Transport Phenomena in a Physical World

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Our physical world is changing, populations are growing, climate changes, new products are constantly being developed, new technologies and concepts emerge. And we have even landed on Mars! All this calls for constant education, especially within natural science.

This book is written to you who have an interest in natural science and especially in understanding some basics within transport phenomena. You either i) study to become or ii) works as a physicist, a chemist or an engineer. As outline above: A lot is changing in our physical world but what is described in this book is not changing! It is not new! And I surely did not invent it! However, as the topics I will cover in this book gave me some fundamental insights into physics and transport phenomena when I was studying, my hope is that it will do the same to you.

This book gives an overview of some analogies between these basic fundamentals:

- Diffusivity, D
- Thermal conductivity, k
- Dynamic viscosity, μ
- Permeability, L_p
- Electrical conductance, σ

These terms are associated with the transport of mass, energy, momentum, volume and electrical charges (electricity). Many analogies can be extruded from these different phenomena which should be clear from reading this book. Knowledge about transport phenomena in general is essential in many technologies. I hope you will see and understand these analogies and benefit from it – just as I have done myself. Understanding the basics is fundamental and a prerequisite for all development!

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1   Introduction

Things only move when they are forced to move! A bicycle only moves when a force is applied in the form of pedaling. A cloud on the sky only moves when a force is applied in the form of a storm or a wind. Electrons only move in a power cable when a force is applied in the form of an electrical field. All sorts of transport only take place when a force, called a *driving force*, is applied.

Transport of mass, energy, momentum, volume, and electricity only takes place when a driving force is applied. Transport is generally expressed as a flux *J*, which is defined by the amount of mass, energy, momentum, volume, or charges that are being transported pr. area pr. time. The transport is proportional to the applied driving force and can be expressed by a linear phenomenological equation:

$$J = -A \cdot \frac{dX}{dx}$$ (1)

In this book we are only dealing with one-dimensional cases where the transport is in the *x*-direction. Analogies to two and three dimensional cases can be found in teaching books about transport phenomena. The driving force is expressed as the gradient of *X* (concentration, temperature, velocity, pressure, or voltage) along the *x*-axis in the transport direction. Since transport always goes "downhill" from high concentration, temperature, velocity etc. to low concentration, temperature, velocity etc., a minus-sign is placed on the right side of the equation as the gradient *dX/dx* is negative and the flux should be positive. The proportionality constant *A* is called a phenomenological coefficient and is related to many well known physical terms associated with different kinds of transport. Table 1 lists different kinds transport together with the driving forces, phenomenological flux equations, names of the phenomenological coefficients, units for the different fluxes and the common name for the transport phenomena.

| Transport of	Driving Force	Flux Equation	Phenomenological Coefficient	Flux SI Unit	Common Name
Mass (kg)	Concentration gradient	$J_m = -D \cdot \frac{dc}{dx}$	Diffusion coefficient D [m^2/s]	$\left[\frac{kg}{m^2 \cdot s} \right]$	Fick's law of diffusion
Energy/heat (J)	Temperature gradient	$J_h = -k \cdot \frac{dT}{dx}$	Thermal conductivity k [J/(s·K·m)]	$\left[\frac{J}{m^2 \cdot s} \right]$	Fourier's law of heat conduction
Momentum (kg·m/s)	Velocity gradient (shear rate)	$J_n = -\mu \cdot \frac{dv}{dx}$	Dynamic viscosity μ [Pa·s]	$\left[\frac{kg \cdot (m/s)}{m^2 \cdot s} \right]$	Newton's law of viscosity
Volume (m^3)	Pressure gradient	$J_v = -L_p \cdot \frac{dP}{dx}$	Permeability coefficient L_p [m^2/(Pa·s)]	$\left[\frac{m^3}{m^2 \cdot s} \right]$	Darcy's law
Electrical charges (C)	Voltage gradient (electrical field)	$J_e = -\sigma \cdot \frac{dE}{dx}$	Electrical conductance σ [C^2/(s·J·m)]	$\left[\frac{C}{m^2 \cdot s} \right]$	Ohm's law

Table 1: Different kinds of transport
Driving forces are specified and flux equations are given for different kinds of transport. SI units for the phenomenological coefficients and the fluxes are given together with the common names for the different transport phenomena.

