C 20645 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Define continuity of a function. Show that the constant function $f(x) = b$ is continuous on $\mathbb{R}$.
2. State Boundedness theorem. Is boundedness of the interval, a necessary condition in the theorem ? Justify with an example.
3. If $f : A \to IR$ is uniformly continuous on $A \subseteq \mathbb{R}$ and (x_n) is a Cauchy sequence in A. Then show that $f(x_n)$ is a Caychy sequence in $\mathbb{R}$.
4. Define Riemann sum of a function $f : [a,b] \to \mathbb{R}$.
5. Suppose f and g are in $\mathbb{R}[a,b]$ then show that $f + g$ is in $\mathbb{R}[a,b]$.
6. State squeeze theorem for Riemann integrable functions.
7. If f belong to $\mathbb{R}[a,b]$, then show that its absolute value $|f|$ is in $\mathbb{R}[a,b]$.
8. Define pointwise convergence of a sequence (f_n) of functions.
9. Discuss the uniform convergence of $f_n(x) = x^n$ on $(-1,1]$.
10. If $h_n(x) = 2nxe^{-nx^2}$ for $x \in [0,1], n \in \mathbb{N}$ and $h(x) = 0$ for all $x \in [0,1]$, then show that :

    $$\lim \int_0^1 h_n(x)\,dx \neq \int_0^1 h(x)\,dx.$$

11. State Cauchy criteria for uniform convergence series of functions.

Turn over

12. Evaluate $\int_{-1}^{0} \frac{dx}{\sqrt[3]{x}}$.

13. What is Cauchy principle value. Find the principal value of $\int_{-1}^{1} \frac{dx}{x}$.

14. State Leibniz rule for differentiation of Ramann integrals.

15. State that $\Gamma(p+1) = p\Gamma p$ for $p > 0$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Show that the Dirichlet's function :

$$f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$$ is not continuous at any point of $\mathbb{R}$.

17. State and prove Bolzano intermediate value theorem.

18. Show that the following functions are not uniformly continuous on the given sets :

 (a) $f(x) = x^2$ on $A = [0, \infty]$.

 (b) $g(x) = \sin 1/x$ on $B = (0, \infty)$.

19. If $f : [a,b] \to \mathbb{R}$ is continuous on $[a,b]$, then show that $f \in \mathbb{R}[a,b]$.

20. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Then show that f is continuous on A.

21. Let $f_n : [0,1] \to \mathrm{IR}$ be defined for $n \geq 2$ by :

$$f_n(x) = \begin{cases} n^2 x & , 0 \leq x \leq \frac{1}{n} \\ -n^2(x - 2/n), & 1/n \leq x \leq \frac{2}{n} \\ 0 & , 2/n \leq x \leq 1. \end{cases}$$

Show that the limit function is Riemann integrable. Verify whether $\lim \int_0^1 f_n(x) = \int_0^1 f(x)\,dx$.

22. Given $\iint_{R^2} e^{-(x^2+y^2)}\, dx\, dy = \pi$, find the value of $\int_0^\infty e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$.

23. Show that $\forall p > 0, q > 0$ $B(p,q) = \frac{\Gamma p \, \Gamma q}{\Gamma(p+q)}$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Location of roots theorem.

25. State and prove Additivity theorem.

26. Evaluate (a) $\lim \frac{x^n}{1+x^n}$ for $x \in \mathbb{R}, x \geq 0$. (b) $\lim \frac{\sin nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.

    Discuss about their uniform convergence.

27. (a) Show that $\forall q > -1$, $\int_0^1 x^q e^{-x} dx$ converges.

    (b) Show that $\forall q \leq -1$, $\int_0^1 x^q e^{-x} dx$ diverges.

(2 × 10 = 20 marks)