CHAPTER 5

LAWS OF MOTION

5.1 Introduction

In the last chapter we had discussed displacement, velocity and acceleration of a body during its motion. But we had not considered the causes that produce the motion and the changes in motion. In the present chapter we will consider these aspects. You already know that this branch of Physics in which the motion of a body is discussed along with the causes of motion and the properties of the moving body is called dynamics.

5.2 Force and Inertia

Let us consider our few observations about the motion of different bodies found in our day-to-day life. (1) A book lying on the table remains steady as it is, if we do not apply any force on it from outside. (2) In order to throw a ball upward; we have to exert a push in the upward direction. (3) To bring a lorry into motion from rest, a person has to push it. (4) To stop the ball rolling down the inclined plane we apply force in the direction opposite to the direction of its motion, using our hand. Considering these observations we realise that to bring the body into motion from its rest position and also to slow down or stop its motion; some **external agency** providing some force is required. In all the above cases the external force is in contact with the body. Such a force which is acting (or is applied) by remaining in contact with the body is called a **contact force.** Some illustrations of motion are also found in which the external agency is not in contact with the body and yet it exerts force on the body. e.g. A body released from the top of a building performs an accelerated motion towards earth. Here the Earth is not in contact with that body but the force on the body due to gravitational field of Earth is responsible for its accelerated motion. When a nail of iron is placed a little away from a magnet it moves towards the magnet due to attraction. Here the force on the nail due to the magnetic field of the magnet is responsible for its motion. Such forces (due to fields like gravitational field, electric field, magnetic field.) are called **field forces.** Thus external agencies can exert force on the body even from away, without coming in contact.

From this, we understand that the external agency producing a force that affects the motion of a body may be or may not be in contact with the body. In the cases discussed above, the body either comes in motion from rest position or its velocity changes during the motion. But a question may arise as to-"Does a body in uniform motion (that is, a motion in the same straight line with constant speed) need an external force to continue its uniform motion ?"

Aristotle (384 B. C. to 322 B. C.), a Greek philosopher, was of the opinion that if the body is in motion - uniform or non-uniform- then it needs 'something' external - that is an external force - to maintain its motion. Before we know the truthfulness of this opinion, let us consider an illustration : A bicycle, moving, in the same straight line with uniform speed on a horizontal road that appears smooth will stop after sometime if pedalling is stopped. If we want to keep the bicycle in uniform motion we have to apply external force by pedalling it. This observation may appear to support Aristotle's concept; but actually it does not.

In fact, we all know that the external force of friction due to the road opposes the motion of the bicycle. Hence it comes to a stop. Pedalling is required if we want to continue its motion. Then, what was Aristotle's fallacy ? He had considered his practical experience as a basic law of nature and there he happened to be wrong. To understand the law of nature about forces and motion, we have to imagine a situation in which the frictional force opposing the motion is not present. Galileo did this and acquired a deeper understanding about motion. Most of the Aristotle's concepts of motion are found to be wrong today.

You have studied about Galileo's experiments in standard-9. Galileo observd that

(i) objects moving downward along an inclined plane are accelerated - that is their velocity increases.

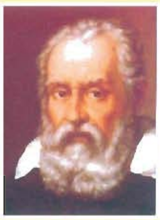
(ii) objects moving upward along an inclined plane are retarded - that is their velocity decreases.

But the motion on the horizontal plane is the intermediate condition of the above two cases. Hence Galileo suggested that a body moving in a straight line on a frictionless horizontal plane should not have either acceleration or retardation. Thus it should move with a constant velocity and it does not need any external force for this.

Friction is an external force on a body, which opposes the motion of the body. If we apply another sufficient external force opposing friction then the net force on the body becomes zero and then its velocity will be maintained constant. Moreover, in the steady state of the body when it remains as it is, also no net external force acts on it. Thus the steady state and the state of uniform motion (motion with constant velocity), are equivalent, as far as force is concerned. This is because in both these states the net external force on the body is zero.

If an external force acts on a body, the state of the body will change. A body does not (and cannot) change of its own, its steady state or the state of uniform motion. This property of the body not to change its state, by itself is called "inertia" of the body. Inertia means to oppose the change. The mass of a body is a measure of its inertia. Out of the two given objects the one with a greater mass is said to have greater inertia also.

Galileo Galilei (1564 - 1642)



Galileo Galilei, born in Pisa of Italy in 1564, was the main figure in the scientific revolution in Europe. He proposed the concept of acceleration. From his studies on the motion of bodies on inclined plane and freely falling bodies, he contradicted the prevailing Aristotelian concepts that to continue a motion some force is needed and heavy bodies fall quickly as compared to lighter bodies under gravity. His idea of inertia later became the starting point of Newton's work.

With the help of a telescope disigned by himself, he made astronomical observations of dark spots on the sun, mountains and depressions on the surface of moon, the moons of Jupiter and phases of Venus. He also proposed that the luminosity of the milky way is due to the light coming from a large number of stars which are not visible to the naked eye.

In his excellent book 'Dialogue on the two Chief World Systems' he advocated the heliocentric theory proposed by Copernicus for the solar system. To day also it is universally accepted.

Due to his great discoveries he is respected and honoured in the world of science.

Isaac Newton

Newton was born in Woolsthorpe, England in 1642, the year Galileo died. During his study at Cambridge University a plague epidemic forced him to return to his mother's farm. Here he had ample scope for thinking deeply, and made many discoveries in mathematics and physics. Calculus, binomial theorem for negative and fractional exponents, inverse square law of gravitation, spectrum of white light...etc. are discoveries due to Newton. On returning to Cambridge he devised a reflecting telescope.

His own book 'The Principia Mathematica' is considered as one of the greatest scientific work. It included his three laws of motion, universal law of gravitation, basic principles of fluid mechanics, mathematics of wave motion, calculation of masses of Sun, Earth and other planets, theory of tides and many other important topics.

His discoveries about light and colours are summarized in his another book 'Opticks'. The scientific revolution initiated by Copernicus, Kepler & Galileo was carried forward and brought to completion by Newton. He proposed that the same laws govern the terrestrial phenomena and the celestial phenomena. e.g. In the fall of an apple on Earth and in the revolution of the moon around Earth the same type of mathematical equations are found applicable. Newton died in 1727.

5.3 Newton's First Law of Motion

Starting with Galileo's logical thoughts, the task of development of mechanics was accomplished almost single-handedly by Isaac Newton, one of the greatest scientists of all times. The three laws of motion developed by Newton form the base of mechanics.

Newton's first law of motion is written as :

"As long as no net external force is acts on the body, a stationary body remains stationary and a body in motion continues to move with constant velocity."

In fact, this is Galileo's law of inertia only. In some cases, we know that the net (resultant) external force on a body is zero and hence we conclude that the acceleration of the body is zero and its velocity is constant. Conversely, sometimes we see a body without acceleration (either in steady state or in motion with constant velocity) and hence we infer that the net external force on the body must be zero.

On applying a net force on a body, it comes into motion if it is stationary and if it is aleady in motion, its velocity changes. Thus in both the cases, acceleration is produced in it. Therefore, force comes out to be the cause of producing acceleration.

From Newton's first law we can say that force is a physical quantity due to which, a stationary body comes into motion and a moving body changes its velocity. Thus Newton's first law of motion gives us the definition of force – but it does not give information about the value (magnitude) of force.

5.4 Momentum

The product of the mass (m) of a body and its velocity ($\vec{v}$) is called its momentum ($\vec{p}$).

That is, $\vec{p} = m\vec{v}$ (5.4.1)

Momentum is a vector quantity and its direction is in the direction of velocity. The SI unit of momentum is kg m s^{-1} or Ns. Its dimensional formula is $[M^1L^1T^{-1}]$.

The momentum of a moving body gives some more information than its velocity. We understand this by examples. Which one out of a bicycle and a car moving with the same velocity can do more harm in colliding with us ? Clearly the car – because due to greater mass its momentum is more.

Thus mementum is an important physical quantity.

5.5 Newton's Second Law of Motion

Newton's second law of motion gives the magnitude (value) of the external force acting on the body.

When force is applied on a body, its velocity changes, hence its momentum also changes. When equal force is applied for equal time on two bodies–one heavier and the other lighter–then it is found that the lighter body acquires a larger velocity but both of them acquire equal momentum.

Suppose you are moving on a bicycle with uniform velocity $\vec{v}$. Now, if you are not in a hurry to stop, you will apply the brakes gently (this produces a small force). Hence the bicycle will gradually slow down, and will stop after some time. But if you want to stop the bicycle quickly, then you will apply the brakes very strongly (this produces a greater force) and then only the bicycle can stop quickly. In both these cases the change in momentum is the same (becomes zero from $m\vec{v}$, where m = mass of yourself + bicycle). But in the second case, greater force had to be applied because that change was to be done faster (Remember that when brakes are applied, pedalling is stopped !)

Thus force has relation with the change in momentum and the time taken for that change; and that relation is obtained from Newton's second law of motion, stated as under :

"The time–rate of change of momentum of a body is equal to the resultant external force acting on it, and this change is in the direction of the force."

Hence, if force $\vec{F}$ is applied on a body of mass m and momentum $\vec{p}$ (= $m\vec{v}$), then,

$$\vec{F} = \frac{d\vec{p}}{dt} \quad (5.5.1)$$

$$= \frac{d}{dt}(m\vec{v}) \quad (5.5.2)$$

If the mass of body remains constant, then

$$\vec{F} = m\frac{d\vec{v}}{dt} \quad (5.5.3)$$

$$\therefore \ \vec{F} = m\vec{a} \qquad \left(\because \ \frac{d\vec{v}}{dt} = \vec{a}\right)$$

Thus force,

$$\vec{F} = \text{mass m} \times \text{acceleration } \vec{a} \quad (5.5.4)$$

The SI unit of force is newton (= N)

The force which produces an acceleration of 1 m s^{-2} in a body of mass 1 kg is called 1 N force.

[1 N = 1 kg m s^{-2}]

(The unit of force in CGS system is dyne and 1 N = 10^5 dyne).

The dimensional formula of force is [$M^1L^1T^{-2}$].

From equation (5.5.4), the **value** of the resultant force acting on the body is obtained.

On applying a force $\vec{F}$ for time–interval Δt, on a body of mass m and velocity $\vec{v}_1$; if its velocity becomes $\vec{v}_2$, then from the measured values of $\vec{v}_1$, $\vec{v}_2$, and Δt; we can find $\vec{a} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

and then from m and $\vec{a}$; force $\vec{F}$ can be calculated with the help of equation (5.5.4).

We note a few important points about this law :

(1) If $\vec{F}$ = 0 (that is, the resultant external force is zero), then $\vec{a}$ = 0, $\therefore$ $\vec{v}$ = constant. This fact is consistent with Newton's first law.

(2) This law is a vector law. Hence for the three components F_x, F_y, F_z of the force $\vec{F}$, three equations are obtained as under :

$$\left.\begin{aligned} F_x &= \frac{d\,p_x}{dt} = m\,a_x \\ F_y &= \frac{d\,p_y}{dt} = m\,a_y \\ F_z &= \frac{d\,p_z}{dt} = m\,a_z \end{aligned}\right\} \quad (5.5.5)$$

(3) The acceleration $\vec{a}$ of the body at a given point at a given instant is determined by the external force $\vec{F}$ acting on it at that point at that instant, and the body has no memory of its acceleration at previous moments. A body released from an accelerating train does not have any acceleration in the horizontal direction at a moment just after the release – (neglecting the resistance of air).

(4) In equation (5.5.1) the value of $\vec{p}$ is not important but the change in $\vec{p}$ is important. If the body is steady in the beginning then initially $\vec{p} = 0$. But if force $\vec{F}$ acts on it, $\vec{p}$ will change and that change is important in this equation.

Illustration 1 : A body of mass 40 kg is moving in a straight line on a smooth horizontal surface. Its velocity decreases from 5.0 m s^{-1} to 2.0 m s^{-1} in 6 seconds. Find the force acting on this body. How much distance would it travel during this time ?

Solution : Taking the direction of motion of the body as X–axis, we can write $F_x = m\,a_x$ for the value of force.

From $v_x = v_{0x} + a_x t$.

$2 = 5 + a_x(6)$

$\therefore\ a_x = -\ 0.5$ m s^{-2}

$\therefore\ F_x = m\,a_x = (40)\,(-\ 0.5) = -20$ N

Thus, this much force is acting opposite to direction of motion (i. e. acting in negative X – direction)

From, $v_x^2 - v_{0x}^2 = 2a_x x$

$4 - 25 = 2(-0.5)x$

$\therefore\ x = 21$ m.

Illustration 2 : When 45 N force is applied on a body of mass m, acceleration of 4.5 m s^{-2} is produced in it. The same force when applied on a body of mass m', acceleration of 9.0 m s^{-2} is produced. Find the acceleration produced by the same force applied on these two bodies tied together.

