n the normal hours per job. The utilization of the workforce within firms is given by $u_w = h/h^n$, while with L being the labor force, the (outside) employment rate is $e = E/L$. We assume the existence of a so-called natural rate of employment, e^o, whose foundations are not explained within the model and which directly serves as a benchmark in several of the behavioral functions below. Potential output Y^p derives from the current labor force, corrected for the natural employment rate, labor supply and an exogenously given level z of labor productivity (in hours), $Y^p = e^o\, z\, (h^n L)$. Actual inputs of hours are supposed to be governed by solely technological factors, so that actual output and hours are linked by the same number, $Y/H = z$; $H = Y/z$ might be conceived as a short-period production function.[31]

By virtue of the latter assumption, the utilization of the workforce can be simply expressed as the ratio of capacity utilization and the employment rate. Formally, with $y = Y/Y^p$ for capacity utilization we get $u_w = h/h^n = H/h^n E = (H/Y) \cdot (Y/Y^p) \cdot (Y^p/h^n L) \cdot (L/E) = (1/z) \cdot y \cdot e^o z \cdot (1/e)$,

$$u_w = y\, e^o \,/\, e. \tag{5.29}$$

The utilization of the workforce may play a role in a wage Phillips curve that takes outsider as well as insider effects into account, so that the nominal wage changes depend (positively) on both the employment rate e and the utilization of the workforce u_w. Equation (5.29) points out that then, *ceteris paribus*, a wage increase from a rising employment rate is mitigated by the simultaneous reduction in the average hours worked per job. On the other hand, if a positive demand shock is not sufficient for firms to raise employment (relative to the growing labor force), there is nevertheless still a certain pressure on wages through the insider effect, which in (5.29) shows up directly as a rise in capacity utilization.

While the changes in capacity utilization would be the subject of another part in a full macro model, the adjustments in employment can here be specified in accordance with the features 1–3 listed above. With respect to a benchmark level of the utilization of the workforce, $\tilde{u}_w$, and the growth rate of the labor force as a benchmark for the normal growth of jobs, the first two points can, in continuous time, be described by an equation like $\hat{E} = \hat{L} + \beta_e\,(u_w - \tilde{u}_w)$ ($\beta_e > 0$). The third points suggests making the utilization benchmark an increasing function of the employment rate, $\tilde{u}_w = \tilde{u}_w(e)$ with $d\tilde{u}_w/de \geq 0$. Referring to the changes $\hat{e} = \hat{E} - \hat{L}$ of the employment rate, the recruitment policy of firms is summarized by $\hat{e} = \beta_e\,[\, u_w - \tilde{u}_w(e)\,]$.

[31] This productivity may be conceived of as growing at some constant rate, but since z cancels out in the following discussion and other parts of a full macro model may be constructed likewise, the precise assumption on the evolution of z does not matter.

While as a first idea this equation may appear a plausible approach to delayed adjustments of employment, it has to be checked that it is (broadly) compatible with the (quarterly) data. To this end let $\tilde{u}_w$ be a linear function of the employment rate, which with respect to the steady state employment rate e^o can be written as $\tilde{u}_w(e) = 1 + \gamma_o + \gamma_e(e - e^o)$. For the estimation, $h^n = h^n_t$ entering the definition of $u_{w,t}$ and $e^o = e^o_t$ are obtained as a Hodrick-Prescott trend (with a large smoothing parameter $\lambda = 102{,}400$). The corresponding regression is then given by

$$4\,\Delta e_t = \alpha_o \;+\; \alpha_u\,(u_{w,t} - 1) \;+\; \alpha_e\,(e_t - e^o_t) \;+\; \eta_t, \tag{5.30}$$

$$\eta_t = \rho\,\eta_{t-1} \;+\; \varepsilon_t. \tag{5.31}$$

Neglecting the (in absolute terms) low variability in e_t and multiplying the continuous-time equation for $\hat{e}$ by the model's steady state value e^o, which is a constant, the coefficients are related by the equations $\alpha_u = \beta_e\, e^o$, $\alpha_o = -\beta_e\,\gamma_o\, e^o$ and $\alpha_e = -\beta_e\,\gamma_e\, e^o$. Accordingly, the coefficient α_u should come out positive and significant, while α_e if it turns out significant should be negative. The autoregressive error terms in (5.30) are a short-cut to capture the other effects in the employment adjustments.

In all estimations that we performed the constant γ_o proved to be insignificant and the fit was hardly affected when it was excluded. The goodness-of-fit is, however, heavily dependent on including the AR(1) error process; the constraint $\rho = 0$ reduces R^2 to an order of magnitude as low as 0.23 (at most), with a Durbin-Watson statistic less than one.

Apart from the inconsistency problems from residuals that may be correlated with some of the regressors, an OLS estimation does not prove to be very attractive since it yields an undesired positive estimate of α_e. The correct negative sign is obtained if 2SLS or GMM are employed, where with all sets of instrumental variables that we explored, the GMM fits were clearly superior. As instrumental variables we considered Δe_{t-1} and several lags of $(u_{w,t} - 1)$ and $(e_t - e^o_t)$. It seems that satisfactory fits require up to eight lags of the latter two variables. For $(u_{w,t} - 1)$ we also included the contemporaneous values. As further variations of the lags did not lead to noteworthy improvements, we may settle down on the estimation reported in Table 5.4.

From the J-statistic, the null hypothesis that the overidentifying restrictions regarding the instrumental variables is satisfied can be inferred to be ac-

Table 5.4. GMM estimation of (2) and (3) over 1961:1–2003:1

$u_{w,t} - 1$	$e_t - e_t^o$	ρ	R^2	SER	DW	J
0.85	−0.58	0.56	0.40	1.00	1.93	0.078
(4.31)	(−2.83)	(10.9)				

Note: t-values in parentheses, and standard errors of ε_t in percentage points; for the instrumental variables see text

ceptable.[32] Although (5.30) and (5.31) do not purport to represent the "true" data generation process, the results of Table 5.4 appear sufficiently credible to be employed in a small macrodynamic model.

The structural coefficients of the employment module are recovered from the above relationships between the α, β and γ coefficients. Assuming a steady state employment rate $e^o = 0.94$, they result as $\beta_e = \alpha_u/e^o = 0.90$ and $\gamma_e = -\alpha_e/\beta_e e^o = -\alpha_e/\alpha_u = 0.65$ (slightly rounded). In sum, the simple recruitment policy here proposed and numerically specified, is described by the two equations:

$$\hat{e} = \beta_e[u_w - \tilde{u}_w(e)], \qquad \beta_e = 0.90, \tag{5.32}$$

$$\tilde{u}_w(e) = 1 + \gamma_e(e - e^o), \qquad \gamma_e = 0.65. \tag{5.33}$$

5.6 Gradual Adjustments of Hours and Employment

In this section, we put forward a model where firms gradually adjust employment as well as hours in response to certain gaps that constitute a disequilibrium for them. While the model does not make any direct reference to Okun's law, it will be seen that the first-differences version of Okun's law emerges as a special case when several reaction coefficients are set to zero.

5.6.1 Theoretical Framework

Preliminaries

For the following theoretical discussion a number of new variables are introduced. They are combined with other variables already known, but for

[32] Under the null with $\ell - k = (1 + 9 + 8) - 3$ overidentifying restrictions, the J-statistic times the number of observations (169) is asymptotically χ^2-distributed. The resulting p-value is 0.59.

convenience we list the new symbols together with the old ones that are here relevant.

E	employment, i.e., number of jobs (the workforce);
E^d	desired number of workers corresponding to H^d;
e	employment rate; $e = E/L$;
e^o	normal, or 'natural', rate of employment;
g_y^e	rate at which firms expect their output to grow (over the next one or two years, the medium-term);
H	total hours worked;
H^d	desired hours by firms to produce current output;
h	average hours per job, $h = H/E$;
h^n	normal hours per job, which change at the growth rate $g_h = \hat{h}^n$;
L	labor force, which grows at the rate $g_\ell = \hat{L}$;
u	utilization with respect to potential output, $u = Y/Y^p = 1 + y$;
u_w	utilization of the workforce;
Y^p	potential output;
y	the output gap, $y = (Y - Y^p)/Y^p$;
z	actual labor productivity; $z = Y/H$;
z^o	productivity under "normal conditions" (the trend, representing the state of technology), which grows at rate $g_z = \hat{z}^o$;
ζ	ratio of actual to trend productivity, $\zeta = z/z^o$.

The growth rates g_z and g_ℓ, the normal working time h^n together with its (negative) growth rate g_h, and the normal rate of employment e^o are considered to be exogenous. While in a full-fledged theoretical model they may be supposed to be constant, it here suffices to assume that they are predetermined in the short period, or at the beginning of a quarter.[33] In the empirical work, the trend values of these magnitudes are adopted (again derived from the HP 102,400 trend). Trend and steady state values are designated by a superscript "o".[34]

In the formal analysis below we have recourse to the components of potential output and its growth rate. For the exposition of the model it is useful to

[33] For example, considering the moderate comovements of the output gap and the lagged labor force, the growth rate of the latter might be specified as an endogenous variable that responds (weakly) to the recent output growth. This idea could be conveniently represented by a so-called adaptive expectations mechanism.

[34] In the description of the theoretical model we refer to the steady state rather than the trend values.

specify the rates of change in continuous time. Substituting the steady state values in the output decomposition (5.23) and neglecting possible changes in the normal rate of employment, we obtain

$$Y^p = e^o\, z^o\, h^n\, L, \tag{5.34}$$

$$\hat{Y}^p = g^o \ := \ g_z + g_h + g_\ell. \tag{5.35}$$

Desired hours H^d by firms to produce their current output correspond to production under normal conditions, which is a state where labor productivity attains its normal level z^o. E^d is the corresponding volume of employment, when workers in that state work normal time. Hence,

$$H^d = Y \,/\, z^o, \tag{5.36}$$

$$E^d = H^d \,/\, h^n. \tag{5.37}$$

The model contains two utilization variables. One refers to output and potential output and represents utilization by $u = Y/Y^p$, which takes the role of the output gap y previously considered in this chapter ($u = 1 + y$, of course). The other concept refers to the 'stock' of workers currently employed. Here the utilization of the workforce relates actual hours H to the hours that the employed workers would normally work, which are given by $h^n E$. Accordingly,

$$u_w = H \,/\, (h^n E) \ = \ (H/E) \,/\, h^n \ = \ h \,/\, h^n. \tag{5.38}$$

The Two Adjustment Equations for Employment and Hours

The modelling of employment and hours is based on the assumption that the production decisions are made first. The determinants of the latter are not discussed within the present framework, so the time paths of output Y, utilization u, and in general also the expected output growth rate g_y^e are treated as exogenous. The two other control variables of the firm (besides prices and investment, which are here completely left aside) are (the flow of) hours H and (the stock of) workers E presently employed. Because of (material and immaterial) adjustment costs, which are not made explicit, E and H are mostly different from their normal or (in some sense) optimal levels.

The number of jobs as well as the total number of hours worked are predetermined in the short period. They adjust gradually over time in response to the disequilibria that the firms perceive, and in order to match up with future growth. Formally, this means that we specify the growth rates of employment and hours ($\hat{E}$ and $\hat{H}$) as functions of several benchmark and gap expressions. In detail, the following components are distinguished, which for simplicity all enter in a linear way.

1. Firms increase both employment and hours to account for the general growth trend. Regarding hours, the growth trend is given by the difference between the steady state output growth rate (g^o) and the rate of technological progress (g_z); regarding employment, the growth rate of normal hours per job (g_h) has to be subtracted in addition.[35] All rates are assumed to be known by the firms.[36]
2. The adjustments may exhibit certain inertia, such that the changes occurring in the previous quarter find some reflection in the present quarter, too. This is most conveniently specified by including the respective growth rates of the previous quarter in the terms determining $\hat{H}$ and $\hat{E}$. To be consistent, the growth terms from this and the preceding point enter as weighted averages with weighting factors for the lagged growth rates β_{hg} and β_{heg}, respectively.
3. Firms seek to gradually bridge the gap between desired (H^d) and actual hours (H), which they do with adjustment speed β_{hh}. Likewise, they seek to close the gap between the number of desired (E^d) and actual jobs (E) with adjustment speed β_{ee}.[37] The gaps are specified as percentage deviations from the current levels, $(X^d - X)/X$ for $X = H, E$, and as before, a one-quarter lag is assumed.
4. Firms increase (decrease) both employment and hours if they expect their output to grow faster (more slowly) than the trend, i.e., if g_y^e exceeds (falls

[35] Consider the identities $H = Y\cdot(H/Y) = Y/z$ and $E = Y\cdot(H/Y)\cdot(E/H) = Y/(zh)$ for the steady state values and subject them to logarithmic differentiation.

[36] Generally, firms may have subjective perceptions of what a proper trend growth rate might be, which they cautiously revise in the light of recent observations. Again these adjustments could be conveniently modelled as a (formally) adaptive expectations mechanism with a (very) slow speed of adjustment. The main dynamic properties of a fully formalized model should remain largely unaffected by this device, which may justify the short-cut (which, after all, is a universal type of simplification in macroeconomic modelling).

[37] One might argue that employment could also respond to the gap in hours, or that this additional possibility should be empirically tested. The two gaps in employment and hours are, however, strongly correlated, so that a distinct influence of the two variables cannot be properly identified.

short) of g^o; the corresponding speeds of adjustment are β_{ey} and β_{hy}, respectively.

5. Firms also increase (decrease) the number of jobs if their workforce works on average more (less) than the normal working time, i.e., if hours H exceed (fall short) of $h^n E$; the adjustment speed is designated β_{eh}.

Referring to discrete-time adjustments in a quarterly model, the (annualized) growth rates of hours H and employment E are thus determined by the following equations:

$$\hat{H} = \beta_{hg}\hat{H}_{-1} + (1-\beta_{hg})(g^o - g_z) + \beta_{hh}\Big(\frac{H^d - H}{H}\Big)_{-1} + \beta_{hy}(g_y^e - g^o), \qquad (5.39)$$

$$\begin{aligned} \hat{E} &= \beta_{eg}\hat{E}_{-1} + (1-\beta_{eg})(g^o - g_z - g_h) + \beta_{ee}\Big(\frac{E^d - E}{E}\Big)_{-1} \\ &\quad + \beta_{ey}(g_y^e - g^o) + \beta_{eh}\frac{H - h^n E}{h^n E}. \end{aligned} \qquad (5.40)$$

The equations are also the basis for the estimations below.

The gap between desired and actual hours does not seem to be a familiar variable. It is, however, nothing else than the trend deviations of labor productivity, since $H^d/H = (Y/z^o)\,/\,(Y/z) = z/z^o$. Hence,

$$\frac{H^d - H}{H} = \frac{z - z^o}{z^o} \;=\; \zeta - 1. \qquad (5.41)$$

The gap terms with H^d and E^d can also be characterized the other way round; $(H - H^d)$ and $(E - E^d)$ are the amount of excess hours currently worked, and the amount of excess labor currently on hand, which firms seek to reduce in a gradual manner.

The Adjustments in Continuous Time and Intensive Form

A straightforward way to translate (5.39) and (5.40) into a continuous-time formulation is to interpret the growth rates on the left-hand and right-hand sides as $\hat{H}_t$ and $\hat{H}_{t-\Delta t}$, respectively, which are based on a fixed time unit ($\hat{H}_t = (H_t - H_{t-\Delta t})/(\Delta t\, H)$) and Δt is the length of the adjustment period (the same applies to the growth rates of employment, of course). Regarding the weights, two polar cases are conceivable. First, we may assume that β_{hg} is multiplied by the length of the adjustment period, so that

$$\hat{H}_t = \Delta t \cdot \beta_{hg}\, \hat{H}_{t-\Delta t} \;+\; (1 - \Delta t \cdot \beta_{hg})\,(g^o - g_z) \;+\; \text{rest.}$$

Letting Δt shrink to zero, the inertia dissolve completely and we obtain

$$\hat{H} = (g^o - g_z) + \beta_{hh}\frac{H^d - H}{H} + \beta_{hy}(g_y^e - g^o), \tag{5.42}$$

$$\hat{E} = (g^o - g_z - g_h) + \beta_{ee}\frac{E^d - E}{E} + \beta_{ey}(g_y^e - g^o) + \beta_{eh}\frac{H - h^n E}{h^n E}. \tag{5.43}$$

On the other hand, the weights β_{hg} and β_{eg} can be supposed to remain unaffected by the length of the adjustment period, that is,

$$\hat{H}_t - \beta_{hg}\,\hat{H}_{t-\Delta t} = (1 - \beta_{hg})\,(g^o - g_z) \;+\; \text{rest}$$

from (5.39). If here Δt tends to zero, we have $\hat{H}_t - \beta_{hg}\,\hat{H}_{t-\Delta t} \;\longrightarrow\; (1-\beta_{hg})\hat{H}_t$. The same kind of equations as (5.42) and (5.43) are obtained, except that the original coefficients β_{hh}, etc., are divided by $(1 - \beta_{hg})$ and $(1 - \beta_{eg})$, respectively. Of course, this presupposes that the estimates of β_{hg} and β_{eg} are less than unity, and do not come close to it, either. On the whole, (5.42) and (5.43) are an appropriate continuous-time counterpart of the discrete-time specification given by (5.39) and (5.40).

For a model analysis it is necessary to set up an intensive form of this building block, such that the state variables could remain constant over time in a state of long-run equilibrium. The intensive-form variables corresponding to H and E are the utilization of the workforce, u_w, and the employment rate, e. They, too, are determined in a dynamic way, i.e., they are predetermined in the short period, and in continuous time their changes over time are governed by differential equations that are not too difficult to derive. Besides, of course, the exogenous time path of utilization u, also labor productivity will enter these relationships. This variable is, however, statically endogenous, i.e., the ratio z/z^o can be expressed as a function of the three variables u, e and u_w.

To establish this relationship, rewrite the workforce utilization as $u_w = H/h^n E = (Y/z)/h^n E = (Y/Y^p)\,(Y^p/h^n L)\,(L/E)\,(1/z) = u\,(e^o z^o)\,(1/e)\,(1/z)$; the last equality sign is based on (5.34). Solving for $\zeta = z/z^o$ gives

$$\zeta \;=\; z/z^o = \frac{e^o\,u}{e\,u_w} \;=\; \frac{u\,/\,u_w}{e\,/\,e^o}. \tag{5.44}$$

If labor productivity is expressed this way, it is no longer so obvious why it should be a procyclical variable with a slight lead. The ratio ζ might react in a fairly sensitive way already to small variations in the lag structure of e and u_w versus utilization u, or in their relative amplitudes.

To derive the changes of the employment rate we note that $E^d/E = H^d/h^n E = (H/h^n E)(H^d/H) = u_w\zeta$ by (5.37) and (5.41). Then, with (5.38), $\hat{e} = \hat{E} - \hat{L} = (g^o - g_h - g_z) + \beta_{ee}(u_w\zeta - 1) + \beta_{ey}(g_y^e - g^o) + \beta_{eh}(u_w - 1) - g_\ell$.

The growth rates cancel out by (5.35), and with $u_w \zeta = e^o u/e$ by (5.44) we have

$$\hat{e} = \beta_{ee}\,(e^o u/e - 1) \;+\; \beta_{eh}\,(u_w - 1) \;+\; \beta_{ey}\,(g_y^e - g^o). \tag{5.45}$$

While the changes in the employment rate are only dependent on the reaction coefficients in the adjustment equation (5.40), the changes in the utilization of the workforce are more involved and include the coefficients of both (5.39) and (5.40), since $\hat{u}_w = \hat{H} - \hat{h}^n - \hat{E}$. This gives us $\hat{u}_w = (g^o - g_z) + \beta_{hh}(\zeta - 1) + \beta_{hy}(g_y^e - g^o) - g_h - (g^o - g_h - g_z) - \beta_{ee}(e^o u/e - 1) - \beta_{ey}(g_y^e - g^o) - \beta_{eh}(u_w - 1)$. Again, the exogenous growth rates cancel out, and we arrive at

$$\begin{aligned} \hat{u}_w = & -\beta_{eh}(u_w - 1) + \beta_{hh}(\zeta - 1) - \beta_{ee}(u_w \zeta - 1) \\ & + (\beta_{hy} - \beta_{ey})(g_y^e - g^o). \end{aligned} \tag{5.46}$$

It is thus seen that the changes in the utilization of the workforce do not only depend on its current level u_w, which when above unity firms seek to reduce by employing more workers (see the origin of the coefficient β_{eh}). In addition, they are influenced by the deviations of productivity from trend, ζ, and the mixed term $u_w \zeta$. Whether expected growth enters positively or negatively depends on the relative size of the respective coefficients β_{hy} and β_{ey} in the hours and job adjustments.

Connection to Okun's Law

It seems at first sight that the differential equation governing the changes in the employment rate does not have anything more to do with Okun's law. Though capacity utilization, or the output gap for that matter, show up on the right-hand side of (5.45), it does this as a level variable, u, and not as a rate of change, $\hat{u}$, as it should if we want to relate (5.45) to the growth rate specification of Okun's law in (5.13).

The growth rate of u can, however, be re-introduced by splitting up the term with the expected growth rate. Taking account of $g^o = \hat{Y}^p$ from (5.35), this gives us $(g_y^e - g^o) = (\hat{Y} - \hat{Y}^p) + (g_y^e - \hat{Y}) = \hat{u} + (g_y^e - \hat{Y})$. Furthermore, we decompose $e^o u - e$, which is the numerator of $e^o u/e - 1 = (e^o u - e)/e$, as $e^o u - e = (u-1)e - (e - e^o)u$. Taken together, (5.45) can be equivalently rewritten as

$$\hat{e} = \beta_{ey}\hat{u} + \{\beta_{ey}(g_y^e - \hat{Y}) + \beta_{eh}(u_w - 1) + \beta_{ee}[(u-1)e - (e - e^o)u]/e\}. \tag{5.47}$$

The way in which the terms governing the changes of the employment rate are organized makes it clear that the two adjustment equations (5.39), (5.40) for hours and jobs contain Okun's Law as a core. As a discrete-time counterpart, the growth rate specification of (5.13) is obtained in the special case $\beta_{ee} = \beta_{eh} = 0$ and if also output is expected to continue its growth at the current rate in the near future. The Okun coefficient β would then be directly given by the structural coefficient β_{ey} in the gradual employment adjustments of the firms. Under these circumstances, their recruitment policy takes trend output and productivity growth into account and, apart from that, increases the number of jobs by β_{ey} percent (per year) if current output grows one percent faster than potential output.

On the other hand, note that considerations of desired levels of employment E^d and the speed at which firms seek to close the gap between E^d and E would play nor role, although informal discussions often seem to see these adjustments reflected in the Okun coefficient. Whether hours currently worked per job are above or below normal would not be taken into account, either..

The curly brackets in (5.47) point out the influence of additional factors that in each period may distort the simple relationship given by (5.13), even if we continue to identify expected and current output growth. Then, the employment rate is comparably higher if the workforce is currently overutilized ($u_w > 1$), or if in a boom output is above its trend and the employment rate is above normal but, as we expect, u has a larger amplitude than e (so that $(u-1)e > (e-e^o)u$). From the latter we could also conclude that in an upper turning point, where $\hat{y} = \hat{u} = 0$, the employment rate would still be rising. Accordingly, (5.47) would predict that the employment rate lags behind the output gap.

5.6.2 Estimation

Adjustments of Employment

Even though the module of the delayed adjustments of hours and employment may make good economic sense, it still has to be confronted with the empirical data. Only a few time series of raw data are needed for that purpose, which we have extracted from the Fair-Parke database. They are listed in Table 5.5, and all of them refer to the firm sector.

Let us begin with the employment adjustments in (5.40). Before resorting to more elaborated econometric methods, we should try how far we can get

Table 5.5. Raw data extracted from the Fair-Parke (FP) database (firm sector)

FP label	Description
HF	hours per job
JF	number of jobs
PROD	output per hour (labor productivity)
Y	real output per quarter

with OLS. This most elementary regression approach can be applied to (5.40) if the equation is rearranged as

$$\begin{aligned}\hat{E} \ &-(g^o - g_z - g_h) = \beta_{eg}[\hat{E} - (g^o - g_z - g_h)]_{-1} \\ + \ &\beta_{ee}\Big(\frac{E^d - E}{E}\Big)_{-1} + \beta_{ey}(g^e_y - g^o) + \beta_{eh}\frac{H - h^n E}{h^n E}. \qquad (5.48)\end{aligned}$$

All of the variables entering here can be constructed from the data in Table 5.5. The three trend rates of growth g^o, g_z, g_h are specified by our method of choice in this chapter, the HP trend line based on the smoothing parameter $\lambda = 102,400$. The gap term $(E^d - E)/E$ is given by $\ln E^d - \ln E = \ln(H^d/h^n) - \ln E = \ln H^d - \ln h^n - \ln E$. The log of normal hours per job, $\ln h^n$, is proxied by the HP 102,400 trend of the log of actual hours per job. The log of desired total hours, $\ln H^d = \ln(Y/z^o)$, according to (5.36), is given by the difference between $\ln Y$ and the HP trend of log productivity.

Regarding the expected growth rate of output g^e_y, it turned out that we already obtained the best results by simply employing the most recent actual output growth rate.[38] So we limit the presentation to $g^e_y = \hat{Y}$ right away. The details of the construction of the variables in (5.48) are summarized in Table 5.6 (the composed variables themselves are indicated by bold face characters). Observe that all of the one-quarter growth rates are annualized.

Table 5.7 contains the essential results of the OLS regressions that we performed. Behind the table are explorations of a variety of lags and lag combinations for the independent variables, which all proved to be distinctly inferior. Though the data available (at the time this chapter was written) extended to quarter 2005:2, the sample period was limited until 2003:4 in order to avoid possible end-of-period effects for the trend variables. A presentable result,

[38] We experimented with a short 4-quarter (backward-looking) moving average of output growth for g^e_y, which is a reasonable makeshift substitute for sales expectations. This variant, however, notably deteriorated the goodness-of-fit in the regressions.

Table 5.6. Time series constructed from the FP raw data

Name in regression	Model notation	Construction
E	E	JF
Hours	H	HF · JF
z	z	PROD
Ln_X_HP1024		HP 102400 trend of ln(X), X = HF, Y, z
d_Ln_X_HP1024		400 · Δ Ln_X_HP1024, X = HF, Y, z
go	g^o	d_Ln_Y_HP1024
gh	g_h	d_Ln_HF_HP1024
gz	g_z	d_Ln_z_HP1024
go_gz	$g^o - g_z$	go − gz
go_gz_gh	$g^o - g_z - g_h$	go − gz − gh
gE	$\hat{E}$	400 · Δ ln(JF)
gH	$\hat{H}$	400 · Δ ln(Hours)
gY	$\hat{Y}$	400 · Δ ln(Y)
gY_go	$\hat{Y} - g^o = g^e_y - g^o$	gY − go
gE_ggg	$\hat{E} - (g^o - g_z - g_h)$	gE − go_gz_gh
gH_gg	$\hat{H} - (g^o - g_z)$	gH − go_gz
Ln_Hd	$\ln H^d = \ln(Y/z^o)$	ln(Y) − Ln_z_HP1024
Hd_H	$(H^d - H)/H$	100 · [Ln_Hd − ln(Hours)]
Ln_hn	$\ln h^n$	Ln_HF_HP1024
Ln_Ed	$\ln E^d = \ln(H^d/h^n)$	Ln_Hd − Ln_hn
Ed_E	$(E^d - E)/E$	100 · [Ln_Ed − ln(E)]
Ln_hnE	$\ln(h^n E)$	Ln_hn + ln(E)
H_hnE	$(H - h^n E)/h^n E$	100 · [ln(Hours) − Ln_hnE]

Note: HP 102400 is the Hodrick-Prescott trend line with smoothing parameter $\lambda = 102{,}400$, Δ denotes the quarterly difference operator

which is shown in the first column (of figures) in Table 5.7, already comes about if lagged employment growth $\hat{E}_{-1}$ is ignored in (5.48), putting $\beta_{eg} = 0$. The coefficients on the other three independent variables have the correct sign and come out significant, while the fit itself is satisfactory. Only the low Durbin-Watson indicates nonnegligible serial correlation in the residuals, but this problem might be tackled with instrumental variable techniques. In any

Table 5.7. OLS regressions of (5.48)

Dependent variable: gE_ggg						
coeff.	Variable	1960:1–2003:4		60:1–83:4	84:1–03:4	90:1–03:4
β_{eg}	$(\text{gE\_ggg})_{-1}$	–	0.47 (9.57)	0.44	0.61	0.67
β_{ee}	$(\text{Ed\_E})_{-1}$	0.44 (5.43)	0.29 (4.52)	0.30	0.25	0.27
β_{ey}	gY_go	0.30 (9.61)	0.27 (10.41)	0.30	0.17	0.14
β_{eh}	H_hnE	0.55 (3.24)	–	–	–	–
	sd(gE_ggg)	2.16	2.16	2.54	1.60	1.67
	SER	1.41	1.17	1.36	0.89	0.94
	R^2	0.58	0.71	0.72	0.70	0.70
	DW	1.00	2.11	2.01	2.41	2.45

Note: 'sd' denotes the standard deviation (of the dependent variable). See Table 5.6 for the acronyms of the variables. *t*-statistics are given in parentheses

way, this regression is a first encouraging outcome concerning the validity of the model.

The fit is substantially improved if the lagged growth rate of employment $\hat{E}_{-1}$ is added to the explanatory variables. In this case, however, the coefficient on the utilization of the workforce, i.e. the gap $(H - h^n E)/h^n E$, turns insignificant. So, in the second column in Table 5.7, it is directly set to zero. As required, the growth rate coefficient β_{eg} is between 0 and 1, and the other two β_{ee} and β_{ey} are both positive. All these coefficients are highly significant. In addition to the smaller standard error of the regression, including lagged employment growth has the merit that the Durbin-Watson statistic moves close to 2. The goodness of the fit is illustrated in Fig. 5.21, which plots the predicted versus the actual values of $\hat{E} - (g^o - g_z - g_h)$.

We should also point out that setting the coefficient β_{eh} equal to zero may be convenient for the properties, or the analysis, of the theoretical model, since the influence of the workforce (u_w) is canceled in the differential equation (5.45) for the employment rate. In a model where the employment rate enters a wage Phillips curve, for example, the complicated (5.44) and (5.46), therefore, need not be considered. The variables u_w and ζ would, however, reappear if the changes in money wages do not (only) refer to a trend rate of productivity

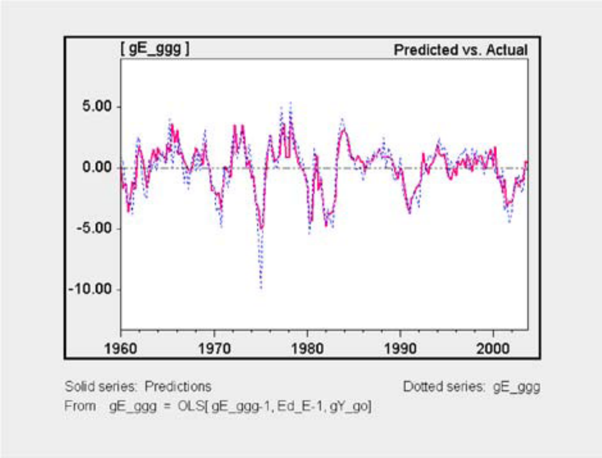

Fig. 5.21. Goodness-of-fit of regression (5.48), from the second column in Table 5.7

growth but (also) to its actual growth; or, of course, if the Phillips curve takes account of insider effects and the utilization of the workforce is introduced through this direct channel.

On the basis of the strong evidence in favor of the model we feel justified in studying possible time variations of the parameters, where it is important to note that they are here no longer of a purely descriptive nature but that they have a direct structural interpretation. Similar to our investigation of cyclical features above, we first subdivide the sample period into the three periods 1960:1–1983:4, 1984:1–2003:4 and 1990:1–2003:4, and re-estimate (5.48) over these subsamples. The outcome is shown in the last three columns in Table 5.7. If we consider the coefficient β_{ee} on the employment gap term $(E^d - E)/E$ to be the central responsiveness of firms to the disequilibria that they perceive, then this responsiveness has not changed very much, and if so, it has slightly decreased as compared to the first 24 years of the sample.

A more detailed examination of possible shifts in the reaction coefficient(s) is, however, possible if we conceive them as time-varying coefficients and estimate these time paths by the method of spline functions. This approach gives a more differentiated, if not different, picture. We distinguish two cases: (1) only $\beta_{ee} = \beta_{ee,t}$ is time-varying and the other two coefficients are fixed; (2) all three coefficients $\beta_{ex} = \beta_{ex,t}$ are time-varying ($x = g, e, y$). The top-left panel of Fig. 5.22 shows the result for $\beta_{ee} = \beta_{ee,t}$ in the first case (its estimation is based on five segments). The responsiveness had somewhat increased until

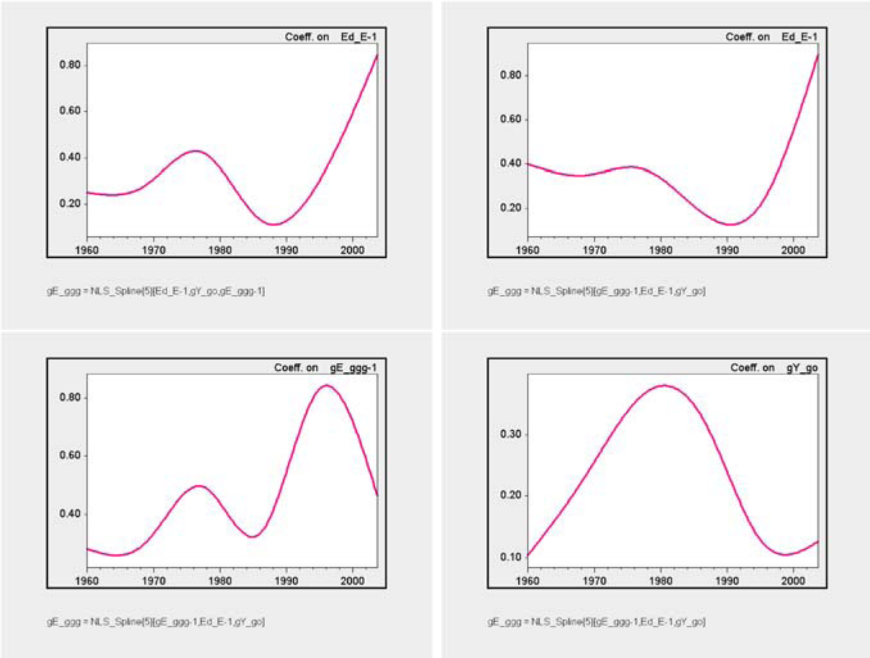

Fig. 5.22. Time-varying coefficients in regression (5.48) (*Note*: Estimation with the method of spline functions, subdividing the sample period in five segments)

the mid-1970s, then it decreased until the end of the 1980s, and from then on $\beta_{ee,t}$ increased to unprecedented levels until the end of the sample period.

It is quite remarkable that, at least from the mid-1970s on, the same behavior and almost the same figures for $\beta_{ee,t}$ result if all three parameters are free to vary over time. The time path of $\beta_{ee,t}$ thus determined is exhibited in the top-right panel in Fig. 5.22. The time paths of the other two coefficients $\beta_{eg,t}$ and $\beta_{ey,t}$ are shown in the two bottom panels (left and right, respectively). The strong variations of these coefficients makes the close resemblance of the evolution of $\beta_{ee,t}$ in the first two panels all the more noteworthy. As a special feature in the time path of $\beta_{ee,t}$ it should be noted that this responsiveness is particularly low during the 1991 recession, whose subsequent recovery is often described as jobless growth. It may be interesting to discuss our notion of desired employment E^d and the finding of the slow adjustment toward it at that time in the light of this phenomenon. It may well be that our business cycle perspective would then have to be complemented with a longer-term

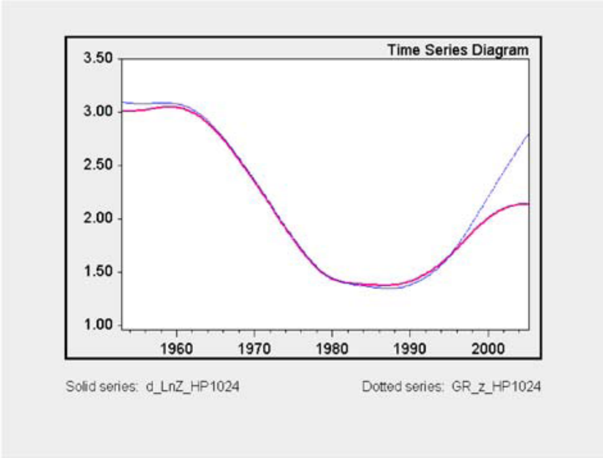

Fig. 5.23. Trend rate of labor productivity growth (see footnote 39 for an explanation of the difference between the two lines)

perspective, where in the first instance one might think of a general increase in labor productivity.

This, however, would open up new issues and new problems. To illustrate this, we conclude this subsection with a plot of the evolution of the trend rate of productivity growth in Fig. 5.23. This rate has indeed substantially increased in recent times. At the beginning of the 1990s, however, its level was still fairly low.[39]

Adjustments of Hours

The estimations of the hours adjustments equation (5.39) can proceed analogously to the preceding subsection. For OLS, we first transform (5.39) to

$$\hat{H} - (g^o - g_z) = b_{hg}[\hat{H} - (g^o - g_z)]_{-1} + \beta_{hh}\Big(\frac{H^d - H}{H}\Big)_{-1} + \beta_{hy}(g_y^e - g^o). \tag{5.49}$$

[39] Incidentally, Fig. 5.23 shows that from the end of the 1990s it makes quite a difference whether trend productivity growth is obtained from computing the trend of the level of productivity and then taking first differences from it (the series gz as described in Table 5.6, the solid line in Fig. 5.23); or whether one takes the actual growth rates of productivity and applies the HP filter directly to them (the dotted line).

Table 5.8. OLS regressions of (5.49)

Dependent variable: gH_gg							
coeff.	Variable	1960:1–2003:4		60:1–83:4	84:1–03:4	90:1–03:4
β_{hg}	$(\text{gH\_gg})_{-1}$	–	0.31 (5.84)	0.24	0.43	0.51
β_{hh}	$(\text{Hd\_H})_{-1}$	0.71 (6.04)	0.55 (4.96)	0.71	0.36	0.33
β_{hy}	gY_go	0.49 (10.70)	0.43 (9.87)	0.44	0.33	0.27
	$sd(gH\_gg)$	2.93	2.93	3.40	2.26	2.31
	SER	2.08	1.90	2.06	1.67	1.72
	R^2	0.50	0.58	0.64	0.47	0.47
	DW	1.39	2.18	2.03	2.49	2.62

Note: 'sd' denotes the standard deviation (of the dependent variable). See Table 5.6 for the acronyms of the variables. t-statistics are given in parentheses

The construction of the variables is detailed in Table 5.6. The regression results for the equation are reported in Table 5.8. While the regression without the growth rate term ($\beta_{hg} = 0$) is satisfactory, including this inertia again leads to a clear improvement. Though R^2 for the unconstrained version (in column 2) is less than its counterpart in Table 5.7, the visual impression of the goodness-of-fit is similar; cf. the plot of predicted versus actual values in Fig. 5.24 and compare it to the fit in Fig. 5.21. Regarding the smaller R^2 for (5.49) note also the greater variability in the dependent variable of Table 5.8: a standard deviation of 2.93% in contrast to the 2.16% in Table 5.7.

The coefficient that represents the speed at which firms adjust actual hours to desired hours is β_{hh}. The last three columns in Table 5.8 with our three subperiods show that the responsiveness has decreased rather than increased. However, this does not necessarily mean that firms are less concerned about the gap in hours, since the growth rate coefficient β_{hg} has changed in the opposite direction and to similar amounts. Therefore, the two effects may be "substitutes": the lower responsiveness to the gap in the level of hours may just offset the increased inertia in the hours growth rates, where the growth rate of the previous period proves to be a good guideline in the present period, too. Computing the time-varying coefficients $\beta_{hh} = \beta_{hh,t}$ and $\beta_{hg} = \beta_{hg,t}$ in the same way as for the employment adjustments in the preceding subsection yields a similar picture to the subsample regressions, although with more

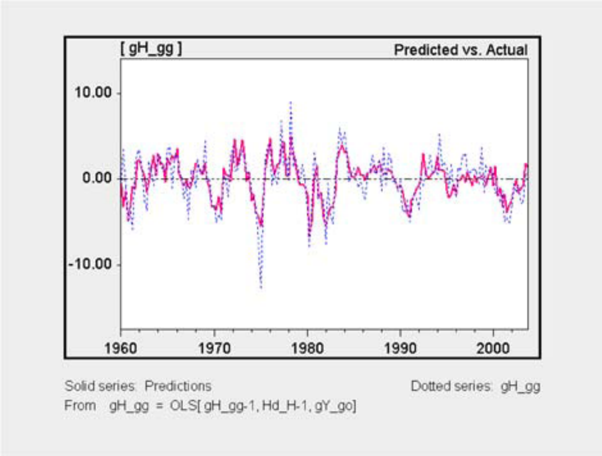

Fig. 5.24. Goodness-of-fit of regression (5.49), from the second column in Table 5.8

medium-term variation; see Fig. 5.25 with $\beta_{hh,t}$ in the two top panels ($\beta_{hh,t}$ as the only time-varying coefficient in the left panel, and $\beta_{hh,t}$ when all three coefficients are free to vary in the panel to the right; the lower two panels display the time paths of $\beta_{hg,t}$ and $\beta_{hy,t}$, respectively). The lower-right panel also shows the changing influence of expected output growth on the growth rate of hours. On the whole, the subsample regressions and the time paths of the coefficients in Fig. 5.25 may be indicative of systematic changes in the reaction pattern of firms. They are, however, not easily characterized and identifying deeper reasons for them is even harder. A discussion approaching this subject might take into account that until the mid-1990s the gap $(H^d - H)/H$ moved more or less together with the growth rate term $\hat{H} - (g^o - g_z)$ (apart from the obvious fact that the growth rates are much noisier, which favors estimation), whereas possibly something has changed in the second half of the 1990s. In any case, going beyond a mere description and attaching economic sense to the estimated time paths in Fig. 5.25 is still an open issue.

The Adjustment Equations Ready for Use

Apart from the perspective on Okun's law and the (possibly time-varying) macroeconomic relationships underlying it, it was our aim in this section to put forward a theoretical building block for the determination of hours and employment, and thus for the utilization of the workforce (giving rise to insider

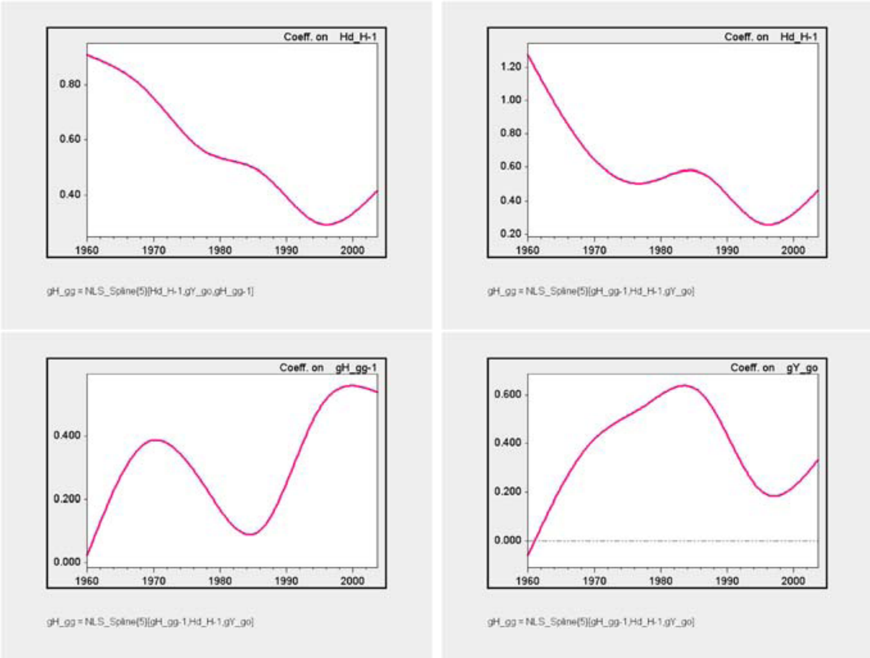

Fig. 5.25. Time-varying coefficients in regression (5.49) (*Note*: Estimation with the method of spline functions, subdividing the sample period in five segments)

effects) and of the labor force (giving rise to outsider effects). The module was meant to be incorporated into broader dynamic models of the macro economy. Since these models will be definitely larger than just two or three dimensions, they have to resort to computer simulations and so have to be specified numerically. For a basic or benchmark version of such a model, our estimations over the entire sample period are sufficiently reliable. For discrete-time models with a quarterly adjustment period we can summarize our main result from Tables 5.7 and 5.8 as follows:

$$\hat{H} = 0.31 \cdot \hat{H}_{-1} + 0.69 \cdot (g^o - g_z) + 0.55 \cdot \left(\frac{H^d - H}{H}\right)_{-1} + 0.43 \cdot (g_y^e - g^o), \tag{5.50}$$

$$\hat{E} = 0.47 \cdot \hat{E}_{-1} + 0.53 \cdot (g^o - g_z - g_h) + 0.29 \cdot \left(\frac{E^d - E}{E}\right)_{-1} + 0.27 \cdot (g_y^e - g^o). \tag{5.51}$$

These equation are now ready for use. (For a continuous-time version the remarks at the beginning of Sect. 5.6.1 may be taken into account.)

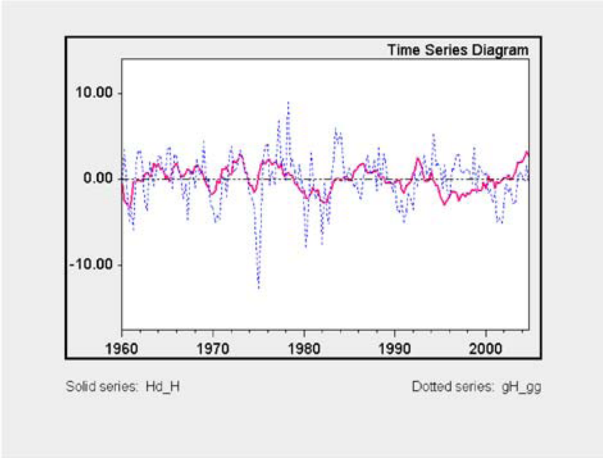

Fig. 5.26. Comovements of $(H^d - H)/H$ (solid) and $\hat{H} - (g^o - g_z)$ (dotted line)

5.7 Conclusion

This chapter has examined the output-employment nexus from different perspectives. On the one hand, it was concerned with an atheoretical summary of the data for the US firm sector that usually goes under the heading of Okun's law. In particular, apparently systematic variations of the famous Okun coefficient over time could here be identified, which partly turned out to be different from other results in the literature. Complementary to that, we considered the cyclical behavior of various macroeconomic variables that contribute to the connection between output and the employment rate and found significant changes over the last 15 years for some of them, too.

On the other hand, this chapter put forward two theoretical models that determine the adjustments of employment and hours by firms in response to expected growth and certain gaps between desired and actual values. The two modules were validated by estimation of the structural parameters, which came out very satisfactorily. In addition, for the more ambitious model regressions with time-varying coefficients were performed. These results allow a more differentiated view on possible "regime shifts" than the variations of the Okun coefficient. Independently of this issue, the two modules and their estimated numerical coefficients can be readily incorporated into larger dynamic macro models.

A side result of the investigations was that the supply side effects, i.e. the cyclical behavior of the labor force, have gained greater importance in the fluctuations of the employment rate. As we have put forward two building blocks determining the demand side in this respect, it would now be desirable to develop one or two small-scale models that could explain adjustments in the labor force along similar lines.

Apart from that, our empirical investigations used U.S. data only so far. In a next step, the significance of our results and their interpretation may be tested with data and economic reasoning from other industrialized countries.

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Part II

The Open Economy

6

Exchange Rate and Stock Market Dynamics in a Two-Country Model

6.1 Introduction

This chapter considers macroeconomic real-financial market interactions in a two country framework with money, bonds, equities and a foreign exchange market with a flexible exchange rate.[1] The model uses conventional dynamic multiplier dynamics in its real part as in Blanchard (1981), augmented by either a simple LM curve or later on a simple Taylor interest rate policy rule, but enriched by a broad spectrum of financial assets, based in their dynamic interaction on the assumption of perfect substitute and perfect foresight throughout. Contrary to Blanchard's (1981) original contribution on stock market dynamics, we however aim to analyze and solve a revised form of the model, in order to avoid the heroical assumption of the jump-variable technique (JVT) of the rational expectations school. In approaching such an objective, the model will first be considered in conventional form, starting from the Dornbusch (1976) overshooting exchange rate model, and will then be integrated with the Blanchard (1981) stock market approach in a two-country setup as in Turnovsky (1986). This will lead us to a dynamic model with five laws of motion and a variety of interacting feedback chains and their (in)stability implications.

We do not show however that the saddle-point dynamics (that will again result in such a framework) with their—as demanded by the rational expectations methodology—three non-predetermined and two predetermined variables allows for a proper application of the JVT by proving the existence of in

[1] This chapter is based on Flaschel and Hartmann (2007): "Perfect Finance-led World Capitalism in a Nutshell". CEM Working Paper 145, Bielefeld University.

fact three unstable and two stable roots of the Jacobian at their steady state, and by handling therewith the cases of unanticipated as well as anticipated shocks. Instead we suggest the principle that Taylor interest rate policy rules should be designed (independently from what is happening in actual monetary policy) in correspondence to the feedback structure of the dynamics that has been assumed to represent the behavior the private sector.

This principle leads us to a Taylor rule that concentrates on the UIP condition of the Dornbusch model. We assume a behavior of the CB's that is opposite to what policymakers would expect a Central Bank to do in the case of an unwanted depreciation of their currency, i.e., we assume as Taylor rule that they lower the nominal rate of interest in such a case (normally viewed as implying capital outflows and therefore giving further momentum to the on-going depreciation of the exchange rate). In the present ideal modeling of financial markets (where we have perfect substitution and myopic perfect foresight everywhere), this inverted policy reaction is however indeed of help and it implies conventional asymptotic stability towards a uniquely determined steady state (a second implication of our choice of the interest rate policy rule). We thus have that all 5 state variables can be considered as predetermined (but not their rates of growth) and therefore not subject to any explosive tendency. this result no longer enforces the need to apply the JVT, as it would be the case in the model with a conventional Taylor rule we have started from.

Such a modification of a conventional RE model thus shows that it can overcome the conundrums that surround the (nowadays generally purely algorithmic) application of the jump-variable technique. We have achieved this by just searching for Taylor rules that do their stability-delivering job in the assumed dynamic environment in a conventionally stabilizing way, with all variables being predetermined (so that only their time rates of change are subject to unanticipated shocks).[2]

From the methodological point of view we proceed as in Turnovsky (1986) from a general two-country approach and its uniquely determined interior steady state position to a (partial) linearization of this model around its steady state and to the assumption of symmetry between the two countries, i.e., to assuming identical parameter values for them. This allows us to decompose the

[2] And with a treatment of anticipated shocks that should be event-specific and not just subject to the mechanic use of explosive bubbles that lead us softly in time towards a state where the stable manifold of the new dynamics comes into being.

dynamics in 2D average dynamics and 3D difference dynamics both of which can be shown to be convergent under standard assumptions on the private sector of the economy if the reaction of the interest rate is exchange-rate oriented in a specific way. Moreover, since Turnovsky (1986) also allows for price dynamics as in Dornbusch (1976) we extend the model in a final section towards an integration of Phillips curves and can again show convergence if the Taylor rule is adjusted to this new situation in an appropriate way.

The results of this chapter question the way rational expectations are exercised in forward looking models, but they also show that the policy that is needed to overcome the saddlepoint instability of perfect foresight models is of a strange type in such a perfect substitute financial world. In the outlook the chapter therefore suggests that realistic models of finance-led world capitalism should consider situations of imperfect substitution between financial assets, take account of heterogeneous expectations formation and admit that adjustment processes may be fast, but not infinitely fast, in particular when the limiting situation of rational expectations is a structurally unstable one (i.e., subject to severe discontinuities in the limit).

A side product of this chapter moreover is that it shows that one can investigate theoretically situations in continuous time that are out of reach for the neo-Wicksellian period models used for example in Woodford (2003). We consider the use of continuous-time models a necessity in macromodels of the real financial markets interaction, see Chap. 1 in this book for the details of such an argument. It can also be used, if extended appropriately by wage-price dynamics and long-term bonds, for a structural comparison with and a theoretical investigation of the empirical DSGE studies, as they are now the fashion in studies of the working of monetary policy rules, see the paper by Smets and Wouters (2003) for an example.

In the next section we briefly consider the Dornbusch exchange rate dynamics for the small open economy. We then reconsider in Sect. 6.3 in detail the Turnovsky two-country version of this model type and its rational expectations solutions. Section 6.4 provides a brief introduction into Blanchard's stock market dynamics for the closed economy. In Sect. 6.5 we provide an integration of the Dornbusch and Blanchard models on a level that is similar to the Turnovsky approach. We briefly discuss there problems of saddlepoint instability, and then reformulate—in Sect. 6.6—the Taylor rule in order to allow for stability in the conventional sense in this model type. 5D stability analysis is carried out in Sect. 6.7 under the assumption of symmetry where it can be shown then that both averages over and differences for the two economies converge to their steady state values. In Sect. 6.9 we conclude showing that inflation dynamics can be added to the model without altering its implications significantly.

6.2 The Dornbusch Exchange-Rate Dynamics

The Dornbusch (1976) exchange rate dynamics reads in the case of the Uncovered Interest Parity condition (UIP) with myopic perfect foresight on the exchange rate dynamic in their simplest form as follows (with e the logarithm of the exchange rate and p the logarithm of the price level, the foreign price level being normalized to 1):

$$\dot{p} = \beta_p(Y^d(e-p) - \bar{Y}),$$
$$\dot{e} = i(p) - \bar{i}^*.$$

We assume that the economy is at its full employment level $\bar{Y}$, but that deviations of aggregate demand Y^d (which only depend on the real exchange rate here) from this level determine with adjustment speed β_p the rate of inflation. The second law of motion is the UIP condition solved for the dynamic of the exchange rate it implies when myopic perfect foresight is assumed. The relationship $i(p)$ is a standard inverted LM curve, i.e., it describes a positive relationship between the price level and the domestic nominal rate of interest. Clearly the steady state is of saddlepoint type (the determinant of the Jacobian is negative) and the implied phase diagram is shown in Fig. 6.1. The rational expectations school solves such a dynamical system in the following way. It assumes that the economy is (if no anticipated shocks are occurring) always on the stable manifold of the given saddlepoint, horizontally to the right of the old steady state 0, in the intersection of the stable manifold (a straight line in this simple model) with the horizontal axis, if an unanticipated shock that moves the steady state to point B has occurred, since the price level can only adjust gradually in this model. We therefore get an overshooting exchange rate (with respect to its new steady state values) and thus an increase in goods demand which increases gradually the price level and the nominal interest rate, which appreciates the exchange rate from its excessively high level until all variables reach the new steady state.

In the case of anticipated shocks the situation that is assumed by the rational expectations school becomes more complicated, since at the time of

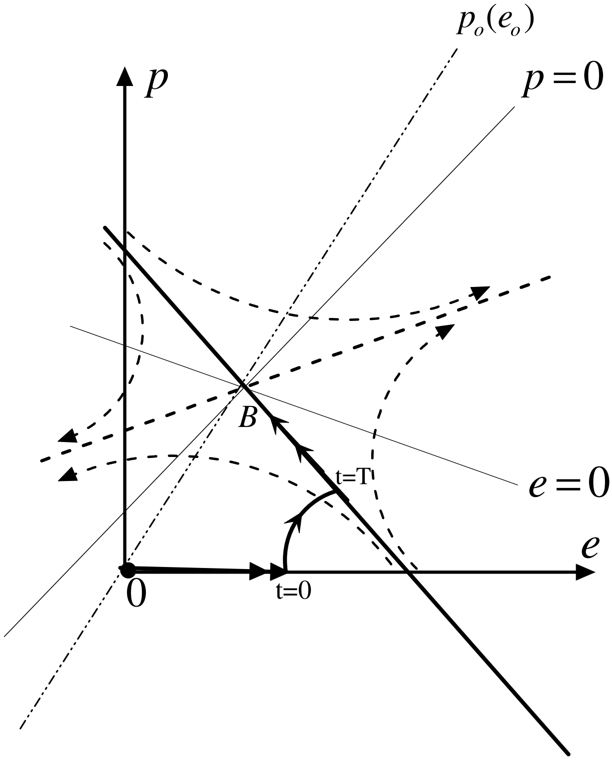

Fig. 6.1. The Dornbusch exchange rate dynamics under myopic perfect foresight

the announcement of the policy we are still in the old phase diagram around the point 0. In this case, the exchange rate jumps at $t = 0$ to the right to a uniquely determined level from where it uses the unstable saddlepoint bubble in the old dynamics that starts at this point and from where it then reaches the stable manifold of the new dynamics exactly at the time T when the announced policy shock actually takes place. We thus have—depending on T—a jump in the exchange rate that may still overshoot its new steady state position and that then switches immediately towards a bubble of length T with both rising prices and exchange rates, from which it departs through a soft landing on the new stable manifold at time T.

This is—when appropriately extended—the rational expectations approach to exchange rate dynamics in the frame of a Dornbusch IS-LM model with a price Phillips curve. Its solution techniques looks attractive, since it provides—in addition to the usual treatments of shocks—also a well-defined answer in the case of anticipated events (within certain bounds, depending on the size of the anticipated shock). But it may also be viewed as a rather heroical solution to the treatment of (un)anticipated demand, supply and policy shocks from a descriptive point of view.

6.3 Symmetric Two-Country Exchange Rate Dynamics

In this section we introduce and make use of a technique that Turnovsky (1986) has employed to analyze Dornbusch type IS-LM-PC analysis in a symmetric two-country setup by means of (mathematical) average and difference considerations, then used to recover the original dynamics from these two hierarchically ordered subdynamics.

Following Turnovsky (1986) we thus consider in this section the following two-country macroeconomic model. It describes two symmetric economies, characterized by the same parameters, with each specializing in the production of a distinct good and international trading of distinct fix-price bonds. All parameters in the following model are assumed to be positive (with $a_1 < 1$ and $\alpha \in (0.5, 1)$ in addition).

$$y = a_1 y^* - a_2(i - \dot{z}) + a_3(p^* + e - p) + \bar{u}, \tag{6.1}$$

$$y^* = a_1 y - a_2(i^* - \dot{z}^*) - a_3(p^* + e - p) + \bar{u}^*, \tag{6.2}$$

$$\bar{m} - z = b_1 y - b_2 i, \tag{6.3}$$

$$\bar{m}^* - z^* = b_1 y^* - b_2 i^*, \tag{6.4}$$

$$i = i^* + \dot{e}, \tag{6.5}$$

$$z = \alpha p + (1 - \alpha)(p^* + e) = p + (1 - \alpha)(p^* + e - p), \tag{6.6}$$

$$z^* = \alpha p^* + (1 - \alpha)(p - e) = p^* - (1 - \alpha)(p^* + e - p), \tag{6.7}$$

$$\dot{p} = \beta_w y, \tag{6.8}$$

$$\dot{p^*} = \beta_w y^*. \tag{6.9}$$

In these equations we make use of the following notation:

y = real output Y (in logarithms) deviation from its natural rate level,
p = price of output, expressed in logarithms,
z = consumer price index, expressed in logarithms,
e = exchange rate (of the domestic economy), measured in logarithms,
i = nominal interest rate,
$\bar{m}$ = nominal money supply, expressed in logarithms,
$\bar{u}$ = real government expenditure, expressed in logarithms.

Domestic variables as usual are unstarred; foreign variables are shown with an asterisk. Equations (6.1) and (6.2) describe goods market equilibrium, or the IS curves, in the two economies. Private goods demand depends upon output in the other country, upon the real rate of interest, measured in terms of consumer price inflation, and the real exchange rate. Because of the assumed symmetry, the corresponding effects across the two economies are identical, with the real exchange rate influencing demand in exactly offsetting ways. The money market equilibrium in the economies is of standard textbook type. It is described by (6.3) and (6.4). These four equations thus provide a straightforward extension of conventional IS-LM block to the case of two symmetric interacting economies.

The perfect substitutability of the domestic and foreign bonds is described by the uncovered interest parity condition described by (6.5). Equations (6.6) and (6.7) define the consumer price index (CPI) at home and abroad. The assumption is made that the proportion of consumption α spent on the respective home good is the same in the two economies. We assume $\alpha > \frac{1}{2}$, so that residents in both countries have a preference for their own good. Finally, (6.8) and (6.9) define the price adjustment in the two economies in terms of simple Phillips curves (which are not expectations augmented). We note that y is already measured as deviation from its steady state level, which is not true for the other variables of the model. Due to the two-country approach here adopted the world interest rate is not a given magnitude, but will be determined by the equations of the model.

The two-country world described by (6.1)–(6.9) represents a linear 3D dynamical system in the domestic and the foreign price levels p, p^*, and the exchange rate, e. Following the methodology developed in the preceding section we assume that the prices p, p^* can only adjust continuously, while the exchange rate is free to jump in response to new information and that it will always jump in such a way that the dynamic responses generated remain bounded away from zero and infinity. The jump variable technique therefore now applies to a 3D phase space and can therefore no longer be depicted graphically in the easy way considered in the preceding section.

Fortunately however, due to the symmetry assumption and the linearity of the considered model, the analysis can be simplified considerably by defining the averages and differences for all variables involved, say x for example, as

$$x^a \equiv \frac{1}{2}(x + x^*),$$
$$x^d \equiv x - x^*.$$

Through elimination of the variables z and z^*, the dynamics can be rewritten in terms of a decoupled system for averages and differences as described below.

The Behavior of the "Average Economy"

Equations (6.10)–(6.12) describe the aggregate world economy. The aggregate IS and LM curves (6.12) and (6.13) determine the average output level and average nominal interest rate in terms of the average price level, the evolution of which is described by the Phillips curve (6.12). Thus

$$(1 - a_1 - a_2\beta_w)y^a = -a_2 i^a + \bar{u}^a, \tag{6.10}$$

$$\bar{m}^a - p^a = b_1 y^a - b_2 i^a, \tag{6.11}$$

$$\dot{p^a} = \beta_w y^a. \tag{6.12}$$

The behavior of this virtual average economy is therefore characterized by virtual IS-LM equilibrium and a virtual Phillips curve, which is not expectations augmented. Note here however that inflation is reflected in aggregate demand in both countries which depends on the actual real rate of interest in both countries where inflation is substituted out by means of the PC's of the model. We assume in this regard

$$b = 1 - a_1 - \beta_w a_2 > 0,$$

i.e., that wages adjust sufficiently sluggishly in order that the resulting IS curve in y^a, i^a space be downward-sloping. We have the stabilizing Keynes-effect present in this formally conventional IS-LM model and no destabilizing Mundell-effect, not however—as in the Dornbusch-Fischer model of Sect. 6.3—by reducing the real rate of interest to a nominal one, but because of the assumption that the Phillips curves are not yet expectations augmented (exhibit stationary expectations). It can therefore be expected that the linear dynamical model for the averages will converge to its steady state solution. Note that this decoupled part of the original model behaves just like a closed economy.

The Dynamics of Differences

The differences in the two economies, together with the exchange rate, are described by

$$(1 + a_1)y^d = a_2(1 - 2\alpha)(\dot{e} - \dot{p}^d) + 2a_3(e - p^d) + \bar{u}^d, \tag{6.13}$$

$$\bar{m}^d - 2(1 - \alpha)e + (1 - 2\alpha)p^d = b_1 y^d - b_2\dot{e}, \tag{6.14}$$

$$\dot{p^d} = \beta_w y^d. \tag{6.15}$$

It is shown below that the virtual dynamics of the state variable e and p^d is of the saddlepoint type that we considered in the preceding section for the case of a small open economy.