2   Different kinds of transport

In the following sub sections the different kinds of transport listed in Table 1 will be described.

2.1   Diffusivity, transport of mass

Diffusion of mass is also known as *mass diffusion*, *concentration diffusion* or *ordinary diffusion*. We are talking about molecular mass transport taking place as diffusion of a component *A* through a medium consisting of component *B*. The *diffusion coefficient D_{AB}* determines how fast the diffusion takes place. The subscript of the diffusion coefficient indicates that the diffusion is associated with the diffusion of *A* through *B*. A diffusion situation is sketched in Figure 1 for the diffusion of a gas component *A* through a plate of silicone rubber.
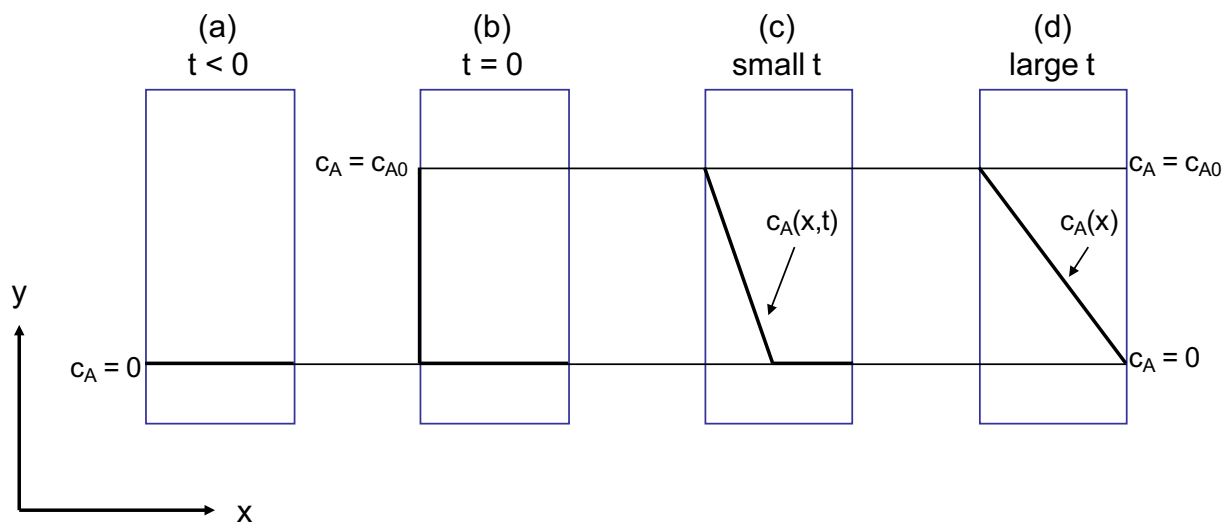
Figure 1: Build-up of concentration profile in a silicone rubber plate
(a) The concentration of *A* at both sides of the silicone rubber plate is zero. (b) At *t* = 0 the concentration of *A* on the left side of the silicone rubber plate is increased to c_{A0}. (c) Component *A* starts to diffuse through the silicone rubber. At small values of *t*, the concentration of *A* in the silicone rubber is thus a function of both time and distance *x*. (d) At large values of *t*, steady state have been established and a linear concentration profile is reached. Thus at steady state the concentration of *A* is only a function of the distance *x* in the silicone rubber plate.

The blue boxes symbolize a barrier consisting of a plate of silicone rubber. The left and right sides are completely separated by the plate. The silicone rubber plate is assumed to consist of component *B*. Initially the concentration of component *A* is zero at both sides of the plate. At time *t* = 0 the concentration of *A* at the left side is suddenly raised to c_{A0} at which it is held constant. Component *A* starts to diffuse through *B* because of the driving force that exists in the form of a concentration difference in the *x*-direction. Thus the concentration of *A* increases in the silicone rubber as a function of the distance *x* inside the rubber and the time *t*. The concentration of *A* at the right side is kept at zero by continually removing the amount of *A* that diffuses through the silicone rubber. At large values of *t*, a steady state linear concentration profile will be established. At this stage the concentration of *A* is only a function of the distance *x* inside the silicone rubber plate.

