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Stefan Seifert

Posted Price Offers in Internet Auction Markets

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With 47 Figures
and 21 Tables

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Karlsruhe, July 2006 *Stefan Seifert*

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Introduction

On eBay—currently one of the world's most successful internet marketplaces—the seller of an item can choose among three basic mechanisms: first, she can offer the item at a fixed or *posted* price $\bar{p}$; second, she can conduct an auction with a reserve price $\underline{p}$; or, third—and this is the main issue of this study—, she can set up an auction with a reserve price $\underline{p}$ and at the same time grant an interested bidder the option to buy the item at a posted price $\bar{p}$.[1]

An interesting question is which of these selling mechanisms a rational seller decides upon and why. In order to answer this question, the behavior of the potential buyers in the respective mechanisms must be taken into consideration. The present analysis addresses this issue and studies an auction format which is similar to the one offered by eBay. In particular, the following questions arise and are investigated:

- When does a bidder accept a posted price offer? What are the key determinants for a bidder's decision?
- How does a posted price offer impact the seller's revenue, the bidders' payoffs, and the generated total surplus?
- Does it pay for the seller to also offer a posted price when conducting an auction? What is an optimal, i. e. revenue maximizing, posted price offer?

1.1 Overview

The analysis applies a two-fold approach. In Chapter 2, a game theoretical model of an auction with a posted price offer is developed. The model considers the auction as a two-stage game. In the first stage, the seller decides on the

[1] To simplify the use of pronouns, the seller of an item is considered female and the potential buyers (generally bidders) are referred to by male pronouns.

amount of the posted price—if offered at all—and offers it to one of the potential bidders. In the second stage, the bidder to whom the offer is made decides whether or not to accept it. If a posted price is not offered or if it is rejected, the item is auctioned among all bidders. The game is solved by backward induction: Firstly, the equilibrium strategies of the bidders are derived given a posted price offer by the seller. Secondly, the equilibrium outcome of the second stage, subject to the seller's choice in the first stage, then allows for the computation of the seller's optimal strategy, i. e. the posted price offer that maximizes her revenue.

After the theoretical analysis, an experimental investigation is conducted in order to compare the theoretical predictions with the behavior of human participants in the lab. In the experiment, subjects take on the roles of both the sellers and the bidders. In parallel to the theoretical analysis, the sellers may opt to offer a posted price and choose the respective amount. In addition, the behavior of bidders in auctions with a posted price offer is observed. The design of the experiment is presented in Chapter 3 and the results are reported in Chapter 4. Chapter 5 discusses the limitations of the model and concludes with an outlook on future work.

Auctions and posted price markets have both been intensively studied in economic theory.[2] Moreover, the use of experiments to investigate the two market institutions is widely spread.[3] Yet, relatively little is known about hybrid mechanisms which combine an auction with a posted price offer.

It is only recently that theoretical literature on hybrid institutions combining auctions with a posted price has started to evolve.[4] The theoretical model of the present analysis ties in with this discussion. It applies the independent private values assumptions, yet places no further restrictions on the number of bidders or their attitudes towards risk, and also allows for almost arbitrary distributions of valuations. Thus, it is more general than most of the existing papers. Moreover, the author is not aware of any experimental study investigating an auction augmented by an additional posted price offer. The experimental analysis presented in this paper bridges this gap.

[2] An overview of auction theory is given, e. g., in the survey by Klemperer (1999) as well as the textbooks by Krishna (2002) or Milgrom (2004). The theory of pricing in a monopoly or in an oligopoly is discussed in most microeconomics textbooks (e. g. Kreps 1990, Chapters 9 and 10; Mas-Colell et al. 1995, Chapter 12). Wang (1993) compares auctions versus posted-price selling.

[3] See, e. g., Kagel (1995) and Davis und Holt (1993, Chapter 4), respectively, for an overview of auction and posted price experiments.

[4] These papers are discussed in Section 2.6.

1.2 General Background

The analysis of an auction with a posted price offer (henceforth abbreviated as APPO) is of a microeconomic nature. One investigates how a given set of individual agents behaves in a particular mechanism. The rules of the mechanism are kept simple, the set of strategic alternatives is small, and outside options are usually not considered.

The micro-level analysis constitutes the foundations for an evaluation of the APPO market institution at the business level. Being able to estimate, for example, how much a posted price offer is worth to the seller allows the operator of a market to set the corresponding fees accordingly. Thus, the analysis may provide a market operator with valuable information regarding an optimal pricing scheme.

The structured design, implementation, and evaluation of markets is subsumed under the term *market engineering*. According to Weinhardt et al. (2003), the research program market engineering comprises a systematic and theoretically grounded approach to the analysis, design, implementation, quality assurance, and further development of electronic markets. Holistic market engineering focuses not only on the design of *new* markets; rather, the study of institutions is considered one of the core activities (Neumann, 2004, p. 127ff). In this sense, market engineering refers to an ongoing process which continuously seeks to adapt to changes in the market environment, to better serve the customers' needs, or to take advantage of new technological developments.

Thus, the analysis of existing markets or market institutions like an APPO may give valuable hints for potential improvement or the development of new products and features. Within this framework, the present study pertains to the evaluation of the economic performance of a market whilst the two-fold analysis relates to the *axiomatic* and the *experimental approaches* (Neumann, 2004, p. 170ff).

Clearly, auctions and posted price offers differ significantly as market institutions and in terms of their characteristics. Nevertheless, they are not as contrarian as they may seem at first glance. Rather, both a pure auction and a pure posted price offer can be interpreted as special cases of the hybrid APPO institution. Wang et al. (2004) point out that in the former institution, the posted price is set so high that it is never accepted whereas in the latter case it is set so low that it is never rejected.[5] In fact, combined auction/posted price mechanisms are neither new nor only observed at internet marketplaces. Budish und Takeyama (2001, p. 326, fn. 2) note that an auction with a posted price "bears some resemblance to the '$100-or-best-offer' pric-

[5] This paper is discussed in more detail in Section 2.6.

ing mechanism often found in secondhand markets." According to Mathews (2003), the phenomenon of auctions augmented with a buyout option was first noted explicitly by Lucking-Reiley (2000).

1.3 The Buy It Now Option on eBay

Several versions of auctions with a posted price offer can be found at different internet marketplaces. The APPO model considered in the present analysis most closely resembles eBay's auction with a *Buy It Now* option.[6] Before the model is introduced in Chapter 2, its eBay archetype is described in the following.[7]

Consider first a pure fixed price offer on eBay. The marketplace refers to such an offer as the *Buy It Now* price. The offer is listed on eBay's internet page for a certain period of time. As soon as a buyer accepts the offer, he acquires the item and pays the price the seller has asked for.

As with the posted price, an auction on eBay lasts for a pre-defined period during which interested bidders are invited to submit (maximum) bids. The auction starts at a reserve price $\underline{p}$ that the seller has set.[8] Bidders then enter so-called "maximum bids" that cannot be lower than the reserve price. At any given time, the pseudonym of the current *high bidder*,[9] i. e. the bidder who has submitted the highest maximum bid, as well as the *current price* of the item is publicly revealed.[10] If no or only one bid has been entered, the current price equals the reserve price $\underline{p}$ that the seller has set. Once two or more bids have been submitted, the current price is the lesser of the highest maximum bid and the second highest maximum bid plus a given *bid increment*. In case of a tie, i. e. if the two highest maximum bids are equal, the high bidder is the bidder who submitted his bid first and the current price is equal to his maximum bid. The rules of eBay further require that any new bid must exceed the current price by at least the bid increment. The bidder who is listed as highest bidder

[6] See `http://www.ebay.com` or `http://www.ebay.de`.

[7] Note that variations of the basic mechanism also exist. An important feature on ebay.com is, for example, the option to specify a secret reserve price that is not revealed to the bidders. This option as well as other variants are not addressed in this analysis.

[8] On eBay the reserve price is called "starting bid."

[9] The pseudonym is a self-chosen user name that a bidder (or seller) uses to log on to the eBay platform. There is not necessarily a one-to-one relationship between user names and persons, since several individuals can share one account (e. g. families) or one person might maintain several accounts.

[10] Different rules apply to so-called private auctions.

at the end of the auction acquires the item and pays the auction's closing price.

If, finally, the seller decides to conduct an auction with a posted price, a bidder can either bid in the auction or acquire the item for the posted price. The latter option, however, is only available as long as no bid has been entered. If the bidder opts for the posted price, the auction closes immediately and the bidder acquires the item for the posted price. If, on the other hand, the bidder submits a bid in the auction, the option to acquire the item for the fixed price expires. Thenceforward, bidders may only bid in the auction, and the above rules of an eBay auction apply to both the winner and the price determination.[11]

1.4 A First Assessment

By means of an auction, the seller of an item is always able to generate revenues at least as high as those generated by using a fixed price offer. To see this, consider a fixed price offer $\bar{p}$ and compare it with an auction in which the seller sets the reserve price $\underline{p} = \bar{p}$. If the item is sold for the fixed price, the seller collects $\bar{p}$. In the same situation, the seller would collect *at least* $\underline{p} = \bar{p}$ in an auction because the closing price of the auction could well be above the reserve price, but never below. If, on the other hand, the item is not sold in the auction, no bidder values the item above the reserve price. Thus, the item would not be sold for the posted price either, and the revenues are zero in both institutions.

One might ask whether the performance regarding the revenues of the hybrid institution, which combines an auction with a posted price offer, lies somewhere in between a pure auction and a pure fixed price offer. Example 1.1, however, shows that—at least ex–post—this is not true in general.

Example 1.1. Consider the situation of two risk neutral bidders, 1 and 2, whose independent private valuations are uniformly distributed over $[0; 1]$ and let the reserve price be zero. Further assume that in a specific case the valuations of the bidders 1 and 2 are $v_1 = 0.9$ and $v_2 = 0.2$, respectively. The

[11] Interesting variations of this mechanism can be found on Yahoo! (`http://auctions.shopping.yahoo.com`) and LabX (`http://www.labx.com`). On the former site, the option to acquire the item for the posted price does not expire after the first bid has been entered. Moreover, the auction closes immediately once bidding reaches the posted price. Thus, the posted price also constitutes an upper bound of the auction's revenue. Similarly, on LabX the posted price does not expire upon the submission of an auction bid. Rather, the option expires 24 hours before the scheduled closing time of the auction.

probability distribution of the valuations is common knowledge; the actual valuations, however, are private information of the respective bidders.

Neglecting the bid increment, an auction generates revenues of 0.2 because bidder 2 will quit the auction if the current price reaches that amount.

Bidder 1, who does not know the valuation v_2 of bidder 2, wins the auction if v_2 does not exceed his own valuation. In this case he acquires the item at a price equal to v_2 and gains $v_1 - v_2$. If bidder 1 does not win the auction, his payoff is 0. Since valuations are uniformly distributed, the ex-ante expected gain of bidder 1 calculates to $\int_0^{0.9}(0.9 - x)\,dx = \frac{81}{200}$.

Assume now that bidder 1 can choose between a posted price offer $\bar{p} = 0.4$ and participating in the auction. If he accepts the posted price offer, he gains $v_1 - \bar{p} = \frac{1}{2}$, which is higher than the expected gain of $\frac{81}{200}$ from participating in the auction. Thus, the bidder will accept the posted price offer and the seller collects revenues of $\bar{p} = 0.4$, which are higher than the revenues of 0.2 generated by the alternative auction. □

Given a particular set of valuations, in Example 1.1 the auction combined with a posted price offer outperforms a (pure) auction in terms of the seller's revenue. The following chapters investigate under which conditions this holds more generally.

2

Model of the APPO Market Institution

In this chapter, a model of an auction with a posted price offer (APPO) will be developed. As is common in the auction literature, an APPO is modeled as a game with incomplete information (e. g. Wilson, 1992). In such a game, a player i has a certain type $\theta_i \in \Theta_i$. Each player knows his own type, but does not know the types of the other players. However, the ex-ante (joint) probability distribution of the players' types is common knowledge, as are the players' payoff functions contingent upon their types.[1]

To analyze games with incomplete information, Harsanyi (1967, 1968a,b) introduces the concept of a Bayesian equilibrium, which is a natural extension of the Nash equilibrium.[2] Unless otherwise stated, the term equilibrium refers to a Bayesian equilibrium in the subsequent analysis.

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[1] See, e. g., Fudenberg und Tirole (1991) for an introduction to the theory of games with incomplete information.

[2] Technically, a game with incomplete information can be transformed into an extensive form game with imperfect information. In the transformed game, an additional move by an artificial player "nature" is introduced prior to any action by a player of the original game. In this first move (or the first n moves), nature assigns each of the players a type. The information sets of the players are such that they can observe only their own types, and not those of the other bidders. In the transformed game, a player's incomplete information becomes imperfect information about nature's moves. Thus, the concept of a Nash equilibrium can be applied to the transformed game (see Appendix A.1 for the notion of a Nash equilibrium). A Bayesian equilibrium characterizes a strategy profile of the incomplete information game that corresponds to a Nash equilibrium in the transformed or expanded game with imperfect information.

2.1 Preliminaries

2.1.1 Basic Assumptions

The APPO model is based on the *symmetric independent private valuations* assumptions (SIPV, cf. e. g. McAfee und McMillan, 1987; Wolfstetter, 1999; Krishna, 2002). In such a setting, a bidder's type is simply a number that represents his (monetary) valuation of the item for auction.

- **(A1)** All bidders know with certainty their own monetary valuation of an item put up for auction. However, bidders do not know the other bidders' valuations of the item.
- **(A2)** Bidders consider all individual valuations but their own as random variables. The (ex-ante) distribution function of a bidder's valuation is common knowledge. All valuations are stochastically independent.
- **(A3)** Bidders are symmetric, i. e. the distribution function of the valuations is identical for all bidders.

Most authors who analyze auctions in an SIPV setting also assume that bidders are risk neutral. The analysis of this chapter forgoes this assumption. In fact, when analyzing the posted price offer, the possibility of risk averse bidders will explicitly be taken into account.

2.1.2 Modeling the eBay Auction

The pure variant of the eBay auction, i. e. an auction in which the item for sale is not also offered at a posted price, has similarities to both the English and the second-price auctions. The similarity to the English auction relates to its open bidding process and the fact that bidders are not restricted to only one bid. In principle, they could repeatedly bid the minimum required bid, i. e. the current price plus the bid increment, and quit the auction if that minimum bid reaches or exceeds their valuation. Such a bidding process does not differ from an English auction.[3]

On its help pages, however, eBay suggests another strategy to bidders:[4]

[3] In an English auction, the sketched strategies even constitute an equilibrium if the required minimum increment is assumed to be small enough to be neglected in the analysis (cf., e. g., Krishna, 2002, p. 5; Milgrom und Weber, 1982a; Milgrom, 1989; Smith, 1987).

[4] See `http://pages.ebay.com/help/buy/proxy-bidding.html` (July 8, 2004).

Here's how bidding on eBay works:

1. When you place a bid, you enter the maximum amount you'd be willing to pay for the item. Your maximum amount is kept confidential from other bidders and the seller.
2. The eBay system compares your bid to those of the other bidders.
3. The system places bids on your behalf, using only as much of your bid as is necessary to maintain your high bid position (or to meet the reserve price). The system will bid up to your maximum amount.
4. If another bidder has a higher maximum, you'll be outbid. But, if no other bidder has a higher maximum, you win the item. And you could pay significantly less than your maximum price! This means you don't have to keep coming back to re-bid every time another bid is placed.

According to eBay, the advantage of the above strategy is that it makes "bidding on auctions more convenient and less time-consuming for buyers."[5] The description on ebay.com, however, remains rather vague on what it means by "as is necessary to maintain your high bid position." Tests with a real auction carried out on the site show that—if at least two valid bids have been placed—the current price is always equal to the lesser of the highest maximum bid and the second highest maximum bid plus the required bid increment.[6]

The technique in which a bidder submits his maximum bid to the platform and allows the system to bid on his behalf is also referred to as *proxy bidding*. The suggested procedure obviously resembles a second-price auction. Since, however, the final price is generally higher than the second highest bid, eBay's auction format does not precisely conform to a second-price auction and the following Example 2.1 shows that bidding one's true maximum willingness to pay is not a dominant strategy.[7] In the example, the timing of bids is also considered—an issue that has been neglected so far and that is investigated in more detail in Section 2.1.3.

Example 2.1. Consider an auction that starts with a reserve price of \$ 25 and let three bidders $i = 1, 2, 3$ participate in that auction. The three bidders are willing to pay a maximum of \$ 35, \$ 31, and \$ 30, respectively. As on eBay, the bid increment is \$ 1 and bidders can bid any dollar and cents amount.

Assume first that all bidders follow eBay's advice: Bidder 1 starts the bidding process by submitting a bid of \$ 35. He becomes the high bidder and the current price is \$ 25. Now bidder 2 bids \$ 31. This causes the current price

[5] Ibid.

[6] On the respective German site ebay.de, eBay states more clearly "Sie bezahlen niemals mehr als Ihr Maximalgebot, sondern stets nur so viel, dass Sie den Zweitbieter mit einem Erhöhungsschritt überbieten" (`http://pages.ebay.de/help/buy/proxy-bidding.html`, July 8, 2004).

[7] See Appendix A.1 for the notion of a dominant strategy.

to jump to \$ 31 + \$ 1 = \$ 32 and bidder 3 to refrain from bidding in the auction. Bidder 1 wins the auction at the final price of \$ 32.

Bidder 1 can, however, do better. An example is the following bid sequence: Suppose that first bidder 3 places a bid of \$ 30. The current price then yields the reserve price of \$ 25. If bidder 1 now enters a bid of \$ 30.01, the current price increases to \$ 30.01 and no further bids will be placed. Bidder 1 again wins the auction but pays only \$ 30.01, i. e. almost two increments less than according to the strategy suggested by eBay. □

The bids in Example 2.1 constitute a (Nash) equilibrium, i. e. no bidder can increase his payoff by deviating from his strategy given that all other bidders choose the presented strategies.[8] In fact, many more (Nash) equilibria exist, but there is no equilibrium that fulfills stronger requirements such as ex-post robustness or even dominance of strategies.[9]

Clearly, the required minimum bid increment and eBay's rule for determining the current price are the main differences between a second-price auction and the auction institution applied by eBay. The minimum increment also defines the magnitude of the maximum difference between the bidders' equilibrium payoffs in a second-price auction and in an auction on eBay.[10] For simplicity, in the remainder of the text it is assumed that the required minimum bid increment is small and that it can be neglected in the analysis. If one further restricts the strategy space to only one bid, an auction on eBay is in fact equivalent to a second-price auction: the bidder who submits the highest bid wins the auction and pays a price equal to the second highest bid. It has been known since Vickrey (1961) that in such a situation, it is a (weakly) dominant strategy for bidders to bid their true valuation (cf. also Krishna, 2002). Thus, the bidder who values the auctioned item most will win the auction and pay the second highest valuation.

Clearly, the open and iterative format of the eBay auction, i. e. the option to submit multiple bids and to react to other bidders' actions, allows

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[8] Example 2.1 also constitutes a counter-example to the claim by Ockenfels und Roth (forthcoming, p. 6, fn. 8) that in an auction on eBay "bidding one's true value is a strategy that always yields a payoff not more than the minimum increment s below the supremum achievable by any other strategy, regardless of the strategies chosen by the other bidders."

[9] The definition of a Nash equilibrium as well as an equilibrium in dominant strategies is given in Appendix A.1. A strategy profile $(\sigma_1(v_1), \sigma_2(v_2), \ldots, \sigma_n(v_n))$ is said to form an ex-post equilibrium if and only if for every bidder i and for every profile of valuations $v = (v_1, v_2, \ldots, v_n)$ the strategy $\sigma_i(\cdot)$ is a best response to $\sigma_{-i}(v_{-i})$ (cf., e. g., Holzman und Monderer, 2004).

[10] Smith (1987) notes that in an English auction, the equilibrium price must lie in the interval $[v_{(2)} - \delta, v_{(2)} + \delta]$ if the minimum increment is denoted by δ. The same applies to eBay's proxy auction.

for a much wider scope of strategic alternatives. The possibility for bidders to act as in an English auction is only one of them. Note, however, that in a private values setting, the revelation of information about other bidders' valuations during the bidding process is of no relevance for the bidders' strategies. Regardless of his attitude towards risk, a rational bidder will bid up to his valuation and quit the auction once the current price reaches that amount (cf. e. g. Kagel, 1995; Milgrom, 1989). If the minimum bid increment is assumed to be small or negligible, again the bidder with the highest valuation wins the auction and pays a price equal to the second highest valuation. Thus, extending the strategy space to that of an iterative auction format does not alter the outcome.

The following analysis of eBay's auction with a posted price offer builds on that result. Whenever the posted price offer is *not* available to a bidder, the auction is modeled as a sealed-bid second-price auction. It is assumed that all bidders follow their dominant strategies and bid their true valuations. Consequently, the auction's revenues are assumed to equal the second highest valuation.

2.1.3 Timing of Bids

In neither of the two stylized interpretations of an eBay auction presented above is the concise timing of a bid of strategic significance—as long as a bid is submitted at all. Nonetheless, the process of an auction on eBay and the timing of bids have been subjects of academic investigations (e. g. Bajari und Hortaçsu, 2003; Ockenfels und Roth, forthcoming; Roth und Ockenfels, 2002). A particularly interesting area of research relates to the ending rule of an eBay auction. Remember that an auction on eBay closes at a pre-determined point in time (*hard close*) as opposed to accepting additional bids for at least a minimum time interval after the receipt of every new bid (*soft close*).[11] A central result of the above investigations is that there are equilibria in which bidders only submit their bids at the very last moment of an auction—a phenomenon that is commonly referred to as *sniping*.

In the following, the line of reasoning for sniping is summarized. It is then argued that the incentives for sniping do not exist if a posted price offer is available to a bidder. Thus, when modeling a hybrid institution which combines an auction with a posted price offer in the following sections, it will

[11] The terms "hard close" and "soft close" were introduced by Roth and Ockenfels (cf. e. g. Roth und Ockenfels, 2002). Examples of auctions with a soft close include the internet auctions conducted by Amazon.com (`http://auctions.amazon.com`) or Yahoo! (`http://www.auctions.yahoo.com`).

be assumed that a bidder acts immediately if he can choose between accepting the posted price offer and bidding in an auction.

In order to specifically address the issue of eBay's strict ending rule, modeling the eBay auction (with no posted price offer) as a two-stage game has been suggested. Bajari und Hortaçsu (2003), for example, consider an auction that closes at time T. According to their model, the first stage consists of an open ascending auction that stretches until $T - \epsilon$. During the first stage, bidders can submit bids, drop out of the auction, observe other bidders drop out, and rejoin the auction without restrictions. The second stage resembles the last moment of the eBay auction. It lasts from $(T - \epsilon)$ until T—a time interval too short for bidders to react to other bids. This stage reflects a second-price sealed-bid auction in which all bidders—including those who dropped out or did not bid at all in the first stage—are invited to submit a final bid.

In the cited paper, Bajari und Hortaçsu specify an econometric model for analyzing empirical data. Investigating a setting which is based on Wilson's (1977) symmetric common value model, they claim that "[b]idding zero (or not bidding at all) in the first stage of the auction and participating only in the second stage of the auction is a symmetric Nash equilibrium of the eBay auction" (ibid., p. 338). Thus, according to their model, bids may be submitted only shortly before the auction closes.

The main driver for Bajari und Hortaçsu's claim is that in the analyzed common value setting, bidders have an incentive not to reveal their private signal or estimate of the true common value of the item to other bidders in the open auction of the first stage.[12]

Ockenfels und Roth (forthcoming) use a similar approach. In their model, the first stage lasts over the time interval $[0; T)$ and the final stage takes place precisely at T. However, they further stipulate that a bid submitted at the final stage is only successfully transmitted with a positive probability $\epsilon < 1$. With the probability $(1 - \epsilon)$, the bid gets lost or does not reach the auction site in time due to erratic delays in internet traffic. Ockenfels und Roth show that in their model, sniping pays even in a private values setting—the setting that has also been chosen for this analysis.

To illustrate profitable sniping, the authors consider an example in which two bidders, 1 and 2, both value the item for auction at h. If both bid in an

[12] The incentive of bidders not to reveal private information is a well-known result of auction theory. Cf., e. g., Milgrom und Weber (1982a,b) for a discussion on revealing private information. Milgrom und Weber (1982a) show in particular that if the item for sale has a common value component, the expected price in an English auction is not less than in a second-price sealed-bid auction due to the revelation of information during the bidding process (linkage principle).

auction up to their valuation, they both receive a payoff of zero.[13] Another symmetric equilibrium that yields higher expected payoffs is given by the following strategy profile: On the equilibrium path, neither bidder submits a bid prior to T. At time T, both players submit a bid of h. Off the equilibrium path, a bidder immediately bids h as soon as the other player submits a bid prior to T. If the reserve price is given by $\underline{p} < h$, the expected payoff of a bidder is $\epsilon\,(1-\epsilon)\,(h-\underline{p}) > 0$.

Ockenfels und Roth claim that the above example can be extended to the case with n bidders and non-degenerated distributions of bidders' valuations. A strategy profile of late bidding (sniping) may still constitute an equilibrium. The result, however, is sensitive to the parameters $\underline{p}$ and ϵ, the number of bidders, and the distribution of valuations.

Note that in the above auction setting there are other equilibria that yield even higher expected payoffs. The following strategy profile is an example. Following Ockenfels und Roth, for once the bid increment s is taken into consideration in order to precisely establish the properties of an equilibrium. The presented strategies, however, differ from those of the cited authors.

Assume that h is much larger than $\underline{p}$ and s (in particular, assume that $\underline{p} + 2s < h$) and let bidder 1 join the auction slightly before bidder 2, i. e. bidder 1 can submit a bid before bidder 2 is able to act. Now consider the following strategies: Bidder 1 submits a bid b slightly above the reserve price $\underline{p}$ plus one increment s, but below $\underline{p}+2s$ (i. e. $\underline{p}+s < b < \underline{p}+2s$) before bidder 2 enters the scene. The current price is then the reserve price $\underline{p}$. Now bidder 2 submits a bid of $\underline{p}+s$. Due to eBay's rules of incrementing the current price, it will jump to b, but bidder 1 remains the high bidder. On the equilibrium path, no further bids will be submitted before T, but at T, both bidders again bid h. As above, off the equilibrium path both bidders will immediately bid h if the other bidder submits a bid before T. Because $b < \underline{p}+2s$, and because bidder 1 remains the high bidder, bidder 2's bid reveals to all bidders that neither bidder has bid his full valuation h. This allows the enforcement of the strategies. Applying this strategy profile, bidder 1 receives a payoff of $(h-b)$ with a probability of $(1-\epsilon)\,(1-\epsilon)$, (i. e. the case in which none of the bids at T reaches the auction platform in time) and a payoff of $(h-\underline{p}-2s)$ with a probability of $\epsilon\,(1-\epsilon)$ (i. e. the case in which only bidder 1's bid is successfully transmitted). Bidder 2 obtains a positive profit if only his bid is successfully transmitted. Thus, his expected payoff yields $\epsilon\,(1-\epsilon)\,(h-\underline{p}-2s-\epsilon)$. If both bidders submit their bid successfully at T, they both receive a payoff of 0.

[13] Ockenfels und Roth do not ignore the minimum bid increment. In that respect their analysis is more precise than the summary above. The simplification of the presentation, however, does not affect the main result.

Suppose the ex-ante probability of becoming bidder 1 or bidder 2 is 0.5. Since h is assumed to be much larger than $\underline{p}$ and s, the expected payoffs of both bidders then exceed the expected payoffs in the cited example from Ockenfels und Roth.

For a (pure) auction on eBay, both of the above strategy profiles are examples of equilibria with private valuations (subject to suited parameters, such as the reserve price, the bid increment, or the probability distribution of valuations). By analyzing the properties of a perfect equilibrium, it will be established that an immediate action dominates waiting if a posted price offer is available to a bidder.[14] Note that this does not rule out sniping: after a bidder has submitted a very low bid, the posted price offer expires and the bidders might continue to bid according to strategies that involve sniping. The strategy profile given in the previous paragraph is an example.

For the next proposition, it is helpful to denote the history of an auction at time τ by a set H_τ, which contains all information about the auction process up until τ that has been made available by eBay's bidding platform.

Proposition 2.1 (Immediate action). *Consider an auction on eBay in which the item is also offered at a posted price. Let the bidders* $i = 1, 2, \ldots, n$ *have private valuations for the auctioned item and let the auction close at* T. *At any point in time* $\tau \leq T$, *a bidder may either wait, i. e. do nothing, place a bid* $b \in \mathbb{R}_+$, *or—as long as the history* H_τ *of the auction is the empty set* $\emptyset$*—accept the posted price offer. A strategy* σ_i *of bidder* i *maps any point of time* τ *and any history* H_τ *into an action* $a = \sigma_i(\tau, H_\tau)$ *with* $a \in \{\text{‘wait’}\} \cup \mathbb{R}_+ \cup \{\text{‘accept’}\}$.

If, at time τ, *the posted price offer is available to a (risk neutral or risk averse) bidder* i *who values the item higher than the reserve price, then in a perfect equilibrium* σ^* *bidder* i *either places an immediate bid of some amount* b *or immediately accepts the posted price offer, i. e.* $\forall\, i, \tau : \sigma_i^*(\tau, \emptyset) \neq \text{‘wait’}$.

Proof. (Sketch.) Assume to the contrary that at τ a bidder decides to wait, even though no bids have been submitted by that time, i. e. $\exists\, i, \tau : \sigma_i^*(\tau, \emptyset) =$ ‘wait’. If the strategy of the waiting bidder is part of a strategy profile σ^* which constitutes a perfect equilibrium, then a sequence of (mixed) strategy profiles s^z and a sequence of trembling functions η^z exist with $\lim_{z\to\infty} \eta^z = 0$ and $\lim_{z\to\infty} s^z = \sigma^*$.[15] Moreover, for all z the profile s^z constitutes a

[14] See Berninghaus et al. (2002) or van Damme (1991) for the notion of a perfect equilibrium. The concept goes back to Selten (1975).

[15] Note that the notation is somewhat vague. Strictly speaking, a strategy profile which constitutes a perfect equilibrium is itself a mixed strategy profile. Addi-

Nash equilibrium of the perturbed game $\Gamma(\eta^z)$ in which the strategy space is restricted to the set of mixed strategies that assign each pure strategy σ_i a positive density of at least $\eta^t(\sigma_i)$. Thus, within the next time interval, both the acceptance of the posted price offer and the submission of a bid occur with positive probability.

In order to sketch the remainder of the proof, it will be assumed for simplicity's sake that in the perturbed game only those bidders competing with the considered bidder are restricted to the perturbed mixed strategies. In that case, the strategy σ_i^* of the considered waiting bidder must be a best reply, even if the other bidders choose the action 'accept' or bid some amount b with positive probability within the next time interval.

Because the considered bidder values the item above the reserve price, his expected utility from participating in the auction is at least zero.

Assume first that at time τ, the utility of the considered bidder from accepting the posted price offer is higher than his expected utility from participating in the auction. If he does not accept the offer immediately, then in the perturbed game, another bidder either accepts the posted price offer, or places a bid in the auction during the next small time interval with positive probability. In the former case, the utility of the considered bidder is reduced to zero; in the latter case, the bidder is forced into an auction because the posted price offer expires. According to the assumption, however, the auction yields lower expected utility than the posted price offer. Thus, the action 'wait' cannot belong to an equilibrium strategy.

Assume now that the considered bidder prefers an auction to the posted price offer. The bidder realizes the maximum payoff in the auction if he obtains the item for the reserve price. In the chosen setting with private valuations there is no disadvantage of placing such a bid immediately. The considered bidder i wins the auction if his valuation v_i is the highest valuation among all bidders. Denote the bidder with the second highest valuation by j and his valuation by v_j. In the perturbed game, bidder j will accept the posted price offer with positive probability. Because $v_j < v_i$, bidder i is well-advised to act quickly. By bidding immediately he will extinguish the posted price offer and win the auction. ■

According to Proposition 2.1, the posted price offer is available to a bidder in only two cases:

1. The bidder is the first to become aware of the auction.

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tional notational difficulties arise because both the time and the amount of a bid are continuous variables. This sketch of proof, however, reflects the idea of the reasoning behind the proposition.

2. All bidders who became aware of the auction before the considered bidder valued the item below the reserve price.

The APPO model focuses on the first case. For the sake of clarity, in an APPO there is only one bidder to whom the posted price offer is available. If that bidder rejects the option, an auction will be conducted. Thus, the APPO model differs slightly from the mechanism applied by eBay.

Still, an APPO is very similar to its archetype, and the analysis of the APPO model provides a valuable insight into the mechanism applied by eBay. The model thus helps to better understand the strategies and decisions of bidders and sellers. From the buyer's point of view the difference mentioned above can be neglected: If a bidder 0 becomes aware of an auction on eBay at time τ_0, he estimates the number n of bidders $j = 1, \ldots, n$ who will also become aware of the auction before it closes at T. He will also estimate the distribution of the highest valuation among all these bidders and calculate his expected utility from participating in the auction. In an SIPV setting, the valuations and strategies of bidders $j = 1, \ldots, n$ or, more precisely, the expected utility of bidder 0 from participating in the auction is independent of how many bidders with a valuation lower than the reserve price became aware of the auction before τ_0.[16]

2.1.4 Definition of an APPO

Based on the preparatory remarks in the previous sections, this section introduces a formal model of an auction with a posted price offer. A fundamental assumption of the model is that the posted price option is offered to exactly one randomly selected bidder. If this bidder rejects the offer, an auction will be conducted and the posted price offer will not be offered to any other bidder. The underlying assumption is that the bidder who is the first to become aware of an APPO's archetype—a real auction with a posted price offer on eBay—is in an advantageous position: he can choose to accept the posted price offer or he can submit an auction bid. According to Proposition 2.1, the bidder will submit a bid immediately if he favors an auction to a posted price offer. In this case, the posted price offer expires on eBay as it does in an APPO. There are, however, two differences: First, in an APPO, a bidder cannot relinquish his right to exercise the posted price offer so that it can be

[16] Of course, this does not hold for the seller: Imagine that a bidder -1 becomes aware of an auction at time τ_{-1}. Since the bidder's valuation is lower than the reserve price, he not only rejects the posted price offer but also declines to bid in the auction. In this case, it makes a difference to the seller whether the next bidder 0 is again granted the posted price offer or not.

exercised by another bidder. Second, if a bidder on eBay decides not to bid in an auction at all because the reserve price exceeds his valuation, the posted price offer does not expire. It does expire, however, in an APPO.

Due to the hard ending rule of eBay, its proxy bidding mechanism, and its rule of incrementing the current price, there is no common agreement among researchers how to model an auction on eBay (cf. Section 2.1.2). In this analysis, the approach taken by Roth und Ockenfels (2002) is adopted and eBay's auction format is modeled as a standard second-price auction.

Definition 2.2 (APPO market institution). *In an* auction with a posted price offer (APPO)*, the seller of an item sets both a reserve price* $\underline{p}$ *and a posted price* $\bar{p}$*. A randomly selected bidder is then asked whether he wants to buy the item at the posted price. If the selected bidder accepts this offer he acquires the item at the price* $\bar{p}$ *and no auction will be conducted. If, on the other hand, the selected bidder rejects the offer, the item is auctioned among all bidders by means of a second-price auction with the reserve price* $\underline{p}$*.*

The bidder who is offered the item at the posted price is called the decisive bidder *and the posted price offer is abbreviated as* PPO*.*

Definition 2.3 (Corresponding auction). *Let A be an APPO with a reserve price* $\underline{p}$*. A second-price auction with the same reserve price* $\underline{p}$ *but no PPO is called the* corresponding second-price auction *of A or* corresponding auction *for short.*

Throughout this chapter it is assumed that a finite number n of bidders participate in an APPO. Let $\mathcal{N}$ be the set of bidders. A bidder is referred to by $i \in \mathcal{N}$ and the decisive bidder by $\hat{\imath} \in \mathcal{N}$. Note that the number n of bidders may not be known in advance. In this case, n is considered the realization of a random variable N whose probability distribution is known to all bidders and the seller. If $n \geq 1$, there is exactly one decisive bidder in each APPO.

The analysis refers to a symmetric independent private values setting. Each bidder $i \in \mathcal{N}$ values the item at v_i. The valuations v_i are independent draws of a random variable V with a cumulative probability distribution function (cdf) $F\colon \mathbb{R} \to [0;1]$ that has a non-empty, convex support $\mathcal{M} \subset \mathbb{R}$. Moreover, V is assumed to have a probability density function (pdf) that is referred to by $f := \mathbb{R} \to \mathbb{R}_+$.

Let $(v_{(1)}, v_{(2)}, \ldots, v_{(n)})$ denote the ordered vector of n independent drawings v_i of V with $v_{(1)} \geq v_{(2)} \geq \ldots \geq v_{(n)}$. The valuations $v_{(1)}$ and $v_{(2)}$ can then be interpreted as the realizations of random variables $V_{(1),n}$ and $V_{(2),n}$, respectively, or $V_{(1)}$ and $V_{(2)}$ for short. A realization of $V_{(i),n}$ is the ith highest value of a sample of size n. If the number n of drawings is known in advance, the distribution and the density of $V_{(1)}$ and $V_{(2)}$ are referred to by $F_{(1),n}(v), f_{(1),n}(v), F_{(2),n}(v)$, and $f_{(2),n}(v)$.[17] If n is not known in advance but is itself the realization of a random variable, or if the number of bidders is not of interest in a particular context, the distribution and the density of $V_{(1)}$ and $V_{(2)}$ are denoted by $G_{(1)}(v), g_{(1)}(v), G_{(2)}(v)$, and $g_{(2)}(v)$, respectively. The highest valuation of all bidders but bidder i is denoted by $v_{(1),-i} = \max_{j\neq i}\{v_j\}$ and its distribution and density by $G_{(1),-i}(v)$ and $g_{(1),-i}(v)$, respectively.

2.2 Equilibrium Strategies and the Acceptance Threshold

It is a well-known result of auction theory that in a second-price auction with private valuations, an equilibrium in (weakly) dominant strategies exists (e. g. Krishna, 2002, p. 15). Obviously, this also holds in the corresponding auction of an APPO if the decisive bidder has rejected the PPO. Proposition 2.4 establishes that result.

Proposition 2.4 (Equilibrium in the corresponding auction). *Consider an APPO in which the decisive bidder has rejected the PPO and let i be a bidder with valuation v_i.*

In the corresponding auction, it is a (weakly) dominant strategy for bidder i to bid his valuation v_i if $v_i \geq \underline{p}$ and not to bid if $v_i < \underline{p}$.

Proof. Note that an APPO can be considered as a two-stage game. In the first stage, the decisive bidder chooses whether or not to accept the PPO. If the decisive bidder accepts the PPO, the game ends and the bidders will take no further actions. If, however, the decisive bidder rejects the PPO, a second-price auction will be conducted as a second stage. Thus, the equilibrium of the corresponding auction is the equilibrium of the subgame of the APPO that starts with the rejection of the PPO. A proof that the strategy given in the proposition in fact constitutes an equilibrium in dominant strategies of the corresponding second-price auction is omitted here and the reader is referred to the literature (cf., e. g., Krishna, 2002, Milgrom, 1989). ■

In a second-price auction, the equilibrium in dominant strategies is independent of the bidders' attitudes towards risk (Kagel, 1995). A bidder's attitude towards risk, however, does affect his decision whether or not to accept an APPO's posted price offer. Let $u_{\hat{\imath}} \colon \mathbb{R} \to \mathbb{R}$ denote bidder $\hat{\imath}$'s (von Neumann–Morgenstern) utility function and assume that the utility function is strictly

[17] For an introduction to order statistics cf. Appendix A.3.

increasing and differentiable.[18] Given that the decisive bidder $\hat{\imath}$ values the item at $v_{\hat{\imath}}$, his utility is

$$u_{\hat{\imath}}(v_{\hat{\imath}} - \bar{p}) \tag{2.1}$$

if he accepts the PPO. If, on the other hand, bidder $\hat{\imath}$ rejects the offer and an auction is being conducted, several cases must be distinguished: Firstly, $u_{\hat{\imath}}(0)$ indicates bidder $\hat{\imath}$'s utility if he is not awarded the item. Secondly, with a probability of $G_{(1),-\hat{\imath}}(\underline{p})$ bidder $\hat{\imath}$ obtains the item for the reserve price $\underline{p}$, in which case his utility is $u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})$. Thirdly, if $\hat{\imath}$ values the item highest among all bidders and if the highest valuation $v_{(1),-\hat{\imath}}$ of all other bidders is higher than the reserve price, his utility yields $u_{\hat{\imath}}(v_{\hat{\imath}} - v_{(1),-\hat{\imath}})$. Remember that the density of $v_{(1),-\hat{\imath}}$ is given by $g_{(1),-\hat{\imath}}(v_{(1),-\hat{\imath}})$ and assume—without loss of generality—that $u_{\hat{\imath}}(0) = 0$.[19] Then the expected utility of bidder $\hat{\imath}$ is

$$Eu_{\hat{\imath}} = \begin{cases} 0 & \text{if } v_{\hat{\imath}} < \underline{p}, \\ u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, g_{(1),-\hat{\imath}}(x)\, dx & \text{otherwise} . \end{cases} \tag{2.2}$$

Equations (2.1) and (2.2) shed light on the optimal strategy of the decisive bidder, i. e. the strategy that maximizes his expected utility. The decisive bidder will accept the PPO only if the utility from accepting the PPO is at least as high as the expected revenues from the auction. If $v_{\hat{\imath}} \geq \underline{p}$, the condition for accepting the PPO yields

$$u_{\hat{\imath}}(v_{\hat{\imath}} - \bar{p}) \geq u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, g_{(1),-\hat{\imath}}(x)\, dx \ . \tag{2.3}$$

Applying the inverse of the utility function to (2.3), one obtains[20]

$$\begin{aligned} & v_{\hat{\imath}} - \bar{p} \geq u_{\hat{\imath}}^{-1}\left(u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, g_{(1),-\hat{\imath}}(x)\, dx \right) \\ \iff & \bar{p} \leq v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, g_{(1),-\hat{\imath}}(x)\, dx \right) . \end{aligned} \tag{2.4}$$

[18] Cf. e. g. Mas-Colell et al. (1995, p. 170ff) for the notion of von Neumann-Morgenstern utility (von Neumann and Morgenstern, 1944).

[19] Von Neumann-Morgenstern utility representations of a preference relation $\prec$ over lotteries are unique up to affine transformations $\alpha u(\cdot) + \beta$ with $\alpha > 0$ and $\beta \in \mathbb{R}$ (Kreps, 1990, p. 76f). If necessary, a preference-preserving transformation of the utility function with an appropriately chosen β ensures the assumption.

[20] Since $u_{\hat{\imath}}(\cdot)$ is strictly increasing, the inverse $u_{\hat{\imath}}^{-1}(\cdot)$ of the utility function is well-defined.

In order to further elaborate on the strategy which maximizes expected revenues, Definition 2.5 introduces the concept of a bidder's acceptance threshold, which plays a crucial role in the remainder of the analysis.

Definition 2.5 (Acceptance threshold). *Consider an APPO with a reserve price $\underline{p}$ and a posted price offer $\bar{p}$ and let the decisive bidder $\hat{\imath}$ value the item at $v_{\hat{\imath}} > \underline{p}$. Then the term*

$$t_{\hat{\imath}}(v_{\hat{\imath}}) := v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left(u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, g_{(1),-\hat{\imath}}(x)\, dx \right)$$

is called the <u>acceptance threshold</u> *of the decisive bidder and the inequality*

$$\bar{p} \leq t_{\hat{\imath}}(v_{\hat{\imath}})$$

is called the <u>APPO threshold condition</u>.

The acceptance threshold of Definition 2.5 is a rather complex term. Lemma 2.6 introduces an alternative representation.

Lemma 2.6 (Alternative form of acceptance threshold). *Equivalently to Definition 2.5, the acceptance threshold of the decisive bidder in an APPO can be written as*

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left(\int_{\underline{p}}^{v_{\hat{\imath}}} u'_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x)\, dx \right) . \qquad (2.5)$$

Proof. According to Definition 2.5, the acceptance threshold is given by

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left(u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, g_{(1),-\hat{\imath}}(x)\, dx \right) .$$

Integration by parts yields

$$\begin{aligned} t_{\hat{\imath}}(v_{\hat{\imath}}) &= v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \Bigg(u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \Big[u_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x) \Big]_{\underline{p}}^{v_{\hat{\imath}}} \\ &\qquad + \int_{\underline{p}}^{v_{\hat{\imath}}} u'_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x)\, dx \Bigg) \\ &= v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left(u_{\hat{\imath}}(0)\, G_{(1),-\hat{\imath}}(v_{\hat{\imath}}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u'_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x)\, dx \right) \\ &= v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left(\int_{\underline{p}}^{v_{\hat{\imath}}} u'_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x)\, dx \right) . \end{aligned}$$

■

For illustration, Figure 2.1 plots a decisive bidder's threshold versus his valuation, which is given on the abscissa. Remember that the acceptance thres–

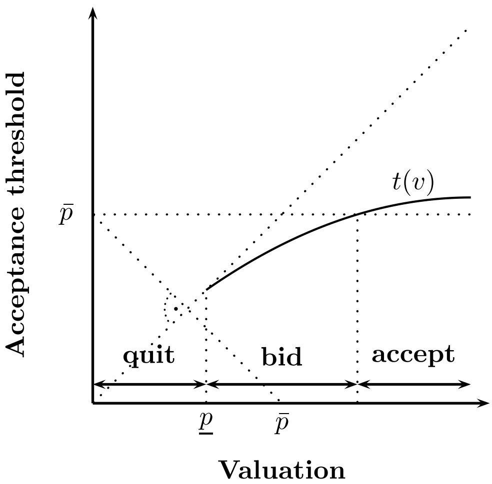

Figure 2.1. Strategy choices of a bidder

hold is only defined for valuations higher than the reserve price $\underline{p}$. In the figure, different valuation areas are mapped to the strategic options 'quit', 'bid', and 'accept'. Based on the preceding calculations, the following Theorem 2.7 establishes that the bidder accepts the PPO if his threshold exceeds the posted price $\bar{p}$. From Proposition 2.4 one knows that the bidder does not submit a bid if his valuation is lower than the reserve price $\underline{p}$. In all other cases, the bidder submits a bid equal to his valuation. As will be shown in Section 2.5, the bidding area is not always a convex set.

Theorem 2.7 (Acceptance of the PPO). *A decisive bidder $\hat{\imath}$ who values the item for auction in an APPO at $v_{\hat{\imath}} \geq \underline{p}$ accepts a posted price offer $\bar{p}$ only if the threshold condition holds. He strictly prefers the PPO over the corresponding auction if the threshold condition holds strictly and he is indifferent between accepting the PPO and bidding in the auction if the threshold condition holds with equality.*

Proof. The "only–if"–part follows from the Equations (2.1)–(2.4). The proofs of the "if"–part and the "indifference"–part run analogously. ■

A bidder who is offered a posted price option compares the utility of accepting the PPO with the expected utility of rejecting it, i. e. the expected utility from participating in the corresponding auction. The acceptance threshold indicates the maximum posted price offer that a bidder with a given valuation would accept, or, more precisely, the threshold is the posted price at which a bidder is indifferent between accepting the offer and rejecting it. If offered the posted price option, a bidder $\hat{\imath}$ accepts any $\bar{p} < t_{\hat{\imath}}(v_{\imath})$ and rejects any $\bar{p} > t_{\hat{\imath}}(v_{\hat{\imath}})$. Given the parameter $\underline{p}$ of an APPO, the cdf $G_{(1),-\hat{\imath}}(\cdot)$, and bidder $\hat{\imath}$'s utility function $u_{\hat{\imath}}(\cdot)$, the acceptance threshold is a function of $v_{\hat{\imath}}$.

Example 2.2. Consider an APPO with n symmetric bidders and independent private valuations that are uniformly distributed over the interval $[0; 1]$. The cdf of the valuations is

$$F(v) = \begin{cases} 0 \text{ if } & v < 0, \\ v \text{ if } & 0 \leq v \leq 1, \\ 1 \text{ if } & 1 < v, \end{cases} \tag{2.6}$$

and the density is given by

$$f(v) = \begin{cases} 0 \text{ if } & v < 0, \\ 1 \text{ if } & 0 \leq v \leq 1, \\ 0 \text{ if } & 1 < v\ . \end{cases} \tag{2.7}$$

Thus, the first-order statistic's density is $f_{(1),n}(v) = n\ F^{n-1}(v)\ f(v)$ (cf. Appendix A.3).

