Constantinos G. Vayenas
Stamatios N.-A. Souentie

Gravity, Special Relativity, and the Strong Force

A Bohr-Einstein-de Broglie Model for the Formation of Hadrons

$$E = 3\gamma m_o c^2$$

$$R = \frac{\hbar}{\gamma m_o v}$$

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To Julia-Alkestis, Vassia-Evangelia,
George-Philippe and Anna-Thalia

Preface

This book presents a novel Bohr-type rotating particle model which describes how gravity can cause confinement of highly energetic neutrinos in bound rotational states and thus, very surprisingly, lead to the formation of hadrons and nuclei. The approach is deterministic, simple, and easy to follow by those trained as physicists or chemists, including physical chemists, or chemical engineers, as it follows exactly the steps of the Bohr treatment of the H atom. The model contains no adjustable parameters.

Consequently this book is addressed to those natural scientists, physicists, chemists, or engineers, who once enjoyed reading in their freshman Physics or Chemistry class about the simple Bohr model description of the H atom, and to those who once, in their early Physics courses, got excited about the elegance of Einstein's special relativity and with its initially surprising and perhaps counterintuitive results, and also to those who, perhaps as early as high school, were impressed when they first encountered the de Broglie wavelength expression, i.e., $\lambda = h/p$, linking the particle momentum p with its equivalent wavelength λ via the Planck constant, h, and thus expressing in a concise manner the dual, corpuscular and ondular (i.e., wave), nature of matter.

Via the synthesis of these long established concepts, the book shows how neutrinos can be trapped by gravity in rotational states which surprisingly have all the properties of baryons, such as neutrons and protons.

No matter how interesting all this may sound, a physicist might still ask why it is worthwhile to read a book on a deterministic Bohr-type model which is, exactly, a century-old approach as this book is written. There are two good reasons: The first is the excellent agreement of experiment with the model, which contains no adjustable parameters, shown briefly in Table 1.1 (p. 11). The second reason is that, as shown, e.g., in Table 6.3 (p. 79) or in Fig. 6.7 (p. 81) or on the cover of this book, the problem solved here is, similar to the Bohr H atom model, a problem in classical mechanics, coupled with an extra algebraic equation, the de Broglie wavelength equation, which ensures conformity with the basis of quantum mechanics. And when it comes to mechanics, physicists, engineers, and also physical chemists, can be equally effective.

Physical chemists and engineers, on the other hand, no matter how impressed with Bohr's H atom model, may wonder why it may be worthwhile to read a book on a variation of this model using gravity as the attractive force, accounting for special relativity and taking us from the comfortable *nm* scale down to the *fm* scale in the exotic world of high-energy physics with relativistic speeds, fast neutrinos and perhaps with the key mysterious players of the strong force, quarks, gluons, and color charge, all of them somehow keeping protons and neutrons together. A first reason is that, aside from the rather general scientific interest of some of the questions touched in this book, the physical chemist or engineer may be happy to realize that his/her modeling may easily be extended to the subatomic world. A second reason is that in the first five chapters of the book they will find a very elementary and practically oriented introduction to these topics and to special relativity, written by two colleagues formally trained as engineers and physical chemists. These chapters aspire to familiarize them with the terminology and basic concepts of these areas and to prepare them for the not only very simple but also central Chapter 6 which presents the Bohr-type rotating neutrino model, leading surprisingly to the formation of hadrons, i.e., to the formation of ordinary matter.

Similar to the Bohr treatment of the H atom, the model contains two parts, a classical mechanical part and a second part where the de Broglie wavelength equation is used to select, among the infinity of solutions obtained in the classical part, the ones which are consistent with the de Broglie wavelength equation and thus with the basis of quantum mechanics. The first part of the model accounts for the corpuscular nature of neutrinos and the second part accounts for the ondular nature of the same particles.

In the same way that a proton and a rotating electron form an H atom in the Bohr model, the present model shows how three very fast rotating neutrinos attracted by gravity can form a baryon, e.g., a neutron.

The only difference from the Bohr treatment of the H atom is that in the present case, due to the relativistic particle velocities involved, the special relativity theory has to be used in the classical mechanical derivation part, coupled with the equivalence principle of inertial and gravitational mass.

A first conclusion emerging from the analysis is that neutrinos are apparently the basic building blocks of baryons, such as neutrons and protons. A second equally surprising conclusion is that the strong interaction force has all the features of the Newtonian gravitational force exerted between two particles when accounting for their relativistic velocities and for special relativity.

One may righteously ask where the extra mass comes from when three neutrinos with rest masses of the order of $0.1\ \mathrm{eV/c^2}$ or 10^{-37} kg each form a baryon, such as a neutron or proton with a mass of the order of $1\ \mathrm{GeV/c^2}$ or 10^{-27} kg. The answer is simple and easy to understand: It is the very large relativistic kinetic energy of the rotating neutrinos which constitutes the rest energy, thus the rest mass, of the rotating bound baryon state. To much more than 99%, the mass of baryons is the kinetic energy of their constituents, a conclusion reached recently by some theoretical physicists using much more complex approaches.

Chapter 1 provides an introduction to the main theme of this book and is addressed to all interested readers. Chapters 2–4 are addressed primarily to those trained as chemists and engineers, as they provide an introduction to the concepts of mass (Chapter 2), of the strong force (Chapter 3), and of elementary particles (Chapter 4). Theoretical physicists may want to jump directly to Chapter 5 which discusses the equivalence principle, special relativity, Newton's gravitational law, and the synthesis of these three cornerstones of Physics to produce an analytical expression for the gravitational force under relativistic conditions. The relation of this expression to general relativity is addressed both in this and in a subsequent chapter.

Chapter 6 presents the classical mechanical relativistic treatment of the two-neutrino and three-neutrino circular motion problems posed above and determines those solutions of the classical problem which are consistent with the de Broglie wavelength expression and thus with the basis of quantum mechanics.

Chapter 7 discusses the properties of the gravitational bound states and compares with experiment to show that these bound states correspond, surprisingly, to hadrons. The three-neutrino model describes baryons and the two-neutrino model produces mesons.

Chapter 8 presents how rest mass is generated from the high kinetic energy of the rotating neutrinos via the formation of hadrons, while Chapter 9 compares the three-neutrino model with the main experimental features of the strong force and with the standard model.

Chapter 10 discusses the usefulness of the deterministic Bohr–Einstein–de Broglie approach to tackle problems involving both gravitational and electromagnetic forces from the *eV* to the *GeV* range, while Chapter 11 focuses on discussing some implications of the generalized gravitational Newtonian expression presented in this book about the nature of dark matter and also possibly of dark energy. Chapter 12 discusses model computed coupling constants and their good agreement with the basic expectations of unification theories. It also summarizes the discussion of the two main questions emerging from the analysis of the simple Bohr-type model presented in this book: "Is the strong force simply relativistic gravity?" and "Are relativistic neutrinos really the main building blocks of hadrons?"

We hope that all readers, including physicists who may start directly from Chapter 5, will then enjoy the comparison with experiment and associated topics in Chapters 7–12.

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Sincerest thanks are also expressed to our parents, George and Julia, and Nizar-Antonios and Zacharoula, and to all the members of our families for their patience and understanding.

Contents

List of Symbols

a_o	Bohr radius
a	Acceleration
c	Speed of light
e	Unit electric charge
E	Relativistic energy, $\gamma m_o c^2$
f	Final state
F	Helmholz free energy
h	Planck constant
$\hbar$	Reduced Planck constant, $h/2\pi$
$\mathcal{H}$	Hamiltonian, $E+V$
i	Initial state
K	Kelvin
L	Angular momentum
$\mathcal{L}$	Luminosity
m	Mass
m_o	Rest mass
γm_o	Relativistic mass
$\gamma^3 m_o$	Longitudinal mass, inertial mass
N	Number of constituent particles
p	Momentum
q	Electrical charge
Q	Dimensionless charge defined in Eq. (10.3)
r	Distance
R	Radius
$\mathcal{S}$	Entropy
S	Laboratory inertial frame
S'	Instantaneous frame moving with particle under examination
T	Translational energy
t	Time
V	Potential energy
v	Velocity

Greek Symbols

α Fine structure constant, $e^2/\varepsilon c\hbar$
γ Lorentz factor
γ_n γ value corresponding to the neutron $(n = 1)$ confined rotational state
ε_o Permitivity of vacuum
ε $4\pi\varepsilon_o$
Θ Absolute temperature in K
μ Magnetic moment

Acronyms

GR General relativity
QCD Quantum chromodynamics
QED Quantum electrodynamics
SR Special relativity

Chapter 1
1905–1930: The Golden Age of Physics

1.1 The Three Major Breakthroughs

Between 1905 and 1925 three of the most spectacular breakthroughs in the history of natural sciences took place: Albert Einstein's special relativity in 1905 [1], Niels Bohr's model of the H atom in 1913 [2] and Louis de Broglie's expression of the equivalent wavelength of moving particles in 1924 [3], an expression which formed eventually the basis of quantum mechanics.

These remarkable theoretical advances involved no unknown parameters and were confirmed by experiment thousands of times. This was a time of great scientific euphoria and it was reasonable to expect that several more breakthroughs would follow.

1.2 Open Problems

A century later one must admit that, despite the enormous experimental progress and the development of the general relativity theory for gravity and of the field of quantum mechanics on the basis of the Schrödinger [4, 5] or of the Dirac equation [6], progress in theoretical Physics and Chemistry has been much slower than expected, at least in producing the same type of remarkable and unquestionable theoretical breakthroughs. Thus there is increasing experimental evidence for the need for "Physics beyond the Standard Model of elementary particles." Not only this model contains too large a number, twenty-six, of adjustable parameters, but also experiment has gradually revealed several problems, e.g. that neutrinos are not massless [7, 8] and that it is rather doubtful if the Higgs boson really exists [9]. Thus the need to develop "Physics beyond the Standard Model" is becoming of paramount importance.

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Another major problem waiting for its solution is that of force unification, originally posed by Albert Einstein. Today we know that there are four types of forces, strong forces, weak forces, electromagnetic forces, and gravitational forces.

For the last two we have long established laws due to Coulomb and Newton which describe the magnitude of the electrostatic or gravitational forces:

$$F_C = \frac{q_1 q_2}{\varepsilon r^2}, \tag{1.1}$$

$$F_G = -\frac{G m_1 m_2}{r^2}. \tag{1.2}$$

The latter is believed to fail under highly relativistic conditions which is the domain of the general relativistic theory of gravitation.

Much less is unfortunately known about the strong force and it looks as if the Standard Model and quantum chromodynamics (QCD) are quite far from providing a simple expression, such as those of Eqs. (1.1) and (1.2), to describe its magnitude. We know, however, that the strong force is very significant only at short distances ($<fm$) and that in this range it increases with distance, a behavior known as color confinement. At even shorter distances it becomes quite weak, a behavior known as asymptotic freedom.

It is believed that the strong force is mediated by gluons in the same way that electromagnetic forces are mediated by virtual photons and gravitational forces are mediated by gravitons. About the latter we know very little.

The gravitational force is much weaker than the strong force or the electrostatic force. The three types of force are believed to become comparable in magnitude at distances of the order of the Planck length, $r_{\rm Pl} = (\hbar G/c^3)^{3/2} \approx 1.6 \cdot 10^{-35}$ m or energies of the order of the Planck mass $m_{\rm Pl} = (\hbar c/G)^{1/2} \approx 1.2 \cdot 10^{19}$ GeV/c^2.

Equally important with the problem of force unification is that of reconciling and perhaps synthesizing General Relativity with Quantum mechanics (Fig. 1.1).

The quest for this synthesis started in the fifties with the work of Wheeler who introduced the concept of geons, i.e. of gravitational or electromagnetic waves held together gravitationally. Geons are classical entities based on general relativity and small geons have been explored as classical models for elementary particles [10]. Today the quest for the synthesis of general relativity and quantum mechanics is known widely as Quantum Gravity [11] (Fig. 1.1).

Special Relativity was historically one of the foundations of General Relativity, the gravitational theory developed by Albert Einstein [12, 13]. Special relativistic mechanics have provided all the necessary generalizations and corrections of Newtonian mechanics at particle velocities near the speed of light.

On the other hand, Quantum Mechanics started historically from the de Broglie wavelength expression

$$\lambda = h/p = h/\gamma m \mathrm{v}, \tag{1.3}$$

which connects, via the Planck constant h, the corpuscular properties of particles, i.e. momentum, p, mass, m, velocity, v, and Lorentz factor, $\gamma (= (1 - \mathrm{v}^2/c^2)^{-1/2}$, with their ondular (wave) aspects. Today this extremely important equation is

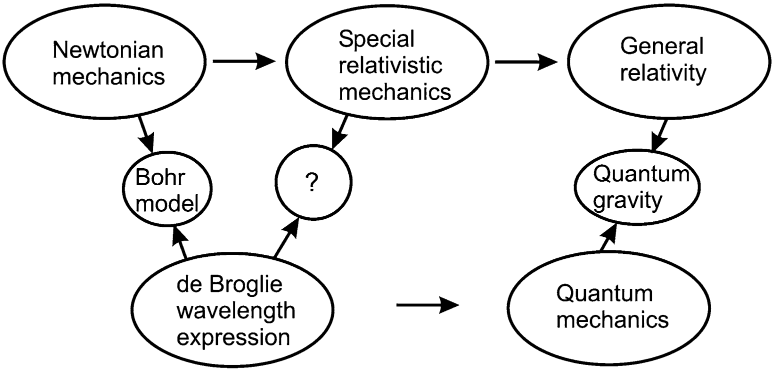
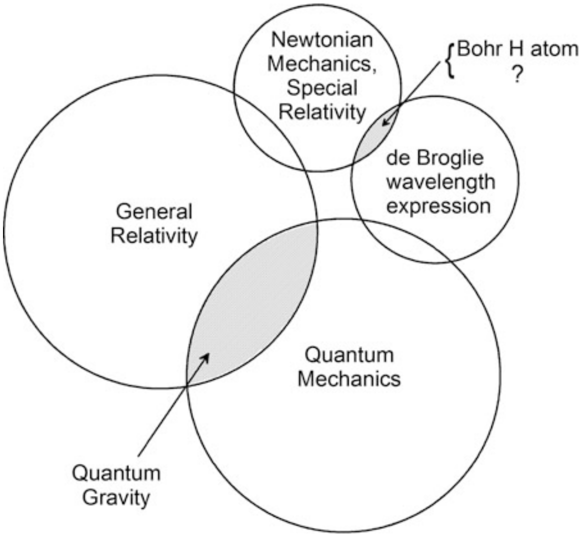


Fig. 1.1 Synthesis of Newtonian mechanics or special relativistic mechanics with the de Broglie wavelength expression; sought synthesis of general relativity and quantum mechanics; the area marked with a questionmark is the topic of this book

Fig. 1.2 Sought synthesis of general relativity with quantum mechanics; synthesis of Newtonian and relativistic mechanics with the de Broglie wavelength expression



sometimes termed “old quantum mechanics” and can be hardly considered to be part of modern quantum mechanics since neither velocity nor momentum play a role within the formalism of the Schrödinger or of the Dirac equation (Fig. 1.1).

While the synthesis of classical mechanics with the de Broglie wavelength equation has been quite fruitful in producing, already back in 1913, the Bohr model of the H atom, there has been so far no similar synthesis of the de Broglie wavelength equation with special relativistic mechanics (Fig. 1.1).

Thus while quantum gravity seeks for the synthesis of general relativity and quantum mechanics (Figs. 1.1 and 1.2), it may be also worth examining the synthesis of special relativistic mechanics with the de Broglie wavelength expression. This is the area marked with a questionmark in Figs. 1.1 and 1.2.

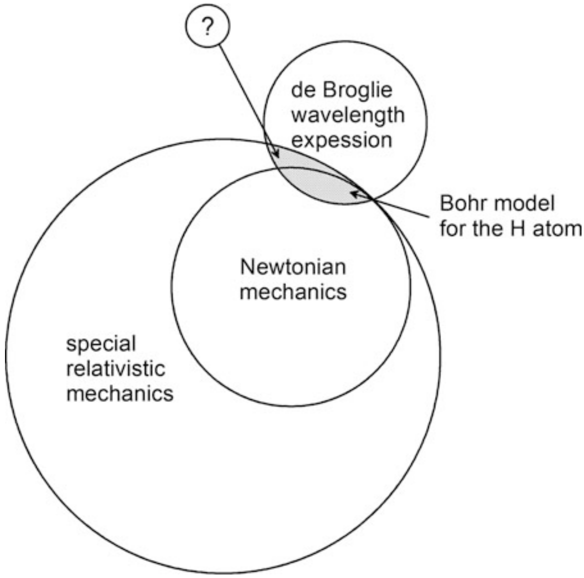


Fig. 1.3 Overlap areas of Newtonian and relativistic mechanics with the de Broglie wavelength expression

This is better illustrated in Fig. 1.3 which underlines that Newtonian mechanics is a subset of special relativistic mechanics corresponding to the limit $\gamma \sim 1$, $\mathrm{v} \ll c$ and that the Bohr model of the H atom is a product of the synthesis of Newtonian mechanics and the de Broglie wavelength expression. This model has been a real triumph of science a century ago. But what can we say about the hitherto unexplored region labeled with a questionmark in Fig. 1.3? Can the Bohr model approach be useful in studying bound states involving light particles with relativistic velocities?

1.3 A Common Starting Point for Natural Scientists: The Bohr Model for the H Atom

The Bohr model of the H atom, developed by the great Danish physicist in 1913, was an important scientific milestone of the early twentieth century and is a model which both physicists and chemists consider to be "their own," i.e. to lie within their own scientific domain.

In the Bohr model the electron is modeled as a negatively charged particle rotating around the proton in a circular orbit [3].

Since the proton mass is a factor of 1836 larger than the electron mass, one can use with a very good approximation the electron mass, m_e, in the equation of motion:

$$F = m_e \frac{\mathrm{v}^2}{R}. \tag{1.4}$$

The centripetal force, F, is the electrostatic attraction between the proton and the electron, which according to Coulomb's law is given by:

$$F = \frac{e^2}{\varepsilon R^2}, \tag{1.5}$$

where $\varepsilon = 4\pi\varepsilon_r\varepsilon_o$. From (1.4) and (1.5) one obtains:

$$R = \frac{e^2}{\varepsilon m_e \mathrm{v}^2}. \tag{1.6}$$

Since both R and v are unknown, a second equation is needed in order to solve the problem. Based on the spectral data of Lyman, Balmer, and Paschen and on his great intuition, Niels Bohr obtained a second equation by assuming quantization of the electron angular momentum, L, in the form:

$$L = m_e \mathrm{v} R = n\hbar, \tag{1.7}$$

where n is an integer and $\hbar$ is the reduced Planck constant ($\hbar = h/2\pi$ where $h = 6.626 \cdot 10^{-34}$ Js). The same result is, of course, obtained by assuming quantization of the action, $(m_e \mathrm{v}^2 \Delta t)$, where Δt is the period.

This was done in 1913, some 10 years before de Broglie proposed his famous wavelength equation:

$$\bar{\lambda} = \frac{\hbar}{p} = \frac{\hbar}{m\mathrm{v}}, \tag{1.8}$$

where p is the momentum.

Thus if one assumes that:

$$R = n\bar{\lambda} = \frac{n\hbar}{m_e \mathrm{v}}, \tag{1.9}$$

i.e. that the radius of rotation of the electron is an integer multiple of the electron de Broglie wavelength, then Eqs. (1.7) and (1.8) are identical.

Consequently the, initially criticited, angular momentum or action quantization condition of Bohr [Eq. (1.7)] is equivalent to assuming that the radius of rotation of the electron equals, for $n = 1$, its reduced de Broglie wavelength $\bar{\lambda}(= \lambda/2\pi)$.

Combining Eqs. (1.6) and (1.9) one obtains the well-known results:

$$\mathrm{v} = \frac{e^2}{n\varepsilon\hbar} \tag{1.10}$$

and therefore

$$\frac{\mathrm{v}}{c} = \frac{e^2}{n\varepsilon c\hbar} = \frac{\alpha}{n}, \tag{1.11}$$

where $\alpha(= e^2/\varepsilon c\hbar = 1/137.035)$ is the famous fine structure constant, and the rotational radius, R, is given by:

$$R = \frac{n^2\hbar}{m_\mathrm{e} c\alpha} = n^2\alpha_o, \tag{1.12}$$

which corresponds for $n = 1$ to the Bohr radius $(\alpha_o = 0.5292 \cdot 10^{-10}\,\mathrm{m})$.

Thus from (1.11) the electron kinetic energy, T, is computed from:

$$T = \frac{1}{2}m_\mathrm{e}\mathrm{v}^2 = \frac{1}{2}m_\mathrm{e}c^2\frac{\alpha^2}{n^2}, \tag{1.13}$$

and the potential energy, V, is computed from:

$$V = -\frac{e^2}{\varepsilon R} = -\frac{\alpha c e^2 m_\mathrm{e}}{n^2\varepsilon\hbar} = -\frac{\alpha^2 m_\mathrm{e}c^2}{n^2}. \tag{1.14}$$

Consequently the total kinetic plus potential energy, $\mathcal{H}$, of the electron, i.e. the Hamiltonian of the electron, is given by:

$$\mathcal{H} = T + \mathrm{V} = -\frac{1}{2}\frac{\alpha^2 m_\mathrm{e}c^2}{n^2} = -\frac{13.6}{n^2}\,\mathrm{eV}, \tag{1.15}$$

which, in view of the definition of α, can also be written as:

$$\mathcal{H} = -\frac{1}{2n^2}\cdot\frac{m_\mathrm{e}e^4}{\varepsilon^2\hbar^2} = \frac{-13.6\,\mathrm{eV}}{n^2}. \tag{1.16}$$

Equations (1.15) or (1.16) provided an excellent quantitative description of all the spectral data obtained previously by Lyman et al.

Similar was the success of the quantum mechanical treatment of the H atom via the Schrödinger equation some 20 years later which led to the same basic result [Eq. (1.16)] [4, 5, 14].

For some years both the Bohr treatment, enriched by Sommerfeld to include elliptical orbits [15], and the Schrödinger treatment were considered as viable alternative descriptions of the same physical reality. However gradually the Bohr–Sommerfeld treatment was abandoned in favor of the Schrödinger treatment and its wavefunctional description of the probability $\Psi^*\Psi$ of finding the electron at some position in space. What we frequently tend to forget is that the deterministic Bohr–Sommerfeld treatment, which is based on the de Broglie wavelength equation and thus on the historical basis of quantum mechanics, provides as successful a mathematical description of the H atom as the quantum mechanical treatment based on the solution of the Schrödinger equation.

Figure 1.4 provides a graphical solution to the Bohr model, i.e. of Eqs. (1.6) and (1.9). The graph shows the curve corresponding to the classical mechanical problem and the curve obtained from the de Broglie wavelength expression (1.9) or, historically, from the Bohr quantization condition [Eq. (1.7)]. The usefulness of Fig. 1.4 will become apparent in Chap. 10 and its figures.

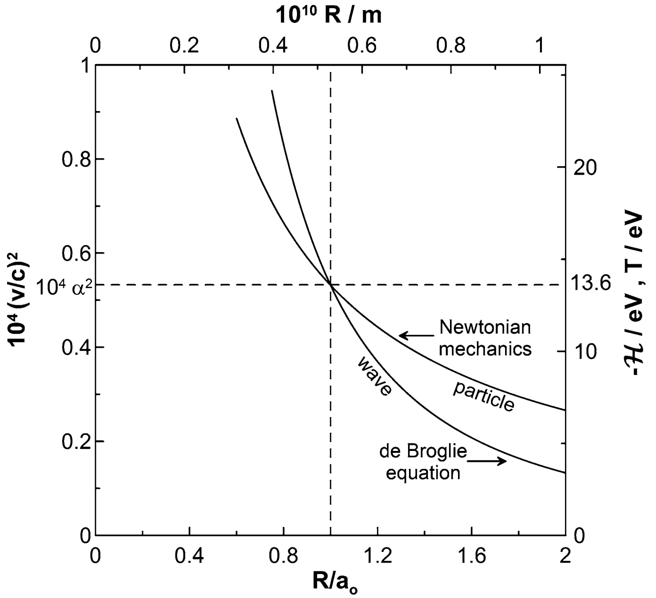


Fig. 1.4 Bohr H atom model: Graphical solution of Eqs. (1.6) and (1.9). The curve labeled "Newtonian mechanics" is Eq. (1.6) rewritten in the form $v^2/c^2 = e^2/\varepsilon m_e c^2 R$. The curve labeled "de Broglie equation" is Eq. (1.9) for $n = 1$ rewritten in the form $v^2/c^2 = \hbar^2/m_e^2 c^2 R^2$; α is the fine structure constant ($e^2/\varepsilon c\hbar = 1/137.035$); and the kinetic energy, T, is computed from $T = (1/2) m_e v^2$

One aspect of Fig. 1.4 and of the Bohr model in general which is worth remembering is that the solution is obtained by giving equal importance both to the corpuscular and to the ondular nature of the electron.

1.4 Deterministic and Probabilistic Models

Was the amazing success of Bohr's model, integrated with de Broglie's equation, a numerical coincidence, a fortuitous curiosity? After the views of Max Born on the probabilistic interpretation of the wavefunction were introduced and gradually accepted [16], the vast majority of physicists and chemists are taught that the success of Bohr's classical model was a good coincidence.

We came to believe that quantum mechanics, as described by the Schrödinger equation, governs everything in our micro-cosmos, to the point that particle velocities and trajectories are very seldom discussed anymore in modern physics and all we hope to get as a glimpse of the reality is some wavefunction and concomitant probability $\Psi^*\Psi$ obtained by solving the Schrödinger or Dirac equation via a series of elaborate numerical approximations. We came to believe that only probabilities of finding a particle somewhere can be computed and that deterministic treatments, even whenever highly successful as is the case of the Bohr–Sommerfeld model, do not provide a correct description of the reality. But in abandoning particle velocities we have lost our only chance for utilizing the equations of special relativity in a direct and forceful way. Albert Einstein was very excited with the de Broglie wavelength expression: "Now I can see some light at the end of the tunnel" he had exclaimed right after reading the PhD Thesis of Louis de Broglie. He was, however, highly skeptical about the Schrödinger equation and its probabilistic interpretation by Born. "God does not play with dice" he had stated bluntly.

In recent years the potential usefulness of deterministic models to gain some insight beneath quantum mechanics has been emphasized by Nobel laureate G. t' Hooft among others [17–20].

Even if Bohr's success were a coincidence, and one can show mathematically by comparing with the Schrödinger equation that it was not, it is fair to ask if other physical problems exist, besides the H atom problem, where this classical mechanical approach coupled at the end with the de Broglie wavelength equation can provide a satisfactory mathematical description of the physical reality.

This is one of the central questions tackled in this book. Can this Bohr–de Broglie-type approach, coupling a deterministic classical mechanical treatment with the de Broglie wavelength equation, which can be viewed in essence as an alternative expression of Heisenberg's uncertainty principle, be also effective in the subatomic world, the world of light particles and generation of hadrons, thus generation of mass, i.e. the world of hadronization, via the condensation of quark-gluon plasma?

It is thus the purpose of this book to explore the region marked with a questionmark in Figs. 1.1, 1.2, and 1.3 and to show that the strong forces holding together hadrons, mesons, and nuclei are relativistic gravitational forces. The approach is, as we shall see, quite straightforward and very similar to the deterministic Bohr approach to the H atom.

In order to explore this idea one may consider a model involving two or three relativistic $(\mathrm{v} \approx c)$ rotating particles in order to simulate quarks and antiquarks in hadrons and mesons. In following this approach two, at first insurmountable, difficulties have to be faced: The treatment of the strong force and the treatment of the relativistic effects.

1.5 Newton's Gravitational Law, Special Relativity, and the Equivalence Principle

It is rather straightforward to show, and it is proven in Chaps. 2, 5, and 6 that, according to special relativity [21, 22] and the equivalence principle, if two particles of rest mass m_{o} each, move with a high velocity v relative to a laboratory observer, their inertial and thus gravitational mass equals $\gamma^3 m_{\mathrm{o}}$ and thus, according to Newton's universal gravitational law, the gravitational force between them goes as:

$$-F_{\mathrm{G}} = \frac{Gm_o^2\gamma^6}{r^2}, \tag{1.17}$$

where $\gamma(= (1 - \mathrm{v}^2/c^2)^{-1/2})$ is the Lorentz factor. This equation can be derived rigorously from the synthesis of special relativity, the equivalence principle and Newton's gravitational law. Consequently one cannot ignore Eq. (1.17) in treating fast particles such as neutrinos, unless one is willing to abandon special relativity or, perhaps more reasonably, abandon Newton's gravitational law in the ultra-relativistic (high Lorentz factor γ) regime. The latter might be a legitimate view, but as shown in this book, if one sticks to Newton's law and thus to Eq. (1.17) then a whole new world opens: Since γ is unbound as v approaches c, then it follows that the gravitational attraction is also unbound and can thus in principle exceed in magnitude any other force, Coulombic or strong, as the velocity v of the two particles approaches c.

This implies that gravity can in principle confine any fast moving particle in bound states. It may confine protons and neutrons to form nuclei and perhaps also gluons and quarks and, why not, even fast neutrinos, to form protons, neutrons, and other hadrons. This is the problem treated in this book.

That gravity can become as strong as the strong interaction force is at first a very counterintuitive idea. The gravitational attraction between two protons at rest is 36 orders of magnitude smaller than their Coulombic repulsion. And if one were to examine the hypothetical case of two similarly charged neutrinos, then the gravitational force between them at rest is 58 orders of magnitude smaller than their Coulombic repulsion. Yet all this is computed from Eq. (1.17) and Coulomb's law for $\gamma = 1$. When the particle velocity, v, approaches c, the Lorentz factor γ becomes unbound and thus the ratio between the gravitational and Coulombic force, i.e.

$$\frac{F_{\mathrm{G}}}{F_{\mathrm{C}}} = \frac{\varepsilon G m_o^2 \gamma^6}{e^2}, \tag{1.18}$$

where $\varepsilon = 4\pi\varepsilon_o$, can in principle reach and even exceed unity.

1.6 Relativistic Rotating Particle Models for Hadrons

One is thus compelled to think that it may be worthwhile to formulate a Bohr-type model for the gravitational confinement of two or three light rotating particles of rest mass m_{o} into a bound rotational state corresponding to a hadron such as a baryon (Fig. 1.5).

The rest mass of the bound state, m_{B}, equals $2\gamma m_{\mathrm{o}}$ or $3\gamma m_{\mathrm{o}}$, respectively, for the two- or for the three-rotating particle model and thus if γ is sufficiently large, i.e. $\mathrm{v} \approx c$, this model provides a simple hadronization mechanism, i.e. a mechanism for generation of rest mass $2\gamma m_{\mathrm{o}}$ or $3\gamma m_{\mathrm{o}}$ starting from an initial rest mass of $2m_{\mathrm{o}}$ or $3m_{\mathrm{o}}$.

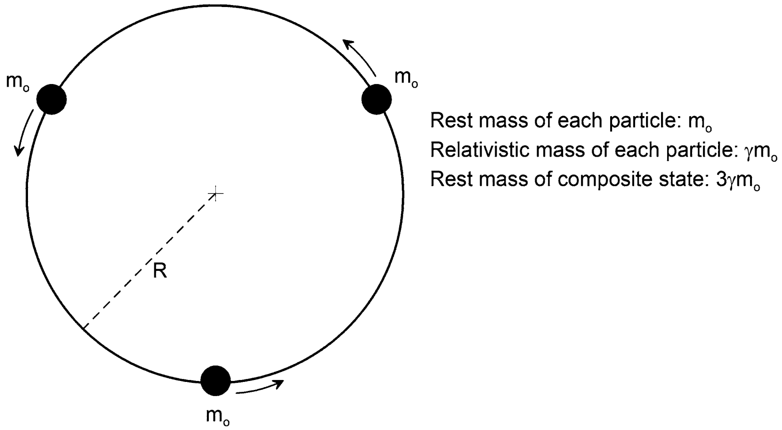


Fig. 1.5 A simple mass generation baryosynthesis mechanism. The mass increases from $3m_o$ for the three unbound particles to $3\gamma m_o$ when the bound rotational state is formed

In exact analogy with the Bohr model of the H atom, there are two unknowns in the model of Fig. 1.5: The radius R and the velocity, v, or equivalently the Lorentz factor γ. There are also two equations: One is obtained via Newton's second Law and special relativity by accounting for the corpuscular nature of the rotating particles. The second equation is obtained via the de Broglie wavelength equation and accounts for the ondular (wave) nature of the rotating particles. By solving, in fact analytically, these two equations for the two unknowns one is faced with a truly exciting double surprise! First the radius R, is in the fm range. Second the velocity, v, is so near to c that the Lorentz factor, γ, is $7.163 \cdot 10^9$. Thus when m_o is chosen to equal the mass of neutrinos, i.e. $\sim 0.04\,\mathrm{eV/c^2}$, then the mass, $3\gamma m_o$, of the bound rotational state is found to equal 885 MeV/c^2, which lies within 8% from the neutron mass of 939.565 MeV/c^2! Exact agreement with the neutron mass is obtained for $m_o = 0.043723\,\mathrm{eV/c^2}$, which practically coincides with the currently computed maximum neutrino mass [7, 8]. This is a truly exciting result. Three neutrinos with total rest mass of $0.131 \cdot 10^{-2}\,\mathrm{eV/c^2}$ form a baryon with a rest mass a factor of $7.163 \cdot 10^9$ times larger, i.e. 939.565 MeV/c^2, the rest mass of a neutron. And this baryosynthesis surprise (Chap. 6) comes without any adjustable parameter!

There is a lot of additional experimental evidence in support of a rotating particle model. For example the model predicts that the masses of the light baryons should follow a $(2n-1)^{1/6}$ law, which is reminiscent of the $-n^{-2}$ law of the energy levels of the H atom. Indeed the masses of the light baryons are found to satisfy this equation within 3%, as discussed in Chap. 6.

Strong additional experimental evidence supporting the rotating neutrino model comes from magnetic moment data. Thus the magnetic moments, μ_p and μ_n, of protons and neutrons, respectively, are given by:

Table 1.1 Properties of the gravitationally confined neutrino states (computed with a neutrino rest mass of $0.043723\,\mathrm{eV/c^2}$) [7, 8]

Property	Model predicted value	Experimental value
Neutron rest mass	$939.565\,\mathrm{MeV/c^2}$	$939.565\,\mathrm{MeV/c^2}$
Proton rest mass	$938.245\,\mathrm{MeV/c^2}$	$938.272\,\mathrm{MeV/c^2}$
Baryon binding energy	208 MeV	160 ± 10 MeV QCD transition energy 217 ± 25 MeV QCD scale
Radius of ground state	0.630 fm	$\sim$0.7 fm
Minimum lifetime	6.6×10^{-24} s	5.6×10^{-24} s
Proton magnetic moment	$15.14\cdot10^{-27}$ J/T	$14.10\cdot10^{-27}$ J/T
Neutron magnetic moment	$-10.09\cdot10^{-27}$ J/T	$-9.66\cdot10^{-27}$ J/T
Gravitational mass, $\gamma^3 m_o$	$1.607\cdot10^{19}\,\mathrm{GeV/c^2}$	$1.221\cdot10^{19}\,\mathrm{GeV/c^2}$ (Planck mass)
Angular momentum	$1.13\,\hbar$	$\sim\hbar$

$$\mu_{\rm p} = 14.10 \cdot 10^{-27} \quad \mathrm{J/T} = 2.79\,\mu_{\rm N}, \tag{1.19}$$

$$\mu_{\rm n} = -10.09 \cdot 10^{-27} \quad \mathrm{J/T} = -1.913\,\mu_{\rm N}, \tag{1.20}$$

where $\mu_{\rm N}(= 5.05 \cdot 10^{-27}\,\mathrm{J/T})$ is the nuclear magneton. Recalling the definition of magnetic moment, i.e.

$$\mu = (1/2)qR\mathrm{v}, \tag{1.21}$$

where q is the rotating charge, R is the rotational radius and v is the velocity, and taking as an example $q = (2/3)e$, the charge of a u quark, $R = 1$ fm, which is considered to correspond to the size of a proton, and $\mathrm{v} \approx c$ one computes from Eq. (1.21) that:

$$\mu = 16.02 \cdot 10^{-27}\,J/T = 3.17\,\mu_{\rm N}, \tag{1.22}$$

which is surprisingly close to the value of $\mu_{\rm p}$ ($2.79\,\mu_{\rm N}$). This shows that there must exist one or more charged particles (such as partons or quarks) rotating in protons and neutrons with a rotational speed close to c. How electrical charge can be introduced in these rotating neutrino states is discussed in Chaps. 7 and 10.

The very good, semiquantitative, agreement between the rotating neutrino model and experiment is shown in Table 1.1. The agreement is quite impressive since the model contains no adjustable parameters.

As shown in Table 1.1 and analyzed in Chaps. 7 and 10, it is not only the masses of light baryons and their magnetic moments which are described by the rotating neutrino model with great accuracy, but also the radii, spins, angular momenta, and reduced Compton wavelengths [23].

Could all this be a great coincidence? A close look at Table 1.1, which compares the model predictions with experiment, leaves little room for any reasonable doubt. The agreement is semiquantitative. Furthermore the model, as will be shown in Chaps. 6 and 7, also predicts both confinement and asymptotic freedom which are the two key characteristics of the strong force.

None of numerous physicists we discussed our findings with, some very prominent ones, could find any mathematical or logical error. Some were ready to abandon Newton's gravitational law in the highly relativistic region and thus Eq. (1.17). But how can we know that Newton's law really fails in this region when using the proper value of the inertial and gravitational mass $(\gamma^3 m_o)$ instead of the inertial mass m_o? Some others said that Eq. (1.17) may be theoretically correct but is not in agreement with experimental data coming from distant galaxies. But couldn't it simply be that our consistent underestimation of the gravitational force at these distant galaxies by using Eq. (1.17) with $\gamma = 1$ for stars and galaxies receding from us with relativistic velocities, is what caused us to postulate the existence of the dark matter?

