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GARCH Models with Fat-Tailed Distributions and the Hong Kong Stock Market Returns

Zi-Yi Guo[1]

[1] Corporate Market & Institution Risk Group, Wells Fargo Bank, USA

Correspondence: Zi-Yi Guo, Corporate Market & Institution Risk Group, Wells Fargo Bank, N.A., 301 S College St, Charlotte, NC28202, USA. E-mail: zachguo0824@gmail.com.

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Abstract

As one of the world's largest securities markets, the Hong Kong stock market plays a significant role in facilitating the development of Chinese economy. In this paper, we investigate a suite of widely-used models, the GARCH models in risk management of the Hong Kong stock market returns. To account for conditional volatilities, we consider a new type of fat-tailed distribution, the normal reciprocal inverse Gaussian distribution (NRIG), and compare its empirical performance with two other popular types of fat-tailed distribution, the Student's *t* distribution and the normal inverse Gaussian distribution (NIG). We show that the NRIG distribution performs slightly better than the other two types of distribution. Also, our results indicate that it is important to introduce both GJR-terms and the NRIG distribution to improve the models' performance. Our results illustrate that the asymmetric GARCH NRIG model has practical advantages in quantitative risk management, and serves as a very useful tool for industry participants.

Keywords: fat-tailed distribution, GJR-GARCH model, model comparison

JEL classifications: C22; C52; G17

1. Introduction

The autoregressive conditional heteroskedasticity (ARCH) and generalized autoregressive conditional heteroskedasticity (GARCH) models have had great success in modeling volatilities of asset prices series in the past three decades. Although GARCH models themselves can predict unconditional excess kurtosis of the data, to account for conditional leptokurtosis one usually assumes the innovations to be Student's *t* distributed as in Bollerslev (1987). Recently, there has been growing interests in other types of fat-tailed distribution in the finance literature, such as generalized hyperbolic distribution (Barndorff-Nielsen, 1977), and GARCH models with the generalized hyperbolic distribution have gained more and more interests, e.g. Andersson (2001), Jensen and Lunde (2001), Venter and de Jongh (2002), Forsberg and Bollerslev (2002), Stentoft and (2008). To explain the non-zero skewness observed in the financial data, in addition to assuming the distribution to be asymmetric another approach is to assume conditional volatilities respond differently to negative and positive innovations. A non-exhaustive list includes Engle (1990), Glosten, Jagannathan and Runkle (1993) and Zakoian (1994). Nevertheless, most of the above literature assumes either the responses of the conditional volatilities or the distribution to be symmetric.

Here, we incorporate the generalized hyperbolic distribution into symmetric and asymmetric GARCH models. We specify the generalized hyperbolic distribution as in Prause (1999) and focus on two special cases: the normal inverse Gaussian (NIG) distribution and the normal reciprocal inverse Gaussian (NRIG) distribution. As discussed in Prause (1999), the main factor for the speed of the estimation is the number of modified Bessel functions to compute. By fixing the index parameter in the generalized hyperbolic distribution to be -1/2 or 1/2, the probability density function only contains one Bessel function instead of two, which significantly reduces the time to estimate the distribution parameters. As the generalized hyperbolic distribution, both the NIG distribution and the NRIG distribution could generate fat tails and skewness. Currently, the NIG distribution is one of the most popular special cases among the generalized hyperbolic distribution family (see Figueroa-Lopez, Lancette, Lee and Mi, 2011, for a survey). Recent studies show that the GARCH-NIG model provides accurate and parsimonious representations in modeling a variety of asset series, such as exchange rate dynamics in Andersson (2001) and Forsberg and Bollerslev (2002), stock returns in Jensen and Lunde (2001), option pricing in Stentoft

(2008), and oil future prices in Hansen and Lunde (2015). Nevertheless, to the best of our knowledge this paper is the first one which takes the NRIG distribution into consideration with GARCH models. We apply GARCH models with the NRIG distribution to the Hong Kong Hang Seng Index (HSI) return series and compare them with GARCH models with the NIG distribution or the Student's *t* distribution. Our results show that the NRIG distribution performs slightly better than the other two types of distribution.