At steady state the flux of component *A* through the silicone rubber is given by the flux equation from Table 1, which is called *Fick's law of diffusion*:

$$J_A = -D_{AB} \cdot \frac{dc_A}{dx} \qquad\qquad (2)$$

The flux J_A is the diffusive flux of component *A* in the direction *x* through the silicone rubber plate. The gradient dc_A/dx is the concentration gradient of component *A* inside the rubber plate which is the driving force. D_{AB} is as mentioned earlier the diffusion coefficient of *A* in *B*. The value of the diffusion coefficient D_{AB} determines how fast the linear steady state concentration profile is established:

> *The larger the diffusion coefficient D_{AB}, the faster the linear steady state concentration profile is established. If D_{AB} is small, the flux of A is small and the time before steady state is reached is large.*

The diffusion coefficient has the units of length^2 pr. time:

$$D, \qquad SI-units: \quad \left[\frac{m^2}{s}\right]$$

At constant temperatures and constant low pressures the diffusion coefficient for a binary gas mixture is almost independent of the composition and can thus be considered a constant. It is inversely proportional to the pressure and increases with the temperature. For binary liquid mixtures and for high pressures the behavior of the diffusion coefficient is more complicated and will not be discussed in this book.

2.2   Thermal conductivity, transport of energy

Energy in the form of heat can be transported when a driving force in the form of a temperature difference is applied. The flux of heat is proportional to the applied driving force and the proportionality constant is called the *thermal conductivity k*. We are talking about molecular energy transport, and a situation with transport of heat through a one layer window is sketched in Figure 2.
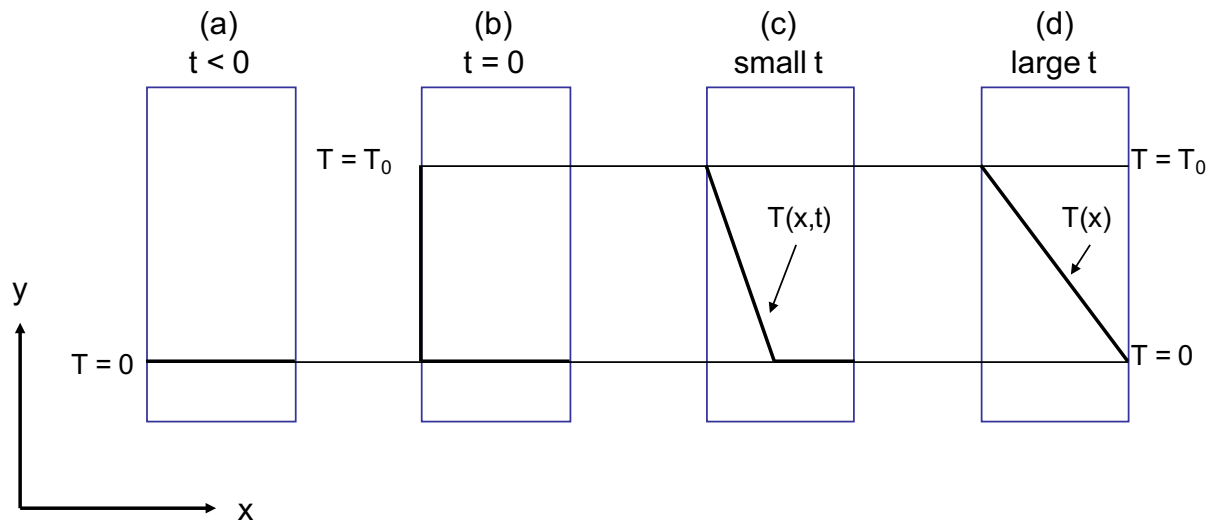
Figure 2: Build-up of temperature profile in a window

(a) The temperature on both sides of the window is zero. (b) At *t* = 0 the temperature on the left side of the window is increased to T_0. (c) Energy/heat starts to flow through the window. At small values of *t*, the temperature in the window is thus a function of both time and distance *x* inside the window. (d) At large values of *t*, steady state is established and a linear temperature profile in the window is reached. Thus at steady state the temperature is only a function of the distance *x* inside the window.

