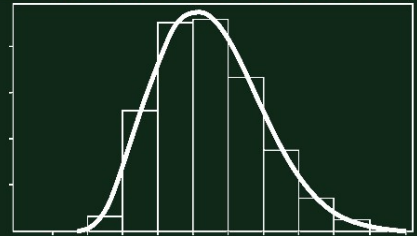Anthonie Van Laar

Alparslan Akça

Forest Mensuration

$$f(x;\alpha,\beta,\gamma) = \frac{\gamma}{\beta}\left[\left(\frac{x-\alpha}{\beta}\right)^{\gamma-1}\cdot\exp^{-\left(\frac{x-\alpha}{\beta}\right)^{\gamma}}\right]$$

Springer

FOREST MENSURATION

Managing Forest Ecosystems

Volume 13

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Aims & Scope:

Well-managed forests and woodlands are a renewable resource, producing essential raw material with minimum waste and energy use. Rich in habitat and species diversity, forests may contribute to increased ecosystem stability. They can absorb the effects of unwanted deposition and other disturbances and protect neighbouring ecosystems by maintaining stable nutrient and energy cycles and by preventing soil degradation and erosion. They provide much-needed recreation and their continued existence contributes to stabilizing rural communities.

Forests are managed for timber production and species, habitat and process conservation. A subtle shift from *multiple-use management* to *ecosystems management* is being observed and the new ecological perspective of *multi-functional forest management* is based on the principles of ecosystem diversity, stability and elasticity, and the dynamic equilibrium of primary and secondary production.

Making full use of new technology is one of the challenges facing forest management today. Resource information must be obtained with a limited budget. This requires better timing of resource assessment activities and improved use of multiple data sources. Sound ecosystems management, like any other management activity, relies on effective forecasting and operational control.

The aim of the book series **Managing Forest Ecosystems** is to present state-of-the-art research results relating to the practice of forest management. Contributions are solicited from prominent authors. Each reference book, monograph or proceedings volume will be focused to deal with a specific context. Typical issues of the series are: resource assessment techniques, evaluating sustainability for even-aged and uneven-aged forests, multi-objective management, predicting forest development, optimizing forest management, biodiversity management and monitoring, risk assessment and economic analysis.

The titles published in this series are listed at the end of this volume.

Forest Mensuration

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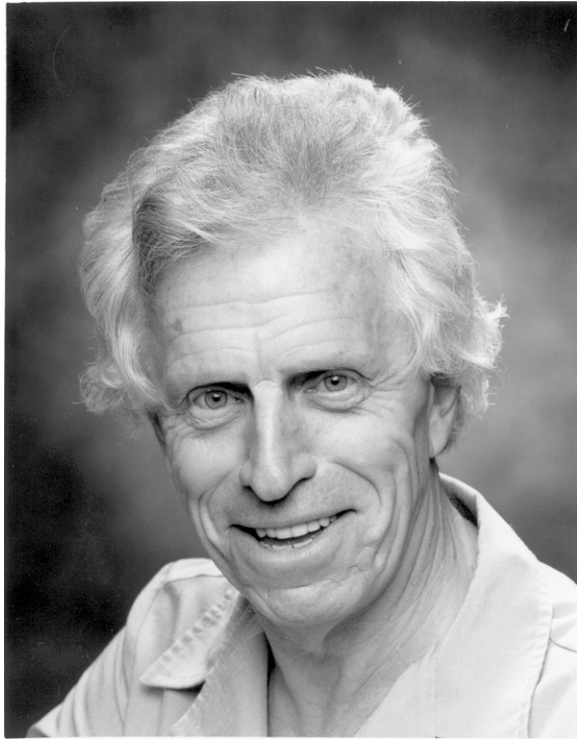
Anthonie van Laar was born in the Netherlands in 1923. He studied forest science at the University and Research Centre Wageningen between 1941 and 1949. In 1958 he emigrated to South Africa and obtained his D.Sc. degree in Forest Science at the University of Stellenbosch (1961), thereafter Dr.oec.pub (1973), and Dr.hab. (1979) at the University of Münich. His theses dealt with forest biometry and growth modeling. Since his retirement in 1988, van Laar continued his involvement in these subjects, more particularly in growth models for *Eucalyptus grandis*.

Alparslan Akça (born 1936) studied forestry at the University of Istanbul, Turkey and at the University of Freiburg, Germany. He received his doctorate in forestry at the University of Freiburg on identification of land use classes and forest types by means of microdensitometer and discriminant analyses in 1970, for his habilitations in photogrammetry and geodesy at the University of Istanbul in 1976, and in forest inventory and forest management at the University of Göttingen in 1981. He was Professor for Forest Management, Forest Inventory, and Remote Sensing at the University of Göttingen. He retired in 2001. His main research interest is in forest mensuration, forest inventory, and remote sensing in forestry.

PREFACE

Forest mensuration is one of the most fundamental disciplines within forest and related sciences. It deals with the measurement of trees and stands and the analysis of the resultant information. During the early days of sustained forest management simple measurement and estimation methods and with the analysis of inventory and research data were available. The middle of last century, however, witnessed a worldwide increase in the need for more quantitative information about trees and stands. This generated the need for more sophisticated methods to obtain and analyze forest data. This development was followed by a phenomenal explosion of information.

During the past decades there has been fruitful cooperation between the Institute of Forest Inventory and Forest Growth, formerly "Institute of Forest Management and Forest Yield Sciences" of the University of Göttingen, Germany and the Faculty of Forestry of the University of Stellenbosch, South Africa. This book is one of the results of this fruitful cooperation between these institutions.

The first edition of this book was published in 1997 by Cuvilliers in Göttingen. It was completely revised and supplemented with a presentation and review of recently developed tools and methods to measure and analyze forestry-related data. The purpose of this book is to provide information about the subject for those readers who are involved in this category of quantitative methods. Since the middle of last century the increased availability of personal computers, and software and the consensus that statistical methods are indispensable for estimating tree and stand parameters and for testing statistical hypotheses, had a considerable impact on the progress of forest mensuration as a research and management tool.

It has been written for forestry students and for practical foresters, and does not presuppose a more than elementary knowledge of mathematics and statistics. However, because of the notable influence of statistics on forest mensuration methods and techniques, and the crucial role of sampling techniques, it reviews and evaluates elementary statistical concepts. At the same time it was thought to be imperative to discuss spatial structure and diversity and to add a chapter on conventional and digital remote sensing. Numerous practical examples have been included in this edition. They are thought to be particularly useful for university and college students. Additional information about specific topics has been added for the benefit of the advanced reader and was written in italics.

Forest mensuration as a scientific tool originated in Europe and has always played an important role in the practice of forest management. In addition to conventional terrestrial methods, quantitative remote-sensing techniques form an integral part of the forest mensuration curriculum at the University of Göttingen. Countries in the southern hemisphere can derive considerable benefits from this longtime experience. Conversely, short-rotation plantation forestry in the southern hemisphere and more particularly in South Africa acted as a stimulus for the application of statistics and other quantitative methods as a decision-making tool.

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May 2007

SUMMARY

The management of forests and tree plantations requires a quantitative esti-mate of the current and future volume and biomass of timber and by-products, at national, regional, and local levels. Such information is also needed for forest-policy decisions. Forest and forestry research requires a great deal of additional information, for example about the density of forests and stands, diversity, spatial distribution of trees within stands, the size and size distribu-tion of trees within stands, and the expected growth of trees and stands. For-est mensuration is the discipline which deals with these topics. During recent years, considerable progress has been made to develop methods for measuring tree and stand characteristics, but also in instrumentation and in the statistical analysis of forest mensurational information. Sampling, based on inferential statistics, plays a dominant role in forest mensuration and forest inventory, pri-marily because of the high cost of collecting and processing field data. Mod-ern sampling methods make it possible to find an optimum sampling strategy which produces sufficiently accurate estimates at the lowest cost. Quantitative information is primarily obtained by ground surveys, aerial photographs, and satellite imagery are increasingly applied to obtain basic information about the spatial distribution of forests, possibly also to classify these forests according to specified categories and to supplement ground surveys.

In this book the authors, who are emeritus professors at the Universities of Stellenbosch and Göttingen in South Africa and Germany respectively, summa-rize and review currently used forest mensuration and forest inventory methods. A large number of worked examples have been added, primarily for the benefit of undergraduate and postgraduate students.

CONTENTS

Chapter 1

INTRODUCTION

Tree growth results from a sequence of physiological processes, consisting of the formation of new cells, cell enlargement, and cell differentiation. Tree physiology deals with the study of these processes and has made a major contribution towards a better understanding of the causal relationship between the production of dry matter and many influential interacting variables.

The science of forest growth and yield emphasizes the construction of models describing the relationship between growth parameters and influential predictor variables, and is based on forest botany, ecology, climatology, and soil science. It makes extensive use of forest mensurational techniques and inferential statistics to model tree and forest growth. To a large extent, these studies are of a phenological nature, indispensable to the forest manager in observing and quantifying growth phenomena in relation to time, site, genetic factors, and stand treatment.

Forest mensuration provides the methods and tools to conduct such studies. It concentrates primarily on the quantitative assessment of tree and stand characteristics at a given point in time during the life of the tree and stand, and provides the data required for efficient forest management. In line with the North American literature, the authors of this book contend that a discussion of empirical and analytical growth models, because of their technical nature, falls within the discipline of forest mensuration. It is not the purpose of this book to discuss advances in process models.

In conclusion, forest mensuration deals with the technical aspects of tree and forest stand measurements, such as:
- Measurement of tree and stand variables, e.g., diameter, height, basal area, bark parameters, and volume of standing and felled trees
- Determination of form and age of trees and forest stands
- Determination of the volume of standing and felled trees
- Measurements of the live crown and quantity of foliage

- Estimation of biomass and biomass components of individual trees and stands
- Estimation of the total and merchantable stand volume and its size class distribution
- Estimation of the diameter, basal area, height, and volume growth of single trees and forest stands
- Estimation of the damages to and the quality of individual trees and forest stands

In addition, it has to deal with the development of models for the construction of tree volume, taper and biomass functions, the construction of stand tables, as well as the development of growth and yield models.

Traditionally, terrestrial methods have been used to measure tree and stand variables. More recently, large-scale aerial photography has been applied successfully to replace some of the methods used in conventional ground surveys and it is for this reason that remote sensing methods have been incorporated in this book. In a broader sense, forest mensuration also deals with the estimation of volume and growth of large forest tracts, for example, in regional and national forest inventories, which are needed as a basis for forest policy decisions. This implies the application of modern sampling concepts and sampling methods, which make it possible to draw inferences about the relevant populations. Since so many excellent text books about forest inventory are available already, this book will not deal in detail with the methodology of regional and national forest inventories.

A surplus of wood and a limited demand for forest products occurred during the early Middle Ages; therefore, there was no direct need to measure the growing stock at periodic intervals. Towards the end of the Middle Ages, however, the increasing demand for timber necessitated some form of yield regulation. Felling concessions were very much limited to designated areas and replaced single-tree forest exploitation. In Central Europe, the early decades of the 18th century witnessed an increased involvement in a more scientific approach to forest measurements. Attempts were made to classify trees and forests according to their dimensions and their usefulness to the local population, although no exact measurements were carried out. During the first half of the 18th century, foresters made a beginning to improve the customary ocular methods for estimating standing timber. In France Duhamel Du Monceau (1764) initiated dendrometry as an independent scientific discipline and in Germany, Oettelt (1765) issued descriptions for the determination of the volume of felled trees and stacked wood. The last decades of the 18th century and the entire 19th century witnessed a relatively rapid development of forest mensurational methods. Hennert (1791) developed xylometric methods to determine the volume of tree

sections by measuring the amount of water displaced by the timber. Hennert also introduced sampling as a device to assess the volume of entire stands. Paulsen (1795) developed the first stem form theory and constructed the first yield tables. Cotta (1804) introduced the caliper and constructed the first volume tables. These early developments paved the way to a scientific basis for forest management and forest yield studies.

During the 20th century, there has been an emphasis on the construction and further development of better and more reliable instruments. In recent years, electronic devices for measuring tree dimensions and ring widths have been developed. At the same time, the application of more sophisticated sampling methods made it possible to obtain better and unbiased estimates at lower cost. In 1948, Bitterlich introduced the revolutionary *angle count concept*, initially to estimate the basal area per hectare. Some years later, Grosenbaugh (1952, 1958) redefined Bitterlich's variable radius method as Sampling Proportional to Size.

After early and largely unsuccessful attempts to use medium-scale aerial photographs for forest surveys, Bickford et al. (1963) introduced a two-phase sampling procedure, which combined information from a large sample of photo-plots with that obtained from a subsample of plots, which were remeasured with conventional terrestrial methods. During the last decades, large-scale color photography was introduced for forest mensurational studies, with emphasis on the measurement of the effect of stresses on needle losses and discoloration. During this period also, satellite images were used for a variety of purposes, but primarily to classify forests according to forest type and to measure or to estimate the areas covered by forest. High resolution satellite imagery, together with digital data processing, opened a new era in forest mensuration on a global scale.

The rapid development of electronic data processing and the increasing availability of powerful microcomputers has been of immense importance for mathematical and statistical advances in forest mensuration, particularly because of the inherent possibility of data storage and high-speed processing of quantitative information. Peripheral equipment and computer software to carry out stem analysis was developed and widened the practical usefulness of stem analysis (Johann 1977; Nagel and Athari 1982).

In Central Europe, the concept forest mensuration ("Holzmesskunde") is synonymous with dendrometry and stand measurements. It covers primarily the methods for measuring trees and stands, whereas the concept forest inventory ("Waldinventur") although being based on dendrometrical methods, deals with estimations and inferences of the volume and growth of larger tracts. Tischendorf's *Lehrbuch der Holzmassenermittlung*, Prodan's classic

Holzmesskunde, Akça and Kramer's *Waldmesslehre*, Pardés's *Dendrometrie*, and Anuchin's *Forest Mensuration* were structured in line with these ideas. In North America, Bruce and Schumacher's classic *Forest Mensuration*, Spurr's *Forest Inventory*, Meyer's *Forest Measurements*, Husch, Miller, and Beers's *Forest Mensuration*, and Avery and Burkhart's *Forest Measurements* combined forest mensuration with inferential statistics, sometimes with modeling, although the emphasis on statistics and modeling varied.

Chapter 2

STATISTICAL PREREQUISITES

1   INTRODUCTION

Forest mensuration deals with the measurement of trees and stands. They are mathematical variables representing different physical entities, and statistical variables with a probability distribution. Processing information about these variables requires the application of statistics and computer technology.

Some characteristics are *continuous* variables, which implies that they can theoretically take on infinitely many values. The diameter of the bole, for example, may be measured in millimetres but continuity implies that a further subdivision can continue indefinitely, although it makes no sense to measure the diameter in 1/10 mm. *Discrete* variables can assume a countable number of values. The number of trees within fixed-radius sample plots, the number of branches within a tree and the number of needles within a branch are discrete variables.

The aggregate of individuals (trees, stands forests), for which information is required, is denoted as *population*. In management inventories, information is required about the single compartment, in regional and national forest inventories the population is defined as the forest in its entirety, although it may be stratified on the basis of tree species or species groups, age class and site quality. The population is described by *parameters*, which are fixed quantities, not subject to variation. They may be size parameters, for example, the mean diameter or mean height of a stand, but also parameters of the diameter distribution, those of functions, which describe the relationship between diameter and height, between diameter and stem volume, etc. The concept population refers to a certain point in time. The parameters of a regression equation, which predicts the mean annual increment of a given species from site variables, may change because of tree breeding which produces hybrids or clones with a higher growth potential. Similarly, the mean annual volume increment of all 10-year-old *Eucalyptus* stands within a region is influenced by climatic cycles, new silvicultural techniques, etc.

Because of the high cost involved, it is impractical to measure all trees within a stand or all stands within a forest. Forest mensuration relies heavily on sampling procedures to obtain quantitative information about the resources at reasonable cost. A sample is defined as the subset of actual measurements within a given population. It is a random sample if each sampling unit has the same chance of being included into the sample. In forest inventories, the *n* plots or individual trees, which constitute the sample, are measured without replacing these sampling units after being drawn. Since this implies that a given sampling unit cannot be drawn more than once, the relevant population, contrary, for example, to controlled experiments, is finite. The construction of volume, taper, biomass functions and the development of growth models assumes that sampling, which is required to estimate the parameters of the model, can continue indefinitely. Conceptually, this population represents an infinite universe. It ignores the fact that climatic cycles or a permanent change of the physical environment of the trees may have a profound effect on the coefficients of a given equation.

2 SCALES AND UNITS OF MEASUREMENT

2.1 Scales of measurement

Different scales of measurement may be used for measuring tree and stand characteristics.

- The *nominal* scale, used for attributes, represents the weakest scale of measurement. The observation is assigned to one out of *k discrete categories*. Species, provenance, forest type and soil type, for example, are *discrete variables* which cannot be arranged in a certain order.
- The next-strongest *ordinal* scale is a *ranking* scale characterized by ordered categories and is used for *ranked variables* (*discrete categorical variables*) The scale is characterized by classes of different but unknown width. Forest soils, for example, could be categorized as poor, medium or good, the vitality of trees as healthy, sick, dying or dead, social tree classes as dominant, co-dominant, dominated and suppressed.
- Almost all forest mensurational characteristics, such as diameter, height, basal area, volume and increments, are *continuous variables*, measured on a *metric* scale.
- *The metric scale* is sometimes subdivided into an *interval* scale, without a natural zero-point and a *ratio* scale, which assumes the existence of a natural zero-point. Temperature represents the classical example of a variable which

is measured on an interval scale when expressed in degrees centigrade or Fahrenheit and on a ratio scale when measured in degrees Kelvin.

- In many cases, the *ranking scale* is converted into a metric scale by assigning numerical values to the class midpoints. Forest soils, for example, might be measured on the basis of soil depth or moisture-storage capacity or by some linear combination of these variables, with different weights being assigned to each of them.

2.2 Units of measurement

Quantitative variables are measured either in the metric or in the English system. The latter was originally used in Great Britain, in countries of the British Commonwealth and in the USA, but many countries within the Commonwealth have converted to the metric system. The most important linear, square, cubic and weight measures in the metric system and their equivalent in the English, Russian and Japanese system are presented in Appendix D.

Data representing continuous variables should be recorded with an appropriate number of significant digits. This number is obtained by counting the number of digits between the first nonzero number on the left and the last digit on the right. A tree diameter with a recorded diameter of 56 cm has two significant digits and implies that the tree has a diameter anywhere between 55.499 and 56.5 cm. A record of 56.4 cm implies that the diameter has some value between 55.349 and 56.45 cm. In consequence, when diameters are recorded in centimetres, no digits should be written to the right of the decimal point and when recording the diameter in millimetres, there should not be more than one digit to the right of the decimal point. Reproducing the data with too many significant digits gives misleading information and suggests a precision which was not achieved. However, when tree diameters (in centimetres) and tree heights (in metres) are recorded with one digit to the right of the decimal point, it is justified to record the sample mean with an additional digit to the right of the decimal point.

3 GRAPHICAL PRESENTATION OF DATA

A graphical display and interpretation of survey and research data is useful for different purposes:

- For the forest manager, a graph may be more persuasive than a summary of the results derived from a fitted model.

- It is frequently necessary to calculate confidence and prediction intervals for the true mean of a variable. This calculation is usually based on the assumption of a normal distribution. Confidence and prediction intervals, however, are sensitive to deviations from normality. In consequence, it is necessary to verify whether or not the assumption of normality is satisfied. This can be done either with the aid of histograms or stem-and-leaf plots, but alternatively, by using a statistical package, which provides estimates of the standardized coefficients of skewness and kurtosis.

The *histogram* and *frequency polygon* are usually based on grouped data representing either continuous or discrete variables. In histograms the frequencies are represented by equally wide columns, with heights which are proportional to the observed frequencies, in frequency polygons the frequencies are plotted on the *y*-axis and connected by straight lines. Both serve as a clue for the distribution function to be fitted. The observed frequency distribution, based on such grouped data, is also useful to estimate probabilities, for example,

$$P(a < x < b) \text{ or } P(x > b)$$

where *a* and *b* are selected points of a given diameter distribution. The selection of an appropriate class width is important for the construction of a histogram, which reflects the distribution adequately. Too few classes obscure the true shape of the distribution curve, whereas too many classes induce excessive variability amongst the class frequencies. A rule of thumb is to draw a sample, which is sufficiently large to ensure that 10–12 classes are generated. Sturges (1926) proposed the following function to serve as a guideline

$$k = 1 + 1.444 \ln N$$

where N = number of observations and k = number of classes. For $N = 50$, the number of classes should be between 6 and 7, for $N = 100$ between 7 and 8, whereas 9 classes are adequate for $N = 250$. To reconstruct the diameter distribution of a stand it is impractical to define classes with fractions of 1 cm as class widths. One-centimetre classes are normally used to obtain the frequency table, but 1 mm classes are required for research purposes, whereas a width of 2 or 4 cm is adequate for management inventories.

Example 2.1   The breast height diameters of 253 trees in a *Pinus radiata* stand are given in Appendix B. The trees were measured in millimetres and grouped in 1 and 4 cm diameter classes. The frequency polygons based on these class widths are shown in Figure 2-1.

In recent years, *stem-and-leaf* plots have become increasingly popular, particularly for small samples. The observations are summarized by a number of

Figure 2-1. Frequency polygons for the diameter distribution in *Pinus radiata* of Appendix B.

vertical lines representing the data in a condensed form. The data are split into one part representing the *stem* and a second part, which is denoted as *leaf*. The first item on a line is usually expressed by the digit to the left of the decimal point and is labelled as *starting part*, the additional information is shown in the leaf, above the stem.

Example 2.2   The first 50 diameters were used to obtain a stem-and-leaf display. The numbers to the right of the decimal point have been entered in the sequence in which they were recorded

					3										
					7		1	1							
					3	6	5	5			5				
	5		1		6	1	2	8	7	3	3				4
2	4	3	6	7	4	2	4	5	0	3	0	5	0		1
7	4	4	0	0	0	3	8	5	5	0	5	9	5	0	0
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29

4   DESCRIPTIVE STATISTICS

The parameters of a distribution are used for inferential as well as descriptive purposes. They can be stored in the memory of a computer, to be retrieved for specific purposes. They are categorized as *location, dispersion* and *shape* parameters, which are estimated by sampling unless the entire population was measured. In a complete stand enumeration, for example, no sampling errors are involved in determining the mean-stand diameter. Different symbols are used for the population and sample respectively.

The *arithmetic* mean is a location statistic, estimated from:

$$\bar{x} = \frac{\sum\limits_{i=1}^{n} x_i}{n}$$

where n = sample size. When all individuals of the population of size N have been measured the sample mean $\bar{x}$ in the above formula is replaced by the population mean μ_x and sample size n by N. The sample mean provides an unbiased estimate of the population mean μ_x, if the sample was drawn at random and no instrument or other errors were involved. To remove the effect of extreme observations, some statistical packages provide the option of estimating the trimmed mean, by excluding data above and below a specific cut-off point.