It is convenient to begin with a characterization of the *steady state* equilibrium. Characterizing steady state values by indexation with zeros, we calculate it from the conditions $\dot{p} = \dot{p^*} = \dot{e} = 0$, and first of all get $y_o = 0$ and $i_o = i_o^*$. Thus the steady state equilibrium in the goods and money markets of the two economies is given by

$$a_2 i_o - a_3(p_o^* + e_o - p_o) = \bar{u},$$
$$a_2 i_o + a_3(p_o^* + e_o - p_o) = \bar{u}^*,$$
$$\bar{m} - p_o - (1 - \alpha)(p_o^* + e_o - p_o) = -b_2 i_o,$$
$$\bar{m}^* - p_o^* + (1 - \alpha)(p_o^* + e_o - p_o) = -b_2 i_o.$$

The solutions to these equations are

$$i_o = \frac{1}{2a_2}(\bar{u} + \bar{u}^*) = \bar{u}/a_2 = i_o^*, \tag{6.16}$$

$$\eta_o \equiv p_o^* + e_o - p_o = \frac{1}{2a_3}(\bar{u} - \bar{u}^*) = \bar{u}^d/(2a_3), \tag{6.17}$$

$$p_o = \bar{m} + \left\{\frac{b_2}{2a_2} + \frac{(1-\alpha)}{2a_3}\right\}\bar{u} + \left\{\frac{b_2}{2a_2} - \frac{(1-\alpha)}{2a_3}\right\}\bar{u}^*, \tag{6.18}$$

$$p_o^* = \bar{m}^* + \left\{\frac{b_2}{2a_2} - \frac{(1-\alpha)}{2a_3}\right\}\bar{u} + \left\{\frac{b_2}{2a_2} + \frac{(1-\alpha)}{2a_3}\right\}\bar{u}^*, \tag{6.19}$$

$$e_o = \bar{m} - \bar{m}^* + \left\{\frac{1-2\alpha}{2a_3}\right\}(\bar{u} - \bar{u}^*). \tag{6.20}$$

We obtain that the steady world rate of interest is independent of monetary policy as well as the real exchange rate. With respect to monetary policy we thus have neutrality results as well as a constant real exchange rates as in the Dornbusch (1976) model.

We investigate the *stability properties of the average economy* first. For its steady state position we get from the above that

$$i_o^a = \frac{1}{a_2}\bar{u}^a,$$
$$p_o^a = \bar{m}^a + \frac{b_2}{a_2}\bar{u}^a,$$
$$y_o^a = 0.$$

We can see that the steady state world interest rate $i_o = i_o^* = i_o^a$ only depends on fiscal policy in the two countries and the interest rate elasticity of the aggregate demand function (of the aggregate investment demand function in particular). For the steady state average price level we get the additional influences of the interest rate elasticity of the money demand functions as well as average world money supply. The steady sate of the average economy is therefore of a very simple type.

In order to discuss its stability we have to solve the IS-LM equations of the average economy for y^a and i^a first. Making use of the abbreviation $b = 1-a_1-\beta_w a_2$ we get from the IS and LM equation for the average economy the linear system

$$\begin{pmatrix} b & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} y^a \\ i^a \end{pmatrix} = \begin{pmatrix} \bar{u}^a \\ \bar{m}^a - p^a \end{pmatrix}.$$

This in turn gives

$$\begin{pmatrix} y^a \\ i^a \end{pmatrix} = \frac{1}{zb_2 + a_2 b_1} \begin{pmatrix} b_2 & a_2 \\ b_1 & -b \end{pmatrix} \begin{pmatrix} \bar{u}^a \\ \bar{m}^a - p^a \end{pmatrix}.$$

Setting $d = 1/(zb_2 + a_2 b_1)$ we therefore get

$$\begin{pmatrix} y^a \\ i^a \end{pmatrix} = d \begin{pmatrix} b_2\bar{u}^a + a_2(\bar{m}^a - p^a) \\ b_1\bar{u}^a - b(\bar{m}^a - p^a) \end{pmatrix},$$

which has the expected signs in front of the coefficients that characterize fiscal and monetary policy. Inserting the expression obtained for the output gap y^a into the average PC then leads us to the linear differential equation in the average price level p^a,

$$\dot{p}^a = \beta_w d(b_2\bar{u}^a + a_2\bar{m}^a - a_2 p^a),$$

which shows that the steady state level $p_o^a = \bar{m}^a + \frac{b_2}{a_2}\bar{u}^a$ is obviously a global attractor for the average price level. The average world economy is therefore globally asymptotically stable in a very straightforward way.

A Remark

However, the PC that is employed in Turnovsky (1986) is *not expectations augmented* and may still allow for stability results in contrast to what was is known about destabilizing Mundell inflationary expectations effects. We therefore briefly discuss the case when expectations augmented PC's are used in the two-country approach here under consideration. Expectations of workers concern the consumer price indices z and z^* in the present context. Due to the definition of the consumer price index (by way of functions of Cobb-Douglas type) we get for the derivative of its logarithm (and thus for the growth rate of the consumer price index)

$$\dot{z} = \alpha\dot{p} + (1-\alpha)(\dot{p}^* + \dot{e}), \quad \dot{z}^* = \alpha\dot{p}^* + (1-\alpha)(\dot{p} - \dot{e}).$$

As expectations augmented Phillips Curves we now define[3]

$$\dot{p} = \beta_w y + \pi, \quad \dot{p}^* = \beta_w y^* + \pi^*,$$

where π and π^* denote the expected growth rates for the consumer price indices of the two countries. For these expected rates we now assume an adaptive expectations mechanism, which in the present context and for the two countries considered must be of the form

$$\dot{\pi} = \beta_\pi(\dot{z} - \pi), \quad \dot{\pi}^* = \beta_\pi(\dot{z}^* - \pi^*),$$

by employing again the symmetry assumption for the considered two-country model.

Note that Turnovsky (1986) and our above presentation of his approach make use of myopic perfect foresight with respect to price inflation as well (in the aggregate demand function), but disregard the fact that this might fix the output level at its NAIRU level. The PC of the above model can therefore be positively sloped, since it has not been augmented by inflationary expectations in the usual way. We now depart from this procedure by inserting expected consumer price inflation into the aggregate demand function in the place of actual consumer price inflation in order to be in line with the conventional IS-LM-PC model type. We therefore now consider a mixed situation with respect to expectations formation: rational ones in the financial markets and adaptive ones with respect to goods markets, labor markets and wage and price inflation. We justify the choice of such a mixed situation with reference

[3] These equations are based on level representations of the type $\hat{p} = \beta_w \ln(Y/\bar{Y}) + \pi$.

to applied models such as the one of McKibbin and Sachs (1991), see also the IMF Multimod Mark III model, where however a more complicated situation is considered, since inflationary expectations are there based on forward and backward looking elements and not included directly in the investment or consumption demand function.

In terms of averages the equations just discussed give rise to

$$\dot{\pi}^a = \beta_\pi(\dot{z}^a - \pi^a) = \beta_\pi(\dot{p}^a - \pi^a),$$
$$\dot{p}^a = \beta_w y^a + \pi^a.$$

There is thus an immediate extension of the model by adaptive inflationary expectations such that an IS-LM-PC analysis is established for the average economy that is of the type considered in Chiarella et al. (2000, 2.2) for the case of a closed economy. Note here however that the IS-LM part of the model is now given by

$$(1 - a_1)y^a = -a_2(i^a - \pi^a) + \bar{u}^a,$$
$$\bar{m}^a - p^a = b_1 y^a - b_2 i^a,$$

whose solution for the variable y^a, r^a now gives [with $d = 1/((1-a_1)b_2 + a_2 b_1)$]

$$\begin{pmatrix} y^a \\ i^a \end{pmatrix} = d \begin{pmatrix} b_2 a_2 \pi^a + b_2 \bar{u}^a + a_2(\bar{m}^a - p^a) \\ b_1 a_2 \pi^a + b_1 \bar{u}^a - (1 - a_1)(\bar{m}^a - p^a) \end{pmatrix},$$

again with the expected signs in front of the coefficients that characterize fiscal and monetary policy (and the role of inflationary expectations now). Inserting the expression for output y^a into the revised dynamical system then finally gives

$$\dot{p}^a = \beta_w y^a + \pi^a = \beta_w d(b_2 a_2 \pi^a + b_2 \bar{u}^a + a_2(\bar{m}^a - p^a)) + \pi^a,$$
$$\dot{\pi}^a = \beta_\pi \beta_w y^a = d(b_2 a_2 \pi^a + b_2 \bar{u}^a + a_2(\bar{m}^a - p^a)).$$

These IS-LM-PC dynamics are of the same qualitative type as the one investigated in Chiarella et al. (2000, 2.2). They therefore now contain the destabilizing Mundell effect (represented by the coefficient db_2a_2) besides the stabilizing Keynes effect (represented by the coefficient -da_2) and thus will not be locally asymptotically stable if the Mundell-effect works with sufficient strength. However, the present analysis is strictly local in nature, since aggregate demand $Y^d = C + I + G$ has been approximated by a loglinear expression of the type $a_1 y^a - a_2(i^a - \pi^a) + \bar{u}^a$. The completion of the analysis by means of a kinked PC and the proof of global stability of these average dynamics is therefore not possible here, but demands a level form representation of the whole model which is necessarily nonlinear in nature—to which the averaging method of this section can then no longer be applied. The analysis of this section therefore becomes considerably more complicated when a global IS-LM-PC approach is attempted that generalizes Chiarella et al. (2000, 2.2) to the case of two (symmetric) interacting open economies.

For the loglinear approximation of this section and the use of positively sloped PC's (static expectations of wage earners) and myopic perfect foresight with respect to price inflation by investors, we have however shown that the average economy is (locally) monotonically and asymptotically stable and thus behaves much simpler than even the traditional monetarist base model and its extension to IS-LM-PC analysis.

Let us now consider *the dynamics of differences* which when slightly reformulated is given by:[4]

$$(1+a_1)\dot{p}^d/\beta_w = a_2(1-2\alpha)(\dot{e}-\dot{p}^d) + 2a_3(e-p^d) + \bar{u}^d,$$
$$\bar{m}^d - 2(1-\alpha)(e-p^d) - p^d = b_1\dot{p}^d/\beta_w - b_2\dot{e}.$$

Rearranging these equations appropriately and using the auxiliary variable $k = e - p^d$ then gives

$$2a_3k + \bar{u}^d = (1+a_1)/\beta_w\dot{p}^d - a_2(1-2\alpha)\dot{k},$$
$$2(1-\alpha)k + p^d - \bar{m}^d = (b_2 - b_1/\beta_w)\dot{p}^d + b_2\dot{k}.$$

In matrix notation this in turn gives with respect to the signs involved in these two equations[5]

$$\begin{pmatrix} + & + \\ - & + \end{pmatrix}\begin{pmatrix} \dot{p}^d \\ \dot{k} \end{pmatrix} = \begin{pmatrix} 0 & + \\ + & + \end{pmatrix}\begin{pmatrix} p^d \\ k \end{pmatrix}.$$

Since the determinant of the matrix on the left hand side of this matrix equation is positive (and thus also the determinant of the inverse of this matrix) and the determinant of the matrix on the right hand side is negative, we get that this implicit differential equation system gives rise to a negative system determinant when solved explicitly (by multiplication of the right hand

---

[4] Note that the third equation is solved for y^d and inserted into the first two equations of the difference dynamics.

[5] If β_w is again assumed to be sufficiently small and considering that $\alpha \in (0.5, 1)$.

side with the inverse of the matrix on the left-hand side). Not surprisingly we therefore get (for adjustment speeds of wages chosen sufficiently small) that the difference dynamics is of saddlepoint type with respect to its unique steady state solution which is given by

$$p_o^d = \bar{m}^d + (1-\alpha)/a_3\bar{u}^d, \quad s_o = e_o - p_o^d = -\bar{u}^d/(2a_3) \quad (\text{and } y_o^d = 0).$$

Of course, reformulating the dynamics in terms of p^d and e provides us with the same result, see Turnovsky (1986, p. 143) in this regard. We thus have now the situation that the jump-variable technique of the rational expectations school must be applied to the differences between the two countries and be translated back to the individual countries thereafter in order to discuss the consequences of monetary or fiscal shocks in such a two-country framework.

In this way Turnovsky (1986) obtains, for example, the following two results on unanticipated and anticipated monetary shocks, specifically an increase in the domestic money supply by one unit, with foreign money supply held constant, which raises the steady state values of p and e by one unit and leaves the steady state value of p^* unaltered. For both situations we quote directly from Turnovsky (1986) in order to describe the results of this analysis in the spirit of the originally chosen presentation and explanation.[6]

Unanticipated Monetary Expansion

> *The formal solution to the model in the case of an unanticipated monetary disturbance is illustrated in [Fig. 6.2]. In this case, the exchange rate overshoots its long-run response, on impact, thereafter appreciating toward its new steady state level. The price of domestic output gradually increases, while domestic output initially increases, thereafter falling monotonically towards its natural rate level. The monetary expansion causes an immediate fall in the domestic interest rate, which thereafter rises monotonically toward its equilibrium. All these effects are familiar from the Dornbusch model or its immediate variants. The effects of the domestic monetary expansion on the foreign economy are less clear cut. The rate of inflation p^* of foreign goods and the level of output abroad will rise or fall on impact, depending on an eigenvalue relationship.*

[6] Note we have changed references to figures and their notation to conform with the numbering and notation of this chapter.

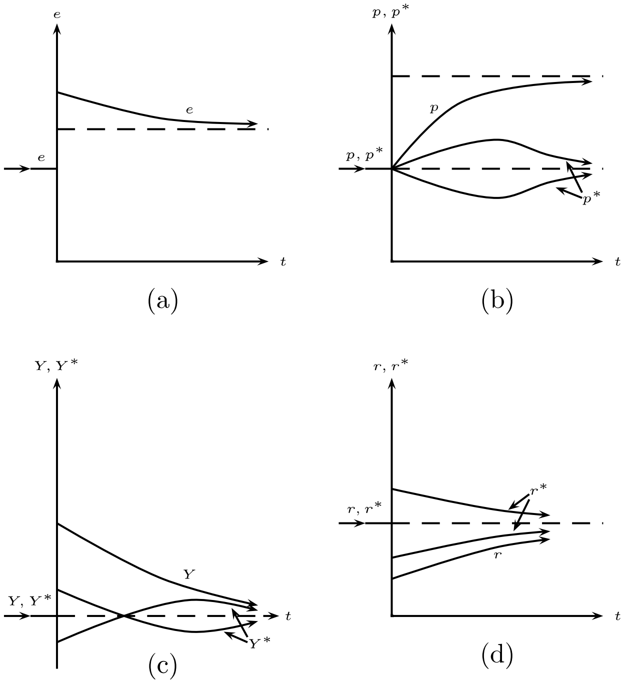

Fig. 6.2. Unanticipated monetary expansion: **(a)** exchange rate; **(b)** prices; **(c)** outputs; **(d)** interest rates

Announced Monetary Expansion

Consider now the behavior of the economy in response to a monetary expansion which the authorities announce at time zero to take place at some future time $T > 0$*. The time paths for the relevant domestic and foreign variables are illustrated in Fig. 6.3. At the time of announcement* $(t = 0)$ *the domestic currency immediately depreciates in anticipation of the future monetary expansion. Whether the initial jump involves overshooting of the exchange rate depends upon the lead time* T*. Following the announcement, the domestic currency continues to depreciate until time* T*, when it reaches a point above the new long-run equilibrium. Thereafter, it appreciates steadily until the new steady state equilibrium is reached. This behavior is identical with that in the Gray and Turnovsky (1979) model. The anticipation of the future monetary expansion causes the domestic price level to begin rising at time zero. The inflation rate increases during the pe-*

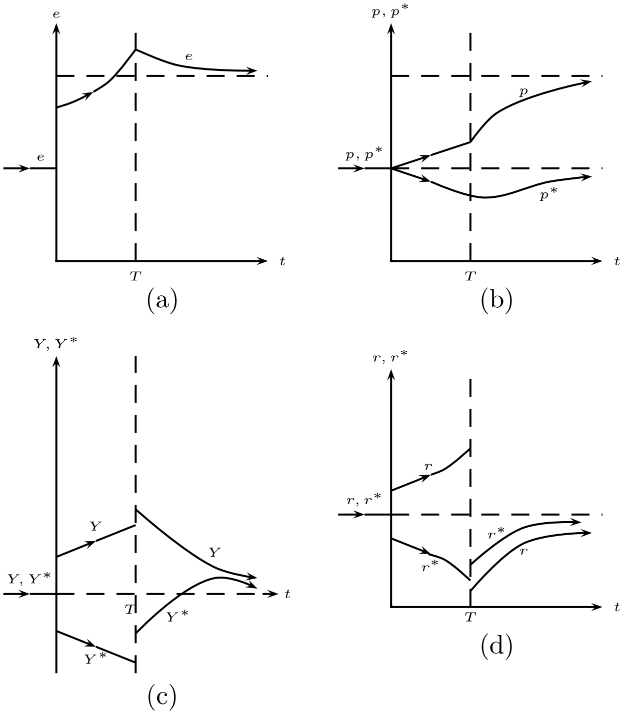

Fig. 6.3. Announced monetary expansion: **(a)** exchange rate; **(b)** prices; **(c)** outputs; **(d)** interest rates

riod 0 to T, when the monetary expansion occurs. This expansion causes a further increase in the inflation rate, which thereafter begins to slow down as the new equilibrium price level is approached. The behavior of the inflation rate is mirrored in the level of output. The positive inflation rate generated by the announcement is accompanied by an immediate increase in output, which increases continuously until the monetary expansion occurs at time T. At that time a further discrete increase in output occurs. [...] At the same time, the initial increase in domestic real output, stimulated by the depreciation of the domestic currency as a result of the announcement, increases the demand for real money balances. In order for domestic money market equilibrium to be maintained, the domestic nominal interest rate must rise. As the price of domestic output increases during the period prior to the monetary expansion, the real domestic money supply contracts further, while the increasing real income causes the real money de-

mand to continue rising. In order for money market equilibrium to be maintained, the domestic nominal interest must therefore continue to rise. At time T, when the anticipated monetary expansion takes place, the domestic interest rate drops, falling to a level below its long-run equilibrium. Thereafter, it rises steadily back towards its (unchanged) long-run equilibrium.

This only partial discussion of the very detailed and numerous results obtained in Turnovsky (1986) shows that interesting conclusions can be drawn from the application of the jump variable technique to unanticipated and even more to anticipated policy shocks. The reader is referred to this very rigorous article for further details on the formal and the verbal explanation of these and other policy studies.

6.4 The Blanchard Stock-Market Dynamics

The Blanchard (1981) output and stock market dynamics reads in its basic form as follows:

$$\dot{Y} = \beta_y(Y^d(Y,q) - Y), \tag{6.21}$$

$$i(Y) = r(Y)/q + \hat{q}: \quad \Rightarrow \quad \dot{q} = i(Y)q - r(Y). \tag{6.22}$$

We here combine a standard dynamic multiplier story for the dynamics of the output level Y (where however aggregate demand Y^d depends positively on Tobin's q in place of a negative dependence on the real rate of interest) with the situation that bonds and equities are perfect substitutes, including myopic perfect foresight concerning the capital gains on equities. We here identify Tobin's average $q = p_eE/pK$ with the share price p_e by assuming that the number of equities E to the value of the capital stock pK is constant and set equal to 1. We denote by $i(Y)$ the inverted LM curve and by $r(Y)$ the profit rate function of the economy (profits per unit of capital) with all profits paid out as dividends to equity owners. The ratio $r(Y)/q$ is then the dividend rate of return on equities (since pK cancels) and $\hat{q} = \dot{q}/q$ are the capital gains per unit of equity, i.e., $r(Y)/q + \hat{q}$ is the (total) rate of return on equities, set equal to the interest rate here due to the perfect substitute assumption.

The two laws of motion of the Blanchard output and stock-market dynamics give rise to two isoclines, an example of which is shown in Fig. 6.4. Moreover, they typically imply saddlepoint dynamics for their intersections

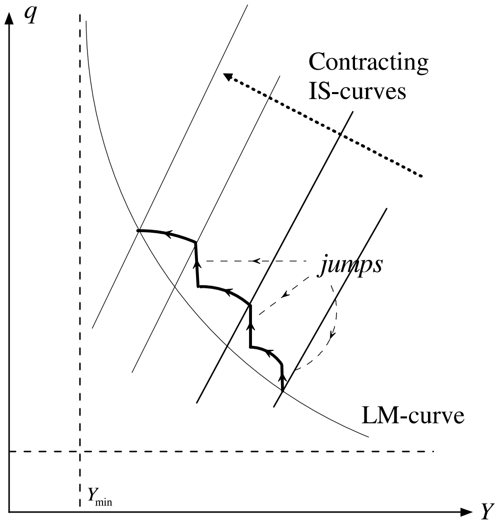

Fig. 6.4. The Blanchard stock-market dynamics in the bad news case

(for the exceptions, see below). Their phase diagram and the stable arm that is directed towards the saddlepoint steady state can therefore be used as usually to apply the JVT of the rational expectations school. This is extensively done in Blanchard's (1981) original paper and need not be repeated here. There are however some problems with this approach to a real-financial market interaction. Even in the case where all behavior is assumed as linear we have obtained here at a 2D non-linear dynamical system as indicated in Fig. 6.4 for Blanchard's bad news case (with a decreasing LM curve). In Blanchard's good news case (where the interest rate is more sensitive to output changes than the profit rate) the LM curve of Fig. 6.4 is positively sloped and it allows for situations of 2, 1 or no steady state, with one steady state being a stable node or focus in the first case. The Jump Variable Technique (JVT) of the Rational Expectations School is therefore then not always applicable which again shows one of its limitations.

We here however concentrate on the case shown in Fig. 6.4 where the relevant stable manifold of the then uniquely determined steady state of saddlepoint type is an upward sloping curve. We assume that the economy is hit by three severe and unanticipated contractive demand shock which shift the IS curve of this model significantly to the left as shown in Fig. 6.4. Equity-financed firms thus experience severe underutilization problems with respect to their productive capacity, but the stock market shows a strong stock-price rally until the final steady state is reached with low rates of capacity utilization of firms, low dividend payments, but high stock prices. This is a situation which surely needs some explanation from the proponents of the RE approach, in the justification of the economic meaningfulness of the JVT.

6.5 Interacting Blanchard Stock Market and Dornbusch Exchange Rate Dynamics: A Two-Country Framework

Since the preceding model types are well documented in the literature on real-financial markets interaction, we do not go into a further investigation of the Blanchard and Dornbusch model here, see however Chiarella et al. (2007) in this respect. Instead we ask the question what the outcome will be if we synthesize the Dornbusch and Blanchard model into a two country model of the world economy. This is a big step forward in the dynamics to be considered and therefore simplified here somewhat by considering for the time being prices (and wages) as given, and normalized to 1. Phillips-curve driven price dynamics is however the next step that has to be considered (see the Sect. 6.8 in this chapter).

As Keynesian aggregate demand functions we postulate now the relationship (all prices = 1, but the nominal exchange rate $s[EUR/US]$ is perfectly flexible):

$$\begin{aligned} Y^d &= C(Y, s) + I(q, s) + G(s) + X(Y^*, s), \\ Y^{d*} &= C^*(Y^*, -s) + I^*(q^*, -s) + G^*(-s) + X^*(Y, -s) \end{aligned}$$

concerning the two goods markets that exist in this two-country two-commodity world.[7] These functions represent the demand side of the two considered economies and they are to be inserted again into the dynamic multiplier process of the Blanchard model:

$$\dot{Y} = \beta_y(Y^d(Y, Y^*, q, s) - Y), \tag{6.23}$$

$$\dot{Y}^* = \beta_{y^*}(Y^{d*}(Y^*, Y, q^*, s) - Y^*) \tag{6.24}$$

in order to provide a full description of the dynamics of the real part of the economy. We note that domestic demands, as well as exports, depend of course on the exchange rate s (positively in the domestic economy and negatively in

[7] C consumption, I investment, G government expenditure and X exports.

the foreign one). We have made this latter negative dependency an explicit one, since we can then simply say, that the case of symmetric economies is characterized by identical demand functions and adjustment speeds in the two economies. In the local stability analysis we shall later on work with their linearizations and thus with identical corresponding parameter values in order to allow to calculate average as well as difference economies from the model of this section.

Besides these two laws of motion we have now six financial assets in our world economy (2 types of money, 2 types of bonds and two types of equities) which under the assumptions of perfect substitution and myopic perfect foresight give rise to three independent laws of motion now:[8]

$$i(Y) = r(Y)/q + \hat{q}: \quad \Rightarrow \quad \hat{q} = i(Y) - r(Y)/q, \tag{6.25}$$

$$i^*(Y^*) = r^*/q^* + \hat{q}^*: \quad \Rightarrow \quad \hat{q}^* = i^*(Y^*) - r^*(Y^*)/q^*, \tag{6.26}$$

$$i(Y) = i^*(Y^*) + \hat{s}: \quad \Rightarrow \quad \hat{s} = i(Y) - i^*(Y^*). \tag{6.27}$$

We interpret the inverted LM-functions $i(Y), i^*(Y^*)$ now as interest rate policy rules which here only exhibit the output gap as an argument, since inflation is still excluded from the considered dynamics.

Concerning steady state calculations we here refer to the later on considered situation of symmetric countries which implies that averages behave exactly as the closed economy case considered in Blanchard (1981). All difficulties of the original Blanchard model with respect to its steady state determination are therefore also present in the two-country case. The Jacobian of the considered 5D dynamical system in the state variables Y, Y^*, q, q^*, s reads in its sign structure as follows:

$$J_0 = \begin{pmatrix} - & + & + & 0 & + \\ + & - & 0 & + & - \\ \pm & 0 & + & 0 & 0 \\ 0 & \pm & 0 & + & 0 \\ + & - & 0 & 0 & 0 \end{pmatrix}.$$

The Dornbusch and Blanchard type feedback chains (between Y, s and Y, q) are clearly visible, but this does not easily imply that this matrix will have

[8] The following equations imply that all rates of return on the internationally traded asset must be the same (and therefore equal to $r^*(Y^*) + \hat{q}^* + \hat{s} = r(Y) + \hat{q} - \hat{s}$ in addition).

three unstable and two stable roots as it would be needed for a successful application of the JVT. The task for the rational expectations school is moreover, since the model has in correspondence to the characteristics of its eigenvalues, three forward looking, non-predetermined variables q, q^*, s and two predetermined ones, Y, Y^*, that are only gradually adjusting, to determine a 2D stable manifold (and a 3D unstable manifold) in the 5D phase space such that the non-predetermined variables can always jump to a unique point of the stable manifold with predetermined (temporarily given) output levels of the two countries. This must hold in the case of unanticipated shocks, while in the case of anticipated ones one has to find in addition the single bubble of length T in the old 5D dynamics through a jump to a position in the 5D phase space in the old dynamics such that this bubble has a soft landing on the new stable manifold exactly at time T (where the anticipated shock would occur).

The solution to these problems (with significant calculation costs for the theorist as well as the assumed type of economic agents) assumes calculation capabilities that are much beyond anything that can be characterized as "rational expectations". We view such a procedure as a wrong axiomatization of what is actually happening in the factual world, with an inherent tendency to use more and more complicated constructions or eventually purely mathematical (economically seen: black-box) iteration mechanisms in order to get by assumption the result that the economic dynamics of such models—in their deterministic core—are always of the type of a shock absorber (or occupied with finding the correct bubble that leads them safely to the new shock absorber (coming into existence at time T). It is obvious that we do not regard this as a promising route for further macrodynamic investigations. We therefore will now reconsider the structure of the private sector in order to find an interest rate policy rule that can stabilize it in the conventional sense of this word.

6.6 A Model-Oriented Reformulation of the Taylor Interest Rate Rule

Instead of pursuing the JVT methodology here any further, we raise the question whether the interest rate policy rule of the Central Banks should not consider the given structure of the economy first before one decides on the economic signals that should guide their choice of an interest rate in such a world. We thus stress that the Taylor rule should be tailored in view of the structure of the economy in which it is supposed to work. Since output dynamics are of stable multiplier type, and since repelling forces only concern the financial sector of the economy (tamed by assumption through the JVT of the RE school), it appears plausible to look at the evolution of either Tobin's q or the UIP exchange rate dynamics to find the variables that should steer the interest rate setting strategy of the CB's. After some experimentation with these possibilities we propose here to use and test the rules:

$$i = i_o + a_i(s_o - s), \tag{6.28}$$

$$i^* = i_o^* - a_i^*(s_o - s). \tag{6.29}$$

When these rules are applied to the UIP condition of the private sector

$$i \overset{UIP}{=} i^* + \hat{s}$$

they deliver the astonishingly simple result:

$$\hat{s} = i_o + a_i(s_o - s) - (i_o^* - a_i^*(s_o - s)) = g(s)$$
$$\text{with} \quad g(s_o) = i_o - i_o^*, \; g'(s) < 0.$$

We thus get that the exchange rate dynamics become independent of the rest of the economy under these choices of policy rules in the two countries and give rise to monotonic convergence to the steady state value s_o jointly set by the two CB's if policy coordination implements the condition $i_o = i_o^*$.

Policy formulation of the described kind (that exhibits private sector structure awareness) therefore removes all repelling forces from the exchange rate dynamic as considered in Fig. 6.1. The only troubling aspect may here be that the above implies that the CB's should do just the opposite of what they might be induced to do in the real world, since—for example the ECB—should lower here the interest rate in the case of an exogenous upward jump in the nominal exchange rate s, a depreciation of the Euro. Leaning against a depreciation in actual economies may however mean that one should attract capital inflows and thus raise the domestic rate of interest, but in the model we are considering this would imply instability and not convergence to the steady state value s_o.

In order to investigate the full dynamics of the model one has to consider of course the remaining four laws of motion in addition. These laws of motion are fully interacting and thus from the perspective of the Routh-Hurwitz stability conditions somewhat demanding. In the next section we will therefore consider the case of symmetric economies, as in Turnovsky (1986), and will linearize the model around its (indeed now uniquely determined) steady state position to prove the local asymptotic stability of the steady state of such a two-country real and financial markets interaction.

The chosen modified form of the Taylor rule does not only simplify the dynamics that is implied by the model, but it also implies that there is in general a unique steady state solution, in contrast to the Blanchard model type considered beforehand. We here use, as in Blanchard (1981), linear aggregate demand functions and ignore the already given steady state position s_o in their formulation.[9] On this background they read:

$$Y^d = a_y Y + b_y q + G + c_y Y^* + const.,$$
$$Y^{d*} = a_{y^*} Y^* + b_{y^*} q^* + G^* + c_{y^*} Y + const.$$

with the propensities to consume, invest and export all given parameters. For Tobin's q we in similar way get (assuming a linear profit rate function as in Blanchard (1981)):

$$q = (a_r Y + b_r)/i_o, \qquad q^* = (a_{r^*} Y^* + b_{r^*})/i_o^* \quad (i_o = i_o^*).$$

Inserting these two equations, which are now linear ones, into the two goods-market equilibrium conditions, provides us with two linear equations for the state variables Y, Y^* which in general have a uniquely determined solution. Of course, the parameters of the model have to be chosen in addition such that the steady state levels of the outputs of the two countries are positive and imply positive profit rates for both of them. We therefore in sum get that coordinated monetary policy provides us with the steady state values of both i and s via the UIP condition, while stationary financial market equilibrium allows us to remove Tobin's q from the goods market equilibrium conditions which then imply the steady state values for both Y and Y^*, on the basis of which the steady state values of the $q's$ can then be determined.

Linearizing also the above 5D dynamics (with the exception of the intrinsically nonlinear q-dynamics) in this way (around the just determined steady state position) gives the dynamical system:

[9] Note however that we will apply this linearity assumption also in the next section (where s is a variable). This means that we have to linearize there the goods market demand function with respect to the s-influence and thus will get local results there only.

$$
\begin{aligned}
\dot{Y} &= \beta_y[-(1-a_y)Y + b_y q + c_y Y^* + d_y s + e_y],\\
\dot{Y}^* &= \beta_{y^*}[-(1-a_{y^*})Y^* + b_{y^*} q^* + c_{y^*} Y - d_{y^*} s + e_{y^*}],\\
\dot{q} &= (i_o + a_i(s_o - s))q - r, \quad r = a_r Y + b_r,\\
\dot{q}^* &= (i_o^* - a_i^*(s_o - s))q^* - r^*, \quad r^* = a_r^* Y^* + b_r^*,\\
\hat{s} &= i_o + a_i(s_o - s) - (i_o^* - a_i^*(s_o - s)).
\end{aligned}
$$

The Jacobian matrix of this linearized dynamics exhibits the following sign structure:

$$
J_0 = \begin{pmatrix} - & + & + & 0 & + \\ + & - & 0 & + & - \\ - & 0 & i_o & 0 & - \\ 0 & - & 0 & i_o & + \\ 0 & 0 & 0 & 0 & - \end{pmatrix}.
$$

The stability of its unique steady state solution will be investigated in the following section by means of an important simplifying device that can be applied to study the interaction of large economies that are sufficiently similar to each other in their real as well as their financial parts.

6.7 Symmetric Countries: Stability Analysis

We consider now the artificial variables $Y^a = (Y+Y^*)/2, q^a = (q+q^*)/2$, the averages of the GDP's and Tobin's q's as well as $Y^\delta = Y - Y^*, q^\delta = q - q^*$, the differences of the GDP's and Tobin's q's and will use in the following the above (partial) linear representation, which includes the export functions $X(Y^*, s), X^*(Y, -s)$. Moreover we now use the pair of Taylor rules $i = i_o + a_i(s_o - s), i^* = i_o - a_i(s_o - s)$ for our stability investigations of the symmetric two-country case. It is obvious from the preceding section that the steady values of the case of a difference economy are zero as far as outputs and Tobin's q's are concerned, i.e., the steady state average values share symmetry with the parameters of the model and are given just by the unique values Y_o, q_o of the case of the average economy.

6.7.1 The Average Economy

In order to show the stability of the full 5D dynamics we assume the case of two symmetric large open economies, in which case all corresponding parameters of the two countries are of the same size, with opposite signs if the exchange rate is involved (also in the policy rules). Taking averages $Y^a = (Y + Y^*)/2, q^a = (q+q^*)/2$ therefore allows to combine the goods market equilibrium conditions into a single world market equation, where the exchange rate effect has been canceled. The stock market dynamics can in the same way be aggregated into a single equation. This in sum gives a 2 dimensional dynamical system as it is shown below. This average economy represents a hypothetical economy that is of the Blanchard (1981) closed economy type (without the intrinsic nonlinearities in the equity dynamic that have complicated the Blanchard stock market model, since the product iq is now simply given by $i_o q$).

$$\begin{aligned}\dot{Y}^a &= \beta_y[(a_y + c_y - 1)Y^a + b_y q^a + const.],\\ \dot{q}^a &= i_o q^a - a_r Y^a + const.\end{aligned}$$

For the Jacobian matrix of this economy we immediately get the result:

$$J = \begin{pmatrix} \beta_y(a_y + c_y - 1) & \beta_y b_y \\ -a_r & i_o \end{pmatrix} = \begin{pmatrix} - & + \\ - & i_o \end{pmatrix}$$

if we have multiplier stability ($a_y + c_y < 1$) and if the output adjustment speed is sufficiently large ($\beta_y[1 - (a_y + c_y)] > i_o$ is solely needed). These conditions are of conventional type in the first case and not at all restrictive in the second case, implying that the average economy is generally asymptotically stable and thus always convergent to its steady state position. Since the interest rate i_o is indeed a small number, setting it 0 in addition exemplifies, that adjustment to the steady state is cyclical for an intermediate range of the output adjustment speed.

6.7.2 The Difference Economy

Taking differences $Y^\delta = Y - Y^*, q^\delta = q - q^*$ implies a three dimensional dynamics (with these state variables and the exchange rate s). These dynamics are, formally seen, of the type of the small open economy as considered in Dornbusch (1976). Note that we have a $d_y s$ term in the aggregate demand equation now and that the domestic Taylor rule is given by: $i = a_i(s_o - s) + i_o$, while the one of the foreign economy has a negative sign in front of the adjustment speed a_i.

$$\begin{aligned}\dot{Y}^\delta &= \beta_y[(a_y - c_y - 1)Y^\delta + b_y q^\delta + 2d_y s],\\ \dot{q}^\delta &= [-2a_i(s - s_o) + i_o]q^\delta - a_r Y^\delta,\\ \hat{s} &= -2a_i(s - s_o),\end{aligned}$$

$$J_0 = \begin{pmatrix} \beta_y[(a_y - c_y - 1) & \beta_y b_y & \beta_y 2d_y \\ -a_r & i_o & -2a_i q_o^\delta \\ 0 & 0 & -2a_i s_o \end{pmatrix} = \begin{pmatrix} - & + & + \\ - & i_o & - \\ 0 & 0 & - \end{pmatrix}.$$

It is obvious from the structure of this Jacobian that asymptotic stability and convergence here holds under the same (and even weaker) conditions as in the case of averages.

6.7.3 Summary

Summing up, we have that differences must converge to 0 (and s to s_o) and that averages must converge to their common steady state values implying that the full 5D dynamics is also characterized by convergence towards their steady state position. This is a convenient short-cut to the analysis of the full 5D system when the parameters of the two countries differ from each other. We thus have an economy now that is stable in the conventional sense of this word. These should be of interest to policy makers if they could accept that leaning against the wind means in this setup just the opposite of what they might be inclined to do intuitively.

6.8 Dornbusch Inflation Dynamics

As in Turnovsky (1986) we now add inflation dynamics in the two countries in the form of the following two Phillips curves (assuming again symmetry between the two countries):

$$\hat{p} = \beta_w(Y - \bar{Y}), \quad \hat{p}^* = \beta_w(Y^* - \bar{Y}).$$

We thus assume that the current output gaps drive current inflation rate, but do not yet consider acceleration terms in these two Phillips curves.

Linearizing these equations around the steady state position (for local stability analysis) gives rise to:[10]

$$\dot{p} = \beta_w p_o(Y - \bar{Y}), \quad \dot{p}^* = \beta_w p_o^*(Y^* - \bar{Y}).$$

Since the price levels in the two country are now moving in time we have to use the real exchange rate $\sigma = sp^*/p$ in the goods markets dynamics behind the

[10] Note here and in the following that we do not use logarithms, but only use first order Taylor approximations for the terms we want to linearize.

parameter d_y now which when linearized by first order Taylor approximation gives rise to the following modification of the model considered in Sect. 6.6 (with symmetry now assumed in addition):

$$\begin{aligned}
\dot{Y} &= \beta_y\left[-(1-a_y)Y + b_y q + c_y Y^* + d_y\sigma_o\left(\frac{s}{s_o} + \frac{p^*}{p^*_o} - \frac{p}{p_o}\right) + e_y\right],\\
\dot{Y}^* &= \beta_y\left[-(1-a_y)Y^* + b_y q^* + c_y Y - d_y\sigma_o\left(\frac{s}{s_o} + \frac{p^*}{p^*_o} - \frac{p}{p_o}\right) + e_y\right],\\
\dot{q} &= (i_o + a_i(s_o - s))q - r, \quad r = a_r Y + b_r,\\
\dot{q}^* &= (i_o - a_i(s_o - s))q^* - r^*, \quad r^* = a_r Y^* + b_r,\\
\hat{s} &= 2a_i(s_o - s),\\
\dot{p} &= \beta_w p_o(Y - \bar{Y}),\\
\dot{p}^* &= \beta_w p^*_o(Y^* - \bar{Y}).
\end{aligned}$$

Considering the averages of these 7D dynamical system gives rise to the same 2D dynamics as already investigated in Sect. 6.7. Convergence to the steady state averages is therefore ensured in this extension of the model of Sect. 6.6. With respect to differences we now however get the following 4D dynamics (under the assumption that $p_o = p^*_o$ holds, see below):

$$\begin{aligned}
\dot{Y}^\delta &= \beta_y[-(1-a_y-c_y)Y^\delta + b_y q^\delta + 2d_y\sigma_o\frac{s}{s_o} - 2d_y\sigma_o/p_o\cdot p^\delta],\\
\dot{q}^\delta &= [-2a_i(s-s_o) + i_o]q^\delta - a_r Y^\delta,\\
\hat{s} &= -2a_i(s-s_o),\\
\dot{p}^\delta &= \beta_w p_o Y^\delta.
\end{aligned}$$

We assume again that the autonomous dynamic of the nominal exchange rate has already settled down at its steady state position s_o and need therefore only investigate the stability of the remaining 3D dynamics in the state variables $Y^\delta, q^\delta, p^\delta$. The Jacobian matrix of this reduced dynamics exhibits the following sign structure:

$$J_0 = \begin{pmatrix} - & + & - \\ - & i_o & 0 \\ + & 0 & 0 \end{pmatrix}.$$

It is easy to show that the Routh-Hurwitz stability conditions are all fulfilled (since i_o is small) with the exception of the determinant of J_o which is positive in this case (but small, due to its multiplicative dependence on i_o). The system is therefore slightly explosive and needs further stabilizing effort from the side of monetary policy in order to allow full convergence to its steady state position.

Moreover, also this steady state position needs some further discussion, since output values are now equal to the NAIRU value $\bar{Y}$ in the steady state and therefore no longer determined through goods market equilibrium (but through labor market equilibrium instead). From the (nonlinearized) equations of the model we can first of all conclude that s must be equal to the value s_o (not yet determined) via the UIP condition. This in turn implies by means of the postulated Taylor rules $i = i^* = i_o$, i.e., interest rate equality with the steady state rate of interest rate set by the Central Banks. We then get for the values of Tobin's q given expressions of the type $r(\bar{Y})/i_o$ which when inserted into the goods market equilibrium equations (which are identical for the two countries) determine the steady state value of the real exchange rate σ, since output must be equal to its natural level. We now assume that the Central Banks know this natural level of σ and base their interest rate policy on the (so far undetermined) condition $s_o = \sigma_o$. This then finally implies that $p_o = p_o^*$ must hold true in the steady state, a condition that is needed for the application of the symmetric country assumption and the mathematical methodology based on it (concerning $\dot{p}^\delta$ here).

In order to get full convergence in the above two country model with Dornbusch inflation dynamics we now augment the coordinated Taylor rules of the two countries as follows:

$$i = i_o + a_i(s_o - s) + b_i\hat{p}, \tag{6.30}$$

$$i^* = i_o - a_i(s_o - s) + b_i\hat{p}^*. \tag{6.31}$$

This gives for the law of motion for s now the equation:

$$\begin{aligned}\hat{s} = i - i^* &= 2a_i(s_o - s) + b_i\dot{p} - b_i\dot{p}^* = 2a_i(s_o - s) + b_i\dot{p}^\delta \\ &= 2a_i(s_o - s) + b_i\beta_w p_o Y^\delta.\end{aligned}$$

This extended Taylor rule modifies the Jacobian of the difference economy for the now 4 interacting state variables Y^δ, q^δ, s, p^δ as follows:

$$J_0 = \begin{pmatrix} - & + & + & - \\ - & i_o & 0 & 0 \\ 0 & 0 & - & + \\ + & 0 & 0 & 0 \end{pmatrix}.$$

Since i_o is small, the trace of the matrix J_o is surely negative. Using the same argument again, the sum of principal minors of order two is easily shown to be positive. It is also easily shown that the determinant of the 4D Jacobian is positive (and small). For the sum of principal minors of order three we finally get a negative value if the value of the new parameter b_i in the Taylor rule is chosen sufficiently small. We thus have that the coefficients a_i of the characteristic polynomial of the matrix J_o are all positive as required by the Routh-Hurwitz stability conditions. According to Asada et al. (2003, Theorem A.6) we have to show however in addition for the validity of asymptotic stability that there holds: $a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$. Since the determinant of J_o is small we have that a_4 is close to zero. There thus remains to be shown that $a_1a_2 - a_3 > 0$ holds true for the trace and the principal minors of order 2 and 3. Using again the conditions that i_o is small and b_i chosen with care then implies also this result, since the expressions in a_1a_2 are then dominating the principal minors of order three of the Jacobian matrix J_o.

We thus get that a cautious anti-inflationary interest policy rule can stabilize the economy where Phillips curve dynamics has been added to the goods markets and asset markets dynamics we have considered in the preceding sections. This makes our model comparable to the Turnovsky (1986) IS-LM two country model, but now with Tobin's q in the aggregate demand function in place of the real rate of interest of conventional textbook analysis. We here however arrive at the result that the Central Banks should use conventional gaps as well as an unconventionally signed nominal exchange rate gap in order to stabilize the economy in the presence of perfect substitution between bonds and equities and perfect foresight on capital gains through stock price and exchange rate dynamics.

6.9 Outlook: Imperfect Capital Markets

This chapter has shown—if one accepts the perfectness assumptions made with respect to asset substitution and expected capital gains—that a better solution to the then implied instability problems (the centrifugal exchange rate as well as stock price dynamics) may be to choose the Taylor interest rate policy rule appropriately in the light of the structure of the private sector, rather than to try to enforce three unstable roots and two stable ones on the dynamics by a conventional type of Taylor rule in order to allow for the application of the JVT of the RE school. Yet, even if one can generate stability in this way, this solution procedure nevertheless shows that the extremely perfect structure of the financial sector implies the need for a—from an applied point of view—fairly strange interest rate reaction function (as far as financial markets are concerned), where it has to be assumed that a monetary policy intended to counteract the country's exchange rate depreciation should lower the interest in this country, but not increase it as conventional wisdom might imply.

The reason for this strange result can be easily detected if exchange rate dynamics are formulated with some sluggishness in their reaction to interest rate differentials. A simple illustration is provided by the following example, where we assume given exchange rate expectations $\hat{s}^e$ for the time being:

$$\hat{s} = \beta_s(i^*(s) + \hat{s}^e - i(s)).$$

Clearly, a positive reaction of the domestic interest rate and a negative reaction of the foreign to the exchange rate, the opposite of what we have used in the preceding section, would contribute now to exchange rate stability, if it is (of course) assumed that exchange rates are increasing if expected returns on foreign bonds are higher than the ones on domestic bonds. This postulated exchange rate reaction is compatible with the direction of capital flows behind the assumed adjustment equation. Yet, going from such fast to infinitely fast exchange rate reactions implies $i(s) = i^*(s) + \hat{s}^e$ which together with the assumption of myopic perfect foresight gives

$$\hat{s} = \hat{s}^e = i(s) - i^*(s),$$

i.e., a sign reversal with respect to the role played by the interest rate differential. This is the reasons why also policy must accept a sign reversal in its orientation in order to be successful in this limit case. It also suggests that approaching the limit case is producing a discontinuity in the behavior of the economy. More generally speaking, we claim here that the limit case is structurally unstable with respect to any model that allows for imperfect substitution and imperfect foresight, i.e., it cannot be approached by considering degrees of imperfections that are shrinking to zero. But if the limit is of strictly isolated importance only, it may be questioned whether it is useful for policy advice. This also holds for the models of Sects. 6.3 and 6.5 which are close in spirit to the nowadays popular DSGE approach which is in fact used for applied policy analysis, see Smets and Wouters (2003) for an example.

The conclusion we draw from this is that model's of portfolio choice with only imperfect asset substitution augmented be heterogeneous (necessarily imperfect) expectation formation and somewhat delayed adjustment processes are the better choice if one wants to model the world in a (mathematically seen) robust and descriptively seen relevant way, in place of the extremely idealized stock market (long-term bond markets) and exchange rate dynamics of today's DSGE models, where determinacy and convergence is enforced, but not proved. Such modifications of the perfectness assumptions of the present chapter will be the topics investigated in the remaining chapters of this part of the book.

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7

Macroeconomic Imbalances and Inflation Dynamics in a Mundell-Fleming-Tobin Framework

7.1 Introduction

Over the last decade, the unprecedented increase in the internal and external imbalances of two of the largest economies in the world, the U.S. and China, has raised serious concerns about the dangers of such macroeconomic imbalances for the stability of the world economy. Nevertheless, a correction of these imbalances by means of a significant readjustment concerning the U.S. dollar is not very probable in the near future, due to the reluctance to deal with this issue not only of both governments but also because many other countries do not make steps into this direction. In this light, national wage-price adjustments as well as foreign asset in- and outflows have become even more important as macroeconomic adjustment mechanisms to these imbalances.

In view of such developments, we consider in this chapter in a Mundell-Fleming-Tobin framework the dynamics of government deficits or surpluses as well the direction of foreign assets accumulation, here for a small open economy, when financial assets are assumed to be imperfect substitutes (where therefore the UIP condition does not hold) and where inflation dynamics is driven by an open-economy Phillips curve.[1] Initially we assume, as in Rødseth (2000), that domestic bonds are non-tradables, i.e., the accumulation of foreign assets by the domestic economy can only occur through surpluses in the current account and not via balanced exchanges of domestic against foreign

[1] The first part of this chapter (without international capital flows) is based on Flaschel et al. (2006): "Twin Deficits and Inflation Dynamics in a Mundell-Fleming-Tobin Framework". CEM Working Paper 143, Bielefeld University.

assets on the international capital markets, to later investigate the differences that the incorporation of international capital movements implies.

Concerning the goods markets in the domestic economy, we introduce a Keynesian demand constraint in place of the assumption of Say's Law as in Flaschel (2006), where the world market delivered or consumed everything not present or not needed in the domestic economy. This will be done by adding an export function (and now also an investment function) to the description of aggregate goods demand in the domestic economy. Finally, we will now assume regressive exchange rate expectations formation in place of rational expectations which, on the one hand, helps to avoid the questionable variable "jumps" associated wit the rational expectations assumption (which deliver stability by assumption in an otherwise unstable saddlepoint environment) and which, on the other hand, allows for maximum stability of the considered dynamics (since only fundamentalist and thus in principle only converging expectations revisions are allowed for). This simplifying assumption will be helpful for the central objective of the chapter which is to focus on the fundamental destabilizing forces contained in the two accumulation equations for internal and external deficits or surpluses, caused by the government budget equation and the evolution of the current account of the considered economy.

We thus allow, compared to Flaschel (2006), for underutilized labor and capital due to insufficient effective demand on the goods market,[2] as they can result from Keynesian consumption and investment demand, augmented by a net export schedule in this open economy, and the goods market equilibrium condition based on these aggregate demand functions. We consider on this basis a Keynesian approach to the business cycle in the real part of the model (based on an IS curve that equates savings to investment). Due to our use of a standard open economy money wage Phillips curve, this real cycle is accompanied by labor market driven inflation or deflation dynamics which—in combination with the capital account and government budget dynamics of the type considered in Flaschel (2006)—provides a dynamic model that goes significantly beyond standard Mundell-Fleming type approaches. Finally, since we are now considering exports and imports simultaneously, we are of course now using a two commodity (but single small country) approach in contrast to the paper of Flaschel (2006), where primarily only one commodity cases were considered.

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[2] And of course also overutilized labor and capital depending on the state of aggregate demand.

Despite the intrinsic government budget and capital account dynamics of the model, we have also a rich set of feedback channels present in it: Hicksian disposable income effects, Pigou price level effects, Keynes price level effects, the Mundell-Tobin effect of inflationary expectations in both the consumption and the investment function, Dornbusch exchange rate effects, portfolio effects, and the stated stock-flow interactions. The interaction of these effects allows for a variety of (in-)stability results, too numerous in order to allow their investigation in a single paper of this model type. We therefore concentrate in this chapter on basic studies of a regime with pegged interest as well as exchange rate and contrast this situation with a regime where the exchange rate is perfectly flexible and the money supply a given magnitude, under the control of the Central Bank of the domestic economy.

In the next section we derive the model of this chapter and the stock-flow interaction that characterize its intrinsically generated dynamics (which includes also a Phillips curve approach to inflation dynamics), calculate the steady state position of the dynamics and perform some preliminary stability considerations. Section 7.3 investigates the case of an interest rate peg coupled with an exchange rate peg and derives stability as well as instability results for such a monetary regime. In Sect. 7.4 we do the same for the regime of perfectly flexible (overshooting) exchange rates and a given money supply. Section 7.5 concludes.

7.2 The General Framework

As point of departure of our theoretical analysis we employ a standard small open economy Mundell-Fleming-Tobin model, as discussed in detail in Rødseth (2000, Chap. 6).[3] On this basis we intend to analyze rigorously the stock-flow dynamics that are generated by the capital account in the balance of payments (with respect to the foreign bond accumulation by private households) and by the government budget constraint (with respect to their domestic bond holdings), when these dynamics are linked (and interact) with macroeconomic activity levels through price level adjustments they imply.

[3] For a comprehensive discussion of the Mundell-Fleming model, see Gandolfo (2001, Chaps. 10–11).

7.2.1 Budget Equations and Saving/Financing Decisions

We start with the budget equations of the three relevant sectors, households, the government and the central bank. With respect to firms, we assume that all of their income is transferred to the household sector and that households then provide the credit needed to allow them to finance their investment expenditures (which is by and large the same as the assumption of a direct investment decision by the household sector). Concerning the Central Bank (CB) we note that it may change its government bond holdings by means of an open market policy $dB_c = dM$, and similarly its foreign bond holdings through $dF_c = dM$, without influencing the flow budget equations to be discussed below, since all interest income from these bond holdings is transferred back to the government sector which therefore only has to pay interest on the bonds B held by the private sector. The domestic bond holdings of the CB can therefore be neglected in the following and an additional indexation of the magnitude that is held by the household sector can thus be avoided.

The budget restrictions for the three sectors of the domestic economy are given by:

$$p(Y-T) + iB + ei^*F_p \equiv pC + pI + \dot{M} + \dot{B} + e\dot{F}_p, \tag{7.1}$$

$$pT + ei^*F_c + \dot{B} + \dot{M} \equiv pG + iB, \tag{7.2}$$

$$\dot{M} \equiv \dot{B}_c. \tag{7.3}$$

We denote here by B, F_p the domestic and foreign bonds held by the household sector and by F_c the foreign assets held by the central bank (its currency reserves, that can only be changed by open market operations on the domestic market for foreign assets). For domestic bonds we assume, as in Flaschel (2006), that they are non-tradables. We use i for the nominal rate of interest and e for the nominal exchange rate and index by $*$ foreign set variables, which are obviously not under the control of the domestic economy. All other symbols are fairly standard and thus need no explanation here. We stress again that domestic and foreign interest incomes of the CB are transferred to the government sector so that the central bank can only change its asset position by printing new money. In the normal course of events it will channel this money into the economy by buying domestic government bonds as shown in the third budget equation, but it can also rearrange its portfolio of domestic and foreign bonds (not considered explicitly here) by open market operations on the domestic financial markets (as already considered above).

These budget equations imply for the evolution of domestic and foreign bonds held by the private sector of the domestic economy:

$$\dot{B} = iB + p(G - T) - ei^* F_c - \dot{M}, \tag{7.4}$$

$$e\dot{F}_p = e\dot{F} = p(Y - C - G - I) + ei^* F. \tag{7.5}$$

These equations show the nominal evolution of government debt implied by its budget constraint and of the country's foreign debt (or foreign surplus) position as implied by the balance of payments (which—due to the situation assumed to hold for the budget equations of the three sectors of our economy—is here always balanced, independently of the exchange rate and monetary regime that is to be investigated).[4]

Considering the given situation of a balanced balance of payments (without the intervention of the CB) from the viewpoint of savings we can write:

$$pS_p = p(Y - T) + iB + ei^* F_p - pC = pI + \dot{M} + \dot{B} + e\dot{F}_p,$$
$$pS_g = pT + ei^* F_c - iB - pG = -\dot{B} - \dot{M}$$

which gives for the total savings pS of the economy (NX net exports):

$$pS = pY - pC - pG + ei^*(F_p + F_c) = pNX + pI + ei^* F = pI + e\dot{F}_p$$
$$= pI + e\dot{F}, \quad F = F_p + F_c.$$

This is again the formulation of the fact that the balance of payments must be balanced in the considered situation without any further adjustment processes, based on the assumption that the issue of new money $\dot{M}$ and new government bonds $\dot{B}$ is in fact always accepted by the household sector (as it is implicitly made in the above formulation of the three budget equations of our economy).

7.2.2 Real Disposable Income and Wealth Expressions

We next derive expressions for the real disposable income of households and the government, respectively, as well as the corresponding real wealth or debt position. Disposable income is defined here in the conventional Hicksian way as the level of income which when consumed just preserves the current level of wealth of the considered sector. This is done first for the aggregate government

[4] This is due to assumption that the new issue of money and domestic government bonds is always voluntarily absorbed by private households.

sector (including the foreign interest income of the central bank) and thereafter for the sector of private households (cf. Flaschel (2006) in this regard). Since the rate of inflation[5] $\hat{p} = \dot{p}/p$ is a variable in the following completion of the model we will use this expression here already in the definition of the real rates of interest to be employed in the subsequent calculations.

By definition, the aggregate real income of the government sector Y_g^a is composed of the tax and interest rate payments and receipts of the government plus the inflation and capital gains on government debt and central bank reserves,

$$\begin{aligned} Y_g^a &:= T - \frac{iB}{p} + \frac{ei^* F_c}{p} + \hat{p}\frac{M+B}{p} + (\hat{e} - \hat{p})\frac{eF_c}{p} \\ &= T - (i - \hat{p})\frac{M+B}{p} + i\frac{M}{p} + (i^* + \hat{e} - \hat{p})\frac{eF_c}{p} \\ &= T + r\frac{M}{p} - (i - i^* - \hat{e})\frac{M+B}{p} + (i^* + \hat{e} - \hat{p})\frac{-(M+B) + eF_c}{p} \\ &= T + i\frac{M}{p} + \xi\frac{M+B}{p} + r^* W_g^a, \end{aligned} \tag{7.6}$$

where real aggregate government wealth is defined as

$$W_g^a := \frac{-(M+B) + eF_c}{p} = -\frac{(M+B)}{p} + \frac{eF_c}{p} = W_g + W_c, \tag{7.7}$$

and $\xi = i^* + \hat{e} - i$ and $r^* = i^* + \hat{e} - \hat{p}$, represent the actual risk premium on foreign bonds and the actual real rate of return on foreign bonds, respectively. Note that the third term in (7.6) can be ignored if the uncovered interest rate parity condition is assumed to hold. This is however only possible if international trade in domestic and foreign bonds is allowed (as it will be the case in the next section).

Concerning the Hicksian real disposable income of the private sector, it is defined as

$$Y_p = Y - T + (i^* + \hat{e} - \hat{p})\frac{eF_p}{p} + (i - \hat{p})\frac{B}{p} - \hat{p}\frac{M}{p},$$

or, after some manipulations,

$$Y_p = Y - T + r^*(W - W_g^a) - \xi\frac{M+B}{p} - i\frac{M}{p}, \tag{7.8}$$

where the actual wealth of the private sector is defined as the difference between the wealth of the whole economy W and the aggregate wealth of the government sector W_g^a, i.e.

[5] We use in this text $\hat{x}$ to denote the growth rate of a variable x.

$$W_p = \frac{M + B + eF_p}{p} = W - W_g^a. \tag{7.9}$$

From the calculations of the disposable income of households and the government, we finally get:

$$Y_p = Y - Y_g^a + r^* W \quad \text{or} \quad Y_p + Y_g^a = Y + r^* W \quad (\text{with} \quad W = \frac{e(F_p + F_c)}{p})$$

as a relationship between the country's total disposable income, its domestic product and the real interest on domestically held foreign bonds.

7.2.3 Temporary Equilibrium: Output, Interest and Exchange Rate Determination

Having described the budget restrictions and disposable income equations of the different sectors in the domestic economy, we follow again Rødseth (2000) in his description of the temporary equilibrium relationships on the goods and the asset markets, which are given by:

$$\begin{aligned}
Y &\overset{IS}{=} C(Y_p, W - W_g^a, r, r^{*e}) + I(Y, r, r^{*e}) + G + NX(\,\cdot\,, \eta, Y^*), && (7.10)\\
M/p &\overset{LM}{=} m^d(Y, i), m_Y^d > 0, m_i^d < 0, && (7.11)\\
W &\equiv eF/p \overset{FF}{=} e(F_p^d + F_c^d)/p = f^d(\xi^e, W - W_g^a) + eF_c^d/p, && (7.12)\\
& \quad f_{\xi^e}^d > 0, f_{W_p}^d \in (0,1), && \\
B/p &= W_p - f^d(\xi^e, W_p) - m^d(Y, i), \quad \xi^e = i^* + \epsilon(e) - i && (7.13)
\end{aligned}$$

with $r = i - \hat{p}$ and $r^{*e} = i^* + \epsilon(e) - \hat{p}$ representing the real domestic interest rate and the expected real return on foreign bonds (with $\epsilon(e)$ representing a regressive expectations mechanism for example). Furthermore, $\eta = \frac{ep^*}{p}$ denotes the real exchange rate. While (7.10) represents an IS-curve of an advanced traditional type,[6] the money market equilibrium, the standard textbook LM relationship, is described by (7.11), where money demand is as usual assumed to depend positively on the level of output and negatively on the domestic interest rate. Equation (7.12), the FF-curve, which represents the equilibrium on the market for foreign bonds (foreign exchange), is determined primarily by the reaction of private household with respect to the risk premium ξ and the marginal wealth effect $f_{W_p}^d$ in foreign (and domestic) bond demand, as

[6] The component "·" in the net export function stands for the variables that determine the domestic absorption in terms of consumption and investment demand.

in Rødseth (2000), with $F = F_p^d + F_c^d$. The market for domestic bonds, on the other hand, is always cleared by Walras's Law of Stocks, when the money market and the market for foreign bonds are in equilibrium.

These equations are to be solved (by means of the implicit function theorem) for the variables considered as statically endogenous (depending on the monetary and exchange rate regimes that are assumed) and to be inserted into the laws of motion of p, B and F_p in order to obtain an autonomous system of ordinary differential equations, describing domestic price level dynamic, the dynamic of the government budget constraint, and of the foreign position of the domestic economy. Of course, the definitions of Y_p, r, r^{*e}, η have also to be inserted to achieve this end. Note that the variables M, B, F, i, e may become policy parameters depending on the monetary and exchange regime that is under consideration.[7]

Concerning the interest rate, by applying the implicit function theorem to (7.11) and assuming that money demand is based on the full employment output level $\bar{Y}$, we get for its level in its reduced form representation the formula:

$$i = i(p, M, \bar{Y}) \quad \text{with} \quad i_1 > 0, i_2 < 0, i_3 > 0.$$

The theory of the nominal rate of interest is thus the standard or even textbook one of Keynesian macroeconomics and closely related to the working of the so-called Keynes-effect whereby money wage decreases stimulate the economy when they imply price level changes in the same direction and on this basis lower the interest rate which then works on consumption and investment via the real rate of interest.

Concerning the nominal exchange rate, inserting the above equation into (7.12) gives us an equation in the endogenous variables e and p and thus provides us with a theory of the nominal exchange rate in its dependence on the dynamically endogenous stock variables B, F_p and the price level p.

$$F_p = pf^d\left(i^* + \epsilon(e) - i(p, M, \bar{Y}), \frac{M + B + eF_p}{p}\right)\Big/ e. \tag{7.14}$$

We recall that f_1^d, $f_2^d > 0$ is assumed to hold and that we have $\epsilon'(e) < 0$ due to the assumption of a regressive formation of expectations of exchange rate de- or appreciation.

[7] In the case of a flexible exchange rate regime and a given money supply we will for example get jumps in the variables e, η, W, W_g^a when there is a shock in money supply occurring.