Initially the temperature on both sides of the window is zero (or room temperature) which means that no heat flow through the window. Suddenly at *t* = 0 the temperature on the left side of the window is increased to T_0. Because of the temperature difference between the two sides of the window heat starts to flow through the window from the warm side to the cold side. The temperature on the right side is kept at zero. Before steady state is reached the temperature inside the window is a function of both distance *x* and time *t*. Depending on how good or bad an isolator the window is, a linear steady state temperature profile is reached after a period of time.

At steady state the flux of heat/energy through the window is given by the flux equation from Table 1, which is called *Fourier's law of heat conduction*:

$$J_h = -k \cdot \frac{dT}{dx} \tag{3}$$

The flux J_h is the flux of heat/energy in the direction *x* through the window. The gradient *dT/dx* is the temperature gradient which is the driving force. The term *k* is as mentioned earlier the thermal conductivity of the window. The value of the thermal conductivity together with other factors determines how fast the linear steady state temperature profile develops. These other factors are the density ρ and the heat capacity C_p of the window. The thermal conductivity, the density and the heat capacity can together be expressed at *the thermal diffusivity, α*:

$$\alpha = \frac{k}{\rho \cdot C_p}, \qquad SI-units: \quad \left[\frac{J}{s \cdot K \cdot m} \cdot \frac{m^3}{kg} \cdot \frac{kg \cdot k}{J} \right] = \left[\frac{m^2}{s} \right] \tag{4}$$

It is seen form equation (4) that the thermal diffusivity has the same units as the ordinary diffusivity D (see section *2.1 Diffusivity, Transport of mass*). Thus the thermal diffusivity can be thought of as a diffusion coefficient for energy/heat. The thermal diffusivity of the window thus determines how fast the steady state temperature profile is established:

> *The larger the thermal diffusivity α (of the window), the faster the linear steady state temperature profile is established. If α is small, the flux of heat/energy is small and the time before steady state is reached is large.*

The thermal conductivity of gasses is obviously dependent on the pressure but also on the temperature. Thermal conductivities of liquids and solids are also temperature dependent but almost pressure independent in the pressure range where they are almost incompressible. Further discussion about pressure and temperature dependence will not be discussed in this book.

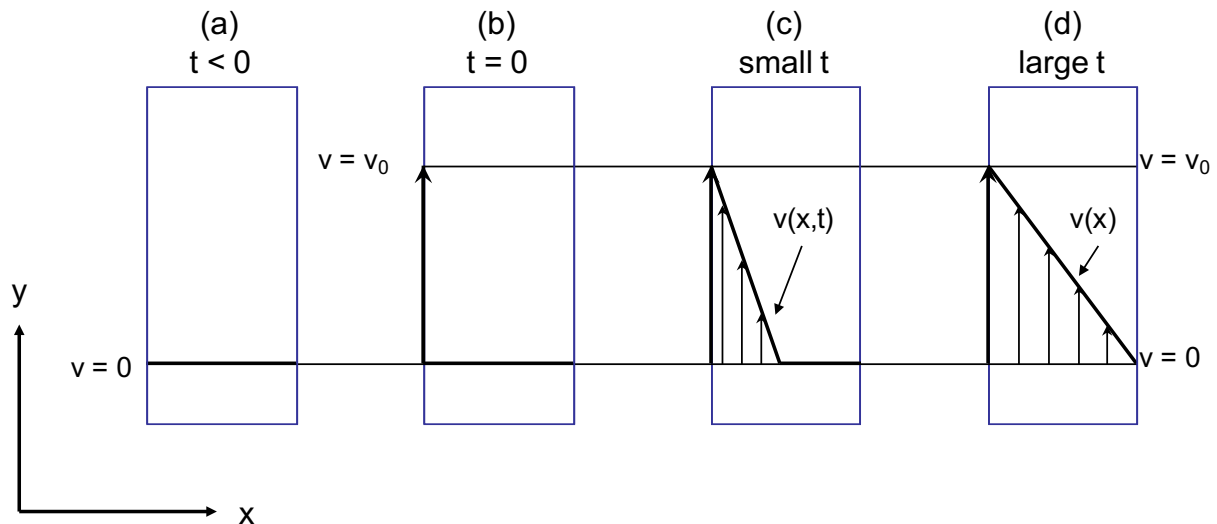## 2.3   Dynamic viscosity, transport of momentum

Momentum can be transferred when a driving force in the form of a velocity difference exists. This can be explained by describing the situation sketched in Figure 3, which shows an example of molecular momentum transport.