The *median* (x_M), which partitions the frequency distribution into two equal halves, is a useful descriptive statistic in excessively asymmetric distributions, since it ensures that 50% of the sample data are located above and below the median, respectively. The sample median is calculated as follows:

$$x_M = x_L + \frac{\left(\sum\limits^{k} n_j\right)/2 - \sum n_{x=x_L}}{f_{x_M}} \cdot w$$

where

x_L = lower limit of the class containing the median

$\sum\limits_{j=1}^{k} n_j$ = total frequency

$\sum n_{x=x_L}$ = cumulative frequency below $x = x_L$

f_{x_M} = number of observations in the class containing the median

w = class width

The *geometric mean* of a sample of size n is defined as follows:

$$x_G = \sqrt[n]{x_1 \cdot x_2 \ldots \ldots x_n}$$

and is calculated as the antilog of the mean of the logarithm of the n observations:

$$x_G = e^{\left(\sum \ln x_i\right)/n}$$

Percentiles (x_P) are location statistics indicating the value of x in the ordered set of data, associated with p per cent $(0 < p < 100)$ of them being smaller

than the pth percentile. The median, for example, represents the 50th percentile of the distribution. Percentiles are useful descriptive statistics when the real distribution function is unknown. The *mode* of a distribution is found as the midpoint of the class with the highest frequency. A more accurate formula is:

$$\text{Mode} = d_l + \frac{f_M - f_{M-1}}{2 f_M - f_{M-1} - f_{M+1}} \cdot w$$

where d_l = lower limit of the diameter class with the highest frequency and w = class width.

The *variance* of the population is defined as the mean of the squared deviation of the variable x from the population mean:

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

where N = population size. The formulae for the sample variance, calculated from ungrouped and grouped observations respectively, are:

$$s^2 (\text{ungrouped}) = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}; \quad s^2 (\text{grouped}) = \frac{\sum_{j=1}^{k} f_j (x_j - \overline{x})^2}{\sum f_j - 1}$$

The corresponding working formulae are:

$$s^2 (\text{ungrouped}) = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n - 1} \quad s^2 (\text{grouped}) = \frac{\sum_{j=1}^{k} f_j x_j^2 - \frac{\left(\sum_{j=1}^{k} f_j x_j\right)^2}{\sum_{j=1}^{k} f_j}}{\sum_{j=1}^{k} f_j - 1}$$

where k = number of classes. The sample variance s^2 represents an unbiased estimate of the population variance σ^2, if the sample was drawn at random. The standard deviations σ and s are defined as the square root of the population and sample variance, respectively. The coefficient of variation is obtained by expressing the standard deviation s as a percentage of the sample mean:

$$s_x (\%) = 100 \frac{s}{\overline{x}}$$

The *range* is defined as the difference between the largest and smallest observation:

$Range = x_{max} - x_{min}$ and the *interquartile* range as the difference between the 75th and the 25th percentile of the distribution:

$Interquartile\ range = x_{75} - x_{25}$ In order to obtain the percentiles for grouped data, we assume a uniform distribution of the observations within each class.

Example 2.2   Descriptive statistics are to be calculated for the observed diameters, measured in millimetres in the *Pinus radiata* stand of Appendix B. The sample mean, sample variance, standard deviation and coefficient of variation are:

$$\bar{x} = 21.8 \quad s_x^2 = 24.60 \quad s_x = 4.96 \quad s_x(\%) = 22.8$$

Trimming the sample, with the 10th and 90th percentile of the distribution as lower and upper limit gives:

$$\bar{x} = 21.9\,\text{cm} \quad s_x^2 = 10.35\,\text{cm}^2 \quad s_x = \pm 3.22\,\text{cm} \quad s_x(\%) = 14.6$$

The lowest and highest recorded diameters are 7.9 and 34.8 cm, respectively. The range calculated from the ungrouped data is therefore 26.9 cm. When based on 1 cm diameter classes, the range is $35.5 - 7.5 = 28.0$ cm. The 25th and 75th percentile of the diameter distribution, based on ungrouped data are 18.9 and 25.2, respectively. Hence, the interquartile range is 6.3 cm. The median of the distribution is 22.0 cm. When calculated for grouped data with 1 cm diameter classes, we obtain:

$$d_M = 21.5 + \frac{126.5 - 119}{20} \cdot 1 = 21.9\,\text{cm}$$

The standardized skewness and kurtosis are -0.23 and 0.27 and do not indicate a non-normal diameter distribution.

The mean and variance are useful numerical expressions for the location and dispersion of a distribution. The distribution of the subject variable, for a given population mean and variance, can then be recovered by using the table of the normal distribution. When based on sampling, however, the distribution reflects the expected and not the true distribution. In a practical application, the proportion of trees within a stand, which fall within a specified-height class, can be determined, if the total number of trees or the number of trees per hectare is known and the mean and variance are either estimated by sampling or from regression equations with influential independent variables as predictors. This remains an approximation, because the true mean and variance are unknown. In many instances, more particularly in diameter distributions, the frequency

curve is drawn out to the right because of thinning from below, which remove trees in the lower tail of the distribution, or it is skewed to the left, because of selection thinning which remove the dominants within a stand.

5   PROBABILITY DISTRIBUTIONS

The probability distribution of a variable x is a function or rule, which defines the probabilities $P(x = x_i)$ for discrete variables and $P(a < x_i < b)$ for continuous variables. The present chapter deals with some distributions which play a role in inferential statistics. Others, used to describe the size distribution of tree characteristics within tree populations, are discussed in Chapter 5.

5.1   Normal distribution

The *normal distribution* is uniquely defined by its mean μ variance σ^2. The normal curve is symmetric and bell shaped, has two inflection points and intersects the abscissa at positive and negative infinity. If the observed values of such a variable were represented by histograms of ever decreasing class width, the step function, which characterizes the histogram of this frequency distribution, would gradually approach a smooth normal curve with infinitely many classes. The density function

$$f\left(x; \mu, \sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left[\frac{x-\mu}{\sigma}\right]_2}$$

gives the ordinates of $f(x)$ for different values of x. Each normal curve is defined by its specific mean and variance. For inferential purposes, for example, to calculate the limits of the interval containing 90% of the values of the subject variable, it is necessary to express x as a deviation from the population mean, with standard deviation as unit of measurement. The variable:

$$z = \frac{x-\mu}{\sigma}$$

is called the *unit normal variate z* and has a normal distribution with zero mean and unit variance. Plotting the cumulative distribution $F(x)$ as a function of x or $F(z)$ as a function of z produces a sigmoid curve, extending between positive and negative infinity. The probability $P(z < z_i)$, which is found as the antiderivative of the density function, has been tabulated for z values between -3 and $+3$. The table can be found in any elementary textbook on statistics and shows, for example, that 68.27% of the z-values fall between -1 and $+1$,
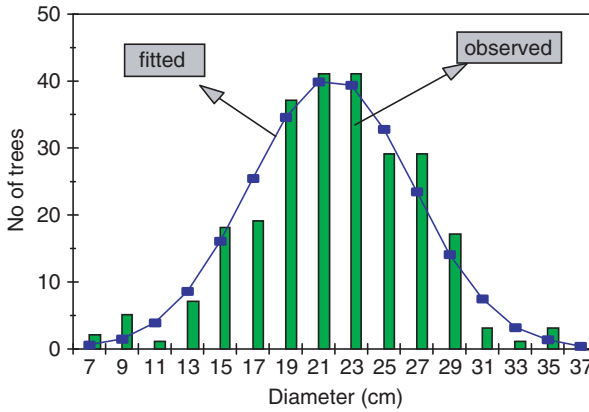
Figure 2-2. Fitted normal distribution.

whereas 95.45% are located between -2 and $+2$ and 99.73% fall between -3 and $+3$. It implies that about 68.3% of the observations of the distribution of x lie within the interval $\mu \pm \sigma$, whereas 95.4% lie within the interval $\mu \pm 2\sigma$ and 99.7% within the interval $\mu \pm 3\sigma$. To some extent, this property can be used to test for normality. For example, if less than 2.5% of the data are smaller than $\mu - 2\sigma$ and more than 2.5% are greater than $\mu + 2\sigma$, there is some evidence of an extended right tail. However, there are more efficient methods to tests for normality, for example, the *Shapiro–Wilk test* and the test based on the distribution of Fisher's g statistics.

   The distribution of many tree characteristics within a given population, for example, that of crown length, breast height diameter, tree height, sapwood area, root and leaf biomass within an even-aged stand can frequently be approximated as a normal distribution (Figure 2-2). The model assumes that random factors have an additive effect on the subject variable and specifies that the distribution of the variable consists of many independent, but not necessarily normal distributions. In biological populations, perfectly normal distributions seldom occur, if ever. When the effect of random factors is multiplicative, the variable x follows the lognormal distribution, in which case the transformed variable, $\ln(x)$, has a normal distribution. The asymmetry, which is apparent in the frequency curve or histogram of the experimental data of a lognormal distributed variable, is then eliminated by a logarithmic transformation of the subject variable.

Example 2.3   A normal distribution was fitted to the data set of 253 tree diameters of Appendix B. The unit's normal deviate was calculated for the lower and upper limit of each 2 cm diameter class (Table 2-1). For the diameter class with a class midpoint of 11 cm, their values are:

Table 2-1. Observed and fitted frequencies based on the assumption of a normal distribution

dbh	n_{obs}	n_{fit}	dbh	n_{obs}	n_{fit}
7	2	0.6	23	41	39.4
9	5	1.5	25	29	32.8
11	1	3.9	27	29	23.5
13	7	8.6	29	17	14.1
15	18	16.1	31	3	7.5
17	19	25.5	33	1	3.3
19	37	34.6	35	3	1.4
21	41	39.9	37	–	0.4

$$z_1 = \frac{10 - 21.79}{4.96} = -2.377 \quad P(z < -2.377) = 0.0087$$

$$z_2 = \frac{12 - 21.79}{4.96} = -1.974 \quad P(z < -1.974) = 0.0242$$

The expected number of trees in this class is:

$$n_{fit} = 253 \cdot (0.0242 - 0.0087) \cong 4$$

5.2 Binomial distribution

Suppose that a survey is carried out to estimate the probability of a tree being alive for k number of months after stand establishment. The proportion of living plants within a random sample of size n is determined at the preselected point in time. The number of plants alive follows the binomial distribution, if the events are independent, i.e., if the probability of a given tree being alive is statistically independent of the status of its neighbor. The assumption of an infinite population is approximately satisfied if the population is large. When the proportion of trees alive is known for a given population, and equal to p, the probability that a sample of size n, with the trees being "replaced" after their selection, contains x living trees ($0 \leq x :< n$) is given by the distribution function

$$f(x; n, p) = \binom{n}{p} p^x (1-p)^{n-x} \quad \text{where}: \binom{n}{p} = \frac{n!}{p!(n-p)!}$$

The sample estimate $\hat{p}$ estimates the population proportion p. Either exact tables are used to obtain confidence intervals for the parameter p, or the binomial distribution is approximated as a normal distribution. The asymmetric step
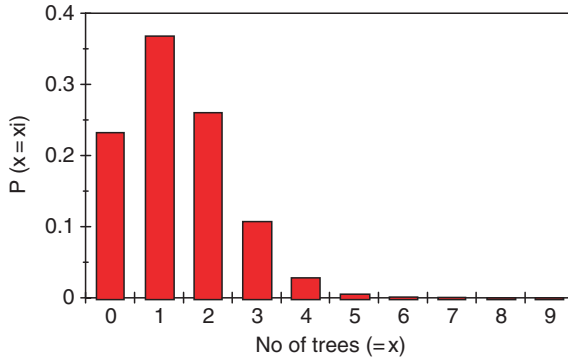
Figure 2-3. Fitted binomial distribution.

function, which characterizes a specific binomial distribution, converges to the normal distribution as p tends to 0.5 and sample size tends to infinity. The approximation is justified if either np or $n(1 - p)$ is greater than 15.

In case of *contagion*, i.e., when the occurrence of the event in question influences the probability of a second occurrence and a given tree is more likely to be dead if its neighbour is dead, other distribution functions, for example, the *negative binomial* or the *Neyman* distribution, should be fitted (Figure 2-3).

Example 2.4   The trees in a 5-year-old *Pinus radiata* plantation are classified as alive or dead, respectively. The true proportion of trees alive ($= p$) is 0.85. We assume a random spatial distribution of mortality and calculate the probability of a 3×3 rows plot containing 0, 1, ..., 9 trees alive. The probabilities are obtained by substituting $x = 0, 1, \ldots, 9$ into the above formula:

$P (n = 9) = 0.85^9 = 0.232$

$P (n = 8) = 9 \cdot 0.85^8 \cdot 0.15 = 0.368$

$P (n = 7) = 36 \cdot 0.85^7 \cdot 0.15^2 = 0.260$

....

$P (n = 0) = 0.15^9 = 0.000\ldots$

5.3   Poisson distribution

The Poisson distribution represents the limiting case of the binomial distribution which occurs:
- If one of the two events occurs rarely ($p \to 0$)
- If a large sample is drawn ($n \to \infty$, so that $M = n \cdot p$ is finite)
- If the assumption of independence is not violated

Table 2-2. Observed and fitted frequencies based
on the assumption of a Poisson distribution

No. of trees	No. of plots	
(= x)	Observed	Fitted
0	1045	1039
1	61	73
2	9	3

The probability of the event occurring x times is given by the function:

$$P(x, \mu) = \frac{\mu^x}{x!} e^{-\mu}$$

Example 2.5   The number of trees of the species *Ocotea bullata* in $200\,\text{m}^2$ sample plots in the indigenous forests of the Cape Province of South Africa was counted in 1115 sample plots (Table 2-2). The sample mean is:

$$\bar{x} = \frac{0 \cdot 1045 + 1 \cdot 61 + 2 \cdot 9}{1115} = 0.07085$$

The expected number of plots containing 0, 1, 2, 3, … trees is determined by multiplying the probabilities obtained from the Poisson function by the total number of trees.

5.4   Distribution of χ^2

When random samples of size n are drawn from a normal distribution, the statistic

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma} \right)^2$$

follows the χ^2 distribution with n degrees of freedom, whereas

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma} \right)^2$$

has a χ^2 distribution with $n - 1$ degrees of freedom.

Applications of the χ^2 distribution:
- The trees within a mixed stand are classified according to species and their social class. Sample trees are selected at random and cross-classified into an r × c contingency table. The test hypothesis is: species and social class are statistically independent
- Suppose that n trees are selected at random in a mixed forest, which contains three species. The χ^2 distribution is applied to test the hypothesis that these species occur in a specified ratio, for example 1:3:2

5.5   Distribution of student's t

The *t-distribution*, which is closely linked with the normal distribution, is eminently important for many inferential purposes, for example, to calculate a confidence interval for the estimated mean of a distribution, if the true variance is unknown and the sample relatively small (s. Chapter 10) and for testing hypotheses. The statistic t is defined as follows:

$$t = \frac{\overline{x} - \mu}{s_{\overline{x}}}$$

Each t-distribution is uniquely defined by the number of degrees of freedom (df). The latter is found by subtracting the number of parameters involved from the number of sampling observations: df $= n - 1$. The t-distribution for different degrees of freedom is shown in Figure 2-4.

The sample variance s^2 estimates the population variance σ^2. With increasing sample size, the sample variance converges to the population variance and the t-distribution tends to the distribution of the unit normal variate z. The two distributions are identical for infinite degrees of freedom.
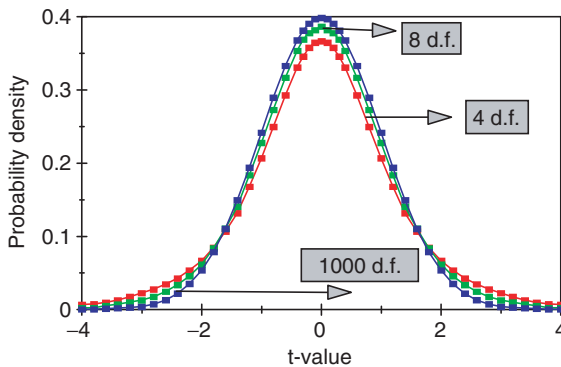
Figure 2-4. Distribution of t for *wit*h different degree of freedom.

Applications of the t-distribution

- *Suppose that it is required to establish whether a new make of a hypsometer, when properly used to reduce operator-bias, gives unbiased estimates of the tree height. A random sample of n trees is selected and the tree heights are measured from two opposite directions. Their mean estimates the true height. The sample trees are felled and their true length is measured with a tape. The test hypothesis is that the mean of the differences between the true and observed heights is zero. Sampling generates a population of differences between observed and true heights. Assuming that this population has a normal distribution, the mean of the differences is divided by the standard error of the mean and compared with the two-sided t-values for $n - 1$ degrees of freedom*

- *The breast height diameter of n randomly selected trees is measured with a tree fork as well as a caliper. The measurements are made from random directions. In this situation, the true diameter is unknown and it not feasible to test for instrument-bias. The hypothesis of a zero difference between the means, however, is testable. The calculation of the test statistic is identical with that in the previous example*

- *In a clonal study, it is required to test the hypothesis that the mean wood density of clone A does not differ from clone B. Two random samples are drawn from stands of a given age and site index. However, there may be contributory factors which influence wood density, in addition to age and site index. For this reason, it is necessary to select k stands of each clone and n trees within each stand. The t-test for independent sampling serves to test the differences between the clone means. The appropriate t-value is obtained by dividing the difference between the means by its standard error, with the mean square for error being calculated as a pooled within-stands mean square. The t-statistic follows the t-distribution with $2n - 2$ degrees of freedom if the assumption of variance homogeneity holds true.*

- *In a study of the effect of drought on radial growths, the hypothesis is tested that tree size and severity of stresses, measured in terms of estimated needle loss are unrelated. The study is based on measurements within a single stand. Assuming a linear relationship between the two variables, the test hypothesis is that the true value of the regression coefficient is zero. The observed parameter estimate is divided by its standard error to produce a t-value, which has n-2 degrees of freedom. Alternatively and equivalently, the test hypothesis could have been formulated that the population correlation coefficient is zero*

- *If the test hypothesis in the previous case is rejected, the t-distribution is used to calculate a confidence interval for the parameter(s) of the equation*

- *In a multiple regression situation, site index is estimated from five site variables, which have been selected as being statistically significant predictor variables. A confidence interval is calculated for each of the five regression coefficients, in which case $(1 - \alpha)100\%$ of the intervals, generated by repeated sampling, contain the true parameter*

6   ESTIMATION

Methods and procedures in inferential statistics deal with two important inter-related topics which have different objectives:
- Estimating parameters
- Testing hypotheses

6.1   Bias, precision and accuracy

Suppose that the mean height of 20 trees is estimated by sampling, with $n = 5$. The number of distinct samples of size 5 is given by

$$\binom{20}{5} = \frac{20!}{5!15!} = 15504$$

The sample mean produces an unbiased estimate of the population mean, if the mean of the 15504 sample means is equal to the population mean. Random sampling ensures unbiasedness, but only if no instrument or operator bias is involved. Instrument bias includes instrument errors such as using a worn diameter tape to measure tree diameters, faulty hypsometer or a caliper with the movable arm not being at a right angle to the fixed beam. Operator bias occurs when over- or underestimating the number of trees, which are counted "in" when applying Bitterlich's method, when measuring the tree heights from a single direction, on sites with a prevailing wind direction and the distance between operator and the tree is not adjusted when measuring tree heights on slopes.

Methodology-related errors, for example rounding-off the observed heights and diameters in upward or downward direction, using ratio estimators when regression estimators are more appropriate (Chapter 10), failure to adjust the variance in sampling without replacement in finite populations (Chapter 10), estimating the tree volume or tree biomass from a regression equation with log-transformed diameter as the independent variable and log-transformed volume or weight as the dependent variable (Chapter 7 and 8), assigning unit weight in a regression analysis, for example, with volume (or weight) as the target variable and d^2h as the predictor, when there is evidence of variance heterogeneity.

Other sources of bias may be involved as well, for example, applying a certain growth model to trees of different genetic material or in stands of different silviculture. In growth modeling, the expected yield at the end of the rotation, or at any other point in time, is usually estimated from age, site index and stand density. Because of the effect of external and frequently unknown factors, however, the growth model may be valid and produce unbiased estimates in certain regions, but not in others. Furthermore, an incorrect model may have been used if too few influential variables were included as predictor variables.

The objective of a forest inventory is primarily to obtain estimates for population aggregates, for example, the total or merchantable volume of a compartment or a group of compartments, or to forecast growth per unit area. Possible sources for obtaining biased estimates are discussed in different chapters of this book. In general, bias should be avoided or minimized, although there are situations where biased estimates of population parameters are closer to the true values.

Precision is synonymous with repeatability and expresses the closeness of the measurements to their mean. Precision is conveniently measured by sample variance and coefficient of variation. Such estimates, however, are sensitive to sample size and outliers in the data space.

Accuracy combines bias and precision and expresses the closeness of the observed measurements to their true values. Unbiasedness, combined with high precision, produces accurate estimates. Using a worn diameter tape with a 2% systematic deviation between the observed and true diameter produces inaccurate estimates of the diameter, although repeated measurements on the same tree vary moderately. Measuring the same tree from a single random direction (in millimetres) with a caliper manufactured from suitable material may produce unbiased estimates, but repeated measurements on the same tree at the same position reveal a much greater variability, because of the irregular cross section of the bole (see Figure 2-5).

| Bias | | | → | *Low accuracy* |
| Low precision | | | | |

| Zero bias | | | → | *Medium accuracy* |
| Low precision | | | | |

| *Medium* bias | | | → | *Medium accuracy* |
| High precision | | | | |

| Zero bias | | | → | *High accuracy* |
| High precision | | | | |
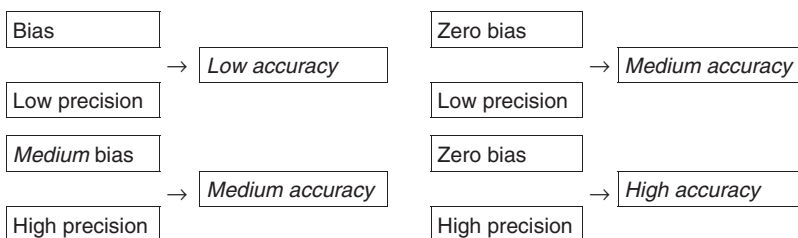
Figure 2-5. Relationships between bias, accuracy and precision.

6.2   Estimators

Because of the necessity to apply sampling in order to obtain information about the tree and stand variables and their distribution at reasonable cost, the estimation of relevant parameters plays a dominant role in forest mensuration. The sample mean and its variance estimate the population mean and variance. They are expressed in a single figure and called *point estimators*. Repeating the sampling procedure under the same conditions produces different estimates. The performance of the estimator is measured by the closeness of the estimates, which result from repeated sampling. This in turn, is partly controlled by sample size, partly by other largely unknown factors. In studies of the relationship between site and growth, for example, the amount of unexplained variability is frequently unacceptably high because the model does not include influential predictor variables. This in turn, has an adverse effect on the quality and the practical usefulness of the model.