In the case of a perfectly flexible exchange rate e where M, F_c are given magnitudes that are under the control of the central bank, and $F = F_p + F_c$ is also assumed to be a given magnitude, we obtain by means of the implicit function theorem the following expressions for the partial derivatives with respect to the dynamically endogenous variables:

$$\frac{\partial e}{\partial F_p} = -\frac{e(1-f_2^d)}{F_p(1-f_2^d) - pf_1^d \epsilon'} < 0,$$
$$\frac{\partial e}{\partial B} = \frac{f_2^d}{F_p(1-f_2^d) - pf_1^d \epsilon'} > 0,$$
$$\frac{\partial e}{\partial p} = \frac{f^d - pf_1^d i_1 - f_2^d W_p}{F_p(1-f_2^d) - pf_1^d \epsilon'} = \frac{eF_p(1-f_2^d)/p - pf_1^d i_1 - f_2^d \frac{M+B}{p}}{F_p(1-f_2^d) - pf_1^d \epsilon'} < 0$$

with $0 < f_2^d < 1$ (the portfolio choice condition). We note that the last partial derivative is negative if the degree of capital mobility with respect to the risk premium is chosen sufficiently high (which is what we do in the following). The signs of the partial derivatives shown above will also apply to the real exchange rate in the place of the nominal one, due to its definition $\eta = e(p)p^*/p$. Note finally that the first two partial derivatives will approach zero if capital mobility approaches infinity and that in this case the limit of the partial derivative with respect to p is simply given by i_1/ϵ'. The result $\frac{\partial e}{\partial p} < 0$ reflects the message of the Dornbusch (1976) model, according to which an increase in the domestic interest rate (caused by shrinking real balances) leads to higher depreciation gains expectations which in a regressive expectations environment demands for a decrease, an appreciation of the nominal exchange rate.

Since the dynamic behavior of prices enters the IS-equilibrium condition through the real interest rate, we already define here the law of motion for the price level p, which is determined, under the assumption of constant markup-pricing, by a standard, expectations augmented, open-economy Phillips curve[8]

$$\hat{p} = \beta_w(Y - \bar{Y}) + \gamma\hat{p} + (1-\gamma)(\pi^* + \epsilon(e)) = \beta_w(Y - \bar{Y})/(1-\gamma) + \epsilon(e), \quad (7.15)$$

with $\gamma \in (0,1)$ and where the output gap $Y - \bar{Y}$ measures the demand pressure on the labor market. Note that this form of a Phillips curve derives from an expectations augmented one where the cost pressure item (concerning the consumer price index) is initially given by a weighted average of domestic and import price inflation of the form: $\gamma\hat{p} + (1-\gamma)(\pi^* + \epsilon(e))$ and where myopic

[8] See Rødseth (2000, Chap. 6) for its motivation.

perfect foresight is assumed with regard to the evolution of the domestic inflation rate. The foreign rate of inflation, π^*, is assumed to be zero for sake of simplicity.

By inserting the open-economy Phillips curve equation in the IS equilibrium equation described by (7.10), we can calculate the following signs of the partial derivatives, which for the case where the speed of adjustment of money wages β_w is chosen sufficiently low, are:

$$\frac{\partial Y}{\partial F_p} > 0, \qquad \frac{\partial Y}{\partial B} > 0, \qquad \frac{\partial Y}{\partial p} < 0.$$

This holds again only in the case where in addition capital mobility is assumed as sufficiently high. We leave the lengthy calculations of the involved partial derivatives here to the reader and only state that increasing wealth of private households with respect to domestic and foreign bond holdings stimulate economic activity, while and increasing price level will reduce economic activity through various channels in the model, in particular through real wealth effects in consumption demand, but also through interest rate increases and real exchange rate decreases.

In the case where capital mobility is nearly perfect ($f_\xi^d \approx \infty$) and wages nearly rigid with respect to demand pressure ($\beta_w \approx 0$) we may summarize the comparative static properties—as far as the dynamically endogenous variables are concerned—approximately as follows:

$$i = i(p), i' > 0, \quad e = e(p), e' < 0, \quad Y(F_p, B, p), \quad Y_1 > 0, Y_2 > 0, Y_3 < 0.$$

7.2.4 Dynamics and the Steady State of the Economy

Our dynamical representation of the Mundell-Fleming-Tobin model in its general form (with F_c a given magnitude in the considered case of flexible exchange rates and with $p^* = 1$ for simplicity),[9] expressed in nominal terms, consists of the following differential equations

$$\dot{F}_p = i^*(F_p + F_c) + NX(\cdot, \eta, Y^*)/\eta, \quad \eta = \frac{e}{p}, \; p^* = 1, \tag{7.16}$$

$$\dot{B} = iB + pG - pT - ei^*F_c \tag{7.17}$$

[9] Such an approach is further justified through the observation that the dynamics of the nominal stock magnitude B is involved in the above considered real dynamics as long as $\xi \neq 0$ holds true which shows that the real dynamics is then in fact a hybrid as far as the distinction between real and nominal variables is concerned.

to be coupled with the law of motion for the price level p:

$$\dot{p} = [\beta_w(Y - \bar{Y})/(1-\gamma) + \epsilon(e)]p \tag{7.18}$$

and with the description of the temporary equilibrium given by (7.10)–(7.12) and the definitions of private wealth and real disposable income discussed in the previous section.

In order to allow for a sequential determination of the steady state values of this dynamical system, we proceed as follows.[10] We assume that the steady state value i_o of i is given by i^*, i.e., we have $\xi = 0$ in the steady state. Due to the Phillips curve we furthermore know that $Y = Y_o$ must hold true. The given quantity of money M then allows to determine the steady state value of p as $p_o = M/m^d(Y_o, i^*)$. Setting $\dot{B} = 0$ furthermore gives a simple positive relationship between B_o and e_o, which represent the equilibrium in the domestic bond market. The FFB-curve implies on this basis:

$$\begin{aligned} F_{po} &\overset{FFB}{=} p_o f^d\left(0, \frac{M + B_o(e_o) + e_o F_{po}}{p_o}\right) \\ &= p_o f^d\left(0, \frac{M}{p_o} + \frac{T-G}{i^*} + \frac{e_o(F_{po} + F_c)}{p_o}\right), \end{aligned}$$

which defines a positive relationship between e_o and F_{po}, as the Fig. 7.1 (where we use linear curves for reasons of simplicity).

From $\dot{F} = 0$, the equilibrium in the foreign currency bond market, we furthermore get

$$i^* F_o + p_o NX(\cdot, e_o/p_o, Y^*) = 0,$$

that is, a negative relationship between e_o and F_{po}, represented in Fig. 7.1 by the $\dot{F} = 0$-curve. If we assume that $M + B_o$ is nonnegative (compare Fig. 7.1), we have a nonnegative demand for foreign bonds at $e_o = 0$ and thus a nonnegative value of F_{po} associated with it. In the case of linearity (which need not hold in the large) we will then get an intersection of the $\dot{F} = 0$ and the FFB curve in the positive half plane of the (F, e) space which gives us positive steady state values of both F_p and e (since one can assume that there is a positive value e where $NX = 0$ holds). Under the stated conditions

[10] Note that this procedure is only one possibility. More generally, if i_o is not fixed at the level i^* in advance, the steady state can be determined without any requirements for T and G as postulated below. The steady state value of i will then usually differ from i^*, which is, however, no problem because the uncovered interest parity need not to be fulfilled in this model.

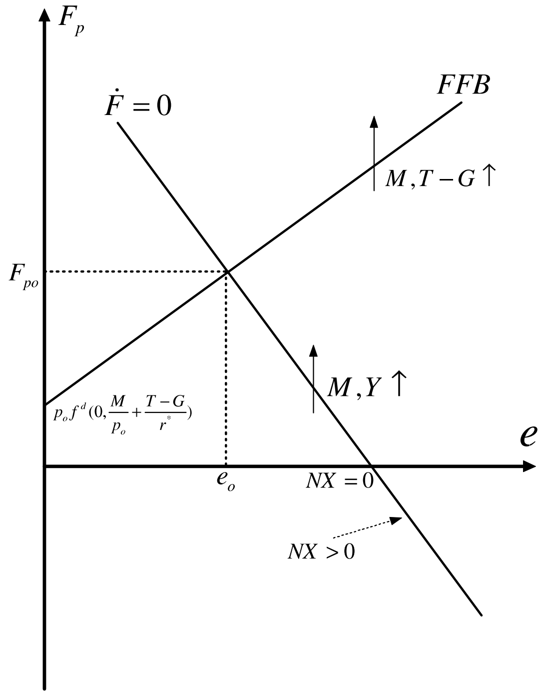

Fig. 7.1. Steady state stock equilibrium ($B_o > 0$)

we thus have that the private agents of the domestic economy will hold a positive amount of foreign bonds in the steady state and that net exports must therefore be negative in the steady state due to a value of the exchange rate that is below the one that balances the trade balance. In the general situation where both curves in Fig. 7.1 are nonlinear, one must show that the $\dot{F} = 0$ curve is not too flat in view of the intersection of the FFB-curve with the vertical axis and both curves are not approaching $+\infty$ for finite values of the nominal exchange rate.

Figure 7.1 shows on the basis of the above the determination of the steady state values of both the nominal exchange rate and the value of foreign debt held domestically, and also indicates how these values can be influenced by fiscal and monetary policy in particular. It shows that fiscal consolidation will make the currency stronger and increases the amount of foreign currency denominated debt held domestically. The latter effect is also produced by a monetary expansion, whose effect on the value of e may however be ambiguous, depending on the competitiveness of the considered economy, i.e., on the elasticity of the net export function with respect to the real exchange rate at the steady state of the economy. Note that at this steady state we have a negative trade account and a positive interest rate account and of course a balanced capital account in the balance of payments.

Inspecting the IS curve on the basis of what has been determined so far shows however that no endogenous variable is then left that allows for IS-equilibrium at the full employment level Y and the foreign interest rate i^*. We therefore now adjust the values T, G such that $T-G$ remains unchanged (that is without impact on the left hand side of the government budget constraint) until also the IS-curve passes through the determined steady state values. This holds if G, T are—on the basis of a given value for $G-T$—determined by:

$$\bar{Y} = C(\bar{Y} - T_o + i^* e_o F_{po}/p_o + i^* B_o/p_o, i^*, i^*) + G_o + NX(\cdot, e_o/p_o, Y^*)$$

if $I = 0$ holds in the steady state (see below).

Note here also that B_o/p_o as given by $(T-G)/i^* + \eta_o F_c$ need not be positive and in fact must be negative in the case of a value of $T-G$ that is sufficiently negative. This is due to the fact that interest payments rB_o are positive in the steady state in the case of a positive value of government debt, implying that real government income $T + e_o F_c/p_o$ must be sufficiently high relative to G to allow for the given interest obligation of the government (since there is no issue of new government debt in the steady state). Note also that we assume the behavioral functions and parameters of the model to be such that disposable income Y_p is positive at the steady state. Note finally that our regressive expectation function for exchange rate de- or appreciations is always assumed to fulfill $\epsilon(e_o) = 0$ in the steady state (is assumed to be asymptotically rational) and is therefore shifting with the steady state solution for e_o.

We have determined in advance the steady state values for p, i, Y, r, r^* and then simultaneously the steady state values for e, η, F_p, B basically from stock equilibrium conditions. We have so far not mentioned the capital stock K here whose growth rate is to be determined by I/K. The model's dynamics are in fact treated without any consideration of the law of motion of the capital stock. We justify this here by assuming as further consistency condition that $0 = I(Y, r, r^*)$ holds in the steady state. The capital stock will then converge to a certain finite value which is of no importance in the model as it has been formulated so far (since we have constant markup pricing by firms, i.e., prices do not react to demand pressure on the market for goods). Measuring this demand pressure by way of Y/K for example would suggest that this term should enter the investment function in the place of just Y. The dynamics of the capital stock would then feed back into the goods market and become interdependent with the rest of the dynamics, even if prices do not react to such a demand pressure term.

We now consider some features of the 3D MFT dynamics of this chapter close to the steady state as determined in the preceding subsection by means of the Jacobian matrix of the dynamics calculated at the steady state position of the economy.

7.2.5 Local Stability Analysis

In the presently considered policy regime we will first of all derive a situation where the steady state of the model is surrounded by centrifugal, that is repelling, forces. This case will then be contrasted with a situation where an attracting steady state is given. The calculations needed to show these two results will demonstrate that there exists indeed a multiplicity of situations where either stability or instability may prevail around the steady state. The conclusions will be that empirical and numerical methods are needed in order to get a more complete picture of the stock-flow dynamics of the considered open MFT economy and that it is likely that policy must be more active than it is currently assumed (in particular with respect to the government budget) to enforce convergent behavior around the steady state of such a small open economy.

Inserting the functions obtained from our short-term comparative static analysis into the laws of motion for the variables F_p, B, p gives rise to the following eigen-feedbacks of the considered state variables (if $\frac{\partial Y}{\partial F_p}$ is used in explicit form in the first partial derivative):

$$\begin{aligned}
\frac{\partial \dot{F}_p}{\partial F_p} &= i^*(1-C_Y)\frac{-NX_Y}{1-C_Y-I_Y-NX_Y} - C_{W_p}\frac{-NX_Y}{1-C_Y-I_Y-NX_Y} > 0, \\
&\quad \text{if} \quad 1-C_Y-I_Y > 0 \quad \text{and} \quad C_{W_p} \quad \text{sufficiently small}, \\
\frac{\partial \dot{B}}{\partial B} &= i^* > 0, \\
\frac{\partial \hat{p}}{\partial p} &= \epsilon'(e)e'(p) > 0.
\end{aligned}$$

We thus have in such a situation that the trace of the Jacobian of the dynamics at the steady state is positive (as is the determinant). The three state variables taken in isolation are therefore subject to repelling forces as far as their own steady state position is concerned:

$$J = \begin{pmatrix} + & - & \pm \\ 0 & + & \pm \\ 0 & 0 & + \end{pmatrix}$$

with in fact all eigenvalues of this Jacobian being real and positive and thus destabilizing.

Even if one assumes that the first of the above partial derivatives is negative, by allowing a high wealth effect on consumption C_{W_p}, and furthermore that the second partial derivative (in the diagonal of J) is sufficiently low (i.e., if the accumulation of foreign and domestic bonds is not by itself subject to strong destabilizing forces), we get for the considered Jacobian approximately still the following sign structure:

$$J = \begin{pmatrix} - & - & \pm \\ 0 & 0 & \pm \\ 0 & 0 & + \end{pmatrix}$$

due to the assumed nearly perfect capital mobility. High capital mobility is therefore problematic for the stability of the balance of payments adjustment process, the evolution of the government debt and for the dynamic price level of the considered small open MFT type economy. There are some forces, by contrast, that appear to be stabilizing, but more detailed analysis is in fact needed in order to get clear-cut results on local asymptotic stability and thus local convergence towards the steady state. Such stability issues will be approached in the next sections by means of two basic scenarios in the case of fixed as well as flexible exchange rate regimes.

7.2.6 Real Twin Deficit Accumulation and Inflation Dynamics

In order to highlight the role of price level adjustments, as well as the complications they imply for the theoretical modelling of the dynamics of the economy, we now reformulate the model in real terms, with the evolution of the real aggregate wealth of the economy and the government sector instead of the law of motions for F_p and B. On this basis we show in particular that the aggregate wealth of the government sector W_g^a is in its time rate of change—as in the case of the private sector—determined by deducting from its real disposable income the real consumption of this sector. This then provides us with a simple law of motion for real aggregate wealth of the government sector, besides the one we have determined for the total (foreign) wealth of the domestic economy in Flaschel (2006), to be reconsidered below. These two laws describe on the one hand the evolution of surpluses or deficits in the government sector and the evolution of current account surpluses or deficits, and thus in particular allow the joint treatment of the issue of twin deficits in an open economy with a government sector, but not yet with real capital accumulation and economic growth.

For the time rate of change of the aggregate wealth of the government, we get from (7.7):

$$\begin{aligned}
\dot{W}_g^a &= \frac{-(\dot{M}+\dot{B})+\dot{e}F_c}{p} - \frac{\dot{p}}{p}\frac{-(M+B)+eF_c}{p} \\
&= \frac{-(pG+iB-pT-i^*eF_c)+\hat{e}eF_c}{p} - \hat{p}\frac{-(M+B)+eF_c}{p} \\
&= T - i\frac{B}{p} + \hat{p}\frac{M+B}{p} + (i^*+\hat{e}-\hat{p})\frac{eF_c}{p} - G \quad \text{which finally gives} \\
\dot{W}_g^a &= Y_g^a - G = r^*W_g^a + i\frac{M}{p} + \xi\frac{M+B}{p} + T - G
\end{aligned} \tag{7.19}$$

as the law of motion for the real aggregate wealth or debt that characterizes the government sector as a whole.

Since this debt position is no longer constant, as in Flaschel (2006), we adjust next from this text, but in a self-contained way, the equation for the evolution of total wealth W of the economy to this case and then consider again private disposable income Y_p and private wealth W_p in its interaction with the evolution of aggregate government debt and the foreign position of the economy. Recall that we assume goods market equilibrium $Y-C-I-G = NX$ in the following derivations.

Concerning the aggregate (foreign) wealth of the domestic economy, we start defining it as

$$W := W_p + W_g + W_c = \frac{eF}{p}, \quad \text{with} \quad F = F_p + F_c$$

or in growth rates

$$\hat{W} = \hat{e} + \hat{F} - \hat{p} \quad \text{or} \quad \dot{W} = \hat{e}W + \frac{e\dot{F}}{p} - \hat{p}W. \tag{7.20}$$

Making use of (7.5), we obtain

$$\begin{aligned}
\dot{W} &= \hat{e}W + \frac{p(Y-C-G-I)+ei^*F}{p} - \hat{p}W \\
&= (\hat{e}-\hat{p})W + i^*\frac{eF}{p} + Y - C - G - I,
\end{aligned}$$

or alternatively,

$$\dot{W} = (i^* + \hat{e} - \hat{p})W + Y - C - G - I$$
$$= r^*W + Y - C - G - I, \quad r^* = i^* + \hat{e} - \hat{p}.$$

In the following we set foreign inflation equal to zero ($\pi^* = 0$),[11] and assume given policy parameters (M, T, G). Inserting the behavioral equations given by (7.10)–(7.12), we obtain, together with the law of motion for the price level described by (7.15), the following 3D dynamical system[12]

$$\dot{W} = r^*W + Y - C(Y_p, W - W_g^a, r, r^{*e}) - I(Y, r, r^{*e}) - G \tag{7.21}$$
$$= r^*W + NX(\,\cdot\,, \eta, Y^*), \tag{7.22}$$
$$\dot{W}_g^a = r^*W_g^a + \xi\frac{M+B}{p} + i\frac{M}{p} + T - G, \tag{7.23}$$
$$\hat{p} = \beta_w(Y - \bar{Y})/(1-\gamma) + \epsilon(e), \quad \gamma \in (0,1). \tag{7.24}$$

Note that we assume myopic perfect foresight with respect to inflation on the market for goods, but allow for errors in exchange rate expectations here. Note also that actual laws of motions are to be based on actual rates of changes in the exchange rate e, but that we have to use the expected depreciation rate inside behavioral relationships. We have assumed above—following Rødseth (2000, Chap. 6)—that total private consumption of domestic and foreign goods depends (positively) on disposable income, private wealth and (negatively) on the real rate of return expected for the two types of bonds, but not on the real exchange rate $\eta = ep^*/p$. Similarly it is assumed that total net investment depends (positively) on domestic economic activity and (negatively) on the same real rates of return of bonds. The real exchange rate $\eta = ep^*/p$ enters the goods market equilibrium condition only via exports by way of an export function of the type $X = X(\eta, Y^*)$.

Note again that the above laws of motion for W, W_g^a are based on actual rates of return and thus actual changes in the exchange rate, while some arguments in the consumption function and the investment function are expected ones (relying on our use of a regressive expectations scheme later on) and have thus to be characterized by an index e for "expected". Furthermore, we have assumed myopic perfect foresight with respect to the inflation dynamics and

[11] And consider of course all foreign variables as given for the small open economy.

[12] Note that—due to the assumed goods market equilibrium—$Y - C - I - G$ can always be replaced by NX if this is convenient for certain calculations of the model's implications.

thus do distinguish there between expected and actual inflation rates, with the former to be used in the behavioral relationships later on while the latter apply to the actual laws of motion for the considered wealth variables. The distinction between actual and perceived rates of return will become important when exchange rate dynamics is considered later on (in our representation of the Dornbusch model in a MFT approach to financial markets). Note finally that we may extend the consumption function of Rødseth (2000, Chap. 6) later on, if we want to distinguish between the consumption of the domestic and the foreign commodity and thus have to include then the real exchange rate into the consumption function explicitly.

7.3 Capital Account and Inflation Dynamics under Interest and Currency Pegs

We consider in this section the dynamic implications of a particular regime among the ones that are economically possible in the considered MFT framework. We here follow Rødseth (2000, Chap. 6.6) and choose a case where in fact the asset markets are sent into the background of the model, a case which therefore solely studies the interactions of the IS-curve with a conventional type of Phillips curve and the dynamics of the capital account. The conventional type of IS-PC analysis (without an LM-curve) is therefore here augmented by the change in foreign assets resulting from the excess of domestic savings over domestic investment (or v.v.). The case considered in this section may be applicable—after some modifications—to an economic situation as represented by the Chinese economy (at least for certain time periods of this economy).

7.3.1 Assumptions

The assumptions we employ in order to derive this special case from our general framework are the following ones:

1.) Given Y^*, p^*, i^* : The small open economy assumptions
2.) $i = i^*$: An interest rate peg by the central bank (via an accommodating monetary policy)
3.) $\bar{e} = \text{const}\ (= 1)$: A fixed exchange rate via an endogenous supply of dollar denominated bonds by the central bank (which is never exhausted)

4.) W_g^a : A tax policy of the government that keeps the aggregate wealth of the government fixed
5.) $G_2, I_2 = 0$: Only consumption goods are import commodities which are never rationed
6.) ω : The real wage is fixed by a conventional type of markup pricing
7.) ρ_f^n : The normal rate of return of firms is fixed (since the real wage is a given magnitude) and set equal to i^* for simplicity
8.) $Y^p = y^p K, L^d = Y/x$: Fixed proportions in production (y^p, x capital and labor productivity, respectively)
9.) $K = const$: The capacity effect of investment is ignored. Potential output $Y^p (= 1)$ is therefore a given magnitude
10.) $\bar{Y} = x\bar{L} (= 1)$: A given level of the full employment output

On the basis of these assumptions we get that the real rates of interest are equalized for the domestic economy: $r^* = i^* + \hat{e} - \hat{p} = i - \hat{p} = r$. Furthermore, the risk premium ξ is zero in the considered situation. Finally, due to the assumed tax policy we have for the disposable income in the household sector: $Y_p = Y + r^* W - G$. Private wealth W_p is given by $W - W_g^a$ in the considered situation.

7.3.2 The Model

We define again the real exchange rate by $\eta = (ep^*)/p$, i.e., the amount of domestic goods that is exchanged for one unit of the foreign good. This rate reduces to $1/p$ due to the above normalization assumptions. Households directly buy investment goods for their firms and use only the normal rate of profit in order to judge their performance, which is a given magnitude here due to the above assumptions (normal output times the profit share). We moreover need only consider one real rate, the rate r, in the following formulation of the consumption decisions (for domestic and foreign goods) and the investment decisions of the household sector now. Note that we now distinguish imported from domestic goods in domestic consumption demand and include the real exchange rate on this substructure of total private consumption now.

$$C_1 = C_1(Y_p, W_p, r, \eta) : \quad \text{consumption demand for the domestic good}$$

$$C_2 = C_2(Y_p, W_p, r, \eta) : \quad \text{consumption demand for the foreign good}$$

$$C = C_1(Y_p, W_p, r, \eta) + C_2(Y_p, W_p, r, \eta)/\eta : \quad \text{total consumption}$$

$$I = I(Y, r) : \text{investment demand, for domestic goods solely}$$

On this basis the goods market equilibrium or the IS-curve of the model is given by:

$$C(Y_p, W_p, r, \eta) + I(Y, r) + G + NX(\cdot) = Y,$$

where net exports are based on a standard export function and import demand as determined by C_2. Imports can be suppressed in this equation by reformulating it as follows

$$Y = C_1(Y + rW - G, W - W_g^a, r, \eta) + I(Y, r) + G + X(Y^*, \eta),\ \eta = 1/p, r = i^* - \hat{p}.$$

We have assumed here that exports X depend as usually on foreign output and the real exchange rate.

We note that we have to add the law of motion for $K : \dot{K} = I(Y, i^* - \frac{1}{1-\gamma}\beta_w(Y-1))$ for a complete treatment of the dynamics of this example of a small open economy. Since however the analysis without such capacity effects of investment is already complicated enough we do not go into such an extended dynamic analysis here. Furthermore, we may also return to an endogenous treatment of the variable W_g^a which would increase the complexity of the analysis further, despite the simple framework that is here chosen (where portfolio choice does not matter for the analysis of the dynamics of the real part of the model, T, G given magnitudes):

$$\begin{aligned}
\hat{p} &= \beta_w(Y-1)/(1-\gamma), \\
\dot{W}_g^a &= (i^* - \hat{p})W_g^a + i^* m^d(Y, i^*) + T - G, \\
\dot{W} &= (i^* - \hat{p})W + X(Y^*, 1/p) \\
&\quad - C_2\left(Y + (i^* - \hat{p})(W - W_g^a) - i^* m^d(Y, i^*) - T, W - W_g^a, i^* - \hat{p}, 1/p\right)/p.
\end{aligned}$$

This extension would again allow for the discussion of the occurrence of twin deficits and other situations of domestic and foreign debt/surpluses.

7.3.3 Steady State Determination

For reasons of simplicity we return here, however, to the situation where aggregate government wealth (basically the government deficit) stays constant in time (by choosing T appropriately) and thus investigate now the steady state solution and the dynamics of the following system:

$$\begin{aligned}
\hat{p} &= \frac{1}{1-\gamma}\beta_w(Y(p, W) - 1), \\
\dot{W} &= (i^* - \hat{p})W + X(Y^*, 1/p) \\
&\quad - C_2(Y(p, W) + (i^* - \hat{p})W - G, W - W_g^a, (i^* - \hat{p}), 1/p)/p,
\end{aligned}$$

where the properties of the IS-equilibrium are characterized by the standard partial derivatives we have discussed above.

With respect to the steady state solution of this dynamical system we assume[13] that the government pursues—in addition to its tax policy—a constant government expenditure policy that is aimed at fixing the steady state value of exports at the level $\bar{X}$. This implies that the steady state value of the price level, p_o, is to be determined from $\bar{X} = X(Y^*, 1/p_o)$. Assuming that this equation has a (uniquely determined) positive solution for p_o we then can obtain the steady state value of W from the labor market equilibrium equation $1 = \bar{Y} = Y(p_o, W_o)$. The solution of this equation may be positive or negative and is again uniquely determined, since the right hand side of this equation is strictly increasing in W_o. The level of government expenditure G that allows for this solution procedure, finally, is then given by

$$0 = i^* W_o + \bar{X} - C_2(\bar{Y} + i^* W_o - G, W_o - W_g^a, i^*, 1/p_o)/p_o.$$

In this equation, this expenditure level is adjusted such that net imports are equal to the foreign interest income of domestic residents, i.e. an excess of imports over exports is needed in the case of a positive foreign bond holdings in the domestic economy in the steady state. Note that the above is also based on the assumption that $I(\bar{Y}, i^*) = 0$ holds in the steady state, since there must be a stationary capital stock in the steady state of the model.

7.3.4 Stability Analysis

Due to our simplifying assumption on the goods-market equilibrium equation $Y = Y(p, W)$ which guarantees that price level increases reduce economic activity and thus provide a check to further inflationary tendencies and which induce economic activity to increase with dollar denominated wealth due to its effects on consumption we have a straightforward sign structure in the partial derivatives of the first law of motion. The second law of motion is however much more difficult to handle. Its partial derivative with respect to W is much more involved than the one in Flaschel (2006) and given by (due to $\hat{p}_o = 0$):

$$\frac{\partial \dot{W}}{\partial W} = i^*(1 - \eta_o C_{2Y_p}) - \eta_o C_{2Y_p} \frac{\partial Y}{\partial W} - \eta_o C_{2W_p} + \hat{p}_W [(\eta_o C_{2Y_p} - 1) W_o + \eta_o C_{2\rho}],$$

[13] As in the previous section, this procedure is only chosen to ensure $i_o = i^*$. In the more general case where i_o is allowed to differ from this value, no assumption has to be made concerning the level of G.

where we have denoted by $\hat{p}_W$ the partial derivative of the first law of motion with respect to W. We have now considerably more terms in the feedback of foreign debt on its time rate of change than was the case in Flaschel (2006) for the there considered perfectly open economy. These additional terms seem to provide more support for a negative feedback chain compared to Flaschel (2006) (if the assumptions on $Y(p, W)$ hold true) where we simply had that this partial feedback mechanism became positive (destabilizing) when wealth effects in consumption are sufficiently weak.

The remaining partial derivative for local stability analysis is given by

$$\frac{\partial \dot{W}}{\partial p} = ((\eta_o C_{2Y_p} - 1)W_o + \eta_o C_{2\rho})\hat{p}_p - \eta_o C_{2Y_p}\frac{\partial Y}{\partial p} + (C_2 + \eta_o C_{2\eta} - X_\eta)/p^2).$$

The first two expressions in this equation are positive in sign while the last term in brackets—the quantitative reaction of net imports to price level changes via the real exchange rate channel—is generally assumed as being negative ($\eta = (ep^*)/p = 1/p$). For the Jacobian J we thus in sum get as sign structure:

$$J = \begin{pmatrix} \frac{\partial \hat{p}}{\partial p}p & \frac{\partial \hat{p}}{\partial W}p \\ \frac{\partial \dot{W}}{\partial p} & \frac{\partial \dot{W}}{\partial W} \end{pmatrix} = \begin{pmatrix} - & + \\ \pm & \pm \end{pmatrix}.$$

The easiest case for a stability result is

$$J = \begin{pmatrix} \frac{\partial \hat{p}}{\partial p}p & \frac{\partial \hat{p}}{\partial W}p \\ \frac{\partial \dot{W}}{\partial p} & \frac{\partial \dot{W}}{\partial W} \end{pmatrix} = \begin{pmatrix} - & + \\ - & - \end{pmatrix},$$

i.e., the case where interest effects do not dominate the capital account adjustment process and where the normal reaction of the trade balance (based on the so-called Marshall-Lerner conditions) dominates the income, wealth and interest rate effects generated by the general form of a consumption function used in this chapter and Flaschel (2006). The steady state is in this case obviously locally asymptotically stable (since trace $J < 0$, det $J > 0$). Graphically, we get in this situation the phase diagram shown in Fig. 7.2.

In view of this figure we must however keep in mind the very restrictive assumptions we have made with respect to the IS-curve and its replacement by the evolution of the state variables of the dynamics, the reaction of the balance of payments and the dynamics of the capital account and also on the reaction of foreign bond accumulation with respect to price level changes. It is therefore by no means clear how dominant the case of stable price level and capital account dynamics are in the set of all possible stability scenarios even

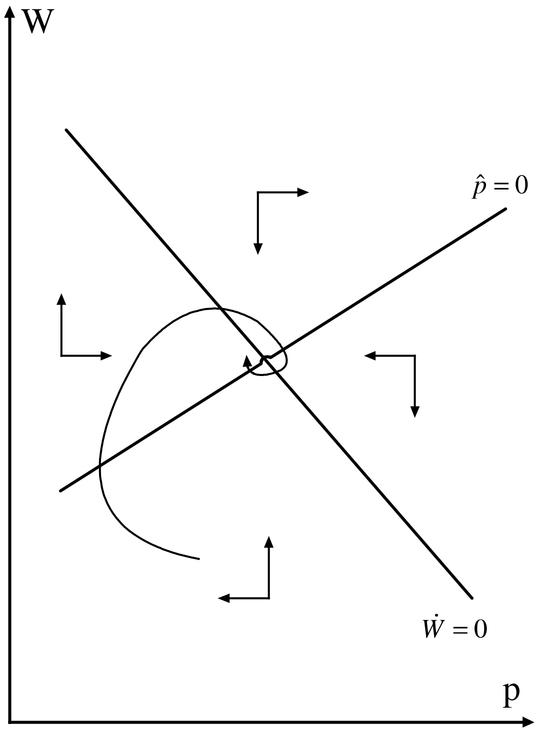

Fig. 7.2. The special example of stable inflation and capital account dynamics

in the special case of a MFT economy here under consideration. In the case of divergence the question is of course what mechanisms in the private sector may then keep the dynamics bounded or what policy actions are needed to ensure this.

The depicted dynamical implications are by and large of the type considered in Rødseth (2000, Chap. 6.6), though our consumption and investment functions differ to some extent from the ones used by there. Note that Rødseth (2000, Chap. 6.6) is using the negative of W in order to characterize the international credit or debt position of the domestic economy. Rødseth (2000, Chap. 6.6) also considers a variety of further issues for this case of a stable steady state of the Mundell-Fleming regime with an interest and ab exchange rate peg. The reader is referenced to this analysis for further aspects of this stable monetary and exchange rate regime.

By contrast, the worst case scenario (for instability) is given by the situation

$$J = \begin{pmatrix} \frac{\partial \hat{p}}{\partial p} p & \frac{\partial \hat{p}}{\partial W} p \\ \frac{\partial \dot{W}}{\partial p} & \frac{\partial \dot{W}}{\partial W} \end{pmatrix} = \begin{pmatrix} - & + \\ + & + \end{pmatrix},$$

in which case the steady state clearly is of saddlepoint type. This situation is depicted in Fig. 7.3.

Advocates of the jump variable technique will not be able to apply this technique here under consideration, since both the price level p and the foreign position W of the economy are predetermined variables here (despite myopic perfect foresight as far as domestic inflation rates are concerned). The solution to the instability shown in Fig. 7.3 can thus not be found in an ad hoc imposition of appropriate jumps in the price level, but must be found through the consideration of private sector or public policy behavioral changes when the economy has departed too much from its steady state position. We consider this a descriptively relevant approach to observed instabilities in the adjustment of the balance of payments in particular.

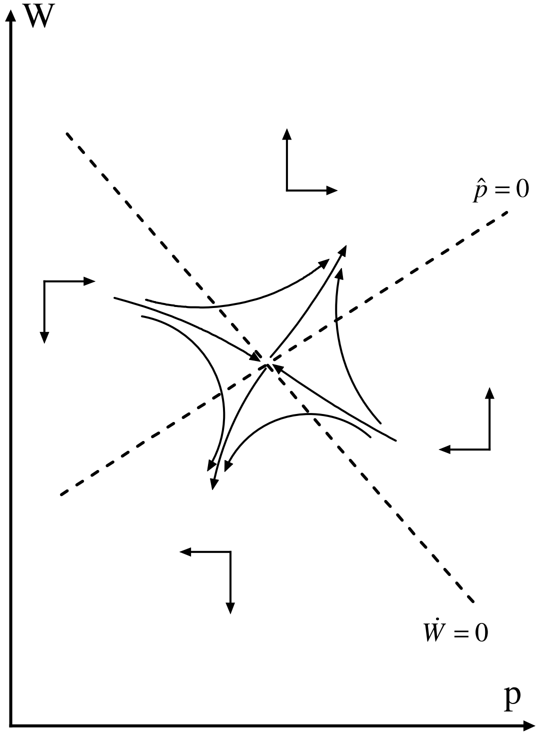

Fig. 7.3. The case of centrifugal inflation and capital account dynamics

7.3.5 Twin Deficit or Surplus Accumulation

In order to highlight the important role of the price dynamics for the stability of the system, as discussed above, we again set foreign inflation equal to zero ($\pi^* = 0$) and assume also constant price levels for the domestic economy ($p = p^* = 1$ for notational simplicity). We assume given policy parameters (M, T, G). Inserting the behavioral equations given by (7.10)–(7.12), we obtain in the presently considered case the 2D dynamical system[14]

$$\dot{W} = i^*W + Y - C(Y_p, W - W_g^a, i^*) - I(Y, r^*) - G = i^*W + NX(\cdot), \quad (7.25)$$

$$\dot{W}_g^a = i^*W_g^a + i^*\frac{M}{p} + T - G. \quad (7.26)$$

The corresponding Jacobian at the steady state of the dynamics is characterized by:

$$J = \begin{pmatrix} \frac{\partial \dot{W}}{\partial W} & \frac{\partial \dot{W}}{\partial W_g^a} \\ \frac{\partial \dot{W}_g^a}{\partial W} & \frac{\partial \dot{W}_g^a}{\partial W_g^a} \end{pmatrix} = \begin{pmatrix} (i^* - C_W) & -C_{W_g^a} \\ 0 & i^* \end{pmatrix} = \begin{pmatrix} \pm & + \\ 0 & + \end{pmatrix}$$

with $\det J < 0$ if $i^* < C_W$ and $\mathrm{tr}\ J > 0$ else. In the absence of price adjustments, therefore, the dynamics of the system become intrinsically unstable in any case (with a saddlepoint in the first and an unstable equilibrium in the second situation), even if the wealth effect on consumption influences in a larger extent the dynamics of the economy's wealth position than the interest payments. We conclude that the dynamics of twin deficits (or surpluses) is far from being self-correcting.

7.4 Overshooting Exchange Rates and Inflation Dynamics for Perfectly Flexible Exchange Rate Regimes

As a second case study we now consider a regime of perfectly flexible exchange rates and given money supply. This implies that supply of foreign bonds at each moment in time is just given by the stock of these bonds held in the private sector, since domestic bonds are assumed to be non-tradables in this chapter. The supply of financial assets is thus fixed in each moment of time

[14] Note that—due to the assumed goods market equilibrium—$Y - C - I - G$ can always be replaced by $NX(Y, Y^*, e)$ if this is convenient for certain calculations of the model's implications.

and can only change in time via the flows induced in the capital account. The central bank may use open market operations in domestic bonds to change the composition of these bonds and money in the households portfolio, but does not issue money otherwise (to buy foreign bonds from domestic residents in particular). The case considered in this section may be applicable—after some modifications—to an economic situation as represented by the Australian economy (at least for certain time periods of this economy). We stress that we reconsider here the Dornbusch (1976) overshooting exchange rate dynamics for imperfect asset substitutability in the case of the empirically questionable case of the uncovered interest rate parity condition.

7.4.1 Equilibrium Conditions

In our reformulation of the Dornbusch overshooting exchange rate dynamics[15] within the framework of a MFT model we assume for simplicity also—just as in the original Dornbusch (1976) approach—that transactions demand in the money demand function is based on full employment output $\bar{Y} = 1$, i.e., we get from the LM curve of our Tobinian portfolio approach the result that the domestic rate of interest is solely dependent on the price level (positively) and on money supply (negatively), i.e.:

$$i = \frac{\ln p - \ln M}{\alpha} + const$$

as in the case of the Cagan money demand function considered in Flaschel (2006). For full asset markets equilibrium we need only consider the market for foreign bonds in addition which in the considered exchange rate regime reads:

$$\frac{eF_p}{p} = f^d\left(i^* + \epsilon(e) - i, \frac{M + B + eF_p}{p}\right) = f^d\left(r^{*e} - r, \frac{M + B + eF_p}{p}\right),$$
$$r^{*e} = i^* + \epsilon(e) - \hat{p}, \quad r = i - \hat{p}$$

with $F_p, M + B, p$ given magnitudes in each moment of time. Since $0 < f_2^d < 1$ holds true in a Tobinian portfolio model, we get from this equilibrium condition that the exchange rate e depends negatively on i, F_p and positively on p (when the effect of the price level on the nominal interest rate is ignored). The theory of the exchange rate of the considered Mundell-Fleming regime thus can be represented as follows:

[15] See Asada et al. (2003, Chap. 5) for a detailed presentation of the Dornbusch model and its various extensions.

$$e = e(i(p, M), p, F_p), \quad e_1 < 0, e_2 > 0, e_3 < 0, \quad i_1 > 0, i_2 < 0.$$

Note that the overall effect of price level changes on the exchange rate may be an ambiguous one.

Next we consider the IS-equilibrium curve of the presently considered situation:

$$Y = C_1(Y + r^{*e}W - G, W - W_g^a, r, r^{*e}, \eta) + I(Y, r, r^{*e}) + G + X(Y^*, \eta)$$

with $\eta = e/p, p^* = 1$ and G, W_g^a again given magnitudes. One has to use our regressive expectations regime, the dependence of the nominal rate of interest and the real exchange rate on the price level and the functional dependence of the nominal exchange rate on i, p derived above in order to derive conclusions on how the equilibrium output level depends on the price level p. The outcome is however ambiguous, but pointing up to a certain degree to a negative overall dependence of Y on p (as usual). We shall assume that this holds true in our following discussion of overshooting exchange rates, since the opposite case would imply a destabilizing feedback of the price level on its rate of change via the Phillips curve mechanism. The dependence of Y on W is obviously a positive one, though we will have ambiguity in the movement of W later on.

7.4.2 Dynamics and Steady State Determination

We have by now determined the statically endogenous variables of the considered MFT regime i, e, Y by the three equilibrium relationships that now characterize the model. The state variables of the model are again p, W (while the movement of the capital stock is still neglected). The laws of motion for these variables are now given by:

$$\dot{W} = r^*W + X(Y^*, \eta) - \eta C_2(Y + r^{*e}W - G, W - W_g^a, r, r^{*e}, \eta),$$
$$\hat{p} = \beta_w(Y - 1)/(1 - \gamma),$$

where $r^* = i^* + \hat{e} - \hat{p}$ and $r^{*e} = i^* + \epsilon(e) - \hat{p}$ and $r = i - \hat{p}$. The law of motion for W is now a very complicate one, since the static relationships have to be inserted into it in various places. We will therefore not discuss its (in-)stability implications in the following, but just assume for the time being that this variable is placed into its new long-run equilibrium position after a shock and kept constant there. We therefore only study the adjustment of the price level p after an open market operation of the central bank (which leaves

$M + B$ unchanged). We note however that this implies a jump in the variable $W = (eF)/p$ that is neglected in our following analysis of such shocks.

In the construction of a steady state reference solution[16] we proceed as follows. We again assume that the government pursues an export target $\bar{X}$ by means of its expenditure policy G, besides the tax policy that keeps W_g^a at a constant level $\bar{W}_g^a$. The steady state real exchange rate η_o is then uniquely determined by the equation $\bar{X} = X(Y^*, \eta)$. On this basis we can use the equilibrium condition on the market for foreign bonds to determine the steady state level of W, since this equilibrium condition can be rewritten as

$$f^d(\xi, W - W_g^a) = W - \eta_o F_c \quad \text{or} \quad 0 = W - \eta_o F_c - f^d(\xi, W - W_g^a) = g(\xi, W)$$

since money supply and thus F_c is held constant by the central bank. Since $\epsilon(e_o)$ and ξ_o should be zero in the steady state (to be further justified below) the above equation can be assumed to have a uniquely determined positive solution if the function f^d is chosen appropriately. We note that we only consider positive values of W_o here, since we assume that households and the central bank hold such assets and since firms do not finance their investment expenditures abroad.

Due to the Phillips curve we get next that $Y_o = \bar{Y}$ must hold in the steady state ($\hat{p}_o = 0$). Furthermore, the regressive expectations scheme is built such that $\epsilon(e_o) = 0$ holds for the steady state value e_o of the nominal exchange rate (that remains to be determined still). We thus have $r_o = i_o$, $r_o^* = r^{*e} = i^*$ in the steady state and postulate that $I(Y, i^*, i^*) = 0$ holds for the investment function used in this MFT model. We assume on this basis in addition that the government chooses the level of G and thus $\bar{X}$ such that $i_o = i_o^*$ is enforced by the IS-equation in the steady state:[17]

$$\bar{Y} = C_1(\bar{Y} + iW_o - G, W_o - \bar{W}_g^a, i_o, i^*, \eta_o) + I(\bar{Y}, i_o, i^*) + G + \bar{X}.$$

[16] Again, as in the sections before, no assumptions concerning G are required once deviations of i_o from i^* are allowed for. Recall in this regard, that according to the Tobinian portfolio approach used here domestic and foreign bonds are only imperfect substitutes so that the uncovered interest parity normally does not hold.

[17] In principle, the government here enforces two things in the steady state, namely that the domestic interest rate must be at the international level and that the level of exports is such that goods market equilibrium is then assured, with a zero level of net investment by the assumption on the investment function. The real exchange rate is then a consequence of the level of $\bar{X}$ needed for goods market equilibrium.

On this basis we can then get the steady state value of the price level p_o from the LM-curve $p_o = M/m^d(\bar{Y}, i^*)$ and thus also the steady state value of the exchange rate $e_o = p_o \eta_o$.

We thus have quite a different "causality" now in the determination of the steady state, where fiscal policy has to provide the necessary anchor for a meaningful steady state solution, whereas in the short-run we have that IS-LM-FF curves determine the variables Y, i, e in principle in this order, while the Phillips curve and the balance of payments determine the dynamics of the price level and of domestically held foreign bonds (in real terms). Note again that the dynamics of the capital stock still remains excluded from consideration here. Note also that we will simplify in the following even further, since we shall also exclude the complicated adjustment process for $W = eF/p$ from consideration and instead assume that this magnitude will immediately jump to its new steady state value after any shock and will be kept frozen there. The aim of the following simplified presentation instead will be to reconsider the Dornbusch (1976) model of overshooting exchange rates in the context of a Tobinian portfolio model of the financial sector and somewhat advanced formulations of consumption and investment behavior.

7.4.3 Dornbusch (1976) Exchange Rate Dynamics

Let us now consider an open market operation of the central bank $dM = -dB$, that increases the money holdings of private agents by reducing their holdings of domestic bonds (which therefore keeps $M + B$ and F_c constant). Our way of constructing a steady state for the considered dynamics immediately implies then that all real magnitudes remain the same (in particular W_o) and that we have as sole steady state changes the following ones:

$$dM/M = dp_o/p_o = de_o/e_o \quad (\eta_o = const, i_o = i^*).$$

The long-run reaction of the dynamics is therefore as in the original Dornbusch (1976) model a very straightforward one, strict neutrality of money and the relative form of the PPP, i.e., there is no change in the real exchange rate caused by the monetary expansion that is undertaken.

In the short-run, prices are fixed and the burden of adjustment in the money market falls entirely on the nominal rate of interest i which is decreased below i^* through the monetary expansion. Since the new steady state value e_o' of the nominal exchange rate is above the old level now and since the assumed regressive expectations mechanism is completely rational in this respect, we would have that $\epsilon(e'_o/e), \epsilon' < 0$ would become positive (generate the expectation of a depreciation) if the short-run exchange rate would remain unchanged. Since we ignore—as described above—adjustments in the value of W we however must have that the current exchange rate depreciates beyond e'_o in order to imply the expectation of an appreciation of the currency such that $\xi = i^* + \epsilon(e'_o/e) - i = 0$ can remain unchanged (due to the unchanged value of W_o,[18] since the adjustment process of W is here ignored).[19] We thus get that i decreases and e increases under the assumed monetary expansion and expect that goods market equilibrium output increases through these two influences, since $i^* + \epsilon = i$ has decreased and since η has been increased (the price level still being fixed at p_o). Again there may be an ambiguous reaction possible, but we assume here—as before—that goods markets behave normally in this respect.

The nominal exchange rate therefore overshoots in the short-run its new long-run level as in the original Dornbusch model. Due to the increase in the output level beyond its normal level we have now for the medium-run that the price level starts rising according to

$$\hat{p} = \beta_w(Y-1)/(1-\gamma) \quad \text{with} \quad Y = Y(p, W_o),\ \partial Y/\partial p < 0$$

according to what has been shown above. The dynamics therefore converges back to its steady state position with output levels falling back to their normal level, prices rising to their new steady state value p'_o, the exchange rate appreciating back to its risen steady state value e'_o and the nominal rate of interest rising again to the unchanged international level i^*. All this takes place in a somewhat simplified portfolio approach to financial markets (in place of the UIP condition under rational expectations) and without any complications arising from possibly adverse adjustments in the balance of payments. It is therefore to be expected that a treatment of the full model along the lines of Flaschel (2006), there for the extremely open economy, will reveal a variety of more complicated situations and in the worst case an unstable dynamics for which then actions of households or the government have again to be found that bound their trajectories to economically meaningful domains.

[18] And also $\bar{W}^a_g$, $\eta_o F_c$.

[19] The argument must be more detailed if the short run reaction in the value of W is taken into account, but would then of course demand a thorough discussion of the conditions under which dynamics drives this variable back to its steady state level.

The present discussion is therefore only the beginning of a detailed discussion of the Dornbusch mechanism in a full-fledged MFT model with inflation and balance of payments adjustment dynamics.

The dynamic adjustment processes just discussed are summarized in Fig. 7.4. We have the simple LMFF curve describing full portfolio equilibrium by way of

$$f^d(i^* + \epsilon(e) - i(p), W_o - \bar{W}_g^a) = W_o - \eta_o F_c,$$

where all revaluation effects of assets have been ignored, where transactions balances are based on normal output still and where most importantly the dynamics of the state variable W is set aside. The true LMFF curve is of course shifting with the changes in W, η and the output level Y. We have furthermore the curve along which the price level is stationary and which also in general is not as simple as shown in this graph (where $\partial Y/\partial e$, $\partial Y/\partial p < 0$

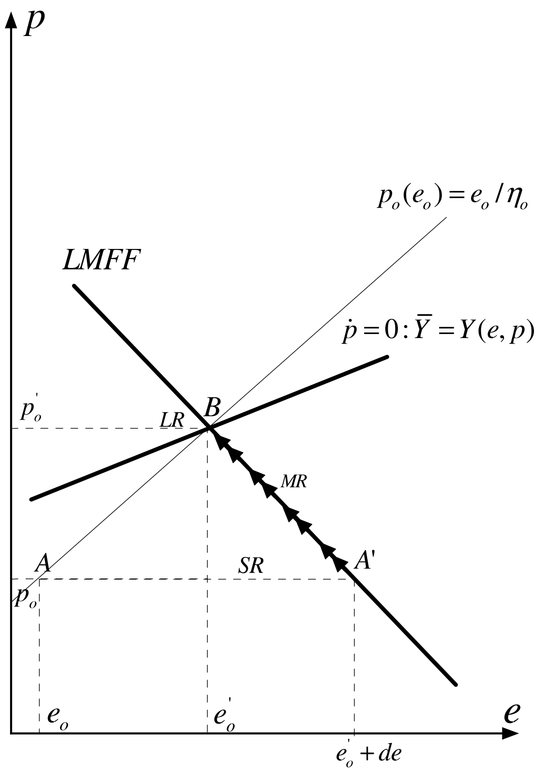

Fig. 7.4. The Dornbusch overshooting exchange rate dynamics

is assumed), but may be quite complicated due to the assumed consumption and investment behavior.

If the old steady state A is disturbed by an expansionary monetary shock that shifts the old LMFF curve (not shown) into the new position (shown) we have as immediate response (SR) that the exchange rate adjusts (to A') such that full portfolio equilibrium is restored. In the medium run (MR) we then have rising price levels, rising interest rates and falling exchange rates until the new steady state position (LR) is reached at point B. We note that the variables W, W_g^a may be subject to jumps in a regime with flexible exchange rates, but are then following their laws of motion if no further shocks occur (in the case of W), respectively remain then fixed at their new level (in the case of W_g^a).

7.4.4 Capital Account and Budget Deficit Dynamics

Furthermore, we may also return to an endogenous treatment of the variable W_g^a (by making use again of given lump sum taxation T again). This would increase the complexity of the analysis even further and lead us to the consideration the following 3D dynamics:

$$\begin{aligned}
\hat{p} &= \beta_w(Y-1)/(1-\gamma),\\
\dot{W}_g^a &= (i^*+\hat{e}-\hat{p})W_g^a + (i^*+\hat{e}-i)\frac{M+B}{p} + i\,m^d(Y,i) + T - G,\\
\dot{W} &= (i^*+\hat{e}-\hat{p})W + X(Y^*, ep^*/p)\\
&\quad -(ep^*/p)C_2\Big(Y + (i^*+\epsilon(e)-\hat{p})(W-W_g^a) - i\,m^d(Y,i)\\
&\quad -(i^*+\epsilon(e)-i)\frac{M+B}{p} - T, W-W_g^a, i-\hat{p}, i^*+\epsilon(e)-\hat{p}, ep^*/p\Big).
\end{aligned}$$

This extension would again allow for the discussion of the occurrence of twin deficits and other situations of domestic and foreign debt/surplus accumulation. Note however that the system has become a very complicated in this case of a perfectly flexible exchange rate, where the statically endogenous variables i, e have to be obtained from the portfolio part of the model, in interaction with the IS-curve of the model, and where the growth rate of e has then to be calculated on this basis in order to insert it into the 3D laws of motion where necessary. This is not at all an easy task and makes an analysis of the model nearly untractable. In view of this, ways have to be found that model the dynamics of the exchange rate directly, so that a differentiation of the portfolio equilibrium equations for this purpose can be avoided.

7.5 International Capital Flows in the MFT Model

So far, we have considered a small open economy of the Mundell-Fleming-Tobin type, exhibiting on the market for goods a Keynesian demand constraint. Moreover, we have also assumed imperfect substitutability of financial assets in place of an UIP condition. This imperfectness was coupled with the assumption that domestic bonds are non-tradables, i.e., the amount of foreign bonds held domestically was only changed to the extent that there is a surplus or a deficit in the current account.

In the next step, this assumption is now relaxed, i.e. also domestic bonds are now assumed to be traded on the international capital market. The main purpose of this extension is to check in how far our previous results are affected by this change, especially whether the dynamics turn out to alter significantly or not. In the following, we thus again consider a small open economy and thus assume again that foreign interest and inflation rates, i^*, π^*, as well as foreign output, Y^*, are given magnitudes. We review in this section the budget equations of the three relevant sectors in our economy, households, the government and the central bank. With respect to firms we assume again that all of their income is transferred to the household sector and that households give them credit to finance their investment expenditures. The other previous assumptions, especially concerning the central bank and its operations, are maintained as well.

7.5.1 Budget Restrictions

The savings decision of households, based on their PBR, the private budget restraint, the government budget constraint (GBR) and the portfolio readjustment of the private sector, via international capital flows (ICF), read as follows

$$p(Y - T) + iB_p + ei^* F_p \equiv pC + pI + \dot{B}_p + e\dot{F}_{p1} \quad \text{PBR} \tag{7.27}$$

$$pT + ei^* F_c + \dot{B} \equiv pG + iB \qquad \text{GBR} \tag{7.28}$$

$$e\dot{F}_{p2} = \dot{B}^* \qquad \text{ICF} \tag{7.29}$$

The first identity represents the budget restriction of the private sector. It shows how disposable income from production and interest income from the government and from abroad are allocated to consumption, (direct) investment and changes in interest bearing asset holdings. The GBR is by and large as it is usually formulated in the literature, see Rødseth (2000, Chap. 6) for example, but it exhibits no money financing of government expenditures and includes interest transfers from the central bank. Compared to the previous section, (7.29) represents the new equation (for the international capital flows (ICF)) which for the moment only symbolizes the flow exchange between foreigners and domestic residents of foreign currency denominated bonds against domestic ones (an equilibrium condition in fact). Note that these equations assume that the allocation of households' savings are based on their consumption and investment decisions and on the assumption that they in fact absorb the new bond issue of the government (which represents a simplifying consistency condition in this modeling framework). The change implied by their savings in their holdings of foreign bonds is then determined residually. We therefore consider their savings decision as only a flow condition in addition to their portfolio choice decision on the international capital markets. In the next "period", they then decide how to reallocate their holdings of domestic and foreign bonds on the international capital markets by either selling or buying domestic bonds on these markets (with behavioral equations to be introduced below).

Their full portfolio change is therefore characterized by:

$$e\dot{F}_p + \dot{B}_p = e(\dot{F}_{p1} + \dot{F}_{p2}) + \dot{B} - \dot{B}^*. \tag{7.30}$$

We make these assumptions here in order to separate clearly in this extension of the standard MFT model, see Rødseth (2000, Chap. 6), the savings decision of households from their portfolio choice decision, where the latter—in contrast to the savings decision—may give rise to significant international capital flows in interaction with the behavior of foreign asset holders. Savings and portfolio rearrangements are now formulated—in contrast to the previous section—in terms of flows, while the cash management decision of households (between money and fix-price domestic bonds) are still modeled in the traditional way by means of an LM-curve which is mirrored by an equivalent stock condition on the market for domestic bonds. Cash management is therefore here still separated (and precedes) rate of return driven portfolio reallocations. In case of an open market operation of the CB ($dM = -dB$) it is moreover assumed that this occurrence precedes the other ones and that portfolio adjustments are then performed in the light of this change.

We get as implications for the evolution of domestic and foreign bonds held by the domestic economy ($F = F_p + F_c$):

$$\dot{B}_p = \dot{B} - \dot{B}^* = iB + p(G - T) - e\bar{i}^* F_c - \dot{B}^*, \tag{7.31}$$

$$e\dot{F}_p - \dot{B}^* = e(\dot{F}_{p1} + \dot{F}_{p2}) - \dot{B}^* = p(Y - C - G - I) + ei^* F - iB^*, \tag{7.32}$$

which (on the basis of a goods market equilibrium condition) is just the statement that the balance of payments is balanced under the assumed budget equations, since output minus domestic absorption is just net export NX and since the other two items in the balance of payments are representing net capital exports NCX and net interest rate flows.

Considering the same situation from the viewpoint of savings we can write:

$$pS_p = p(Y - T) + iB_p + ei^* F_p - pC = pI + \dot{B} + e\dot{F}_{p1}, \tag{7.33}$$

$$pS_g = pT + ei^* F_c - iB - pG = -\dot{B}, \tag{7.34}$$

which gives for the total savings pS of the economy:

$$pS = pY + ei^* F - iB^* - pC - pG = pI + e\dot{F}_{p1} = pI + e\dot{F}_p - \dot{B}^*.$$

This is again a formulation of the fact that the balance of payments must be balanced in the here assumed situation without any further adjustment processes, solely based on the assumption that the new issue of government bonds $\dot{B}$ is accepted by the household sector as it is implicitly made in the above formulation of the budget equations of our economy.

7.5.2 Real Disposable Income and Wealth Accounting

In analogy to the Hicksian definition of private disposable income we now define and rearrange this concept for the aggregate government sector (including the foreign interest income of the central bank) and show on this basis in particular that the aggregate wealth of this sector W_g^a is in its time rate of change—as in the case of the private sector—determined by deducting from its disposable income the consumption of this sector. This then also provides us with a law of motion for real aggregate wealth of the government sector, besides the one we have already determined for the total wealth of the economy. These two laws describe on the one hand the evolution of surpluses or deficits in the government sector and the evolution of current account surpluses or deficits, and thus in particular allow the joint treatment of the issue of twin deficits in an open economy with a government sector, but not yet with capital accumulation and economic growth.

The Government Sector

The following calculations concern the sources of income and consider as disposable (in the Hicksian sense) that income that when consumed just preserves the current level of wealth of the considered sector, here the government sector. The resulting definitions of Y_g^a (the government's disposable income) and W_g^a (the government's real wealth position) are the same as in Sect. 7.2.2 of the present chapter and therefore only briefly repeated here:

$$\begin{aligned} Y_g^a &= T + i\frac{M}{p} + \xi\frac{M+B}{p} + r^* W_g^a, \quad \xi = i^* + \hat{e} - i, \quad \text{with} \\ W_g^a &:= \frac{-(M+B)+eF_c}{p} = -\frac{(M+B)}{p} + \frac{eF_c}{p} = W_g + W_c. \end{aligned}$$

This implies

$$\begin{aligned} \dot{W}_g^a &= \frac{-(\dot{M}+\dot{B}) + \dot{e}F_c + e\dot{F}_c}{p} - \frac{\dot{p}}{p}\frac{-(M+B)+eF_c}{p} \\ &= \frac{-(pG + iB - pT - i^* eF_c) + \hat{e}eF_c}{p} - \hat{p}\frac{-(M+B)+eF_c}{p} \\ &= T - i\frac{B}{p} + \hat{p}\frac{M+B}{p} + (i^* + \hat{e} - \hat{p})\frac{eF_c}{p} - G \quad \text{which finally gives} \\ \dot{W}_g^a &= Y_g^a - G = r^* W_g^a + i\frac{M}{p} + \xi\frac{M+B}{p} + T - G. \end{aligned}$$

The Evolution of Total Domestic Wealth and Domestic Bonds Held Internationally

Since this debt position is currently not assumed as constant, we repeat next the equations for the evolution of total wealth W of the economy in this case and then consider again private disposable income Y_p and private wealth W_p in its interaction with the evolution of aggregate government debt. Note that we assume goods market equilibrium $Y - C - I - G = NX$ in the following derivations (and set $F = F_p + F_c$).

$$\begin{aligned} \hat{W} &= \hat{e} + \hat{F} - \hat{p}, \\ \dot{W} &= \hat{e}W + \frac{e\dot{F}_p}{p} - \hat{p}W = \hat{e}W + \frac{p(Y-C-G-I) + ei^*F - iB^* + \dot{B}^*}{p} - \hat{p}W \\ &= (\hat{e} - \hat{p})W + i^*\frac{eF}{p} + Y - C - G - I - iW^* + \frac{\dot{B}^*}{p} \\ &= (i^* + \hat{e} - \hat{p})W + Y - C - G - I - iW^* + \frac{\dot{B}^*}{p}, \\ \dot{W} &= r^* W + Y - C - G - I - iW^* + \frac{\dot{B}^*}{p}, \\ \dot{W}^* &= \frac{\dot{B}^*}{p} - \hat{p}W^*. \end{aligned}$$

Private Wealth and Income

We have for the definition of private wealth and disposable income:

$$
\begin{aligned}
W_p &= \frac{M + B_p + eF_p}{p} = W - W_g^a - W^*, \\
Y_p &= Y - T + (i^* + \hat{e} - \hat{p})\frac{eF_p}{p} + (i - \hat{p})\frac{B}{p} - \hat{p}\frac{M}{p} \\
&= Y - T + (i^* + \hat{e} - \hat{p})W_p - (i^* + \hat{e} - \hat{p})\frac{M + B}{p} + (i - \hat{p})\frac{B}{p} - \hat{p}\frac{M}{p} \\
&= Y - T + r^* W_p - (i^* + \hat{e} - i)\frac{M + B}{p} - i\frac{M}{p} \\
&= Y - T + r^*(W - W_g^a - W^*) - \xi\frac{M + B}{p} - i\frac{M}{p}.
\end{aligned}
$$

From the results on the disposable income of households and the government we finally also get:

$$
Y_p = Y - Y_g^a + r^*(W - W^*) \quad \text{or} \quad Y_p + Y_g^a = Y + r^*(W - W^*)
$$

as relationship between total disposable income, domestic product and real interest on domestically held foreign bonds minus foreign holdings of domestic bonds.

7.5.3 The Four Laws of Motion of the MFT Open Economy with International Capital Flows

Foreign inflation is now set equal to zero and we assume fixed policy parameters M, T, G:

1.) $\dot{W} = r^* W + Y - C(\cdot) - I(\cdot) - G - iW^* + \dot{B}^*/p$
2.) $\dot{W}_g^a = r^* W_g^a + \xi\frac{M+B}{p} + i\dfrac{M}{p} + T - G$
3.) $\dot{W}^* = \dot{B}^*/p - \hat{p}W^*$

to be coupled with the law of motion for the price level p:

4.) $\hat{p} = \hat{w} = \beta_w(Y - \bar{Y})/(1-\gamma) + \hat{e} + \pi^*, \quad [\hat{w} = \beta_w(Y - \bar{Y}) + \gamma\hat{p} + (1-\gamma)\widehat{ep}^*]$

which again represents the reduced form of a standard open economy Phillips curve, here both with myopic perfect foresight on the domestic price level and on the expected growth rate of the exchange rate (γ the weight of the domestic price level in the domestic consumer price index, see Rødseth (2000, Chap. 6) for details).[20] Note that the above laws of motion still assume myopic perfect foresight with respect to the exchange rate and inflation dynamics and thus do not yet distinguish between expected rates of return and actual ones, with the former to be used in the behavioral relationships later on while the latter apply to the actual laws of motion for the considered wealth variables. The distinction between actual and perceived rates of return will become important when exchange rate dynamics is considered later on (in our representation of the Dornbusch model in a Tobinian approach to financial markets). Note also that we have extended the consumption and investment function of Rødseth (2000, Chap. 6) slightly, since we intend in particular to distinguish between the consumption of the domestic and the foreign commodity later on and thus have to include the real exchange rate into the consumption function explicitly.