There are quite extensive studies about GARCH models on the Hong Kong stock market returns. For instance, Kim and Mei (2001) employed a GARCH-jump process to investigate the possible market impact of political risk. The paper showed that political developments in Hong Kong have a significant impact on its market volatility and returns. Qiao, Chiang and Wong adopted a novel FIVECM-BEKK GARCH approach to examine the bilateral relationships among the A-share and B-share stock markets in China and the Hong Kong stock market. Qiao, et al. showed that these stock markets are fractionally cointegrated with spillover effects. Wang, Rui and Firth (2002) investigated how returns and volatilities of stocks are correlated for dually-traded stocks on two non-synchronous international markets, the Hong Kong and London stock markets. Using daily data for the period from October 1996 to July 2000, Wang, Rui and Firth found evidence of returns and volatility spillovers from Hong Kong to London, and from London to Hong Kong, and the results are especially astonishing during the Asian financial crisis. In this paper, we focus on risk management and apply Value at Risk as the risk measure. Our backtesting results show that the GJR-GARCH NRIG model could generate satisfactory VaRs. In this article, we are interested in the Hong Kong stock market. Similar results have been obtained in other financial markets in Guo (2017a).

The remainder of the paper is organized as follows. In Section 2, we discuss GARCH models and the three types of fat-tailed distribution. Section 3 summarizes the data. The estimation results are in Section 4. In Section 5, we compare numerical performance of GARCH models with the three types of fat-tailed distribution. Section 6 concludes.

2. The Models

We consider a simple GARCH(1,1) process as:

$$\varepsilon_t = \mu + \sigma_t e_t \tag{1}$$

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\ 2} + \beta_1 \sigma_{t-1}^{\ 2} \tag{2}$$

Where the three positive numbers α_0, α_1 and β_1 are the parameters of the process and $\alpha_1 + \beta_1 < 1$. The assumption of a constant mean return μ is purely for simplification and reflects that the focus of the paper is on dynamics of return volatility instead of dynamics of returns. To account for asymmetric responses of conditional volatilities to negative and positive innovations observed in the data, we consider replacing equation (2) to extend the GARCH process as in Glosten, Jagannathan and Runkle (1993). The GJR (1,1,1)-GARCH model is specified as:

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\ 2} + \gamma \varepsilon_{t-1}^{\ 2} I(\varepsilon_{t-1} < 0) + \beta_1 \sigma_{t-1}^{\ 2} \tag{3}$$

Where $I(\cdot)$ denotes the indicator function and $\alpha_1 + \gamma E[I(\varepsilon_{t-1} < 0)] + \beta_1 < 1$. The variable e_t is identically and independently distributed (*i.i.d.*). Three types of fat-tailed distribution are assumed: the Student's *t* distribution, the normal inverse Gaussian (NIG) distribution, and the normal reciprocal inverse Gaussian (NRIG) distribution. The density function of the standard Student's *t* distribution with ν degrees of freedom is given by:

$$f(e_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^{\ 2}}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \ \nu > 4. \tag{4}$$

with excess kurtosis $\frac{6}{\nu-4}$, where ψ_{t-1} denotes the σ-field generated by all the available information up through time $t-1$. The conditional density function of ε_t then can be written as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi\sigma_t^{\,2}]^{1/2}} \left(1 + \frac{\varepsilon_t^{\,2}}{(\nu-2)\sigma_t^{\,2}}\right)^{-\frac{\nu+1}{2}}, \ \nu > 4. \tag{5}$$