Since point estimates do not provide information about their variability in repeated sampling, a confidence interval for the parameter has to be calculated. The expected variability of the point estimates is controlled by its variance and sample size. A confidence interval is associated with a quantifiable uncertainty that the interval will indeed contain the parameter. The confidence coefficient $(1 - \alpha)$ implies that $(1 - \alpha)\%$ of the intervals, obtained by repeated sampling, contain the population parameter. The confidence coefficient $(1-\alpha)$ is the compliment of α, the two-sided level of significance in hypothesis testing. The table of the t-distribution is consulted to obtain a confidence interval for the population mean, but assumes a normal distribution of the subject variable. However, with increasing sample size, the distribution of sample means converges to the normal distribution, reliable estimates of the confidence interval are obtained unless such estimates are based on small samples from extremely skewed distributions. Cochran (1953) proposed the rule:

$$n \geq 25g_1^2$$

where $g_1 =$ Fisher's standardized measure of skewness

For discrete variables which follow the binomial distribution, the large-sample approximation of confidence intervals is also based on the model of the normal distribution. The normal approximation is permissible for large samples, if the population proportion does not differ too much from 0.5. Some authors apply the crude rule to prescribe that np as well as $n(1 - p)$ are greater than 15. For proportions below and above 0.3, however, a larger sample should be drawn to ensure reliable confidence intervals for the population mean. The χ^2-distribution is used to obtain a confidence interval for the population

variance of continuous distributions. Their reliability is not seriously affected by sample size, unless they are based on excessively small samples.

Example 2.6   The following sample of size 20 was drawn at random from a population of tree diameters within a given stand:

| 53.6 | 46.0 | 48.2 | 38.8 | 32.6 | 50.0 | 33.8 | 44.5 | 46.1 | 49.5 |
| 44.1 | 50.0 | 48.0 | 57.1 | 41.7 | 48.1 | 47.8 | 42.5 | 47.6 | 41.6 |

The assumption is that the subject variable has a normal distribution. The sample mean and variance are 45.58 and 35.6, respectively. In order to obtain the 95% confidence interval for the population mean, we require the 97.5th percentile of the t-distribution with 19 degrees of freedom: $t_{0.05;19} = 2.093$. Ignoring the correction for the finite population (see Chapter 10), the confidence interval is as follows:

$$45.58 \pm 2.093 \sqrt{\frac{35.6}{20}}; \quad 44.25 - 46.91$$

It ensures a 95% chance that the calculated interval contains the parameter in repeated random sampling from the same population. In the present case, the population mean is known to be 47.38 and is indeed located within the calculated interval. The same sampling observations are used to calculate a 95% confidence interval for the population variance σ^2. The confidence interval is as follows:

$$\frac{(n-1) \cdot s^2}{\chi^2_{0.025;19\,df}} < \sigma^2 < \frac{(n-1) \cdot s^2}{\chi^2_{0.975;19\,df}}; \quad 42.42 - 48.74$$

6.3   Estimating accuracy

In many situations, it is necessary to test the accuracy of a model against new data, not used to estimate the parameters of the model, for example to test the accuracy of an existing tree volume equation, in order to decide whether or not to update the volume function. In other cases, it might be necessary to evaluate a new measuring technique, for example, to test a new instrument for measuring upper-stem diameters. Freese (1960) approached this research question as a hypothesis testing problem and suggested to proceed as follows:
- Specify the required accuracy
- Determine the accuracy attained by the model
- Apply a method to test whether the required accuracy was achieved

The observed values in a sample of size n are compared with those being considered as being "true" values. When testing the accuracy of a tree volume table or a stand level growth equation, for example, observed values will be compared with those from the volume table or growth model, respectively. When testing a new instrument for measuring upper stem diameters, the true value is obtained by measuring this diameter accurately, for example, by climbing the tree to reach the point of measurement. The standard procedure for determining sample size prescribes that a specified maximum error of E units will not be exceeded with a probability greater than 1% or 5%, or any other selected probability. The squared maximum error is found as the product of the appropriate unit normal variate z, with $z = 1.96$ for $(1 - \alpha) = 0.95$ and $z = 2.576$ for $(1 - \alpha) = 0.99$. The requirement will be met if:

$$s_{\bar{x}}^2 \leq \frac{E}{z_{1/2\alpha}^2}$$

The conventional

$$\chi^2 = \sum_{i=1}^{n} \frac{(x_i - \mu_i)^2}{\sigma^2}$$

with μ_i being the "true" value of the ith sampling unit and σ^2 representing the required accuracy in terms of variance, has a χ^2-distribution with n degrees of freedom. In order to decide whether or not a new technique or a new model meets the specified accuracy, the value σ^2 in the above formula is replaced by $E/z_{1/2\alpha}^2$. A high value of χ^2 would indicate that the new technique or model is not sufficiently accurate. This may be due to bias, to lack of precision or both. In order to remove the effect of bias, defined as the mean of the differences between the estimated and observed values of x, it was suggested to introduce the modified χ^2-formula

$$\chi^2 = \sum_{i=1}^{n} (x_i - \mu_i)^2 - \frac{Bias^2}{\sigma^2}$$

which is approximately χ^2-distributed, with $n - 1$ degrees of freedom. The bias is estimated from the mean deviation of the observed values from model predictions.

Example 2.7   The stem diameter of 10 trees was measured with a diameter tape and caliper respectively. The tape was assumed to represent a nearly error-free measurement. In both instances the diameter was measured in millimetres. The caliper was used with a single measurement per tree, which was obtained from a random direction. The measurement error can therefore be expected

to be substantial. The (fictitious) data for tape (y_1) and caliper (y_2) and the observed differences (d) are as follows:

					Diameter					
y_1	22.5	26.3	19.5	19.8	23.7	26.3	21.5	28.3	26.2	30.1
y_2	22.1	26.9	19.0	19.4	24.1	24.9	22.3	26.9	27.3	31.6
d	+0.4	−0.6	+0.5	+0.4	−0.4	+1.4	−0.8	+1.4	−1.1	−1.5

A maximum error of 9 mm is tolerated, with a specified probability of 5% that the error will be exceeded. The hypothesized variance is:

$$\sigma^2 = \frac{0.9^2}{1.96^2} = 0.211$$

and

$$\chi^2 = \frac{9.11}{0.211} = 43.2, \text{ with 9 degrees of freedom } (\chi^2_{0.05} = 16.9)$$

The estimated bias is $\sum d/n = 0.11$ cm and the adjusted χ^2 is

$$\chi^2_{\text{adj.}} = \frac{9.11 - 10 \cdot 0.11^2}{0.211} = 42.7$$

In conclusion, the caliper measurements fail to produce the specified accuracy.

7 REGRESSION AND CORRELATION ANALYSIS

7.1 Simple linear regression

Many forest mensuration studies require the estimation of a dependent variable y from a single predictor variable x, for example, to estimate tree height from diameter at breast height, wood density from tree age, timber yield per hectare from spacing. In spite of the remarkable progress in computerized processing of forest mensuration data, it remains essential to prepare a graph, with the dependent variable being plotted over the predictor variable, to establish whether the relationship is linear and explore a suitable transformation of scales in instances of nonlinearity. The modern computer packages such as EXCEL are eminently suited for this purpose.

The two-parameter simple regression equation is based on the model

$$y_i = b_0 + b_1 x_i + e_i$$

with x and y representing the predictor and target variable respectively. In case of nonlinearity, they are replaced by a function of x and y. Alternatively a
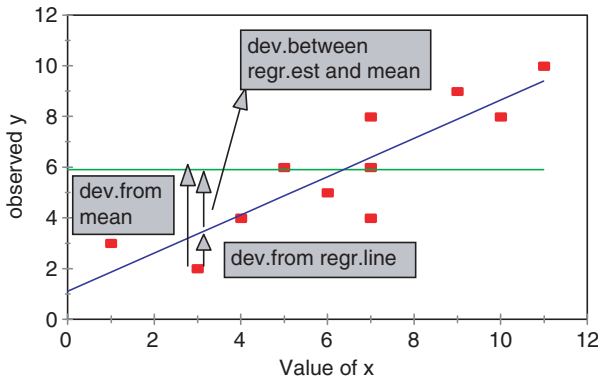
Figure 2-6. Partition of deviations from the mean in regression analysis.

second-degree equation is fitted or an intrinsically nonlinear model is used. The coefficient b_0 expresses the intercept, i.e., the value of y for $x = 0$ and b_1 represents the slope of the regression line, i.e., increase or decrease of y per unit increase of x. The residual e_i is the deviation between the ith observation of y (for $x = x_i$) and the regression estimate. The two coefficients are calculated by least squares analysis which ensures that the sum of the squared deviations from the fitted regression line is minimized. See Figure 2-6.

Computer programs are available to estimate the two parameters of the model, together with confidence intervals and to draw inferences about predicted y values.

7.2   Correlation analysis

In many situations there is no clear distinction between predictor and target variable, but it is necessary to know whether two variables, x_1 and x_2 are related. The correlation coefficient $r((|r| \leq 1)$ which assumes a linear relationship between the two variables can be tested for its statistical significance. The computer program for regression will also provide information about the correlation coefficient and its confidence interval.

7.3   Multiple regression analysis

Normally more than one predictor variable is used to estimate the target variable. The basic principle which underlies regression analysis with one predictor variable is retained, but extended to k predictor variables. The computer

program for multiple regressions also provides various options to determine the "best" model. It will also estimate the multiple correlation coefficient between y and the group of k predictors. In multiple regression the k predictors are usually continuous variables, but in many cases it is necessary to regress the target variable on one or more than one continuous variables but in addition on a nominal variable, for example to regress tree height on bark thickness at 50% of the height, for k number of different species. An analysis of covariance, with the aid of dummy variables, is then necessary to decide whether tree height is significantly related to bark thickness in presence of species as a discrete predictor variable. A comprehensive discussion of multiple regression and covariance analysis falls outside the scope of this book. Several textbooks are available which deal with such models and fitting procedures (Draper and Smith 1981; Ott 1988; Kleinbaum and Kupper 1978). Practical examples of multiple regression will be provided in later chapters.

7.4   Nonlinear regression

The standard simple or multiple regression model deals with those situations where the equation can be linearized either by a transformation of variables or by the addition of functions of the predictor variables. There are many cases, more particularly in growth modeling, where the equation is not be linearizable, for example $y = b_1(1 + b_2 \exp(b_3 x)^{b_4}$. Computer programs, for example PROC NLIN in the SAS system provide algorithms to obtain least squares parameter estimates for such equations. The program requires that the user specifies the model and provides initial estimates of the parameters. The program then searches for a vector of parameter estimates which produces the smallest sum of squared residuals.

8   MOVING AVERAGE

The moving average defines a new variable y as a linear combination of p consecutive observations of the time series or those of observations which are sequential in space

$$y_1 = a_1 x_1 + a_2 x_2 + \ldots \ldots . a_k x_k$$

with $a_1 = a_2 = \ldots . ak = 1/k$. The first element of the new time series with $k = 4$ therefore is

$$y_1 = (x_1 + x_2 + x_3 + x_4)/4$$

The second element is obtained by deleting x_1 and adding $x_k + 1$:

$$y_2 = (x_2 + x_3 + x_4 + x_5)/4$$

The new time series contains $(n - k + 1)$ observations. The method of moving averages induces a smoothening of the time series by removing short-term fluctuations. The smoothening process is more severe when the means are calculated for large k, but at the same time the occurrence of meaningful fluctuations may not be detected when too many observations are used to calculate a mean. A moving average, based on low values for k, removes short-term fluctuations and is called a low-pass filter, whereas a large value of k removes long-term fluctuations and is described as a high-pass filter. In some cases, the time series is characterized by oscillations with a constant period, for example, when measuring the radial increment of single trees at hourly intervals. This type of oscillation is completely removed by selecting $k = 24$. In other cases, the oscillations contain irregular elements and the calculation of moving averages merely smooths the time series.

9   SMOOTHENING BY FITTING EQUATIONS

In a study of the impact of stresses on radial growth, it might be appropriate to fit a negative exponential or another model to filter the age effect. In other cases, a polynomial or a spline function is fitted to the observed ring widths. The next step will then be to determine the differences or the percentage deviations from the trend curve. This procedure converts the time series of observed y-values into a times sequence of deviations, which can be tested for serial correlations.

The sequence of deviations shows a cyclical pattern. They could possibly be removed by fitting a polynomial including higher powers of age, but there is no biological explanation for such a model.

Example 2.8   Sample plots consisting of 2×2 rows of trees were established along a transect in a *Pinus radiata* stand, with no buffer zone between adjoining plots. Moving averages with $k = 3$ and $k = 5$ were calculated to smooth the data (Figure 2-7) and to detect a possible pattern. The resultant stand density pattern can be ascribed to soil variability within the stand.

Example 2.9   The annual diameter for a 144 year beech tree was regressed on age. The following equation was fitted:

$$\text{dbh} = 45.566 \left(1 - e^{-0.00972 \text{ age}}\right)^{0.87939}$$
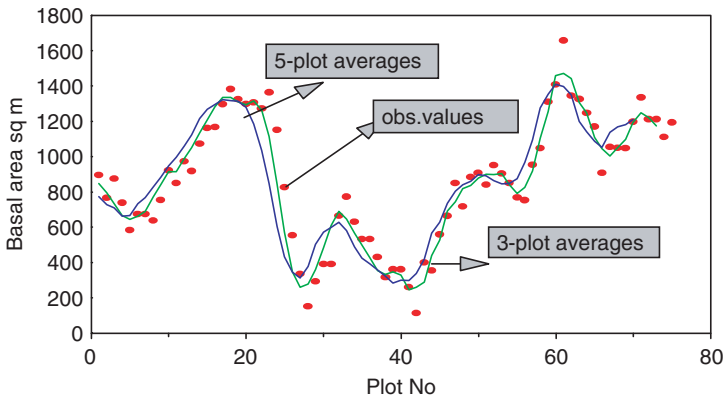
Figure 2-7. Moving averages based on 3 and 5 plot means.
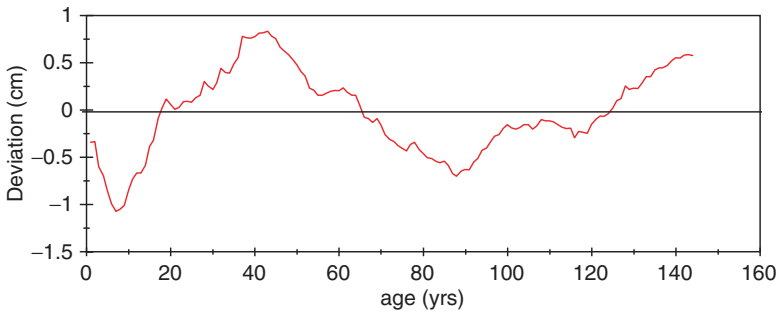
Figure 2-8. Deviations from trend curve.

Plotting the deviations from the trend curve over age (Figure 2-8) indicates that the time series was not successfully converted into a stationary time series.

10 FREEHAND FITTING

Due to the worldwide phenomenal expansion of computer technology, the use of graphical methods to explore the relationship between variables, relevant to forestry, has been drastically reduced. If, however, the infrastructure for electronic data processing is not available, a graphical method may be more successful either to fit a curve and to obtain estimates. It is particularly useful when biological variables are involved, for which no suitable statistical model can be

found and expert knowledge about the nature of the relationship is available. In the past is has been successfully applied for data sets:

- With dbh or age as predictor variable and either tree height, volume or biomass as target variable
- With dbh or relative vertical position along the bole of the tree as predictor and bark thickness as dependent variable
- With age as predictor variable and mean height, top height or other stand characteristics as dependent variable

The following method for graphically fitting is usually successful:

1. The observations of the independent variable are arranged in ascending order and the data are subsequently subdivided into k classes with approximately the same number of observations in each class. The number of observations in each class should not be less than 5 and the number of classes also should preferably not be less than 5. More accurate estimates are obtained by increasing the number of classes to approximately 10–12.
2. The arithmetic mean of the dependent and independent variable are calculated in each class.
3. The class means of the dependent variable are plotted over the mean of the independent variable and a freehand curve is drawn and eventually adjusted to ensure that the mean deviation is zero. This required expert knowledge of the form of the relationship.

Example 2.10   The observed dbh and heights of the trees in Appendix C serve to illustrate the graphical method. The data set contains 55 observations which were sorted for increasing dbh. They were subsequently subdivided into 5 classes with 11 observations within each class. The class means are given below:

Diameter class	Class midpoint	
	dbh (cm)	height (m)
1	10.4	10.8
2	16.2	14.5
3	19.6	15.9
4	22.3	17.3
5	27.3	18.6
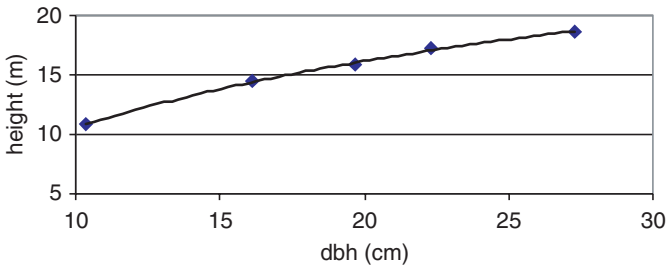
Figure 2-9. Graphically fitted height curve.

The mean heights are plotted over mean dbh and observations which are identi-
fied as outliers are removed before calculating the means (Figure 2-9). A curve
is fitted and possibly adjusted to ensure that the mean deviation from the curve
is zero.

Chapter 3

INSTRUMENTS

1 DIAMETER-MEASURING INSTRUMENTS

1.1 Calipers

The *caliper* consists of a fixed arm mounted perpendicularly to a graduated beam and a movable arm, parallel to the former and sliding along the fixed beam (Figure 3-1).

The caliper is used to measure stem diameters on felled trees and the over bark breast height diameter of standing trees. In order to minimize instrument errors, a rigid construction of the caliper is imperative. The early wooden calipers were subject to wear and tear, steel calipers are rigid and reliable but heavy and uncomfortable during cold weather. Aluminum calipers have increased in popularity, but they should be regularly checked for their accuracy and, if necessary, calibrated at least once annually. In general, calipers have to meet the following quality specifications:

1. The graduated beam must be perfectly straight, of sufficient length for measuring large-dimension and sturdy trees. To eliminate recording errors the graduations should be clearly visible.
2. The movable and fixed arm should run exactly parallel, two arms of the caliper should be located on a plain. If the movable arm is not at a right angle to the fixed beam (Figure 3-2), the resultant systematic positive error is dependent upon the angle of deviation (α) and the diameter of the tree.
3. The point of measurement is consistently incorrectly positioned. The resultant operator bias is two-sided. In research plots, the error may be negligible by permanently marking the breast height position.
4. The graduated beam is not held at an angle of 90° to the stem. For an angle of deviation of α degrees, the observed diameter is approximately equal to $d(1 - \tan(\alpha)/2)$ and the percentage error is equal to $50(\tan(\alpha))$.

Figure 3-1. Caliper.

$$\Delta d = \frac{d}{2} \cdot \tan\alpha \left(1 - \tan\frac{\alpha}{2}\right)$$

or

$$\Delta d \cong \frac{d}{2} \cdot \tan\alpha$$
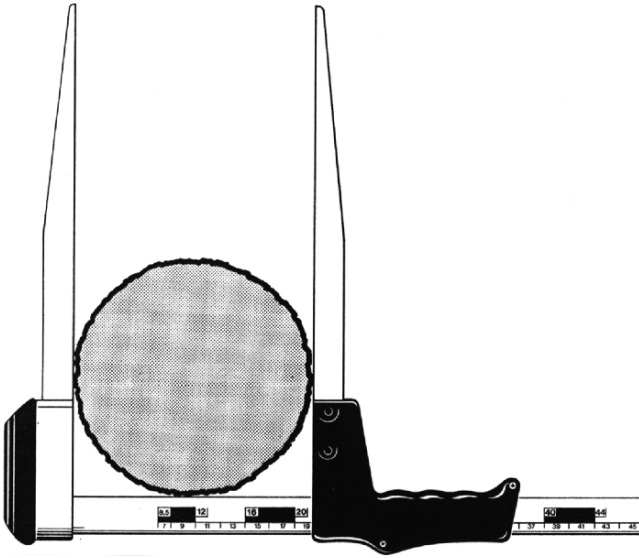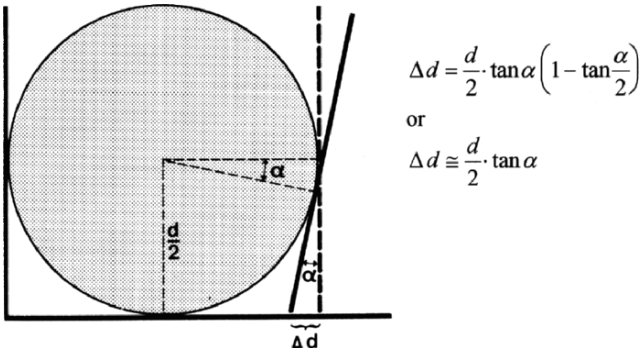
Figure 3-2. Incorrect angle of the movable arm.

The corresponding error in the estimated basal area is approximately twice as high.

5. The operator sometimes tends to exert too much pressure during measuring, in which case a systematic, negative operator-bias is introduced.
6. On slopes, a positive bias is likely to occur by not consistently measuring from the uphill position.

7. On terrain with a heavy and tough ground vegetation of grasses and other plants, the point of measurement is usually located above its true position, in which case a negative bias is introduced. A similar situation may arise during winter, if the ground is covered by snow. In Continuous Forest Inventories, the measuring point is usually permanently marked, partly to establish whether or not a particular tree was measured at the previous occasion. It has the additional advantage of reducing random or systematic errors associated with inaccurately positioning the tape or caliper.

When using calipers for remeasuring permanent sample plots, they should be checked regularly for the occurrence of instrument-related errors. In forest inventories, they are to be calibrated annually. Calipers used to measure research plots require a 1 mm graduation. Those with a 1, 2 or 4 cm graduation are adequate and more practicable for management, regional, and national forest inventories. The diameters are marked on the graduated beam, and show the midpoint of the diameter class. When using self-rounding calipers, these midpoints are always printed at the lowest point of the diameter class.

1.2   Biltmore stick

The *Biltmore stick* is occasionally used to obtain quick and rough estimates of the mean diameter of standing trees, but its use is restricted to North America. It consists of a graduated rule, which is held against the stem at breast height. It is usually calibrated for a distance of 25 in. between the operator and the tree. The observer aligns the zero mark of the stick with the left edge of the stem and at this point reads off the diameter. The calibration formula for a distance of 60 cm is

$$\text{distance from zero point} = d\sqrt{\frac{60}{60+d}}.$$

The tree fork consists of two fixed arms, mounted on a handle at a certain angle between the arms. The diameter is read out on the graduated arm, at the contact point with the stem. In order to avoid an excessive length of the instrument, forks are constructed with different fixed angles between the arms, for example, an angle of 60° for young and medium-aged trees, with a diameter below 30 cm. An angle of 90° should be used for mature stands, with a mean diameter above 30 cm. For an angle of 60°, the stem diameter is 1.154 times the recorded distance between the contact point and the zero mark, for an angle of 90°, the tree diameter equals twice this distance.