The Dynamics of the Private Sector

We assume now that consumption and investment behavior is determined as before, i.e., consumption depends on real disposable income, the domestic and the foreign real rate of interest and on real wealth (and in addition to Rødseth (2000, Chap. 6) on the real exchange rate η), while investment is determined by capacity utilization and the domestic and the foreign real rate of interest. The partial derivatives with respect to these variables are assumed to exhibit the usual signs in such a Keynesian framework to a small open economy. We furthermore maintain our previous assumption that taxes are varied endogenously such government debt as measured by W_g^a stays constant in time. In this case the above dynamical system is reduced to the following simpler form:

$$\dot{W} = r^*W + Y - C(Y_p, W_p, r, r^{*e}, \eta) - I(Y, r, r^{*e}) - G - iW^* + \dot{B}^*/p, \quad (7.35)$$

$$\dot{W}^* = \dot{B}^*/p - \hat{p}W^*, \quad (7.36)$$

$$\hat{p} = \beta_w(Y - \bar{Y})/(1-\gamma) + \epsilon(e) + \pi^* \quad (7.37)$$

based on the following further definitions and further relationships:

$$r = i - \hat{p}, \quad (7.38)$$

$$r^* = i^* + \hat{e} - \hat{p} \quad [\text{expected rate} \quad r^{*e} = i^* + \epsilon(e) - \hat{p}], \quad (7.39)$$

$$Y_p = Y + r^{*e}(W - W^*) - G, \quad (7.40)$$

$$W_p = W - W_g^a - W^*, \quad (7.41)$$

$$\eta = e/p \quad [p^* = 1]. \quad (7.42)$$

[20] In the case of regressively formed expectations one has to use the scheme $\epsilon(e), \epsilon'(e)$ in place of $\hat{e}$.

This dynamical system is to be supplemented by the temporary equilibrium relationships for the goods, the money and the foreign exchange market:

$$Y = C_1(Y_p, W_p, r, r^{*e}, \eta) + I(Y, r, r^{*e}) + G + X(Y^*, \eta), \quad (7.43)$$

$$M/p = m^d(Y, i), \quad [m^d = kY \exp(\alpha(i^* - i)) \text{ for example}], \quad (7.44)$$

$$e\dot{F}_{p2}/p = f^d(\xi^e, W_p) = -f^{d*}(\xi^{*e}, W^*) = \dot{B}^*/p \quad (7.45)$$

with $\xi^e = i^* + \epsilon(e) - i, \xi^{*e} = i^* + \epsilon^*(e) - i$. As far as goods market equilibrium is concerned we have canceled in this equation imports against the part of domestic absorption that is served by imports and thus consider in this equation domestic goods (type 1) solely. We assume that only consumption goods (goods type 2) are imported and thus have to replace in the considered situation $C(Y_p, W_p, r, r^{*e}, \eta)$ by $C_1(Y_p, W_p, r, r^{*e}, \eta)$ simply and of course to reduce net exports NX to exports X. The market for imported goods thus behaves in a passive fashion only (whereby consumption demand of imported commodities is always served and thus not a restriction for the working of domestic economy).

The new relationship in these equations, compared to our previous considerations in the first part of this chapter, is the equation describing exchange equilibrium on the international capital market of domestic against foreign bonds:

$$f^d(\bar{i}^* + \epsilon(e) - i, W - W_g^a - W^*) = -f^{d*}(\bar{i}^* + \epsilon^*(e) - i, W^*).$$

It exhibits on its left-hand side the flow demand of domestic residents for foreign bonds (if positive, otherwise the supply of such bonds) measured in terms of domestic goods. On its right hand side, the expression $f^{d*}(i^* + \epsilon^*(e) - i)$ provides the demand of foreigners for domestic bonds (if positive, otherwise the supply of such bonds) also measured in terms of domestic goods.[21] This equilibrium equation describes the capital flows in the capital account that are not caused by the savings decisions within the domestic economy. In the case of uniform expectations of de- or appreciation in the world economy, i.e., when $f^{d*}(\bar{i}^* + \epsilon(e) - i, \cdot) = f^d(\bar{i}^* + \epsilon(e) - i, \cdot)$ holds (where therefore domestic and foreign asset holders have the same demand schedule), we assume that this implies $f^{d*} = f^d = 0$ since both parties are then expecting the same risk

[21] One may argue here that this real foreign bond demand should be measured in foreign goods and thus has to be multiplied by the real exchange rate $\eta = ep^*/p$ before it can be added to the real foreign bond demand of domestic residents. We leave this alterative approach for future investigations here.

premium and should therefore both be demanders or both suppliers of foreign bonds, causing a reaction of the exchange rate that reduces the resulting excess demand (or supply) to zero.

Steady State Considerations

The first thing to be noted here is that the regressive expectations mechanisms must fulfill some consistency requirement in order to allow for a meaningful steady state consideration, namely $\epsilon(e_o) = \epsilon^*(e_o) = 0$ for the steady state value of the exchange rate. Furthermore we make use of the law of motion for the capital stock in the following steady state consideration and assume $\hat{K} = I/K = I(Y/K, r, r^{*e})/K = 0$ as side condition for these considerations $I = 0!$. From the Phillips curve we get $\hat{p} = 0, Y_o = \bar{Y}$ and thus $r_o^* = r_o^{*e} = i^*$ and $r_o = i_o$ as well as $\xi_o = i^* - i = \xi_o^*$.

Next, the conditions $\dot{W}^*, B^* = 0$ imply $f^{d*}(\xi^*, W^*) = 0$. We assume here as simplified form for the function $f^{d*}(\xi^*, W^*) = f^{d*}(\xi^*)W^*$ a multiplicative expression with $f^{d*}(0, W^*) = 0$. The disappearance of international capital flows in the steady state therefore implies that $i_o = i^*$ must hold true in the steady state. From the LM-curve we then get in the case of the regime of a given money supply $p_o = M/m^d(\bar{Y}, i^*)$. International capital market equilibrium $f^d(0, W - W^* - W_g^a) = -f^{d*} = 0$ furthermore implies a steady state value for $W - W^*$ which when inserted into the IS-curve (together with the other steady state values already determined) implies a steady state value for the real exchange rate $\eta = e/p$ and thus also a steady state value for the nominal exchange rate e.

Since we already had $\dot{B}^* = 0$ we finally get from the established equation for total savings that $\dot{F}_p$ must be zero as well, i.e., $\dot{W}$ will be guaranteed automatically. This however means that there is zero root hysteresis in the levels of both W and W^* since only their difference is uniquely determined in the steady state. The steady state value of the capital stock K is finally uniquely determined through the condition $0 = I(\bar{Y}/K, i, i^*)$.

A Digression

The assumed behavior of domestic agents on the international capital markets for domestic and foreign bonds can be related to the stock portfolio approach of Chiarella et al. (2008, Chap. 12) as follows. We assume that money market equilibrium is already ensured and consider the function $f^d(\xi, W_p)$ as defined in their Chap. 12, i.e., as real stock demand for foreign bonds eF_p^d/p. The stock demand for domestic bonds is then characterized residually by $W_p - M/p - f^d(\xi^e, W_p)$. We now assume however that there are adjustment costs with respect to these desired stock changes and that therefore only a portion δ is currently realized as flow demand giving rise to

$$\frac{e\dot{F}_{p2}^d}{p} = \delta(f^d(\xi^e, W_p) - f_p), \quad f_p = eF_p/p,$$
$$\frac{e\dot{B}_{p2}^d}{p} = \delta\left([W_p - \frac{M}{p} - f^d(\xi^e, W_p)] - \frac{B_p}{p}\right).$$

It is immediately obvious that these induced flows add up to zero and thus transform the initial form of a Walras law of stocks (including money holdings) to a partial Walras law of flows on the interest bearing asset markets. We stress again that the flow of savings is considered separately from these reallocations in the portfolio of the private agents (which are to be associated with the index 1 in the place of the index 2 used above). This brief sketch that transforms stock into flow demands suggests that we should use f_p in addition to W_p in the above presentation of the general MFT model when stock related capital flows are considered (and in fact W^* in the demand function of foreigners for domestic bonds). For simplicity however we use only the aggregate W_p in the flow demand function and characterize this function by the same symbolic expression as the stock demand function.

A Summing Up

Summarizing the above MFT model extension towards the integration of international capital flows, we can recapitulate that we assume in this extension that cash management comes first and is always characterized by the partial stock money market equilibrium condition $M/p = m^d(Y, i) \quad [B/p = B^d/p]$. On this background domestic agents then plan to reallocate their interest-bearing assets according to their behavioral relationships $f^d(\xi^e, W_p)$ $[f^d(\xi^{*e}, W^*)]$. The flow of savings (with $\dot{M} = 0$ by assumption) is then added to these portfolio changes as far as financial assets are concerned (with resulting additional flows of foreign bonds being determined by $e\dot{F}_{p1} = pNX + ei^*F_p$). This stylized ordering of stock and flow conditions is intended to avoid any confusion between international capital flows and the allocation of savings. In the model, the goods, the money and the international capital market are of course considered simultaneously and—for example—used to determine the variables Y, i, e (in the case of flexible exchange rates and a given money supply in the economy).

The money market is here of course of traditional LM-curve type still and can thus represented graphically in the usual way. A graphical representation of the international bond markets is provided in Fig. 7.1. In this figure we consider a regime of flexible exchange rates that are then to be determined from the interaction of demand and supply on the international capital markets.

Figure 7.5 shows the level of the exchange rate e_o where the domestic demand curve for foreign bonds (the supply curve for domestic bonds) and the foreign demand curve for domestic bonds (the supply curve for foreign bonds) intersect and where therefore capital flow equilibrium is established. The equilibrium exchange rate now also depends on foreign characteristics in contrast to the case considered previously in this chapter where domestic bonds were considered as non-traded goods and where therefore the supply of such bonds (in the case of a flexible exchange rate regime) was just given by the bonds held domestically, F_p. In this case the equilibrium in the domestic

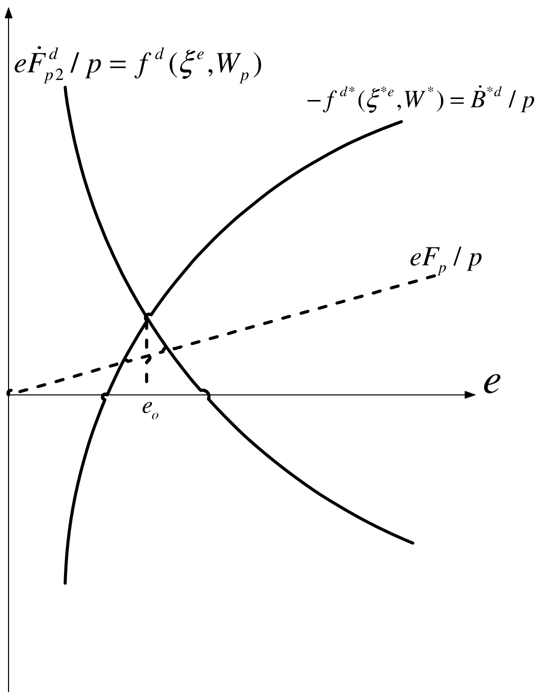

Fig. 7.5. Equilibrium on the international capital market

market for foreign bonds would be determined by the intersection of the f^d curve with the straight line shown in the above figure and would thus be independent from foreign asset demands, definitely a situation that is too simple to characterize today's international financial system. Note that the $-f^{d*}$ curve is a supply curve of foreign bonds, while in the form f^{d*} it would represent a demand curve for foreign bonds. In the case it is identical in Fig. 7.5 to the f^d curve there shown, the intersection of the f^d, f^{d*} must lie on the horizontal axis, i.e., the equilibrium exchange rate then implies that there is no international capital flow existing in such a situation.

In comparison to the original model of the previous section, we observe that the model has become more complicated from the economic point of view, but remains similar to the original MFT approach with respect to its comparative static results as we shall show later (since the formal properties of the equilibrium condition for the market for foreign bonds have remained the same). However, the evolution of foreign bonds held domestically as well as abroad can now change radically in the course of time and is no longer a simple reflection of the savings decision in the domestic economy.

7.6 International Financial Dynamics: Some Basic Results

7.6.1 Inflation Dynamics and International Capital Flows under Interest and Exchange Rate Pegs

In the following we base our considerations on the same assumptions as in Sect. 7.3 of this chapter which are briefly repeated here for sake of convenience:

1.) $i = i^*$: An interest rate peg by the central bank (via an accommodating monetary policy)
2.) $e = 1$: A fixed exchange rate via an endogenous supply of dollar denominated bonds by the central bank
3.) Given $Y^*, p^*(i^*)$: The small country assumption
4.) W_g^a : A tax policy of the government that keeps the aggregate wealth of the government fixed
5.) $G_2, I_2 = 0$: Only consumption goods are import commodities which are never rationed
6.) $\bar{\omega}$: The real wage is fixed by a conventional type of markup pricing

7.) $\bar{\rho}_f^n$: The normal (capacity utilization) rate of return of firms is fixed (since the real wage is a given magnitude) and set equal to i^* for simplicity
8.) $Y^p = \bar{y}^p K, N^d = Y/x$: Fixed proportions in production
9.) $\bar{K}$: The capacity effect of investment is ignored. Potential output $\bar{Y}^p = 1$ is therefore a given magnitude
10.) $\bar{Y} = x\bar{N} = 1$: A given level of the full employment output

On the basis of the above assumptions we get again that the real rates of interest are equalized for the domestic economy: $r^* = i^* - \hat{p} = i - \hat{p} = r$. Furthermore, the risk premium ξ is zero in the considered situation. Finally, due to the assumed tax policy we have again for the disposable income in the household sector the term: $Y_p = Y + r^*(W - W^*) - G$. Private wealth W_p is given by $W - W^* - W_g^a$ in the considered situation.

The real portfolio stock demand for money and the excess flow demand for foreign bonds of the private sector (including foreigners) are now given by:

$$M^d = pm^d(Y, i), \tag{7.46}$$

$$-\dot{F}_c/p = f^d(0, W - W_g^a - W^*) + f^{d*}(0, W^*), \tag{7.47}$$

and are to be satisfied by an accommodating monetary policy and out of the stocks of foreign bonds held by the central bank. They are therefore not of importance for the dynamics of the model under normal circumstances (non-exhausted reserves). The second equation distinguishes the present model from our former approach (analyzed in Sect. 7.3 of this chapter) and it of course becomes crucial when the stability of the fixed exchange rate system is in question.

The real exchange rate $\eta = (ep^*)/p$, i.e., the amount of domestic goods that are exchanged for one unit of the foreign good, reduces to $1/p$ due to the above normalization assumptions. Households directly buy investment goods for their firms and use the normal rate of profit in order to judge their performance, which is a given magnitude due to the above assumptions (normal output times the profit share). We therefore consider only one real rate, the rate r, in the following formulation of the consumption decisions (for domestic and foreign goods) and the investment decisions of the household sector (which again coincide with those in Sect. 7.3):

$C_1 = C_1(Y_p, W_p, r, \eta)$: consumption demand for the domestic good
$C_2 = C_2(Y_p, W_p, r, \eta)$: consumption demand for the foreign good

$$C = C_1(Y_p, W_p, r, \eta) + C_2(Y_p, W_p, r, \eta)/\eta : \quad \text{total consumption}$$
$$I = I(Y, r) : \quad \text{investment demand, for domestic goods solely}$$

On this basis the goods market equilibrium is again given by:

$$Y = C_1(Y + r(W - W^*) - G, W - W^* - W_g^a, r, \eta) + I(Y, r) + G + X(Y^*, \eta).$$

The dynamic equations of the model are now the following ones (the growth of the capital stock is neglected by assumption):

$$\hat{p} = \hat{w} = \beta_w(Y - \bar{Y}) + \gamma\hat{p} + (1-\gamma)\widehat{ep}^*, \tag{7.48}$$
$$\dot{W} = rW + Y - C(Y + r(W - W^*) - G, W - W^* - W_g^a, r, 1/p)$$
$$-I(Y, r) - G - i^*W^* + \dot{B}^*/p, \tag{7.49}$$
$$\dot{W}^* = \dot{B}^*/p - \hat{p}W^*. \tag{7.50}$$

The second and third law of motion are our new representation of the balance of payments in real terms, while the first one is again the standard money wage Phillips curve. On the basic of constant markup pricing, this equation can be again reformulated as follows:

$$\hat{p} = \frac{1}{1-\gamma}\beta_w(Y - \bar{Y}), \tag{7.51}$$
$$d(W - W^*)/dt = r(W - W^*) + Y$$
$$-C(Y + r(W - W^*) - G, W - W^* - W_g^a, r, 1/p)$$
$$-I(Y, r) - G. \tag{7.52}$$

We also show here a single reduced form expression for the second and the third law of motion into which the first law has to be inserted in two places in order to arrive at a system of differential equations which on its right hand side only depends on p, $W - W^*$ and the statically endogenous variable $Y(\cdot)$. Note that also the position of the IS-curve only depends on the difference $W - W^*$ between real domestic holding of foreign bonds and foreign holdings of domestic bonds, since the temporary equilibrium output Y depends only on the two state variables p, $W - W^*$ by means of the following goods market equilibrium condition ($\eta = 1/p$):

$$Y = C_1\left(Y + \left(i^* - \frac{1}{1-\gamma}\beta_w(Y - \bar{Y})\right)(W - W^*) - G, W - W^* - W_g^a, \right. \tag{7.53}$$
$$\left. i^* - \frac{1}{1-\gamma}\beta_w(Y - \bar{Y}), 1/p\right) + I\left(Y, i^* - \frac{1}{1-\gamma}\beta_w(Y - \bar{Y})\right) + G$$
$$+X(Y^*, 1/p). \tag{7.54}$$

The Core Dynamics

We assume for the time being that the parameters of this equilibrium condition are such the conventional dependency of IS-equilibrium output on the price level results: $\partial Y/\partial p < 0$. With respect to the wealth term $W - W^*$ it is obvious that $\partial Y/\partial(W - W^*) > 0$ holds true. The resulting dynamical system in the state variables $p, \Omega = W - W^*$ is by and large of the same type as the one considered in Chiarella et al. (2008, Chap. 12) and can be treated in the same way as the one that is considered in this earlier work.

For reasons of simplicity we maintain the assumption that aggregate government wealth (basically the government deficit) stays constant in time and investigate now the steady state solution and the dynamics surrounding it:

$$\hat{p} = \frac{1}{1-\gamma}\beta_w(Y(\Omega, p) - \bar{Y}),$$
$$\dot{\Omega} = r\Omega + X(Y^*, 1/p) - \eta C_2(Y + r\Omega - G, \Omega - W_g^a, r, 1/p),$$

where the properties of the IS-equilibrium are characterized by the standard partial derivatives just discussed. Since the system is formally equivalent to the one in Sect. 7.3.3 (with Ω only replacing W here), we also get the same results as obtained there. It seems that the introduction of bilateral international capital flows does not lead to any changes within the concrete framework considered here.

Given the above dynamical system, we can recover the dynamics of the two wealth variables W, W^* in the following way:

$$\dot{W}^* = f^{d*}(0, W^*) - \hat{p}(Y(p, \Omega)W^*,$$
$$W = \Omega + W^*$$

which completes the picture. In comparison to Chiarella et al. (2008, Chap. 12) we thus now have to consider the variables W, W^* in place of the treatment of only the law of motion for W. Moreover, the central bank has now also to take account of what foreigners are doing on the market for foreign exchange and is thus now much more vulnerable with respect to its foreign reserve holdings, even if the adjustment process of interacting W, W^*, p dynamics is stable. The next steps in a fuller treatment of the situation of an interest and exchange rate peg surely is the inclusion of the dynamics of the GBR as well as the instability scenarios that are possible in this simple as well as in a more extended framework.

7.6.2 The Case of Flexible Exchange Rates

Similar to our considerations in Sect. 7.4 of this chapter we now consider the opposite case of perfectly flexible exchange rates and flexible interest rates, i.e. the case of a given money supply and a central bank that allows international capital markets to determine the exchange rate through the supply (demand) and demand (supply) of foreign (domestic) bonds. The IS-, LM-, FF-schedules now in general determine simultaneously output Y, the domestic rate of interest i and the exchange rate e. We simplify this equilibrium portion of the model again by assuming that money demand depends on normal output $\bar{Y}$ instead of actual output. This implies that the rate of interest r depends only on the state variable p of the dynamics. In the case of the Cagan money demand function $m^d = kY\exp(\alpha(i^* - i))$, e.g., we get:

$$i = i^* + \frac{\ln p - \ln M + \ln k + \ln \bar{Y}}{\alpha} = i(p).$$

Furthermore, inserting this dependency into the equilibrium condition for the international capital market then implies a dependency of the exchange rate e on the state variable of the dynamics that is independent of the characteristics of goods market equilibrium. This latter equilibrium condition can subsequently be solved for output Y in order to obtain the comparative statics of the IS-curve with respect to the three state variables p, W, W^*. Again we assume that the central bank may use open market operations in domestic bonds to change the composition of these bonds and money in the household's portfolio, but does not issue money otherwise. We also again assume that taxes are determined endogenously such that the real debt of the government W_g^a remains constant.

Temporary Equilibrium

For full asset markets equilibrium as characterized by the LMFF-curve, we only need to consider the international flow market for bonds in addition, which in the considered exchange rate regime reads ($e\dot{F}_{p2}/p = \dot{B}^*/p$):

$$f^d(i^* + \epsilon(e) - i(p, M), W - W^* - W_g^a) \overset{LMFF}{=} -f^{d*}(i^* + \epsilon^*(e) - i, W^*). \tag{7.55}$$

It is easy to see, compare Fig. 7.5, that a decrease in i shifts both curves in this figure to the right, which implies that the exchange rate will increase in such a case (the opposite will occur in the case of an increasing foreign interest rate i^*). Furthermore, it is also easy to see that an increase in the foreign wealth of domestic residents W will increase the exchange rate, while an increase in the domestic bond holding W^* of foreigners will have ambiguous effects. Finally, an increase in money supply M as component of private wealth $W_p = W - W^* - W_g^a$ (which increases the steady state value of the exchange rate—the point of reference for the assumed regressive expectation mechanism in the foreign exchange market—by the same percentage) must increase the current exchange rate beyond this level and therefore leads to overshooting exchange rate reactions, see below.

Next we consider, on the basis of the given capital market schedule $e = e(W, W^*, p)$ the IS-equilibrium curve of the presently considered situation:

$$\begin{aligned} Y \overset{IS}{=} C_1 \big(&Y + (i^* + \epsilon(e) - \hat{p}(Y))(W - W^*) - G, W - W^* - W_g^a, \\ &i^* + \epsilon(e) - \hat{p}(Y), i(p) - \hat{p}(Y), e/p) + I(Y, i^* + \epsilon(e) - \hat{p}(Y), i(p) - \hat{p}(Y)) \\ &+G + X(Y^*, e/p). \end{aligned} \tag{7.56}$$

We assume for the time being that the parameters of this equilibrium condition are such that the conventional dependency of IS-equilibrium output on the price level results: $\partial Y/\partial p < 0$. With respect to the wealth terms W, W^* it is obvious that $\partial Y/\partial(W) > 0$ and $\partial Y/\partial(W^*) < 0$ holds true. One has of course to use our regressive expectations regime, the dependence of the nominal rate of interest and the real exchange rate on the price level and the functional dependence of the nominal exchange rate on p, W, W^* derived above to derive conclusions on how the equilibrium output level depends on the price level p and on W, W^*. The outcome is however ambiguous, but pointing to a certain degree to a (conventional) negative overall dependence of Y on p. We shall assume that this holds true in our following discussion of overshooting exchange rates, since the opposite case would imply a destabilizing feedback of the price level on its rate of change via the Phillips curve mechanism. We thus have from the above also a schedule $Y = Y(p, W, W^*)$ characterizing goods market equilibrium on the basis of full asset markets equilibrium. We note that shocks to the exchange rate e will give rise to shocks in the variables W, W^* in particular, but that the situation after such shocks is then determined by a smooth evolution according to the laws of motion discussed in the next subsection.

Laws of Motion

We have by now determined the statically endogenous variables of the considered MFT regime (r, e, Y) by the three equilibrium relationships that now characterize the model. The state variables of the model are p, W, W^* (while the movement of the capital stock is still neglected). The laws of motion for these variables are in the present case given by:

$$\dot{W} = r^*(W - W^*) + Y - C(Y_p, W - W^* - W_g^a, r, r^{*e}, \eta) - I(Y, r, r^{*e}) - G + \dot{W}^*, \tag{7.57}$$

$$\dot{W}^* = \dot{B}^*/p - \hat{p}(Y)W^* = f^{d*}(\xi^{*e}, W^*) - \hat{p}(Y)W^*, \tag{7.58}$$

$$\hat{p} = \beta_w(Y - \bar{Y})/(1 - \gamma) + \epsilon(e) + \pi^*, \quad \pi^* = 0 \tag{7.59}$$

based on the following further definitions and further relationships:

$$r = i(p) - \hat{p}(Y), \tag{7.60}$$

$$r^* = i^* + \hat{e} - \hat{p}(Y) \quad [\text{expected rate} \quad r^{*e} = i^* + \epsilon(e) - \hat{p}(Y)], \tag{7.61}$$

$$\xi^* = i^* + \hat{e} - i(p) \quad [\text{expected rate} \quad \xi^{*e} = i^* + \epsilon(e) - i(p)], \tag{7.62}$$

$$Y_p = Y + r^{*e}(W - W^*) - G, \tag{7.63}$$

$$\eta = e/p \quad [p^* = 1] \tag{7.64}$$

and the equilibrium schedules $Y(W, W^*, p), e(W, W^*, p)$.

The law of motion for W is now a very complicated one, since the static relationships $e(W, W^*, p), Y(W, W^*, p)$ have still to be inserted into it in various places. A full treatment of the model is thus almost impossible so that one has to recourse to simplifications like the one considered in Sect. 7.4. Further modifications of the above approach could, however, include a direct modelling of the exchange rate dynamics in order to avoid the difficulties just mentioned and to allow for a better comparison between the case of bilateral international capital flows with the case of non-tradable domestic bonds which was considered before. But even models of this kind may include many scenarios where the steady state of the economy is indeed surrounded by repelling forces in place of attracting ones and where therefore nonlinearities in economic behavior have to be found that keep the dynamics bounded.

7.7 Conclusions

In this chapter, we have considered a small open economy of the Mundell-Fleming-Tobin type, containing a Keynesian demand constraint on the goods market as well as imperfect substitutability of financial assets in place of an UIP condition. In a first step, this imperfectness was coupled with the assumption that domestic bonds are non-tradables, i.e., the amount of foreign bonds held domestically was only changed to the extent that there is a surplus or a deficit in the current account. Additionally, we also assumed regressive exchange rate expectations. This simplifying assumption was helpful for the central objective of the chapter which was to isolate the fundamental destabilizing forces contained from the two accumulation equations of the model, concerning internal and external deficits or surpluses caused by the government budget equation and the evolution of the current account of the considered economy. Due to our use of a standard open economy money wage Phillips curve, the Keynesian business fluctuations approach was accompanied by labor market driven inflation or deflation dynamics which—in combination with the capital account and government budget dynamics—provided a dynamic model that goes significantly beyond standard Mundell-Fleming type approaches.

In a second step, we then introduced bilateral international capital flows. Although a thorough reconsideration of the budget constraints was necessary, the resulting dynamics turned out not to differ significantly from the previous case (where domestic bonds were non-tradables). It should be mentioned, however, that a relaxation of certain simplifying assumptions (common for both scenarios considered) might alter these findings, so that there is still a promising field for future research.

On the other hand it should be noted, that already the present setup reveals a considerable degree of complexity, especially with regard to the rich set of feedback channels present in the model: Hicksian disposable income effects, Pigou price level effects, Keynes price level effects, the Mundell-Tobin effect of inflationary expectations in both the consumption and the investment function, Dornbusch exchange rate effects, portfolio effects, and the stated stock-flow interactions. The interaction of these effects allowed for a variety of (in-)stability results, too numerous to allow their investigation in a single chapter of this model type. We therefore concentrated here on a regime with pegged interest rate as well as exchange rate and contrasted this situation with a regime where the exchange rate is perfectly flexible and the money supply a given magnitude under the control of the monetary authorities of the domestic economy. The (in-)stability results that were obtained suggested that this type of approach is rich in implications, but unfortunately poor in providing simple and unambiguous answers to those who prefer simple economic conclusions and on this basis also simple advices for policy interventions.

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8

Currency Crises, Credit Rationing and Monetary Policy in Emerging Markets

8.1 Introduction

Since the breakdown of the Bretton-Woods exchange rate system and the subsequent liberalization of the international capital markets, a striking proliferation of currency and financial crises throughout the world has been observed, the majority of them though taking place in the so called "emerging market economies". Among these episodes of financial turmoil, the 1994–95 Mexican and the 1997–98 East Asian crises are quite important chapters not only due to the suddenness and extent of the resulting nominal exchange rate depreciations but also because of the severity by which the real side of the economy was affected in the concerned countries.

A variety of studies such as Krugman (2000a), Aghion et al. (2001, 2004), and Céspedes et al. (2003), for example, have elaborated on the mechanisms of how a currency crisis can trigger a financial crisis that may lead to a severe economic slowdown in such type of economies.[1] This research has focused on the credit market problems that arise for firms after a strong currency devaluation in a country where credit market frictions exist and where a significant fraction of domestic banks and firms possesses unhedged foreign currency denominated liabilities. Yet, in those models the wage and the price levels have usually been assumed to remain constant over time.

The main contribution of this chapter to the currency crises literature is to introduce a macroeconomic framework with gradually adjusting domestic wages and prices which shows how not only nominal, but also (through do-

[1] For an excellent discussion of first-, second- and third- generation currency crisis models, see Gandolfo (2001, Chap. 16).

mestic wages and price level changes) real exchange rate adjustments affect the dynamics of the macroeconomic activity after the occurrence of a currency crisis in the medium run. In our view wage and price dynamics are the missing link to explain the medium run dynamics of a country that has experienced—through a currency and financial crisis—a severe slowdown in its economic activity.

Our study is organized as follows: In Sect. 8.2 we present a general version of the currency crises theoretical framework discussed in Flaschel and Semmler (2006) and Proaño et al. (2005). We discuss the dynamics of output and the nominal exchange rate after a currency crisis first, in Sect. 8.3, under the assumption of fixed domestic wages and goods prices and then, in Sect. 8.4, under the assumption of a gradual adjustment of these variables. Furthermore, in Sect. 8.5, we discuss the more general case where nominal exchange rates as well as domestic prices adjust to the state of the economy, showing the different mechanisms that allow the economy to recover after a sharp credit tightening due to the occurrence of a sudden currency mismatch. The empirical results of a VARX analysis are presented and discussed in Sect. 8.6. Section 8.7 draws some concluding remarks.

8.2 The General Framework

In order to analyze the effects of sharp currency devaluations in economies with unhedged dollarized liabilities in graphical though analytically nontrivial manner, we build on a modified version of the Mundell-Fleming-Tobin developed by Rødseth (2000) as the one discussed in Flaschel and Semmler (2006). We thus analyze a small open economy where output is basically demand-driven and, in the most general case, wages/prices as well as nominal exchange rates adjust gradually to disequilibrium situations in the labor and the financial markets, respectively. For now we abstract from international capital inflows as well as significant changes in the capital stock of the domestic economy K, in the domestic households' financial wealth W_h as well as in the foreign and domestic currency debt of the domestic entrepreneurial sector, assuming thus that the analyzed time span is short enough to allow considering these variables as basically unchanging despite the presence of positive or negative net investment and households' savings. Furthermore we assume a constant foreign price level p^*, which we normalize to one ($p^* = 1$) for simplicity. The real exchange rate is defined as $\eta = ep^*/p = e/p$, with e as the nominal exchange rate and p as the domestic price level.

8.2.1 The Goods Markets

As in Proaño et al. (2006), we assume that the domestic entrepreneurial sector can finance its investment projects through the issuance of bonds denominated in domestic (with capital costs i) as well as in foreign currency (with capital costs i^*) B_f and F_f, respectively (where $F_f < 0$ and $B_f < 0$, indicating a negative foreign and domestic currency bond stock held by domestic firms, or in other words, that firms are indebted). We assume that the domestic firms cannot hedge their exchange rate exposure, so that the domestic currency value of their dollarized liabilities eF_f evolves in a one-to-one manner with the evolution of the exchange rate. For simplicity we assume that each firm chooses just one investment financing option, so that in the aggregate the domestic entrepreneurial sector can be divided into two fractions: The one fraction, denominated by υ, finances its investment projects solely through foreign currency denominated credits, while the other fraction of domestic firms $(1 - \upsilon)$ borrows only in domestic currency.

As usually done in similar currency crises models which analyze the observed output decline after the 1997–98 East Asian currency and financial crisis such as Aghion et al. (2001, 2004) and Céspedes et al. (2003), we assume the existence of asymmetric information between the domestic firms (the potential borrowers) and the lending institutions which forces the latter to determine the level of credit awarding on the basis of some notion about the creditworthiness of the domestic firms, such as their net worth. In our theoretical framework the net worth of a firm is defined as the difference between its assets—which we assume to consist only of fixed capital K—and its liabilities $B_f + eF_f$ (both expressed here in domestic currency). Note that while the exchange rate influences the net worth of the firms with foreign currency liabilities, it does not affect the net worth of the firms indebted in domestic currency $(1 - \upsilon)$. A glance at the balance sheet of the fraction of domestic firms υ can clarify why this holds: A rise of the nominal exchange rate (or an decrease of the domestic price level) leads to an increase in the nominal (and here also real) value of the liabilities of this group and therefore to a decrease in its net worth, without affecting the net worth of the other group of firms.[2]

[2] Again, we assume that the totality of the foreign currency debt is unhedged: As discussed e.g. in Röthig et al. (2007), with increasing currency hedging by the domestic firms, the fragility of the real side of the economy to unexpected exchange rate depreciation decreases.

Table 8.1. Business Sector's Balance Sheets

	Business Sector's Balance Sheets	
Firms' Fraction	Assets	Liabilities
υ	pK_υ	eF_f
$1-\upsilon$	$pK_{1-\upsilon}$	B_f

Note that despite the fact that the share υ of domestic firms borrowing in foreign currency is kept constant here, one could think of it as being a function of the risk premium ξ. For now, though, we just assume that due to financial technology differences or firm size factors not the totality but only a constant fraction of the domestic entrepreneurial sector can actually obtain loans denominated in foreign currency.

We assuming that the lending institutions evaluate the creditworthiness of the domestic firms based on their respective debt-to-capital ratio

$$\tilde{q}_\upsilon = \frac{eF_f}{p\bar{K}_\upsilon} = \tilde{q}_\upsilon(\eta) \quad \text{and} \quad \tilde{q}_{1-\upsilon} = \frac{B_f}{p\bar{K}_{1-\upsilon}}.$$

Note that even though both groups might be subject to the imposition of credit constraints by the financial sector, only the debt-to-capital ratio of the fraction of firms indebted in foreign currency is affected by nominal exchange rate fluctuations.

Under the assumption that $\bar{K} = \bar{K}_\upsilon + \bar{K}_{1-\upsilon} = \text{const.}$, together with $B_f = \text{const.}$ and $F_f = \text{const.}$, we can represent the nonlinear relationship between the real exchange rate level and the extent of credit rationing in the economy through[3]

$$\mu_F = f(\eta) = \frac{1}{1+(\eta-1)^2} - 1 = -\frac{(\eta-1)^2}{1+(\eta-1)^2}. \tag{8.1}$$

According to (8.1), the extent of credit rationing by the lending institutions depends on the actual deviation of the real exchange rate from its PPP consistent level: For $\eta \approx 1$, $\tilde{q}_\upsilon = eF_f/pK_\upsilon \approx F_f/K_\upsilon$, and $\mu_F \approx 0$. In the contrary

[3] In our previously cited works we followed Krugman (2000a) by assuming that the elasticity of the investment function with respect to e (η) was high for "intermediate values" and low for extreme "low" and "high" exchange rate values. This specification, though maybe not completely theoretically founded, allowed us to analyze the consequences of a currency breakdown for the dynamics of output in a graphical manner. The nonlinear specification of the financial accelerator of this paper allows us to still do so, with the advantage of being more theoretically grounded and computationally feasible.

case, as η increases, the measure of creditworthiness of the domestic firms $\tilde{q}_v = eF_f/pK$ becomes more and more relevant for the credit awarding decisions by the lending institutions, and consequently for the level of aggregate investment in the economy, because

$$\lim_{\eta \to \infty} \mu_F = -1.$$

A sharp currency devaluation (and therefore an increase in the debt-to-capital ratio of the domestic firms indebted in foreign currency) leads to the activation of credit constraints by the lending institutions (the "balance sheet channel" introduced by Bernanke et al. (1994)) and thus to a decrease in the aggregate investment, since

$$\mu_F'(\eta) = -\frac{2(\eta - 1)}{(2 - 2\eta + \eta^2)^2} < 0 \quad \text{for} \quad \eta > 1.$$

This effect, nevertheless, is not unbounded, as the second partial derivative of I with respect to η shows:

$$\mu_F''(\eta) = \frac{2(2 - 6\eta + 3\eta^2)}{(2 - 2\eta + \eta^2)^3}$$

since

$$\lim_{\eta \to \infty} \mu_F''(\eta) = 0.$$

As $\mu_F(\eta)$ is specified, it allows us to model the nonlinear nature of the credit rationing by the financial institutions as a reaction to a decline of the firms' net worth caused by a sudden sharp devaluation of the domestic currency. We sketch the dependence of the aggregate investment on the nonlinear financial accelerator term μ_F in Fig. 8.1.

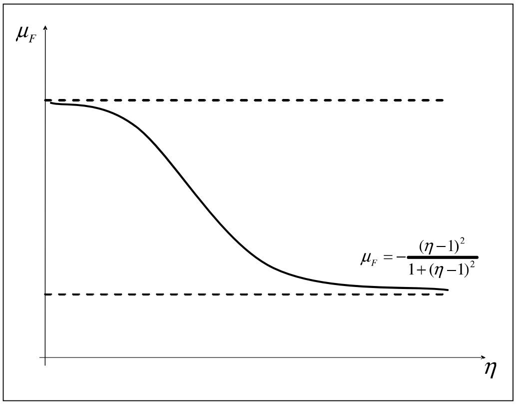


Fig. 8.1. A real exchange rate state-dependent financial accelerator term

Note that even though the theory behind Fig. 8.1 is of a very simple nature, it allows us to incorporate the basic implications of the theory of imperfect capital markets developed by Akerlof (1970) and Stiglitz and Weiss (1981) and to discuss manner the operation of the Bernanke et al. (1994) financial accelerator effect in a graphical in our framework.

Furthermore, such a specification opens up the possibility of multiple equilibria and, therefore, to the existence of "normal" and "crisis" steady states, respectively. Note that the magnitude of the balance sheet effect depends in a great manner on υ, that is on the degree of liability dollarization of the economy: For $\upsilon = 1$, i.e. in the case of total liability dollarization (as in Flaschel and Semmler 2006), the balance sheet effect alone (except changes in the foreign interest rate) determines the level of aggregate investment. For $\upsilon = 0$, on the contrary, changes in the real exchange rate do not directly affect the financial state of the domestic firms.

In the aggregate, the investment function is thus determined by

$$\begin{aligned} I &= i_o - (1-\upsilon)i_1(r) + \upsilon\mu_F \\ &= I((1-\upsilon)\underset{-}{r}, \upsilon\underset{-}{\mu_F}) \end{aligned} \tag{8.2}$$

with $r = i - \hat{p}$ as the real domestic interest rate and i_1 as the interest rate sensitivity of aggregate investment.

Concerning the rest of the economy, we assume quite standard consumption and net exports functions, so that the goods market equilibrium in the small open economy can be expressed as

$$Y = C(Y - \bar{T} - \delta\bar{K}) + I((1-\upsilon)r, \upsilon\mu_F) + \delta\bar{K} + \bar{G} + NX(Y^*, \eta, Y) \tag{8.3}$$

with $Y - \bar{T} - \delta\bar{K}$ denoting the disposable income, $\bar{T}$ lump sum taxes, $\bar{G}$ the government consumption and $\delta\bar{K}$ the capital depreciation. Net exports NX depend in a standard way positively on the foreign output level (assumed for simplicity to be at its natural level) and on the real exchange rate $\eta = e/p$ (the foreign goods price still set equal to one), and negatively on Y, the domestic output level.

As for example in Blanchard and Fischer (1989), we assume the following dynamic adjustment process in the goods markets:

$$\begin{aligned} \dot{Y} = \beta_y(Y^d - Y) = \beta_y \big[& C(Y - \delta\bar{K} - \bar{T}) + I((1-\upsilon)r, \upsilon\mu_F) + \delta\bar{K} + \bar{G} \\ & + NX(Y^*, \eta, Y) - Y \big] \end{aligned} \tag{8.4}$$

with Y^d as the aggregate goods demand and β_y as a parameter representing the speed of adjustment in the goods markets.

Using the Implicit Function Theorem, it follows for the displacement of the DD-Curve with respect to nominal and real exchange rate increases

$$\left.\frac{\partial Y}{\partial \eta}\right|_{\dot{Y}=0} = -\frac{\partial_\eta I(\cdot)\upsilon\mu_F'(\eta) + \partial_\eta NX(\cdot)}{C'(Y) + \partial_Y NX(\cdot) - 1} \gtrless 0.$$

Here we can observe one of the essential points of our model. The effect of a devaluation of the domestic currency on economic activity depends on the relative strength of the exports reaction as compared to the reaction of aggregate investment.[4] In the "normal case", where firms are not wealth constrained, the exchange rate effect on investment is supposed to be very weak and thus dominated by the exports effect. Then we have

$$\partial_\eta NX(\cdot) > |\partial_\eta I(\cdot)\upsilon\mu_F'(\eta)| \Longrightarrow \left.\frac{\partial Y}{\partial \eta}\right|_{\dot{Y}=0} > 0.$$

In the "fragile case", i.e. in a middle range for the exchange rate, the balance-sheet effect of a devaluation of the domestic currency is assumed to be large so that it overcomes the positive exports effect:

$$|\partial_\eta I(\cdot)\upsilon\mu_F'(\eta)| > \partial_\eta NX(\cdot) \Longrightarrow \left.\frac{\partial Y(\cdot)}{\partial \eta}\right|_{\dot{Y}=0} < 0.$$

The resulting DD-curve is depicted in Fig. 8.2.

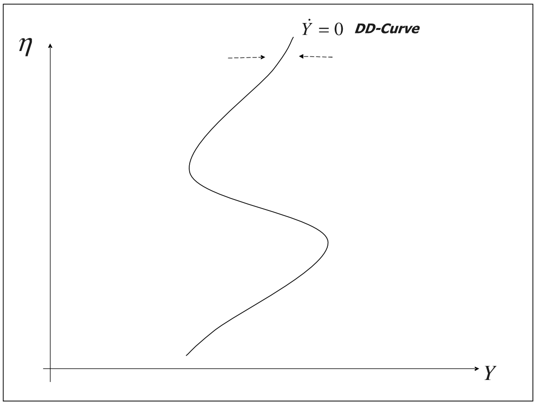


Fig. 8.2. The DD-Curve under state-dependent credit awarding

[4] The denominator is assumed to be unambiguously negative, so that the sign of the numerator is decisive for the slope of the DD-Curve.

8.2.2 The Financial Markets

Following Rødseth (2000), we follow a portfolio approach of Tobin type when modeling the financial markets in our framework. The defining financial market equations are:

$$\text{Households' Financial Wealth} \quad W_h = \frac{M + B_h + eF_h}{p} \tag{8.5}$$

$$\text{Risk Premium} \quad \xi = i(Y, \bar{M}/p) - \bar{i}^* - \epsilon \tag{8.6}$$

$$\text{Foreign Currency Bond Market} \quad eF_h = pf(\underset{-}{\xi}, \underset{+}{W_h}, \underset{+}{\kappa}), \tag{8.7}$$

$$\text{Money Market Equilibrium} \quad \bar{M}/p = kY + h_o - h_1 i = m(\underset{+}{Y}, \underset{-}{i}), \tag{8.8}$$

$$\text{Domestic Currency Bond Market} \quad B_h/p = W_h - f\,(\xi, W_h, \kappa) \tag{8.9}$$

$$\text{Equilibrium Condition} \quad F_h + F_c + \bar{F}_* = 0 \quad \text{or}$$

$$F_h + F_c = -\bar{F}_*. \tag{8.10}$$

Equation (8.5) describes the financial wealth of the private sector consisting of domestic money M, bonds in domestic currency B_h and bonds in foreign currency F_h, all expressed in domestic currency. Domestic and foreign currency denominated bonds are allowed to deliver different rates of return, meaning that the Uncovered Interest Rate Parity does not hold, at least in the short run.

The resulting risk premium of holding domestic currency bonds, i.e. the expected rate of return differential between the two interest bearing financial assets, is represented by (8.6), with ϵ denoting the expected rate of currency depreciation, which is assumed to be determined by

$$\epsilon = \hat{e}^e = \beta_\epsilon(1 - \eta).$$

The agents in the financial markets are assumed to form their expectations concerning the future depreciation rate of the nominal exchange rate based on the long run validity of the Purchasing Power Parity (PPP) postulate: If η, the real exchange rate is below one (is overvalued), the long run PPP consistent level, the agents will expect a nominal depreciation of the domestic currency. In the contrary case, where $\eta > 1$, that is, when the domestic currency is undervalued, the economic agents will expect an appreciation of the nominal exchange rate. For the steady state real exchange rate level $\eta = 1$, $\epsilon(\eta_o) = 0$ obviously holds. This expectation formation mechanism can be considered as purely forward looking and in this respect asymptotically rational, since the economic agents, having perfect knowledge of the long run steady state real exchange rate level η_o, expect the actual exchange rate to converge monotonically to it after each shock that affects the economy.

Equation (8.7) represents the market equilibrium for foreign currency denominated bonds. Hereby the demand for this type of financial assets is assumed to depend negatively on the risk premium, positively on the private financial wealth and positively on the parameter κ, which acts as a catch-all variable of foreign market pressures like political instability or speculative herding behavior. Equation (8.8) represents the domestic money market equilibrium with the usual reactions of the money demand to changes in interest rates and output. The domestic bond market given by (8.9) is then in equilibrium via Walras' Law of Stocks, if this holds for the bonds denominated in foreign currency.

The last equation describes the equilibrium condition for the foreign exchange market. It states that the aggregate demands of the three sectors—domestic private sector, the monetary authority and foreign sector—sum up to zero, see Rødseth (2000, p. 18). Following the assumption that the supply of foreign-currency bonds from the foreign sector is constant ($-\bar{F}_*$), the additional amount of foreign-currency bonds available to the private sector (besides his own stocks) is solely controlled by the monetary authorities (This assumption can be justified by assuming as in Rødseth (2000) that domestic bonds cannot be traded internationally). The prevailing exchange rate regime thus depends on the disposition of the central bank to supply the private sector with foreign-currency bonds.

By inserting the money market equilibrium interest rate (the inverse function of (8.8))

$$i(Y, \bar{M}/p) = \frac{kY + h_o - \bar{M}/p}{h_1}$$

in (8.7) the Financial Markets Equilibrium- or AA-Curve is derived

$$eF_h/p = f\left(i(Y, \bar{M}/p) - \bar{i}^* - \beta_\varepsilon \left(1 - \eta\right), \frac{M + B_h + eF_h}{p} \right). \tag{8.11}$$

This equilibrium equation can be interpreted as a representation of the $\dot{e} = 0$-isocline. Under the assumption that the exchange rate does not adjust automatically to foreign exchange market disequilibria, one may postulate as exchange rate dynamics:

$$\dot{e} = \beta_e \left[f\left(i(Y, \bar{M}/p) - \bar{i}^* - \beta_\varepsilon \left(1 - \eta\right), \frac{M + B_h + eF_h}{p}, \kappa \right) - \frac{eF_h}{p} \right]. \tag{8.12}$$

The slope of the $\dot{e} = 0$-isocline is determined by the Implicit Function Theorem in the following way:

$$\left.\frac{\partial e}{\partial Y}\right|_{\dot{e}=0} = -\frac{\partial_\xi f(\cdot\,)\,\partial_Y i(\cdot\,)}{\partial_\xi f(\cdot\,)\,\epsilon'(\eta) + (\partial_{W_h} f(\cdot\,) - 1)\,F_h/p} < 0.$$

8.3 Dynamics under Fixed Prices and Flexible Exchange Rates

As a starting point we analyze the case where domestic (and foreign) goods prices are fixed ($p = p^* = 1$), and the nominal exchange rate, once set to float, adjusts according to (8.12).

8.3.1 Local Stability Analysis

The case of fixed domestic goods prices ($p = 1$), where $\dot{\eta} = \dot{e}$ holds, was analyzed in Flaschel and Semmler (2006). In this case the differential equations of the currency crisis model are:

$$\dot{Y} = \beta_y \left[C(Y - \delta\bar{K} - \bar{T}) + I((1-v)r, v\mu_F) + \delta\bar{K} + \bar{G} + NX(Y^*, e, Y) - Y\right],$$
$$\dot{e} = \beta_e \left[f\left(i(Y, \bar{M}) - \bar{i}^* - \beta_\varepsilon(1-e), M + B_h + eF_h, \kappa\right) - eF_h\right].$$

The Jacobian of this system is

$$J = \begin{bmatrix} \beta_y\,[C'(Y) + \partial_Y NX(\cdot\,) - 1] & \beta_y\,[\partial_e I(\cdot\,)\mu'_F + \partial_e NX(\cdot\,)] \\ \beta_e(\partial_\xi f(\cdot\,)\,\partial_Y i(\cdot\,)) & \beta_e(-\partial_\xi f(\cdot\,)\,\epsilon'(e) + (\partial_{W_p} f(\cdot) - 1)F_h) \end{bmatrix}.$$

Because of the nonlinearity of the $\dot{Y} = 0$-isocline there exist three economically meaningful steady states in the considered situation, whose local stability properties can easily be calculated:

$$J_{E1} = \begin{bmatrix} - & + \\ - & - \end{bmatrix} \Longrightarrow \mathrm{tr}(J_{E1}) < 0, \quad \det(J_{E1}) > 0 \Longrightarrow \text{stable steady state}$$

$$J_{E2} = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \Longrightarrow \mathrm{tr}(J_{E2}) < 0, \quad \det(J_{E2}) < 0 \Longrightarrow \text{saddle point}$$

$$J_{E3} = \begin{bmatrix} - & + \\ - & - \end{bmatrix} \Longrightarrow \mathrm{tr}(J_{E3}) < 0, \quad \det(J_{E3}) > 0 \Longrightarrow \text{stable steady state.}$$

Steady state $E1$ represents the "normal" steady state, where the economy's output is high as well as the domestic investment activity. In this steady state, the standard case $|\partial_e I(\cdot)\upsilon\mu_F'| < \partial_e NX(\cdot)$ holds. Steady State $E2$ represents the fragile case with $|\partial_e I(\cdot)\upsilon\mu_F'| > \partial_e NX(\cdot)$: Because a slight deviation of the output level from this steady state level can lead the economy to a short-run investment boom or to a decline in the economic activity, this equilibrium point is unstable. Steady State $E3$ constitutes the "crisis equilibrium". At this equilibrium point the investment activity is highly depressed due to the high value of e. Nevertheless, the slope of the $\dot{Y}$-isocline is again positive because of $|\partial_e I(\cdot)\upsilon\mu_F'| < \partial_e NX(\cdot)$ describing the dominance of exports over (the remaining) investment demand in the considered situation.

Figure 8.3 shows the phase diagram of the complete DD-AA system for a floating nominal exchange rate regime.

Note that the AA-curve becomes only binding if the monetary authorities choose a flexible exchange rate regime. In a pegged exchange rate system the output level defined by the $\dot{Y} = 0$-isocline at the given exchange rate level fully defines the unique equilibrium of the system. In such a currency system the $\dot{e} = 0$-isocline can be interpreted as a "shadow curve" which represents the exchange rate level which would prevail if the currency was left to float freely. Because the monetary authorities are committed to remove any foreign exchange market disequilibria by buying or selling any amount of foreign currency bonds needed to defend the prevailing exchange rate level, the $\dot{e} = 0$-isocline represents the over-demand or over-supply with which the central bank is confronted. Consequently, the larger the difference between

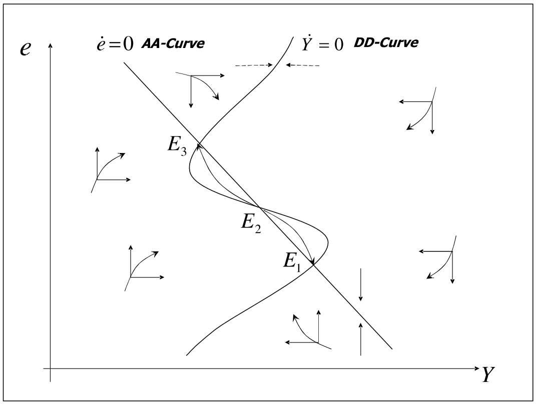


Fig. 8.3. The DD-AA model

the exchange rate level given by the "shadow curve" and the currency peg level, the higher is the demand for foreign currency bonds that the domestic central bank has to satisfy, and vice versa.

8.3.2 The Baseline Currency Crisis Scenario

In this section the macroeconomic dynamics resulting from a currency and financial crisis in economies with the totality of liabilities denominated in foreign currency under an initial fixed exchange rate regime and constant wages and prices (as it was assumed in the simple theoretical model by Flaschel and Semmler (2006)) will be discussed in order to give a first intuition in the way the model works. The results presented here will be useful to highlight the results of the different model extensions to be discussed below.

Assume the economy is initially at steady state $E1$ in Fig. 8.4 and that The prevailing exchange rate system is a currency peg which is fully backed by the domestic central bank. Now suppose that the demand for foreign-currency bonds increases due to a rise in the "capital flight" parameter κ. As long as the central bank is disposed to defend the prevailing currency peg by selling foreign currency bonds there are no real effects on the domestic economic activity. In the case that the domestic monetary authorities decide to give in to the speculative pressures and to let the exchange rate float, the AA-Curve becomes the binding curve in the model. The exchange rate then jumps from the initial equilibrium $E1$ to the corresponding point at the AA-Curve (with a still unchanged output level). From this point on, two

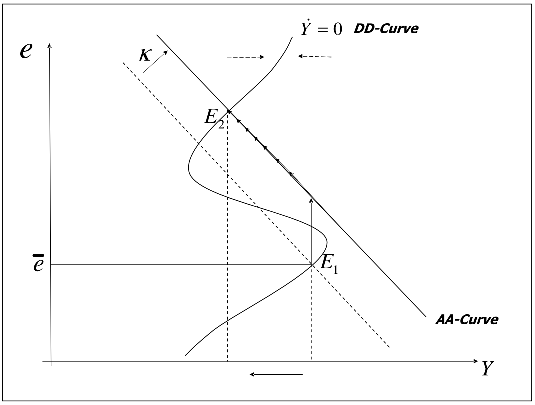


Fig. 8.4. The macroeconomic effects of a currency and financial crisis under fixed domestic goods prices

different scenarios which depend on the actual extent of the nominal exchange rate depreciation (or, in other words, the position of the AA-Curve by the time of the currency peg breakdown) are possible. In the first scenario, the corresponding point of the AA-Curve by the time of the depreciation lies below the backwards-sloped section of the DD-Curve. In this case, the depreciation and the resulting tightening of the credit conditions by the financial sector will not be of significant proportions so that the economy will, due to the temporary gain in competitiveness resulting from the depreciation, return to the equilibrium point $E1$.

In the second scenario, on the other hand, an increase in κ leads to a significant shift to the right of the AA-Curve, so that when the abandonment of the currency peg takes place, the corresponding point on the AA-Curve lies above the backwards-sloped section of the DD-Curve. As the trajectories sketched in Fig. 8.3 show, if this shift is large enough, the equilibrium point $E2$ will be the attractor of the dynamics and the economy will converge to it. As it can be clearly observed in Fig. 8.4, this convergence, which will take place along the AA-Curve, will imply further nominal exchange rate depreciations as long as $E2$ lies left to $E1$. Indeed, location of the new steady state $E2$ is central for the resulting dynamics not only of the nominal exchange rate but also of aggregate investment and output. If the new $E2$ lies left from the initial $E1$, the sharp nominal exchange rate depreciation will indeed lead to a breakdown of the economy due to the predominant effect of the depreciation on the credit awarding by banks and therefore on aggregate investment (and further nominal depreciations). But if on the contrary the new equilibrium point lies right from the old one, the gain in competitiveness and the resulting increase in the net exports will predominate the negative investment reaction. In this case the initial sharp depreciation of the currency will be followed by a revaluation of the currency and an increase in investment and output.

8.4 Dynamics under Gradually Adjusting Prices in Fixed Currency Regimes

Although the basic model just discussed is capable of describing the main sequence of events observed in the 1994–95 Mexican and 1997–98 East Asian currency and financial crises, it still neglects two central factors: the role of the price adjustments and the role of the domestic nominal interest rate. Indeed, because the domestic interest rate does not influence the level of domestic investment in a direct manner because the totality of domestic firms was assumed to borrow in foreign currency, the domestic monetary authorities are not confronted with an exchange-rate policy dilemma during a currency crisis. Theoretically, they are capable to increase the domestic interest rates indefinitely in order to defend the currency peg without producing any negative effects on the domestic economy. The decision to give in to a speculative attack on the domestic currency here relies more on the level of the foreign exchange reserves that the country has at the time of the currency run which may of course, dissipate during the currency run.

Despite the fact that during a currency and financial crisis price fluctuations probably do not play an important role because of the short time span in which such a twin crisis takes place, they surely are of great importance for the economic recovery process of such an economy. Real as well as nominal exchange rate fluctuations determine the international competitiveness of the domestic goods and the volume of the exports of the economy.

The assumption of constant domestic commodity prices in the context of sharp exchange rate fluctuations is also problematic because the exchange rate can affect the domestic price level through the following channels (see Svensson 2000):

- The exchange rate level affects the domestic currency prices of imported goods, and therefore the CPI inflation rate.
- The exchange rate affects domestic currency prices of intermediate inputs.
- The exchange rate can also influence the nominal wage determination through CPI inflation and thus again the domestic inflation.

Given the above shortcomings of the basic model, we discuss now a first variant of the baseline currency crisis model of the previous section. The purpose of this extended model is to show how besides the exchange rate-also domestic price level fluctuations can influence the macroeconomic performance of a small open economy with dollarized liabilities and credit market constraints.

The core of this extended model still is the balance-sheet state-dependent investment function. Since domestic price fluctuations are now included, besides the role of the exchange rate, changes in the domestic price level p now also influence the aggregate investment level of the small open economy (while K, F_f are still kept constant). The inclusion of price fluctuations into the considered dynamics through a standard Phillips-Curve, coupled with the price level dependent investment function indeed brings considerable complexity into the system.

The aggregate consumption and export functions remain unchanged, with the only difference that the real exchange rate $\eta = e/p$ (the foreign goods price still set equal to one for simplicity) is now mainly driven by domestic price and not nominal exchange rate changes. The same holds also for the reaction of aggregate investment to changes in η.

On the assumption that a fixed exchange rate system prevails ($\bar{e} = 1$), the slope of the $\dot{Y} = 0$-isocline is described in the extended phase space (Y, p) (for output and the domestic price level now) by:

$$\left.\frac{\partial Y}{\partial p}\right|_{\dot{Y}=0} = -\frac{\partial_p I(\,\cdot\,)((1-\upsilon)i' + \upsilon\mu_F') + \partial_\eta NX(\cdot)}{C'(Y)\partial_Y NX(\cdot) - 1}$$

It can easily be seen that the slope of the $\dot{Y} = 0$-isocline depends on which of the two opposite effects is the dominant one: the balance-sheet-effect $\partial_\eta I(\,\cdot\,) > 0$ or the competitiveness effect $\partial_\eta NX(\,\cdot\,) < 0$:

$$\partial_\eta I(\,\cdot\,)((1-\upsilon)i' + \upsilon\mu_F') > |\partial_\eta NX(\,\cdot\,)| \Longrightarrow \left.\frac{\partial Y}{\partial p}\right|_{\dot{Y}=0} > 0$$

and

$$\partial_\eta I(\,\cdot\,)((1-\upsilon)i' + \upsilon\mu_F') < |\partial_\eta NX(\,\cdot\,)| \Longrightarrow \left.\frac{\partial Y}{\partial p}\right|_{\dot{Y}=0} < 0.$$

From the shape of the assumed investment function there results (if its interior part is sufficiently steeper than its counterpart in the export function) that for intermediate values of $\tilde{q}$ the creditworthiness (the balance-sheet) effect is stronger than the "normal" competitiveness effect, changing the slope of the $\dot{Y} = 0$-isocline and therefore opening up the possibility of multiple equilibria now in the (Y, p) phase space. The structure of the financial markets is as described in the baseline model.

Concerning the domestic price dynamics, we use a simple expectations augmented, open-economy wage-price Phillips-curve (on the assumption of a constant productivity production function and mark-up pricing) which can be written as

$$\hat{p} = \beta_p(Y - Y^n) + \hat{p}_c^e \quad \text{with } p_c = p^\theta (ep^*)^{1-\theta},\ \theta \in (0,1). \tag{8.13}$$

We now use p_c for the consumer price level, based on a geometric mean of the domestic and the foreign price level, both expressed in the domestic currency. Superscript e denotes expectations, implying that marked up domestic wage inflation is explained in this Phillips Curve by the output gap and the expected consumer price inflation rate. By keeping the foreign price level constant and assuming perfect foresight concerning domestic price level inflation and asymptotically rational nominal exchange rate depreciation expectations (implying thus that the domestic agents incorporate their expected exchange rate changes in their price setting behavior) we can reformulate the open-economy wage-price Phillips-curve as

$$\hat{p} = \beta_p(Y - Y^n) + \theta\hat{p} + (1-\theta)\beta_\epsilon(1-\eta)$$

or, after rearranging,

$$\hat{p} = \frac{1}{1-\theta}\beta_p(Y - Y^n) + \frac{(1-\theta)}{(1-\theta)}\beta_\epsilon(1-\eta).$$

Defining $\tilde{\beta}_p = \beta_p/(1-\theta)$, we thus obtain

$$\hat{p} = \tilde{\beta}_p(Y - Y^n) + \beta_\epsilon(1-\eta) \quad \text{or} \quad \dot{p} = \left(\tilde{\beta}_p(Y - Y^n) + \beta_\epsilon\,(1-\eta)\right)p, \tag{8.14}$$

what turns to

$$\hat{p} = \left(\tilde{\beta}_p(Y - Y^n)\right)p, \tag{8.15}$$

if the nominal exchange rate as well as the foreign price level are believed to remain constant by the economic agents. More generally, one may simply assume for this case that inflationary expectations are still ignored by this model type in order to save one law of motion and to leave the discussion of destabilizing Mundell type effects for later extensions of the model, see our discussion of kinked Phillips curves below.

The implied $\dot{p} = 0$-isocline turns out to consist of two parts: for $p > 0$ the $\dot{p} = 0$-isocline is the straight vertical line $Y = Y^n$. Along the horizontal axis ($p = 0$), additionally, $\dot{p} = 0$ also holds: As discussed in the next section, a further—though economically not meaningful—steady state exists at the intersection of the $\dot{Y} = 0$-isocline and the horizontal axis.

8.4.1 Local Stability Analysis

The extended currency crisis model (under a fixed exchange rate regime) is fully described by the following differential equations:

$$\dot{Y} = \beta_y\left[C(Y - \delta\bar{K} - \bar{T}) + I((1-\upsilon)r, \upsilon\mu_F) + \delta\bar{K} + \bar{G} + NX(Y^*, 1/p, Y) - Y\right],$$
$$\dot{p} = \left[\tilde{\beta}_p(Y - Y^n)\right]p$$

with the nominal exchange rate depreciation expectations still ignored.