To allow possible skewness of innovations, we extend the above symmetric distribution to the skewed Student's *t* distribution as in Hansen (1994):

$$f(e_t \mid \nu, \beta) = \begin{cases} bc\left(1 + \frac{1}{\nu-2}\left(\frac{be_t + a}{1-\beta}\right)^2\right)^{-(\nu+1)/2} & e_t < -a/b \\ bc\left(1 + \frac{1}{\nu-2}\left(\frac{be_t + a}{1+\beta}\right)^2\right)^{-(\nu+1)/2} & e_t \geq -a/b \end{cases} \tag{6}$$

where the constants a, b and c are given by $a = 4\beta c\left(\frac{\nu-2}{\nu-1}\right)$, $b^2 = 1 + 3\beta^2 - a^2$, and $c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The density function has a mode of $-a/b$, a mean of zero, and a unit variance. The density function is skewed to the right when $\beta > 0$, and vice-versa when $\beta < 0$. The skewed Student's *t* distribution specializes to the standard Student's *t* distribution by setting the parameter $\beta = 0$.

GARCH models with the Student's *t* distribution have been extensively investigated to analyze volatilities of asset returns. In this paper, we also consider the NIG distribution and the NRIG distribution as special cases of the generalized hyperbolic distribution. The generalized hyperbolic distribution is specified as in Prause (1999):

$$f(e_t \mid \lambda, \mu, \alpha, \beta, \delta) = \frac{(\sqrt{\alpha^2 - \beta^2}/\delta)^{\lambda} K_{\lambda-1/2}(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2 + (e_t - \mu)^2}/\alpha)^{1/2-\lambda} K_{\lambda}(\delta\sqrt{\alpha^2 - \beta^2})} \exp(\beta(e_t - \mu)) \tag{7}$$

where $K_\lambda(\cdot)$ is the modified Bessel function of the third kind and index $\lambda \in \mathbb{R}$ and: $\delta > 0$, $0 \leq |\beta| < \alpha$.

When $\lambda = -\frac{1}{2}$, the Bessel function in the denominator has a closed-form solution, and we have the NIG distribution as:

$$f(e_t \mid \mu, \alpha, \beta, \delta) = \frac{\alpha\delta K_1(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\pi\sqrt{\delta^2 + (e_t - \mu)^2}} \exp(\delta\sqrt{\alpha^2 - \beta^2} + \beta(e_t - \mu)) \tag{8}$$

The NIG distribution is normalized by setting $\mu = -\frac{\delta\beta}{\sqrt{\alpha^2 - \beta^2}}$ and $\delta = \frac{\left(\sqrt{\alpha^2 - \beta^2}\right)^3}{\alpha^2}$, which implies $E(e_t) = 0$ and $Var(e_t) = 1$. When $\beta = 0$, the NIG distribution is symmetric with excess kurtosis $\frac{3}{\alpha^2}$ and the conditional density function of ε_t can be written as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\alpha^2 K_1(\frac{\alpha}{\sigma_t}\sqrt{\alpha^2\sigma_t^{\ 2}+\varepsilon_t^{\ 2}})}{\pi\sqrt{\alpha^2\sigma_t^{\ 2}+\varepsilon_t^{\ 2}}}\exp(\alpha^2)\,. \tag{9}$$

When $\lambda = \frac{1}{2}$, we have the NRIG distribution as:

$$f(e_t \mid \mu,\alpha,\beta,\delta) = \frac{\sqrt{\alpha^2-\beta^2}K_0(\alpha\sqrt{\delta^2+(e_t-\mu)^2})}{\pi}\exp(\delta\sqrt{\alpha^2-\beta^2}+\beta(e_t-\mu))\,. \quad 10)$$

The NRIG distribution is normalized by setting $\mu = -\left(\frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}+\frac{\beta}{\alpha^2-\beta^2}\right)$ and $\delta = \frac{1}{\alpha^2\sqrt{\alpha^2-\beta^2}}[(\alpha^2-\beta^2)^2-\alpha^2-\beta^2]$, which implies $E(e_t)=0$ and $Var(e_t)=1$. When $\beta=0$, the NRIG distribution is symmetric with excess kurtosis $\frac{3}{\alpha^2}+\frac{3}{\alpha^4}$ and the conditional density function of ε_t can be written as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\alpha K_0(\sqrt{(\alpha^2-1)^2+\frac{\alpha^2\varepsilon_t^{\ 2}}{\sigma_t^{\ 2}}})}{\pi\sigma_t}\exp(\alpha^2-1) \tag{11}$$