Equation (1.17) is based entirely on Newton's gravitational law, special relativity and the well-proven equivalence principle of Einstein and Eötvös. If we are not willing to abandon any of these well-tested and proven concepts, then this equation is valid and everything falls into place. The formation of hadrons and nuclei from neutrinos can be rationalized immediately and described with great accuracy, color confinement and asymptotic freedom can be explained directly and practically all the experimental facts regarding the strong force can be readily rationalized (Table 1.1) as shown in Chaps. 6 and 7.

1.7 Synopsis

The Bohr model for the H atom is deterministic and gives equal weight to the corpuscular and to the ondular (wave) nature of the electron. The combination of classical mechanics coupled with the Coulomb equation on the one hand and of the de Broglie wavelength equation on the other, provides excellent agreement with experiment. No similar deterministic model has been developed yet, coupling classical mechanics, also accounting for special relativity and for the equivalence principle, gravitational attraction as the confining force and the de Broglie wavelength expression. Such a model involving three rotating neutrinos is presented in this book. The ground bound rotational state is found, as shown in this book, to have all the properties of the neutron. There is no new theory and no adjustable parameters.

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Chapter 2
Mass, Special Relativity and the Equivalence Principle

2.1 The Concept of Mass

With the exception of energy there is no other concept or natural quantity more important than mass in the natural sciences and yet its definition still poses some interesting questions.

There are two ways to define mass and they lead to the definition of the gravitational mass and of inertial mass.

The gravitational mass of a body, m_g, is responsible for the gravitational force. Upon considering the gravitational force, $\mathbf{F}$, between the body under consideration with mass m_g and a second body, e.g. the earth, of mass M_g one can write according to Newton's universal gravitational law:

$$\mathbf{F} = G\frac{m_g M_g}{r^3}\mathbf{r} \tag{2.1}$$

or more commonly:

$$F = G\frac{m_g M_g}{r^2}. \tag{2.2}$$

The inertial mass of a body, m_i, is defined from Newton's second law of motion, i.e. from:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m_i\mathbf{a} = m_i\frac{d\mathbf{v}}{dt} \tag{2.3}$$

and provides a measure of the resistance that a body offers to changes in its state of motion. According to Newton, inertia is an inherent property of matter and thus the inertial mass of a body, m_i, does not depend on other bodies or events in the Universe.

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2.2 The Equivalence Principle

According to the equivalence principle the two mass quantities defined above in Eqs. (2.2) and (2.3) are equal, i.e.

$$m_{\mathrm{g}} = m_{\mathrm{i}}. \tag{2.4}$$

To be more precise this is the weak equivalence principle, first formulated by Isaac Newton and whose validity has been confirmed thousands of times, starting from the observations of Philoponus (sixth century AD) about the time of fall of falling balls, continuing with the similar observations of Simon Stevin and Galileo in the sixteenth century and with the more recent and sophisticated ones by Eötvös (1876) using the torsion balance. The more modern torsion balance measurements have established the validity of the weak equivalence principle, i.e. $m_{\mathrm{i}} = m_{\mathrm{g}}$, to at least 1 part in 10^{12} [1–4].

How do the equal falling time observations lead to the equivalence principle equation (2.4)? The answer goes as follows: From Eqs. (2.2) and (2.3), one obtains that:

$$\frac{m_{\mathrm{g}}}{m_{\mathrm{i}}} = \frac{r^2}{GM_{\mathrm{g}}}\alpha. \tag{2.5}$$

Upon considering another body with inertial mass m_i' and gravitational mass m_g' interacting with the same second body, e.g. the earth, of gravitational mass M_{g}, at the same initial distance r it follows that:

$$\frac{m_g'}{m_i'} = \frac{r^2}{GM_{\mathrm{g}}}\alpha'. \tag{2.6}$$

Since, for the same initial distance r, the falling time, Δt, is a monotonously decreasing function of the initial acceleration, α, it follows that the equality $\Delta t = \Delta t'$ leads to $\alpha = \alpha'$. Thus from (2.5) and (2.6) it follows that the ratio $m_{\mathrm{g}}/m_{\mathrm{i}}$ is the same constant for all bodies.

That this constant equals unity follows e.g. by considering the equation for the terminal velocity of a free falling body $\mathrm{v} = (2m_{\mathrm{g}}gh/m_{\mathrm{i}})^{1/2}$ obtained by equating the terminal kinetic energy $(1/2)m_{\mathrm{i}}\mathrm{v}^2$ with the initial potential energy $m_{\mathrm{g}}gh$. The fact that the experimentally observed value of v is given by $\mathrm{v} = (2gh)^{1/2}$ shows that indeed the constant $m_{\mathrm{g}}/m_{\mathrm{i}}$ ratio equals unity.

A more detailed discussion of the equivalence principle and its two other and stronger formulations, i.e. the Einstein equivalence principle and the strong equivalence principle is given in Chap. 5. However, it is the weak equivalence principle, the validity of which has been thoroughly confirmed experimentally thousands of times, which suffices for the purposes of the present book.

2.3 Rest, Relativistic, Inertial, and Gravitational Mass in Special Relativity: Some Questions

The above definitions of the gravitational and inertial mass had been formulated by Newton long before Einstein developed the theory of special relativity, a theory which has by now also been confirmed thousands of times.

The question thus arises about what happens with these definitions, as well as with the weak equivalence principle, when the velocities of the bodies involved are relativistic, i.e. their velocities with respect to a laboratory observer in some frame of reference S approach the speed of light c.

In relativistic mechanics equation (2.3) takes the more general form [5]:

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \gamma m_{\mathrm{o}} \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \gamma^3 m_{\mathrm{o}} \frac{1}{c^2} \left(\mathbf{v} \cdot \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \right) \mathbf{v}, \tag{2.7}$$

where m_{o} is the rest mass. For colinear $\mathbf{F}$ and $\mathbf{v}$ one obtains:

$$F = \left[\gamma + \gamma^3 \frac{\mathrm{v}^2}{c^2} \right] m_{\mathrm{o}} \frac{\mathrm{dv}}{\mathrm{d}t} = \left[\gamma + \gamma^3 (\gamma^2 - 1)/\gamma^2 \right] m_{\mathrm{o}} \frac{\mathrm{dv}}{\mathrm{d}t} = \gamma^3 m_{\mathrm{o}} \frac{\mathrm{dv}}{\mathrm{d}t} \tag{2.8}$$

and thus the inertial mass, m_{i}, which is the ratio of F and $\frac{\mathrm{dv}}{\mathrm{d}t}$, equals $\gamma^3 m_{\mathrm{o}}$.

This is very important to consider, since while in Newtonian mechanics we have to deal with only one particle mass, m, equal to m_{i} and to m_{g}, here in the relativistic mechanics we have to deal with three different masses (Table 2.1), i.e. the rest mass m_{o}, the relativistic mass γm_{o} (corresponding to the total particle energy $E = \gamma m_{\mathrm{o}} c^2$) and the inertial mass, m_{i}, which at least for linear particle motion equals $\gamma^3 m_{\mathrm{o}}$ [5,6], as already shown.

Thus while for Newtonian mechanics ($\gamma \approx 1$) the differences between these three masses are negligible, the situation changes dramatically when considering relativistic ($\mathrm{v} \approx c$) velocities where γ can take very large values.

Assuming that the equivalence principle ($m_{\mathrm{g}} = m_{\mathrm{i}}$) remains valid under relativistic conditions, and this is something which has never been challenged in the context of any gravitational theory, including general relativity [7–10], it follows that it is the inertial mass, $m_{\mathrm{i}} (= \gamma^3 m_{\mathrm{o}})$, and not the rest mass m_{o} or the relativistic mass γm_{o} which has to be used in Newton's gravitational law.

Based on the theory of general relativity, which is a gravitational theory (the most successful and well-known one, but not the only one [11, 12]) one can obtain Newton's gravitational law at the limit of low particle energies, so many people

Table 2.1 Masses of a particle

m_{o}	Rest mass
γm_{o}	Relativistic mass
$\gamma^3 m_{\mathrm{o}}$	Inertial mass Gravitational mass

believe that Newton's universal gravitational law does not remain valid in the highly relativistic regime. This, however, has not been proven experimentally using $\gamma^3 m_o$ rather than m_o in Newton's universal gravitational law.

2.4 Newton's Gravitational Law, Velocity and General Relativity

Gravity or gravitation is one of the four fundamental interactions of nature, together with electromagnetism, the nuclear strong force and the nuclear weak force.

In the gravitational phenomenon physical bodies attract each other with a force proportional to their mass. Isaak Newton formulated in *Principia* his law of universal gravitation [Eq. (2.2)] and noted in his own words that "the forces which keep the planets in their orbits must be reciprocally as the squares of their distances from the centers about which they resolve."

For more than two centuries Newton's universal gravitation law was extremely successful in explaining all gravitational phenomena and even predicting the existence of Neptune on the basis of the orbit of Uranus.

However by the end of the nineteenth century some observations on the perihelion behavior of the orbit of mercury were made which could not be accounted for exactly by Newton's gravitational law, at least when assuming a perfectly uniform distribution in the sun mass. These observations were described with great accuracy by Albert Einstein's theory of general relativity published in 1915.

Today modern physics describes gravitation almost exclusively in terms of the general relativity theory according to which gravitation is a consequence of the curvature of spacetime resulting from the presence of matter. According to the general relativity theory this matter-induced curvature of spacetime governs the motion of inertial objects.

Einstein developed the field equations of general relativity which relate the presence of matter with the curvature of spacetime. The Einstein field equations are commonly written in the tensor equation form:

$$G_{ik} = (8\pi G/c^4)T_{ik}, \tag{2.9}$$

which relates the Einstein tensor G_{ik} with the stress-momentum-energy tensor T_{ik} [8, 9].

This equation looks simple but in reality represents a set of ten simultaneous and nonlinear differential equations. The solutions of these Einstein field equation are the components of the metric tensor of spacetime which describes the geometry of spacetime. From the metric tensor one can calculate the geodesic paths, which are the, locally straight, paths followed by free-falling objects in curved spacetime [8, 9, 13].

Thus in general relativity gravitation is ascribed to spacetime curvature (a flat spacetime is a Minkowski spacetime) and not to a force, which is the case in Newtonian physics. General relativity uses the equivalence principle to equate free fall with inertial motion and describes free-falling inertial objects as being accelerated relative to a noninertial observer standing on the ground. In Newtonian physics, however, no such accelerated motion can take place in absence of a force acting on an object. It may be interesting to note that, although general relativity does not use forces to describe motions, the inverse of the key parameter $8\pi G/c^4$ which appears in the field equations (2.9) has units of force.

Thus one naive way to look at Eq. (2.9) in one dimension is that it describes the manner in which a Newtonian force of magnitude $c^4/8\pi G$ causes a spacetime distortion proportional to the local value of G_{ik} in presence of matter with local energy content T_{ik}.

Since the first experimental tests of general relativity [14], this area of research has remained quite active [15–18]. Cases where the predictions of special and general relativity differ have also been discussed [19].

As already noted, the 10 simultaneous nonlinear differential equations resulting from the Einstein field equation (2.9) are very difficult to solve and analytical solutions have been obtained only in special cases such as:

- The Schwarzschild solution which describes spacetime around a spherically symmetric nonrotating uncharged massive body of mass M. For distances shorter than the Schwarzschild radius, $r_S (= 2GM/c^2)$, the solution describes a black hole. For radial distances from the center which are much longer than the event horizon, i.e. the Schwarzschild radius, r_S, of the black hole, the Schwarzschild solution practically coincides with the behavior predicted by Newton's universal gravitational law.
- The Kerr solution of rotating spherically symmetric uncharged massive bodies. This solution produces black holes with multiple event horizons.
- The Reissner–Nordström solution in which the central nonrotating massive body carries an electrical charge. This solution also describes black holes with two event horizons.
- The Kerr–Newman solution for charged rotating massive bodies which also predicts black holes with multiple event horizons.

Thus in the context of general relativity, significant deviations from Newton's gravitational law are expected under highly relativistic conditions resulting from the curvature of spacetime. If the curvature is small and the actual spacetime geometry may be approximated by a small perturbation of the flat Minkowski spacetime, then one may linearize the Einstein field equations (2.9) and obtain Newton's gravitational law as a limiting case, as also described in Chap. 5. In summarizing this section it should be stated that most nonrelativistic gravitational calculations are still made with Newton's gravitational law which is much simpler to use than general relativity.

It is also fair to say that experimental deviations from Newton's gravitational law, whose validity has been confirmed to distances as short as a few *mm* [3], are very small and rare and their existence needs closer examination if one uses the proper definition, $\gamma^3 m_o$, of inertial mass which accounts for the particle velocity.

2.5 Quantum Gravity

Quantum gravity is the field of theoretical physics which attempts to unify quantum mechanics with general relativity [20–22]. Such a unification appears to be quite difficult since it was realized from the beginning of the development of the general relativity theory and of quantum mechanics in the form of the Schrödinger equation that the two theories are not compatible with each other. After all, general relativity is a deterministic theory describing the trajectories (geodesics) of individual objects in spacetime while quantum mechanics in the form of the Schrödinger equation is a probabilistic theory which abandons the concept of particle trajectory and velocity and instead computes probabilities of finding a given particle in a given position in space.

This is the reason that even the combination of quantum mechanics in the form of the Schrödinger equation with special relativity is highly problematic. In special relativity the velocity, $\mathbf{v}$ of a particle relative to an observer in a frame of reference, e.g. in the frame of a laboratory observer, plays a key role in describing all its properties including its inertial mass and thus gravitational mass. However the concept of particle velocity is out of use in quantum mechanics, as described by the Schrödinger equation.

Thus quantum gravity (QG) attempts to develop a theory which reduces to ordinary quantum mechanics in the limit of weak gravity, i.e. when the gravitational potential, Φ, $(= GM/r)$, is much smaller than the ratio, c^2, of relativistic energy and rest mass, i.e. when

$$\Phi = GM/r \ll c^2 \tag{2.10}$$

or equivalently:

$$r \gg \frac{GM}{c^2} = r_S/2, \tag{2.11}$$

where r_S is the Schwarzschild radius of "event horizon" of a black hole with mass M. The same theory has to reduce to general relativity, or at least to special relativity, in the limit of large actions, i.e. actions much larger than the reduced Planck constant $\hbar$, i.e. when:

$$GM^2 > \hbar c; \quad M > (\hbar c/G)^{1/2} \tag{2.12}$$

or equivalently when:

$$\frac{\hbar c}{Mc^2} < r_S/2; \quad Mc^2 > \frac{2\hbar c}{r_S}. \tag{2.13}$$

Table 2.2 Forces and the corresponding exchange particles

Force	Exchange particle
Electromagnetic force	Virtual photon
Strong force	Gluon
Weak force	Vector boson
Gravitational force	Graviton

Table 2.3 Planck scale

	Definition	Value
Planck mass:	$m_{\mathrm{Pl}} = (\hbar c/G)^{1/2}$	$1.221 \cdot 10^{19}$ GeV/c^2
Planck distance:	$r_{\mathrm{Pl}} = (\hbar G/c^3)^{3/2}$	$1.615 \cdot 10^{-35}$ m
Planck time:	$t_{\mathrm{Pl}} = (\hbar G/c^5)^{1/2}$	$5.383 \cdot 10^{-44}$ s

The theory must be able to describe adequately phenomena where both strong field gravity and quantum effects are important.

Quantum mechanics is commonly used to describe phenomena at very short ($r < 10^{-9}$ m) distances.

On the contrary, general relativistic effects show up only for very large bodies, such as collapsed stars, since the gravitational fields of ordinary stars and planets are well described by Newtonian gravity.

On the other hand, it is possible to describe gravity in the framework of quantum field theory as is the case with the other fundamental forces (Table 2.2) by assuming that the attractive gravitational force is due to the exchange of virtual gravitons, which are predicted to be spin-2 massless objects.

The existence of gravitons has not yet been shown experimentally. Also there is yet no obvious experimental evidence calling for the introduction of quantum gravity since classical deterministic physics (Newtonian and Einsteinian) cover successfully gravitational effects over a very large range of masses (10^{-23}–10^{50} kg) and distances (10^{-2}–10^{26} m).

Nevertheless quantum gravity appears to be necessary for understanding nature at the Planck scale (Table 2.3) where gravitational and strong force effects are thought to become of equal importance. This is addressed in Chap. 12.

2.6 Synopsis

According to special relativity and to the equivalence principle, the mass of linearly moving particles to be used in Newton's universal gravitational law is neither the rest mass m_o nor the relativistic mass, γm_o, but rather the inertial and thus gravitational mass, $\gamma^3 m_o$. This result is generalized for arbitrary particle motion in Chap. 5. General relativity is a very successful gravitational theory but general relativistic effects are usually very small and become important only for very strong gravitational fields, such as those of neutron stars. The gravitational fields of ordinary stars are well described by Newton's universal gravitational law which has been shown to be valid for distances as short as 10^{-3}m.

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Chapter 3
The Strong Force: From Quarks to Hadrons and Nuclei

3.1 The Strong Force

The *strong force* is the force which binds together the constituents of protons, neutrons, and other hadrons which are currently thought to be quarks and gluons. It is also the force which binds together protons and neutrons to form nuclei.

The strong force is also known under the names *strong interaction*, *strong nuclear force* and *color force*. When refering to the binding of protons and neutrons to form nuclei the strong force is called the *nuclear force* or the *residual strong force*.

The strong force is thought to be mediated by gluons acting upon quarks, antiquarks, and the gluons themselves. This is analyzed in detail in the theory of quantum chromodynamics (QCD).

3.1.1 Classical and Quantized Fields

Noncontact forces, such as the strong force, the electromagnetic force, the weak force or the gravitational force used in the past to be described exclusively by classical *fields*. Thus the electrical repulsion of two electrons is in classical electrodynamics attributed to the electric field surrounding the two electrons and created by the two electrons themselves.

In quantum electrodynamics (QED), however, the electric field is assumed to be quantized in the form of photons and the repulsive interaction is modeled as due to the exchange of photons between the two electrons. Thus the electromagnetic force is mediated via the exchange of virtual photons.

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3.1.2 The Mediation Mechanism

For the nonphysicist reader it is probably useful to describe in qualitative terms a little more what is meant by mediation. A qualitative picture can be obtained by examining two ice skaters on a frozen lake throwing and catching snowballs back and forth between them. It is evident that this ball exchange causes the two skaters to move apart with the succession of recoils and this gives the appearance that a repulsive force is exerted between the two skaters. The effect of this repulsive force can be described by the exchange of the snowballs. One might question how an attractive force could be described by this type of particle exchange mechanism but there are of course limits to this analogy and more generally one might think of the exchanged particles as "messagers" for moving apart or for moving closer. Thus this simple physical picture provides a qualitative description of a mediation mechanism.

All natural forces are currently thought to be mediated by the exchange of real or virtual particles. Electromagnetic forces are described by the exchange of photons, strong forces are described by the exchange of gluons and gravitational forces are described by the exchange of gravitons. While a lot is known about photons, little is known about gluons and even less, if anything, is known about gravitons. One may wonder how this exchange or mediation type of mechanism became over the years so widely accepted on the basis of so little rigid experimental information. The reason can be sought in the great success of QED which is based on the photon exchange mechanism.

3.1.3 History and the Postulate of Color Charge

The stability of natural nuclei, consisting of nucleons, i.e. protons and neutrons, can only be explained if in addition to the Coulombic repulsion between protons there exists a second force acting between nucleons which is attractive and in fact stronger than the Coulombic repulsion. This hypothesized force was termed the *strong force* and was believed for several decades, roughly between 1940 and 1970, to be a fundamental force acting on the nucleons, i.e. on the protons and the neutrons.

When deep inelastic electron scattering led to the discovery that protons and neutrons are not fundamental particles but consist of constituent particles, termed originally partons by Feynman and later quarks by Gell–Mann, it became evident that the force between nucleons is the result, or side effect, of a stronger and more fundamental force which binds together quarks in protons and neutrons. This is the force known today as the *strong force*, while the resulting weaker force acting between nucleons is known as the *nuclear force* or the *residual strong force*.

In order to rationalize the existence of the strong force it was postulated in the theory of QCD that quarks carry a property named color charge (without of course any relation to visible color), and that the strong force is exerted between quarks carrying different color charge (blue, red, green). It was also postulated that quarks and gluons are the only fundamental particles carrying a nonvanishing amount of color charge and thus participating in strong interactions. Gluons mediate the strong force exerted between quarks and this keeps quarks bound in protons and neutrons. This is the picture of the standard model of elementary particles [1, 2].

3.1.4 Properties of the Strong Force

The strong force is the strongest of the four fundamental forces. At a distance of 1 fm it is a factor of 100 or α^{-1} ($\sim$137.035, where $\alpha = e^2/\varepsilon c\hbar$ is the fine structure constant) stronger than the electromagnetic force, a factor of 10^{12} stronger than the weak force and a factor of 10^{39} stronger than gravity.

Unlike the electromagnetic and gravitational forces, which are governed by the Coulomb and Newton gravitational laws, and despite the color charge postulate, there is no simple analytical expression describing the dependence of the strong force on color charge and/or distance.

In fact the strong force exhibits two unique features which are totally different from all other forces, electromagnetic, weak, and gravitational. These two features are known as asymptotic freedom and confinement.

Asymptotic freedom means that, below a certain distance of the order of 0.5 fm, the force between two particles with color charge does not increase but on the contrary decreases with decreasing distance and in fact becomes negligible at very short distances.

Confinement means that the strong force does not diminish but instead increases with increasing distance, thus causing the particles (quarks and gluons) to remain confined (Fig. 3.1). In QCD this phenomenon is termed color confinement.

The strong force is currently described by QCD, which is part of the standard model of particle physics. In mathematical terms QCD is a non-Abelian gauge theory based on a gauge symmetry group called SU(3) [3–5].

So far, despite very intense experimental efforts, all searches for isolating and studying free quarks, or gluons, have failed. Confinement is too strong and only at temperatures exceeding $3.3 \cdot 10^{12}$ K, i.e. energies exceeding 200 MeV, *confinement* gives way to a quark-gluon plasma [6–8].

When in particle accelerator experiments a quark in a proton is struck by a very fast quark of an impacting proton then jets of newly created hadrons appear. This phenomenon will be discussed in Chap. 9.

3.1.5 The Residual Strong Force

The *residual strong force*, also known as the *nuclear force*, is the force exerted between nucleons, i.e. protons and neutrons in nuclei. This force is also known as nucleon–nucleon force (NN force).

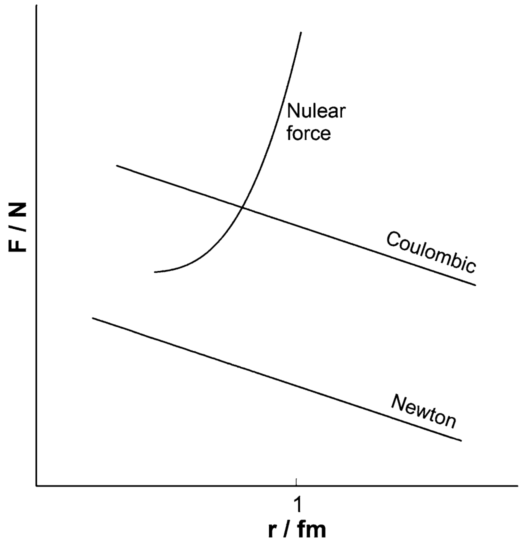

Fig. 3.1 Force dependence on distance for the electromagnetic, gravitational, and strong force

The residual strong force is a small residuum of the *strong force* which binds quarks together in protons and neutrons. The residual strong force is much weaker (roughly a factor of 100) than the strong force in the same way that the van der Waals force is much weaker than the Coulombic force from which it originates. This can be rationalized if the strong force is mostly neutralized within the nucleons in the same way that the electromagnetic force is mostly neutralized inside molecular structures held together by van der Waals forces.

Unlike the more fundamental strong force, the nuclear force *decreases* with *increasing* distance and in fact this decrease can be approximated by a Yukawa potential, i.e. by a negative exponential power of distance. This expression is mathematically identical with that of the Poisson–Boltzmann expression for the electrical potential around an ion surrounded by oppositely charged counterions.

This rapid decrease in the magnitude of the nuclear force with distance, which is sharper than the decrease in the repulsive electromagnetic force between protons in a nucleus, is considered to be the main cause for the instability of large nuclei, such as those with atomic numbers exceeding 82.

The nuclear force is only exerted between hadrons, e.g. protons and neutrons. At very short distances, less than $\sim$0.5 fm between the hadron centers, the NN force is strongly repulsive. This can be understood in terms of the Pauli exclusion principle for identical nucleons (i.e. two protons or two neutrons) and by the Pauli exclusion principle between quarks of the same type for proton–neutron pairs. The NN force has also a tensor component depending on the parallel or anti-parallel spin orientation of the nucleons. At distance below $\sim$1.2 fm the NN force is stronger than the Coulombic repulsion between protons. At distances, however, above 1.7 fm the NN force practically vanishes and the Coulomb repulsion dominates (Fig. 3.1).

Due to the nuclear force the formation of stable nuclei from nucleons, i.e. from protons and neutrons, is exoergic, i.e. energy is released when a nucleus is created. This energy release is typically $\sim$7 MeV per nucleon which happens to be in the range of $\alpha m_\mathrm{p}c^2$, where m_p ($\sim$938 MeV/c^2) is the proton mass and α is the fine structure constant. In view of Einstein's famous $\Delta E = \Delta mc^2$ formula, a negative ΔE yields a negative Δm (mass deficit), i.e. the nucleus mass is smaller than the total mass of the individual nucleons. This negative Δm is typically of the order of $-\alpha m_\mathrm{p}(\approx -7\,\mathrm{MeV/c^2})$ per nucleon for nuclear reactions.

The situation is not different in Chemistry where Δm is typically of the order $\pm\alpha^2 m_\mathrm{e}$, i.e. a few eV/c^2, and thus is immeasurably small but present in all chemical reactions. Thus in view of Einstein's famous formula there is a mass change associated with any reaction, be it nuclear or chemical.

The nuclear force is practically the same for proton–proton, neutron–neutron and proton–neutron pairs, a property called charge independence. Its magnitude depends strongly on whether the nucleon spins are parallel or antiparallel and, as previously noted, it has a tensor (noncentral) component. This part of the force does not obey orbital angular momentum conservation, as is the case with central force motions.

Nucleons have no net color charge and consequently the nuclear force does not involve directly the action of the force carriers of QCD, i.e. the action of gluons. In the same way, however, that electrically neutral atoms attract each other via electrical polarization effects, i.e. via dipole–dipole or dipole-induced dipole or induced dipole-induced dipole (London or van der Waals) forces, it is believed that by analogy "color-neutral" neucleons can attract each other by a type of "color polarization" which allows for some gluon-mediated attractive effects to be transferred from one color-neutral nucleon to another.

The study of NN forces is commonly carried out by formulating potentials, such as the Yukawa potential, for the nucleons and using the Schrödinger equation. The mathematical form of the potential is derived phenomenologically and the model parameters are obtained by fitting to experimental data, such as binding energies, NN elastic scattering cross sections, etc. The original Yukawa potential is nowadays seldom used and more elaborate potentials, such as the Paris potential, the Argonne Av18 potential, and Nijmegen potentials are used. Two- and three-nucleon potentials have been investigated thoroughly for nuclear masses up to 12.

3.1.6 Quantum Chromodynamics

QCD is a theory of the strong force based on the concept of color charge. It focuses on the study of the symmetry unity (3), $SU(3)$, Yang–Mills theory of quarks, which are fermions carrying color charge in the same way that electrons carry normal electrical charge. QCD is a quantum field theory of a particular type called a nonabelian gauge theory. QCD is a central part of the Standard Model of particle physics [1, 2, 9, 10].

QCD is a successful theory in that it has been able to describe two key aspects of the strong force, i.e. *confinement* and *asymptotic freedom*.

Confinement means that the attraction between quarks does not decrease as their distance increases but on the contrary increases in a pronounced manner so that an infinite amount of energy is needed to separate two quarks. This implies that they will stay for ever bound in hadrons such as protons. Confinement can explain the repeated failure of searches for free quarks [1, 2, 10].

Asymptotic freedom means that in very high energy reactions and thus at very short (<0.5 fm) distances the interaction between quarks and gluons becomes very weak. QCD is able to predict this behavior, as proven by D. Politzer, F. Wilczek, and D. Gross in the early 1970 [3, 4], work for which they won the 2004 Nobel Prize in Physics.

During the last few decades the string theory [11–13] has been also forcefully deployed to describe the strong force. Strings are envisioned to have dimensions in the Planck length scale ($\sim 10^{-35}$ m). Progress and current directions have been reviewed recently [13].

3.2 Synopsis

The strong force keeps the constituents of hadrons bound together. The residual strong force keeps hadrons (protons and neutrons) bound in nuclei.

The strong force exhibits two striking features relative to Newtonian gravity or Coulomb's electrostatics: It exhibits confinement and asymptotic freedom. This means that the hadron constituents, named originally partons by Feynman and later on quarks by Gell-Mann, are bound together by a force which increases with distance (confinement) and practically vanishes at very short (<1 fm) distances (asymptotic freedom). The strong force is attributed to a property of quarks and gluons called color charge. Gluons are believed to mediate the strong force exerted between quarks. For particle energies above the QCD scale (~ 200 MeV) hadrons are believed to decompose to a quark-gluon plasma. It is the condensation of this quark-gluon plasma which is believed to have caused the genesis of hadrons. Despite intensive experimental efforts, no quarks or gluons have ever been isolated and studied.

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Chapter 4
The World of Particles and the Standard Model

4.1 Elementary Particles

4.1.1 History

In particle physics *elementary* particles or *fundamental* particles are those which are not known to have a substructure.

In the history of science, since the days of Democritos, atoms were thought to be elementary particles. This view changed significantly in 1897 with the discovery of the electron by J.J. Thomson. Until 1932 protons, neutrons and electrons were the only known elementary particles. This early period is frequently called *classical* period of elementary particles [1].

During that period and via the work of Planck, de Broglie, Einstein, and Compton the *photon* also gradually emerged as a massless elementary particle [1, 2].

The period 1932–1960 witnessed the gradual discovery of several new particles originally proposed on theoretical grounds. These were Yukawa's meson which was thought to mediate the strong force, Dirac's positron, which is the antiparticle of an electron (i.e. a particle with the opposite electrical charge) and Pauli's neutrino ν, a particle with an extremely small mass, originally and until recently [3, 4] (1998) thought to be zero, which was necessary to maintain momentum and energy conservation in the β-decay reaction which transforms neutrons into protons and electrons:

$$n \rightarrow p^{+} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}. \tag{4.1}$$

The symbol $\bar{\nu}_{\mathrm{e}}$ stands for an electron antineutrino which is the antiparticle of the electron neutrino ν_{e}. Since the neutrino carries no electrical charge there is still a discussion if ν_{e} and $\bar{\nu}_{\mathrm{e}}$ are different particles, in which case they are termed Dirac neutrinos, or different states of the same particle, in which case they are called Majorana neutrinos.

In the years after 1950 several new particles were discovered which included several *mesons*, such as the $pi(\pi)$, the miu (μ), and the K^{o} (Kaon). Such particles

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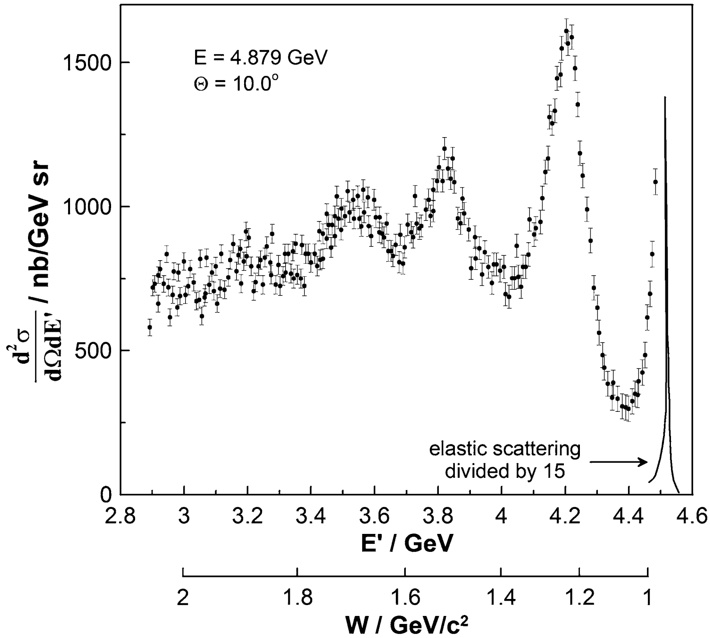


Fig. 4.1 Spectrum of scattered electrons from electron–proton scattering at an electron energy of $E = 4.96$ GeV and a scattering angle of $\theta = 10°$C (from [5], Fig. 1, reprinted with permission from Elsevier)

were called mesons (from $\mu\acute{\varepsilon}\sigma o\varsigma$, the intermediate one) because their masses (typically 100–600 MeV/c^2) lie between the masses of light *leptons* (from $\lambda\varepsilon\pi\tau\acute{o}\varsigma$, the thin one) (such as the electron with a rest mass of 0.51 MeV/c^2) and the heavier *baryons* (from $\beta\alpha\rho\acute{\upsilon}\varsigma$, heavy), such as the proton (mass 938.2 MeV/c^2) and the neutron (mass 939.5 MeV/c^2).

Nowadays baryons and mesons are collectively called *hadrons* (the big ones). A very important discovery at the beginning of the modern period of elementary particles (after 1960) was that hadrons have substructure and therefore are *not* elementary particles. This we know from inelastic electron–proton and electron–neutron scattering experiments, such as the ones shown in Fig. 4.1 [5] which revealed that baryons (such as protons and neutrons) contain *three* point-like constituents, while mesons contain *two* such constituents (Fig. 4.2). These constituent point-like particles were initially termed *partons* by Feynmann and later on *quarks* by Gell–Mann. It was postulated that these spin 1/2 constituent particles are held together by the *strong* force which is mediated by massless zero spin particles, termed *gluons*. As already mentioned in Chap. 3, it was also postulated that the strong force is due to the differences in a property of quarks termed *color*, which comes in three types (red, green, and blue). Quarks were also assumed to come in three *flavors* (u for "up," d for "down", and s for "strange") [1, 2, 6]. As already mentioned, quarks cannot be separated and studied individually.

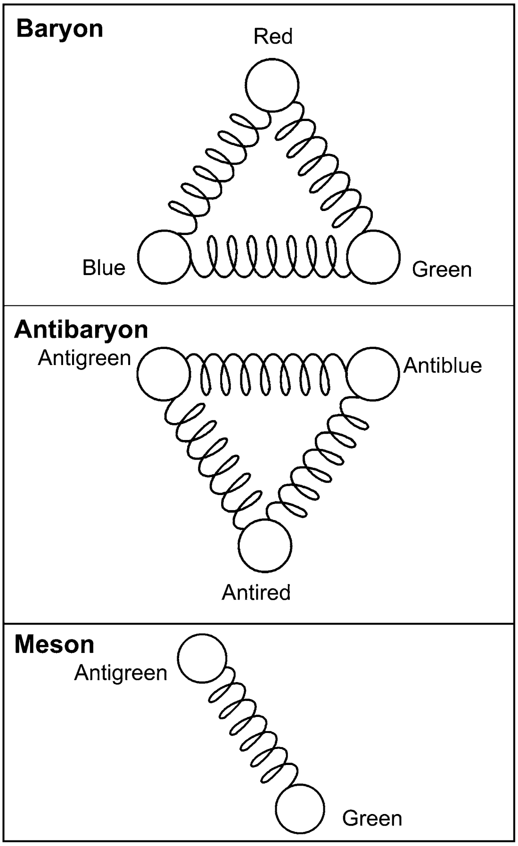


Fig. 4.2 Hadrons: Baryons and mesons. They consist of elementary point-line constituents (three for baryons, two for mesons) initially termed partons by Feynmann and later on quarks by Gell–Mann. The quarks and antiquarks are assumed to have a property called *color* which is the origin of the strong force

Quarks are also assumed to carry electrical charge, e.g. $(2/3)e$ for u quarks and $-(1/3)e$ for d quarks, so that the proton which is a *uud* particle carries a net charge of $+e$, while the neutron, which is a *udd* particle, carries a zero net charge.

The taxonomy of the quarks and of the other elementary particles of the Standard Model (neutrinos, leptons, and bosons) is shown in Fig. 4.3. One observes that quarks (top two rows), neutrinos, and leptons all come in three "generations" with very significant increase in mass as one moves from the first to the third "generation."

4.1.2 The Standard Model of Elementary Particles

Thus one may summarize the present situation regarding elementary particles by stating that they are of three types, i.e. *leptons* (which include the electron), *quarks*, which form the hadrons, and *mediators*, such as the photons, which mediate the electromagnetic force and the $W^{\pm}$ and Z^o *bosons*, which mediate the weak interaction.