The Jacobian of this system at the steady state is given by:

$$J = \begin{bmatrix} \beta_y \left(C'(Y) + \partial_Y NX(\,\cdot\,) - 1\right) & \beta_y \left(\partial_\eta I(\,\cdot\,)((1-\upsilon)i' + \upsilon\mu'_F) + \partial_\eta NX(\,\cdot\,)\right) \\ \tilde{\beta}_p p_0 & 0 \end{bmatrix}.$$

Because of the nonlinear shape of the $\dot{Y} = 0$-isocline there exist three possible economically meaningful steady states,[5] whose local stability properties can easily be calculated:

$$J_{E1} = \begin{bmatrix} - & - \\ + & 0 \end{bmatrix} \Longrightarrow \mathrm{tr}(J_{E1}) < 0, \quad \det(J_{E1}) > 0 \Longrightarrow \text{a stable steady state}$$

$$J_{E2} = \begin{bmatrix} - & + \\ + & 0 \end{bmatrix} \Longrightarrow \mathrm{tr}(J_{E2}) < 0, \quad \det(J_{E2}) < 0 \Longrightarrow \text{a saddle point}$$

$$J_{E3} = \begin{bmatrix} - & - \\ + & 0 \end{bmatrix} \Longrightarrow \mathrm{tr}(J_{E3}) < 0, \quad \det(J_{E3}) > 0 \Longrightarrow \text{a stable steady state.}$$

The resulting dynamics of our extended currency crisis model (still a fixed exchange rate) are described in Fig. 8.5.

Steady state $E1$ represents the "normal" situation, where the domestic price level is high and therefore (under the assumption of a fixed exchange rate) η is low. In this steady state the economy's output is at its full employment level, the investment activity is high (due to the low $\tilde{q}_\upsilon$) and the exports are low due to a strong currency—relative to the high domestic price level—and thus low competitiveness.[6] In this steady state the standard situation $\partial_\eta I(\,\cdot\,)((1-\upsilon)i' + \upsilon\mu'_F) < |\partial_p NX(\,\cdot\,)|$ holds.

Steady State $E2$ represents the fragile case where $\partial_\eta I(\,\cdot\,)((1-\upsilon)i' + \upsilon\mu'_F) > |\partial_\eta NX(\,\cdot\,)|$ holds: A slight deviation of the output level from this steady state level can lead the economy to a short-run investment boom (and to an over-employment situation) or to an economic slowdown or a recession, since this

[5] Due to the nonlinearity of the second law of motion there exists a fourth steady state at the point where the $\dot{Y} = 0$-isocline cuts the horizontal axis. This fourth steady state is not relevant however, since it cannot be approached by the trajectories in the presently considered dynamics.

[6] Such a situation could be observed in the years preceding the East Asian Crisis, see Corsetti et al. (1999).

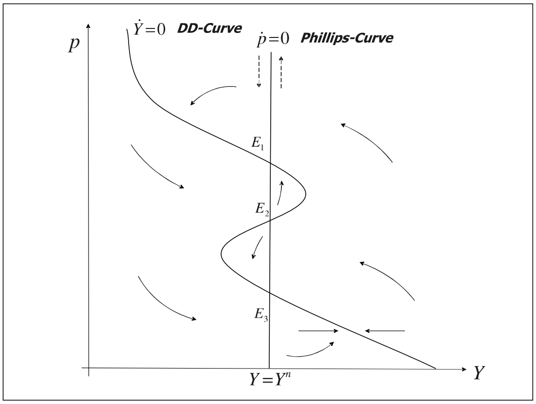


Fig. 8.5. Phase space representation of the DD-PC dynamics with a normal and a bad equilibrium, E_1 and E_3, respectively

equilibrium point is unstable up to the two stable arms that are shown as dotted lines.

Steady State $E3$ constitutes the "crisis equilibrium". At this equilibrium point the investment activity is severely depressed due to the high value of η $(\tilde{q}_v)$. The economy, though, is situated at its full-employment level because of the strong export of goods production induced by the very low level of domestic prices, and thus by the implied competitiveness of the economy.[7]

8.4.2 Dynamics with a Standard Phillips-Curve

Next, by employing a standard Phillips-Curve we discuss a situation where, after a one-time devaluation of the domestic currency as a result of a speculative attack, the economy experiences a financial crisis and can, eventually, shift from a "normal" equilibrium with high investment and low exports to a "crisis" equilibrium with depressed investment activity and high exports due to the included domestic price level adjustments. Assuming again a constant exchange rate after the devaluation, the model dynamics will be sketched in the $(p-Y)$ space instead of the $(e-Y)$ space as in the Flaschel and Semmler (2006) model.

Assume the economy is initially situated at its upper full employment level E_1 in Fig. 8.6. A significant flight into foreign currency can take place in the

[7] For proofs for the global stability of the system, see Proaño et al. (2005).

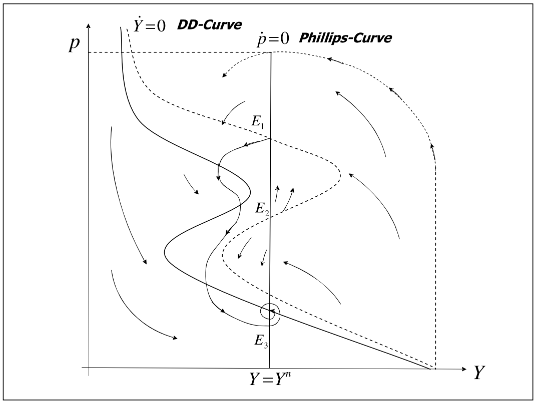


Fig. 8.6. The macroeconomic effects of a currency and financial crisis under gradually adjusting domestic goods prices

model through an increase of the κ parameter in the foreign currency bond demand. As long as the monetary authorities can defend the old currency peg, the flight into foreign currency does not have any effects besides the reduction of the foreign exchange reserves of the central bank (because a constant money supply is assumed, a full sterilization of money base changes by the central bank is also implicitly assumed). Now suppose that the central bank gives in to the foreign market pressure after a while, because of the dangerous lowering of foreign reserves. In order to release the pressure from the foreign exchange market, see (8.11), it carries out a one-time devaluation (for simplicity we assume in this section that the new exchange rate is then considered by the economic agents as “sustainable”). The effect of the currency devaluation on the $\dot{Y} = 0$-isocline, i.e. the direction of the shift of the $\dot{Y} = 0$-isocline depends on the strength of the balance-sheet and the competitiveness effects. To understand this point, consider the original model of Sect. 8.3 where prices were held constant: There too the output reaction to the exchange rate change depended on the strength of the exchange rate effect on investment and exports.

The direction of the shift of the $\dot{Y} = 0$-isocline in the extended 2D model (in the $(p - Y)$ space) is analogous to the exchange rate effect on output in the original model (in the $(e - Y)$ space): If $\partial_\eta I(\cdot)\upsilon\mu_F > |\partial_\eta NX(\cdot)|$ holds, the nominal devaluation of the domestic currency will shift the $\dot{Y} = 0$-isocline to the left and vice-versa.

The resulting system dynamics after a severe nominal devaluation of the currency under the first situation are sketched in Fig. 8.6.

Directly after the currency crash, the banks take into account the sharp deterioration of the balance sheets of the business sector due to the strong devaluation of the domestic currency. As a means of hedging themselves against “bad creditors”, they implement credit constraints and cut the volume of the granted loans. The industrial sector, put now in a dramatic financial situation, must cancel the majority of the investment projects either voluntarily or due to the credit constraints, leading to a reduction of aggregate demand, employment and, due to the Phillips-curve, also of the domestic price level.

We consider now as an example the case where the DD-curve—with the exception of the zero price level situation—unambiguously shifts to the left as a result of the currency devaluation. The steady state E_3 is assumed, due the size of the shift of the DD-curve, to be the only economically meaningful steady state. Falling prices are then at first accompanied by falling output levels. They lead to a further real depreciation, in addition to the initially caused nominal exchange rate shock. Thus in turn leads to further gains in competitiveness on the international goods market. Sooner or later, the increased foreign demand for domestic goods leads to such an increase in exports and that aggregate demand starts rising again, when the DD-curve is crossed from above by the initiated temporary deflationary process. The deflationary spiral, since output cannot rise by so much that the NAIRU level is reached again. Instead output will start falling once again—when the DD-curve is crossed again on the way down to the bad equilibrium—which still further down leads again to increasing output and then either to monotonic adjustment to the steady state E_3 or to damped cycles around it (shown in Fig. 8.6). We thus in sum observe a deflationary process with fluctuating economic activity below the NAIRU output level until the economy is back to normal output levels and smaller cycles around them.

Falling prices have negative effects as well as positive effects on aggregate demand. The latter because of an export increase and the former because they reduce the nominal value of the firms’ capital, deteriorating even more their financial situation. We thus have both counteracting forces operating simultaneously, with a stronger investment effect initially (implying decreasing output levels), and an over time increasingly stronger competitiveness effect, which will lead to improvements in the trade balance and therefore to higher economic activity, which however will still be lower than at its normal level and thus will still be accompanied by further deflationary pressures. The graphical analysis here shows that the economy will fluctuate around a somewhat persistent underemployment situation (with still falling prices) until the competitiveness effect finally dominates (maybe because the investment level has come close to its “floor” level) and the economy returns to its “full”-employment level. This must happen by assumption since it was assumed that the intersection of the DD-curve with the horizontal axis lies to the left of the NAIRU output level. Deflation therefore must come to an end and—maybe also with temporarily increasing price levels—approach the steady state price level at E_3 finally.

Note that the crucial mechanism in this dynamics that allows the economy to return to the full-employment situation is the variation of the domestic price level. Because in basic model sketched in Sect. 8.3.2 there are assumed prices to be constant, one cannot consider the (medium-run) possibility for the economy to return to a “full”-employment situation. In the basic model the country which suffered from a currency and financial crisis is “thrown back” to a lower steady state output level where it remained because the model did not produce any endogenous mechanism that would allow the economy to recover from the twin crisis. In this extended version where we allow prices to be sufficiently flexible the economic performance of the economy can sufficiently improve through a real depreciation of the domestic goods, leading it back to the full-employment level if the ceilings and floors in the investment and the export function are chosen appropriately.

However, in contrast to our basic model discussed in Sect. 8.3, where we could have large output loss in the crisis equilibrium, this extended model may raise the question to what extent the lower equilibrium represents a bad equilibrium. We have full employment in the sense of the NAIRU theory, coupled with high exports, but very low investment activity. The growth rate of the capital stock is thus very depressed or even negative. This, of course, is bad for the future evolution of the economy with respect to income growth and employment growth. Furthermore, if economic activity is moving towards the lower equilibrium, the evolution may be subject to long periods of deflation, in particular if there are downward rigidities in money wage adjustments (a nonlinear PC) as we will consider them briefly below. In the present model, the dangers of deflation are not fully included, and thus do not call for particular attention by the monetary authority—with the exception that monetary policy may be used to speed up the process of convergence to the good equilibrium and to avoid the bad equilibrium. This will be discussed in the next section.

8.4.3 Dynamics with a "Kinked" Phillips Curve

Yet, already Keynes (1936, Chap. 2) had recognized that an economy-wide wage deflation with a simultaneous deflation in money wages is rare to be observed. Taking downward money wage rigidity into account the wage-price Phillips-Curve can be modified as follows:

$$\hat{p} = \max\left\{\tilde{\beta}_p(Y - Y^n), 0\right\}. \tag{8.16}$$

This modified Phillips-Curve implies that in under-employment situations prices do not fall but instead remain constant. Price changes can only take place in over-employment situations, where the price level is assumed to rise as before. Adding such a kink in money wage behavior modifies the considered dynamics as follows:

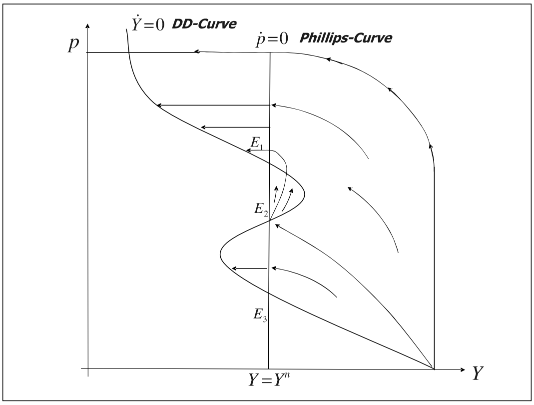


Fig. 8.7. The consequences of downward money wage rigidity

The empirical observation of downwardly rigid wages has important consequences for the dynamics of the extended model. Assume the country finds itself in the same situation as discussed above: After a run on the foreign currency the monetary authorities are forced to devaluate, so that—under the assumption that $\partial_\eta I(\cdot)\upsilon\mu'_F > |\partial_\eta NX(\cdot)|$ holds—aggregate demand declines and the $\dot{Y} = 0$-isocline shifts again to the left. Because in the entire economic domain to the left of the $\dot{p} = 0$-isocline we have that $\hat{p} = 0$ holds true, the $\dot{Y} = 0$-isocline is in this domain an attracting curve representing stable depressions. All points on the $\dot{Y} = 0$-isocline to the left of the $\dot{p} = 0$-isocline are thus now equilibrium points. For each of these steady states, there is no longer a mechanism that allows the economy to return to its "full"-employment level. The "crisis" equilibrium in the basic model of Sects. 8.3.2, derived from Flaschel and Semmler (2006), is now just one of these under-employment equilibria.

One may argue therefore that the assumed wage rigidity is bad for the working of the economy. However, at present deflation is still a too tranquil process in order to allow for any other conclusion. Advocates of downward wage flexibility should therefore not yet interpret the present framework as an developed argument that supports their view. Adding real rate of interest effects (Mundell-effects) to the formulation of aggregate demand, or adding a Fisher debt effect to investment behavior, can easily remove the lower turning point in deflation dynamics from the model. This would therefore imply an economic breakdown, in case the kink in the money wage PC was not there. The kinked wage Phillips Curve prevents or at least delays deflationary processes from working their way. Though stable depressions may be considered a big problem, the remedy to allow for a significant degree of wage flexibility would in such situations not revive the economy, but make things only worse, in particular in situations where a liquidity trap has become established.

In the present form of the model the kink in wage behavior however implies that a stable depression becomes established to the left of the good equilibrium E_1 with no effect on the price level, as it was already considered in Flaschel and Semmler (2006).

8.5 Dynamics under Gradually Adjusting Nominal Exchange Rates and Prices

After having investigated the role of nominal exchange rate and domestic price level adjustments for the medium run dynamics of our model separately, we analyze in this section the dynamics of the model when both the nominal exchange rate and the domestic price level are allowed to adjust to disequilibrium situations in both the goods and the financial markets. Note that the strength of the balance-sheet effect on the aggregate demand depends on the degree of dollarization of liabilities in the economy υ: For $\upsilon = 1$, the balance-sheet effect operates with full strength, while for $0 < \upsilon < 1$ this effect operates only partially.

In the flexible exchange rate regime the model consists of the following differential equations:

$$\begin{aligned}\dot{Y} &= \beta_y \left[C_1(Y-\bar{T}) + I((1-\upsilon)r, \upsilon\mu_F) + \delta\bar{K} + \bar{G} + X(\bar{Y}^*, \eta, Y) - Y\right],\\ \dot{\eta} &= \frac{\beta_e}{p}\left[f\left(i(Y, \bar{M}/p) - \bar{i}^* - \beta_\varepsilon(1-\eta), W_h, \kappa\right) - \eta F_h\right]\\ &\quad - \left(\tilde{\beta}_p(Y-Y^n) + \beta_\epsilon(1-\eta)\right)\eta.\end{aligned}$$

The first differential equation is the standard goods markets adjustment mechanism described by (8.4) in Sect. 8.2. The second equation gathers (8.12) and (8.14) and represents the adjustment of the real exchange rate to changes in Y. As it can easily be observed, for $Y > Y^n$ the real exchange rate appreciates, i.e. $\dot{\eta} < 0$. This development results from two effects: on the one hand, a rise in Y above Y^n leads to a nominal interest rate increase due to the Taylor rule and therefore to a nominal exchange rate appreciation, i.e. $\dot{e} < 0$. On the other hand, for $Y > Y^n$ follows that $\dot{p} > 0$. The additional term of exchange rate expectations only strengthens this effect (but it is not its main determinant). For $Y < Y^n$, the opposite holds.

The Jacobian of this system is

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

with

$$\begin{aligned}J_{11} &= \beta_y \left[C'(Y) + \partial_r I(\,\cdot\,)(1-\upsilon)\partial_Y i(\,\cdot\,) + \partial_Y NX(\,\cdot\,) - 1\right],\\ J_{12} &= \beta_y \left[\partial_\eta I(\,\cdot\,)\upsilon\mu_F' + \partial_\eta NX(\,\cdot\,)\right],\\ J_{21} &= \frac{\beta_e}{p}\partial_\xi f(\,\cdot\,)\partial_Y i(\,\cdot\,) - \tilde{\beta}_p\eta,\\ J_{22} &= \frac{\beta_e}{p}(\partial_{W_p} f(\,\cdot\,) - 1)F_{po}).\end{aligned}$$

Because of the nonlinearity of the $\dot{Y} = 0$-isocline there exist three economically meaningful steady states in the considered situation, whose local stability properties can be calculated easily:

$$J_{E1,\,E3} = \begin{bmatrix} - & + \\ - & - \end{bmatrix} \Longrightarrow \text{tr}(J_{E1}) < 0, \quad \det(J_{E1}) > 0 \Longrightarrow \text{stable steady state}$$

$$J_{E2} = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \Longrightarrow \text{tr}(J_{E2}) < 0, \quad \det(J_{E2}) < 0 \Longrightarrow \text{saddle point}$$

under the assumption of a stable interaction between goods markets, price dynamics and monetary policy, implying that

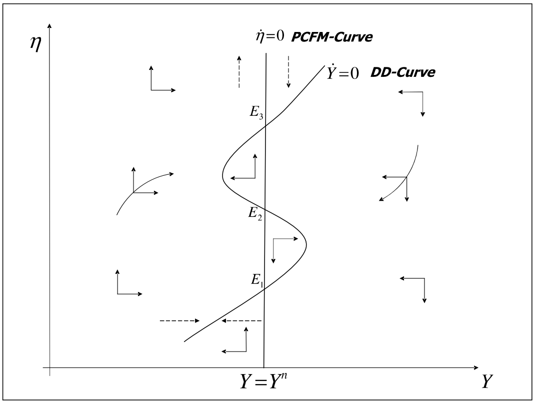


Fig. 8.8. The DD-PCFM model

$$C'(Y) < |\partial_r I(\cdot)(1-\upsilon)\partial_Y i(\cdot) + \partial_Y NX(\cdot) - 1|$$

holds for J_{11}. The resulting phase diagram of this currency crisis model is sketched in Fig. 8.8.

Steady state $E1$ represents the "normal" steady state, where the economy's output is high as well as the domestic investment activity. In this steady state, the standard case $|\partial_\eta I(\,\cdot\,)\upsilon\mu'_F| < \partial_\eta NX(\,\cdot\,)$ holds. Steady State $E2$ represents the fragile case with $|\partial_\eta I(\,\cdot\,)\upsilon\mu'_F| > \partial_\eta NX(\,\cdot\,)$: Because a slight deviation of the output level from this steady state level can lead the economy to a short-run investment boom or to a decline in the economic activity, this equilibrium point is unstable. Steady State $E3$ constitutes the "crisis equilibrium". At this equilibrium point the investment activity is highly depressed due to the high value of e. Nevertheless, the slope of the $\dot{Y} = 0$-isocline is again positive because of $|\partial_\eta I(\,\cdot\,)\upsilon\mu'_F| < \partial_\eta NX(\,\cdot\,)$ describing the dominance of exports over (the remaining) investment demand in the considered situation.

8.5.1 The Case of Total Liability Dollarization $\upsilon = 1$

In order to highlight the implications of the incorporation of the domestic interest rate in the aggregate investment function of our model, we discuss first the resulting dynamics for the complete liability dollarization case ($\upsilon = 1$), as it was the case in Flaschel and Semmler (2006) and Proaño et al. (2005). Assume the economy is initially situated at its NAIRU employment level in steady state E_1. A speculative attack on the domestic currency can be represented in our model by an increase of the parameter κ in the foreign currency bond demand. In such a situation, the domestic monetary authorities can choose between two strategies: Increasing the nominal interest rates or serve the excess demand by the reduction of their foreign exchange reserves. In both cases, the flight into foreign currency does not have any real effects as long as the monetary authorities can defend the old currency peg besides the reduction in the money supply—resulting from the operations in the foreign exchange market—and the reduction of the foreign exchange reserves of the central bank. Since domestic investment is solely financed through foreign currency credit, as long as the nominal exchange rate remains unchanged, the speculative attack on the domestic currency does not affect the creditworthiness of—and therefore the credit awarding to domestic firms. Now suppose that the central bank gives in to the foreign market pressure due to, say, a dangerous approaching of the international reserves to a critical level. As a result of a speculative attack on the domestic currency, the nominal exchange rate devaluates sharply, triggering the activation of credit constraints by the financial sector and depressing the level of aggregate investment and therefore of the entire economic activity. Now, while in the model discussed in the previous section the new exchange rate level after the one-time devaluation was assumed for simplicity to be considered by the economic agents as "sustainable", here, on the contrary, this must not necessarily be the case, as we will discuss below.

In the (η, Y)-space, the short run sharp nominal exchange rate devaluation is represented by a "jump" of the value of η along the $\dot{\eta} = 0$-isocline up to point B.[8] Now, due to the dynamic adjustment mechanism in the goods markets, Y declines so that $Y < Y^n$. This development has two effects, as already discussed in the previous section. On the one hand (assuming now a post-crisis flexible exchange rate regime), a decrease in Y leads to a fall in the domestic interest rates and to a higher demand for foreign bonds and to a further rise in e. On the other hand, the underemployment situation leads to a fall in the domestic price level, i.e. $\dot{p} < 0$. Both effects lead to a strong depreciation of real exchange rate, helping the economy to return to its NAIRU employment level through the expansion of the domestic ex-

[8] We assume that the nominal depreciation is of such magnitude that B lies above the unstable steady state $E2$. We will discuss the case where B lies below $E2$ below.

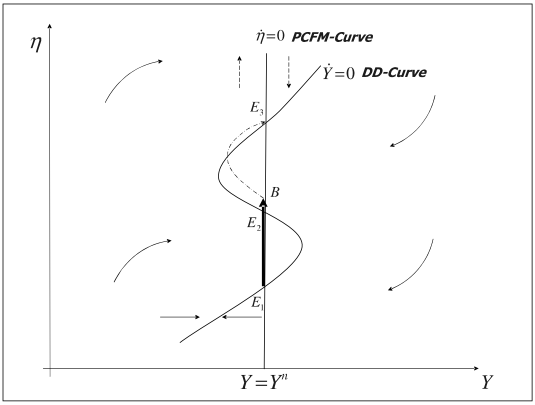


Fig. 8.9. The consequences of a breakdown of the currency peg in the case of total liability dollarization

ports. Obviously, in a post-crisis fixed exchange rate regime, a much more severe deflationary process than in a flexible exchange rate regime is required for the domestic economy to return to its pre-crisis NAIRU level of employment, as discussed in Proaño et al. (2005). These dynamics are sketched in Fig. 8.9.

Since now the nominal exchange rate is allowed to float after the occurrence of the currency crisis, there is no necessity for a severe deflationary process in order to reach the NAIRU consistent output again. Further nominal exchange rate adjustments (in this case depreciations) can also contribute to further real exchange rate devaluations.

As stated before, in the case of total liability dollarization, a defense of the prevailing exchange rate level by the domestic monetary authorities would not have (at least in the short run) a direct effect on the aggregate demand because the domestic firms finance their investment projects completely through foreign currency credit. Theoretically, the monetary authorities could thus indefinitely increase the domestic interest rates in order to reduce the pressure on the exchange rate without directly affecting the real sector of the economy. Such a measure could reduce the magnitude of an eventual nominal depreciation and might even generate a short run expansion due to the expansion of the net exports, if the real exchange rate jumps in the short run only to a point B' below the steady state E_2.

Downward rigidity as discussed by the "kinked" Phillips Curve in Sect. 8.4.3 does not "condemn" here, in contrast, the economy to remain in such underemployment equilibria due to the absence of a way to re-gain competitiveness

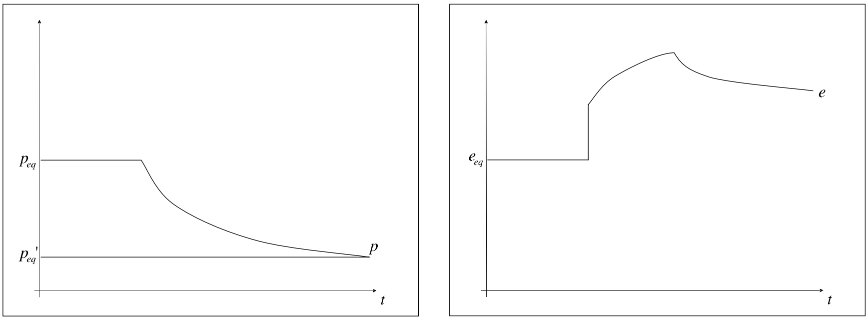


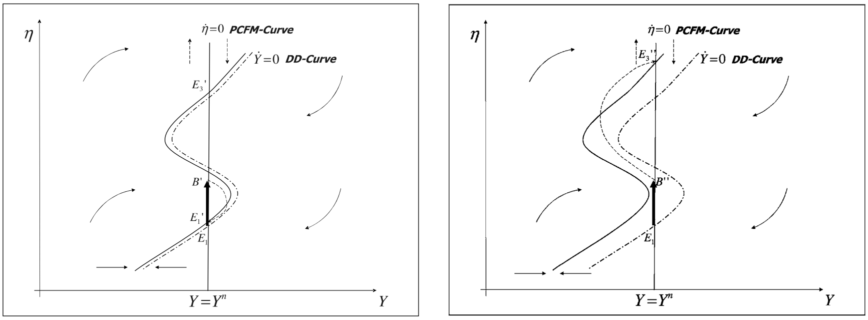
Fig. 8.10. Price level and exchange rate adjustments after a breakdown of the currency peg in the DD-PF model

and therefore to increase the level of exports again. In the presence of downward rigidity the nominal exchange rate assumes the whole weight of the recovery process so that further nominal (and also real) exchange rate depreciations are needed to enhance the competitiveness of the domestic products in the international goods markets and so to return to the NAIRU-production level.

8.5.2 The Case of Partial Liability Dollarization $0 < \upsilon < 1$

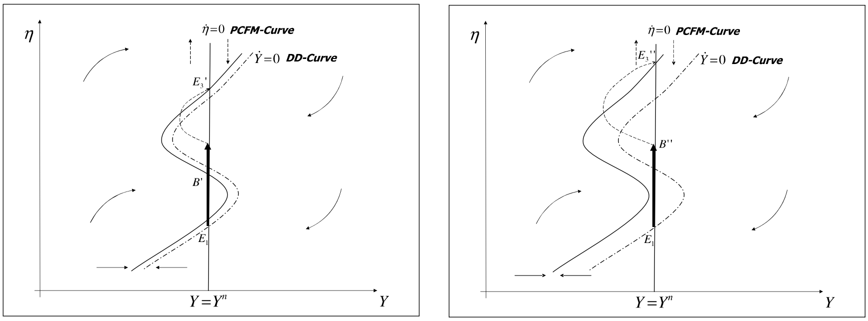
We now analyze the dynamics of the model for the case where a fraction of the domestic firms does not issue foreign-currency debt but instead finances its investment projects by domestic currency denominated credit. Despite the fact that the $\dot{Y} = 0$-isocline has basically the same shape as in the previous section, the magnitude of the balance-sheet effect on the aggregate investment and therefore on the aggregate demand depends on the degree of dollarization of liabilities in the economy.

In an economy with liabilities denominated only partially in foreign currency, during a speculative attack on the domestic currency the monetary authorities are confronted with a lose-lose situation. Exactly this situation is represented when $0 < \upsilon < 1$. In this case an increase of the domestic interest rate has a direct effect on the aggregate investment due to a subsequent decrease of the investments undertaken by the fraction $(1 - \upsilon)$ of domestic firms. In our model, such a response to a speculative attack on the domestic currency does not only influence the dynamics of $\dot{\eta}$, but it also shifts the $\dot{Y} = 0$-isocline to the left reducing the aggregate investment and demand, as sketched in Figs. 8.11 and 8.12. In the next sections we show that this poten-



a) b)

Fig. 8.11. Real exchange rate and output dynamics resulting from a successful defense of the exchange rate level under **(a)** low and **(b)** high interest rate elasticity of the aggregate investment



a) b)

Fig. 8.12. Real exchange rate and output dynamics resulting from a failed defense of the exchange rate level under **(a)** low and **(b)** high interest rate elasticity of the aggregate investment

tial counterproductiveness of an increase in the domestic interest rate by the monetary authorities depends on the elasticity of the aggregate demand with respect to interest rate changes.

Dynamics after a Successful Defense of the Currency Peg

Under the assumption that the increase of the domestic nominal interest rate by monetary authorities succeeds in lowering the pressure in the foreign exchange market so that the prevailing exchange rate (or currency peg in case of a fixed exchange rate regime) remains at its former level (or just slightly deviates from it), two scenarios are possible. If the elasticity of the aggregate investment with respect to domestic interest rate changes is low (and/or the degree of liability dollarization in the economy is particularly high), the $\dot{Y} = 0$-isocline will not significantly shift to the left and the economy will return to an equilibrium point very similar to the pre-crisis equilibrium situation after a short term period of slight over-production and -employment and a moderate domestic inflation, as sketched in Fig. 8.11a. If, on the contrary, the elasticity of the aggregate demand to interest rate changes is high (and/or the degree of liability dollarization is low), the DD-Curve will significantly shift to the left and the $E1$ equilibrium point might be missed, as sketched in Fig. 8.11b.

In this second case an increase in the domestic interest rate will thus lead to a severe economic slowdown in the short run due to the fall in the aggregate demand, despite of the successful defense of the nominal exchange rate. In the medium run, the equilibrium point E_3 is the only steady state to which the economy can converge to. Since $Y < Y^n$, the domestic price level will fall and $\dot{\eta} > 0$, even though the nominal exchange rate might remain fixed. This decrease in the domestic price level will enhance the competitiveness of the domestic products in the international markets, expanding the export volume and leading the economy in the medium run to the full-employment level E_3, with a different composition though, namely high exports and depressed investments. Note nevertheless that this recovery process might only take place if the domestic wages and prices fall sufficiently to enhance in a significant way the competitiveness of the domestic goods. If the domestic nominal wages and prices are downwardly rigid as empirically is the case in the majority of modern economies, then the economy might stay in an unemployment situation where the nominal exchange rate is constantly under pressure for a longer period of time. In this case, two alternatives to return to Y^n exist: either the government pursues an expansionary fiscal policy (shifting the DD-Curve back to the right) or the monetary authorities give in to the foreign exchange market pressure and devaluate the domestic currency, accelerating so the recovery process to E_3.

Dynamics After a Failed Defense of the Currency Peg

The two possible scenarios discussed in the previous section were based on the assumption that an interest rate increase by the domestic monetary authorities is successful in the defense of the prevailing exchange rate (or currency peg in case of a fixed exchange rate regime). Nevertheless, as the majority of currency crises in the last decades have demonstrated, the foreign exchange market pressure can be of such a magnitude that the monetary authorities might be forced to devalue or let the nominal exchange rate float. In such a case the currency mismatch between the assets and liabilities of the fraction of domestic firms which finance their investment projects through foreign currency credit takes place all in all, leading to the activation of credit constraints by the financial sector and to a decrease in the investment of the group of domestic firms indebted in foreign currency. The investment of the remaining firms, which actually does not depend on the level of the nominal exchange rate, is also affected by the domestic interest rate increases undertaken by the monetary authorities in their effort to defend the currency peg. The aggregate consequences for the economy are then catastrophic, since not a fraction, but the complete investment by the entrepreneurial sector is depressed. The extent of the investment decrease depends, of course, on the interest rate elasticity of the aggregate demand, as shown in Fig. 8.12.

Figures 8.12a and 8.12b show an important insight: the higher the interest rate elasticity of the aggregate demand is, the longer will be the recession period the economy will experience after the currency breakdown and the higher will be the equilibrium real exchange rate to which the economy converges in the medium run, as shown in Fig. 8.12b. These results are intuitive: the higher the interest rate elasticity of the aggregate demand, the greater is the investment decrease and therefore the real exchange rate increase which is needed for the economy to return to its NAIRU level of employment and production. We see thus that a failed defense of the prevailing exchange rate by the monetary authorities can have disastrous implications for the short- and medium run performance of the economy.

8.5.3 Short Term Policy Responses and Medium Term Consequences: The Rules vs. Discretion Debate

Due to the potential occurrence of the scenarios discussed above, Furman and Stiglitz (1998), Radelet and Sachs (1998) and Krugman (2000b) have pledged that in certain situations not an *increase*, but a *decrease* in the domestic interest rate might be the right measure during a speculative attack on the domestic currency. Indeed, since the exchange rate does not have *per se* a real meaning for the economic activity, the monetary authorities might decide to bring the foreign exchange market turmoil behind them once and for all and

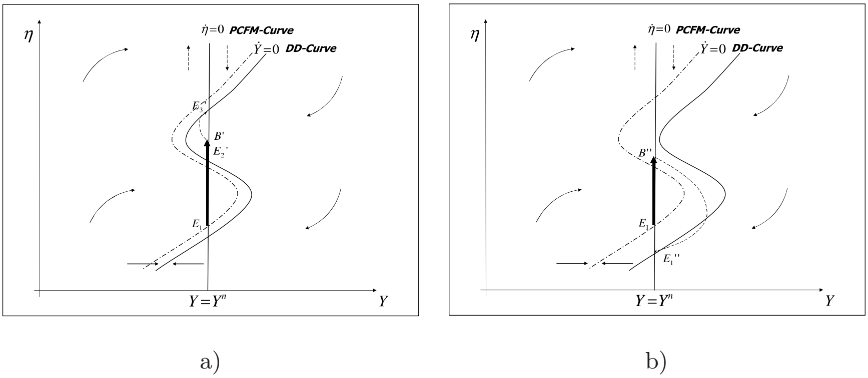


Fig. 8.13. Two possible consequences of an interest rate decrease as a response to a speculative attack a in economy with dollarized liabilities

to stabilize or even to enhance the economic activity by lowering the interest rates, irrespective of the exchange rate level. Such a measure will induce an even greater nominal exchange rate depreciation, i.e. a jump from E_1 to B' as sketched in Fig. 8.13. As shown there, the resulting dynamics depend on whether the new nominal (and real in the short run) exchange rate lies beneath or above the unstable new steady state E_2''.

In Fig. 8.13a, where the interest rate elasticity of the aggregate demand is low, the DD-Curve does not significantly shift to the right after the interest rate decrease, so that B' lies above E_2'' in the short run. In this case the economy will experience a period of underemployment, depressed investment of the fraction of domestic firms indebted in foreign currency (due to the increase of the domestic currency value of the foreign currency liabilities of that group) and falling prices (since $Y < Y^n$) until E_3' is reached. Again, the enhancement of the net exports via real depreciation is the mechanism which enables the economy to return to its NAIRU level of employment. Nevertheless, the output loss in this case will probably be lower and the duration of the economic slowdown shorter than in the previous cases where the exchange rate was successfully defended but the aggregate investment was severely damaged, as sketched in Fig. 8.12.

In Fig. 8.13b, where the interest rate elasticity of the aggregate demand is high, the aggregate investment rises due to the decrease of the domestic interest rate, shifting the DD-Curve significantly to the right (perhaps even so much that E_3 disappears as sketched above), so that B' lies below E_2'' in the short run (if E_2 still exists). In this case the positive effect of the domestic interest rate decrease overcomes the negative balance sheet effect resulting form the nominal depreciation of the domestic currency. In this scenario, thus, the final outcome is a period of over-production and -employment of the economy caused by higher aggregate investment and net exports, despite of the decline in investment of the fraction of firms indebted in foreign currency. Nevertheless, due to the resulting increase in the domestic price level (since $Y > Y^n$), over time the domestic products will loose competitiveness and the net exports will decrease again, leading the economy to its NAIRU production level at E_1'.

In the previous section we showed that while the defense of the currency peg through interest rate increases is likely to have devastating effects on the real side of economy, even if it is successful, in determinate situations the alternative strategy of interest rate decreases, as proposed by Furman and Stiglitz (1998) and Krugman (2000b), among others, might be a better strategy to follow from a medium-run output-stabilization point of view.

From the intertemporal point of view and more specifically, in the context of the rules-vs.-discretion debate, the choice of the "adequate" strategy to follow depends on additional factors. Indeed, if as for example in Burnside et al. (2001) the mere existence of a currency peg regime is considered as dependent on the credibility of the monetary authorities as "peg defenders", the only available, and from an intertemporal point of view, sustainable strategy to be implemented would be the defense of the currency peg by means of domestic interest rate increases. Indeed, if the domestic monetary authorities would not follow such a strategy, the resulting lack of credibility in the financial markets would enhance and even encourage further attacks on the domestic currency, due to the (justified) expected capital gains from such a handling. This would make it impossible for the attacked economies to remain in a fixed exchange rate regime, even if the domestic monetary authorities peg the exchange rate at a new (higher) level after a successful speculative attack.

On the contrary, if the monetary authorities decide during a speculative attack in a discretionary manner to abandon the system of currency pegs and let the nominal exchange rate float indefinitely and therefore to fulfill a radical macroeconomic regime change, then considerations about their credibility as "peg defenders" will not be of much importance anymore. In the new flexible exchange rate regime, the domestic monetary authorities would not be directly accountable for nominal exchange rate fluctuations. Their commitment to price stability and therefore to a stable purchasing power of the domestic currency is what would be of much more importance for the determination of the nominal exchange rate in such a floating regime.

The decision concerning the response to a speculative attack, and more generally, concerning the old rules-vs.-discretion debate, depends thus not only on the relative impact of currency mismatches and domestic interest rates effects, but also on the intertemporal pros and cons of the abandonment of a fixed exchange rate system for a floating regime for the domestic economy.

8.6 Econometric Analysis

As stated before, the East Asian as well as the Mexican (1994–95) currency and financial crises are quite particular episodes in financial history due to the abruptness and extent to which the real side of the economy was affected by the sharp devaluations of the domestic currency. As previously discussed, the fact that in the majority of countries the domestic monetary authorities tried to defend—though unsuccessfully—the prevailing nominal exchange rate by raising the domestic short term interest rates (a measure which was backed up by the IMF) raises the question whether that strategy was not in part responsible for the subsequent sharp decline in the aggregate demand, and especially in the aggregate investment, as discussed in the model scenario sketched in Fig. 8.12.

In Fig. 8.14 we show time series data of Mexico and selected East Asian countries for the 1990s, the decade when currency and financial crises took place in those countries. In all cases a sharp nominal exchange rate depreciation, caused by a successful speculative attack, preceded an abrupt and severe decline in the aggregate investment activity in the following quarters.

While the collapse of the aggregate investment in nearly all attacked East Asian countries has been explained by the majority of researchers by means of an open-economy version of the financial accelerator concept introduced by Bernanke et al. (1994), whereafter the domestic currency value of the foreign currency liabilities soared due to the sharp nominal depreciation of the domestic currencies, leading to a credit crunch by the financial intermediaries, the sharp increase in the nominal domestic interest rates might also have been an important source of the breakdown of investment, as discussed in the theoretical model in the previous sections. In order to address the relative importance of these two effects for the behavior of aggregate investment after the currency crisis, in this section we investigate the dynamic interaction of the

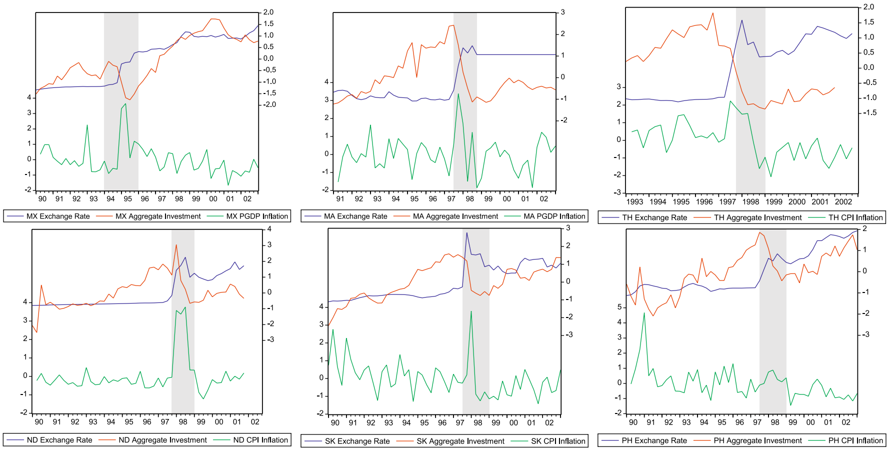


Fig. 8.14. Real aggregate investment, nominal exchange rates and price inflation (normalized data)

nominal exchange rate, the domestic nominal interest rate and the aggregate investment in a sample of East Asian countries by means of an unrestricted VARX (vector autoregressive model with exogenous terms) estimation.

For our analysis we use time series data of aggregate investment (in constant 1995 prices, seasonally adjusted), the respective nominal bilateral US-Dollar exchange and domestic interest rates of Mexico (MX), Malaysia (MA), Thailand (TH), Indonesia (ND), South Korea (SK) and the Philippines (PH) and the CPI/PGDP (depending on their availability) inflation stemming from the International Statistical Yearbook 2003. As endogenous variables in the VARX model we use the logs of these variables, with the exception of the respective price indices, of which we use the growth rates.

The question whether the analyzed time series were integrated and eventually cointegrated was not pursued in the following econometric analysis because of two reasons: First, no economic interpretation or theory is available (as for as we know) about a "steady state" relationship between the *level* of dollarized debt of a country and its aggregate investment: A possible cointegrating relationship between these variables would face difficulties to be interpreted in form of economic theory. Second, because of the small size of the time series samples, the results of unit root and cointegration tests would not be reliable enough to make a definitive assertion about their order of integration and cointegration. Because of these reasons we perform our VARX analysis in level form.

Table 8.2. Descriptive statistics

Country	Variable	Sample	Obs	Mean	Min	Max	Std.Dev	JB-Prob.
	exchrate_lg	1990Q1-2003Q1	53	1.764	0.989	2.022	0.518	0.023
MX	tbillrate_lg	1990Q1-2003Q1	53	2.917	1.880	4.100	0.523	0.917
	investment_lg	1990Q1-2003Q1	53	5.971	5.613	6.274	0.175	0.269
	PGDP inflation	1990Q2-2003Q1	52	0.039	−0.015	0.160	0.032	0.000
	exchrate_lg	1991Q1-2003Q1	49	1.123	0.901	1.400	0.197	0.024
MA	lendrate_lg	1991Q1-2003Q1	49	2.080	1.854	2.502	0.167	0.367
	investment_lg	1991Q1-2003Q1	49	9.828	9.498	10.32	0.231	0.135
	PGDP inflation	1991Q2-2003Q1	48	0.008	−0.025	0.068	0.017	0.046
	exchrate_lg	1993Q1-2002Q1	37	3.471	3.203	3.852	0.247	0.084
TH	lendrate_lg	1993Q1-2002Q1	37	2.375	1.945	2.724	0.256	0.180
	investment_lg	1993Q1-2002Q1	37	5.729	5.286	6.186	0.289	0.158
	CPI inflation	1993Q2-2003Q1	39	0.009	−0.009	0.029	0.008	0.777
	exchrate_lg	1990Q1-2001Q4	48	7.742	7.502	9.413	0.695	0.023
ND	moneyrate_lg	1990Q1-2001Q4	48	2.566	1.900	4.306	0.577	0.000
	investment_lg	1990Q1-2001Q4	48	10.282	9.586	10.77	0.208	0.009
	CPI inflation	1990Q1-2003Q1	47	0.032	−0.022	0.201	0.04	0.000
	exchrate_lg	1990Q1-2003Q1	53	6.861	6.554	7.435	0.248	0.064
SK	tbillrate_lg	1990Q1-2003Q1	53	2.259	1.386	3.173	0.524	0.064
	investment_lg	1990Q1-2003Q1	53	10.35	10.00	10.54	0.126	0.342
	CPI inflation	1990Q1-2003Q1	53	0.012	−0.002	0.052	0.010	0.000
	exchrate_lg	1990Q1-2003Q1	53	3.489	3.118	3.990	0.284	0.041
PH	lendrate_lg	1990Q1-2003Q1	53	2.505	1.531	2.541	0.396	0.630
	investment_lg	1990Q1-2003Q1	53	4.676	4.379	4.676	0.128	0.306
	CPI inflation	1990Q2-2003Q1	52	0.018	0.000	0.079	0.012	0.000

The restricted data availability does not allow us, unfortunately, to check for a highly probable nonlinear relationship between investment and sharp nominal exchange increases by means of econometric methods, forcing us to assume a log-linear relationship in our estimations. Concerning the lag order of the VARX models, it was chosen according to the Schwarz information criterion.

We analyze the dynamic response of aggregate investment to nominal exchange rate and interest rate shocks by calculating impulse-response functions based on the estimated parameters of the corresponding VARX models. As discussed in Bernanke et al. (1997), VARs are used to analyze the systematic responses of an economic system, for example, to unanticipated shocks (like exchange rate shocks). Now, since large and abrupt nominal exchange and interest rate increases as the ones observed during the Mexican and East Asian crises are likely to have been unexpected by the economic agents, we interpret both exchange rate and interest rate impulses as representing unanticipated shocks, even though the interest rate increases are probably in line with the

Table 8.3. Schwarz Bayesian VAR lag length selection criteria

Lags	MX	MA	TH	ND	SK	PH
0	1.474	−1.591	−0.848	2.999	−0.700	−0.917
1	−7.739	−7.510*	−7.175*	−3.373*	−7.659	−6.671*
2	−7.905*	−7.215	−6.872	−3.240	−8.117*	−46.314
3	−7.577	−6.971	−6.905	−3.142	−8.094	−5.896
4	−7.434	−6.921	−6.567	−3.039	−7.583	−5.354
5	−6.856	−6.370	−5.967	−2.836	−7.044	−4.899
6	−6.370	−5.762	−5.725	−2.528	−6.690	−4.544

Table 8.4. Residual serial correlation LM-tests
H0: No serial correlation at lag order h. Probs from chi-square with 9 DFs

Lags	MX	MA	TH	ND	SK	PH
1	0.1479	0.0049	0.1349	0.0034	0.0002	0.1346
2	0.8761	0.0070	0.1832	0.3669	0.2340	0.0595
3	0.6915	0.3359	0.0195	0.3207	0.8823	0.8778
4	0.7882	0.2362	0.9898	0.3292	0.8882	0.1344
5	0.7046	0.6483	0.0031	0.0173	0.2753	0.2362
6	0.0283	0.3015	0.0288	0.8298	0.6693	0.0910
7	0.4765	0.5400	0.2478	0.9164	0.8256	0.0066
8	0.2146	0.8300	0.8809	0.6179	0.2686	0.4955
9	0.9168	0.7342	0.5277	0.7469	0.8055	0.1958
10	0.9653	0.3624	0.4262	0.9871	0.6591	0.6801

reaction function or at least with the preferences of the domestic monetary authorities.

Figure 8.15 shows the estimated dynamic responses of the real aggregate investment and price inflation in the analyzed countries to a generalized one standard deviation shock in the respective nominal exchange rates calculated according to Pesaran and Shin (1998): As discussed there, the principal advantage of this calculation procedure is that the resulting impulse response functions (generated by an orthogonal set of innovations) are independent of the ordering of the endogenous variables in the VARX model, in contrast to the standard Cholesky orthogonalization procedure. As it can be observed there, in all economies an important and statistically significant negative reaction of aggregate investment to a positive shock in the log nominal exchange rate. Concerning the reaction of price inflation, in nearly all countries an initially positive reaction (caused probably by a pass-through effect) is followed by a later negative reaction, which nevertheless is almost insignificant from the statistical point of view, leading to the presumption that the deflationary pressures discussed previously, though theoretically possible, probably did not play an important role in the medium run dynamics in the analyzed economies.

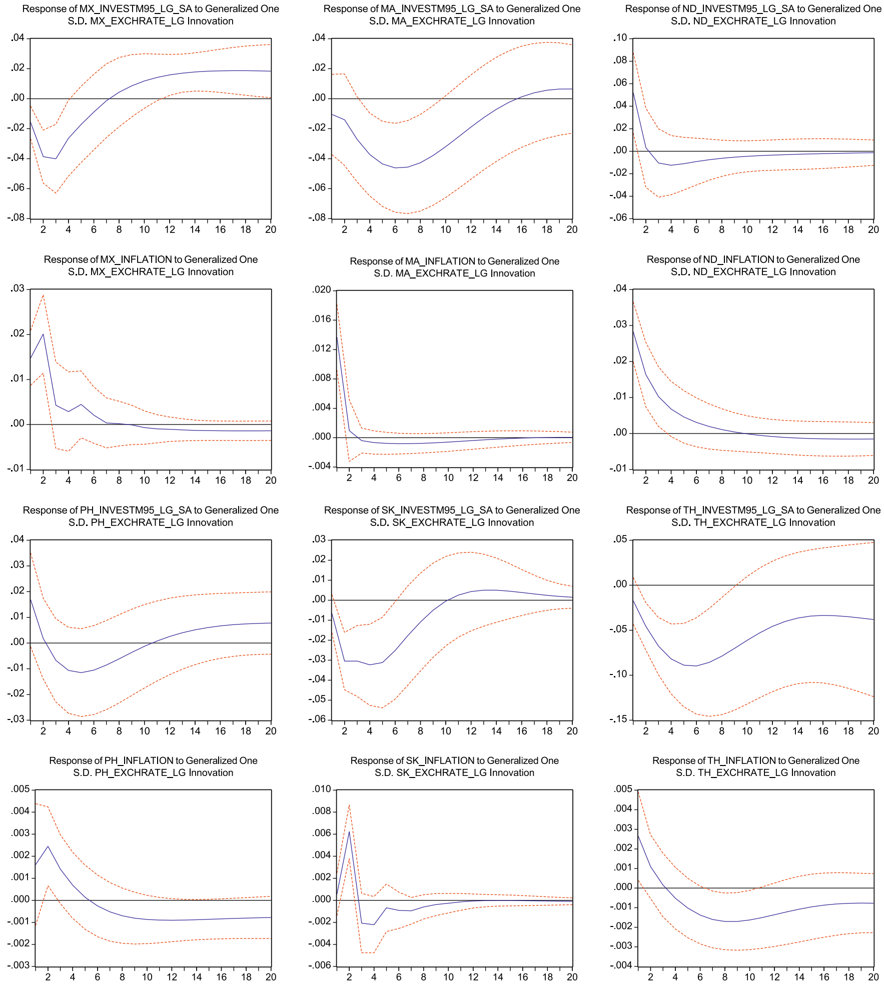


Fig. 8.15. Response of the log of aggregate investment and of price inflation to a generalized one std. dev. innovation in the logs nominal exchange

In the following we analyze the dynamic response of aggregate investment to an unexpected one standard deviation shock in the log nominal exchange rate and alternatively, in the domestic nominal interest rate.

As Fig. 8.16 shows, the reaction of aggregate investment to a shock in the nominal exchange rate and interest rate varies significantly across countries. While in Mexico the extent and duration of both responses is relatively low (what would explain the fast recovery of the Mexican economy after the 1994–1995 Tequila crisis), in the other countries the recovery of aggregate

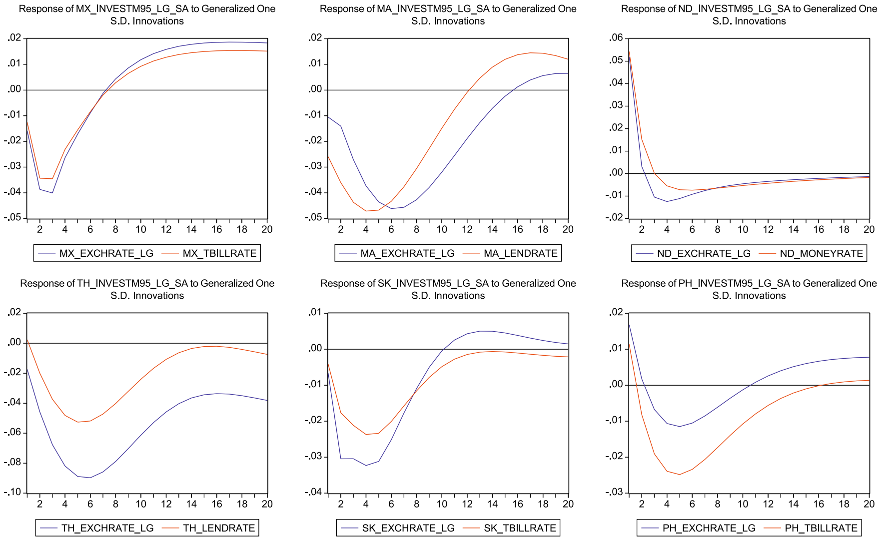


Fig. 8.16. Response of the log of aggregate investment to a generalized one std. dev. innovation in the log nominal exchange and the nominal interest rate

investment has a much longer duration. An additional and probably more interesting finding is the heterogeneous relative strength of the nominal exchange and interest rates across countries. While in Thailand and South Korea the exchange rate shock affects the respective aggregate investment levels in a more significant manner, in Indonesia and the Philippines the interest rate effect seems to have predominated during the estimation period. Now, even if these empirical findings are not definitive in their implications with respect to the main determinant (if there was only one) of the investment decline in the East Asian countries, they nevertheless open up the question whether, at least in some countries, the defense of the currency pegs through interest rate increases might perhaps not have been the most adequate response to the speculative attacks on the domestic currencies.

8.7 Concluding Remarks

This chapter builds on our previous work on the short run dynamics which are triggered by a sharp depreciation of the domestic currency and the resulting credit rationing in economies with dollarized liabilities. Going beyond the scope of the Flaschel and Semmler (2006) framework, where a rather short run perspective disregarding wage and price dynamics was presumed, we developed and discussed here two variants of the Mundell-Fleming-Tobin currency crisis model which focused on the medium run recovery processes after the occurrence of such a currency and financial crisis. These two model variants, which consisted of the incorporation of domestic price level adjustments and the general formulation of the real exchange rate dynamics, allowed us to highlight the importance of nominal exchange rate and domestic price level adjustments for the medium run recovery process in economies which have suffered from a sharp depreciation of their domestic currency and the subsequent breakdown of aggregate investment caused by the activation of credit constraints.

Besides highlighting the fact the net short run effect of a sharp nominal exchange rate depreciation and the resulting medium run dynamics depend on the relative strength of the real interest rate and the credit rationing by the financial institutions on aggregate investment, we showed that in the short run either the defense of the currency peg or the more unorthodox decrease of the nominal interest rate as proposed by Radelet and Sachs (1998), Furman and Stiglitz (1998) and Krugman (2000b) can turn out either successful or unsuccessful, depending on the different macroeconomic characteristics of the concerned countries. In this line, the empirical evidence on the diversity of the currency crises transmission mechanisms across the Mexican and East Asian economies provided by the econometric VARX analysis of Sect. 8.6 supported the differentiated scenario analysis of Sect. 8.5.

We conclude this chapter by pointing out that models where currency shocks trigger a financial crisis and a severe economic slowdown need to take into account some specification of the wage- and price inflation dynamics in order to also study medium run scenarios where inflation or deflationary pressures may arise and where the effects of monetary policy are to be considered. The wage and price dynamics have not been made an issue in the work of the third generation currency crisis models. The research there has mainly focused on the mechanisms that transmit large currency shocks to the financial sector and to real economic activity.

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9

Keynesian Dynamics and International Linkages in a Two-Country Model

9.1 Introduction

During the last decade and especially after the prominent contribution by Obstfeld and Rogoff (1995), an important paradigm change concerning the theoretical modeling approach of open economies. After the long-lasting predominance of Mundell-Fleming-Dornbusch type of models in the academic as well as in the more policy-oriented literature, the so-called "New Open Economy Macroeconomics" approach has become the workhorse framework in the mainstream academic literature for the analysis of open-economy issues in recent years.

As in their closed-economy DSGE counterparts, such as the ones discussed Erceg et al. (2000) and Smets and Wouters (2003) a central feature in this type of models is the assumption of rational expectations. However, even though theoretically appealing, the notion of fully rational agents is still quite controversial in the academic literature, and especially in the literature on nominal exchange rate dynamics. As pointed out e.g. by De Grauwe and Grimaldi (2005), efficient markets rational expectations models are unable to match empirical data on foreign exchange rate fluctuations as well as the occurrence of speculative bubbles, herding behavior and runs. "Non-rational" models, that is models which feature heterogenous beliefs by the economic agents or different types of agents with different attitudes or trading schemes seem much more successful in this task. Such models, on the other hand, often constrain themselves on the analysis of the FX markets and do not analyze the effects of such non-rational behavior by the FX market participants for the dynamics stability at the macroeconomic level.

In this chapter we attempt to fill in this gap in this alternative literature by setting up a two-country semi-structural macroeconomic model with a baseline formulation of the nominal exchange rate dynamics, which however could be easily reformulated and expanded by means of a chartists/fundamentalists module, as done for example in Proaño (2008). As we formulate the present model, it reacts to disequilibrium situations in both goods and labor markets in a sluggish manner primarily due to the only gradual adjustment of nominal wages and prices to such situations. This is the first logical step for the understanding of real effects of monetary and fiscal policy in economies which are highly interrelated with each other through a variety of markets and channels, when one allows for the non-clearing of markets at every point in time and for gradual adjustments to such market disequilibrium situations.

To do so we reformulate the theoretical disequilibrium model of AS-AD growth investigated in Chen et al. (2006b) and Proaño et al. (2006), for the case of two large open economies, first each in isolation and then in their interaction as two subsystems within a large closed dynamical system. The proposed model structure is similar in spirit to the two-country KMG model considered in Asada et al. (2006), but is appropriately simplified in order to have a framework more suitable for empirical estimation and also for the study of the role of contemporary interest rate policy rules.

The remainder of the chapter is organized as following: In Sect. 9.2 we describe the theoretical two-country semi-structural framework for the case of an open economy. In Section 9.3 we integrate two open economies, discuss in more detail the linking channels between both economies, as well as the dynamics of the nominal exchange rate (the financial link) and the steady state conditions. Additionally we estimate the model and discuss the resulting dynamic adjustments of the variables of the calibrated framework. In Sect. 9.4 we investigate by means of eigen-value analysis the consequences of wage and price flexibility as well as of monetary policy for the stability of the dynamical system. Section 9.5 draws some concluding remarks.

9.2 The Baseline Open-Economy Framework

In this section we describe the macroeconomic module of our theoretical framework by extending the closed economy, semi-structural macroeconomic model discussed in Chen et al. (2006a, 2006b) and Proaño et al. (2006) through the incorporation of trade, price and financial links between two similar economies with imperfectly flexible nominal wages and prices. Hereby we assume that both economies have the same macroeconomic structure and are additionally conducted with the same type of monetary policy. Therefore we discuss in this section only the structure of the domestic economy, denoting with the superscript f foreign economy variables and assuming equivalent formulations for the foreign economy (with the effect of the log real exchange rate $\eta = s + \ln(p^f) - \ln(p)$ adequately adjusted).

9.2.1 The Goods and Labor Markets

Concerning the real part of the economy, we follow a semi-structural approach assuming that the dynamics of output and employment can be summarized by the following laws of motion:

$$\hat{u} = -\alpha_{uu}(u - u_o) - \alpha_{uv}(v - v_o) - \alpha_{ur}(i - \hat{p} - (i_o - \pi_o)) \\ +\alpha_{u\eta}\eta + \alpha_{uu^f}\hat{u}^f, \tag{9.1}$$

$$\hat{e} = \alpha_{e\hat{u}}\hat{u} - \alpha_{ev}(v - v_o). \tag{9.2}$$

The first law of motion is of the type of a dynamic backward-looking open economy IS-equation, here represented by the growth rate of the capacity utilization rate of firms. Concerning the closed economy dimension, it has three important domestic characteristics; (i) it reflects the dependence of output changes on aggregate income and thus on the rate of capacity utilization by assuming a negative, i.e., stable dynamic multiplier relationship in this respect, (ii) it shows the joint dependence of consumption and investment on the domestic income distribution, which in the aggregate in principle allows for positive or negative signs before the parameter α_{uv}, depending on whether consumption, investment or the next exports are more responsive to relative real wage and wage share changes),[1] and (iii) it incorporates the negative influence of the real rate of interest on the evolution of economic activity. Additionally, in contrast to the closed economy model discussed investigated in Proaño et al. (2006), we incorporate (iv) the positive effect of foreign goods demand (represented by the *growth rate* of capacity utilization in the foreign economy, (v) the positive influence of the deviation of the log real exchange rate $\eta = s + \ln(p^f) - \ln(p)$ (s being the log nominal exchange rate, which law

[1] We will, however, not engage into this debate here but rather adopt the most traditional view according to which $\partial\hat{u}/\partial v$ is unambiguously negative.

of motion will be defined below) from its PPP consistent steady state level $\eta_o = 0$.

In the second law of motion, for the growth rate of the rate of employment, we assume that the employment policy of firms follows—in the form of a generalized Okun's Law—the growth rate of capacity utilization (with a weight $\alpha_{e\hat{u}}$).[2] Moreover, we additionally assume that an increasing wage share has a negative influence on the employment policy of firms. Employment is thus in particular assumed to adjust to the level of current activity since this dependence can be shown to be equivalent to the use of a term $(u/u_o)^{\alpha_{e\hat{u}}}$ when integrated, i.e., the form of Okun's law in which this law was originally specified by Okun (1970) himself.

9.2.2 The Wage-Price Dynamics

As for example Barro (1994) observes, perhaps the most important feature that theoretical Keynesian models should comprise is the existence of imperfectly flexible wages as well as prices. This is a common characteristic between our approach and advanced New Keynesian models such as Erceg et al. (2000) and Woodford (2003). However, even though the resulting structural wage and price Phillips Curves equations of our approach resemble to a significant extent those included in those theoretical models, their microfoundations are completely different. Indeed, instead of assuming monopolistic power in the price and wage setting of forward-looking, purely rational firms and households under a Calvo (1983) pricing scheme,[3] our wage and price inflation adjustment equations are based on the more descriptive structural approach proposed by Chiarella and Flaschel (2000) and Chiarella et al. (2005), which, being a Keynesian framework of aggregate demand fluctuations which allows for under- (or over-)utilized labor *as well as* capital, is based on gradual adjustments to disequilibrium situations of all real variables of the economy.

[2] Despite of being largely criticized due to its "lack of microfoundations", in a large number of microfounded, "rational expectations" models such as Taylor (1994), Okun's law is used to link production with employment.

[3] Recently, the overly unrealistic assumption in DSGE models as Erceg et al. (2000) of wages set by the households in a monopolistic manner has been replaced through more realistic wage setting schemes based on job search wage bargaining considerations by Trigari (2004) and Gertler and Trigari (2006).

By allowing for disequilibria in both goods and labor markets, we can discuss the dynamics of wages and prices separately from each other in their structural forms, assuming that both react to their own measure of demand pressure, namely $e - e_o$ and $u - u_o$, in the market for labor and for goods, respectively.[4] Here we denote by e the rate of employment on the labor market and by e_o the NAIRU-equivalent level of this rate, and similarly by u the rate of capacity utilization of the capital stock and u_o the normal rate of capacity utilization of firms.