Solution :

$F = ma \quad \therefore\ 45 = m\,(4.5) \ \therefore\ m = 10$ kg

$F = m'a' \ \therefore\ 45 = m'\,(9.0) \ \therefore\ m' = 5$ kg

If after tying these two bodies together, the same force produces acceleration a'', then

$F = (m + m')\,a''$

$45 = (10 + 5)\,a'' \ \therefore\ a'' = 3$ m s^{-2}

Illustration 3 : A car of mass 1000 kg. is moving with a velocity of 30 m s^{-1} on a horizontal straight road. On seeing red light of a traffic signal, the driver applies brakes to produce a constant braking force of 4 kN. (i) Find the deceleration (or retardation) of the car. (ii) What time will the car take to stop ? (iii) How much distance would it travel during this motion ?

Solution : (i) Taking X–axis in the direction of the car; the force applied on it would be in negative X–direction. 4kN = 4×10^3 N

From $F_x = m\,a_x$; $-4 \times 10^3 = (1000)a_x$

$\therefore\ a_x = -4$ m s^{-2}

(ii) The velocity of the car becomes zero when it stops.

$\therefore$ From $\quad v_x = v_{0x} + a_x t$

$0 = 30 + (-4)t$

$\therefore\ t = 7.5$ s

(iii) From $v_x^2 - v_{0x}^2 = 2\,a_x x$

$0 - 900 = 2\,(-4)x$

$\therefore\ x = 112.5$ m

5.6 Impulse of Force

The product of force $\vec{F}$ acting on a body and the time–interval for which it acts is called impulse of force. From equation (5.5.1) representing Newton's second law, we can write,

impulse of force $\vec{F}\,dt = d\vec{p}$ = change in its momentum (5.6.1)

When a force of large value acts for a very short time, it is difficult to get the values of force and the time interval. But the change in momentum is measurable. e.g. When a ball of mass m moving with velocity $\vec{v}$ hits a wall and rebounds with velocity $\vec{v}\,'$, the force exerted on the ball, by the wall acts for a very small time. By measuring the velocities of the ball $\vec{v}$ and $\vec{v}\,'$ the change in its momentum can be known.

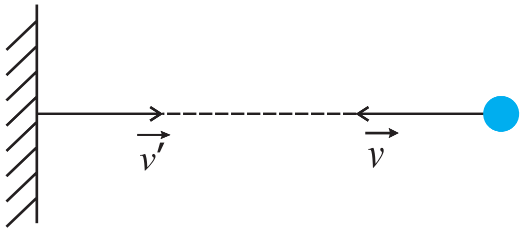


Figure 5.1

Such a force that acts for a very short time is called an impulsive force.

Illustration 4 : A ball of mass 150 g and velocity 12 m s^{-1} coming towards a batsman is hit by him with a force of 480 N in such a way that the ball moves with velocity 20 m s^{-1} in the direction opposite to its original one. Find the time of contact between the ball and the bat.

Solution : If we take the original direction of the ball as negative X–direction, then $\vec{v}_1 = -12\hat{i}$ m s^{-1}, $\vec{v}_2 = 20\hat{i}$ m s^{-1} and $\vec{F} = 480\hat{i}$ N.

Change in momentum of the ball

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

$$= m\vec{v}_2 - m\vec{v}_1 = m(\vec{v}_2 - \vec{v}_1)$$

$$= (0.150)\,[\,20\hat{i} - (-\,12\hat{i})\,]$$

$$= 4.8\hat{i} \text{ kg m s}^{-1}$$

From, $|\vec{F}| = \dfrac{|\Delta\vec{p}|}{\Delta t}$, $\quad 480 = \dfrac{4.80}{\Delta t}$

$\therefore \Delta t = 0.01$ s.

5.7 Newton's Third Law of Motion

Newton's second law of motion gives the relation between the resultant external force acting on a body and the acceleration of the body, ($\vec{F} = m\vec{a}$). But what is the cause of such a force acting on the body ? Actually the force on the body is exerted due to another body (or bodies). Hence a question may arise – "if the other body applies a force on the given body, does the given body apply force on that another body or not ?" The answer to this question is obtained from Newton's third law of motion, which is stated as under.

"To every action there is always an equal and opposite reaction."

Press a spring with your hand. You will experience that the spring also exerts a force in the opposite direction on the hand. Here the spring and the hand were in contact with each other; so we could feel the force. But when a stone attracted by Earth falls towards it, does the stone also exert a force on the Earth ? And does the Earth also rises up towards the stone ? – Such questions may arise in our mind. The answer to this question, according to Newton is – Yes, the stone also exerts the same force on Earth in opposite direction as Earth has on it. But because of the very large mass of Earth the effect of this force on its motion is extremely small. Therefore we are not able to see or detect or feel it, that is, such an effect is negligible.

Thus from Newton's third law of motion, it comes out that there exists no isolated force in nature. Forces are produced only due to interaction between two bodies. Forces are produced only in pairs, and the forces in a pair are equal in magnitude and opposite in direction. Newton's own wording of this third law of motion as given above, is so crisp and beautiful that it has become a part of day to day conversations. We make a few clarifications about this law to avoid any misunderstanding.

(1) In the interaction between two bodies, any one force is called 'action' and then the other is called 'reaction'.

(2) It is not correct to suppose that first the 'action' takes place and then as a result of it 'reaction' occurs. Such cause-effect relationship is not implied in this third law. The force on A by B and the other on B by A both get simultaneously applied.

(3) Action and reaction act on two different bodies. If we represent the force on A by B as $\vec{F}_{AB}$ and the one on B by A as $\vec{F}_{BA}$, then according to this third law,

$$\begin{bmatrix} \vec{F}_{AB} \\ \text{i.e. Force on A} \\ \text{by B} \end{bmatrix} = -\begin{bmatrix} \vec{F}_{BA} \\ \text{i.e. Force on B} \\ \text{by A} \end{bmatrix}$$

Therefore, if we want to discuss the motion of only one body (e.g. of A) then we have to take the force on that body ($\vec{F}_{AB}$) only, the other force $\vec{F}_{BA}$ is irrelevant and it is not to be taken into account. In discussing the motion of only one body, A or B, if we say that "adding the two forces, the net force obtained is zero", then it is wrong, because these two forces are acting on two different bodies. But if we consider the motion of the system as a whole, consisting of these two bodies, then these two forces ($\vec{F}_{AB}$ and $\vec{F}_{BA}$) will become internal forces and their total force will be zero (How will it become zero will be seen in detail in the chapter on Dynamics of system of particles, in future) Therefore they are not to be considered for the motion of the system as a whole. It is due to this fact that Newton's second law of motion can be applied to the system of particles also.

For the motion of the system as a whole, the force within the system is not responsible. We cannot drive a car by pushing the car by sitting inside the car. For the motion of the car, an external force is required to act on it. Now, you may perhaps feel as to how are we able to drive a car by starting its engine, which is an internal part of the car ! Actually, the external force needed to run the car is provided in the form of friction with the road. This may appear surprising to you, but it is true and you will learn it in future.

5.8 Law of Conservation of Momentum

Newton's second and third laws of motion lead to an important result "the law of conservation of momentum". Let us consider an example of a bullet fired from a rifle. As the bullet fired from the rifle moves forward, the rifle is pushed backward (it is called the recoil of a rifle). If the force on the bullet by the rifle is $\vec{F}$, then the force exerted on the rifle by the bullet is $-\vec{F}$. Both these forces act for the same time interval Δt. Before the bullet is fired, both the bullet and the rifle were steady. Therefore their respective momenta $\vec{p}_b$ and $\vec{p}_r$ both were zero. Hence their total initial momentum,

$$\vec{p}_b + \vec{p}_r = 0 \qquad (5.8.1)$$

Now according to the equation (5.6.1) obtained from Newton's second law of motion,

$$\text{change in momentum of bullet} = \vec{F}\Delta t \qquad (5.8.2)$$

$$\text{change in momentum of rifle} = -\vec{F}\Delta t \qquad (5.8.3)$$

As the initial momentum of each one of them is zero, their respective final momenta ($\vec{p}'_b$ and $\vec{p}'_r$), will be equal to the change in their respective momenta.

$$\text{Thus } \vec{p}'_b = \vec{F}\Delta t \text{ and } \vec{p}'_r = -\vec{F}\Delta t \qquad (5.8.4.)$$

From equations (5.8.4) and (5.8.1), we get

$$\vec{p}'_b + \vec{p}'_r = 0 = \vec{p}_b + \vec{p}_r \qquad (5.8.5)$$

$$\text{i.e.} \begin{bmatrix}\text{Final momentum} \\ \text{of (bullet + rifle)}\end{bmatrix} = \begin{bmatrix}\text{initial momentum} \\ \text{of (bullet + rifle)}\end{bmatrix}$$

Here no external force has acted on the system of (rifle + bullet), hence this system is called an isolated system. The forces which act are only the internal forces; and their resultant is always zero. All these facts are included in the law of conservation of momentum which is stated as under :

"The total momentum of an isolated system remains constant."

This law is fundamental and universal just like the law of conservation of energy and the law of conservation of charge. Moreover, it is equally true for the interactions between large bodies like planets and stars and interactions between micro particles like electrons and protons. No phenomenon or process can occur which violates this law.

Illustration 5 : A soldier fires bullets, each of mass 50 g, from his automatic rifle with a velocity of 1000 m s^{-1}. If he can bear a maximum force of 200 N on his shoulder, find the maximum number of bullets which he can fire in a second.

Solution : Suppose mass of each bullet = m and maximum n bullet are fired per second. Before firing the total momentum of bullets and rifle = 0. After firing, the momentum of every bullet, $p = mv$

∴ The momentum imparted to bullets per second = $(nmv - 0) = nmv$

Since no external force acts during the process of firing, the system consisting of (bullet + rifle) can be considered as an isolated system and so its total momentum should remain constant.

∴ The momentum received by the rifle, in opposite direction, in 1 second = nmv

Now, since the change in momentum per second is equal to force, we can say that the force on the rifle and hence on the shoulder of the soldier = nmv

$\therefore\ nmv = 200\text{N}$

$\therefore\ n\ (50 \times 10^{-3}\ \text{kg}) \times 100\ \text{m/s} = 200\ \text{N}$

$\therefore\ n = 4\ \text{s}^{-1}$

Illustration 6 : A person of mass 60 kg is standing on a raft of mass 40 kg in a lake. The distance of the person from the bank is 30 m. If the person starts running towards the bank with velocity 10 m/s, then what will his distance be from the bank after one second ?

Solution : When the person starts running towards the bank the system (raft + person) moves backwards.

Suppose, mass of person = m_P

mass of raft = m_R

and, velocity of person w.r.t. raft = $\vec{v}_{PR}$

velocity of raft w.r.t. the bank = $\vec{v}_{RB}$

velocity of person w.r.t. the bank = $\vec{v}_{PB}$

Taking the direction of running of person as X–axis,

$\vec{v}_{PR} = 10\hat{i}$ m/s

It is clear that $\vec{v}_{PB} = \vec{v}_{PR} + \vec{v}_{RB}$ (1)

Since no external force is acting on this system of (person + raft) according to the law of conservation of momentum,

$$\begin{bmatrix}\text{initial momentum} \\ \text{of (person + raft)}\end{bmatrix} = \begin{bmatrix}\text{final momentum} \\ \text{of (person + raft)}\end{bmatrix}$$

$$\therefore\ 0 = m_P\vec{v}_{PB} + m_R\vec{v}_{RB}$$

$$= m_P(\vec{v}_{PR} + \vec{v}_{RB}) + m_R\vec{v}_{RB}$$

$$= m_P\vec{v}_{PR} + (m_P + m_R)\vec{v}_{RB}$$

$$\therefore\ 0 = 60\ (10\hat{i}\ ) + (60 + 40)\vec{v}_{RB}$$

$$\therefore\ \vec{v}_{RB} = -\ 6\hat{i}\ \text{m/s.}$$

∴ From equation (1)

$$\vec{v}_{PB} = 10\hat{i} - 6\hat{i} = 4\hat{i}\ \text{m/s}$$

Thus the person travels an effective distance of 4 m in one second, towards the bank (because he moves forward by 10 m and the raft moves backward by 6 m)

∴ After 1 s, his distance from the bank is 30 − 4 = 26 m.

Illustration 7 : A bomb in the steady state explodes into three fragments. Two fragments of equal masses move with velocity 30 m/s in mutually perpendicular directions. The mass of the third fragment is equal to three times the mass of each of these two fragments. Find the magnitude and direction of the velocity of this third fragment.