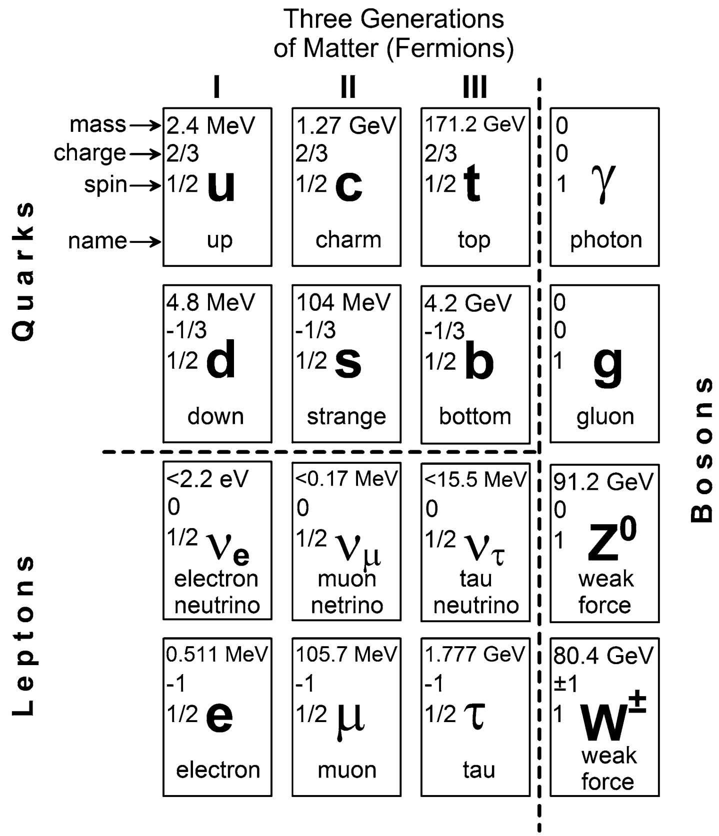


Fig. 4.3 Taxonomy of the elementary particles according to the standard model

All elementary particles are either bosons or fermions [1, 2, 7]. Bosons follow Bose–Einstein statistics (allowed multiple occupancy of the same state), carry integer spin, and are associated with forces. Fermions have half-integer spin, follow Fermi-Dirac statistics (disallowed multiple occupancy of the same state), and are associated with normal matter.

Table 4.1 lists the currently known elementary particles in a way similar to Fig. 4.3. It should be noted that Table 4.1 contains both the classification of leptons and quarks by generations according to the Standard Model of elementary particles but also experimental data regarding masses, lifetimes, and principal decays.

This classification is also shown in Table 4.2 which introduces additionally the twelve antiparticles corresponding to the twelve elementary fermions.

Thus according to the Standard Model of particle physics there exist 12 flavors of elementary fermions plus their corresponding antiparticles (Tables 4.1 and 4.2). There are also four bosons which mediate the forces and also the undiscovered Higgs boson [8–10] which is hypothesized to mediate the creation of mass and

Table 4.1 Elementary particles [1]; masses in MeV/c^2, lifetimes in s

Generation	Flavor	Charge	Mass	Lifetime	Principal decays
Leptons (spin 1/2)					
First	e (electron)	-1	0.510999	∞	–
	ν_e (e neutrino)	0	0	∞	–
Second	μ (muon)	-1	105.659	2.19703×10^{-6}	$e\nu_\mu \bar{\nu}_e$
	ν_μ (μ neutrino)	0	0	∞	–
Third	τ (tau)	-1	1776.99	2.91×10^{-13}	$e\nu_\tau \bar{\nu}_e, \mu\nu_\tau \bar{\nu}_\mu,$ $\pi^- \nu_\tau$
	ν_τ (τ neutrino)	0	0	∞	–
Quarks (spin 1/2)					
First	d (down)	$-1/3$	7		
	u (up)	2/3	3		
Second	s (strange)	$-1/3$	120		
	c (charm)	2/3	1,200		
Third	b (bottom)	$-1/3$	4,300		
	t (top)	2/3	17,400		
Bosons (spin 1)					
Strong	g (8 gluons)	0	0	∞	–
Electromagnetic	γ (photon)	0	0	∞	–
Weak	$W^\pm$ (charged)	± 1	80,420	3.11×10^{-25}	$e^+\nu_e, \mu^+\nu_\mu,$ $\tau^+\nu_\tau,$ $cX \to$ hadrons
	Z^0 (neutral)	0	91,190	2.64×10^{-25}	$e^+e^-, \mu^+\mu^-,$ $\tau^+\tau^-,$ $q\bar{q} \to$ hadrons

Table 4.2 Fundamental fermions

First generation	Second generation	Third generation
Particles		
Electron: e^-	Muon: μ^-	Tau lepton: τ^-
Electron-neutrino: ν_e	Muon-neutrino: ν_μ	Tau-neutrino: ν_τ
Up quark: u	Charm quark: c	Top quark: t
Down quark: d	Strange quark: s	Bottom quark: b
Antiparticles		
Positron: e^+	Positive muon: μ^+	Positive tau lepton: τ^+
Electron-antineutrino: $\bar{\nu}_e$	Muon-antineutrino: ν_μ	Tau-antineutrino: $\bar{\nu}_\tau$
Up antiquark: $\bar{u}$	Charm antiquark: $\bar{c}$	Top antiquark: $\bar{t}$
Down antiquark: $\bar{d}$	Strange antiquark: $\bar{s}$	Bottom antiquark: $\bar{b}$

whose existence has become highly questionable after two years of intensive search for it at the LHC of CERN near Geneva [10]. The Standard Model contains a total of 26 adjustable parameters, a number which is too large for a mature theory. It is thus widely recognized that, despite its successes, the Standard Model, which is fundamentally incompatible with general relativity, is a provisional theory and not a fundamental one. For example, it is now well established [3, 4] that neutrinos are not massless as assumed in the Standard Model. Also the Standard Model does not deal at all with gravity and with the associated hypothesized mediating particles termed gravitons. The need for "Physics beyond the Standard Model" is becoming increasingly evident [4, 10].

4.2 Leptons

4.2.1 Charged Leptons

As shown in Fig. 4.3 and Tables 4.1 and 4.2 there are three charged leptons, i.e. the electron (mass 0.511 MeV/c^2) the muon (μ, mass 105.7 MeV/c^2) and the more recently discovered and much heavier tau lepton (τ, mass 1.777 GeV/c^2). Their corresponding antiparticles are the positron (e^+), the positive muon (μ^+), and the positive tau lepton (τ^+).

The muon has a relatively long lifetime ($2.2 \cdot 10^{-6}$ s) and behaves in every respect as a heavy electron. It decomposes to an electron, a miu-neutrino (ν_μ) and an electron antineutrino ($\bar{\nu}_e$). The tau has a significantly shorter lifetime ($1.9 \cdot 10^{-13}$ s) and also decomposes to an electron, a muon, a negatively charged pion, an electron antineutrino, a muon antineutrino and a tau neutrino. It is difficult to understand how a particle with such a complex decay spectrum can be considered as an elementary particle.

It is also impressive and somehow difficult to understand how the lepton family contains some extremely light particles, i.e. the neutrinos with masses of the order of 0.1 eV/c^2 and at the same time particles as heavy as the muon (105.7 MeV/c^2) and the tau lepton (1.777 GeV/c^2). The latter is ten orders of magnitude heavier than the neutrino and it is indeed very difficult to imagine how both can belong to the same family and be elementary particles.

4.2.2 Neutrinos

Neutrinos are the lightest known leptons. As their name suggests they are electrically neutral. They usually travel close to the speed of light and pass through ordinary matter almost unaffected, thus making them very difficult to detect.

Until 1980 they were believed to have no rest mass and this assumption was built in the Standard Model. It is now firmly established that they do have rest mass which is quite small, i.e. in the 0.04–0.4 eV/c^2 range [3,4]. This makes them a factor $2.4 \cdot 10^9$–$2.4 \cdot 10^{10}$ lighter than protons and neutrons and 10^6–10^7 times lighter than electrons.

Neutrinos are extremely abundant in our universe. They are produced or consumed with practically every nuclear reaction occurring in nature, so that supernovae, distant stars, our sun but also human manufactured and operated nuclear reactors are sources of neutrinos. Most neutrinos passing through the Earth originate from the sun. These solar neutrinos are so abundant that every second in the region of the Earth about sixty five billion, i.e. $6.5 \cdot 10^{10}$ solar neutrinos, pass through every square centimeter perpendicular to the direction of the sun. Yet it is estimated that only one of them may interact with a nucleon in a human body over a lifetime.

Due to their very large abundance it is estimated that the total neutrinos mass is equivalent to 10–100% of the total baryonic mass of the universe.

Since neutrinos are overall electrically neutral they are not affected significantly by electromagnetic forces, although it was shown recently that neutrinos can have an effective charge radius in presence of fermionic masses [11, 12] and that neutrinos have magnetic dipole moments of the order of $10^{-19}\ \mu_B$, where $\mu_B = e/2m_e$ is the Bohr magneton.

Neutrinos can be used as beam particles [2] in scattering experiments studying the quark distribution in nucleons. Neutrinos are believed to couple to the weak charge of the quarks via the weak interaction.

4.2.2.1 Rest Masses and Total Energies of Neutrinos

Although the rest masses of neutrinos are in the 0.04–0.4 eV/c^2 range, their energy can be very high. Thus supernova neutrinos have typical energies, E, of 10–200 MeV.

Since the total energy of a particle, E, is related to its rest mass, m_o, via the Einstein equation:

$$E = \gamma m_o c^2, \tag{4.2}$$

where γ, the Lorentz factor, equals $(1 - v^2/c^2)^{-1/2}$, it follows that such highly energetic neutrinos (e.g. $E = 200$ MeV) travel with γ values as high as $5 \cdot 10^9$, i.e.

$$\gamma \approx 5 \cdot 10^9 \tag{4.3}$$

for $m_o = 0.04$ eV/c^2.

These extremely high γ values lead to very high inertial, m_i, and thus gravitational, m_g, masses. Thus, as already discussed in Chap. 2 and further analyzed in Chap. 5, one has:

$$m_i = m_g = \gamma^3 m_o. \tag{4.4}$$

Consequently if we take $m_o = 0.04$ eV/c^2 and $E = 200$ MeV it is $\gamma = 5 \cdot 10^9$ and therefore:

$$m_i = m_g = (1.25 \cdot 10^{29}) m_o = 5 \cdot 10^{18}\,\text{GeV/c}^2 \tag{4.5}$$

which, very surprisingly, is almost half the value of the Planck mass, i.e. $1.221 \cdot 10^{19}$ GeV/c^2 or 21.8 μg. The Planck mass definition and value was first presented in Chap. 2 (Table 2.3). This is a huge mass for a single particle and it corresponds to a particle energy of $1.221 \cdot 10^{19}$ GeV. At such extremely high particle energies, termed Planck energies, it is anticipated, in the context of the Grand Unification Theories (GUT) [1], that the magnitudes of gravitational and strong forces merge. It thus becomes evident that gravitational forces between such highly energetic neutrinos may be quite significant, i.e. as strong as the strong force. This will be discussed more quantitatively in Sect. 6.2. The strong gravitational attraction between highly relativistic neutrinos may also be closely related to neutrino trapping [13, 14] and is quite close to the central theme of this book, presented in Chap. 6.

4.2.2.2 Neutrino Flavours

There are three types, or "flavors," of neutrinos, i.e. electron neutrinos, muon neutrinos and tau neutrinos. They are symbolized ν_e, ν_μ and ν_τ (Table 4.1).

The corresponding antiparticles are symbolized $\overline{\nu}_e$, $\overline{\nu}_\mu$, and $\overline{\nu}_\tau$, respectively (Table 4.2). Since known neutrinos carry no net charge, it is possible that these antineutrinos are identical to the corresponding neutrinos. Particles with this property are known as Majorana particles.

A fourth type of neutrino, the sterile neutrino, is also being theorized in an effort to account for neutrino oscillations.

Neutrinos play an important role in astrophysical events such as supernovae which take place when an old massive star collapses after depletion of its nuclear fuel. During this collapse, the star becomes for a few ($\sim$10) seconds a neutrino star. This demonstrates the ability of neutrinos to form condensed structures. During these seconds the neutrinos totally dominate in number of particles and carry out most of the energy of the implosion which leads to the formation of a neutron star. The energy released during this implosion is much larger than that radiated during the whole life of the star. Approximately 99% of the energy of the supernova is released during this 10s burst of neutrinos which are the only particles to escape the advancing core of the collapse phase of the supernova. Neutrinos are produced in all exoergic events taking place in the cosmos (supernovae) or in the microcosmos (nucleosynthesis, nuclear reactions, β-decay). They also play an important role in energy transport since with their short range forces they escape all but the most dense objects.

As already noted, it is estimated that the total mass of neutrinos in the universe is very significant and equivalent to 10–100% of the baryonic mass of the universe. If the neutrino masses are of several eV total, then it is estimated that neutrinos may be the dominant mass in the universe.

There have been propositions that neutrinos might account for the "dark matter problem" but according to some theoretical calculations neutrinos do not cluster enough for such a purpose. In any event, due to their estimated huge number and total mass, neutrinos must play a very important role in the formation, structure, and ultimately in the fate of our universe.

4.2.2.3 Neutrino Detection

Since neutrinos are thought to interact very weakly with matter, neutrino detectors are very large in order to capture significant numbers of neutrinos. Neutrino detectors are often constructed deep in the earth in order to isolate the detector from cosmic rays and other radiation.

Antineutrinos were first detected near a nuclear reactor in the 1950s by Reines and Cowan. They used two large targets containing an aqueous solution of $CdCl_2$ and two scintillation detectors each next to the Cd targets. Antineutrinos with an energy above 1.8 MeV interacted with protons in the water, producing positrons and neutrons. The concomitant positron annihilations with electrons produced photons with an energy around 0.5 MeV. In this way pairs of photons could be detected simultaneously by the two scintillation detectors above and below the target.

The neutrons were captured by Cd nuclei thus producing γ rays of around 8 MeV. These gamma rays were detected a few microseconds after each positron annihilation event.

During the last 50 years several others detection methods have been developed.

The Super-Kamiokande in the Japanese Alps is based on the detection of Cherenkov radiation emitted when a neutrino creates an electron or muon (of positive or negative charge) in the water and this charged particle moves some distance (up to 5 m) in the H_2O with a velocity higher than the speed of light in H_2O (which is roughly 3/4 of the speed of light in vacuum). The resulting Cherenkov radiation is detected by neighboring photomultiplier tubes [3].

Similar is the detection principle at the Sudbury Neutrino Observatory where D_2O (rather than H_2O) is used as the detecting medium. This also allows for the neutrino induced photodissociation of deuterium which leads to a free neutron. This is then detected from the gamma radiation emitted after chlorine capture.

Other detectors utilize large volumes of chlorine or gallium which interact with electron neutrinos to yield argon or germanium, respectively. The IceCube Neutrino Observatory uses 1 km^3 of Antarctic ice with photomultiplier tubes distributed throughout the volume.

4.2.2.4 Neutrino Oscillations

Since the late 1960s it was found that the number of electron neutrinos arriving from the sun was between one third and one half the number predicted by the Standard Solar Model (SSM). This became known as the solar neutrino problem and remained unresolved for 30 years.

After 1998 it was found both in the Super-Kamiokande [3] and in the Sudbury Neutrino observatories [4] that solar and atmospheric neutrinos change flavors. This could explain the solar neutrino problem, i.e. electron neutrinos produced in the sun change into other flavors which cannot be detected.

Thus neutrino oscillations is the phenomenon where neutrinos change flavor (e.g. $\nu_\mu \leftrightarrow \nu_e$) and at the same time change mass. This permits a neutrino, originally produced as an electron neutrino at a given location, to be detected as a muon neutrino or as a tau neutrino after traveling to some other location. According to the standard model the existence of flavor oscillations directly implies nonzero differences between the neutrino masses, commonly denoted m_1, m_2, and m_3. This is because the amount of mixing between neutrino flavors depends on the differences in their squared masses [4, 14].

Neutrino oscillation experiments have already provided reliable values for the neutrino mass-squared differences, $\Delta m_{ij}(\Delta m_{ij}^2 = m_j^2 - m_i^2)$, and for the mixing angler, θ_{ij}, which are the angles in the usual particle data group definition of a unitary mixing matrix.

The allowed ranges at the 3σ confidence level are:

$$
\begin{aligned}
\Delta m_{12}^2 &= 7.9(\pm 2.8) \times 10^{-5} (\mathrm{eV/c^2})^2 \\
|\Delta m_{23}^2| &= 2.6(\pm 0.2) \times 10^{-3} (\mathrm{eV/c^2})^2 \\
\theta_{12} &= 33.7(\pm 1.3) \\
\theta_{23} &= 43.3(\pm 9.8) \\
\theta_{13} &= 0(\pm 2.6).
\end{aligned}
$$

These lead to the following possible arrangements of the neutrino masses [3, 4]:

1. Normal mass hierarchy, i.e. $m_1 < m_2 \ll m_3$. In this case $\Delta m_{23}^2 = m_3^2 - m_2^2 > 0$ and $m_3 \approx \sqrt{\Delta m_{23}^2} \approx 0.051(\pm 0.01)\,\mathrm{eV/c^2}$. The solar neutrino oscillations occur between the two lighter levels.
2. Inverted mass hierarchy, i.e. $m_1 \approx m_2 \gg m_3$ with $m_1 \approx m_2 \approx \sqrt{\Delta m_{23}^2} \approx 0.051(\pm 0.01)\,\mathrm{eV/c^2}$. In this case solar neutrino oscillations take place between the two heavier levels.

The best current estimates of the neutrino masses are shown in Figs. 4.4 and 4.5 on the basis of the Kamiokande experiments. These lead to an estimate of $m_o = 0.051 \pm 0.01\,\mathrm{eV/c^2}$ for the mass of the heaviest neutrino. Very useful information is expected from a new neutrino observatory in Karlsruhe, called KATRIN, which is planned to start full operation in 2013 and is expected to measure the mass of the electron antineutrino with sub-eV precision by examining the spectrum of electrons emitted in tritium β-decay.

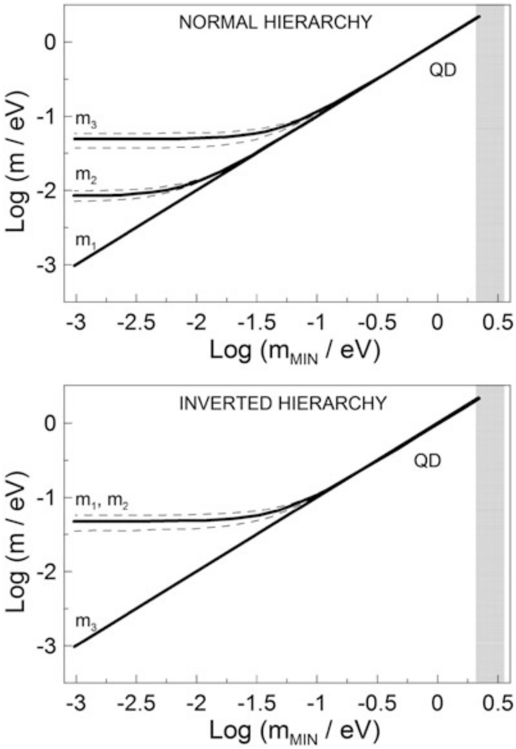


Fig. 4.4 The three light neutrino masses as a function of the lightest mass for the normal (*top plot*) and inverted (*bottom plot*) hierarchy, reprinted from [4]

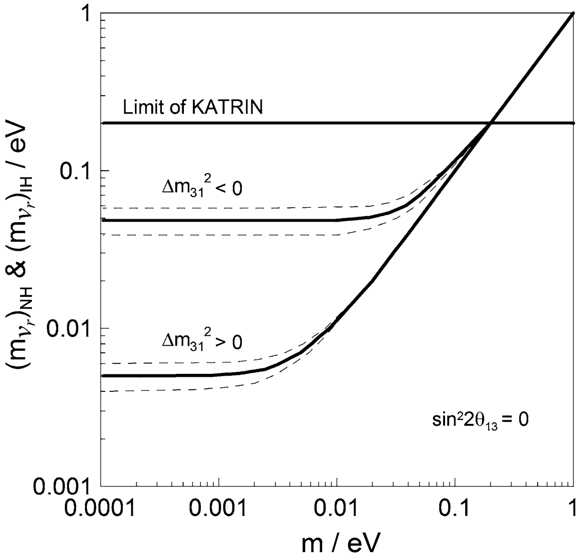


Fig. 4.5 The observable neutrino effective mass m_{ν_r} [4] as a function of the lightest mass for the normal (*bottom*) and inverted (*upper*) mass ordering. The currently allowed 3σ ranges of the oscillation parameters were used. Comparison with the detection limit of KATRIN [4]

4.2.2.5 Neutrino Spins and Electromagnetic Dipole Moments

Neutrinos are fermions with a spin of 1/2 and it is not yet clear if they are Dirac or Majorana particles. In the latter case the neutrino and the antineutrino would in fact be the same particle. The neutrino could transform into an antineutrino by changing the orientation of its spin state. Such a change in spin would require both the neutrino and antineutrino to have nonzero masses, and therefore to travel slower than light. This is because such a spin change, caused only by the change in point of observation, is possible only if there exist reference inertial frames moving faster than the particle. Such a particle has a spin of one orientation when observed from a frame moving slower than the particle and the opposite spin when observed from a frame moving faster than the particle.

The still open question regarding the Dirac or Majorana nature of neutrinos has some significant implications regarding their magnetic and electric dipole moments. Thus a massive neutrino can have a diagonal magnetic dipole moment. It can also have a CP-violating electric dipole moment. CP-symmetry refers to the combination of C-symmetry (charge conjugation) and parity symmetry [1, 4]. Both Dirac and Majorana neutrinos can have nondiagonal, magnetic, and electric dipole moments, termed transition moments.

If the Standard Model is extended to include massive Dirac neutrinos, then the neutrino magnetic moment is given by [4]:

$$\mu_{\nu_i} = \frac{3eG_F m_{\nu_j}}{8\sqrt{2}\pi^2} = 1.6 \cdot 10^{-19} \left(\frac{m_{\nu_i}}{\mathrm{eV}}\right) \mu_\mathrm{B}, \tag{4.6}$$

where $\mu_\mathrm{B} = e/2m_\mathrm{e}$ is the Bohr magneton. Neutrino scattering measurement has provided an upper limit for the neutrino magnetic moment, i.e.

$$\mu_{\nu_i} \leq 1.3 \times 10^{-10} \mu_\mathrm{B}. \tag{4.7}$$

The observation of nonzero neutrino magnetic moments can be considered as evidence for new physics at the TeV scale.

4.3 Hadrons

Hadrons are composite particles consisting of quarks and antiquarks. Baryons and mesons belong to the hadron family. Baryons consist of three quarks, while mesons consist of a quark and an antiquark (Fig. 4.2).

Quarks are confined in hadrons and cannot be separated from them and studied independently. As already discussed in Chap. 3, they are held together by the strong force due to their differences in a property called color (Fig. 4.2).

The most common baryons are the protons and neutrons which make up most of the mass in the visible matter in the universe. Electrons, the other major component of atoms, molecules and living organisms, are leptons. Each baryon

Table 4.3 Quark masses (MeV/c^2) [1]

Quark flavor	Bare mass	Effective mass
u (up)	2	336
d (down)	5	340
s (strange)	95	486
c (charm)	1,300	1,550
b (bottom)	4,200	4,730
t (top)	174,000	177,000

has a corresponding antiparticle (antibaryon) where quarks are replaced by their corresponding antiquarks. As an example the proton is a *uud* particle, i.e. it consists of two *u* (up) quarks and a *d* (down) quark. Its corresponding antiparticle, the antiproton, contains two up antiquarks and one down antiquark. Interestingly, although quarks and antiquarks are universally believed to be the building blocks of protons, neutrons, and other baryons, they have never been isolated and studied independently. Thus there exist, for example, quite large uncertainties about their actual masses, as shown in Table 4.3. Rigorous definitions of "bare" and "effective" masses shown in this table are difficult to find but in a broad sense they can be considered equivalent to rest and relativistic masses, respectively, as discussed in Chap. 8.

As already noted, the existence of quarks has been inferred from inelastic electron–proton and electron–neutron scattering spectra, such as the one shown in Fig. 4.1. In addition to the dominant elastic scattering peak, there exist at least three more inelastic scattering peaks. These spectra show conclusively that baryons have an internal structure and thus consist of fundamental constituents which in fact [2] are pointlike and have a spin 1/2. As already noted, these fundamental point-like constituents were called partons by Feynman and later on quarks by Gell–Mann [2, 15].

Also, as already discussed, there are six types of quarks according to the Standard Model (Fig. 4.3), which include the *u* (up), *d* (down), *c* (charm), *s* (strange), *t* (top), and *b* (bottom). Quarks participate in the strong interaction via the action of gluons and form hadrons.

When examining the Standard Model quark content of baryons and mesons (Table 4.4), one notes in these tables that with the exception of the proton, the electron and the neutrinos (Table 4.1), all other elementary particles are unstable, with lifetimes varying between 10^{-6} and 10^{-25} s. The neutron is also unstable with a lifetime of 885.7 s and decomposes to a proton, an electron and a $\bar{\nu}_e$ electron antineutrino as already discussed. However the neutron becomes quite stable when it is bound with protons, forming nuclei via the residual strong force. It is worth noting that the lifetimes of some baryons and mediators is so short (10^{-25} s, which corresponds to the period of rotation of a relativistic particle on a circular orbit of radius 1 fm) that the distinction between a particle and a *resonance* becomes rather vague.

Table 4.4 Hadrons [1]; masses in MeV/c^2

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
Baryons (spin 1/2)					
N p	uud	1	938.272	∞	–
N n	udd	0	939.565	885.7	$pe\bar{\nu}_e$
Λ	uds	0	1,115.68	2.63×10^{-10}	$p\pi^-, n\pi^0$
Σ^+	uus	1	1,189.37	8.02×10^{-11}	$p\pi^0, n\pi^+$
Σ^0	uds	0	1,192.64	7.4×10^{-20}	$\Lambda\gamma$
Σ^-	dds	-1	1,197.45	1.48×10^{-10}	$n\pi^-$
Ξ^0	uss	0	1,314.8	2.90×10^{-10}	$\Lambda\pi^0$
Ξ^-	dss	-1	1,321.3	1.64×10^{-10}	$\Lambda\pi^-$
Λ_c^+	udc	1	2,286.5	2.00×10^{-13}	$pK\pi, \Lambda\pi\pi, \Sigma\pi\pi$
Baryons (spin 3/2)					
Λ	uuu, uud, udd, ddd	$2,1,0,-1$	1,232	5.6×10^{-24}	$N\pi$
Σ^*	uus, uds, dds	$1,0,-1$	1,385	1.8×10^{-23}	$\Lambda\pi, \Sigma\pi$
Ξ^*	uss, dss	$0,-1$	1,533	6.9×10^{-23}	$\Xi\pi$
Ω^-	sss	-1	1,672	8.2×10^{-11}	ΛK^-, $\Xi\pi$
Pseudoscalar Mesons (spin 0)					
$\pi^\pm$	$u\bar{d}, d\bar{u}$	$1,-1$	139.570	2.60×10^{-8}	$\mu\nu_\mu$
π^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	0	134.977	8.4×10^{-17}	$\gamma\gamma$
$K^\pm$	$u\bar{s}, s\bar{u}$	$1,-1$	493.68	1.24×10^{-8}	$\mu\nu_\mu, \pi\pi, \pi\pi\pi$
$K^0, \bar{K}^0$	$d\bar{s}, s\bar{d}$	0	497.65	$K_S^0: 8.95\times10^{-11}$ $K_L^0: 5.11\times10^{-8}$	$\pi\pi$ $\pi e\nu_e, \pi\mu\nu_\mu, \pi\pi\pi$
η	$(u\bar{u}+d\bar{d}-2ss)/\sqrt{6}$	0	547.51	5.1×10^{-19}	$\gamma\gamma, \pi\pi\pi$
η'	$(u\bar{u}+d\bar{d}-ss)/\sqrt{3}$	0	957.78	3.2×10^{-21}	$\eta\pi\pi, \rho\gamma$
$D^\pm$	$c\bar{d}, d\bar{c}$	$1,-1$	1,869.3	1.04×10^{-12}	$K\pi\pi, K\mu\nu_\mu, Ke\nu_c$
$D^0, \bar{D}^0$	$c\bar{u}, u\bar{c}$	0	1,864.5	4.1×10^{-13}	$K\pi\pi, Ke\nu_c, K\mu\nu_\mu$
$D_s^\pm$	$c\bar{s}, s\bar{c}$	$1,-1$	1,968.2	5.0×10^{-13}	$\eta\rho, \phi\pi\pi, \phi\rho$
$B^\pm$	$u\bar{b}, b\bar{u}$	$1,-1$	5,279.0	1.6×10^{-12}	$D^*\ell\nu_\ell, D\ell\nu_\ell, D^*\pi\pi\pi$
$B^0, \bar{B}^0$	$d\bar{b}, b\bar{d}$	0	5,279.4	1.5×10^{-12}	$D^*\ell\nu_\ell, D\ell\nu_\ell, D^*\pi\pi$
Vector Mesons (spin 1)					
ρ	$u\bar{d}, (u\bar{u}-d\bar{d})/\sqrt{2}, d\bar{u}$	$1,0,-1$	775.5	4×10^{-24}	$\pi\pi$
K^*	$u\bar{s}, d\bar{s}, s\bar{d}, s\bar{u}$	$1,0,-1$	894	1×10^{-23}	$K\pi$
ω	$(u\bar{u}+d\bar{d})/\sqrt{2}$	0	782.6	8×10^{-23}	$\pi\pi\pi, \pi\gamma$
ψ	$c\bar{c}$	0	3,097	7×10^{-21}	$e^+e^-, \mu^+\mu^-, 5\pi, 7\pi$
D^*	$c\bar{d}, c\bar{c}, u\bar{c}, d\bar{c}$	$1,0,-1$	2,008	3×10^{-21}	$D\pi, D\gamma$
Υ	$b\bar{b}$	0	9,460	1×10^{-20}	$e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

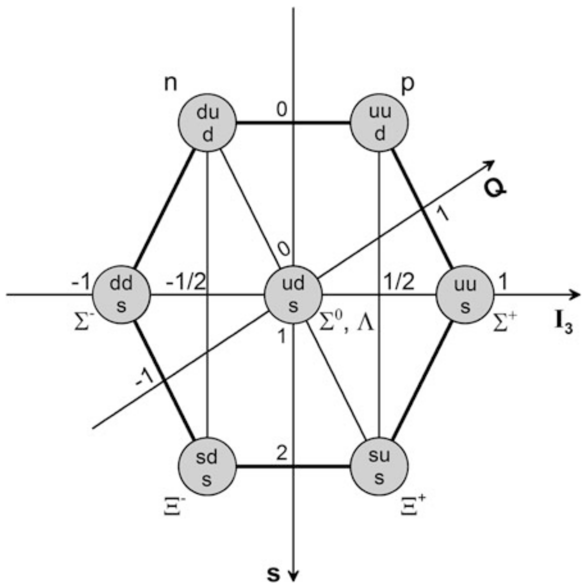


Fig. 4.6 Combinations of three u, d, or s quarks forming baryons with a spin-$1/2$ create the *uds baryon octet*. Here Q is the charge in units of e, s is the strangeness, and I_3 is the isospin. For example the proton has charge 1, isospin 1/2, and strangeness zero

When examining Table 4.4, please note that with the exceptions of the proton, the neutron, and the Kaon $(K^{\pm}, K^0, \bar{K}^0)$ all other baryons and mesons are called as the letter of their symbol, e.g. pi for $(\pi^{\pm}, \pi^0)$. Quark symbols stand for up (u), down (d), strange (s), charm (c), and bottom (b).

4.3.1 The Standard Model Taxonomy of Hadrons

The present Standard Model taxonomy of hadrons in terms of their charge (Q), strangeness (S), and isospin, (I_3) is due primarily to Gell–Mann. Strangeness is a measure of the number of strange quarks (s) present in the baryon and isospin is a quantity analogous conceptually to spin, used first by Heisenberg to differentiate between a proton and a neutron which have very similar masses. Two examples of this Gell–Mann taxonomy, which actually led to a Nobel prize in Physics in 1969 [15], are shown in Figs. 4.6 and 4.7.

Thus Fig. 4.6. shows combinations of three u, d, or s quarks forming baryons with spin 1/2. This is called the *uds* baryon octet.

Combination of three u, d, or s quarks forming baryons with spin 3/2 are shown in Fig. 4.7. They are called the *uds* baryon decuplet.

4.3.2 Hadron Masses

There have been continuous efforts to find regularities regarding the hadron masses. The most well-known correlation is due to Gell–Mann and Okubo [16, 17].

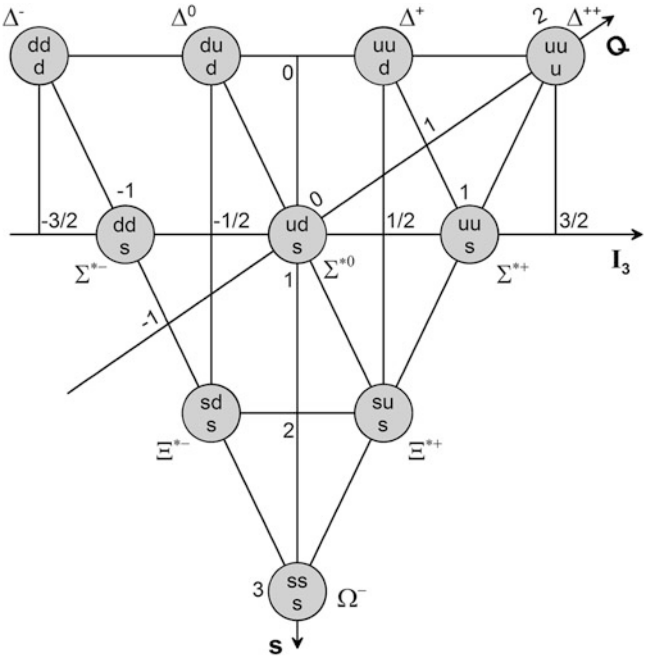


Fig. 4.7 Combinations of three u, d, or s quarks forming baryons with a spin-3/2 create the *uds baryon decuplet*

Another recently found correlation is shown in Fig. 4.8. The masses of uncharmed baryons follow a $(2n-1)^{1/6}$ law [18]. A possible reason for this is presented in Chap. 6.

4.3.3 Hadron Angular Momenta

There exists a very interesting observation regarding the angular momenta, L, of hadrons and of their excited states. This is shown in Fig. 4.9.

The normalized, with respect to $\hbar$, angular momenta of hadrons and of their excited states are of the order of, and seem to be bounded by, the square of their mass expressed in GeV^2 [19]. The reason for this is not yet obvious. It is interesting however to note that the angular momenta of baryons are very similar, $\sim\hbar$, to that of an electron in a H atom. Since the radius of baryons ($\sim$fm) are much shorter than that of the H atom ($\sim$0.1 nm), it follows that the baryons must contain very fast rotating constituents.

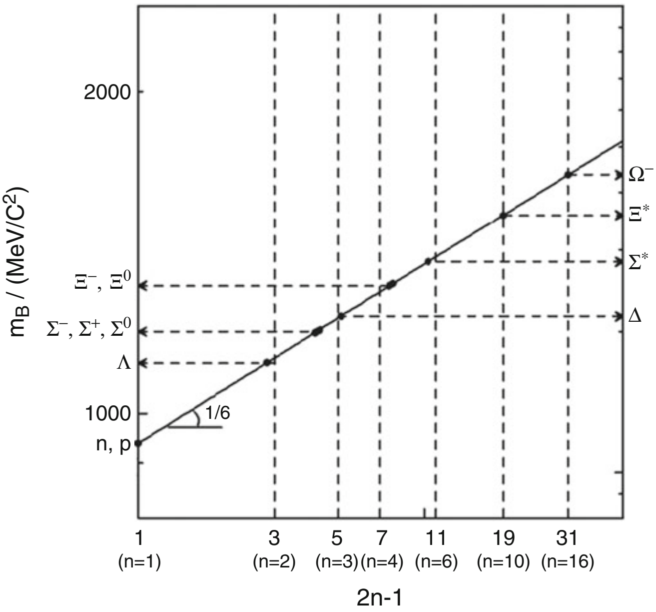
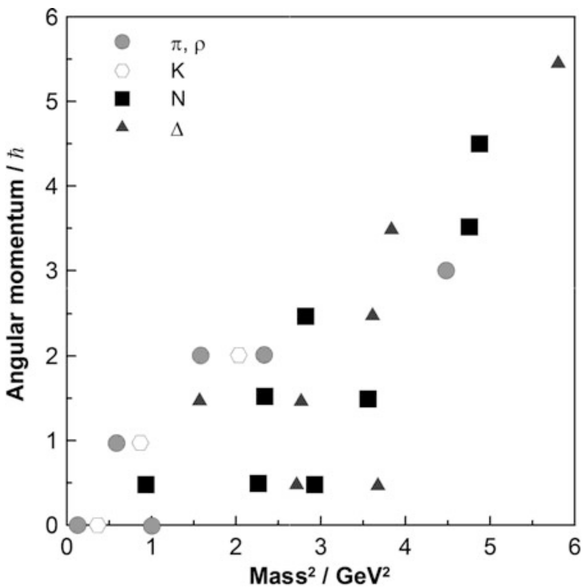


Fig. 4.8 Comparison of the masses, m_B, of the uncharmed baryons, consisting of u, d, and s quarks, with equation $m_B = m_n(2n-1)^{1/6}$, where m_n is the neutron mass [18]

Fig. 4.9 Dependence of the normalized by $\hbar$ angular momentum of some hadrons and of their excited states (resonances) on their normalized by 1 GeV mass [19]. Reprinted with permission from Nature McMillan



This can be deduced approximately as follows: Assuming that a fraction, f, of the baryon mass, m_b, is rotating with a speed $\mathrm{v_b}$ on a circular orbit with radius R_b, it follows that the baryon angular momentum, L_b, is given by:

$$L_\mathrm{b} = f m_\mathrm{b} \mathrm{v_b} R_\mathrm{b} \approx \hbar. \tag{4.8}$$

Similarly for the electron in the ground state of the H atom it is $\mathrm{v_e} = \alpha c$ and thus:

$$L_\mathrm{e} = m_\mathrm{e} \alpha c R_e \approx \hbar. \tag{4.9}$$

From Eqs. (4.8) and (4.9), it follows:

$$\frac{\mathrm{v_b}}{c} = \frac{m_\mathrm{e}}{f m_\mathrm{b}} \cdot \frac{\alpha R_e}{R_\mathrm{b}}. \tag{4.10}$$

Considering a neutron as the baryon ($m_\mathrm{b} = 939.565\,\mathrm{MeV/}^2$) using $m_\mathrm{e} = 0.511$ MeV/c^2, $R_e = 0.53 \cdot 10^{-10}$ m, $R_\mathrm{b} \approx 0.63 \cdot 10^{-15}$ m, $\alpha = 1/137.035$ and assuming $f = 1/3$ one computes from Eq. (4.10) that indeed $\mathrm{v_b}/c \approx 1$. This point will be addressed more exactly in Chaps. 6 and 7.