The likelihood function for a sample of T observations $\{\varepsilon_1,\varepsilon_2,...,\varepsilon_T\}$ is given by:

$$f(\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_T) = f(\varepsilon_T \mid \varepsilon_1,\cdots,\varepsilon_{T-1})f(\varepsilon_{T-1} \mid \varepsilon_1,\cdots,\varepsilon_{T-2})\cdots f(\varepsilon_1) \tag{12}$$

3. Data and Summary Statistics

The empirical performance of GARCH models with fat-tailed distribution using the Hong Kong stock market returns series is investigated. Three different time periods of the HSI are selected: HSI (1), HSI (2) and HSI (3), which span from September 11, 2011, August 9, 2007 and December30, 1986 to November 6, 2013 respectively, indicating a recent 500 rolling window, the Subprime crisis, and the longest available time period when the data was collected for the research. All the data are collected from Yahoo Finance.

Table 1 summarizes simple statistics of the data. The data present the standard set of well-known stylized facts of asset prices series: non-normality, limited evidence of short-term predictability in return and strong evidence of predictability in volatility. All series are presented in daily percentage growth rates/returns. The Bera–Jarque test conclusively rejects normality of raw returns in all series, which confirms our assumption that the model selected should account for the fat-tail phenomenon. The smallest test statistic is much higher than the 5% critical value of 5.99. The market index is negatively skewed and has fat tails. The asymptotic SE of the skewness statistic under the null of normality is $\sqrt{6/T}$, and the SE of the kurtosis statistic is $\sqrt{24/T}$, where T is the number of observations. Almost all series show statistically significant leptokurtosis, indicating that accounting for fat-tailness is more pressing than skewness in modelling asset prices dynamics.

Table 1. Summary statistics

Series	Obs.	Mean	Std.	Skewness	Kurtosis	BJ	Q(5)	QARCH(5)	Q2(5)

HSI (1)	500	0.056	0.744	-0.004	5.668**	95.1**	10.72**	5.41	101.28**
HSI (2)	1542	0.018	1.352	-0.251**	7.751**	189.4**	17.31**	5.15	135.33**
HSI (3)	6637	0.036	1.084	-0.762*	8.433**	355.1**	25.33**	6.32	187.45**

Notes. BJ is the Bera-Jarque statistic and is distributed as chi-squared with 2 degrees of freedom, Q(5) is the Ljung-Box Portmanteau statistic, $Q^{ARCH}(5)$ is the Ljung-Box Portmanteau statistic adjusted for ARCH effects following Diebold (1986) and $Q^2(5)$ is the Ljung-Box test for serial correlation in the squared residuals. The three Q statistics are calculated with 5 lags and are distributed as chi-squared with 5 degrees of freedom.

* and ** denote a skewness, kurtosis, BJ or Q statistically significant at the 5% and 1% level respectively.

We use the Ljung-Box portmanteau, or Q, statistic with five lags to test for serial correlation in the data, and adjust the Q statistic for ARCH models following Diebold (1986). The results that no serial correlation is found for almost all the series confirm our assumption of a constant mean return μ in Equation (1). The evidence of linear dependence in the squared demeaned returns, which is an indication of ARCH effects, is significant for all the series.