Figure 3: Build-up of velocity profile in a Newtonian fluid

One plate to the left side and one plate to the right side separate a Newtonian. (a) The velocity of both plates is zero. (b) At $t = 0$ the left plate is set at motion with a constant velocity v_o while the right plate is kept at rest. (c) The fluid just next to the moving plate start to move. This fluid in motion then starts to move the fluid to its the right which is initially at rest. Thus as velocity is propagated, momentum is transferred in the x direction. At small values of t, the velocity in the fluid between the plates is a function of both time and distance x. (d) At large values of t, steady state is established and a linear velocity profile in the fluid is reached. Thus at steady state the velocity is only a function of the distance x in the fluid.

A Newtonian fluid (the term Newtonian will be elaborated on in a moment) is contained between two plates. It could be water or ethanol for example. Initially the plates and the fluid are a rest. At time $t = 0$ the plate to the left is suddenly set at motion with a constant velocity v_0 in the y-direction. The fluid just next to the left plate will then also start to move in the y-direction. That way the fluid throughout the whole distance between the plates will eventually be set at motion. The right plate is kept at rest. The fluid just next to the right plate will all the time not move because "no slip" is assumed between the fluid an the plate. At small values of t the velocity in the y-direction is a function of both the time and distance x in the fluid. After a while a linear steady state velocity profile is established and the velocity in the y-direction is then only a function of the distance x in the fluid.

A constant force is required to keep the left plate at motion. This force is proportional to the velocity v_0, the area of the plate and inversely proportional to the distance between the two plates. The *force pr. area* ratio can be thought of as a flux of y-momentum (momentum in the y-direction) in the x-direction. The proportionality constant is the *dynamic viscosity μ* of the fluid (the dynamic viscosity can also be denoted with the symbol η). The viscosity of a fluid is then associated with a resistance towards flow.

At steady state the momentum flux through the fluid is given by the flux equation from Table 1, which is called *Newton's law of viscosity*:

$$J_n = -\mu \cdot \frac{dv}{dx} \tag{5}$$

Equation (5) only applies to fluids with molecular weights less than about 5000 Da. Such fluids are called *Newtonian fluids* because they are described by Newton's law of viscosity. The viscosity of such fluids is independent of the velocity gradient which is not the case for non-Newtonian fluids (ketchup for example).

The flux J_n is the flux of *y*-momentum in the direction *x* through the fluid. The gradient *dv/dx* is the velocity gradient which is the driving force. Remember that the velocity is in the *y*-direction and the momentum flux is in the *x*-direction. This driving force is often referred to as the *shear rate*. The term μ is as mentioned earlier the dynamic viscosity of the fluid. The value of the dynamic viscosity together with density of the fluid determines how fast the linear steady state velocity profile develops. The dynamic viscosity and the density of the fluid can together be expressed at the *kinematic viscosity, v*:

$$\nu = \frac{\mu}{\rho}, \qquad SI-units: \quad \left[Pa \cdot s \cdot \frac{m^3}{kg} \right] = \left[\frac{kg}{m \cdot s} \cdot \frac{m^3}{kg} \right] = \left[\frac{m^2}{s} \right] \tag{6}$$

It is seen form equation (6) that the kinematic viscosity has the same units as the ordinary diffusivity *D* (see section *2.1 Diffusivity, Transport of mass*) and the thermal diffusivity *α* (see section *2.2 Thermal conductivity, Transport of energy*). Thus the kinematic viscosity can be thought of a diffusion coefficient for velocity. The kinematic viscosity of the fluid thus determines how fast the steady state velocity profile is established:

> *The larger the kinematic viscosity v (of the fluid), the faster the linear steady state velocity profile is established. If v is small, the flux of momentum is small and the time before steady state is reached is large.*

The dynamic viscosity is very temperature and pressure dependent. For liquids the dynamic viscosity decreases with increasing temperature while for low density gasses the viscosity increases with increasing temperature. The dynamic viscosity normally increases with increasing pressure. Further temperature and pressure dependency will not be discussed in this book.