Assume further that the decisive bidder $\hat{\imath}$ is risk neutral and let his utility of an auction's payoff x be $u_{\hat{\imath}}(x) = x$. Applying Lemma 2.6, the decisive bidder's acceptance threshold with respect to his valuation $v_{\hat{\imath}}$ is then given by

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(\int_{\underline{p}}^{v_{\hat{\imath}}} u'_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x)\, dx\right) \tag{2.8}$$

$$= v_{\hat{\imath}} - \int_{\underline{p}}^{v_{\hat{\imath}}} x^{n-1}\, dx \tag{2.9}$$

$$= v_{\hat{\imath}} + \frac{1}{n}\, \underline{p}^{\,n} - \frac{1}{n}\, v_{\hat{\imath}}^{\,n}\ . \tag{2.10}$$

Figure 2.2 on the next page illustrates the acceptance threshold of the example for $\underline{p} = 0$ and $n = 3$ as well as $n = 5$ bidders. □

2.3 Properties of the Acceptance Threshold

Figure 2.2 shows that the threshold functions lie below the first bisecting line. This, of course, is a general result because accepting a PPO at a price that is

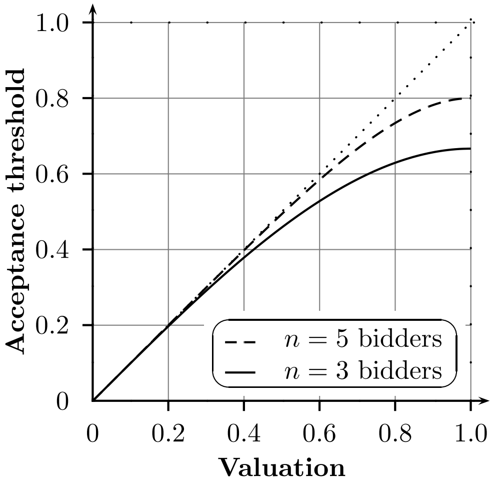

Figure 2.2. Acceptance threshold for risk neutral bidders with [0; 1] uniformly distributed valuations and $\underline{p} = 0$

higher than a bidder's valuation leads with certainty to a loss of the respective bidder.[21] The first bisecting line thus constitutes an upper bound of the acceptance threshold. In fact, the bisecting line is the acceptance threshold of an extremely risk averse bidder who follows a *maximin*-strategy. Such a bidder (weakly) prefers any posted price offer that does not exceed his valuation over the corresponding auction, since in the auction the bidder bears the risk of not obtaining the item at all, which would yield a payoff of zero.

The threshold function of a risk neutral bidder, on the other hand, constitutes the lower bound of the threshold of a risk averse bidder: Such a bidder prefers obtaining the item with certainty to the random outcome of the corresponding auction. In order to ensure the certain outcome at the given price $\bar{p}$, he is willing to pay a (risk) premium on top of the risk neutral threshold. Thus, for any given valuation, the threshold of a risk averse bidder is higher than the threshold of a risk neutral bidder. The opposite is the case for risk loving bidders whose threshold lies below the risk neutral threshold. In the case of an extremely risk loving bidder, i. e. a bidder who always prefers a lottery over any certain outcome of that same lottery, the threshold is given by the valuation-axis. Such a bidder would never accept a positive posted price offer if there is a chance of obtaining the item for less in the corresponding auction.

[21] More formally, the result follows immediately from Lemma 2.6.

Proposition 2.9 confirms the above argument by comparing the threshold functions of two bidders who differ in their degree of risk aversion in the sense of Arrow and Pratt (cf. A.2). Prior to stating the proposition, however, a lemma is introduced that will be useful for the proof of the proposition.

Lemma 2.8 (Variation of Jensen's inequality). *Let f, g, and h be three functions with domain and range $\mathbb{R}_+$ and $a, b \in \mathbb{R}_+$ (or $b = \infty$) with $a < b$. Assume further that $\int_a^b f(x)\,dx \leq 1$ and $h(0) = 0$. If h is concave, then*

$$h\left(\int_a^b f(x)\,g(x)\,dx\right) \geq \int_a^b f(x)\,h(g(x))\,dx \enspace .$$

Proof. Define $\alpha := \int_a^b f(x)$. Note that $0 < \alpha \leq 1$. If h is concave, then for all x

$$\begin{aligned}
&h\left(\int_a^b f(x)\,g(x)\,dx\right) \\
&= h\left(\alpha\left(\frac{1}{\alpha}\int_a^b f(x)\,g(x)\,dx\right) + (1-\alpha)\,0\right) \\
&\geq \alpha\,h\left(\frac{1}{\alpha}\int_a^b f(x)\,g(x)\,dx\right) + (1-\alpha)\,h(0) \\
&= \alpha\,h\left(\int_a^b \frac{1}{\alpha}\,f(x)\,g(x)\,dx\right) \enspace .
\end{aligned}$$

By construction $\int_a^b \frac{1}{\alpha} f(x)\,dx = \frac{1}{\alpha}\int_a^b f(x)\,dx = 1$ and Jensen's inequality, as stated e. g. in Sydsæter et al. (2000, p. 42), can be applied. One obtains

$$\begin{aligned}
&\alpha\,h\left(\int_a^b \frac{1}{\alpha}\,f(x)\,g(x)\,dx\right) \\
&\geq \alpha\int_a^b \frac{1}{\alpha}\,f(x)\,h(g(x))\,dx \\
&= \int_a^b f(x)\,h(g(x))\,dx
\end{aligned}$$

which proves the lemma. ■

Since Lemma 2.8 does not require $\int_a^b f(x)\,dx = 1$, it is more general than the assumptions of Jensen's inequality in that respect. Note, however, that the lemma requires $h(0) = 0$. In that sense it is more restrictive than the latter.

Proposition 2.9 (Acceptance threshold and risk aversion). *Let $i, j \in \mathcal{N}$ be two bidders, of whom i is the more risk averse, i. e.*

$$\frac{u_i''(x)}{u_i'(x)} \geq \frac{u_j''(x)}{u_j'(x)} \quad \forall\, x \in \mathbb{R} \ .$$

Then the threshold functions $t_i(\cdot)$ and $t_j(\cdot)$ of the two bidders satisfy

$$t_i(v) \geq t_j(v) \quad \forall\, v \in \mathcal{M} \ .$$

Proof. Let $u_i(\cdot)$ and $u_j(\cdot)$ denote the utility functions of the two bidders i and j and assume that bidder i is more risk averse than bidder j. According to Pratt's Theorem, a concave function $z\colon \mathbb{R} \to \mathbb{R}$ exists such that $u_i(v) = z(u_j(v))\ \forall\, v$ (see Appendix A.2). Since $u_i(0) = u_j(0) = 0$ and $u_i(x), u_j(x) > 0\ \forall\, x > 0$, one has $z(0) = 0$ and $\lambda z(x) \geq z(\lambda x)\ \forall\, \lambda \geq 1$.

With symmetric distributions of bidders' valuations $G_{(1),-i} \equiv G_{(1),-j}$ and $g_{(1),-i} \equiv g_{(1),-j}$. For convenience, define $G(x) := G_{(1),-i}(x)\ \forall\, x$ and $g(x) := g_{(1),-i}(x)\ \forall\, x$ as well as $\xi := \frac{\int_{\underline{p}}^{v} u_j(v-x)\, g_{(1),-j}(x)\, dx}{1-G_{(1),-j}(\underline{p})}$.

To be shown is

$$\begin{aligned}
t_i(v) \qquad &\geq \qquad t_j(v) \qquad \forall\, v \in \mathcal{M} \\
\iff v - u_i^{-1}&\left(u_i(v-\underline{p})\, G_{(1),-i}(\underline{p}) + \int_{\underline{p}}^{v} u_i(v-x)\, g_{(1),-i}(x)\, dx \right) \\
&\geq v - u_j^{-1}\left(u_j(v-\underline{p})\, G_{(1),-j}(\underline{p}) + \int_{\underline{p}}^{v} u_j(v-x)\, g_{(1),-j}(x)\, dx \right) \\
\iff u_i(v-\underline{p})&\, G_{(1),-i}(\underline{p}) + \int_{\underline{p}}^{v} u_i(v-x)\, g_{(1),-i}(x)\, dx \\
&\leq z\left(u_j(v-\underline{p})\, G_{(1),-i}(\underline{p}) + \int_{\underline{p}}^{v} u_j(v-x)\, g_{(1),-i}(x)\, dx \right) \ . \qquad (2.11)
\end{aligned}$$

Starting with the right-hand side of Inequality (2.11), one obtains

$$\begin{aligned}
& z\left(u_j(v-\underline{p})\, G_{(1),-j}(\underline{p}) + \int_{\underline{p}}^{v} u_j(v-x)\, g_{(1),-j}(x)\, dx \right) \\
= \;& z\left(u_j(v-\underline{p})\, G_{(1),-j}(\underline{p}) + \xi \left(1 - G_{(1),-j}(\underline{p})\right)\right) \\
\geq \;& G_{(1),-j}(\underline{p})\, z(u_j(v-\underline{p})) + \left(1 - G_{(1),-j}(\underline{p})\right)\, z(\xi) \qquad \text{(concavity of } z\text{)}
\end{aligned}$$

$$
\begin{aligned}
&= G_{(1),-j}(\underline{p})\, z(u_j(v-\underline{p})) \\
&\quad + \left(1-G_{(1),-j}(\underline{p})\right)\, z\left(\int_{\underline{p}}^{v} u_j(v-x)\, \frac{g_{(1),-j}}{1-G_{(1),-j}(\underline{p})}\, dx\right) \\
&\geq G_{(1),-j}(\underline{p})\, z(u_j(v-\underline{p})) \\
&\quad + \left(1-G_{(1),-j}(\underline{p})\right) \int_{\underline{p}}^{v} z(u_j(v-x))\, \frac{g_{(1),-j}}{1-G_{(1),-j}(\underline{p})}\, dx \quad \text{(Lemma 2.8)} \\
&= G_{(1),-j}(\underline{p})\, u_i(v-\underline{p}) + \int_{\underline{p}}^{v} u_i(v-x)\, g_{(1),-j}\, dx \qquad \text{(Pratt's theorem)} \\
&= u_i(v-\underline{p})\, G_{(1),-i}(\underline{p}) + \int_{\underline{p}}^{v} u_i(v-x)\, g_{(1),-i}\, dx \qquad \text{(symmetry)}
\end{aligned}
$$

which equals the left-hand side of Inequality (2.11) and thus proves the proposition. ■

Example 2.2 also suggests that a bidder's threshold increases with his valuation. While this is true if bidders are risk neutral, it does not hold in general. The following Proposition 2.10 establishes the former and the subsequent (counter-) Example 2.3 proves the latter.

Proposition 2.10 (Monotonicity of the threshold in the valuation). *The threshold function $t_i(v_i)$ of a risk neutral bidder i is strictly increasing in v_i.*

Proof. Note first that the utility function $u_i(x) = x$ captures a risk neutral bidder's preferences (cf. fn. 19 on p. 19). The Equation (2.5) of Lemma 2.6 can thus be simplified to

$$
t_i(v_i) = v_i - \int_{\underline{p}}^{v_i} G_{(1),-i}(x)\, dx
$$

and the threshold's derivative is

$$
\begin{aligned}
t_i'(v_i) &= 1 - G_{(1),-i}(v_i) \\
&> 0 \qquad\qquad \forall\, v_i < \sup(\mathcal{M}) \ .
\end{aligned}
$$

■

The fact that the acceptance threshold may be decreasing in the valuation is somewhat counter-intuitive. Example 2.3, however, shows that this may be the case if bidders are risk averse. In light of Proposition 2.10, the example is even more surprising. Remember that the acceptance threshold of a risk averse bidder lies in between the risk neutral threshold and the first bisecting line, which are both strictly increasing on $\mathcal{M}$.

Example 2.3. Let $\hat{\imath}$ be a risk averse bidder with $u_{\hat{\imath}}(x) = \sqrt{x}$ and assume further that a total of $n = 2$ bidders participate in the APPO. As in Exam– ple 2.2, $\underline{p} = 0$ and valuations are independently and uniformly distributed on $[0; 1]$. The threshold function of the decisive bidder is[22]

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - \frac{4}{9} v_{\hat{\imath}}^3 \ . \tag{2.12}$$

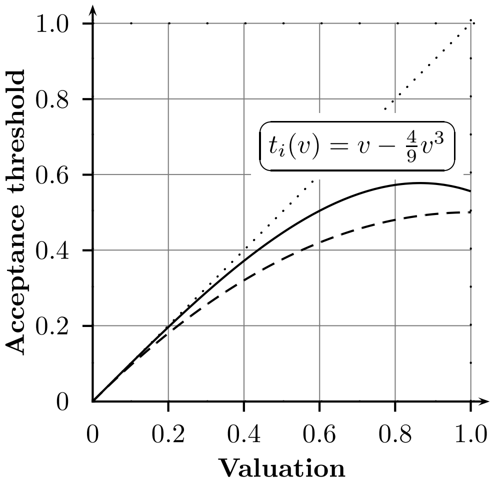

Figure 2.3. Acceptance threshold of a risk averse bidder

In Figure 2.3 the solid line represents the threshold function $t_{\hat{\imath}}(v_{\hat{\imath}})$. For low valuations, the threshold rises with the valuation. It declines, however, for valuations that are larger than $\frac{\sqrt{3}}{2}$. The reason is that the risk premium declines sharply as the bidder's valuation approaches the supremum of $\mathcal{M}$. The dotted and the dashed lines, respectively, contrast the acceptance threshold against the first bisecting line and the corresponding threshold of a risk neutral decisive bidder. □

Example 2.2 in Section 2.2 also suggests that the threshold increases with the number of bidders. Ceteris paribus, this is true regardless of the decisive bidder's risk attitude.

[22] The calculation of the threshold function is rather mechanical. In Appendix A.4 the threshold function for the general case with n bidders and a decisive bidder with a utility function $u(v) = v^{\alpha}$ $(\alpha > 0)$ is derived. Equation (A.7) in the Appendix can be used as a formula here.

Proposition 2.11 (Effect of the number of bidders on the threshold). *Let i be a bidder in an APPO with a reserve price $\underline{p}$ and a known number of (symmetric) bidders. For any given valuation v_i, the threshold $t_i(v_i)$ rises with the number n of bidders.*

Proof. Let F denote the common cdf of the bidders' valuations and u_i bidder i's utility function. Apply Equation (A.3) in the Appendix A.3 to Lemma 2.6. Then the acceptance threshold $t_i(v_i)$ yields

$$t_i(v_i) = v_i - u_i^{-1}\left(\int_{\underline{p}}^{v_i} u_i'(v_i - x)\, F^{n-1}(x)\, dx\right) \ .$$

Since $u_i(x)$ is strictly increasing in x, u_i^{-1} is strictly increasing too, and $u_i'(\cdot) > 0$. Further, $F^{n-1}(x)$ decreases in n, which proves the proposition. ■

Proposition 2.12 finally states that a bidder's acceptance threshold rises with the reserve price. Again, this result holds no matter what the bidder's risk attitude is.

Proposition 2.12 (Effect of the reserve price on the threshold). *For any given valuation v_i, the acceptance threshold $t_i(v_i)$ of bidder i increases with the APPO's reserve price $\underline{p}$ independent of the bidder's attitude towards risk.*

Proof. Again, the proof starts with Lemma 2.6:

$$t_i(v_i) = v_i - u_i^{-1}\left(\int_{\underline{p}}^{v_i} u_i'(v_i - x)\, G_{(1),-i}(x)\, dx\right) \ .$$

The derivative $g'(z)$ of the inverse $g \equiv f^{-1}$ of an invertible function f defined on $I \subset \mathbb{R}$ is $g'(z) = \frac{1}{f'(g(z))}$ whenever $f'(g(z)) \neq 0$ (see, e. g., Simon und Blume, 1994, p. 79). Thus, the derivative of $t_i(v_i)$ with respect to $\underline{p}$ yields

$$\begin{aligned}\frac{d}{d\underline{p}} t_i(v_i) &= \frac{u_i'(v - \underline{p})\, G_{(1),-i}(\underline{p})}{u_i'\left(\int_{\underline{p}}^{v_i} u_i'(v_i - x)\, G_{(1),-i}(x)\, dx\right)} \\ &> 0 \ .\end{aligned}$$

■

Proposition 2.12 shows that the reserve price and the PPO are not inde– pendent of each other. The higher the seller sets the reserve price, the higher the likelihood that the decisive bidder will accept the PPO. The reason is obvious: the higher the reserve price is, the less attractive the corresponding auction for the bidder is.

2.4 Revenues in an APPO

When analyzing auctions, the expected revenues of the auctioneer are of particular interest. In this section, the expected revenues of an APPO are investigated and compared to those of a (pure) second-price auction.

First intuition suggests that offering a posted price option favors the bidders at the expense of the seller. The decisive bidder accepts the PPO if that offer is more attractive than the corresponding second-price auction. In fact, a risk neutral bidder accepts the PPO if it is sufficiently low compared to the expected price in the corresponding auction. Since, however, these two numbers also correspond to the seller's revenues, giving the bidder the possibility to choose between the two alternatives conflicts with the sellers' goal to maximize revenues.

Theorem 2.13 shows that offering a posted price option in fact lowers the seller's expected revenues if the decisive bidder is risk neutral.

Theorem 2.13 (Revenues from an APPO with risk neutral bidders). *If bidders are risk neutral (or risk loving) and if there is a positive probability that the decisive bidder will accept the posted price offer, the expected revenues from an APPO are lower than those from the corresponding second-price auction.*

Proof. If the decisive bidder rejects the PPO, a second-price auction with the reserve price $\underline{p}$ will be conducted. In this case the revenues from the APPO are equal to the revenues from the corresponding auction.

If, on the other hand, the decisive bidder accepts the PPO, the APPO revenues are given by $\bar{p}$ and—because of Theorem 2.7—the threshold condition holds. Note that $u_{\hat{\imath}}(v_{\hat{\imath}}-\underline{p})\, G_{(1),-\hat{\imath}}(\underline{p})+\int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}}-x)\, g_{(1),-\hat{\imath}}(x)dx$ is the expected utility of the decisive bidder in the corresponding auction, conditional on $v_{\hat{\imath}} \geq \underline{p}$. If bidder $\hat{\imath}$ is risk neutral (or risk loving) this expected utility is at least as high as his utility of the expected outcome of the corresponding auction, conditional on $v_{\hat{\imath}} \geq \underline{p}$. One obtains

$$\begin{aligned}\bar{p} &\leq v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(u_{\hat{\imath}}(v_{\hat{\imath}}-\underline{p})\, G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}}-x)\, g_{(1),-\hat{\imath}}(x)\, dx\right)\\ &\leq v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(u_{\hat{\imath}}\left((v_{\hat{\imath}}-\underline{p})G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} (v_{\hat{\imath}}-x)\, g_{(1),-\hat{\imath}}(x)\, dx\right)\right)\\ &= v_{\hat{\imath}} + \underline{p}G_{(1),-\hat{\imath}}(\underline{p}) - v_{\hat{\imath}}G_{(1),-\hat{\imath}}(v_{\hat{\imath}}) + \int_{\underline{p}}^{v_{\hat{\imath}}} x\, g_{(1),-\hat{\imath}}(x)\, dx \ . \end{aligned} \tag{2.13}$$

Consider now the alternative revenues R^C from the corresponding auction, conditional on both the threshold condition and $v_{\hat{\imath}} \geq \underline{p}$, i.e. consider the revenues the seller could obtain by a pure auction, given that the decisive bidder $\hat{\imath}$'s valuation is so high that he would accept a PPO. Two mutually exclusive cases can be distinguished: (i) bidder $\hat{\imath}$ wins the auction and (ii) bidder $\hat{\imath}$ does not win the auction. In the first case $R^C_{(i)} = \max\{\underline{p}, v_{(1),-\hat{\imath}}\}$ and in the second case $R^C_{(ii)} \geq v_{\hat{\imath}}$ holds. Thus, the expected value of R^C is

$$\begin{aligned} E[R^C] = &\qquad E[R^C_{(i)}] \qquad + \qquad E[R^C_{(ii)}] \\ &> \underline{p}G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} x\, g_{(1),-\hat{\imath}}(x)\, dx + v_{\hat{\imath}}(1 - G_{(1),-\hat{\imath}}(v_{\hat{\imath}})) \\ &= v_{\hat{\imath}} + \underline{p}G_{(1),-\hat{\imath}}(\underline{p})) - v_{\hat{\imath}}G_{(1),-\hat{\imath}}(v_{\hat{\imath}}) + \int_{\underline{p}}^{v_{\hat{\imath}}} x\, g_{(1),-\hat{\imath}}(x)\, dx \qquad (2.14) \\ &\geq \bar{p}\ . \qquad (2.15) \end{aligned}$$

Put another way, if the decisive bidder accepts the PPO with positive prob– ability, the expected revenues from the APPO are lower than those of the corresponding second–price auction. ■

In the case of risk neutral bidders, Theorem 2.13 confirms the intuitive conjecture that the seller does not gain from offering a posted price option (cf. p. 29). Of course, the seller can set the PPO high enough so that it does not damage revenues. If, for example, she sets the PPO above the supremum of the support of the bidders' valuations, it will never be accepted and consequently the PPO does not affect revenues. Example 1.1 has shown that in certain cases a rational bidder accepts the PPO even though he would have won the alternative corresponding auction at a lower price, i.e. the corresponding auction would have generated less revenue than the acceptance of the PPO. Theorem 2.13, however, states that this does not hold for the ex–ante expected values: the revenues of an APPO are lower than the *expected* revenues of an alternative (pure) auction if the PPO is accepted. For the expected revenues of an APPO, the result holds, even if the PPO is only accepted with positive probability and not with certainty.

The above argument does not hold if the decisive bidder $\hat{\imath}$ is risk averse. To see this, consider again the case of an extremely risk averse bidder who follows the maximin strategy. The bidder accepts any PPO $\bar{p}$ that does not exceed his valuation $v_{\hat{\imath}}$. Note that the first–order statistic of the valuations of all bidders but the decisive bidder constitutes an upper bound of the expected revenues independent of $v_{\hat{\imath}}$. Since any PPO $\bar{p} < \sup(\mathcal{M})$ will be accepted with positive probability by a decisive bidder applying the maximin strategy, a PPO with $E[V_{(1),-\hat{\imath}}] < \bar{p} < \sup(\mathcal{M})$ increases the expected revenues.

Interestingly, offering a posted price option does not only pay if bidders (or at least the decisive bidder) are strongly risk averse. According to Theorem 2.14, it is possible for the seller to set a PPO $\bar{p}$ such that the expected revenues in the APPO are higher than in the corresponding auction even if the decisive bidder is only slightly risk averse.

Theorem 2.14 (Revenues from an APPO with risk averse bidders). *If the decisive bidder is risk averse, then a PPO $\bar{p}$ exists such that the expected revenues in the APPO are higher than those from the corresponding second-price auction.*

Proof. Assume first that the decisive bidder $\hat{\imath}$ has maximum valuation, i. e. $v_{\hat{\imath}} = \sup(\mathcal{M})$ and let the expected revenues from the corresponding auction be denoted by $E[R^C \mid v_{\hat{\imath}} = \sup(\mathcal{M})]$. In this case, bidder $\hat{\imath}$ wins the corresponding auction with certainty and $E[R^C \mid v_{\hat{\imath}} = \sup(\mathcal{M})] = E[V_{(1),n-1}]$. If the decisive bidder were risk neutral, the expected revenues from the corresponding auction would equal the bidder's acceptance threshold. Since, however, bidder $\hat{\imath}$ is risk averse, he strictly prefers a PPO $\bar{p} = E[V_{(1),n-1}]$ over the uncertain price in the corresponding auction. Thus, $t_{\hat{\imath}}(\sup(\mathcal{M})) > E[V_{(1),n-1}]$ holds.

Now, consider a PPO $\bar{p}$ with $E[V_{(1),n-1}] < \bar{p} < t_{\hat{\imath}}(\sup(\mathcal{M}))$, i. e. a PPO that lies in between the expected revenues of an auction if the decisive bidder had maximum valuation and the acceptance threshold of the decisive bidder in that same case. Since $\bar{p}$ is lower than $t_{\hat{\imath}}(\sup(\mathcal{M}))$, such a PPO will be accepted with positive probability, and not only then if the decisive bidder has maximum valuation. Since, in addition, $\bar{p}$ is higher than $E[V_{(1),n-1}]$, the expected revenues are higher than in the corresponding auction. ∎

Example 2.4 (Continuation of Example 2.3). Consider again an APPO in which $n = 2$ bidders participate. As in Example 2.3, valuations are independently and uniformly distributed over $[0; 1]$ and the decisive bidder's utility function is given by $u_{\hat{\imath}}(x) = \sqrt{x}$.

Assume first that the seller sets no or a very high PPO, e. g. $\bar{p} > 1$. In that case, the PPO will never be accepted and the APPO is equivalent to a standard second-price auction. If, in addition, the seller sets no reserve price (i. e. $\underline{p} = 0$), the expected revenues equal the expected value of the second-order statistic $E[V_{(2),2}] = \frac{1}{3}$.

How does the situation change if the seller sets a (reasonable) PPO? The proof of Theorem 2.14 suggests the choice of a PPO $\bar{p}$ with $E[V_{(1),n-1}] < \bar{p} < t_{\hat{\imath}}(\sup(\mathcal{M}))$. Note that this condition is sufficient, but not necessary for the APPO to outperform a (pure) auction.[23] In this example, a PPO

[23] In fact, the revenue maximizing PPO of the given example is not in that interval.

$\bar{p} = t_{\hat{\imath}}(\sup(\mathcal{M})) = t_{\hat{\imath}}(1) = \frac{5}{9}$ yields the desired result and is computationally manageable. Equation (2.12) states the decisive bidder's acceptance threshold $t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - \frac{4}{9}v_{\hat{\imath}}^3$. Thus, the decisive bidder will accept the PPO if

$$v_{\hat{\imath}} - \frac{4}{9}v_{\hat{\imath}}^3 \geq \frac{5}{9} \ .$$

Since, by construction, $v_{\hat{\imath}} = 1$ solves the cubic equation

$$-\frac{4}{9}v_{\hat{\imath}}^3 + v_{\hat{\imath}} - \frac{5}{9} = 0 \ , \tag{2.16}$$

a polynomial division by $(v_{\hat{\imath}} - 1)$ simplifies (2.16), which can then be easily solved. One obtains

$$-\frac{4}{9}v_{\hat{\imath}}^2 - \frac{4}{9}v_{\hat{\imath}} + \frac{1}{3}\underline{p}^2 + \frac{5}{9} \ = \ 0 \tag{2.17}$$

$$\Longleftrightarrow v_{\hat{\imath}_{1/2}} \ = \ -\frac{1}{2} \pm \frac{\sqrt{6 + 3\underline{p}^2}}{2} \ . \tag{2.18}$$

Thus, (2.16) has three real valued solutions, $-\frac{1}{2}\left(\sqrt{6}+1\right)$, $\frac{1}{2}\left(\sqrt{6}-1\right)$, and 1, but only the second solution is an interior point of the support of the decisive bidder's valuation. For this reason, define $v^* := \frac{1}{2}\left(\sqrt{6}-1\right)$ and distinguish two cases for the calculation of the expected revenues: (a) the decisive bidder accepts the PPO, i. e. $v_{\hat{\imath}} \geq v^*$, and (b) the decisive bidder rejects the PPO, i. e. $v_{\hat{\imath}} < v^*$. In the latter case, an auction is being conducted in which either the decisive bidder $\hat{\imath}$ or the other bidder $-\hat{\imath}$ obtains the item. Let the expected revenues of the aforementioned cases be denoted by R_a and R_b with $R_b = R_b^{\hat{\imath}} + R_b^{-\hat{\imath}}$, respectively. The expected revenues $E[R]$ of the APPO are then given by $R_a + R_b$:

$$\begin{aligned}
R_a &= \bar{p}\,(1 - v^*) \\
&= \frac{5}{9}\,\left(1 - \frac{1}{2}\left(\sqrt{6} - 1\right)\right) \\
&= \frac{5}{6} - \frac{5\sqrt{6}}{18} \\
R_b &= \qquad R_b^{\hat{\imath}} \qquad + \qquad R_b^{-\hat{\imath}} \\
&= \int_0^{v^*}\!\!\int_0^{v} x \; dx \; dv + \int_0^{v^*} v \int_v^1 1 \; dx \; dv \\
&= \frac{1}{2}(v^*)^2 - \frac{1}{6}(v^*)^3 \\
&= \frac{61}{48} - \frac{7}{16}\sqrt{6}
\end{aligned}$$

$$\begin{aligned} E[R] &= R_a + R_b \\ &= \frac{101}{48} - \frac{103}{144}\sqrt{6} \\ &\approx .3521 > \frac{1}{3} \ . \end{aligned}$$

The expected revenues of the APPO in fact exceed the expected revenues of a pure auction. □

2.5 Optimal APPOs

Finally, the reserve price deserves a closer look. It is a well-known result that by setting a suited reserve price $\underline{p} > 0$, the seller can increase expected revenues even though she risks not selling the item at all. In a second-price auction with independent private valuations, the seller optimally sets the reserve price $\underline{p}$ such that

$$\underline{p} = \frac{1 - F(\underline{p})}{f(\underline{p})} \tag{2.19}$$

is satisfied (e. g. McAfee und McMillan, 1987; Myerson, 1981).[24] Interestingly, the optimal reserve price depends only on the distribution F of the bidders' valuations but not on the number of bidders. Thus, the optimal reserve price equals the optimal *take-it-or-leave-it* price a seller would offer if there were exactly one interested buyer.

Example 2.5 shows that by means of an appropriately chosen reserve price, the seller can also increase expected revenues in an APPO.

Example 2.5 (Continuation of Example 2.4). In contrast to the previous examples, the reserve price is no longer fixed at zero. Solving Equation (2.19) yields the optimal reserve price $\underline{p}^* = \frac{1}{2}$ of a second-price auction. Remember that F denotes the cdf of the bidders' valuations and $f_{(i),n}$ the density of the ith-order statistic. The expected revenues $E[R]$ of an optimal second-price auction evaluate to

[24] If the hazard rate $f(v)/(1 - F(v))$ is not strictly increasing, Equation (2.19) may have multiple roots. In this case, the seller chooses the solution of (2.19) that yields the highest expected revenues of all solutions (cf. Wolfstetter, 1999, p. 213).

$$\begin{aligned}E[R] &= 2\,\underline{p}\,F(\underline{p})\left(1 - F(\underline{p})\right) + \int_{\underline{p}}^{1} x\, f_{(2),2}(x)\,dx \\ &= 2\,\underline{p}^2(1-\underline{p}) + \int_{\underline{p}}^{1} x\ (2-2x)\ dx \\ &= \frac{1}{4} + \left[x^2 - \frac{2}{3}x^3\right]_{\frac{1}{2}}^{1} \\ &= \frac{5}{12}\ .\end{aligned}$$

Thus, $\frac{5}{12}$ are the maximum of the expected revenues in a second–price auction.

Now consider an APPO in which the reserve price is set to $\underline{p} = \frac{1}{2}$, as in the optimal second–price auction. According to Theorem 2.14, in an APPO with risk averse bidders, the seller can increase expected revenues by offering an appropriately chosen posted price. Taking the reserve price $\underline{p} = \frac{1}{2}$ as given, in the following the *optimal*, i. e. the revenue maximizing, PPO is derived. Then the expected revenues are calculated. It will be shown that these revenues not only exceed the expected revenues of the optimal second–price auction above, but also the revenues of the APPO in Example 2.4.

In order to determine the revenue maximizing PPO $\bar{p}^*$, one starts with the decisive bidder's acceptance threshold, which is given by Definition 2.5

$$\begin{aligned}t_{\hat{\imath}}(v_{\hat{\imath}}) &= v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(u_{\hat{\imath}}(v_{\hat{\imath}} - \underline{p})\,G_{(1),-\hat{\imath}}(\underline{p}) + \int_{\underline{p}}^{v_{\hat{\imath}}} u_{\hat{\imath}}(v_{\hat{\imath}} - x)\,g_{(1),-\hat{\imath}}(x)\,dx\right) \\ &= v_{\hat{\imath}} - \left(\sqrt{v_{\hat{\imath}} - \underline{p}}\ \underline{p} + \int_{\underline{p}}^{v_{\hat{\imath}}} \sqrt{v_{\hat{\imath}} - x}\,dx\right)^2 \\ &= v_{\hat{\imath}} - \left(\sqrt{v_{\hat{\imath}} - \underline{p}}\ \underline{p} + \left[\frac{2}{3}\ (v_{\hat{\imath}} - x)^{\frac{3}{2}}\right]_{\underline{p}}^{v_{\hat{\imath}}}\right)^2 \\ &= v_{\hat{\imath}} - \frac{4}{9}v_{\hat{\imath}}^3 + \frac{1}{3}v_{\hat{\imath}}\underline{p}^2 + \frac{1}{9}\underline{p}^3\ .\end{aligned} \tag{2.20}$$

Applying $\underline{p} = \frac{1}{2}$ in (2.20) yields

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = -\frac{4}{9}v_{\hat{\imath}}^3 + \frac{13}{12}v_{\hat{\imath}} + \frac{1}{72}\ . \tag{2.21}$$

Figure 2.4 illustrates the example. The solid line shows the acceptance thres– hold of the decisive bidder and the horizontal dotted line represents the reserve price $\underline{p} = \frac{1}{2}$. Remember that the threshold is only defined for valuations higher than the reserve price. Note that the threshold has an interior maximum and decreases for large valuations. We will focus the search for the optimal $\bar{p}^*$ on

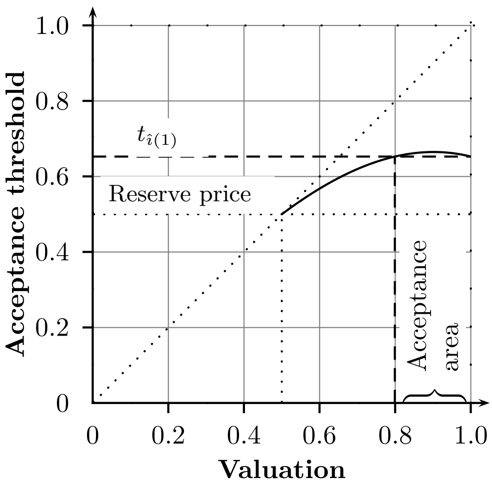

Figure 2.4. Acceptance threshold of a risk averse bidder if $\underline{p} > 0$

values smaller than $t_{\hat{\imath}}(1) = \frac{47}{72}$—indicated in the figure by the dashed hori– zontal line. This simplifies the calculation since it allows the assumption that there exists a valuation v^* above which the decisive bidder will accept the PPO (otherwise bidders with large valuations would reject the PPO). The assumption also narrows down the interval of feasible values for v^*. Because $t_{\hat{\imath}}(-\frac{1}{2} + \frac{3}{4}\sqrt{3}) = \frac{47}{72}$, the value $v_{\hat{\imath}} = -\frac{1}{2} + \frac{3}{4}\sqrt{3}$ constitutes an upper bound for v^*.[25] Moreover, $v^* > \underline{p}$ must hold. Thus, we search for a $v^* \in \left[\frac{1}{2}; -\frac{1}{2} + \frac{3}{4}\sqrt{3}\right]$. The results will later show that these assumptions were feasible.[26]

In order to determine v^*, one needs to solve

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = \bar{p}$$
$$\Longleftrightarrow -\frac{4}{9}v_{\hat{\imath}}^3 + \frac{13}{12}v_{\hat{\imath}} + \frac{1}{72} = \bar{p} \ . \tag{2.22}$$

25 As in Example 2.4, the cubic equation $t_{\hat{\imath}}(v_{\hat{\imath}}) = t_{\hat{\imath}}(1) \Longleftrightarrow -\frac{4}{9}v_{\hat{\imath}}^3 + \frac{13}{12}v_{\hat{\imath}} - \frac{46}{72} = 0$ is reduced to a quadratic form through a polynomial division by $(v_{\hat{\imath}} - 1)$. The value $v_{\hat{\imath}} = -\frac{1}{2} + \frac{3}{4}\sqrt{3}$ is one of the solutions of the remaining quadratic equation $-\frac{4}{9}v_{\hat{\imath}}^2 - \frac{4}{9}v_{\hat{\imath}} + \frac{46}{72} = 0$.

26 Strictly speaking, the argument only shows that the identified $\underline{p}^*$ is a local extremum. The proof that $\underline{p}^*$ is also a global maximum is rather technical and left out here.

Cardano's formula[27] offers three real valued solutions of Equation (2.22); selecting the one that lies within the above interval, one obtains

$$v^* = \frac{1}{4}\sqrt{13}\left(-\cos\left(\frac{\pi}{3} - \frac{1}{3}\cos^{-1}\left(\frac{72\bar{p}-1}{169}\sqrt{13}\right)\right)\right.$$
$$\left. + \sqrt{3}\sin\left(\frac{\pi}{3} - \frac{1}{3}\cos^{-1}\left(\frac{72\bar{p}-1}{169}\sqrt{13}\right)\right)\right) \quad . \tag{2.23}$$

In Example 2.4, several cases were distinguished for the calculation of expected revenues. Taking the reserve price into account, the classification of cases must be refined. Case (a), in which the decisive bidder accepts the PPO, remains unchanged. If (*b*) the decisive bidder rejects the PPO, it is now further required that the reserve price is not binding in the corresponding auction. Two additional cases arise: (*c*) the item is sold for the reserve price and (*d*) the item is not sold at all. Analogously to Example 2.4, the expected revenues of these cases are denoted by R_a, R_b, R_c, and R_d, respectively, and $E[R] = R_a + R_b + R_c + R_d$ holds for the total expected revenues. Because the decisive bidder accepts the PPO if his valuation exceeds v^*, the formula for expected revenues yields

$$\begin{aligned}
R_a &= \bar{p}\,(1 - v^*) \\
R_b &= \int_{\underline{p}}^{v^*}\int_{\underline{p}}^{v} x \; dx \; dv + \int_{\underline{p}}^{v^*} v \int_{v}^{1} 1 \; dx \; dv \\
R_c &= \underline{p}\left(\underline{p}\,(1-\underline{p}) + \underline{p}\,(v^* - \underline{p})\right) \\
R_d &= 0 \\
E[R] &= R_a + R_b + R_c + R_d \\
&= \bar{p} - \bar{p}\,v^* - \frac{1}{6}(v^*)^3 + \frac{1}{2}(v^*)^2 + \frac{1}{2}\underline{p}^2 v^* - \frac{4}{3}\underline{p}^3 + \frac{1}{2}\underline{p}^2 \quad .
\end{aligned} \tag{2.24}$$

With $\underline{p} = \frac{1}{2}$ one obtains from Equation (2.24)

$$E[R] = \bar{p} - \bar{p}\,v^* - \frac{(v^*)^3}{6} + \frac{(v^*)^2}{2} + \frac{v^*}{8} - \frac{1}{24} \quad . \tag{2.25}$$

In order to maximize expected revenues, differentiate (2.25) with respect to $\bar{p}$. Note that v^* is a function of $\bar{p}$ with derivative $v^{*\prime}(\bar{p})$:

[27] For a description of Cardano's formula, see Gellert et al. (1969). The formula goes back to Tartaglia and was first published by Cardano in his mathematical work *Ars Magna* (1545).

$$\frac{dE[R]}{d\bar{p}} = 1 - \bar{p}\, v^{*\prime}(\bar{p}) - v^* + \left(-\frac{1}{2}(v^*)^2 + v^* + \frac{1}{8}\right) v^{*\prime}(\bar{p}) \stackrel{!}{=} 0$$
$$\iff \frac{1 - v^*}{v^{*\prime}(\bar{p})} - \bar{p} - \frac{1}{2}(v^*)^2 + v^* + \frac{1}{8} = 0 \ . \quad (2.26)$$

Equation (2.26) yields the first-order condition for a revenue maximizing PPO $\bar{p}$. One obtains $v'(\bar{p})$ from Equation (2.23). Plugging both Equation (2.23) and its derivative into Equation (2.26) finally yields the optimal PPO. The actual calculations of the last two steps, however, are spared the reader but have been passed on to a computer algebra program.[28] The solution comprises complex trigonometric terms and the numerical value is

$$\bar{p}^* \approx 0.6523 \ . \quad (2.27)$$

Applying (2.27) to Equations (2.23) and (2.25) yields $v^* \approx 0.7971$, and the expected revenues with the optimally chosen PPO are

$$E[R] \approx 0.4236 \ . \quad (2.28)$$

Note that in fact $v^* \in \left[\frac{1}{2}; -\frac{1}{2} + \frac{3}{4}\sqrt{3}\right]$, which justifies the above assumption. Further, as indicated at the beginning of the example, the expected revenues $E[R] = 0.4236$ exceed both the expected revenues of $\frac{5}{12}$ in the (pure) second-price auction as well as those of an APPO with no reserve price, as in Example 2.4.[29] □

The following Example 2.6, however, shows that Equation (2.19), which determines the optimal reserve price in a pure auction, does not hold for the optimal reserve price of an APPO. Instead, the seller can do better by setting a slightly different reserve price. One might be tempted into thinking that a higher reserve price favors the seller since a higher reserve price also increases the decisive bidder's threshold (cf. Proposition 2.12). This increases the probability that the decisive will bidder accept the PPO or allow the seller to set a higher PPO. However, the opposite is true. The optimal reserve price balances the (expected) increase in revenue if the second highest valuation is low with the opportunity costs in the event that the item remains unsold because the highest valuation is also low. In an APPO, however, the decisive bidder accepts the PPO if his valuation is high. These cases can therefore be taken out of consideration when determining the optimal reserve price. Thus,

[28] The computations have been performed with Maple version 9.51.

[29] Strictly speaking, the comparison is not appropriate since in Example 2.4 the revenues of an APPO with an arbitrarily chosen PPO are calculated, as opposed to a revenue maximizing PPO. However, selecting a revenue maximizing PPO in Example 2.4 increases the expected revenue from 0.3521 to only 0.3522.

the increase in revenue by means of a given reserve price is lower in an APPO than in a second-price auction while the risk of not selling the item remains the same. The optimal reserve price of a second-price auction therefore exceeds the optimal reserve price of an APPO.

To maximize an APPO's revenue, one needs to simultaneously optimize over both the reserve price and the posted price. Let $E[R(\underline{p}, \bar{p})]$ denote the expected revenue for a given reserve price $\underline{p}$ and a posted price $\bar{p}$. The optimization problem of the seller then reads

$$\max_{\underline{p},\, \bar{p}\, \in \mathcal{M}} \{E[R(\underline{p},\, \bar{p})]\} \ . \tag{2.29}$$

Example 2.6 solves the problem (2.29) for the setting of Example 2.5.

Example 2.6 (Continuation of Example 2.5). Calculating the optimal combination of the reserve price and the posted price runs analogously to Example 2.5. The difference is that the expected revenue must be differentiated with respect to both $\underline{p}$ and $\bar{p}$. Then, instead of Equation 2.19, the first-order condition is given by a system of two equations, which again can be easily solved by a computer algebra program. The tricky part remains inverting the threshold condition—in our case again a cubic equation—and picking the right solution (which also satisfies the second-order condition) for the remaining calculations. Depending on the parameters $\underline{p}$ and $\bar{p}$, quite a few cases must be distinguished.

In our example, the solution is

$$\begin{aligned} \underline{p}^* &\approx 0.4839 \\ \bar{p}^* &\approx 0.6456 \\ E[R]_{\underline{p}^*, \bar{p}^*} &\approx 0.4238 \ . \end{aligned}$$

Note that the expected revenue is higher compared to the case with a reserve price of $\underline{p} = \frac{1}{2}$. The additional gain, however, is rather marginal. □

Theorem 2.15 (APPO outperforms second-price auction). *If bidders are risk averse, then a combination of an APPO's reserve price $\underline{p}$ and a posted price $\bar{p}$ exists such that the respective APPO generates higher expected revenues than any second-price auction.*

Proof. Assume to the contrary that a second-price auction with a reserve price $\underline{p}'$ yields higher expected revenue than the revenue maximizing APPO with reserve price $\underline{p}$ and PPO $\bar{p}$. Because of Theorem 2.14, a posted price $\bar{p}'$ exists such that an APPO with the reserve price $\underline{p}'$ and the PPO $\bar{p}'$ generates higher revenue than the second-price auction. This contradicts the assumption that the above APPO is revenue maximizing among all APPOs. ■

2.6 Discussion and Related Literature

Several authors have investigated hybrid market institutions that combine auctions with posted price offers and are similar to the model of an APPO presented in the previous sections. Worth mentioning in particular are the works by Budish und Takeyama (2001), Hidvégi et al. (2003), Reynolds und Wooders (2003), Mathews (2002, 2003, 2004a, 2004b), Mathews und Katzman (forthcoming), and Wang et al. (2004). In this section, these papers will be summarized and contrasted to the APPO model. An outlook on an extension to multi-unit demand by Kirkegaard und Overgaard (2004) completes the section.

It has become common in the literature to refer to a posted or fixed price offer supplementing an auction as a *buy price*—irrespective of the concise rules of the auction and the rules regarding the buy price. To ease discussion, this labeling has been adopted in this section. Unfortunately, however, the term *Buy Price* is also employed by Yahoo! as the name for its buy price variant. In the text, capital initials are used to indicate the Yahoo! variant.

2.6.1 The Model of Budish und Takeyama

Budish und Takeyama (2001) are the first to analyze buy price auctions. They investigate a variation of eBay's hybrid mechanism in which—in contrast to the APPO model and its eBay archetype—the posted price offer does not expire once a bidder submits an auction bid. The model is motivated by the institutions that are available to sellers on the auction sites of Yahoo! or Amazon.com. As mentioned above, Yahoo! refers to its counterpart to eBay's *Buy It Now* feature as *Buy Price*. Amazon.com calls it *Take-It Price*.

Since the buy prices on Yahoo! and Amazon.com do not expire if a bidder enters an auction bid, the buy price is not only an option for a bidder to acquire the item with certainty; it also constitutes a maximum price of the auction. Yahoo! explicitly states that an auction closes automatically once the bids reach the *Buy Price*.[30] Thus, an auction price that exceeds the *Buy Price* is not possible, and by offering a *Buy Price*, the seller rules out the possibility of generating higher revenues.

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[30] See `http://help.yahoo.com/help/us/auct/asell/amerc/amerc-11.html` (October 13, 2004).

Budish und Takeyama analyze Yahoo!'s *Buy Price* in a two-bidder setting with private valuations. Each bidder is of one of two types, i. e. a bidder may either value the item high or low. The probability that a bidder is of the low valuation type L, in which case his valuation is v^L, is ϵ. With a probability of $(1-\epsilon)$ the bidder is of the high valuation type H and values the item at v^H. If a bidder of type i is awarded the item for a price p, his utility is given by $u(v^i - p)$.[31]

Consider the case that no buy price is available. In Budish und Takeyama's model, the bidders then bid up to their valuation. If the bidders differ in type, the bidder with the high valuation is awarded the item. If both bidders are the same type, a coin is tossed in order to determine which bidder obtains the item. In all cases, the auction price equals the valuation of the bidder who loses the auction. Thus, the seller's expected revenues $E[R^A]$ of a pure auction evaluate to

$$E[R^A] = \epsilon^2\, v^L + 2\,\epsilon\,(1-\epsilon)v^L + (1-\epsilon)^2\, v^H \quad . \tag{2.30}$$

The utility of a high valuation bidder is $u(0) = 0$ if the other bidder is also a high valuation bidder, whilst the utility is $u\left(v^H - v^L\right)$ if the other bidder has a low valuation. Thus, the expected utility of a high valuation bidder is given by

$$\epsilon\, u(v^H - v^L) \quad . \tag{2.31}$$

Assume now that the seller offers a buy price B. The utility of a high valuation bidder who accepts the buy price is then $u\left(v^H - B\right)$. As above, if both bidders seek to accept the buy price, tossing a coin determines which bidder is awarded the item. If high valuation bidders follow a strategy of always accepting the buy price, the expected utility of a high valuation bidder yields

$$\begin{aligned} &\epsilon\, u(v^H - B) + \frac{1-\epsilon}{2}\, u\left(v^H - B\right) \\ &= \frac{1+\epsilon}{2}\, u\left(v^H - B\right) \quad . \end{aligned} \tag{2.32}$$

Since the auction price can never be lower than v^L, Budish und Takeyama argue that the seller will only offer a buy price $B > v^L$. Consequently, the low valuation bidder will never accept the buy price. In fact, in the two-bidder model with two bidder types there is no equilibrium in which the low valuation bidder earns a positive payoff.

