D 30563 **(Pages : 3)** Name.......................................

Reg. No...................................

FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY

(2019 Admission only)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 20.

1. Find focus, vertex and directrix of the parabola $y^2 = 2x$.

2. Determine the equation of the tangent at the point P with parameter t on the rectangular hyperbola with parametric equations $x = t,\ y = \frac{1}{t}$.

3. Find the equation of the normal to the parabola with parametric equations $x = 2t^2$, $y = 4t$ at the point with parameter $t = 3$.

4. Write the equation of the conic $11x^2 + 4xy + 14y^2 - 4x - 28y - 16 = 0$ in matrix form.

5. Prove that if t_1 is an Eucledean transformation of $\mathbb{R}^2$ given by $t_1(\mathrm{X}) = \mathrm{UX} + a, \mathrm{X} \in \mathbb{R}^2$, then :

   i) The transformation of $\mathbb{R}^2$ given by $t_2(\mathrm{X}) = \mathrm{U}^{-1}\mathrm{X} - \mathrm{U}^{-1}a, \mathrm{X} \in \mathbb{R}^2$ is also a Euclidean transformation.

   ii) The transformation t_2 is the inverse of t_1.

6. Prove that Euclidean congruence is an equivalence relation.

7. Determine the affine transformation which maps the points (0, 0), (1, 0) and (0, 1) to the points (3, 2), (5, 8) and (7, 3) respectively.

8. Prove that an affine transformation maps parallel straight lines to parallel straight lines.

Turn over

9. State Ceva's theorem.

10. The triangle ΔABC has vertices $A(1,3), B(-1,0)$ and $C(4,0)$ and the points $P(0,0), Q\left(\frac{8}{3},\frac{4}{3}\right)$ and $R\left(\frac{-2}{3},\frac{1}{2}\right)$ lie on BC, CA and AB respectively :

    (a) Determine the ratios in which P, Q and R divide the sides of the triangle.

    (b) Determine whether the lines AP, BQ and CR are concurrent.

11. Find the equation of the line that passes through the point [2, 5, 4] and [3, 1, 7].

12. Determine whether the points [1, 2, 3], [1, 1, –2] and [2, 1, –9] are collinear.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Ceiling is 30.

13. Derive the standard form of the equation of the hyperbola.

14. State and prove reflection property of the ellipse.

15. Show that a perpendicular from a focus of a parabola to a tangent meets the tangent on the auxiliary circle of the parabola.

16. Determine the image of the line $3x - y + 1 = 0$ under the affine transformation

$$t(X) = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ -1 & 2 \end{pmatrix} X + \begin{pmatrix} \frac{-3}{2} \\ 4 \end{pmatrix}, X \in \mathbb{R}^2.$$

17. Determine the affine transformation which maps the points (1, – 1), (2, – 2) and (3, – 4) to the points (8, 13), (3, 4) and (0, – 1) respectively.

18. Prove that an affine transformation preserves ratios of length along parallel straight lines.

19. Determine the point of $\mathbb{RP}^2$ at which the line through the points [1, 2, – 3] and [2, – 1, 0] meets the line through the points [1, 0, – 1] and [1, 1, 1].

Section C

Answer any **one** *question.*

The question carries 10 *marks.*

Maximum marks 10.

20. Classify the conic $x^2 - 4xy + 4y^2 - 6x - 8y + 5 = 0$ in $\mathbb{R}^2$ and also determine its centre and axis.

21. State and prove Menelau's theorem.

D 10670 **(Pages : 3)** **Name.......................................**

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FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find the equation of the tangent at the point with parameter t to the parabola with parametric equations $x = at^2, y = 2at$ where $t \in \mathbb{R}$.

2. Let E be a parabola with parametric equations $x = t^2, y = t, t \in \mathbb{R}$. Find focus, vertex axis and directrix of E.

3. Prove that the equation of the tangent at the point (x_1, y_1) to an ellipse is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

4. Write the equation of the conic $x^2 - 4xy + 4y^2 - 6x - 8y + 5 = 0$ in matrix form.

5. Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal for each real number θ.

6. Let the Euclidean transformations t_1 and t_2 of $\mathbb{R}^2$ be given by :

$$t_1(X) = \begin{bmatrix} \frac{3}{5} & \frac{-4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} X + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \text{ and}$$

$$t_2(X) = \begin{bmatrix} \frac{-4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} x + \begin{bmatrix} -2 \\ 1 \end{bmatrix}. \text{ Find } t_2 \circ t_1.$$

Turn over

7. Find the inverse of the affine transformation $t(\mathrm{X}) = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \mathrm{X} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

8. State fundamental theorem of affine geometry.

9. Prove that an affine transformation maps straight lines to straight lines.

10. State Desargue's theorem.

11. Find the equation of the line that passes through the point $[1, 2, 3]$ and $[2, -1, 4]$.

12. Find the point of intersection of the lines in $\mathbb{RP}^2$ with equations $x + 6y - 5z = 0$ and $x - 2y + z = 0$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Let PQ be an arbitrary chord of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let M be the midpoint of PQ. Prove that the following expression is independent of the choice of P and Q : Slope of OM × Slope of PQ.

14. State and prove reflection properties of the ellipse.

15. Prove that the set of all affine transformations A(2) forms a group under the operation of composition of functions.

16. Determine the image of the line $y = 2x$ under the affine transformation $t(\mathrm{X}) = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} \mathrm{X} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathrm{X} \in \mathbb{R}^2$.

17. Determine the affine transformation which maps the points $(2, 3), (1, 6)$ and $(3, -1)$ to the points $(1, -2), (2, 1)$ and $(-3, 5)$ respectively.

18. Prove that affine transformations maps ellipses to ellipses, parabolas to parabolas and hyperbolas to hyperbolas.

19. Determine the point of $\mathbb{RP}^2$ at which the line through the points $[1, 2, -3]$ and $[2, -1, 0]$ meets the line through the points $[1, 0, -1]$ and $[1, 1, 1]$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Prove that the conic E with equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ is a hyperbola. Determine its centre, and its major and minor axes.

21. State and prove Ceva's theorem.

(1 × 11 = 11 marks)