2.4   Permeability, transport of volume

A volume flux is induces when a driving force in the form of a pressure difference is applied. The flux of volume is proportional to this driving force and the proportionality constant is called the *permeability L_p*. This is often used in pressure driven membrane processes where a pressure difference across a membrane induces a volume flux through the membrane. Proportionality between the applied pressure and the flux is seen when pure water permeates through the membrane. A pressure difference situation across a membrane is sketched in Figure 4.
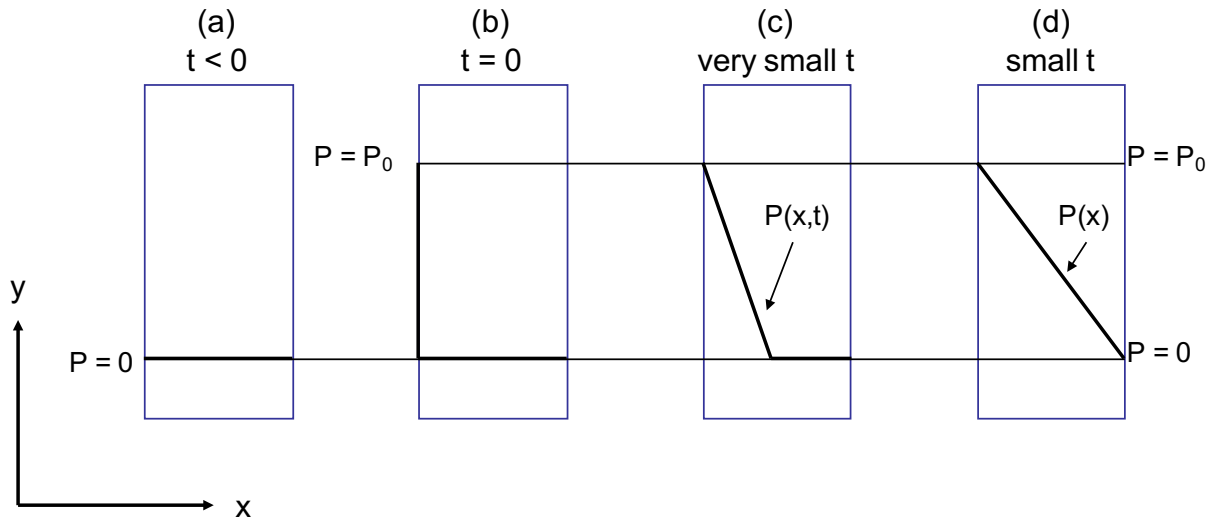
Figure 4: Build-up of pressure profile in a membrane

(a) The pressure on both sides of the membrane is zero. (b) At $t = 0$ the pressure on the left side of the membrane is increased to P_0. (c) Water starts to flow through the membrane because of the pressure difference. Thus a very small t values, the pressure profile is not fully established. At this stage the pressure inside the membrane is a function of the distance x and the time t. (d) Very short after the pressure on the left side has been raised to P_0, steady state is reached and a linear pressure profile in the membrane is established. Thus at steady state the pressure is only a function of the distance x inside the membrane.

Initially the pressure is zero (atmospheric pressure) at both sides of the membrane. No water will then flow through the membrane. At time $t = 0$ the pressure on the left side is suddenly raised to P_0 at which it is held constant. Water starts to flow through the membrane because of the pressure difference. The pressure thus increases in the membrane as a function of the distance x and the time t. The pressure on the right side is kept at zero. Because membranes are usually very thin, a steady state linear pressure profile is reached very fast, and thus the pressure is then only a function of the distance x inside the membrane.