1.3   Diameter tapes

Before the beginning of the 19th century, European foresters used *diameter tapes* to measure the diameter of felled roundwood. They were gradually replaced by the more convenient caliper. The tape is graduated on both sides, with the linear scale on one side giving the true circumference of the tree, whereas on the other side it is scaled to give the corresponding diameter. The earlier steel tapes were gradually replaced by those manufactured from weave material, reinforced with wire.

The diameter tape produces slightly biased estimates, if the stem cross section is not exactly circular. The following sources of errors have been noted:

- Systematic errors occur when the measuring position is consistently located above or below 1.30 m.
- The tape is slanted around the tree and sags on one side, in which case a positively biased estimate is obtained. The error increases with the size of the tree and may be substantial for large-sized trees, when it is inconvenient for the operator to verify the position of the tape on the backside of the tree.
- Excessive pressure induces a negative operator bias. In general the resultant bias remains within acceptable limits.
- The occurrence of loose bark, for example, in *Eucalyptus* plantations generates a positive error unless these bark sections are removed prior to measurement.

The diameter tape is usually considered to produce an almost error-free estimate of the tree diameter. Because of the smaller random error involved in successive tape measurements on the same tree, the diameter increment is also obtained free of bias and more accurately, compared with caliper measurements.

Some studies have been carried out to compare the two instruments. Kennel (1959) reported on the accuracy of diameter tape and caliper in estimating the tree basal area. The caliper produced estimates with a mean difference of 2.14% below that obtained with the diameter tape, which indicated a positive instrument-bias for the diameter tape.

1.4   Permanent diameter tapes

Hall (1944) introduced the vernier tree-growth band to measure short-term growth responses at breast height. It consisted of an aluminum band, which was held in place by a coil spring. The band was graduated in inches and 1/10inches and fitted with a vernier to permit more accurate readings. Aluminum has the advantage of being light and easy to work with, but has a relatively high factor
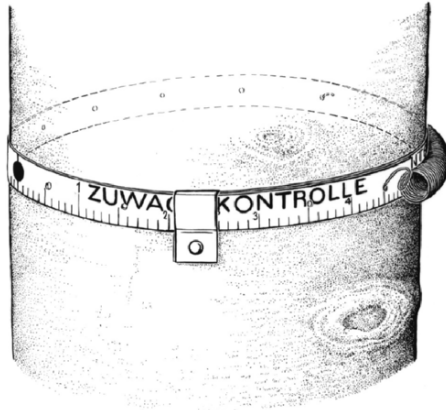
Figure 3-3. Permanent diameter tape.
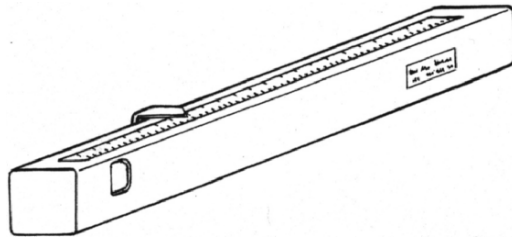
Figure 3-4. Wheeler's pentaprism.

of expansion when exposed to the sun in outdoor conditions. The permanent band is widely used in growth studies in Germany. When graduating in centimeters and millimeters, the use of a vernier is not necessary (see Figure 3-3).

Other devices to measure short-term radial growth responses are discussed in section 6.

1.5   Wheeler's pentaprism

Wheeler's pentaprism (Figure 3-4) consists of a fixed and a movable pentaprism, which is mounted on or moves along a graduated beam. After sighting the point of measurement on the upper stem, the movable prism is adjusted in such a way that the right side of the stem coincides with its left, which is directly viewed. Up to a measuring height of 15 m, the measurement error lies between 0.5 and 1.2 cm (Avery et al. 1983). Van Laar (1984) investigated the accuracy of upper stem measurements obtained with Wheeler's pentaprism and the Finnish optical caliper. The latter produced the best results.

1.6   **Finnish parabolic caliper**

The *Finnish parabolic caliper* is used to measure the stem diameter at those positions on the stem, which cannot be reached from the ground. In Finland, Germany, and Switzerland, for example, they are used to measure the stem diameter at 6 or 7 m above the base of the tree. The latter serves as an additional predictor variable to estimate tree volumes from volume functions with three predictor variables. The instrument consists of a parabolically curved arm with a 1 cm graduation, which is mounted on a hand-held 5–7 m aluminum pole. In order to improve visibility, the diameter class limits enclose 1 cm wide strips of different colors. During measuring the straight section of the caliper is in contact with the stem. In order to eliminate a positive operator-bias, the operator stands exactly vertically underneath the caliper arm. In comparison with the standard type of calipers, diameters are measured less accurately, but it is feasible to classify the stem diameter in classes of 1 cm. (Figure 3-5).
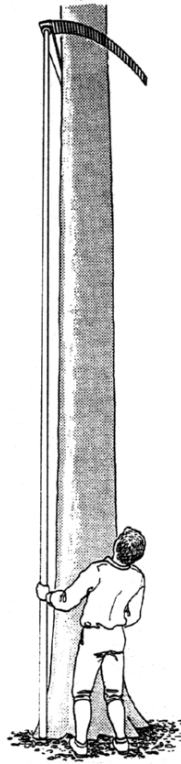
Figure 3-5. Finnish caliper.

1.7   Barr and stroud optical dendrometer

The Barr and Stroud optical dendrometer, which is no longer manufactured, has been used extensively in conjunction with Grosenbaugh's 3P sampling and has a confirmed accuracy of 2.5 mm for upper stem diameters below 25 cm. The instrument is a split-image magnifying rangefinder and is used primarily to measure upper diameters. However, it can also be used to measure tree heights and distances. The readings obtained with the dendrometer are transformed, either by using tables or with the aid of a computer program. The dendrometer is still used in 3P sampling.

2   RELASCOPES AND PRISMS

The principle of Bitterlich's method, based on angle count sampling, is discussed in Chapter 10. All instruments in this group have in common that the angle subtended between the sampling point and the stem at breast height is evaluated.

2.1   Angle gauges

The early *angle gauges* consisted of a 50 cm or 1 m long hand-held stick, with a metal blade 1 cm wide for the 50 cm stick and 2 cm for the 1 m stick being mounted on one side. The trees surrounding the sampling point were sighted at breast height in a 360° sweep. The tree is counted if it subtends an angle which exceeds the critical angle of the instrument.

2.2   Kramer's dendrometer

The multipurpose instrument incorporates the basic principle of the measuring blade, which is 1 cm wide (Figure 3-5) and generates such an angle that each tree counted corresponds with $1\,m^2$ basal area per hectare, i.e., it represents a basal area factor (BAF) of 1. When using a width of either 2 or 4 cm ("op" or "mn" in Figure 3-6), the corresponding BAF are 2 and 4, respectively. The dendrometer is held vertically at a distance of 50 cm to determine the basal area per hectare. The right edge is equipped with a scale for measuring heights, which is similar to the *Vorkampff–Laue hypsometer*. The observer seeks a position where the top and base of the scale (k and h in Figure 3-5) exactly covers the tree and measures the height of the point on the stem which superimposes the mark i on the instrument. Tree height can be calculated by multiplying the distance
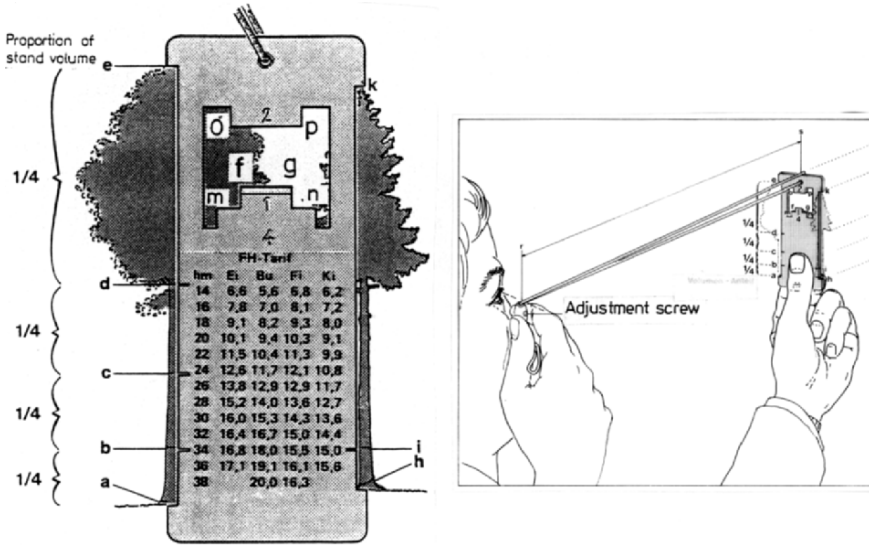
Figure 3-6. Kramer's dendrometer.

from *i* on the tree to tree base by 10. The scale on the left edge indicates the position on the stem, which corresponds with one-fourth of the stem volume. The printed table is based on form heights and used to estimate stand volumes.

2.3   Bitterlich's mirror relascope

The mirror relascope (Figure 3-7) is a small hand-held instrument, which can be used for a variety of purposes:
- Estimation of the basal area per hectare
- Optical distance measurements, adjusted for slope
- Measurement of tree height either for distances of 15, 20, 25, and 30 m or for arbitrary distances
- Measurement of upper-stem diameter, from fixed distances
- Combined height and diameter measurements
- Estimation of relative form heights, to determine absolute form heights, factors, and the volume of the standing tree
- Measurement of slopes
- Estimation of Hirata's stand mean height, based on vertical point sampling

The instrument is based on the principle of a drum pendulum, which is released when measurements are made. The relascope is equipped with a peephole to be used for viewing the object of measurement and lateral windows to admit
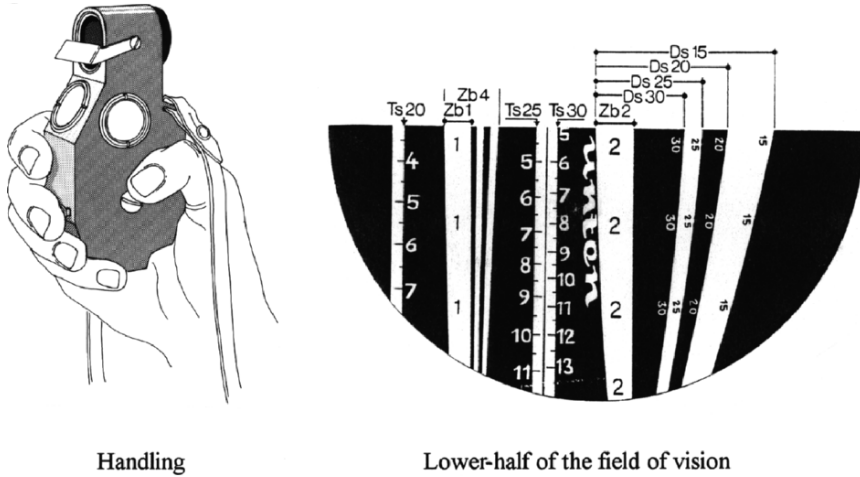
Handling   Lower-half of the field of vision

Figure 3-7. Mirror relascope.

light. The instrument is usually held with the right hand, and the left hand may be used to give the instrument extra support and the middle fingers to press the button. Alternatively, the mirror relascope can be mounted on a tripod or monopod, in order to reduce erratic movements of the instrument during viewing, although this restricts the freedom of movement of the operator. The drum pendulum is equipped with a number of measuring bands, which are mounted on roller bearings to ensure that the pendulum is in a vertical position during the measurements. The pendulum wheel is provided with a brake to dampen the movements of the pendulum. A built-in lens projects the magnified measuring bands onto a mirror. The image is visible in the lower half of the field of vision, with a horizontal line separating the lower from the upper half of the field of vision, which is used to view the object. The width of the measuring bands is adjusted for slope. The adjustment factor is equal to the cosine of the angle of slope. This property is used when measuring upper-stem diameters and when evaluating stems at breast height on sloping terrain. The lower half-field of vision, which reveals a number of white and black bands, is shown in Figure 3-7.***

The "count" bands 1 and 2, which correspond with Zb1 and Zb2 in Figure 3-6, are used to estimate the basal area per hectare for the BAFs 1 and 2 in the metric system. Adding the two white and black bands on the right of Zb1 gives band 4 (Zb4 in Figure 3-6) to estimate the BAF of 4. The distance bands Ds15, Ds20, Ds25, and DS30 are required for optical distance measurements with the aid of a 2 m vertical staff and correspond with horizontal distances of

15, 20, 25, and 30 m from the object. The tangent scale Ts is used for height measurements, for combined diameter and height measurements and to determine Hirata's stand height. They are located to the left of Zb1, (20 m scale), and between Zb2 and Zb4 (25 m and 30 m scale).

- *Estimating the basal area per hectare at breast height*
  In a 360° sweep, the number of trees is counted with an apparent diameter, which exceeds one of the selected "count" bands. The number of trees counted gives the estimated basal area in square meter per hectare if band 1 is used, multiplied by 2 and by 4 when using band 2 and 4, respectively.
- *Distance measurements with the aid of a horizontal staff*
  Band Zb4 is to be used for measuring horizontal distances. A staff of a fixed length, for example, 80 cm is held against the tree, in a horizontal position. The operator locates the point where band Zb4 exactly covers the 80 cm staff. The distance is found as the product 0.80*25 = 20 m.
- *Distance measurements with the aid of vertical staff*
  The distance bands Ds 15 to Ds 30 are used in combination with a vertical 2 m staff to determine one of the fixed distances of 15, 20, 25 or 30 m (See Figure 3-7). Before determining the exact distance, the latter is estimated ocularly, ignoring slope. Releasing the pendulum, the relascope is pointed at the halfway point of the horizontal staff and arrested in this position, in order to adjust for slope. The relascope is subsequently rotated counterclockwise at an angle of 90°. The operator moves forward and backward in order to ensure that the lower terminal point of the vertical staff coincides with the lower edge of band 2 and the upper terminal point of the vertical staff coincides with the appropriate distance band.
- *Measuring tree height*
  The tangent scales are used in combination with the distance bands. The tangent scales are provided for each of the horizontal distances 20, 25, and 30 m. The 30 m scale can be used for the measuring distance 15 m by multiplying the recorded tree heights by 0.5. The height scale for 20 m is found at the far left of the instrument (Ts20), whereas Ts25 and Ts30 are located between ZB4 and ZB2.
- *Estimating an upper-stem diameter*
  Band Zb1 and the adjoining Zb4 are used to measure an upper diameter. Band Zb1 and Zb4 correspond with a ratio object width: horizontal distance of 1:50 and 1:200, respectively. The relascope unit is defined as the band width Zb4, so that count-band Zb1 contains 4 and ZB4 contains 8 relascope units. For a horizontal distance of 10 m, one relascope unit corresponds with a width of 5 cm of the object. When measuring upper diameters from a fixed horizontal distance, the diameter is measured in relascope units and then
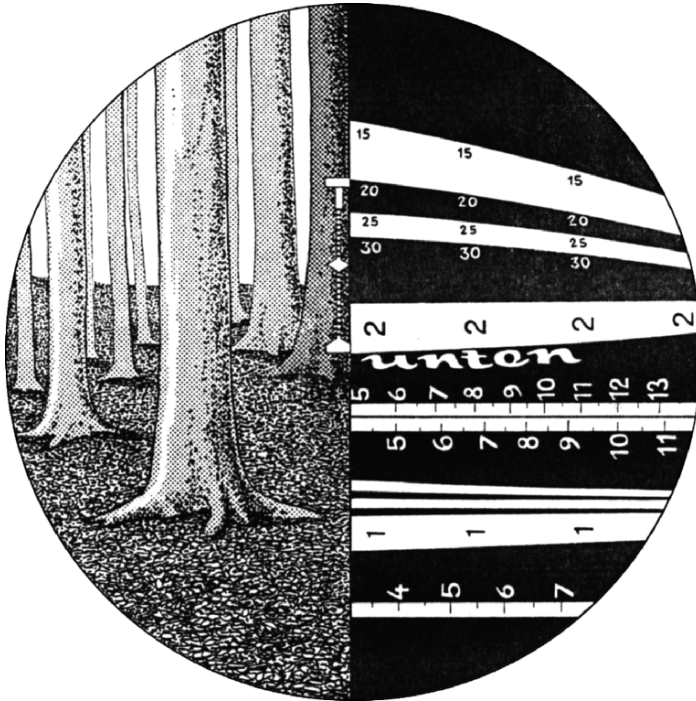
Figure 3-8. Distance measurement with the aid of a vertical staff.

converted to obtain the estimated diameter. For example, when 6.3 relascope units are measured at a distance of 10 m, the estimated upper diameter is 31.5 cm (see Figure 3-8).

2.4　Wide-scale mirror relascope

At a later stage, Bitterlich constructed the wide-scale mirror relascope for measuring upper diameters of large trees and to apply the relascope technique to estimate the basal area per hectare for large BAF (see Figure 3-9). The instrument is equipped with slope scales for degrees (G) and percentage (P), respectively, with four narrow black–white bands to measure upper diameters and to estimate the basal area per hectare for low basal-area factors, with a white band 1 (Zb1), which corresponds with BAF = 1 and distance factor 50 as well as with five black and five white bands, which correspond with BAF = 1. The zero mark is located at the right edge of the Zb1 band. Those units, which are completely covered by the tree to the left and right, are counted and converted into basal area per hectare by using the appropriate conversion factors
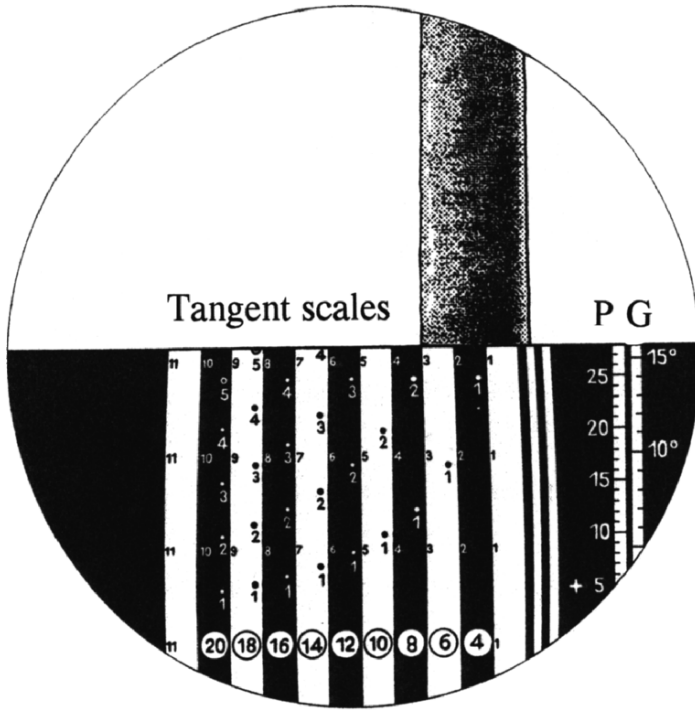
Figure 3-9. Measuring an upper-stem diameter.
(Horizontal distance $= 10\,\text{m}$, Relaskop units (Ru) $= 12.4$, Diameter $= 5\,\text{cm} \cdot 13.5$

for those at the left and right of the zero mark, respectively. The instrument is also suitable for measuring distances and upper-stem diameters.

2.5   Bitterlich's telerelascope

The *telerelascope* represents a vastly improved version of Bitterlich's *mirror relascope*, primarily to estimate upper-stem diameters and their corresponding heights above the base of the tree, from arbitrary sighting distances. The following steps are required:

- The instrument is mounted on a tripod, either with a movie head adapter or with a micrometer head with a fixed avallactical point, i.e., with a fixed sighting-angle vertex.
- The left edge of the tree and that of a white band, which corresponds with one tachymetric unit are aligned. The number of tachymetric units is determined in 1/10 units. This gives the upper-stem diameter in tachymetric units.

- A second reading is made on a horizontal base rod, positioned aside the tree on which as many full tachymetric units as possible are read out. The reading on the base rod, divided by the number of full units, gives the base reading, which is multiplied by the number of tachymetric units obtained for the upper stem diameter.
- The reading obtained on the base rod gives the horizontal distance in meters.
- The left graduation on the instrument is used to obtain percentage readings for the position of the upper diameter and that of the base rod. The algebraic difference is multiplied by the base reading and gives the height above ground level of the upper diameter.

Sterba (1976) summarized errors involved in the estimation of stem volume with the aid of the telerelascope. Instrument errors were associated with movements of the vertex of the sighting angle, which affected the estimation of the operator to tree distance. The distance errors varied between 0.5% and 1% and necessitated upper-stem diameter adjustments between 1% and 2%, whereas height estimates should be adjusted upward by 20–40 cm. The volume estimates were furthermore associated with errors due to the formulae being used. An upward adjustment of 3–5% was required to remove this source of bias.

2.6 Prisms

The *prism* is a thin wedge made of glass or plastic, which deflects the incoming rays through an angle that is constant for a given prism. Deflection causes displacement of the tree when viewed through the prism, the amount of displacement being dependent on the diopter strength of the prism, which in turn is a function of the angle between the two surfaces of the prism. A strength of one diopter is the equivalent of a displacement of 1 unit per 100 units distance. The displacement of the image produces a critical angle, similar to that established by the relascope. When the left edge of the stem, viewed through the prism, is aligned with the right edge of the stem, viewed over the prism, the corresponding stem diameter is 1/100 of the distance to the tree, for a prism with a diopter strength of one. During a 360° sweep with the sampling point as center, the line between the two surfaces of the prism is held vertically above the sampling point and should remain in this position. Trees which are displaced less than the apparent diameter are counted, those which are displaced fall outside the imaginary plot and are not counted (see Figure 3-10). The borderline trees, i.e., those trees for which the amount of displacement is equal to the apparent diameter, should be checked by measuring the stem diameter and distance from sampling point. Because of the amount of time involved in checking, it is customary to assign a count of one-half to each borderline tree,
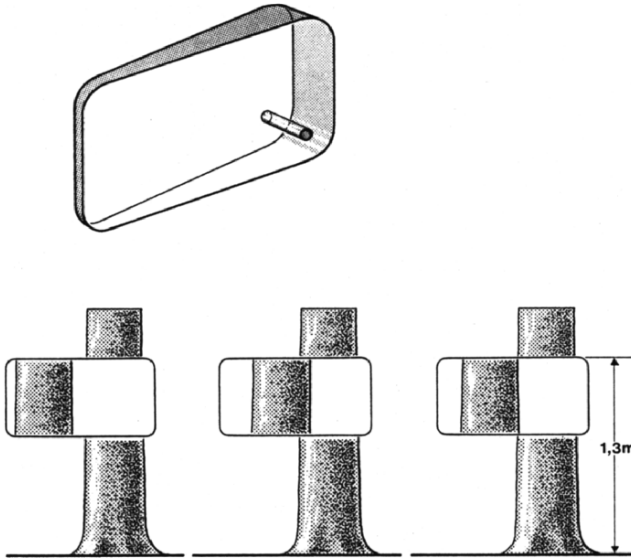
Figure 3-10. Decision on counting a tree with a wedge prism.

although this might produce operator-bias. On a sloping terrain, the number of trees counted produces a biased estimate of the basal area, since slope distance instead of horizontal distance is observed. Corrections are necessary for slopes of more than 10°. The basal-area estimate, corrected for slope is calculated as follows:

$$G_{ha} = \frac{\text{BAF} \cdot n}{\cos \alpha}$$

where n = tree count, α = angle of slope. In order to calibrate the prism, a target of known width, between 30 cm and 1 m is set up. The observer moves towards and away from the target until the image seen through the prism and the right side of the stem viewed over the prism are exactly aligned. The distance between the observer and the target in measured with a tape. Because of measurement errors, the calibration exercise should be repeated three or four times. The BAF is calculated from

$$\text{BAF} = \frac{10000}{1 + 4 (L/w)^2}$$

where L = distance in meters and w = width of target in meters. The more expensively calibrated prisms, however, eliminate the necessity of field calibrations.