4.4 Synopsis

Things are changing significantly every decade in the fascinating world of particles. The nature of quarks and gluons still remains an exciting mystery. There have been some very important recent discoveries regarding neutrinos, which have been shown to have nonzero masses and also to be able to form, even though for short times, (only $\sim$10 s), stable structures, such as neutrino stars during the onset of supernovae. Neutrino trapping is also a related interesting phenomenon. Neutrinos are emitted in practically all nuclear reactions. The angular momenta of baryons and their excited states is of the order of $\hbar$, similarly with the angular momentum of an electron in the H atom Bohr model. The exact physical meaning of quark color, strangeness, and isospin is not yet fully understood.

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Chapter 5
The Equivalence Principle, Special Relativity, and Newton's Gravitational Law

5.1 The Weak Equivalence Principle

The equivalence principle has always played a central role in the development of Physics and in our efforts to understand gravity.

The weak equivalence principle is the simplest and less demanding form of the equivalence principle and states simply that inertial mass equals gravitational mass. The first experimental observations leading eventually to the weak equivalence principle can be traced back to John the Philoponus in the sixth century AD who described correctly the negligible effect of different masses on the falling time of dropping balls.

Some ten centuries later, in 1586, Simon Stevin had reached the same conclusion by dropping lead balls of different masses off the Delft churchtower: All balls would reach the ground simultaneously. Even today this remains the simplest experimental way to check for the validity of the equivalence principle.

Around 1610 Galileo Galilei made similar observations upon rolling balls down inclined planes. He found that they all fell simultaneously.

Galileo expressed his and previous observations in a few important lines: "The acceleration of a test mass due to gravitation is independent of the amount of mass being accelerated."

This was a first expression of the weak equivalence principle and Isaac Newton, adding his own experimental observations on the identical periods of pendulums with different masses and identical length, proceeded to lay the foundations of the gravitational theory in which the inertial and gravitational masses are identical.

Thus upon considering a falling body, with inertial mass m_{i}, and upon using Newton's second law, it is:

$$F = m_{\mathrm{i}} \frac{\mathrm{d}\mathrm{v}}{\mathrm{d}t}. \tag{5.1}$$

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DOI 10.1007/978-1-4614-3936-3_5,

If the force acting on the body is the gravitational attraction of a second body of mass M_g (e.g. the earth), then according to Newton's universal gravitational law, it is:

$$F = F_G = G\frac{m_g M_g}{r^2}. \tag{5.2}$$

Upon combining with Eq. (5.1) one obtains:

$$\frac{m_i}{m_g}\frac{dv}{dt} = \frac{GM_g}{r^2}. \tag{5.3}$$

A second falling body with inertial mass m_i' and gravitational mass m_g' will satisfy the same equation, i.e.

$$\frac{m_i'}{m_g'}\frac{dv'}{dt} = \frac{GM_g}{r^2}. \tag{5.4}$$

Since experiment shows that the acceleration of both particles is the same, it follows that:

$$\frac{m_i}{m_g} = \frac{m_i'}{m_g'}, \tag{5.5}$$

i.e. the gravitational mass, passive or active, has to be proportional to the inertial mass for all objects.

This leads to the following common expression of the *weak equivalence principle*, also known as the *universality of free fall*: "The trajectory of a falling test particle depends only on its initial position and velocity and is independent of its composition and rest mass."

The weak equivalence principle has been confirmed many thousands of times. Most of the confirmations during the last century involve the torsion balance, first designed in 1876 by the Hungarian Baron Loränd Eötvös. Using torsion balances with aluminium and platinum test masses and measuring acceleration towards the sun, it is now firmly established that any difference between gravitational and inertial mass is less than one part in 10^{12} [1–3].

In summary, the weak equivalence principle is proven beyond any reasonable doubt and has been at the center of Newtonian mechanics for more than three centuries. This central position of the weak equivalence principle has never been challenged after the introduction and wide acceptance of special relativity and of relativistic mechanics. The development by Einstein of the gravitational theory of general relativity has also never challenged the validity of the weak equivalence principle. In fact the weak equivalence principle is always considered as one of the cornerstones of the theory of general relativity.

Before proceeding to discuss the two other and stronger versions of the equivalence principle, i.e. the Einstein equivalence principle and the strong equivalence principle, it is useful for our analysis and subsequent synthesis to first discuss special relativity, the theory first published by Einstein in 1905 [4–6]. There are two reasons for this:

- First because special relativity, like the weak equivalence principle, has been confirmed experimentally many thousands of times and there can be no reasonable doubt about its exact validity.
- Second because the synthesis of special relativity and of the weak equivalence principle creates some fascinating possibilities which have not been yet fully exploited. The reason we believe is that the general relativity theory, which is one but not the only theory of gravity, was developed by Albert Einstein only 10 years after his pioneering special relativity paper.

With such a powerful theory for gravity as general relativity, published in 1916 [7, 8] and first tested in 1919 [9], very few researchers would invest in examining what other possibly simpler and perhaps to some extent similar approaches may result from the synthesis of special relativity and of the equivalence principle, which is a key theme of the present book.

5.2 Special Relativity

It was in 1905 that Albert Einstein published his famous paper "On the electrodynamics of moving bodies" [4] and presented the foundations of Special Relativity on the basis of two very simple and very powerful postulates:

1. All physical laws valid in one frame of reference, S, are equally valid in any other frame of reference, S', moving uniformly relative to the first.
2. The speed of light (in vacuum) is the same in all inertial frames of reference, regardless of the motion of the light source.

The second postulate is very counterintuitive at the beginning, but is firmly based on the results of the famous and ingenuous Michelson–Morley experiments, which established that the ether does not exist.

5.2.1 Implications of the Special Relativity: Length Contraction and Time Dilation

We consider two inertial frames, S and S'. The latter has a velocity, v, relative to frame S as measured in S. For simplicity one may consider that S is the frame of a laboratory observer.

A key aspect of special relativity are the Lorentz–Einstein transformations [10, 11].

Table 5.1 Lorentz–Einstein transformations

$x' = \gamma(x - \mathrm{v}t)$	$x = \gamma(x' + \mathrm{v}t')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \gamma(t - \mathrm{v}x/c^2)$	$t = \gamma(t' + \mathrm{v}x'/c^2)$
where γ is the Lorentz factor, i.e. $\gamma = (1 - \mathrm{v}^2/c^2)^{-1/2}$ and v is the velocity of S' as measured in S	

A first and striking result is that length, e.g. of an object, and time interval as measured in frame S (denoted $\ell (= x_2 - x_1)$ and $\Delta t (= t_2 - t_1)$ respectively) are different from those measured in frame S' (denoted $\ell' (= x_2' - x_1')$ and $\Delta t' (= t_2' - t_1')$ respectively).

Thus it follows from Table 5.1 that

$$x_1' = \gamma(x_1 - \mathrm{v}t_1), \tag{5.6}$$

$$x_2' = \gamma(x_2 - \mathrm{v}t_2), \tag{5.7}$$

and therefore considering two measurements of the distance for $t_1 = t_2$ it is

$$\ell' = x_2' - x_1' = \gamma(x_2 - x_1) = \gamma\ell. \tag{5.8}$$

Since $\gamma \geq 1$ Eq. (5.8) describes length contraction, i.e. the observer at S perceives a length which is shorter than that perceived in frame S' where the body actually resides.

An equally interesting result is obtained for time interval Δt. Considering two events corresponding to two different readings of the clock, t_1' and t_2', at the same point $x_o' = x_1' = x_2'$ in frame S', one obtains from Table 5.1:

$$t_1 = \gamma(t_1' + \mathrm{v}x_o'/c^2) \tag{5.9}$$

$$t_2 = \gamma(t_2' + \mathrm{v}x_o'/c^2) \tag{5.10}$$

and thus

$$t_2 - t_1 = \gamma(t_2' - t_1') \tag{5.11}$$

i.e.

$$\Delta t = \gamma \Delta t'. \tag{5.12}$$

This is usually referred to as time dilation, i.e. the time interval measured in S between two events taking place in S', is the longest.

Thus it follows by observing Eqs. (5.8) and (5.12) that the measured length of a body is greater in its rest frame than in any other frame while the time interval between two events is the shortest in the rest frame where the two events take place.

One way to memorize these observations is to remember that the laboratory observer S always perceives that events in S' proceed too slowly (short distances,

ℓ, are covered and actually this takes long time intervals Δt). On the contrary an observer in S', where the body actually resides, perceives a very busy environment, i.e. long distances are covered in short time intervals. This is common practice among humans too, who commonly perceive that they (in S') live in a busy environment while others (in S) think that the people (in S') enjoy a much slower pace of life.

5.2.2 *Transformation of Velocities*

Once the Lorentz transformations for distance and time have been obtained, one can extract the time derivatives of displacement as measured in the two different inertial frames.

We define by x the direction of relative motion of the two reference frames and we use Cartesian coordinates with y and z axes perpendicular to each other and to the direction x. Since the transformations for the z direction can be obtained directly from those for the y direction, we treat the vectors as if they have only x and y components.

Starting from (Table 5.1):

$$x = \gamma(x' + \mathrm{v}t') \qquad x' = \gamma(x - \mathrm{v}t) \tag{5.13}$$

$$y = y' \qquad y' = y \tag{5.14}$$

$$t = \gamma(t' + \mathrm{v}x'/c^2) \qquad t' = \gamma(t - \mathrm{v}x/c^2) \tag{5.15}$$

and considering, e.g., an object with velocity components $u_{x'}$, $u_{y'}$ measured in S', i.e.

$$u_{x'} = \frac{\mathrm{d}x'}{\mathrm{d}t'}; \quad u_{y'} = \frac{\mathrm{d}y'}{\mathrm{d}t'} \tag{5.16}$$

one obtains after some simple algebra:

$$u_x = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{u_{x'} + \mathrm{v}}{1 + \mathrm{v}u_{x'}/c^2} \quad u_{x'} = \frac{u_x - \mathrm{v}}{1 - \mathrm{v}u_x/c^2}. \tag{5.17}$$

$$u_y = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{u_{y'}/\gamma}{1 + \mathrm{v}u_{x'}/c^2} \quad u_{y'} = \frac{u_y/\gamma}{1 - \mathrm{v}u_y/c^2}. \tag{5.18}$$

Equations (5.17) and (5.18) represent the relativistic law of addition of two velocities.

For example if we denote:

$$\mathrm{v} = \beta_1 c, \tag{5.19}$$

$$u_{x'} = \beta_2 c, \tag{5.20}$$

one obtains:

$$\frac{u_x}{c} = \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \tag{5.21}$$

or equivalently:

$$1 - \beta = \frac{(1-\beta_1)(1-\beta_2)}{1 + \beta_1 \beta_2}. \tag{5.22}$$

5.2.3 Accelerated Motions

Special relativity can deal not only with inertial motions, but also with accelerated motions in a very straightforward manner [10, 11]. In this case the frame S is again the laboratory frame and the frame S' is an *instantaneous* inertial frame usually taken to be moving together with the object under study with a velocity $\mathbf{v}$ relative to the laboratory frame S.

The longitudinal and transverse accelerations with respect to the direction of relative motion of the two inertial frames can be readily obtained starting from Eqs. (5.17), (5.18), and (5.15), i.e.

$$u_x = \frac{u_{x'} + \mathrm{v}}{1 + \mathrm{v} u_{x'}/c^2} \tag{5.23}$$

$$u_y = \frac{u_{y'}/\gamma}{1 + \mathrm{v} u_{x'}/c^2} \tag{5.24}$$

$$t = \gamma(t' + \mathrm{v}x'/c^2) \tag{5.25}$$

and differentiating at fixed instantaneous velocity v, i.e.

$$\begin{aligned} \mathrm{d}u_x &= \frac{\mathrm{d}u_{x'}}{1 + \mathrm{v}u_{x'}/c^2} - \left[\frac{u_{x'} + \mathrm{v}}{(1 + \mathrm{v}u_{x'}/c^2)^2} \cdot \frac{\mathrm{v}\mathrm{d}u_{x'}}{c^2}\right] \\ &= \frac{(1 - \mathrm{v}^2/c^2)\mathrm{d}u_{x'}}{(1 + \mathrm{v}u_{x'}/c^2)^2} = \frac{\mathrm{d}u_{x'}}{\gamma^2(1 + \mathrm{v}u_{x'}/c^2)^2} \\ \mathrm{d}t &= \gamma(\mathrm{d}t' + \mathrm{v}\mathrm{d}x'/c^2) = \gamma(1 + \mathrm{v}u_{x'}/c^2)\mathrm{d}t', \end{aligned} \tag{5.26}$$

which lead to:

$$a_x \equiv \frac{\mathrm{d}u_x}{\mathrm{d}t} = \frac{\mathrm{d}u_{x'}/\mathrm{d}t'}{\gamma^3(1 + \mathrm{v}u_{x'}/c^2)^3}. \tag{5.27}$$

Consequently:

$$a_x = \frac{\alpha_{x'}}{\gamma^3(1 + \mathrm{v}u_{x'}/c^2)^3}. \tag{5.28}$$

If the particle under consideration is moving together with the instantaneous frame S', then it is $u'_x = 0$ and Eq. (5.28) reduces to:

$$a_x = \frac{a_{x'}}{\gamma^3}. \tag{5.29}$$

This, as we shall see, is a very important result. *The acceleration measured in the laboratory frame S is γ^3 times smaller than the acceleration measured in the instantaneous frame S'.*

Similarly from Eq. (5.24) one obtains:

$$\mathrm{d}u_y = \frac{\mathrm{d}u_{y'}}{\gamma(1+\mathrm{v}u_{x'}/c^2)} - \frac{u_{y'}}{\gamma(1+\mathrm{v}u_{x'}/c^2)^2} \cdot \frac{\mathrm{v}du_{x'}}{c^2}. \tag{5.30}$$

Consequently

$$a_y = \frac{\mathrm{d}u_y}{\mathrm{d}t} = \frac{\mathrm{d}u_{y'}/\mathrm{d}t'}{\gamma^2(1+\mathrm{v}u_{x'}/c^2)^2} - \frac{u_{y'}}{\gamma^2(1+\mathrm{v}u_{x'}/c^2)^3} \cdot \frac{\mathrm{v}\mathrm{d}u_{x'}\mathrm{d}t'}{c^2}, \tag{5.31}$$

and therefore:

$$a_y = \frac{a_{y'}}{\gamma^2(1+\mathrm{v}u_{x'}/c^2)^2} - \frac{(\mathrm{v}u_{y'}/c^2)a_{x'}}{\gamma^2(1+\mathrm{v}u_{x'}/c^2)^3}. \tag{5.32}$$

If the body is instantaneously at rest in S' then it is $u_{y'} = u_{x'} = 0$ and thus:

$$a_y = \frac{a_{y'}}{\gamma^2}. \tag{5.33}$$

Thus the acceleration components measured in the laboratory frame S are decreased by a factor γ^3 for the x direction and γ^2 for the y direction in comparison with the acceleration components measured in the instantaneous frame S'. The laboratory observer sees that things are moving and accelerating very slowly regarding the particle under consideration, and thus, to the extent that the force is invariant, i.e. the same force is perceived in S and S', then he/she is perceiving that the object has a very large mass.

This becomes more clear in the next section.

5.2.4 Forces in Relativistic Mechanics

Similar to Newtonian mechanics, force in relativistic mechanics is defined as the time derivative of momentum **p**, i.e.

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} \tag{5.34}$$

and since $\mathbf{p} = \gamma m_o \mathbf{v}$ it follows:

$$\mathbf{F} = m_o \frac{d(\gamma \mathbf{v})}{dt}. \tag{5.35}$$

The common approach to the problem of the study of the relativistic motion of a particle is the following: At any instant the particle has a well-defined velocity $\mathbf{v}$ as measured in the laboratory reference frame S. The particle is being instantaneously in a rest frame S' which has the same velocity $\mathbf{v}$ with respect to the laboratory frame S.

5.2.4.1 Force Invariance and the Inertial Mass

We imagine that as measured in frame S', a test force $\mathbf{F}_{ox}$ is applied *parallel* to $\mathbf{v}$ causing an acceleration a_{ox}. Since the particle is at rest in S' it follows that the particle mass measured in this frame is just the rest mass m_o. Consequently it is:

$$F_{x'} = m_o a_{x'}. \tag{5.36}$$

In the laboratory frame S the particle is judged to have a momentum, p_x, given by:

$$p_x = \gamma m_o \mathrm{v} = \frac{m_o \mathrm{v}}{(1 - \mathrm{v}^2/c^2)^{1/2}} \tag{5.37}$$

and consequently the force judged in S is given by:

$$F_x = \frac{dp_x}{dt} = \frac{m_o}{(1 - \mathrm{v}^2/c^2)^{1/2}} \frac{d\mathrm{v}}{dt} + m_o \mathrm{v} \frac{d}{dt} \left[(1 - \mathrm{v}^2/c^2)^{-1/2} \right]. \tag{5.38}$$

Denoting $a_x = d\mathrm{v}/dt$, which is the acceleration observed in the laboratory, one obtains:

$$F_x = \frac{m_o a_x}{(1 - \mathrm{v}^2/c^2)^{1/2}} + \frac{m_o (\mathrm{v}^2/c^2) a_x}{(1 - \mathrm{v}^2/c^2)^{3/2}}, \tag{5.39}$$

which upon collecting terms and using the definition of γ becomes:

$$F_x = \gamma^3 m_o a_x. \tag{5.40}$$

There is, however, a very simple equation between a_x and $a_{x'}$, i.e. Eq. (5.29)

$$a_x = \frac{1}{\gamma^3} a_{x'}. \tag{5.41}$$

Consequently from (5.36), (5.40) and (5.41) it follows:

$$F_x = \gamma^3 m_o a_x = m_o a_{x'} = F_{x'}. \tag{5.42}$$

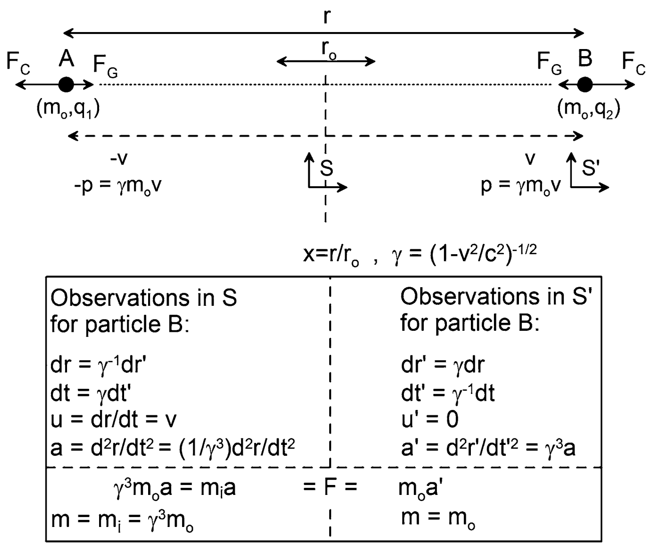

Fig. 5.1 Schematic of the two charged particles with rest mass m_o, repelled by the Coulombic force F_C and attracted by the gravitational force F_G. The figure shows the symmetry axis and two reference frames S and S'. *Inset* summarizes the key [10, 11] relativistic relationships between the observations in frames S and S'; $\gamma^3(r)m_o$ is the longitudinal and inertial mass [11] at displacement r; it is the mass judged in S. The same force value, F, is observed in both frames, but the two observed accelerations, a′ and a, have a ratio of γ^3 [10, 11]

This is a very important result. Despite the different values of mass and acceleration perceived in the two frames, the measure of the x component of force is the same. Force in the x direction is *invariant*.

As a result of (5.42) the mass of the moving particle perceived in S is a factor of γ^3 larger than the rest mass, m_o, perceived in S'. A simple example involving linear motion is shown in Fig. 5.1.

Returning to the more general case (Fig. 5.2) one observes that since the direction x is that defined by the instantaneous velocity **v** and since the test force $\mathbf{F}_{ox}$ is also parallel to **v**, it follows that the instantaneous direction x is the only relevant one for describing the motion. The vertical directions y (and z) are unimportant, since neither the velocity **v** nor the force $\mathbf{F}_{ox}$ have any nonzero components in these directions. This is an important and subtle point: Regardless of the direction of the actual force **F** causing the particle motion, when using the instantaneous frame S' one can consider that the particle motion is due to the action of an instantaneous force $\mathbf{F}_{ox}$ parallel to **v** and *that this can be done at any point of the actual particle trajectory*.

Consequently it is:

$$\mathbf{F}_x = \mathbf{F}_{x'} = \mathbf{F}_{ox} \tag{5.43}$$

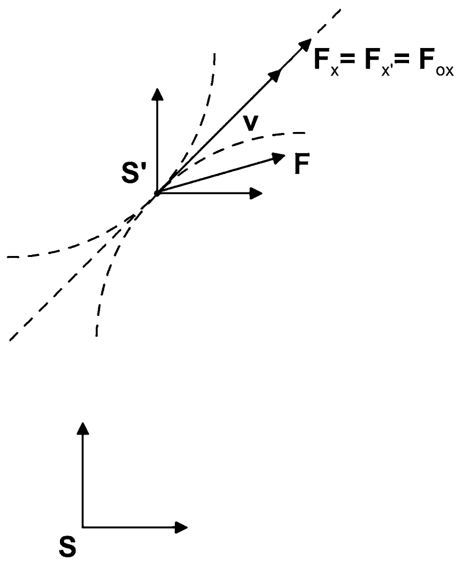

Fig. 5.2 Laboratory frame S and instantaneous inertial frame S', the latter moving with the particle under consideration. The frame S' is uniquely defined by the vector **v** alone regardless of the motion (e.g. linear or circular) performed by the particle. The test force $\mathbf{F}_{ox}$ is applied parallel to the instantaneous velocity **v**

and the instantaneous inertial mass of the particle, m_i, in the direction x (or **v**) which is the only direction relevant to the motion, is obtained directly from (5.42), i.e.:

$$m_i = \frac{\mathbf{F}_x}{\mathbf{a}_x} = \gamma^3 m_o. \tag{5.44}$$

This is an important and hitherto not fully exploited result: Regardless of the actual motion performed by the particle, its inertial mass, m_i, is always equal to $\gamma^3 m_o$.

But this being the case, it follows that due to the equivalence principle, the gravitational mass, m_g, must also equal m_i, thus:

$$m_g = m_i = \gamma^3 m_o. \tag{5.45}$$

The very significant implications of Eq. (5.45) regarding Newton's gravitational law under relativistic conditions will be discussed in Sect. 5.4. It is useful here to make some observations regarding Eq. (5.42).

For $\gamma \gg 1$ an observer at S' perceives a small mass, m_o, and a very large acceleration $a_{x'} (= \gamma^3 a_x)$.

More importantly, for $\gamma \gg 1$, the laboratory observer at S observes a very small acceleration $a_x (= a_{x'}/\gamma^3)$ and a huge inertial mass, $\gamma^3 m_o$.

Thus, due to the invariance of F [Eqs. (5.42) and (5.43)] it follows that, for the laboratory observer, $\gamma^3 m_o$ is the inertial mass of a particle under relativistic conditions, regardless of the actual motion that the particle is performing. For the laboratory observer a particle in S' moving on a circular path and having an instantaneous velocity **v** is indistinguishable from a particle with the same rest mass and velocity **v** moving on a straight line (Fig. 5.2).

5.2.4.2 Transverse Forces

In order to appreciate the importance of the previous results and of the force invariance Eqs. (5.42) and (5.43), it is worth making a similar calculation for the transverse force in which case one finds that the invariance of Eq. (5.42) does not hold. Thus one has to consider here a force, F, such that its y and z components do not vanish because if they do, then the previous analysis is applicable. In the instantaneous frame S' it is in general:

$$F_{y'} = m_o a_{y'}. \tag{5.46}$$

We focus on the case where the force, F_y, perceived in the laboratory frame S, is applied perpendicular to the momentum vector $m\mathbf{v}$ and thus:

$$F_y \Delta t = \gamma m_o \Delta u_y. \tag{5.47}$$

Therefore at the limit $\Delta t \to 0$ one obtains:

$$F_y = \gamma m_o a_y. \tag{5.48}$$

We have already found [Eq. (5.33)] that a_y and $a_{y'}$ are related via:

$$a_y = \frac{a_{y'}}{\gamma^2} \tag{5.49}$$

and thus:

$$F_y = \gamma m_o \frac{a_{y'}}{\gamma^2} = \frac{1}{\gamma} m_o a_{y'}. \tag{5.50}$$

Consequently:

$$F_y = \frac{1}{\gamma} F_{y'}, \tag{5.51}$$

and therefore the force invariance condition does not hold.

5.3 Newton's Universal Gravitational Law

The law of gravity was first presented by Newton in 1610. The gravitational attraction is proportional to the product of the gravitational masses of the two attracting bodies and inversely proportional to the square of the distance:

$$F_G = -\frac{G m_{1,g} m_{2,g}}{r^2}. \tag{5.52}$$

The gravitational constant G was first measured via torsion balances by Cavendish around 1830 and the currently recommended value is $6.676 \cdot 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\mathrm{s}^{-2}$ [2,3].

Do violations from Newton's gravitational law exist? Equation (5.52) has been confirmed down to submillimeter distances [1,12,13]. It is anticipated, but not really proven, that deviations may exist at even shorter distances and that eventually the gravitational force becomes of the same magnitude as the strong force at distances of the order of the Planck length.

$$r_{\mathrm{Pl}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.616 \cdot 10^{-35}\,\mathrm{m}. \tag{5.53}$$

This has not yet been confirmed experimentally.

It is also anticipated in the context of general relativity, but also yet not proven experimentally, that deviations from Newton's gravitational law can exist under highly relativistic conditions resulting from the curvature of spacetime [14–17]. In special relativity the spacetime geometry is described by the flat Minkowski spacetime. By assuming that the actual spacetime geometry can be approximated by a small perturbation of the flat Minkowski spacetime one may linearize the Einstein field equations of general relativity and obtain [18]:

$$\nabla^2 \Phi_{\mathrm{G}} = 4\pi G \mu(x), \tag{5.54}$$

where Φ_{G} is a scalar gravitational potential, G is the Newton constant, and $\mu(x)$ is mass density. This equation is equivalent to (5.52).

Thus in the context of general relativity significant deviations from Newton's gravitational law are anticipated for highly curved spacetime geometry. In discussing such deviations, however, there is an important point which has to be clarified: Deviations from Newton's universal gravitational law may be of two kinds:

1. *Genuine* deviations, i.e. deviations observed despite the proper use of the gravitational mass $m_{\mathrm{g}}(= m_{\mathrm{i}} = \gamma^3 m_{\mathrm{o}})$ in Eq. (5.52).
2. *Artificial* deviations obtained by the inappropriate use of rest mass, m_{o}, or relativistic mass, γm_{o}, in Eq. (5.52). This point does not appear to have been discussed in the literature and, to the best of our knowledge, no *genuine* deviations from Newton's gravitational law under relativistic conditions have ever been reported.

5.4 The Synthesis of Newton's Gravitational Law, Equivalence Principle, and Special Relativity

Before embarking to discuss possible genuine or artificial deviations from the classical universal Newton's gravitational law due to the theory of general relativity or other gravitational theories, it appears worthwhile to examine what deviations may be anticipated on the basis of the equivalence principle and Special Relativity alone, since we know that these two principles have never been challenged and have been experimentally confirmed beyond any possible doubt.

Thus, according to the weak equivalence principle, gravitational mass equals inertial mass, i.e.

$$m_{1,g} = m_{1,i}; \quad m_{2,g} = m_{2,i} \tag{5.55}$$

and thus Newton's gravitational law is written as

$$F_{\mathrm{G}} = -G\frac{m_{1,i}m_{2,i}}{r^2}. \tag{5.56}$$

On the other hand we have already shown in Sect. 5.2, Eq. (5.42), that the inertial mass, m_{i}, perceived by the laboratory observer, i.e. the ratio of force F_x and acceleration a_x is given by:

$$m_{\mathrm{i}} = \gamma^3 m_{\mathrm{o}}. \tag{5.57}$$

Consequently Newton's gravitational law takes the form:

$$F_{\mathrm{G}} = -G\frac{m_{1,o}m_{2,o}\gamma_1^3\gamma_2^3}{r^2}, \tag{5.58}$$

where γ_1 and γ_2 are the Lorentz factors corresponding via:

$$\gamma_1 = (1 - \mathrm{v}_1^2/c^2)^{-1/2}; \quad \gamma_2 = (1 - \mathrm{v}_2^2/c^2)^{-1/2} \tag{5.59}$$

to the velocities, v_1 and v_2, of the two particles under consideration relative to the laboratory observer S.

For $\mathrm{v}_1 = \mathrm{v}_2$, thus $\gamma_1 = \gamma_2$, Eq. (5.59) becomes

$$F_{\mathrm{G}} = -G\frac{m_{1,o}m_{2,o}\gamma^6}{r^2}. \tag{5.60}$$

Is this a genuine deviation from Newton's gravitational law, i.e. from the form valid for low particle velocities relative to the observer?

$$F_{\mathrm{G}} = -\frac{Gm_{1,o}m_{2,o}}{r^2}. \tag{5.61}$$

The answer is clearly negative. The deviation is artificial and is introduced via the inappropriate use of rest mass rather than gravitational mass in Eq. (5.52). When using Eq. (5.60) the mathematical and physical essence of Newton's gravitational law and of the equivalence principle remain untouched, but the gravitational force increases dramatically as v approaches c and thus becomes unbound.

In fact the gravitational force described by (5.60) can exceed the magnitude of any other force for sufficiently high particle velocity and thus sufficiently high Lorentz factor γ.

Once Eq. (5.60) is derived and accepted, then the rest of the hadron formation model described in this book is a simple algebraic exercise.

But objecting equation (5.60) implies objecting one of the three thoroughly proven by experiment cornerstones of mechanics and physics, i.e.

1. The equivalence principle
2. Special relativity
3. Newton's universal gravitational law at low particle velocities

Why has this equation not been derived or used before? One can think of several reasons but perhaps the most plausible one is that after 1920 gravity related research has been dominated almost entirely by the theory of general relativity where the concepts of force and velocity lose a lot of the importance they have in Newtonian and relativistic mechanics. It is not obvious how Eq. (5.60) can be derived in the context of the general relativity theory of gravity. On the other hand to the extent that the general relativity theory encompasses both special relativity and the equivalence principle, then it should also in principle contain Eq. (5.60) as a limiting case. This point is discussed in Sect. 7.3.

The basic question is how does the general relativity theory cope with the effect of particle velocity on the gravitational attraction exerted between two particles. It appears to be commonly thought that this can be done by considering that the mass of the two particles is equal to their relativistic mass γm_o. This however cannot be the case, since as already noted, special relativity dictates that the inertial mass, m_i, equals $\gamma^3 m_o$ and the equivalence principle dictates that the inertial mass, m_i, equals the gravitational mass, m_g, thus implying that $\gamma^3 m_o$ rather than γm_o is the gravitational particle mass, m_g, to be used under relativistic conditions.

This appears to imply that the appropriate mass to be used in the field equation of general relativity is neither the rest mass m_o, nor the relativistic mass γm_o but rather the inertial and thus gravitational mass $\gamma^3 m_o$.

One may thus conclude that with this, apparently unavoidable, choice of gravitational mass the rotational neutrino model analyzed in this book can be accommodated within the theory of general relativity, a point further discussed in Sect. 7.3 of Chap. 7.

5.5 Einstein's Equivalence Principle and Strong Equivalence Principle

For reasons of completeness we present also here briefly the two stronger versions of the equivalence principle, i.e. the Einstein equivalence principle and the strong equivalence principle. Since both of these versions accept the validity of the weak equivalence principle, i.e. that gravitational mass equals inertial mass, they do not affect the model and results presented in this book in any direct or indirect way, except that they demand the constancy of the natural constants in space and time, an assumption inherent in this model and in the vast majority of physical models.

The Einstein equivalence principle accepts the validity of the weak equivalence principle and additionally demands that "the outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime." The term "local" implies that there is no part of the experiment taking place outside the laboratory. The Einstein equivalence principle implies that dimensionless quantities such as the proton to electron mass ratio and the fine structure constant do not depend on where in space and time they are measured.

The strong equivalence principle demands the validity of the weak equivalence principle expressed in the form "the gravitational motion of a small test body depends only on its velocity and initial position in spacetime and not on its constitution" and also that "the outcome of any local experiment, gravitational or not, in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime." This is the only form of the equivalence principle which is applicable to self-gravitating bodies, such as stars, which have strong internal gravitational interactions. The strong equivalence principle demands that Newton's gravitational constant G is also constant in spacetime. Estimates obtained from orbits in the solar system and from studies of big bang nucleosynthesis suggest that this version of the principle is also valid, as G cannot have varied by more than 10% after the big bang.

5.6 Synopsis

Both special relativity and the weak equivalence principle have been confirmed thousands of times by experiment. Their exact validity has never been challenged. Yet their straightforward combination leads to the surprising result that the inertial and thus gravitational mass of a particle with a velocity v relative to a laboratory observer is $\gamma^3 m_o$. Thus for v approaching the speed of light, c, the Newtonian gravitational force between two such particles becomes stronger than any other force, including the Coulombic force. This can lead to particle, e.g. neutrino, confinement in rotational orbits with relativistic velocities, as analyzed in the next chapter.

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Chapter 6
The Three and Two Rotating Neutrino Models: Particle Confinement by Gravity

6.1 Requirements for a Satisfactory Hadron Formation Model

A satisfactory model for hadron formation should fulfil several requirements:

1. It should involve *three constituent particles* for the case of baryons (protons, neutrons, etc.) and *two constituent particles* for the case of mesons. The presence of these constituent particles was first deduced experimentally from inelastic hadron–electron scattering experiments (Fig. 4.1). These constituent particles are known as *quarks*, a term first introduced by Gell–Mann. The term partons had been introduced earlier by Feynmann for the same particles.
2. It should provide an explanation about why quarks (whose masses are estimated to be in the 4–400 MeV/c^2 range vs 938.272 MeV/c^2 for the proton) *cannot be isolated* and studied independently.
3. It should provide a *binding mechanism* for quarks which are currently considered to be held together by the exchange of *gluons*. This binding mechanism is the cause of the *strong force*.
4. This binding mechanism should be able to explain both *asymptotic freedom* and *color confinement*, i.e. the fact that the strong force is weak at very short distances (<1 fm, asymptotic freedom) and increases dramatically with increasing distance, thus causing quark confinement.
5. It should be able to predict that the strong force is roughly a factor of α^{-1}($\approx$137.035) stronger than the Coulombic force in the fm distance range. The constant $\alpha(=e^2/\varepsilon c\hbar)$ is the *fine structure constant*.
6. It should be able to predict that the condensation of these constituent particles (quarks and gluons) to form hadrons, occurs at a temperature termed QCD *quantum chromodynamics transition temperature* of $\sim$ 160–220 MeV in the kT scale which correspond to an individual constituent particle energy of about 160–220 MeV at the QCD condensation transition. This energy is also frequently termed QCD scale.

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7. It should predict the existence of excited states and of the baryon mass spectrum, i.e. the existence of Λ, Ξ, Ω etc. particles.
8. The model should also be able to predict hadron magnetic moments in the $\pm 2-3$ μ_B (nuclear magnetons) range, i.e. in the range of $\pm 10^{-26}$ J/T.
9. It should also be consistent with the experimental hadron spins of 1/2, 3/2 etc. [1].
10. The magnitude of the dipole moments, L, of the hadrons and all their excited states should be of the order of $\hbar(m_B/MeV)^2$ [1].
11. The lifetimes (fast decay) of the unstable baryons should be around $5 \cdot 10^{-24}$ s [1].
12. Ideally such a model should also provide a mechanism for mass generation, i.e. a mechanism for baryosynthesis or, equivalently, for hadronization, i.e. a rationalization about how the mass of baryons and other hadrons, and thus the mass of our universe was created starting from some initial conditions involving practically no mass and only high, very high, energy.

It will be shown that all these requirements are met, without any adjustable parameters, by the model presented in this chapter.

6.2 The Inertial and Gravitational Mass of Fast Neutrinos

During the last two decades the study of neutrino oscillations [2, 3] has shown conclusively that all the three types, or flavors, of neutrinos (ν_e, ν_μ, ν_τ) have small but nonzero rest masses [1–3]. While neutrinos have very small rest masses ($\sim$0.04 to 0.4 eV/c^2) [1–3] they have typically quite large (as high as 200 MeV) total energies [2, 3]. This implies that their velocities are very near the speed of light and that their Lorentz factors $\gamma(=(1-\text{v}^2/c^2)^{-1/2})$ are very large. Since the total energy, E, is related to the rest mass, m_o, via the Einstein equation:

$$E = \gamma m_o c^2 \tag{6.1}$$

it follows, as an example, that for $E = 200$ MeV and $m_o = 0.04$ eV/c^2 it is $\gamma m_o = 200$ MeV/c^2 and thus $\gamma = 5 \cdot 10^9$.