At steady state the volume flux through the membrane is given by the flux equation from Table 1, which is often called *Darcy's law*:

$$J_v = -L_p \cdot \frac{dP}{dx} \tag{7}$$

The flux J_v is the flux of volume in the direction x through the membrane. The gradient dP/dx is the pressure gradient inside the membrane which is the driving force. L_p is as mentioned earlier the permeability coefficient which depends on the resistance towards mass transport in the membrane (the membrane resistance) and the dynamic viscosity of the fluid that flows through the membrane. The resistance inside the membrane depends on many parameters such as pore size distribution, pore radius, torosity of the pores, hydrophilic/hydrophobic nature of the membrane material compared to the fluid etc. The permeability depends on the temperature since the dynamic viscosity is a function of the temperature, and the pressure dependence of the permeability can also play a role if the membrane material is compressible.

2.5   Electrical conductance, transport of electricity

A flux of charges is induced when a driving force in the form of an electrical field is applied over a medium or a material that is able to conduct electrical charges. An electrical field has the unit of [V/m] corresponding to a voltage drop. In a wire or power cable the electrical flux consists of moving electrons – either in one direction (DC, direct current) or in alternating direction (AC, alternating current). The flux multiplied with the cross sectional area of the wire corresponds to the current measured in amperes [A]. The build-up of a voltage profile in a short piece of wire is sketched in Figure 5.
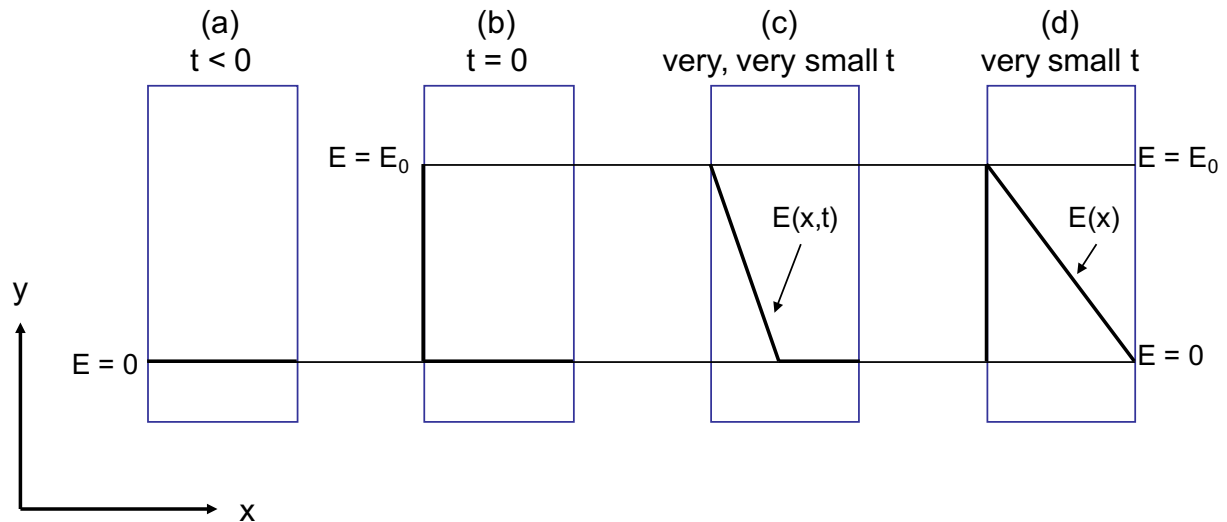
Figure 5: Build-up of voltage profile in a short piece of wire
(a) The voltage *E* in both ends of the wire is zero. (b) At *t* = 0 the voltage in the left end of the wire is increased to E_0 while the voltage in the right end is kept a zero. (c) Electrical charges (electrons) start to migrate through the piece of wire because they are in an electrical field. At very, very small values of time *t*, the voltage in the wire is a function of both time and distance *x*. (d) Very fast after the voltage in the left end is increased, steady state is reached and a linear voltage profile in wire is established. Thus at steady state the voltage is only a function of the distance *x* in the wire.