3   TREE HEIGHT

3.1   Telescopic poles

The *telescopic poles*, manufactured from aluminum or fiberglass are conveniently used to measure the height of young trees, up to 15 m. The pole sections, of constant length but decreasing diameter, are connected with a coupling device. The top of the coupled-pole sections is positioned at the same height as the top of the tree with the exact height being read out on the lowest-pole section. When care is taken to eliminate or at least reduce parallax between the telescopic pole and the apex of the tree, tree heights can be determined accurately. It is impractical to use poles with a total length of more than 15 m.

3.2   Hypsometers

The standard practice is to measure tree heights with hypsometers. The conventional hypsometers are classified according to the principle of their construction. The *Blume–Leiss, Suunto*, and *Haga hyposometer* and the *Abney level* are based on trigonometric relationships, the *Christen, Merritt, Chapman*, and *Vorkampff–Laue hypsometer* have a geometric basis.

3.3   Hypsometer according to the trigonometric principle

Hypsometers based on the trigonometric principle measure, from eye level, the vertical angles between the baseline and the top and base of the tree, respectively (Figure 3-11). The tree height is obtained from measurements of the angle subtended by the top and base of the tree with the horizontal:

$$h = e(\tan \alpha_1 - \tan \alpha_2)$$

The sign of the angle α is positive if the aim is located above eye level and negative, if not (Figure 3-11). The instruments differ in respect of the presence or absence of built-in range-finders to determine the horizontal distance between the tree and the operator. The type 7 Blume–Leiss hypsometer has a single pointer, which can be locked in the desired position and is not equipped with a built-in range-finder (Figure 3-12).

The *Abney level* consists of a 10 cm square sighting-tube, with a mirror inside a rotatable level indicator, ensuring that a horizontal position of the tube can be maintained during measuring. After sighting the top of the tree, the
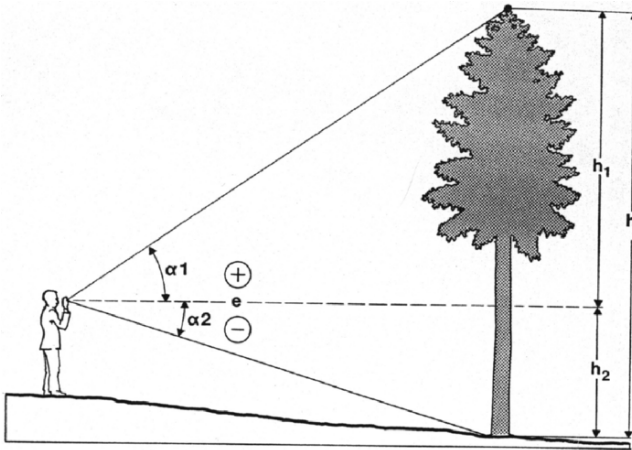
Figure 3-11. Trigonometric principle.
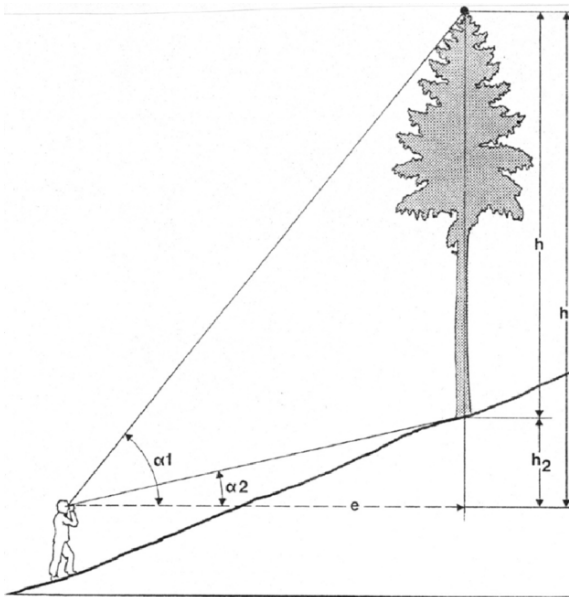
Figure 3-12. Measuring uphill.

vertical angle subtended with the horizontal tube is read out on an arc under-
neath the latter. This is repeated for the base of the tree. The graduation of the
arc is given either in degrees or in percentages. Trigonometric tables are used
for readings in degrees. In the case of percentages, the latter are multiplied by
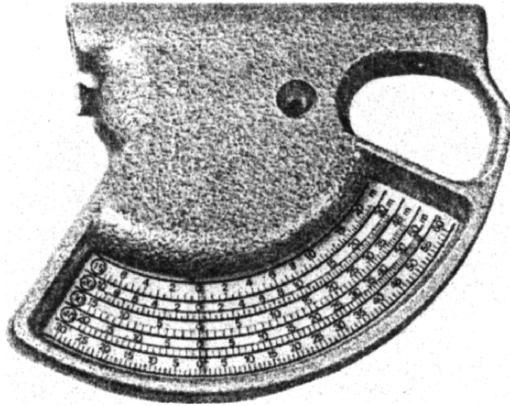
Figure 3-13. Blume–Leiss hypsometer.
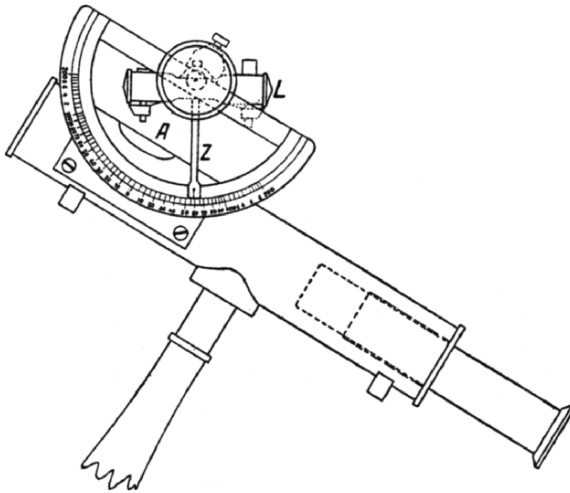
Figure 3-14. Abney-level.

the horizontal distance to obtain the height above the baseline. The results are added if eye level is located above the base of the tree and subtracted, if not.

The basic constructional principle of the Haga altimeter is similar to that of the Blume–Leiss hypsometer and the Abney level, but the optical measurement of distances differs slightly from the Blume–Leiss (Figures 3-13–3-15). Fixed scales for distances of 15, 20, 25, and 30 m are provided and shown separately by rotating a knob, which serves to select a specific scale. This eliminates the risk of reading on an incorrectly selected scale. The instrument is also equipped with a scale to obtain a vertical angle expressed as a percentage, with

Figure 3-15. Haga hypsometer.
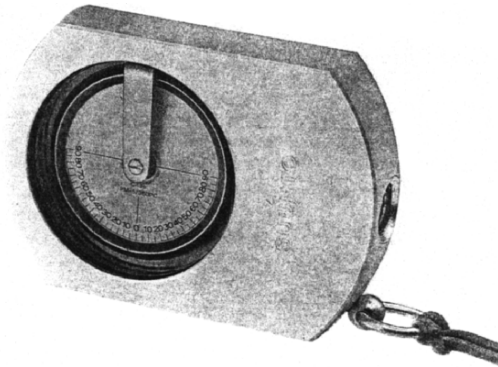
Figure 3-16. Suunto clinometer.

a range extending between −40% and +50%. Hence, it gives the tree height as a percent of the baseline distance. The Haga hypsometer replaces the multi-range target of the Blume–Leiss with a tape, which is provided with a fixed and a detachable target blade, which correspond to fixed distances.

The *Suunto clinometer* is a small, hand-held, compact instrument, equipped with a freely movable scale card, which is surrounded by a damping liquid and supported by a bearing assembly (Figure 3-16). The instrument has two built-in scales to obtain tree heights for fixed distances of 15 and 20 m. When measuring tree heights from a distance of 30 and 40 m respectively, the readings obtained with the 15 and 20 m scales, respectively are doubled. A third scale within the instrument serves to measure slopes as percentages. The table on the back of the instrument converts percentages to degrees. The slope of the

terrain can be measured in degrees, by using the 20 m scale and sighting a point at eye level. The optical measurement of the distance to the tree is similar to that for the Blume–Leiss, but the operation of the instrument differs slightly from it. When looking through the lens, the top of the tree is sighted with one eye and the scale is simultaneously read with the other eye. This is repeated for the base of the tree. A nomogram on the back of the instrument is used to correct tree heights for a sloping terrain. The instrument has the advantage of dampening the movement of the pointer during sighting. Its main disadvantage is the necessity to aim at the top or base of the tree and simultaneously obtain a reading on the scale of the instrument.

The errors involved in height measurements are partly of a random nature, for example, because the pointer moves during sighting, or when measuring leaning trees (assuming that there is no dominating direction of lean), or because the top of the tree is obscured in dense stands. Other sources induce systematic errors, for example, when the baseline distance is not corrected for slope. In addition, operator-bias may occur. Two error sources associated with leaning and round-topped hardwoods, respectively are particularly important. A positive error occurs when the tree leans towards the operator, a negative error when it leans away from the operator. A nearly error-free estimate for tree height is obtained by averaging the heights recorded from two opposite directions. Alternatively, the height may be measured at an angle of 90° with the direction of the lean. The resultant error is then equal to $h(1 - \cos\alpha)$. Grosenbaugh (1980) emphasized the occurrence of biased height estimates when the degree of lean is greater than 8°.

3.4   Baseline slope correction

The Blume–Leiss hypsometer is equipped with a table, which gives the correction factors for uphill measurements (Figure 3-17). For example, for a baseline slope of 25°, and a measured tree height of 30.6 m, the correction factor is 0.18 and the measurement error is $30.6 \cdot 0.18 = 5.5$ m. The adjusted height is $30.6 - 5.5 = 25.1$ m. Alternatively, it is calculated as $30.6 \cdot \cos^2 25 = 25.1$ m.

Measurements errors, associated with round-topped trees tend to be positive. Their magnitude depends upon the shape of the crown and decreases with increasing distance between the operator and the tree.

In dense, unthinned or lightly thinned stands, or when light conditions are poor, it may be disadvantageous to measure tree heights from a fixed distance from the tree. In such situations the instruments are used at variable distances, from such measuring positions where the top of the tree is visible.
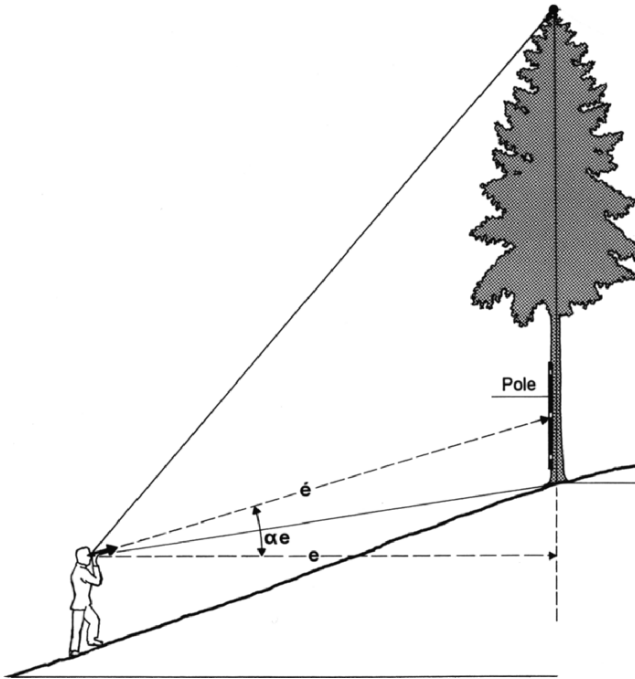
Figure 3-17. Height correction of trees on slopes (measuring baseline slope).

The scale of the instrument allows for accurately reading slopes in 1/10 degrees. The instrument is operated as follows. A pole is placed against the tree. The vertical angles with the top of the tree, the top of the pole and the base of the tree are measured (see Figure 3-18). The relevant angles are α_3, α_2, and α_1. When calculating the height, the algebraic sign is taken into account similar to other instruments in this group.

$$h = \frac{L \cdot (\tan \alpha_3 - \tan \alpha_1)}{\tan \alpha_2 - \tan \alpha_1} = \frac{L \cdot (p_3 - p_1)}{p_2 - p_1}$$

where L = pole length, p_i = inclination (in percent).

Example 3.1

$\alpha_1 = -10.7°$ $p_1 = -18.9\%$

$\alpha_2 = -3.5°$ $p_2 = -6.1\%$

$\alpha_3 = 45.3°$ $p_3 = 101.1\%$

$L = 4\,\text{m}$
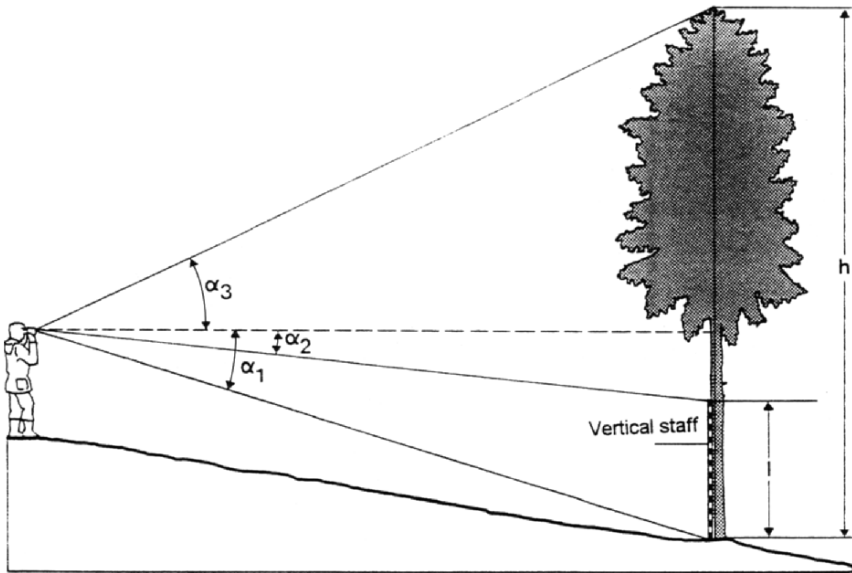
Figure 3-18. Measuring tree height with a vertical staff.
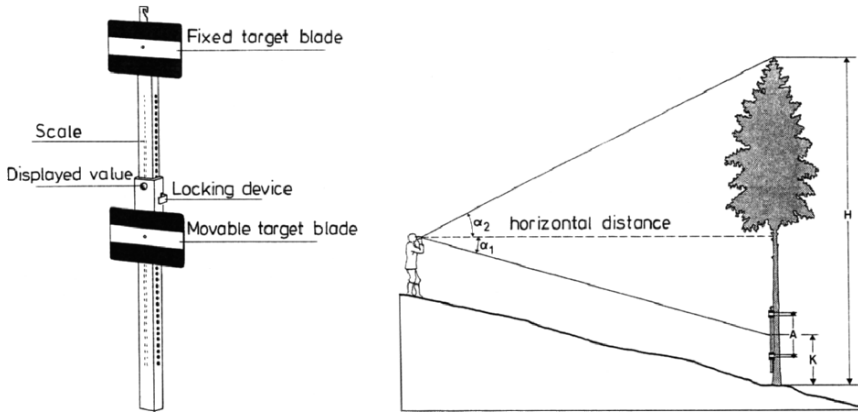
Figure 3-19. System Johann.

$$h = \frac{4 \cdot (\tan 45.3 - \tan(-10.7))}{\tan(-3.5) - \tan(-10.7)} = 37.5\,\text{m} \qquad h = \frac{4 \cdot (101.1 - (-18.9))}{-6.1 - (-18.9)} = 37.5\,\text{m}$$

The *System Johann* uses a vertical staff with a variable size (Figure 3-19), provided with a scale, which is equipped with one target blade in a fixed position and a second freely movable one, which can be locked at any position.

The target staff has to be used in combination with an optical range-finder. It serves to find the horizontal distance to the tree. This depends on the distance between the target blades and the angle of the optical range-finder. The measurements are carried out by two persons. The operator determines the position from which the top and base of the tree are clearly visible. The second person holds the pole against the tree, taking care to hold it in a vertical position. The target blades on this pole must be clearly visible. For measuring the horizontal distance, the operator uses the optical range-finder, while the second person moves the lower blade into a position where the upper target blade of the virtual image coincides with the lower blade of the real image. The movable target blade is then locked in its correct position and the distance between this blade and the upper blade is read. Two angles are measured, the vertical angle with a point halfway between the target blades and the vertical angle to the top of the tree. The constant K (see Figure 3-19) is added. The height of the tree is calculated as follows:

$$h = A \cdot C \cdot \cos^2 \alpha_1 \cdot (\tan \alpha_2 - \tan \alpha_1) + K$$

where

> h = height in meters
>
> A = distance between the target blades
>
> C = 1/3 for Blume–Leiss and Suunto
>
> α_1 = vertical angle with the point halfway between the. target blades
>
> α_2 = vertical angle with the top of the tree
>
> K = height above ground of the point halfway between the blades

The constant K is equal to $(1.3 - A/200)$ if the upper target blade on the pole coincides with breast height. Either a programmable pocket calculator or tables are used to obtain tree height. The instrument produces sufficiently accurate estimates, no fixed distance between the operator and the tree is prescribed and no corrections for slope are required. It is ideal for measuring heights in dense research plots. However, the user needs an assistant for operating the pole with its adjustable target blade and the two angles involved must be measured accurately. Furthermore, the height is calculated and not directly read on the instrument.

3.5   Hypsometer according to the geometrical principle

Hypsometers derived from geometric relationships are based on the similarity of triangles. The *Christen hypsometer* consists of a folding blade with a fixed length of 30 cm, which isequipped with a nonlinear scale. A pole with a fixed
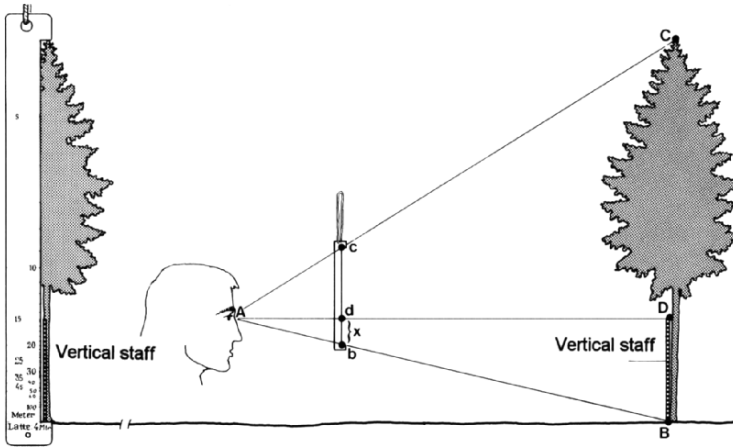
Figure 3-20. Christen hypsometer.

length of 4 m is held against the tree. The operator moves towards and away from the tree to obtain a position where the top of the pole, the top of the tree and the base of the tree are visible. The height is then read at point *d* of Figure 3-20. Because of the similarity of the triangles ABC and Abc and that of the triangles ABD and Abd, we have

$$\frac{BC}{bc} = \frac{AB}{Ab} \quad \text{and} \quad \frac{BC}{bd} = \frac{AB}{Ab}$$

It follows, that

$$BC = BD \cdot \frac{bc}{Ab}$$

and, for BD = 4 m and bc = 0.3 m we have: $h = \dfrac{1.2}{bc}$

With increasing height, the scale distance, per unit increase in height, decreases. The instrument is compact and cheap to manufacture. Tree heights can be measured quickly because no distance measurements are required and a single reading produces the estimated tree height. Its disadvantage is the necessity to cover the entire tree during measuring. Additionally, due to the nonlinear scale, the accuracy decreases with increasing tree height. To some extent, this can be overcome by increasing the fixed length of the instrument.

The *Chapman hypsometer* is based on the same geometric principle, but uses a fixed point on the scale, for example, at 3 cm from the zero mark, as well as a fixed length of the pole, for example, 3 m. The height of the tree is read out on the scale. The Vorkampff–Laue hypsometer is based on a fixed length

of the staff and a fixed mark on the scale. The tree height is read out on the pole along the tree. The Merritt hypsometer does not require a pole to be held against the tree. The operator stands at a given distance from the tree and holds the instrument at a fixed reach distance.

4   BLUME–LEISS RANGE-TRACER DRUM

The Blume–Leiss range-tracer drum has been constructed by the Institute of Forest Management at Göttingen, Germany (Figure 3-21). It is used in combination with the Blume–Leiss, Suunto, or Haga hypsometer to optically determine the boundaries of plots with a radius of 17.84 m and 12.62 m, corresponding with a plot area of 0.1 and 0.05 ha, respectively. On slopes, it automatically adjusts for the slope effect on recorded height. The operator releases the adjustment screw (I) of the lower drum and slides it down the pipe until the marked position of the required plot radius has been reached. Thereafter, the operator releases the adjustment screw (II) and slides the upper pipe
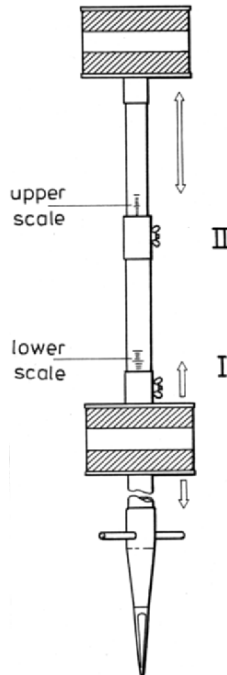
Figure 3-21. Range-tracer drum.

until the appropriate mark on the upper scale has been reached. The maximum slope is used for each adjustment setting. The range-tracer drum is set up in a vertical position in the center of the circular sample plot. To trace the border of the plot, the white strips of the upper and lower drums are brought into coincidence with the aid of the distance measuring device of the Blume–Leiss or Suunto clinometers.