[31] The two-bidders two-valuations setting was introduced by Maskin und Riley (1985).

The cited authors derive a symmetric equilibrium in which a high valuation bidder immediately accepts the buy price and a low valuation bidder bids up to v^L in the auction. From Equations (2.31) and (2.32) one obtains a condition necessary for a symmetric equilibrium:

$$\frac{1+\epsilon}{2}\, u\left(v^H - B\right) \geq \epsilon\, u\left(v^H - v^L\right)$$
$$\iff \quad B \leq v^H - u^{-1}\left(\frac{2\epsilon}{1+\epsilon}\, u\left(v^H - v^L\right)\right) \quad . \tag{2.33}$$

The inequality (2.33) corresponds to the threshold condition of the APPO model: if the condition holds, a high valuation bidder prefers accepting the buy price rather than participating in an auction. Thus, in a symmetric equilibrium a high valuation bidder accepts the buy price if the inequality (2.33) holds. In all other cases bidders bid up to their valuation.[32]

Note that the inequality (2.33) captures the analyzed scenario with two bidders and two discrete valuations v^H and v^L. It also reflects the fact that both bidders might seek to acquire the item for the buy price, as is possible on Yahoo! but not in an APPO. However, the model fails to take the complete structure of possible actions as well as the information feedback during the bidding process into account. The given strategies would work well if an auction with a buy price were modeled as a sealed bid institution in which a bidder enters a two-part strategy before the bidding begins: firstly, he indicates whether or not he is willing to buy the item at the buy price B and, secondly, he indicates how far he would bid in an English auction if the item is not sold at the buy price. In such a setting, an auction would be conducted only if no bidder were willing to accept the buy price.

Budish und Takeyama, however, explicitly seek to address an English auction which is augmented with a buy price that does not expire once a bidder submits a bid. In such a situation, consider an auction with one low and one high valuation bidder and assume that the low valuation bidder starts the bidding process by submitting a very low bid. If the strategy profile described above does in fact constitute an equilibrium, the high valuation bidder would *know* that the competing bidder has a low valuation. Thus, in an auction the high valuation bidder would win the item for the price v^L. According to the above strategy profile, however, the high valuation bidder accepts the buy price B. Clearly, since $B > v^L$, this cannot be optimal.

Nonetheless, the model is interesting and highly relevant for analyzing auctions with an additional fixed price component. Despite their goal to analyze an English auction augmented with a non-expiring buy price, the model of

[32] According to Budish und Takeyama this is also the only symmetric equilibrium.

Budish und Takeyama rather refers to the following two-stage scenario: In the first stage, bidders are asked whether they are willing to accept the buy price. If the item remains unsold in the first stage, a second stage follows in which the item is auctioned off by means of a second-price auction. The difference to the APPO model is that all and not only one bidder may accept the posted price offer.

In Budish und Takeyama's model the seller sets the buy price B^* optimally so that (2.33) is fulfilled with equality. Thus, B^* is the highest buy price that will be accepted by a high valuation bidder. Any lower buy price would also lower the seller's revenues. The authors note that "[s]trictly speaking, the optimal buy price should actually be understood to be some arbitrarily small amount, ϵ, less than that defined [by B^*]" (p. 328, fn. 7).

Based on the above findings, Budish und Takeyama derive three main results:

1. For the case of risk neutral bidders, the expected revenues of a pure auction and an English auction augmented with the buy price B^* are equal.
2. If bidders are risk averse, the seller is strictly better off by offering the buy price B^*.
3. With risk averse bidders, the expected revenues of an auction augmented with a buy price B^* may even be higher than those of a first-price sealed-bid auction.

The first two results are basically identical to those of the APPO model. Note that the optimal buy price B^* is defined such that the high value holder is indifferent between accepting the buy price and bidding in the auction. If, analogously, the PPO $\bar{p}$ in an APPO were set such that a risk neutral bidder with maximum valuation were indifferent between the PPO and bidding in the auction, this would not affect the seller's expected revenues, even if the PPO were in fact accepted by bidders with full valuation. However, taking Budish und Takeyama's comment into account, according to which the optimal buy price is some ϵ smaller than B^*, their first result is indeed equivalent to Theorem 2.13 of Section 2.4: With risk neutral bidders, neither the seller in their model nor the seller in an APPO can profit from offering a buy price. If the buy price is set such that a bidder with maximum valuation is indifferent between the buy price and bidding in an auction, the expected revenues in both institutions are equal to the revenues in a pure second-price (or English) auction.

A similar link can be established between the second result above and Theorem 2.14 of Section 2.4: if bidders are risk averse in both Budish und Takeyama's model of an auction with a buy price and an APPO, the seller can increase expected revenues by offering a suited buy price.

The issue of Budish und Takeyama's third result addresses the comparison between a standard first-price (or Dutch) auction and an English auction augmented with a buy price. It is a well-known result that with private valuations and risk averse bidders, a first-price sealed-bid auction yields higher revenues than an English (or a second-price) auction (e. g. Maskin und Riley, 1984; McAfee und McMillan, 1987; Klemperer, 1999). In order to see this, remember that the equilibrium strategies of an English auction do not depend on the bidders' attitude towards risk. By choosing a bid in a first-price auction, however, a bidder trades the probability of winning the auction against the price he has to pay should he in fact win the auction. Compared to a risk neutral bidder, a risk averse bidder is willing to pay a higher price in order to increase the probability of winning. From the *revenue equivalence theorem* (Milgrom, 1989; Klemperer, 1999) it is known that with risk neutral bidders and symmetric independent private valuations, the expected revenues of an English and a first-price auction are equal. Since in a first-price auction (but not in an English auction) a risk averse bidder bids more aggressively than a risk neutral bidder, a first-price auction yields higher revenues than an English auction if bidders are risk averse.

Interestingly, Budish und Takeyama find that in their model, the English auction augmented with a buy price may even yield higher revenues than a standard first-price sealed bid-auction. It is not known whether an APPO or a first-price sealed-bid auction yields higher expected revenues. This remains an interesting question for future research. Since the model of an APPO has not yet been compared to a first-price sealed-bid auction, a more detailed description of Budish und Takeyama's line of reasoning is not included here and the reader is referred to the original paper.

The paper by Budish und Takeyama is innovative in being the first to address fixed price components as complementary features of standard auctions. By differentiating between risk neutral and risk averse bidders, the authors show in an analysis of Yahoo!'s *Buy Price* that "this seemingly irrational auction mechanism" can in fact "improve the seller's profits" in terms of expected revenues (p. 325). Still, restricting attention to only two bidders and only two possible bidder types rather limits the scope of the model. The authors themselves note that introducing a larger number of bidder types "admits the possibility of inefficient outcomes" and speculate that "the effectiveness of the buy price to enhance sellers' profits when bidders are risk averse may be diminished" (p. 328). The APPO model shows that with more than two bidder types an outcome may in fact be inefficient. However, Theorem 2.14 in Section 2.4 also shows that the expected revenues of an APPO still exceed the revenues of a pure auction.

2.6.2 Thorough Analysis of Yahoo!'s Buy Price

In a barely observed paper, Hidvégi et al. (2003) relax the restriction imposed by Budish und Takeyama (2001) to only two bidders and only two bidder types. Rather, they consider a model with n bidders and arbitrarily distributed valuations. With respect to the distribution of valuations, the paper is more general in terms of methodology than most of the papers that will be presented in the following sections. It is, however, only cited by Wang et al. (2004) and appears to be unknown to the other authors investigating buy price options.[33]

Hidvégi et al. further investigate Yahoo!'s buy price auction. The paper is rather comprehensive regarding the description of the auction setup. For example, the authors explicitly state that in their model, the seller "has committed not to list her item if she receives no valid bid", or that each bidder knows the total number of bidders at the opening of the auction but that they "do not know how many bidders remain active" during the auction (p. 2). These assumptions are crucial for the model and are also made, e. g., by Reynolds und Wooders (2003)—without, however, being explicitly stated.

The paper proves that under certain conditions Yahoo!'s buy price auction yields the same expected revenue as the standard auctions—even if the buy price is accepted with positive probability: They show that in a symmetric equilibrium, a bidder i with valuation v_i accepts the buy price once the auction price reaches a certain critical level $s(v_i)$. Then they prove that the function $s(\cdot)$ is monotonically decreasing and conclude that the auction is efficient since the bidder with the highest valuation will accept the buy price first.[34] If bidders are risk neutral, the *revenue equivalence theorem* ensures that all efficient auctions, i. e. also Yahoo!'s buy price auction, yield the same expected revenues.

Hidvégi et al. derive other results too. In summary, their findings are:

1. If either the seller or the bidders are risk averse, the seller can gain higher expected utility by offering a buy price.
2. If the buy price is set sufficiently high, Yahoo!'s buy price auction is efficient, i. e. the bidder with the highest valuation wins the item even if the buy price is accepted with positive probability.

[33] Wang et al. (2004) actually refer to an earlier draft of that paper from 2002. The results of Hidvégi et al. (2003) and the earlier draft from 2002 are basically identical; only the presentation has been rearranged and some sections were rewritten. The 2002 draft is available online (`http://ruby.bus.utexas.edu/~gengxj/dss/session9/BuyPrice.pdf`, October 20, 2004).

[34] They further require that the buy price is set high enough so that it will only be executed *after* the bidding process has started and the current auction price exceeds the auction's reserve price. Efficiency is not guaranteed if a bidder would accept the buy price immediately at the start of the auction.

3. Under the assumptions of 2. above, bidders with constant absolute risk aversion are indifferent between a buy price auction and a standard English auction.[35]

As will be shown in the next section, the last result is also supported by Reynolds und Wooders (2003), who make the same observation not only with respect to the English (or second-price) auction, but all standard auction formats. Without being stated by either Hidvégi et al. or Reynolds und Wooders, another direct and interesting consequence of this result is that the expected utility of bidders with constant absolute risk aversion is independent of the buy price B.

2.6.3 Comparison of Yahoo!'s and eBay's Buy Price Variants

Reynolds und Wooders (2003) do not only consider Yahoo!'s variant of an auction with a *Buy Price* but also compare that auction with eBay's variant of an auction with a *Buy It Now* price. They commonly refer to both of these auction types as *buy price auctions.*

Following Budish und Takeyama (2001), Reynolds und Wooders consider a model with only two bidders. However, they allow valuations to be uniformly distributed on the interval $[\underline{v}; \bar{v}]$.

For the eBay variant, Reynolds und Wooders characterize a bidder's strategy by a so-called *cutoff value.* They derive a symmetric equilibrium in which the cutoff value c represents the lowest valuation of a bidder who accepts a given buy price B. A bidder i, following a strategy characterized by a cutoff value c, accepts the buy price if his valuation $v_i \in [\underline{v}; \bar{v}]$ is at least as high as the cutoff value ($v_i \geq c$); the bidder rejects the buy price if his valuation is below the cutoff value ($v_i < c$).[36] In the latter case, the bidder bids up to his valuation in the auction. Note that the cutoff value is a function of the buy price B.[37] Thus, the cutoff value is the inverse of the bidder's threshold $t_i(v_i)$ in an APPO, which is a function of a bidder's valuation, indicating up to which amount a bidder i with valuation v_i would accept a PPO $\bar{p}$.

Since the buy price offer on Yahoo! does not expire once a bidder places a bid, a strategy for bidding in that auction is somewhat more complex. Reynolds und Wooders model the Yahoo! auction as an ascending clock auction in which the auction price is raised continuously from $\underline{v}$ to B. At any

[35] See Definition A.5 on page 141 for the notion of absolute risk aversion.

[36] Actually, Reynolds und Wooders do not consider the case of a bidder with a valuation equal to the cutoff value.

[37] In the cited paper, the buy price is technically taken as given and the cutoff value is simply denoted as a number.

point during the rise of the auction price, a bidder can either wait (i. e. remain in the auction), drop out of the auction, or claim the item. If a bidder drops out, the other bidder obtains the item at the current auction price. If a bidder claims the item, it is awarded to him at the buy price B and the auction closes. Given that framework, a bidding strategy is characterized by a function $f\colon [B, \bar{v}] \to [\underline{v}; B]$ which maps a bidder's valuation to the auction price at which he claims the item by accepting the buy price offer. A bidder with a valuation below B never exercises that option. Instead, he remains in the auction until the auction price reaches his valuation, at which point he drops out.

Reynolds und Wooders analyze above settings both for risk neutral bidders and for bidders with constant absolute risk aversion. They derive the following results:

1. If bidders are risk neutral, a buy price may be set so high that it is not exercised in the eBay variant. The same buy price, however, is exercised in the Yahoo! variant with positive probability. Moreover, in this case, the two buy price auctions and a standard English auction yield the same expected revenues.
2. If bidders are risk neutral, eBay's and Yahoo!'s variants of buy price auctions with the same buy price B are revenue equivalent.
3. If a buy price B is exercised with positive probability in the eBay variant by risk neutral bidders, the eBay and the Yahoo! variants with the same buy price B yield lower revenues than a standard English auction.
4. For bidders with constant absolute risk aversion, the Yahoo! variant yields higher revenues than the eBay variant with the same buy price B. The expected utility of a bidder, however, is equal in both auction formats.[38]
5. There are buy prices such that for bidders with constant absolute risk aversion, both the eBay and the Yahoo! variants yield higher expected revenues than a standard English auction.

Throughout the analysis, Reynolds und Wooders take the buy price B as given. They show that for a wide range of buy prices, offering such an option is profitable for the seller. However, they do not seek to identify a buy price that maximizes the seller's expected revenue. The papers that will be presented in the following sections address this issue too. Moreover, the restriction to only two bidders is lifted.

[38] It is interesting to note that bidders with constant absolute risk aversion are indifferent among all standard auctions (cf. Matthews, 1987). Even though these auctions (may) differ with respect to the revenues of the seller, they are equivalent with regard to the expected utility of the bidders. Thus, in this case the two buy price auctions yield the same expected bidder utility as all four standard auctions.

2.6.4 Impatience and Risk Aversion

While Budish und Takeyama (2001) motivate their model with Yahoo!'s auction augmented with a buy price and Reynolds und Wooders (2003) compare the buy price formats of Yahoo! and eBay, Mathews (2002) suggests a model in his doctoral dissertation that focuses on eBay's *Buy It Now* feature, which he refers to as a *buyout option.* Based on this dissertation, Mathews has published a family of four papers (one together with Katzman) that address the issue of the *Buy It Now* price. The papers are closely related and build on each other. Note, however, that the publication dates do not reflect the dependencies of the papers with respect to the development of the line of reasoning.

Mathews (2004a) introduces the basic model of an auction with a buyout option and analyzes time impatience on the side of either the seller or the buyers. Mathews (2003) focuses on a scenario in which the seller is risk averse rather than impatient and in which the bidders are indifferent as to when a transaction takes place. Both the 2004a and the 2003 paper assume that bidders' valuations are independently and uniformly distributed on $[0; 1]$. Mathews und Katzman (forthcoming) further investigate the model of Mathews (2003) by allowing the independent private valuations to be arbitrarily distributed and by also taking a reserve price into account. Finally, Mathews (2004b) elaborates on the bidder welfare in an auction with a buyout option.

Mathews diverges from Budish und Takeyama (2001) or Reynolds und Wooders (2003) by considering auctions in which $n \in \mathbb{N}$ participate, rather than having only two bidders. Moreover, bidder valuations are drawn from a continuous distribution. In all of Mathews' papers, however, bidders are assumed to be risk neutral. His model and its main results are summarized in the following.

Mathews (2004a) assumes that an auction lasts from time 0 to 1 and that each bidder i arrives at the auction at a random time $t_i \in [0; 1]$. After arriving at the auction, a bidder may either accept the buyout option, submit a (proxy) bid or wait, i. e. do nothing. If the bidder submits a bid, the buyout option expires not only for him but also for all other bidders. The item is then auctioned off by means of an ascending price auction with proxy bidding. Mathews analyzes the model for bidder valuations v_i and arrival times t_i that are both independently and uniformly distributed over $[0; 1]$.

The utility of a bidder i^* who is awarded the item for a price p at time $t \geq t_{i^*}$ is $u(v_{i^*} - p, t) = (v_{i^*} - p)\,\delta^t$ with $\delta \in (0; 1]$ representing the common degree of the bidders' impatience. Thus, if a bidder i^* with valuation v_{i^*} accepts a buyout option of B at $t \in [t_{i^*}; 1]$, his utility is given by $(v_{i^*} - B)\,\delta^t$. If, alternatively, the buyout option is not accepted, the item is awarded in an auction that lasts until $t = 1$. The utility of the winning bidder i^* is then $(v_{i^*} - p)\,\delta$ with p denoting the final price of the auction.

Mathews shows that in equilibrium, a bidder will either accept the buyout option or submit a proxy bid immediately upon arriving at the auction. He further shows that in equilibrium, bidder i accepts the buyout option if

$$B \leq v_i - \frac{v_i^n}{n}\delta^{1-t_i} \tag{2.34}$$

holds.

The model by Mathews is very similar to the APPO model. In both models, there is exactly one bidder who has the power to decide whether an auction is conducted or not. Moreover, Equation (2.34) resembles the APPO's threshold condition. In fact, in scenarios that are covered by both Mathews (2004a) and the APPO model, the condition (2.34) and the threshold condition are equivalent. To see this, note that Mathews restricts the analysis to risk neutral bidders with uniformly distributed valuations.[39] Moreover, he does not take a possible reserve price into consideration. Equation (2.10) in Example 2.2 gives the respective threshold of a risk neutral bidder i in an APPO with uniformly distributed valuations. Setting the reserve price $\underline{p} = 0$ yields the APPO's threshold condition

$$\bar{p} \leq v_i - \frac{1}{n}v_i^n \ . \tag{2.35}$$

In order to account for bidders who are indifferent as to when a transaction occurs, set $\delta = 1$ in Equation (2.34) and the equivalence is obvious.

Based on the above model, Mathews (2004a) derives the following results:

1. If either the seller or the bidders are time impatient, the seller can profit from offering a buyout option that is accepted with positive probability.
2. Allowing an impatient seller to offer a buyout option results in an increase in all bidders' (ex-ante) expected payoffs compared to a pure second-price (or English) auction.

The second result above is particularly noteworthy because the buyout option gives rise to outcomes which are possibly inefficient. This aspect is considered later.

Mathews (2004a) does not consider positive reserve prices. Interestingly, however, he conjectures that "[s]uch a minimum opening bid would likely increase the probability that the option is successfully exercised" (p. 15). He

[39] Since the utility functions $u_i(x,t) = x\,\delta^t$ are linear in the payoff $x = v_i - p$, bidders are risk neutral with respect to the monetary payoff $x = v_i - p$.

also provides empirical data from eBay that support his hypothesis. Note that the hypothesis is confirmed by—or at least congruent with—Proposition 2.12 of the APPO model.

A slightly different approach is taken in Mathews (2003). In contrast to the 2004a paper, both the bidders and the seller are indifferent as to when a transaction occurs. Instead, the seller is assumed to be risk averse.

The results obtained are very similar to those above:

1. If bidders are risk neutral and indifferent as to when a transaction takes place, a risk averse seller can profit from offering a buyout option that is accepted with positive probability.
2. Allowing a risk averse seller to offer a buyout option results in an increase in all bidders' (ex-ante) expected payoffs compared to a pure second-price (or English) auction.

Moreover, Mathews finds that a seller s_1 who is more risk averse than a seller s_2 offers a buyout option B_1 that is not higher than a buyout option B_2 seller s_2 would offer in the same situation. As a consequence, the probability that a buyout option is exercised as well as the probability of an (ex-post) inefficient outcome (weakly) increases as the seller becomes more risk averse.

Mathews's results and those of the APPO model complement each other. While Mathews concentrates on varying the characteristics of the seller, the analysis of an APPO focuses on the bidders and their attitudes towards risk. The models show that an additional fixed price option (the buyout option or the PPO) enhances the attractiveness of an English (or second-price) auction if either the seller or the bidders are risk averse. Neither side of the transaction, however, is able to realize all of the additional (ex-ante) expected surplus. Instead, the increase in (ex-ante) expected utility is shared among all participants. In all cases, the gain in ex-ante expected utility is linked to a loss in the sum of ex-post payoffs since exercising the buyout option or the PPO may yield inefficient outcomes, i. e. the item might not be awarded to the bidder who values it highest.

Mathews und Katzman (forthcoming) extend the analysis of the previous paper by allowing for any distribution of valuations rather than only the uniform distribution. Moreover, a reserve price set by the seller is now taken into consideration. The authors observe that the first result above, i. e. the fact that the seller can profit from offering a buyout option, does not depend on the distribution of valuations. However, they find that the second result, i. e. the (ex-ante) gain in expected profits by all bidders, depends crucially on that distribution. Note that the buyout option allows for inefficient outcomes. In these cases the high bidder is clearly worse off ex-post compared to a standard second-price auction. Mathews und Katzman establish that if the distribution of valuations is convex, all bidders gain ex-ante from a buyout option even though a bidder with a high valuation may suffer from the option ex-post.

The impact of the valuations' distribution function on the ex-ante bidder welfare is further investigated in Mathews (2004b). Denote the interval of feasible valuations by $M \subset \mathbb{R}$ and the distribution and the density function by $F : M \rightarrow [0; 1]$ and $f : M \rightarrow \mathbb{R}_+$, respectively. Mathews restricts his attention to the case $M = [0; 1]$ and confirms the result from Mathews und Katzman (forthcoming) by proving that the offer of a buy price raises all bidders' ex-ante expected payoffs compared to a standard English auction if $f'(x) \geq 0 \;\forall x \in [0; 1]$. Moreover, he shows that if $f'(x) < 0 \;\forall x \in [0; 1]$, a bidder with a sufficiently high valuation would prefer the seller not to offer a buyout option.

2.6.5 Participation Costs

Wang et al. (2004) examine eBay's variant of a buy price auction by taking bidders' participation costs into account. The authors argue that when bidding in an online auction, bidders incur costs associated with waiting and following the bidding process. A bidder does not face these costs when exercising the buyout option, but only when bidding in the auction. When bidding, however, the bidder has to bear these costs irrespective of whether he wins the auction or not.[40]

Once more, the model by Wang et al. builds on the assumption that bidders' valuations are independently and uniformly distributed on [0; 1]. Moreover, both the bidders and the seller are considered risk neutral.[41] In their model, Wang et al. let the bidders arrive sequentially at the auction and allow only the first bidder to exercise the *Buy It Now* option. The point of time at which a bidder arrives, however, has no further consequences. This feature is thus equivalent to randomly selecting the decisive bidder in the APPO model and captures the property that the *Buy It Now* option expires once a bidder submits a bid in an auction on eBay.

Given the buy price B and the participation costs c for bidding, Wang et al. calculate a symmetric equilibrium. The respective equilibrium strategies can be described by two characteristic values, the so-called participation

[40] Rather than participation costs, one might more appropriately label these costs as bidding costs.

[41] The authors argue that "[a]lthough some auctioneers could be risk averse, many large retailers that have opened eBay stores, like Dell, IBM, Sun Microsystems, and Sony are unlikely to be risk averse." Even though many models (including the APPO model) assume that sellers are risk neutral, this reasoning is certainly questionable.

threshold s_a and the buy threshold s_b. The participation threshold indicates the valuation at which the respective bidder is indifferent between bidding in the auction and not participating at all whilst the buy threshold is the valuation at which the bidder is indifferent between bidding in the auction and exercising the *Buy It Now* option. Several cases arise of which the most interesting is

$$0 \leq s_a \leq s_b \leq 1 \text{ and}$$
$$s_a < B \ .$$

In this case, the complete strategy of a bidder i with valuation v_i is not to bid if $0 \leq v_i < s_a$, to bid in the auction if $s_a \leq v_i < s_b$, and to exercise the *Buy It Now* option if $s_b \leq v_i \leq 1$.

Clearly, the participation threshold is mainly driven by the participation costs c. The higher the costs c, the more likely it is that a bidder will not bid in an auction. In equilibrium the participation threshold evaluates to

$$s_a = \sqrt[n]{c} \ .$$

Thus, the participation threshold also increases with the number of bidders n.

According to Wang et al. the buy threshold solves

$$s_b - B = \frac{s_b^n - c}{n} \ . \tag{2.36}$$

Rearranging (2.36) and setting $c = 0$ yields $B = s_b - \frac{s_b^n}{n}$, i. e. if there are no participation costs, the buy threshold of Wang et al. (2004) equals the threshold of a bidder in an APPO. The authors show that the buy threshold decreases in the participation costs c. Thus, ceteris paribus, the acceptance of the *Buy It Now* option becomes more attractive as the participation costs increase.

Wang et al. also seek to derive the *Buy It Now* price B^* that maximizes the seller's revenue. The result stated in Proposition 3 of their paper is, however, inconsistent with the other papers discussed so far. According to Wang et al. (2004) the optimal *Buy It Now* price B^* is

$$B^* = \frac{2 + R_a}{3} - \frac{2(1 - c)}{3n} \tag{2.37}$$

with R_a denoting the expected revenues of a pure English auction with no buy price. Setting the participation costs $c = 0$, Equation (2.37) suggests to the seller offering a *Buy It Now* price B that is accepted with positive probability.

The analyses by Reynolds und Wooders (2003), Mathews (2004a, 2003), as well as the APPO model, however, show that offering such a *Buy It Now* price does not pay for a risk neutral seller in the setting investigated by Wang et al. One finds the solution of that discrepancy in the proof of the proposition that Wang et al. give in their appendix: "In order to get an analytically tractable result for the optimal buy-it-now price we use a second-order Taylor series expansion to approximate s^n around the upper bound of the value distribution. [...] Based upon a comparison with simulated numerical results this approximation is very accurate" (p. 33). Apparently, the approximation is not accurate enough for numerical examples and it is definitely not analytically correct as claimed in the text.

The approximation does, however, give some qualitative insight into the behavior of the seller. For example, the optimal buy price B^* increases with the number of bidders n.[42] This leads Wang et al. to conjecture that, if the number of bidders increases, the *Buy It Now* option will be offered less frequently—a hypothesis that they examine with empirical data. They report that they find their hypothesis affirmed by data that they collected from eBay in four categories (memory sticks, iPod players, and two different kitchen mixers).

2.6.6 Extension: Multi-unit Demand

Kirkegaard und Overgaard (2004) extend the analysis of auctions with a buy price to the case in which bidders have multi-unit demand.[43] They analyze a situation with n bidders and two sellers, 1 and 2. Each of the sellers initially owns one item which she values at 0. Bidders have positive but decreasing marginal valuations for the two items. More precisely, Kirkegaard und Overgaard assume that the bidders' valuations of the first item are independently distributed on an interval $[0, \bar{v}]$. Further, there is some k with $0 < k < 1$ and the valuation of the second item of a bidder i who values the first item at v_i is given by kv_i.

The authors refer to a benchmark model by Black und de Meza (1992) who show that in the above situation a sequence of two second-price sealed-bid auctions yields an efficient outcome and increasing revenues from the first to the second auction.

[42] Remember that the revenue R_a is also a function of n.

[43] Kirkegaard und Overgaard refer to their paper as "incomplete", yet the current draft comprises 48 single-spaced pages.

In contrast to the benchmark model, Kirkegaard und Overgaard allow the first (but not the second) seller to offer a buy price. The following schedule describes the bidding procedure:[44]

1. Seller 1 announces a buy price B.
2. Bidders indicate whether they wish to accept the buy price. If exactly one bidder accepts the buy price, he is awarded the item. If more than one bidder is willing to accept the offer, the item is awarded by a random method. In both cases, the bidder who is awarded the item pays the buy price B.
3. If no bidder is willing to accept the buy price, seller 1 auctions her item by means of a second–price auction.
4. Seller 2 auctions the second item through a second–price auction.

Kirkegaard und Overgaard consider risk neutral bidders and compute a symmetric equilibrium for bidding in that auction sequence. Based on the bidding equilibrium they also derive the optimal buy price of seller 1 in the first auction. They obtain the following results:

1. Seller 1 can gain from offering a buy price, i. e. a carefully selected buy price increases the expected revenues in the first auction.
2. If seller 1 offers a buy price, the auction may result in an inefficient out–come.
3. If seller 1 offers a buy price, the expected revenue decreases from the first to the second auction.
4. Further, a buy price in the first auction reduces seller 2's expected revenues in the second auction. Moreover, even the sum of the revenues in the first and the second auction is lower in comparison to the case without a buy price.

The results by Kirkegaard und Overgaard are particularly noteworthy for two reasons. Firstly, in the multi–unit case, the buy price reverses the ranking of the auctions' revenues. Remember that according to Black und de Meza (1992), revenues increase from the first to the second auction in the benchmark case without buy prices. Kirkegaard und Overgaard show that with a buy price, it is the first auction that yields higher revenues. Secondly, in the case of an isolated one–unit auction with risk neutral (and not impatient) bidders, the APPO model, as well as the papers presented in the previous sections

[44] Interestingly, Kirkegaard und Overgaard choose a setting similar to that of Budish und Takeyama (2001). While Budish und Takeyama claim that this setting resembles Yahoo!'s variant of a buy price auction, Kirkegaard und Overgaard relate this setting to "a buy-out price of the eBay-variety."

shows that the seller cannot increase expected revenues by means of offering a buy price. Kirkegaard und Overgaard give an explanation as to why offering a buy price could be rational for a risk neutral seller even if bidders are also risk neutral.

2.7 Summary

All models that were discussed in this chapter are based on the *symmetric independent private values* assumptions. Thus, in the APPO model as well as in the presented alternative approaches that are suggested in the literature, the bidders know their own valuation for the object but not those of the other bidders. Moreover, all bidders are ex-ante symmetric and their valuations are independent of each other. All models analyze an auction with a buy price as a non-repeated game, i. e. the auction is conducted only once and the agents will not meet again in the future. A seller who does not sell the item may not relist it and the bidders have no alternative source of acquiring the item for a price below their private valuation.

In all models, some kind of a threshold strategy is derived and the characteristics of this threshold strategy are discussed in varying degrees of detail. All models investigate the expected revenues (or the expected utility) of the seller. Table 2.1 summarizes the different assumptions made by the respective authors and contrasts them to the APPO model.[45] The table classifies the models with respect to the distribution of valuations, the number of bidders, and the characteristics of the agents. An agent's characteristic is determined in particular by its attitude towards risk. Mathews (2003) and Wang et al. (2004) also consider impatient agents and bidders that incur participation costs, respectively. The table also lists the format of the auction if the buy price is not accepted and states whether a reserve price $\underline{p} > 0$ is taken into consideration. Finally, the table shows whether the buy price is offered to all bidders or only to one randomly selected bidder. In the column "variant", an entry "eBay" indicates that the buy price expires once the bidding has started. The entry "Yahoo!" indicates that the buy price may be accepted even after bidders have submitted bids.

Table 2.2 summarizes the main results of the models. For ease of notation, in the results column any remark with respect to the revenues or the agents' utility refers to the respective expected values. A result "offering a posted price does not pay for the seller" means that for any posted price offer the *expected*

[45] The model by Kirkegaard und Overgaard (2004) is not included in the table due to its focus on multi-unit demand.

Table 2.1. Overview of model assumptions

Model	Distribution of valuations	No. of bidders	Risk attitudes		Corresponding auction		Buy price	
			bidders	seller	format	res. price	offered to	variant
APPO	convex support, arbitrary F	n	arbitrary	risk neutral	second-price	yes	one bidder	eBay
Budish/ Takeyama	two types, $V \sim B(1,p)$	2	arbitrary	risk neutral	second-price[a]	no	all bidders	eBay[a]
Hidvégi et al.	convex support, arbitrary F	n	arbitrary	arbitrary	English	yes	all bidders	Yahoo!
Reynolds/ Wooders	$V \sim U[a,b]$	2	risk neutral / CARA	risk neutral	ascending clock	no	all bidders	eBay / Yahoo!
Mathews (2004a)	$V \sim U[0,1]$	n	risk neutral, impatient	risk neutral, impatient	English with proxy bidding	no	one bidder	eBay
Mathews (2003)	$V \sim U[0,1]$	n	risk neutral	risk averse	English with proxy bidding	no	one bidder	eBay
Mathews (2004b) Mathews/Katzman	convex support, arbitrary F	n	risk neutral	risk averse	English with proxy bidding	yes	one bidder	eBay
Wang et al.	$V \sim U[0,1]$	n	risk neutral, bidding costs[b]	risk neutral	second-price	yes	one bidder	eBay

[a] Budish und Takeyama (2001) seek to address an open English auction with the Yahoo! variant of a buy price. However, their model more closely resembles a second-price auction with the eBay variant.

[b] In the model by Wang et al. (2004) the agents incur costs when bidding in an auction—independent of whether they win the auction or not. They do not incur these costs when accepting the buy price.

revenues of the seller are lower than the *expected* revenues of the corresponding pure auction. There may well be particular cases, i. e. particular realizations of valuations, for which the opposite is true.

Table 2.2: Overview of the results

<table>
<tr><th>Model</th><th>Results</th></tr>
<tr><td>APPO</td><td>• In equilibrium, the strategy of the decisive bidder $\hat{\imath}$ with valuation $v_{\hat{\imath}}$ is characterized by a threshold function $t_{\hat{\imath}}(v_{\hat{\imath}})$.
• The decisive bidder accepts a buy price (PPO) $\bar{p}$ if $\bar{p} \leq t_{\hat{\imath}}(v_{\hat{\imath}})$.
• The acceptance threshold increases with
– the number of bidders,
– the reserve price, and
– the degree of risk aversion of the decisive bidder.
• If bidders are risk neutral (or risk loving),
– offering a buy price does not pay for the seller.
• If bidders are risk averse,
– the seller can profit from offering a buy price,
– an optimal APPO outperforms an optimal second-price auction with respect to revenues.</td></tr>
<tr><td>Budish/ Takeyama</td><td>• If bidders are risk neutral,
– offering a buy price does not pay for the seller.
• If bidders are risk averse,
– the seller can profit from offering a buy price,
– the revenues of second-price auction with a buy price exceed even the revenues of a first-price auction.</td></tr>
</table>

Hidvégi et al.	• Bidders accept a (given) buy price once the auction price reaches a certain level. This level decreases with the bidders' valuations. • The seller can profit from offering a buy price if either the seller or the bidders are risk averse. • If bidders are risk neutral (and the buy price is set sufficiently high), – a buy price auction yields the same revenue as a (pure) standard auction—this holds even if the buy price is accepted with positive probability. • If bidders are risk averse (and the buy price is set sufficiently high), – the seller can profit from offering a buy price, – a buy price auction is efficient, – bidders with constant absolute risk aversion are indifferent between a (pure) English auction and a buy price auction.
Reynolds/ Wooders	• If bidders are risk neutral, – the revenues of the buy price variants of eBay and Yahoo! are the same, – a sufficiently high buy price might not be accepted in the eBay variant but it will in the Yahoo! variant—in this case the two buy price auctions yield the same revenues as a pure English auction, – if the buy price is exercised with positive probability in the eBay variant, the two buy price auctions yield lower revenue than a pure English auction. • If bidders have constant absolute risk aversion, – the bidders are indifferent between the two auction formats, – the Yahoo! variant yields higher revenue than the eBay variant, – the buy price can be set such that both variants yield higher revenues than a (pure) English auction.
Mathews (2004a)	• The seller can profit from offering a buyout option if either the seller or the bidders are impatient. • A buyout option offered by an impatient seller also increases the bidders' payoffs.

Mathews (2003)	• A risk averse seller can profit from offering a buyout option. • The more risk averse a seller is, the lower is the buyout option that this seller optimally offers. • A buyout option offered by a risk averse seller also increases the bidders' payoffs.
Mathews (2004b) Mathews/ Katzman	• A risk averse seller can profit from offering a buyout option (see also Mathews, 2003). • Whether the bidders also profit from a buyout offer depends on the distribution of bidders' valuations.
Wang et al.	• Equilibrium strategy of bidder i with valuation v_i is characterized by the participation threshold s_a and the buy threshold s_b. • If $s_a \leq v_i < s_b$, bidder i bids in the auction. • If $s_b \leq v_i$, bidder i accepts the buy price. • Participation threshold increases with – the bidding costs and – the number of bidders. • Buy threshold decreases with the bidding costs. • Optimal buy price increases with the number of bidders.

3

Design of the APPO Experiment

3.1 Motivation and Research Questions

There may be many potential explanations why a seller would also offer a posted price when conducting an auction. From a theoretical perspective that builds on the notion of rational bidders and sellers, the main drivers for posted price offers are risk aversion or impatience on either side of the transaction, as well as explicit or implicit transaction costs. Of course, many more explanations are possible.

The theoretical analysis of the APPO model has focused on the bidders' attitudes towards risk. Neither impatience nor transaction costs have been modeled. Moreover, the analysis has only investigated the expected revenues of the seller rather than her expected utility with respect to some utility function. Thus, throughout the theoretical analysis, the seller is considered risk neutral.

The theoretical analysis has resulted in two main findings:

1. If bidders are risk neutral (or risk loving), offering a PPO is not worthwhile for the seller (Theorem 2.13).
2. If bidders are risk averse, the seller can set the PPO $\bar{p}$ such that the expected revenues of an APPO with the PPO $\bar{p}$ will be higher than those of a (pure) second-price auction (Theorem 2.14).

The second result also holds in a more general environment. For a wide range of settings, it is sufficient to assume that the decisive bidder is risk averse with positive probability. The theorem even holds if there are bidders with different attitudes towards risk. If the decisive bidder is risk loving or risk neutral, the seller's revenue is not affected. Such a bidder will simply reject a PPO if it is appropriately set. The seller can still gain if the decisive bidder turns out to be risk averse.

In fact, bidders on eBay most likely differ in their risk attitudes. There might be some bidders who consider bidding fun and who appreciate in particular the gambling component of online auctions—these bidders are risk loving. However, it appears reasonable to assume that a core of bidders exists that is risk averse rather than risk neutral.

Can one conclude from the above that the APPO model sufficiently explains the agents' behavior in auctions on eBay? Is the phenomenon of sellers offering *Buy It Now* options on eBay simply due to the existence of risk averse bidders? Do sellers correctly anticipate the bidding strategies of these bidders?

Unfortunately, the analysis of real-world phenomena is not that simple. As with any model, the results of the theoretical analysis of the APPO model leave some questions unanswered. The most fundamental issues are:

1. The APPO model is based on the assumption of rational bidders and sellers. However, actual bidders and sellers may not always behave in accordance with the theory.
2. The APPO market institution is similar to an auction with a *Buy It Now* price on eBay. It is, however, not *exactly* the same as an auction on eBay. Thus, bidders and sellers may behave differently in an auction on eBay and in an APPO.

Regarding the first issue above, remember that calculating the theoretical solution is rather challenging. An auction participant might not be able or willing to exert such mental effort within the short time frame when deciding on his or her strategy.[1] The second issue relates to the fact that the APPO model is built on several artificial assumptions that do not hold in a real-world setting. To name but a few, the model assumes that bidders are symmetric and that the number of bidders as well as the probability distribution function of their valuations is common knowledge. Moreover, in the model, bidders demand only one unit of an item and sellers try to sell a particular item only once. Relaxing these assumptions might alter the behavior of the agents and in turn the outcome of an auction.

In order to shed more light on the posted price option, a computer-based experiment has been conducted. The experiment aims to analyze the APPO model empirically, i. e. it mainly addresses the first of the two drawbacks of the theoretical model that were mentioned above. The key questions that guide the analysis are:

[1] The use of gender-specific pronouns has nothing to do with the actual sex of the subjects in the experiment. Rather, the notation follows the previous use of pronouns: bidders are referred to by the pronouns "he" or "his" and the sellers by the female counterparts "she" or "her". In the experiment, the participants were randomly selected and the group of subjects was of mixed gender.

1. Is the bidders' behavior in an APPO consistent with the theory developed in Chapter 2?
2. Do sellers offer a posted price? Are they in fact able to extract additional revenue by offering a posted price?
3. Besides the seller's revenue, how does the existence of the PPO affect the bidders' surplus and the social surplus?

3.2 Treatments

The research questions require a two-fold analysis that investigates the behavior of both the bidders and the sellers. In order to address the two groups of agents, the present analysis applies an experimental design with separate sessions for bidders and sellers.

In principle, one could have observed bidders and sellers simultaneously by having them participate in the same sessions. As will be shown in the following, however, the chosen design yields several advantages. By separating bidders and sellers,

- the treatments without a PPO and the treatments with a PPO can be set up very similarly,
- more data are generated at lower cost, and
- distortions of the incentive-compatibility of the payoff function are minimized.

Regarding the above advantages, it should first of all be noted that in order to compare auctions with and without a PPO, the respective experimental treatments must be set up analogously. They should only differ in the presence of absence of the PPO, but—as far as possible—not in any other parameter. Note that settings in which a seller is present and those in which she is not might be perceived quite differently by the participants. In the latter case, the individual participants compete not only against each other for the item; as a group, they also play against the (anonymous) auctioneer, and by means of their bids, they determine the size of the total bidder surplus that is shared among them. In the former case, such an interpretation might shift towards splitting the total surplus of an auction between the bidders and the seller, who is now part of the group. Issues that are difficult to control in an experiment (but which are not the object of this investigation), like envy and fairness, might play a different role in the two settings.

Consequently, if a seller participates in a treatment with a PPO, she should also be present if auctions without a PPO are under investigation. In the latter case, however, the seller would have nothing to do. Having a subject in an experiment who has nothing to do, is not just a cost issue for the experimenter. More importantly, it might be confusing for some participants, as they might be distracted by the presence of a participant who is not actively involved in the experiment. The separation of bidders and sellers in the chosen design avoids this problem because bidders and sellers are never physically present in the lab at the same time.

Second, one of the research questions is whether or not the seller offers a PPO. In the lab, one wants to observe what the seller actually does. If she decides, however, not to offer this option, nothing can be learned about the behavior of the bidders with respect to a PPO. When the behavior of the bidders is under investigation, it is more valuable for the analysis to ensure that a PPO is in fact being offered. By analyzing bidders and sellers separately, this can easily be achieved.

Finally, there is also a practical aspect: If the number of bidders in an auction is not too low, the expected revenue of the seller will be much higher than the expected payoffs of the bidders.[2] This, again, might give rise to several distortions. From ultimatum game experiments it is known, for example, that the proposer, who is in a more advantageous position than the responder, generally offers the responder a much larger share of the pie than the theory suggests (e. g. Camerer und Thaler, 1995; Güth, 2000).[3] Similarly, the seller in an APPO experiment might set the PPO rather low in order to share her profit with the bidders. Moreover, ultimatum game experiments show that the responder often punishes a proposer who offers only a small share of the money by rejecting the offer (ibid). If the seller does not offer a low enough PPO, might bidders in the experiment refuse to bid aggressively? Of course, this is highly speculative. However, the argument illustrates the risk of distortional effects that stem from the interplay of bidders and sellers. Again, the separation of bidders and sellers avoids these issues.

As mentioned earlier, the sessions with bidders were conducted with auctions both with and without a PPO in order to study its effect. Since in auctions without a posted price offer there is nothing for sellers to decide, only auctions in which they could set a PPO were conducted with them.

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[2] If valuations are independently and uniformly distributed on an interval $[0; \alpha]$, this holds if at least two bidders participate in an auction.

[3] In the ultimatum game, two players, called the proposer and the responder, obtain the opportunity to share an amount of money. First, the proposer offers some share of the money to the responder. If the responder accepts the offer, she is paid that share and the proposer keeps the rest. If, however, the responder rejects the offer, both players receive nothing. According to the theory, the proposer would offer the smallest currency unit and the responder would accept that offer.

Thus, there are three treatment families that will hereafter be referred to by A, B, and S.

Treatment family A constitutes the benchmark case. In this treatment, auctions without a PPO are conducted and the behavior of bidders is observed. Treatment family B deals with bidders in auctions in which a PPO is available, and treatment family S focuses on the PPOs that the sellers set.

All treatment families were conducted with both three and five bidders. This results in a total of six different treatments: A3, A5, B3, B5, S3, and S5 (see Table 3.1). All treatments were conducted separately and each subject participated in only one of the treatments.

Table 3.1. Overview of treatments in the experiment

Treatment	Role of participants	Description
A3	bidder	no PPO, 3 bidders per auction
A5	bidder	no PPO, 5 bidders per auction
B3	bidder	PPO available, 3 bidders per auction
B5	bidder	PPO available, 5 bidders per auction
S3	seller	PPO available, 3 bidders per auction
S5	seller	PPO available, 5 bidders per auction

3.3 Collecting Data: the Strategy Method

The separation of bidders and sellers poses a challenge for the design of the APPO experiment. On the one hand, one seeks to confront the sellers with a realistic decision problem, i. e. a situation in which actual bidders decide whether or not to accept a PPO. On the other hand, however, in the S–treatments bidders are not present in the lab.

The APPO experiment addresses this challenge by applying the strategy method. According to this method, participants are asked for their entire strategy as opposed to observing individual actions during the course of the experiment. The term *strategy* follows the notion of game theory by referring to a complete and concise plan that defines an action of a participant for any possible state of the experiment. Recording the bidders' strategies in the B–treatments and mapping the sellers in each S–treatment auction to one of the observed bidders' strategy profiles allows the evaluation of the auctions in the S–treatments based on actual bidder behavior.

The strategy method was introduced by Selten (1967) and has been used by several authors since then.[4] According to Brosig et al. (2003) "[t]he obvious advantage of this method is that it immediately generates data for all of the information sets in a game and greatly reduces the cost of experimental research." The fact that the strategy method collects data for all nodes of a game (and not only those that are on the actual path of the chosen strategy profiles) is crucial for the APPO experiment. The separation of bidders and sellers is only possible because the strategy method provides the answer of the decisive bidder to *any* PPO as well as his bid in an auction if the seller decides not to tender a PPO.

The above quote by Brosig et al. also mentions the cost advantage of the strategy method. This holds true for the APPO experiment as well because the strategy method greatly enhances the sufficiency of the data collected for the analysis. Consider, for example, an APPO in which the seller does not offer a PPO: if only the actions of the participants were recorded, the experiment would not generate any data with respect to the reaction of the decisive bidder to the PPO. Note that based on the theory, a seller is expected to set the PPO, if she offers one at all, rather high. Only a small fraction of the bidders are then candidates for accepting the offer. Note, also, that one needs n bidders for each auction—and only one of these bidders, the decisive bidder, is potentially offered a PPO. Moreover, what can be learned if a decisive bidder with a very high valuation accepts a rather low PPO, and what if a bidder with a low valuation rejects a high PPO? If only the actual actions of the participants were observed and recorded, a large number of participants might be necessary to obtain a data set that is sufficiently large for the analysis. By not only recording the 'accept' / 'reject' decisions of the decisive bidder with respect to a given PPO but also the acceptance threshold of all bidders, i. e. the maximum PPOs the bidders are willing to accept, the APPO experiment yields more valuable data.

From a theoretical perspective, the outcomes of experiments conducted using the strategy method should not differ from those in which the protocol method is applied.[5] Often, however, experiments (such as the APPO experiment) are used to test whether theoretical predictions hold in a laboratory setting and significant differences are frequently found. This raises the question of whether the strategy and protocol methods in fact lead to identical outcomes. Potential distortions may arise because under the strategy method subjects are forced to think about the game as a whole and to submit an entire strategy rather than to concentrate on their individual actions. Differences of

[4] See Brosig et al. (2003) for an overview of strategy method experiments.

[5] The term "protocol method" was introduced by Selten (1967) in reference to the traditional method of recording the actual actions of the participants.

this kind, however, are per se neither an advantage nor a disadvantage of the strategy method (Roth, 1995).