4. Estimation Results

We investigate three different models: GARCH (1,1) model with symmetric innovations, GJR (1,1,1)-GARCH model with symmetric innovations, and GJR (1,1,1)-GARCH model with skewed innovations. The maximum likelihood estimator of the parameter vector θ ($\theta = (\alpha_0, \alpha_1, \beta_1, \nu, \beta)$ for the Student's *t* distribution and $\theta = (\alpha_0, \alpha_1, \beta_1, \alpha, \beta)$ for the NIG distribution and the NRIG distribution) is defined by maximizing the log-likelihood function of equation (12):

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} \log(f(\varepsilon_t \mid \varepsilon_1, \cdots, \varepsilon_{t-1})) \tag{13}$$

Table 2 reports estimation results of GARCH models with the three types of fat-tailed distribution for all the HSI index series. Almost all the parameters are significantly different from zero except β for the time period from September 11, 2011 to November 6, 2013. The significance of parameter $1/\nu(1/\alpha)$ indicates necessity of fat-tailed innovations in explaining leptokurtosis of the data. The significance of either γ or β indicates non-zero skewness of the market index series. Furthermore, both positive values of γ and negative values of β predict the negative skewness observed in our preliminary analysis in Table 1.

Table 2. Estimation results for the HIS series with different time periods for GARCH models with fat-tailed innovations

		alpha0	alpha1	gamma	beta1	1/nu (1/alpha)	beta	log-likelihood
Sep. 27, 2011 - Nov. 6, 2013								
Benchmark								
	Student's t	0.048**	0.123**		0.821**	0.131**		-621.3
	NIG	0.049**	0.123**		0.820**	0.710**		-620.2
	NRIG	0.050**	0.124**		0.819**	0.612**		-619.6
with GJR terms								
	Student's t	0.042**	0.002**	0.294**	0.815**	0.131**		-611.5
	NIG	0.043**	0.003**	0.295**	0.814**	0.671**		-610.7
	NRIG	0.043**	0.003**	0.297**	0.813**	0.583**		-610.6
with GJR and skewed innovations								

	Student's t	0.043**	0.002**	0.298**	0.813**	0.121**	-0.081	-610.6
	NIG	0.043**	0.002**	0.302**	0.811**	0.657**	-0.210	-609.8
	NRIG	0.043**	0.002**	0.303**	0.811**	0.569**	-0.205	-609.6
Aug. 9, 2007 - Nov. 6, 2013								
Benchmark								
	Student's t	0.024**	0.101**		0.892**	0.151**		-2416.5
	NIG	0.024**	0.101**		0.890**	0.770**		-2412.8
	NRIG	0.025**	0.102**		0.890**	0.656**		-2411.5
with GJR terms								
	Student's t	0.028**	0.002**	0.185**	0.895**	0.131**		-2363.2
	NIG	0.028**	0.002**	0.185**	0.895**	0.699**		-2360.5
	NRIG	0.028**	0.002**	0.186**	0.894**	0.593**		-2369.5
with GJR and skewed innovations								
	Student's t	0.028**	0.002**	0.188**	0.894**	0.135**	-0.152**	-2340.6
	NIG	0.028**	0.002**	0.187**	0.894**	0.632**	-0.402**	-2337.2
	NRIG	0.029**	0.002**	0.189**	0.893**	0.566**	-0.376**	-2336.5
Dec. 31, 1986 - Nov. 6, 2013								
Benchmark								
	Student's t	0.013**	0.062**		0.933**	0.161**		-10158.3
	NIG	0.014**	0.065**		0.931**	0.772**		-10161.2
	NRIG	0.014**	0.065**		0.930**	0.637**		-10161.5
with GJR terms								
	Student's t	0.018**	0.011	0.111**	0.922**	0.141**		-10096.2
	NIG	0.019**	0.010	0.114**	0.921**	0.725**		-10099.3
	NRIG	0.019**	0.010	0.114**	0.920**	0.601**		-10099.7
with GJR and skewed innovations								
	Student's t	0.018**	0.011*	0.112**	0.922**	0.142**	-0.071**	-10084.6
	NIG	0.019**	0.011*	0.112**	0.921**	0.704**	-0.173**	-10083.2
	NRIG	0.019**	0.011*	0.113**	0.920**	0.594**	-0.175**	-10083.1

Note. * and ** denote statistical significance at the 5% and 1% level respectively.