Initially the voltage (energy pr. charge) is zero in both ends of the piece of wire. No charges will then migrate through the wire. At time *t* = 0 the voltage in the left end is suddenly raised to E_0 at which it is held constant. Charges start to migrate through the wire because of the voltage difference. The voltage in the right end is kept at zero. The voltage thus increases in the wire as a function of the distance *x* and the time *t*. Because the wire is an electrical conductor, a steady state linear voltage profile in the wire is established very fast, and thereafter the voltage is only a function of the distance *x* in the wire.

At steady state the electrical flux (current density) through the wire is given by the flux equation from Table 1, which is called *Ohm's law*:

$$J_e = -\sigma \cdot \frac{dE}{dx} \tag{8}$$

The flux J_e is the flux of charges in the direction *x* in the wire. The gradient *dE/dx,* which is the voltage gradient inside the wire, is the driving force and is also called the electrical field. In an electrical field a particle with a charge *q* will experience a force *F* equivalent to its charge *q* multiplied with the electrical field strength. The conductance *σ* of the wire is inversely proportional to the electrical resistivity of the wire. The resistivity tells how much the material or medium opposes the flow of the electrical current.

For a typical metal the resistivity normally increases linearly with the temperature which means that the electrical conductance decreases with the temperature.

3   Dimensionless numbers

In this section the first three transport phenomena (mass, energy, and momentum) will be included in the explanation of different dimensionless numbers often used in physics and chemical engineering. For those three transport phenomena certain "diffusion coefficients" are able to describe how "easy" the transport takes place:

- *Ordinary diffusion coefficient, D* [m²/s]: Tells how "easy" transport of mass takes place as diffusion of "kilo grams" of component *A* through a media of component *B*.
- *Thermal diffusivity, α* [m²/s]: Tell how "easy" transport of energy/heat takes place as diffusion of "joules" through a material.
- *Kinematic viscosity, v* [m²/s]: Tells how "easy" transport of momentum takes place as diffusion of velocity though a fluid.

Since these terms all have the same units [m²/s] they can be used to calculate certain dimensionless numbers describing properties of different fluids. These dimensionless numbers and their physical meaning are summed up in Table 2.

Dimensionless number	Definition	Physical meaning
Lewis number	$Le = \dfrac{\alpha}{D_{AB}}$	The Lewis number tells how fast energy/heat propagates through the fluid compared to how fast mass (component *A*) propagates (diffuses) though the fluid (component *B*).
Schmidt number	$Sc = \dfrac{v}{D_{AB}}$	The Schmidt number tells how fast velocity propagates through the fluid compared to how fast mass (component *A*) propagates (diffuses) though the fluid (component *B*).
Prandtl number	$Pr = \dfrac{v}{\alpha}$	The Prandtl number tells how fast velocity propagates through the fluid compared to how fast energy propagates through the fluid.

Table 2: Dimensionless numbers
Definition and physical meaning of the Lewis, Schmidt, and Prandtl number.

The dimensionless numbers in Table 2 can by used in different dimensionless equations for describing systems in which competing transport processes occur.

4 Summary

In this book it has been shown that different kinds of transports (mass, energy, momentum, volume, and electrical charges) can be described by simple phenomenological equations of the same kind. The flux is in all cases proportional to a driving force which is the gradient of concentration, temperature, velocity, pressure, or voltage, respectively. The proportionality constant is called a phenomenological coefficient and corresponds to well know physical terms such as the diffusion coefficient, thermal conductivity, dynamic viscosity, permeability, or electrical conductance.

For the transport of energy and momentum, the ease at which the transport takes place, not only depends on the phenomenological coefficient but also on other physical properties such as heat capacity and density. By combining these terms with the phenomenological coefficient, the thermal diffusivity and the kinematic viscosity are defined. These terms have the same units as the ordinary diffusion coefficient and can thus be used in the comparison between competing transport processes in fluids. For this purpose three dimensionless numbers (Lewis, Schmidt, and Prandtl number) are defined as the different ratios between the ordinary diffusion coefficient, the thermal diffusivity and the kinematic viscosity – associated with the molecular transport of mass, energy, and momentum, respectively. These numbers can be used in dimensionless equations describing systems in which competing transport phenomena occur.