5 TREE CROWN AND FOLIAGE

Johansson (1985) examined the accuracy of the estimation of crown canopy with the vertical tube method. A 20 cm long and 1 cm wide hand-held tube is mounted on a universal joint to ensure a vertical position during measuring. Cross wires are mounted at the upper end and a mirror at the lower end. At each sample point, the number of measuring points covered by sky is recorded and estimates the "crown-free projection."

A *crown mirror* to project the tree crown onto a horizontal surface was constructed at the Institute of Forest Management and Yield sciences of the University of Göttingen. A mirror is attached to the lower-notched section of a hollow cylinder, at an angle of 45° to the cylinder. Cross wires are mounted in the upper and lower sections of the cylinder, above the mirror. A handle keeps the cylinder in a vertical position. A marker-release device is attached to the bottom of the cylinder, below the mirror. It is triggered by a button located on one side of the cylinder. During viewing, the center of the mirror should be held at eye level. The position of the cross wires is forced to coincide with the position of the projected edge of the crown. The marker released by pushing the button, drops to the ground. The crown radius is found as the distance between the center of the stem and the ground position of the marker.

6 SHORT-TERM RADIAL GROWTH RESPONSES

Reineke (1932) introduced a *precision dendrometer* for measuring short-term growth responses, which was subsequently improved by Daubenmire (1945). The tips of three wooden screws are driven into the stem, to a depth of 1 cm. The heads form a plane, which remains in a fixed position on the stem. A dial gauge is fastened onto a platform, which is held against the screw heads. The dial registers 0.001in. distance. Only one dial gauge is required to measure the growth of a large number of trees. Furthermore, it is not necessary to make use of wooden or copper screws, which are in a more permanent position and

damage the cambium and xylem. Stimulated cambial activity around the screws also tends to produce positively biased readings. Bormann et al. (1962) compared the dial gauge dendrometer with vernier bands to estimate the weekly radial growth of eastern white pine. The two instruments produced similar growth patterns, but ring bands were inefficient in recording stem shrinkage. Although dendrometers has a high mechanical precision, the measurement of growth on a single point sometimes produces more erratic estimates than ring bands. In studies to explain short-term diameter growth reactions in terms of weather conditions with the dial gauge dendrometer, Kern (1961) stated the necessity of distinguishing between fluctuations due to swelling and shrinkage of the stems and real growth.

Auchmoody (1976) examined the accuracy of growth estimates obtained from vernier bands, which were compared with approximately error-free measurements and found that nearly unbiased estimates were obtained whenever the diameter growth was more than 1.25 cm. Fritts et al. (1955) designed a *dendrograph*, which was a greatly improved version of a similar instrument, used by McDougal, which had a magnification of 22. The frame of reference was identical with Daubenmire's dendrometer platform. The Fritts dendrograph had a 100-fold magnification, with the readings being recorded on a 7-day rain-gauge clock. A similar dendrograph was used for measuring the daily radial growth of Pinus radiata (van Laar 1966). Dendrographs are more expensive than dendrometers, but have the advantage of being able to record radial growth and shrinkage during a 24 h period. It was found however, that a large proportion of the radial increment, recorded during a period of rainfall, is primarily attributable to swelling of the bark. It is also possible that real growth during dry spells is clouded by shrinkage. By installing instruments on trees, cut at a height of 2 m, radial changes of the severed stem section can be recorded and the growth records on the living trees can be corrected for swelling and shrinkage of the bark. Kinerson (1973) constructed a transducer which converts the magnitude of applied stimulus into a proportional electrical signal. The instrument contains a linear-motion potentiometer, which is fixed to a band of a nickel-steel alloy.

7   INCREMENT CORES

The Swedish increment borer consists of a handle, a hollow cutting bit and an extractor, which is pressed into the cutting bit. The core is extracted after a sharp turn of the extractor. The following measurement errors occur.

- Errors due to deformation and more particularly to compression of the cores.
- Positively biased measurements due to an incorrect off-center boring direction. This is sometimes unavoidable due to the center of the cross section not coinciding with the pith.
- Negative bias due to shrinkage of the cores between the time of extraction and measuring.

Cole (1977) recommended storing increment cores in plastic straws sealed with cellulose-acetate tape, thereby providing protection against shrinkage for several days, but freezing the straws is recommended if storage time exceeds 3 days. Liu (1986) developed a geometric model off-center increment cores, to be used to rectify radii. Hall et al. (1984) designed a portable power-driven increment borer, constructed from a 58 cc gasoline chainsaw, which produced sufficient power for extracting samples from trees up to a diameter of 80 cm. Johann (1977) describes the two types of the S*ystem Digitalpositiometer*, manufactured by the Austrian firm Kutschenreuter, suitable for measuring increment cores and stem cross sections, respectively.

8   BARK THICKNESS

The Swedish *bark gauge* is used to measure the bark thickness on standing trees. It consists of a semicircular chisel, which is forced into the xylem and extracted. The bark thickness is read out on a graduated scale on the chisel. The assumption is that the instrument will not be driven into the sapwood, but this may be difficult for species with hard barks. A penetration of the sapwood, overestimates the bark thickness.

9   RECENT DEVELOPMENTS IN INSTRUMENTATION

A measuring device which uses polar coordinates and consists of an x, y, ρ coordinate measuring table and has an accuracy of 1/1000 mm and 1/100°, in addition to a turntable for stem discs, two optical systems with CCD video camera and computer, as well as processing equipment, was developed by researchers in the of Section Forest Biometry at the University of Göttingen (Taube et al. 1992). Although being a multipurpose instrument, it is used primarily to measure ring widths, either on stem discs or on photographs.

Bräker et al. (1992) introduced an instrument for measuring ring widths, similar to Eklunds device, but with the advantage that no adjustment of

the position of the increment core is necessary. Lega (1992) introduced the *dendrochronograph SMIL 3*, which can be used to measure ring widths on stem discs up to a diameter of 1 m as well as on increment cores. The instrument has a confirmed accuracy of 0.01 mm and is equipped with a telegoniometer which eliminates those measurement errors associated with an inclination of the rings at the point of measurement.

Roth et al. (1992) developed the computer package *ARISA* (Automated Ring Sequence Analysis) based on digital image processing and pattern recognition. It enables the user to simultaneously measure a large number of radials on stem cross sections, which in turn reduces the standard error of the estimated means. The technique is thought to be particularly useful in analyzing disturbed ring patterns.

9.1   Laser dendrometer LEDHA

The Zeiss *range finder* is a new instrument, with a weight of 2.2 kg, which operates on the basis of travel-time measurements of laser pulses by diffuse backscattering of such pulses. The instrument is suitable for stationary as well as mobile applications and has a storage capacity of 4000 measured values, which are delivered to a printer or personal computer. It has the capacity to measure azimuths, distances, vertical angles, and heights and was developed and marketed by the optical firm Zeiss.

The laser dendrometer was developed for specific applications in the practice of forestry. The incorporation of modern laser technology permits the user to measure and store data such as distance, tree height, slope, azimuth and diameter. The device resembles a pair of binoculars, with the optical and electrical unit parts being enclosed in a sturdy housing, to which the four push-button controls – two of them operated through the oculars, the other two for program selection and "measuring" are attached (see Figure 3-22). Tree numbers (between 1 and 999) and tree species (numerical code between 1 and 99) can be entered digitally and then stored for each subject tree. Distance and the magnetic azimuth of a tree can be either specified in reference to the actual point of measurement or to its site. Tree height measurements are performed by locating the relevant points (an arbitrary point on the tree trunk for distance measurements, others at the base and top of the tree for recording the angle of slope). The stem diameter is determined optically with the aid of a graduated scale. All values can be stored either directly or they can be digitally transmitted to the system. The stored data can be transferred to a PC via a connecting cable or can be printed.
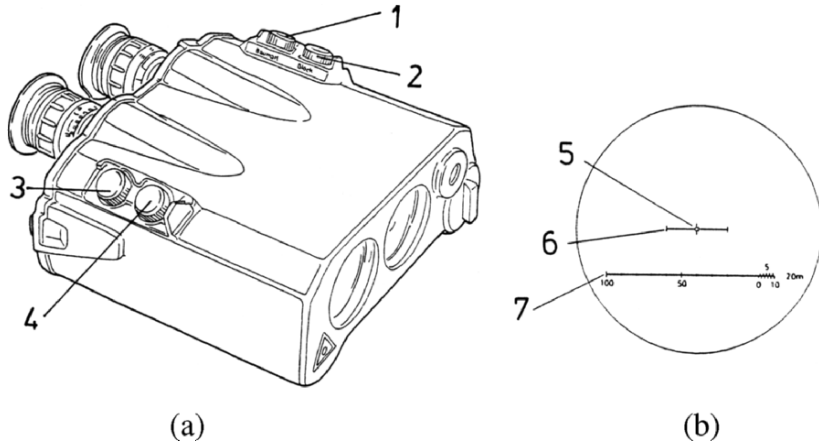
Figure 3-22. Laser dendrometer (a) and graduated dial in the right ocular, for angle-count sampling and diameter measurements (b). 1. Push-button control for entering tree species; 2. Push-button control for entering tree number; 3. Push-button control for program selection 4. Push-button control for "measurement"; 5. Visor mark; 6. Scale for angle-count sampling: 1:50; 7. Optical-diameter scale, 0–100 cm, accuracy ±1 cm (verified for a distance of 20 m to the object)

Table 3-1. Unit functions of laser dendrometers

Model	LEDHA 100	LEM 300W	LEDHA Geo
Enter tree species	+	+	+
Enter tree number	+	+	+
Distance, direct	+	+	+
Distance, reduced	−	+	+
3-point tree height	+	+	+
2-point tree height	−	+	+
Angle of slope	−	+	+
Azimuth	+	−	+
Diameter	−	−	+

At present, three models of the instrument are available. The LEDHA 100 is primarily suited for dendrometric records on individual trees or in sampling studies for forest inventories, whereas the LEM 300W is provided with excellent geodetic menus. The third model, LEDHA Geo, is a combination of the LEHDA 100 and LEM 300W. The units' functions are depicted in Table. 3-1.

9.2   Criterion 400 laser dendrometer

The dendrometer incorporates a ranging facility which makes use of laser technology to determine the distance between the instrument and the object. When coupled with a telescopic sight and a graduated reticle, it allows the measurement of tree diameters from a remote position. The dendrometer consists of a laser-range sensor, a fluid-tilt sensor, and a fluxgate compass. The entire data capture process is conveniently carried out by a single operator, since the Criterion stores data, which are downloaded to a computer or similar data storage device.

9.3   Digital hypsometer forestor vertex

The Swedish *Forestor Vertex* is a new measuring instrument used to determine distances, heights, and angles (Figure 3-23). The unit is based on the ultrasonic determination of distance, as well as the measurement of angle slope. It calculates the object's height from both measured values. The instrument is comprised of two components: the actual measuring device and the ultrasonic transponder, DME 201. The Forestor Vertex can also be used without the transponder. In that case, the distance to the measured object must be entered by hand.

If there is a natural source for ultrasonic waves during measuring, distances cannot be measured with the Vertex. In consequence it is not possible to measure tree heights, especially in tropical natural forests, with some species of the fauna-emitting ultrasonic waves.
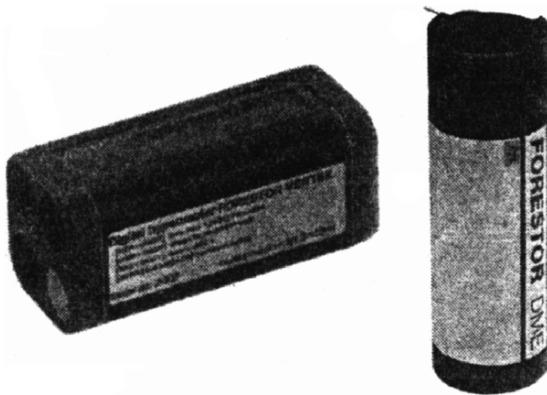
Figure 3-23. Digital hypsometer Forestor Vertex.
(the hypsometer (left) and the transponder)

Chapter 4

SINGLE-TREE MEASUREMENTS

1   MEASUREMENTS ON STANDING TREES

Age, diameter, basal area, total and merchantable height, total and merchantable volume, stem form, bark thickness and growth are important single-tree characteristics, which generate information about the growing stock of stands. They may include other tree characteristics, such as species, crown form, branchiness and damage caused by insects and pathogens.

1.1   Age

The age of a tree is defined as the period of time which elapsed since germination and in some cases, especially in commercial forestry, since the time of planting. On trees with recognizable annual layers of wood, age can be determined either by felling the tree, counting the number of annual rings at stump height and adding the estimated period of time the tree requires to reach stump height, or by counting the number of rings on increment cores extracted at breast height or lower, with the aid of an increment borer, and adding the estimated number of years required to reach this point of extraction, or by counting the number of internodes, whenever identifiable, for example in poplars and some uninodal conifers.

False rings are formed when a dry spell during the growing season induces the formation of latewood and is followed by a period of high rainfall, which is conducive to the formation of earlywood. On stem discs false rings can be identified more easily than on increment cores (see Chapter 9, section 2.2). The age is overestimated when false rings are incorrectly identified as true annual growth layers. In exceptional cases, for example under extremely dry weather conditions or on damaged trees, no distinct growth layers may be visible, and the age is underestimated. The estimation of the age of mature trees, obtained from ring counts on increment cores, is usually difficult and inaccurate because

of off-center boring and the presence of false rings. On the other hand, in many instances, missing rings can occur either due to senescence or because of the effect of environmental pollution or pathogens stresses. In order to eliminate these error sources, it is necessary to synchronize the annual ring series similarly cross-dating in dendrochronological studies (see Chapter 9, Example 9.1 and Figure 9-3).

1.2   Diameter and basal area

In nursery and regeneration studies, the diameter of a plant is always measured at its base and is described as *collar diameter*. In forest stands, tree diameter usually refers to the over bark diameter at a fixed distance from the base of the tree. In most countries, the point of measurement is located at 1.30 m above ground level, but a height of 4.5 ft($= 1.37$ m) is used in the USA and a height of 1.2 m in Japan and Korea. The relevant diameter is described as *breast height diameter*. Throughout this book, the International Union of Forestry Research Organizations (IUFRO) recommendation to use the symbol *d* for *breast height diameter is* applied in equations, the notation *dbh* is occasionally used in the text. In addition the following symbols are used:

- The subscripted diameters $d_{o.b.}$ and $d_{u.b.}$ refer to the over and under bark breast height diameter, respectively
- The subscript *i* added to the lower case letter d_i denotes the over or under bark stem diameter at a height of *i* m above the base of the tree
- The subscript *i*% is used to indicate a point of measurement at *i*% of the tree height above the base of the tree

The following rules for measuring breast height diameter were formally adopted by IUFRO:

- On slopes, the diameter is measured on the uphill side of the tree
- In tropical forests, the zero reference mark is located at some point above the buttress of the tree to eliminate the variability caused by excessive butt swell. In absence of fixed rules, the exact position varies with operators and regions, but in many cases the point of measurement is located well above 1.3 m.
- In the case of irregular stem cross sections, for example, due to protruding branch stumps, two diameters are measured, at *a* cm above and below the correct position respectively. The average of the two readings estimates the true diameter
- In the Austrian National Forest Inventory, the rules for measuring diameters have been modified for crooked stems.
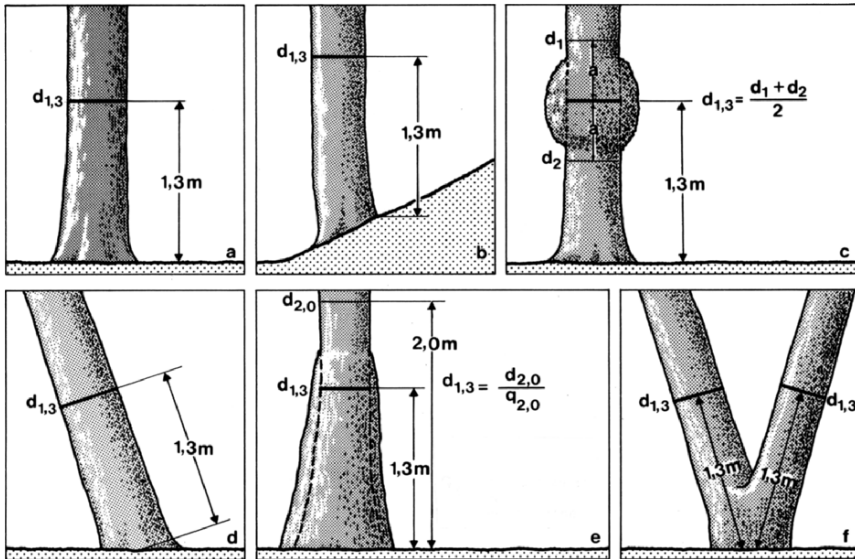
Figure 4-1. Locating breast height.

- The rules, although formally adopted by member institutes of IUFRO, are not universally applied. Bruce (1980) summarized commonly used definitions for the base of the tree:
  - An average derived from measurements at different points around the tree
  - The base of the tree is synonymous with root collar
  - The measuring position is located on the uphill side
  - The measuring position is located 4.15 cm beneath stump height

Figure 4-1 illustrates the rules applied in Germany and some other European countries. They make provision for trees growing on slopes, for trees with irregular bole shapes at breast height, for leaning and forked trees and for trees with excessive butt swell.

The basal area of a tree is defined as the cross-sectional area of the stem, either at breast height or at a specified height above the base of the tree. It is either derived from the tree diameter, measured with a caliper, or from the stem circumference measured with a tape. In both cases, the calculation assumes a circular shape of the cross section of the stem. An elliptical cross section occurs occasionally:

1. In regions with a prevailing wind direction either during the growing season or throughout the year. It may induce an elliptical cross section of the crown with the longest axis coinciding with the dominant wind

direction. The radial increment along the perimeter of the stem and corresponding crown radius are correlated, possibly because of the impact of the eccentricity of the live crown on the transport of photosynthate.

2. It occurs on steep slopes, since the longest axis of the cross section tends to coincide with the direction of slope.

3. When trees are planted in a rectangular spacing pattern, with a large distance between and a much shorter distance within the rows. The effect of this spacing pattern should not be exaggerated. In *Eucalyptus grandis* Bredenkamp (1982) did not detect a statistically significant effect of such planting patterns on the ratio of the stem diameter in the direction of the planting rows over that at an angle of 90°. Similar results were obtained in Germany (Akça 1995).

The cross-sectional area is usually calculated as the area of the circle, which corresponds with the recorded stem diameter. On elliptical stem cross sections, the resultant error may be reduced by measuring the stem diameter from two directions, one in the direction of the longest axis of the ellipse and the other at an angle of 90° to the former.

Several studies have been undertaken to examine the accuracy of breast height diameter measurements. Kennel (1959) compared the basal area estimates of single trees, obtained from tape and caliper measurements. The former were about 2% above those obtained with the caliper. The standard deviation of the sampled diameter distribution was not affected by obtaining two caliper readings, at an angle of 90° to one another. Gregoire et al. (1990) investigated the accuracy of basal area estimates on stem discs. The average error, based on a single-tape measurement, was +3.1%, that based on two caliper measurements at an angle of 90° to one another was −2.5%. Noncircularity had a profound effect on the observed bias. The positive bias associated with calipers and tapes produced biased growth estimates, if the trees were measured with calipers on the first and with tapes on the second occasion. Chacko (1961) compared cross-sectional area estimates for individual trees, based on a single randomly selected diameter, on the average of the largest diameter and a second one at an angle of 90° to the former, on the geometric mean of a randomly selected diameter and a second one at an angle of 90° to the first one and on the average of the smallest diameter and a second one at an angle of 90°. All four estimators produced positively biased estimates of the cross-sectional area.

Matern (1956) characterized out-of-roundness of stem cross sections in terms of convex deficit and isoperimetric deficit. When a diameter tape is tightly wrapped around a tree with an irregular stem form, it encloses an area which is denoted as its convex closure. The measured circumference is always smaller than the true circumference and the true cross-sectional area is smaller than

the estimated area. The difference is called convex deficit. An isoperimetric deficit occurs when a convex closure is not a circle, but can be described by mathematical functions. When the corresponding area is calculated as a circle, but derived from the circumference, the true area is overestimated because the circle represents the geometric figure with the smallest circumference for a given area and conversely, with the largest area for a given circumference. The difference was described as isoperimetric deficit. Müller (1958) investigated the effect of the isoperimetric deficit on the estimated basal area, Biging et al. (1988) that of eccentricity on the basal area and basal area increment, with estimates being obtained from one diameter measurement on each cross section and from the mean of two measurements at an angle of 90°. The commonly used estimators usually overestimated the basal area. When using a single-diameter measurement, the length of the minor axis of the ellipse produced a more accurate estimate of the cross-sectional area. Goetz et al. (1987) tested four photographic techniques to measure the cross-sectional areas of stem discs on standard prints. For slow-growing species, black-and-white enlarged prints of black-and-white photographs of unplaned discs produced the best results.

1.3   Tree height

Tree height is required to determine the site class or site index of a stand and to estimate the volume of standing trees. Total tree height is defined as the distance between the top and base of the tree, measured along a perpendicular, dropped from the top (see Figure 4-2).
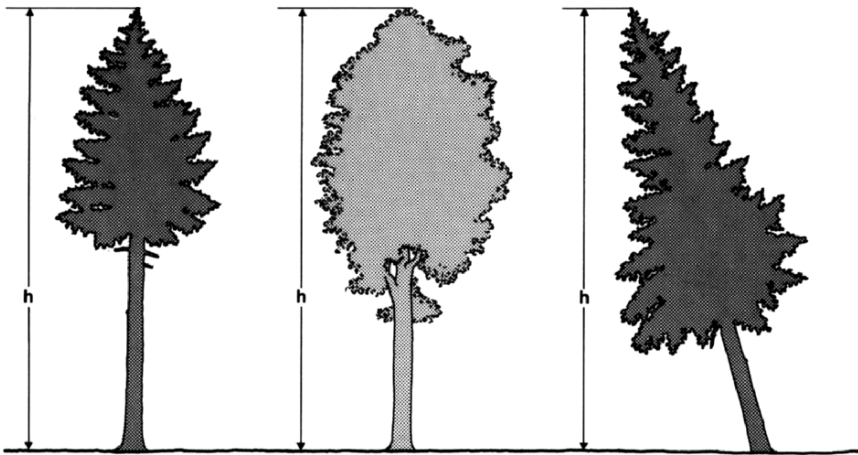
Figure 4-2. Defining tree height.

   In the case of perfectly straight stems of exactly vertical trees, tree height and stem length are identical. The merchantable height uses an upper point of measurement, which coincides with the limit of merchantability. The latter depends primarily upon the minimum over bark or under bark stem diameter, which may vary from country to country.