Assessing the method, Roth (1995) acknowledges its above-mentioned advantage of generating much more data than the protocol method. However, he also identifies a disadvantage in pointing out that the strategy method "removes from experimental observations the possible effects of the timing of decisions" (p. 322). In reference to ultimatum games, Roth argues that "it will not be possible to observe any effects that may be due to the accepter/rejecter making her decision *after* the proposer has made his decision, knowing what has been proposed" (ibid, original italics). A similar critique may apply to the APPO experiment. Note that in a real setting, the seller decides on a PPO and a bidder then knows whether he is being offered a PPO or not. Moreover, the bidder knows the amount of the PPO if it is offered to him. The APPO experiment turns this order around. First, a bidder submits his acceptance threshold and learns only thereafter (*i*) whether he was offered a PPO or not and (*ii*) the amount of the PPO.[6] Then, second, the seller decides about a PPO in a separate session. However, this PPO, if offered, is not presented to a bidder who accepts or rejects it. Rather, the PPO is matched with bidder strategies which have been recorded days before the sellers are invited to the lab.

While Roth (1995) only speculates about differences between the strategy and the protocol methods, Brandts und Charness (2000) analyze whether such differences actually exist. They investigate sequential versions of the prisoner's dilemma and the chicken game. Both games are played in two treatments. In the so-called "hot" treatment, the protocol method is applied and the "cold" treatment is played according to the strategy method. Interestingly, Brandts und Charness find no differences between the two treatments. Similar results are also reported e. g. in Cason und Mui (1998).

Güth et al. (2001), on the other hand, observe quite the opposite. They compare three implementations of the mini-ultimatum game, which only differ in the allocation implied by the "high" offer.[7] Applying the protocol method, Güth et al. (2001) find that the proposer offers "high" more often if this option leads to exactly equal shares. The "high" option is chosen significantly less frequently if the resulting allocation is not exactly equal—regardless of whether this allocation would favor the proposer or the responder. Moreover,

[6] Note also that this PPO is randomly drawn by a computer and not by a human subject.

[7] In contrast to the ultimatum game, in a mini-ultimatum game the proposer can choose between only two options, "high" and "low", rather than proposing *any* allocation. Often, the "high" option translates to an allocation of roughly equal shares and the "low" option yields only a marginal share to the responder.

the responder rejects a "low" offer more often if an alternative offer of exactly equal shares has been available. The phenomenon diminishes if the strategy method is applied. Another example of different results between the protocol and the strategy method is shown in Hoffman et al. (1998), who study trust in a bargaining experiment.

Revert now to this study's experiment. Given a bidder's valuation, his strategy in an APPO is the maximum PPO $\bar{p}$ that the bidder will accept if the PPO is offered to him and the amount b that he will bid in the corresponding auction if the PPO is rejected by the decisive bidder.[8] In the benchmark case without a PPO, i. e. the treatments A3 and A5, a bidder's strategy is the bid he submits.[9] Figures 3.1 and 3.2 display screenshots of the participants' user interface in the A and the B–treatments.[10] The screens show the difference between the two treatments. In the A–treatments, a bidder's strategy in an auction consists of only one number, namely his bid (Figure 3.1). In the B–treatments, bidders have to enter two numbers: the maximum posted price they are willing to accept and their bid in the corresponding auction in case the decisive bidder rejects the PPO (Figure 3.2).

In the B–treatments, a PPO is offered in every auction. The amount $\bar{p}$ of the PPO is chosen randomly before the start of the experiment. A computerized random number generator determined a PPO for every round of the experiment.[11] The same PPO is offered in all simultaneous auctions of a session and the same sequence of PPOs is applied in all sessions of the B–treatments.[12]

After all bidders have entered their strategies, the auction round is evaluated and only then is the amount $\bar{p}$ of an auction's PPO announced to the participants in the B–treatments. In treatments A3 and A5, the winning bidder and the auction price are determined according to the rules of a second–price auction (see p. 73 for details). In treatments B3 and B5, the auction bids are only evaluated if the item is not awarded to the decisive bidder, i. e. if the pre–

[8] Strictly speaking, in a sequence of twelve auctions a game theoretical strategy covers an agent's concise and complete plan for the behavior in the whole sequence of auctions. With regard to the setup as a stranger experiment (see Section 3.4), however, in this paper, the term strategy refers to only one auction.

[9] One could also argue that a bidder's strategy consists of the threshold and the bidding functions $t(v)$ and $b(v)$. In the experiment, however, a bidder was informed about his valuation before he had to decide on a strategy.

[10] The experiment was conducted in German. Figures 3.1 and 3.2 are translations of the original screens. In Appendix D.2 the screens that were actually used in the experiment are shown.

[11] In the experiment, the subjects participate in a sequence of auctions. These auctions are also referred to as rounds (see Section 3.5 for details).

[12] The PPOs offered in the experiment are given in Table B.6 of Appendix B.

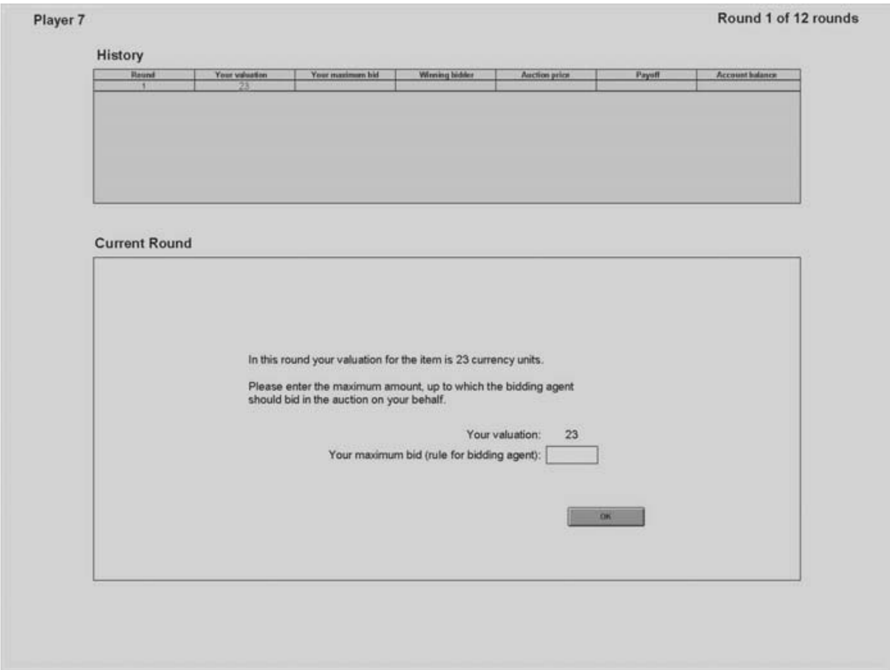

Figure 3.1. Bidding screen in treatments A3 and A5

determined PPO is higher than the maximum PPO that the decisive bidder is willing to accept. The decisive bidder (or more precisely his identification number) in each auction is randomly selected before the experiment is started and remains identical for the two sessions of each of the two treatments B3 and B5. The identification numbers of the decisive bidders of all auction rounds are indicated by a bold font face in Tables B.1 and B.2 in Appendix B. In neither the A nor the B-treatments is a seller physically present. Rather, a (virtual) seller is built into the software.

The behavior of the seller is investigated in treatments S3 and S5. Her strategic options are illustrated in Figure 3.3: she first indicates whether or not she wishes to offer a PPO and, if so, she specifies, secondly, the respective amount. Since the seller's decision depends on her expectation regarding the bidders' responses to a PPO, it is crucial for the analysis to map the seller with actual rather than fictitious bidder behavior. It has been mentioned above that in order to do that, the strategies of the bidders in treatments B3 and B5 are recorded and saved in the system. The auctions in treatments S3 and S5 are then evaluated based on both the PPOs set by the sellers, i. e. the actual subjects in these treatments, and the strategies of the bidders which

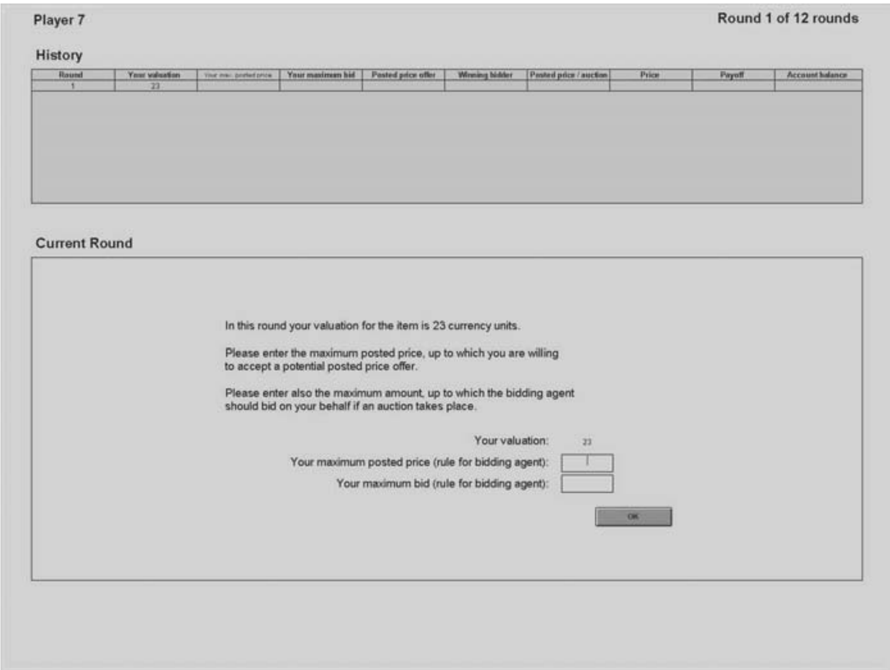

Figure 3.2. Bidding screen in treatments B3 and B5

have been recorded in the B-treatments. Section 3.5 elaborates on the details of the implementation of the experiment and describes how the sellers are matched to the recorded strategies of the bidders.

3.4 Setup as a Stranger Experiment and Independency of Observations

The numerical calculations in Chapter 2 show that the differences between the expected revenues of an APPO and a pure auction can be rather small. As a consequence, one of the difficulties in designing the experiment is generating a data set that is large enough to identify these differences—if they exist at all.

In order to solve this difficulty, the experiment is conducted as a stranger experiment, i. e. the formation of the groups changes from round to round. There is one rotating schedule for the sessions with three bidders and one for the sessions with five bidders. These rotating schedules are determined before the start of the first session and they are kept identical over all sessions with

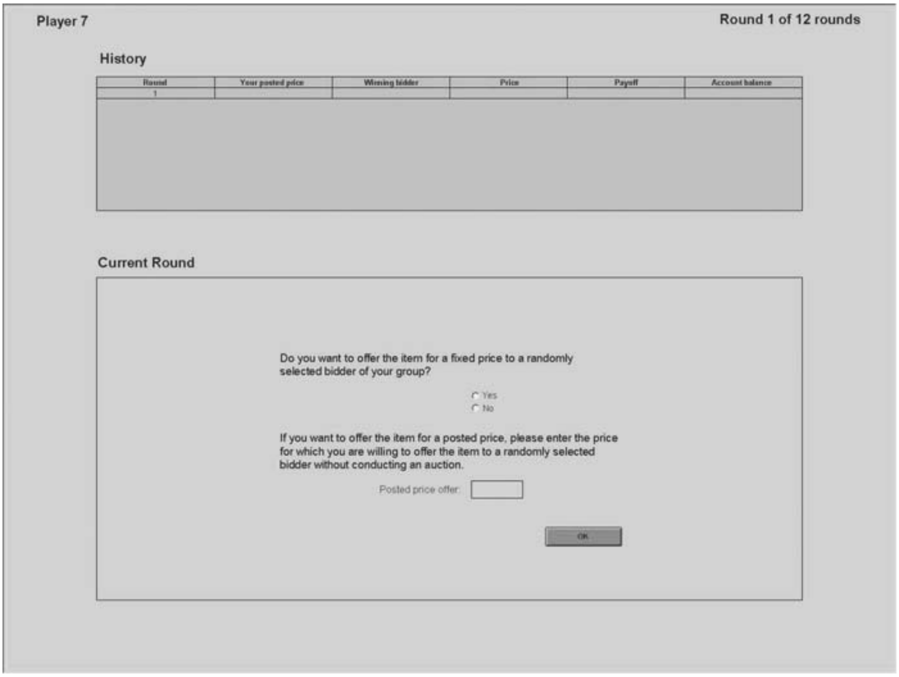

Figure 3.3. Seller's screen in treatments S3 and S5

three and five bidders, respectively. The concise composition of the groups is given in Tables B.1 and B.2 in Appendix B. Note that the experiment is not set up as a perfect stranger experiment in which the probability that two subjects are mapped to the same group more than once is zero—with 15 bidders, twelve rounds and more than two bidders per auction, this is not possible. Yet the number of re-matchings is minimized and the design ensures that the same group never meets twice. In the experiment, the bidders are not informed with whom they are bidding in a particular auction. Only after an auction they learn the identification number of the bidder who was awarded the item.

According to Davis und Holt (1993, p. 528), the stranger method allows "a researcher to generate multiple observations with a single cohort, while simultaneously maintaining the independence of these observations. [...] [T]he idea is to rotate participants in such a manner that each person in a cohort meets each of the others only once. [...] If understood and believed by the subjects, it will induce a series of single-period games." The authors add, however, that the stranger method does not guarantee independence. In fact, they argue that a single participant might behave so bizarrely that he confuses all the peers in his group. This bizarre behavior could then affect all later rounds in which the peers participate. Even worse, due to the rematching, this participant would infect all (or at least many) subjects within the cohort. In this case, the individual outcomes observed in different rounds and/or groups of the cohort are clearly not independent of one another. In general, however, the setup as a stranger experiment reduces the dependency of single observations, which will be very helpful in the analysis.

In the chosen design of the experiment, each subject participates in a sequence of twelve auctions (cf. Section 3.5). Because the same subjects are observed several times, the outcomes of these auctions are, of course, not independent. At least three sources of dependency exist: First of all, the sample of subjects is smaller than the number of observations. Each participant is observed in several auctions and certain characteristics of the individual participants are thus repeatedly reflected in the data. Secondly, subjects gain experience over time and may systematically alter their behavior during the course of the experiment. Finally, repeated games may give rise to strategic patterns that are not typical for one-shot games. Collusive bidding, i. e. a tacit "agreement" to submit only very low bids and to extract high payoffs, could be an example. In this case, the behavior in a single auction is not a strategy in itself, but only a component of a more extensive strategy. As such a strategy embraces several rounds and may depend heavily on the history of past auctions.

In spite of the above issues, the analysis seeks as many data points as possible. Justified by the setup as a stranger experiment, the collected observations are referred to as *quasi-independent* and treated as if they were independent. Nonetheless, the analysis acknowledges that strictly speaking, this is not true. As a consequence, the results of the statistical tests based on the quasi-independent observations are more liberal than the reported p-values imply. With respect to the three critical issues raised above, the following paragraphs once more discuss the implications of the quasi-independent observations for the analysis of the APPO experiment.

First of all, there is not much one can do with respect to the dependency of the auction results on the small set of experimental subjects. Note that a bidder's induced valuation changed from round to round and, in some sense, a bidder was assigned a new task in every round. However, this does not completely solve the issue of dependent observations and this dependency thus remains a flaw of the experimental design. This flaw was tolerated, however, to keep the costs of the experiment under control.[13]

[13] Having chosen a stranger setup, it is not possible to analyze the data based on aggregate group results as, for example, the average results of independent cohorts.

Second, as discussed in Section 3.8, paired tests are performed in order to compare the experimental results. This reduces the effect of trends over time (e. g. learning by subjects). The analysis assumes that if a systematic trend exists, this trend is reflected similarly in the results of the A and the B–treatments. Since a paired test considers the differences between the results, the effect of the trend diminishes. The design ensures that a matched pair always comprises results of the same round, i. e. a result of an A–treatment observed in round i is only compared with a result of the same round i of the corresponding B–treatment.

Third, the setup as a stranger experiment also minimizes a potential bias caused by (collusive) effects within the groups. Changing the composition of groups from round to round limits the cohorts' possibilities of developing and applying long–term strategic behavior.

Another issue that relates to the (in–)dependence of observations concerns the setup of treatment S5. The objective is to generate a large data set without inflating the costs of the experiment. Thus, one seeks to observe a sufficiently large number of sellers. If each seller were to be matched with a unique set of bidding data, however, the experiment would be very expensive due to the costs of inviting and paying additional bidders. This is particularly the case for auctions with five bidders.

The problem is solved by assigning each of the profiles of bidding strategies which are recorded in treatment B5 not only to one but to two different sellers. The chosen design doubles the size of the seller sample without incurring additional costs for bidders. When analyzing the auction outcomes, however, the difficulty arises that, again, the data are not independent. Because each bidding strategy observed in treatment B5 is duplicated, it is incorporated twice in the outcomes of the auctions in the treatment S5.[14] Thus, it is not appropriate to base a statistical test on all individual auctions of treatment S5.

For the above reason, the outcomes of S5 are reported in two separate sets of data labeled "Observation I" and "Observation II". The two observations differ in the sellers, i. e. the subjects of treatment S5, but are based on pair–

Due to the re-matching of the bidders, all auctions are ultimately dependent and independent cohorts do not exist.

[14] Strictly speaking, not only the outcomes, i. e. the revenues, the winning bidders' payoffs, and the social surplus, but also the observations with respect to the behavior of the sellers are dependent. In each instance there are two sellers that share the same sequence of strategy profiles of bidders. Such a pair of sellers therefore experiences a similar history of auction outcomes. Moreover, even the strategy profiles of different bidders are not independent but only quasi-independent. This kind of dependency, however, is considered marginal and is neglected in the analysis of the behavior of the sellers.

wise identical bidding data. The statistical tests comparing the results of the treatments with and without a PPO in auctions with five bidders are each run twice: the first run is based on "Observation I" and the second run is based on "Observation II". The p-values of both runs are given. Because the two runs are based on the same bidding data not only of treatment B5 but also of treatment A5, they are not independent. Rather, they should be interpreted as substitutes.

An alternative possibility for testing would be to calculate the arithmetic mean of all pairs in Observations I and II. This approach, however, has been rejected because it would significantly alter the structure of the data. It will be shown in Section 4.2, for example, that the data is characterized on the one hand by a large number of observations consistent with the theoretical benchmark and on the other hand by a substantial share of observations which deviate strongly from this benchmark. There are relatively few observations with medium deviations from the theory. Averaging pairs of observations would overstate the fraction of medium deviations from the theory and reduce the impact of outliers.

3.5 Further Design Parameters

Each of the treatments A3, A5, B3, and B5 is conducted with two sessions with 15 subjects participating in each of these sessions. Thus, a total of 30 bidders is observed in each treatment. Within a session, the bidders take part in a sequence of twelve consecutive auctions that are also referred to as rounds. Since there are three bidders per auction in the sessions of treatments A3 and B3, in every round of these sessions the 15 participants are mapped to one of five auctions that are conducted simultaneously. Analogously, in the sessions of the treatments A5 and B5, three simultaneous auctions are conducted in each round. Bidders who bid in the same auction are also said to constitute a group in that round.

In the experiment, the participants do not trade real objects. Rather, in every round a virtual item is auctioned for which the bidders' private valuations are induced: if a participant is awarded the item, the difference between his valuation and the price to pay is credited to the bidder's experimental account. The bidders' valuations in the experiment are randomly and independently drawn from the discrete uniform distribution over the support $M = \{1, 2, \ldots, 100\}$. The ex-ante probability distribution is made known to the participants.

The data set of the valuations is composed of a (12×15) matrix that assigns each of the 15 bidders a valuation for each round. All valuations are drawn before the start of the first session of the experiment. They are shown in Table B.5 in Appendix B. Note that the same table of valuations is used in all sessions with bidders. Thus, the analysis allows for a pairwise comparison of strategies between the A and the B-treatments as well as a comparison between auctions with three and with five bidders. Friedman und Sunder (1994, pp. 68, 100) argue in favor of designs that produce matched-pair data because they sharpen the test when analyzing the data.

The treatments S3 and S5 are investigated in one session each with ten and twelve participants, respectively. Remember that there are five simultaneous auctions in every round of treatment B3. Because two sessions of B3 are conducted, the bidders' strategies in a total of ten auctions are observed per round. Thus, in every round each seller can be mapped to a separate set of bidders' strategies. Treatment S5 is set up accordingly. Since there are five bidders per auction, in the two sessions of treatment B5 only six bidder groups can be observed per round. When the behavior of sellers is being investigated, these data sets are duplicated, i. e. the data set of every auction in treatment B5 is assigned to two different sellers, which allows treatment S5 to be conducted with twelve sellers. Following the design principle of a stranger experiment, the assignment of bidder groups of treatment family B to sellers in treatment family S is randomized.

Finally, the rules of the auction in the A-treatments as well as the rules of the corresponding auction in the B-treatments deserve a detailed look. In the APPO model, the corresponding auction is modeled as a second-price or Vickrey auction (Vickrey, 1961). Following the APPO model, a second-price auction is (internally) implemented in the experiment. This auction format is known to perform poorly in experimental settings, however. Several authors observe a significant share of bidders who deviate from their dominant strategy, with a majority bidding more aggressively than according to the theory (e. g. Kagel und Levin, 1993; Kagel, 1995; Harstad, 2000).[15] In order to prevent these distortions, the participants are instructed to enter the bidding limit of a computerized agent who bids on their behalf in an English auction. Technically, the auction institution of the experiment is thus an English proxy auction. The authors cited above report that the English auction performs much better than the second-price auction in experimental settings, and theoretically, the implemented proxy version is equivalent to a second-price auction. Much care was devoted to the wording of the instructions to precisely

[15] In a specific setting of a multi-attribute procurement auction, Seifert und Strecker (2003) observe to the contrary significantly more defensive than aggressive bids compared to the dominant strategy.

describe the auction rules and to avoid any strategic guidance. Moreover, two examples are given in the instructions for illustration.[16]

In the experiment, the reserve price is set to one currency unit and the integer numbers $\mathbb{N}$ are the set of feasible bids.[17] Thus, there is a positive probability of tied bids which are resolved by a random method.

Table 3.2 once more summarizes the design parameters which differ among the sessions of the experiment. All treatments share the following properties:

- Model framework: induced symmetric independent private valuations
- Bidder valuations: uniformly distributed over $\{1, 2, \ldots, 100\}$
- Auction institution: English proxy auction
  - Feasible bids: integer values
  - Reserve price: 1 currency unit
  - Minimum increment: 1 currency unit

Table 3.2. Summary of design parameters

	Treatment					
Parameter	**A3**	**A5**	**B3**	**B5**	**S3**	**S5**
Number of Rounds	12	12	12	12	12	12
Bidders per auction	3	5	3	5	3	5
Auctions per round	5	3	5	3	10	12
Subjects per session	15	15	15	15	10	12
Number of sessions	2	2	2	2	1	1
Total subjects	30	30	30	30	10	12
Total auctions	120	72	120	72	120	144
Data gathered	bids		thresholds; bids		PPOs y/n; $\bar{p}$	

3.6 Conducting the Experiment

The experiment was conducted at the experimental laboratory of the Informa– tion Management and Systems research group at the University of Karlsruhe

[16] The complete instructions are available for download from `http://www.stefanseifert.de/downloads/appo`.

[17] A difficult but important issue is the minimum increment of bids. The instructions lay out that a computerized auctioneer will start the auction by offering the item for a price of one currency unit to the bidding agents. As long as at least two bidding agents indicate their willingness to buy, the price is increased by one unit. If only one bidder remains in the auction, he is awarded the item and pays the last price at which two (or more) bidders were still active in the auction.

in the fall of 2003 and was carried out in German. Subjects in the experiment were students from various disciplines, with a majority from the Department of Economics and Business Engineering. The students were randomly selected for participation from a large subject pool of more than 2,000 volunteers. None of the subjects had ever participated in a forward auction experiment at the University of Karlsruhe before.[18]

The experiment was computerized. It was programmed and conducted with the software z-Tree (Fischbacher, 1999). The participants entered their decisions on computer terminals that were set up in individual, visually isolated cabins (Figures D.1 and D.2 in Appendix D show the experimental lab and a participant cabin). These cabins or seats were labeled with the letters 'A', 'B', ..., 'O'. Upon arrival at the experimental lab, the participants randomly drew a card that revealed the letter of their cabin. In addition to the computer terminal, participants found a pen, a ruler, and two sheets of blank paper in the cabin. The paper was intended for the participants' personal use and was not collected or analyzed. Communication between the participants was not permitted.

After the subjects had been seated, they received written instructions, which were also read aloud by a research assistant.[19] To ensure that they had understood the rules of the experiment, the participants had to answer an extensive online questionnaire before the actual experiment was started. The questionnaire comprised—depending on the treatment—12 to 19 questions covering all the rules given in the instructions.

Once the questionnaires had been processed, the software assigned identification numbers to the participants. These numbers served as the participants' names throughout the experiment. In the A and B-treatments the numbers ranged from 1 to 15 and in treatments S3 and S5 they ranged from 1 to 10 and 1 to 12, respectively. The identification numbers were randomly mapped to the participants in every session and had nothing to do with their seat labels.

The subjects participated in twelve auction rounds. There were no trial rounds. At the beginning of each round, the participants were informed about their assigned valuation for the (virtual) item in the current round. The valuation was displayed on the bidding screens on the participants' computer terminal (see the figures in Appendix D.2). If a participant was awarded the item, the difference between his valuation and the price he had to pay was credited to his experimental account. The current account balance as well as

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[18] Some of the students might have participated in a multi-attribute procurement auction experiment in the fall of 2002 (cf. Strecker, 2004).

[19] For details on the instructions see `http://www.stefanseifert.de/downloads/appo`.

the history of the past auction rounds were visible to the participants throughout the entire session.[20]

Reading the instructions and answering the questionnaire took about half the time of an experimental session. On average, a session lasted just under an hour. At the end of an experimental session, the subjects were paid in cash. Valuations and prices in the experiment were calculated in currency units. The conversion mechanism was explained in the instructions and thus known to the participants. In treatments A3 and B3 the bidders' final account balance was divided by 10 and then yielded the payoff in euros. Thus, the bidders received 10 euro cents for every currency unit in the experiment. In treatments A5 and B5, the final account balance was divided by 4, i. e. the participants were paid 25 euro cents per currency unit in the experiment. In treatments S3 and S5 the conversion factor from the currency units in the experiment to euros was 0.02 and 0.015, respectively.

There was no show-up fee or up-front payment. Table 3.3 shows the average as well as the minimum and the maximum payoffs of the participants in the different treatments. In one of the two sessions of treatment B5, the calculated payoff of three participants was negative. For legal reasons, however, the rules of the experiment guaranteed that participants could not incur losses. Thus, these subjects were paid a zero payoff. In all other sessions, all participants received positive payoffs.

Table 3.3. Payoffs of the participants in euros

Treatment	**Participants**	**Payoff [€]**		
		Minimum	**Maximum**	**Mean**
A3	30	0.90	24.60	10.85
A5	30	2.25	18.00	9.95
B3	30	3.20	25.30	11.15
B5	30	−19.00	24.50	8.73
S3	10	7.16	13.12	10.85
S5	12	10.16	12.81	11.95
overall	142	−19.00	25.30	10.37

[20] For each completed auction in which a subject participated, the history showed the valuation and chosen strategy of this subject as well as the auction's outcome.

3.7 Analytic Solution

In order to obtain a benchmark for the experimental results, in this section the theoretical solution of the setting described in the previous sections is derived.

Let V denote the random variable whose realizations yield the bidders' valuations. Its distribution F has the support $\mathcal{M} = \{1, 2, \ldots, 100\}$ and all valuations are ex-ante equally likely. Thus, the probability $f(v)$ that a bidder has valuation v is

$$\begin{aligned} f(v) &= \Pr(V = v) \\ &= \begin{cases} \frac{1}{100} & \text{if } v \in \{1, 2, \ldots, 100\} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

and for integer values v the cdf $F(v)$ is given by

$$F(v) = \begin{cases} 0 & \text{if } v < 1 \\ \frac{v}{100} & \text{if } v \in \{1, 2, \ldots, 100\} \\ 1 & \text{if } v > 100 \ . \end{cases}$$

Remember that the APPO model in Chapter 2 builds on continuously distributed valuations. Since valuations in the experiment are realizations of a discrete rather than a continuous random variable, the formulae for the distribution of the first–and the second-order statistic given in Section A.3 need a few adjustments. In the following, the values of v are restricted to the support $\{1, 2, \ldots, 100\}$ of the distribution F. One obtains for the first-order statistic

$$\begin{aligned} F_{(1),n}(v) &= F^n(v) \\ &= \left(\frac{v}{100}\right)^n \\ f_{(1),n}(v) &= F_{(1),n}(v) - F_{(1),n}(v-1) \\ &= \left(\frac{v}{100}\right)^n - \left(\frac{v-1}{100}\right)^n \end{aligned}$$

and for the second-order statistic

$$\begin{aligned}
F_{(2),n}(v) &= F_{(1),n}(v) + n\ F^{n-1}(v)\ \ (1 - F(v)) \\
&= \left(\frac{v}{100}\right)^n + n\left(\frac{v}{100}\right)^{n-1}\left(1 - \frac{v}{100}\right) \\
f_{(2),n}(v) &= F_{(2),n}(v) - F_{(2),n}(v-1) \\
&= \left(\frac{v}{100}\right)^n + n\left(\frac{v}{100}\right)^{n-1}\left(1 - \frac{v}{100}\right) \\
&\quad - \left(\frac{v-1}{100}\right)^n - n\left(\frac{v-1}{100}\right)^{n-1}\left(1 - \frac{v-1}{100}\right) \ .
\end{aligned}$$

Thus, if no PPO is being offered, the expected valuation $E[V_{(1)}]$ of the winner of an auction and the expected revenues $E[R]$ of the seller calculate to

$$E[V_{(1)}] = \sum_{v=1}^{100} v\, f_{(1),n}(v) \tag{3.1}$$

and

$$E[R] = \sum_{v=1}^{100} v\, f_{(2),n}(v) \ \ , \tag{3.2}$$

respectively.

Denote the payoff of bidder i in a particular auction by Π_i and the payoff of the winning bidder by $\Pi_{(1)}$. The expected payoff $E[\Pi_i|v_i]$ of a bidder i with valuation v_i is then given by

$$E[\Pi_i|v_i] = \sum_{j=1}^{v_i} (v_i - j)\ f_{(1),n-1}(j) \tag{3.3}$$

and the ex-ante expected payoff $E[\Pi_i]$ of a bidder is

$$E[\Pi_i] = \sum_{v=1}^{100} E[\Pi_i|v] f(v) \ \ . \tag{3.4}$$

Finally, one obtains the expected payoff $E[\Pi_{(1)}]$ of the winner of an auction by

$$E[\Pi_{(1)}] = n\ E[\Pi_i] \ \ . \tag{3.5}$$

For the treatments with $n = 3$ bidders, i. e. the treatments A3, B3, and S3, one obtains with the above formulae (3.1), (3.2), (3.4), and (3.5)

$$\begin{aligned} E[V_{(1)}] &= \frac{30199}{400} \\ &\approx 75.50 \\ E[R] &= 50.5 \\ E[\Pi_i] &= \frac{3333}{400} \\ &\approx 8.33 \\ E[\Pi_{(1)}] &\approx 25.00 \end{aligned}$$

and for the treatments with $n = 5$ bidders, i. e. the treatments A5, B5, and S5,

$$\begin{aligned} E[V_{(1)}] &= \frac{335316667}{4000000} \\ &\approx 83.83 \\ E[R] &= \frac{134333333}{2000000} \\ &\approx 67.17 \\ E[\Pi_i] &= \frac{66650001}{20000000} \\ &\approx 3.33 \\ E[\Pi_{(1)}] &\approx 16.66 \end{aligned}$$

holds.

Remember that the above formulae are based on the assumption of risk neutral bidders and sellers. In such a setting, the seller does not quote a PPO that is accepted with positive probability. Note also that the given values $E[V_{(1)}]$, $E[R]$, $E[\Pi_i]$, $E[\Pi_{(1)}]$ refer to the ex-ante expected values. In Section B.4 of the Appendix, the theoretical outcomes of all auctions in the A-treatments are summarized given the parameters of the experiment.

Now consider the case that a PPO is being offered. Since a risk neutral bidder $\hat{\imath}$ with valuation $v_{\hat{\imath}}$ accepts a PPO $\bar{p}$ if $v_{\hat{\imath}} - \bar{p} \geq E[\Pi_{\hat{\imath}}|v_{\hat{\imath}}] \iff \bar{p} \leq v_{\hat{\imath}} - E[\Pi_{\hat{\imath}}]$, the acceptance threshold $t_{\hat{\imath}}(v_{\hat{\imath}})$ is given by

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - E[\Pi_{\hat{\imath}}|v_{\hat{\imath}}] \ .$$

Using Equation (3.3), one obtains for the treatments with $n = 3$ bidders

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = \frac{59999}{60000} v_{\hat{\imath}} + \frac{1}{20000} v_{\hat{\imath}}^2 - \frac{1}{30000} v_{\hat{\imath}}^3 \tag{3.6}$$

and for the treatments with $n = 5$ bidders

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = \frac{3000000001}{3000000000} v_{\hat{\imath}} - \frac{1}{300000000} v_{\hat{\imath}}^3 + \frac{1}{200000000} v_{\hat{\imath}}^4 - \frac{1}{500000000} v_{\hat{\imath}}^5 \tag{3.7}$$

3.8 Statistical Tests for the Analysis of the Experiment

In order to identify major characteristics of the experimental results, the statistical analysis applies tests for differences between the treatments with respect to both the central tendency and the dispersion of the observations. Moreover, tests for trend and tests for the goodness of fit with respect to a common or the normal distribution are performed.

The statistical computations are conducted using the software package R: A Language and Environment for Statistical Computing, version 2.0.0 (The R Development Core Team, 2004).[21] R provides ready-to-use implementations for all tests applied in the analysis. In particular, the functions *wilcox.test* (Wilcoxon rank sum test and Wilcoxon signed ranks test), *fligner.test* (Fligner-Killeen test), *cor.test* (test for association), *chisq.test* (χ^2-test), and *shapiro.test* (Shapiro-Wilk goodness-of-fit test) have been used.

The Fligner-Killeen test (1976) is a non-parametric alternative to the F-test investigating the homogeneity of variances. It is preferable to the F-test because the latter not only assumes that the data is normally distributed but is also sensitive to violations of this assumption (e. g. Pearson, 1931). In the APPO experiment, normality cannot be guaranteed. In fact, it will be shown in Section 4.2 that—based on the data of the experiment—the hypothesis of normality should be rejected. The Fligner-Killeen test is barely discussed in standard textbooks. A study by Conover et al. (1981) compares fifty-six tests for homogeneity of variances and argues in favor of the Fligner-Killeen procedure by highlighting both its power and its robustness.

Using the parameter *method = "spearman"*, the procedure *cor.test* tests for independence (or, more precisely, the correlation) of data based on Spearman's rank correlation coefficient r_s. If the correlation is computed between the observed series of size m and the position numbers $(1, 2, \ldots, m)$ of the observations within the series, the correlation coefficient is equivalent to the rank-based statistic D used to test against trend within a series of data.[22] In fact, Lehmann (1975, p. 300) shows that

$$r_s = 1 - \frac{6D}{m^3 - m} \quad .$$

The R procedure *cor.test* supplies both the rank correlation coefficient r_s and the statistic D.

The χ^2-test needs no further comment. It is discussed in most statistics textbooks (e. g. Wonnacott und Wonnacott, 1990; Hartung et al., 1999). In

[21] In addition, most of the figures are generated by R.

[22] Because the correlation is computed based on ranks, the vector of the positions can consist of any date information, as e. g. $(1950, 1951, \ldots, 1964)$.

the analysis, it is performed to test whether classified observations stem from the same distribution.

The procedure *shapiro.test* is based on Shapiro und Wilk's (1965) W statistic. This statistic is invariant with respect to both the mean and the variance of the underlying distribution. Hence, it is appropriate for a test of normality if neither the true mean nor the true variance is known.

The main focus of the statistical analysis, however, is whether offering a PPO affects the revenues, the bidders' payoffs, or the social surplus. Thus, these variables are tested for differences in central tendency and the Wilcoxon signed ranks test (WSR) is applied for this purpose. Because the WSR plays a prominent role in the analysis, this test and its assumptions are discussed in more detail in the remainder of this section. The WSR is contrasted to its alternatives to test for differences in central tendency and the reasoning behind the selection of this particular test is laid out.

The tests most commonly applied to investigate differences in central tendency are the *median test*, the *t-test*, and the *Wilcoxon rank sum* test, which is also often referred to as the *Mann-Whitney U-test*. As indicated by its name, the median test compares the medians of two distributions. In the APPO experiment, however, one is more interested in the distributions' means than the medians. A seller, for example, might want to know which mechanism yields higher revenues on average, as opposed to which mechanism more often yields higher revenues. The t-test straightforwardly investigates the difference in the means of two populations. However, this test also assumes that the populations from which the samples are drawn are normally distributed.[23] The Wilcoxon rank sum test, finally, forbears from the normality assumption but is not free of assumptions, as will be shown later in this section.[24]

In the controlled setting of the experiment, the same set of random bidder valuations was used in the A and the B-treatments. This design results in matched pairs rather than independent samples and allows the use of one-sample tests on the differences of the paired observations. The respective coun-

[23] Note, however, that several authors have argued that the t-test is robust to violations of the normality assumption. The discussion goes back as far as Rider (1929). Glass et al. (1972) hold a similar view and call "[t]he flight to non-parametrics [...] unnecessary" (p. 237).

[24] Further discussions of the tests mentioned here can be found in almost any statistics textbook. For the t-test see, e. g. Moore und McCabe (2003) or Wonnacott und Wonnacott (1990). Siegel und Castellan (1988) provide an overview of non-parametric tests. A detailed discussion of non-parametric tests, including a comparison with the t-test, is given in Lehmann (1975).

terparts for matched pairs of the above-mentioned tests are the *sign test*,[25] the *paired* or *one-sample t-test*, and the *Wilcoxon signed ranks test.* The paired versions of the tests are generally more powerful (e. g. Davis und Holt, 1993, p. 30f, p. 547ff; Friedman und Sunder, 1994, p. 100f).

Using a paired test also has a technical advantage. It will be shown in Section 4.2 that the treatments with and without a PPO differ significantly in dispersion. In this case, the performance of a pure t-test is negatively affected (Hsu, 1938; Scheffé, 1959). More recently, Zimmerman (2000) found that the Wilcoxon rank sum test suffers from a similar flaw. If two populations differ in their variability, the Type I error of the Wilcoxon rank sum test markedly exceeds the stated p-value.[26] Based on the dispersion tests of Section 4.2, the pure variants of neither the t-test nor the Wilcoxon test are suitable for the analysis of the APPO experiment.[27]

Thanks to the work of Welch (1938, 1947) and Satterthwaite (1946), adaptations of the t-test for the case of heterogeneous variances exist and are commonly available in most statistical software packages. Yet the t-test also assumes that the data is normally distributed—an assumption that is not fulfilled, as will also be shown in Section 4.2. In analogy to the adaptations by Welch and Satterthwaite, Fligner und Policello (1981) suggest a modification of the Wilcoxon rank sum test, the so-called *robust rank order test*, as a non-parametric two-sample test for populations with heterogeneous variances. Unfortunately, however, the robust rank order test is rarely implemented in statistical standard software and critical values are only known for small sam-

[25] The proposed analogy stems from a practical view. Clearly, the difference of the median of two groups of data is not the same as the deviation from zero of the median of paired differences of the same data.

[26] Loosely speaking, the Wilcoxon rank sum test may identify differences in the shape rather than a difference in the location of the two populations' distributions. Cf. the discussion on page 85.

[27] Box (1953) criticizes preliminary tests on the homogeneity of variances because he considers the t-test for comparing the means of two populations as more robust than traditional dispersion tests: "To make the preliminary test on variances is rather like putting to sea in a rowing boat to find out whether conditions are sufficiently calm for an ocean liner to leave port!" Zimmerman (1996) raises a similar concern. He finds that two-stage procedures, i. e. a preliminary test on variances in combination with either the t-test or the Wilcoxon test, are ineffective and distortive with respect to the significance level. The bias between the Type I error and the reported p-value is particularly high if the samples are of unequal size and, interestingly, if the difference of the variances is small. Note that the Fligner-Killeen test, which is used in this analysis for the comparison of variances, was developed only after Box's study. Since it is a distribution-free test, the critique of the former author should not apply. Zimmerman's objections are met by noting that the sample sizes are equal and the differences of the variances are large.

ples (Feltovich, 2003).[28] Moreover, Feltovich notes that the convergence of the robust rank order statistic to the normal distribution is rather slow.

Due to the above reasons, the following analysis applies the WSR to test for differences in central tendency. As its analogon for independent samples, the WSR considers ranks rather than the cardinal values of the data. It is a distribution-free test, which differs from the t-test by not requiring that the observations stem from a normal distribution. Being a paired test, the WSR also avoids the issue of heterogeneous variances because only one sample—the differences of the matched pairs—is being considered. Since the WSR is based on ranks, the test uses not only the direction of the differences within pairs but also their relative magnitude. Compared to the sign test, this has the advantage of taking more information into account (Siegel und Castellan, 1988, p. 87; Hartung et al., 1999, p. 243).

According to Schaich und Hamerle (1984) or Sheskin (2003), the WSR requires that the underlying distribution is symmetric with respect to some ξ.[29] The hypothesis being tested reads

$$H_0 : \xi = 0 \quad \text{versus} \quad H_1 : \xi \neq 0 \ .$$

In this case the rejection of H_0 is straightforward. It follows from the assumption that both the median and the mean are ξ. Thus, if H_0 is rejected, the central tendency is different from ξ.

Unfortunately, however, the symmetry assumption is rather strong.[30] Moreover, it is unlikely to hold in the APPO experiment. Consider, e.g., the social surplus: By construction, it is bounded by the maximum of the bidders' valuations and one would expect that low deviations from the maximum occur more frequently than large deviations. Thus, if two treatments differ in the average surplus, the distribution of this difference will be skewed rather than symmetric.

Hartung et al. (e.g. 1999, p. 243) forbear from the symmetry assumption. Instead, these authors *test* whether observations can be interpreted as drawings from a symmetric distribution F, i.e. whether $F(\xi + x) = 1 - F(\xi - x)$ holds for all x. Testing for differences in paired observations, one sets $\xi = 0$ and the null hypothesis is then stated as

[28] Fligner und Policello (1981) present critical values for sample sizes up to 12. Feltovich (forthcoming) provides critical values for different significance levels for sample sizes up to 40. Available software is not yet that advanced.

[29] Often it is also required that the distribution is continuous.

[30] Note that the distribution of matched pairs of a certain variable is, of course, symmetric if two treatments do *not* differ with respect to this variable. The reversal, however, does not hold.

$$H_0 : F(x) = 1 - F(-x) \; \forall x \; . \tag{3.8}$$

The rejection of the null hypothesis is now more difficult to interpret.[31] Siegel und Castellan (1988, p. 87) suggest that "the researcher can make the judgment of 'greater than' between any pair's two values" if the test is run on the differences of matched pairs of two treatments. The following example shows that such a judgement ignores the fact that H_0 might be rejected simply because F is not symmetric if the null hypothesis is stated according to (3.8).

Clearly, both the mean and the median of a random variable with distribution F are zero if in fact $F(x) = 1 - F(-x)$ holds for all x. Thus, two treatments do not differ in central tendency if the above null hypothesis is true. With respect to the quote by Siegel und Castellan, however, note that the reversal does not hold in general. Neither the mean nor the median must be non-zero if the distribution is not symmetric with respect to zero. For illustration, consider the vector

$$x = (-300, -221, -220, -219, -218, -217, -216, -215, -214, -213, -212, \\ -1, 0, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211)$$

of hypothetical observations. Based on this sample, the WSR rejects the (two-sided) null hypothesis at a 10% confidence level but both the mean and the median of the sample *are* zero.[32] Of course it is possible to construct continuous distributions with a true mean and median at zero that are likely to yield outcomes similar to the above sample. If samples of such a distribution are tested by the WSR, the Type I error will be much higher than the p-values given by the test.

The opposite may occur as well. Consider, e. g., a random variable with a density that has four narrow peaks at $-5, -1, 3$, and 4 of approximately the same probability mass. This density is not symmetric with respect to zero. Moreover, both the expected value and the median are greater than zero. Still, the WSR is likely to fail to reject the null hypothesis (Type II error). In fact, as the sample size grows the p-value converges to 1.

[31] Moran und Solomon (2002, p. 316) make a similar comment on the Wilcoxon rank sum test: "[...] the interpretation of the null hypothesis being tested with the [Wilcoxon rank sum] test is not easily described; conventional interpretation (including examples provided by statistical packages) would have it that the null hypothesis is one of equal group medians, but such is not the case, rather it is a test for equality of group mean ranks."

[32] The procedure *wilcox.test* (and similarly the procedure *wilcox.exact*) of the R software computes a p-value of 8.9% and suggests that the "true μ is not equal to 0."

The above examples illustrate failures of the WSR in investigating differences in central tendency. Similar weaknesses of its two-sample counterpart, the Wilcoxon rank sum test, have been the subject of a variety of studies and are well documented.[33] It is argued, e. g., that the Wilcoxon rank sum test—despite being a distribution-free test—is not a test without assumptions (see, e. g., Moran und Solomon, 2002, and the references therein). Feltovich (2003) notes that the Wilcoxon rank sum test investigates differences of the first moment of a population assuming that higher-order moments are the same. "[I]ts interpretation as a test of medians is valid when the only distributional difference is a shift in location" (Moran und Solomon, 2002, p. 316). Of course, a difference in medians is equivalent to a difference in means and even to first-order stochastic dominance if two distributions are in fact equally shaped and differ only in their location. For many applications, it is convenient to relax the assumptions of the test and to only require that the respective distribution functions do not intersect. In such a setting, the Wilcoxon rank sum test tests for first-order stochastic dominance. In fact, the test is introduced in this vein, e. g. in Bosch (1998, p. 711f).

In the literature, the WSR is not discussed as intensely as the Wilcoxon rank sum test. Presumably, however, the above argument holds similarly for the WSR. Rather than comparing the shape and the location of the distributions of two populations, one considers the distributions $F(x)$ and $G(x) := 1 - F(-x)\ \forall x$ constructed from the same sample. Under the assumption that F and G do not intersect, i. e.

$$\nexists\, x_1, x_2 : F(x_1) > G(x_1) \wedge F(x_2) < G(x_2) \;, \tag{3.9}$$

it follows from the rejection of the null hypothesis

$$H_0 : F(x) = G(x) \quad \forall x$$

that the mean (and generally also the median) of F are different from zero. In fact, the two examples which were given above to illustrate Type I and Type II errors of the WSR involve distribution pairs $F(x)$ and $G(x) = 1-F(-x)$ which intersect several times.

Alternatively, the test assumption (3.9) can equivalently be stated as

$$F(x) \geq 1 - F(-x)\ \ \forall x \quad \text{or} \quad F(x) \leq 1 - F(-x)\ \ \forall x \;.$$

[33] In addition to the weaknesses of the Wilcoxon rank sum test itself, Bergmann et al. (2000) find differences in the implementation of the test in various statistics packages. The reported p-values range "from significant to nonsignificant at the 5 % level" (p. 72).

Note that this is much less restrictive than the symmetry assumption generally mentioned in the literature.

4

Results of the Experiment

This chapter presents the results of the experiment. First, Section 4.1 gives a rather descriptive overview. The data are presented in more detail in the subsequent Section 4.2 and an inferential analysis which tests the significance of the results with respect to the auction outcomes is provided in Section 4.3. The behavior of the bidders is discussed in Section 4.4 and the behavior of the sellers is investigated in Section 4.5. Section 4.6 summarizes the main findings.

4.1 Overview

To start the analysis, Figures 4.1, 4.2, and 4.3 give an overview of the experimental results by plotting the average revenue, the average winning bidder's payoff, and the average social surplus, respectively. Each figure contrasts—for both the auctions with three (a) and five bidders (b)—the equilibrium outcomes (bars labeled "Theory") with the experimental results of the auctions without a PPO ("A3" and "A5") and the auctions with a PPO ("B/S3" and "B/S5").