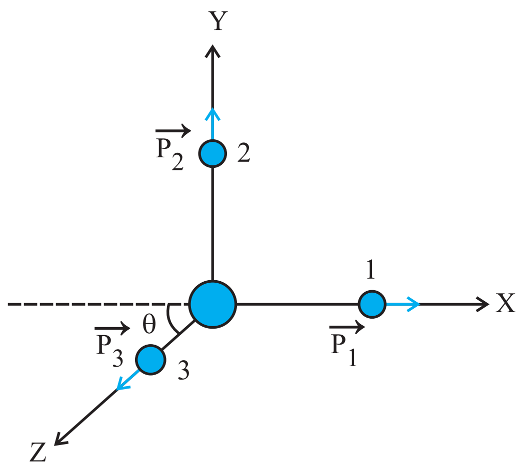


Figure 5.2

Solution : Before explosion, the bomb is steady. ∴ its initial momentum = 0. No external force acts on the bomb during the explosion. Hence, according to the law of conservation of momentum, the vector sum of the momenta of all the fragments after explosion must be zero. Here the mass of each of two fragments is equal to m. ∴ Mass of the third fragment = 3 m. If their respective momenta are $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_3$.

then, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

Taking X– and Y– axes according to Fig. 5.2,

$$\vec{p}_1 = m(30)\hat{i}\ ,\ \vec{p}_2 = m(30)\hat{j}$$

$$\therefore\ m(30)\hat{i} + m(30)\hat{j} + (3m)\vec{v}_3 = 0$$

$$\therefore\ 3m\,(\vec{v}_3) = -30m\,(\hat{i} + \hat{j})$$

$$\therefore\ \vec{v}_3 = -10\hat{i} - 10\hat{j}$$

$$\therefore\ |\vec{v}_3| = \sqrt{(-10)^2 + (-10)^2}$$

$$= 10\sqrt{2}\ \text{m/s}.$$

And, $tan\ \theta = \dfrac{v_{3y}}{v_{3x}} = \dfrac{-10}{-10} = 1$

$\therefore\ \theta = 45^{o}$

Thus the third fragment moves at an angle of 45^{o} with negative X axis and nagative Y axis.

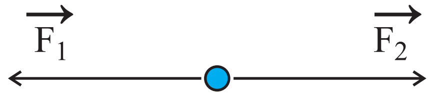
5.9. Equilibrium of Concurrent Forces

The forces, of which the lines of action pass through the same point, are called the concurrent forces.

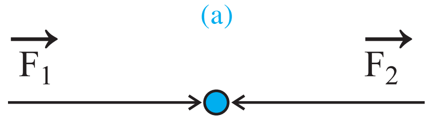
The condition, in which the resultant (net) force of all the external forces acting on a particle, becomes zero, is called equilibrium. Looking from this view point, the steady state and the state of motion with uniform velocity of the body are both equilibrium states.

Thus, for equilibrium $\Sigma\vec{F} = 0$.

If only one external force $\vec{F}$ acts on a particle, then acceleration will be produced in it according to $\vec{F} = m\vec{a}$ and the particle cannot remain in equilibrium. If two external forces $\vec{F}_1$ and $\vec{F}_2$ act on the particle, then for equilibrium (i. e. for $\Sigma\vec{F} = 0$), $\vec{F}_1 = -\vec{F}_2$. Fig. 5.3 (a, b)



(a)



(b)

Figure 5.3

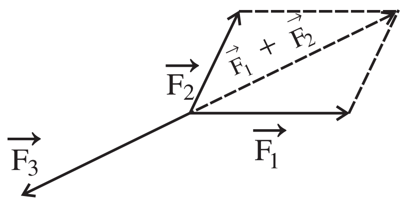
If more than two external forces are acting, then for equilibrium, their vector sum must be zero i.e. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + ... = 0.$

Moreover, since force is a vector quantity, the sum of the corresponding components of all the forces should also become zero. i.e.

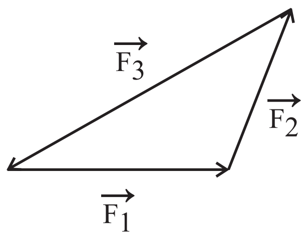
$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0.$

For a particle remaining in equilibrium, under the effect of three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$, the vector sum of the two forces $(\vec{F}_1 + \vec{F}_2)$ has the magnitude equal to that of the third force and direction opposite to it so that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. [See Fig. 5.4. (a)]

i.e. $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$



(a)



(b)

Figure 5.4

In other words, if all the three force vectors are arranged tail to head, they should form a closed figure [see Fig. 5.4. (b)] so, the resultant force becomes zero.

Illustration 8 : As shown in Fig. 5.5., two strings AO and BO are tied with a rigid support and a body of 20 kg. mass is suspended with a third string OC. In the equilibrium condition of this entire system, the strings AO and BO make angles of 60° and 30° respectively with the horizontal. Assuming all these strings as massless, find the tensions produced in these strings. [Take $g = 10$ m s^{-2}]

Solution : Here, as the strings are massless, the force applied at one end of the string is communicated undiminished at the other end. (Thus a massless string works like a postman).

In equilibrium condition, the body and the point O are steady.

Suppose the tensions in the strings are T_1, T_2, T_3 as shown in the Fig. 5.5 in equilibrium condition.

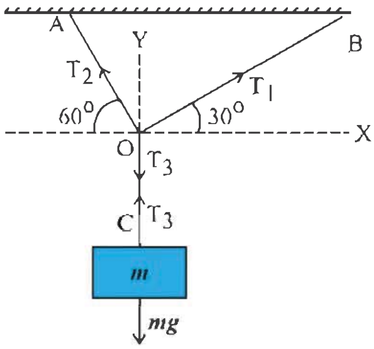


Figure 5.5

For equilibrium of point C, $T_3 - mg = 0$

$\therefore \quad T_3 = mg = (20)(10)$

$= 200$ N (1)

Take X–axis in horizontal direction and Y–axis perpendicular to it.

From Fig., x component of $T_1 = T_1 \cos 30^\circ$

y component of $T_1 = T_1 \sin 30^\circ$

x component of $T_2 = T_2 \cos 60^\circ$

y component of $T_2 = T_2 \sin 60^\circ$

For equilibrium of point 0,

$\Sigma F_x = 0$ gives, $T_1 \cos 30^\circ - T_2 \cos 60^\circ = 0$

$$\therefore \frac{\sqrt{3}}{2} T_1 - \frac{1}{2} T_2 = 0$$

$$\therefore \sqrt{3}\, T_1 - T_2 = 0 \qquad (2)$$

$\Sigma F_y = 0$ gives, $T_1 \sin 30^\circ + T_2 \sin 60^\circ - T_3 = 0$

$$\therefore \frac{1}{2} T_1 + \frac{\sqrt{3}}{2} T_2 - 200 = 0$$

[from eqn. (1)]

$$\therefore T_1 + \sqrt{3}\, T_2 = 400 \qquad (3)$$

Multiplying equation (2) with $\sqrt{3}$ and then adding in equation (3), we get,

$$(3T_1 - \sqrt{3}\, T_2) + T_1 + \sqrt{3}\, T_2 = 400$$

$$\therefore T_1 = 100 \text{ N}$$

Putting this value in equation (3), we get $T_2 = 173$ N.

5.10 Friction

When the bodies are in contact, mutual contact, forces are generated in every pair of particles at their contact surface.This contact forces obey Newton's third law of motion. Consider two components of this contact force – (i) the component perpendicular to the contact surfaces is called **normal reaction N** (sometimes in short, called the normal force) (ii) the component parallel to the contact surfaces is called **frictional force *f*,** or in short, the **friction.**

Such contact forces and frictional forces are determined by the roughness of the contact surfaces at the molecular level. Surfaces of objects may appear extremely smooth, but when viewed through a microscope 'ridges' and 'valleys' are found all over the surface. Ridges of one surface and valleys of the other surface get interlocked with each other and a 'cold welding' takes place. Hence when one surface tries to shift on the other, a force opposing it comes into play – which is called the **frictional force *f*.**

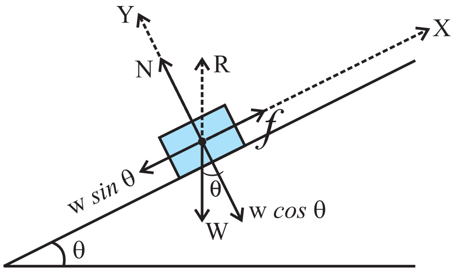
We consider an illustration. Suppose a block is at rest on a slope inclined at an angle θ, to the horizontal. The block exerts force equal to its weight $\vec{W}$ on the surface of slope. This surface exerts an equal and opposite force $\vec{R}$ on the block. $\vec{R}$ is the contact force.

For convinience we choose the X-axis parallal to the inclined surface (see Fig. 5.6(a)) Two mutually perpendicular components of this force $\vec{R}$ are as under :

(1) the component perpendicular to the surface in called the normal force $\vec{N}$

(2) the component parallel to the surface is called the frictional force *f*. In the absence of such a frictional force *f*, the block would have slipped down the slope due to the force W sin θ Such a motion (which actually does not take place

due to the presence of frictional force) is called impending motion.



5.6 (a)

Now, $|\vec{R}|^2 = |\vec{N}|^2 + |\vec{f}|^2$ since the block is in equillibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, $\Sigma F_x = 0$ gives $|\vec{f}| - w \sin\theta = 0$ (1)

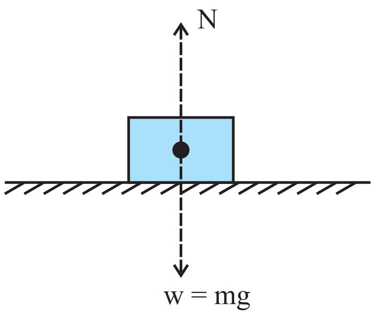
and $\Sigma F_y = 0$ gives $|\vec{N}| - w\cos\theta = 0$ (2)

From eqns. (1) and (2),

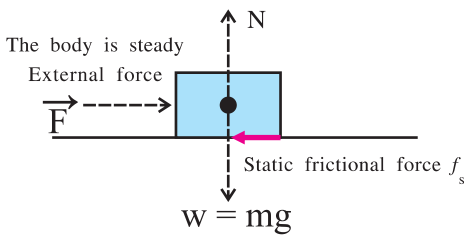
$$\frac{f}{N} = \tan\theta \quad (3)$$

(a) Static friction : Consider a body of mass m lying steady on the horizontal surface of a table. It exerts a force equal to its weight W (= *mg*) on the surface, and the surface exerts the normal reaction N on the body. From the equillibrium condition of the body, we can say that

$$\begin{bmatrix} \text{gravitational force} \\ \text{on the body} \\ \text{in downward direction} \\ W(= mg) \end{bmatrix} = \begin{bmatrix} \text{The normal force N} \\ \text{by the surface on} \\ \text{the body in upward} \\ \text{direction} \end{bmatrix} \quad (5.10.1)$$



(b)



(c)

Figure 5.6

Now suppose a small force $\vec{F}$ is applied on the body in +*x*– direction and yet the body does not come into motion but remains steady. See Fig. 5.6 (b). If this small force would be the only force on the body, then it would come into motion, even with a small acceleration (= F/*m*). But the body is still steady. Hence some other force must be acting on the body in –*x* direction which balances this force $\vec{F}$ and makes total external force zero and thus keeps the body steady.

This other force is the component of contact force parallel to surface and it is called the frictional force f_s. It is also called friction or static friction. This static friction does not exist by itself, but comes into play when the external force acts on the body.

Now on increasing the external force slightly and gradually, the body still does not move. Hence it is clear that the frictional force also must be increasing in the same proportion. Thus this static friction is a self–adjusting force. This happens only upto a certain limit. This static friction opposes the impending motion. The impending motion means, the motion that would have occured (but actually does not occur) under the effect of the external force, if friction were to be absent.

On still increasing the applied external force $\vec{F}$, the value of frictional force increases only up to a certain limit. If the applied external force becomes slightly more than this maximum frictional force, immediately the body starts moving. The frictional force acting on the body when it is on the verge of starting the motion, is called the maximum static friction $f_{s(max)}$ or the limiting frictional force. Experiments show that – (1) maximum static friction does not depend on the contact area of the surfaces. (2) The maximum static friction $f_{s(max)}$ is proportional to the normal reaction N. i.e. $f_{s(max)} \propto N$.

The above facts [stated in (1) and (2)] are called the laws of static friction.

Here it is clear that $f_{s(max)} = \mu_s N$ (5.10.2)

Where μ_s is the constant of proportionality and it is called the **coefficient of static friction.** Its value depends on the nature of surfaces, the materials of surfaces and the temperature. The value of μ_s is smaller for a smooth surface than that for a rough surface. The value of μ_s is in the range between 0.01 to 1.5. As long as the steady body is stationary, we can say that $f_s \leq \mu_s N$.

Equation (5.10.2) is the relationship between only the values of $f_{s(max)}$ and N, while their directions are mutually perpendicular.

Illustration 9 : A block of mass 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle of 15° with the horizontal, the block just begins to slide. What is the co–efficient of static friction between the block and the surface ?