As in Chiarella and Flaschel (2000) and Chiarella et al. (2005), we model the expectations in both wage and price Phillips curve in a hybrid way, with crossover myopic perfect foresight (model-consistent) expectations with respect to short-run wage and domestic price inflation on the one hand and an adaptive updating inflation climate expression (symbolized by π_c) concerning the evolution of the CPI inflation ($\hat{p}_c$), on the other hand. Note that, through this specification, our model features while not rational, nevertheless model consistent expectations concerning the evolution of the wage and price inflation and also incorporates a similar degree of inertia obtained in New Keynesian models only through also ad-hoc "rules-of-thumb" or price indexation assumptions, see e.g. Galí and Gertler (1999) and Galí et al. (2001).

More specifically, we assume concerning the wage Phillips curve that the short-run price level considered by workers in their wage negotiations is set by the producer, so that producer price inflation gives the rate of inflation that is perfectly foreseen by workers as their short-run cost-push term. Additionally, in order to incorporate the role of import price inflation for the dynamics of the economy, we assume that the measure that is taken by workers to judge the medium-run evolution of prices in their respective economies is the Consumer Price Index, defined as

$$p_c = p^{\gamma}(Sp^f)^{1-\gamma},$$

the geometric average of domestic and import prices—with p^f being foreign price level and S the nominal exchange rate.

Consequently, the CPI inflation $\hat{p}_c$ includes both domestic inflation (with a specific weight γ) and imported goods price inflation (with weight $1 - \gamma$), so that

$$\hat{p}_c = \gamma\hat{p} + (1 - \gamma)(\dot{s} + \hat{p}^f), \tag{9.3}$$

[4] As pointed out by Sims (1987), such strategy allows to circumvent the identification problem which arises when both wage and price inflation equations have the same explanatory variables.

with $s = \ln(S)$. Because of the uncertainty linked with nominal exchange rate movements, we assume for both workers and firms' decision taking processes, that CPI inflation is updated in an adaptive manner according to[5]

$$\dot{\pi}_c = \beta_{\pi_c}(\hat{p}_c - \pi_c) = \beta_{\pi_c}\gamma(\hat{p} - \pi_c) + \beta_{\pi_c}(1-\gamma)(\hat{p}^f + \dot{s} - \pi_c). \tag{9.4}$$

We thereby arrive at the following two Phillips Curves for wage and price inflation, which in this core version of Keynesian AS-AD dynamics are—from a qualitative perspective—formulated in a fairly symmetric way.

The structural form of the wage-price dynamics:

$$\hat{w} = \beta_{we}(e - e_o) - \beta_{wv}\ln(v/v_o) + \kappa_{wp}\hat{p} + (1-\kappa_{wp})\pi_c + \kappa_{wz}\hat{z}, \tag{9.5}$$

$$\hat{p} = \beta_{pu}(u - u_o) + \beta_{pv}\ln(v/v_o) + \kappa_{pw}(\hat{w} - \hat{z}) + (1-\kappa_{pw})\pi_c, \tag{9.6}$$

where $\hat{z}$ denotes the growth rate of labor productivity (which we assume here just to be equal to $g_z = \hat{z} =$ const. (g_z denoting the trend labor productivity growth).

Note that as the wage-price mechanisms are formulated, the development of the CPI inflation does not matter for the evolution of the domestic wage share $v = (w/p)/z$, measured in terms of producer prices, the law of motion of which is given by (with $\kappa = 1/(1-\kappa_{wp}\kappa_{pw})$):

$$\hat{v} = \kappa\left[(1-\kappa_{pw})f_w(e,v) - (1-\kappa_{wp})f_p(u,v) + (\kappa_{wz}-1)(1-\kappa_{pw})g_z\right], \tag{9.7}$$

with

$$f_w(e,v) = \beta_{we}(e - e_o) - \beta_{wv}\ln(v/v_o) \quad \text{and}$$
$$f_p(y,v) = \beta_{pu}(u - u_o) + \beta_{uv}\ln(v/v_o)$$

which follows easily from the following obviously equivalent representation of the above two PC's:

$$\hat{w} - \pi_c = \beta_{we}(e - e_o) - \beta_{wv}\ln(v/v_o) + \kappa_{wp}(\hat{p} - \pi_c),$$
$$\hat{p} - \pi_c = \beta_{pu}(u - u_o) + \beta_{pv}\ln(v/v_o)) + \kappa_{pw}(\hat{w} - \pi_c)$$

by solving for the variables $\hat{w} - \pi_c$ and $\hat{p} - \pi_c$. It also implies the following two across-markets or *reduced form Phillips Curves*:

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[5] In the empirical applications of this adaptive revision of the CPI inflation we will simply use a moving average of the CPI inflation with linearly declining weights.

$$\hat{w} = \kappa\,[\beta_{we}(e - e_o) - \beta_{wv}\ln(v/v_o) + \kappa_{wp}(\beta_{pu}(u - u_o) + \beta_{pv}\ln(v/v_o))$$
$$+(\kappa_{wz} - \kappa_{wp}\kappa_{pw})g_z] + \pi_c,$$
$$\hat{p} = \kappa\,[\beta_{pu}(u - u_o) + \beta_{pv}\ln(v/v_o) + \kappa_{pw}(\beta_{we}(e - e_o) - \beta_{wv}\ln(v/v_o))$$
$$+\kappa_{pw}(\kappa_{wz} - 1)g_z] + \pi_c,$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market, as used in the majority of New Keynesian models.

Equation (9.7) shows the ambiguity of the stabilizing role of the real wage channel, already discussed by Rose (1967) which arises—despite of the incorporation of specific measures of demand and cost pressure on both the labor and the goods markets—if the dynamics of the employment rate and the workforce utilization are linked to the fluctuations of the firms' capacity utilization rate via Okun's law. Indeed, as sketched in Fig. 9.1, a real wage increase can act, taken by itself, in a stabilizing or destabilizing manner, depending among others on whether the dynamic of the capacity utilization rate depends positively or negatively on the real wage (i.e. on whether consumption reacts more strongly to real wage changes than investment and, in an open economy, net exports, or vice versa) *and* whether price flexibility is greater than nominal wage flexibility with respect to their own demand pressure measures.

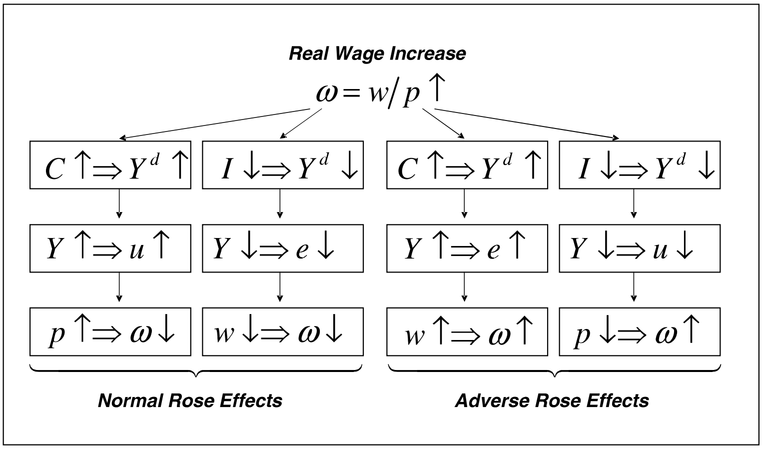

Fig. 9.1. Normal (convergent) and adverse (divergent) Rose effects: The real wage channel of Keynesian open economy macrodynamics

9.2.3 Monetary Policy

As standard in modern macroeconomic models, we assume that money supply accommodates to the interest rate policy pursued by the central bank and thus does not feedback into the core laws of motion of the model. As interest rate policy we assume the following classical type of Taylor rule:

$$i_{\mathrm{T}} = (i_o - \pi_o) + \hat{p} + \phi_{ip}(\hat{p} - \pi_o) + \phi_{iu}(u - u_o). \tag{9.8}$$

The target rate of the central bank i_{T} is thus assumed to depend on the steady state real rate of interest—augmented by actual inflation back to a nominal rate—, on the inflation gap and on the capacity utilization gap (as a measure of the output gap). We assume furthermore that the monetary authorities, when pursuing this target rate, do not react automatically but rather adjust to it in a smooth manner according to

$$\dot{i} = \alpha_{ii}(i_{\mathrm{T}} - i), \tag{9.9}$$

with α_{ii} determining the adjustment speed of the nominal interest rate.[6] Inserting i_{T} in and rearranging terms we obtain from this expression the following dynamic law of motion for the nominal interest rate

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \pi_o) + \gamma_{iu}(u - u_o), \tag{9.10}$$

where we have $\gamma_{ii} = \alpha_{ii}, \gamma_{ip} = \alpha_{ii}(1+\phi_{ip})$, i.e., $\phi_{ip} = \gamma_{ip}/\alpha_{ii} - 1$ and $\gamma_{iu} = \alpha_{ii}\phi_{iu}$. Note that the actual (perfectly foreseen) rate of inflation $\hat{p}$ is used to measure the inflation gap with respect to the inflation target π_o of the central bank. Note also that we could have included (but have not done this here yet) a new kind of gap into the above Taylor rule, the wage share gap, since we have in our model a dependence of aggregate demand on income distribution and the real wage. The state of income distribution matters for the dynamics of our model and thus might also play a role in the decisions of the central bank.[7]

[6] In the academic literature there is an ongoing and still not solved debate about whether there is indeed an interest smoothing parameter in the monetary policy reaction rule of the central banks or whether the observed high autocorrelation in the nominal interest rate is simply the result of highly correlated shocks or only slowly available information, see e.g. Rudebusch (2002) and Rudebusch (2006) for a throughout discussion.

[7] All of the employed gaps are measured relative to the steady state of the model, in order to allow for an interest rate policy that is consistent with it.

9.2.4 The Nominal Exchange Rate Dynamics

A common procedure in the open-economy DSGE type of models is to assume that the dynamics of the nominal exchange rate are driven by the validity of the purchasing power parity (PPP) postulate, see e.g. Obstfeld and Rogoff (1995). Through a log-linearization around the general equilibrium "rational expectations" steady state of the system, the—correctly—expected depreciation rate of the nominal exchange rate between two economies is simply determined by

$$E_t[s_{t+1} - s_t] = \pi_t - \pi_t^f,$$

with s_t denoting the log of the nominal exchange rate and π_t and π_t^f the domestic and foreign price inflation rates, respectively. Under the assumption that the price inflation rate is determined by the difference between money- and consumption growth differentials, the actual nominal exchange rate can be expressed (applying the no-bubbles condition) as (see Walsh 2003, p. 277)

$$s_t = \frac{1}{1+\delta} \sum_{i=0}^{\infty} \left(\frac{1}{1+\delta} \right)^i [(m_{t+i} - m_{t+i}^*) - (c_{t+i} - c_{t+i}^*)]$$

with δ as the intertemporal discount rate; m and m^* as the money supplies; and c and c^* as the consumption levels in the domestic and foreign economies. Thus, in the New Keynesian framework, the actual nominal exchange rate between two countries depends on the current and future paths of the nominal money supply- and consumption differentials between both economies.

Though straightforward in a theoretical rational expectations general equilibrium framework, this solution implies nevertheless the existence of (solely) purely rationally handling agents in the financial markets, an assumption that has been proven to be unable to explain major stylized facts of the nominal exchange rate dynamics. As shown for example in Ehrmann and Fratzscher (2005), the volatility of fundamentals (modeled in that study through an index of interest rate and output growth differentials and current account deficits) is by far not as large as the dynamics of the corresponding nominal exchange rates.

Due to the empirical failure of rational expectations models, a large literature based on the assumption of heterogenous expectations or beliefs among the traders in the foreign exchange market has arisen in the last decade. The inclusion of such heterogeneity, and therefore of a somewhat "non-rational" behavior by the economic agents has proven quite valuable in providing insights and explanations concerning some of the "puzzles" which arise when "rationality" is assumed.[8]

In the most basic heterogenous expectations framework, see e.g. Frankel and Froot (1990), two basic types of traders with different belief patterns (or expectations) concerning the future behavior of the nominal exchange rate are modeled: the fundamentalists and the chartists. The fundamentalists typically believe that the nominal exchange rate is driven by macroeconomic fundamentals such as interest rate differentials, different developments of production and employment and/or the validity of the PPP postulate and consequently trade conforming to this belief. In contrast, the chartists are assumed to follow the market tendencies, acting thus in principle in a destabilizing manner. The dynamics and stability of the resulting nominal exchange rate, therefore, depend on the relative strength and proportion of these two groups in the foreign exchange market.

In more advanced theoretical frameworks about heterogenous beliefs a wide variety of extensions concerning the endogenous determination of the trader groups composition can be found: in Kirman (1993) for example the determination of the two groups is determined by a purely stochastic factor; in Lux (1995) the "contagion" effect, that is, the change in the trading strategy, depends on the overall "mood" of the market and on the observed realized returns. De Grauwe and Grimaldi (2005), in a similar manner, assume the *group change* probability as a function of the relative probability of the forecasting rules of the two groups and the *risk* associated with their use.[9]

In our theoretical framework though we will leave these possible model extensions for future research and assume for simplicity a delayed adjustment of the nominal exchange rate based on the Uncovered Interest Rate Parity (UIP) postulate, namely

$$\dot{s} = \beta_s(i^f - i + \hat{s}^e) \tag{9.11}$$

with

$$\hat{s}^e = \beta_{s\eta}(-\eta)$$

[8] See De Grauwe and Grimaldi (2006, Chap. 1) for an extensive discussion of the advantages of the heterogenous agents-approach with respect to the rational-expectations approach in the explanation of empirical financial market data.

[9] See Samanidou et al. (2007) for a comprehensive survey article on this strain of research.

denoting the expected nominal depreciation rate (specified here the expected equation (9.11)). This law of motion, together with the price inflation adjustment equations for the domestic and the foreign economies deliver

$$\begin{aligned}\dot{\eta} &= \dot{s} + \hat{p}^f - \hat{p} \\ &= \beta_s(i^f - i + \hat{s}^e) + \hat{p}^f - \hat{p}. \qquad (9.12)\end{aligned}$$

Taken together the model of this section consists of the following six laws of motion (with the derived reduced form expressions as far as the wage-price spiral is concerned and with reduced form expressions by assumption concerning the goods and the labor market dynamics):[10]

The One-Country Sub-Module

$$\begin{aligned}
\hat{u} &\overset{Dynamic\ IS}{=} -\alpha_{uu}(u-u_o) - \alpha_{ur}(i-\hat{p}-(i_o-\pi_o)) \\
&\quad -\alpha_{uv}(v-v_o) + \alpha_{u\eta}\eta + \alpha_{uu^f}\hat{u}^f \qquad (9.13) \\
\hat{e} &\overset{Okun's\ Law}{=} \alpha_{e\hat{u}}\hat{u} - \alpha_{ev}(v-v_o) \qquad (9.14) \\
\hat{v} &\overset{Wage\ Share}{=} \kappa[(1-\kappa_{pw})(\beta_{we}(e-e_o) - \beta_{wv}\ln(v/v_o)) \\
&\quad -(1-\kappa_{wp})(\beta_{pu}(u-u_o) + \beta_{pv}\ln(v/v_o)) + \rho g_z], \qquad (9.15) \\
&\quad \text{with } \rho = (\kappa_{wz}-1)(1-\kappa_{pw}) \\
\dot{\pi}_c &\overset{CPIClimate}{=} \beta_{\pi_c}(\hat{p}_c - \pi_c), \quad \hat{p}_c = \gamma\hat{p} + (1-\gamma)(\dot{s} + \hat{p}^f) \qquad (9.16) \\
\dot{i} &\overset{Taylor\ Rule}{=} -\gamma_{ii}(i-i_o) + \gamma_{ip}(\hat{p}-\pi_o) + \gamma_{iu}(u-u_o) \qquad (9.17) \\
\dot{\eta} &\overset{Real\ Exchange}{=} \beta_s(i^f - i - \beta_{s\eta}\eta) + \hat{p}^f - \hat{p}. \qquad (9.18)
\end{aligned}$$

The above equations represent, in comparison to the baseline model of New Keynesian macroeconomics, the IS goods market dynamics, here augmented by Okun's Law as link between the goods and the labor market, and of course the Taylor Rule, and now also a law of motion for the wage share $\hat{v}$ that makes use of the same explaining variables as in the New Keynesian model with both staggered prices and wages (but with inflation rates $\hat{p}, \hat{w}$ in place of their time rates of change and with no accompanying sign reversal concerning

[10] As the model is formulated we have no real anchor for the steady state rate of interest (via investment behavior and the rate of profit it implies in the steady state) and thus have to assume here that it is the monetary authority that enforces a certain steady state values for the nominal rate of interest.

the influence of output and wage gaps), and finally the law of motion that describes the updating of the inflationary climate expression. We have to make use in addition of the reduced form expression for the price inflation rate or the price PC, our law of motion for the price level p in the place of the New Keynesian law of motion for the price inflation rate π^p:

$$\hat{p} = \kappa[\beta_{pu}(u-u_o)+\beta_{pv}\ln(v/v_o)+\kappa_{pw}(\beta_{we}(e-e_o)-\beta_{wv}\ln(v/v_o))]+\pi_c \quad (9.19)$$

which has to be inserted into the above laws of motion in various places in order to get an autonomous nonlinear system of differential equations in the state variables: capacity utilization u, the rate of employment e, the nominal rate of interest i, the wage share v, and the inflationary climate expression π_c. We stress that one can consider (9.19) as a sixth law of motion of the considered dynamics which however—when added—leads a system determinant which is zero and which therefore allows for zero-root hysteresis for certain variables of the model (in fact in the price level if the target rate of inflation of the monetary authorities is zero and if interest rate smoothing is present in the Taylor rule). We have written the laws of motion in an order that first presents the dynamic equations also present in the baseline New Keynesian model of inflation dynamics, and then our formulation of the dynamics of income distribution and of the inflationary climate in which the economy is operating.

In sum, therefore, our dynamic AS-AD growth model exhibits a variety of features that are much more in line with a Keynesian understanding of the characteristics of the trade cycle than is the case for the conventional modeling of AS-AD growth dynamics or its radical reformulation by the New Keynesians (where—if non-determinacy can be avoided by the choice of an appropriate Taylor rule—only the steady state position is a meaningful solution in the related setup we considered in the preceding section).

9.2.5 Local Stability Analysis: The Small-Open Economy Case

We start our analysis of the stability properties of the system with the small-open economy case, assuming that the foreign economy is and remains at its steady state level ($u^f = u^f_o$, $e^f = e^f_o$, $v^f = v^f_o$). We note that the steady state of the 5D subdynamics, due to its specific formulation, can be supplied exogenously. As this submodule is formulated it exhibits five gaps, to be closed in the steady state and has five laws of motion, which when set equal to zero, exactly imply this result (assuming that the foreign economy stays at its steady state level).

Since we assume the same structure for both economies, the local stability of one subsystem would imply the same for the other subsystem, assuming that similar parameter dimensions.

As discussed in Chen et al. (2006a, 2006b), the steady state of the dynamics of the closed-economy version of this model is asymptotically stable under certain sluggishness conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability cyclically (by way of so-called Hopf-bifurcations) if the system becomes too flexible, and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high. If the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior—like downward money wage rigidity—to manifest themselves at least far off the steady state in order to bound the dynamics to an economically meaningful domain in the considered 5D state space. Chen et al. (2006a) provide a variety of numerical studies for such an approach with extrinsically motivated nonlinearities through detailed numerical investigation.

In order to investigate the role of heterogenous expectations in the foreign exchange market as well as more traditional international transmission channels for the stability of the whole macroeconomic system in an analytical manner, we reduce the dimensions of our theoretical framework through the following simplifying assumptions:

- The monetary authorities do not pursue an interest rate smoothing strategy, so that $i = i_{\mathrm{T}}$ always holds. This is the case when $\alpha_{ii} \to \infty$.
- $\beta_{\pi^c} = 0$. In this case the inflationary climate is constant (hereby we assume that $\pi_c = 0$).
- We can replace e through $\alpha_{eu} u$ in the wage and price inflation adjustment equations without loss of generality.

Under the simplifying assumptions, the initial 5D dynamical system can be reduced to the following 3D subsystem

$$\begin{aligned}\hat{u} &= -\alpha_{uu}(u-u_o) - \alpha_{ur}(\phi_{ip}(\hat{p}-\pi_o) + \phi_{iu}(u-u_o)) \\ &\quad -\alpha_{uv}(v-v_o) + \alpha_{u\eta}\eta, \qquad (9.20)\\ \hat{v} &= \kappa[(1-\kappa_{pw})(\beta_{we}(\alpha_{eu}u - e_o) - \beta_{wv}\ln(v/v_o)) \\ &\quad -(1-\kappa_{wp})(\beta_{pu}(u-u_o) + \beta_{pv}\ln(v/v_o)) + \rho g_z], \qquad (9.21)\\ \dot{\eta} &= \beta_s(i_o^f - (i_o + (1+\phi_{ip})(\hat{p}-\pi_o) + \phi_{iu}(u-u_o)) - \beta_{s\eta}\eta) - \hat{p} \qquad (9.22)\end{aligned}$$

with

$$\hat{p} = \kappa[\beta_{pu}(u-u_o)+\beta_{pv}\ln(v/v_o)+\kappa_{pw}(\beta_{we}(\alpha_{eu}u-u_o)-\beta_{wv}\ln(v/v_o))] \quad (9.23)$$

to be inserted in several places.

The corresponding Jacobian of this reduced 3D subsystem

$$J_{3D} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

with

$$J_{11} = \frac{\partial \hat{u}}{\partial u} = -\alpha_{uu} - \alpha_{ur}\left(\phi_{ip}\kappa\left(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu}\right) + \phi_{iu}\right) < 0, \quad (9.24)$$

$$J_{12} = \frac{\partial \hat{u}}{\partial v} = -\alpha_{uv} - \alpha_{ur}\phi_{ip}\kappa\left(\frac{\beta_{pv} - \kappa_{pw}\beta_{wv}}{v_o}\right) < 0, \quad (9.25)$$

$$J_{13} = \frac{\partial \hat{u}}{\partial \eta} = \alpha_{u\eta} > 0, \quad (9.26)$$

$$J_{21} = \frac{\partial \hat{v}}{\partial u} = \kappa((1-\kappa_{pw})\beta_{we}\alpha_{eu} - (1-\kappa_{wp})\beta_{pu}), \quad (9.27)$$

$$J_{22} = \frac{\partial \hat{v}}{\partial v} = -\kappa\left(\frac{(1-\kappa_{pw})\beta_{wv} + (1-\kappa_{wp})\beta_{pv}}{v_o}\right) < 0, \quad (9.28)$$

$$J_{23} = \frac{\partial \hat{v}}{\partial \eta} = 0, \quad (9.29)$$

$$J_{31} = \frac{\partial \dot{\eta}}{\partial u} = -\beta_s((1+\phi_{ip})\kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu}) + \phi_{iu}) \\ -\kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu}) < 0, \quad (9.30)$$

$$J_{32} = \frac{\partial \dot{\eta}}{\partial v} = -\beta_s(1+\phi_{ip})\kappa\left(\frac{\beta_{pv} - \beta_{wv}\kappa_{pw}}{v_o}\right) - \kappa\left(\frac{\beta_{pv} - \beta_{wv}\kappa_{pw}}{v_o}\right), \quad (9.31)$$

$$J_{33} = \frac{\partial \dot{\eta}}{\partial \eta} = -\beta_s\beta_{s\eta} < 0 \quad (9.32)$$

has the following sign structure

$$J_{3D} = \begin{bmatrix} - & - & + \\ ? & - & 0 \\ - & ? & - \end{bmatrix}.$$

According to the Routh-Hurwitz stability conditions for a 3D dynamical system, asymptotic local stability of a steady state is fulfilled when

$$a_i > 0, \quad i = 1,2,3 \quad \text{and} \quad a_1a_2 - a_3 > 0,$$

where $a_1 = -\text{trace}(J)$, $a_2 = \sum_{k=1}^{3} J_k$ with

$$J_1 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}, \qquad J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}, \qquad J_3 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}.$$

and $a_3 = -\det(J)$.

Our reduced 3D dynamical system is stable around its interior steady state, if following proposition are fulfilled:

Proposition 9.1. *Assume that* (i) $\beta_{we}\alpha_{eu} > \beta_{pu}$*, that is, that wage inflation reacts more strongly to changes in capacity utilization than price inflation and additionally, assume that* (ii) κ_{pw} *is of a sufficiently small dimension, so that* $(1-\kappa_{pw})\beta_{wv} + (1-\kappa_{wp})\beta_{pv} > 0$ *and* $\partial\hat{u}/\partial v < 0$ *is fulfilled.*

Then: The Routh-Hurwitz conditions are fulfilled and the unique steady state of the reduced 3D *dynamical system is locally asymptotic stable.*

Proof. As it can be easily observed, according to the formulation of the dynamics of the nominal exchange rate, these are unambiguously asymptotically stable, since $\partial\dot{\eta}/\partial\eta < 0$. Under Proposition 9.1 ($\partial\hat{v}/\partial v < 0$), and the trace of J is then unambiguously negative (and $a_1 > 0$ holds), since

$$\text{tr}(J) = J_{11} + J_{22} + J_{33} < 0. \tag{9.33}$$

Condition (i) additionally ensures the partial derivative of $\dot{\eta}$ with respect to v to be negative, that is $\partial\dot{\eta}/\partial v < 0$. Condition (ii) assures that $\partial\hat{v}/\partial u > 0$.

If conditions (i) and (ii) hold, the sign structure of the Jacobian matrix is given by

$$J_{3D} = \begin{bmatrix} - & - & + \\ + & - & 0 \\ - & - & - \end{bmatrix}.$$

Under such a sign structure, J_1, J_2 and J_3, the second-order minors of J, are given by

$$\begin{aligned} J_1 &= J_{22}\cdot J_{33} - J_{32}\cdot J_{23} \\ &= \beta_s\beta_{s\eta}\kappa\left(\frac{\beta_{wv}(1-\kappa_{pw}) + \beta_{pv}(1-\kappa_{wp})}{v_o}\right) > 0, \end{aligned} \tag{9.34}$$

$$\begin{aligned} J_2 &= J_{11}\cdot J_{33} - J_{31}\cdot J_{13} \\ &= \beta_s\beta_{s\eta}[\alpha_{uu} + \alpha_{uv}(\phi_{ip}\kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu}) + \phi_{iu})] \\ &\quad +\alpha_{u\eta}[\beta_s((1+\phi_{ip})\kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu}) + \phi_{iu}) + \kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu})] > 0, \end{aligned} \tag{9.35}$$

$$\begin{aligned} J_3 &= J_{11} \cdot J_{22} - J_{21} \cdot J_{12} \\ &= [\alpha_{uv} + \alpha_{ur}(\phi_{ip}\kappa(\beta_{pv} - \beta_{wv}\kappa_{pw}) + \phi_{iu})] \cdot \kappa \left(\frac{(1-\kappa_{pw})\beta_{wv} - (1-\kappa_{wp})\beta_{pv}}{v_o} \right) \\ &\quad + \kappa[(1-\kappa_{pw})\alpha_{eu}\beta_{we} - (1-\kappa_{wp})\beta_{pu}] \left[\alpha_{uv} + \alpha_{ur}\phi_{ip}\kappa \left(\frac{\beta_{pv} - \kappa_{pw}\beta_{wv}}{v_o} \right) \right] > 0. \end{aligned} \tag{9.36}$$

It can be easily confirmed that $a_2 = \sum_{k=1}^{3} J_k > 0$ and $a_3 = -\det(J) > 0$, as well as the critical condition $a_1 a_2 - a_3 > 0$ for local asymptotic stability of the steady state of the system hold under the assumed parameter constellation.

Concerning the determinant of J, from the sign structure of the 3D Jacobian it can be easily seen that it is negative, so that $a_3 = -\det(J) > 0$.

Concerning the local asymptotic stability properties of the 6D subsystem, we can infer without an analytical proof that it will lose stability if a) the conditions (i)–(ii) in Proposition 9.1 are no longer fulfilled, b) the adjustment speed of the inflationary climate β_{π^c} approaches infinity or c) the nominal interest rate does not adjust sufficiently fast to the target rate pursued by the monetary authorities, that is, when the interest rate smoothing parameter α_{ii} is insufficiently low.

9.3 The Two-Country Framework: Estimation and Evaluation

After having set up the basic structure of an open-economy of Keynesian nature, in this section we integrate two economies (and therefore, two small-open-economy dynamic models if considered separately) with similar characteristics (as the Eurozone and US) into a consistent whole.

Considering both economies as a single macroeconomic framework, the resulting 11D dynamical system comprises 11 dynamic variables with the gaps

$$u - u_o, \quad e - e_o, \quad v - v_o, \quad i - i_o, \quad \hat{p} - \pi_o, \quad \eta - \eta_o,$$

plus the five ones for the foreign economy that correspond to the first (domestic) five of the list shown above.

For the unique determination of the steady state position we set $\hat{u}, \hat{e}, \hat{v}, \dot{i}$ equal to zero (and of course have the same situation for the foreign economy). This holds only when all gaps are zero simultaneously, what additionally delivers (for $\eta = \eta_o = 0$) $\dot{s} = 0$.

Assuming a constant steady state nominal exchange rate s, we moreover get from the reduced form price Phillips curves

$$\begin{aligned}
\hat{p}_o &= \pi_{co} = \gamma \pi_{co} + (1-\gamma)\pi_{co}^f \\
\hat{p}_o^f &= \pi_{co}^f = \gamma^f \pi_{co}^f + (1-\gamma^f)\pi_{co} \Longleftrightarrow \\
\pi_c &= \pi_c^f.
\end{aligned}$$

By inserting again (9.3) and its foreign economy counterpart, we obtain

$$\gamma \hat{p}_o + (1-\gamma)\hat{p}_o^f = \gamma^f \hat{p}_o + (1-\gamma^f)\hat{p}_o^f$$

what only holds true for $\hat{p} = \hat{p}^f$. At the steady state, thus, both countries share the same inflationary climate and equilibrium inflation rate, independently of the actual composition of the CPI index in both economies. Under this condition, the nominal exchange rate equation (9.11) delivers indeed a constant nominal exchange rate at the steady state, and therefore also a constant real exchange rate, since $\eta = \eta_o$.

The structure of the 11D dynamical system is summarized in Fig. 9.2. This figure shows at its top the interaction of the foreign exchange market with the two economies and towards the bottom the interaction of both economies through their goods markets.

As this diagrammatic exposition of quantity and price trade channels linking the two economies shows, the macroeconomic interaction between them seems apparently intrinsically stable, and the sole obvious source of instability or even chaos is laid on the foreign exchange markets. Indeed, in the absence of predominant unstable nominal exchange rate dynamics, the dynamics of the two-country framework seem to be of a self-regulating nature through the interaction of quantity and price trade linkages. This, however, is not necessarily the case: So for example leads an exogenous increase in the foreign demand ($u^f \uparrow$) on the one hand to an increase of price and (through the related increase in foreign employment) wage inflation abroad, which in turn leads to a loss of competitiveness ($\eta \uparrow$) and to a cooling down of the economy. On the other hand, though, an increase in u^f leads (through the "locomotive" effect) to an increase in the domestic level of economic activity, to an increase in domestic wage and price inflation and subsequently to a fall of η, which, in turn, is likely to boost furthermore the economic activity abroad. The net effect of these two opposite effects and therefore the stability of the system depends thus to an important degree on the degree of wage and price flexibility in both economies. However, since a throughout analytical calculation

The Two-Country Model

$$\hat{u} = -\alpha_{uu}(u-u_o) - \alpha_{ur}(i-\hat{p}-(i_o-\pi_o)) - \alpha_{uv}(v-v_o) + \alpha_{u\eta}\eta + \alpha_{uu^f}\hat{u}^f$$

$$\hat{e} = \alpha_{e\hat{u}}\hat{u} - \alpha_{ev}(v-v_o)$$

$$\hat{v} = \kappa[(1-\kappa_{pw})f_w(e,v) - (1-\kappa_{wp})f_p(u,v) + \rho g_z],$$

$$\dot{\pi}_c = \beta_{\pi_c}(\hat{p}_c - \pi_c), \quad \hat{p}_c = \gamma\hat{p} + (1-\gamma)(\dot{s}+\hat{p}^f)$$

$$\dot{i} = -\gamma_{ii}(i-i_o) + \gamma_{ip}(\hat{p}-\pi_o) + \gamma_{iu}(u-u_o)$$

$$\dot{\eta} = \beta_s^f(i^f - i - \eta) + \hat{p}^f - \hat{p}$$

$$\hat{u}^f = -\alpha_{uu}(u^f-u_o) - \alpha_{ur}(i^f-\hat{p}^f-(i_o-\pi_o)) - \alpha_{uv}(v^f-v_o) - \alpha_{u\eta}\eta + \alpha_{uu^f}\hat{u}$$

$$\hat{e}^f = \alpha_{e\hat{u}}\hat{u}^f - \alpha_{ev}(v^f-v_o)$$

$$\hat{v}^f = \kappa[(1-\kappa_{pw})f_w(e^f,v^f) - (1-\kappa_{wp})f_p(u^f,v^f) + \rho g_z^f]$$

$$\dot{\pi}_c^f = \beta_{\pi_c}(\hat{p}_c^f - \pi_c^f), \quad \hat{p}_c^f = \gamma\hat{p}^f + (1-\gamma)(-\dot{s}+\hat{p})$$

$$\dot{i}^f = -\gamma_{ii}(i^f-i_o) + \gamma_{ip}(\hat{p}^f-\pi_o) + \gamma_{iu}(u^f-u_o)$$

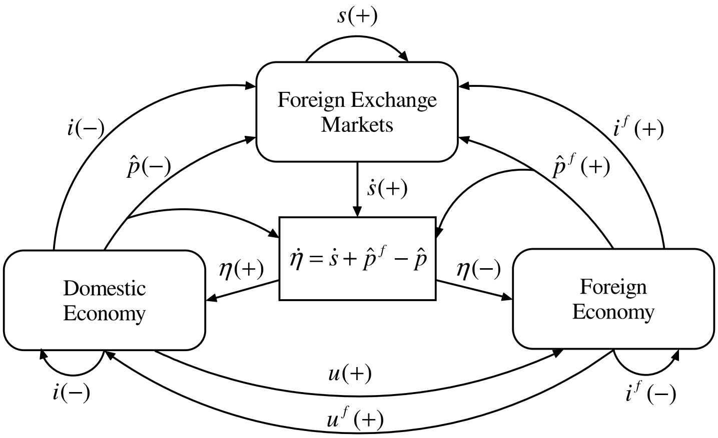

Fig. 9.2. The real and financial links of the two country model

of the Routh-Hurwitz local stability conditions for a 11D system would be an extremely complicated and more importantly, nontransparent task, we will investigate the stability of the system in a numerical manner focusing on the role of the wage and price flexibility for the stability of the system by means of an eigen-value analysis in Sect. 9.4.

9.3.1 Stylized Facts of Monetary Policy

Since the seminal contribution by Sims (1980), vector autoregressive (VAR) models have become a standard tool for the study of the transmission of monetary policy in industrialized economics.

In the majority of existing studies the VAR analysis is performed under the implicit assumption that the studied economies have "closed" or "small open economy" characteristics due to the possible collinearity and identification problems which can arise if a large number of variables is incorporated in the VAR model. From the econometric perspective, this means that foreign variables, if included in the estimated VAR model, are assumed to be exogenously determined. Indeed, most of the prominent studies on monetary policy transmission such as Bernanke and Blinder (1992), Bernanke and Mihov (1998) and Christiano et al. (1999) for the U.S. economy, Kim (1999) for the G-7 countries and Peersman and Smets (2003) for the Euro area are based on a "small open economy" assumption.

Under such a specification the main stylized facts concerning the monetary policy transmission mechanism can be summarized as follows:

- An unexpected increase in the U.S. nominal interest rate (a contractionary monetary policy shock) leads to a slowdown of economy activity, which reaches its peak after five quarters, approximately.
- The response of employment resembles the output reaction, though in a somewhat delayed manner.
- Price inflation initially increases (the price puzzle discussed, e.g., by Sims (1992)), but, after some quarters, an unambiguously negative effect can be observed.
- The domestic currency appreciates due to, among other things, the interest rate parity.

Concerning the international transmission of monetary policy, Kim (2001) discusses two main findings from his VAR estimations: First, that monetary policy in the non-US G-6 countries follows U.S. monetary policy shocks (a result which corroborates the findings of Eichenbaum and Evans (1995) concerning the dynamic behavior of spread between foreign and U.S. interest rates after such types of shocks). Second, that U.S. monetary expansions have a positive spill-over effect on the remaining G-7 countries primarily due to the resulting reduction in the world interest. This result is also found by Bluedorn and Bowdler (2006) and Eickmeier (2007), the latter concerning the effect of U.S. monetary shocks on Germany.

The empirical evidence on the reaction of nominal exchange rates to monetary policy shocks is, on the contrary, not as undisputed. While Eichenbaum and Evans (1995, p. 976) for example find that "the maximal effect of a contractionary monetary policy shock on U.S. exchange rates is not contemporaneous; instead the dollar continues to appreciate for a substantial period of time [a finding which] is inconsistent with simple rational expectations overshooting models of the sort considered by Dornbusch (1976)", Kim and Roubini (2000), Kalyvitis and Michaelides (2001) and Bluedorn and Bowdler (2006) find little evidence on such a behavior for the G-7 nominal exchange rates after the inclusion of alternative measures of monetary policy shocks as well as of relative output and prices in their specifications.

Next the strength of international transmission channels between the U.S. and the euro area, two large economies which are likely to indeed influence each other by a variety of macroeconomic channels are investigated by means of econometric methods.

9.3.2 Data Sources and Descriptive Statistics

In order to analyze the interaction of two economies which indeed are of sufficiently large dimension to significantly influence each other, we take as examples the U.S. and the euro area economies. The empirical data of the corresponding time series stem from the Federal Reserve Bank of St. Louis data set (see http://www.stls.frb.org/fred) and the OECD database for the U.S. and the euro area, respectively. The data are quarterly, seasonally adjusted and concern the period from 1980:1 to 2004:4.

The logarithms of wages and prices are denoted $\ln(w_t)$ and $\ln(p_t)$, respectively. Their first differences (backwardly dated), i.e. the current rate of wage and price inflation, are denoted $\hat{w}_t$ and $\hat{p}_t$ as in the theoretical framework. The inflationary climate π^c of the theoretical part of this chapter is approximated here in a very simple way by a linearly declining moving average of price inflation rates with linearly decreasing weights over the past twelve quarters, denoted π_t^{12}.

Figure 9.3 shows the time series of both the U.S. and the Euro Area described in Table 9.1. As it can be observed there, the U.S. and the Euro Area have featured in the last two decades a remarkable similarity in their respective wage and price inflation developments, as well—to a somewhat

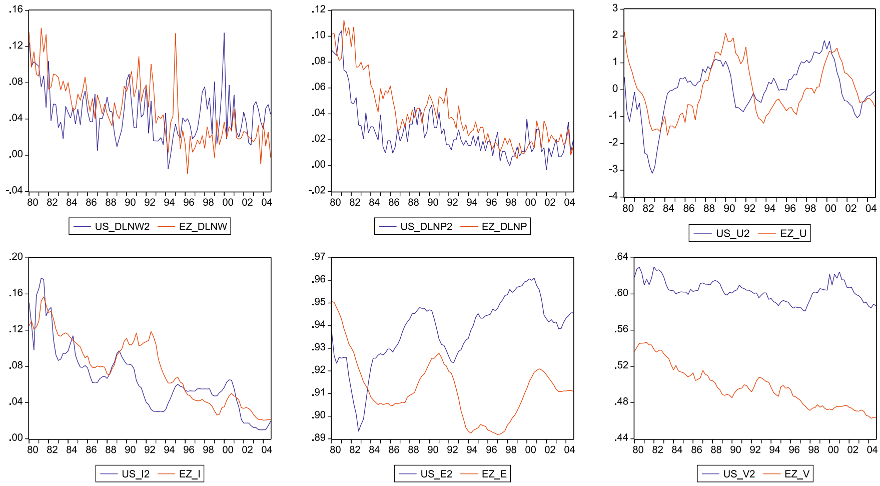

Fig. 9.3. U.S. and Euro area aggregate time series

Table 9.1. Data set

Variable	Description of the original series
e	U.S. : Employment Rate
	Euro Area : Employment Rate (HP cyclical component, $\lambda = 640000$)
u	U.S. : Capacity Utilization: Manufacturing, Percent of Capacity
	Euro Area : Output Gap
w	U.S. : Nonfarm Business Sector: Compensation Per Hour,
	$1992 = 100$
	Euro Area : Business Sector: Wage Rate Per Hour,
p	U.S. : Gross Domestic Product: Implicit Price Deflator, 1996=100
	Euro Area : Gross Domestic Product: Implicit Price Deflator, 2000=100
z	U.S. : Nonfarm Business Sector; Output Per Hour of All Persons,
	$1992 = 100$
	Euro Area : Labor Productivity of the business economy,
v	U.S. : Nonfarm Business Sector: Real Compensation Per Output Unit,
	$1992 = 100$
	Euro Area : Business Sector: Real Compensation Per Output Unit
	(HP cyclical component, $\lambda = 640000$)
i	U.S. : Federal Funds Rate
	Euro Area : Short Term Interest Rate
s	EUR/USD Nominal Exchange Rate

lesser extent—in the dynamics of the capacity utilization and the output gap, respectively.

This, however, does not hold for the dynamics of the employment rate and the wage share of both economies. As it can be observed in Fig. 9.3, while the U.S. unemployment rate has fluctuated, roughly speaking, around a constant level over the last two decades, the European employment (unemployment) rate described a persistent downwards (upwards) trend over the same time period. This particular European development has been explained by Layard et al. (1991) and Ljunqvist and Sargent (1998) by an over-proportional increase in the number of long-term unemployed (i.e. workers with an unemployment duration over 12 months) with respect to short term unemployed (workers with an unemployment duration of less than 12 months) and the phenomenon of hysteresis especially in the first group. Because long-term unemployed become less relevant in the determination of nominal wages (since primarily the short-term unemployed are taken into account), the potential downward pressure on wages resulting from the unemployment of the former diminishes, with the result of a higher level of the NAIRU, see Blanchard and Wolfers (2000). When the long-term unemployment is high, the aggregate unemployment rate of an economy thus, "becomes a poor indicator of effective labor supply, and the macroeconomic adjustment mechanisms—such as downward pressure on wages and inflation when unemployment is high—will then not operate effectively" (OECD 2002, p. 189).

Since time series data for long-term unemployment in the Euro area is not available for the analyzed sample period, we used the adjusted cyclical component of the unemployment rate as a proxy for the short-term unemployment. This series was calculated as the difference between the actual unemployment rate and the HP trend series obtained on the basis of a smoothing factor $\lambda = 640000$ (interpretable as a proxy for the actual development of long-term unemployment in the Euro Area), normalized to zero in 1970:1, where unemployment (and also long-term unemployment) was extremely low in the European continent.[11] In our econometric estimation, thus, we implicitly assume the existence of a variable NAIRU in the Euro area, despite the fact that we did not explicitly model it in the theoretical framework of the previous section.

In order to check the stationarity of the analyzed time series, Phillips-Perron unit root tests were computed in order to account, besides of residual autocorrelation as done by the standard ADF Tests, also for possible resid-

[11] See Proaño et al. (2006) for a detailed description of this procedure.

Table 9.2. Data set: Descriptive statistics

	Euro area						U.S.						
	u	e	dln(w)	dln(p)	v	i	u	e	dln(w)	dln(p)	v	i	nxr
Mean	0.893	0.977	0.049	0.040	0.599	0.077	0.987	0.948	0.046	0.026	0.604	0.066	0.899
Median	0.891	0.975	0.045	0.034	0.599	0.079	0.988	0.947	0.042	0.019	0.604	0.058	0.860
Max.	0.929	0.997	0.140	0.112	0.617	0.157	1.027	0.964	0.135	0.104	0.629	0.178	1.370
Min.	0.864	0.958	−0.020	0.005	0.578	0.020	0.920	0.911	−0.015	−0.003	0.581	0.010	0.620
Std. Dev.	0.017	0.012	0.034	0.026	0.009	0.037	0.022	0.012	0.027	0.021	0.012	0.037	0.157
J.B. Prob.	0.058	0.013	0.085	0.000	0.453	0.060	0.000	0.000	0.003	0.000	0.487	0.000	0.004
Sum	89.32	97.71	4.995	4.049	59.98	7.714	98.78	94.84	4.605	2.639	60.37	6.615	89.92
Sum Sq.Dv.	0.029	0.014	0.117	0.068	0.009	0.135	0.047	0.014	0.073	0.044	0.013	0.139	2.424
Obs.	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 9.3. Phillips-Perron unit root test results. Sample: 1980:1–2004:4

Country	Variable	Lag Length	Determ.	Adj. Test Stat.	Prob.*
	$\hat{w}$	–	const.	−6.7769	0.0000
U.S.	$\hat{p}$	–	const.	−2.7647	0.0671
	$\hat{u}$	–	–	−7.0655	0.0000
	$\hat{e}$	–	–	−4.8206	0.0000
	i	–	–	−1.8553	0.0608
	$\hat{w}$	–	const.	−3.4982	0.0100
Euro area	$\hat{p}$	–	none	−2.3617	0.0183
	$\hat{u}$	–	const.	−8.0891	0.0000
	$\hat{e}$	–	–	−3.1516	0.0019
	i	–	–	−1.4810	0.1290

*MacKinnon (1996) one-sided p-values

ual heteroskedasticity. The Phillips-Perron test specifications and results are shown in Table 9.3.

The applied unit root tests reject the hypothesis of a unit root for all series with exception of the euro area nominal interest rate i. However, we interpret these results as only providing a hint that the nominal interest exhibit a strong autocorrelation due to the known low power of the unit root tests.

9.3.3 Structural Estimation Results

We discuss now the system estimations of both countries carried out based on the parameter restrictions stemming from the theoretical model discussed in Sect. 9.2.

As discussed in the previous section, the law of motion for the real wage rate, given by (9.7), represents a reduced form expression of the two structural equations for $\hat{w}_t$ and $\hat{p}_t$. Noting again that the inflation climate variable is defined in the estimated model as a linearly declining function of the past twelve price inflation rates, the dynamics of the system (9.13)–(9.16) can be reformulated as

$$
\begin{aligned}
\hat{w}_t^j &= \beta_{we}(e_{t-1}^j - e_o^j) - \beta_{wv}\ln(v_{t-1}^j/v_o^j) + \kappa_{wp}\hat{p}_t^j + \kappa_{w\pi^{12}}\pi_t^{12,\mathrm{j}} + \kappa_{wz}\hat{z}_t^j + \epsilon_{wt},\\
\hat{p}_t^j &= \beta_{pu}(u_{t-1}^j - u_o^j) + \beta_{pv}\ln(v_{t-1}^j/v_o^j) + \kappa_{pw}(\hat{w}_t^j - \hat{z}_t^j) + \kappa_{p\pi^{12}}\pi_t^{12,\mathrm{j}} + \epsilon_{pt},\\
\ln u_t^j &= \ln u_{t-1}^j - \gamma_{uu}(u_{t-1}^j - u_o^j) - \alpha_{ur}(i_{t-1}^j - \hat{p}_t^j) \pm \alpha_{uv}(v_t^j - v_o^j)\alpha_{u\eta}\eta_{t-4} + \epsilon_{ut},\\
\hat{e}_t^j &= \alpha_{eu-1}\hat{u}_{t-1}^j + \alpha_{eu-2}\hat{u}_{t-2}^j + \alpha_{eu-3}\hat{u}_{t-3}^j + \epsilon_{et},\\
i_t^j &= \phi_i i_{t-1}^j + (1-\phi_i)\phi_\pi\hat{p}_t^j + (1-\phi_i)\phi_y u_{t-1}^j + \epsilon_{it}, \quad \text{with} \quad j = us, ez,\\
s_t &= i_{t-1}^{us} - i_{t-1}^{ez} + \alpha_{ss}s_{t-1} - \lambda\beta_s^f\eta_t + (1-\lambda)\beta_s^c\hat{s}_{t-1}
\end{aligned}
$$

with $\gamma_{uu} = 1 - \alpha_{uu}$ and sample means denoted by a subscript o.

In order to investigate the differences between a single-country system estimation and a two-country system estimation for the values of the parameters of the model for the U.S. and Euro Area, we estimated the structural equations of both countries separately and jointly by means of Three-Stage-Least-Squares (3SLS), in order to account for a possible regressor endogeneity and heteroskedasticity.

As it can be observed in Table 9.4, we find a wide support for the theoretical formulation discussed in the previous section. In the first place we find similar and statistically significant coefficients for $\ln(v/v_o)$, the Blanchard-Katz error correction terms, in both the wage and price adjustment equations of both the U.S. and the euro area.

In the second place, our cross-over formulation of the inflationary expectations cannot be rejected statistically in the wage and price inflation equations of both economies. As Table 9.4 shows, the inclusion of the market specific demand pressure terms (the capacity utilization in the price- and the employment rate in the wage Phillips curve equations) is also corroborated by our estimations, as well as the fact that wage flexibility is higher than price flexibility (concerning their respective demand pressure measures) in both the U.S. and the Euro Area, a result in line with the findings of Chen and Flaschel (2006), Proaño et al. (2006) and Flaschel et al. (2007).

Concerning the estimated open economy IS equation, the 3SLS estimations summarized in Table 9.4 show, as expected, the negative influence of the expected real interest rate on the dynamics of capacity utilization in both economies. The same holds true for the effect of $v - v_o$ in both U.S. and Euro Area, the deviation of the labor share from its steady state level, showing that a relatively high labor share (or real average unit labor costs) has a negative impact on the domestic rate of capacity utilization, something that holds for a profit led economy. The coefficient α_{uu^f}, which represents the effect of foreign goods demand on the dynamics of the domestic capacity utilization rate, are

Table 9.4. 3SLS parameter estimates: One-country specification

Estimation Sample: 1980:1–2004:4							
$\hat{w}_t$	β_{we}	β_{wv}	κ_{wp}	$\kappa_{w\pi^{12}}$	κ_{wz}	$\bar{R}^2$	DW
Euro Area	0.481	−0.424	0.878	0.254	0.238	0.705	1.608
	[2.669]	[-3.643]	[3.584]	[1.091]	[2.843]		
U.S.	0.684	−0.352	0.685	0.634	0.376	0.317	1.828
	[3.425]	[−2.744]	[2.618]	[2.462]	[5.239]		
$\hat{p}_t$	β_{pu}	β_{pv}	κ_{pw}	$\kappa_{p\pi^{12}}$		$\bar{R}^2$	DW
Euro Area	0.274	0.136	0.083	0.864		0.898	1.518
	[4.644]	[2.471]	[2.081]	[23.240]			
U.S.	0.250	0.097	0.085	0.833		0.774	1.354
	[4.604]	[1.769]	[2.311]	[18.084]			
$\ln u_t$	γ_{uu}	α_{ur}	α_{uv}	$\alpha_{u\eta}$	α_{u_uf}	$\bar{R}^2$	DW
Euro Area	−0.136	−0.059	-0.203	0.012	0.070	0.927	1.839
	[−3.896]	[−2.905]	[−3.292]	[2.183]	[0.988]		
U.S.	−0.069	−0.044	−0.048	−0.001	0.185	0.904	1.495
	[−2.454]	[−1.804]	[−1.557]	[−1.458]	[1.845]		
$\hat{e}$	α_{eu-1}	α_{eu-2}	α_{eu-3}			$\bar{R}^2$	DW
Euro Area	0.139	0.129	0.071			0.616	1.121
	[7.284]	[6.791]	[3.881]				
U.S.	0.138	0.092	0.045			0.357	1.377
	[4.763]	[3.097]	[1.541]				
i	ϕ_i	ϕ_{ip}	ϕ_{iu}			$\bar{R}^2$	DW
Euro Area	0.926	1.519	1.468			0.981	1.364
	[43.669]	[10.705]	[2.524]				
U.S.	0.820	2.217	0.611			0.927	1.887
	[29.764]	[15.661]	[2.578]				
s	α_{ss}	β_s	$\beta_{s\eta}$			$\bar{R}^2$	DW
Euro Area	0.903	0.340	0.145			0.917	1.409
	[17.192]	[1.737]	[0.578]				
U.S.	0.908	0.309	0.048			0.917	1.413
	[17.322]	[1.578]	[0.678]				

both positive and significant (with the U.S. coefficient of an unexpectedly high value) for both economies.

The parameter estimates in the dynamic Okun's law and Taylor rule equations of both economies are positive, statistically significant and of reasonable dimension, with nevertheless a much higher reaction coefficient to inflation than output in the U.S. than in the euro area for the analyzed sample period. Concerning the law of motion of the log nominal exchange rate, both the log real exchange rate as well as the interest rate differential influence the level of the log nominal exchange rate, the former in a negative and the latter in a positive manner.

Besides the one-country 3SLS estimations just discussed, we estimated both countries as a single system by means of 3SLS.[12] Compared with the

[12] The set of instrumental variables in the 3SLS estimation consisted on the same lagged values of the two countries used in the previous estimations.

Table 9.5. 3SLS parameter estimates: Two-country specification

Estimation Sample: 1980:1–2004:4							
$\hat{w}_t$	β_{we}	β_{wv}	κ_{wp}	$\kappa_{w\pi 12}$	κ_{wz}	$\bar{R}^2$	DW
Euro Area	0.462	−0.412	0.888	0.244	0.234	0.705	1.615
	[2.594]	[−3.589]	[3.672]	[1.059]	[2.836]		
U.S.	0.661	−0.386	0.484	0.582	0.352	0.341	1.830
	[4.158]	[−2.933]	[1.797]	[3.107]	[5.112]		
$\hat{p}_t$	β_{pu}	β_{pv}	κ_{pw}	$\kappa_{p\pi 12}$		$\bar{R}^2$	DW
Euro Area	0.252	0.115	0.076	0.870		0.898	1.522
	[4.334]	[2.135]	[1.965]	[23.907]			
U.S.	0.130	0.138	0.107	0.579		0.789	1.391
	[2.757]	[2.450]	[3.168]	[19.324]			
$\ln u_t$	γ_{uu}	α_{ur}	α_{uv}	$\alpha_{u\eta}$	α_{uu^f}	$\bar{R}^2$	DW
Euro Area	−0.109	−0.064	−0.161	0.012	0.101	0.927	1.746
	[−3.412]	[−3.264]	[−2.693]	[2.408]	[1.540]		
U.S.	−0.094	−0.040	−0.118	−0.008	0.161	0.906	1.529
	[−3.485]	[−1.937]	[−2.193]	[−1.330]	[1.634]		
$\hat{e}$	α_{eu-1}	α_{eu-2}	α_{eu-3}			$\bar{R}^2$	DW
Euro Area	0.137	0.129	0.076			0.616	1.132
	[7.237]	[6.886]	[4.199]				
U.S.	0.153	0.101	0.045			0.371	1.444
	[5.334]	[3.418]	[1.559]				
i	ϕ_i	ϕ_{ip}	ϕ_{iu}			$\bar{R}^2$	DW
Euro Area	0.925	1.534	1.769			0.981	1.358
	[48.649]	[11.237]	[3.048]				
U.S.	0.823	2.157	0.375			0.928	1.890
	[31.627]	[15.271]	[1.756]				
s	α_{ss}	β_s	$\beta_{s\eta}$			$\bar{R}^2$	DW
	0.909	0.383	0.111			0.917	1.412
	[17.628]	[1.995]	[0.534]				

single-country 3SLS estimations just described, Table 9.5 delivers quite similar values concerning all estimated parameters, corroborating the robustness of our results. There are nevertheless two remarkable differences: While in the 3SLS estimations we obtained a quite high coefficient for α_{uu^f} in the U.S. equation (representing the role of the growth rate of capacity utilization in the Euro Area for the dynamics of the same variable in the U.S.), in Table 9.5 we obtained a parameter estimate of more reasonable dimension (though still to high if compared with the coefficient in the Euro Area $\ln u$ equation, if one takes into account that the U.S. is probably more important for the Euro Area than otherwise).[13] The second remarkable difference between the one-country and the two-country 3SLS estimations concerns the influence of the log real exchange rate on the log nominal exchange rate: While in the estimations summarized in Table 9.4 its coefficient was highly significant and in line with

[13] In the dynamic adjustments simulations of the next section we will calibrate this coefficient to be if not *lower*, at least *equal* to that of the Euro Area.

our theoretical formulation, in the second estimation described in Table 9.5 that coefficient seems to be statistically insignificant.

9.3.4 Dynamic Adjustments

In order to evaluated the empirical plausibility of our theoretical framework, we simulate an approximate discrete time version of the semi-structural model discussed in Sect. 9.2 based on the estimated 3SLS structural model parameters discussed in the last section.[14] Additionally, we calibrate the parameters concerning the theoretical CPI inflationary climate for both countries with the following values:

$$\beta_{\pi_c} = 0.25, \qquad \kappa_{\pi_c} = 0.5, \qquad \gamma = 0.85.$$

Both countries have thus the same degree of inflation climate inertia (represented by β_{π_c}, the adjustment coefficient of the CPI inflationary climate), whereafter each new (quarterly) CPI inflation rate observation updates with only a 0.25 weight the inflationary climate. Both countries have also the same degree of credibility in the monetary policy target (κ_{π_c}) as well as the same composition of domestic and foreign goods in the CPI index.[15]

A U.S. Monetary Policy Shock

In Fig. 9.4 we show the dynamic adjustments to a one percent (100 basis points) monetary policy shock in the U.S. economy of the two countries using the structural parameters estimates of both the U.S. and the Euro Area depicted in Table 9.5. As Fig. 9.4 shows, the numerical simulations of the calibrated theoretical discrete time model resemble to a large extent the stylized facts of monetary policy briefly discussed in the previous section.

As expected, a positive monetary policy shock in the U.S. leads to an depreciation of the EUR/USD nominal exchange rate primarily via the uncovered interest rate parity (UIP) condition comprised in the law of motion of the log nominal exchange rate. This nominal appreciation of the US dollar, together with the effect of the interest rate increase, leads to a slowdown of the U.S. economy, observable in the decrease in capacity utilization. Following the downturn of this variable, employment also falls, as well as wage and

[14] The numerical simulation in this section were performed using MATLAB. The simulation code is available upon request.

[15] We adopt this specific value from Rabanal and Tuesta (2006).

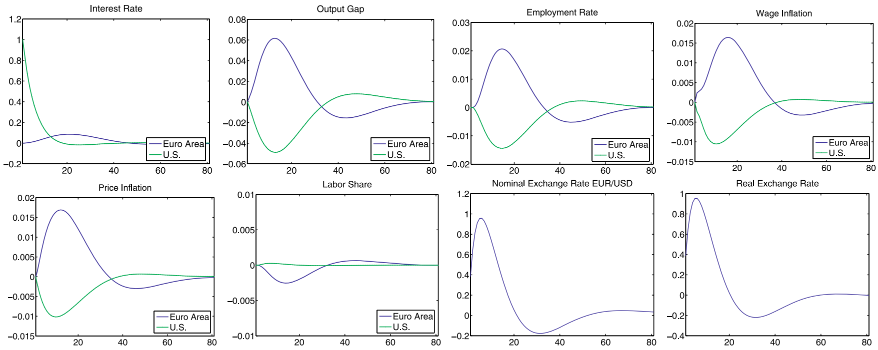

Fig. 9.4. Simulated responses to a one percent U.S. monetary policy shock

price inflation start falling after some quarters below baseline. The Euro Area is affected from the contractionary U.S. monetary policy shock through three macroeconomic channels: the nominal depreciation of the euro, the drop in foreign aggregate demand and the gain of relative competitiveness resulting from an increase in η. As Fig. 9.4 shows, the activation of these three channels leads to an increase in economic activity in the euro area.

A Euro Area Monetary Policy Shock

As a second simulation experiment, we compute the dynamic adjustments of both the U.S. and the Euro Area after a monetary policy shock by the European Central Bank (ECB) with our calibrated model.

The dynamics depicted in Fig. 9.5 resemble to a large extent the dynamic adjustments to a U.S. monetary shock previously discussed. However, we can identify one main important difference: Indeed, while the Euro Area was largely affected by the contractionary monetary policy shock in the U.S., the opposite does not hold by far for the U.S. economy, due to the relatively lower foreign goods demand coefficient α_{uu^f} coefficient as well as due to the absence of a significant influence of the real exchange rate and the relative competitiveness channel.[16]

It should be stressed that the overshooting nominal (and real) exchange rate dynamics observable in Figs. 9.4 and 9.5 arise due to its specific formulation in our model. However, even though this overshooting behavior is

[16] This statement is based on the estimations results previously discussed, which lead to the presumption that, if present, these channels are not statistically important for the dynamics of the capacity utilization in the U.S. given the data set used.

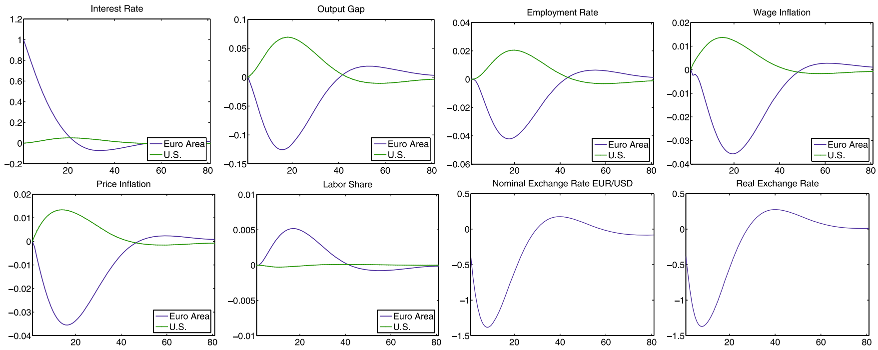

Fig. 9.5. Simulated impulse-responses to a one std. dev. Euro area monetary policy shock

concordant with the findings by Eichenbaum and Evans (1995), this result should not be over-interpreted given the contrary empirical evidence by the alternative studies previously discussed.

9.4 Eigen-Value-based Stability Analysis

As previously mentioned, if the stability of a macrodynamic system is not simply imposed through the rational expectations assumption, the relative strength of the different macroeconomic channels interacting in an economy become central for the local and global stability properties of the system analyzed.

The main purpose of this section is to highlight this issue within the semi-structural two-country macro-framework discussed and estimated in the previous sections. For this an eigen-value stability analysis is used taking as the benchmark parameters the estimated values presented in the previous section. After calibrating the 11D continuous time system, the eigen-values of the system are calculated ceteris paribus for different parameter of the models (mostly in the 0–1 interval) using the SND software.[17]

In Figs. 9.6–9.9 the maximal eigen-values of the system for varying parameter values in the closed-economy- (the one-country submodule under α_{uu^f}, $\alpha_\eta = 0$, $\xi = 1$) calculated with the U.S. parameter estimates of Proaño et al. (2006)—shown in Table 9.6—) and in the open-economy cases are sketched.

[17] Downloadable from Carl Chiarella's website at the UTS in Sydney, Australia.