One interesting fact is that, for all the four models and all the time series, GARCH models with the NRIG distribution return the highest log likelihood among the three types of fat-tailed innovations, followed by GARCH models with the NIG distribution, and GARCH models with the Student's *t* distribution return the lowest log likelihood.

5. Model Comparison

We have introduced four different GARCH models, and each model is associated with three types of fat-tailed distribution. A question would be which model with a particular type of distribution is the most appropriate candidate in application. Our statistical test results from Table 2 suggest that it is important to have fat-tailed innovations for all the series. To explain non-zero skewness, it is of importance to introduce both GJR terms in GARCH processes and asymmetric innovations for the long market index series, but for the short market index series it is only of importance to introduce GJR terms.

The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for model comparison are used for model comparison. The BIC is asymptotically valid when we consider finite-number of parameters. The AIC penalizes the number of parameters less strongly than does the BIC. Table 3 confirms our finding in the hypothesis testing procedures that it is crucial to introduce both GJR terms in GARCH processes and asymmetric innovations in modelling the long market index series according to both criteria. For the short market index series it is still important to introduce GJR terms. Among the three types of fat-tailed distribution, almost all models are in favor of the NRIG distribution in modelling the market index series.

Table 3. Model comparison by the Akaike information criterion (AIC) and the Bayesian information criterion (BIC)

	BIC			AIC		
	Student's T	NIG	NRIG	Student's T	NIG	NRIG
Benchmark						
HSI (1)	1259.1	1237.5	1236.5	1222.6	1220.4	1219.7
HSI (2)	4862.2	4835.3	4832.2	4821.2	4813.2	4810.3
HSI (3)	20593.7	20579.2	20579.4	20545.1	20551.6	20551.5
with GJR terms						
HSI (1)	1240.7	1218.3	1218.1	1199.5	1197.7	1197.5
HSI (2)	4802.2	4778.2	4776.6	4756.2	4751.2	4749.3
HSI (3)	20477.4	20463.4	20463.2	20423.8	20428.5	20428.1
with GJR and skewed innovations						
HSI (1)	1244.1	1222.2	1222.5	1197.1	1197.7	1197.6
HSI (2)	4785.3	4758.5	4756.2	4733.5	4726.3	4724.3
HSI (3)	20463.7	20439.1	20440.4	20401.9	20398.1	20398.2

In summary, we could not find that GJR-GARCH model with the Student's *t* distribution and GJR-GARCH model with the NRIG distribution perform better for most of the series.

6. GJR-GARCH NRIG Model

Our last two sections illustrate that the GJR-GARCH model with the NRIG distribution has relatively better empirical performance. As discussed in the introduction section, both the NIG and NRIG distributions have finite MGF at unity, and when the MGF is finite the NRIG distribution could generate a larger value of excess kurtosis than the NIG distribution. Both the NIG and NRIG distributions have the potential to be used in quantitative risk management modeling, but the NRIG distribution has more freedom to fit asset returns with heavier tails. Considering the advantage of the NRIG distribution in risk management practice, we investigatethe GJR-GARCH model with the NRIG distribution, and fit the model on a rolling 500-day period. We extend the series spanning from January 3, 2005 to November 6, 2013, in total 2228 observations, roughly covering a full business cycle. The estimated parameters of the model are shown in Figure 1. The model is estimated consistently, and we cannot see no big jumps from day to day and the range of attainable values is relatively narrow.