On the other hand, we have seen that special relativity dictates that when a particle moving on an instantaneous frame S' has a velocity $\mathbf{v}$ and concomitant Lorentz factor γ relative to a laboratory observer in a frame S, then the inertial mass, m_i, equals $\gamma^3 m_o$ [4, 5]. This is obtained after some simple algebra from [4, 5]:

$$F = \frac{dp}{dt} = \frac{d(\gamma m_o \text{v})}{dt} = \gamma^3 m_o \frac{d\text{v}}{dt}, \tag{6.2}$$

where p is the momentum.

However, according to the equivalence principle, the inertial mass, m_i, of a particle equals the gravitational mass, m_g [6, 7], and thus one obtains:

$$m_\mathrm{g} = m_\mathrm{i} = \gamma^3 m_\mathrm{o}. \tag{6.3}$$

Upon substituting the above values, i.e. $\gamma = 5 \cdot 10^9$ and $m_\mathrm{o} = 0.04\,\mathrm{eV/c^2}$, one finds:

$$m_\mathrm{g} = m_\mathrm{i} = 5 \cdot 10^{18}\,\mathrm{GeV/c^2} \tag{6.4}$$

which, surprisingly, is almost half of the value of the Planck mass [6], i.e. $1.221 \cdot 10^{19}\,\mathrm{GeV/c^2}$ $(= 2.177 \cdot 10^{-8}\,\mathrm{kg})$. Since the magnitude of gravitational and strong forces are expected to merge at energies close to the Planck energy, $1.221 \cdot 10^{19}$ GeV, it is reasonable to expect that the gravitational force between such fast neutrinos in the fm range can be quite significant, perhaps comparable in magnitude with the strong force at the same distance, and thus can lead to the creation of very strongly bound confined states.

Indeed one can compute as an example that the gravitational potential energy, V_g, of two such fast moving particles when they are at a distance of 1 fm is:

$$V_\mathrm{g} = -\frac{Gm_g^2}{r} = -5.30 \cdot 10^{-12}\,J = -33.09\,\mathrm{MeV}, \tag{6.5}$$

whereas for comparison the Coulombic potential energy of a u and a d quark at the same distance is:

$$V_\mathrm{c} = -\frac{(2/3)(1/3)e^2}{\varepsilon r} = -5.126 \cdot 10^{-14} J = -0.32\,\mathrm{MeV}, \tag{6.6}$$

i.e. the gravitational interaction is, surprisingly, a factor of 100 stronger than the Coulombic interaction.

Since the strong force interaction between quarks is estimated to be a factor of $\alpha^{-1}(= 137.035)$ stronger than the Coulombic interaction at the fm range [1, 6, 8] it follows that the magnitude of the gravitational force between fast neutrinos, when accounting for special relativity and for the equivalence principle, can be comparable to the magnitude of the strong force at the fm range.

This result is at first surprising but stems directly from special relativity, i.e. Eqs. (6.2) and (6.3), from the weak equivalence principle of Eötvös and Einstein [6] [Eq. (6.4)], and from Newton's gravitational law, without making any assumptions.

It thus becomes interesting to examine what type of bound states such a powerful attractive force can create.

6.3 The Three-Neutrino Model

We thus examine the circular motion of three neutrinos (e.g. three electron neutrinos or antineutrinos) on a circle of radius R (Fig. 6.1) under the influence of their gravitational attraction.

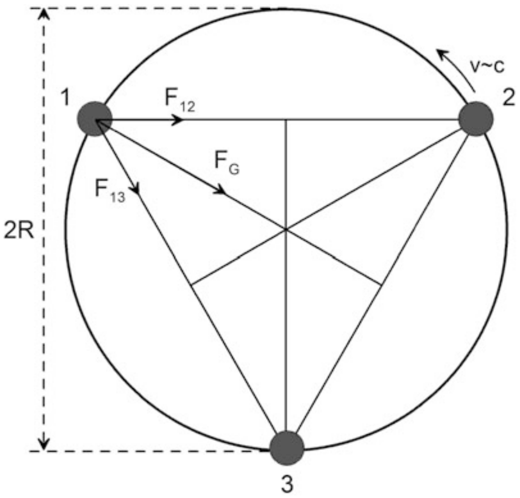

Fig. 6.1 Three particles moving at a constant tangential velocity, **v**, in a circle of radius R around their center of mass. They are equally spaced. F_{12} and F_{13} are two particle attraction forces and F_G is the resultant, radial, force

6.3.1 Equivalence Principle and Inertial Mass

It is important to first examine if Eq. (6.3) for the particle inertial and gravitational mass, i.e. $m_g = m_i = \gamma^3 m_o$, obtained first via Eq. (5.40) or (5.42) or (6.2) for linear particle motion, is also applicable when the particle performs a circular motion. This important point was proven already in Sect. 5.2.4.1, but it is perhaps useful to provide a similar, shorter and equally rigorous proof here.

We thus consider a laboratory frame S and an instantaneous inertial frame S' moving with a particle with an instantaneous velocity **v** relative to frame S (Fig. 6.2).

It is worth noting that the instantaneous inertial frame S' is defined by the vector **v** alone and not by the overall type of motion (e.g. linear or cyclic) performed by the particle [4, 5].

For the laboratory observer in S a particle in the instantaneous frame S' performing a circular motion is indistinguishable from a particle of the same rest mass m_o and velocity, **v**, performing a linear motion (Fig. 6.2).

Thus one can assign to the frame S' and corresponding velocity **v** an inertial particle mass, m_i, by considering a test force, **F**, parallel to **v**, acting on the particle. According to the theory of special relativity the case where **F** and **v** are parallel is the only case where the force is invariant, i.e. the force perceived in S and S' is the same [4].

Starting from the general relativistic equation of motion, i.e. from [4, 5]:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \gamma m_o \frac{d\mathrm{v}}{dt} + \gamma^3 m_o \frac{1}{c^2}\left(\mathrm{v} \cdot \frac{d\mathrm{v}}{dt}\right)\mathrm{v}, \qquad (6.7)$$

and using the fact that the test force **F** is taken to be parallel to **v**, one obtains after some simple algebra that the measures of the force, F, and of the acceleration, $\frac{d\mathrm{v}}{dt}$, are related via:

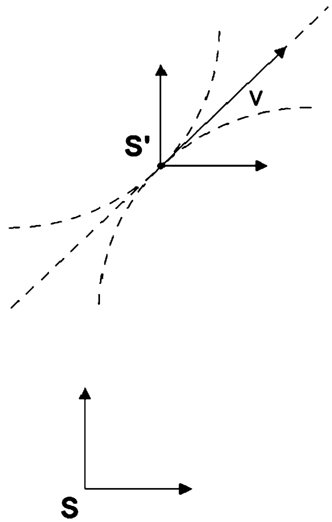

Fig. 6.2 Laboratory frame S and instantaneous inertial frame S', the latter moving with the particle under consideration. The frame S' is uniquely defined by the vector **v** alone regardless of the motion (e.g. linear or circular) performed by the particle

$$\begin{aligned} F &= \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}(\gamma m_\mathrm{o} \mathrm{v})}{\mathrm{d}t} = m_\mathrm{o} \frac{\mathrm{d}[\mathrm{v}(1-\mathrm{v}^2/c^2)^{-1/2}]}{\mathrm{d}t} \\ &= m_\mathrm{o}\left[\left(1-\frac{\mathrm{v}^2}{c^2}\right)^{-1/2} + \frac{\mathrm{v}^2}{c^2}\left(1-\frac{\mathrm{v}^2}{c^2}\right)^{-3/2}\right]\frac{\mathrm{dv}}{\mathrm{d}t} \\ &= m_\mathrm{o}\left[\gamma + \frac{\mathrm{v}^2}{c^2}\gamma^3\right]\frac{\mathrm{dv}}{\mathrm{d}t} = m_\mathrm{o}\left[\gamma + \left(1-\frac{1}{\gamma^2}\right)\gamma^3\right]\frac{\mathrm{dv}}{\mathrm{d}t} \\ &= \gamma^3 m_\mathrm{o}\frac{\mathrm{dv}}{\mathrm{d}t}, \end{aligned} \tag{6.8}$$

which is Eq. (6.2).

This defines the mass $\gamma^3 m_\mathrm{o}$, frequently termed longitudinal mass [5], which is the inertial mass of the particle, m_i, which is always defined as the ratio of force and acceleration [9]. Thus it is $m_\mathrm{i} = \gamma^3 m_\mathrm{o}$. According to the equivalence principle, m_i also equals the gravitational mass, m_g, of the particle [6], i.e.:

$$m_\mathrm{g} = m_\mathrm{i} = \gamma^3 m_\mathrm{o} \tag{6.9}$$

which is Eq. (6.3). As already noted, for given m_o and v, the inertial mass m_i, and thus the gravitational mass m_g are both uniquely determined by Eq. (6.9) and their value does not depend on the type of motion (e.g. linear or circular) performed by the particle.

Thus upon considering a second particle of rest mass m_o and instantaneous velocity measure v relative to the observer at S and at a distance r from the first particle, it follows that the inertial and gravitational mass of the second particle is also given by $\gamma^3 m_\mathrm{o}$, as in Eq. (6.9), and thus one can use these two m_g values in

Newton's gravitational law in order to compute the gravitational force, F_{G}, between the two particles. Thus from:

$$F_{\mathrm{G}}=-\frac{Gm_{1,g}m_{2,g}}{r^2}, \tag{6.10}$$

and from Eq. (6.9), one obtains:

$$F_{\mathrm{G}}=-\frac{Gm_{\mathrm{o}}^2\gamma^6}{r^2}, \tag{6.11}$$

which depends on the 6th power of γ [10] and accounts explicitly for the velocity dependence of the inertial and gravitational mass. It is worth remembering that this equation stems directly from special relativity [Eq. (6.8)], the weak equivalence principle [Eq. (6.9)] and Newton's gravitational law. No other assumptions are involved.

Application of Eq. (6.11) to the circular motion of Fig. 6.1 gives after some simple trigonometry:

$$F_{\mathrm{G}}=-\frac{Gm_{\mathrm{o}}^2\gamma^6(R)}{\sqrt{3}\,R^2}. \tag{6.12}$$

6.3.2 The Classical Mechanical Problem

One may now consider the relativistic equation of motion [Eq. (6.7)] for the case of a circular orbit of radius R. Since in this case due to the circular motion it is $\mathbf{v}\cdot\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=0$ the last term in Eq. (6.7) vanishes and thus one obtains:

$$\mathbf{F}=\gamma m_{\mathrm{o}}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \tag{6.13}$$

and since for circular motion it is always $\left|\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\right|=\mathrm{v}^2/R$ it follows:

$$F=\gamma m_{\mathrm{o}}\frac{\mathrm{v}^2}{R}. \tag{6.14}$$

Upon combining with Eq. (6.12) and $F=-F_{\mathrm{G}}$ one obtains:

$$\frac{Gm_{\mathrm{o}}^2\gamma^6(R)}{\sqrt{3}\,R^2}=\frac{\gamma(R)m_{\mathrm{o}}\mathrm{v}^2}{R} \tag{6.15}$$

i.e., the gravitational force F_{G} given by Eq. (6.12) acts as the centripetal force for the rotational motion.

It must be noted that on the basis of (6.13) one might be tempted to assign the value γm_{o}, commonly termed transverse mass [4, 5], to the inertial and thus gravitational mass of each particle. However, as already noted, the mass $m_{\mathrm{i}}(=m_{\mathrm{o}}\gamma^3)$, and thus also m_{g} is uniquely determined for given m_{o} and $\mathbf{v}$, via the colinear to $\mathbf{v}$ test force $\mathbf{F}$, and does not depend on the type of motion (e.g. linear or circular) performed by the particle.

Upon utilizing $\gamma(R)=(1-\mathrm{v}^2/c^2)^{-1/2}$ in Eq. (6.15) one obtains:

$$R=\frac{Gm_{\mathrm{o}}}{\sqrt{3}\,\mathrm{c}^2}\gamma^5\left(\frac{\gamma^2}{\gamma^2-1}\right) \tag{6.16}$$

or, equivalently:

$$R=(R_S/(2\sqrt{3}))\gamma^5\left(\frac{\gamma^2}{\gamma^2-1}\right), \tag{6.17}$$

where $R_S(=2Gm_{\mathrm{o}}/c^2)$ is the Schwarzschild radius of a particle with rest mass m_{o}.

As shown in Fig. 6.3 bottom, the function R defined by Eq. (6.17) exhibits a minimum, $R_{\min}=2.343R_S$, at $\gamma_{\min}=\sqrt{7/5}=1.1832$ thus $\mathrm{v}_{\min}=\sqrt{2/7}c$. This is the minimum radius for a circular orbit and the corresponding minimum angular momentum is $L_{\min}=\gamma_{\min}m_{\mathrm{o}}\mathrm{v}_{\min}R_{\min}=1.481m_{\mathrm{o}}cR_S=2.963\ Gm_{\mathrm{o}}^2/c$. This condition, i.e. $L>2.963\,Gm_{\mathrm{o}}^2/c$, is similar to the criteria $L>Gm^2/c$ found for circular orbits in special relativity [10, 11] or $L>2\sqrt{3}Gm^2/c$ for the Schwarzschild metric in general relativity [12] with orbits around point masses with r^{-1} potentials (Table 6.1).

Equation (6.17) defines two γ branches (Fig. 6.3), one corresponding to low γ values ($\gamma<1.1832$) the other corresponding to large γ values ($\gamma>1.1832$). The first branch corresponds to common Keplerian gravitational orbits. In this case γ and thus the velocity $\mathbf{v}$ decreases with increasing R, e.g. $\mathrm{v}=(Gm_{\mathrm{o}}/(\sqrt{3}R))^{1/2}$ in the nonrelativistic case ($\gamma\approx 1$).

The second branch which leads to relativistic velocities defines rotational states where γ and thus v *increase with increasing R*. These states with $\gamma\gg 1$ are the states of primary interest to the present model. For $\gamma\gg 1$, e.g. $\gamma>10^2$, Eq. (6.17) reduces to:

$$R=(R_S/(2\sqrt{3}))\gamma^5;\quad \gamma=(2\sqrt{3})^{1/5}(R/R_{\mathrm{S}})^{1/5}. \tag{6.18}$$

As shown in Fig. 6.3 top, γ reaches values of the order $7\cdot 10^9$ for R values in the fm (10^{-15} m) range. Thus for a neutrino mass of $5\cdot 10^{-2}\,\mathrm{eV/c^2}$ (Sect. 4.2.2.4) one observes that the relativistic mass, $3\gamma m_{\mathrm{o}}$, of the rotating neutrinos is of the order of $1\,\mathrm{GeV/c^2}$, as also shown in Fig. 6.3, top. The importance of this point is discussed in the next section.

6.3.3 The de Broglie Wavelength Expression and Consistency with Quantum Mechanics

We then proceed to identify among the infinity of bound rotational states described by Eq. (6.18), each corresponding to a different R, those rotational states where R is an integer multiple of the reduced de Broglie wavelength $\lambdabar(=\hbar/p)$ of the light rotating particles.

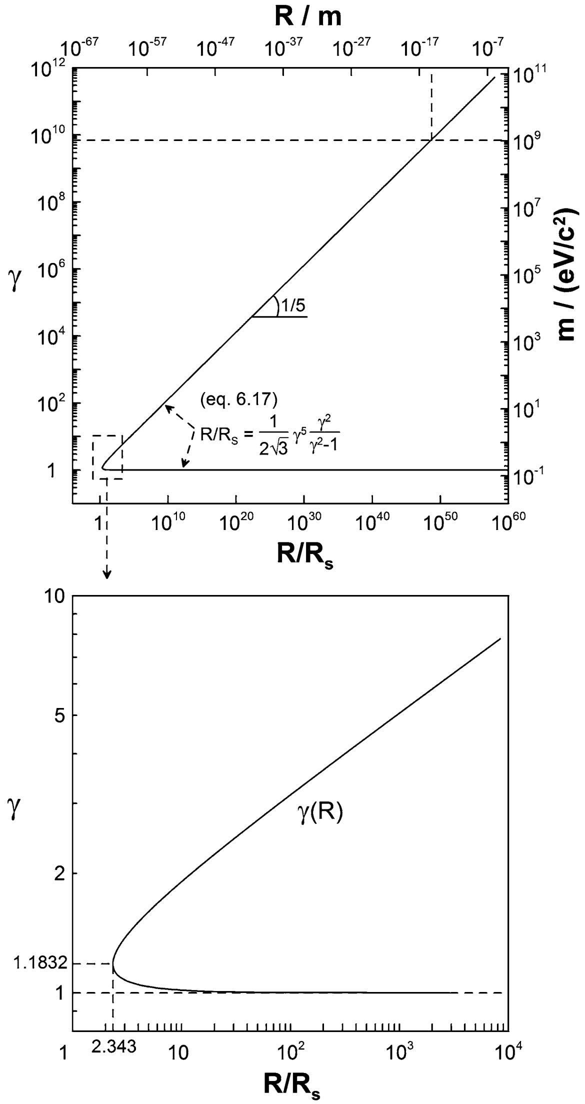

Fig. 6.3 Plot of Eq. (6.17) for R values up to 10^{-5} *m* (*top*) and near the minimum R, denoted $R_{\min}$ (*bottom*). The *m* axis is constructed from $m = 3\gamma m_o$ with $m_o = 5 \cdot 10^{-2}$ eV/c^2

Table 6.1 Stability criteria for circular orbits

Special relativity	$L > Gm^2/c$	(r^{-1} potential, [11])
Special relativity	$L > 2.963\ Gm^2/c$	(present model, [10])
General relativity	$L > 3.46\ Gm^2/c$	(Schwarzschild metric, r^{-1} potential, [12])

Similar to the Bohr model of the H atom, this can be done by introducing quantization of the angular momentum of the light particles in the form:

$$L = \gamma m_o R c = (2n-1)\hbar. \tag{6.19}$$

Solving for R one obtains:

$$R = \frac{(2n-1)\hbar}{\gamma m_o c} = \frac{3(2n-1)\hbar}{mc}, \tag{6.20}$$

where $m(= 3\gamma m_o)$ is the rest mass of the bound rotational state formed.

For $n = 1$ this is mathematically equivalent to assuming that the rotational radius, R, equals the reduced (rotational) de Broglie wavelength, $\lambda\!\!\!^{-}$, of the rotating neutrino. In this way consistency with the "old quantum mechanics," i.e. with the de Broglie wavelength equation, is guaranteed.

Using the definition of $R_S(= 2Gm_o/c^2)$ one can thus introduce the ratio R/R_S in Eq. (6.20). It is:

$$\frac{R}{R_S} = \frac{(2n-1)\hbar c}{2\gamma G m_o^2}. \tag{6.21}$$

This ratio is also given by Eq. (6.18), i.e.

$$\frac{R}{R_S} = \frac{\gamma^5}{2\sqrt{3}}. \tag{6.22}$$

Consequently from (6.21) and (6.22) one obtains:

$$\gamma^6 = \frac{3^{1/2}(2n-1)\hbar c}{G m_o^2} = 3^{1/2}(2n-1)\frac{m_{\text{Pl}}^2}{m_o^2}, \tag{6.23}$$

or equivalently:

$$\gamma = 3^{1/12}(2n-1)^{1/6}\frac{m_{\text{Pl}}^{1/3}}{m_o^{1/3}}, \tag{6.24}$$

where $m_{\text{Pl}} = (\hbar c/G)^{1/2}$ is the Planck mass. Recalling that the rest mass, m, of the bound rotational state equals $3\gamma m_o$ one thus obtains:

$$m = 3\gamma m_o = 3^{13/12}(2n-1)^{1/6} m_o^{2/3} m_{\text{Pl}}^{1/3} \tag{6.25}$$

and consequently the rest mass, m, of the rotational composite state has been expressed in terms of the neutrino mass m_o and of other natural constants. This completes the solution of the three rotating neutrino problem.

6.3.4 *Numerical Substitutions*

Setting $n = 1$, $m_o = 0.04\,\text{eV/c}^2$ and using $m_{\text{Pl}} = 1.221 \cdot 10^{19}\,\text{GeV/c}^2$ in Eq. (6.25), one obtains:

$$m = 885.43\,\text{MeV}/\text{c}^2 \tag{6.26}$$

which, surprisingly, is in the hadrons mass range and in fact differs less than 6% from the rest mass of the proton (938.272 MeV/c^2) and of the neutron (939.565 MeV/c^2). Exact agreement with the neutron mass, m_n, is obtained for:

$$m_o = 0.043723\,\mathrm{eV/c^2} = 7.7943 \cdot 10^{-38}\,\mathrm{kg}, \tag{6.27}$$

which corresponds via Eq. (6.24) or, easier via the first equation (6.25), i.e. $m = 3\gamma m_o$, to:

$$\gamma = \gamma_n = 7.163 \cdot 10^9, \tag{6.28}$$

where we use the subscript n to denote that this specific γ value corresponds to the formation of a neutron.

The m_o value presented in Eq. (6.27) is the value computed from Eq. (6.25) for $n = 1$, which is assumed to correspond to a neutron, i.e.:

$$m_o = \frac{(m_n/3)^{3/2}}{3^{1/8} m_{Pl}^{1/2}}. \tag{6.29}$$

The thus computed m_o value is in quite good agreement with the current best estimate of $m_o = 0.051(\pm 0.01)\,\mathrm{eV/c^2}$ for the mass of the heaviest neutrino [2] extracted from the Super-Kamiokande data [2]. As discussed in Chap. 4 this value is computed from the square root of the $\left|\Delta m_{23}^2\right|$ value of $2.6(\pm 0.2) \times 10^{-3}(\mathrm{eV/c^2})^2$ extracted from the Super-Kamiokande data for the $\nu_\mu \longleftrightarrow \nu_\tau$ oscillations [2].

Actually as shown in Fig. 6.4 the m_o value of 0.043723 eV/c^2 [Eqs. (6.27) and (6.29)] practically coincides with the currently computed maximum neutrino mass value both for the normal mass hierarchy ($m_3 \gg m_2 > m_1$) and for the inverted hierarchy ($m_1 \approx m_2 \gg m_3$) [3].

This is also shown in Fig. 6.5 which compares Eq. (6.27) with the Super-Kamiokande data in terms of the effective neutrino mass m_ν [3].

With this m_o value Eq. (6.25) can also be written as:

$$m = (2n-1)^{1/6} m_n, \tag{6.30}$$

where m_n is the neutron mass. As shown already in Fig. 4.8 this expression is also in very good agreement with experiment regarding the masses of baryons consisting of u, d, and s quarks [1, 13] which follow the $(2n-1)^{1/6}$ dependence of Eq. (6.30) with an accuracy better than 3% [10]. This is shown more clearly in Table 6.2.

A summary of the rotating neutrino model presented in this chapter is given in Table 6.3. The model is in retrospect quite simple.

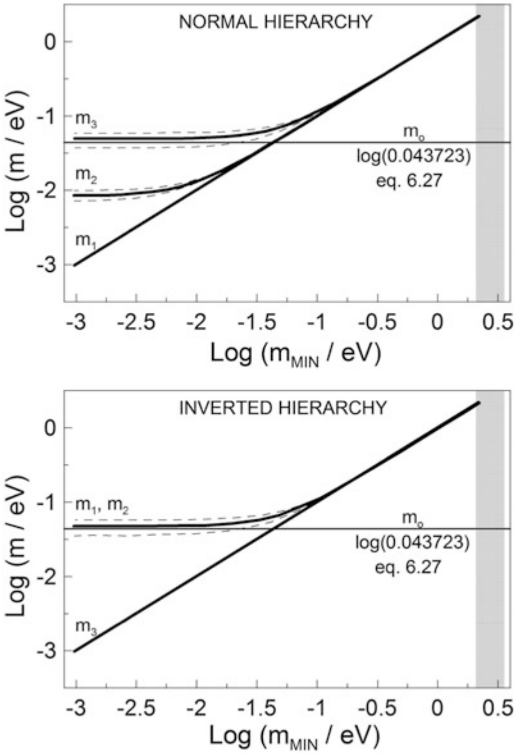

Fig. 6.4 The three light neutrino masses as a function of the lightest mass for the normal (*top plot*) and inverted (*bottom plot*) hierarchy, reprinted from [3] and comparison with Eq. (6.27) or (6.29), i.e. $m_o = (m_n/3)^{3/2}/(3^{1/8} m_{Pl}^{1/2}) = 0.043723\,eV/c^2$

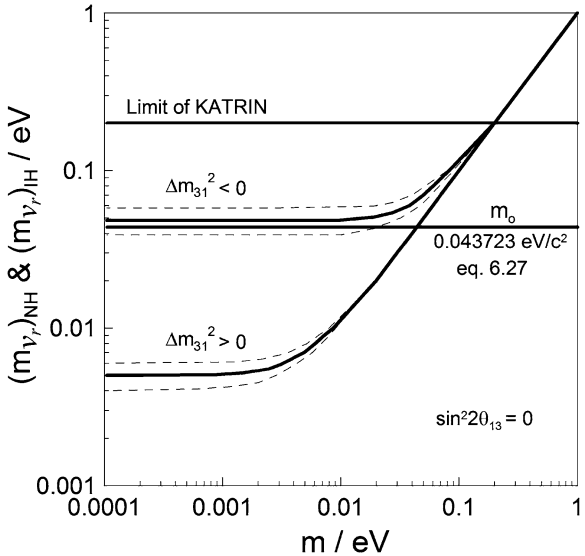

Fig. 6.5 The observable effective mass m_{ν_r} as a function of the lightest mass for the normal (*bottom*) and inverted (*upper*) mass ordering [3] and comparison with Eq. (6.27) or (6.29). The currently allowed 3σ ranges of the oscillation parameters were used [3]. The estimated detection limit of KATRIN is also shown

Table 6.2 Experimental [1] and computed [Eq. (6.30)] baryon masses

Baryon	Experimental mass value MeV/c^2	$m_n(2n-1)^{1/6}$	n
N p	938.272	939.565	1
N n	939.565		
Λ	1115.68	1128.3	2
Σ^+	1189.37	1228.6	3
Σ^o	1192.64		
Σ^-	1197.45		
Δ	1232		
Ξ^o	1314.8	1299.5	4
Ξ^-	1321.3		
$\Sigma*$	1385	1401.2	6
$\Xi*$	1533	1534.7	10
Ω^-	1672	1665.3	16

6.4 The Two-Neutrino Model

Before proceeding to compare the properties of the bound three-neutrino states, it is useful to also derive the corresponding equations for the two-neutrino model (Fig. 6.6). In this case Eq. (6.15) takes the form:

$$\frac{Gm_o^2\gamma^6(R)}{4R^2} = \frac{\gamma(R)m_o\mathrm{v}^2}{R} \tag{6.31}$$

and upon utilizing $\gamma(R) = (1-\mathrm{v}^2/c^2)^{-1/2}$ and the definition, $R_S = 2Gm_o/c^2$, of the Schwarzschild radius one obtains:

$$R = \frac{Gm_o}{2c^2}\gamma^5\left(\frac{\gamma^2}{\gamma^2-1}\right) \tag{6.32}$$

or equivalently:

$$R = (R_S/4)\gamma^5\left(\frac{\gamma^2}{\gamma^2-1}\right). \tag{6.33}$$

For $\gamma \gg 1$ this reduces to

$$R = (R_S/4)\gamma^5; \quad \gamma = 4^{1/5}(R/R_S)^{1/5}. \tag{6.34}$$

Upon introducing quantization of angular momentum in the form:

$$L = \gamma m_o Rc = (2n-1)\hbar, \tag{6.35}$$

one obtains:

$$R = \frac{(2n-1)\hbar}{\gamma m_o c} = \frac{2(2n-1)\hbar}{mc}, \tag{6.36}$$

where $m(=2\gamma m_o)$ is the rest mass of the bound rotational state formed.

Table 6.3 Summary of the three rotating neutrino model

a Special relativity

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \gamma^3 m_o \frac{dv}{dt} \Rightarrow$$

$$m_i = \gamma^3 m_o \qquad (1)$$

b Equivalence principle

$$m_g = m_i \qquad (2)$$

c Newton's gravitational law

$$F_G = \frac{G m_g^2}{r^2} \qquad (3)$$

(a, b, c) $\Rightarrow$ **d**

$$F_G = \frac{G m_o^2 \gamma^6}{r^2} \qquad (4)$$

For radius R and 3 particles:

$$F_G = \frac{G m_o^2 \gamma^6}{\sqrt{3} R^2} \qquad (5)$$

e Newton's second law for circular motion

$$F_G = \frac{\gamma m_o v^2}{R} \qquad (6)$$

(d, e) $\rightarrow$ **f**

$$\frac{G m_o^2 \gamma^6}{\sqrt{3} R^2} = \frac{\gamma m_o v^2}{R} \qquad (7)$$

g

$$(7) \Rightarrow \frac{R}{R_S} = \frac{1}{2\sqrt{3}} \gamma^5 \frac{\gamma^2}{\gamma^2 - 1} \qquad (8)$$

$$R_S = \frac{2 G m_o}{c^2}$$

For $R >> R_S$

$$\frac{R}{R_S} = \frac{1}{2\sqrt{3}} \gamma^5 \qquad (9)$$

h De Broglie wavelength $\bar{\lambda}$

$$\bar{\lambda} = R = \frac{(2n-1)\hbar}{p} = \frac{(2n-1)\hbar}{\gamma m_o c}$$

$$\Rightarrow \frac{R}{R_S} = \frac{(2n-1)\hbar c}{2\gamma G m_o^2} \qquad (10)$$

i

$$\left.\begin{matrix}(9)\\(10)\end{matrix}\right\} \Rightarrow \gamma = 3^{1/12}(2n-1)^{1/6} \frac{m_{Pl}^{1/3}}{m_o^{1/3}} \quad \text{with} \quad m_{Pl} = \left(\frac{\hbar c}{G}\right)^{1/2}$$

$$\Rightarrow m = 3\gamma m_o = 3^{13/12}(2n-1)^{1/6} m_o^{2/3} m_{Pl}^{1/3}$$

For $m_o = 0.043723\ \text{eV}/c^2$ and $n = 1$ it is $m = 939.565\ \text{MeV}/c^2$

Using the definition of the Schwarzschild radius, $R_S = 2Gm_o/c^2$, one can rewrite Eq. (6.36) in the form:

$$\frac{R}{R_S} = \frac{(2n-1)\hbar c}{2\gamma G m_o^2}. \qquad (6.37)$$

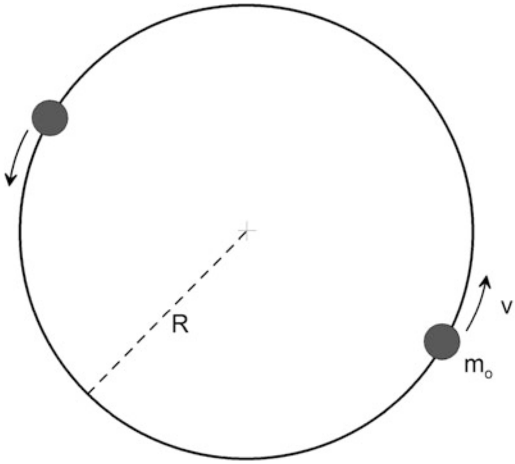

Fig. 6.6 Two-neutrino model

Upon combining with Eq. (6.34), i.e.

$$\frac{R}{R_S} = \frac{\gamma^5}{4}, \tag{6.38}$$

one obtains:

$$\gamma^6 = \frac{2(2n-1)\hbar c}{Gm_o^2} = 2(2n-1)\frac{m_{\text{Pl}}^2}{m_o^2}, \tag{6.39}$$

where $m_{\text{Pl}}(= (\hbar c/G)^{1/2})$ is the Planck mass. Recalling that $m = 2\gamma m_{\text{o}}$ is the mass of the bound rotational state it follows:

$$m = 2^{7/6}(2n-1)^{1/6} m_{\text{o}}^{2/3} m_{\text{Pl}}^{1/3}. \tag{6.40}$$

Setting $n = 1$, $m_{\text{o}} = 0.043723\,\text{eV/c}^2$ and using $m_{\text{Pl}} = 1.221 \cdot 10^{19}\,\text{GeV/c}^2$ one obtains that $\gamma = 7.337 \cdot 10^9$ and

$$m = 641.5\,\text{MeV/c}^2, \tag{6.41}$$

which interestingly lies in the meson mass range. Agreement with the $\pi^{\pm}$ mass of $139.57\,\text{MeV/c}^2$ is obtained exactly with $m_{\text{o}} = 4.437 \cdot 10^{-3}\,\text{eV/c}^2$ which is a factor of 9.85 smaller than the m_{o} value of $4.3723 \cdot 10^{-2}\,\text{eV/c}^2$ obtained from the baryon data and which is exactly a factor of two smaller than the value of $m_2 = 8.88 \cdot 10^{-3}\,\text{eV/c}^2$ extracted from the Kamiokande data (Fig. 4.4 top), i.e. from the Δm_{12}^2 value of $7.9(\pm 2.8) \cdot 10^{-5}(\text{eV/c}^2)$. Since there are three types (or flavors) of neutrinos (ν_e, ν_μ, ν_τ) it is possible that other combinations by two or by three neutrinos can lead to the formation of various mesons and baryons, respectively.

Interestingly there are ten combinations of (ν_e, ν_μ, ν_τ) neutrinos taken by three and there are also ten baryons which do not contain a charm quark. Also there are six combinations of (ν_e, ν_μ, ν_τ) neutrinos taken by two, and there are also six pseudoscolar mesons and six vector mesons as already discussed in Chap. 4.

6.5 Summarizing Remarks

The three and two neutrino models presented in this chapter are rather simple and contain no adjustable parameters. Figure 6.7 summarizes the physical model for the three-neutrino case. The corresponding mathematical model presented in this chapter is summarized in Table 6.3. It takes only ten simple algebraic equations to reach the final result for the mass of the confined rotational state.

An interesting aspect of the three rotating neutrino model is presented in Fig. 6.8 which depicts the effect of the relativistic neutrino mass, γm_o, on the inertial or gravitational mass, $\gamma^3 m_o$. Note that an eV/c^2 scale is used for the relativistic mass and a GeV/c^2 scale is used for the gravitational mass. For $\gamma = 1$ the two masses coincide (at $m_o = 0.043723\,\text{eV/c}^2$). With increasing γ both masses increase (due to the log–log scale the plot is linear) and when γm_o reaches $\gamma_n m_o$, i.e. one third of the neutron mass ($939.565\,\text{MeV/c}^2$) and thus the mass $3\gamma m_o = 3\gamma_n m_o$ of the rotational three-neutrino state equals the neutron mass, then at this point the inertial and gravitational mass, $\gamma^3 m_o$, practically coincides with the Planck mass, $m_{Pl} = (\hbar c/G)^{1/2} = 1.221 \cdot 10^{19}$ GeV/c^2. This very interesting result stems directly from Eq. (6.24) which can be rewritten as:

$$\gamma^3 m_o = 3^{1/4}(2n-1)^{1/2} m_{Pl}, \tag{6.42}$$

i.e. for $n = 1$ the gravitational mass of each rotating neutrino differs less than 32% from the Planck mass of the $1.221 \cdot 10^{19}$ GeV/c^2 or, equivalently, $2.1765 \cdot 10^{-8}$ kg or 21.765 μg. The Planck scale can apparently be reached by nature, in fact here in our surroundings, much easier than usually thought.

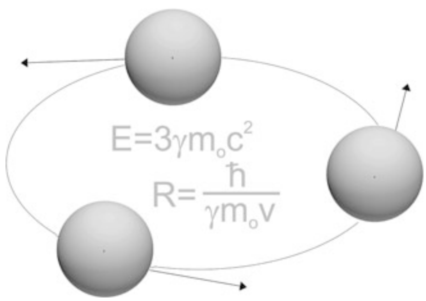

Fig. 6.7 Schematic of the three rotating neutrino model with two of the three corresponding model equations, i.e. $m = 3\gamma m_o$ and Eq. (6.20) for $n = 1$. The third model equation is Eq. (6.18), which can be also rewritten as $R = G m_o \gamma^5/(\sqrt{3}c^2)$

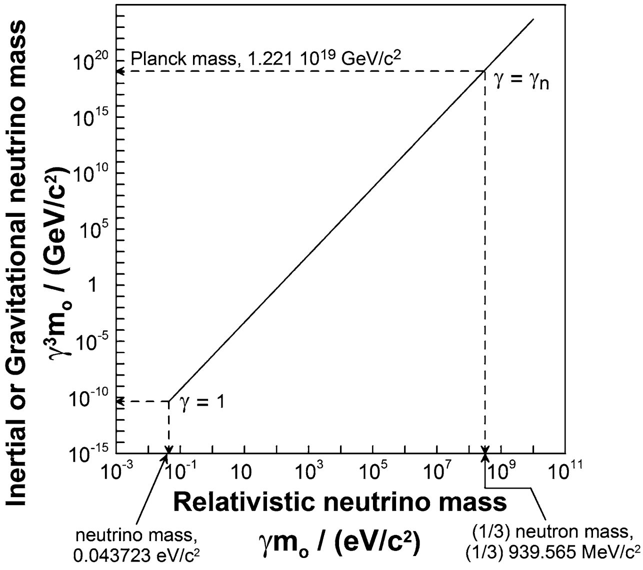

Fig. 6.8 Effect of relativistic neutrino mass (γm_o) on the inertial or gravitational neutrino mass, $\gamma^3 m_o$. When the latter reaches the Planck mass, the former reaches one third of the neutron mass

6.6 Synopsis

Gravity can confine three or two neutrinos in rotational states having the masses of hadrons and mesons, respectively. This astounding result is obtained without developing any new theory or using any adjustable parameters.

The analysis also provides a straightforward mechanism for baryosynthesis, or more generally for hadronization: The baryon mass is due to 99.99999999% to the kinetic energy of the rotating neutrinos. Only the remaining 10^{-8}% corresponds to the neutrinos mass.

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Chapter 7
Energy and Other Properties of the Rotational States

7.1 Potential, Translational, and Total Energy of the Neutrinos

In order to compute the gravitational potential energy of the rotating neutrinos in the confined state, and thus also the system Hamiltonian, it is necessary to return to the force expression (6.12) in Chap. 6.