Solution : The forces acting on this block are :

(i) Its weight (gravitational force) acting downwards = mg

(ii) Normal reaction exerted by the surface = N and

(iii) Static frictional force = f_s (parallel to the inclined plane).

see Fig. 5.7

Since the block is in equilibrium, the resultant of all these forces would be zero. Taking two mutually perpendicular components of weight mg, as shown in the figure, we can write

$$mg \sin\theta = f_s \text{ and} \quad ...(1)$$

$$mg \cos\theta = N \quad ...(2)$$

Taking the ratio of these two equations

$$\tan\theta = \frac{f_s}{N} \quad ...(3)$$

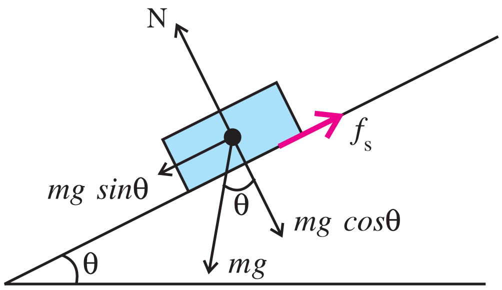


Figure 5.7

As the value of θ increases, the value of $\tan\theta$ also increases and to balance it, the magnitude of frictional force f_s will also increase upto certain value. Suppose for $\theta = \theta_{max}$ the magnitude of static friction becomes maximum, viz. $f_{s\,(max)}$. Hence from eqn. (3).

$$\tan\theta_{max} = \frac{f_{s(\,max\,)}}{N} = \mu_s$$

$$\therefore \theta_{max} = \tan^{-1} \mu_s$$

When the value of θ becomes little more than θ_{max}, the block starts sliding down because of (resultant) unbalanced forces acting on it. It is given that $\theta_{max} = 15°$

$$\therefore \mu_s = \tan 15° = 0.27$$

Note that θ_{max} depends only on μ_s, not on the mass of the block.

(b) Kinetic Friction :

In the illustration of body on the table in article (a) discussed above, if the applied external force $\vec{F}$, becomes even slightly more than $f_{s(max)}$, the body immediately comes into motion. Experiments show that as soon as this relative motion starts, the value of frictional force becomes less than that of maximum static friction $f_{s(max)}$. See Fig. 5.8.

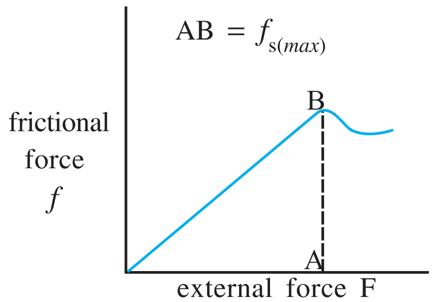


Figure 5.8

The frictional force opposing the relative motion of the contact surfaces is called the kinetic friction. See Fig. 5.9. Like the static frictional force this kinetic friction also does not depend on the contact area and it is proportional to the normal reaction (N). Moreover it is almost independent of the velocity.

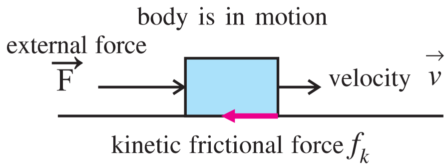


Figure 5.9

Here, it is clear that $f_k = \mu_k$ N (5.10.3)

where μ_k is the constant called coefficient of kinetic friction. Its value depends on the nature of contact surfaces. Experiments show that $\mu_k < \mu_s$.

We have to remember that to bring a body into motion from steady state, the external force has to overcome the maximum static friction but once the body has come into motion, the external force has to face the kinetic friction. The kinetic friction is always less than maximum static friction $f_{s(max)}$.

Illustration 10 : A block of mass 15 kg is lying on an inclined plane of angle 20°. In order to make it move upward along the slope with an acceleration of 25 cm/s², a horizontal force of 200 N is required to be applied on it. Calculate (i) frictional force on the block and (ii) co-efficient of kinetic friction.

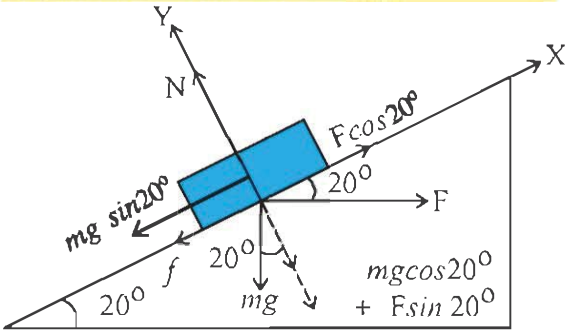


Figure 5.10

Solution : The situation described here is shown in Fig. 5.10. We take X–axis in the direction parallel to the surface of the slope and so Y–axis would be in the direction perpendicular to the slope. Remember that the block performs accelerated motion in X–direction and so the resultant force on it is not zero.

Hence there is no equilibrium in X–direction.

From $\Sigma F_x = ma_x$

$F\cos 20^\circ - f - mg \sin 20^\circ = (15)(0.25)$

$\therefore$ $(200)(0.9397) - f - (15)(9.8)(0.3420) = 3.75$

$\therefore$ $f = 134$ N

Since the block is in equilibrium in the Y–axis

$\Sigma F_y = 0$

$\therefore$ $N - mg \cos 20^\circ - F \sin 20^\circ = 0$

$\therefore$ $N - (15)(9.8)(0.9397) - (200)(0.3420) = 0$

$\therefore$ N = 207 N

Now,

$$\mu_k = \frac{f}{N} = \frac{134}{207} = 0.65$$

Note here that due to the force F the (effective) normal force increases.

(c) Rolling Friction : When a disc, ring or a sphere is rolling without sliding on a surface, the line (or the point) that touches the surface remains momentarily steady. On such a body static or kinetic friction does not act. Then, why does it stop after rolling over a certain distance ? Why do they not continue the motion with constant velocity ? In such motions of the body rolling friction is acting and hence to continue the motion some external force has to be applied. For a body of a given mass and a given surface, the rolling friction is much less (sometimes of thousandth part) than the static and the kinetic friction.

In such rolling phenomena, the contact surfaces are slightly deformed. Hence instead of a line or a point a **small area** comes in contact with the surface. Hence, a component of contact force parallel to the surface (which we call the rolling friction) comes into play, that opposes this relative motion. It depends on the radius, speed and the nature of material of body.

(d) Advantages and Disadvantages of Friction : Under certain conditions, friction is undesirable. Due to the friction opposing the relative motion between different parts in machines, power is dissipated in the form of heat. To decrease the kinetic friction in them, lubricants (e. g. grease, oil, soap, air.... etc.) are used. Another way is to use ball-bearings. Rolling friction will act in them which is very much less than static and kinetic friction. Hence power dissipation is decreased.

In certain conditions friction is necessary also. The kinetic friction dissipates power but it is needed to stop vehicles. It is used in brakes in machines and automobiles (What will happen if we drive a vehicle without brakes ?) It is only due to friction that we are able to walk. It is not possible for a car to move on a very slippery road. Normally the friction between the tyres and road is the necessary external force needed to accelerate the vehicle. Laws of friction are not simple and precise like those of gravitation and electric forces. Laws of friction are empirical and are only apporixamately true.

However, they are useful in solving the problems of mechanics.

The process of friction is very complex at the micro level.

5.11 Dynamics of Uniform Circular Motion

(a) Centripetal force : We have seen in Chapter 4 that the acceleration of a body of mass m, moving with a uniform (constant) speed v on a circular path of radius r, is $\frac{v^2}{r}$ directed towards the centre. It is called centripetal acceleration a_c. Hence according to Newton's second law of motion, a force $F_c = \frac{mv^2}{r}$ (5.11.1)

is required to be acting on it towards the centre for such a motion. This force is called the **centripetal force**. See Fig. 5.11

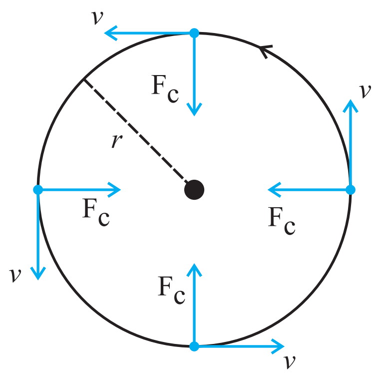


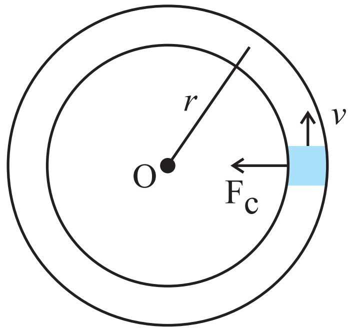
Figure 5.11

The required centripetal force for the circular motion of a planet around the sun is provided by the gravitational force on the planet by the sun.

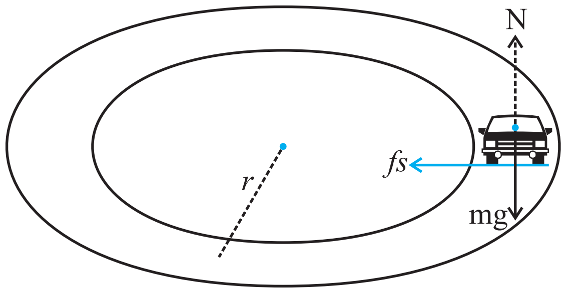
In the case of an electron moving in a circular path around the nucleus, the required centripetal force is provided by the Coulomb attractive force on the electron by the nucleus.

Uniform circular motion is different from the uniform motion mentioned many times earlier in this chapter. In the uniform motion the velocity-vector ($\vec{v}$) of the body is constant and its acceleration is zero. But in the uniform circular motion the speed of the body on the circular path is constant, its velocity vector ($\vec{v}$) changes and the acceleration is $\frac{v^2}{r}$ towards the centre of the circle.

(b) Motion of a Vehicle on a Level Circular Path : In Fig. 5.12 (a, b) a vehicle is shown moving on a horizontal curved path (such path can be considered as a part of a circle). It can move safely only if sufficient centripetal force is acting on the vehicle on this path, (otherwise it will be thrown towards outside). Here the forces acting on the vehicle are :



(a)



(b)

Figure 5.12

(1) Weight of the vehicle (mg) – in downward direction (2) the normal reaction (N) by the road – in upward direction (3) the frictional force (f_s) by the road – parallel to the surface of the road.

Since the vehicle has no acceleration in vertical direction,

$N - mg = 0$

$\therefore N = mg$ (5.11.1)

The required centripetal force F_c for the circular motion of the vehicle on this road must be provided by the frictional force f_s.

$\therefore F_c = f_s = \frac{mv^2}{r}$ (5.11.2)

This frictional force is the static frictional force, which opposes the impending motion of the vehicle away from the centre of the circle. If the maximum frictional force by the road is $f_{s(max)}$

$$f_{s(max)} = \mu_s N \text{ (from equation 5.10.1)}$$

$$= \mu_s mg \text{ (from equation 5.11.1)} \quad (5.11.3)$$

Where μ_s = the coefficient of static friction between the tyres of the vehicle and the road.

From this, we can say that if the speed v of the vehicle is such that

$$\begin{bmatrix} \text{required centripetal} \\ \text{force } \frac{mv^2}{r} \end{bmatrix} \leq \begin{bmatrix} \text{maximum frictional} \\ \text{force } \mu_s \; mg \end{bmatrix} \quad (5.11.4)$$

then only the vehicle will move safely on this road.

$$\therefore \; v^2 \leq \mu_s r g \quad (5.11.5)$$

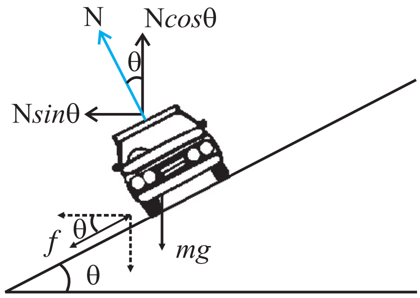
and the maximum speed for the safe motion

$$v_{max} = \sqrt{\mu_s \; r \; g} \quad (5.11.6)$$

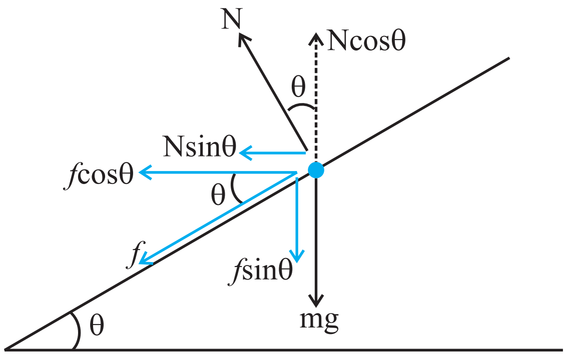
If the speed of the vehicle is more than this v_{max}, it will be thrown away from the road. This v_{max} has the same value for all vehicles – light or heavy. See that mass does not appear in equation (5.11.6)

From this discussion you would be able to understand as to why do we slow down the vehicle while turning on a road.