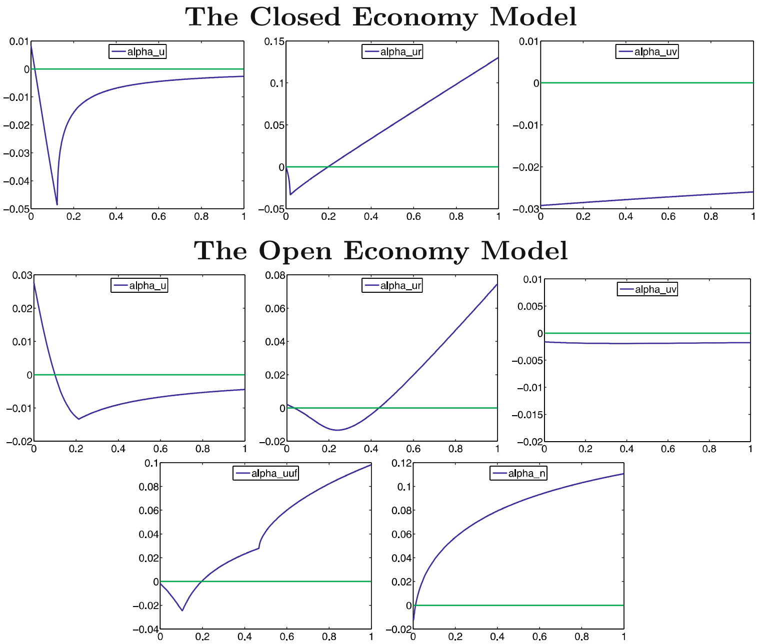

Fig. 9.6. Eigen-value-based stability analysis: The real economy

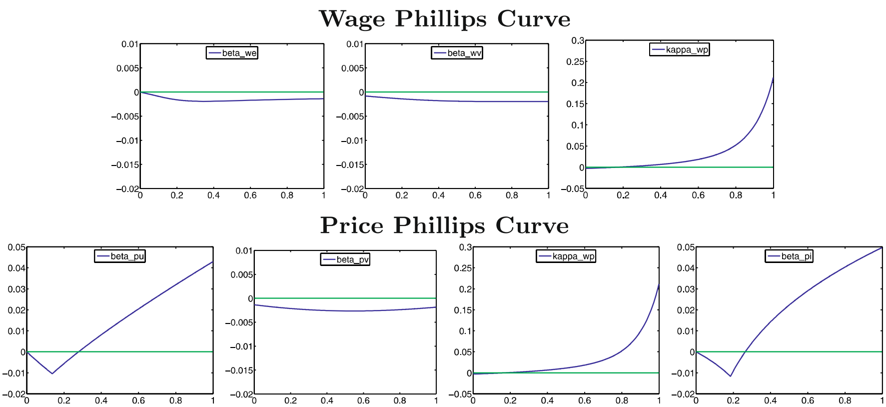

Fig. 9.7. Eigen-value-based stability analysis Ib: Wage-price dynamics (the open economy case)

The comparison between the eigen-value diagrams of the closed-economy and open-economy cases depicted in Figs. 9.6–9.8 reveals by and large the same qualitative implications of a variation of the analyzed coefficients for the stability of the system (and the two- and the one-country case): So, while

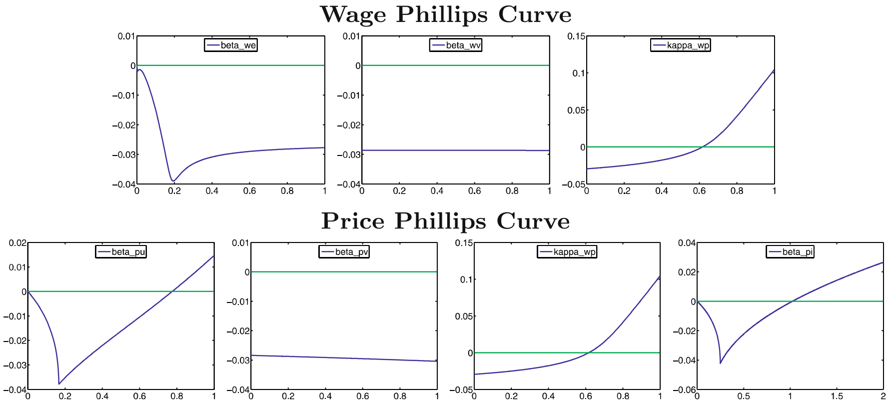

Fig. 9.8. Eigen-value-based stability analysis: Wage-price dynamics (the closed economy case, using the parameter values estimated in Proaño et al. (2006))

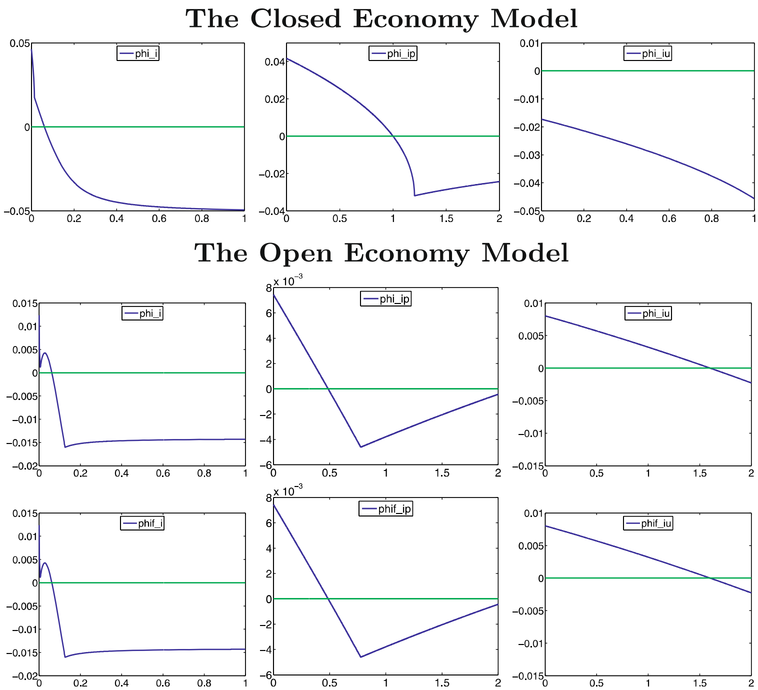

Fig. 9.9. Eigen-value-based stability analysis: Monetary policy

the stability properties of the respective systems seem to be invariant for different parameters of α_{uv} (the reaction strength of capacity utilization to an increase in the wage share), β_{we} (the wage inflation reactiveness parameter

Table 9.6. Closed-Economy Model – Calibration Parameters (Proaño et al. 2006)

Goods Markets	γ_{uu}	α_{ur}	α_{yv}	
	0.077	0.042	−0.173	
Labor Markets	α_{eu1}	α_{eu2}	α_{eu3}	α_{ev}
	0.201	0.113	0.039	0.100
Wage Phillips Curve	β_{we}	β_{we}	κ_{wp}	$1-\kappa_{wp}$
	0.679	0.208	0.420	0.580
Price Phillips Curve	β_{we}	β_{we}	κ_{wp}	$1-\kappa_{wp}$
	0.294	0.113	0.044	0.956
Monetary Policy Rule	α_{ii}	ϕ_{ip}	ϕ_{iu}	
	0.830	2.17	0.423	

with respect to labor marker disequilibrium situations), as well as β_{wv} and β_{pv} (the Blanchard-Katz error correction terms in both the wage and price inflation adjustment equations), the same does not hold for the remaining real economy parameters. Indeed, high coefficients of α_{ur}, the real interest rate reactiveness of the capacity utilization, both κ_{wp} and κ_{pw}, the cross-over inflation terms in the wage and price Phillips Curve equations, a high price flexibility with respect to goods market disequilibrium situations (represented by the parameter β_{pu}) as well as a high adjustment of the inflationary climate π_c, determined by β_{π_c} seem to induce instability in the system.

Concerning the open-economy dimension of the model, Fig. 9.6 shows that both a high reactiveness of capacity utilization towards the real exchange rate *and* the dynamics of the foreign economy (determined by α_η and α_{uu^f}, respectively), are likely to induce instability of the system due to an eventual over-synchronization of both economies which might feature reinforcing properties.

Figure 9.9 shows the eigen-value diagrams resulting from variations in the monetary policy parameters. As expected, while an increase in the α_{ii} (that is, a lower degree of interest rate smoothing in the nominal interest rate law of motion, or in other words, the faster adjustment speed of the actual nominal interest rate with respect to i_{T}) induces stability into both the closed and the open economy systems, the steady state stability properties seem to be invariant to changes in ϕ_{iu} (the reaction coefficient of the monetary policy instrument with respect to the output gap).

This, however, does not hold for ϕ_{ip}, the reaction coefficient with respect to the inflation gap. Indeed, consistently with the academic literature on monetary policy, we find for the closed economy case that the steady state of the economic system is be stable only if $\phi_{ip} > 1$, that is, only if monetary policy reacts in a sufficiently active manner with respect to inflationary developments, as discussed for example in Walsh (2003) and Woodford (2003). In the open economy case, however, the eigen-value diagram of ϕ_{ip} shows that the threshold value for stability lies much lower than in the closed-economy case, relativizing up to a certain extent the validity of the prominent Taylor Principle, at least for large economies such as the U.S. and the euro area. This result, though somewhat surprising at first sight, is actually quite reasonable: In contrast to the closed economy case, in an open economy the monetary policy transmission mechanism is, additionally to traditional transmission channels such as the credit and the balance sheet channels, enriched by other transmission channels such as the nominal exchange rate and the competitiveness channels. So leads for example an interest rate increase not only to higher borrowing costs and therefore to a lower consumption and investment demand, but also, in an open economy, to a nominal (and real) appreciation of the domestic currency, which in turn leads to a decrease in the net exports. In an open economy, thus, monetary policy can rely on the activation of more transmission channels and therefore needs not to be as aggressive as in the closed-economy case.

9.5 Concluding Remarks

In this chapter we studied a basic theoretical two-country framework based on the disequilibrium approach by Chiarella and Flaschel (2000) and Chiarella et al. (2005), where two large open economies interacted with each other and indeed influenced each other through trade, price and financial channels.

Despite of the straightforwardness of the theoretical formulation of this semi-structural two-country model, we were able to perform an insightful analysis of the macroeconomic interaction of two large economies at both the theoretical and empirical level. At the theoretical level, we were able to identify the stability conditions of the continuous time dynamical system, highlighting primarily the role of wage flexibility for macroeconomic stability. At the empirical level, the econometric estimations of the euro area and the U.S. economy (two large open economies which are in fact highly interrelated through a variety of macroeconomic channels) showed, on the one hand, the empirical plausibility of our theoretical framework, corroborating the results of the closed economy model discussed in Part I of this book. On the other hand, they showed the remarkable similarities between the euro area and the U.S. economy not only in the wage and price inflation equations, but also in the dynamics of the goods and labor markets. Furthermore, using the parameter estimates of the euro area and the U.S. economy, we were able to generate dynamic impulse response functions quite concordant with the VAR evidence discussed in the academic literature.

An important issue worth being highlighted is the eigen-value analysis performed in the previous section. Given the actual predominance of rational expectations models where the model stability is given by assumption and by the associated model solution method, the present analysis shows an alternative—and also valid—perspective on the analysis of model stability. This alternative approach allowed us to identify and to highlight, among other things, the role of wage and price stability, as well as the importance of an active monetary policy, for the stability of the system. Additionally, we could investigate, in a graphical and insightful manner, the differences in the stability conditions between closed and open economies. Concerning this last point, a remarkable result of the eigen-value analysis was the different threshold values of ϕ_{ip}, the inflation gap coefficient in the Taylor rule, in the closed- and open economy cases. As previously discussed, our analysis showed that the coefficient value dividing a "passive" from an "active" monetary policy is, in our theoretical formulation and given our parametrization, lower in the open- than in the closed economy case. Though still preliminary, this result stresses the necessity to incorporate open economy factors in macroeconomic models when studying the effectiveness and adequacy of different monetary policy rules.

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10

Outlook: Supply Constraints in Demand-Driven Macromodels

10.1 Introduction

In this chapter we reconsider the microfounded approaches to macro-statics and -dynamics that started from the so-called fix-price models (with quantity adjustments in the place of price adjustments) for their determination of temporary equilibrium positions. We shall reconsider these approaches here purely on the macrolevel and from the perspective of descriptive macrodynamics in order to investigate the contributions of these approaches to macrodynamics to a further improvement of the general KMG model of monetary growth originally developed in Chiarella and Flaschel (2000).

Non-Walrasian macroeconomics has provided many significant contributions to macrostatics as reviewed for example in Benassy (1993). Important approaches in this regard are the models of Clower (1965), Barro and Grossman (1971), Benassy (1977, 1984, 1986) and Malinvaud (1977, 1980, 1984), to mention only a few of them. The common starting point of these theories can be seen in a critique of the main assumption of Walrasian market-clearing models, that transactions do not occur before the market clearing price vector has been found. If, however, in contrast to this, transactions at "wrong prices" are taken into account, they are typically connected with rationing on different markets. The crucial point is now, that supply and demand on a single market depend no longer only on the price vector (including not only goods prices but also wages, interest rates etc.), but also on the rationing, that agents have experienced on other markets. This type of demand, originally formulated by Clower (1965) and thus usually referred to as "Clower demand”,[1] already appeared in the KMG model type formulated in Chiarella et al. (2000, Part I), where firms reacted with regard to their emission of new equities on the disequilibrium perfectly perceived by them at the beginning of each period. On this basis many approaches like, e.g. the one of Malinvaud (1980), came to a division of the state space of the economy into different regimes in dependence on the short side of the goods market and the labor market. Thus, if the temporarily given wage-price constellation led to aggregate demand being the binding constraint on the goods market and the resulting demand for labor being the short side on the labor market, the corresponding regime was called “Keynesian”. On the other hand, if supply constraints due to capacity or profitability constraints represented the short side on the goods as well as on the labor market, the economy found itself in a “Classical” regime. A situation with excess demand on both markets finally was called a regime of “repressed inflation”. Malinvaud (1980)—as many other approaches in this respect—now formulated laws of motion for the temporarily fixed wages and prices in order to study the dynamic adjustment processes of the economy considered. In addition, however, he also took explicitly into account the dependence of investment on profitability as well as on capacity utilization and thus elements also playing an important role in our KMG framework. On this basis, a business cycle could be derived which, however, got stuck in the Keynesian regime with a stable depression, the size of which furthermore depended on the initial point, at which the dynamics took its point of departure.

Thus, although non-Walrasian theories considerably improved the description of the short-run, when prices are temporarily fixed, there are however far less contributions of this approach to the theory of business fluctuations and only very few to the theory of economic growth (in real or monetary economies). There is indeed only one footnote in Benassy (1993) with respect to these important subjects of macrodynamics, which (though not mentioning all contributions in this area) nevertheless provides the correct impression that there is not much to say about the non-Walrasian modeling of monetary growth. The explanation for this theoretical deficit in non-Walrasian macrodynamics is not difficult to provide if one looks at the papers in Hénin and Michel (1982) for example: Regime switching scenarios, as they are investigated thoroughly in the non-Walrasian statical analyses of general economic interdependence and spill-over of disequilibria between markets, can be man-

[1] This designation was introduced by Benassy (1977).

aged in the static context. However, they become nearly untractable in the analysis of macroeconomic fluctuations and growth in monetary economies, in particular if the considered dynamics is no longer planar, due to laws of motion for real wages, real money balances, factor endowments, and more.

In this respect the paper by Picard (1983) has made significant progress since it provides a monetary growth model of non-Walrasian type with its typical switches of regimes (Keynesian or classical unemployment and repressed inflation), giving rise to a three-dimensional dynamics in the above named variables, which, however, is not easy to analyze (as the long appendices containing the proofs of Picard's (1983) propositions show).

In Sect. 5.2 we will reconsider a slightly modified version of the original Picard model. Section 5.3 shows for the Picard approach to monetary growth that this analysis—and the model formulation on which it is based—can be considerably simplified if some basic aspects in the formulation of wage-price dynamics in the macrodynamic literature are taken into account, i.e., the facts (for which various rationalizations may be offered) that wages and prices start rising before the level of absolute full employment in the labor market and absolute capacity utilization within firms has been reached. These simple additions to the wage-price module (which are used here as an example solely) suffice to show that the environment of the steady state of such models of monetary growth is completely Keynesian (with no regime switching to classical unemployment or repressed inflation). The dynamics around the steady state is thereby radically simplified and propositions as in Picard (1983) are now easily proved and extended.

With this background the KMG model of Chap. 4 is reconsidered in Sect. 5.4, where now various boundaries concerning the supply side of the economy are included. As will be shown there, however, the explicit modeling of inventories and other buffers like overtime work prevent the economy from leaving the Keynesian regime (where demand is never rationed) for quite a large region around the steady state, although there are several subregimes within the Keynesian one, which have to be regarded. On the other hand, a switch out of the Keynesian regime can occur, when inventories become exhausted. This case and the resulting rationing of demand is also considered in Sect. 5.4 which is closed by a number of simulation runs to investigate the stabilizing and destabilizing effects of different parameter constellations. In this context, a further important nonlinearity in form of a kinked Phillips-curve, which prevents nominal wages from falling when unemployment is high, is taken into account. After a summing up of the main results in Sect. 5.5 the subsequent appendix shows, that one of the earliest contributions to models exhibiting regime switching, namely the one of Solow and Stiglitz (1968) can be interpreted as a special case of the KMG model.

We conclude that macroeconomic applications of the non market-clearing approach are predominantly Keynesian in nature and thus do not need the heavy machinery of nonlinear differential inequalities for most of its propositions.

10.2 A Non-Walrasian Model of Monetary Growth

To demonstrate the working of a typical and already comparatively elaborated model of the non-Walrasian variety, the model of Picard (1983) shall be considered in this section. Picard (1983) distinguishes three commodities and four agents. Since however the banking sector and the rate of interest ascribed to it play no explicit role in the static or dynamic part of the model, it suffices for our purposes to assume as framework the following set of markets and sectors:

The Commodities: goods, labor, money.

The Sectors: households, firms, government.

The adoption of this framework changes the interpretation of the growth model of Picard (1983), but it does so without changing the temporary equilibrium positions nor the assumed laws of motion, but instead provides an interesting alternative view on the contents of these constituent parts of the model.

The Households: Households are represented as a single aggregate agent with their planned consumption given by

$$C = c(1-\tau)Y + dM/p, \quad c \in (0,1), d > 0$$

and an inelastic labor supply L that grows with the natural rate n = const > 0 over time. The income Y (before taxes τY) will be determined later on. The budget constraint of the household sector is

$$C + S = (1-\tau)Y + \dot{M}/p, \quad S = \dot{M}^d/p.$$

The Government: The government sector exhibits the following budget constraint

$$pI + pG = \tau pY + \dot{M},$$

i.e. the state finances real investment I and real public consumption G via real taxes τY and the real change in money supply $\dot{M}/p$. It is assumed that all income of the firms (wages and profits) is distributed to households (and then taxed) and that firms are organizing production as well as formulating investment plans. The amount of public consumption then adjusts so that the above budget constraint is fulfilled.

The Firms: Firms have a linear production technology of the form

$$Y^p = y^p K, \quad y^p = \text{ const} \quad (U_c = Y/Y^p),$$
$$Y = xL^d, \quad x = \text{ const} \quad (V = L^d/L),$$

where as usual Y^p denotes potential output and Y actual output and thus $L^d = Y/x$ actual employment.

In order to determine actual output Y several intermediate concepts for the description of the supply and demand of goods are needed. We denote by $\widetilde{Y}^d$ the unconstrained demand for goods

$$\widetilde{Y}^d = C + I + G.$$

Furthermore

$$\widetilde{L}^d = \min\{\widetilde{Y}^d/x, Y^p/x\}, \quad \check{Y} = \min\{Y^p, xL\}$$

denote the minimum labor demand (when aggregate demand $\widetilde{Y}^d$ and potential output Y^p are taken into account as constraining labor demand) and the minimum production when potential output and the full employment output act as constraints on production [for $\omega = w/p < x$]. The above two magnitudes are called effective labor demand and the effective supply of goods in Picard (1983).

Finally, desired production $\overline{Y}$ is defined by Picard (1983) by

$$\overline{Y} = \min\{\widetilde{Y}^d, xL\}$$

and used as an argument in the investment behavior of firms which in its qualitative form is determined by

$$I/K = i_1 \overline{Y}/K + i_2(x - w/p)$$

as long as real wages do not exceed labor productivity: $w/p < x$ (I/K is zero otherwise). This type of investment function (which allows for positive investment in the face of excess capacity) is justified in Picard (1983) in two different ways.

Note finally that we do not have to specify a budget restraint for firms since all profits are distributed to households and all investment is financed by the government.

Fix Price Equilibria: We have already defined effective demands and supplies on the labor market: $\widetilde{L}^d, L$ as well as on the market for goods: $\widetilde{Y}^d, \check{Y}$. We now assume that realized transactions on each of the two markets are given by the minimum of demand and supply:

$$Y = \min\{\widetilde{Y}^d, \check{Y}\}, \quad L^d = \min\{\widetilde{L}^d, L\} = Y/x.$$

These two magnitudes thus describe actual output and actual employment and are the basis of wage payments and profit transfers. They describe the temporary equilibrium position of the economy as far as output and employment are concerned. This temporary equilibrium position can be of one of the following three types:

$$\begin{aligned} \text{Keynesian unemployment (KU)}: Y &= \widetilde{Y}^d \leq \check{Y} \qquad (\widetilde{L}^d \leq L), \\ \text{Classical unemployment (CU)}: Y &= y^p K \leq \widetilde{Y}^d \qquad (\widetilde{L}^d \leq L), \\ \text{Repressed inflation (RI)}: Y &= xL \leq \widetilde{Y}^d \qquad (L \leq \widetilde{L}^d). \end{aligned}$$

In the Keynesian case there is excess supply in both the labor and the goods market, in the Classical case there is excess demand in the market for goods and excess supply in the market for labor and under repressed inflation there is excess demand in both the market for labor and the market for goods.[2]

Using the intensive form variables $k = K/L$ and $m = M/(pL)$ we can represent the domains of the validity of the three regimes as shown in Fig. 10.1, see Picard (1983, pp. 272ff.) for details.

Under classical unemployment real balances m and thus effective demand are high and capital per unit of labor low, while the opposite is the case under Keynesian unemployment. Repressed inflation occurs in the economy if both magnitudes are high, leading to excess demand for goods as well as for labor.

If one now assumes that firms and the government are never rationed in the market for goods, so that this rationing only concerns the household

[2] The fourth possible combination of such market constellations is not available in the present type of model.

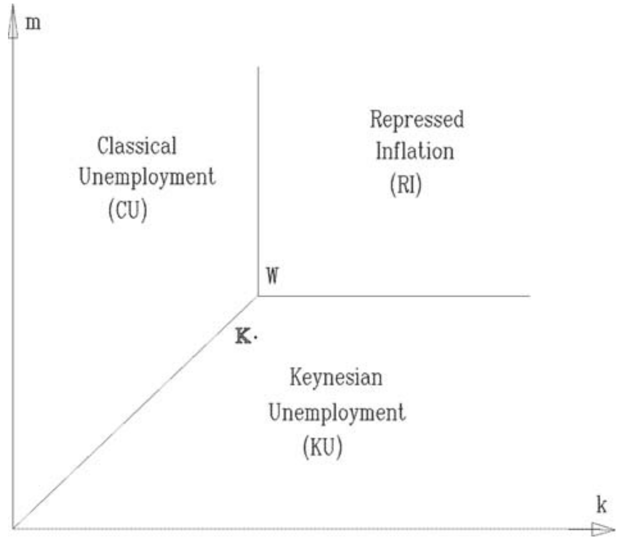

Fig. 10.1. The three regimes in the k, m state space

sector, one can immediately progress from the given description of temporary equilibrium positions to dynamics, since the magnitudes I and G then describe realized magnitudes that can be used in conjunction with Y, L^d (and $\widetilde{L}^d, \widetilde{L}, \check{Y}$) to formulate the laws of motion of such an economy.

The Laws of Motion of the Economy: Picard (1983) here assumes two specific types of Phillips curves, one for money wages w and one for the price level p, based on demand–pull as well as cost–push considerations of the following form ($\omega = w/p$, $\widetilde{\omega}$ the given target real wage of workers):

$$\hat{w} = \beta_{w_1}\left(\frac{\widetilde{L}^d - L}{\widetilde{L}^d}\right) + \beta_{w_2}(\widetilde{\omega} - \omega) + \kappa_w \pi^e, \quad \hat{p} = \beta_p\left(\frac{\widetilde{Y}^d - \check{Y}}{\widetilde{Y}^d}\right) + \kappa_p \pi^e$$

with positive β's as adjustment speeds and both κ_w and κ_p in the interval $[0, 1]$. And for capital stock growth we of course have:

$$\widehat{K} = i_1 \overline{Y}/K + i_2(x - \omega)$$

or $\dot{K} = 0$ for $\omega \geq x$. These three laws of motion imply the following nonlinear autonomous dynamical system in the intensive form variables $\omega = w/p$, $m = M/(pL)$ and $k = K/L$ if we assume as in Picard (1983) that $\mu = \dot{M}/M = \text{const}$ holds and if expected inflation $\pi^e = \mu - n = \widehat{M} - \widehat{L}$ is fixed at the steady state value of the rate inflation $\mu - n$. For simplicity we assume $\mu = n$ in the following:

$$\widehat{\omega} = \beta_{w_1}\left(\frac{\widetilde{L}^d - L}{\widetilde{L}^d}\right) + \beta_{w_2}(\widetilde{\omega} - \omega) - \beta_p\left(\frac{\widetilde{Y}^d - \check{Y}}{\widetilde{Y}^d}\right), \tag{10.1}$$

$$\widehat{m} = -\beta_p\left(\frac{\widetilde{Y}^d - \check{Y}}{\widetilde{Y}^d}\right), \tag{10.2}$$

$$\widehat{k} = i_1\overline{Y}/K + i_2(x - \omega) - n \tag{10.3}$$

with
$$\frac{\widetilde{L}^d - L}{\widetilde{L}^d} = \frac{\widetilde{l}^d - 1}{\widetilde{l}^d}, \qquad \frac{\widetilde{Y}^d - \check{Y}}{\widetilde{Y}^d} = \frac{\widetilde{y}^d - \check{y}}{\widetilde{y}^d}, \qquad \frac{\overline{Y}}{K} = \frac{\overline{y}}{k},$$
where all variables have been transformed to intensive form by dividing through labor supply L.

Since $\check{y} = \min\{y^p k, x\}$, $\overline{y} = \min\{\widetilde{y}^d, x\}$, $\widetilde{l}^d = \min\{\widetilde{y}^d/x,\ y^p k/x\}$ and $y = \min\{\widetilde{y}^d,\ \check{y}\}$, $l^d = y/x$ there remains $\widetilde{y}^d$ to be calculated. In the Keynesian regime one gets for this variable:

$$\widetilde{y}^d = \frac{1}{1 - c(1-\tau) - \tau}(d + \mu)m = f^1(m) \tag{10.4}$$

which can be interpreted as a Keynesian multiplier, while $\widetilde{y}^d$ is given in the Classical regime by

$$\widetilde{y}^d = (c(1-\tau) + \tau)y^p k + (d + \mu)m = f^2(k, m) \tag{10.5}$$

and in the regime of repressed inflation by

$$\widetilde{y}^d = (c(1-\tau) + \tau)x + (d + \mu)m = f^3(m). \tag{10.6}$$

This shows that the above dynamical system can indeed be reduced to the three state variables ω, m and k.

Steady States and Stability: We define $\overline{\omega}$ by the condition

$$I/K = i_1 y^p + i_2(x - \overline{\omega}) = n, \quad \overline{\omega} < x$$

i.e. the wage rate that equalizes capital stock growth with natural growth at (desired) full capacity growth ($i_1 y^p < n$!). With respect to this expression the following proposition then holds, see Picard (1983, p. 278):

Proposition 10.1. *If $\widetilde{\omega}$, the target real wage, belongs to an appropriately chosen neighborhood U_1 of $\overline{\omega}$, the above dynamical system has a unique stationary point (ω_0, m_0, k_0) which coincides with the Walrasian equilibrium $(\overline{\omega}, W)$, see Fig. 10.1, when $\widetilde{\omega} = \overline{\omega}$. Furthermore, when $\widetilde{\omega} > \overline{\omega}$ $[\widetilde{\omega} < \overline{\omega}]$ we have for the steady state (m_0, k_0) in the projected phase plane of Fig. 10.1: $(m_0, k_0) \in KU \cap CU$ $[(m_0, k_0) \in KU \cap RI]$, respectively.*

Proof of Proposition 10.1. (For $\overline{\omega} = \widetilde{\omega}$):

1. $\dot{m} = 0 \quad (m \neq 0)$ implies $\widetilde{Y}^d = \check{Y} = \min\{Y^p, xL\}$.
2. In the case $\widetilde{Y}^d = Y^p \leq xL$ we then get $\overline{Y} = Y^p = \widetilde{Y}^d$, i.e., $\omega_0 = \widetilde{\omega} = \overline{\omega}$. Therefore: $L = \min\{\widetilde{Y}^d/x, Y^p/x\}$ because of $\dot{\omega} = 0$ $(\omega \neq 0)$, i.e., $\widetilde{Y}^d = xL$, i.e. we are in the situation W of Fig. 10.1, the Walrasian general equilibrium.
   We then get from the expressions for $\widetilde{y}^d$ first
   $$x = \widetilde{y}^d_0 = \frac{1}{1 - c(1-\tau) - \tau}(d + \mu)m_0,$$
   the steady state value of real balances m_0, and from the expression for $\widetilde{y}^d$ of the classical equilibrium
   $$x = (c(1-\tau) + \tau)y^p k_0 + (d + \mu)m_0,$$
   the steady state value of the capital intensity k_0.
3. In the case $\widetilde{Y}^d = xL = \bar{Y} \leq Y^p$ we, on the other hand, have that we are on the borderline between KU and RI, so that $\widetilde{y}^d = x \leq y^p$ must hold true. Therefore:
   $$m_0 = (1 - c(1-\tau) - \tau)x/(d + \mu)$$
   and $\omega_o \leq \bar{\omega} = \widetilde{\omega}$ due to $\dot{k} = 0$ $(k \neq 0)$. We thus have $\beta_{w_2}(\widetilde{\omega} - \omega) > 0$ and due to $\dot{\omega} = 0$: $\widetilde{L}^d \leq L$ or $\min\{\widetilde{Y}^d/x, Y^p/x\} \leq L$ which again implies $\widetilde{Y}^d = Y^p = xL$. The remaining calculations are then as in the preceding case. ■

There are further steady state calculations in Picard (1983) and also one proposition on the local asymptotic stability of steady states in situations of interior steady states (where no minimum operator is operative, so that the dynamical system is then not only continuous, but also continuously differentiable). Since we are, however, in this chapter concerned mainly with the structure of non-Walrasian monetary growth models and not so much with the results that can be obtained from them, we can stop at this point.

Summing up the preceding presentation of this type of disequilibrium monetary growth theory we can state that

- its behavioral equations are simpler, but not unrelated to the Keynes–Metzler model discussed in Chiarella and Flaschel (2000, Chap. 6) (with a stabilizing Pigou-effect in place of the stabilizing Keynes-effect of the latter model)

- it describes a situation that is closer to a description of a planned than of a market economy
- its richness of implication is based on the numerous regimes it allows for (in fact there are two further subregimes in each of three regimes we have considered in this section)
- it represents a complete model of monetary growth with a very complicated dynamical structure due to the various differential inequalities that have to be considered in general.

In view of this we shall demonstrate in the following section that a much richer structural form of Keynesian monetary growth (which is much closer to the description of a market economy than the model of this section) will be much simpler to treat from the viewpoint of dynamical systems (steady state and stability analysis) due to the fact that this flexible model of monetary growth of a market economy rarely undergoes switching of regimes and even much less often will allow for the establishment of Classical unemployment or repressed inflation in the sense of a rationing of the aggregate demand $\widetilde{Y}^d = C + I + G$. Therefore much of the effort that has gone into the analysis of laws of motions based on differential inequalities can simply be avoided by paying attention to the fact that market economies have a variety of mechanisms and flexibilities that allow them to avoid the rationing of consumers, or investors or the government on the macroeconomic level.

10.3 From Non-Walrasian to Keynesian Modeling of Monetary Growth

The most basic critique of the non-Walrasian dynamics of Sect. 9.2 is that its two Phillips-curves for money wages and the price level are misspecified with respect to the actual working of market economies. This is due to their neglect of NAIRU levels of rates of employment and also of rates of capacity utilization which may be difficult to rationalize from a microeconomic perspective, but which are surely relevant on the macroeconomic level.[3]

Whatever motivation is offered for the NAIRU rate of employment $\bar{V}$, it is generally agreed that money wages are subject to an upward pressure before everybody in the workforce is employed. The obvious and necessary change

[3] See also Flaschel (1999a) with respect to the following reformulation of Non-Walrasian rationing regimes.

in the money wage PC thus is

$$\widehat{w} = \beta_w \left(\frac{\tilde{\bar{L}}^d - \bar{V}L}{\tilde{\bar{L}}^d} \right) + \beta_{w_2}(\widetilde{\omega} - \omega), \quad \bar{V} \in (0,1), \tilde{\bar{L}}^d = \min\{\widetilde{Y}^d/x, \bar{U}_c Y^p/x\}.$$

Likewise, the price level starts rising before either the capacity constraint or the labor supply constraint becomes binding, i.e., the law for price dynamics should be modified as follows

$$\widehat{p} = \beta_p \left(\frac{\widetilde{Y}^d - \check{\check{Y}}}{\widetilde{Y}^d} \right), \quad \check{\check{Y}} = \min\{\bar{U}_c Y^p, \bar{V}Lx\}, \quad \bar{U}_c \in (0,1),$$

where $\bar{U}_c$ represents a normal degree of capacity utilization below 100%. Finally, firms should now use $\bar{\bar{Y}} = \min\{\widetilde{Y}^d, \bar{V}Lx\}$ in the place of $\bar{Y} = \min\{\widetilde{Y}^d, Lx\}$ as the capacity constraint for their investment decisions which gives $\widehat{k} = i_1(\bar{\bar{Y}}/K) + i_2(x - \omega) - n$ as the third law of motion.

Up to the use of $\check{\check{Y}}, \bar{\bar{Y}}, \bar{V}L$ in the place of $\check{Y}, \bar{Y}$ and L in the dynamical system (10.1)–(10.3) the model is the same as before.

Proposition 10.2. *The unique interior steady state of the revised dynamical system*

$$\widehat{\omega} = \beta_{w_1} \left(\frac{\widetilde{L}^d - L\bar{V}}{\widetilde{L}^d} \right) + \beta_{w_2} (\widetilde{\omega} - \omega) - \beta_p \left(\frac{\widetilde{Y}^d - \check{\check{Y}}}{\widetilde{Y}^d} \right), \tag{10.7}$$

$$\widehat{m} = -\beta_p \left(\frac{\widetilde{Y}^d - \check{\check{Y}}}{\widetilde{Y}^d} \right), \tag{10.8}$$

$$\widehat{k} = i_1 \left(\bar{\bar{Y}}/K \right) + i_2 (x - \omega) - n \tag{10.9}$$

is given by

$$\omega_0 = \widetilde{\omega}, \quad m_0 = (1 - c(1-\tau) - \tau)x\bar{V}/(d+\mu), \quad k_0 = \frac{x\bar{V}}{y^p \bar{U}_c},$$

if we assume that $\widetilde{\omega} = \bar{\omega}$ *holds,* $\bar{\omega}$ *given by* $i_1 \bar{U}_c y^p + i_2(x - \bar{\omega}) = n$. *At this steady state we have*

$$\widetilde{Y}^d = \bar{U}_c Y^p = \bar{V}Lx < \min\{Y^p, xL\},$$

i.e., this steady state K, see Fig. 10.1, belongs to the region of Keynesian unemployment (which also holds true for all steady states belonging to a target real wage $\widetilde{\omega}$ *in a neighborhood of* $\bar{\omega}$*).*

Proof. The proof of this proposition is based on the following simple observations:

1. $\dot{m} = 0 \quad (m \neq 0)$ implies $\widetilde{Y}^d = \check{\check{Y}} = \min\{\bar{U}_c Y^p, \bar{V} L x\}$.
2. In the case $\widetilde{Y}^d = \bar{U}_c Y^p \leq \bar{V} L x$ we then get $\bar{\bar{Y}} = \bar{U}_c Y^p = \widetilde{Y}^d$, i.e., $\omega_0 = \bar{\omega} = \widetilde{\omega}$. Therefore: $\bar{V} L = \min\{\widetilde{Y}^d/x, Y^p/x\}$ because of $\dot{\omega} = 0 \quad (\omega \neq 0)$, i.e., $\widetilde{Y}^d = x\bar{V}L$ or in sum
$$\widetilde{Y}^d = x\bar{V}L = \bar{U}_c Y^p.$$
Since we are thus always in the Keynesian regime we have
$$\widetilde{y}^d = x\bar{V} = \frac{1}{1 - c(1-\tau) - \tau}(d + \mu) = m_0$$
as equation for m_0 and
$$x\bar{V} = \bar{U}_c y^p k_0$$
as equation for k_0 $[= x\bar{V}/(\bar{U}_c y^p)]$.
3. In the other case $\widetilde{Y}^d = \bar{V} L x = \bar{\bar{Y}} \leq \bar{U}_c Y^p$ (see 1.), we have $\widetilde{y}^d = x\bar{V} \leq \bar{U}_c y^p$. We therefore again get $\omega_0 \leq \bar{\omega}$ and thus $\widetilde{L}^d \leq L\bar{V}$ or $\min\{\widetilde{Y}^d/x, \bar{U}_c Y^p/x\} \leq L\bar{V}$. Thus
$$\widetilde{y}^d = x\bar{V} = \bar{U}_c y^p.$$
Therefore $\omega_0 = \bar{\omega}$ and m_0, k_0 are determined as in the preceding case 2. ■

We thus get that $Y = \widetilde{Y}^d$ holds always in the neighborhood of the steady state and $L^d = \widetilde{Y}^d/x$, i.e., employment is always demand determined sufficiently close to the steady state. The KU-regime is therefore the only relevant one at least in the vicinity of the steady state solution of the dynamical system (10.7)–(10.9).

Remark. This result on the dominance of the Keynesian regime can be made much stronger if overtime work of insiders, smooth factor substitution, excessive production (with respect to the profit maximizing output) in order to satisfy customers' demand and inventories are taken into account as in Flaschel (1999a, 1999b).

Instead of pursuing this line of approach further, let us reexamine here the dynamical laws for $\widehat{w}$ and $\widehat{p}$ with respect to their meaningfulness. Since we have the Keynesian demand regime $(Y = \widetilde{Y}^d)$ close to the steady state we get for $\widetilde{\widetilde{L}}^d/L$ the expression $\min\{V, \bar{U}_c y^p k/x\}$ with $V = L^d/L = Y/(xL)$ the actual rate of employment. But how does the expression $\bar{U}_c y^p k/x$ influence money wage dynamics as proposed by the expression

$$\beta_{w_1}\left(\frac{\widetilde{\widetilde{L}}^d - \bar{V}L}{\widetilde{\widetilde{L}}^d}\right)$$

in the above money wage Phillips-curve? Furthermore, why this choice of a denominator? In our view it is sufficient to use (as is customary):

$$\beta_{w_1}\left(\frac{V-\bar{V}}{\bar{V}}\right) = \beta_{w_1}\left(V/\bar{V}-1\right), V = L^d/L$$

in place of the above expression in order to describe (the demand pull component of) the dynamics of money-wages. Similarly (because of $\widetilde{Y}^d = Y$):

$$\beta_p\left(\frac{\widetilde{Y}^d - \min\{\bar{U}_c Y^p, \bar{V}Lx\}}{\widetilde{Y}^d}\right) = \beta_p\left(\frac{U_c - \min\{\bar{U}_c, \bar{V}x/(y^p k)\}}{U_c}\right).$$

But why $\bar{V}x/(y^p k)$ and U_c in the denominator of this expression? Again the term

$$\beta_p\left(\frac{U_c - \bar{U}_c}{\bar{U}_c}\right) = \beta_p\left(\frac{U_c}{\bar{U}_c} - 1\right)$$

is fully sufficient to express the demand pull component, now in the market for goods. Taken together, the dynamical system (10.7)–(10.9) should therefore be rewritten as

$$\widehat{\omega} = \beta_{w_1}\left(V/\bar{V}-1\right) + \beta_{w_2}\left(\widetilde{\omega}-\omega\right) - \beta_p\left(U_c/\bar{U}_c - 1\right), \tag{10.10}$$

$$\widehat{m} = -\beta_p\left(U_c/\bar{U}_c - 1\right), \tag{10.11}$$

$$\widehat{k} = i_1 y_p U_c + i_2\left(x-\omega\right) \tag{10.12}$$

since $Y^d = Y, L^d = Y/x$ are the relevant expressions for the actual position of the economy, below or above $\bar{V}L$ and $\bar{U}_c Y^p$, but below L and Y^p.

Remark. The interior steady-state of the system (10.10)–(10.12) is the same as for the system (10.7)–(10.9).

Proposition 10.3.

1. *The steady-state of the dynamical system* (10.10)–(10.12) *is locally asymptotically stable if* $\beta_{w_1} < \beta_p$ *holds true.*[4]

[4] This assertion is similar to one in Picard (1983, p. 279), but does not allow for alternative regimes in the present framework.

2. *The steady-state of the dynamical system* (10.10)–(10.12) *loses its stability in a cyclical fashion at the unique Hopf-bifurcation point:*

$$\beta_{w_1}^H = \frac{a_1 a_2}{\beta_p i_2 (V'/\bar{V})(-U_{ck}/\bar{U}_c)\omega_o m_o k_o}$$

through the birth of a stable limit cycle or the death of a stable corridor.[5]

Proof.

1. Due to the prevalence of the Keynesian regime around the steady-state we have

$$\widetilde{y}^d(m) = y = \frac{1}{1 - c(1-\tau) - \tau}(d+\mu)m$$

for $V = L^d/L = Y/(xL) = y/x$ and $U_c = Y/Y^p = y/(y^p k)$. Hence, $V = V(m), V' > 0$ and $U_c = U_c(m,k)$, $U_{cm} > 0, U_{ck} < 0$.
According to the Routh–Hurwitz conditions, see Benhabib and Miyao (1981), one has to show

$$a_1 = -\text{trace}\, J > 0, \quad a_3 = -\det J > 0, \quad a_1 a_2 - a_3 > 0 \quad (a_2 > 0)$$

where J is the Jacobian of the above dynamical system at the steady-state and where a_2 is given by the sum of principal minors of this matrix.
It is easy to show that $\det J < 0$ must hold, since linearly dependent expressions can be removed from J without altering its determinant which simplifies the calculation of this determinant significantly. Thus:

$$a_3 = -\det J = \beta_{w_1}\beta_p i_2 (V'/\bar{V})(-U_{ck}/\bar{U}_c)\omega_o m_o k_o > 0.$$

Quite obviously, also

$$a_1 = -\text{trace}\, J = \beta_{w_2}\omega_0 + \beta_p (U_{cm}/\bar{U}_c) m_0 + i_1 y_p (-U_{ck}) k_0 > 0.$$

Furthermore, we also immediately get:

$$a_2 = \beta_{w_2}\beta_p (U_{cm}/\bar{U}_c)\omega_o m_o + \beta_{w_2} i_2 y^p (-U_{ck})\omega_o k_o + \beta_p i_2 (-U_{ck}/\bar{U}_c)\omega_o k_o$$

Due to $V(m) = \tilde{y}^d(m)/x, U_c(m,k) = \tilde{y}^d(m)/(y^p k)$ we finally get (by setting the positive β_{w_2}-terms all equal to zero)

$$a_1 a_2 - a_3 > \beta_p i_2 (-U_c/\bar{U}_c)\omega_o m_o k_o \tilde{y}^{d'}(m_o)(\beta_p - \beta_{w_1}),$$

since $i_1 y^p(-U_{ck})k_o > 0$ and $y^p \bar{U}_c k_o = x\bar{V} \quad [\bar{U}_c Y^p = x\bar{V}L]$ at the steady state.

[5] See the proof for the expressions that define $a_1 a_2$.

2. Since the parameter β_{w_1} only appears in the determinant of the Jacobian J the calculation of the Hopf-bifurcation point is an easy task, since it is characterized by $b = a_1a_2 - a_3 = 0$, the only stability condition which can change its sign in the present situation. Furthermore, the value of b is a linear (negatively sloped) function of the parameter β_{w_1} which implies as in Benhabib and Miyao (1981) that the eigenvalues cross the imaginary axis with positive speed and thus allows the application of the Hopf-bifurcation theorem. ■

We thus have that price flexibility that is larger (with respect to its demand pull component) than wage flexibility (with respect to the demand pull component) is good for economic stability, which is not too surprising due to the assumed Pigou- or real balance-effect on aggregate demand. Furthermore, this stability is increased through increases in the parameter β_{w_2}, since this adjustment parameter only appears in a_1a_2 and there always with positive signs as the above calculations have shown. We therefore have definite reasons to expect that the local asymptotic stability result holds also for $\beta_p \ll \beta_{w_1}$ if (e.g.) β_{w_2} is chosen sufficiently high. Nevertheless, there is a limit to this stability result if the parameter β_{w_1} is made sufficiently large (all others held constant), where the stability of the system gets lost in a cyclical fashion.

We conclude that the non-Walrasian approach to monetary growth of Sect. 5.2 can be made a (very) special case of our general Keynes-Metzler model with only minor or—if less secondary—very improbable possibilities of a change in the generally Keynesian regime that governs its evolution. Regime switches and rationing arbitrarily close or even less close to the steady state of market economies is neither empirically plausible nor analytically convincing, since it is based on too strict inequalities in contrast to the many flexible adjustment procedures that are imaginable for developed market economies.

Thus, for example, the situation of repressed inflation will only happen far off the steady state if overtime work of the workforce of firms is taken into account, see Sect. 5.1. The classical regime furthermore is much less likely if account is taken of smooth factor substitution and of the fact that firms will temporarily serve their customers in a Keynesian environment (where firms are not price-takers), beyond the point where prices equal marginal costs—should their inventories be exhausted, see Sect. 5.3. The barriers to serving aggregate demand for goods are thus much less rigid than assumed in non-Walrasian macroeconomics which invests high technical competence in analyzing complicated growth dynamics which have not much in common with the macrodynamics of market economies. Reworking their structural equations in view of this not only will improve the model's relevance, but also simplifies its analysis radically as we hope to have shown in this chapter.

These results are in line with Benassy's (1984) model of a Keynesian limit cycle, where also an appropriate form of the Phillips-curve, that led (nominal) wages converge to infinity as the employment rate converged to one, ensured that the dynamics could not leave the boundaries of the Keynesian regime. The same is valid for the KMG model of the preceding chapter, which will be further extended in this regard in the following with the consequence, that switches away from the Keynesian regime, though possible in principle, will not occur for a large range of parameter values.

In general, we assume that the model's dynamics stays in domains where $\rho^e, M^d, I+\delta K, (1-\tau_c)r-\pi^e$ and all stock variables remain positive and where moreover the share of wages $u = \omega L^d/Y$ remains below 1. To ensure these side conditions it may be necessary to introduce further and extrinsic nonlinearities into our dynamic model which so far rests on unavoidable or intrinsic nonlinearities solely. These are the only side-conditions for the variables of the model that have to be checked in this regard, since all other variables automatically remain restricted to economically meaningful values in finite time. We return to these side-conditions later on in this chapter.[6]

Non-Walrasian theory made us aware of the fact that all side-conditions of models of fluctuating growth must be specified and proved to hold but it vastly overstated the importance and actual relevance of these side conditions for the ordinary working of market economies.

10.4 Regime Switching in KMG Growth

In the following we now want to apply the Non-Walrasian methodology just discussed to the KMG model developed in Chiarella et al. (2000, Chap. 3) in order to describe possible regime switches also in this context and to evaluate their relevance. As will be shown, supply bottlenecks can indeed occur in situations, when inventories become exhausted. On the other hand, the various

[6] Note here however, that the conditions $\rho^e > 0, u < 1$ can be ensured simultaneously in the following completions of the above version of the model (with smooth factor substitution) when there is no difference between expected demand Y^e and actual demand Y^d and when intended inventories are neglected as in the usual IS-LM approach of the traditional Keynesian macrodynamics.

buffers available in the economy as well as the fact mentioned above, that wages begin to rise already before full employment is reached, will make such situations quite exceptional and let them appear only far off the steady state. The next subsection shows, that as a consequence of the introduction of two upper bounds for production some changes due to the supply side already take place within the Keynesian regime without however changing its main characteristic, i.e. the full satisfaction of aggregate demand. Thus, even if actual production falls short of demand, inventories will prevent demand from being rationed. This constellation will not change before inventories become exhausted, a situation which is then analyzed in Sect. 5.4.2. A numerical analysis of different scenarios of the whole model (with and without rationing) will be considered at the end of this section; there, the model will be enriched by a further extrinsic nonlinearity, i.e. a kinked Phillips-curve for nominal wages, which will have the effect of stabilizing the economy even for such parameter constellations, which would otherwise lead to explosiveness and regime switching.

10.4.1 Supply Bottlenecks with Positive Inventories

In this subsection we consider the reference KMG growth model of the preceding chapter, now for sluggishly adjusting prices, wages and quantities throughout, and as basis for a general analysis of the Non-Walrasian regime switching approach to economic growth, which we presented in Sect. 5.2 in its basic format. Since it is of importance for the evaluation of the role of supply side bottlenecks (classical regimes or regimes of repressed inflation) we shall include into the KMG model neoclassical smooth factor substitution, overtime work of the employed coupled with a sluggish employment policy of firms, and also endogenous natural growth, building on the presentation of these additions considered in Chap. 4, Sects. 4.3 and 4.7. However, in order not to overload the presentation we shall not consider here technical change (neither endogenous nor exogenous one), since this has been treated extensively in the preceding chapter and is not of central interest in the consideration of the relevance of supply side constraints, which are in the center of interest of the present chapter. This section therefore marries aspects of disequilibrium KMG growth with the classical viability investigation of Sect. 4.7, but leaves the integration of endogenous technical change for later reconsiderations of the employed model.

Before we start with the presentation of the model we have to define the two (here fairly soft) upper bounds for the productive activities of firms (arising from labor supply and profitability considerations) that are the basis of the regime switching approach in the KMG framework considered below. Such bounds are here introduced in a way as simple as possible in order to demonstrate in a basic way that there exist important buffers created by the behavior of firms that generally allow to avoid the rationing of *aggregate* goods demand in market economies even for larger deviations from their steady state growth path. These assumed bounds to production can be described in formal terms as follows:

$$\bar{Y}^w = F(K, (1+\bar{o})L^w) > 0 \quad \text{if } L^w > 0, \tag{10.13}$$

$$\bar{Y}^p = F(K, \bar{L}^p) = \omega \bar{L}^p + \delta K > 0. \tag{10.14}$$

Note that there is a unique solution $\bar{L}^p$ to the second condition for neoclassical production functions with the usual properties.

Equation (10.13) states (in slightly more general terms than in Sect. 4.7) that there is a fixed limit to overtime work supplied by the employed workforce L^w of firms, given by the ratio term $\bar{o}$, which may be justified by legal restraints for example, but which in the end should of course be modeled as a flexible barrier. Normal hours worked, L^w, can therefore be at most augmented by $100 \cdot \bar{o}$ percent of overtime work of the workforce L^w of firms, where the latter magnitude is fixed at each moment in time, since the employment of additional workers follows the path of overtime work only with a time delay. This is a limitation on labor supply within firms which already indicates that the regime of repressed inflation, in the form it was described in Sect. 5.2, will here only be established as a hard constraint in periods of heavily booming economies.

Equation (10.14) adds the constraint that arises from the current size of the capital stock, which when operative gives rise to the so-called classical regime of Non-Walrasian economics. We here go to the extreme that firms extend their production in periods of high aggregate goods demand up to the point where profits become zero, i.e., where the real wage cost line ωL^d (augmented by capital stock depreciation δK) intersects the production function for a given value of K. Again this is only a first example for such an economic limit to the output of firms, which from the technological perspective could even be infinite in the case of a standard neoclassical production function. Non-Walrasian economics generally assumes that this limit is given by the profit maximizing output of firms and thus is very narrowly determined. However, from the perspective of descriptive macrodynamics, it is acceptable to assume that firms will go beyond the point where given output prices equal marginal wage costs by a variety of reasons that have to do with market shares, customer satisfaction and the like. Of course, the behavior of firms, in particular their pricing decision, may change significantly (in a nonlinear way) when the profit maximizing output is crossed from below. Such change in behavior is however not yet considered here, due to our general methodological approach of considering such behavioral nonlinearities only after the basic form of the model has been developed and investigated.

Equation (10.14) may of course be based also on any other magnitude between Y^p, the profit maximizing output at current wages and prices, and $\bar{Y}^p$ the zero profit output, that can be supplied as a systematic equation based on profitability considerations internal to the firm or external profitability comparisons. A possible procedure could be to assume that employment of the actual labor force is at most extended to the level where firms would use their potential (= profit-maximizing) output Y^p just for paying this employment of the presently employed workforce (which at this point produce more than just Y^p). Yet, in order to be at first as generous as possible regarding the production decisions of firms we will stick to the above very soft constraint on the behavior of firms. In order to defend their market shares and to satisfy their customers as far as possible it is thus assumed in the following that firms expand production beyond the point where prices are equalized to marginal wage costs up to the situation where prices equal average wage costs (based on net output). Such situations indeed characterize actual firm behavior in booming economies. The only question here is the determination of the limit to this type of behavior which of course must be less than ∞ since total profits would be $-\infty$ then.

The following set of equations describes the working of the economy in situations for all cases of constrained production of firms where inventories are positive and where aggregate goods demand can therefore be fulfilled by firms and is indeed fulfilled by assumption. Note, however, that there are now several subregimes within the Keynesian regime. So, e.g., there is a region, where actual production suffices to serve demand, so that no reduction of inventories is required for this purpose. On the other hand, there is a subregime where exactly this is necessary, because production is limited for some reason, i.e. there is an upper bound to production due to $Y^e + \mathcal{I}$, $\bar{Y}^p$ or $\bar{Y}^w$, so that demand can only be satisfied by selling additional goods from inventories; note, that the three possible boundaries to production just mentioned also provide a basis for a classification of further subregimes. The case, however, where no inventories are available any more to fill the gap between production and demand, represents another main regime, which will be analyzed in the next subsection. At this stage, we only want to point to the fact, that dividend payments now depend on the respective subregime and approach their lower bound zero in cases, where production costs of Y^e and $\mathcal{I}$ exceed expected (real) revenues Y^e of firms.

1. Definitions (remunerations and wealth):

$$\omega = w/p, \quad \tilde{\rho}^e = \max\{(Y^e - \delta K - \omega L^d)/K, 0\}, \tag{10.15}$$

$$W = (M + B + p_e E)/p, \quad p_b = 1. \tag{10.16}$$

2. Households (workers and asset-holders):

$$W = (M^d + B^d + p_e E^d)/p, \tag{10.17}$$

$$C = \omega L^d - T_w + (1 - s_c)[\tilde{\rho}^e K + rB/p - T_c], \quad s_w = 0, \tag{10.18}$$

$$T = T_w + T_c, \tag{10.19}$$

$$S_p = \omega L^d + \tilde{\rho}^e K + rB/p - T - C = s_c[\tilde{\rho}^e K + rB/p - T_c]$$
$$= (\dot{M} + \dot{B} + p_e \dot{E})/p, \tag{10.20}$$

$$\hat{L} = n = \bar{n}_o + n_1(V^w - 1) + n_2(V - \bar{V}), n_1, n_2 > 0. \tag{10.21}$$

3. Firms (production-units and investors):

$$\omega = F_L(K, L^p), \ Y^p = F(K, L^p), \ \rho^p = (Y^p - \delta K - \omega L^p)/K, \tag{10.22}$$

$$U_c = Y/Y^p, \quad U_c^e = (Y^e + \mathcal{I})/Y^p, \tag{10.23}$$

$$L^d \quad \text{via } Y = F(K, L^d), \ V = L^w/L \leq 1, \ V^w = L^d/L^w, \tag{10.24}$$

$$\dot{L}^w = \beta_v(L^d - L^w) + \gamma L^w \gamma = n, \tag{10.25}$$

$$I/K = i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) + \gamma, \ \gamma = n, \ I + \delta K \geq 0, \tag{10.26}$$

$$S_f = Y - \tilde{\rho}^e K - \delta K - \omega L^d = I + \dot{N} - p_e \dot{E}/p, \tag{10.27}$$

$$I = Y^d - \max\{Y^e, \delta K + \omega L^d\} + p_e \dot{E}/p, \tag{10.28}$$

$$I^a = I + \dot{N}, \tag{10.29}$$

$$\hat{K} = I/K. \tag{10.30}$$

4. Government (fiscal and monetary authority):

$$T = T_w + T_c,\ T_w = \tau_w \omega L^d,\ t_c^n = (T_c - rB/p)/K = \text{const} < 0, \quad (10.31)$$

$$G = \tau_w \omega L^d + \bar{g}K,\ \bar{g} \geq 0, \quad (10.32)$$

$$S_g = T - rB/p - G\ \ [= (t_c^n - \bar{g})K = -(\dot{M} + \dot{B})/p], \quad (10.33)$$

$$\hat{M} = \bar{\mu} = n, \quad (10.34)$$

$$\dot{B} = pG + rB - pT - \dot{M}. \quad (10.35)$$

5. Equilibrium Conditions (asset-markets):

$$M = M^d = h_1 pY + h_2 pK(r_o - r),\ \ [B = B^d, E = E^d], \quad (10.36)$$

$$B, E \geq 0,\ r > 0,$$

$$r = \frac{\tilde{\rho}^e pK + \dot{p}_e E}{p_e E}. \quad (10.37)$$

6. Disequilibrium Situation (goods-market adjustments):

$$S = S_p + S_g + S_f = Y - \delta K - (C + G) = I + \dot{N} = I^a, \quad (10.38)$$

$$Y^d = C + I + \delta K + G, \quad (10.39)$$

$$\dot{Y}^e = \beta_{y^e}(Y^d - Y^e) + \gamma Y^e,\ \gamma = n, \quad (10.40)$$

$$N^d = \beta_{n^d} Y^e, \quad (10.41)$$

$$\mathcal{I} = \beta_n(N^d - N) + \gamma N^d,\ \gamma = n, \quad (10.42)$$

$$Y = \min\{Y^e + \mathcal{I}, \bar{Y}^p, \bar{Y}^w\}, \quad Y^e + \mathcal{I} > 0, \quad (10.43)$$

$$\dot{N} = Y - Y^d, \quad N > 0. \quad (10.44)$$

7. Wage-Price-Sector (adjustment equations):

$$\hat{w} = \beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^e, \quad (10.45)$$

$$\hat{p} = \beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2} U_n + \kappa_p \hat{w} + (1 - \kappa_p)\pi^e, \quad (10.46)$$

$$U_n = (N^d - N)/K,$$

$$\dot{\pi}^e = \beta_{\pi_1^e}(\hat{p} - \pi^e) + \beta_{\pi_2^e}(\bar{\mu} - n - \pi^e). \quad (10.47)$$

Note that the two assumed limits to production depend on the current state (variables) of the economy and are only needed in (10.43) of the model. Note further that aggregate demand Y^d is always served from production or from inventories $N > 0$ in the presently considered case. However, there are further changes necessary to the model concerning the dividend policy of firms, the choice of a measure of capacity utilization directing investment and pricing decisions of firms, and a few further changes concerning in particular investment and asset market behavior. All these changes will now be justified in detail by going through the above equations of the model step by step.

Concerning block 1. of the model we have as necessary change here, as compared to Sects. 4.2, 4.3 and 4.7 of the preceding chapter, that the rate of return used to characterize dividend payments to asset holders must of course always stay nonnegative, since there are no negative dividend payments. This constraint has been neglected so far due to the local character of the analysis supplied in the preceding chapter. Dividend payments are still based here on expected sales and current production costs (which generally include the production of inventories), that is on the expected level for the current cash flow of firms. Due to the assumed positive level of inventories expected sales can always be fulfilled by firms and thus can still be assumed as basis of their dividend policy, up to, for example, the exceptional case where

$$Y^e < \bar{Y}^p < \min\{\bar{Y}^w, Y^e + \mathcal{I}\}$$

holds or more generally where inventory production is so large that actual production is so much higher than expected sales that expected sales do not cover production costs. These examples show that the situation $\tilde{\rho}^e = 0$ is not likely to be met in simulation runs of the dynamics implied by the model. Of course this completed characterization of the dividend payments of firms has now to be applied when the income of asset holders, underlying their consumption and savings decision, is specified.

Considering next block 3 of the model, the sector of firms as far as their employment and investment decisions are concerned, we have to introduce specific measures now which underlie their investment decision and which modify the investment function to some extent, compared to the one of Chap. 4. We define in (10.22) again potential output and employment, Y^p, L^p, by the profit maximizing levels and now also consider the rate of return ρ^p that corresponds to these levels (disregarding inventory production here).[7] Beyond this rate of return, rates of profits are falling. Excessive and still further increasing expected demand Y^e therefore would lead in such cases via falling rates of return

[7] Making use of $\tilde{\rho}^p = y^p\bar{U}_c/(1+\gamma\beta_{n^d})-\delta-\omega l^p$ would be more appropriate here, since we have so far neglected inventory holdings and excess capacity in rate of return calculations, see the definition of the rate ρ^e in Chap. 4. We will circumvent this problem in the following investigation of the model, by assuming $\bar{U}_c = 1, \gamma = 0$ for reasons of simplicity.

to reductions in investment if this rate would determine the relative profitability measure so far used in the investment function. We therefore propose now to base this measure on the target rate of profit of firms, which for given wages and prices is measured by ρ^p.[8] Furthermore we have two measures of capacity utilization now, the actual one, U_c, and the one that is based on desired output levels of firms derived from their sales expectations and intended inventory changes. We consider the latter measure as the more appropriate one for representing demand pressure and use it for the investment and the pricing decisions of firms. Actual employment L^d is determined by actual production Y which is defined in (10.43) based on the three constraints that firms face with respect to goods demand, labor supply and a profitable use of the capital stock in existence. Hiring and firing of workers is again based on the comparison of actual hours worked with normal working conditions L^w and is thus not made dependent on expected sales in a direct way as in the case of investment.

The budget constraint of firms now reads $Y^d - \max\{Y^e, \delta K + \omega L^d\} + p_e\dot{E}/p = I$ where the first term denotes the income of firms based on their wage payments and their dividend policy and which is of the type of windfall profits and the second term represents again the issuing of new equities in order to close the gap between this income and the intended fixed business investment expenditures of firms. Note that this budget equation can be rewritten as

$$\begin{aligned} Y - & \max\{Y^e, \delta K + \omega L^d\} + p_e\dot{E}/p \\ & = Y - Y^d + Y^d - \max\{Y^e, \delta K + \omega L^d\} + p_e\dot{E}/p \\ & = I + \dot{N} \end{aligned}$$

which represents an accounting budget equation of firms that now also includes actual inventory changes $\dot{N}$ as the other factual investment activity of firms.

There is no change in the government sector of the model which again is played down as much as possible, since we want to concentrate on the private sector in this part of the book. The asset market equilibrium is also the same as before (see (10.37)). Note, that the capital gains captured by the term $\dot{p}_e E$ ensure, that the assumption of perfect substitutability between equities and

[8] Inventories, though a necessity, are considered as a secondary issue and thus neglected in this expression, i.e., are not added to the capital stock of firms when their desired or other rates of profits are calculated.

bonds can be maintained also in such cases, where dividend payments are zero.[9]

We have already taken notice of the production decision of firms which is given by the minimum of intended production $Y^e + \mathcal{I}$, of maximum production $\bar{Y}^w$ of the workforce employed by firms, and of $\bar{Y}^p$ zero profit output as the absolute limit to firms willingness to supply their customers with goods in the case of a booming economy. Up to this modification, the inventory adjustment mechanism remains the same as before, since aggregate demand is never rationed in the present subdynamics of the complete model where actual inventory changes are of course again given by the excess of actual production Y over actual demand Y^d. Finally, the pricing decision is now based on two measures for disequilibrium within firms, the deviation of desired output $Y^e + \mathcal{I}$ from the normal level $\bar{U}_c Y^p$ and the deviation of desired inventories from their actual level, measured relative to the size of the capital stock.