Equation (6.18) can be used to eliminate R or γ in the force expression of Eq. (6.12). In the former case (i.e. elimination of R) one obtains:

$$F_{\mathrm{G}} = -\frac{\sqrt{3}c^4}{\gamma^4 G} \tag{7.1}$$

and thus, interestingly, for any fixed value of γ or R, the attractive force is uniquely determined by the familiar G/c^4 parameter of the gravitational field equations of general relativity [1–3], i.e. $G_{ik} = (8\pi G/c^4)T_{ik}$, which relates the Einstein tensor G_{ik} with the stress-momentum-energy tensor T_{ik} [1–3].

In the latter case, i.e. elimination of γ, one obtains:

$$F_{\mathrm{G}} = -m_{\mathrm{o}}c^2 \left(\frac{2\sqrt{3}}{R_S}\right)^{1/5} \frac{1}{R^{4/5}}. \tag{7.2}$$

The force equation (7.2) refers to circular orbits only and thus defines a certain conservative force, since the work done in moving the particles between two points R_1 and R_2, corresponding to two rotational states with radii R_1 and R_2, is independent of the path taken. The force vector direction is also defined, as it is always pointing to the center of rotation and thus a conservative vector field is defined which is the gradient of a scalar potential, denoted $V_{\mathrm{G}}(R)$. The latter is the gravitational potential energy of the three rotating particles when accounting for their rotational motion and corresponds to the energy associated with transfer of the

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particles from the minimum circular orbit radius R_{min} (Fig. 6.3) to an orbit of radius of interest, R. The function $V_G(R)$ is obtained via integration of Eq. (7.2). Thus, denoting by R' the dummy variable, one obtains:

$$\begin{aligned} V_G(R) - V_G(R_{\text{min}}) &= \int_{R_{\text{min}}}^{R} F_G \mathrm{d}R' \\ &= -5m_o c^2 \left(\frac{2\sqrt{3}}{R_S}\right)^{1/5} \left(R^{1/5} - R_{\text{min}}^{1/5}\right). \end{aligned} \tag{7.3}$$

Noting that $R_{\text{min}} = 2.343 R_S$ (Fig. 6.3) and that the value of the Schwarzchild radius, R_S, $(= 2Gm_o/c^2)$ for neutrinos is extremely small $(\sim 10^{-63}\,\text{m})$ it follows that for any realistic R value (e.g. above the Planck length value of $10^{-35}\,\text{m}$) Eq. (7.3) reduces to:

$$V_G(R) = -5m_o c^2 (2\sqrt{3})^{1/5} (R/R_S)^{1/5}. \tag{7.4}$$

Thus while the magnitude of the gravitational force acting on the rotating particles increases with decreasing radius, R [Eq. (7.2)], the absolute value $|V_G(R)|$ of the gravitational potential energy increases with increasing R as shown by Eq. (7.4). This behavior is reminiscent of asymptotic freedom [4–7], i.e. the attractive interaction energy is small at short distances and increases significantly with increasing distance R.

In view of Eq. (6.18), i.e. $R/R_S = \gamma^5/(2\sqrt{3})$, one can rewrite Eq. (7.4) as:

$$V_G(R) = -5\gamma m_o c^2. \tag{7.5}$$

On the other hand the kinetic energy, T, of the three rotating neutrinos is:

$$T(R) = 3(\gamma - 1) m_o c^2. \tag{7.6}$$

Thus one may now compute the change, $\Delta\mathcal{H}$, in the Hamiltonian, $\mathcal{H}$, i.e. in the total energy of the system, upon formation of the rotational bound state from the three originally free neutrinos. The Hamiltonian, $\mathcal{H}$, is the sum of the relativistic energy, $E = 3\gamma m_o c^2$, and of the potential energy V_G. The relativistic energy is the sum of the rest energy, $3m_o c^2$, and of the kinetic energy T. Denoting by f and i the final and initial states (i.e. the three free non-interacting neutrinos at rest and the bound rotational state) and by (RE) the rest energy, one obtains:

$$\begin{aligned} \Delta\mathcal{H} &= \mathcal{H}_f - \mathcal{H}_i \\ &= \left[(RE)_f + T_f + V_{G,f}\right] - \left[(RE)_i + T_i + V_{G,i}\right] \\ &= \left[3m_o c^2 + 3(\gamma - 1) m_o c^2 - 5\gamma m_o c^2\right] - 3m_o c^2 \\ &= \Delta T + \Delta V_G = -(2\gamma + 3) m_o c^2 \approx -2\gamma m_o c^2, \end{aligned} \tag{7.7}$$

where the last equality holds for $\gamma \gg 1$ as is the case of interest here.

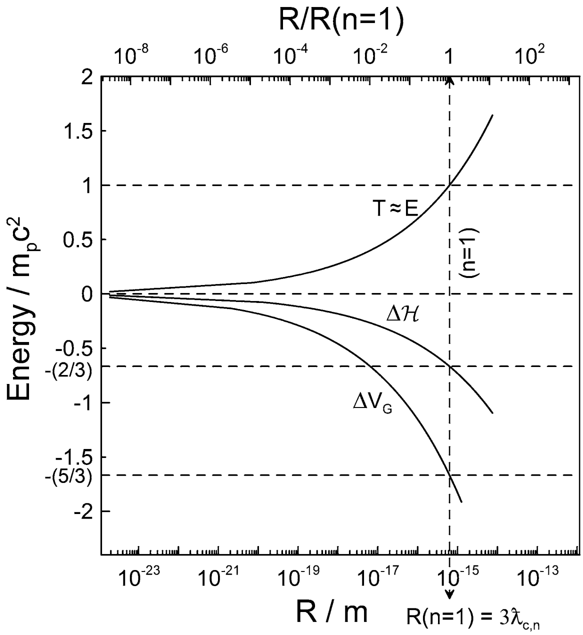

Fig. 7.1 Plot of Eqs. (7.8), (7.9), (7.10), and (6.20), the latter for $n = 1$, showing the dependence on rotational radius, R, of the kinetic energy, T, of the potential energy ΔV_G and of the Hamiltonian (total energy) $\Delta\mathcal{H}$. This is negative for all R, indicating that the formation of the bound state is exoergic ($\Delta\mathcal{H} < 0$) and thus occurs spontaneously. The total energy, E, of the bound state equals $3\gamma m_o c^2$ and thus practically coincides with T which equals $3(\gamma - 1)m_o c^2$. Note that the E vs R curve exhibits both asymptotic freedom ($\Delta V_G, \Delta\mathcal{H} \to 0$ for $R \to 0$) and color confinement ($-\Delta V_G, -\Delta\mathcal{H} \to \infty$ for $R \to \infty$)

The same $\Delta\mathcal{H}$ expression is, of course, obtained regardless of the choice of the reference potential energy state. Thus in view of Eqs. (6.18), (7.4), (7.5), and (7.7), one can summarize the dependence of ΔT, ΔV_G, and $\Delta\mathcal{H}$ on γ and R for $\gamma \gg 1$ as:

$$\Delta T = T = 3(\gamma - 1)m_o c^2 \approx 3\gamma m_o c^2 = 3m_o c^2 \left(\frac{2\sqrt{3}R}{R_S}\right)^{1/5} \tag{7.8}$$

$$\Delta V_G = -5\gamma m_o c^2 = -5m_o c^2 \left(\frac{2\sqrt{3}R}{R_S}\right)^{1/5} \tag{7.9}$$

$$\Delta\mathcal{H} = -(2\gamma + 3)m_o c^2 \approx -2\gamma m_o c^2 = -2m_o c^2 \left(\frac{2\sqrt{3}R}{R_S}\right)^{1/5}. \tag{7.10}$$

The negative sign of $\Delta\mathcal{H}$ shows that the formation of the bound rotational state starting from the three initially free neutrinos happens spontaneously, is exoergic ($\Delta\mathcal{H} < 0$), and the binding energy $BE(= -\Delta\mathcal{H})$ equals $2\gamma m_o c^2$.

The dependence of T, ΔV_G, and $\Delta\mathcal{H}$ on rotational radius is shown in Fig. 7.1, which also shows the reduced de Broglie wavelength of the rotating neutrino and thus the radius of the rotational baryon state.

7.2 Properties of the Bound States

7.2.1 *Rest Energy and Binding Energy*

As already noted [e.g. Eq. (6.25)] the total rest plus kinetic energy of the three rotating particles equals $3\gamma m_o c^2$ and constitutes at the same time the rest energy, mc^2, of the composite particle formed, i.e. of the rotational bound state:

$$mc^2 = 3\gamma m_o c^2. \tag{7.11}$$

It is useful to note that in the model the rest mass of the three particles, i.e. $3m_o c^2$, does not change when the bound state is formed. The transformation of the kinetic energy of the three rotating particles into rest energy of the bound state is associated with the change in choice of the boundaries of the system. In the former case (three individual rotating particles) the boundaries are geometrically disconnected and encompass each particle individually, in the latter case the system boundary contains all three particles and the center of mass is at rest with respect to the observer. Thus the formation of the bound rotating state by the three particles provides a simple hadronization mechanism, i.e. generation of rest mass, m, starting from an initial rest mass $3m_o$, according to Eq. (7.11).

It follows from (7.7) and (7.11) that:

$$BE = -\Delta\mathcal{H} = (2/3)mc^2. \tag{7.12}$$

Thus the binding energy per light particle is $(2/9)mc^2$, which for $m = m_p = 938.272\,\text{MeV/c}^2$, the proton mass, gives an energy of 208 MeV, in good qualitative agreement with the estimated particle energy of 150–200 MeV at the transition temperature of QCD [8] and in even better agreement with the QCD scale of $217 \pm 25\,\text{MeV}$ [9].

One may note here that since the potential energy expression (7.3) does not depend on the number, N, of rotating particles but the kinetic energy, T, does, i.e. $N(\gamma - 1)m_o c^2$, it follows from (7.10) that, according to the model, stable rotational states cannot be obtained for $N > 5$ since they lead to positive $\Delta\mathcal{H}$. The case $N = 2$ is interesting, as it leads to composite masses, m, in the range of mesons, i.e. in the $0.5\,\text{GeV/c}^2$ range, as already shown in Sect. 6.4.

7.2.2 *Radii and Lorentz Factors γ*

The hadron radius computed from Eq. (6.20) for $n = 1$, i.e. from:

$$R(n = 1) = \frac{3\hbar}{m_n c} = 0.630\,\text{fm} \tag{7.13}$$

equals three times the neutron Compton wavelength and is in very good agreement with the experimental proton and neutron radii values which lie in the 0.6–0.7 fm range.

For $n > 1$ the corresponding $R(n)$ values can be computed from Eq. (6.20), i.e. from:

$$R = \frac{(2n-1)\hbar}{\gamma m_o c}. \tag{7.14}$$

By accounting for the γ dependence on $(2n-1)$ given by Eq. (6.24), i.e.

$$\gamma(n) = (2n-1)^{1/6}\gamma(n=1) = 7.163 \cdot 10^9 (2n-1)^{1/6}, \tag{7.15}$$

one obtains:

$$R(n) = (2n-1)^{5/6} R(n=1) = 0.630(2n-1)^{5/6}\,\text{fm}. \tag{7.16}$$

The $\gamma(n)$ values are in the range computed in the Sect. 6.2 for 200 MeV neutrinos. The radii $R(n)$ also lie, interestingly, in the range of hadron, e.g. proton or neutron, radii.

7.2.3 *Lifetimes and Rotational Periods*

The period of rotation $\tau(n)$ of the neutrinos within the composite state, $2\pi R/\text{v} \sim 2\pi R/c$, is, using Eq. (7.16),

$$\tau(n) = (2n-1)^{5/6}\tau_p = (2n-1)^{5/6} 6.6 \cdot 10^{-24}\,\text{s}, \tag{7.17}$$

where $\tau_p = 2\pi R_p/c = 6.6 \times 10^{-24}$ s is the rotation period for the proton or the neutron. The time interval $\tau(n)$ provides a rough lower limit for the lifetime of the composite particles, interpreted as baryons, as they can be defined only if the neutrinos complete at least a revolution. Indeed all the known lifetimes of the baryons are not much shorter than that estimate. The lifetime of the Δ baryons, which is the shortest, is $5.6 \cdot 10^{-24}$ s [10].

7.2.4 *Spins and Charges*

Neutrinos are fermions with spin 1/2 [10] and thus one may anticipate spin of 1/2 or 3/2 for composite states formed by three neutrinos. Indeed most baryons have spin 1/2 and some, as shown in Table 4.4, have spin 3/2 [10].

Several baryons are charged, e.g. the proton or the Ξ^+. The differences in mass, m, from their neutral brethren (i.e. the n or the Ξ^o) is small and of the order of αm, where $\alpha (= e^2/\varepsilon c\hbar = 1/137.0359)$ is the fine structure constant. Thus the rotating neutrino model discussed here can describe with reasonable accuracy (e.g. Fig. 4.8 and Table 6.2) the masses of both neutral and charged baryons. However, since neutrinos are electrically neutral, the question arises about how charged baryons can be formed within the rotating neutrino model.

One possibility is that in the distant past charged neutrinos existed. Their stronger interaction among themselves and with other particles led to their extinction via formation of hadrons, mesons, and neutral neutrinos. A more likely explanation is that neutral hadrons were first formed (e.g. neutrons) and then protons and electrons were formed via the β-decay [10], i.e. $n \rightarrow p^+ + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$, which has a half-life of 885.7 s.

One can assume as an example that in the final state the charges of the constituent particles are equal to those of u and d quarks, i.e. $(2/3)e$ and $-(1/3)e$. This leads as shown in the next section to very good agreement with experiment regarding magnetic moments.

The Coulombic forces between charged particles with relativistic velocities have been studied in detail [11, 12]. It is well established that Coulomb's law correctly gives the force on the test charge for any velocity of the test charge provided the source charge is at rest [11,12]. In the simplified geometry of Figs. 6.1 or 6.7 the distance between the two particles remains constant, thus in the reference frame of the source charge the test charge is also at rest, thus Coulomb's law remains valid without any relativistic corrections.

It is thus possible to estimate the Coulomb interaction energy between the rotating particles. In the simplified geometry of Fig. 6.1 the total Coulomb potential energy for the proton (assumed Standard Model charges 2/3, 2/3, $-1/3$) vanishes, i.e. denoting $\varepsilon = 4\pi\varepsilon_o$ one obtains:

$$V_{\mathrm{C,p}} = \frac{e^2}{\varepsilon\sqrt{3}R}\left[(4/9) - (2/9) - (2/9)\right] = 0, \tag{7.18}$$

while for the neutron (assumed Standard Model charges $-1/3, -1/3$, 2/3) it is negative:

$$V_{\mathrm{C,n}} = \frac{e^2}{\varepsilon\sqrt{3}R}\left[(1/9) - (2/9) - (2/9)\right] = -\frac{(e^2/\varepsilon)}{3\sqrt{3}R}, \tag{7.19}$$

i.e. there is an overall attractive Coulombic interaction. Upon substituting R from Eq. (7.14) for the case of the neutron $(n = 1)$, one obtains:

$$V_{\mathrm{C,n}} = -\frac{e^2}{9\sqrt{3}\varepsilon c\hbar} m_\mathrm{p} c^2 = -\frac{\alpha}{9\sqrt{3}} m_\mathrm{n} c^2 = -4.69 \cdot 10^{-4}\, m_\mathrm{n} c^2 = -0.44\,\mathrm{MeV/c^2}, \tag{7.20}$$

which confirms that the Coulombic interaction energy is negligible in comparison with the relativistic gravitational interaction energy and is of the same order of magnitude as the difference in the rest energies ($\sim$1.3 MeV/c^2) of neutrons and protons.

Nevertheless if the Coulomb interaction is taken into consideration, the symmetry of the configuration of Figs. 6.1 or 6.7 is broken as not all three charges are the same. Although the deviation from threefold symmetry is small, since the Coulombic energy is small, and thus one may still use with good accuracy Eq. (7.20) to estimate the attractive interaction between the three particles forming a neutron, it is conceivable that this broken symmetry may be related to the relative instability of the neutron (lifetime 885.7 s) vs the proton (estimated lifetime $\sim 10^{32}$ s [10]).

7.2.5 Magnetic Moments

It is interesting to compute the magnetic dipole moments, μ, of these bound rotational states. Using the definition of $\mu (= (1/2)qR\text{v})$ and considering the case $n = 1$, corresponding to a proton (which is a uud baryon) with charge $2e/3$ for u and $-e/3$ for d it is:

$$\mu_{\text{p}} = (1/2)eRc\,[(2/3) + (2/3) - (1/3)] = (1/2)eRc. \tag{7.21}$$

Upon substituting $R = R(n = 1) = 0.630\,\text{fm}$ one obtains:

$$\mu_{\text{p}} = 15.14 \cdot 10^{-27}\,\text{J/T} \quad (= 3\mu_{\text{N}}), \tag{7.22}$$

where μ_N is the nuclear magneton $(5.05 \cdot 10^{-27}\ \text{J/T})$. This value differs less than 8% from the experimental value of $14.10 \cdot 10^{-27}\ \text{J/T}$ (i.e. 2.79 μ_{N}) [13].

Equivalently, the same result can be obtained by assuming that the three constituents (partons) of the proton have charges $2e$, $-e$ and zero, or e,e and $-e$. This point will be further addressed in Chap. 10.

In the above computation [Eqs. (7.21) and (7.22)] one assumes that the spin vectors of the three small particles are parallel to the vector of rotation of the rotating proton state. If one considers the neutron which is a udd particle and assumes that the spin of one of the two d quarks is parallel with the rotation vector of the rotating neutron state and the spins of the other two particles are antiparallel to the neutron rotation vector, then one obtains:

$$\mu_{\text{n}} = (1/2)eRc\,[(-2/3) + (1/3) - (1/3)] = -(1/3)eRc, \tag{7.23}$$

and upon substitution of $R = 0.630\ fm$, one obtains:

$$\mu_{\text{n}} = -10.09 \cdot 10^{-27}\,\text{J/T} = -2\mu_{\text{N}}, \tag{7.24}$$

which is in excellent agreement with the experimental value of $-9.66 \cdot 10^{-27}\,\text{J/T}\,(= -1.913\mu_{\text{N}})$.

This good agreement seems to imply that the spin contribution of the light particles to the magnetic moment of the rotating state is small and only the spin vector orientation (parallel or antiparallel to the baryon rotation vector) is important. Thus other parton charge combinations which are consistent with zero total charge, and with the experimental μ_n value, are of the type 0, $-(1/3)e$ and $(1/3)e$. In this case the spin vector of the positively charged parton is antiparallel to the vector of rotation of the neutron state. This point is further discussed in Chap. 10.

7.2.6 Inertial Mass and Angular Momentum

As already noted in Sect. 6.5 and Fig. 6.8, it follows from Eq. (6.24) that in the case of the neutron or proton $(n = 1)$ the inertial and gravitational mass of each rotating particle, $\gamma^3 m_o$, is related to the Planck mass, $m_{Pl} = (\hbar c/G)^{1/2}$, via a very simple equation, i.e.

$$\gamma^3 m_o = 3^{1/4} m_{Pl} = 3^{1/4}\left(\frac{\hbar c}{G}\right)^{1/2} = 1.607 \cdot 10^{19}\,\mathrm{GeV/c^2}, \tag{7.25}$$

which provides an interesting direct connection between the Planck mass and the rotating neutrino model. Gravity is generally expected to reach the level of the strong force at energies approaching the Planck scale ($\sim 10^{19}$ GeV) [14] which is in good agreement with the model results [Eq. (7.25)].

It is worth reminding here Wheeler's concept of geons [1, 2, 15, 16], i.e. of electromagnetic waves or neutrinos held together gravitationally, which had been proposed as a classical relativistic model for hadrons [1]. Similar to the present case [Eq. (7.25)] the minimum mass of a small geon formed from neutrinos had been estimated [1] to lie in the Planck mass range.

It is interesting to note here that when using the inertial or gravitational mass, $\gamma^3 m_o$, in the definition of the Compton wavelength, λ_c of the particle $(= h/mc)$ then one computes the Planck length ($\sim 10^{-35}$ m), but when using the mass corresponding to the total energy of the particles, $3\gamma m_o$, then one computes the proton Compton wavelength ($\sim 10^{-15}$ m), which is close to the actual distance between the rotating particles. This point is further discussed in Chap. 12 and its figures.

Another positive feature of the model is that it is qualitatively consistent with a central experimental observation about the strong force [17], i.e. that the normalized angular momentum of practically all hadrons and their excited states is roughly bounded by the square of their mass measured in *GeV* [17]. Indeed from Eqs. (6.19) and (6.30) one obtains:

$$(L/\hbar)/(m/GeV)^2 = 1.13(2n-1)^{2/3}, \tag{7.26}$$

which is in reasonable qualitative agreement with experiment for small integer n values (Fig. 7.2).

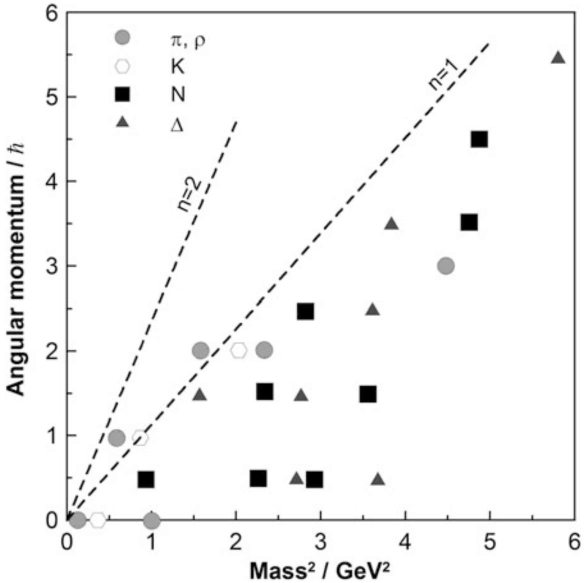

Fig. 7.2 Comparison of Eq. (7.26) for $n = 1$ and 2 with the data of Fig. 4.9, i.e. with the experimental dependence of the normalized by $\hbar$ angular momentum of some hadrons and of their resonances on the square of their normalized by *GeV* mass [17]

7.2.7 Gravitational Force

The strong force is generally estimated to be a factor of $\alpha^{-1}(\approx 137.035)$ stronger than the Coulombic force at distances of the order of 1 fm [10]. It is therefore interesting to compute the gravitational force in the actual rotational states whose radius, R, is given by Eq. (6.20), i.e.

$$R = \frac{(2n-1)\hbar}{\gamma m_o c}. \tag{7.27}$$

From Eq. (6.14) and accounting for $\mathrm{v} \approx c$ one obtains:

$$-F_G R = \gamma m_o c^2, \tag{7.28}$$

which is valid for any value of R. Combining with (7.27) which is valid only for those R values which satisfy the angular momentum quantization condition (6.20) one obtains:

$$-F_G R^2 = (2n-1)\hbar c, \tag{7.29}$$

which using the definition of $\alpha(= e^2/\varepsilon c\hbar)$ can also be written as:

$$-F_G = \frac{(2n-1)\hbar c}{R^2} = \frac{(2n-1)\alpha^{-1}e^2}{\varepsilon R^2}. \tag{7.30}$$

Table 7.1 Properties of the bound state (with $m_o = 0.043723\,\text{eV}/c^2$)

	Theory	Experiment	Neutrino model
Rest energy	–	939 MeV	939 MeV
Binding energy	160–220 MeV	–	208 MeV
Radius	–	0.6–0.8 fm	$0.630(2n-1)^{1/6}$ fm
Lifetimes and rotational periods	–	$5.6 \cdot 10^{-24}$ s	$6.6 \cdot 10^{-24}(2n-1)^{5/6}$ s
Spins	1/2, 3/2	1/2, 3/2	1/2, 3/2
Magnetic moments	–	$14.10 \cdot 10^{-27}$ J/T(p) $-9.66 \cdot 10^{-27}$ J/T(n)	$15.14 \cdot 10^{-27}$ J/T $-10.09 \cdot 10^{-27}$ J/T
Inertial mass	$\sim 10^{19}$ GeV/c^2	$\sim 10^{19}$ GeV/c^2	$1.607 \cdot 10^{19}$ GeV/c^2
Angular momentum	–	$(L/\hbar)/(m/GeV)^2 \leq 1$	$(L/\hbar)/(m/GeV)^2 = 1.13(2n-1)^{3/2}$

This equation shows that the magnitude of the relativistic gravitational force at the radius of the rotational state is a factor of α^{-1} (137.035) larger than the Coulombic force between two unit charges at the same distance. Since this is known to be the case for the strong force, it follows that the relativistic gravitational force behaves in the same way as the strong force, i.e. its coupling constant in the rotational baryon state is a factor of α^{-1} stronger than the Coulombic coupling constant.

7.2.8 Summary of the Comparison with Experiment

The satisfactory agreement between the three neutrino model and experiment is shown in Table 7.1. The model presented in Chaps. 6 and 7 includes no adjustable parameters and predicts very well in a semiquantitative and sometimes quantitative manner all the important properties of light baryons, i.e. masses, binding energies, radii, lifetimes, spins, charges, magnetic moments, and angular momenta. The model also shows that the magnitude of the relativistic gravitational force at the radius of the rotational state is a factor of α^{-1} (137.036) larger than the magnitude of the Coulombic force at the same distance, i.e.

$$-F_G/F_C = \alpha^{-1}. \tag{7.31}$$

This is in very good agreement with theoretical expectations and with experiment [14], regarding the ratio of the magnitudes, i.e. of the coupling constants, of the strong and the Coulombic forces in the fm range. This point is further discussed in Chap. 12.

7.2.9 Gravitational Constant

It is interesting to note that, given the values of m_{n} and m_{o}, one can use Eq. (6.29) to derive a simple formula for the gravitational constant. Thus one obtains:

$$m_{\mathrm{Pl}} = \frac{(m_{\mathrm{n}}/3)^3}{3^{1/4} m_o^2} \tag{7.32}$$

and using the definition of the Planck mass ($m_{\mathrm{Pl}} = (\hbar c/G)^{1/2}$) it follows:

$$G = \frac{3^{1/2} m_{\mathrm{o}}^4}{(m_{\mathrm{n}}/3)^6} \hbar c. \tag{7.33}$$

Upon substitution ($m_{\mathrm{o}} = 0.043723\,\mathrm{eV/c^2}$, $m_{\mathrm{n}} = 939.565\,\mathrm{MeV/c^2}$, $\hbar = 1.056 \cdot 10^{-34}\,\mathrm{Js}$, $c = 2.997 \cdot 10^8\,\mathrm{m/s}$) one finds the experimental G value, i.e. $6.673 \cdot 10^{-11} m^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ [13, 18, 19]. When more exact values of the mass m_{o} become known in the future, it is possible that Eq. (7.33) may provide more exact values of G than those obtained currently via torsion balance measurements [13, 18, 19].

7.3 Energy-Curvature Dependence and General Relativity

Since the present Bohr-type model is based on the combination of special relativity and the equivalence principle, which was the basis of the theory of general relativity, it is worthwhile to explore if the simple mathematical equations of the present model may have some similarity with some limiting form of the field equations of general relativity.

Thus by using Eq. (6.18) to eliminate R in the force expression of Eq. (6.12) one obtains:

$$F_{\mathrm{G}} = -\sqrt{3}\frac{c^4}{\gamma^4 G} \tag{7.34}$$

and thus, interestingly, for any given value of γ, and thus, via Eq. (6.18), for any given value of R, the attractive force is uniquely determined by the familiar G/c^4 parameter of the gravitational field equations of general relativity, i.e.:

$$G_{ik} = 8\pi \left(\frac{G}{c^4}\right) T_{ik}, \tag{7.35}$$

which relates the Einstein tensor G_{ik} with the stress-momentum-energy tensor T_{ik} [1, 2, 20].

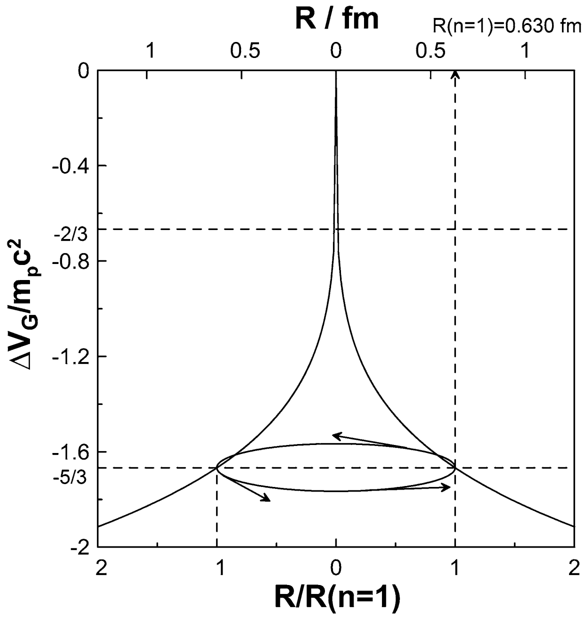

Fig. 7.3 Plot of ΔV_G *vs* R from Eq. (7.9)

In view of Eq. (7.3) the force, F_G, can also be expressed as:

$$F_G = -\frac{dV_G}{dR} \tag{7.36}$$

and thus combining with (7.34) one obtains:

$$dR = \frac{\gamma^4}{\sqrt{3}}\left(\frac{G}{c^4}\right)dV_G. \tag{7.37}$$

The actual dependence of V_G and ΔV_G on R, obtained from Eq. (7.9), was given in Fig. 7.1 and is also shown in Fig. 7.3. For small variations in γ and thus R and V_G Eq. (7.37) gives:

$$\Delta R = \frac{\gamma^4}{\sqrt{3}}\left(\frac{G}{c^4}\right)\Delta V_G. \tag{7.38}$$

Upon comparing with the field equations (7.35) one observes that (7.38) is similar to a limiting one-dimensional analogue of (7.35) with the change in radius, or in curvature ΔR, being analogous to the spacetime curvature due to the presence of mass and the change in gravitational energy, ΔV_G, being analogous to the stress-momentum-energy tensor T_{ik}.

7.4 Model Consistency with General Relativity: Kerr Black Holes

During the last 30 years there have been some suggestions in the context of "strong gravity" that hadrons can be viewed as microscopic black holes. Strong gravity is a no-main-stream area of physics which assumes that the gravitational constant G, which equals $6.673 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$ [13, 18, 19], has a second value, of the order of $10^{28}\,\mathrm{m^3kg^{-1}s^{-2}}$ at very short distances [21]. This rather arbitrary assumption has been considered as a viable alternative to QCD in the past [22]. It thus becomes interesting to examine whether the rotational neutrino states obtained in Chap. 6 and corresponding to hadrons can be viewed as microscopic Kerr black holes. That hadrons may be viewed as microscopic black holes is an idea studied from a geometric general relativistic approach in the past [23] and also discussed again more recently [24].

In general relativity the Kerr metric (or Kerr vacuum) is an exact solution of the Einstein field equations and describes the geometry of spacetime around a rotating body. The Kerr metric is often used to describe rotating black holes [1–3, 24].

A necessary condition for the existence of a solution and thus for the stability of such a rotating black hole is that:

$$\rho < r_s/2, \tag{7.39}$$

where ρ is a characteristic length defined from:

$$\rho = \frac{J}{Mc}, \tag{7.40}$$

where J is the spin angular momentum and r_s is the Schwarzschild event horizon:

$$r_s = \frac{2GM}{c^2}. \tag{7.41}$$

Thus the condition (7.39) is written as:

$$\frac{Jc}{GM^2} \leq 1. \tag{7.42}$$

In the case of equality, the black hole is termed extreme black hole.

When the rotating black hole is charged, the solution of the Einstein–Maxwell equations is the Kerr–Newman metric and the black hole is stable for:

$$\rho^2 \leq \left(GM^2 - \frac{q^2}{\varepsilon}\right)\left(\frac{G}{c^4}\right). \tag{7.43}$$

In the present rotating neutrino model case it is:

$$M = \gamma^3 m_o; \quad GM^2 = Gm_o^2\gamma^6 \tag{7.44}$$

and thus inequality (7.42) becomes:

$$\frac{Jc}{G\gamma^6 m_o^2} \leq 1. \tag{7.45}$$

Upon using $J = L = (2n-1)\hbar$ from Eq. (6.19) one obtains:

$$\frac{(2n-1)\hbar c}{\gamma^6 m_o^2 G} \leq 1. \tag{7.46}$$

Upon introducing Eq. (6.23), i.e.

$$\gamma^6 = (2n-1)3^{1/2}\left(\frac{\hbar c}{G}\right)/m_o^2, \tag{7.47}$$

one obtains:

$$\frac{(2n-1)\hbar c}{3^{1/2}(2n-1)\frac{\hbar c}{G}G} \leq 1; \quad \frac{1}{3^{1/2}} \leq 1, \tag{7.48}$$

i.e. inequality (7.42) is satisfied, which implies that the confined neutral rotational state corresponding to the baryon can be viewed as a stable Kerr black hole.

If the rotational state is charged, then substituting in (7.43) one obtains that an additional constraint for the stability of the Kerr–Newman black hole is:

$$(q^2/e^2) \leq 3^{1/2}(2n-1)\alpha^{-1} \tag{7.49}$$

which is again clearly satisfied in the model for any realistic q values, i.e. $\pm e$ or $\pm 2e/3$ or $\pm e/3$. Thus one may view the rotational state also as a stable Kerr–Newman black hole. This might provide an explanation for the extremely long ($\sim 10^{32}$ s) estimated lifetime of protons but cannot provide any rationalization for the much shorter (885.7 s) lifetime of neutrons.

7.5 Synopsis

The three rotating neutrino model is in semiquantitative agreement with experiment regarding the rest and binding energies, radii, lifetimes, spins, magnetic moments, and angular momenta of baryons. The inertial and gravitational mass of the rotating neutrinos practically coincides with the Planck mass. The relativistic gravitational coupling constant is found to be a factor of $\alpha^{-1}(137.035)$ larger than the Coulombic coupling constant, exactly as anticipated for the strong force in the Standard Model. The rotating neutrino hadron states are not inconsistent with general relativity and can indeed be viewed as Kerr or Kerr–Newman black holes, as suggested during the last few decades. This however does not appear to provide any immediate additional information regarding the properties of these states.

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Chapter 8
Gravitational Hadronization: How Mass Can Be Produced from Gravity

8.1 The Generation of Rest Mass by the Kinetic Energy of the Constituents of a Confined State

This section examines the changes induced in the total energy (Hamiltonian) of a system as well as to its relativistic energy and rest mass for two cases: First when a H atom is formed via the Coulombic attraction of a proton and an electron and second when a rotational state corresponding to a baryon (neutron) is formed via the relativistic gravitational attraction of three neutrinos.

The total energy, i.e. the Hamiltonian, $\mathcal{H}$, of a system consisting of N particles ($i = 1, 2, \ldots, N$) is the sum of the total relativistic energy, E, and of the potential energy of the system, V:

$$\mathcal{H} = E + V. \tag{8.1}$$

The relativistic energy, E, of the system is the sum of its rest energy, $\sum m_{\mathrm{o,i}}c^2$, plus its kinetic energy T, which equals $\sum(\gamma_i - 1)m_{\mathrm{o,i}}c^2$, i.e.

$$E = \sum_{i=1}^{N} m_{\mathrm{o,i}}c^2 + \sum_{i=1}^{N}(\gamma_i - 1)m_{\mathrm{o,i}}c^2. \tag{8.2}$$

Thus in the case of a H atom, neglecting the (rather small) kinetic energy of the proton, we can write with good accuracy:

$$E = m_{\mathrm{p}}c^2 + m_{\mathrm{e}}c^2 + (\gamma_e - 1)m_{\mathrm{e}}c^2 \tag{8.3}$$

$$\mathcal{H} = m_{\mathrm{p}}c^2 + m_{\mathrm{e}}c^2 + (\gamma_e - 1)m_{\mathrm{e}}c^2 + V_{\mathrm{e}}, \tag{8.4}$$

where V_{e} is the electrostatic energy of the electron. Since $\mathrm{v} = \alpha c$, thus $\mathrm{v} \ll c$, the kinetic energy term reduces to $(1/2)m_{\mathrm{e}}\mathrm{v}^2 = (1/2)\alpha^2 m_{\mathrm{e}}c^2$. Also for the circular orbit of the Bohr model it is $V_{\mathrm{e}} = -2T$, thus one obtains:

$$E = m_{\mathrm{p}}c^2 + m_{\mathrm{e}}c^2 + (1/2)\alpha^2 m_{\mathrm{e}}c^2 \tag{8.5}$$

$$\mathcal{H} = m_{\mathrm{p}}c^2 + m_{\mathrm{e}}c^2 - (1/2)\alpha^2 m_{\mathrm{e}}c^2. \tag{8.6}$$

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DOI 10.1007/978-1-4614-3936-3_8,

In the initial state (a proton and an electron at rest and "infinite" distance), both E and $\mathcal{H}$ equal $(m_p + m_e)c^2$, thus upon formation of the bound H atom state it is:

$$\Delta E = (1/2)\alpha^2 m_e c^2 (= T) \tag{8.7}$$

$$\Delta\mathcal{H} = -(1/2)\alpha^2 m_e c^2 = -13.6\,\text{eV}. \tag{8.8}$$

Thus while the system Hamiltonian has decreased by 13.6 eV, and therefore the formation of the H atom occurs spontaneously and is exoergic, i.e. energy is being released, at the same time the relativistic energy of the system, ΔE, has increased by the Bohr energy $(1/2)\alpha^2 m_e c^2$, which actually equals T, i.e. the kinetic energy of the electron. Consequently for a laboratory observer who cannot detect the circular motion, the apparent *rest* energy of the system has increased by $(1/2)\alpha^2 m_e c^2$.