(c) Motion of a Vehicle on Banked Curved Road : We have seen that the required centripetal force for safe motion of a vehicle on a level circular path is provided by only the friction of the road. But at the curvature of the road (at the turning of the road) if the road is banked (such that the inner edge of the circular road is low and the outer edge is high), then, in the required centripetal force for circular motion; a certain contribution can be obtained from the normal force (N) by the road and the contribution of friction can be decreased, to that extent. In the Fig. (5.13), the section of the road with the plane of paper is shown. This road is inclined with the horizontal at an angle θ. The forces acting on the vehicle are also shown in the Fig. 5.13(b).



(a)



(b)

Figure 5.13

The forces acting on the vehicle are :

(1) weight (mg), in downward direction

(2) normal force (N), perpendicular to road, upward

(3) frictional force (f), parallel to the road.

As the acceleration of the vehicle in the verticle direction is zero,

$$N\cos\theta = mg + f\sin\theta \quad (5.11.7)$$

$$\therefore \; mg = N\cos\theta - f\sin\theta \quad (5.11.8)$$

In the horizontal direction, the vehicle performs a circular motion. Hence it requires centripetal force $\frac{mv^2}{r}$, which is provided by the horizontal components of N and f.

$$\therefore \; \frac{mv^2}{r} = N\sin\theta + f\cos\theta \quad (5.11.9)$$

Dividing equation (5.11.9) by equation (5.11.8), we get,

$$\frac{v^2}{rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} \quad (5.11.10)$$

We put $f = f_{s(max)} = \mu_s N$ in the above equation to get the maximum safe speed v_{max}, on this road.

$$\frac{v^2_{max}}{rg} = \frac{N\, sin\theta + \mu_s\, N\, cos\theta}{N\, cos\theta - \mu_s\, N\, sin\theta} \quad (5.11.11)$$

$$\therefore\ v^2_{max} = rg\left[\frac{sin\theta + \mu_s\, cos\theta}{\cos\theta - \mu_s\, sin\theta}\right] \quad (5.11.12)$$

Dividing the numerator and the denominator by cosθ; we get

$$v^2_{max} = rg\left[\frac{tan\theta + \mu_s}{1 - \mu_s\, tan\theta}\right] \quad (5.11.13)$$

$$\therefore v_{max} = \sqrt{rg\left[\frac{\mu_s + tan\theta}{1 - \mu_s\, tan\theta}\right]} \quad (5.11.14)$$

Comparing equations (5.11.6) and (5.11.14), we find that the maximum safe speed on a banked curved road is more than that on horizontal curved road, since *tan*θ is positive here.

By knowing the radius of curvature of the road *r*, by determining the maximum safe speed (e.g. 100 km/h) of the vehicle on this road, and by knowing the coefficient of static friction μ_s between the tyre and the road, the equation (5.11.14) enables us to find the necessary angle of banking θ, of the road and accordingly such roads shoul be constructed. Moreover a board indicating such maximum safe (v_{max}) should be placed at a proper place.

Let us consider following two special cases in this discussion.

(i) In equation (5.11.14), for $\mu_s = 0$ (i. e. friction does not act at all),

$$v_0 = \sqrt{rg\ tan\theta} \quad (5.11.15)$$

If we drive the vehicle at this speed on the banked curved road the contribution of friction becomes minimum in the required centripetal force, and hence wear and tear of the tyres can be minimised. This speed v_0 is called the optimum speed.

(ii) If $v < v_0$, then the frictional force will act towards the higher (upper) edge of the banked road [See that in the above figure *f* acts towards the lower edge of the road]. The vehicle can be kept stationary i.e. can be parked – on the banked road, only if $tan\theta \leq \mu_s$

Illustration 11 : On a smooth, horizontal surface of a table, a body of mass *m* is connected, with a help of a light string passing through the hole on the surface, to a body of mass M suspended at the other end. (See Fig. 5.14)

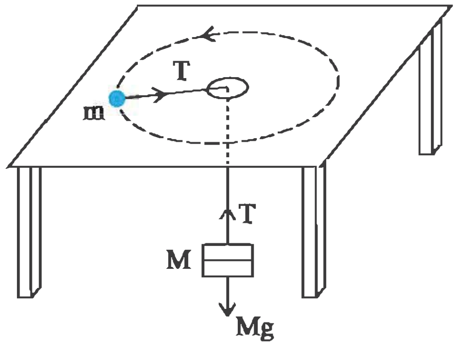


Figure 5.14

(a) In order that the body of mass M remains stationary obtain the condition for the circular motion of the body of mass *m* in terms of *v* and *r*.

(b) In the above case to maintain a uniform circular motion of a body of mass 10 kg, with a speed of 5 m/s, on the path of radius 2m, what should be the mass suspended at the other end ?

(Take $g = 10$ m/s^2)

Solution :

(a) In this circular motion if the tension in the string is T, then

the required centripetal force $\frac{mv^2}{r} = T$ (1)

Where v = speed, r = radius of path

And for the body of mass M, to remain steady,

$$Mg = T \quad (2)$$

$$\therefore\ \frac{mv^2}{r} = Mg \quad (3)$$

$\therefore\ \frac{v^2}{r} = \frac{M}{m}g$ is the required condition.

(b) In equation (3) above

$$(10)\frac{(5)^2}{2} = M(10)$$

$\therefore$ M = 12.5 kg

Illustration 12 : A disc is rotating around its centre, in a horizontal plane at the rate of $\frac{100}{3}$ rotations / minute. A coin is placed at a distance of 5 cm and another similar coin at 25 cm from its centre. The coefficient of static friction between the disc and the coins is 0.2. Which coin will be thrown away from the disc ? Which coin will keep rotating with the disc ? (Take g = 10 m/s^2, π^2 = 10)

Solution :

Suppose the mass of each coin = m.

For uniform circular motion of the coin, if

required centripetal force $\frac{mv^2}{r}$	$\leq$	maximum static friction $f_{s(max)}$

then the coin can keep rotating with the disc.

Moreover $f_{s(max)} = \mu_s N = \mu_s mg$

($\therefore$N = normal force = mg)

Here time for $\frac{100}{3}$ rotations = 60 s

$\therefore$ time for 1 rotation (= T) = ?

$\therefore$ Periodic time T = $\frac{60 \times 3}{100}$ = 1.8 s

we know $v = \frac{2\pi r}{T}$

$\therefore$ From above condition

$$\frac{m}{r}\left(\frac{4\pi^2 r^2}{T^2}\right) \leq \mu_s mg$$

$$\therefore\ r \leq \frac{\mu_s\ gT^2}{4\pi^2}$$

$$\therefore\ r \leq \frac{(0.2)(10)(1.8)^2}{(4)(10)}$$

$\therefore$ $r \leq$ 0.162 m

$\therefore$ $r \leq$ 16.2 cm

The coin near the centre will rotate with the disc and the one which is away will be thrown away.

Illustration 13 : A cyclist speeding at 18 km/h on a level road takes a sharp circular turn (without bending) of radius 3 m without reducing the speed. The co–efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ?

Solution :

Here $v = \frac{18000}{3600}$ = 5 m/s,

r = 3 m and μ_s = 0.1

The formula for maximum safe speed, on a horizontal curved road is $v_{max} = \sqrt{\mu_s\ rg}$

$$\therefore\ v_{max} = \sqrt{(0.1)(3)(9.8)}$$

$$= 1.714 \text{ m s}^{-1}$$

Since the speed of the cyclist (5 m s^{-1}) is more than this value he will slip while negotiating the turn.

Illustration 14 : In Illustration 13 what should the cyclist do in order to prevent slipping while passing over the same road ?

Solution : Here, friction does not provide the necessary centripetal force. If the cycle is inclined at an angle θ with the vertical, the component of contact force towards the centre of the circular path would provide the necessary centripetal force. This is shown in Fig. 5.15.

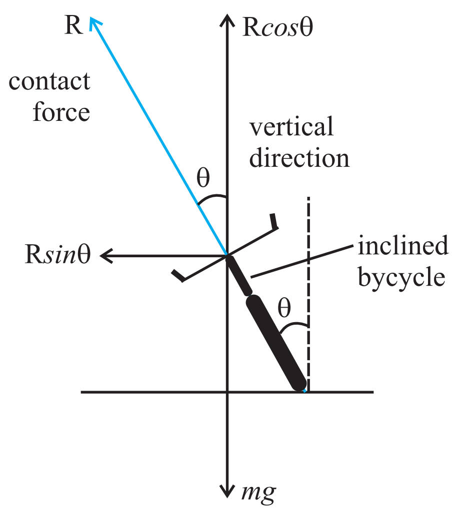


Figure 5.15

Here, R is the contact force between the cycle and the road. Out of two mutually perpendicular components R cosθ and R sinθ, R sinθ provides the necessary centripetal force.

$$\therefore\ R\sin\theta = \frac{mv^2}{r} \qquad (1)$$

From the figure, Rcosθ = mg (2)

Dividing eqn, (1) by eqn. (2),

$$tan\theta = \frac{v^2}{rg} \Rightarrow \theta = tan^{-1}\left(\frac{v^2}{rg}\right) = 40^o23'$$

(What do you think can be the reason for this much inclination ?)

Note : This problem can also be solved by taking moments of the force.

Illustration 15 : A circular racetrack of radius 300 m is banked at an angle of 15° . If the co–efficient of friction between the wheels of a race–car and the road is 0.2, calculate (i) the optimum speed of the race–car to avoid wear and tear on its tyres, and (ii) the maximum permissible speed to avoid slipping ?

Solution :

(i) Normally, on a banked road the sum of the horizontal components of normal force and frictional force provides the necessary centripetal force to keep a car moving on a circular turn without slipping. At the optimum speed, the horizontal component of normal reaction alone is enough to provide the required centripetal force and the frictional force is not needed.

From the formula $v_0 = \sqrt{rg\ tan\theta}$ for optimum speed,

$$v_0 = \sqrt{(300)(9.8)(tan\,15^o)}$$

= 28.1 m/s.

(ii) From the formula

$$v_{max} = \sqrt{rg\left[\frac{\mu_s + tan\theta}{1 - \mu_s\ tan\theta}\right]}$$

$$v_{max} = \sqrt{(300)(9.8)\left[\frac{0.2 + tan\,15^o}{1 - 0.2 tan\,15^o}\right]}$$

= 38.1 m/s

5.12 Inertial and Non–inertial Frames of Reference

We had seen in Chapter 3 that the place and the situation from where an observer takes his observation is called the frame of reference. Such a frame of reference may be steady, may be moving with a constant velocity or may be moving with an acceleration.

Suppose you are sitting in a bus at rest. When the bus starts with a jerk, you are pushed backward. Now when the same bus is moving with a constant velocity, at that time no such push is experienced by you. When the bus was steady then also you had not experienced the push. If the driver applies brakes suddenly, you are pushed forward. Thus during the accelerated or retarded (decelerated) motion of the bus, you experience such a push, although no apparent force is applied on you. Here, it seems that Newton's first law is not holding good, because according to this law, as long as an external force is not acting on the body, its state of motion should not change.

From this discussion it is clear that Newton's first law of motion holds good in a frame of reference which is either stationary or moving with a constant velocity, but it is not obeyed in an accelerated frame of reference. The frame of reference, in which Newton's first law is obeyed is called the **inertial** frame of reference; and the one in which it is not obeyed is called the **non-inertial** frame of reference. A rotating reference frame is an illustration of non–inertial frame of reference. In the obove illustration, when the bus remains at rest or is moving with a constant velocity, it is an inertial frame of reference; but at the time of accelerated (or retarded) motion, the same bus is a non-inertial reference frame.

In the non-inertial frame of reference, we need to consider one more additional force acting on a body in discussing its motion. This force is called **pseudo or fictitious force F_p**. A force is actually exerted during the interaction between two bodies; but for this pseudo force mentioned here, no other body seems to interact with the given body. This force appears to act only due to the accelerated motion of the frame of reference. Hence it is called fictitious or pseudo force F_p. The direction of this force F_p is opposite to that of acceleration of the reference frame.

A body of mass m, in an accelerated reference frame is given an additional acceleration equal but opposite to the acceleration of the reference frame.

This acceleration is called the pseudo or fictitious acceleration a_p. The motion of body is analyed by giving this force $\vec{F}_p = m\,\vec{a}_p$ in the direction opposite to the acceleration of reference frame, and also considering other forces which are actually acting on the body. We imagine such an additional fictitious force in order to solve such problems in the accelerated reference frame, using Newton's second law of motion. In the inertial frame of reference such a fictitious force F_p is not to be considered.

A merry - go - round is also an accelerated (hence non - inertial) reference frame. For a man sitting in it and rotating with it, a centripetal force is required for circular motion which is provided by the friction between the man and the seat (and / or normal force by the support at the back). This is the actual force.