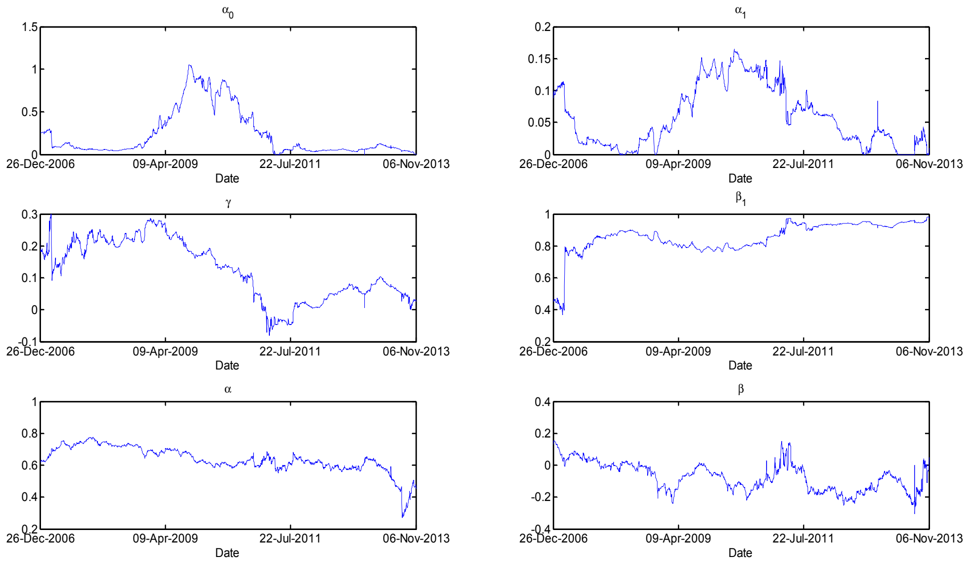

Figure 1. Estimated parameters of GJR-GARCH model with NRIG distribution from January 3, 2005 to November 6, 2013

One important aspect of the evaluated model performance was comparing actual profits & losses with projected VaR estimates from one-day ahead simulation forecast. We consider the market indexspanning from February 8, 2007 to November 6, 2013, in total 1700 observations. In Table 4, we summarize the daily exceedances over 95%, 97.5%, 99%, and 99.5% for both a long and a short positions. Most of the Kupiec's test results show that one could not reject the model proposed for VaR calculation.

Table 4. Backtesting

Stock	VaR95long	VaR95short	VaR975long	VaR975short	VaR99long	VaR99short	VaR995long	VaR995short
HSI	5.87%	6.01%	3.47%*	2.80%	1.27%	1.13%	0.47%	0.47%

Note. * and ** denote statistical significance for the Kupiec's test at the 5% and 1% level respectively.

7. Conclusion

The empirical performance of GARCH and GJR-GARCH models with three types of fat-tailed innovations has been evaluated: the Student's *t* distribution, the normal inverse Gaussian distribution, and the normal reciprocal inverse Gaussian distribution using the Hong Kong Hang Seng Index return series. By checking several commonly used criteria, the rules of thumb for practical implementation obtained from our experiments are as follows:

(1) It is important to incorporate fat-tailed innovations in fitting all the asset prices series to account for leptokurtosis, while for non-zero skewness it is helpful to incorporate both GJR-terms into GARCH processes and asymmetry into innovations for the market index series;

(2) The performance of the three types of fat-tailed distribution in fitting the financial data is quite similar, and the NRIG distribution dominates the other two types of distribution;

(3) GJR-GARCH models with the NRIG distribution perform very well for VaR calculations, and therefore could be valuable for quantitative risk management and serves as a very useful tool for industry participants.

The paper could be extended in several directions. First, some other market index in addition to the HSI and some other types of financial assets, for instance commodities as in Guo (2017b), may be studied. Second, the paper only considers asymmetric response of conditional volatilities to negative and positive shocks as in Glosten, Jagannathan and Runkle (1993). It would be valuable to consider other types of models which share a similar feature, such as in Zakoian (1994). Finally, there are some other special types of fat-tailed distribution among the generalized hyperbolic family, such as the hyperbolic distribution. It would be interesting to compare the NRIG distribution with other special types of fat-tailed distribution.

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