Thus the rest energy increase is equal to the kinetic energy of the electron (or of the proton–electron pair) in the confined H atom state.

Similar is the situation when a composite particle (in this case a hadron) is formed by three light particles (in this case neutrinos).

In this case in the initial state, i.e. three noninteracting neutrinos at rest, it is $E_o = \mathcal{H}_o = 3m_o c^2$ and upon formation of the bound state it is:

$$E = 3m_o c^2 + 3(\gamma - 1)m_o c^2 = 3\gamma m_o c^2 \tag{8.9}$$

$$\mathcal{H} = 3\gamma m_o c^2 + V_G = 3\gamma m_o c^2 - 5\gamma m_o c^2 = -2\gamma m_o c^2, \tag{8.10}$$

where in the last equation we have expressed V_G via Eq. (7.5).

Thus again it is $\Delta\mathcal{H} < 0$, i.e. the reaction is exoergic and occurs spontaneously, but at the same time it is $\Delta E > 0$, i.e. the relativistic energy of the system increases. Consequently for a laboratory observer the apparent rest energy increase is $\Delta E = 3(\gamma - 1)m_o c^2$. Thus again the rest energy increase equals the kinetic energy of the three rotating particles. But in this case, when comparing this rest energy increase with the rest energy of the initial state, the ratio is enormous. Thus the ratio of the masses of the final and initial states is $1 + (1/2)\alpha^2 (\approx 1)$ in the case of the H atom and is $m/3m_o (= 7.163 \cdot 10^9)$ in the case of hadron formation. This is due to the huge $(= 7.163 \cdot 10^9)$ value of γ in the rotational state. Thus in this case apparent rest energy and apparent rest mass are generated simply by the action of gravity, which causes particle confinement in a high kinetic energy, thus high γ value state. According to this simple mechanism there appears to be no need to hypothesize the action of a boson in order to rationalize the creation of rest mass.

The results are summarized in Fig. 8.1 and in Tables 8.1 and 8.2.

It is useful to define a parameter, ξ, which expresses the ratio of new mass created divided by the initial mass, i.e.

$$\xi = \frac{\text{new mass created}}{\text{initial mass}} \tag{8.11}$$

$$\xi + 1 = \frac{\text{final mass}}{\text{initial mass}}. \tag{8.12}$$

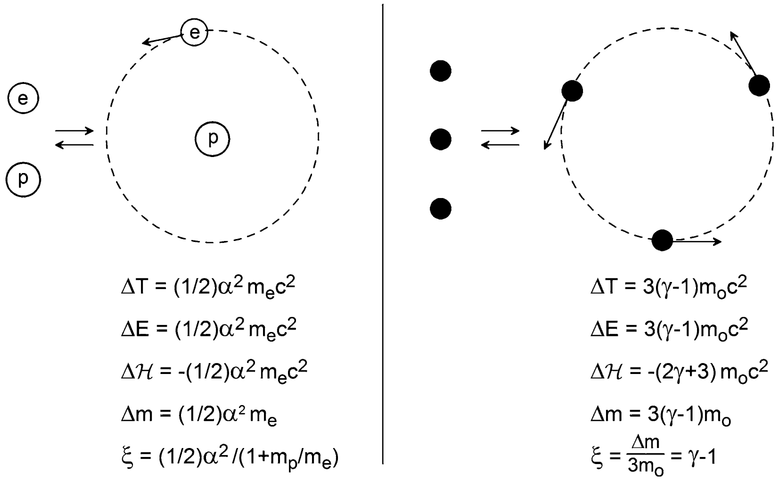

Fig. 8.1 Apparent mass generation mechanism during formation (**a**) of the H atom from a proton and an electron (*left*) (**b**) of a neutral baryon (e.g. a neutron) from three neutrinos (*right*)

Table 8.1 Mass generation in H atom and Baryon formation

	H atom	Baryon
Initial rest mass	$m_{\mathrm{p}}+m_{\mathrm{e}}$	$3m_{\mathrm{o}}$
Final rest mass	$m_{\mathrm{p}}+m_{\mathrm{e}}+(1/2)\alpha^2 m_{\mathrm{e}}$	$3\gamma m_{\mathrm{o}}$
Rest mass increase	$(1/2)\alpha^2 m_{\mathrm{e}}c^2$	$3(\gamma-1)m_{\mathrm{o}}$
Binding energy (BE)	$(1/2)\alpha^2 m_{\mathrm{e}}c^2$	$2\gamma m_{\mathrm{o}}c^2$
Potential energy of BS[a]	$-\alpha^2 m_{\mathrm{e}}c^2$	$-5\gamma m_{\mathrm{o}}c^2$

[a]Bound state

Table 8.2 Rest, relativistic, inertial, and gravitational mass of the neutrino constituents of baryons

	Symbol	Value
Rest mass	m_{o}	$0.043723\,\mathrm{eV/c^2}$
Relativistic mass	γm_{o}	$313.188\,\mathrm{MeV/c^2}$
Inertial mass or gravitational mass	$\gamma^3 m_{\mathrm{o}}$	$1.60692\cdot 10^{19}\,\mathrm{GeV/c^2}$
Confined state baryon mass ($n=1$)	$m=3\gamma m_{\mathrm{o}}$	$939.565\,\mathrm{MeV/c^2}$

Thus in the case of the H atom it is:

$$\xi = \frac{(1/2)m_{\mathrm{e}}\alpha^2}{m_{\mathrm{p}}+m_{\mathrm{e}}} \approx 1.45\cdot 10^{-8}, \tag{8.13}$$

while for the case of the neutron it is:

$$\xi = \frac{3(\gamma_n-1)m_{\mathrm{o}}}{3m_{\mathrm{o}}} = \gamma_n - 1 \approx 7.163\cdot 10^{9}. \tag{8.14}$$

This powerful means of mass generation is very simple and quite different from other proposed mass generation schemes [1–5]. It is similar in its conclusions with the QCD results of Dürr et al. [3] who showed that even if the quark masses vanished, the baryon mass would not change much, a phenomenon sometimes called "mass without mass" [4, 5].

8.2 Thermodynamics of Neutrino and Quark-Gluon Plasma Condensation

The formation of a baryon by three neutrinos is an exothermic process, as already discussed in Chap. 7. The potential energy, $V_{\rm G}$, decreases by $5\gamma m_{\rm o}c^2$ [Eq. (7.9)], i.e.:

$$\Delta V_{\rm G} = -5\gamma m_{\rm o}c^2 = -(5/3)mc^2, \tag{8.15}$$

where m is the baryon (neutron) mass. At the same time it is $\Delta T = \Delta(RE) = (m - 3m_{\rm o})c^2 \approx mc^2$ where ΔT is the increase in kinetic energy of the neutrinos upon formation of the bound state and $\Delta(RE) = \Delta T$ is the increase in rest energy of the confined state. Consequently the total Hamiltonian energy change $\Delta\mathcal{H}$ is given by:

$$\Delta\mathcal{H} = \Delta T + \Delta V_{\rm G} = mc^2 - (5/3)mc^2 = -(2/3)mc^2. \tag{8.16}$$

Thus the binding energy per particle, $-\Delta\mathcal{H}/3$, is 208 MeV which is in good agreement with the QCD scale of 217 ± 25 MeV.

The change in Helmholz free energy, F, can be computed from:

$$\Delta F = \Delta\mathcal{H} - \Theta\Delta\mathcal{S}, \tag{8.17}$$

where Θ is the absolute temperature in K and $\Delta\mathcal{S}$ in the entropy change associated with baryon formation from the three neutrinos. The sign of $\Delta\mathcal{S}$ is negative, as three translational degrees of freedom are being lost upon formation of the confined state. Thus to a good approximation it is:

$$\Delta\mathcal{S} = -k_{\rm b} ln3, \tag{8.18}$$

where $k_{\rm b} = 1.38 \cdot 10^{-23}\,{\rm J/K} = 8.617 \cdot 10^{-5}\,{\rm eV/K}$ is the Boltzmann constant.

Upon combining Eqs. (8.16), (8.17), and (8.18), one obtains that the free energy vanishes, i.e. $\Delta F = 0$, at:

$$k_{\rm b}\Theta_{\rm cr} = \frac{(2/3)mc^2}{ln3} = 570.15\,{\rm MeV}, \tag{8.19}$$

where Θ_{cr} is the critical temperature corresponding to equilibrium between condensed (i.e. confined) and free neutrinos. This temperature is similar to the condensation temperature of the quark-gluon plasma [6–8].

Consequently the critical kinetic (thermal) energy per particle is:

$$T_{cr} = k_b \Theta_{cr}/3 = 190.05\,\mathrm{MeV}, \tag{8.20}$$

and consequently

$$\Theta_{cr} = 2.206 \cdot 10^{12}\,\mathrm{K}. \tag{8.21}$$

The above computed critical or condensation energy is in good agreement with the predictions of the QCD Theory about the QCD transition energy and temperature (i.e. 160–200 MeV) [6–10]. Consequently the predictions of the rotating neutrino model are in good agreement with experiment both regarding the QCD scale and the QCD transition energy and temperature.

8.3 Synopsis

The formation of bound rotational states by relativistic neutrinos provides a very efficient mechanism for hadronic mass generation. The high neutrino kinetic energy becomes the rest energy and thus the rest mass of the hadrons. There is a critical particle temperature ($\sim 2.2 \cdot 10^{12}$ K) and equivalent energy (190 MeV) below which this mass creating condensation can take place. These values practically coincide with those of the transition temperature and transition energy of the QCD quark-gluon condensation.

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Chapter 9
Model Comparison with the Main Experimental Features of the Strong Interaction Force

9.1 Quarks, Gluons, and Color Charge

Within the *standard model* and quantum *chromodynamics* (QCD) baryons consist of three *quarks* or *antiquarks* which are bound together by the *strong force* which is mediated via the exchange of *gluons*. The magnitude of the strong force is dictated by the *color charge* of quarks, antiquarks, and gluons. Quarks cannot be isolated and studied independently due to *confinement* which is the property of the strong force to become unbound as the distance between quarks increases (above $\sim$1 fm). Another key feature of the strong force is *asymptotic freedom* which is the property of the strong force to become very weak at very short distances [1, 2].

Furthermore, an important aspect of the standard model is the existence of the Higgs boson which has mediated, via the Higgs mechanism, the creation of matter [3–5].

It therefore becomes important to examine how the rotating neutrino model discussed in this book can fit with these theoretical concepts (quarks, gluons, color charge, Higgs bosons) and experimental observations (confinement, asymptotic freedom).

An answer to these questions may have been reached already by the reader after reading Chaps. 6–8. It is time now to try to formulate and summarize some of these answers as they emerge from the rotating neutrino model and the very good agreement between model and experiment.

A summary of such emerging possible answers is presented in Table 9.1 according to the rotating neutrino model in parallel with those of the Standard Model. Some of these questions and answers are further discussed below.

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Table 9.1 Standard model for particles and neutrino model

	Standard model	Neutrino model
Force	Strong force	Relativistic gravitational force
Force expression	–	$F_G = -Gm_{1,g}m_{2,g}/r^2$ $= -Gm_{1,o}m_{2,o}\gamma_1^3\gamma_2^3/r^2$ [a]
Hadron constituent	Quark	Fast neutrino
Property determining the force magnitude	Color charge	Inertial(=gravitational) mass $\gamma^3 m_o$
Mediating particle	Gluon	–
Predicted baryon mass	(Model parameter)	939 MeV/c^2
Neutrino mass	zero	0.043723 eV/c^2

[a] $m_{1,g}$ and $m_{2,g}$ are the gravitational, thus also inertial, masses of two bodies; $m_{1,o}$ and $m_{2,o}$ are the corresponding rest masses and γ_1 and γ_2 are the corresponding Lorentz factors computed from the velocities v_1 and v_2 of the two particles relative to the laboratory observer

Table 9.2 Quark masses and relativistic rotating neutrino mass for n=1, 2 and 3 (MeV/c^2)

Quark flavor	Bare mass [6]	Effective mass [6]	Relativistic neutrino mass, γm_o,from Eq. (6.25)
u	2	336	313.2 ($n = 1$)
d	5	340	376.1 ($n = 2$)
s	95	486	409.6 ($n = 3$)
c	1,300	1,550	
b	4,200	4,730	
t	174,000	177,000	

Table 9.3 Masses of fast neutrinos

Rest mass	m_o	0.043723 eV/c^2
Relativistic massin a baryon for $n = 1$	γm_o	313.2 MeV/c^2
Inertial-gravitational mass in a baryon for $n = 1$	$\gamma^3 m_o$	$1.607 \cdot 10^{19}$ GeV/c^2

9.1.1 Quarks

Within the rotating neutrino model, quarks are just fast rotating neutrinos, caught on circular or elliptical orbits via (relativistic) gravitation. This can explain:

a. *Why there is a very large uncertainty about the masses of quarks and antiquarks* which range from a few (∼2–6) MeV/c^2, called *bare* masses, to a few hundreds (∼300–500) MeV/c^2, called *effective* masses as already discussed in Sect. 4.3 (Table 9.2, [6]).

According to the rotating neutrino model there are three masses to consider, rest, relativistic, and inertial-gravitational, of which the last two are by far the most important (Table 9.3).

The *effective* masses of u, d, and s quarks [6] appear to correspond to the relativistic masses of the rotating neutrinos. It is also worth keeping in mind that there are three neutrino flavors (ν_e, ν_μ, ν_τ) with different masses, so their combinations can lead to a variety of hadrons with different gravitational masses for their constituent components.

b. *Why quarks cannot be isolated and studied independently*: First, because the rotating neutrinos are confined, i.e., the gravitational potential energy goes to negative infinity with increasing radius of rotation [Eq. (7.4)].

Second, because if somehow a stable baryon is forced to decompose (e.g. via extremely energetic γ-rays) the resulting single particles have lost their rotating partners and thus the mechanism via which their large relativistic and gravitational mass was sustained. If they lose their high kinetic energy, they are just neutrinos with a rest mass ten billion times smaller than the baryon mass, thus in principle barely detectable and in practice not detectable in synchrocyclotron hadron colliders [6–9].

9.1.2 Gluons

In the rotating neutrino model there is no need for the postulate of gluons. In the context of the model, the strong force is just relativistic gravity (i.e. gravitational attraction between relativistic particles) and if any mediator is really needed, this would be the graviton. In a broad sense one may say that the gluon (i.e. the means of binding quarks or partons and in reality neutrinos together) is just the high speed of the rotating neutrino which, via the γ^3 term, creates a huge inertial and gravitational neutrino mass, i.e. $\gamma^3 m_o$, which inside hadrons is in the Planck mass range ($\sim 10^{19}$ GeV/c^2).

9.1.3 Color Charge

There is apparently no need for postulating a color charge in the rotating neutrino model. The magnitude of the attractive strong force is determined by the magnitude of the gravitational and inertial mass $\gamma^3 m_o$. Thus in a broad sense the gravitational mass $\gamma^3 m_o$ may be considered to have the same function as the color charge.

9.1.4 Confinement and Asymptotic Freedom

Within the rotating neutrino model, both confinement and asymptotic freedom are described by Eqs. (7.9) and (7.10), i.e.

$$\Delta V_G = -5 m_o c^2 \left(\frac{2\sqrt{3}R}{R_S} \right)^{1/5} \tag{9.1}$$

$$\Delta \mathcal{H} = -2 m_o c^2 \left(\frac{2\sqrt{3}R}{R_S} \right)^{1/5} . \tag{9.2}$$

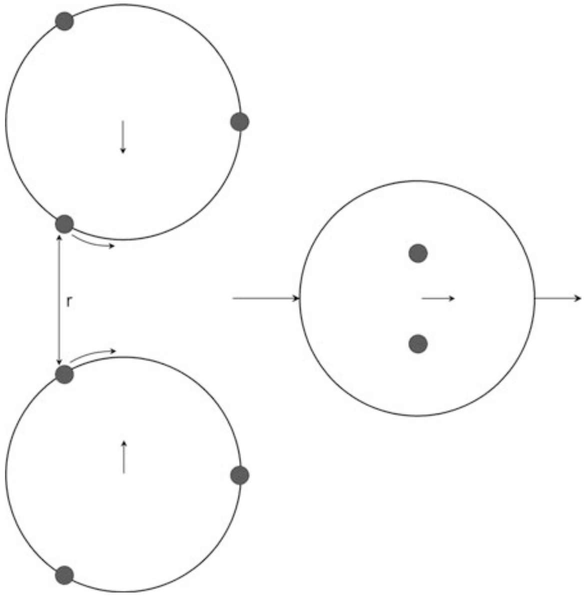

Fig. 9.1 Schematic of a possible newly created hadrons jet generation mechanism via the approach of two baryons leading to the formation of a jet in a direction perpendicular to the hadrons motion

Equation (9.2) shows that the binding energy, $-\Delta\mathcal{H}$, is negligible for small R values and becomes unbound with increasing R. The former is consistent with asymptotic freedom, the latter is consistent with confinement.

9.1.5 Scattering Cross Sections and Hadron Jets

It is possible that the rotating neutrino model may also be able to provide some input regarding the anomalous behavior exhibited by the elastic scattering cross sections of polarized proton beams, i.e. depending on whether they are parallel or oppositely polarized [10]. For example the cross section for parallel beams, i.e. polarized in the same direction are up to a factor of four larger than that observed with oppositely polarized beams in the 10 GeV scale [10]. This behavior has been attributed to spin–torsion interactions in the context of supergravity models [10].

The rotating neutrino model may also provide a qualitative scheme to account for the emission of jets of newly created hadrons when highly energetic hadrons are forced to collide with each other, such as in the LHC experiments [8]. These jets appear in a direction vertical to the direction of the colliding protons. This behavior could be rationalized as follows: If two rotating neutrinos, one in each colliding baryon, come close to each other, then the gravitational attraction between them can become very large due to their high rotational velocity and small distance, r, and in this way the two neutrinos may escape together in a direction vertical to the direction of the colliding baryons (Fig. 9.1) as experimentally observed.

9.2 Synopsis

It appears feasible to obtain a straightforward, one-to-one correspondence between the key elements of the rotating neutrino model and the key elements of the Standard Model regarding the structure and properties of baryons and those of the strong force acting inside the baryons. It appears that quarks may be associated with relativistic neutrinos, that gluons may be associated with the confining action of the relativistic gravitational force, that color charge may be associated with the relativistic gravitational mass, and that the strong force may be associated with the relativistic gravitational force.

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Chapter 10
The Bohr–de Broglie Approach in Physics: The Dual Nature of Matter

10.1 Merits

The successful description of the formation and of the major properties of baryons by the deterministic three-neutrino model discussed in this book provides support to the idea that the remarkable success of the Bohr [1–3] and Bohr–Sommerfeld model [4, 5] for the description of the H atom was not coincidental.

It also provides support to the idea that the deterministic coupling of relativity with the "old quantum mechanics" as expressed by the de Broglie wavelength equation can be quite fruitful. This combination is based after all on the very old conclusion that the corpuscular and ondular (wave) properties of matter are equally important.

In essence the three-neutrino model for the description of baryons is the same as the Bohr model for the H atom (Fig. 10.1), i.e. it consists of two parts:

1. A classical mechanical part which leads to an infinity of acceptable solutions.
2. The use of the de Broglie wavelength expression in order to ensure that the solution chosen is consistent with the dual particle-wave nature of matter (Table 10.1 and Figs. 10.2 and 10.3).

Using the de Broglie wavelength expression is mathematically equivalent to assuming quantization of the action or of the angular momentum. It underlines the dual, corpuscular, and ondular nature of matter.

Thus the only difference between the two models is in the treatment of the special relativistic corrections to the Newtonian mechanics. These corrections are negligible and of the order of $\alpha(\approx 1/137.035)$ in the case of the H atom, and are quite important in the case of the three rotating neutrino model (Fig. 10.1).

The potential importance and usefulness of deterministic models in the study of subatomic phenomena has been discussed repeatedly by Nobel Laureate G. t' Hooft in recent years [6–10]. Einstein himself had questioned several times whether the quantum mechanical description of the physical reality can be considered complete [11, 12] or whether some level of determinism may be necessary beneath

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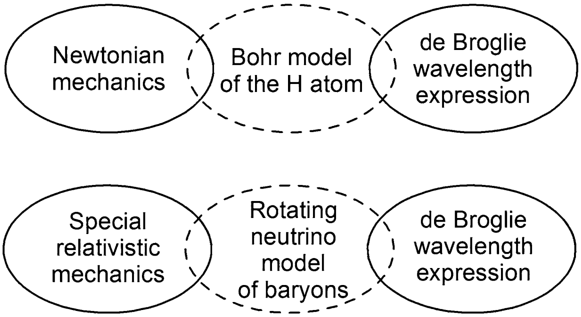

Fig. 10.1 Conceptual basis of the Bohr model for the H atom and of the three-neutrino model for the formation of baryons

Table 10.1 Deterministic models for the formation of the H atom and of the neutron

Bohr model for the H atom	**Present Bohr-Einstein model for the neutron**
Electron as *particle* $m_e v^2/R = e^2/\varepsilon R^2$ ↑ Newton's 2nd law ↑ Coulomb law	Neutrino as *particle* $\gamma m_o \frac{v^2}{R} = \frac{G m_o^2 \gamma^6}{\sqrt{3}\, R^2}$ ↑ Relativistic equation of motion for circular motion ↑ Newton's gravitational law accounting for special relativity ($m_i = \gamma^3 m_o$) and for equivalence principle ($m_g = m_i$)
Electron as *wave* $\frac{\hbar}{m_e v} = R$ De Broglie (for n=1)	Neutrino as *wave* $\frac{\hbar}{m_o v} \approx \frac{\hbar}{m_o c} = R$ De Broglie (for n=1)
<u>Results</u> $v = \alpha c \quad ; \gamma \approx 1$ $R = \frac{\hbar}{\alpha m_e c} = 0.5292 \cdot 10^{-10}$m $V_e = -27.2$ eV $T = 13.6$ eV $\mathcal{H} = T + V_e = -13.6$ eV $\Delta(RE) \equiv \Delta mc^2 = T = (1/2)\alpha^2 m_e c^2 = 13.6$ eV	<u>Results</u> (with m_o=0.043723 eV/c^2) $v \approx c$ $R = \frac{\hbar}{\gamma m_o c} = 0.630 \cdot 10^{-15}$m $V_G = -(5/3)m_n c^2 = -1565.9$ MeV $T = m_n c^2 = 939.565$ MeV $\mathcal{H} = T + V_G = -(2/3)m_n c^2 = -626.4$ MeV $\Delta(RE) = \Delta mc^2 = T = m_n c^2 = 939.565$ MeV Per particle: $T_p = 313.2$ MeV $\mathcal{H}_p = -208.8$ MeV

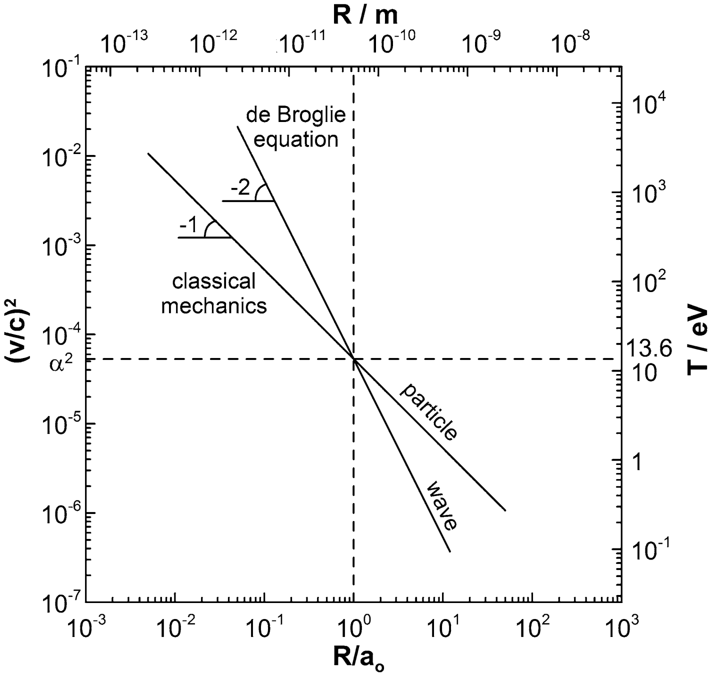

Fig. 10.2 Graphical solution of the two equations of the Bohr model, i.e. of the classical mechanical equation $v^2 = e^2/\varepsilon m_e R$ [Eq. (1.6)] and of the de Broglie wavelength equation, for $n = 1$, $v^2 = \hbar^2/m_e^2 R^2$ [Eq. (1.9)]. The kinetic energy, T, is computed from $T = (1/2)m_e v^2$, a_o is the Bohr radius $\hbar/m_e\alpha c$

the common probabilistic quantum mechanical description introduced by Born [13]. This idea has led to Bohmian mechanics [14] which interpret the quantum mechanical theory in terms of "hidden" deterministic variables [14]. Experiments have repeatedly supported the deterministic views of Einstein [15].

The exact analogy between the Bohr model for the H atom and the present Bohr–Einstein model for the neutron is shown in Table 10.1. In both cases the Hamiltonian, $\mathcal{H}(= T + V)$, is negative, thus the rotational state is stable. The mass increase, $\Delta(RE) = \Delta mc^2$, equals in both cases the translational energy, T, in the rotating state. Thus T is negligible ($\sim 10^{-4}$ of the electron rest energy) in the case of the H atom and very large, equal to the rest energy of the baryon formed (939.565 MeV), in the case of the rotating neutrino model.

The great similarity between the two models can also be seen by comparing Fig. 10.2 (Bohr model, as in Fig. 1.4) and Fig. 10.3 (three-neutrino model). In both cases the model solution lies at the intersection of the de Broglie equation line with the classical mechanics curve which is a straight line in the case of the Bohr model (Fig. 10.2) and a more complex curve consisting essentially of two straight lines in the case of the three-neutrino model (Fig. 10.3). This curve was first discussed

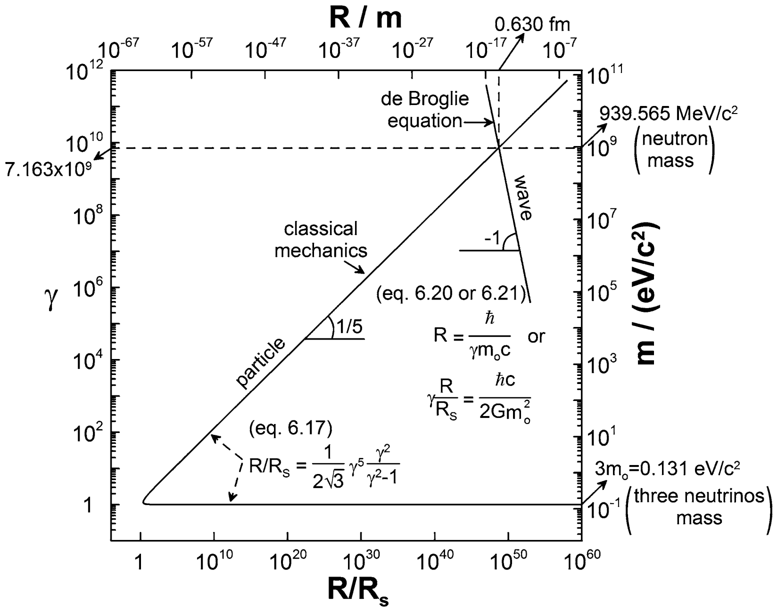

Fig. 10.3 Graphical solution of the three-neutrino model equations, i.e. of the classical mechanical equation (6.17) with its Keplerian ($\gamma < 1.1832$) and non-Keplerian ($\gamma > 1.1832$) branch, and of the de Broglie wavelength equation (6.20). The energy or mass axis is obtained from the γ axis and the Einstein equation $E = 3\gamma m_o c^2 = mc^2$

in Chap. 6 and is a plot of Eq. (6.17), also shown inside the figure. In both figures the *y-axis* provides the kinetic energy of the particle(s) and thus the increase in rest energy of the system. Thus in the case of the rotating neutrino model (Fig. 10.3) one observes how the energy $E = 3\gamma m_o c^2$ and thus the mass $3\gamma m_o$ of the rotating system varies with varying R in view of Eq. (6.17), shown in the figure, which, as already discussed in Chap. 6, describes both Keplerian ($\gamma \approx 1$) and relativistic ($\gamma \gg 1$) orbits. Other than this relativistic complication, there is complete analogy between Figs. 10.2 and 10.3.

10.2 Limitations

Perhaps it is reasonable to anticipate that the same practical limitations faced by the Bohr or Bohr–Sommerfeld deterministic model when attempting to describe many-electron atoms or molecules [3] will be also encountered by the deterministic rotating neutrino model when seeking to describe the formation of nuclei, i.e. when seeking to describe the residual strong force rather than the strong force itself.

Although magnetic moment data can provide some useful guidelines, it is difficult to predict qualitatively the actual neutrino orbits even for simple nuclei, such as the deuteron or the ^{4}He nucleus. Thus the classical mechanical problem by itself becomes quite difficult while the implementation of the de Broglie wavelength condition appears in general rather cumbersome and problematic as the nucleus size increases. It is difficult to assess how difficult these problems will turn out to be and to what extent they can be successfully overcome in the future.

10.3 Charged Baryons

Before concluding this chapter it is worth examining the possibility that the light rotating particles may also acquire a charge, e.g. via the β-decay electron abstraction, to form charged baryons such as the proton:

$$n \to p^+ + \mathrm{e}^- + \bar{\nu}_\mathrm{e}. \tag{10.1}$$

In this case the equation of motion for such a rotating light particle carrying an electric charge q_1 takes the form:

$$\frac{(Gm_o^2\gamma^6 - Qe^2/\varepsilon)}{\sqrt{3}R^2} = \gamma m_\mathrm{o}\mathrm{v}^2 R, \tag{10.2}$$

which is a generalization of Eq. (6.15) with:

$$Q = (q_1q_2 + q_1q_3)/e \tag{10.3}$$

and q_1, q_2, and q_3 are the charges of the three hadron constituents.

In writing Eq. (10.2), with a single R value, one assumes that either $q_1 = q_2 = q_3$ or that the Coulombic force is much smaller than the relativistic gravitational force. The latter is always the case near the intersection with the de Broglie wavelength line, as already discussed in Chap. 7 and as further examined here and in Chap. 12.

After some algebra one obtains from Eq. (10.2) that:

$$R = (R_\mathrm{S}/2\sqrt{3})\left(\frac{\gamma^2}{\gamma^2-1}\right)\left[\gamma^5 - \frac{QR_\mathrm{c}}{\gamma R_\mathrm{s}}\right], \tag{10.4}$$

where R_s is the Schwarzschild radius, $2Gm_\mathrm{o}/c^2$, and $R_\mathrm{c}(= 2e^2/\varepsilon m_\mathrm{o}c^2)$ provides a measure of the effective charge radius of the rotating light particle. For $Q = 0$, Eq. (10.4) reduces to Eq. (6.17). For $R_\mathrm{c} \gg R_\mathrm{s}$ and $\gamma \approx 1$ it reduces to the classical nonrelativistic Coulombic rotational state limit:

$$R = \frac{Qe^2}{\varepsilon m_\mathrm{o}\mathrm{v}^2}. \tag{10.5}$$

Figure 10.4 provides plots of the $\gamma(R)$ relation defined by Eq. (10.4) for various values of Q. The same figure provides the graphical solution to the three-charged

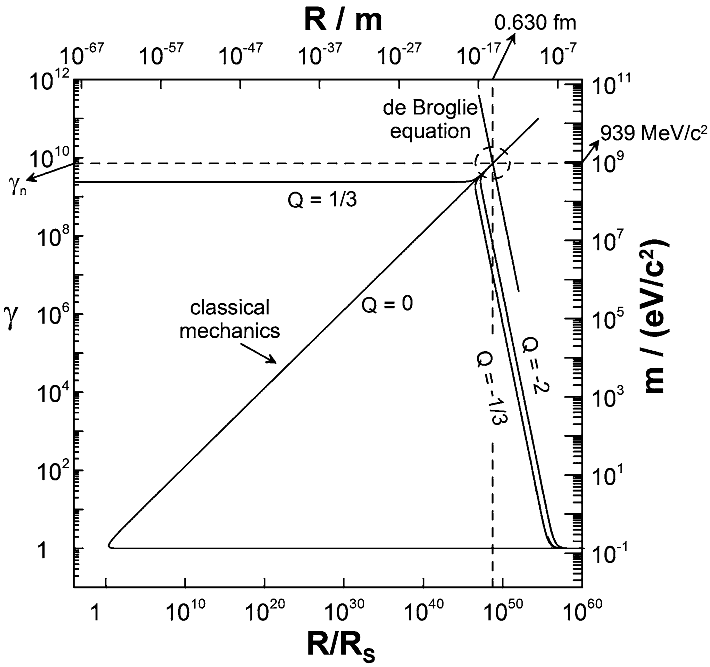

Fig. 10.4 Graphical solution of the three rotating charged particle model [Eq. (10.4)] coupled with the de Broglie wavelength equation $R = \hbar/\gamma m_o c$ [Eq. (10.6)] for $Q = 1/3, Q = 0, Q = -1/3$, and $Q = -2$. The *circled* area is magnified in Fig. 10.5

rotating particle model via the intersection of the classical mechanics $\gamma(R)$ curve with the de Broglie wavelength equation (6.20) for $n = 1$, i.e. with:

$$R = \hbar/\gamma m_o c. \tag{10.6}$$

One observes that the effect of Q is rather small in the vicinity of the intersection with the de Broglie equation line, but the γ value at the intersection increases with more positive Q, i.e. with electrostatic repulsion, a point shown better in Fig. 10.5.

Thus Fig. 10.5 focuses on the circled area of Fig. 10.4. This figure shows that for $Q = -2$ the rest energy, $3\gamma m_o c^2$, of the confined state matches quite well that of the proton, $m_p = 938.27\,\text{MeV}$. In view of Eq. (10.3) this suggests $q_1 = 2e$, $q_2 = -e$ and $q_3 = 0$, or $q_1 = -e$, $q_2 = e$ and $q_3 = e$. These are charge combinations which in Chap. 7 were found to provide an excellent fit to the proton magnetic moment, i.e.

$$\mu = (1/2)qR\text{c} = (1/2)eR\text{c} = 3\mu_\text{N} \tag{10.7}$$

vs 2.79 $\mu_\text{N}(14.10 \cdot 10^{-27}\,\text{J/T})$ which is the experimental value.

Consequently Eq. (10.4) with $Q = -2$ provides a very good fit not only to the proton mass but also to its total charge and magnetic moment of the proton.

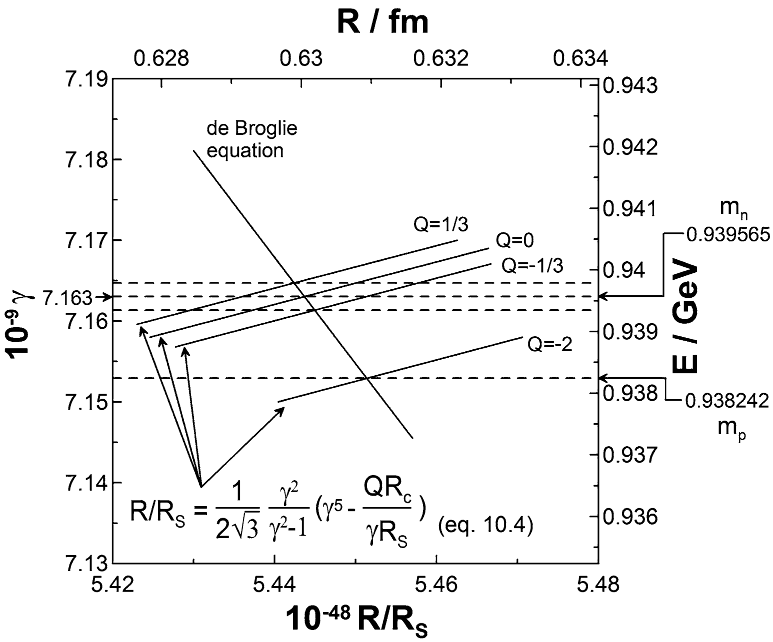

Fig. 10.5 Graphical solution of the three rotating charged particle model [Eq. (10.4)] coupled with the de Broglie equation $R = \hbar/\gamma m_o c$ [Eq. (10.6)] for $Q = 1/3, Q = 0, Q = -1/3$, and $Q = -2$ focusing on the circled area of Fig. 10.4; $m_o = 0.043723$ eV/c^2. The intersection corresponding to $Q = 0$ gives the neutron mass ($m = 939.565$ MeV/c^2) and the intersection corresponding to $Q = -2$ gives the proton mass ($m = 938.24$ MeV/c^2), the former within less than 10^{-4}%, the latter within 0.023%

As also shown in Fig. 10.5, Eqs. (10.2) or (10.4) with $Q = 0$ provides a very good fit to the neutron mass (939.565 MeV/c^2). This is consistent with $q_1 = 0$, $q_2 = e/3$, $q_3 = -e/3$ which also provides a good fit $(-2\ \mu_N)$ to the neutron magnetic moment data $(-1.913\ \mu_N)$ as discussed already in Sect. 7.2.5 of Chap. 7.

Why the individual charges of the constituent particles appear, from this fit, to be different in the case of the proton (integer multiples of e) and of the neutron (integer multiples of $e/3$ as in the Standard Model) is not obvious but could perhaps be related to some charge redistribution during the β-decay reaction [Eq. (10.1)] and to the metastable nature of the free neutron which has a lifetime of 885.7 s.

10.4 Synopsis

It appears that the deterministic Bohr–de Broglie approach which gives equal weight to the corpuscular and ondular nature of matter can be quite useful for exploring at least a few important problems as shown in Table 10.1. By including the Coulombic term in the rotating light particle model one can compute not only the neutron mass, but also the proton mass with an accuracy better than 0.0023%.