According to the figures, a PPO appears to slightly lower both the revenues and the social surplus. In the auctions with three bidders, the average revenue and social surplus decrease from 49.0 to 45.2 and from 76.1 to 74.8, respectively. With five bidders, the revenue and the social surplus average 67.1 and 83.6 in treatment A5, but only 66.4 and 80.8 if the seller can choose to propose a PPO in B/S5. Interestingly, the effect of the PPO on the winning bidders' payoffs seems to depend on the number of bidders. With three bidders, the winning bidders' payoffs in treatment B/S3 (29.6) are on average higher than both the equilibrium payoffs (28.1) and the bidders' payoffs in treatment A3 (27.1). With five bidders the opposite is observed: in this case the average of the winning bidders' payoffs in treatment B/S5 (14.4) is more

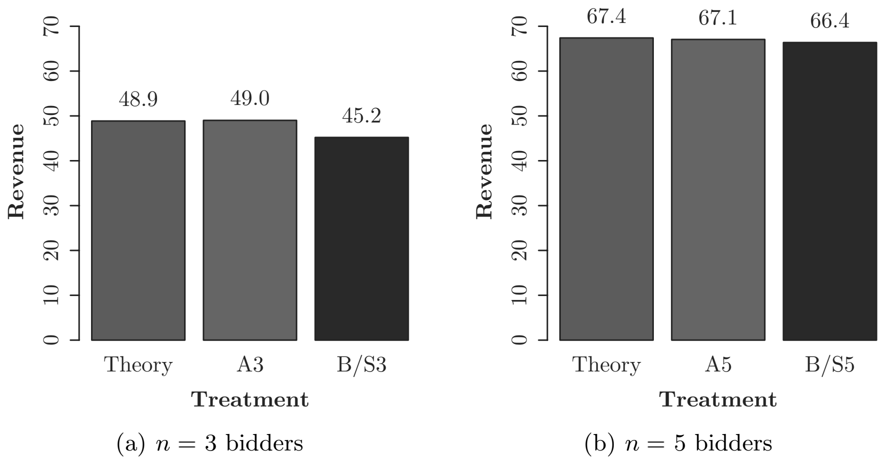


Figure 4.1. Average revenue

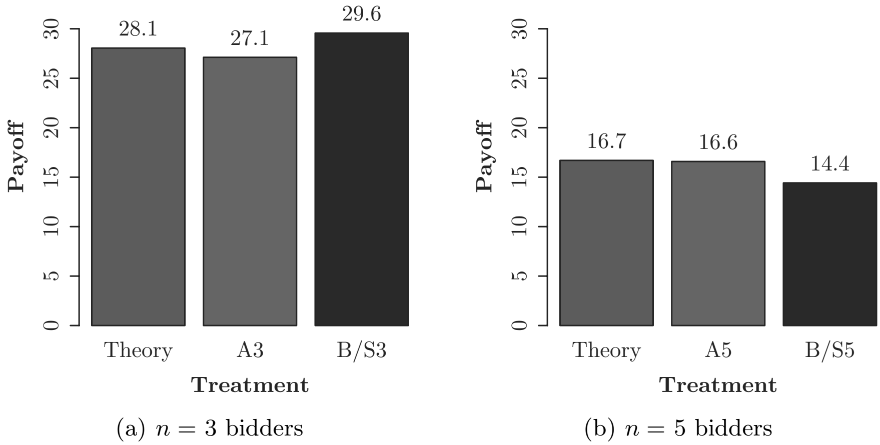
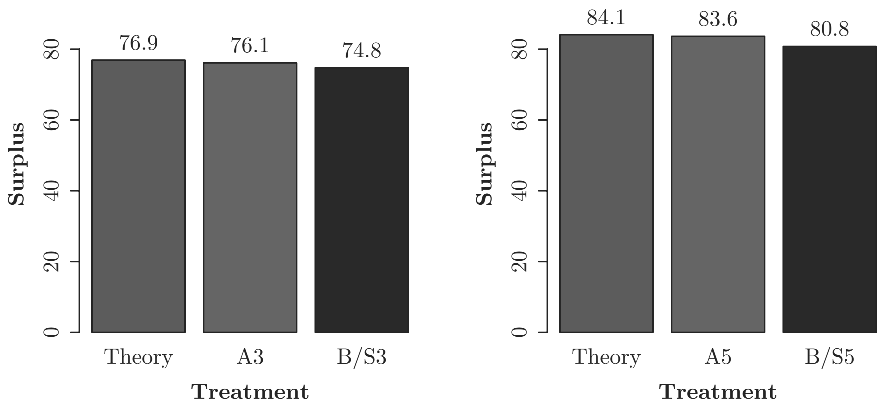


Figure 4.2. Average winning bidder's payoff

than 10% lower than in treatment A5 (16.6) or the theoretical benchmark (16.7).

The theoretical benchmark in Figures 4.1–4.3 refers to the equilibrium outcomes of an auction in which a PPO does not exist or a scenario in which a risk neutral seller faces risk neutral bidders. In the latter case the seller will opt against offering a PPO that would be accepted with positive probability. The calculations of the average theoretical benchmark are based on the actual realizations of the bidders' valuations in the experiment. For each auction, i. e.



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.3. Average social surplus

for each set of bidder valuations in the experiment, the equilibrium outcome is derived and the auctioneer's revenue, the winning bidder's payoff as well as the social surplus are determined. The equilibrium calculations comprise 60 auctions (twelve rounds with five groups each) for the treatments with three bidders and 36 auctions (twelve rounds with three groups each) for the treatments with five bidders.[1]

The calculations with respect to the results of treatments A3 and A5 are straightforward. Since two sessions of each treatment were conducted, the given results average over 120 (= 2×60) and 72 (= 2×36) auctions, respectively. Some comments should be made, however, regarding the auctions with a PPO. The auction outcomes of the treatments B3 and B5 cannot be analyzed meaningfully on their own. These treatments provide insight into the behavior of the individual bidders and will be discussed in more detail in Section 4.4. With respect to the revenue, the payoffs, and the social surplus, however, the results are significantly distorted since the PPOs in the B-treatments were randomly set.[2] Meaningful outcomes of auctions with a PPO can be obtained by matching the recorded strategy profiles of the bidders in treatments B3 and B5 with the strategies of the sellers in treatments S3 and S5, respectively. The outcomes constructed in this vein are indicated

[1] The equilibrium outcomes of all individual auctions are given in Tables B.7 and B.8 in Appendix B.

[2] See Sections 3.3 and 3.4 for details on the setup of the B-treatments. The PPOs used in the experiment are given in Table B.6 in Appendix B.

by B/S3 and B/S5.[3] Again, the results of treatment B/S3 are based on 120 individual auctions. In treatment S5, however, each of the recorded bidders' strategy profiles of the 72 auctions of treatment B5 was mapped to two different sellers. Thus, treatment B/S5 covers the decisions of sellers with respect to setting a PPO in a total of 144 individual auctions, of which every other is based on the same bidding data.

Tables 4.1 and 4.2 summarize the information of Figures 4.1–4.3. In contrast to Figures 4.1–4.3, the tables provide two theoretical benchmarks. The first row in each table refers to the expected analytical solution as calculated in Section 3.7. The second row is based on the actual valuations of the bidders in the experiment and yields the numbers of the above figures. The differences between the first and second rows reflect the influence of random when determining the valuations. In order to obtain meaningful results, the experimental observations must be compared to the second row of the tables.[4] The theoretical benchmarks are labeled A3 and A5 in the first column in order to indicate that a PPO is not available.

Table 4.1. Theoretical solution and experimental results with three bidders

Treatment	Description	Revenue	Payoff	Surplus
A3	Theoretical solution (ex-ante expected values)	50.5	25.0	75.5
A3	Theoretical solution (mean) with valuations of experiment	48.9 (100%)	28.1 (100%)	76.9 (100%)
A3	Experimental results (mean)	49.0 (100.3%)	27.1 (96.7%)	76.1 (99.0%)
B/S3	Experimental results (mean)	45.2 (92.5%)	29.6 (105.4%)	74.8 (97.2%)

The third and fourth rows show the experimental results. In order to facilitate the comparison of the treatments, Tables 4.1 and 4.2 also display the relative performance of the experimental outcomes with respect to the theo-

[3] Since the recorded behavior of bidders in treatments B3 and B5 was used to parameterize and to evaluate the auctions in treatments S3 and S5, the outcomes of B/S3 and B/S5 can be derived from the respective S-treatments alone. The notations B/S3 and B/S5 have been chosen to indicate that these results incorporate the observed behavior of both bidders and sellers.

[4] Remember that in all sessions of both the A and B-treatments the same set of bidder valuations was used (cf. Section 3.5). The valuations are listed in Table B.5 in Appendix B.

retical benchmark.[5] Note that both the auctioneer's revenue and the bidders' payoffs in the experiment can be lower or higher than those predicted by the theory. The social surplus, i. e. the sum of the auctioneer's revenue and the bidders' payoffs, however, cannot be higher than the surplus in equilibrium. If the surplus of an auction in the experiment equals the equilibrium surplus, the auction is efficient. If it is lower than the equilibrium surplus, the auction is not efficient.

Table 4.2. Theoretical solution and experimental results with five bidders

Treatment	Description	Revenue	Payoff	Surplus
A5	Theoretical solution (ex–ante expected values)	67.2	16.7	83.8
A5	Theoretical solution (mean) with valuations of experiment	67.4 (100%)	16.7 (100%)	84.1 (100%)
A5	Experimental results (mean)	67.1 (99.5%)	16.6 (99.3%)	83.6 (99.5%)
B/S5	Experimental results (mean)	66.4 (98.5%)	14.4 (86.4%)	80.8 (96.1%)

The two tables show that the experimental results of the treatments without a PPO are very close to the theoretical benchmark. In fact, in both A3 and A5 only six out of a total of 120 and 72 auctions (i. e. 5% and 8%, respectively) are not efficient. In all other auctions the item is awarded to the bidder with the highest valuation. The auctions in the B–treatments are not as efficient. In treatment B/S3 18 out of 120 (15%) auctions and in treatment B/S5 25 out of 144 (17%) auctions are not efficient. This is consistent with the observation that the B–treatments result in lower social surplus than the A–treatments. An inferential analysis of the different treatments is provided in Section 4.3.

[5] The percentages are calculated based on the average outcomes. Note that on an individual basis, the percentage may not exist since the winning bidder's payoff may be zero. This case does in fact occur in the theoretical benchmark in round 4 of group 1 (treatment A3) since the two highest valuations are equal in this auction.

4.2 Individual Auction Outcomes

After the introductory overview in the last section, this section presents the experimental results with respect to the auction outcomes in more detail. The individual behavior of the bidders and the sellers is investigated in Sections 4.4 and 4.5, respectively.

In order to precisely render the results, a few preliminary remarks on the notation are necessary. In Chapters 2 and 3 the symbols R, $\Pi_{(1)}$, and $V_{(1)}$ denote the equilibrium outcomes of an auction. Since in the experiment the winning bidder is not necessarily the bidder with the highest valuation, a slightly different notation is chosen to denote experimental outcomes. The symbols $\hat{R}$, $\hat{\Pi}$, and $\hat{V}$ indicate the auctioneer's revenue, the payoff of the winning bidder, and the social surplus of an auction in the experiment. Because the winning bidder's payoff is the difference between his valuation and the price paid, the social surplus $\hat{V} = \hat{R} + \hat{\Pi}$ equals the valuation of the winning bidder even if the item is not awarded to the bidder with the highest valuation.

Tables 4.3, 4.4, 4.5, and 4.6 display the outcomes of all individual auctions in the treatments A3, A5, B/S3, and B/S5. Table 4.3 provides the results of treatment A3. The table is arranged in two sections. The top section shows the results of the first and the bottom section the results of the second session. Each section provides five triple-columns—one for each of the five groups observed in that session—documenting the revenue, the winning bidder's payoff, and the social surplus of that group.[6] In addition to the results of each round, the table also provides the groups' average results as well as the respective minima and maxima. Finally, the overall mean and the sample standard deviation of the results are given.[7]

The results of treatment A5 are shown in Table 4.4. What differs from treatment A3 is that in each session of treatment A5 only three groups per round were observed. Thus, the two sections of the table are arranged side by side rather than one below the other. Apart from that the table is set up analogously to Table 4.3.

Table 4.5 displays the results of the combined treatment B/S3. Containing five bidder groups per round, the table follows the layout of Table 4.3. In this

[6] Remember that the experiment was set up as a stranger experiment and that participants were assigned to different groups in every round (cf. Section 3.4).

[7] As is common in descriptive statistics, the sample standard deviation is calculated by taking the square root of the sample variance $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$. While s^2 is an unbiased estimator of the true variance σ^2, i. e. $E[s^2] = \sigma^2$, this does not hold for the standard deviation. Due to Jensen's inequality, $E[\sqrt{s^2}] < \sqrt{E[s^2]} = \sigma$. Thus, $\sqrt{s^2}$ understates the true standard deviation σ (cf. e. g. Bosch, 1998, p. 425).

treatment the posted price option was available. A bold font face indicates those auctions in which the PPO was accepted. If the item was nevertheless awarded to the bidder who submitted the highest maximum bid, only the revenue is set in bold face. An example is the entry (**50**, 41, 91) in round 1 of group 2. In this case, the auction's closing price, 50, is determined by the PPO set by the seller rather than by the second highest bid.[8] The social surplus of 91, however, is not affected by the acceptance of the PPO. Further, if the item was awarded to a bidder who did not submit the highest bid, the winning bidder's payoff as well as the social surplus is also highlighted by bold numbers, e. g. (**40**, **9**, **49**) in round 5 of group 1.

Remember that every strategy profile of bidders that was observed in treatment B5 has been assigned to two different sellers in treatment S5. Thus, Table 4.6—summarizing the outcomes of the combined treatment B/S5—contains twice as many individual auctions as observed in treatment B5 and stretches over two pages. The overall layout of the table is similar to Table 4.4, i. e. the bidder groups of the two sessions of treatment B5 are arranged side by side. The logic of highlighting individual outcomes by using a bold font face to indicate the acceptance of a PPO is the same as in Table 4.5.

The first page (labeled "Observation I", p. 97) and the second page (labeled "Observation II", p. 98) of Table 4.6 differ in terms of sellers but are based on the same bidding data.[9] This means, e. g., that the two triple-entries for group 1 in round 1 on pages 97 and 98, i. e. the data (**68**, 23, 91) and (**70**, 21, 91), respectively, refer to the same bidders and their strategy profiles, but to two different sellers. Apparently, both sellers offered a PPO that was accepted by the decisive bidder, who had a valuation of 91. The respective PPOs were 68 and 70. For illustration purposes, consider two further examples: firstly, round 1 of group 5 (which refers to the bidder group 2 in session 2 of treatment B5) of Observations I and II and, secondly, round 3 of group 3 of the two observations. In the first case, i. e. the two identical triples (40, 24, 64), neither of the two sellers offered a PPO that was accepted by the decisive bidder. Thus, the two outcomes are equivalent: both auctions closed at a price of 40, the winning bidder's payoff was 24, and a social surplus was 64. Recall that the underlying strategy profiles of the bidders are the same. In the latter

[8] Remember that in the theory laid out in Chapter 2, the corresponding auction of an APPO is modeled as a second-price auction. In the experiment, an English proxy auction is implemented. From a theoretical perspective the two institutions are equivalent. Thus, in the following 'bid' and 'maximum bid' are not differentiated.

[9] Consequently, the statistical analysis of these data is difficult. Because the observed bidding behavior is duplicated in the table, the calculated standard deviation most likely underestimates the true standard deviation.

Table 4.3. Treatment A3 — Outcomes of individual auctions in the experiment

Session 1

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	70	7	77	64	27	91	19	15	34	44	52	96	42	24	66
2	28	22	50	23	43	66	72	6	78	47	45	92	45	23	68
3	56	43	99	20	35	55	35	48	83	71	7	78	63	36	99
4	82	1	83	35	46	81	77	12	89	15	39	54	38	41	79
5	49	4	53	77	−57	20	24	72	96	12	51	63	50	21	71
6	62	4	66	50	24	74	50	3	53	30	13	43	35	64	99
7	47	38	85	61	30	91	35	49	84	89	3	92	80	14	94
8	53	20	73	35	59	94	60	38	98	79	21	100	73	4	77
9	73	4	77	59	1	60	32	19	51	14	26	40	45	46	91
10	23	65	88	30	31	61	81	−3	78	7	76	83	52	6	58
11	62	15	77	60	28	88	64	24	88	50	18	68	41	58	99
12	35	17	52	62	13	75	34	47	81	25	50	75	74	21	95
mean	53.3	20.0	73.3	48.0	23.3	71.3	48.6	27.5	76.1	40.3	33.4	73.7	53.2	29.8	83.0
min	23	1	50	20	−57	20	19	−3	34	7	3	40	35	4	58
max	82	65	99	77	59	94	81	72	98	89	76	100	80	64	99

Session 2

Round	Group 1			Group 2			Group 3			Group 4			Group 5		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	70	7	77	64	27	91	20	14	34	40	56	96	48	18	66
2	35	15	50	20	46	66	73	5	78	45	47	92	63	5	68
3	56	43	99	20	35	55	26	57	83	61	17	78	80	17	97
4	75	8	83	45	36	81	70	19	89	20	34	54	38	41	79
5	52	−3	49	46	32	78	24	72	96	14	49	63	45	26	71
6	66	−3	63	50	24	74	53	25	78	34	9	43	35	64	99
7	57	28	85	86	5	91	27	57	84	90	2	92	80	14	94
8	58	15	73	34	60	94	54	44	98	80	20	100	72	5	77
9	72	5	77	43	17	60	40	11	51	14	26	40	44	47	91
10	23	65	88	31	30	61	78	3	81	7	76	83	57	1	58
11	62	15	77	54	34	88	80	8	88	51	17	68	41	58	99
12	43	9	52	60	15	75	35	46	81	26	49	75	74	21	95
mean	55.8	17.0	72.8	46.1	30.1	76.2	48.3	30.1	78.4	40.2	33.5	73.7	56.4	26.4	82.8
min	23	−3	49	20	5	55	20	3	34	7	2	40	35	1	58
max	75	65	99	86	60	94	80	72	98	90	76	100	80	64	99

overall mean of $\hat{R}$: 49.0 overall mean of $\hat{\Pi}$: 27.1 overall mean of $\hat{V}$: 76.1

std. dev. of $\hat{R}$: 20.6 std. dev. of $\hat{\Pi}$: 21.6 std. dev. of $\hat{V}$: 17.4

Table 4.4. Treatment A5 — Outcomes of individual auctions in the experiment

Round	Session 1									Session 2								
	Group 1			Group 2			Group 3			Group 1			Group 2			Group 3		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	68	23	91	40	24	64	77	19	96	60	31	91	42	22	64	76	20	96
2	63	10	73	65	1	66	66	26	92	65	8	73	66	2	68	78	14	92
3	71	28	99	72	11	83	66	31	97	56	43	99	83	16	99	78	19	97
4	53	24	77	83	6	89	71	8	79	53	24	77	89	−6	83	79	2	81
5	71	7	78	55	41	96	52	11	63	71	7	78	77	19	96	52	11	63
6	40	26	66	74	25	99	53	25	78	35	31	66	73	26	99	52	26	78
7	48	43	91	92	2	94	84	2	86	48	43	91	90	2	92	84	2	86
8	98	2	100	73	4	77	80	14	94	98	2	100	77	−4	73	80	14	94
9	77	14	91	45	28	73	35	16	51	77	14	91	45	28	73	36	15	51
10	48	30	78	61	22	83	81	7	88	49	29	78	61	22	83	81	7	88
11	64	13	77	88	11	99	68	20	88	63	14	77	84	15	99	68	20	88
12	74	21	95	75	6	81	61	14	75	74	21	95	75	6	81	61	14	75
mean	64.6	20.1	84.7	68.6	15.1	83.7	66.2	16.1	82.3	62.4	22.3	84.7	71.8	12.3	84.2	68.8	13.7	82.4
min	40	2	66	40	1	64	35	2	51	35	2	66	42	−6	64	36	2	51
max	98	43	100	92	41	99	84	31	97	98	43	100	90	28	99	84	26	97

overall mean of $\hat{R}$: 67.1 **overall mean of $\hat{\Pi}$: 16.6** **overall mean of $\hat{V}$: 83.6**

std. dev. of $\hat{R}$: 15.4 **std. dev. of $\hat{\Pi}$: 11.3** **std. dev. of $\hat{V}$: 12.2**

Table 4.5. Treatment B/S3 — Outcomes of individual auctions in the experiment

Groups 1–5 (bidder groups 1–5 of session 1 of treatment B3)

Round	Group 1			2			3			4			5		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	17	52	69	**50**	41	91	15	19	34	**25**	71	96	35	31	66
2	26	24	50	20	46	66	65	8	73	**40**	52	92	**60**	**3**	**63**
3	56	43	99	10	10	20	26	57	83	71	7	78	75	22	97
4	**65**	18	83	30	51	81	65	24	89	15	39	54	38	41	79
5	**40**	**9**	**49**	15	63	78	22	74	96	14	49	63	50	21	71
6	**10**	**53**	**63**	41	33	74	30	23	53	34	9	43	**45**	54	99
7	46	39	85	48	43	91	**45**	39	84	89	3	92	30	64	94
8	**60**	**13**	**73**	**5**	**30**	**35**	49	49	98	75	5	80	71	6	77
9	73	4	77	43	17	60	31	20	51	11	29	40	40	51	91
10	22	66	88	31	30	61	78	3	81	7	76	83	28	30	58
11	62	15	77	53	35	88	63	25	88	25	43	68	41	58	99
12	35	17	52	56	19	75	**48**	33	81	26	49	75	74	21	95
mean	42.7	29.4	72.1	33.5	34.8	68.3	44.8	31.2	75.9	36.0	36.0	72.0	48.9	33.5	82.4
min	10	4	49	5	10	20	15	3	34	7	3	40	28	3	58
max	73	66	99	56	63	91	78	74	98	89	76	96	75	64	99

Groups 6–10 (bidder groups 1–5 of session 2 of treatment B3)

Round	Group 6			7			8			9			10		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	68	9	77	**72**	19	91	8	12	20	44	52	96	33	33	66
2	26	24	50	23	43	66	**70**	8	78	**70**	22	92	35	28	63
3	50	49	99	19	36	55	21	62	83	**49**	**22**	**71**	96	3	99
4	80	3	83	34	47	81	75	2	77	35	19	54	37	42	79
5	49	4	53	44	34	78	24	72	96	12	51	63	**66**	**−16**	**50**
6	**45**	**18**	**63**	45	29	74	55	23	78	**40**	3	43	30	69	99
7	47	38	85	73	18	91	**47**	37	84	89	3	92	**66**	**15**	**81**
8	**55**	18	73	33	61	94	**50**	48	98	79	21	100	73	4	77
9	**50**	27	77	60	−17	43	30	21	51	13	27	40	37	54	91
10	21	67	88	31	30	61	72	9	81	**63**	20	83	52	6	58
11	61	16	77	52	36	88	62	26	88	63	5	68	41	58	99
12	35	17	52	51	24	75	**65**	16	81	54	21	75	74	21	95
mean	48.9	24.2	73.1	44.8	30.0	74.8	48.3	28.0	76.3	50.9	22.2	73.1	53.3	26.4	79.8
min	21	3	50	19	−17	43	8	2	20	12	3	40	30	−16	50
max	80	67	99	73	61	94	75	72	98	89	52	100	96	69	99

overall mean of $\hat{R}$: 45.2 overall mean of $\hat{\Pi}$: 29.6 overall mean of $\hat{V}$: 74.8

std. dev. of $\hat{R}$: 20.9 std. dev. of $\hat{\Pi}$: 20.3 std. dev. of $\hat{V}$: 18.1

Table 4.6. Treatment B/S5 — Outcomes of individual auctions in the experiment

Observation I

Round	Groups 1–3 (bidder groups 1–3 of session 1 of treatment B5)									Groups 4–6 (bidder groups 1–3 of session 2 of treatment B5)								
	1			**2**			**3**			**4**			**5**			**6**		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	**68**	23	91	49	15	64	71	25	96	**56**	35	91	40	24	64	77	19	96
2	62	11	73	62	6	68	77	15	92	55	18	73	66	2	68	**60**	**18**	**78**
3	**25**	74	99	90	9	99	**65**	**13**	**78**	**85**	14	99	83	16	99	**65**	**13**	**78**
4	53	24	77	**80**	**3**	**83**	78	3	81	**50**	**4**	**54**	79	10	89	78	3	81
5	70	8	78	42	54	96	52	11	63	**70**	8	78	45	51	96	53	−3	50
6	35	31	66	73	26	99	61	17	78	34	32	66	74	25	99	74	4	78
7	47	44	91	**75**	**6**	**81**	75	9	84	47	44	91	90	4	94	84	2	86
8	97	3	100	76	−3	73	79	15	94	80	20	100	74	3	77	80	14	94
9	76	15	91	55	18	73	35	16	51	77	14	91	45	28	73	51	−40	11
10	49	29	78	**80**	3	83	80	8	88	78	−47	31	60	23	83	81	7	88
11	63	14	77	80	19	99	67	21	88	61	16	77	93	−5	88	68	20	88
12	73	22	95	77	4	81	70	5	75	74	21	95	**70**	**5**	**75**	64	11	75
mean	59.8	24.8	84.7	69.9	13.3	83.3	67.5	13.2	80.7	63.9	14.9	78.8	68.3	15.5	83.8	69.6	5.7	75.3
min	25	3	66	42	−3	64	35	3	51	34	−47	31	40	−5	64	51	−40	11
max	97	74	100	90	54	99	80	25	96	85	44	100	93	51	99	84	20	96

Continued on page 98 ...

Table 4.6 (continued . . .)

Observation II

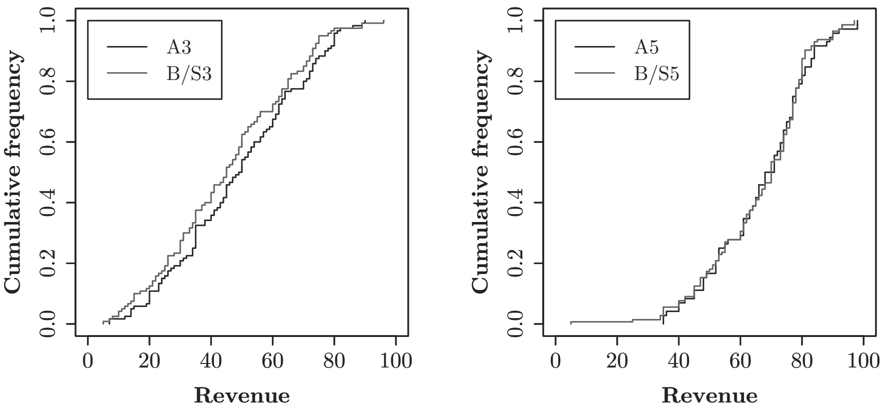
Round	Groups 1–3 (bidder groups 1–3 of session 1 of treatment B5)									Groups 4–6 (bidder groups 1–3 of session 2 of treatment B5)								
	1			**2**			**3**			**4**			**5**			**6**		
	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$	$\hat{R}$	$\hat{\Pi}$	$\hat{V}$
1	**70**	21	91	**40**	24	64	**5**	**61**	**66**	**60**	31	91	40	24	64	77	19	96
2	62	11	73	62	6	68	77	15	92	55	18	73	66	2	68	92	−44	48
3	81	18	99	90	9	99	78	19	97	**45**	54	99	83	16	99	**55**	**23**	**78**
4	53	24	77	**67**	**16**	**83**	78	3	81	54	23	77	79	10	89	78	3	81
5	**70**	8	78	42	54	96	52	11	63	70	8	78	45	51	96	**70**	−20	50
6	35	31	66	73	26	99	61	17	78	34	32	66	74	25	99	74	4	78
7	47	44	91	**65**	**16**	**81**	75	9	84	47	44	91	90	4	94	84	2	86
8	97	3	100	76	−3	73	79	15	94	80	20	100	74	3	77	80	14	94
9	76	15	91	55	18	73	35	16	51	77	14	91	45	28	73	51	−40	11
10	49	29	78	**70**	13	83	80	8	88	78	−47	31	60	23	83	81	7	88
11	63	14	77	80	19	99	67	21	88	61	16	77	93	−5	88	68	20	88
12	73	22	95	77	4	81	70	5	75	74	21	95	81	−29	52	64	11	75
mean	64.7	20.0	84.7	66.4	16.8	83.3	63.1	16.7	79.8	61.3	19.5	80.8	69.2	12.7	81.8	72.8	−0.1	72.8
min	35	3	66	40	−3	64	5	3	51	34	−47	31	40	−29	52	51	−44	11
max	97	44	100	90	54	99	80	61	97	80	54	100	93	51	99	92	23	96

overall mean of $\hat{R}$: 66.4 **overall mean of $\hat{\Pi}$: 14.4** **overall mean of $\hat{V}$: 80.8**

std. dev. of $\hat{R}$: 16.0 **std. dev. of $\hat{\Pi}$: 18.1** **std. dev. of $\hat{V}$: 16.2**

example, i. e. the triples (**65**, **13**, **78**) and (78, 19, 97), only one of the two sellers offered a PPO that was accepted by the decisive bidder. Moreover, the decisive bidder was not the participant who submitted the highest bid. Thus, the two outcomes differ in all three components.

In order to facilitate the examination and interpretation of the data provided in Tables 4.3–4.6, Figures 4.4–4.6 plot the empirical distribution of the revenue, the winning bidder's payoff, and the social surplus. The graphical representations are helpful for a comparison of the individual auction outcomes. Figure 4.4(a) shows, for example, that for the auctions with three bidders the distribution of the revenues of treatment A3 lies below the respective curve of treatment B/S3 for (almost) all revenue values. This suggests that a PPO significantly lowers the revenues which is, in fact, the case—the statistical test is given in Section 4.3.

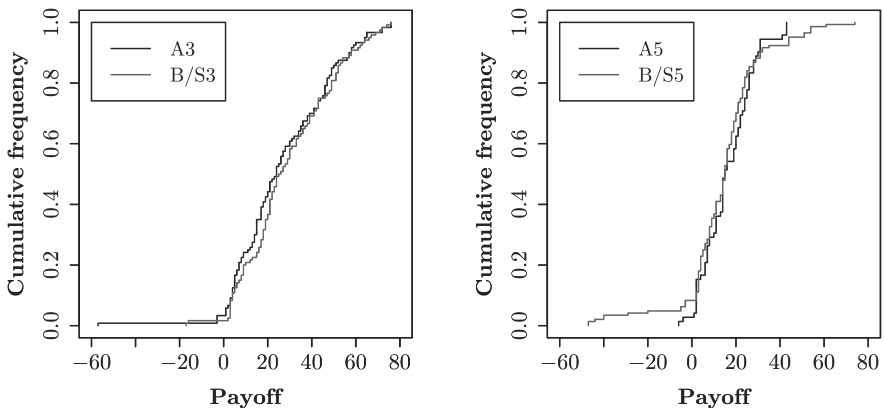


(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.4. Empirical distributions of revenues

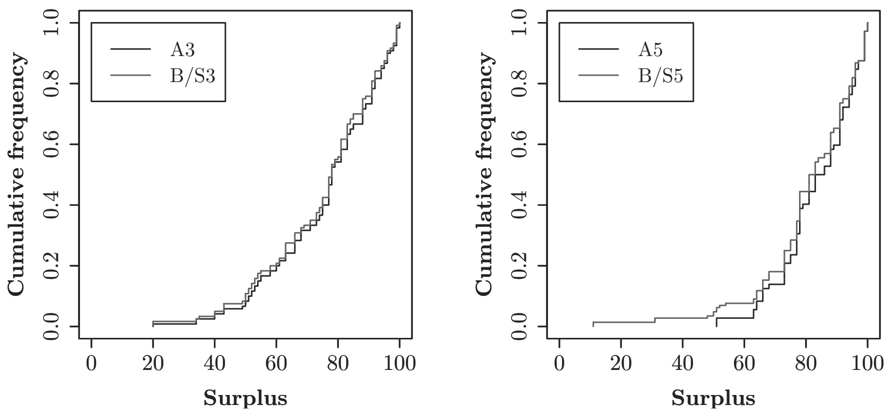
Similar observations can be made with respect to the social surplus (Figure 4.6). For both auctions with three and five bidders the distributions of the A-treatments lie below the B/S-treatments. The difference is not very large and sometimes the curves lie one upon the other, but the distribution of the A-treatment is never greater than that of the respective B/S-treatment. It will be shown in Section 4.3 that the difference in surplus is in fact significant.

Figure 4.5(a) shows a reverse order of the treatments' distributions. In analogy to first-order stochastic dominance, one observes that the winning bidders' payoffs of the B/S-treatment dominate the payoffs of the pure auctions. Thus, the figure supports the hypothesis that in the auctions with three



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.5. Empirical distributions of winning bidders' payoffs



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.6. Empirical distributions of social surplus

bidders the bidders benefit from a PPO—an observation that has already been made in Section 4.1.

In the case of the auctions with five bidders, Figures 4.4(b) and 4.5(b) show no indication regarding the central tendency of the revenues and the bidders' payoffs in treatments A5 and B/S5. Interestingly, however, Figure 4.5(b) does suggest that the treatments A5 and B/S5 differ in the dispersion of the bidders' payoffs. For low payoffs the distribution of B/S5 is greater and for large payoffs it is smaller than the respective distribution of A5.

In Section 3.8, the statistical methods used in the analysis were briefly introduced. Inter alia, the choice of the selected tests was accounted for by the presumption that the experimental data might be non-normal and that the treatments might differ in dispersion. In the remainder of this section, it is shown that both of these presumptions in fact hold. Moreover, the applicability of the WSR is investigated by scrutinizing whether the empirical distribution functions and their respective mirror images intersect.[10]

The discussion starts with a closer look at the differences in dispersion. Tables 4.3–4.6 show—in addition to the outcomes of all individual auctions—the range of each group's results as well as the overall standard deviation within each treatment. The variability of the results, however, is not only an effect of varying bidders' behavior but is primarily due to the dispersion of the bidders' random valuations. Thus, the range and the standard deviation are of limited meaning and difficult to compare.

In order to reduce the impact of the random bidder valuations and to facilitate a comparison of the variability of the auction outcomes, one can compute the difference between the observed outcomes and the theoretical benchmark.[11] Such a transformation eliminates the influence of random insofar as a transformed outcome is zero if the observed result equals the equilibrium outcome. Non-zero values indicate deviations from the theoretical benchmark. Moreover, any systematic constant deviation from the benchmark would lead to a zero variance of the transformed experimental outcomes.

Table 4.7 displays the standard deviation of the transformed experimental outcomes for each treatment, i. e. the standard deviation of the differences between the experimental results and the theoretical benchmark, rather than the standard deviation of the results themselves. The table shows that

[10] Another issue with respect to the applicability of the WSR relates to the independence of the data. Remember that all observations are treated as if they were independent (quasi-independence). The independence assumption does not hold if the data reveal a trend during the course of the experiment (e. g. learning, experience, . . .). It will be shown in Section 4.4.1 that the bids do in fact become more aggressive over time. In Section 3.4, it was argued, however, that even if such a trend exists, one would expect a similar trend to exist in all treatments. Thus, the trend diminishes if one considers the differences of matched pairs on which the WSR is performed. In order to verify the argument, for each treatment and for each round, the average results with respect to the revenues, the bidders' payoffs, and the social surplus are computed. Testing for trend over the series of the matched differences between any pair of treatments reveals that the null hypothesis, that there is no trend within these differences, cannot be rejected at a 5% significance level. Thus, the trend in the individual data does not seriously restrict the application of the WSR.

[11] The theoretical benchmark is given by the equilibrium outcomes of a second-price auction. These outcomes are listed in Tables B.7 and B.8 in Appendix B.

Table 4.7. Comparison of variability of experimental results

Standard deviation of difference between experimental and equilibrium outcomes		A-treatment	B/S-treatment
3 bidders	Revenues	6.5	14.3
	Winning bidder's payoff	10.3	14.7
	Social surplus	5.8	7.5
5 bidders	Revenues	5.3	10.9[a]
	Winning bidder's payoff	5.4	15.1[a]
	Social surplus	2.1	9.4[a]

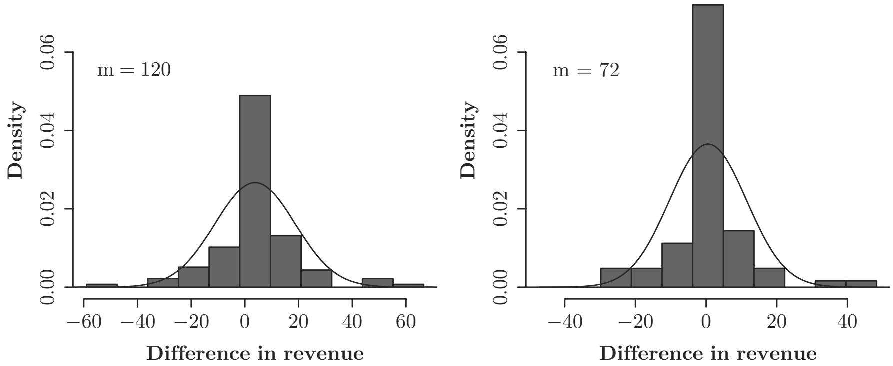
[a]Remember that bidding behavior is duplicated in treatment B/S5. Thus, this calculation underestimates the true standard deviation. Cf. fn. 9 on page 93.

the variability—if measured by the standard deviation of the transformed outcomes—is higher in the B/S-treatments than in the A-treatments. In fact, this difference is statistically significant.[12] Thus, the existence of the PPO gives rise to a stronger variability of the auction outcomes.

The goodness of fit of the experimental data with respect to the normal distribution is illustrated in Figure 4.7 using the observed revenues as an example. For both the treatments with three bidders (a) and five bidders (b), the figure shows a histogram of the paired differences in revenues and a fitted normal curve.[13] Each of the two histograms is based on eleven classes of equal width and the middle class is centered around the mean of the sample. The two normal curves are plotted using the maximum likelihood estimators of

[12] To test for significance, the Fligner-Killeen test is performed. The test translates rank scores into a χ^2-statistic with one degree of freedom. For the treatments with three bidders the sample size $m_{A3} = m_{B3} = 120$ and the test statistic for the revenues, the winning bidders' payoffs, and the social surplus are $\chi^2_{\hat{R}} = 26.6$, $\chi^2_{\hat{\Pi}} = 22.6$, and $\chi^2_{\hat{V}} = 6.7$. The respective p-values are all smaller than 1%. For the treatments with five bidders, the analysis is complicated because the bidding data is duplicated in B5. In order to avoid artificially overstating the number of observations, the test is conservatively estimated by averaging over the Observations I and II of B5. This yields lower standard deviations than those reported in Table 4.7 of $s_{\hat{R}} = 8.9$, $s_{\hat{\Pi}} = 13.6$, and $s_{\hat{V}} = 8.8$. The sample sizes are now $m_{A3} = m_{B3} = 72$, and $\chi^2_{\hat{R}} = 14.0$, $\chi^2_{\hat{\Pi}} = 18.3$, and $\chi^2_{\hat{V}} = 5.5$. The p-values with respect to the standard deviation of the revenues and the winning bidders' payoffs are again smaller than 1% and for the social surplus p-value = 1.9%. Note that the results are conservative due to the systematic underestimation of the variances in the treatment B/S5.

[13] Figure 4.7(b) is based on Observation I of treatment B/S5 (cf. Section 4.2). The respective graph based on Observation II looks very similar.



(a) $n = 3$ bidders (b) $n = 5$ bidders (Observation I)

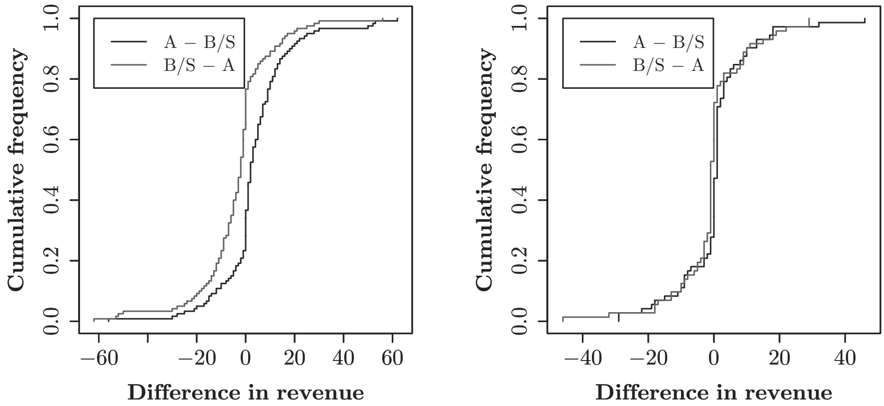
Figure 4.7. Histogram of experimental outcomes compared to density of normal distribution (the figure shows the differences in revenues between the A and B/S-treatments)

the mean and the standard deviation.[14] The figure shows that—compared to the normal distribution—the experimental data is characterized by high peaks around zero and a larger probability mass at its tails. Meanwhile, the observed data has a relatively low density for medium deviations from the mean.

To test the goodness of fit of the paired differences in the experimental outcomes with respect to a normal distribution, a Shapiro-Wilk test is performed. The test shows that the null hypothesis, that the data is normally distributed, must be rejected.[15] The result confirms the rejection of the t and the F-tests for investigating differences in central tendency and dispersion of the experimental data.

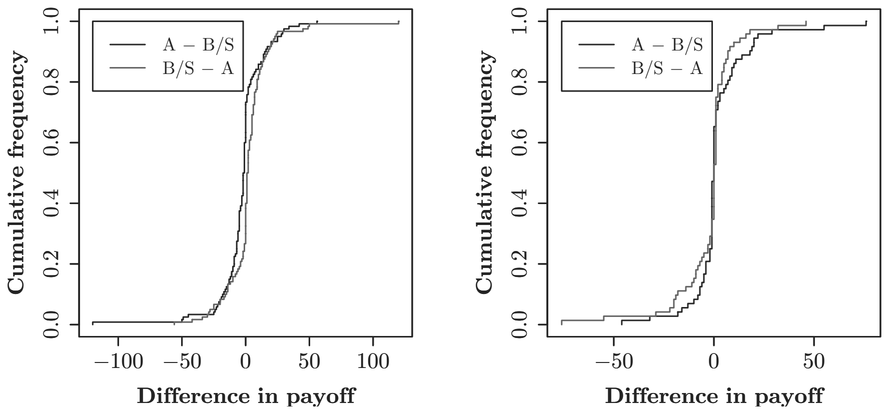
[14] For three bidders the two maximum-likelihood estimators are $\tilde{\mu} = 3.8$ and $\tilde{\sigma} = 14.9$, and for five bidders the respective estimators are $\tilde{\mu} = 0.6$ and $\tilde{\sigma} = 10.9$.

[15] For $n = 3$ bidders, the sample size $m_3 = 120$ and the Shapiro-Wilk statistics with respect to the differences in revenues, the winning bidders' payoffs, and the social surplus are $W_{\hat{R}} = 0.88$, $W_{\hat{\Pi}} = 0.78$, and $W_{\hat{V}} = 0.36$, respectively. Since the bidding data of treatment B5 was duplicated in treatment S5, two separate tests have been conducted to test whether the differences in treatment A5 and the combined treatment B/S5 are normally distributed. The two separate tests are based on identical bidder data in treatments A5 and B5, but independent seller groups in treatment S5 (Observation I and Observation II). In all cases the sample size $m_5 = 72$. The test statistics are $W_{\hat{R},I} = 0.84$, $W_{\hat{R},II} = 0.77$, $W_{\hat{\Pi},I} = 0.74$, $W_{\hat{\Pi},II} = 0.74$, $W_{\hat{V},I} = 0.52$, and $W_{\hat{V},II} = 0.50$. All reported p-values are smaller than 1%—this holds for the differences in revenues, the winning bidders' payoffs, and the social surplus, both for the experiments with three and with five bidders.



(a) $n = 3$ bidders (b) $n = 5$ bidders (Observation I)

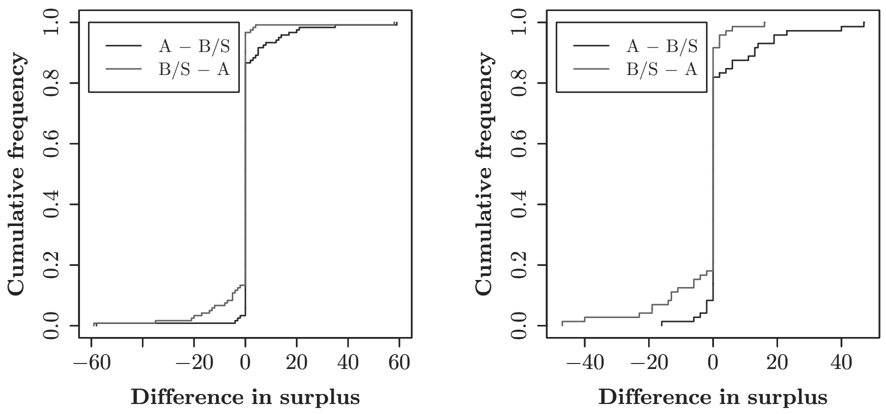
Figure 4.8. Empirical distributions of the differences in revenues



(a) $n = 3$ bidders (b) $n = 5$ bidders (Observation I)

Figure 4.9. Empirical distributions of the differences in payoffs

Finally, Figures 4.8–4.10 link the data of the experiment back to the assumptions of the WSR (cf. Section 3.8). Each plot contains two distribution functions: one of them is based on the differences derived by subtracting the results of the B-treatment from the results of the A-treatment and the other one is based on the reversal differences. Note that if the former distribution is denoted by F and the latter by G, $G(x) = 1 - F(-x)$ holds for all x. According to the discussion on pages 83f, these two functions should not intersect in order for the WSR to be applicable.



(a) $n = 3$ bidders (b) $n = 5$ bidders (Observation I)

Figure 4.10. Empirical distributions of the differences in surpluses

The figures show that the respective pairs of the distribution functions are similarly shaped. More important is the fact that the distributions in Figures 4.8(a), 4.10(a), and 4.10(b) do not intersect (for some values they lie upon each other). There are no major intersections in 4.9(a) either. In Section 4.3 it is shown that these are exactly those cases in which the null hypothesis $F(x) = G(x)$ is rejected. Thus, the assumptions of the WSR hold and the danger of obtaining false positive results due to technical reasons, i. e. a systematic underestimation of the Type I error by the p-value, is low.

4.3 Comparison of Auction Outcomes

In Section 4.1 the average results of the four treatments A3, A5, B/S3, and B/S5 were presented. The preliminary findings indicate that if the PPO is available, both the average revenue and the average social surplus are lower than in the auctions without a PPO. The results with respect to the bidders' payoffs are not that clear: in the auctions with three bidders, the bidders gain from the existence of the PPO but the opposite holds for the auctions with five bidders. The graphs of the empirical distributions in Section 4.2 on the one hand call into question the significance of the results with respect to the revenues and the bidders' payoffs in the auctions with five bidders. On the other hand, they strengthen the assumption that in the auctions with three bidders, auctions with a PPO yield lower revenues for the seller but higher bidder payoffs. Moreover, pure auctions appear to be more efficient than auctions with a PPO regardless of the number of bidders. In this section, the statistical significance of these findings is more thoroughly investigated.