Taken together we therefore have as changes revised budget restrictions of asset holders and firms, certain revisions in the behavioral assumptions for firms which only to some extent generalize earlier presentations of them, an improved price Phillips curve and of course a supply decision of firms that now takes into account further limits to production than only the aggregate level of goods demand. As the model is currently formulated there are however further economic side conditions that may be violated which are:

1. A nonnegative amount of gross investment $I + \delta K$ (only checked numerically)
2. A positive nominal interest rate r (only checked numerically)
3. Nonnegative stocks of financial assets $(B, E \geq 0)$ (only checked numerically)
4. Levels of desired production $Y^e + \mathcal{I}$ that are bounded by a positive magnitude from below (only checked numerically)
5. Rates of employment $V \leq 1$ (imposed)
6. Nonnegative inventories $N \geq 0$ (imposed)

The first two conditions demand for nonlinearities in investment and money demand which will not be considered here (these side conditions therefore have to be checked in all simulations that are performed in order to exclude

[9] We continue to use actual output of firms as measure for transactions in the money demand equation, though Y^d, the actual sales, might be a better measure for this purpose in the considered situation.

from consideration all trajectories where they are violated). Condition 3 will generally not be endangered in practicable applications of the model, but should of course also be checked for validity in actual simulation runs of the model. The same holds true for condition 4. Note here that expected sales Y^e must remain positive if actual sales Y^d remain positive throughout, which we shall show to be the case in the next paragraph. Condition 4 is therefore only violated when stocks of inventories are so high that desired inventory reductions exceed expected sales. Condition 5 is simply imposed onto the dynamics of the model, just as in the model considered in Sect. 4.7. As in this earlier model it does not prevent further increases of the output of firms as long as the constraint $\bar{Y}^w$ is not yet operative. Note in this respect that our approach does not demand that the capacity rates U^c, U^e_c must be smaller or equal to 1 since firms will go up to the limit $\bar{Y}^p$ in their use of their productive capacity, which generally is much more than the desired capacity output Y^p on which their measuring of rates of capacity utilizations are based.

From the side conditions just considered we get that actual output Y must always be positive, if the number of employed persons L^w remains positive. This latter number cannot however approach zero, since the constraint $\bar{Y^w}$ would then become binding which would imply $L^d = (1 + \bar{o})L^w > L^w$ and thus $\dot{L}^w > 0$ before $L^w = 0$ can be reached. Next, aggregate demand Y^d is bounded by a positive number from below, since this holds true for employment L^d and thus for wage income and the consumption of wage earners, while the consumption of asset holders remains positive, since their income remains positive due to the assumption $t^n_c < 0$ and since gross investment and government expenditures must remain nonnegative by assumption. If Y^d remains positive we get that Y^e remains positive, due to the assumed adaptive mechanism for the formation of sales expectations. The expected real rate of interest, however, in contrast to the other magnitudes considered, may become negative in which case capital losses $\hat{p}_e < 0$ must be such that the dividend rate of return becomes equalized through their addition with this negative rate of return on government bonds.

Condition 6, finally, could be weakened if backlog orders are allowed for. We will however consider it as a strict condition and investigate in the next section how the model's structure has to be changed when this floor in inventory holdings is reached. The model will therefore be made complete in this respect, in contrast to our present treatment of nominal interest, gross investment and desired production levels. We thus have to check in numerical simulations of the model's dynamics that these latter magnitudes stay in domains of economic relevance. Note here that all other variables, not explicitly considered here, automatically remain restricted to economically meaningful values in finite time, due to the employed growth law formulations and other types of adjustment rules. With respect to dividend payments and the rate $\tilde{\rho}^e$ on which they are based we expect furthermore for relevant trajectories of the dynamics that the situation:

$$Y^e < \bar{Y}^p \leq Y^e + \mathcal{I},$$

where they are zero does not occur for such high activity levels of the economy which generally should not be approached even approximately.

Summing up we assert that there is essentially only one significant change in the model, namely that production may now also be limited by the amount of overtime work that is available to firms or by the additional level of production that price-setting producers are willing to supply to their customers in situations of a booming economy at costs that exceed proceeds and that thus diminishes their profit sum. Despite these additional constraints on the level of production far off the steady state, actual sales are, in the presently considered situation, always equal to aggregate demand which also continues to determine expected demand and on this basis by and large the dividend payments that are made by firms. In short, the Keynesian regime here remains in full power with respect to the results that are achieved on the market place—up to the situation where aggregate demand exceeds intended or constrained production levels coupled with inventory changes that lead to zero inventories. This situation will be considered in the following section.

Note finally that the above model formulations imply that the Keynesian regime prevails around the steady state of the economy even if there were no possibilities for overtime work and for production at prices lower than marginal wage costs (and also if there were no increasing natural rate of growth (of the labor supply) for high levels of the employment rate). This is true since we have assumed in line with most of the macroeconomic literature that there is a NAIRU-level of the rate of employment $\bar{V}$ less than one and also a NAIRU-level of the rate of capacity utilization $\bar{U}_c$ less than one at which prices can become stationary and to which the economy will converge (if locally asymptotically stable).

Equations (10.45) and (10.46) can be rearranged in the usual way, now giving rise to

$$\hat{w} - \pi^e = \beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1) + \kappa_w(\hat{p} - \pi^e),$$
$$\hat{p} - \pi^e = \beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2}U_n + \kappa_p(\hat{w} - \pi^e).$$

Solving for the two variables $\hat{w} - \pi^e$, $\hat{p} - \pi^e$ these two equations then in turn imply (with $\kappa = (1 - \kappa_w\kappa_p)^{-1}$)

$$\hat{w} - \pi^e = \kappa[\beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1) + \kappa_w(\beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2}U_n)],$$
$$\hat{p} - \pi^e = \kappa[\beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2}U_n + \kappa_p(\beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1))].$$

Subtracting the second from the first equation finally again provides the law of motion for the real wage

$$\hat{\omega} = \kappa[(1 - \kappa_p)(\beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1)) + (\kappa_w - 1)(\beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2}U_n)] \quad (10.48)$$

which gives the first of the differential equations to be employed in the following in qualitatively the same form as in the earlier models without supply side bottlenecks.

The law of motion for the full employment labor intensity L/K reads in the present model:

$$\hat{l} = -(i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c)) \quad (10.49)$$

and thus solely reflects the slightly changed type of investment behavior of this section.

The next two laws of motion for real balances $m = M/(pK)$ per unit of capital and inflationary expectations π^e are obtained from the expressions:

$$\hat{m} = -(\hat{p} - \pi^e) - \pi^e - (i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c)), \quad (10.50)$$
$$\dot{\pi}^e = \beta_{\pi_1^e}(\hat{p} - \pi^e) + \beta_{\pi_2^e}(\bar{\mu} - n - \pi^e) \quad (10.51)$$

by inserting the expression we obtained for $\hat{p} - \pi^e$ above (note that we have assumed $\bar{\mu} = n$ for reasons of simplicity).

The laws of motion for expected sales and inventories per unit of capital are as in the KMG models of Chiarella et al. (2000) and thus read in qualitative terms:

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + \hat{l}y^e, \quad (10.52)$$
$$\dot{\nu} = y - y^d + (\hat{l} - n)\nu, \quad \nu > 0. \quad (10.53)$$

Finally, we now have a further law of motion as in Sect. 4.7, concerning the employment policy of firms, which in the present situation is given by the following differential equation for the workforce–capital ratio $l^w = L^w/K$:

$$\hat{l}^w = \beta_v(V^w - 1) - (i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c)). \tag{10.54}$$

In order to obtain an autonomous system of differential equations from the above laws of motion one has to supply the definitions for V, V^w, U_c^e, U_n, ρ^p, r, y^d, y and their components in addition. For these expressions we obtain from the above model (10.15)–(10.46) the following set of equations (where f as usual denotes the given production function on the intensive form level):[10]

$$\begin{aligned}
V &= l^w/l,\\
V^w &= l^d/l^w, \quad f(l^d) = y,\\
U_c^e &= (y^e + \beta_n(\beta_{n^d} y^e - \nu) + n\beta_{n^d} y^e)/y^p,\\
U_n &= \beta_{n^d} y^e - \nu,\\
\rho^p &= f(l^p) - \delta - f'(l^p)l^p, \quad f'(l^p) = \omega,\\
r &= r_o + (h_1 y + m)/h_2,\\
y^d &= \omega l^d + (1 - s_c)(\tilde{\rho}^e - t_c^n) + i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c)\\
&\quad + n + \delta + \bar{g},\\
y &= \min\{y^e + \beta_n(\beta_{n^d} y^e - \nu) + n\beta_{n^d} y^e, \bar{y}^p, \bar{y}^w\},\\
y^p &= f(l^p), \quad l^p = (f')^{-1}(\omega),\\
\bar{y}^p &= f(\bar{l}^p) = \omega\bar{l}^p + \delta,\\
\bar{y}^w &= f((1 + \bar{o})l^w),\\
\tilde{\rho}^e &= \max\{y^e - \delta - \omega l^d, 0\},\\
n &= \bar{n}_o + n_1(V^w - 1) + n_2(V - \bar{V}).
\end{aligned}$$

This dynamical system describes the evolution of the KMG growth dynamics with supply bottlenecks as long as $\nu > 0$ remains true, with no regime switching caused due to the varying size of aggregate demand y^d until inventories get exhausted.

Proposition 10.4. *Assume (for notational simplicity) that $\bar{U}_c = 1, n = 0$ holds true. Then: There is a uniquely determined interior steady state solution of the dynamics of this subsection which is given by:*

[10] Note here that the measure U_n is closely linked to the measure U_c^e and thus does not add something that is qualitatively very different from the effects of the rate of capacity utilization on the evolution of price inflation.

$$
\begin{aligned}
&\tilde{\rho}^e = \rho^p = \bar{g}/s_c - (1-s_c)t_c^n/s_c > 0,\\
&y^e > 0 \quad \textit{given by the solution to} \quad \rho^p = y^e - \delta - f'(f^{-1}(y^e))f^{-1}(y^e),\\
&y^e = y^d = y = y^p,\\
&\omega = f'(f^{-1}(y^e)),\\
&l^p = f^{-1}(y^e) = l^d = l^w, \quad l = l^w/\bar{V},\\
&r = r_o = \rho^p,\\
&m = h_1 y^e,\\
&\pi^e = \hat{p} = \hat{w} = \hat{p}_e = \bar{\mu} - n = 0,\\
&\nu = \beta_{n^d} y^e > 0.
\end{aligned}
$$

Note with respect to this proposition that the steady state lies in the interior of the phase space of the dynamics considered in this subsection. Note furthermore that aggregate savings (and net investment) $S/K(= I/K)$ has to be zero in the steady state which is guaranteed by the following equation:

$$s_c(\rho^p - t_c^n) = g - t_c^n > 0$$

stating the equality between private savings and government dissavings in the steady state. Note finally that the equation $\tilde{\rho}^e = (n+\bar{g})/s_c - (1-s_c)t_c^n/s_c$ also holds true in the general case, and is obtained from goods market equilibrium in both cases, which thus determines the dividend rate of return of asset holders ($p_e E = pK$ in the steady state) as based on the size of natural growth, government expenditure and capital taxation rules, and the savings propensity of asset holders. This rate of return can therefore in particular be increased by increasing government expenditures per unit of capital and is completely independent from the marginal productivity theory of income distribution (which however determines the real wage in the steady state).

Proof. To be based on the steady state relationships $V = V^w = U_c^e = 1$, $U_n = 0$, $\rho^p = r_o$, $\pi^e = 0$, $y = y^d = y^p = y^e$ that can be obtained by setting the differential equations of this section equal to zero. ∎

We conjecture, but cannot prove this in this subsection, that the system will be locally asymptotically stable for all adjustment speed parameters $\beta_x' s$ chosen sufficiently low (up to the adjustment speed of sales expectations, which must be chosen sufficiently high and up to one exception to be considered below). The basis for this conjecture is that the system is basically of the type considered in Sects. 4.2, 4.3, since the constraints that were added to it are not in operation close to the steady state. However, we have now a seventh law of motion, the employment policy of firms, and have also added a second demand pressure term in the wage as well as in the price inflation equation the role of which has not yet been investigated. Furthermore the case of smooth factor substitution has not yet been considered very carefully so far. Due to the high dimension of the considered dynamics we cannot offer any proven proposition at this stage of the investigation.

The same holds true with respect to the assertion that the determinant of the system at the steady state will always be negative, so that the system can only gain or lose stability by way of Hopf-bifurcations. Stability losses will come about if either wage or price flexibility is chosen sufficiently high, if adaptively formed inflationary expectations are moving with sufficient speed, and if inventories are adjusted with sufficient speed. Again, this is but a conjecture based on the experience gained with closely related, but lower dimensional dynamic systems in the last chapter and in particular in Chiarella and Flaschel (2000). We also expect that the adjustment parameter in the employment policy of firms will be destabilizing when chosen sufficiently high, but have to leave all these conjectures for numerical investigations for the time being. Note that the instability assertions made are not easy to prove as they will not operate through the trace of the Jacobian of the dynamics at the steady state, but only via higher principal minors of this Jacobian, the calculation of which however can be approached via the numerous linear dependencies that characterize components of the rows of this Jacobian. The following proposition provides an example for such a calculation.

Proposition 10.5. *The dynamics of this subsection will be unstable at their interior steady state if the parameters $\beta_{\pi_1^e}$, β_{y^e} are chosen sufficiently high.*

Proof. Considering the interaction of the state variables in isolation gives rise to the following subdynamics of the full 7d dynamics as far as the parameters $\beta_{\pi_1^e}, \beta_{y^e}$ are concerned:

$$\dot{\pi}^e = \beta_{\pi_1^e}(\hat{p} - \pi^e) + \beta_{\pi_2^e}(\bar{\mu} - n - \pi^e), \tag{10.55}$$

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + \hat{l}y^e. \tag{10.56}$$

It is easy to see that

$$\hat{p} - \pi^e = \kappa[\beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2}U_n + \kappa_p(\beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1))]$$

depends positively on y^e, since the expressions U_c^e, U_n, V^w all depend positively on y^e, while there is no direct influence of π^e on this equation, which drives inflationary expectations as far as the adaptive component is concerned as is shown above. The rate of change $\dot{y}^e$ in addition depends positively on inflationary expectations π^e, since y^d depends on them in this way. This gives for the corresponding $2 \star 2$ submatrix $J(2,2)$ of the full Jacobian J the qualitative result:

$$J(2,2) = \begin{vmatrix} J_{44} & J_{45} \\ J_{54} & J_{55} \end{vmatrix} = \begin{vmatrix} 0 & +const\ \beta_{\pi_1^e} \\ +const\ \beta_{y^e} & ? \end{vmatrix} = -const\ \beta_{\pi_1^e}\beta_{y^e}.$$

The sum of all principal minors of dimension two can thereby made negative by choosing the parameter pair $\beta_{\pi_1^e}, \beta_{y^e}$ jointly sufficiently large, since this pair does only appear together in the submatrix here considered. Therefore one of the conditions of the Routh-Hurwitz theorem that are sufficient for local asymptotic stability is violated in a way that implies the existence of at least one eigenvalue with positive real part. ■

We thus have local divergence with respect to at least a submanifold of dimension one which raises the question what the forces are that may nevertheless make the considered dynamics bounded in a domain that is of economic relevance. In order to investigate this question we have however first of all to complete the description of the dynamics for the case where inventories have been run down to zero. This task will be solved in the now following subsection.

10.4.2 Exhausted Inventories and Excessive Aggregate Demand

With respect to the notation of the KMG model of the preceding section we now have the situation that

$$Y^d = C_w + C_c + I + \delta K + G > Y = \min\{Y^e + \mathcal{I}, \bar{Y}^p, \bar{Y}^w\}, \quad N = 0$$

hold simultaneously, i.e., aggregate demand Y^d can no longer be fulfilled from the production decisions made by firms and from their inventory holdings.

In order to show how the model needs change in such a situation we measure the extent of rationing that will then occur with respect to the planned demands of economic agents by the coefficient $\alpha \in (0,1)$ defined by:

$$\omega L^d + \delta K + \alpha(C_c + I + \bar{g}K) = Y.$$

Thus, we assume that rationing concerns the consumption of asset owners, net investment in fixed capital and a part from government consumption, whereas investment compensating for depreciation δK as well as the consumption of workers ($C_w = (1-\tau)\omega L^d$) and the part of government expenditures equal to $\tau\omega L^d$ remain unaffected by the supply shortage.[11] In this context it is important to note the (downward) jump in dividend payments that will occur, when production is limited due to the boundary $\bar{Y}^w$ or $\bar{Y}^p$ and inventories become exhausted, so that the part of planned revenues Y^e, that was realized by sells from inventories before, vanishes. As, on the other hand, dividend payments represent a part of the income of asset holders, effective demand and thus also the value for α has to be calculated after this jump in dividend payments has appeared.[12] Note furthermore that $\alpha > 0$ is indeed guaranteed in the considered model, since we have $Y \leq \bar{Y}^p$ by assumption and thus $Y - \delta K - \omega L^d \geq 0$, leading to $Y - \delta K - C_w > 0$ due to $C_w = (1-\tau_w)\omega L^d < \omega L^d$. Note here also that actual inventory investment $\dot{N} = Y - Y^d > 0$ will lead us back to the situation of the preceding section. We here assume however that $\alpha \leq 1$ holds in the following modification of the preceding less restricted model.

The following set of equations describe the working of the economy in such extreme situations of a booming economy.

1. Definitions (remunerations and wealth):

$$\omega = w/p, \quad \tilde{\rho}^e = \max\{(\min\{Y^e, Y\} - \delta K - \omega L^d)/K, 0\}, \tag{10.57}$$

$$W = (M + B + p_e E)/p. \tag{10.58}$$

2. Households (workers and asset-holders):

$$W = (M^d + B^d + p_e E^d)/p, \tag{10.59}$$

$$C = \omega L^d - T_w + (1 - s_c)[\tilde{\rho}^e K + rB/p - T_c], \tag{10.60}$$

$$C^a = \omega L^d - T_w + \alpha(1 - s_c)[\tilde{\rho}^e K + rB/p - T_c], \tag{10.61}$$

$$\begin{aligned} S_p &= \omega L^d + \tilde{\rho}^e K + rB/p - T - C^a \\ &= (s_c + (1-\alpha)(1 - s_c))[\tilde{\rho}^e K + rB/p - T_c] \\ &= (\dot{M} + \dot{B} + p_e \dot{E})/p, \quad T = T_w + T_c, \end{aligned} \tag{10.62}$$

$$\hat{L} = n = \bar{n}_o + n_1(V^w - 1) + n_2(V - \bar{V}), \quad n_1, n_2 > 0. \tag{10.63}$$

---

[11] Wage earners are here excluded from this proportional reduction of the demand plans of economic agents in order to avoid unnecessary complications for a first presentation of the case of rationing (since one would have to consider their forced savings in such a situation and thus add money holdings of workers for example).

[12] In general, it can not be excluded, that at least in some cases the jump in dividend payments just described is large enough to push aggregate demand again below the level of production, so that the economy immediately returns to the former regime after inventories have become exhausted.

3. Firms (production-units and investors):

$$\omega = F_L(K, L^p),\ Y^p = F(K, L^p),\ \ \rho^p = (Y^p - \delta K - \omega L^p)/K, \quad (10.64)$$

$$U_c^e = (Y^e + \mathcal{I})/Y^p, \quad (10.65)$$

$$L^d \quad \text{via } Y = F(K, L^d),\ \ V = L^w/L \leq 1,\ \ V^w = L^d/L^w, \quad (10.66)$$

$$\dot{L}^w = \beta_v(L^d - L^w) + \gamma L^w, \quad \gamma = n, \quad (10.67)$$

$$I/K = i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) + \gamma, \quad \gamma = n, \quad (10.68)$$

$$I^a/K = \alpha(i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) + \gamma), \quad \gamma = n, \quad (10.69)$$

$$I^a + \delta K \geq 0,$$

$$I^a = Y - \tilde{\rho}^e K - \delta K - \omega L^d + p_e \dot{E}/p = S_f + p_e \dot{E}/p, \quad (10.70)$$

$$\hat{K} = I^a/K. \quad (10.71)$$

4. Government (fiscal and monetary authority):

$$T = T_w + T_c,\ T_w = \tau_w \omega L^d,\ t_c^n = (T_c - rB/p)/K = \text{const} < 0, \quad (10.72)$$

$$G = \tau_w \omega L^d + \bar{g}K, \quad \bar{g} \geq 0, \quad (10.73)$$

$$G^a = \tau_w \omega L^d + \alpha \bar{g} K, \quad (10.74)$$

$$S_g = T - rB/p - G^a, \quad (10.75)$$

$$\hat{M} = \bar{\mu} = n, \quad (10.76)$$

$$\dot{B} = pG^a + rB - pT - \dot{M}. \quad (10.77)$$

5. Equilibrium Conditions (asset-markets):

$$M = M^d = h_1 pY + h_2 pK(r_o - r), \quad [B = B^d, E = E^d], \quad (10.78)$$

$$B, E \geq 0,\ r > 0,$$

$$r = \frac{\tilde{\rho}^e pK + \dot{p}_e E}{p_e E}. \quad (10.79)$$

6. Disequilibrium Situation (goods-market adjustments):

$$S = S_p + S_g + S_f = Y - \delta K - (C^a + G^a) = I^a, \quad (10.80)$$

$$Y^d = C + I + \delta K + G, \quad (10.81)$$

$$\dot{Y}^e = \beta_{y^e}(Y^d - Y^e) + \gamma Y^e, \quad \gamma = n, \quad (10.82)$$

$$N^d = \beta_{n^d} Y^e, \quad (10.83)$$

$$\mathcal{I} = \beta_n(N^d - N) + \gamma N^d, \quad \gamma = n, \quad (10.84)$$

$$Y = \min\{Y^e + \mathcal{I}, \bar{Y}^p \bar{Y}^w\}, \quad Y^e + \mathcal{I} > 0, \quad (10.85)$$

$$\dot{N} = \max\{Y - Y^d, 0\} \geq 0 [> 0, \text{if regime switching takes place}]. \quad (10.86)$$

7. Wage-Price-Sector (adjustment equations):

$$\hat{w} = \beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^e, \tag{10.87}$$

$$\hat{p} = \beta_{p_1}(U_c^e - \bar{U}_c) + \beta_{p_2} U_n + \kappa_p \hat{w} + (1 - \kappa_p)\pi^e, \tag{10.88}$$

$$U_n = (N^d - N)/K, \tag{10.89}$$

$$\dot{\pi}^e = \beta_{\pi_1^e}(\hat{p} - \pi^e) + \beta_{\pi_2^e}(\bar{\mu} - n - \pi^e). \tag{10.90}$$

As already described above, the fundamental change that has taken place now consists in the formulation of situations, where demand is larger than actual production and where at a certain point in time there are no inventory stocks available any more, by which the difference between Y^d and Y could be bridged. Before commenting on these changes in more detail we want to stress that the economy switches back to Regime 1 (considered in the preceding subsection) if and only if $\dot{N} = \max\{Y - Y^d, 0\}$ becomes positive again. Conversely, regime 1 is left and gives way to the regime considered in the present subsection if and only if the situation $Y^d > Y(\dot{N} < 0)$ is given and $N = 0$ is reached. We thus have in this extension of the KMG growth model by means of relevant macroeconomic supply bottlenecks only two regimes, a Keynesian (Regime 1) one where aggregate demand is always fulfilled and a Non-Keynesian one (Regime 2) where aggregate demand is rationed and where this rationing may be due to too low sales expectations and intended inventory changes or to exhausted possibilities for overtime work or to the zero profit bound on production. Therefore, in this latter case of exhausted inventories, the three regimes of Non-Walrasian macroeconomics reappear so to speak en miniature, but now as an event that will rarely occur in the investigated economy.

Turning now to the changes that differentiate regime 2 from regime 1, we have to note first that actual dividend payments are now formulated in a narrower way in (10.57), since firms will not pay dividends according to expected sales when they know that their actual production will be lower than expected sales, in which case only the amount $Y - \delta K - \omega L^d$ will be distributed. There are also the obvious changes due to rationed goods demand of asset holders, the firms and the government, which also modifies their savings which are now based on disposable income flows of type $\tilde{\rho}^e K + rB/p - T$ for asset holders, $Y - \tilde{\rho}^e K - \delta K - \omega L^d$ of firms and of $T - rB/p$ for the government. These are however already all changes when we go from regime 1 to regime 2 implying that there will only be few changes to the laws of motion of the preceding subsection when such a regime switch occurs. The following provides a list of these changes without presenting again the equations of the model in intensive form that have remained unchanged between the two regimes. The law of motion for the full employment labor intensity L/K now reads:

$$\hat{l} = n - \alpha(i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) + n) \tag{10.91}$$

and the one for real balances $m = M/(pK)$ per unit of capital is changed by the same reason as follows

$$\hat{m} = \bar{\mu} - (\hat{p} - \pi^e) + \pi^e - \alpha(i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) + n) \tag{10.92}$$

as is the one for the workforce—capital ratio $l^w = L^w/K$:

$$\hat{l}^w = \beta_v(V^w - 1) + n - \alpha(i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) + n). \tag{10.93}$$

Since $\hat{l}$ is to be inserted into the law of motion for y^e there is therefore also an implicit change there, while we have $\nu = 0$ as long as regime 2 applies (in which case $U_n = \beta_{n^d} y^e$ holds). Concerning the static equations of the model we only have to note that aggregate demand is implicitly changed, since disposable income is different in regime 2:

$$\begin{aligned} y^d &= \omega l^d + (1 - s_c)(\tilde{\rho}^e - t_c^n) + i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - \bar{U}_c) \\ &\quad + n + \delta + \bar{g}, \\ \tilde{\rho}^e &= \max\{\min\{y^e, y\} - \delta - \omega l^d, 0\}. \end{aligned}$$

All other static expressions remain the same which in sum means that the formal change to the model is a very limited one.

We expect that the case of an economy-wide or global rationing[13] does play no role at all or only occurs under rare circumstances in the KMG model of this chapter, see our later numerical investigations of this model. Introducing in addition flexible wage, price, employment and labor supply adjustments, and also flexible investment and inventory adjustment mechanisms when aggregate demand becomes rationed will make such a case even more unlikely in a market economy. A simple model where exactly such situations are excluded from occurring is provided through a nonlinear Phillips curve mechanism in Benassy (1984), an approach which is easily generalized to the high-dimensional dynamics here under consideration (but not easy to

[13] This describes a situation, where all shops and factories have nothing left after each trading period.

be analyzed here, where there are many buffers and adjustment mechanisms available in order guarantee the general prevalence of the Keynesian regime).

As described above the case of empty inventories is characterized by three different possibilities for the constraint that acts on aggregate demand, leading to its rationing of the type

$$\omega L^d + \alpha C_c + \alpha I + \delta K + \alpha \bar{g} K = Y = \min\{Y^e + \mathcal{I}, \bar{Y}^p, \bar{Y}^w\}$$

with $\alpha \in (0,1)$. As in the Non-Walrasian approach considered in Sect. 5.2, but now only for zero inventory stocks, we have a "Keynesian" regime[14] $Y^e + \mathcal{I} = \min\{Y^e + \mathcal{I}, \bar{Y}^p, \bar{Y}^w\}$, a classical regime $\bar{Y}^p = \min\{Y^e + \mathcal{I}, \bar{Y}^p, \bar{Y}^w\}$ and a regime of repressed inflation $\bar{Y}^w = \min\{Y^e + \mathcal{I}, \bar{Y}^p, \bar{Y}^w\}$ which of course overlap with each other at their boundaries.

The first of these regimes is characterized by the situation where desired output $Y^e + \mathcal{I}$ is restricting actual production and is below aggregate demand Y^d. It is to be expected then that sales expectations will be faster and faster adjusted towards actual aggregate demand (by way of a nonlinearity in the adjustment mechanism not considered in the above model) and that inventories are also adjusted faster than in the way given by the model so that the situation where desired production is the cause of demand rationing should soon be overcome by such flexibilities in actual market economies.

In the next regime, where $\bar{Y}^p$ and thus zero profitability due to excessive use of productive capacity is the cause of demand rationing, one would expect and add to the model strong nonlinearities in the price adjustment mechanism (much earlier than this barrier is in fact reached) which speed up the rate of increase in the price level by so much that the resulting fall in real wages (since labor markets are not yet too tight) will increase this barrier $\bar{Y}^p$ so strongly that it will no longer represent the limiting item. Of course, this is only a partial description of the whole dynamics and may not always hold in the phase space in this straightforward way, but may interact with tight labor in certain situations in a more complicated way.

In the third regime, where the limit to overtime work is the barrier for serving aggregate demand, we should expect that the parameter characterizing the employment policy of firms, β_l, is increased significantly. If, however,

[14] This designation should not be taken too literally and not be confused with our regime 1. While in regime 1 demand is always served, its rationing is the main characteristic of regime 2 and thus also of the so-called "Keynesian" subregime just mentioned.

the labor market is in the state of absolute full employment with respect to the currently available labor force, the rate of growth of labor supply will be increased, in particular by active search of firms for laborers across the border of the considered economy so that this limit for serving aggregate demand can be overcome. There are of course also real wage increases to be expected in such a situation which however will not help in the case where aggregate demand depends positively on real wages, yet it is to be expected that such real wage increases will have larger and larger effects on investment behavior leading to a negative dependence of aggregate demand on real wages eventually (by parameter changes not considered in the model).

All these observations suggest that the regime 2 introduced and considered in the present subsection will not last very long. We expect this to be true even in the framework of the model itself, an assertion which at present can only be confirmed by means of numerical simulations which are the topic of the next subsection.

At this stage, our model has already reached a high degree of completeness, as supply bottlenecks due to exhausted inventories are now explicitly taken into account. On the other hand, there are still a number of items to be checked numerically, especially the nonnegativity constraints on gross investment, interest rates, stocks in bonds, equities and output.[15] Furthermore, it should be checked, for which parameter constellations there is convergence to the (Keynesian) steady state and how, on the other hand, ceilings as $Y \leq 1$ and buffers like the possibility of overtime word help to keep the economy in meaningful bounds in the case of local explosiveness. The simulations to be considered in the next subsection will also treat the inclusion of a further nonlinearity, namely a kink in the Phillips-curve, by which a downward rigidity of money wages is described. As will be shown, for quite a large set of parameter values this leads to stable depressions with $V < \bar{V}$ instead of limit cycles, which would occur otherwise.

10.4.3 Numerical Analysis

In this section we provide some numerical illustrations in particular with respect to Proposition 10.5 on the destabilizing role of adaptively formed expectations if the speed of adjustment of these expectations is increased. We

[15] Compare in this respect also the treatment of these problems in Laxton et al. (1998).

will see that the system's dynamics is indeed a converging one for low adjustment speed parameters, which becomes locally divergent already for relatively low speeds of adjustments of inflationary expectations. This local instability gives rise to global limit cycle behavior for a certain range of this adjustment speed beyond the region of local asymptotic stability and gives way to pure explosiveness and economic breakdown thereafter. The issue of economic viability is therefore not completely resolved by the supply side constraints we have added to the KMG model in this chapter. We therefore will add another extrinsic nonlinearity to the model at the end of this subsection which is of a very fundamental nature and which will indeed guarantee economic viability for a range of adjustment speeds that is significantly larger than the one without it.

In order to simulate the dynamics for both regime 1 and 2 of the preceding subsections we have to integrate these two subsections into a single dynamical model and have to make use of specific production functions in addition. We shall make use of a Cobb-Douglas production function where we have to keep in mind that marginal productivity theory will be valid in our model only in the steady state, i.e., the model allows for fluctuations in the share of wages despite the assumption of a production technology with elasticity of substitution equal to one. For reasons of simplicity we shall also assume in the following that the natural rate of growth is zero and that the normal rate of capacity utilization of firms is given by one. Based on the assumptions

$$Y = F(K, L^d) = K^{1-a}(L^d)^a, a \in (0,1),$$
$$n \equiv 0 (= \bar{\mu}) \ (L = const, \hat{l} = -\hat{K}), \quad \bar{U}_c = 1$$

the dynamical system to be investigated in this subsection is therefore the following one:

$$\begin{aligned}\hat{\omega} &= \kappa[(1-\kappa_p)(\beta_{w_1}(V-\bar{V}) + \beta_{w_2}(V^w - 1)) \\ &\quad + (\kappa_w - 1)(\beta_{p_1}(U_c^e - 1) + \beta_{p_2}U_n)], \end{aligned} \tag{10.94}$$

$$\hat{l} = -g_k, \tag{10.95}$$

$$\hat{m} = -(\hat{p} - \pi^e) - \pi^e - g_k, \tag{10.96}$$

$$\dot{\pi}^e = \beta_{\pi_1^e}(\hat{p} - \pi^e) + \beta_{\pi_2^e}(-\pi^e), \tag{10.97}$$

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) - g_k y^e, \tag{10.98}$$

$$\dot{\nu} = y - y^d - g_k \nu, \tag{10.99}$$

$$\hat{l}^w = \beta_v(V^w - 1) - g_k \tag{10.100}$$

now supplemented by the following algebraic equations and regime switching conditions:[16]

$$\begin{aligned}
\hat{p} &= \kappa[\beta_{p_1}(U_c^e - 1) + \beta_{p_2} U_n \\
&\quad + \kappa_p(\beta_{w_1}(V - \bar{V}) + \beta_{w_2}(V^w - 1))] + \pi^e, && (10.101)\\
V &= l^w/l \le 1, && (10.102)\\
V^w &= l^d/l^w, \quad y = f(l^d) = (l^d)^a, \quad l^d = y^{1/a}, && (10.103)\\
U_c^e &= (y^e + \beta_n(\beta_{n^d} y^e - \nu))/y^p, && (10.104)\\
U_n &= \beta_{n^d} y^e - \nu, && (10.105)\\
\omega &= f'(l^p) = a(l^p)^{a-1}, \quad l^p = (a/\omega)^{\frac{1}{1-a}}, && (10.106)\\
\rho^p &= f(l^p) - \delta - f'(l^p) l^p = (1-a) y^p - \delta, \quad y^p = (l^p)^a, && (10.107)\\
r &= r_o + (h_1 y + m)/h_2 > 0, && (10.108)\\
y &= \min\{y^e + \beta_n(\beta_{n^d} y^e - \nu), \bar{y}^p, \bar{y}^w\}, && (10.109)\\
\bar{y}^p &= (\bar{l}^p)^a = \omega \bar{l}^p \rightarrow \bar{l}^p = (1/\omega)^{\frac{1}{1-a}} > l^p = (a/\omega)^{\frac{1}{1-a}}, && (10.110)\\
\bar{y}^w &= ((1+\bar{o}) l^w)^a, && (10.111)\\
\tilde{\rho}^e &= \max\{y^e - \delta - \omega l^d, 0\}, \quad \text{if} \quad \nu > 0, && (10.112)\\
\tilde{\rho}^e &= \max\{\min[y^e, y] - \delta - \omega l^d, 0\}, \quad \text{if} \quad \nu = 0, && (10.113)\\
y^d &= \omega l^d + (1 - s_c)(\tilde{\rho}^e - t_c^n) \\
&\quad + i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - 1) + \delta + \bar{g}, && (10.114)\\
\alpha &= \min\left\{ \frac{y - \omega l^d}{(1 - s_c)(\tilde{\rho}^e - t_c^n) + I/K + \delta + \bar{g}}, 1 \right\}, && (10.115)\\
I/K &= i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - 1) \ge 0, && (10.116)\\
g_k &= i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - 1), \quad \text{if} \quad \nu > 0, && (10.117)\\
g_k &= \alpha(i_1(\rho^p - (r - \pi^e)) + i_2(U_c^e - 1)), \quad \text{if} \quad \nu = 0. && (10.118)
\end{aligned}$$

As the model is specified now it represents a complete dynamic model as long as the side conditions $r > 0, I/K \ge \delta, y^e + \beta_n(\beta_{n^d} y^e - \nu) > 0$ (and also $B, E \ge 0$) are not violated during the fluctuations its dynamics will generally generate. These side conditions are always checked in the following simulation studies. Should they become violated the model must be developed further, by introducing further extrinsic nonlinearities, which avoid negative nominal rates of interest and the like. Surely the model is still much too linear in this respect in order to allow the expectation that the stated side

[16] The steady state level of output, demand and sales expectations is given by $y = y^d = y^e = (\rho^p + \delta)/(1 - \alpha)$.

conditions will be guaranteed for reasonable parameter ranges. We add that the rationing coefficient α will always be positive as long as gross investment stays nonnegative, due to the assumptions made on the parameters $\bar{g}, t_c^n$. Furthermore, $\alpha = 1$ will always imply that $y^d \leq y$ must hold true.

Note next that when inventories get exhausted we may have a discontinuity (a downward jump) in dividend payments to asset holders if

$$y = \min\{\bar{y}^p, \bar{y}^w\}$$

holds, since sales expectations y^e may be larger than y in such a situation. However the bound $\bar{y}^w$, where overtime work gets exhausted, will rarely become operative and even more so the bound $\bar{y}^p$ where gross profits are zero. Therefore we will generally have $y = y^e + \beta_n(\beta_{n^d} y^e - \nu)$, and of course $y^d > y$, when a zero inventory level is reached[17] in which case $\beta_n(\beta_{n^d} y^e - \nu)$ will be positive and therefore $y^e < y$. In this case there is thus no discontinuity in dividend payments, but just a continuous switch in the measure on which the dividend policy of firms rests. As already mentioned above, in case of a discontinuous switch there will be a downward jump in aggregate demand as well (due to the consumption function of asset holders) which may in extreme situations be so large that demand rationing is completely avoided and therefore a return to positive inventory levels immediately. If this description does not apply we however have to use the factor α (based on reduced consumption demand of asset holders) to scale down actual investment, the only rationed goods demand that influences the real dynamics of the model in its present form.

The simulations of the dynamics in the Figs. 10.2ff., with the possibility of supply side restrictions, are based on the following base parameter set:

Table 10.1. Calibration parameters

$s_c = 0.8$,	$\delta = 0.1$,	$t_c^n = -0.1$,	$\bar{g} = 0.1$,	$\bar{o} = 0.6$		
$h_1 = 0.1$,	$h_2 = 0.05$,	$i_1 = 1$,	$i_2 = 0.2$			
$\beta_{w_1} = 1$,	$\beta_{w_2} = 0$,	$\beta_{p_1} = 0.2$,	$\beta_{p_2} = 0$	$\kappa_w = 0.5$,	$\kappa_p = 0.5$	
$\beta_{\pi_1} = 0.09$,	$\beta_{\pi_2} = 0$,	$\beta_{n^d} = 0.1$,	$\beta_n = 0.5$,	$\beta_y^e = 2$,	$\beta_v = 0.5$	
$a = 0.7$,	$\bar{V} = 0.93$,	$\bar{\mu} = n = 0$				

We have a 3% money supply shock at $t = 1$ and a horizon of 50 years. Note that the consumption and investment parameters have been chosen such that the

[17] The same holds when inventories return to positive levels by way of an aggregate demand that does not absorb total production.

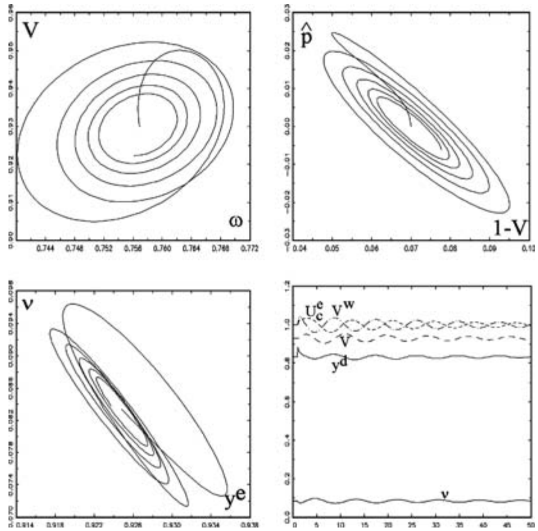

Fig. 10.2. Convergence for sluggish adjustment speeds of inflationary expectations

cost effect of real wage increases is the dominant effect in aggregate demand, because otherwise an upper turning point in the boom would not be necessarily guaranteed. Figure 10.2 now displays a situation, where the steady state is local asymptotically stable due to low adjustment speeds, especially concerning the expected rate of inflation. Note, that the choice of a $\beta_{w_2} > 0$ would enlarge the region of parameter values, for which asymptotic stability prevails.

The next simulation run, shown in Fig. 10.3, now with a larger adjustment speed of inflationary expectations ($\beta_{\pi_1} = 0.3$), results in a Keynesian limit cycle. In this case, the full employment ceiling is reached in the boom phase of the business cycle without however representing a limit to production due to the possibility of overtime work, which clearly can be seen when looking at the time series for V^w in the lower right picture of Fig. 10.3. The lower left picture of Fig. 10.3 shows furthermore, that the whole dynamics takes place in regime 1, i.e. inventories are positive throughout and demand is never rationed here.

The situation in Fig. 10.3 is easily extended to include periods of demand rationing, when β_{π_1} is increased further, as Fig. 10.4 shows. For $\beta_{\pi_1} = 0.37$ inventories now reach their lower bound zero in the boom, so that supply bottlenecks become effective and demand rationing occurs. Furthermore, there is then also a downward jump of dividend payments in this upper region, as can be guessed from the corresponding kinks in the y^d-curve at $t \approx 27$ and $t \approx 39$, i.e., at those points in time, where inventories have approached zero. Note,

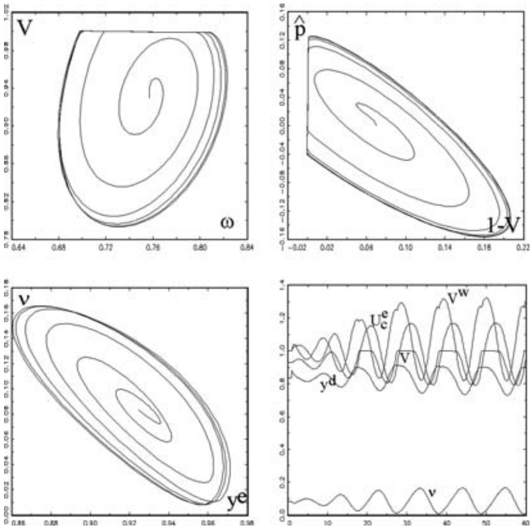

Fig. 10.3. Destabilizing inflationary expectations and a supply side generated limit cycle: $\beta_{\pi_1} = 0.3$ (only $V \leq 1$)

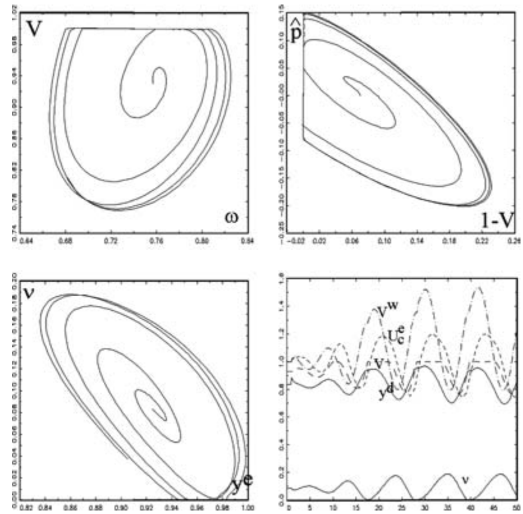

Fig. 10.4. Increased local instability and regime 2 switches: $\beta_{\pi_1} = 0.37$

that without the bounds on employment and inventories the development of the economy considered would be much more explosive in its deviation from its interior steady state. Notice also that the slump is now characterized by significantly higher deflation compared to the situation considered in Fig. 10.3.

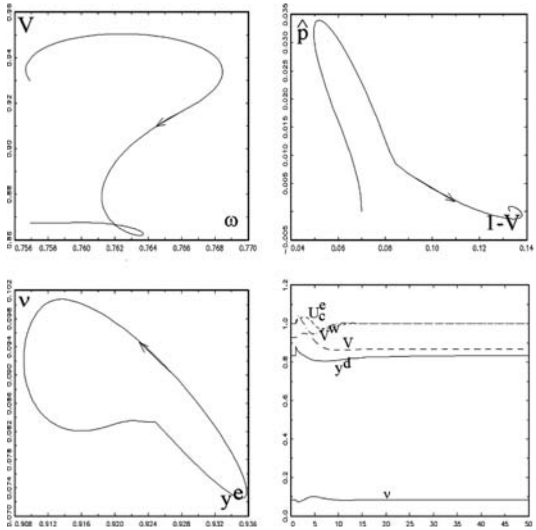

Fig. 10.5. Reestablishing convergence, but stable depressions by absolute downward rigidity of nominal wages

The next simulation, shown in Fig. 10.5, now makes use of a new element which has not been taken account of so far. With regard to reality it is now assumed, that downward flexibility of the nominal wage is a phenomenon that usually cannot be observed, so that the corresponding part of the nominal wage Phillips curve is now replaced by a horizontal line. Formally, the equations for wage and price inflation now read:

$$\hat{p} = \beta_{p_1}(.) + \beta_{p_2}(.) + (1 - \kappa_p)\pi^e + 0,$$

$$\hat{\omega} = -(\beta_{p_1}(.) + \beta_{p_2}(.)) - (1 - \kappa_p)\pi^e.$$

The consequences of this change in the wage-price sector are dramatical, as Fig. 10.5 illustrates. Instead of a limit cycle now convergence to a stable depression occurs with an employment rate V below the natural one $\bar{V}$. Although this is indeed the most important consequence, another one should also be regarded, namely the fact that now the economy again stays within the limits of regime 1, so that the supply bottlenecks of Fig. 10.4 do not appear any more. This might be surprising at the first look, as the downward rigidity of wages in the first place prevents a severe deflation as considered in Fig. 10.4. On the other hand, however, not only the phase of deflation is avoided, but also the whole business cycle and thus phases of an overheated economy, too.

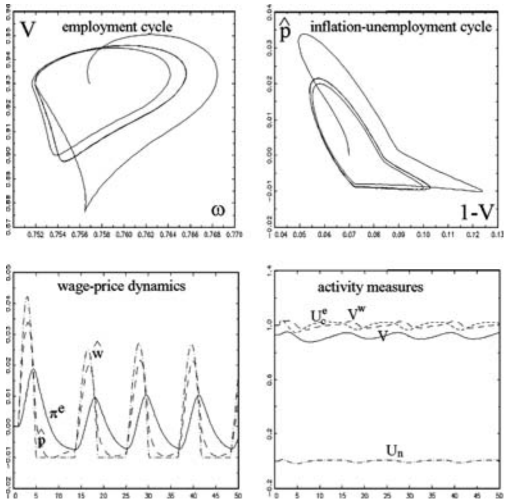

Fig. 10.6. Mild limit cycle fluctuations for sluggishly decreasing nominal wages

In Fig. 10.6, finally, we consider a situation with a less severe rigidity on wage adjustment. We assume now, that downward flexibility of nominal wages is possible, but that there is still a lower boundary for wage cuts. In other words: the floor for wage changes is no longer zero but given by a certain negative number, which we have chosen in the simulation underlying Fig. 10.6 as 0.01. Thus, nominal wages can fall as a reaction to high unemployment, but by not more than 1% per period.

As a result, the stable depressions of the preceding case now disappear, and again a limit cycle occurs, yet diminished in size in comparison to Fig. 10.3 or 10.4. Furthermore, the Keynesian regime 1 is never left. This example demonstrates, how additional nonlinearities, inserted into the model at appropriate places, can help to avoid instability scenarios like the one of Fig. 10.4, which are unlikely to provide a satisfactory description of the economy from an empirical point of view, if one takes into account, that situation of demand rationing rarely are observed in developed market economies. In contrast to this Fig. 10.6 already yields, e.g., a relationship between unemployment and inflation which is not too far from empirical patterns.

10.5 Summary

In this chapter we have shown, how the KMG model developed in Chiarella et al. (2000) could be appropriately extended in order to take account of possible regime switches as they are well-known from non-Walrasian disequilibrium theories. On the other hand, the question was raised, how relevant these regime switches actually are. Starting with Picard's (1983) model we have shown, that already in this context the importance of actual regime switches will be significantly reduced, if one takes into account, that wages and prices begin to rise much earlier than full employment of the labor force and full capacity utilization are reached. This result was further underlined by the consideration of inventory changes and buffers like overtime work or overuse of productive capacity as they are now in the KMG model.

Here, the possibility for firms to serve their demand by selling from inventory stocks, or by going temporarily beyond the point where prices equal marginal wage costs, considerably enlarged the region, in which supply bottlenecks and demand rationing do not occur. On the other hand, at least from a more theoretical point of view, the possibility of regime switches cannot a priori be excluded and thus has to be modeled carefully, especially with regard to the interdependencies between the single markets concerned. This was done in Sect. 5.3, where the case of exhausted inventories and resulting demand rationing as well as the possible feedbacks on dividend payments were explicitly considered. A number of simulation runs finally demonstrated, under which conditions regime switches become relevant. Furthermore, the stabilizing potential coming about by a kinked Phillips-curve, leading to a constant nominal wage in regions of high unemployment, has again been added and evaluated numerically.

This last point will again be taken up in the next chapter, where the importance of market imperfections on the labor market as well as on the goods market and their role for wage and price formation will be investigated. Before however turning to this point, we shortly want to consider in the subsequent appendix one of the earliest contributions made to Non-Walrasian disequilibrium theory, namely the model of Solow and Stiglitz (1968).

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Mathematical Appendix: Some Useful Theorems

1. The Concepts of Local Stability and Global Stability in a System of Differential Equations

Let $\dot{x} \equiv \frac{dx}{dt} = f(x),\ x \in R^n$ be a system of n-dimensional differential equations that has an equilibrium point x^* such that $f(x^*) = 0$, where t is interpreted as 'time'. The equilibrium point of this system is said to be *locally asymptotically stable*, if every trajectory starting sufficiently near the equilibrium point converges to it as $t \to +\infty$. If stability is independent of the distance of the initial state from the equilibrium point, the equilibrium point is said to be *globally asymptotically stable*, or *asymptotically stable in the large*, see Gandolfo (1996, p. 333).

2. Theorems that Are Useful for the Stability Analysis of a System of Linear Differential Equations or the Local Stability Analysis of a System of Nonlinear Differential Equations

Theorem A.1 (Local stability/instability theorem, see Gandolfo (1996, pp. 360–362)). *Let* $\dot{x}_i = f_i(x),\ x = [x_1, x_2, \ldots, x_n] \in R^n \mid (i = 1, 2, \ldots, n)$ *be an* n*-dimensional system of differential equations that has an equilibrium point* $x^* = [x_1^*, x_2^*, \ldots, x_n^*]$ *such that* $f(x^*) = 0$*. Suppose that the functions* f_i *have continuous first-order partial derivatives, and consider the Jacobian matrix evaluated at the equilibrium point* x^*

$$J = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1} & f_{n2} & \cdots & f_{nn} \end{bmatrix},$$

where $f_{ij} = \partial f_i / \partial x_j$ $(i, j = 1, 2, \cdots, n)$ *are evaluated at the equilibrium point.*

(i) *The equilibrium point of this system is locally asymptotically stable if all the roots of the characteristic equation* $|\lambda I - J| = 0$ *have negative real parts.*

(ii) *The equilibrium point of this system is unstable if at least one root of the characteristic equation* $|\lambda I - J| = 0$ *has positive real part.*

(iii) *The stability of the equilibrium point cannot be determined from the properties of the Jacobian matrix if all the roots of the characteristic equation* $|\lambda I - J| = 0$ *have non-positive real parts but at least one root has zero real part.*

Theorem A.2 (See Murata 1977, pp. 14–16). *Let* A *be an* $(n \times n)$ *matrix such that*

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}.$$

(i) *We can express the characteristic equation* $|\lambda I - A| = 0$ *as*

$$|\lambda I - A| = \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \cdots + a_r \lambda^{n-r} + \cdots + a_{n-1}\lambda + a_n = 0, \tag{10.1}$$

where

$$a_1 = -(traceA) = -\sum_{i=1}^{n} a_{ii}, \quad a_2 = (-1)^2 \sum_{i<j} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}, \ldots,$$

$$a_r = (-1)^r \sum_{i<j<\cdots<k} \underbrace{\begin{vmatrix} a_{ii} & a_{ij} & \cdots & a_{ik} \\ a_{ji} & a_{jj} & \cdots & a_{jk} \\ \vdots & \vdots & \ddots & \vdots \\ a_{ki} & a_{kj} & \cdots & a_{kk} \end{vmatrix}}_{(r)}, \ldots, \quad a_n = (-1)^n \det A.$$

(ii) *Let* λ_i $(i = 1, 2, \ldots, n)$ *be the roots of the characteristic equation* (10.1). *Then, we have*

$$traceJ = \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i, \quad \det A = \prod_{i=1}^{n} \lambda_i.$$

Theorem A.3 (Routh-Hurwitz conditions for stable roots in an n-dimensional system, cf. Murata (1977, p. 92), Gandolfo (1996, pp. 221–222)).[1] *All of the roots of the characteristic equation* (10.1) *have negative real parts if and only if the following set of inequalities is satisfied:*

$$\Delta_1 = a_1 > 0, \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix} > 0, \quad \Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0, \ldots,$$

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & a_7 & \cdots & 0 \\ 1 & a_2 & a_4 & a_6 & \cdots & 0 \\ 0 & a_1 & a_3 & a_5 & \cdots & 0 \\ 0 & 1 & a_2 & a_4 & \cdots & 0 \\ 0 & 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_n \end{vmatrix} > 0.$$

The following Theorems A.4–A.6 are corollaries of Theorem A.3.

Theorem A.4 (Routh-Hurwitz conditions for a two-dimensional system). *All of the roots of the characteristic equation*

$$\lambda^2 + a_1\lambda + a_2 = 0$$

have negative real parts if and only if the set of inequalities

$$a_1 > 0, \quad a_2 > 0$$

is satisfied.

Theorem A.5 (Routh-Hurwitz conditions for a three-dimensional system). *All of the roots of the characteristic equation*

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

have negative real parts if and only if the set of inequalities

$$a_1 > 0, \quad a_3 > 0, \quad a_1a_2 - a_3 > 0 \tag{10.2}$$

is satisfied.

[1] See also Gantmacher (1954) for many details that can be associated with and Brock and Malliaris (1989) for a compact representation of these conditions.

Remark on Theorem A.5. The inequality $a_2 > 0$ is always satisfied if the set of inequalities (10.2) is satisfied.

Theorem A.6 (Routh-Hurwitz conditions for a four-dimensional system). *All roots of the characteristic equation*

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0,$$

have negative real parts if and only if the set of inequalities

$$a_1 > 0, \quad a_3 > 0, \quad a_4 > 0, \quad \Phi \equiv a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0, \tag{10.3}$$

is satisfied.

Remark on Theorem A.6. The inequality $a_2 > 0$ is always satisfied if the set of inequalities (10.3) is satisfied.

3. Theorems that Are Useful for the Global Stability Analysis of a System of Nonlinear Differential Equations

Theorem A.7 (Liapunov's Theorem, cf. Gandolfo (1996, p. 410)). *Let* $\dot{x} = f(x), x = [x_1, x_2, \ldots, x_n] \in R^n$ *be an* n*-dimensional system of differential equations that has the unique equilibrium point* $x^* = [x_1^*, x_2^*, \ldots, x_n^*]$ *such that* $f(x^*) = 0$*. Suppose that there exists a scalar function* $V = V(x - x^*)$ *with continuous first derivatives and with the following properties* (1)–(5)*:*

(1) $V \geqq 0$,
(2) $V = 0$ *if and only if* $x_i - x_i^* = 0$ *for all* $i \in \{1, 2, \ldots n\}$,
(3) $V \to +\infty$ *as* $\|x - x^*\| \to +\infty$,
(4) $\dot{V} = \sum_{i=1}^n \frac{\partial V}{\partial(x_i - x_i^*)}\dot{x}_i \leqq 0$,
(5) $\dot{V} = 0$ *if and only if* $x_i - x_i* = 0$ *for all* $i \in \{1, 2, \ldots, n\}$*. Then, the equilibrium point* x^* *of the above system is globally asymptotically stable.*

Remark on Theorem A.7. The function $V = V(x - x^*)$ is called the "Liapunov function".

Theorem A.8 (Olech's Theorem, cf. Olech (1963), Gandolfo (1996, pp. 354–355)). *Let* $\dot{x}_i = f_i(x_1, x_2)$ $(i = 1, 2)$ *be a two-dimensional system of differential equations that has the unique equilibrium point* (x_1^*, x_2^*) *such that* $f_i(x_1^*, x_2^*) = 0$ $(i = 1, 2)$*. Suppose that the functions* f_i *have continuous first-order partial derivatives. Furthermore, suppose that the following properties* (1)–(3) *are satisfied:*

(1) $\frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} < 0$ *everywhere,*
(2) $(\frac{\partial f_1}{\partial x_1})(\frac{\partial f_2}{\partial x_2}) - (\frac{\partial f_1}{\partial x_2})(\frac{\partial f_2}{\partial x_1}) > 0$ *everywhere,*
(3) $(\frac{\partial f_1}{\partial x_1})(\frac{\partial f_2}{\partial x_2}) \neq 0$ *everywhere, or alternatively,* $(\frac{\partial f_1}{\partial x_2})(\frac{\partial f_2}{\partial x_1}) \neq 0$ *everywhere.*

Then, the equilibrium point of the above system is globally asymptotically stable.

4. Theorems that Are Useful to Establish the Existence of Closed Orbits in a System of Nonlinear Differential Equations

Theorem A.9 (Poincaré-Bendixson Theorem, cf. Hirsch and Smale (1974, Chap. 11)). *Let* $\dot{x}_i = f_i(x_1, x_2)$ $(i = 1, 2)$ *be a two-dimensional system of differential equations with the functions* f_i *continuous. A nonempty compact limit set of the trajectory of this system, which contains no equilibrium point, is a closed orbit.*

Theorem A.10 (Hopf Bifurcation Theorem for an n-dimensional system, cf. Guckenheimer and Holmes (1983, pp. 151–152), Lorenz (1993, p. 96) and Gandolfo (1996, p. 477)).[2] *Let* $\dot{x} = f(x; \varepsilon), x \in R^n, \varepsilon \in R$ *be an* n*-dimensional system of differential equations depending upon a parameter* ε*. Suppose that the following conditions* (1)–(3) *are satisfied:*

(1) *The system has a smooth curve of equilibria given by* $f(x^*(\varepsilon); \varepsilon) = 0$,
(2) *The characteristic equation* $|\lambda I - Df(x^*(\varepsilon_0); \varepsilon_0)| = 0$ *has a pair of pure imaginary roots* $\lambda(\varepsilon_0), \bar{\lambda}(\varepsilon_0)$ *and no other roots with zero real parts, where* $Df(x^*(\varepsilon_0); \varepsilon_0)$ *is the Jacobian matrix of the above system at* $(x^*(\varepsilon_0), \varepsilon_0)$ *with the parameter value* ε_0,
(3) $\left.\frac{d\{\mathrm{Re}\,\lambda(\varepsilon)\}}{d\varepsilon}\right|_{\varepsilon=\varepsilon_0} \neq 0$, *where* $\mathrm{Re}\,\lambda(\varepsilon)$ *is the real part of* $\lambda(\varepsilon)$.

Then, there exists a continuous function $\varepsilon(\gamma)$ *with* $\varepsilon(0) = \varepsilon_0$, *and for all sufficiently small values of* $\gamma \neq 0$ *there exists a continuous family of non-constant periodic solution* $x(t, \gamma)$ *for the above dynamical system, which collapses to the equilibrium point* $x^*(\varepsilon_0)$ *as* $\gamma \to 0$. *The period of the cycle is close to* $2\pi/\mathrm{Im}\,\lambda(\varepsilon_0)$, *where* $\mathrm{Im}\,\lambda(\varepsilon_0)$ *is the imaginary part of* $\lambda(\varepsilon_0)$.

Remark on Theorem A.10. We can replace the condition (3) in Theorem A.10 by the following weaker condition (3a) (cf. Alexander and York (1978)).

[2] See also Strogatz (1994), Wiggins (1990) in this regard.

(3a) For all ε which are near but not equal to ε_0, no characteristic root has zero real part.

The following theorem by Liu (1994) provides a convenient criterion for the occurrence of the so called 'simple' Hopf bifurcation in an n-dimensional system. The 'simple' Hopf bifurcation is defined as the Hopf bifurcation in which all the characteristic roots *except* a pair of purely imaginary ones have negative real parts.

Theorem A.11 (Liu's Theorem, see Liu (1994)). *Consider the following characteristic equation with $n \geqq 3$:*

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda_{n-2} + \cdots + a_{n-1}\lambda + a_n = 0.$$

This characteristic equation has a pair of pure imaginary roots and $(n-2)$ roots with negative real parts if and only if the following set of conditions is satisfied:

$$\Delta_i > 0 \text{ for all } i \in \{1, 2, \ldots, n-2\}, \quad \Delta_{n-1} = 0, \quad a_n > 0,$$

where Δ_i $(i = 1, 2, \ldots, n-1)$ are Routh-Hurwitz terms defined as

$$\Delta_1 = a_1, \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix}, \quad \Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}, \ldots,$$

$$\Delta_{n-1} = \begin{vmatrix} a_1 & a_3 & a_5 & a_7 & \cdots & 0 & 0 \\ 1 & a_2 & a_4 & a_6 & \cdots & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & \cdots & 0 & 0 \\ 0 & 1 & a_2 & a_4 & \cdots & 0 & 0 \\ 0 & 0 & a_1 & a_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_n & 0 \\ 0 & 0 & 0 & 0 & \cdots & a_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & a_{n-2} & a_n \\ 0 & 0 & 0 & 0 & \cdots & a_{n-3} & a_{n-1} \end{vmatrix}.$$

The following Theorems A.12–A.14 provide us with some convenient criteria for two-dimensional, three-dimensional, and four-dimensional Hopf bifurcations respectively. It is worth noting that these criteria provide us with useful information on the "non-simple" as well as the "simple" Hopf bifurcations.

Theorem A.12. *The characteristic equation*

$$\lambda^2 + a_1\lambda + a_2 = 0,$$

has a pair of pure imaginary roots if and only if the set of conditions

$$a_1 = 0, \quad a_2 > 0$$

is satisfied. In this case, we have the explicit solution $\lambda = \pm i\sqrt{a_2}$, *where* $i = \sqrt{-1}$.

Proof. Obvious because we have the solution $\lambda = (-a_1 \pm \sqrt{a_1^2 - 4a_2})/2$.

Theorem A.13 (cf. Asada (1995), Asada and Semmler (1995)).
The characteristic equation

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

has a pair of pure imaginary roots if and only if the set of conditions

$$a_2 > 0, \quad a_1a_2 - a_3 = 0,$$

is satisfied. In this case, we have the explicit solution $\lambda = -a_1,\ \pm i\sqrt{a_2}$, *where* $i = \sqrt{-1}$.

Theorem A.14 (cf. Asada and Yoshida (2003), Yoshida and Asada (2007)). *Consider the characteristic equation*

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0. \tag{10.4}$$

(i) *The characteristic equation* (10.4) *has a pair of pure imaginary roots and two roots with non-zero real parts if and only if either of the following set of conditions* (A) *or* (B) *is satisfied:*
   (A) $a_1a_3 > 0, \quad a_4 \neq 0, \quad \Phi \equiv a_1a_2a_3 - a_1^2a_4 - a_3^2 = 0.$
   (B) $a_1 = a_3 = 0, \quad a_4 < 0.$
(ii) *The characteristic equation* (10.4) *has a pair of pure imaginary roots and two roots with negative real parts if and only if the following set of conditions* (C) *is satisfied:*
   (C) $a_1 > 0, \quad a_3 > 0, \quad a_4 > 0, \quad \Phi \equiv a_1a_2a_3 - a_1^2a_4 - a_3^2 = 0.$

Remark on Theorem A.14.

1. The condition $\Phi = 0$ is always satisfied if the set of conditions (B) is satisfied.

2. The inequality $a_2 > 0$ is always satisfied if the set of conditions (C) is satisfied.
3. We can derive Theorem A.14(ii) from Theorem A.11 as a special case with $n = 4$, although we cannot derive Theorem A.14(i) from Theorem A.11.

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