But the man feels that a force is acting on him towards away from the centre (he feels as if he is being thrown away). This is the fictitious force Fp.

Earth is also a non–inertial frame of reference. But due to its very small acceleration due to its rotational motion whatever error occurs in the measurements is extremely small. Hence, for practical purposes Earth is taken as an inertial reference frame.

The outcome of the discussion above is : If the downward acceleration of a non–inertial (means accelerated) reference frame is ($\vec{a}$), then-for an observer inside it to understand the motion of a body of mass m-all actual forces acting on the body are also to be considered and an additional force $\vec{F}_p = m(\vec{a})$ in the upward direction is also to be considered and then Newton's second law of motion should be used.

Illustration 16 : Ramesh with mass of 60 kg stands on a spring–balance in a lift. (a) If the lift moves with an acceleration of 2 m/s^2 (i) upward and (ii) downward what will be his weight as recorded ? (b) If the cable of the lift breakes what will be his recorded weight ? (g = 10 m/s^2)

Solution : The gravitational force of the Earth on a body is called its weight w and $w = mg$.

The weight recorded by the spring balance is the normal reaction by its surface on the body. When the lift is steady or moving with a constant velocity, it is an inertial frame of reference. At that time, his weight recorded will be $w_1 = mg = (60)\ (10) = 600$ N.

In the accelerated motion of the lift, the man and forces on him are shown below in the Fig. 5.16 (a, b, c).

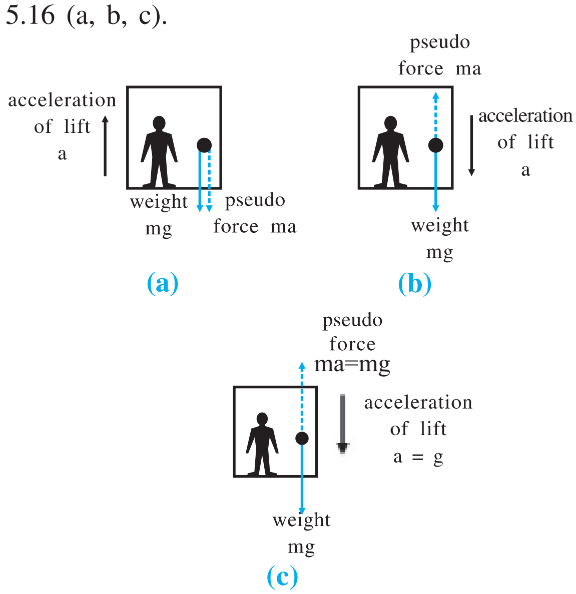


Figure 5.16

(a) (i) Upward acceleration of lift = a

∴ Ramesh inside it is in an accelerated reference frame. Hence, fictitious acceleration ***a*** should be given to Ramesh in the downward direction. Moreover his weight mg also acts in the downward direction. ∴ The resultant force on him in the downward direction is $W_1 = mg + ma = m\ (g + a)$. He will exert this force W_1 on the balance Fig. 5.16 (a) and surface of the balance will apply an equal normal force upwards (normal reaction) on him.

$$\therefore \text{ Weight recorded } = W_1$$
$$= m(g + a)$$
$$= 60(10 + 2) = 720 \text{ N.}$$

(ii) downward acceleration of lift = a.

∴ Ramesh inside it is in an accelerated reference frame. Hence fictitious acceleration **a** should be given to him in the upward direction. Moreover weight mg is acts in the downward direction. Therefore the resultant force on him in downward direction is $W_2 = mg - ma = m\ (g - a)$. He will exert this much force W_2 on the balance Fig. 5.16 (b). The surface of balance will apply an equal normal force upward.

$\therefore$ Weight recorded $= W_2 = m\ (g - a)$

$= 60\ (10 - 2)$

$= 480$ N.

(b) If the cable of the lift breaks, the lift will fall freely with an acceleration, $a = g$.

$\therefore$ recorded weight $W_3 = m\ (g - g) = 0$ N.

This is called the state of weightlessness.

5.13 Guidance for Solving Problems in Dynamics

(Only for information)

(A) In physics many different words like – 'friction', 'normal reaction', 'normal force', 'reaction', 'action', 'air resistance', 'thrust', 'buoyant force', 'pull', 'centripetal force', 'weight', 'tension' etc. – are used in the discussion of many different phenomena and processes. All these terms mean 'force' – in reference to those phenomena and processes.

(B) Tension in a string : When a body (of mass m) is suspended at the end of a string hung from a rigid support, the string becomes tight. In this condition, every region of the string is said to be in tension. The protons – electrons near the lower end of the string exert electromagnetic force on the protons–electrons of the body near the contact point and vise–versa. This force is called the contact force. And due to this contact force; the body does not fall but remains suspended.

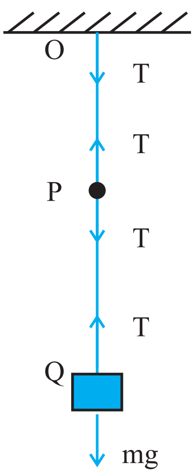


Figure 5.17

Similarly contact forces also act between one region and the other opposite rigion at every point of the string. They are equal and in opposite directions according to Newton's third law of motion. This common magnitude (i.e. equal value) of the contact forces between two regions is called tension produced in the string at each point.

If the string is light (i.e. massless), the tension T is the same at every point in the string, (We shall accept this fact without giving any proof, at present).

In the Figure 5.17 at point P, tension towards PO is T, towards PQ is T. At point Q tension towards QP is T and at O tension towards OP is T.

Moreover, since the body is stationary it is clear that $mg = T$.

(C) Free Body Diagram (FBD) : We can solve different problems of dynamics using Newton's three laws of motion, which we have learned. Sometimes in a given problem more than one body is involved. Such bodies exert force on each other. Moreover every body (object) also experiences gravitational force. In solving such problems, out of the assembly of bodies or systems; we have to take that part as a 'system', of which the motion is to be discussed. And the remaining parts of the assembly and other agencies which affect our system are taken as 'environment'.

To solve the problem, we should proceed acccording to the following steps :

(1) Draw a schematic diagram showing the assembly of different object, other objects connected with them, and those which support these objects.

(2) Select the object (or objects) as 'system', of which we want to discuss the motion.

If the system under consideration is made up of more than one object, then take care that the acceleration vector (magnitude + direction) of all these objects should be the same.

(3) Make a list of the forces acting on the system by the remaining parts of the assembly and by other agencies. In this list, the forces acting inside the system are not to be included.

For example, a coolie standing with a heavy load on his head is shown in Fig. 5.18 (a) (b) (c). The forces acting in this case are as follows : (i) weight of the load mg (acting in downward direction on coolie and on load),

(ii) normal force of reaction (N_1) on the load by the coolie (in the upward direction) (Note clearly that this force is not on the coolie but is on the load) (iii) weight of the coolie, Mg (this force acts in the downward direction on both, the coolie and the ground) (iv) normal force of reaction (N_2) on coolie by the ground (in the upward direction) (v) the force on the ground, (m + M) g. Which of these forces should be considered depends on the choice of the system.

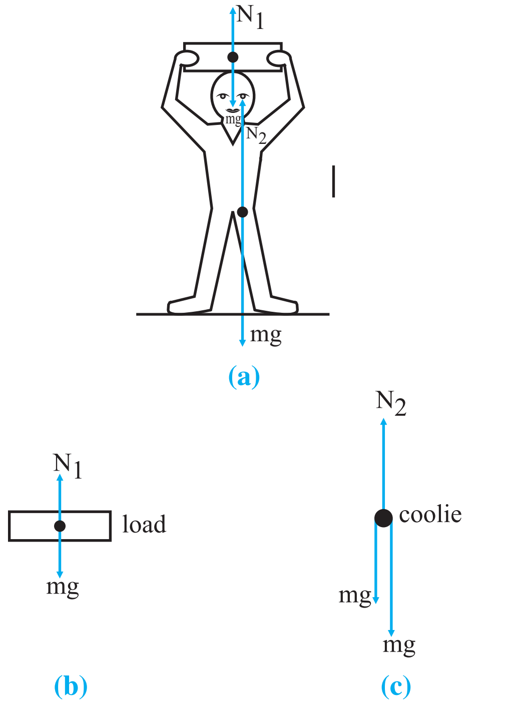


Figure 5.18

If we are interested in the state of motion of the load, we should consider the forces acting on the load only, viz. (i) and (ii).

If we are interested in the state of motion of the coolie only, we should consider coolie as our system and consider the forces acting on the coolie only, viz. (i), (iii) and (iv).

If we are interested in the state of motion of (coolie + load), we should consider the forces acting on both the coolie and the load viz. (a) (m + M) g and (b) N_2. Here, the force by the coolie on the load or the force by the load on the coolie are not considered as they are internal forces between the parts of the system.

(4) Showing a system as a point, all the forces acting on it are depicted as vectors from that point. This figure is called the free body diagram (FBD). This does not mean that the system under consideration is free from forces-actually, only the forces on the system are shown in the figure.

In this figure the forces exerted by the system on the environment are not to be shown.

(5) Now choose X–axis in the direction of actual or likely motion of the system. The direction normal to it becomes the Y–axis.

Now find the resultant of the X–components of the forces on the system. Write an equation showing that this value (resultant) is equal to the product of the mass of the system and its acceleration in X–direction. Similarly the Y–components will give another equation. Such equations are called the equations of motion. By solving such equations we can determine the unknown quantity (or quantities) in them.

(6) If the number of unknown quantities is more than number of equations obtained, then we take another part of assembly as another system, obtain equations from its FBD and hence find the solutions.

Illustration 17 : As shown in Fig. 5.19 two blocks 1 and 2, of the same mass, are in contact with block 3. The co-efficient of friction between the surfaces of 3 and 1 and that between 3 and 2 is μ. The blocks 1 and 2 are tied by a light string and the string is passed over a frictionless pulley. With what minimum acceleration should the block 3 move, in horizontal direction, so that there is no motion of 1 and 2 w.r.t. 3 ?

[This illustration is only for information]

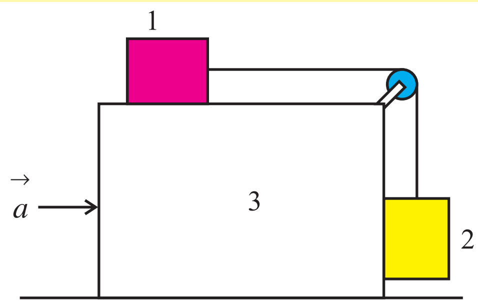


Figure 5.19

Solution : Suppose the minimum acceleration (required) of block 3, in horizontal direction, is a. The forces acting on block 1 are :

(i) The gravitational force of the Earth = mg downward,

(ii) The normal force exerted by the surface of block 3 = N_1 (upward)

(iii) The force of the tension exerted by the string = T (towards right)

(iv) The force of friction = μN_1 (towards left)

(v) Pseudo force = ma (towards left)

The FBD for block 1 (considering it as the system) is shown in Fig. 5.20. Since there is no acceleration in the vertical direction $N_1 = mg$ and in the horizontal direction ma + μN_1 = T

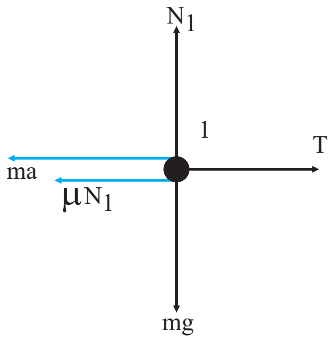


Figure 5.20

$\therefore$ ma + μmg = T (1)

The forces on block 2 are :

(i) The gravitational force of the Earth = mg (downward),

(ii) The normal force exerted by the surface of block 3 = N_2 (towards right),

(iii) The force of tension exerted by the string = T (upward),

(iv) The force of friction = μN_2(upward)

(v) Pseudo force = ma (towards left)

The FBD for block 2 (considering it as the system) is shown in Fig. 5.21. Since there is no acceleration in the horizontal direction $N_2 = ma$

and in the vertical direction $\mu N_2 + T = mg$

$\therefore \ \mu ma + T = mg$

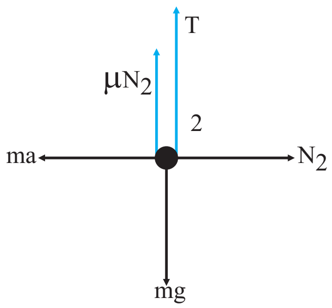


Figure 5.21

Substituting the value of T from eqn. (1),

$\mu ma + ma + \mu mg = mg$

$\therefore \ a(\mu + 1) = g(1 - \mu)$

$\therefore \ a = g\left(\dfrac{1-\mu}{1+\mu}\right)$

Illustration 18 : In an accelerating goods train, 25 wagons of equal mass are attached to the engine. Show by calculation whether the tension in the coupling between the fourth and the fifth wagon is the same as that between twenty first and twenty second wagon.