In light of the preparations of the previous sections, the comparison of the different treatments is relatively straightforward. The individual auction outcomes are considered quasi-independent and the Wilcoxon signed ranks test (WSR) is performed on the paired differences of these outcomes (cf. Section 3.4). For all tests a significance level of 5% (two-sided) is applied.[16]

Table 4.8. Comparison of auction outcomes

	Variable	Observation	Two-sided WSR
3 bidders	Revenue	Treatment A3 outperforms B3	$m' = 76; V^+ = 3{,}822.5;$ p-value $< 1\%$
	Bidder payoff	Treatment B3 outperforms A3	$m' = 32; V^+ = 1{,}994;$ p-value $= 1.7\%$
	Social surplus	Treatment A3 outperforms B3	$m' = 16; V^+ = 180.5;$ p-value $< 1\%$
5 bidders	Revenue	—	$m'_I = 38; V_I^+ = 994;$ p-value $= 28.2\%$ $m'_{II} = 33; V_{II}^+ = 847.5;$ p-value $= 87.0\%$
	Bidder payoff	—	$m'_I = 25; V_I^+ = 934;$ p-value $= 71.3\%$ $m'_{II} = 24; V_{II}^+ = 860.5;$ p-value $= 97.2\%$
	Social surplus	Treatment A5 outperforms B5	$m'_I = 13; V_I^+ = 155;$ p-value $= 1.6\%$ $m'_{II} = 12; V_{II}^+ = 138.5;$ p-value $= 2.2\%$

Table 4.8 summarizes the findings.[17] As conjectured in Section 4.2, the differences between the auctions with and without a PPO are significant in the treatments with three bidders: In this case, a pure auction outperforms an APPO in terms of the sellers' revenues and the social surplus, i. e. a pure auction yields significantly higher revenues and is also more efficient. The reverse holds for the bidders' payoffs, i. e. the bidders profit from a PPO. Moreover, the results are also significant with respect to the surplus in auctions with five bidders. Here again, a pure auction is more efficient. The observed differences with respect to the revenues and the bidders' payoffs, however, are not significant for the auctions with five bidders.

[16] Due to ties or the size of the sample, the software may internally estimate the p-values by a normal approximation of the test statistic. A correction to account for the continuity of the approximation is applied. Moreover, the implementation of the test eliminates equal values (i. e. ties at zero). The reduced sample size is denoted by m'. The original sample sizes for the treatments with three bidders are $m = 120$ and for five bidders $m = 72$.

[17] In the treatments with five bidders the tests have been run twice—once based on Observation I and then based on Observation II. An alternative calculation averaging the two observations yields only slightly different p-values and results in the same classification significant/non-significant.

In the following, the factors that are responsible for the differences identified in Table 4.8 are investigated. It has already been mentioned in Section 4.1 that the results of the A-treatments are very close to the theoretical benchmark given by the equilibrium outcomes of a second-price auction. Tables 4.1 and 4.2 show that the results of the combined B/S-treatment are also close, but not as close, to this benchmark. This is not very surprising since the A-treatments just implement a second-price auction, while in the B-treatments an additional degree of freedom is introduced by the existence of the PPOs.

Interestingly, the following analysis of the impact of the PPO shows that it is not necessarily the PPO itself, i. e. the offer and the acceptance of a PPO that causes the observed differences. Rather—at least for the revenues and the winning bidders' payoffs—the differences are due to a shift in the bids that the participants submit in the case that the PPO is not accepted.

The design of the experiment allows the construction of two additional virtual treatments. Firstly, one can evaluate the bidders' strategies recorded in the treatments B3 and B5 as if the seller had not offered a PPO. These first virtual treatments will be referred to by B3/no PPO and B5/no PPO, respectively. Remember that in the B-treatments the bidders entered two numbers: (*i*) their acceptance threshold up to which they were willing to accept a PPO if offered to them and (*ii*) their bid in a second-price auction in the case that an auction was conducted. Rather than matching the recorded strategy profiles with the PPOs set by the sellers in treatments S3 and S5, respectively, the virtual treatments B3/no PPO and B5/no PPO match the strategies of the bidders with fictitious sellers that never propose a PPO. Thus, the outcomes of the A and the B/no PPO-treatments would have been the same if the bidders in the A-treatments had submitted the same bids as the bidders in the B-treatments.

Secondly, one could assume that the subjects in the B-treatments did in fact submit the same bids as the subjects in the A-treatments. Thus, one constructs fictitious bidding strategies based on the bids submitted in the A-treatments and the acceptance thresholds observed in the B-treatments and matches these strategies with the actual PPOs of the sellers in the S-treatments. The virtual treatments constructed in this manner are referred to by A/S3 and A/S5. Note that from a theoretical perspective, there is no reason why a bidder should submit different bids in the A and the B-treatments.

The relation between the actual and the virtual treatments is illustrated in Figures 4.11–4.14. Along one axis, the bids are kept constant and the effect of the PPO itself is isolated by comparing treatments with and without a PPO. Along the other axis nothing is varied with respect to the PPO. The bids, however, differ along this axis. The arrowed edges start at a node representing a treatment based on the bids of an A-treatment and they end at a node representing a treatment based on the bids of a B-treatment. At the edge labeled “no PPO” two treatments without a PPO are compared and at the edge “with PPO” exactly the same PPOs are offered and accepted. The outcomes at this edge differ only in those auctions in which the PPO is rejected.

The outcome of an auction in the B/S-treatment can be different from the outcome of the respective auction in the A-treatment if the decisive bidder either accepts the PPO in the B/S-treatment or if the submitted bids differ. The Figures 4.11–4.14 separate these two effects. Bold arrowed edges indicate that the effect which the respective edge represents is significant at the 5% level.[18]

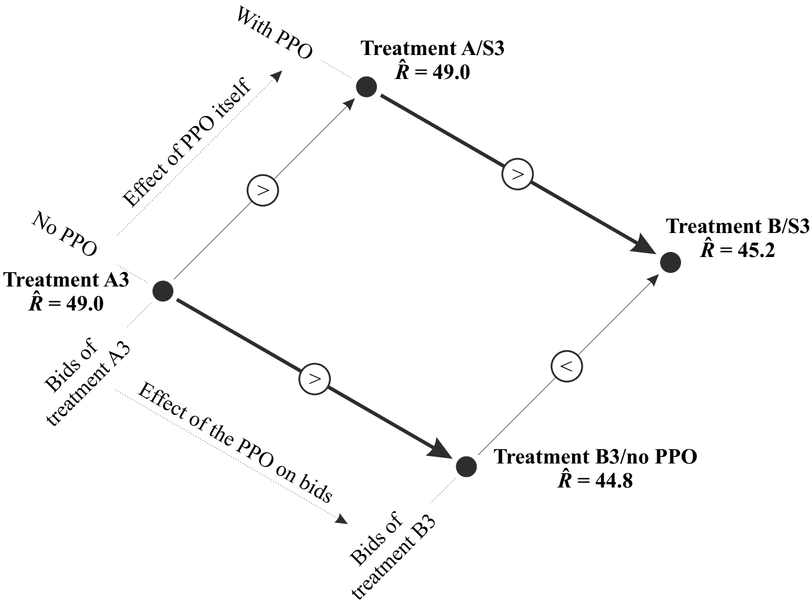


Figure 4.11. Effect of a PPO on the revenues in auctions with $n = 3$ bidders

Consider Figures 4.11 and 4.12. These figures analyze the differences regarding the revenues and the winning bidders’ payoffs in the auctions with three bidders. The thin arrowed edges of the figures show that the null hy-

[18] Again, the WSR has been performed on paired differences. For clarity, the presentation dispenses with listing all individual test statistics and concentrates on the p-values.

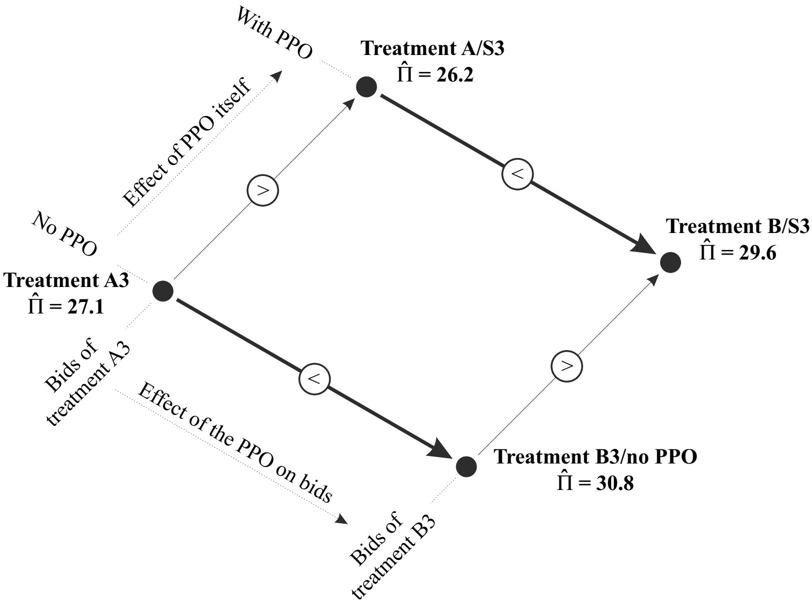


Figure 4.12. Effect of a PPO on the winning bidders' payoffs in auctions with $n = 3$ bidders

pothesis, that the PPO has no direct effect, cannot be rejected.[19] In other words, if auctions are replaced by the acceptance of the PPO, neither the revenues nor the winning bidders' payoffs change significantly. Rather, the figures reveal a shift in the bidding behavior from treatment A3 to treatment B3. This shift—represented by the bold arrowed edges—strongly affects the auction outcomes.[20] The revenues calculated on the basis of the bids of A3 are higher than those based on the bids of B3 and the opposite holds for the winning bidders' payoffs. This raises the presumption that subjects bid more defensively in treatment B3 than in treatment A3. In Section 4.4, this presumption is tested on the basis of individual bids.

Table 4.8 on page 106 not only reports significant differences between the A3 and the B3-treatments with respect to the revenues and the winning bidders' payoffs, but also differences with respect to the social surplus, irrespective of the number of bidders. Analogously to Figures 4.11 and 4.12, Figures 4.13 and 4.14 investigate the differences in the social surplus. It turns out that

[19] The (two-sided) p-values with respect to the revenue are 95% (bids of A3) and 79% (bids of B3) and with respect to the bidders' payoffs 35% (bids of A3) and 28% (bids of B3).

[20] The respective (two-sided) p-values of all four bold edges in Figures 4.11 and 4.12 are smaller than 1%.

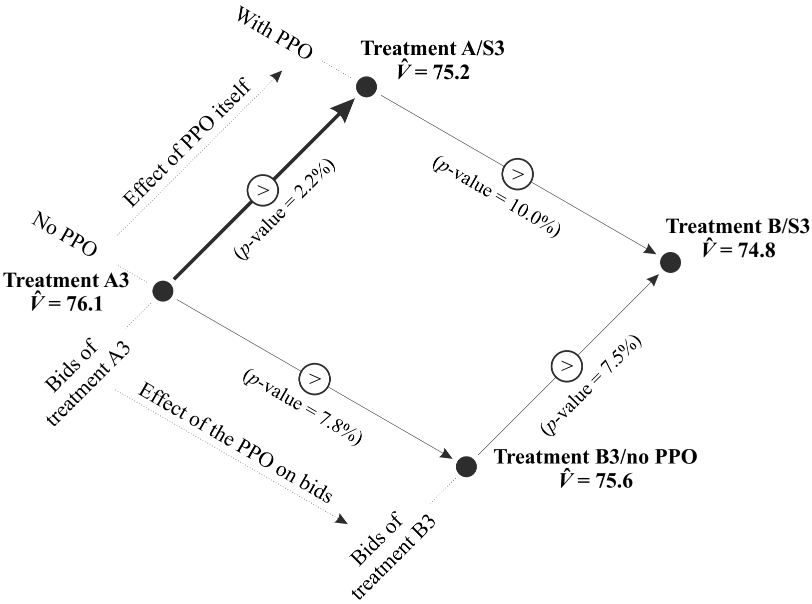


Figure 4.13. Effect of a PPO on the social surplus in auctions with $n = 3$ bidders

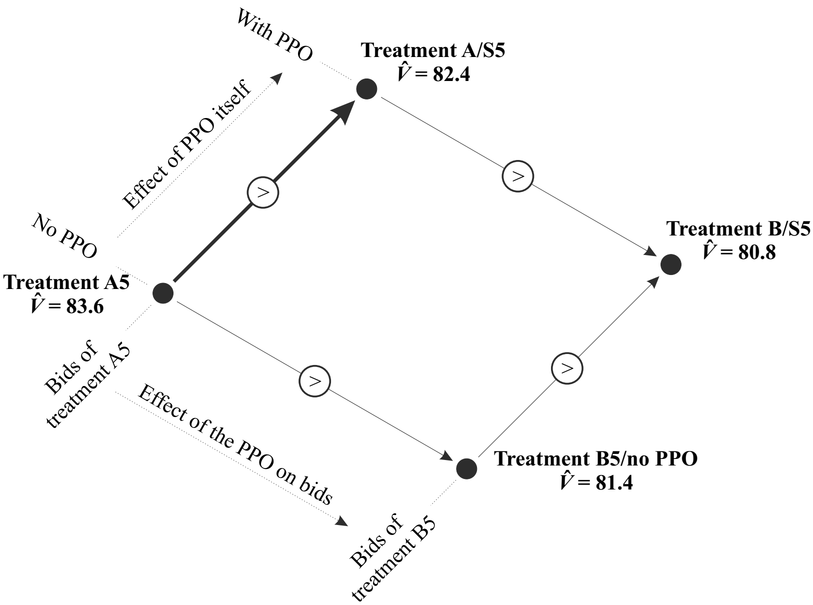


Figure 4.14. Effect of a PPO on the social surplus in auctions with $n = 5$ bidders

no significant differences can be identified if one substitutes the bids of the A–treatments with the bids of the B–treatments—independent of whether the PPOs are taken into consideration or not. However, with both sets of bids, the social surplus decreases when the accepted PPOs are considered. This decrease is significant for the bids of treatments A3 and A5. If the auctions are evaluated based on the bids of the B–treatments, the difference is not significant.[21]

The discrepancies between Figures 4.11 and 4.12, on the one hand, and Figures 4.13 and 4.14, on the other hand, are noteworthy. There is, however, a plausible explanation. Remember that the observed differences in the revenues and the payoffs have opposite signs. Since the social surplus is defined as the sum of the revenues and the bidders' payoffs, the two effects cancel each other out. In fact, if all bids in an auction were similarly reduced, the winner of the auction would not change and, thus, the social surplus would be identical. This explains why the differences caused by the effect of the PPO on bids diminish when looking at the social surplus. Consider now the axis "effect of the PPO itself." There is no ex–ante reason to assume that offering a PPO to a randomly selected bidder will increase the efficiency of the mechanism.[22] Thus, the lower surplus in the treatments with a PPO comes as no surprise. The reason that this difference can be classified as significant based on the bids of the A but not on the bids of the B–treatments by the statistical test may be due to the fact that the bids of the A–treatments are relatively close to the theory and that there is too much noise in the bidding data of the B–treatments. A corresponding presumption has already been made in Section 4.1; it will be proven in the next section. Moreover, the effect of the PPO itself does not appear to be strong enough to substantiate the differences regarding the revenues or the bidders' payoffs.

4.4 Behavior of the Bidders

Having focused in the previous sections on the outcomes of the auctions, i. e. the revenues, the bidders' payoffs, and the social surplus, the analysis now shifts towards the behavior of the individual participants. The question of

[21] Since for the auctions with three bidders the classification significant/not significant is not clear-cut, the respective p-values are incorporated in Figure 4.13. With respect to the auctions with five bidders, only the edge "bids of treatment A5" represents significant differences (the p-values of Observations I and II are 2.2% and 5.8%, respectively; averaging the two observations yields a p-value $< 1\%$).

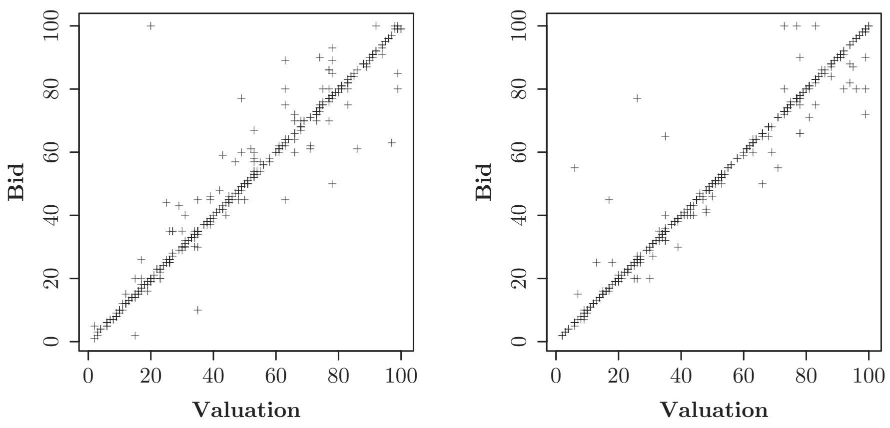
[22] Note that a second-price auction is an efficient mechanism, i. e. in theory a second-price auction awards the item to the bidder who values it highest.

interest is whether the patterns which the theory predicts for the behavior of rational bidders are reflected in the data.

1. Do bidders apply their dominant strategy when bidding in an auction, i. e. do they bid their valuation of the item?
   - Are there differences between the bids in the A and the B–treatments?
   - Does the number of bidders influence the bids?
2. Is the acceptance threshold of the bidders in accordance with the theoretical model presented in Chapter 2?
   - Is the first bisecting line an upper bound of the acceptance threshold?
   - Does the theoretical acceptance threshold of a risk neutral bidder constitute a lower bound of the acceptance threshold, i. e. is the observed behavior consistent with the presumption of risk neutral or risk averse bidders?
   - Does the acceptance threshold increase with the number of bidders?

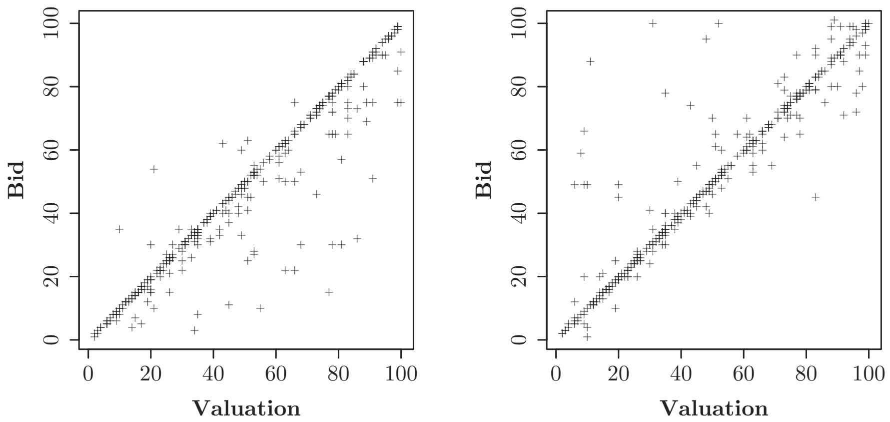
4.4.1 Bidding Behavior

To start the analysis, Figures 4.15 and 4.16 provide an overview of the subjects' bids in the A and the B–treatments, respectively. The two figures relate the bids to the valuation of the respective bidder. Each plot contains 360 data points (30 subjects, 12 rounds). The complete tables with all data are shown in the Appendix C (Tables C.1–C.4).



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.15. Bids in treatments A3 and A5



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.16. Bids in treatments B3 and B5

The figures show that the bids are close to the theoretical benchmark. Particularly in the A-treatments many points lie on or close to the diagonal (not drawn in the figures). There are somewhat more deviations in the plots of the B-treatments. Thus, the figures strengthen the presumption made in Sections 4.1 and 4.3 that there is more noise in the data of the B than the A-treatments.

In order to measure how well the dominant strategy $\tilde{b_i} = v_i$ predicts the observed bids $\hat{b_i}$, the determination coefficients

$$B = 1 - \frac{\sum_i (\hat{b_i} - \tilde{b_i})^2}{\sum_i (\hat{b_i} - \bar{b})^2}$$

with $\bar{b}$ denoting the mean of the observed bids are calculated. The determination coefficient B takes on values in the range $[0; 1]$. Values close to one indicate that the observed variance is well explained by theory. If, otherwise, the value of B is low, the theory provides little insight into what has been observed in the experiment.

Table 4.9 depicts the respective values which show that the concept of the dominant strategy explains more of the observed variance in the A-treatments than in the B-treatments.[23]

[23] The determination coefficient B, also denoted by r^2, is common in regression analysis. Note that the dominant strategy bids $\tilde{b_i} = v_i$ do not stem from a linear regression. Thus, the identity $\frac{\sum_i (\hat{b_i} - \tilde{b_i})^2}{\sum_i (\hat{b_i} - \bar{b})^2} + \frac{\sum_i (\tilde{b_i} - \bar{b})^2}{\sum_i (\hat{b_i} - \bar{b})^2} = 1$ which is known from regression analysis (e. g. Wonnacott und Wonnacott, 1990, Chapter 15) does not

Table 4.9. Determination coefficients relating the observed bids to the dominant strategy

Treatment	**Determination coefficient** B
A3	94.3%
A5	95.6%
B3	86.8%
B5	86.0%

The issue becomes even more apparent if one classifies the data. Consider, e. g., one class that comprises those bids $\hat{b}_i$ which equal the bidder's valuation v_i. The bids that deviate by one currency unit from the valuation constitute a second class, whereas a third class is based on the bids which deviate by more than one currency unit. The respective frequencies of bids falling in these classes are captured by the contingency tables which are shown in Table 4.10. The table reveals that for both $n = 3$ and $n = 5$ bidders, the first class contains more bids in the A–treatments than in the B–treatments. The reverse holds for the third class, which is larger in the B than in the A–treatments. The table also renders the χ^2–statistic, according to which the hypothesis that the bids in the A and the B–treatments originate from the same distribution should be rejected.[24]

One can conclude from the above that bids are in fact closer to the theory in the A–treatments than in the B–treatments. It appears that the additional complexity which accompanies the existence of a PPO makes the auction institution less transparent and veils the strategic characteristics of bidding in the pure auction.

The analysis now shifts to the first question posed at the beginning of this section. It will be investigated whether (i) the existence of a PPO or (ii) the number of bidders has an impact on the bids submitted. According to the theory, one would not expect such differences. In Section 4.3, however, the presumption was made that in the experiment participants bid more defensively in treatment B3 than in A3. Since the bids in the B–treatments deviate from the theory more than in the A–treatments, an additional shift in location would not be surprising. Table 4.11 shows the average bid in each treatment.

hold in the current setting. This confines the interpretation of the determination coefficient as the percentage of explained variance over total variance.

[24] The test, however, should be interpreted with care because the 360 individual data points stem from only 30 different bidders. Thus, the observations are not independent but only quasi-independent (cf. Section 3.4).

Table 4.10. Contingency tables of bids

(a) $n = 3$ bidders

Treatment	$\hat{b}_i = v_i$	$\lvert\hat{b}_i - v_i\rvert = 1$	$\lvert\hat{b}_i - v_i\rvert > 1$	$\sum$
A3	193 (54%)	97 (27%)	70 (19%)	360 (100%)
B3	151 (42%)	100 (28%)	109 (30%)	360 (100%)
$\sum$	344 (48%)	197 (27%)	179 (25%)	720 (100%)

$\chi^2 \approx 13.7$, $df = 2$, p-value $< 1\%$

(b) $n = 5$ bidders

Treatment	$\hat{b}_i = v_i$	$\lvert\hat{b}_i - v_i\rvert = 1$	$\lvert\hat{b}_i - v_i\rvert > 1$	$\sum$
A5	248 (69%)	62 (17%)	50 (14%)	360 (100%)
B5	138 (38%)	114 (32%)	108 (30%)	360 (100%)
$\sum$	386 (54%)	176 (24%)	158 (22%)	720 (100%)

$\chi^2 \approx 68.0$, $df = 2$, p-value $< 1\%$

By construction of the experiment, the average valuations are identical in all treatments.

Table 4.11. Average bids in the different treatments

Treatment	**Mean valuation**	**Mean bid**
A3	50.3	50.7
A5	50.3	50.1
B3	50.3	46.9
B5	50.3	51.3
mean	50.3	49.8

Interestingly, Figure 4.16 reveals a tendency of underbidding (i. e. bidding less than one's private valuation for the item) in B3 but appears to reveal the opposite, i. e. a slight tendency of overbidding, in B5. Table 4.11 supports this observation: The average valuation is 50.3 and the average bid in B3 is only 46.9. In treatment B5, on the other hand, the average bid (51.3) is higher than the average valuation (50.3). It will be shown in the following, however, that overbidding in B5 is not a strong result. The reason is that in this treatment six of the seven strongest deviations from theory stem from only one bidder. As a consequence, there are still more bidders who bid below rather than above their valuation (on average a total of 20 bidders underbid, whereas only 7 bidders overbid).

Thus, for a more thorough look, it is more meaningful to condense the data and to aggregate over individual bidders. In a second step, the data is again classified. For all bidders the average deviation $b_i - v_i$ is calculated and five classes are considered. Table 4.12 displays the frequencies of bidders in these classes.

Table 4.12. Frequencies of bidders deviating from theory with their bids

Treatment	**Average deviation from theory ($b_i - v_i$)**					$\sum$
	Underbidding		Theory	Overbidding		
	$(-\infty, -1)$	$[-1, 0)$	$\{0\}$	$(0, 1]$	$(1, \infty)$	
A3	6 (20%)	5 (17%)	11 (36%)	3 (10%)	5 (17%)	30 (100%)
B3	14 (47%)	7 (23%)	6 (20%)	1 (3%)	2 (7%)	30 (100%)
A5	6 (20%)	6 (20%)	15 (50%)	0 (0%)	3 (10%)	30 (100%)
B5	10 (33%)	10 (33%)	3 (10%)	2 (7%)	5 (17%)	30 (100%)
$\sum$	36 (30%)	28 (23%)	35 (29%)	6 (5%)	15 (13%)	120 (100%)

$\chi^2 \approx 21.7, \quad df = 12, \quad p\text{-value} = 4.1\%$

According to Table 4.12, it is unlikely that bidders behave similarly in all treatments. In particular, the table strengthens the presumption that subjects bid more defensively in B3 than in A3.[25] The table also shows that—in contrast to the first impression suggested by Figure 4.16—underbidding is more common in B5 than in A5. Apart from that, however, the table gives only little support for a pairwise comparison of individual treatments.

Because a χ^2-test based on the contingencies of Table 4.12 is not strong enough for an inferential analysis, a more powerful test is performed. Remember that the table of the bidders' valuations is kept identical over all sessions of the experiment. Thus, one can again consider paired differences of the data. For every treatment, the average bid of each bidder is calculated.[26] A paired test is then based on the differences in the average bids of pairs of bidders with the same sequence of valuations. Table 4.13 shows the results of a Wilcoxon signed ranks test (WSR).[27]

[25] The respective χ^2-test, however, is not strong enough to prove the difference: $\chi^2 \approx 7.4$, $df = 4$, p-value $= 12.1\%$.

[26] The average bids of all treatments are given in Table 4.11.

[27] One might think that the average bids are normally distributed due to the central limit theorem (see e. g. Kempthorne und Folks, 1971, section 6.11). Performing a

Table 4.13. Comparison of treatments with respect to bidding behavior

Treatments	Observation	Mean difference Δ / Two-sided WSR
A3 vs. B3	Bids in A3 are higher than in B3	$\Delta = 3.8$ $m' = 29; V^+ = 333.5;$ p-value $= 1.3\%$
A5 vs. B5	—	$\Delta = -1.2$ $m' = 30; V^+ = 255;$ p-value $= 65.1\%$
A3 vs. A5	—	$\Delta = 0.5$ $m' = 21; V^+ = 129;$ p-value $= 65.1\%$
B3 vs. B5	—	$\Delta = -4.4$ $m' = 27; V^+ = 123;$ p-value $= 11.6\%$

The tests show that a significant difference in the bidding behavior can be identified only between the treatments A3 and B3. Participants bid more defensively in the latter treatment. In fact, the bids in treatment B3 are the lowest of all treatments. This also explains the differences in the average revenue (which is lower in B3 than in A3) and the average bidder payoff (which is higher in B3) which were identified in Section 4.3. In all other cases, no significant differences are found. This is particularly noteworthy because the average bid in B5 is even higher than the average bid in A3. Apparently, however, there is so much noise in the data of the B-treatments that a significant difference between B3 and B5 is not observed.

Finally, the development of the bidding behavior during the course of the experiment is investigated. As an illustration, Figures 4.17–4.22 depict the the average bids per round in relation to the average valuation. The figures on page 118 refer to the A-treatments and the figures on page 119 refer to the B-treatments (since the valuations were kept identical, Figures 4.17 and 4.20 are also identical).

Most interesting are the Figures 4.19 and 4.22. These figures display the difference between the average bid and the average valuation per round, i. e. the deviation from the dominant strategy. The figures also show Spearman's rank correlation coefficient r of the observed differences versus the round numbers. This coefficient is positive in all treatments. Thus, all treatments exhibit an upward trend and the bidding becomes more aggressive. Performing a test for trend on the data of the figures reveals that the increase in the

Shapiro-Wilk test reveals that even for the average deviations from theory, this is not the case. An explanation is that neither the different bids of one bidder are independent, nor are the deviations of the different bidders identically distributed. Thus, the t-test is not appropriate and the WSR is utilized.

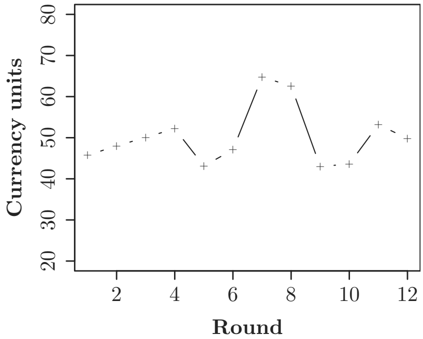
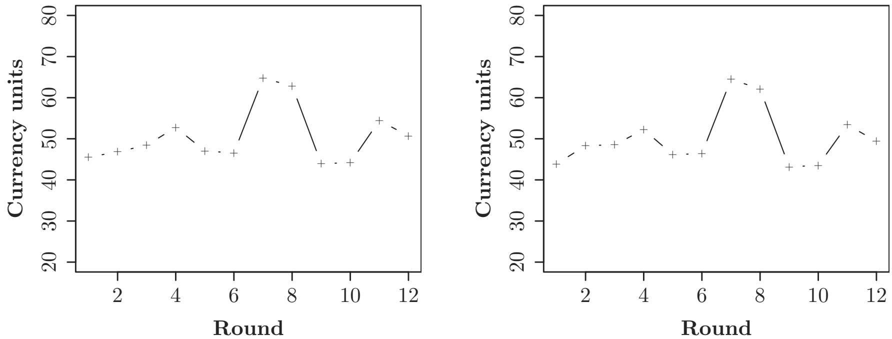
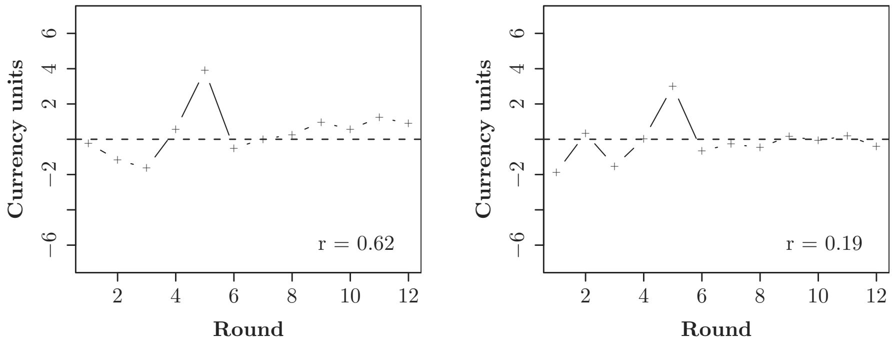


Figure 4.17. Average valuation per round



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.18. Average bid per round in the A-treatments



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.19. Average deviation of bids from theory in the A-treatments

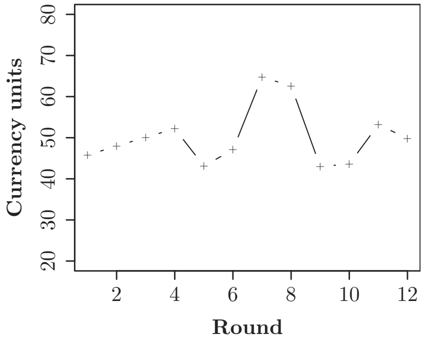
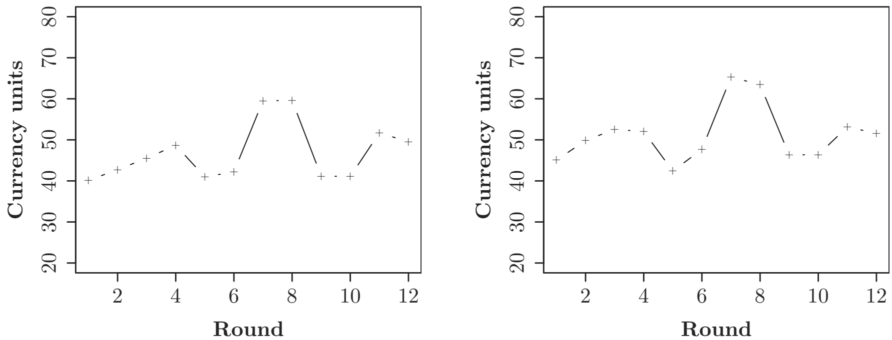
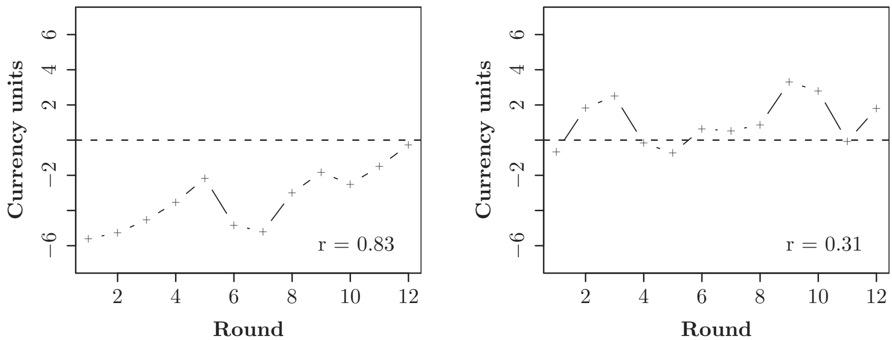


Figure 4.20. Average valuation per round (identical to Figure 4.17)



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.21. Average bid per round in the B-treatments



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.22. Average deviation of bids from theory in the B-treatments

aggressiveness of the bids is significant in the treatments A3 and B3.[28] In the treatments with five bidders, the trend is not strong enough to be verified by statistical methods.[29] Moreover, in all treatments, bidding starts at a level below the valuation (underbidding).

The observed patterns are in accordance with the observations made by Harstad (2000) who investigates the English and the second-price auction using a series of experiments. Harstad finds that in second-price auctions inexperienced bidders initially bid mostly below their valuation. Then "bidding rapidly becomes more aggressive, quickly breaking through the threshold of bidding equal to the value [and] [b]ehavior does settle down to a persistent level of overbidding" (p. 267), Harstad explains the "substantial and persistent overbidding" (p. 261) by the feedback mechanism of the second-price auction: a "subject might overbid, win, and still make money" and concludes that "[s]uch an occurrence may be viewed (mistakenly) as positive feedback" (p. 262).

If Harstad's hypothesis of overbidding due to the feedback mechanism is true, one might conjecture that the observed phenomenon is stronger, when fewer bidders participate in an auction: a lower number of bidders increases the likelihood of winning an auction and, ceteris paribus, reduces the risk of paying more than the valuation when winning with a bid above valuation. This would explain why a strong increase in the bidding level can be observed in the treatments with three bidders, while the increase is weak in the treatments with five bidders. Note that Harstad does not vary the number of bidders. Rather, he conducts all auctions with six bidders.[30] It seems that the effect of increasing bidding levels and persistent overbidding is much stronger in Harstad's experiment. This can easily be explained because in the APPO experiment the second-price auction is introduced as an English proxy auction. The English auction is known to perform much better in experimental settings (e. g. also Harstad, 2000). Since the English proxy auction has elements of both the English and the second-price auction, it is reasonable to assume that its performance is somewhere in between the two framing formats.

[28] Test for trend: $m = 12$, $D = 108$, and p-value $= 3.4\%$ (two-sided) for A3 and $m = 12$, $D = 50$, and p-value $< 1\%$ (two-sided) for B3.

[29] Test for trend: $m = 12$, $D = 232$, and p-value $= 55\%$ (two-sided) for A5 and $m = 12$, $D = 196$, and p-value $= 31\%$ (two-sided) for B5.

[30] In some sessions the number of bidders decreases due to the bankruptcy of individual subjects.

4.4.2 Acceptance Thresholds in the B-treatments

This section investigates the behavior of subjects with respect to the key feature of an APPO: the posted price offer (PPO). In Chapter 2, the concept of the acceptance threshold was introduced. The acceptance threshold is the maximum PPO a bidder is willing to accept. Moreover, a small corridor for the acceptance threshold of rational risk neutral or risk averse bidders was derived.

Figure 4.23 depicts the acceptance thresholds observed in the experiment and contrasts them with the above–mentioned corridor indicated by the dashed lines. The figure shows that most of the observed threshold values lie below the diagonal that constitutes the upper bound of the corridor. Accepting a PPO above this upper bound yields a loss for the respective bidder. The fact that only a few threshold values exceed the upper bound suggests that this is well understood by the participants.

There are, however, many threshold values which even lie below the lower bound of the corridor. Bidders with such low thresholds abstain from accepting a PPO, which would yield a positive and risk–free payoff that is higher than the expected payoff from participating in the corresponding auction. In a model based on rational bidders, this can only be explained if the bidders are risk loving (cf. Section 2.3).

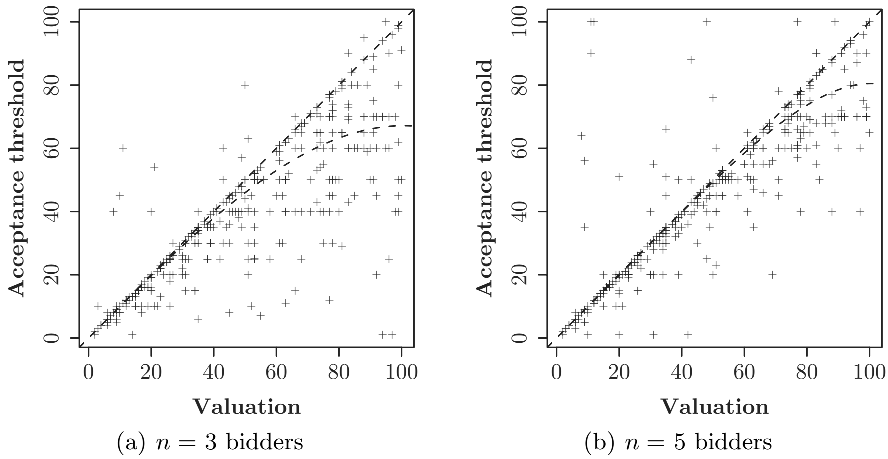


Figure 4.23. Acceptance thresholds in treatments B3 and B5

Table 4.14 once more illustrates the persistency of low threshold values: in both treatments more than half of the observed thresholds are below the risk neutral benchmark. It would be inappropriate, however, to conclude that the experimental participants are risk loving. It is more likely that the individual computation of the threshold exceeds the mental arithmetic skills of the subjects in the quick course of the experiment. Remember that the derivation of the theoretical benchmark involves rather complex computations. Since the observed threshold values are rather low, the participants probably overestimate the payoffs they might earn in the corresponding auction, which constitutes the alternative to accepting a PPO.

Table 4.14. Acceptance threshold: observed values and theoretical corridor

Treatment	**Classification of observed threshold**			$\sum$
	Below corridor	Within corridor	Above corridor	
B3	194 (54%)	147 (41%)	19 (5%)	360 (100%)
B5	209 (58%)	122 (34%)	29 (8%)	360 (100%)
$\sum$	403 (56%)	269 (37%)	48 (7%)	720 (100%)

Nonetheless, Figure 4.23 reveals several characteristics which correspond to the theory. The first one has already been mentioned: only a few threshold values are above the theoretical corridor; of all observations, less than 10% fall in this region (cf. Table 4.14).

Secondly, if one takes into account that the subjects might differ in their estimates regarding the uncertain outcome of the corresponding auction, as well as their preferences over these outcomes, the scatter plots reveal similarities to the overall shape of the theoretical corridor. The plots are rather dense close to the diagonal, and they become sparser towards the bottom right corner. Note that (*i*) the threshold values increase with the valuation and (*ii*) the range of the observations widens for large valuations. The correlation of the observed thresholds with the valuations is substantiated by Pearson's correlations coefficient. The coefficient computes to $r_{\mathrm{B3}} = 0.80$ for treatment B3 and $r_{\mathrm{B5}} = 0.85$ for treatment B5. To verify that the dispersion of the observed thresholds also increases, consider the lower third of the valuations (i. e. the valuations from 1 to 33) and the upper third of the valuations (i. e. the valuations from 67 to 100). In the lower third more than 60% (B3: 64%, B5: 67%) of all thresholds deviate from the respective valuations by at most one currency unit. Looking at the upper third, however, one finds that 59% (B3) and 50% (B5) of the thresholds deviate by at least ten currency units.

Thirdly, Figure 4.23 supports the theoretical claim that the threshold increases with the number of bidders (cf. Proposition 2.11 in Section 2.3). In fact, in treatment B3, the average threshold is 9.9 currency units lower than the average valuation. In treatment B5, the respective difference is on average only 3.7 currency units. The difference between the two treatments is statistically significant.[31]

One can conclude that the experimental observations yield characteristics that are roughly in accordance with the theoretical predictions. The most fundamental difference to the model in Chapter 2 is that the subjects in the experiment reveal a strong tendency towards rather low threshold values. The low thresholds can be explained by overly optimistic estimations regarding the alternative payoffs in the corresponding auction.

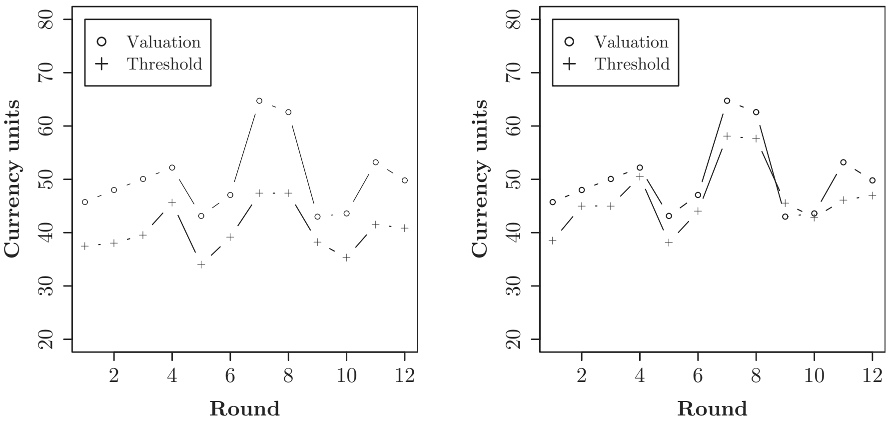
Remember that the seller can only profit from offering a PPO as compared to conducting a pure auction if the acceptance thresholds of the bidders lie above the risk neutral threshold, i. e. above the lower bound of the theoretical corridor. In Section 3.1, it was argued that it need not be the case that all bidders behave as if they were risk averse. Rather, it would be sufficient if a certain share of such bidders existed. The experimental results, however, suggest that this share comprises less than half of the bidders. Thus, substantial gains from offering a PPO cannot be expected.

Analogously to Section 4.4, this section is completed by a brief look at the development of the subjects' acceptance thresholds. The development is illustrated in Figures 4.24 and 4.25. Again, the average threshold and the average valuation are computed for each round. Figure 4.24 comprises the average threshold and the average valuation and Figure 4.25 depicts the difference between the two series. This difference indicates by how much the bidders shade their valuations on average. As a measure for a potential trend, the latter figure also shows Spearman's rank correlation coefficient r of the average shaded amounts versus the respective round numbers.

The figures reveal some variability in the amount by which the bidders' thresholds shade the respective valuations. It appears that the higher the valuation, the larger the gap between the average threshold and the average valuation, but a clear trend over time cannot be identified.[32] Note, however, that the figures nicely illustrate that the thresholds are higher in treatment B5 than in treatment B3. As a consequence, in B5 the gap between the threshold and the valuation is smaller than in treatment B3.

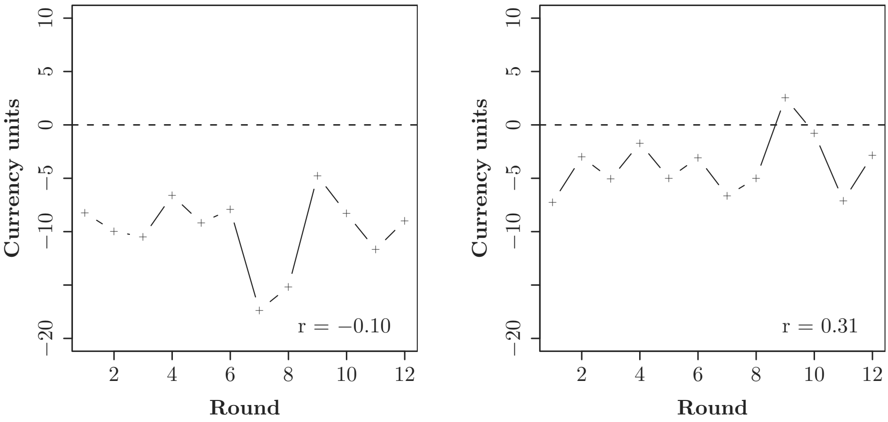
[31] WSR: $m = 30$, $V^+ = 380$, p-value $< 1\%$ (two-sided). In order to conduct the test, the average bid is computed for each subject. Then, the WSR is performed on matched pairs of bidders with the same sequence of valuations.

[32] Test for trend: $m = 12$, $D = 314$, p-value $= 76.3\%$ (two-sided).



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.24. Average threshold and valuation per round in the B-treatments



(a) $n = 3$ bidders (b) $n = 5$ bidders

Figure 4.25. Average gap between valuation and threshold in the B-treatments

4.5 Behavior of the Sellers

Having investigated the bidders' bids as well as their thresholds, the analysis now shifts towards the behavior of the sellers. The results in Section 4.4.2 show that compared to the theory, the bidders' thresholds are rather low. Thus, the potential gains for the sellers from offering a PPO are at most marginal.

For the analysis the following questions are of particular interest:

- Do the sellers offer a PPO?
- What is the amount of an offered PPO?
- Do the sellers choose PPOs which are accepted by the bidders?
- Is the PPO suitable for raising the sellers' revenues?

To start the analysis, Tables 4.15 and 4.16 provide an overview of the data. The tables display the PPOs set by the sellers in the treatments B/S3 and B/S5.[33] The PPOs which were accepted by the decisive bidder are highlighted by a bold font face, whilst a dash indicates that no PPO was offered.

Table 4.15. PPOs set by the sellers in treatment B/S3

	Seller											
Round	1	2	3	4	5	6	7	8	9	10	**mean**	#
1	–	–	50	70	**72**	73	**25**	–	–	**50**	56.7	6
2	88	50	**40**	**70**	–	**70**	45	–	100	**60**	65.4	8
3	–	50	**49**	–	72	66	52	–	100	82	67.3	7
4	–	50	55	–	66	**65**	–	40	100	80	65.1	7
5	–	**40**	55	90	**66**	69	–	–	90	75	69.3	7
6	–	**45**	**45**	80	67	67	**10**	**40**	80	60	54.9	9
7	50	**45**	**47**	80	**66**	60	–	–	70	65	60.4	8
8	**50**	49	**55**	–	67	**60**	**5**	–	65	–	50.1	7
9	–	**50**	60	–	67	64	–	–	65	–	61.2	5
10	50	50	60	85	67	**63**	–	–	65	82	65.3	8
11	–	50	50	80	67	64	55	–	65	–	61.6	7
12	–	**48**	59	80	67	72	55	–	**65**	66	64.0	8
mean	59.5	47.9	52.1	79.4	67.6	66.1	35.3	40.0	78.6	68.9	61.8	
#	4	11	12	8	11	12	7	2	11	9		87

In the treatment B/S3 the seller offered a PPO in 87 (73%) of the 120 auctions and the average offered PPO is 61.8. The decisive bidder accepted 26 (30%) of the offered PPOs and rejected 61 (70%). This means that the PPO was accepted in only 22% of all auctions. The average of the accepted PPOs is 50.0 and the mean of the rejected PPOs is 66.8.