1.4   Stem form

The following methods may be applied to either determine or estimate age-related form changes:
- Measurement of a series of upper-stem diameters – at different points in time – with the aid of *Bitterlich's telerelascope*, the *Barr and Stroud dendrometer* or other recently developed electronic instruments
- Measurement of dbh, height and a single upper-stem diameter in combination with a function, which predicts the breast height form factor. The measurements are to be carried out at different points in time within the rotation period. The *Finnish caliper* is a useful instrument to measure upper-stem diameters up to a height of 7 m above ground (Rhody et al. 1984).
- Measurements on representative trees selected for *stem analysis*. The present and past under bark diameters at different distances from the top of the tree serve to reconstruct the *form factor* or *form quotient k* years ago or to fit an equation with age as independent and a form variable as target variable.
- *Taper functions* are used to predict upper-stem diameters as a function of dbh and height. They produce estimates of dbh and height-related form changes, but give no information about age-related changes, since the effect of site and stand density on dbh and height is ignored

1.4.1   Stem profile and taper

The stem profile of a tree describes the decrease of the over or under bark stem diameter with increasing height above the base of the tree. The stem curve of a specific tree is obtained by plotting the height at *i* m above ground over the corresponding diameter, for different positions along the stem. In taper modeling, however, these variables are scaled by dividing the stem diameter at *i* m by dbh and the corresponding height by total tree height (Figure 4-3). Others express the point of measurement in terms of the absolute or relative distance from the top of the tree.

   To describe the stem curve by mathematical functions, the bole is sometimes subdivided into a lower, a central and an upper section. The solid, represented by the lower section, up to a height of approximately 10% of the
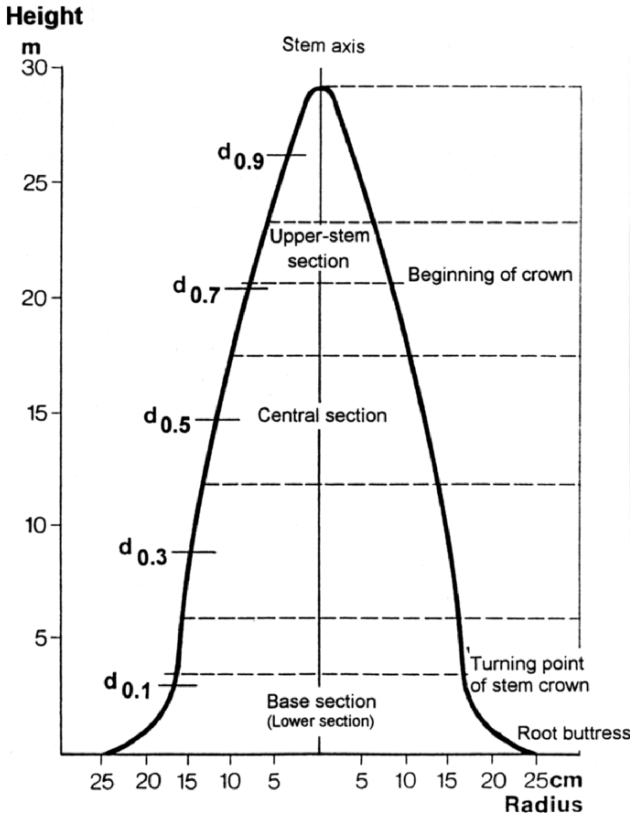
Figure 4-3. Stem profile.

tree height, resembles a *truncated neiloid*, the central section is a *truncated quadratic paraboloid*. The upper section is either approximated as a *cone* or as a *quadratic paraboloid*. Although this general pattern is apparent in open-grown as well as in stand-grown trees, the stem profile of the individual tree is affected by its social position within the stand as well as site, silvicultural treatments, such as stand density, planting espacement and fertilizer, and by genetic parameters.

   Taper is defined as the rate of decrease of the diameter of the bole per unit increase in height above the base of the tree. For practical purposes, it is expressed in centimeter per meter stem length, usually for the stem section between breast height and the merchantable upper diameter. This definition assumes a conical stem profile within this stem section and ignores the nonlin-ear rate of decrease of the diameter within the individual stem. Taper is closely

related to the growing space available to the individual tree and is extremely high for solitaries. Competition amongst trees reduces the rate of diameter growth, but has a negligible effect on height growth, with the exception of overcrowded stands. A relatively high rate of decrease can, therefore, be expected in widely spaced and heavily thinned stands. Taper is also affected by the application of fertilizers, which tend to stimulate radial growth in the upper part of the stem section, but below the base of the live crown, more severely than in the lower stem section.

1.4.2   Stem form theories

Around the end of last century, several forest scientists proposed theories to explain the characteristic form of trees. Metzger (1894) hypothesized that the shape of the stem is controlled by lateral pressures, which are centered at a fixed focal point within the live crown. The stem was thought to be a beam of uniform resistance within the stem section extending between butt swell and this focal point. In that case, the assumption of a cubic paraboloid holds true. Metzger's hypothesis assumed that the relationship between stem diameter and height above ground can be expressed by the equation $d^3 = b_0 + b_1 h$. Hohenadl (1923) suggested that crown weight rather than wind-controlled lateral pressure was the dominant factor controlling stem form. Jaccard (1912) proposed the water conduction theory, which stated that the growth distribution within the stem is controlled by the physiological necessity of water conduction. Larson (1963) formulated the hormonal theory, which emphasized the role of growth regulators for the growth distribution within the tree. There is no convincing evidence that any of the proposed theories explains the form of the stem satisfactorily.

Gray (1956) conducted empirical studies and concluded that the requirements for strength, formulated by Metzger, and expressed by a cubic paraboloid were unnecessarily stringent and produced a tree with a greater resistance against lateral forces than required by the root system, possibly because the roots are embedded in relatively weak material. The author proposed the quadratic paraboloid $d^2 = b_0 + b_1 h$ which has 20% less volume than a cubic paraboloid and was considered to satisfy the strength requirements of the tree. Newnham (1962, 1965), who compared the stem form of open-grown with that of close-grown trees of various species, found that the form of open-grown trees resembled a cone, but occasionally it was neiloid-like. The author's studies confirmed the suitability of Gray's model of a quadratic paraboloid to describe the stem form of conifers in British Columbia. The coefficient b_1 was not affected by age and site index and could be satisfactorily interpreted in terms of its relationship with the combined variable

d^2/h. Burkhart et al. (1985) fitted quadratic as well as cubic paraboloids to loblolly pine. In most trees, the model of a quadratic paraboloid produced a close fit.

Some trials under controlled conditions have been conducted to obtain a better understanding of stem form changes, due to crown characteristics and silvicultural treatments. In a pruning trial in loblolly pine, Labyak et al. (1954) regressed the cross-sectional area growth at nine points along the stem on branch characteristics. Those variables, which expressed the amount of foliage above the point of measurement, were highly significant predictor variables. To some extent, these studies supported the contention that the development of the tree form is controlled by strength requirements of the main stem, as well as by physiological responses.

1.4.3 Form factors and form quotients

The *form factor* of a tree or stem is defined as stem volume, expressed as a proportion of the volume of a cylinder of the same height, with a diameter equal to the stem diameter at the selected reference point:

$$f = \frac{\text{stem volume}}{\text{cylinder volume}}$$

The different reference points being used, produce different types of form factors: The *absolute form factor* is based on the diameter at the base of the tree as a reference point, the *false* or *breast height form factor (f)* locates this point at 1.3 m above the base of the tree and the *true form factor* (λ), which is also known as *Hohenadl's form factor*, is based on a reference point which coincides with a height of 10% of the tree height above ground.

Because of the erratic distribution of radial growth within the annual ring in the lower stem section, due to butt swell, the absolute form factor is seldom if ever used. The *breast height form factor* is conveniently used for computational purposes, for example, to estimate the stem volume from tree basal area, tree height and form factor. The true form factor has the advantage of more appropriately reflecting the average taper of the stem, but necessitates the measurement of the stem diameter at the 10% reference point. The latter cannot be reached with a caliper or tape, if the tree is higher than 17–18 m, and then requires the measurement of the tree height. In Germany, tree volume is expressed either as the total tree volume or as the merchantable volume, up to a fixed upper-stem diameter of 7 cm over bark, both including branch volume. Alternatively, tree volume is expressed as either the total or the utilizable volume of the bole. The corresponding false form factors express the ratio between these volumes and that of a cylinder with breast height as the reference

point. Furthermore, the volume may either represent the over or under bark volume. The total tree volume, including branches, is of considerable interest and increasing importance when trees are grown for the production of wood for energy.

The *form quotient* of a tree is similarly controlled by the rate of decrease in the stem diameter and is expressed by a single number. The *false form quotient* is defined as follows:

$$q_{0.5h} = \frac{d_{0.5h}}{d}$$

A fixed reference point of 5 m above the base of the tree in the numerator was used by Mitscherlich (1942), to construct tables for estimating the merchantable tree form factor from the form quotient:

$$q_M = \frac{d_5}{d}$$

The true form quotient

$$\eta_{0.5h} = \frac{d_{0.5h}}{d_{0.1h}}$$

can be applied to estimate the true form factor. Krenn et al. (1944) related the *true form factor* to the *true form quotient* $d_{1/2h}/d_{0.1h}$:

$$\lambda = -0.038 + 0.777 \cdot \frac{d_{0.5h}}{d_{0.1h}}$$

Hohenadl's form quotient (q_H) is defined as

$$q_H = \frac{dbh}{d_{0.1h}}$$

is greater than one for trees with a height of less than 13 m. Because breast height diameter is measured at a fixed and $d_{0.1h}$ at a variable height, this form quotient is influenced by age. *Hohenadl's form quotient*, however, is useful for converting the true form factor of a tree into its false form factor. Girard (1939) introduced the form quotient:

$$q_G = \frac{d_u (17.3\,\text{ft})}{d_{o.b.}} \cdot 100$$

The upper diameter is measured at 17.3 ft and coincides with the position of the thin end of the bottom log, with 1 ft being added to adjust for stump height and 0.3 ft to allow for log trimming. It was used as a predictor variable for

the construction of volume tables and assumed that there is little variation in the taper above the first log of trees with the same breast height diameter and merchantable height. Jonson (1910, 1913) proposed the function

$$y_i = b_1 \frac{\log (b_2 + x_i - 2.5)}{b_2}$$

with y_i = diameter at the ith percentage distance from the top of the tree, x_i = percentage distance of the point of measurement from the top of the tree.

Behre (1923) examined the validity of Jonson's formula for describing the stem taper and proposed the equation

$$y_i = \frac{x_i}{b_0 + b_1 x_i}$$

Trees were grouped in form classes, defined by the diameter at 50% of the height above breast height, divided by dbh. The parameter estimates were harmonized to remove inconsistencies amongst the taper curves.

Example 4.1 The following bole diameters were measured on a *Pinus patula* tree with a dbh of 45.6 cm, a height of 27.4 m and a total stem volume of 1.782 m^3:

Height (m)	Diameter (cm)	Height (m)	Diameter (cm)	Height (m)	Diameter (cm)	Height (m)	Diameter (cm)
0.3	50.0	6.0	37.7	14.0	30.3	22.0	16.1
1.3	45.6	8.0	35.9	16.0	28.3	24.0	8.7
2.0	43.4	10.0	34.6	18.0	25.6	26.0	3.7
4.0	38.8	12.0	33.0	20.0	21.9	27.4	0.0

The diameters at different relative heights were obtained by interpolation in order to calculate the true form quotients (*TFQ*) less than.

	$i = 10\%$	$i = 30\%$	$i = 50\%$	$i = 70\%$	$i = 90\%$
$d_i\%$	40.2	35.8	30.7	23.5	7.05
TFQ	1	0.89	0.76	0.58	0.18

The false form quotient for $d_{50\%}$ as upper-stem diameter is $30.7/45.6 = 0.673$. The form quotient with $d_{10\%}$ as numerator is $40.2/45.6 = 0.882$. The false and true form factors are:

$$f_{1.3}(\text{false form factor}) = \frac{1.782}{\frac{\pi}{4} \cdot 0.456^2 \cdot 27.4} = 0.398$$

$$\lambda_{0.1}(\text{true form factor}) = \frac{1.782}{\frac{\pi}{4} \cdot 0.402^2 \cdot 27.4} = 0.512$$

1.4.4   Stem profile functions

Ormerod (1973) proposed a power function to describe the profile of stem sections

$$d_i = (A - B) \cdot \left(\frac{t}{T}\right)^r + B$$

where A, $B =$ top and bottom diameter of a given stem section, $t =$ distance between a given point P and the top of the stem section, $d_i =$ stem diameter at point P, $T =$ length of stem section and $r =$ exponent. The author also developed the diameter-point method for modeling the stem profile (Ormerod 1986). The method emphasized the prediction of diameter and height at the two inflection points of the stem form curve, which acted as joins and demarcated the three resultant stem sections. The stem profile between two consecutive points was described by simple functions based on the location of these critical points, which were arbitrarily set at 20% and 65% of the height. A third point, located between the previous two, was selected and, together with the joins, they were used to calculate the coefficients of a simple one-parameter taper function for each of the sections. The submodel was used to predict the three diameters within a stem section. Behre's (1923) hyperbolic function

$$d_i = \frac{(A - B)t}{T - s(T - t)} + B$$

where A, $B =$ end diameters, $t =$ distance from the thin end of the stem section and $T =$ section length, was applied to estimate intermediate diameters. When d is known for given t, the equation can be solved for s. The section volumes are then calculated by integration.

   In 1973, Ormerod introduced the following taper equation for estimating the upper-stem diameter:

$$d_i = d \left(\frac{h - h_i}{h - 1.30}\right)^{b_1}$$
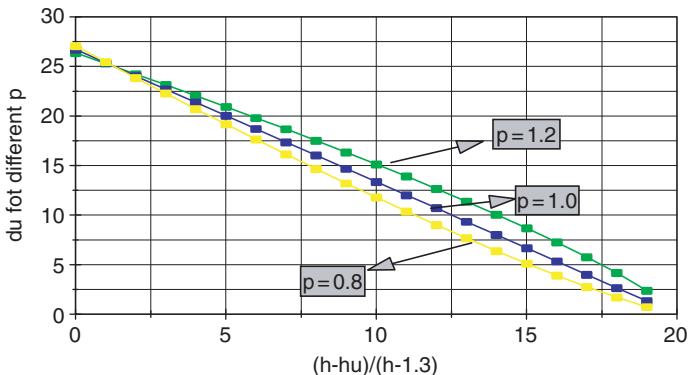
Figure 4-4. Stem profiles, generated by Ormerod's model.

The coefficient b_1 is a stem form parameter which can be estimated by ordinary least squares. The equation is written as $y = b_1 x$ with $x = (h - h_i)/(h - 1.30)$, $y = d_i/d$ and fitted to observed data. A value of 1 for b_1 generates a conical stem form, a value of 0.5 produces a quadratic paraboloid and values above 1 produce neiloidal solids. Based on the above formula, the stem profile of a tree with a dbh of 30 cm, a height of 25 m, and b_1 values of 0.8, 1.0 and 1.2, respectively, are shown in Figure 4-4.

Reed et al. (1985) fitted Ormerod's (1973) taper function to sample trees and proposed the following estimator:

$$b_1 = 1 - \left(\frac{h}{d} - 30\right)/120$$

The coefficient values 30 and 120 were introduced as fixed quantities, based on observed p-values and their relationship with the ratio h/d.

In recent years, a considerable amount of research has been carried out to develop taper functions, which make it possible to reconstruct the stem profile of a tree of given dbh and height. They are dealt with in Chapter 6.

1.5 Tree volume

The volume of the individual standing trees is usually obtained from the equation

$$v = 1/4\pi d^2 \cdot h \cdot f$$

and is usually estimated from equations either with dbh or dbh and height or from dbh, height and an upper-stem diameter as predictor variables (Chapter 7). The volume estimates obtained from these equations or volume tables represent

averages. *Pressler's critical height* and *Tor Jonson's form point method* have been applied to provide direct estimates.

In the early days of forest mensuration, it was suggested to determine the point along the stem where its diameter was 50% of dbh (Pressler 1865). The resultant volume formula was based on the approximation of the two stem sections below and above this point of measurement as a paraboloid and cone, respectively. Bitterlich (1959) recommended the application of this method, with Pressler's critical point being measured with the mirror relascope or with the telerelascope. Jonson (1928) introduced the concept of form point, which represents the center of wind resistance and is located at the center of gravity of the tree crown. The relative height above ground for this form point was used as a predictor variable to estimate the form class for different diameter classes within a stand. It can be seen as a substitute for the absolute form quotient. In dense stands, however, the inaccessibility of the form point within the live crown presents insurmountable problems. The concept was abandoned in favor of more efficient estimators for stem volume. Schöpfer (1976) recommended to measure two stem diameters, located at breast height and 50% of the tree height, respectively. The shape of the upper stem section is represented by a paraboloid and its volume is estimated accordingly. The volume of the central section may be estimated with Smalian's formula, whereas the butt section might be approximated as a cylinder. Alternatively, Bitterlich's telerelascope may be used to estimate upper-stem diameters and the Smalian formula applied for each stem section separately, but this method is too costly for general use.

1.5.1 Importance and centroid sampling

Gregoire et al. (1986) introduced *importance sampling* to obtain unbiased estimates of the tree volumes, based on a single-diameter measurement at a given point on the stem. The latter is selected at random, proportional to the estimated volume distribution within the stem, which is determined with the aid of a proxy taper function. The resultant volume is a biased estimate of the total or merchantable stem volume. In order to obtain an unbiased estimate, it is adjusted by a factor, which is calculated as the ratio of the observed over the estimated cross-sectional area at the point of measurement. The estimator is

$$v_I = v_{pr} \cdot \frac{\sum\limits_{i=1}^{n} \dfrac{g_i}{g_{i(pr)}}}{n}$$

Where

v_I = importance sampling estimator

g_i = true (measured) basal area at height i

$g_{i(pr)}$ = proxy basal area at height i

N = sample size

Wiant et al. (1992) wrote a program in BASIC for determining the point of measurement on the bole for importance as well as for *centroid sampling*. The former requires the selection of a random number between 0 and 1, drawn from a uniform distribution. The assumption that the bole can be approximated as a second-degree paraboloid is introduced to generate the proxy function, whereas a value of 0.5 generates the centroid estimator. The BASIC program consists of two subprograms, one with stump height, merchantable and total height as input variables. It is used to determine the sampling position for importance as well as centroid sampling, the other using the same input variables, but with the addition of dbh, double bark thickness and sampling method (importance versus centroid) as inputs. For reasons of simplicity, Ormerod's taper function was used as proxy function.

Studies carried out by Wiant et al. (1989) showed that the dendrometry required to obtain estimates of the same accuracy as that obtained with three-P sampling can be reduced by as much as 96%. Later studies by Wood et al. (1990) showed that the best results are obtained when the point of measurement coincides with the centroid of the bole, i.e., with the point on the stem, which divides the stem into two sections of equal volume. The latter occurs at a height of 30% of the total tree height above the base of the tree. This paved the way for a specific version of importance sampling, which was denoted as centroid sampling. The formula for this estimator is identical to that of importance sampling, but is based on a fixed value of 0.5 instead of a random number between 0 and 1. This is in line with Van Deusen et al. (1987) who defined centroid sampling as importance sampling at a fixed point on the stem, which reflects the expected position, resulting from repeated sampling. Because of sampling proportional to size, the majority of the points of measurement used in importance sampling are located in the lower part of the stem. Wood et al. (1992) noted two advantages of centroid sampling. No measurements are required within the upper part of the crown with its poor visibility and tree-to-tree predictions are more consistent. Centroid sampling has the disadvantage of not producing unbiased estimates of stem volume, but the above studies carried out by Wood and Wiant showed that the magnitude of bias is negligible. The position of the centroid, however, is not estimated accurately, if a poor proxy function is used. In importance sampling, a poor proxy function has an adverse effect on the precision of the estimates, but the mean is not affected by the quality of this function.

In control-variate sampling, the proxy function is used to model the stem volume. Diameter measurements at points along the bole, selected with the aid

of a certain probability density function are required to estimate the difference between the true volume and the volume, obtained from the proxy taper function. The estimator is

$$v_{CV} = v_{pr.} + \frac{h_1 - h_2}{n} \sum_{i}^{n} \left(g_i - g_{i(pr.)} \right)$$

where h_1 and h_2 refer to the section of the bole for which the volume is estimated, for example, $h_1 = total$ or merchantable height, $h_2 = stump$ height. Hence, contrary to importance sampling, which is based on a multiplicative adjustment of the proxy volume, the control-variate estimator is based on additive adjustments.

Wiant et al. (1995) compared the centroid method with the Newton, Huber, Bruce and Smalain formula to estimate the volume of butt logs and evaluated the results in terms of bias and tolerance intervals. Valentine et al. (1995) compared importance sampling with control-variate sampling to estimate the bole volume or that of parts of the stem, for example, between stump height and merchantable height. Kleinn (1995) compared importance sampling with control-variate sampling to estimate the volume of single-standing trees, based on single as well as multiple measurements along the stem and different taper proxy functions.

1.6   Bark thickness

Bark is usually a waste product, sometimes a utilizable by-product and occasionally, a main product with timber as by-product. Black wattle plantations in South Africa and South America, for example, are planted primarily for the production of tannins, extracted from the bark. The thickness of the bark is a genetic characteristic and for a given species or provenance is related to age, diameter and site. It can be determined accurately by felling the tree and extracting a cross section for further measurements with a ruler.

Bark thickness is frequently converted into a bark factor. Meyer (1942) assumed a linear relationship between the over and under bark diameter within a given stand, which implies a linear relationship between bark thickness and *dbh*, and defined the bark factor k as $k = d_{u.b.}/d_{o.b.}$, Zöhrer et al. (1973) used its inverse. A ratio of means estimator is normally used to estimate k.

In Swedish studies, a regression estimator for bark thickness was proposed, with relative diameter at a given point above the base of the tree as predictor variable, habitat and site index as covariates. (Johnson et al. 1987). Swedish studies, however, indicated that a regression estimator with logtransformed bark thickness as dependent and logtransformed diameter as predictor variable

produces better results (Oestlin 1963). In similar studies, Johnson et al. (1987) proposed an equation for estimating relative bark thickness (RBT) at height i from site index (SI), a dummy variable representing habitat (H) and the relative diameter at height i (RD_i). The prediction equation used H, SI, H.SI, $RD_i.H$, $SI.RD_i$ and $H.RD_i.SI$ as predictor variables. In addition, a nonlinear three-parameter model, based on Grosenbaugh's equation, was tested. Deetlefs (1957) introduced bark thickness to estimate the bark weight of black wattle. The prediction equation, with bark thickness at height i being expressed as a ratio over dbh, was as follows:

$$w = b \cdot \left(\frac{\pi}{2}\right) \cdot \frac{BT_i}{d_i} \cdot d_{1.3}^2 \cdot h$$

where $w = bark\ weight$ and $b = parameter\ of\ the\ equation$. Schönau (1970) developed an equation to estimate the timber yield of black wattle per hectare from bark weight, mean bark thickness at breast height, mean dbh and mean height. Bark thickness was related to latitude, site index mean dbh.