Solution : Suppose the pull of the engine on the first wagon = P.

friction on every wagon = f

mass of every wagon = m

acceleration of entire goods train = a.

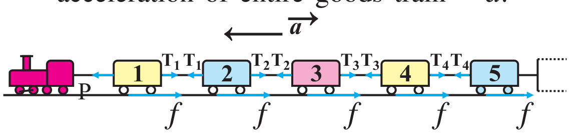


Figure 5.22

Considering the FBD of the first four wagon

$P - 4f - T_4$ = resultant force = $(4m)a$

$\therefore \ T_4 = P - 4(f + ma)$ (1)

This T_4 is the tension between the fourth and the fifth wagon.

Similarly considering FBD of the first 21 wagons,

$T_{21} = P - 21(f + ma)$ (2)

This T_{21} is the tension between 21st and 22nd wagons.

From equations (1) and (2) it is clear that $T_4 \neq T_{21}$ and $T_4 > T_{21}$.

Illustration 19 : A block (A) of 20 kg is put on a frictionless surface and another object (B) of mass 2 kg is placed over it. The co-effecient of friction between the surfaces of A and B is 0.25. A horizontal force of 2 N is applied on B. Calculate (i) the acceleration of block A and that of object B, (ii) frictional force betwen A and B and (iii) Calculate all these quantities again if the force on B is of 20 N. Take $g = 10 \text{ m s}^{-2}$.

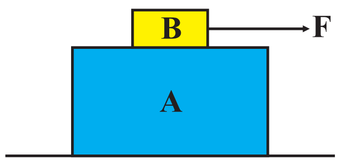


Figure 5.23

Solution : It is clear that as long as the applied force is less than the maximum static force of friction, there would not be any relative motion between A and B this means that both the object would move, under the effect of applied force, as if they are the one object.

In the present case, maximum static frictional force = μmg

= (0.25) (2) (10) = 5 N

(i) When a force of 2 N is applied on B there would not be relative motion between A and B and so the acceleration of both would be the same, say a. mass × acceleration = force

$$\therefore\ (2 + 20)a = 2\ \therefore\ a = \frac{1}{11} = 0.09 \text{ m s}^{-2}$$

(ii) The frictional force between A and B $f = F - ma = 2 - (2)(0.09) = 1.82$

(iii) When a force of 20 N is applied on B, since it is more than maximum static frictional force (5 N), now there will be relative motion between A and B. Hence the magnitudes of their accelerations would be different. In this position the FBD of both A and B will be as shown in Fig. 5.24.

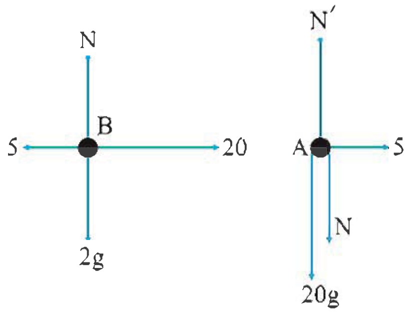


Figure 5.24

From this $20 - 5 = 2\,a_B\ \therefore\ a_B = 7.5 \text{ m s}^{-2}$

and $5 = 20\,a_A\ \therefore\ a_A = 0.25 \text{ m s}^{-2}$.

Illustration 20 : Three blocks of masses m_1 = 1 kg and m_2 = 2 kg and m_3 = 3kg are connected by massless strings and placed on a horizontal frictionless surface as shown in Fig. 5.25. A force F = 12 N is applied to mass m_1 as shown. Calculate (i) the acceleration of the system, (ii) tension (T_2) in the string between m_1 and m_2 and (iii) tension (T_3) between m_2 and m_3.

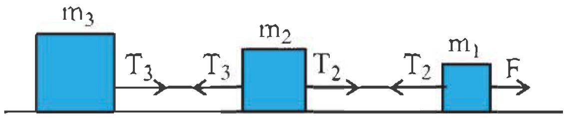


Fig. 5.25

Solution : (i) The acceleration of system

$$a = \frac{\text{Total force}}{\text{Total mass}} = \frac{12}{1+2+3} = 2 \text{ m s}^{-2}$$

(ii) $T_2 = (m_2 + m_3)a = (2 + 3)(2)$

$= 10$ N

(iii) $T_3 = m_3 a = 3 \times 2 = 6$ N

Solve this numerically by assuming that the same force (12 N) acts on m_3 towards left and write your conclusion.

SUMMARY

1. We shall consider the causes of motion and changes in motion.
2. Aristotels concept – that force is required to continue the motion of the body is not true. In practice whatever external force that is required to continue the motion with a constant velocity is only to counter the friction (which is also an external force).
3. The law of inertia given by Galileo was represented by Newton as the first law of motion : "If no external force acts on a body, the body at rest remains at rest and a body in motion continues to move with the same velocity." This law gives us the definition of force.
4. The momentum of a body $\vec{p} = m\vec{v}$ is a vector quantity. It gives more information than the velocity.
5. Newton's second law of motion : The time–rate of change in momentum of a body is equal to the resultant external force applied on the body and is in the direction of the external force.

$\vec{F} = d\vec{p} / dt = m\vec{a}$ is the vector relationship.

The SI unit of force is newton (= N). 1 N = 1 kg m s^{-2}. This law gives the value of force. It is consistent with the first law. ($\vec{F} = 0$ indicates that $\vec{a} = 0$) In this equation the acceleration of the body $\vec{a}$ is that which it has when the force is acting on it. (Not of the past !). $\vec{F}$ is only the resultant external force.

6. The impulse of force is the product of force and the time for which it acts. When a large force acts for a very small time, it is difficult to measure $\vec{F}$ and Δt but the change in momentum can be measured, which is equal to the impulse of force ($\vec{F}\ \Delta t$)

7. Newton's third law of motion : "To every action there is always an equal and opposite reaction."

   Forces always act in pairs, and $\vec{F}_{AB} = -\vec{F}_{BA}$. The action and the reaction act simultaneously. They act on different bodies, hence they cannot be cancelled by adding. But the resultant of the forces between different parts of the same body becomes zero (You will get the explanation on how this happens, when you study the chapter on "Dynamics of system of particles." in future in the next semester.)

8. The law of conservation of momentum is obtained from Newton's second law and the third law. It is written as – "The total momentum of an isolated system remains constant."

9. The concurrent forces are those forces of which the lines of action pass through the same point. For equilibrium of the body, under the effect of such forces, $\Sigma\vec{F}$ must be = 0. Moreover the sum of the corresponding components also should be zero. ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$)

10. Friction is produced due to the contact force between the surfaces in contact. It opposes the impending or the real relative motion.

    Static frictional force $f_s \le f_{s(max)} = \mu_s N$ and

    the kinetic friction is $f_k = \mu_k N$

    μ_s = coefficient of static friction

    μ_k = coefficient of kinetic friction and $\mu_k < \mu_s$.

11. On a body performing uniform circular motion a force equal to mv^2 / r acts towards the centre of the circular path. This is called the centripetal force.

    The maximum safe speed on level curved road is $v_{max} = \sqrt{\mu_s\ rg}$

    The maximum safe speed on a banked curved road is

$$v_{max} = \sqrt{rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$$

12. The reference frame, in which Newton's first law of motion is obeyed is called the inertial frame of reference and the one in which it is not obeyed is called non–inertial frame of reference. The frame of reference with constant velocity is an inertial frame of reference and one which has acceleration is non–inertial frame of reference.

An additional (fictitious) acceleration equal but opposite to that of the non–inertial reference frame is considered on the body to solve problems on motion for accelerated frame of reference.

EXERCISES

Chose the correct option from the given options :

1. When a force acts on a body of mass 100 g, the change in its velocity is 20 cm s^{-1} per second. The magnitude of this force is .... N.

(A) 0.2 (B) 0.02 (C) 0.002 (D) 2.0

2. A bullet of mass m, moving horizontally with velocity v hits and gets embedded in a wooden block of mass M resting on a horizontal frictionless surface. What will be the velocity of this composite system ?

(A) $\frac{mv}{M - m}$ (B) $\frac{Mv}{M - m}$

(C) $\frac{Mv}{M + m}$ (D) $\frac{mv}{M + m}$

3. The accelerated motion of the vehicle on a horizontal road is due to what ?

(A) Engine of the vehicle

(B) Driver

(C) Earth's gravitational force

(D) Friction between the road and the vehicle.

4. An object of mass 8 kg is suspended through two light spring balances as shown in the figure Then,

(A) both the balances will read 8 kg.

(B) both the balances will read 4 kg.

(C) The upper balance will read 8 kg and the lower balance will read zero.

(D) The balances will read any value but their sum will be 8 kg.

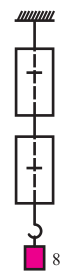


Figure 5.26

5. A block of mass m is placed on a smooth inclined plane of angle θ. The normal force exerted by the surface of the plane would be ....

(A) mg (B) $\frac{mg}{\cos\theta}$ (C) $mg \cos\theta$ (D) $mg \sin\theta$

6. A block of mass m is placed on a smooth slope of angle θ. The whole system (slope + block) is moved horizontally with acceleration a in such a way that the block does not slip on the slope. Hence, a = ...

(A) $g \tan\theta$ (B) $g \sin\theta$ (C) $g \cos\theta$ (D) $g / \sin\theta$

7. A block is placed on the top of a smooth inclined plane of inclination θ, kept on the floor of a lift. When the lift descends with a retardation a, the relative acceleration of the block, parallel to the surface of the slope, is ....

(A) $g \sin\theta$ (B) $a\sin\theta$

(C) $(g - a)\sin\theta$ (D) $(g + a)\sin\theta$

8. Which of the following statement is correct ?

(A) A body has a constant velocity but a varying speed.

(B) A body has a constant speed but a varying value of acceleration.

(C) A body has a constant speed and non-zero constant acceleration.

(D) A body has a constant speed but varying velocity.

9. A reference frame attached with a geostationary satellite moving around the earth can be regarded as ....

(A) non-inertial (B) inertial

(C) any one of them (D) none of above

10. A person standing on the floor of a lift drops a coin. The coin reaches the floor of the lift in time t_1 if the lift is stationary and in time t_2 if it is accelerated in upward direction. Then

(A) $t_1 = t_2$ (B) $t_1 < t_2$

(C) $t_1 > t_2$ (D) cannot say anything.

11. A person standing on the floor of a lift drops a coin. The coin reaches the floor of the lift in time t_1 if the lift is stationary and in time t_2 if it is moving with uniform velocity in upward direction. Then

(A) $t_1 = t_2$ (B) $t_1 < t_2$

(C) $t_1 > t_2$ (D) cannot say anything.

12. N bullets, each of mass m, are fired normally towards a wall at the constant rate of n bullets per second with velocity v. They stop on the wall. Hence, the reaction on bullets by the wall is ....

(A) nmv (B) $\frac{Nmv}{n}$ (C) $\frac{nNm}{v}$ (D) $\frac{nNv}{m}$

13. A force acts on an object of mass 1.5 kg at rest, for 0.5 s. After the force stops acting, the object travels a distance of 5 m in 2 s. Hence, the magnitude of the force will be ....

(A) 5 N (B) 7.5 N (C) 10 N (D) 12.5 N

14. A force of 4 N acts on an object of mass 2 kg in X–direction and another force of 3 N acts on it in Y–direction. Hence, the magnitude of the acceleration of the object will be ....

(A) 1.5 m s^{-2} (B) 2.0 m s^{-2} (C) 2.5 m s^{-2} (D) 3.5 m s^{-2}

15. A rope which can withstand a maximum tension of 400 N hangs from a tree. If a monkey of mass 30 kg climbs on the rope in which of the following cases will the rope break ? (Take g = 10 m s^{-2} and neglect the mass of the rope.)

(A) When the monkey climbs with constant speed of 5 m s^{-1}

(B) When the monkey climbs with constant acceleration of 2 m s^{-2}

(C) When the monkey climbs with constant acceleration of 5 m s^{-2}

(D) When the monkey climbs with constant speed of 12 m s^{-1}

16. A ball with momentum 0.5 *kg* m s^{-1} coming towards a batsman is hit by him such that it goes on the same path in opposite direction with momentum 0.3 *kg* m s^{-1}. If the time of contact of the ball with the bat is 0.02 s, find the force on the ball by the bat.

(A) 10 N (B) 40 N (C) 75 N (D) 30 N

17. A block of mass 1000 *kg* lying steady on the horizontal surface of a table needs 200 N horizontal force to come into motion. What is the coefficient of static friction between the block and the surface of table ?

[take g = 10 m s^{-2}]

(A) 0.2 (B) 0.02 (C) 0.5 (D) 0.05.

18. As shown in the figure, a horizontal force of 70 N is applied on a system of blocks of masses 4 *kg*, 2 *kg* and 1 kg placed on a frictionless horizontal surface. If the tension in one string is T_1 = 60 N, find the tension T_2 in the second string.