In the treatment B/S5 a PPO was offered in 119 (83%) of the 144 auctions, but only 24 (20%) of these offers were accepted. Thus, in this treatment only 17% of all auctions ended with the acceptance of the PPO. As suggested by the theory, the PPOs appear to be higher than in the treatments with three

[33] The notation B/S3 and B/S5 as opposed to S3 and S5 has been chosen in order to emphasize that the sellers' experience with the strategies of the bidders might have influenced their decisions.

Table 4.16. PPOs set by the sellers in treatment B/S5

	Seller													
Round	1	2	3	4	5	6	7	8	9	10	11	12	**mean**	#
1	67	**5**	**40**	**56**	78	–	–	**60**	**68**	**70**	70	–	57.1	9
2	70	–	80	–	66	80	90	70	74	80	**60**	60	73.0	10
3	**65**	**25**	**55**	**45**	72	75	80	–	72	**85**	**65**	–	63.9	10
4	**67**	**50**	–	–	89	–	**80**	–	75	90	70	120	80.1	8
5	**70**	–	90	85	89	**70**	80	**70**	80	95	65	80	79.5	11
6	75	–	–	–	88	75	80	90	80	85	60	90	80.3	9
7	75	755	75	–	82	78	77	80	80	80	**65**	**75**	138.4	11
8	75	100	–	–	84	80	75	70	80	75	65	99	80.3	10
9	75	–	75	–	90	85	75	–	80	–	70	85	79.4	8
10	75	–	–	70	86	75	**70**	80	**80**	80	70	80	76.6	10
11	77	80	80	–	88	80	73	90	88	90	70	75	81.0	11
12	75	200	90	85	92	75	**70**	90	88	75	75	90	92.1	12
mean	72.2	173.6	73.1	68.2	83.7	77.3	77.3	77.8	78.8	82.3	67.1	85.4	82.7	
#	12	7	8	5	12	10	11	9	12	11	12	10		119

bidders: The overall mean equals 82.7 and the average of the rejected PPOs is 88.2. The accepted PPOs average 61.1.

Remember that a seller is able to raise her expected revenue if the PPO is set above the threshold of a risk neutral bidder with maximum valuation. In treatment B/S3 this boundary computes to 67.17 and in B/S5 the respective value is 80.50 (cf. Section 3.7). These values also relate to the upper bound of the expected revenues in a pure auction given any valuation of the decisive bidder. Thus, if a PPO above this boundary is accepted by the decisive bidder (e. g. due to risk aversion), the seller's revenue (which is then equal to the PPO) is higher than her expected revenue given the valuation of the decisive bidder. Note that a risk neutral bidder will never accept such a high PPO (cf. Section 2.2).

By plotting histograms of the chosen PPOs, Figure 4.26 gives a more descriptive summary of the data. The histograms are based on ten classes, each of which is ten currency units wide. In addition, for each treatment a vertical dashed line indicates the above-mentioned boundary. According to the theory, PPOs above this boundary increase the expected revenue if accepted by the decisive bidder. Whether lower PPOs raise or lower the expected revenue depends on the bidders' attitudes towards risk. By offering PPOs below the given boundary, the seller risks that the PPO will be accepted in cases in which a pure auction would yield higher (expected) revenues.

Comparing the data of Tables 4.15 and 4.16 with the revenue increasing boundary shows that in B/S3 only 25 (29%) of the 87 PPOs and in B/S5 only

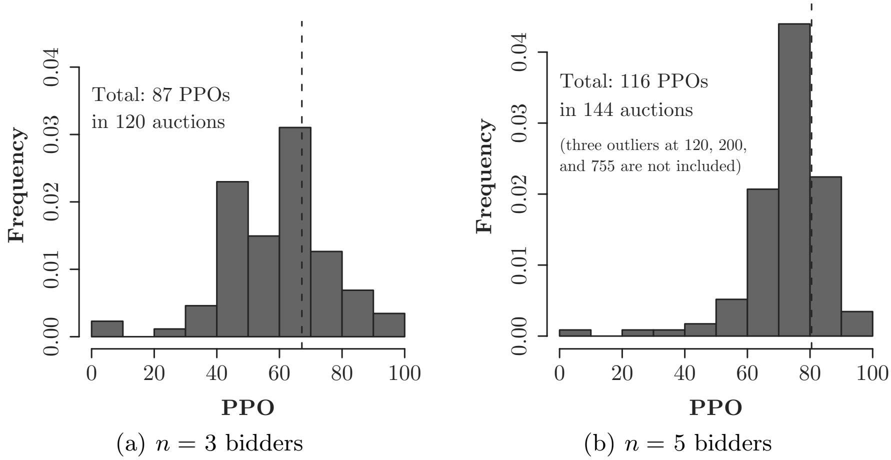


Figure 4.26. Posted price offers set by the sellers in treatments B/S3 and B/S5

33 (28%) of the 119 PPOs are at or above the respective boundary.[34] Taking the fact into account that the acceptance thresholds of the bidders are rather low, the PPOs offered by the sellers do not appear conducive to increasing the revenue.

To further pinpoint this issue, consider the PPOs which are accepted by the decisive bidders. Remember that if a bidder accepts the PPO, his valuation is rather high. In this case, the revenue of an alternative pure auction is expected to exceed the average revenue. Thus, in order to increase expected revenues, the average accepted PPO should be substantially higher than the revenues of an alternative pure auction. However, the data reveal that in treatment B/S3 the average accepted PPO equals 50.0, which is approximately the same as the average revenue in treatment A3 (49.0). In B/S5 the average accepted PPO (61.1) is even lower than the average revenues in A5 (67.1). Thus, the sellers do not employ the posted price offer sophisticatedly enough to take advantage of this mechanism. It appears as if the sellers—similarly to the bidders—underestimate the expected revenue of a pure auction. Alternatively, low PPOs can also be explained by risk averse sellers because such offers reduce

[34] Of the 33 PPOs above the boundary in B/S5, three PPOs are even higher than the maximum valuation. These PPOs will never be accepted by a rational bidder. In the following they are considered as outliers and are taken out of the remainder of the analysis.

the variability of the revenues.[35] Note, however, that an assumption of risk averse participants is not consistent with the observed behavior of the bidders.

Even though the sellers do not extract additional revenue from offering a PPO, Figure 4.26 suggests that the impact of the number of bidders is well understood by the sellers: the histograms show that the frequencies of relatively high PPOs are higher in B/S5 than in B/S3. Performing a Wilcoxon rank sum test verifies that the increase in the amount of the PPOs from three to five bidders is statistically significant at the 5% level.[36]

Analogously to the discussion of the bidding behavior, this section concludes with a look at the development of the sellers' PPOs over time. Two variables are differentiated. Figure 4.27 depicts the fraction of sellers who offer a PPO in each round and Figure 4.28 displays the average amount of the PPOs being offered.

For the treatments with three bidders, the figures show some variability in the fraction of sellers offering a PPO and the PPO itself but they do not reveal a clear trend.[37] A slight trend can, however, be observed in the treatments with five bidders. Both the fraction and the amount of the PPOs increase during the course of the experiment. The latter increase is also statistically significant at the 5% level but the former is not.[38]

If the hypothesis that the sellers offer low PPOs because on average they underestimate the expected revenue in a pure auction, one would expect them to adjust their offers once they have learned better. The increasing PPOs in treatment B/S5 can be interpreted in this sense. Since, however, a significant trend is not observed in B/S3, the information feedback mechanism of an APPO deserves a closer look.

Consider the individual PPOs set by the seller and focus on those PPOs which the seller changes in the following round.[39] First of all, in treatment B/S5 there are 16 accepted PPOs which are followed by a higher PPO

[35] Risk averse sellers are considered in the (theoretical) model by Mathews (2003). See Section 2.6 for a discussion of this model.

[36] Wilcoxon rank sum / Mann-Whitney-U test: $m_{\mathrm{B/S3}} = 10$, $m_{\mathrm{B/S5}} = 12$, $U = 25$, p-value $= 2.0\%$ (two-sided). The three outliers at 120, 200, and 755 in B/S5 were taken out of the data; with respect to the result of the test, this correction keeps the test conservative. Note that there are no matched pairs within the group of sellers.

[37] The observations rest on a test for trend. For the share of sellers offering a PPO: $m = 12$, $D = 232$, p-value $= 55.0\%$ (two-sided); and for the average amount of the PPOs: $m = 12$, $D = 338$, p-value $= 57.3\%$ (two-sided).

[38] Test for trend regarding the fraction of sellers: $m = 12$, $D = 155$, p-value $= 13.4\%$ (two-sided); regarding the amount of the PPOs: $m = 12$, $D = 58$, p-value $< 1\%$ (two-sided).

[39] PPOs followed by either the same or no PPO are not considered.

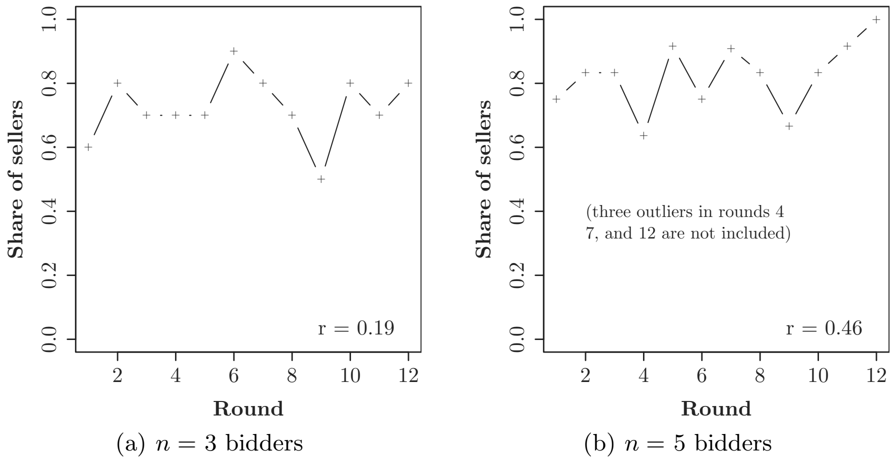


Figure 4.27. Share of sellers offering posted price offers in treatments B/S3 and B/S5

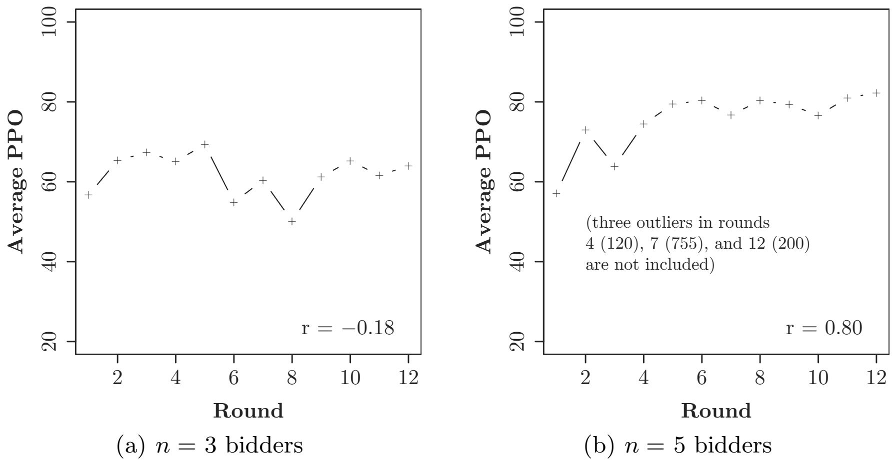


Figure 4.28. Posted price offers set by the sellers in treatments B/S3 and B/S5

in the subsequent round. The opposite, i. e. an accepted PPO followed by a lower offer, is not observed (cf. Table 4.16 on page 126). Clearly, if a PPO is accepted, there is no reason for the seller to lower this offer in the next round. Rather, the accepted PPO may encourage the seller to ask for a higher price in the next round.

Note, however, that the seller actually learns the revenue of an auction only if the PPO is rejected. In B/S5 there are 25 rejected PPOs followed by a higher PPO in the subsequent round and the seller decreases her offer 31 times. Of these 31 reductions of the PPO occur 30 after the seller experienced auction revenues which were smaller than the respective PPO. Apparently, this is bad news for the seller and causes her to offer a lower PPO in the next round.

In some cases, however, the feedback mechanism may also be misleading. Possibly, even the lack of a significant upward trend in treatment B3 can be explained by the misinterpretation of the information feedback in an APPO. Consider, for example, seller 9 in Table 4.15 (p. 125). This seller starts in round 2 by offering a PPO at 100 and offers the same PPO in rounds 3 and 4. From round 5 to round 8, the seller continuously lowers her offer and finally sets it at 65 from round 8 on to the end of the session. The decisive bidders reject all but the final PPO in round 12. This may well be perceived as negative feedback. In addition, from rounds 2 to 6, the seller's auction revenues are lower than her offered PPO.[40]

Similarly, the mechanism may slow down the adjustment of low PPOs towards higher PPOs. This is illustrated by seller 3 in treatment B/S3, who offers rather low PPOs. If no or a rather high PPO had been offered, the revenues of the corresponding auction in rounds 1 to 6 would be 8, 48, 71, 30, 22, and 62. In the respective rounds, the seller sets her PPOs at 50, 40, 49, 55, 55, and 45. In rounds 2, 3, and 6 the PPOs are accepted (cf. Table 4.15, p. 125). Thus, the seller experiences auction revenues only when they are low (8, 30, and 22 in rounds 1, 4, and 5) and receives no feedback with respect to higher auction revenues (namely 48, 71, and 62) in the rounds in which her PPO is accepted.

The above argument shows that the information feedback of a mechanism plays an important role with respect to the evolution of strategies. In this manner, the analysis ties in to work by Harstad (2000).

4.6 Summary

To conclude this chapter, the main findings of the experiment are summarized. With respect to the outcomes of the auctions, i. e. the social surplus, the revenue, and the bidders' payoffs, the following observations are made:

[40] The auction revenues in these rounds are 65, 50, 35, 24, and 34. Compare with the PPOs at 100, 100, 100, 90, and 80, respectively.

1. An APPO yields a lower social surplus than the corresponding English auction with proxy bidding, i. e. a pure auction is more efficient than an APPO. This result holds regardless of the number of bidders.
2. In the treatments with three bidders, the corresponding pure auction generates higher revenues for the seller than an APPO.
3. In accordance with Observation 2, in the treatments with three bidders the payoffs of the bidders are higher in an APPO than in the corresponding pure auction.
4. The variability of the outcomes is higher in the treatments with a PPO.

Further, the experimental results show that only the first of the above results can be attributed to the proposal and the acceptance of a PPO. Another important factor which accounts for the observed differences is a change in the bidding behavior of the participants. This is supported by the individual bidding data.

5. In an APPO, the bidders deviate more strongly and more often from the dominant strategy bids. This explains not only Observation 4 above but also adds to the likelihood of inefficient outcomes in an APPO (Observation 1).
6. Moreover, in the auctions with three bidders, the participants submit more defensive bids in an APPO than in a pure auction. This is the reason for the lower revenue (Observation 2) and the higher bidder payoffs (Observation 3) in an APPO.
7. During the course of the experiment, the APPO bids become more aggressive and the observed underbidding (Observation 6) diminishes over time.

With respect to the bidders' acceptance thresholds, the study finds:

8. Overall, the thresholds of the bidders tend to be lower than predicted by theory.

Possibly, the low thresholds of the bidders can be explained if bidders tend to underestimate the expected final price of an auction.

Finally, the behavior of the sellers is investigated.

9. In the APPOs, sellers quite frequently quote a PPO, i. e. in more than two-thirds of all APPOs.
10. Similar to the low bidders' thresholds (Obseration 8), the sellers are observed to quote rather low PPOs compared to the theoretical predictions.

11. In the APPOs with five bidders, an increasing trend in the amount of the sellers' PPOs can be observed.

The fact that the sellers choose to propose PPOs which lower their average revenue is noteworthy. One possible reason for the low PPOs could be that the subjects in the role of sellers are risk averse. Risk aversion, however, does not explain the observed behavior of the bidders. The low PPOs of the sellers can more plausibly (and in line with the defensive thresholds of the bidders) be explained by rather low estimates on the part of the subjects with respect to the revenue in a pure auction. If this presumption holds, it could also serve as an explanation for the increase in the sellers' PPOs over time. In this case, the observed increase could be a reaction of the sellers to the experience they gain during the course of the experiment.

5

Conclusion and Outlook

Recently, internet marketplaces like eBay and Yahoo! have extended the flexibility of the selling mechanisms available on their platforms. The product features "*Buy It Now*" or "*Buy Price*", for example, allow the seller of an item to offer an additional posted price when conducting an auction. Bidders can then decide whether to bid in the auction or to acquire the item for the fixed price offer.

If a bidder submits a bid in the auction, the posted price offer expires on eBay. Thus, the final price in the auction—which is conducted as an English auction with proxy bidding—may be below or above the posted price. Moreover, a bidder runs the risk of not winning the auction at all. By accepting the fixed price offer, the bidder can eliminate these risks and acquire the item with certainty.

The present study introduces a model of an auction with a posted price offer (APPO) based upon the auction with a *Buy It Now* option on eBay. In the APPO model, the posted price offer (PPO) is extended to exactly one of the n bidders in the auction. The underlying rationale is that on eBay the PPO expires if it is not accepted by a bidder who would rather submit a bid in the auction. The PPO is then no longer available to any other bidder. If the bidder rejects the PPO in the APPO model, a second-price auction is conducted.

5.1 Summary of Main Results

In order to analyze the APPO institution, the concept of a bidder's acceptance threshold, which yields the maximum PPO the bidder is willing to accept, is introduced. The APPO model is then investigated both theoretically and by a lab experiment. Both approaches are based on the independent private values assumptions.

In the theoretical part of this study, an APPO is considered as a Bayesian game and solved by equilibrium analysis. If the PPO is rejected, an equilibrium in dominant strategies exists for the subsequent subgame. The threshold of a bidder with respect to the acceptance of the PPO can be derived by backward induction. It is shown that the threshold increases with the number of bidders, their degree of risk aversion, and the auction's reserve price. Moreover, if a bidder is risk neutral, his threshold rises globally with his valuation.

Applying the bidders' acceptance thresholds, the expected revenue of an APPO is calculated given a PPO set by the seller. Once more using backward induction, the seller's revenue maximizing PPO is determined. The analysis results in two main findings:

1. If the bidders are risk neutral or risk loving, the seller cannot gain from offering an additional PPO when conducting an auction.
2. If the bidders are risk averse, the seller can set a PPO such that the expected revenues exceed those of a pure auction.

Note that the second result also holds if a non-zero reserve price is taken into consideration: if bidders are risk averse, then for any given reserve price $\underline{p}$ a PPO $\bar{p}$ exists such that the respective APPO outperforms the corresponding second-price auction in terms of the expected revenue.

Finally, optimal APPOs are considered, i. e. APPOs in which the seller simultaneously maximizes over the reserve price and the PPO in order to maximize her expected revenue. On the basis of an example, it is also argued that the optimal reserve price in an APPO is lower than the revenue maximizing reserve price in the respective pure second-price auction.

The second part of the study investigates actual bidder and seller behavior in a lab experiment. In the experiment, both the bidders and the sellers are investigated in APPOs with three and five bidders. Moreover, pure auctions without a PPO were conducted as a benchmark.

To a large extent, the lab experiment confirms the predictions of the theoretical model. The experiment, however, also reveals characteristics that cannot be explained by the model.

1. Both the bidders and the sellers appear to underestimate the expected revenue in an auction. As a consequence, the thresholds of the bidders as well as the PPOs set by the sellers are on average lower than predicted by the model.
2. The variability of the bids with respect to the dominant strategy increases if a PPO is available. The increase in dispersion is explained by the addi-

tional complexity that comes along with the existence of the PPO, which makes the auction institution less transparent.

3. Moreover, in the treatments with three bidders, the bids are significantly lower in an APPO than in a benchmark treatment without a PPO.

The low PPOs set by the sellers could also be explained by risk aversion. However, the behavior of subjects in the role of bidders is not coherent with risk averse decision making. Thus, the conjecture that the participants in the experiment underestimate the expected auction price regardless of their roles accounts for the observed results more consistently.

Based on the above findings, the following conclusions stand to reason. First, the low thresholds and the low PPOs observed in the experiment call into question the effectiveness of the PPO as a suitable means for increasing the seller's expected revenue. In fact, the experimental data do not show an increase in revenue due to the bidders' acceptance of PPOs. Rather, the increase in the variability of the bids negatively affects the efficiency of the institution. Because bidders more often deviate from bidding their valuation in comparison to a pure auction, the item is less often awarded to the bidder with the highest valuation. This clearly lowers the total surplus which the bidders and the sellers share. Finally, there is no obvious explanation for the last of the above observations. However, the low bids in an APPO with three bidders have an important consequence: the revenues in an APPO are significantly lower than those in a pure auction. On the other hand, due to defensive bids, the bidders' payoffs are higher in an APPO. The effect is not observed in the treatments with five bidders.

5.2 Limitations of the Study

Both the theoretical and the experimental analyses of an APPO concern a simplified model of auctions with a *Buy It Now* option or a *Buy Price* as available on eBay or Yahoo!. To a certain degree, the generality of the results is limited by the assorted assumptions of the model.

An apparent difference between the APPO model and an auction on the internet is that in a real auction the number of participants is not given or known to all participants. Rather, an auction is listed for a certain time period and—loosely speaking—discovered by an arbitrary number of bidders during the time of its listing.

In addition, the analysis is based on the assumptions of the symmetric independent private values model. In both the theoretical model and the experiment, the distribution function of a bidder's valuation is known to all participants. Moreover, this distribution is the same for all bidders and all valuations are independent. One would presume that this does not hold in real settings: Bidders may not be symmetric and, generally, the distribution functions of their valuations are not known to all bidders. Even more importantly, the quality of an item traded at an online auction is not perfectly observable—nor is the trustworthiness of the seller. Thus, there is a clear unknown common value component.

One of the reasons for offering a PPO might also be that either party prefers an early transaction to waiting for an auction to close. A bidder might be willing to pay a premium and a seller could be willing to offer a discount in relation to the expected final price of an auction if the transaction is initiated early. A similar argument also holds for implicit transaction costs. Particularly for low valued items, a bidder might prefer an immediate transaction as opposed to running the risk of being outbid and possibly having to search for an alternative auction of a similar item.

The last point also relates to another issue. An APPO is considered as a one-shot game. Within the model, a seller can try to sell a particular item in one auction only. Similarly, the bidders can acquire the item in only this auction. A bidder who is not awarded the item receives a zero payoff. Both of these assumptions may not hold in reality: A seller can easily relist an unsold item—this is even facilitated by actual platforms. In addition, several auctions often offer similar items. This substantially increases the strategy space of the bidders.

5.3 Outlook

This study has investigated a market institution which combines an auction with a posted price offer. By proposing a game theoretical model which is more general than those in previous literature and conducting conducting the first known experimental analysis of an auction with a posted price offer, it has laid the basis for further studies which aim to extend the scope of the presented results. Both the model and the employed methodology—game theoretical modeling combined with experimental economics—bear the potential of being deepened and broadened by the market engineer.

Firstly, the model can be extended. In the context of internet auctions, a natural extension of the APPO model is to not fix the number of bidders in advance but to consider the arrivals of bidders as a realization of a stochastic process. Such an extension would not only address the issue of a random number of bidders; explicitly taking the time process of bidding into account would also allow the modeling of both bidders' and sellers' time preferences.

The resulting model is likely to capture still unknown yet important aspects of the actual bidding process. Moreover, it would be interesting to study the effect of bids as value signals in a setting with common or affiliated values. Note that an APPO promotes early bids because by submitting a bid, bidders can disable the PPO. In a common value setting, this is in the interest of the seller.

Secondly, the theoretical and experimental approaches to analyzing market institutions can be supplemented by empirical methods. Both the theoretical model and the experimental investigation have applied a simplified model of reality. In order to increase the external validity of the results, a further direction for future research is to focus on real auctions rather than simplified models. The idea is to analyze field data from auctions with a posted price offer which are actually run at internet marketplaces. Because internet auctions can be monitored not only over their duration but are also available for scrutiny after the closing of the auction, a considerable amount of data is available for evaluation.

Thirdly, the domain of the application can be refined. Consider, for example, a procurement setting. In such a situation, a (corporate) buyer seeks to acquire an item from one of several potential suppliers. Clearly, more complex valuation models may be necessary. The item's value may be uncertain due to, for example, the unknown cost of development and (future) production. It may also have negotiable attributes such as certain technical features or service levels which cannot or should not be specified in advance. In such scenarios, multi-attribute auction protocols may be applied. Moreover, a buyer may want to purchase several different goods or services which may be obtained either from a single supplier or from several different vendors. The delivery of a large number of computers in combination with a service contract could be an example. Since it is not known in advance whether the buyer should package the hardware with the service and tender the bundle or whether he should purchase the goods individually, combinatorial auctions should be considered. Both multi-attribute and combinatorial auctions can be augmented by posted offers.

Finally, the market operator and its business model may be the focus of investigation. The analysis would then shift towards an operator's business model, its tariff structure, and the strategic strength of a particular model in a competitive environment. How well, for example, does a model perform if customers—buyers and sellers—can choose between alternative marketplaces? Again, the same set of methodological approaches is available to the market engineer: Competition of homogeneous or heterogeneous markets can be studied analytically as well as by laboratory experiments. As the complexity of the analytical models grows, computer-based simulations may supplement the tool-box.

A

Mathematical Fundamentals

A.1 Basic Concepts of Game Theory

Let $N = \{1, 2, \ldots, n\}$ be a set of players and denote for all $i \in N$ the set of strategies of player i by Σ_i and the set of strategy profiles by $\Sigma = \Sigma_1 \times \Sigma_2 \times \ldots \times \Sigma_n$. The payoff (utility) of a player i is captured by a function $u_i \colon \Sigma \to \mathbb{R}$ and the payoffs of all players, given a strategy profile $\sigma \in \Sigma$, are in short written as $u(\sigma)$ with $u \colon \Sigma \to \mathbb{R}^n$ and $u(\sigma) = (u_1(\sigma), u_2(\sigma), \ldots, u_n(\sigma))$.

Using the above notation, a game can be described by a triple (N, Σ, u). For convenience we also define $\Sigma_{-i} := \Sigma_1 \times \Sigma_2 \times \ldots \times \Sigma_{i-1} \times \Sigma_{i+1} \times \ldots \Sigma_n$ as the set of strategy combinations of all players but player i.

Definition A.1 (Dominant strategy). *Let $\Gamma = (N, \Sigma, u)$ be a game and $i \in N$ a player. A strategy $\sigma_i^* \in \Sigma_i$ is called a* dominant strategy *of player i if and only if*

$$\forall\, \sigma_i \in \Sigma_i,\ \forall\, \sigma_{-i} \in \Sigma_{-i} : u_i(\sigma_i^*, \sigma_{-i}) \geq u_i(\sigma_i, \sigma_{-i}) \quad \textit{and} \tag{A.1}$$

$$\forall\, \sigma_i \neq \sigma_i^* :\ \exists\, \sigma_{-i} \in \Sigma_{-i} :\ u_i(\sigma_i^*, \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i}) \ . \tag{A.2}$$

An equivalent definition can be found e. g. in Mas-Colell et al. (1995, p. 238).[1]

A weaker form (that could equivalently be used in this dissertation) is common in the mechanism design literature (see, e. g., Jackson, 2003, whose notation, however, is quite different due to the focus of his paper). Here σ_i^* is called a dominant strategy even if it does not weakly dominate a single

[1] Oddly, in Excercise 8.B.2 Mas-Colell et al. (1995, p. 262) consider a game in which “a player has two weakly dominant strategies.” This, of course, is not possible with the given definition of a dominant strategy.

$\sigma_i \in \Sigma_i$. This definition is consistent with Holler und Illing (1996, p. 53) or Osborne und Rubinstein (1998, p. 181).

Yet another definition is given by Berninghaus et al. (2002, p. 20) or Pfähler und Wiese (1998, p. 46). There, the additional constraint reads $\exists\, \sigma_{-i} \in \Sigma_{-i} : \forall\, \sigma_i \neq \sigma_i^* : u_i(\sigma_i^*, \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i})$. This, however, is not consistent with the common notion that bidding one's true valuation is a dominant strategy in the Vickrey auction.

Fortunately, there is no disunity in the literature regarding the definition of a Nash equilibrium.

Definition A.2 (Nash equilibrium). *Let $\Gamma = (N, \Sigma, u)$ be a game. A strategy profile $\sigma^* \in \Sigma$ is called a* Nash equilibrium *or equilibrium for short if and only if*

$$\forall\, i \in N,\ \forall\, \sigma_i \in \Sigma_i : \quad u_i(\sigma^*) \geq u_i(\sigma_i, \sigma_{-i}^*) \ .$$

Using the notion of a dominant strategy, the following definition refines the concept of a Nash equilibrium.

Definition A.3 (Equilibrium in dominant strategies). *Let $\Gamma = (N, \Sigma, u)$ be a game. A strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*) \in \Sigma$ is called an* equilibrium in dominant strategies *if and only if*

$$\forall\, i \in N : \sigma_i^* \textit{ is a dominant strategy of player } i \ .$$

Clearly, Definition A.3 is much stronger than Definition A.2. There are only a few games which have an equilibrium in dominant strategies.

A.2 Risk Measurement by Arrow and Pratt

The concept most commonly applied to measure the degree of an agent's risk aversion was independently suggested by Kenneth J. Arrow and John W. Pratt (cf. Pratt, 1964, and the references therein). The following presentation follows Kruschwitz (1995) and Wolfstetter (1999).

Definition A.4 (Markowitz risk premium). *Let X be a real-valued random variable and w_0 the initial wealth of an agent whose preferences over lotteries are represented by a utility function $u\colon \mathbb{R} \to \mathbb{R}$. The term*

$$\pi(X, w_0) = w_0 + E[X] - u^{-1}\left(E[u(w_0 + X)]\right)$$

is called risk premium.

Definition A.5 (Absolute and relative risk aversion). *Let* $u\colon \mathbb{R} \to \mathbb{R}$ *be a continuous, strictly increasing, and twice differentiable utility function for money. The term*

$$ARA(x) = -\frac{u''(x)}{u'(x)}$$

is called absolute risk aversion *and the term*

$$RRA(x) = ARA(x)\, x$$

is called relative risk aversion*.*

Based on Definition A.5, one can compare two utility functions with re-spect to the degree of risk aversion that they exhibit:

Definition A.6 (Higher degree of risk aversion). *Let* $u_i, u_j : \mathbb{R} \to \mathbb{R}$ *be two continuous, strictly increasing and twice differentiable utility functions for money. The function* u_i *is said to exhibit higher risk aversion than* u_j *in the sense of Arrow and Pratt, if and only if*

$$-\frac{u_i''(x)}{u_i'(x)} \geq -\frac{u_j''(x)}{u_j'(x)} \;\; \forall\, x \in \mathbb{R} \;\; .$$

The following Theorem A.7 originates from Pratt (1964).

Theorem A.7 (Pratt). *Let* $u_i, u_j : \mathbb{R} \to \mathbb{R}$ *be two continuous, strictly increasing and twice differentiable utility functions for money and* X *a real-valued random variable. Then the following conditions are equivalent*

(*i*) $-\frac{u_i''(x)}{u_i'(x)} \geq -\frac{u_j''(x)}{u_j'(x)} \;\; \forall\, x \in \mathbb{R}$

(*ii*) $\exists (z\colon \mathbb{R} \to \mathbb{R}) \;:\; z'(x) \geq 0, z''(x) \leq 0, u_i(x) = z(u_j(x)) \;\; \forall\, x \in \mathbb{R}$

(*iii*) *for any random variable* $X, \forall w_0 : \pi_i(X, w_0) \geq \pi_j(X, w_0)$.

Part (*iii*) of Theorem A.7 intuitively illustrates Definition A.6: an agent i is considered more risk averse than an agent j if at any given (but identi-cal) respective level of initial wealth, the risk premium agent i demands for accepting a lottery X (or the certainty equivalent i is willing to pay in order to avoid the lottery) is higher than that of agent j.

A.3 Order Statistics

Let X be a (continuous) random variable with a cdf $F : \mathbb{R} \rightarrow [0;1]$ and a pdf $f : \mathbb{R} \rightarrow \mathbb{R}_+$ and let $x_1, x_2, \ldots, x_n$ be n independent drawings of that variable.[2] Now consider an ordered list of the realizations and denote the largest realization by $x_{(1)}$, the second largest by $x_{(2)}$, etc. so that

$$x_{(1)} \geq x_{(2)} \geq \ldots \geq x_{(n)} \ .$$

Of course, $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ are again realizations of implicitly defined random variables

$$X_{(1)}, X_{(2)}, \ldots, X_{(n)} \ .$$

The variable X, the cdf F, and the pdf f are respectively called parent (random) variable, parent distribution, and parent density, and $X_{(1)}, X_{(2)}$, and $X_{(i)}$ are referred to as *first-order*, *second-order*, and *ith-order* statistics.[3] In this section, general formulae for the distribution and the density of the order statistics will be derived.

The probability that $x_{(1)}$ is not larger than a given x is equal to the probability that no individual x_i is larger than x. One obtains

$$F_{(1),n}(x) = F^n(x) \tag{A.3}$$

$$f_{(1),n}(x) = n\, F^{n-1}(x)\, f(x) \ . \tag{A.4}$$

Analogously, the probability that $x_{(i)}$ is not larger than x is equal to the probability that not more than $(i-1)$ of the individual realizations of X are larger than x. The probability that exactly j realizations are larger than x calculates to

$$\Pr(\exists_{j!}\, y \in \{x_1, x_2, \ldots, x_n\} \ : y > x) = \binom{n}{j} F^{n-j}(x)\,(1 - F(x))^j \ .$$

---

[2] The following formulae can easily be extended to the case with discrete random variables.

[3] In the statistics literature, the first-order statistic typically refers to the lowest realization and the nth-order statistic to the largest realization (cf. e. g. Arnold et al., 1992; David, 1981). Economists tend to reverse the ranking when analyzing auctions (e. g. McAfee und McMillan, 1987). This paper adopts the reversed notation particularly since it provides a simple notation for the statistic of the largest valuation in an auction if the number n of bidders is not known in advance. Note that the order statistics are well defined even if $n = 0$ is possible with positive probability. The respective distribution function $F_{(1)}(v)$ is then to be read in the sense that $(1 - F_{(1)}(v))$ denotes the probability that there is no valuation that exceeds v.

Thus, the distribution of the nth-order statistic yields

$$F_{(i),n} = \sum_{j=0}^{i-1} \binom{n}{j} F^{n-j}(x)\,(1-F(x))^j \quad . \tag{A.5}$$

One possibility to obtain the density of $X_{(i)}$ is to differentiate Equation (A.5) with respect to x. An alternative form of the pdf is given by Equation (A.6):

$$f_{(i),n}(x) = \frac{n!}{(n-i)!\,(i-1)!} f(x)\; F^{n-i}(x)\; (1-F(x))^{i-1} \quad . \tag{A.6}$$

In fact, this alternative form is much simpler because it entails a closed form rather than the sum of i elements. To prove that Equation (A.6) does in fact yield the density, integrate Equation (A.6) by parts $(i-1)$ times.

A.4 Acceptance Threshold of Bidders in Example 2.3

This section is an addendum to Example 2.3 (page 27) of Section 2.3. In the example, the rather mechanical computation of the acceptance threshold is left out in the main text. It is given in the following.

Consider an APPO with n bidders $i = 1, 2, \ldots, n$ whose valuations v_i are independently and uniformly distributed over the interval $[0;1]$. Let the reserve price be $\underline{p} = 0$. In this section the threshold function $t_{\hat{\imath}}(v_{\hat{\imath}})$ of bidder $\hat{\imath}$ with a utility function $u_{\hat{\imath}}(x) = x^\alpha$ $(\alpha > 0)$ is derived. According to Lemma 2.6 the threshold function is

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(\int_{\underline{p}}^{v_{\hat{\imath}}} u'_{\hat{\imath}}(v_{\hat{\imath}} - x)\, G_{(1),-\hat{\imath}}(x)\, dx\right) \quad .$$

Because valuations are uniformly distributed over $[0;1]$, $G_{(1),-\hat{\imath}}(x) = x^{n-1}$ holds and one obtains

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(\int_0^{v_{\hat{\imath}}} \alpha (v_{\hat{\imath}} - x)^{\alpha-1}\, x^{n-1}\, dx\right) \quad .$$

Integration by parts yields

$$t_{\hat{\imath}}(v_{\hat{\imath}}) = v_{\hat{\imath}} - u_{\hat{\imath}}^{-1}\left(-\left[(v_{\hat{\imath}} - x)^\alpha\, x^{n-1}\right]_0^{v_{\hat{\imath}}} + \int_0^{v_{\hat{\imath}}} (v-x)^\alpha\, (n-1)\, x^{n-2}\, dx\right)$$

and by repeating integration by parts another $(n-2)$ times one obtains

$$
\begin{aligned}
&= v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left(\alpha \int_0^{v_{\hat{\imath}}} \left(\prod_{i=0}^{n-2} \frac{1}{\alpha + i} \right) \, (v_{\hat{\imath}} - x)^{\alpha+n-2} \, (n-1)! \, dx \right) \\
&= v_{\hat{\imath}} - u_{\hat{\imath}}^{-1} \left((n-1)! \, \left[- \left(\prod_{i=1}^{n-1} \frac{1}{\alpha + i} \right) \, (v - x)^{\alpha+n-1} \right]_0^{v_{\hat{\imath}}} \right) \\
&= v_{\hat{\imath}} - \left(\frac{(n-1)!}{\prod_{i=1}^{n-1} (\alpha + i)} \right)^{\frac{1}{\alpha}} v^{1+\frac{n-1}{\alpha}} \quad . \qquad \text{(A.7)}
\end{aligned}
$$

B

Parameters of the Experiment

In this chapter of the Appendix, the parameters of the experimental setup are laid out in detail. A thorough description of the experimental design is given in Chapter 3.

For each of the treatments A3, A5, B3, and B5, two sessions were conducted and in each session 15 bidders participated in a sequence of twelve auctions.[1] Thus, in each of the treatments investigating bidder behavior, a total of 30 bidders participated.

In treatments S3 and S5, the behavior of sellers in an APPO is investigated. There was one session of treatment S3 with ten sellers and one session of treatment S5 with twelve sellers. In fact, the data of each auction in treatment B3 were mapped to exactly one seller in treatment S3 and each auction's data in treatment B5 were mapped to two different sellers in treatment S5, respectively (see Section 3.4 for details).

Note that the identification numbers of the subjects in the following sections refer to their internal names. In the experiment, the identification numbers were randomly assigned to the participants' seat labels (letters from 'A' to 'O'). Participants were only informed about their own identification number and not about the numbers of the other participants in the lab.

B.1 Assignment of Participants to Groups

In treatments A3 and B3, each auction was conducted with three bidders whilst in treatments A5 and B5 five bidders participated in each auction. Since a session was conducted with 15 bidders, in every round of each session

[1] The limitation to 15 participants in each session was due to the capacity constraints of the experimental lab.

of the treatments A3 and B3 there were five groups of bidders and in each session of the treatments A5 and B5 there were three groups of bidders.

The experiment was conducted as a *stranger experiment*, i. e. the composition of the groups varied from round to round. The schedules according to which bidders were assigned to groups are given in Tables B.1 and B.2. In these tables the groups are numbered from 1 to 5 and 1 to 3, respectively. In round 2 of the treatments A3 and B3, for example, bidders 1, 4, and 7 are all assigned to group 1. Thus, these bidders participated in the same auction.

Table B.1. Assignment of bidders to groups in treatments A3 and B3

Bidder	**Round**											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	**1**	1	**1**	1	**1**	**1**	1	1
2	**1**	2	2	2	**2**	**1**	**5**	3	4	**5**	4	3
3	1	3	3	3	**4**	2	5	4	**5**	**4**	3	2
4	2	1	**2**	3	3	1	**3**	**4**	**2**	**2**	1	5
5	**2**	2	**1**	5	4	4	**2**	5	1	2	2	**2**
6	2	3	5	1	2	**2**	1	**5**	5	5	5	**5**
7	3	**1**	**3**	4	2	3	2	**1**	5	**3**	4	4
8	**3**	5	1	2	**3**	5	1	2	**3**	4	**4**	**1**
9	3	4	4	**1**	**5**	**4**	3	**2**	**4**	4	2	5
10	4	**2**	**5**	4	1	2	2	2	2	1	5	**3**
11	**4**	4	2	5	3	4	**4**	1	4	2	**5**	**4**
12	4	5	**4**	**3**	5	**5**	5	5	1	3	3	3
13	5	**3**	4	**4**	4	5	4	**3**	3	1	**3**	4
14	**5**	**5**	5	**5**	5	**3**	4	4	3	5	**1**	2
15	5	**4**	3	**2**	1	3	3	3	2	3	**2**	1

Before the experiment began, the decisive bidder of every auction in the treatments B3 and B5 was determined by a random method. In the Tables B.1 and B.2, the decisive bidder is highlighted by a bold font face.

Note that the group numbers were only used internally in the system. They were not communicated to the participants. Moreover, the participants were neither informed about the schedule assigning bidders to groups nor did they know with whom they were bidding in an auction.

In the sessions of the treatments B3 and B5, the strategies applied by the bidders, i. e. the chosen acceptance thresholds and the chosen bidding limits, were recorded and saved in the system. In treatments S3 and S5, participants in the role of sellers were confronted with the recorded strategies of the bidders in treatments B3 and B5, respectively. Thus, in every round a seller had to

Table B.2. Assignment of bidders to groups in treatments A5 and B5

Bidder	**Round**											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	**1**	1	1	1	1	**1**	1	**1**	1	1
2	3	2	3	**3**	2	**2**	**2**	2	2	3	3	3
3	1	1	2	2	3	3	1	1	2	**2**	**3**	3
4	**2**	**2**	1	2	1	**1**	3	3	1	2	1	1
5	**1**	2	1	3	2	1	1	2	**2**	2	2	2
6	2	1	2	**2**	**1**	1	2	3	**3**	1	3	2
7	2	2	1	3	3	3	3	1	**1**	1	**2**	**1**
8	3	2	2	3	2	1	3	3	1	2	**1**	**3**
9	2	3	3	2	**3**	3	**3**	1	3	1	2	1
10	1	3	2	**1**	3	2	1	**3**	1	3	2	2
11	3	3	**2**	1	2	**3**	2	2	2	3	2	1
12	1	1	1	1	**2**	2	2	**2**	3	**3**	3	3
13	2	**3**	**3**	3	3	2	3	1	3	2	1	3
14	3	**1**	3	2	1	2	2	3	3	3	3	**2**
15	**3**	3	3	1	1	3	**1**	2	2	1	1	2

decide whether to offer a PPO, and, if so, she had to specify its amount. The seller's payoff in an APPO in treatments S3 and S5 was then determined based on her own decisions and the recorded strategies of the bidders in treatments B3 and B5.

As with the bidders, the sellers participated in a sequence of twelve auc– tions. Moreover, an auction that was played, e. g., in round 3 with the bidders was also assigned to a seller in round 3. Following the *stranger* design, however, the assignment of sellers to bidder groups varied from round to round.

In a session of treatment B3, there were five auctions in every round. Because two sessions were conducted, a total of ten auction data sets was available per round. It was thus convenient to investigate treatment S3 in a session with ten sellers and to map in every round the ten different auction data sets to different sellers.

Similarly to Table B.1, Table B.3 shows the assignment of the sellers to the bidder groups. The group numbers 1 through 5 in Table B.3 refer to the bidder groups 1 through 5 of session 1 of treatment B3 and the group numbers 5 through 10 refer to the groups 1 through 5 of session 2 of treatment B3.

Because in treatment B5 five bidders participated in an auction, only three auctions were conducted per session and round in this treatment. Because two sessions were conducted, six different auction data sets were recorded for every round. In order to increase the number of observations, treatment S5 was

Table B.3. Assignment of sellers to bidder groups in treatment S3

Seller	Round											
	1	2	3	4	5	6	7	8	9	10	11	12
1	6	6	10	6	4	10	9	8	2	1	4	6
2	1	7	8	10	1	5	3	7	6	3	3	3
3	8	4	9	2	3	6	8	6	5	7	9	7
4	10	9	5	8	7	2	6	4	3	6	5	1
5	7	10	2	7	10	8	10	3	4	10	1	2
6	3	8	1	1	6	3	1	1	9	9	6	5
7	4	1	7	3	2	1	5	2	8	4	7	10
8	9	2	3	4	5	9	2	5	7	2	8	4
9	5	3	6	9	8	4	4	10	1	8	10	8
10	2	5	4	5	9	7	7	9	10	5	2	9

investigated in one session with twelve sellers and each recorded auction's data set of treatment B5 was assigned to two different sellers. Table B.2 displays the assignment of sellers to bidder groups in that treatment. Similar to Table B.3, in Table B.4 the group numbers 1 through 3 refer to the bidder groups of session 1 of treatment B5 and the group numbers 4 through 6 refer to the bidder groups of session 2 of treatment B5. Note that in all rounds each group number is mapped with two different sellers.

Table B.4. Assignment of sellers to bidder groups in treatment S5

Seller	Round											
	1	2	3	4	5	6	7	8	9	10	11	12
1	6	4	6	2	1	3	1	4	4	3	6	6
2	3	3	1	4	2	6	5	5	2	5	5	5
3	2	2	6	5	6	1	1	2	5	1	1	1
4	4	6	4	3	5	3	5	1	3	6	4	4
5	5	1	5	5	2	1	6	6	1	6	3	3
6	3	5	1	1	4	6	3	1	6	5	6	1
7	6	1	5	2	4	2	4	4	6	2	2	5
8	4	4	2	1	6	4	4	6	2	4	3	6
9	1	5	3	6	5	5	6	2	3	2	5	2
10	1	3	4	3	3	4	3	3	4	4	2	3
11	2	6	3	4	3	5	2	3	5	3	4	2
12	5	2	2	6	1	2	2	5	1	1	1	4

B.2 Bidder Valuations

Table B.5 shows the valuations of the bidders in the experiment. All valuations are random integer numbers from 1 to 100. They were independently drawn from a uniform distribution by a computer algebra program before the start of the experiment. The same set of valuations was used in all sessions of the treatments A3, A5, B3, and B5.[2]

The table also shows the mean of the valuations for each round as well as the overall mean, which at 50.3 is slightly below its theoretical prediction of 50.5.

Table B.5. Bidder valuations

Bidder	**Round**											
	1	2	3	4	5	6	7	8	9	10	11	12
1	69	27	99	25	49	17	47	53	77	12	77	29
2	77	66	55	35	45	63	81	53	14	53	68	34
3	19	39	83	89	15	51	30	100	45	83	2	61
4	64	50	10	61	24	66	84	68	19	12	32	74
5	91	14	56	79	12	25	48	77	37	61	45	39
6	26	73	26	83	78	20	85	15	35	16	16	46
7	20	9	6	10	20	78	86	73	91	49	51	21
8	34	68	17	81	6	35	33	94	18	7	13	35
9	9	48	8	83	50	43	9	35	11	3	88	95
10	44	23	99	54	53	74	91	33	60	88	41	81
11	96	35	20	38	96	34	92	38	40	31	99	26
12	6	35	71	77	26	99	94	73	73	81	88	4
13	42	78	78	15	63	17	74	98	31	22	63	75
14	23	63	97	23	71	31	90	80	51	58	62	75
15	66	92	26	30	39	53	27	49	43	78	53	52
mean	45.7	48.0	50.1	52.2	43.1	47.1	64.7	62.6	43.0	43.6	53.2	49.8
	Overall mean: 50.3											

[2] There are exceptions, though. For technical reasons, in the experiments with five bidders in each auction, i. e. the treatments A5 and B5, the valuations of bidders 2 and 12 were interchanged in round 9 and 10 as were the valuations of bidders 9 and 11 in round 10. The table represents the valuations in treatments A3 and B3.

B.3 Posted Price Offers in Treatments B3 and B5

Table B.6 displays the predetermined PPOs in the treatments B3 and B5. In any given round, all bidder groups were presented with the same predetermined PPO, which was kept equal throughout all sessions. Moreover, no distinction was made with respect to the size of a bidder group. The PPO was only varied from round to round.