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Chapter 11
Gravity at Relativistic Velocities and Dark Matter

11.1 Dark Matter in Galaxies

Although there is strong evidence that the mass of the Universe is dominated by dark matter which exerts gravitational attraction, the exact nature of dark matter is still unknown.

Common spiral galaxies are known to reside in extended dark matter (DM) halos. The rotational speeds of their gas discs do not decline outside the visible body, in contrast with the expectation from Keplerian circular velocities at a radius r about a mass M, i.e. $\mathrm{v} = (GM/r)^{1/2}$. Thus the DM mass within r grows roughly as $M(r) \propto r$ and it dominates the gravitational potential beyond a certain radius.

The existence of dark matter and its special distribution are both determined on the basis of the assumption that the gravitational attraction computed from Newton's gravitational law can always be computed from:

$$F_{\mathrm{G}} = -G\frac{m_1 m_2}{r^2}, \tag{11.1}$$

where m_1 and m_2 are the rest masses of two stars or galaxies and r is their distance.

11.2 Newton's Gravitational Law and Special Relativity

However, as already discussed in this book (e.g. in Chap. 6), it follows from the equivalence principle [1] that the gravitational mass equals the inertial mass and the latter can in general be computed using special relativity [2, 3] from [4]:

$$F = \frac{\mathrm{d}(\gamma m v)}{\mathrm{d}t} = \gamma^3 m \frac{\mathrm{d}\mathrm{v}}{\mathrm{d}t}, \tag{11.2}$$

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where $\gamma(= (1 - \mathrm{v}^2/c^2)^{-1/2})$ is the Lorentz factor. Thus for an observer on earth (laboratory observer) the gravitational attraction force exerted on a star with rest mass m rotating at a distance r around the center of a distant galaxy of rest mass M is given by:

$$F_{\mathrm{G}} = -\frac{GMm\gamma_M^3\gamma_m^3}{r^2}. \tag{11.3}$$

Therefore according to the equivalence principle of Einstein and Eötvös [1] and the theory of special relativity, inertial mass equals the gravitational mass and thus $\gamma_m^3 m$ and $\gamma_M^3 M$ rather than m and M have to be used in Newton's gravitational law as in Eq. (11.3).

11.3 Virial Theorem and Dark Matter

Historically the existence of dark matter was first proposed by Fritz Zwicky in 1933 on the basis of his observations with distant galaxies, such as the Coma Cluster of galaxies [5, 6]. He estimated the cluster's total mass in two different ways. First on the basis of the motions of galaxies near its edge and second on the basis of the number of galaxies and total brightness of the cluster. By applying the *virial theorem* which states that the kinetic energy of the rotating galaxies is one half of their potential energy of the rotating galaxies, Zwicky found experimental evidence that the kinetic energy is much larger [5, 6]. If one assumes that the gravitational mass is only due to the visible matter of the galaxy, then the stars far away from the center of the galaxy have much higher velocities than those predicted by the virial theorem. This is known as the *missing mass* problem.

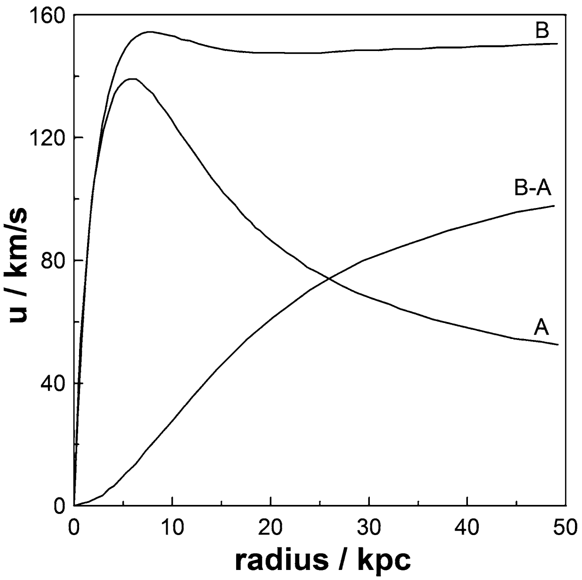
The simplest way to rationalize the missing mass observations was to assume that there exists some type of matter (dark matter) which is not visible. This was the postulate made by Zwicky. Galaxies show signs of consisting mostly of a spherically symmetric halo of dark matter with the visible matter concentrated near the center (Fig. 11.1).

Gravitational lensing, i.e. the bending of light emitted from a very distant bright source, such as a quasar, around a massive object, such as a cluster of galaxies, has also provided strong evidence that in many galaxy clusters there exists significantly more mass than indicated by the cluster's light alone [5, 6].

It is currently estimated that ordinary matter constitutes only 4.6% of the mass of the universe. Dark matter is estimated to account for 23% of the mass of the universe with the remaining 72% theorized to consist of dark energy whose gravitational effect approximates that of Einstein's cosmological constant Λ [7–11].

It is commonly believed that dark matter is primarily nonbaryonic and three types of nonbaryonic dark matter have been proposed, i.e. hot dark matter consisting of ultrarelativistic nonbaryonic particles, warm dark matter consisting of relativistic nonbaryonic particles, and cold dark matter consisting of nonrelativistic nonbaryonic particles [9].

Fig. 11.1 Radial velocity distribution of stars in a galaxy anticipated from Keplerian orbits (**a**) and experimentally observed (**b**). The difference (**b**-**a**) is commonly attributed to dark mass gravitational attraction



11.4 Alternate Explanations

The existence of dark mass and dark energy represents the currently most popular theories among cosmologists to explain the various inconsistencies from the virial theorem that Zwicky and more recent studies have revealed. However there is no direct experimental evidence for the existence of dark matter or dark energy. Consequently various alternative theories have been formulated. These include some quantum mechanical explanations as well as the possibility that dark matter consists of neutrinos of about 1.5 eV [12, 13]. Computations have shown that in this scenario active (left-handed) neutrinos account for 9.5% of the dark matter and sterile (right handed) neutrinos account for 19%. This approach falls within the hot dark matter scenario.

11.5 Gravity Modification

The most common alternative theory to dark matter is that Newton's gravitational law is not obeyed at great distances or in weak fields. One of the proposed models is Modified Newtonian Dynamics (MOND) which adjusts Newton's laws at small accelerations [14]. However developing a successful relativistic MOND theory is still problematic and it is not clear how the MOND theory can account for gravitational lensing measurements of the deflection of light around galaxies. Other gravitational theories are also being tested in connection to the missing mass problem [15–17].

11.6 Gravitational Mass

In Zwicky's analysis and much of the subsequent dark matter work, the mass (M)-to-light (luminosity, $\mathcal{L}$) ratios of galaxies were determined via the following equation:

$$\frac{M}{\mathcal{L}} \approx 1h \left(\frac{M_\odot}{\mathcal{L}_\odot}\right)_{\mathrm{V}}, \tag{11.4}$$

where h is a constant, estimated to be 5.58 in the thirties and currently believed to lie between 0.5 and 1 [7–10] and the subscript $\odot$ denotes the properties of our Sun. From this equation one extracts the value of M which is subsequently substituted in the equation of the gravitational potential energy, i.e.

$$V_{\mathrm{G}} = -a\frac{GM^2}{r}, \tag{11.5}$$

where a is a constant dependent on the density distribution. It is $a = 3/5$ for a uniform sphere and $a = 3/(5-n)$ for a polytropic sphere with polytropic index n.

It is clear from the above analysis that the value of M determined from Eq. (11.4) is not the *gravitational* mass, which must be substituted in the potential energy expression of Newton's gravitational law [Eq. (11.5)] but rather the *rest* mass of the galaxy.

In view of Table 11.1 it follows that the gravitational energy is thus *underestimated* by a factor of γ^6 while the gravitational mass is underestimated by a factor of γ^3.

The currently estimated ratio of dark and light matter mass is $23/4.6 = 5$, i.e.:

$$\frac{M_{\mathrm{D}}}{M_{\mathrm{L}}} = 5. \tag{11.6}$$

On the other hand it follows from Table 11.1 that:

$$\frac{M_{\mathrm{D}}}{M_{\mathrm{L,real}}} = \gamma^3. \tag{11.7}$$

Consequently from (11.6) and (11.7) it follows $\gamma^3 = 5$ which implies $\gamma = 1.71$ and $\mathrm{v} \approx 0.81c$.

Consequently the experimental observations regarding the "missing mass" could in principle be accounted without invoking the existence of dark matter if visible

Table 11.1 Gravitational and rest mass ratios of spiral galaxies

Rest mass	M
Relativistic mass	γM
Gravitational mass	$\gamma^3 M$
Dark-to-light mass ratio	γ^3

matter had on the average a velocity near $0.81c$ relative to the laboratory observer on earth. Although some distant galaxies are receding from our galaxy with velocities very near c, still many other less distant ones have significantly lower velocities as one can also infer from Hubble's law. Although the velocities of stars near the center of such galaxies are relativistic, still this explanation does not appear sufficient to account for the entire missing mass problem [18].

11.7 Neutrinos in Space

As already analyzed in detail in this book, the gravitational mass of fast neutrinos is not to be underestimated. A neutrino with a rest mass of $0.04\,\mathrm{eV/c^2}$ and total energy of $400\,\mathrm{MeV/c^2}$ has a Lorentz factor $\gamma = 10^{10}$ and thus a gravitational mass of $m_\mathrm{g} = \gamma^3 m_\mathrm{o} = 4 \cdot 10^{19}\,\mathrm{GeV/c^2}$ which is a factor of 10^{20} larger than the rest mass of a proton and neutron. Consequently the gravitational attraction of such relativistic or super relativistic neutrinos among themselves and, most importantly, between themselves and the baryonic light-emitting constituents of galaxies cannot be neglected. It is likely that this strong attraction may be sufficient to rationalize not only the phenomena related to dark mass near galaxies but also those related to dark energy far from galaxies [19–21].

11.8 Synopsis

Gravitational attraction between relativistic neutrinos may be important not only in the interior of hadrons but also possibly in space in connection to the experimental observations and to the postulates of dark matter and dark energy.

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Chapter 12
Force Unification: Is the Strong Force Simply Gravity?

12.1 Coupling Constants: Facts and Expectations

A coupling constant is a dimensionless number which describes the strength of an interaction [1–3]. Typical coupling constant values for the four fundamental forces are shown in Table 12.1.

The coupling constant for the strong force is commonly taken to equal unity for energies of the order of 1 GeV, i.e.:

$$\alpha_s \approx 1 \tag{12.1}$$

but decreases with increasing energy [4] and also depends on quark separation [2]. The value of 0.119 is used frequently in the energy range of 10–100 GeV [4]. Thus in reality the coupling constant for the strong force is not a true constant [2, 4].

According to the Standard Model, the strong force is the force acting between the constituent quarks of hadrons. As already discussed, this force is known to decrease inside the hadrons so that the quarks are able to move freely, a behavior known as asymptotic freedom. Quantum chromodynamics provides the following expression for this behavior:

$$\alpha_s(E) = \frac{12\pi}{(33 - 2n_f)ln\left(\frac{E^2}{\Lambda^2}\right)}, \tag{12.2}$$

where n_f is the number of quarks active in attracting pair production and Λ, termed the *QCD scale* is an experimentally determined parameter of the order of 200 MeV. The most commonly used value for the *QCD* scale is 217 ± 20 MeV [4].

The electromagnetic coupling constant, α_e, frequently denoted simply α, equals the fine structure constant α (1/137.0359). This is obtained by considering the electrostatic force energy between two electrons or protons at a distance r, i.e.

$$E_e = \frac{e^2}{\varepsilon r} \tag{12.3}$$

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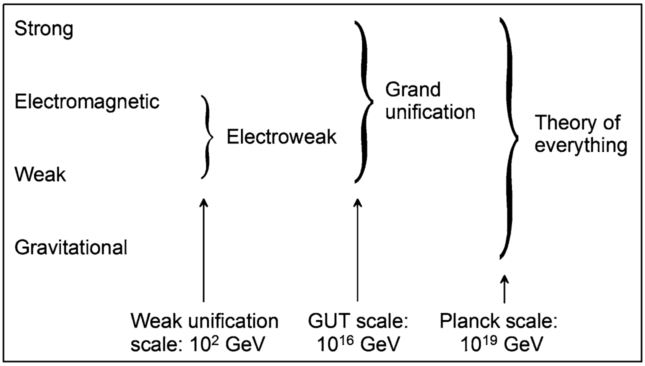
Table 12.1 Coupling constants

Coupling constants		
Strong	α_S	1
Electromagnetic	α	1/137
Weak	α_W	10^{-6}
Gravity	α_g	10^{-39}

Table 12.2 Estimated energy scales for force unification according to the Grand Unification Theories (GUT) and according to the "Theories of everything" [1]



and comparing it with the energy of a photon, E_{ph}, with wavelength λ equal to $2\pi r$, i.e.

$$E_{ph} = hc/2\pi r = \hbar c/r. \tag{12.4}$$

From (12.3) and (12.4) one obtains:

$$\alpha_e = E_e/E_{ph} = \frac{e^2}{\varepsilon c\hbar} = \alpha = 1/137.0359. \tag{12.5}$$

The weak coupling constant α_w is estimated by using the ratio of the lifetimes of particles decaying via the strong interaction ($\sim 10^{-24}$ s) and via the weak interaction ($\sim 10^{-12}$ s).

To a first approximation this ratio is inversely proportional to the square of the coupling constants of the forces which cause the decay. Consequently it is estimated that:

$$\alpha_w \approx (10^{-12})^{1/2} \approx 10^{-6}, \tag{12.6}$$

i.e. the weak coupling constant is some six orders of magnitude smaller than the strong force coupling constant.

As shown in Table 12.2 and Fig. 12.1, strong, electromagnetic and weak interaction coupling constants are expected to merge at the GUT (grand unification theories) scale, i.e. energies per particle of $\sim 10^{16}$ GeV.

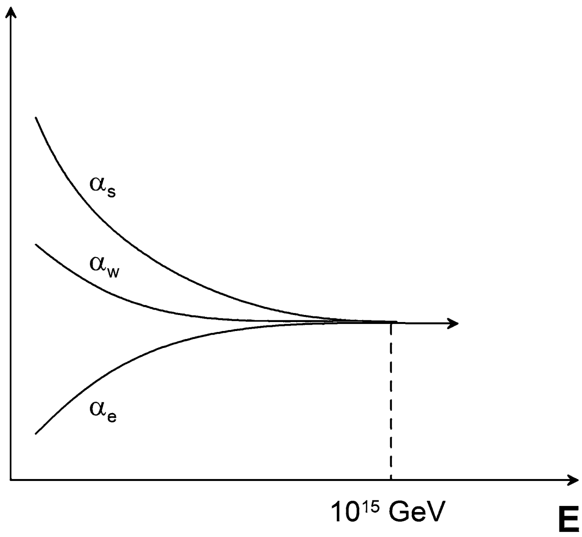


Fig. 12.1 Estimated effect of energy [1] on the coupling constants of the strong, weak, and electrostatic forces. Unification with the, much weaker, gravitational force is anticipated to occur around the Planck energy ($\sim 10^{19}$ GeV/c^2)

12.2 Gravitational Coupling Constants

The gravitational coupling constant, α_g, is computed by considering the ratio of the gravitational force between two charged particles divided by the corresponding Coulombic force. Thus considering two protons of mass m_p each, it is:

$$\frac{\alpha_g}{\alpha_e} = \frac{Gm_p^2}{e^2/\varepsilon} = 8.1 \cdot 10^{-37} \tag{12.7}$$

thus:

$$\alpha_g = \frac{\alpha\varepsilon Gm_p^2}{e^2} = 5.9 \cdot 10^{-39}. \tag{12.8}$$

If the force between two electrons is used, then one computes:

$$\alpha_g = \frac{\alpha\varepsilon Gm_e^2}{e^2} = 1.75 \cdot 10^{-45}. \tag{12.9}$$

Similarly one may consider the gravitational force between two neutrinos at rest with a rest mass m_o each at a distance r and divide it by the Coulombic force between two particles of charge e at the same distance. One thus obtains

$$\alpha_{g,o} = \frac{\alpha\varepsilon Gm_o^2}{e^2} = 1.281 \cdot 10^{-59}, \tag{12.10}$$

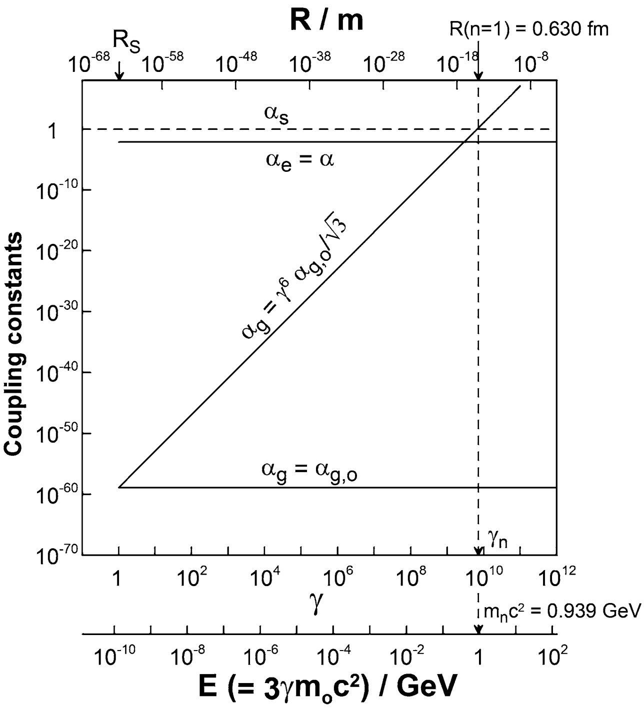


Fig. 12.2 Coupling constants dependence on rotational radius (*top*) and energy (*bottom*) according to the rotational neutrino model

where the superscript "o" is used in "$\alpha_{g,o}$" to remind that this is the value of the gravitational coupling constant when the neutrinos are at rest relative to the laboratory observer.

When the three neutrinos have a velocity v and thus a Lorentz factor γ relative to the laboratory observer, then it follows from Eq. (6.12) in p. 72 that:

$$\frac{\alpha_g}{\alpha_{g,o}} = \frac{F_g}{F_{g,o}} = \gamma^6/\sqrt{3}. \tag{12.11}$$

Figure 12.2 provides a plot of Eqs. (12.10) and (12.11) as a function of γ and also, via Eq. (6.18), i.e., $R = (R_S/(2\sqrt{3}))\gamma^5$, as a function of R.

Upon substituting in Eq. (12.11) $\gamma = \gamma_n = 7.163 \cdot 10^9$ which is the value of γ corresponding to the ground rotational radius, $R(n = 1)$, of the bound state, one finds:

$$\frac{\alpha_g}{\alpha_{g,o}} = 7.7985 \cdot 10^{58} \tag{12.12}$$

and thus using Eq. (12.10):

$$\alpha_g = 1, \tag{12.13}$$

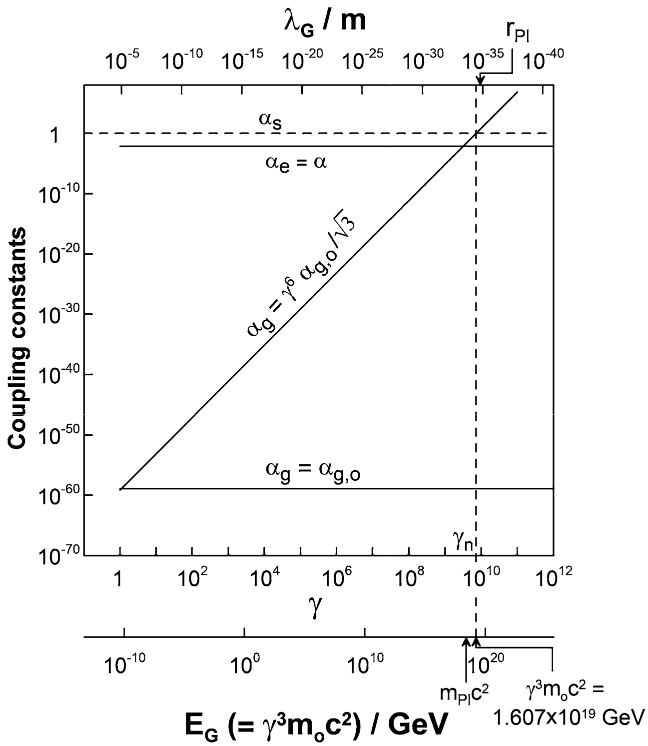


Fig. 12.3 Coupling constants dependence on the energy, E_{G}, associated with the gravitational mass, $\gamma^3 m_{\mathrm{o}}$, and on the corresponding Compton wavelength $\lambda_{\mathrm{G}}(=\hbar/\gamma^3 m_{\mathrm{o}} c)$

i.e. at $R = R(n = 1)$ and thus $\gamma = \gamma(n = 1) = \gamma_n$, it is:

$$\alpha_{\mathrm{g}} = \alpha_{\mathrm{s}} = 1. \tag{12.14}$$

This is an important result which confirms that the relativistic gravitational force is exactly equal to the strong force at $R = R(n = 1)$, a conclusion already reached in Sect. 7.2.7.

This is also shown in Fig. 12.2 which depicts the dependence of α_{g} on the rotational radius R and also on γ and on energy, E. One notes that α_{g} has two branches. One corresponds to common nonrelativistic gravity ($\gamma = 1$, $\alpha_{\mathrm{g}} = \alpha_{\mathrm{g,o}}$) and the other to relativistic gravity with $\alpha_{\mathrm{g}} = \gamma^6 \alpha_{\mathrm{g,o}}/\sqrt{3}$. This branch intersects the $\alpha_{\mathrm{s}} = 1$ line at $R = R(n = 1) = 0.630\,\mathrm{fm}$, $\gamma = \gamma_n$, and $E = m_{\mathrm{n}} c^2 = 0.939\,\mathrm{GeV}$, which is the neutron rest energy.

Figure 12.3 shows the coupling constants dependence on the energy associated with the gravitational mass $\gamma^3 m_{\mathrm{o}}$ (labeled E_{G}) and on the corresponding equivalent Compton wavelength, λ_{G}, computed from $\lambda_{\mathrm{G}} = \hbar/(\gamma^3 m_{\mathrm{o}})c$, i.e.

$$E_{\mathrm{G}} = \gamma^3 m_{\mathrm{o}} c^2 \tag{12.15}$$

$$\lambda_G = \frac{\hbar}{\gamma^3 m_o c}. \tag{12.16}$$

As shown in Fig. 12.3 the three coupling constants nearly converge at $E_G = \sqrt{3} m_{Pl} c^2 = 1.607 \cdot 10^{19}\,\text{GeV/c}^2$ and $R \approx r_{Pl} \approx 10^{-35}\,\text{m}$, i.e. at the Planck energy and Planck distance, as anticipated by the "Theories of everything" [1].

Figure 12.4 is a combination of Figs. 12.2 and 12.3. The key point is that the relativistic gravitational coupling constant reaches unity at $R(n=1) = 0.630\,\text{fm}$ and $E = 0.939\,\text{GeV}$ which is the radius and energy of the neutron rotational state. This is the case when using the proper distance (R) and energy (E) scales. However when using the gravitational mass energy scale, $E_G = \gamma^3 m_o c^2$, and the corresponding equivalent Compton wavelength scale ($\lambda_G = \hbar/\gamma^3 m_o c$) then α_G reaches α_S and unity at the Planck energy ($\sim 10^{19}\,\text{GeV}$) and Planck distance scale ($\sim 10^{-35}\,\text{m}$) as anticipated by most force unification theories [1, 2, 5–7], e.g. Table 12.2.

The fact that relativistic gravity appears to coincide with the strong force comes initially as a surprise, but is strongly reminiscent of Wheeler's geon analysis [8, 9] and with the work of Hehl and coworkers [10] who sought for the development of a common gauge theory for gravity and strong interactions. A unified approach to strong and gravitational interactions using some geometrical methods of general relativity has been also proposed for years by Recami and coworkers [11]. The interrelation of gravity, strong gravity, black holes, and hadrons has been also discussed by Salam coworkers [12].

One important aspect of force unification is Einstein's expectation that elementary particles would be described as solutions of a classical field theory [13]. General relativity itself has a family of particle-like solutions, i.e. the Kerr solution which reduces to the Schwarzschild solution for zero angular momentum as already discussed in Chap. 7. The rotating neutrino model seems to offer a glimpse of such an expectation as presented in Chaps. 6 and 7, where it was seen that different values of the integer n correspond to the formation of different particles.

One may draw the analogy with chemistry where the solutions of the Bohr model of the H atom and its electrostatic field are known as the ground state and the excited states of the H atom. A simple graphical way to obtain them is given in Fig. 12.5 where the two equations of the Bohr model (Chap. 1) have been recast in the form:

$$E = -\frac{e^4}{2\varepsilon^2 m_e (R\text{v})^2} \quad \text{(particle)} \tag{12.17}$$

$$R\text{v} = \frac{n\hbar}{m_e} \quad \text{(wave)}. \tag{12.18}$$

In the case of the rotating neutrino model this can be done in a similar manner if Eqs. (6.18), (7.4), and (7.5) are combined in the form:

$$V_G(\gamma R) = -5 m_o c^2 (2\sqrt{3}/R_S)^{1/6} (\gamma R)^{1/6} \quad \text{(particle)}. \tag{12.19}$$

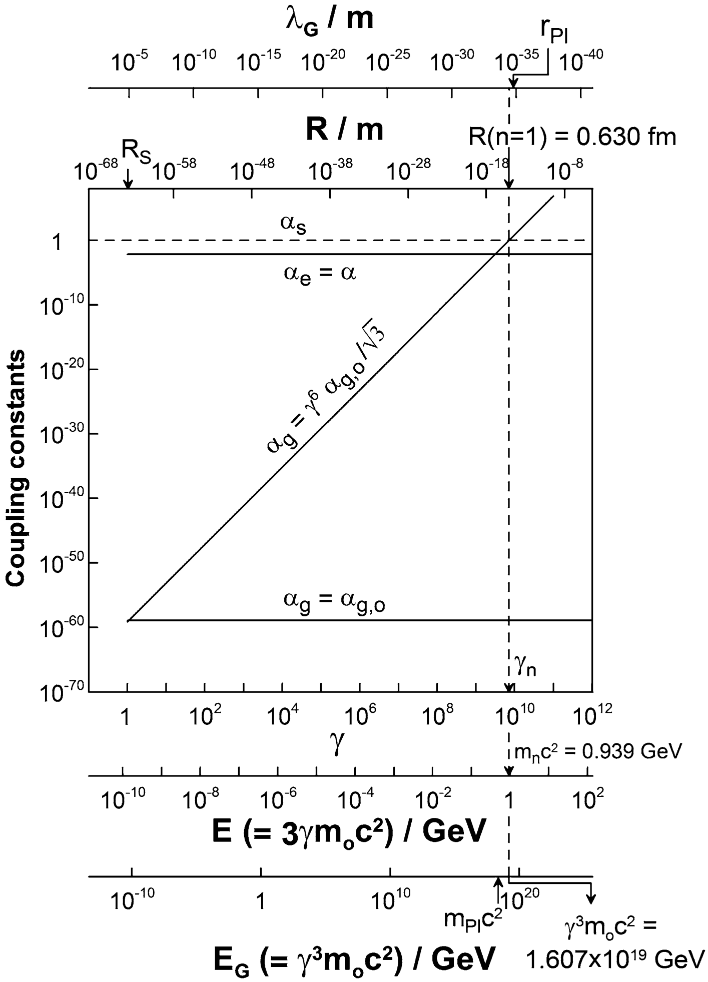


Fig. 12.4 Coupling constants dependence on rotational radius (*top*) or energy (*bottom*) according to the rotating neutrino model, α_e is the Coulombic coupling constant which equals the fine structure constant α(=1/137.035); α_g is the Newtonian gravitational coupling constant; α_s is the strong force coupling constant which is defined as unity; α_g is the relativistic-gravitational coupling constant which equals γ^6, starts from $\alpha_{g,o}$ at $\gamma \approx 1$ and reaches unity, thus α_s, at $\gamma = \gamma_n = 7.163 \cdot 10^9$. At this point it is $R = R(n=1) = 0.630\,\text{fm}$ and $E(= \gamma m_o) = 0.939\,\text{GeV}$, which is the neutron rest energy. The corresponding E_G value, defined from $E_G = \gamma^3 m_o c^2$, is in the Planck energy range $(1.607 \cdot 10^{19}\,\text{GeV})$ and the corresponding Compton wavelength $\lambda_G (= \hbar/\gamma^3 m_o c)$ is in the Planck length range $(10^{-35}\,\text{m})$

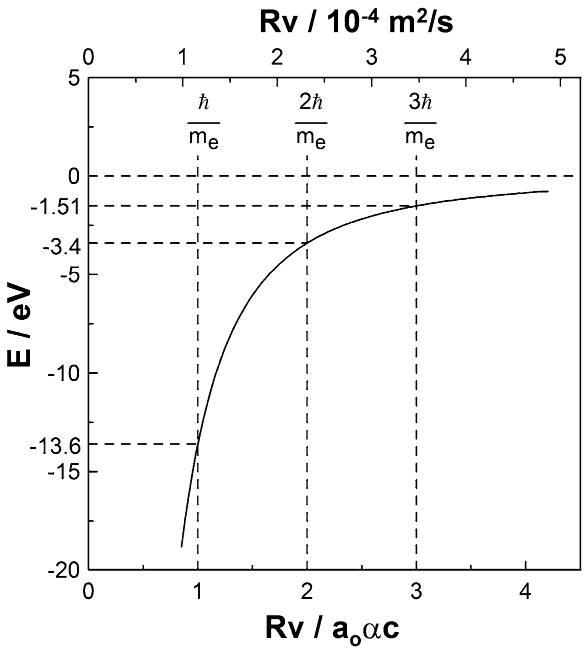


Fig. 12.5 Graphical solution of the Bohr model equations for the H atom written in the form $E = -e^4/2\varepsilon^2 m_e (R\mathrm{v})^2$ (particle) and $R\mathrm{v} = n\hbar/m_e$ (wave); $\alpha (= e^2/\varepsilon c\hbar)$ is the fine structure constant and $a_o (= \hbar/m_e \alpha c)$ is the Bohr radius

On the other hand the de Broglie wavelength quantization condition (6.36) gives:

$$\gamma R = \frac{(2n-1)\hbar}{m_o c} \quad \text{(wave)}. \tag{12.20}$$

As shown in Fig. 12.6 the intersections of the plots of Eqs. (12.19) and (12.20) define in a simple pictorial manner the rotational neutrino states discussed in Chaps. 6 and 7, each corresponding to the mass and energy of different baryons.

12.3 Synopsis

The analysis of the coupling constant behavior of the relativistic gravitational force, α_g, as extracted from the rotating neutrino model, shows that it is negligible at low energies and long distances and that it converges with the coupling constant of the strong force and of the electrostatic force at short, fm, distances and at high, 10^{19} GeV, energies, as anticipated by most force unification theories. It thus appears that the rotational neutrino model provides a useful tool for investigating some aspects of force unification.

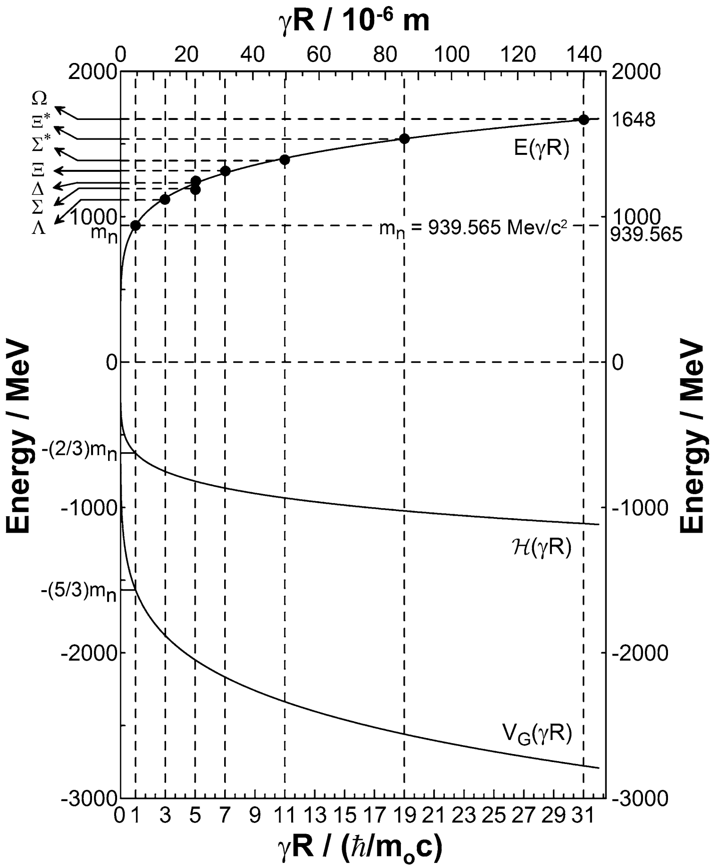


Fig. 12.6 Graphical solution of the rotating neutrino model for the generation of hadrons written in the form $V_G(\gamma R) = -5m_o c^2(2\sqrt{3}/R_S)^{1/6}(\gamma R)^{1/6}$ (particle) and $\gamma R = (2n-1)\hbar/m_o c$ (wave) showing the formation of the neutron ($m_n = 939.565$ MeV) and the other baryons consisting of *u*, *d*, and *s* quarks

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Epilogue

The very good agreement between experiment and the rotating neutrino model presented in this book leads to two interesting and at first surprising suggestions: First that hadrons consist, at least primarily, of rotating neutrinos and their mass is due to the kinetic energy of the rotating neutrinos. And second that the strong force is the relativistic gravitational force.

In retrospect none of these rotating neutrino model suggestions is too surprising: Why would nature have created a separate force, aside from the gravitational and the electromagnetic force, just in order to keep hadrons and nuclei together? And how could neutrinos, which are emitted in all nuclear reactions, not constitute an essential part of nucleons and nuclei?

Another interesting result which emerges from the rotating neutrino model has to do with the concept of rest mass. Apparently what we perceive as rest mass of baryons is to 99.99999999% the kinetic energy of the rotating neutrinos. This is to a significant extent already known, e.g. [1, 2]. If we could somehow directly observe this rotational motion we would never assign the value of 939 MeV to the proton or neutron *rest* mass. We would say that the rest mass inside these baryons is a billion times smaller, i.e. is the rest mass of the rotating neutrinos. Thus our definition of rest mass appears to be relative, i.e. is related to our ability or inability to observe the fm aspects of some phenomena.

We close by reminding to the reader that the model presented in this book *does not constitute any new theory*. The model is based entirely on well-established concepts and experimental facts (special relativity, equivalence principle, Newton's gravitational law, de Broglie wavelength) and, following the steps of the Bohr model, provides an "engineering" solution to an apparently useful problem.

That gravity and the strong force appear to be the same force seems to constitute a useful step towards the long sought force unification.

In his famous book "Dreams of a final theory" published in 1993 [3] Nobel Laureate Steven Weinberg notes: "The century coming to a close has been in physics a dazzling expansion of scientific knowledge. . . ." and a few lines below he adds "But now we are stuck. The years since the mid-1970s have been the most frustrating

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in the history of elementary particle physics. We are paying the price of our own success: theory has advanced so far that further progress will require the study of processes at energies beyond the reach of existing experimental facilities" [3]. And in Chap. 8 of the same book, named "Twentieth century blues," he addresses some of the emerging weaknesses of the standard model and the fact that the model "leaves out a fourth force, actually the first known of all forces, the force of gravitation."

Twenty years later with the successful construction and operation of the LHC and some other advanced experimental facilities, it is indeed "Testing time for theories" [4]. Famous theories such as supersymmetry [3] or the Higgs boson mass generation mechanism [5] are now being put in direct comparison with experiment [6, 7]. These are exciting times and there is certainly a lot more to be found in the next few years to come.

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Appendix A
Natural Constant Symbols and Values

a_o	Bohr radius=0.5292×10^{-10} m
c	Speed of light, 2.997925×10^{8} m/s
e	Unit charge, 1.6021765×10^{-19} C
E	Energy, J or MeV
G	Gravitational constant, 6.6742×10^{-11} $\mathrm{m^3kg^{-1}s^{-2}}$
h	Planck constant, 6.6260693×10^{-34} Js
$\hbar$	$h/2\pi = 1.05457\times10^{-34}$ Js
k_b	Boltzman constant= $1.38\cdot10^{-23}$ J/K= $8.617\cdot10^{-5}$ eV/K
m_{Pl}	Planck mass, $(\hbar c/G)^{1/2}$, $2.1765\cdot10^{-8}$ kg, $1.2209\cdot10^{19}$ GeV/c^2
m_p	Proton mass, 1.67262171×10^{-27} kg= 938.272 MeV/c^2
m_n	Neutron mass, 1.67492728×10^{-27} kg= 939.565 MeV/c^2
m_e	Electron mass, $9.109382\cdot10^{-31}$ kg= 0.511 MeV/c^2
m_o	Neutrino mass, 7.7943×10^{-38} kg= 0.043723 eV/c^2 (computed, eq. 6.27)
r_{Pl}	Planck distance, $(\hbar G/c^3)^{1/2}$, $1.615\cdot10^{-35}$ m
R	Rotational radius, *m*

Greek Symbols

α	Fine structure constant, $e^2/\varepsilon c\hbar$, $1/137.035 = 7.297353\times10^{-3}$
γ	Lorentz factor $(1-\mathrm{v}^2/c^2)^{-1/2}$
γ_n	Neutrino γ value in the neutron $(n=1)$ rotational state $7.16302\cdot10^9$
ε	$4\pi\varepsilon_o\varepsilon_r = 1.112649\times10^{-10}\varepsilon_r$, $\mathrm{C^2/Nm^2}$
ε_o	Permittivity of vacuum, $8.854187817\times10^{-12}\ \mathrm{C^2/Nm^2}$
ε_r	Relative dielectric constant

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