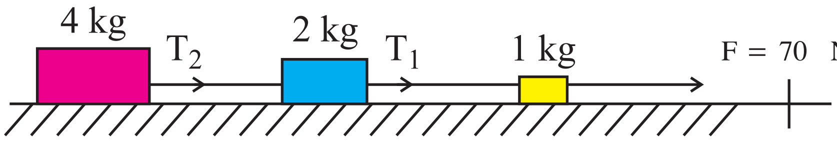


Figure 5.27

(A) 40 N (B) 60 N (C) 20 N (D) 10 N

19. A body of mass 30 *kg* at one end and another of 50 *kg* at the other end of a string passing over a frictionless pully are suspended as shown in the figure. What is the acceleration of this system ?

[take g = 10 m s^{-2}]

(A) 8 m s^{-2} (B) 6 m s^{-2}

(C) 2.5 m s^{-2} (D) 2 m s^{-2}

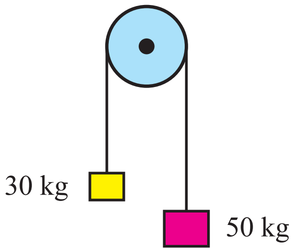


Figure 5.28

20. As shown in the figure blocks of masses 2 kg, 5 kg and 3 kg are arranged with light strings and frictionless pully fitted on frictionless horizontal surface. What is the acceleration of this system ?

[take g = 10 m s^{-2}]

(A) 1 m s^{-2} (B) 2 m s^{-2}

(C) 5 m s^{-2} (D) 8 m s^{-2}

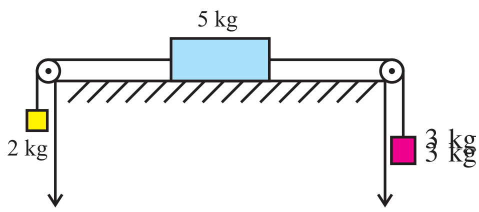


Figure 5.29

21. What is the value of the force $\vec{F}$ to be applied horizontally on a block of mass 5 kg which is in contact with a wall, as shown in the figure. (take g = 10 m s^{-2}) such that it does not fall down. The coefficient of friction between the block and the wall is 0.4.

(A) 200 N (B) 20 N

(C) 12.5 N (D) 125 N

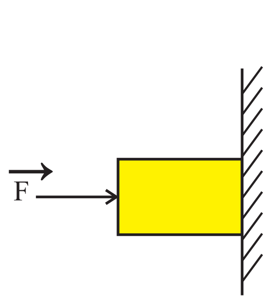


Figure 5.30

22. A stationary bomb explodes into three pieces. If the momenta of two pieces are $2\hat{i}$ unit and $3\hat{j}$ unit respectively, then what is the value of the momentum of the third piece ?

(A) $\sqrt{13}$ unit (B) 5 unit (C) 6 unit (D) 13 unit

23. As shown in the figure, one block of 2.0 kg at one end and the other of 3.0 kg at the other end of a light string are connected. If this system remains stationary, find the magnitude and direction of the frictional force. (take g = 10m s^{-2})

(A) 20 N, downward on slope

(B) 20 N, upward on slope

(C) 10 N, downward on slope

(D) 10 N, upward on slope.

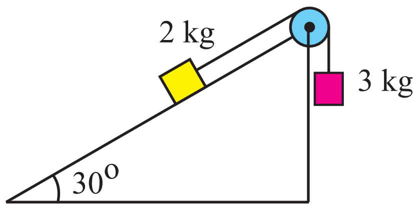


Figure 5.31

24. A man sitting in a train moving with uniform (constant) velocity tosses a coin upward from his hand, which comes back in his hand after sometime. What will be the nature of motion of the coin observed by a steady observer on the ground outside the train ?

(A) parabola (B) horizontal

(C) Straight line upward and then straight line downward. (D) circular

25. Consider a pendulum suspended from the ceiling of a room and oscillating in the vertical plane suppose that the string breaks when (i) the bob is at the end position of its path, (ii) bob is in the mean position of its path. What will be the nature of the path of bob, till it touches the ground.

(A) (i) curved towards downward; (ii) straight line downward

(B) (i) straight line downward; (ii) parabola

(C) (i) straight line upward; (ii) parabola

(D) (i) straight line upward; (ii) curved towards downward

26. Seven blocks each of mass 10 kg are arranged one above the other as shown in Fig. 5.32. What are the values of the contact forces exerted on the third block; by the fourth block and the second block respectively ?

(Take g = 10 m s^{-2})

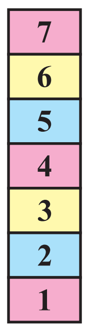


Figure 5.32

(A) 40 N, 50 N (B) 50 N, 40 N
(C) 40 N, 20 N (D) 50 N, 30 N

ANSWERS

1. (B)	2. (D)	3. (D)	4. (A)	5. (C)	6. (A)
7. (D)	8. (D)	9. (A)	10. (C)	11. (A)	12. (A)
13. (B)	14. (C)	15. (C)	16. (B)	17. (B)	18. (A)
19. (C)	20. (A)	21. (D)	22. (A)	23. (A)	24. (A)
25. (B)	26. (A)				

Answer the following questions in short :

1. A ball of mass 0.2 kg is thrown in the vertical direction with a velocity of 2 m s^{-1}. At the top of its path (i) what is the value of its velocity ? (ii) What is the value of its acceleration ? (iii) what is the value of the force acting on it ? [Take g = 10 m/s^2] [**Ans. :** 0, 10 m/s^2, 2N]
2. Define inertia
3. What is meant by non–inertial frame of reference ?
4. What is similar from the dynamics point of view between a book lying stationary on the horizontal table and a raindrop falling downward with constant speed ?
5. F → t graph for a body is shown in the figure. What is the change in the value of momentum in the initial time interval of 0.03 s ?

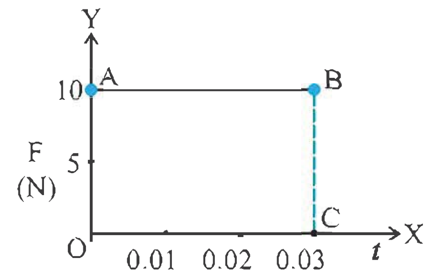


Figure 5.33

[**Ans. :** 0.3 kg m s^{-1}]

6. What is impending motion ?
7. Give the dimensional formula of impulse of force.
8.

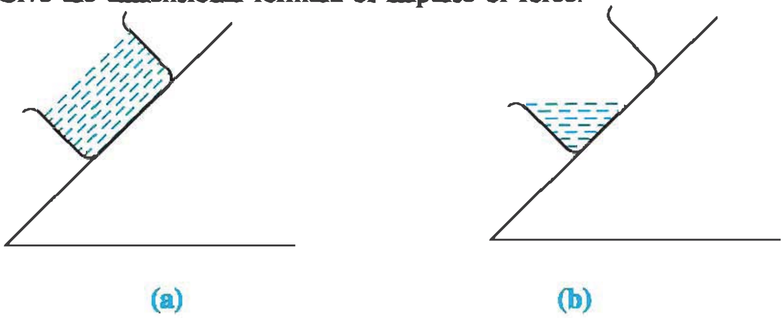


Figure 5.34

Can you tell which beaker is steady and which is coming down with acceleration, in the above figure ? [Hint : The level of a steady liquid remains horiozontal. On the liquid in the accelerating beaker fictitious force acts opposite to acceleration of beaker.]

9. In uniform circular motion (i) only the value of velocity is constant (ii) velocity vector is constant (iii) direction of velocity is constant Select the correct.

[**Ans. :** (i)]

[Similar questions for acceleration, momentum and force can be formed.]

10. Which out of (i) value of velocity, (ii) value of acceleration, (iii)value of force, (iv) the momentum vector of the body is not constant during uniform circular motion ? [**Ans. :** (iv)]

Answer the following questions :

1. Define momentum. Write Newton's second law of motion and hence derive the equation $\vec{F} = m\vec{a}$
2. Write the law of conservation of momentum and explain with an illustration.
3. Give Newton's first and second laws of motion. State which information they provide about force.
4. Explain about the static friction and give its laws.
5. Obtain the formula for the maximum safe speed (v_{max}) of a vehicle on a level curved road.
6. For a vehicle moving on a banked curved road, using free body diagram (FBD), obtain the formula for the maximum safe speed (v_{max})
7. State advantages and disadvantages of friction.

Solve the following problems :

1. Two balls, each of mass 80 g, moving towards each other with a velocity 5 m s^{-1}, collide and rebound with the same speed. What will be the impulse of force on each ball due to the other ? What is the value of change in momentum of each ball ? [**Ans. :** 0.8 N S, 0.8 kg m s^{-1}]
2. Two blocks of masses 6 kg and 2 kg are placed in contact on a horizontal frictionless surface. If a horizontal force of 2 N is applied to mass 6 kg. to move them together, what will be the acceleration of 2 kg block ? What will be the force on this block ? [**Ans. :** 0.25 m s^{-2}, 0.5 N]
3. Three blocks of masses 1 kg, 2 kg and 3 kg are placed in contact with each other on a horizontal frictionless surface as shown in Fig. 4.28. A force of 12 N is applied as shown in the figure.

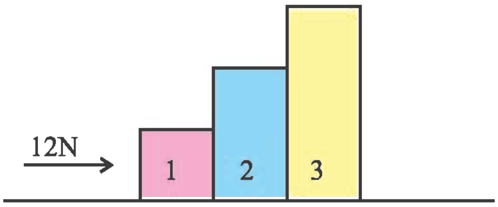


Figure 5.35

Calculate (i) the acceleration of the system of these three blocks, (ii) the contact force acting on 2 kg block by first block of 1 kg and (iii) the contact force on 3 kg block.

[**Ans. :** (i) 2 m s^{-2} (ii) 10 N (iii) 6 N]

4. A block of 50 kg on a smooth plane inclined at 60° and another block of 30 kg on a smooth plane inclined at 30° with horizontal, are cannected by a light string passing over a frictionless pully as shown in the Fig. 5.35.

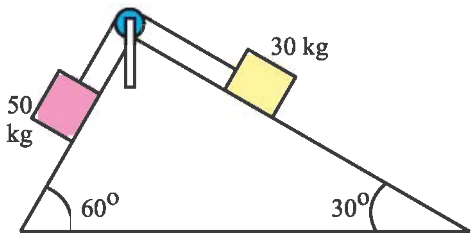


Figure 5.36

Calculate the acceleration of the blocks and tension in the string.

[Take g = 10 m s^{-2}, $\sqrt{3}$ = 1.7] [**Ans. :** 3.437 m s^2 , 253.11 N]

5. Two blocks, each of mass 3 kg, are connected by a light string and are placed on a horizontal surface. If a force of 20 N is applied in the horizontal direction on either of these blocks, the acceleration of each block is 0.5 m s^{-2} Assuming that the frictional forces on the two blocks are equal, calculate the tension produced in the string. [**Ans. :** 10 N]

6. As shown in Fig. 5.37 unequal forces F_1 and F_2 ($F_2 < F_1$) act on a rod of length L. Calculate the tension at a point situated at a distance y from end A.

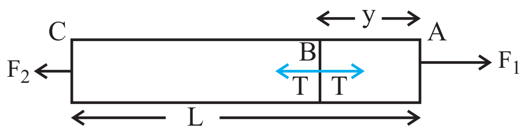


Figure 5.37

[**Ans. :** $T = F_1\left(1 - \frac{y}{L}\right) + F_2\left(\frac{y}{L}\right)$]

7. For a body of mass 2.0 kg moving in a straight line, the graph of its distance x from the starting point $\rightarrow$ time t is shown in the Fig. 5.38 Find the value of impulse of force at. (i) t = 2 s and (ii) t = 6 s for very small time intervals.

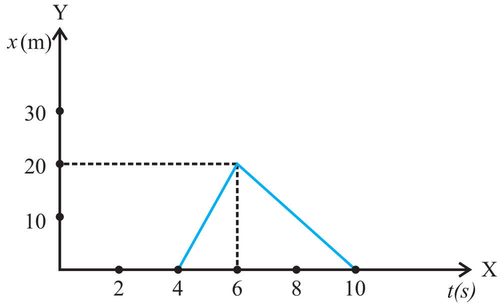


Figure 5.38

[**Ans. :** (i) 0, (ii) 30 N s]

8. Two objects of masses m_1 and m_2; start moving towards each other under the effect of only the gravitational force on each other. If the distances travelled by them are s_1 and s_2 respectively when they meet, find the ratio $\frac{s_1}{s_2}$. [**Ans. :** $\frac{s_1}{s_2} = \frac{m_2}{m_1}$]

9. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest from the top of the plane; if comes back to rest at the bottom, what is the coefficient of friction between the surface of the block and the rough surface of the inclined plane ?

[**Ans. :** $\mu = 2\tan\theta$]