Table B.6. Predetermined posted price offers in treatments B3 and B5

	Round											
	1	2	3	4	5	6	7	8	9	10	11	12
PPO	65	69	80	65	58	67	67	60	56	74	51	74

B.4 Theoretical Solutions of Treatments A3 and A5

It was shown in Section 3.7 that if no PPO is available and if $n = 3$ bidders participate in an auction, the expected values for the seller's revenue $E[R]$, the winning bidder's payoff $E[\Pi_{(1)}]$, and the valuation $E[V_{(1)}]$ of the winner of an auction are

$$\begin{aligned} E[R] &= 50.5 \\ E[V_{(1)}] &\approx 75.50 \\ E[\Pi_{(1)}] &\approx 25.00 \ . \end{aligned}$$

The above numbers refer to the ex-ante *expected* values. Table B.7 lists the respective numbers given the bidders' valuations in the experiment (cf. Table B.5). The table displays the seller's revenues R as well as the payoff $\Pi_{(1)}$ and the valuation $V_{(1)}$ of the winning bidder of all auctions in the experiment with three bidders if—in the absence of a PPO—the participants bid in the auctions according to their dominant strategy, i. e. if they submitted their true valuation as their maximum bid.

Similarly, Table B.8 shows the respective theoretical solution of the experiments with $n = 5$ bidders. For comparison, the ex-ante expected values are

$$E[R] \approx 67.17$$
$$E[V_{(1)}] \approx 83.83$$
$$E[\Pi_{(1)}] \approx 16.66 \ .$$

Table B.7. Treatment A3 — Theoretical solutions of individual auctions

Round	**Group**														
	1			**2**			**3**			**4**			**5**		
	R	$\Pi_{(1)}$	$V_{(1)}$	R	$\Pi_{(1)}$	$V_{(1)}$	R	$\Pi_{(1)}$	$V_{(1)}$	R	$\Pi_{(1)}$	$V_{(1)}$	R	$\Pi_{(1)}$	$V_{(1)}$
1	69	8	77	64	27	91	20	14	34	44	52	96	42	24	66
2	27	23	50	23	43	66	73	5	78	48	44	92	63	5	68
3	56	43	99	20	35	55	26	57	83	71	7	78	97	2	99
4	83	0	83	35	46	81	77	12	89	15	39	54	38	41	79
5	49	4	53	45	33	78	24	72	96	15	48	63	50	21	71
6	63	3	66	51	23	74	53	25	78	34	9	43	35	64	99
7	47	38	85	86	5	91	27	57	84	90	2	92	81	13	94
8	53	20	73	35	59	94	53	45	98	80	20	100	73	4	77
9	73	4	77	43	17	60	31	20	51	14	26	40	45	46	91
10	22	66	88	31	30	61	78	3	81	7	76	83	53	5	58
11	62	15	77	53	35	88	63	25	88	51	17	68	41	58	99
12	35	17	52	61	14	75	34	47	81	26	49	75	74	21	95
mean	53.3	20.1	73.3	45.6	30.6	76.2	46.6	31.8	78.4	41.3	32.4	73.7	57.7	25.3	83.0

overall mean R: 48.9 overall mean $\Pi_{(1)}$: 28.1 overall mean $V_{(1)}$: 76.9

Table B.8. Treatment A5 — Theoretical solutions of individual auctions

Round	Group 1			Group 2			Group 3		
	R	$\Pi_{(1)}$	$V_{(1)}$	R	$\Pi_{(1)}$	$V_{(1)}$	R	$\Pi_{(1)}$	$V_{(1)}$
1	69	22	91	42	22	64	77	19	96
2	63	10	73	66	2	68	78	14	92
3	71	28	99	83	16	99	78	19	97
4	54	23	77	83	6	89	79	2	81
5	71	7	78	45	51	96	53	10	63
6	35	31	66	74	25	99	53	25	78
7	48	43	91	92	2	94	84	2	86
8	98	2	100	73	4	77	80	14	94
9	77	14	91	45	28	73	35	16	51
10	49	29	78	61	22	83	81	7	88
11	63	14	77	88	11	99	68	20	88
12	74	21	95	75	6	81	61	14	75
mean	64.3	20.3	84.7	68.9	16.3	85.2	68.9	13.5	82.4

overall mean R: 67.4
overall mean $\Pi_{(1)}$: 16.7
overall mean $V_{(1)}$: 84.1

C

Strategies of the Bidders in the Experiment

In this appendix, the strategies chosen by the bidders in the experiment are individually listed. The induced valuations of the bidders are denoted by v_i, the observed bids by $\hat{b}_i$, and the acceptance threshold in the B-treatments by $\hat{t}_i$.

C.1 Treatment A3: Three Bidders, no PPO

See Table C.1 on pages 154–155.

C.2 Treatment A5: Five Bidders, no PPO

See Table C.2 on pages 156–157.

C.3 Treatment B3: Three Bidders, PPO

See Table C.3 on pages 158–159.

C.4 Treatment B5: Five Bidders, PPO

See Table C.4 on pages 160–161.

Table C.1: Strategies of the bidders in treatment A3

Session 1

Round	Bidder 1 v_i	Bidder 1 $\hat{b}_i$	Bidder 2 v_i	Bidder 2 $\hat{b}_i$	Bidder 3 v_i	Bidder 3 $\hat{b}_i$	Bidder 4 v_i	Bidder 4 $\hat{b}_i$	Bidder 5 v_i	Bidder 5 $\hat{b}_i$
1	69	70	77	76	19	16	64	64	91	91
2	27	28	66	64	39	37	50	50	14	14
3	99	100	55	54	83	80	10	10	56	56
4	25	26	35	35	89	87	61	61	79	79
5	49	49	45	44	15	2	24	24	12	12
6	17	17	63	62	51	50	66	66	25	25
7	47	47	81	80	30	30	84	84	48	48
8	53	53	53	52	100	99	68	68	77	77
9	77	77	14	14	45	45	19	19	37	37
10	12	12	53	52	83	83	12	12	61	61
11	77	77	68	68	2	5	32	32	45	45
12	29	29	34	34	61	62	74	74	39	39
mean	48.4	48.8	53.7	52.9	51.4	49.7	47.0	47.0	48.7	48.7

Round	Bidder 6 v_i	Bidder 6 $\hat{b}_i$	Bidder 7 v_i	Bidder 7 $\hat{b}_i$	Bidder 8 v_i	Bidder 8 $\hat{b}_i$	Bidder 9 v_i	Bidder 9 $\hat{b}_i$	Bidder 10 v_i	Bidder 10 $\hat{b}_i$
1	26	25	20	19	34	33	9	8	44	44
2	73	72	9	8	68	67	48	47	23	23
3	26	25	6	5	17	16	8	7	99	99
4	83	82	10	9	81	81	83	82	54	54
5	78	77	20	100	6	6	50	50	53	53
6	20	19	78	50	35	35	43	42	74	74
7	85	84	86	61	33	33	9	8	91	91
8	15	14	73	70	94	93	35	35	33	33
9	35	34	91	90	18	18	11	12	60	60
10	16	15	49	48	7	7	3	3	88	88
11	16	15	51	50	13	13	88	88	41	41
12	46	45	21	20	35	35	95	95	81	81
mean	43.3	42.3	42.8	44.2	36.8	36.4	40.2	39.8	61.8	61.8

Round	Bidder 11 v_i	Bidder 11 $\hat{b}_i$	Bidder 12 v_i	Bidder 12 $\hat{b}_i$	Bidder 13 v_i	Bidder 13 $\hat{b}_i$	Bidder 14 v_i	Bidder 14 $\hat{b}_i$	Bidder 15 v_i	Bidder 15 $\hat{b}_i$
1	96	96	6	6	42	42	23	20	66	72
2	35	10	35	35	78	78	63	45	92	100
3	20	20	71	71	78	78	97	63	26	35
4	38	38	77	77	15	15	23	22	30	35
5	96	96	26	26	63	64	71	62	39	46
6	34	30	99	99	17	18	31	30	53	67
7	92	92	94	94	74	75	90	89	27	35
8	38	37	73	73	98	99	80	79	49	60
9	40	39	73	73	31	32	51	50	43	59
10	31	30	81	81	22	23	58	57	78	93
11	99	99	88	88	63	64	62	62	53	60
12	26	25	4	4	75	76	75	75	52	61
mean	53.8	51.0	60.6	60.6	54.7	55.3	60.3	54.5	50.7	60.3

continued on page 155 ...

Table C.1 (continued ...)

Session 2

Round	**Bidder** 1 v_i	1 $\hat{b}_i$	2 v_i	2 $\hat{b}_i$	3 v_i	3 $\hat{b}_i$	4 v_i	4 $\hat{b}_i$	5 v_i	5 $\hat{b}_i$
1	69	70	77	80	19	18	64	64	91	91
2	27	35	66	70	39	38	50	50	14	14
3	99	100	55	57	83	82	10	10	56	56
4	25	44	35	45	89	88	61	61	79	79
5	49	77	45	46	15	14	24	24	12	12
6	17	26	63	75	51	50	66	66	25	25
7	47	57	81	80	30	29	84	84	48	48
8	53	58	53	54	100	99	68	68	77	77
9	77	86	14	14	45	44	19	19	37	37
10	12	15	53	57	83	82	12	12	61	61
11	77	86	68	70	2	1	32	32	45	45
12	29	43	34	35	61	60	74	74	39	39
mean	48.4	58.1	53.7	56.9	51.4	50.4	47.0	47.0	48.7	48.7

Round	**Bidder** 6 v_i	6 $\hat{b}_i$	7 v_i	7 $\hat{b}_i$	8 v_i	8 $\hat{b}_i$	9 v_i	9 $\hat{b}_i$	10 v_i	10 $\hat{b}_i$
1	26	26	20	20	34	34	9	8	44	40
2	73	73	9	9	68	68	48	45	23	20
3	26	26	6	6	17	17	8	7	99	80
4	83	83	10	10	81	81	83	75	54	53
5	78	78	20	20	6	6	50	45	53	52
6	20	20	78	78	35	35	43	42	74	73
7	85	85	86	86	33	33	9	8	91	91
8	15	15	73	73	94	94	35	34	33	33
9	35	35	91	91	18	18	11	10	60	60
10	16	16	49	49	7	7	3	2	88	88
11	16	16	51	51	13	13	88	88	41	41
12	46	46	21	21	35	35	95	95	81	81
mean	43.3	43.3	42.8	42.8	36.8	36.8	40.2	38.3	61.8	59.3

Round	**Bidder** 11 v_i	11 $\hat{b}_i$	12 v_i	12 $\hat{b}_i$	13 v_i	13 $\hat{b}_i$	14 v_i	14 $\hat{b}_i$	15 v_i	15 $\hat{b}_i$
1	96	96	6	5	42	48	23	23	66	60
2	35	35	35	30	78	85	63	63	92	92
3	20	20	71	61	78	89	97	97	26	26
4	38	38	77	70	15	20	23	23	30	30
5	96	96	26	25	63	89	71	71	39	45
6	34	34	99	85	17	20	31	31	53	53
7	92	92	94	91	74	90	90	90	27	27
8	38	38	73	72	98	100	80	80	49	49
9	40	40	73	72	31	40	51	51	43	43
10	31	31	81	81	22	23	58	58	78	78
11	99	99	88	88	63	80	62	62	53	54
12	26	26	4	4	75	80	75	75	52	52
mean	53.8	53.8	60.6	57.0	54.7	63.7	60.3	60.3	50.7	50.8

Table C.2: Strategies of the bidders in treatment A5

Session 1

Round	**Bidder** 1		2		3		4		5	
	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$
1	69	68	77	77	19	19	64	64	91	91
2	27	26	66	66	39	39	50	50	14	14
3	99	98	55	55	83	83	10	10	56	56
4	25	24	35	35	89	89	61	61	79	79
5	49	48	45	45	15	15	24	24	12	12
6	17	16	63	63	51	51	66	66	25	25
7	47	46	81	81	30	30	84	84	48	48
8	53	52	53	53	100	100	68	68	77	77
9	77	77	73	73	45	45	19	19	37	37
10	12	12	81	81	83	83	12	12	61	61
11	77	77	68	68	2	2	32	32	45	45
12	29	29	34	34	61	61	74	74	39	39
mean	48.4	47.8	60.9	60.9	51.4	51.4	47.0	47.0	48.7	48.7

Round	**Bidder** 6		7		8		9		10	
	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$
1	26	26	20	19	34	33	9	7	44	40
2	73	73	9	8	68	65	48	41	23	22
3	26	26	6	5	17	45	8	8	99	72
4	83	83	10	9	81	71	83	82	54	53
5	78	78	20	19	6	55	50	50	53	52
6	20	20	78	77	35	40	43	43	74	74
7	85	85	86	85	33	35	9	9	91	91
8	15	15	73	72	94	82	35	35	33	33
9	35	35	91	90	18	25	11	11	60	60
10	16	16	49	48	7	15	31	31	88	88
11	16	16	51	50	13	25	88	88	41	41
12	46	46	21	20	35	32	95	95	81	81
mean	43.3	43.3	42.8	41.8	36.8	43.6	42.5	41.7	61.8	58.9

Round	**Bidder** 11		12		13		14		15	
	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$	v_i	$\hat{b}_i$
1	96	80	6	6	42	40	23	23	66	50
2	35	35	35	35	78	66	63	63	92	80
3	20	20	71	71	78	66	97	97	26	20
4	38	38	77	77	15	15	23	23	30	20
5	96	96	26	26	63	60	71	71	39	30
6	34	34	99	99	17	17	31	31	53	53
7	92	92	94	94	74	74	90	90	27	27
8	38	38	73	73	98	98	80	80	49	49
9	40	40	14	14	31	31	51	51	43	43
10	3	3	53	53	22	22	58	58	78	75
11	99	99	88	88	63	64	62	62	53	53
12	26	26	4	4	75	76	75	75	52	52
mean	51.4	50.1	53.3	53.3	54.7	52.4	60.3	60.3	50.7	46.0

continued on page 157 ...

Table C.2 (continued . . .)

Session 2

Round	Bidder 1 v_i	$\hat{b}_i$	Bidder 2 v_i	$\hat{b}_i$	Bidder 3 v_i	$\hat{b}_i$	Bidder 4 v_i	$\hat{b}_i$	Bidder 5 v_i	$\hat{b}_i$
1	69	60	77	76	19	19	64	64	91	91
2	27	25	66	66	39	39	50	50	14	14
3	99	90	55	55	83	83	10	10	56	56
4	25	20	35	35	89	89	61	61	79	79
5	49	48	45	45	15	15	24	24	12	12
6	17	17	63	63	51	51	66	66	25	25
7	47	45	81	81	30	30	84	84	48	48
8	53	50	53	53	100	100	68	68	77	77
9	77	77	73	73	45	45	19	19	37	37
10	12	12	81	81	83	83	12	12	61	61
11	77	77	68	68	2	2	32	32	45	45
12	29	29	34	34	61	61	74	74	39	39
mean	48.4	45.8	60.9	60.8	51.4	51.4	47.0	47.0	48.7	48.7

Round	Bidder 6 v_i	$\hat{b}_i$	Bidder 7 v_i	$\hat{bi}$	Bidder 8 v_i	$\hat{b}_i$	Bidder 9 v_i	$\hat{b}_i$	Bidder 10 v_i	$\hat{b}_i$
1	26	26	20	20	34	34	9	8	44	43
2	73	100	9	9	68	68	48	42	23	22
3	26	27	6	6	17	17	8	7	99	98
4	83	100	10	10	81	81	83	75	54	53
5	78	90	20	20	6	6	50	46	53	52
6	20	21	78	78	35	35	43	40	74	73
7	85	86	86	86	33	33	9	10	91	90
8	15	16	73	73	94	94	35	32	33	32
9	35	36	91	91	18	18	11	11	60	59
10	16	16	49	49	7	7	31	27	88	87
11	16	16	51	51	13	13	88	84	41	40
12	46	47	21	21	35	35	95	87	81	80
mean	43.3	48.4	42.8	42.8	36.8	36.8	42.5	39.1	61.8	60.8

Round	Bidder 11 v_i	$\hat{b}_i$	Bidder 12 v_i	$\hat{b}_i$	Bidder 13 v_i	$\hat{b}_i$	Bidder 14 v_i	$\hat{b}_i$	Bidder 15 v_i	$\hat{b}_i$
1	96	96	6	6	42	42	23	23	66	65
2	35	35	35	65	78	78	63	63	92	91
3	20	20	71	55	78	78	97	97	26	25
4	38	38	77	100	15	15	23	23	30	29
5	96	96	26	77	63	63	71	71	39	38
6	34	34	99	80	17	17	31	31	53	52
7	92	92	94	88	74	74	90	90	27	26
8	38	38	73	80	98	98	80	80	49	48
9	40	40	14	14	31	31	51	51	43	42
10	3	3	53	53	22	22	58	58	78	77
11	99	99	88	88	63	63	62	62	53	52
12	26	26	4	4	75	75	75	75	52	51
mean	51.4	51.4	53.3	59.2	54.7	54.7	60.3	60.3	50.7	49.7

Table C.3: Strategies of the bidders in treatment B3

Session 1

Round	Bidder 1			Bidder 2			Bidder 3			Bidder 4			Bidder 5		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	69	68	68	77	12	15	19	16	17	64	52	60	91	75	91
2	27	26	26	66	15	22	39	30	31	50	44	48	14	13	14
3	99	40	98	55	7	10	83	60	65	10	8	10	56	50	56
4	25	24	24	35	6	8	89	59	69	61	56	58	79	50	79
5	49	48	48	45	8	11	15	10	14	24	20	22	12	11	12
6	17	16	16	63	11	22	51	30	41	66	60	65	25	24	25
7	47	40	46	81	29	30	30	25	25	84	80	83	48	40	48
8	53	40	53	53	25	27	100	50	75	68	65	67	77	50	77
9	77	40	77	14	1	4	45	30	40	19	16	19	37	36	37
10	12	12	12	53	25	28	83	70	73	12	10	12	61	50	61
11	77	70	77	68	25	53	2	2	2	32	25	32	45	40	45
12	29	28	29	34	30	31	61	59	56	74	70	74	39	36	39
mean	48.4	37.7	47.8	53.7	16.2	21.8	51.4	36.8	42.3	47.0	42.2	45.8	48.7	39.6	48.7

Round	Bidder 6			Bidder 7			Bidder 8			Bidder 9			Bidder 10		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	26	25	25	20	15	15	34	34	34	9	9	9	44	35	41
2	73	53	72	9	8	8	68	68	68	48	48	48	23	22	20
3	26	25	25	6	5	5	17	17	17	8	8	8	99	50	75
4	83	60	82	10	8	8	81	81	81	83	69	83	54	50	52
5	78	60	77	20	15	15	6	6	6	50	50	50	53	50	52
6	20	19	19	78	50	30	35	35	35	43	43	43	74	65	74
7	85	60	84	86	60	32	33	33	33	9	9	9	91	65	90
8	15	14	15	73	65	46	94	70	94	35	35	35	33	33	35
9	35	34	34	91	70	51	18	18	18	11	11	11	60	45	60
10	16	15	15	49	40	33	7	7	7	3	3	3	88	65	88
11	16	15	15	51	20	25	13	13	13	88	70	88	41	41	41
12	46	40	45	21	10	10	35	35	35	95	60	95	81	65	81
mean	43.3	35.0	42.3	42.8	30.5	23.2	36.8	34.8	36.8	40.2	34.6	40.2	61.8	48.8	59.1

Round	Bidder 11			Bidder 12			Bidder 13			Bidder 14			Bidder 15		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	96	70	96	6	4	5	42	25	35	23	13	22	66	40	50
2	35	30	35	35	35	33	78	45	65	63	62	50	92	60	92
3	20	20	20	71	70	71	78	65	78	97	96	96	26	20	26
4	38	35	38	77	77	65	15	14	15	23	22	22	30	20	30
5	96	60	96	26	10	15	63	50	63	71	50	70	39	30	39
6	34	30	34	99	80	85	17	10	17	31	20	30	53	35	53
7	92	45	92	94	70	90	74	40	74	90	50	89	27	20	27
8	38	35	38	73	60	71	98	40	98	80	40	80	49	35	49
9	40	35	40	73	65	73	31	15	31	51	25	50	43	30	43
10	31	25	31	81	79	81	22	10	22	58	25	58	78	30	78
11	99	45	99	88	70	88	63	40	63	62	30	62	53	30	53
12	26	25	26	4	4	4	75	40	75	75	30	75	52	10	52
mean	53.8	37.9	53.8	60.6	52.0	56.8	54.7	32.8	53.0	60.3	38.6	58.7	50.7	30.0	49.3

continued on page 159 ...

Table C.3 (continued . . .)

Session 2

Round	**Bidder** 1			2			3			4			5		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	69	68	68	77	76	76	19	10	12	64	63	63	91	85	75
2	27	26	26	66	65	65	39	30	32	50	50	50	14	13	13
3	99	98	98	55	54	54	83	73	70	10	10	10	56	50	50
4	25	25	25	35	34	34	89	80	75	61	61	61	79	78	65
5	49	48	49	45	44	44	15	10	7	24	24	24	12	12	12
6	17	16	17	63	62	62	51	41	45	66	66	66	25	24	27
7	47	46	47	81	80	80	30	22	22	84	84	84	48	40	42
8	53	52	53	53	52	52	100	91	91	68	60	68	77	77	77
9	77	70	77	14	13	13	45	36	37	19	19	19	37	37	37
10	12	12	12	53	52	52	83	74	75	12	12	12	61	61	61
11	77	45	77	68	67	67	2	1	1	32	32	32	45	45	45
12	29	29	35	34	33	33	61	30	51	74	65	74	39	39	39
mean	48.4	44.6	48.7	53.7	52.7	52.7	51.4	41.5	43.2	47.0	45.5	46.9	48.7	46.8	45.3

Round	**Bidder** 6			7			8			9			10		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	26	25	25	20	16	16	34	15	3	9	6	8	44	43	44
2	73	43	72	9	5	6	68	40	30	48	38	40	23	23	23
3	26	25	25	6	8	5	17	8	5	8	40	8	99	99	99
4	83	43	82	10	45	35	81	65	57	83	90	80	54	53	54
5	78	50	77	20	40	30	6	5	5	50	80	48	53	52	53
6	20	19	19	78	74	65	35	30	30	43	50	40	74	54	74
7	85	45	84	86	80	73	33	32	26	9	10	8	91	50	91
8	15	14	14	73	71	71	94	1	90	35	40	32	33	33	33
9	35	34	34	91	89	89	18	16	16	11	60	10	60	59	60
10	16	15	15	49	46	46	7	6	6	3	10	2	88	60	88
11	16	15	15	51	63	63	13	12	12	88	95	80	41	40	41
12	46	45	45	21	54	54	35	33	33	95	100	90	81	65	81
mean	43.3	31.1	42.3	42.8	49.3	46.1	36.8	21.9	26.1	40.2	51.6	37.2	61.8	52.6	61.8

Round	**Bidder** 11			12			13			14			15		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	96	90	95	6	6	6	42	37	33	23	22	22	66	70	75
2	35	34	34	35	35	35	78	70	72	63	40	62	92	70	90
3	20	19	19	71	71	71	78	72	75	97	1	96	26	18	21
4	38	25	37	77	77	77	15	13	14	23	22	22	30	26	28
5	96	25	95	26	26	26	63	50	59	71	40	70	39	35	40
6	34	15	33	99	99	99	17	16	16	31	30	30	53	50	55
7	92	20	91	94	94	94	74	62	73	90	60	89	27	30	30
8	38	25	38	73	73	73	98	70	97	80	50	79	49	57	60
9	40	40	40	73	73	73	31	30	30	51	40	50	43	60	62
10	31	31	31	81	81	81	22	21	21	58	40	57	78	72	72
11	99	62	99	88	88	88	63	50	62	62	45	61	53	50	52
12	26	26	26	4	4	4	75	60	75	75	55	74	52	45	45
mean	53.8	34.3	53.2	60.6	60.6	60.6	54.7	45.9	52.3	60.3	37.1	59.3	50.7	48.6	52.5

Table C.4: Strategies of the bidders in treatment B5

Session 1

Round	**Bidder** 1			2			3			4			5		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	69	56	68	77	70	71	19	20	25	64	63	63	91	90	90
2	27	22	26	66	55	62	39	50	50	50	49	49	14	14	14
3	99	70	98	55	50	51	83	90	90	10	9	1	56	50	55
4	25	24	24	35	32	33	89	100	101	61	60	60	79	65	78
5	49	45	48	45	42	42	15	20	21	24	23	23	12	12	12
6	17	16	16	63	60	59	51	60	61	66	65	65	25	24	24
7	47	42	46	81	75	79	30	40	41	84	50	83	48	40	47
8	53	45	52	53	50	52	100	100	111	68	67	67	77	60	76
9	77	61	76	73	70	72	45	45	55	19	18	18	37	36	36
10	12	11	11	81	79	80	83	90	92	12	11	11	61	60	60
11	77	61	76	68	66	67	2	2	2	32	31	31	45	44	44
12	29	25	28	34	30	34	61	65	70	74	69	73	39	38	39
mean	48.4	39.8	47.4	60.9	56.6	58.5	51.4	56.8	59.9	47.0	42.9	45.3	48.7	44.4	47.9

Round	**Bidder** 6			7			8			9			10		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	26	26	26	20	20	49	34	34	34	9	5	5	44	40	43
2	73	73	64	9	9	49	68	68	68	48	25	42	23	20	22
3	26	26	25	6	6	49	17	17	17	8	8	8	99	90	98
4	83	83	83	10	1	49	81	81	81	83	68	79	54	50	53
5	78	78	78	20	1	20	6	6	6	50	20	46	53	50	52
6	20	20	20	78	40	78	35	35	35	43	39	39	74	70	73
7	85	85	85	86	68	75	33	33	33	9	9	9	91	90	90
8	15	15	15	73	66	79	94	94	94	35	30	30	33	30	32
9	35	35	35	91	70	99	18	18	18	11	100	11	60	55	59
10	16	16	16	49	48	49	7	7	7	31	1	35	88	87	87
11	16	16	16	51	23	51	13	13	13	88	70	80	41	40	40
12	46	46	46	21	10	21	35	35	35	95	80	94	81	80	80
mean	43.3	43.3	42.4	42.8	30.2	55.7	36.8	36.8	36.8	42.5	37.9	39.8	61.8	58.5	60.8

Round	**Bidder** 11			12			13			14			15		
	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$	v_i	$\hat{t}_i$	$\hat{b}_i$
1	96	70	98	6	8	12	42	1	41	23	20	22	66	60	60
2	35	30	35	35	45	40	78	70	77	63	35	62	92	85	88
3	20	19	20	71	78	81	78	65	78	97	40	85	26	15	20
4	38	37	38	77	77	90	15	15	15	23	20	22	30	20	24
5	96	70	96	26	26	28	63	60	63	71	45	70	39	33	37
6	34	33	34	99	99	100	17	17	17	31	28	30	53	45	48
7	92	70	92	94	94	99	74	65	73	90	60	80	27	25	25
8	38	37	38	73	73	83	98	75	97	80	60	79	49	45	40
9	40	39	40	14	14	20	31	30	30	51	45	65	43	50	40
10	3	3	3	53	53	60	22	21	21	58	45	65	78	75	77
11	99	70	99	88	88	95	63	55	63	62	58	65	53	53	53
12	26	25	26	4	4	4	75	70	75	75	60	77	52	50	51
mean	51.4	41.9	51.6	53.3	54.9	59.3	54.7	45.3	54.2	60.3	43.0	60.2	50.7	46.3	46.9

continued on page 161 ...

Table C.4 (continued . . .)

Session 2

Round	**Bidder** 1 v_i	$\hat{t}_i$	$\hat{b}_i$	2 v_i	$\hat{t}_i$	$\hat{b}_i$	3 v_i	$\hat{t}_i$	$\hat{b}_i$	4 v_i	$\hat{t}_i$	$\hat{b}_i$	5 v_i	$\hat{t}_i$	$\hat{b}_i$
1	69	20	55	77	57	77	19	10	10	64	63	63	91	72	90
2	27	27	25	66	46	66	39	20	39	50	49	50	14	13	13
3	99	99	90	55	51	55	83	55	83	10	10	10	56	52	55
4	25	25	25	35	33	35	89	65	89	61	61	61	79	67	78
5	49	49	49	45	43	45	15	10	15	24	24	24	12	11	11
6	17	17	17	63	47	63	51	40	51	66	66	66	25	24	24
7	47	47	47	81	69	81	30	20	30	84	84	84	48	44	47
8	53	53	53	53	50	53	100	65	100	68	68	68	77	66	76
9	77	100	77	73	65	73	45	30	45	19	19	19	37	36	36
10	12	100	12	81	69	81	83	61	83	12	12	12	61	51	60
11	77	77	77	68	67	68	2	1	2	32	32	32	45	44	44
12	29	29	29	34	33	34	61	40	64	74	74	74	39	38	38
mean	48.4	53.6	46.3	60.9	52.5	60.9	51.4	34.8	50.9	47.0	46.8	46.9	48.7	43.2	47.7

Round	**Bidder** 6 v_i	$\hat{t}_i$	$\hat{b}_i$	7 v_i	$\hat{t}_i$	$\hat{b}_i$	8 v_i	$\hat{t}_i$	$\hat{b}_i$	9 v_i	$\hat{t}_i$	$\hat{b}_i$	10 v_i	$\hat{t}_i$	$\hat{b}_i$
1	26	26	26	20	15	19	34	20	30	9	35	20	44	43	44
2	73	73	73	9	5	8	68	55	67	48	100	95	23	22	23
3	26	26	26	6	3	5	17	16	16	8	64	59	99	70	99
4	83	83	79	10	8	4	81	80	80	83	150	45	54	51	54
5	78	78	78	20	14	19	6	5	5	50	76	70	53	51	53
6	20	20	20	78	77	77	35	34	34	43	88	74	74	68	74
7	85	85	85	86	70	85	33	32	32	9	56	66	91	70	91
8	15	15	15	73	60	72	94	93	93	35	66	78	33	32	33
9	35	35	35	91	70	90	18	17	17	11	90	88	60	55	60
10	16	16	16	49	35	47	7	6	6	31	55	100	88	68	88
11	16	16	16	51	49	50	13	12	12	88	77	99	41	40	41
12	46	46	46	21	20	20	35	34	34	95	120	99	81	65	81
mean	43.3	43.3	42.9	42.8	35.5	41.3	36.8	33.7	35.5	42.5	81.4	74.4	61.8	52.9	61.8

Round	**Bidder** 11 v_i	$\hat{t}_i$	$\hat{b}_i$	12 v_i	$\hat{t}_i$	$\hat{b}_i$	13 v_i	$\hat{t}_i$	$\hat{b}_i$	14 v_i	$\hat{t}_i$	$\hat{b}_i$	15 v_i	$\hat{t}_i$	$\hat{b}_i$
1	96	90	78	6	5	7	42	36	40	23	19	20	66	60	66
2	35	51	40	35	34	36	78	63	65	63	50	55	92	92	92
3	20	51	45	71	60	72	78	74	70	97	60	90	26	26	26
4	38	35	40	77	60	78	15	14	13	23	20	20	30	30	30
5	96	87	72	26	15	27	63	61	53	71	50	70	39	39	39
6	34	28	33	99	70	100	17	17	15	31	20	31	53	53	53
7	92	70	71	94	80	95	74	73	70	90	70	90	27	27	27
8	38	32	35	73	66	74	98	96	80	80	70	80	49	49	49
9	40	35	38	14	14	15	31	31	28	51	40	51	43	43	43
10	3	3	3	53	53	54	22	21	20	58	50	58	78	78	78
11	99	73	93	88	40	89	63	62	61	62	50	62	53	53	53
12	26	23	25	4	4	5	75	74	71	75	70	75	52	52	100
mean	51.4	48.2	47.8	53.3	41.8	54.3	54.7	51.8	48.8	60.3	47.4	58.5	50.7	50.2	54.7

D

Pictures and Screenshots

D.1 Pictures of the IW Experimental Lab

Figure D.1. Experimental lab at the Institute of Information Engineering and Management, University of Karlsruhe

Figure D.2. Participant cabin in the experimental lab

D.2 Screenshots of the Experimental Software

Figures D.3–D.12 show selected screenshots of the experiment software. The Figures D.3–D.5 refer to the treatment family A and Figures D.6–D.12 show screens of the B-treatments. In both cases, first the bidding screen is presented and then the different result screens are shown.

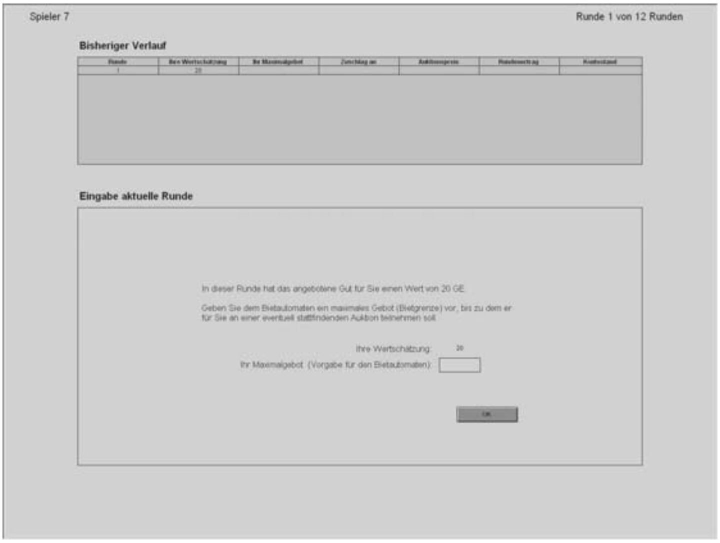


Figure D.3. Bidding screen in treatment A

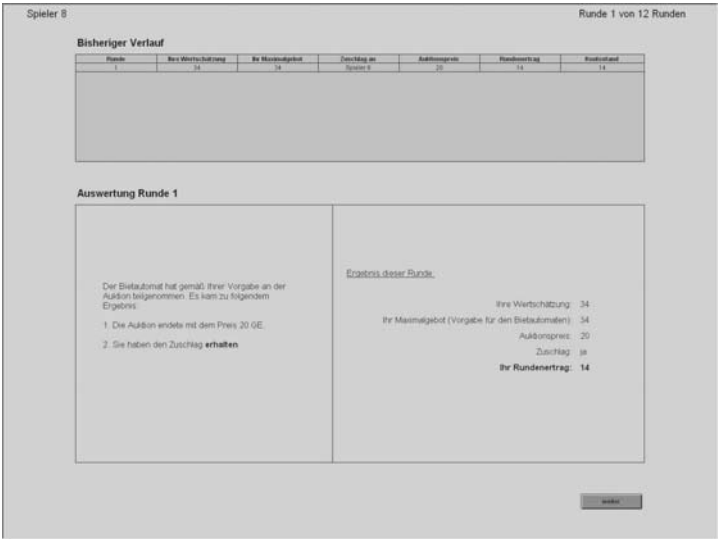


Figure D.4. Result screen of winning bidder in treatment A

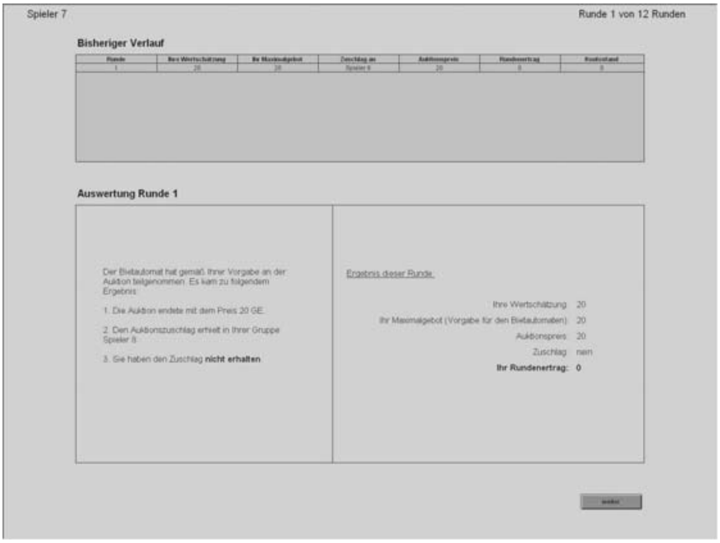


Figure D.5. Result screen of losing bidder in treatment A

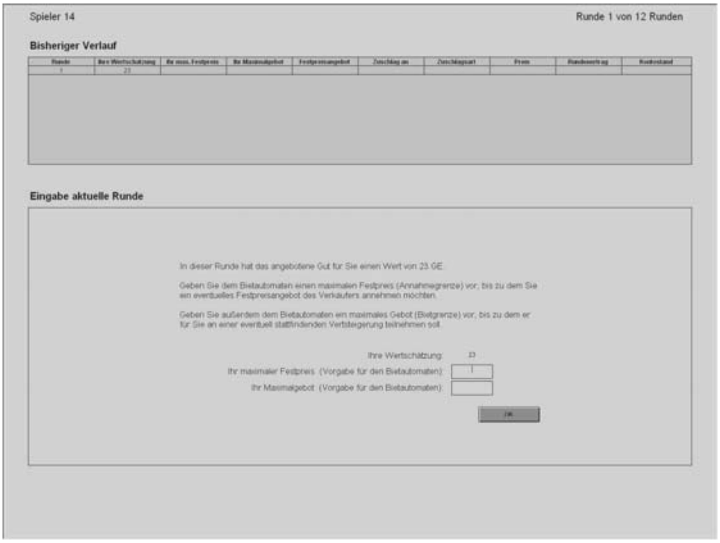


Figure D.6. Bidding screen in treatment B

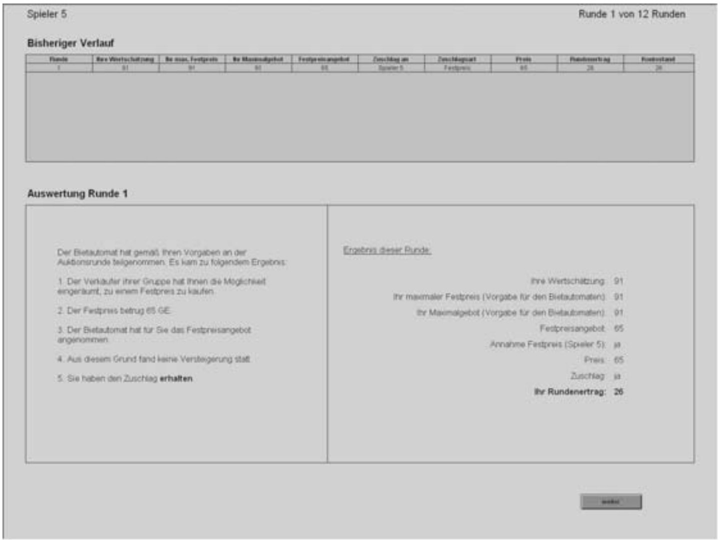


Figure D.7. Result screen of decisive bidder who accepted the PPO in treatment B

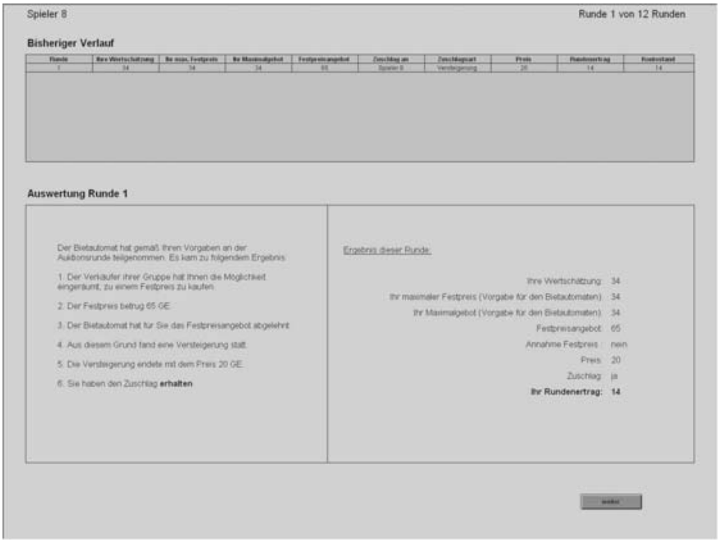


Figure D.8. Result screen of decisive bidder who rejected the PPO and won the corresponding auction in treatment B

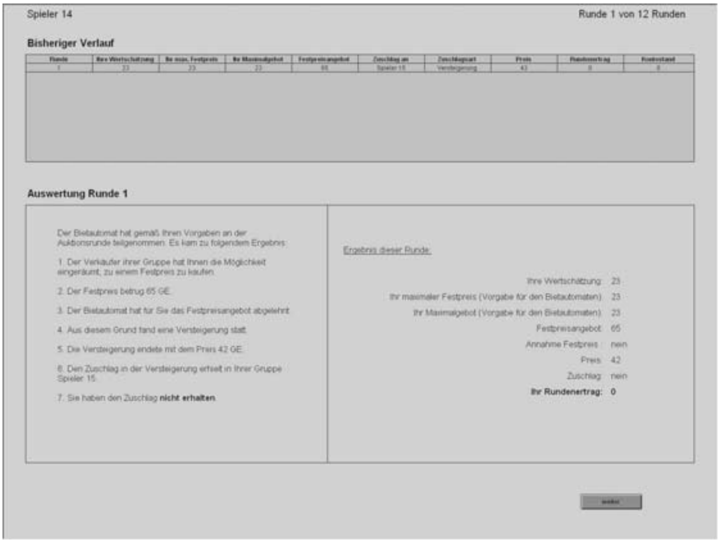


Figure D.9. Result screen of decisive bidder who rejected the PPO and lost the corresponding auction in treatment B

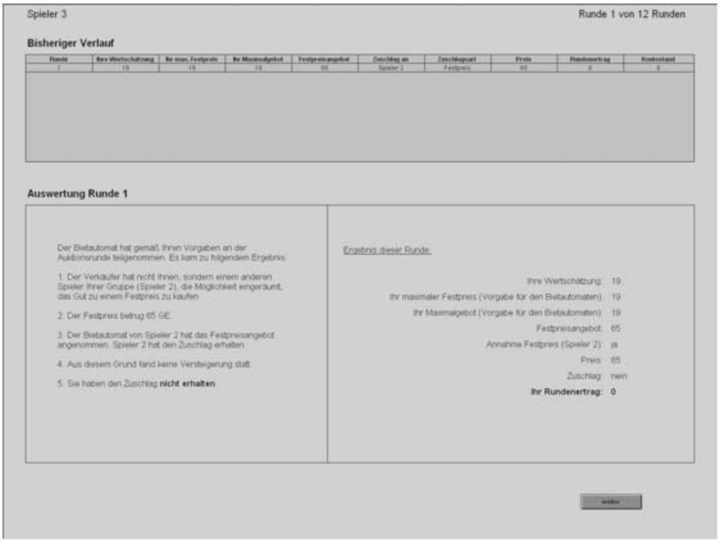


Figure D.10. Result screen of non-decisive bidder if the decisive bidder accepted the PPO in treatment B

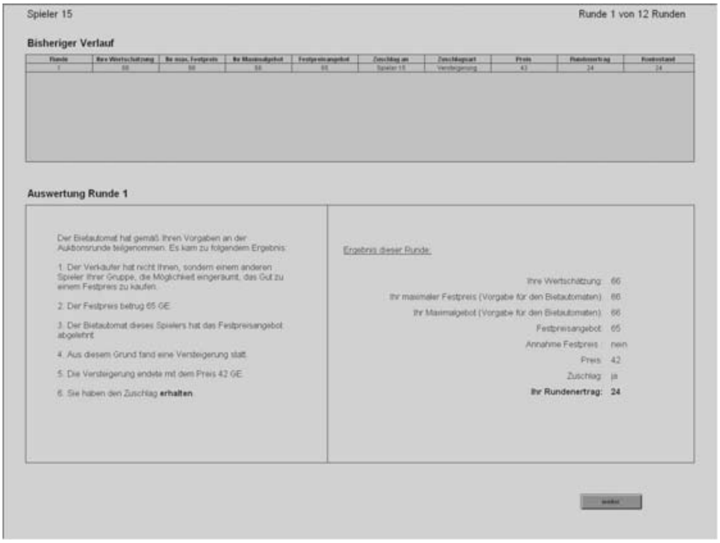


Figure D.11. Result screen of non-decisive bidder who won the corresponding auction treatment B

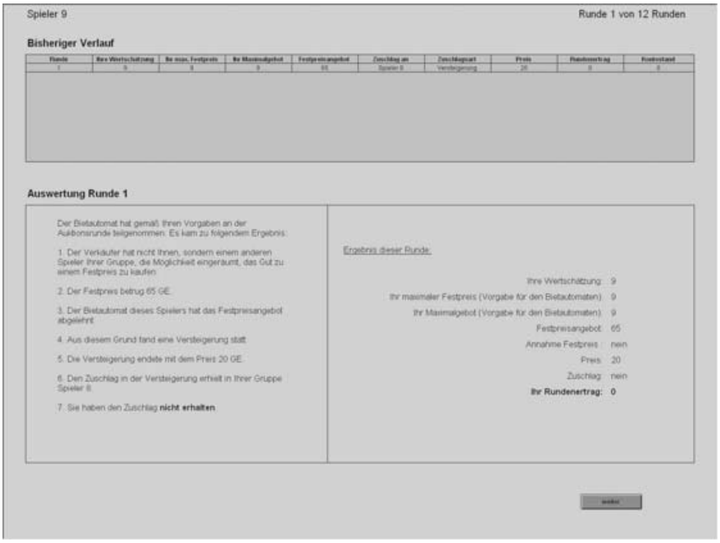


Figure D.12. Result screen of non-decisive bidder who lost the corresponding auction in treatment B

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Abbreviations and Symbols

Abbreviations

APPO	auction with a posted price offer
CARA	constant absolute risk aversion
cdf	cumulative probability distribution function
cf.	confer
fn.	footnote
ibid.	ibidem, at the same place
p., pp.	page, pages
pdf	probability density function
PPO	posted price offer
SIPV	symmetric independent private values model
std. dev.	standard deviation
WSR	Wilcoxon signed ranks test

Symbols

Unless otherwise stated, lower case letters denote numbers, upper case letters indicate random variables, and calligraphic letters refer to sets.

$\exists$	there exists (at least) one ...
$\exists_{i!}$	there exist exactly i ...
$\emptyset$	empty set
$f \equiv g$	$f(x) = g(x)\ \forall x$
$f^n(x)$	$(f(x))^n\ (n \geq 0)$
$f^{-1}(\cdot)$	inverse function: $f^{-1}(f(x)) = x\ \forall x$

$\hat{b}_i$	(maximum) bid of bidder i in the experiment
$\tilde{b}_i$	dominant strategy bid of bidder i
$\bar{b}$	average bid
B	buy price in an auction; also determination coefficient
$E[\cdot]$	expected value
F	distribution function of random variable with density f
$F_{(i),n}$, $f_{(i),n}$	distribution, density of ith-order statistic subject to n drawings
$G_{(i)}$, $g_{(i)}$	distribution, density of ith-order statistic if the total number of drawings is not known or not of interest
$G_{(1),-i}$, $g_{(1),-i}$	distribution, density of first-order statistic, subject to all but the ith-drawing
m	size of a random sample
n	number of bidders
$\mathbb{N}$	set of natural numbers
p	price
$\underline{p}$	reserve price in an auction
$\bar{p}$	posted price offer (PPO)
π	3.141...; also Markovitz risk-premium
$\Pi_{(1)}$	payoff of winning bidder in an auction
$\hat{\Pi}$	payoff of winning bidder in the experiment
Pr	probability
r	Spearman's rank or Pearson's correlation coefficient, depending on context
R	revenue of an auction
$\hat{R}$	revenue of an auction in the experiment
R^C	revenue of the corresponding auction of an APPO
$\mathbb{R}$	set of real numbers
$\mathbb{R}_+$	set of positive real numbers including zero
t_i	acceptance threshold of bidder i
$\hat{t}_i$	acceptance threshold of bidder i in the experiment
T	closing time of an auction
v_i	valuation of bidder i
$V_{(1)}$	valuation of winning bidder, total surplus of an auction
$\hat{V}$	valuation of winning bidder in the experiment, total surplus of an experimental outcome

Definitions, propositions, and theorems are numbered consecutively. Examples are numbered separately. Proofs and examples are terminated by the symbols ■ and □, respectively.

Sellers and buyers are referred to by female and male pronouns, respectively.

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