Example 4.2 The following regression equation was fitted to predict the under bark diameter of *P. patula* from the over bark diameter and upper height:

$$d_{u.b.} = -1.694 + 0.9724 d_{o.b.} - 0.000686 d_{o.b.}^2 + 0.2021 h_u - 0.00653 h_u^2 + 0.00156 d_{o.b.} h_u$$

The equation is used to determine the estimated bark thickness at various heights above ground (Figure 4-5).
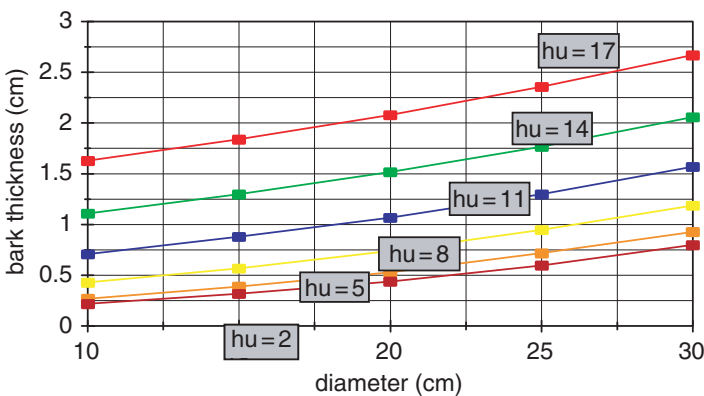
Figure 4-5. Relationship between diameter over bark, upper height and bark thickness.

1.7   Tree crown and foliage

Crown characteristics are useful to predict growth responses in spacing, thinning and fertilizer trials and to relate growth to soil moisture availability. They are frequently required for growth modeling with tree growth being estimated from crown and other tree characteristics. These studies emphasize the close relationship between the size of the crown and the amount of photosynthetically active foliage. The general crown morphology of a spruce tree and the calculation of crown parameters is illustrated in Figure 4-6. The *crown radius* is defined as the distance between the center of the bole and the outer edge of the crown, usually measured at the position of maximum width. *Crown width* is usually defined as twice the radius. In many species, the outer edge can be identified accurately, in others, for example, in Eucalyptus and in Southern pines such as *P. patula* and *P. elliottii* this may prove more difficult. To facilitate the determination of the crown diameter, a *crown mirror* may be used to project the crown onto the ground.

   The increase in crown radius with increasing tree age is primarily controlled by competition from neighboring trees. In dense stands, crown excentricity is quite high, because of unequal competition from surrounding trees. For this reason, the crown radius is usually measured from 4 or more than four directions.

$$\text{Crown ratio} \quad = \frac{Crown\ length}{Tree\ height}$$

$$\text{Crown form index} \quad = \frac{Crown\ length}{Crown\ width}$$

$$\text{Crown thicknes index} = \frac{Crown\ width}{Crown\ length}$$

$$\text{Linear crown index} \quad = \frac{Crown\ width}{dbh}$$

$$\text{Crown spread ratio} \quad = \frac{Crown\ width}{Tree\ height}$$
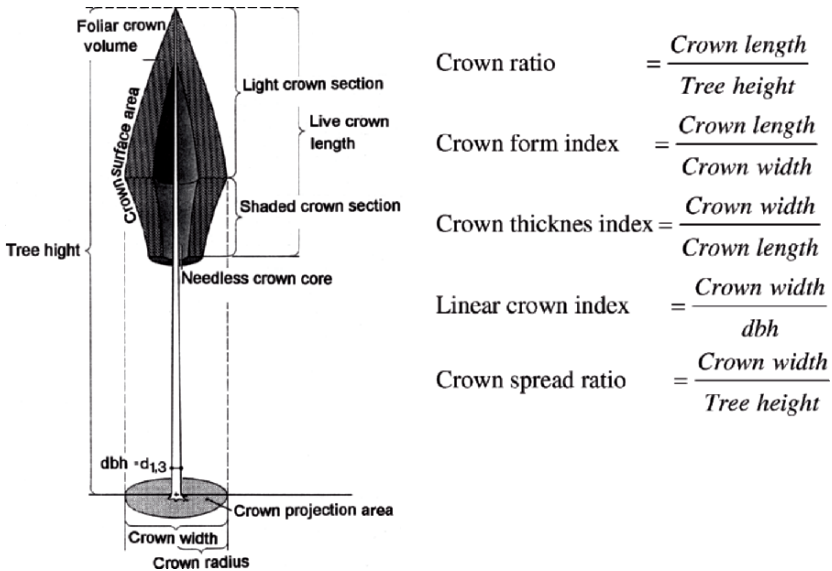
Figure 4-6. Crown morphology of *Picea abies* (Burger 1939).

Huber 1987 measured the crown radius from 8 instead of 4 directions. Röhle et. al. (1985) evaluated the accuracy of the estimation of the horizontal crown projection of the single tree, based on the measurement of 32, 16, 8, 4 and 2 radii per tree, and for different assumptions concerning the spatial distribution of trees within the stand. Between 4 and 8 radii per tree were sufficient to estimate the crown projection for the entire stand, whereas between 8 and 16 radii per tree were required to obtain reliable estimates for individual trees. The same author compared the accuracy of crown projection estimates, resulting from plumb-line projections and from subjective, visually controlled projections, respectively (Röhle 1986). The latter tended to produce inaccurate estimates.

For mapping of crown projections, the crown radius is measured from random directions and recorded as polar coordinates (Figure 4-7).

In growth studies, many researchers prefer to use live crown length instead of crown diameter to represent the tree crown. On standing trees, it is conveniently measured with the hypsometer.

In crown studies of six coniferous species, Biging et al. (1990) defined the live crown length as the distance between the apex of the tree and the base
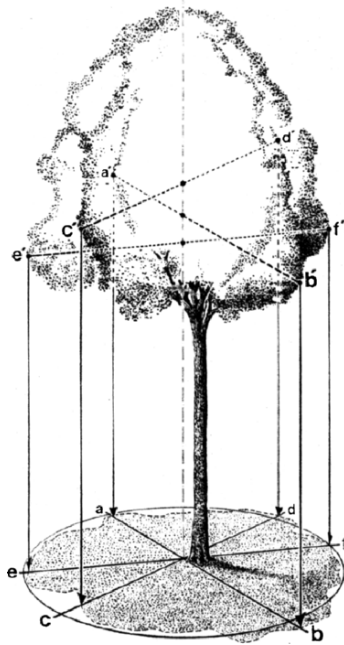
Figure 4-7. Mapping of horizontal crown projections.

of the live crown, whereas similar studies in Germany specify that the base of the live crown should coincide with the whorl which contains at least three live branches. In other studies, the base of the live crown was defined halfway between the first whorl with one or more than one live branch and the whorl with at least four live branches (van Laar 1969, 1972).

In such growth studies, crown length is normally expressed as a crown ratio, i.e., as the length of the live crown length calculated as a per cent of the total tree height. It has been found to be a function of dbh, height, stand density and age. Dyer (1987) proposed the following regression equation for estimating the crown ratio (CR) of loblolly pine:

$$CR = 1 - e^{\left((b_0 + b_1/A)\cdot\frac{d}{h}\right)}$$

Crown ratio decreases with age, whereas the variable d/h controls the effect of crown ratio on taper. Holdaway (1986) developed a model for predicting the crown ratio from tree and stand variables. At tree level, dbh was the most suitable measure of the competitive status of the subject tree. The final model was

$$CR = b_1 \left(\frac{1}{1 + b_2 G}\right) + b_3 \left(1 - e^{-b_4 d}\right)$$

where G = basal area per hectare and d = diameter of the subject tree. The coefficient b_1 estimates the crown ratio in the case of complete absence of competition, b_2 measures the rate of decrease of the crown ratio with increasing competition. The effect of competition, at stand level, was expressed by the quantity $b_1/(1 + b_2 G)$.

Crown surface area (CSA) is defined as the outer surface area of the live crown. Normally, it is assumed that the crown of conifers can be represented either as a cone or as a paraboloid. The crown surface, based on the assumption of a paraboloid, is calculated as follows:

$$CSA = \frac{\pi CW}{12CL^2} \left(4CL^2 + \frac{1}{4}CW^2\right)^{3/2}$$

where CL = crown length and CW = crown width. The calculation of the surface area based on the formula for a paraboloid, assumes that the maximum crown width is found at the base of the live crown. A more accurate estimate of the crown surface area is obtained by measuring the crown radius on different points along felled sample trees (Dong et al. 1985). The crown surface area expresses the area of photosynthetically active needles, although the rate of photosynthesis per unit leaf area is affected by the position within the crown and is higher for the light crown than for the shaded crown section. In studies

to assess the effect of pollution stresses on growth, volume growth per unit crown surface area is a useful target variable (Kramer 1986).

Crown volume is calculated from crown radius and crown length. It is less suitable to predict growth than crown surface area since, the inner core of the crown does not contain photosynthetically active foliage. Crown biomass expresses the sum of the biomass of branches and foliage and is a useful tree characteristic, which is closely related to the total photosynthesis of the tree, particularly when the live crown is partitioned into sections expressing the degree of shading.

1.8   Leaf surface area, leaf weight and sapwood area

Leaf surface area is an important parameter in physiological studies because of its close relationship with photosynthesis. Waring et al. (1980) examined the relationship between leaf area and the rate of volume growth of single trees. The latter was obtained from estimates of the leaf surface area per unit leaf biomass and from the estimated leaf biomass per unit sapwood area. The observed ratio, basal area growth to sapwood area, was consistent with that of volume production to leaf area. Vose (1988) modeled the leaf area distribution within the live crown of loblolly pine with the aid of the two-parameter *Weibull distribution*. The following function was used:

$$y = TLA\left(\frac{\alpha\left(\text{depth}^{(\alpha-1)} \cdot e^{-(\text{depth}/\beta)\alpha}\right)}{\beta^{\alpha}}\right)$$

where y = projected leaf area, TLA = total leaf area m^2/tree, depth = depth within the live crown and α, β = Weibull parameters. Several methods have been developed to determine the surface area of fascicles and the leaf area of the single tree. Beets (1977) compared four methods to obtain the fascicle surface area of *P. radiata*. The most accurate estimates were obtained from a function; which used the square root of the fascicle volume and its length in addition to a shape coefficient. Davies et al. (1980) investigated and described the glass bead method by which needles were coated with glass balls in a fluidized bed. Swank et al. (1974) compared three methods for estimating the fascicle surface area in eastern white pine: diameter-based stratified two-phase sampling, ratio-of-means estimator, and a regression estimator, both obtained in two-phase sampling. Stratified two-phase sampling produced the best results.

Partially due to the high cost of direct measurements, models have been constructed to estimate leaf area from other characteristics. Johnson (1984) proposed a method for estimating the total surface area of pine needles, which required the measurement of the volume of the needle sample, displaced after

immersion in water, the cumulative needle length and the number of needles per fascicle. The needle surface area was calculated from

$$A = 2l \left(1 + \frac{\pi}{n} \right) \cdot \sqrt{\frac{\upsilon n}{\pi l}}$$

where υ = needle volume, l = cumulative needle length and n = number of needles per fascicle. Wood (1971) investigated the solid shape of 1-year-old fascicles of *P. radiata.* Based on the approximation as a cylinder, formulae were derived to express the surface area of single needles as well as for 2-, 3-, 4- and 5-needled fascicle types as a function of needle width and length. Ohmart et al. (1986) applied the following equation for estimating the needle surface area of *P. radiata* from needle weight:

$$\ln(SA) = b_0 + b_1(\text{age}) + b_2 \ln(\text{weight}) + b_3(DC)$$

where SA = surface area and DC = depth in canopy. Bacon et al. (1986) developed equations for hardwood species to predict leaf area from basal area. Shelton et al. (1984) measured the fascicle area in seven loblolly pine stands of different ages. Fascicle samples of two fascicle ages were obtained from the upper and lower canopy, respectively. Log fascicle area was subsequently regressed on log fascicle weight. The fascicle surface area:fascicle weight ratio was greatest for currently produced needles in the lower canopy and highest for the older fascicles in the upper canopy.

The surface area of a fascicle, the total leaf surface of a tree and the *leaf area index* (LAI), defined as the leaf surface area per hectare, expressed as a proportion of the corresponding ground area, are closely related with the production of photosynthate. The leaf surface is also closely correlated with the *sapwood area* of the single tree and, for this reason, many studies use the sapwood area to substitute leaf area or other characteristics. Long et al. (1988) investigated the relationship between leaf area (*LA*) of lodgepole pine and *breast height sapwood area* (*SA*) as well as distance from breast height (*DIST*) to the center of the live crown as predictors

$$LA = b_1 SA^{b_2} DIST^{b_3}$$

A possible effect of stand density and site index was accounted for by their correlation with sapwood area. Studies by Kendall et al. (1978), which were carried out in four tree species, indicated that sapwood area estimates crown biomass better than breast height diameter. Long et al. (1981) found the sapwood cross-sectional area at any point along the bole to be linearly related to the amount of foliage above this point. In large trees, however, the sapwood area, which is required for the supply of water to the transpiring foliage, is insufficient to provide the necessary mechanical support for the stem. Sapwood

and heartwood together however determine stem form in response to the need for mechanical support. Albrektson (1984) applied destructive sampling procedures to obtain foliage mass and sapwood basal area in 153 Scots pine trees from 16 stands in Sweden, and applied regression analysis with foliage weight as the dependent and mean annual ring width in the sapwood zone as the predictor variable. The within-stand correlations were high, but there were substantial differences amongst the regression coefficients of different stands. In *Abies balsamea* and *Picea rubens*, Marchand (1984) found evidence of a linear relationship between sapwood area at breast height, as well as immediately below the live crown and the projected leaf area, as well as foliage mass. Paine et al. (1990) examined the regression of foliage surface area on sapwood area as a follow-up of studies, which showed that the ratio basal area growth to the sapwood cross-sectional area was an indicator for tree vigor. Blanche et al. (1985) found a linear relationship between the sapwood area and leaf area of loblolly pine. Replacing sapwood area at breast height by that at the base of the live crown improved the predictive ability of the equation. Sampling studies during May and August, indicated that the highest correlation coincides with the time of the highest leaf biomass production. Dean et al. (1986) found a linear relationship between the sapwood area at the base of the live crown and leaf area of *P. contorta*. The influence of the development stage (saplings versus mature trees) on the slope of the regression line could be removed by regressing log (sapwood area) on log (leaf area × distance between cross section and the crown center).

2 VOLUME, LOG CLASSES AND WEIGHT OF FELLED TREES

When no volume tables are available, the total or merchantable volume has to be measured on felled trees, in order to estimate the volume of the mean tree within a given stand or to estimate the stem volume for each diameter class separately. Such direct measurements may also be necessary when regional volume tables tend to produce biased volume estimates. In general, however, measurements on felled trees are necessary to construct volume tables and to estimate the parameters of tree volume equations. Weight measurements are a common practice for measuring the quantity of roundwood, utilized as mining timber, pulpwood and for manufacturing chipboard and other similar products. In these cases, the sales price expresses the price per tone as the unit of weight. In other instances, the timber is sold on the basis of its roundwood volume, but the air- or oven-dry weight is measured and subsequently converted to volume.

2.1   Volume

2.1.1   Roundwood volume

The xylometer, which can be used to measure the volume of logs, is a tank filled with water, equipped with a gauge that determines the change of the water level inside the tank. The single log is submerged, with the calibrated gauge being used to read off the volume of displaced water. Xylometer measurements produce accurate estimates of log volumes, although the estimates are not completely unbiased, as some water is absorbed by the log. The main drawbacks are the necessity to build a sufficiently large mobile tank and the high cost of transport.

Planimeter measurements are an acceptable alternative for these direct measurements. The cross-sectional area at a given point of measurement of i m above the base of the tree is plotted over the point of measurement. The area under the stem curve is determined with a planimeter and multiplied by an appropriate scale factor to obtain the tree volume.

A less accurate, but more cost-efficient method, is to subdivide the stem into sections, usually of a fixed length, for example, 1 m for trees below 12 and 2 m for those more than 12 m high. Each of the sections is envisaged as a truncated cone with the volume being obtained by multiplying the cross-sectional area at the midpoint, by the length of the section. The stem volume is obtained by adding the volume of the top section to the sum of the volumes of the equally long sections

$$v = \frac{\pi}{4} \cdot l \cdot \left(d_1^2 + d_2^2 + \ldots d_i^2 . + \ldots \ldots \ldots + d_k^2 \right) + \frac{\pi}{4} \cdot l_t \cdot d_t^2$$

where d_i = diameter at the midpoint of the ith section, l = section length, d_t = diameter at the midpoint of the top section, l_t = length of top section (Figure 4-8).

Alternatively, the stem is subdivided into sections of equal relative lengths. Hohenadl (1936) recommended sections of one-fifth the stem length. A further
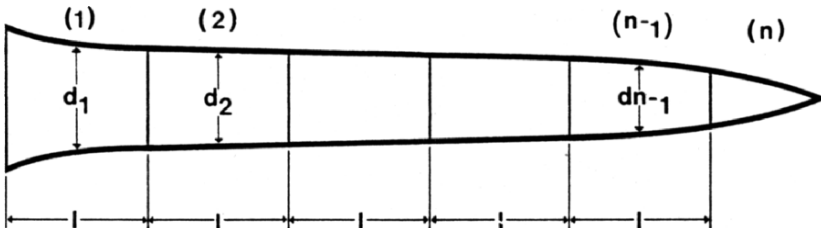
Figure 4-8. Sectionwise measurements on felled trees.

subdivision may be necessary for the lowest section, because it resembles a truncated neiloid, in which case the estimate is negatively biased. Either this subdivision, or the application of the Simony formula may be worthwhile for high-priced timber, e.g., used for veneering.

For practical purposes, these tedious measurements are unnecessarily costly. Formulae have been suggested and are widely applied to estimate the total and merchantable volume of felled trees from a limited number of diameter measurements:

Huber $\quad v = g_m l$

Smalian $\quad v = \dfrac{g_u + g_l}{2} \cdot l$

Newton $\quad v = \dfrac{g_u + 4g_m + g_l}{6} \cdot l$

Hossfeld $\quad v = \dfrac{3g_{1/3} + g_l}{4} \cdot l$

Simony $\quad v = \dfrac{2g_{1/4} - g_m + 2g_{3/4}}{4} \cdot l$

Hohenadl $\quad v = \dfrac{g_{0.1} + g_{0.3} + g_{0.5} + g_{0.7} + g_{0.9}}{5} \cdot l$

With

g_m	= cross-sectional area at the midpoint,
g_1, g_u	= cross-sectional area at the lower and upper end,
$G_{1/3}, g_{1/4}, g_{3/4}$	= cross-sectional area at 1/3rd, 1/4th, 3/4th of the stem and
$G_{0.1}, g_{0.3}, \ldots, g_{0.9}$	= cross-sectional area at 10%, 30%, …, 90% of the total length.

Bruce (1982) compared volume estimates of butt logs obtained from measurements at both ends, from measurements at the midpoint and at the small end as well as some intermediate point. The best volume equation was

$$v = 0.00007854 \cdot L \cdot \left(0.25 \cdot DL^2 + 0.75 \cdot DS^2\right)$$

where L = length of butt log in meters, DL = diameter at thin end in cm and DS = diameter at large end in centimeters. Grosenbaugh (1952) proposed the following equation to estimate the volume of butt logs

$$v = 0.00007854 \cdot DL^2 \cdot \left(DS \cdot DL + (DL - DS)^2 / k\right)$$

with $k = 2$ for a paraboloid, $k = 3$ for a cone and $k = 4$ for a subneiloid. Bruce (1987) combined Grosenbaugh's formula for estimating the butt-log volume with Baisiger's modification and proposed the formula

$$v = 0.00007854 \cdot DL^2 \cdot L \cdot \left(DS/DL + c \cdot \left(1 - (DS - DL)^2\right)\right)$$

with $c = -0.96$. Anuchin (1970) compared the estimates of the Huber, Smalian, Simpson and Gosfeld formulae, applied to single-stem sections with their true values obtained from xylometer measurements. In pines, the mean deviation varied between -1.2% for the Huber formula and $+0.3\%$ for the Smalian formula. Deviations of the same magnitude could be expected for the Hohenadl formula. The errors associated with the latter tended to be negative for the upper and lower section, but the bias for the entire tree was below -2%. The Huber, Smalian and Newton formulae produce unbiased estimates, if the stem represents a cylinder, a quadratic or a truncated paraboloid (Akça et al. 1982). The Huber formula produces a negative error for the cone and neiloid, the expected errors being 25% and 30%, respectively. For these solids, the Smalian formula is positively biased and has the additional disadvantage of being affected by butt swell. Biging (1988) compared the Huber, Smalian and Newton formulae with estimates obtained from fitted cubic splines to determine the log volumes of the entire tree. Taper functions were used to simulate the form of trees. The results were subsequently used to determine bias associated with the three formula. The Newton formula, for example, produced almost unbiased estimates, if the stem taper could be described by the taper equation

$$d_u/dbh = b_1 + b_2 \ln(1 - (h/H)^{1/3})(1 - \exp(-b_1/b_2)).$$

2.1.2   Volume of stacked wood

Pulpwood and firewood is either sold on a weight basis or as stacked wood. If sold as stacked wood, the volume of the pile is determined and a conversion factor applied to adjust for the free space between the roundwood logs. In the USA, the standard cord has a size of $8 \times 4 \times 4$ ft, the volume being $3.624\,\mathrm{m}^3$, but firewood cut to lengths less than 4 ft is sold as short cord, pulpwood with a length greater than 4 ft as long cord. In Germany, the "Raummeter" or "stère" was defined as a $1 \times 1 \times 1$ m pile of stacked wood. The conversion factor depends upon the straightness and length of the logs and for conifers it is higher than for hardwoods. When timber is sold as pulpwood (under bark), a conversion factor of 0.80 is appropriate, in the case of firewood, which is recovered from wood not meeting the quality standards for sawlogs, a conversion factor of 0.70 (over bark) is more appropriate. A factor between 0.68 and 0.70 for converting the standard cord to cubic volume is common in the USA (Avery et al. 1988). The quality of the stacking operation has to meet certain quality specifications. In Europe, a required stack height of 4% above the prescribed height of 1 m is frequently specified.

2.2 Log rules, grades and classes

2.2.1 Log rules

A board foot is the equivalent of the cubic volume of a 1 in. × 1 × 1 ft board. It contains 144 cu in. of timber and is the equivalent of 12 bd ft (broad feet). The North American log rule, is a table or formula, which gives the estimated volume of logs of specified diameters and lengths (Husch et al. 1982). The majority of these rules estimate the volume in board feet of timber.

The construction of a board feet log rule is hampered by a number of factors. The dimensions of the timber recovered from the sawlogs vary, different equipment is used by operators with varying skill, and sawing prescriptions differ. It is therefore necessary to distinguish between the log scale, recovered from the log rule and the mill tally, which shows the actual recovery. An overrun occurs when the mill tally exceeds the log scale, an underrun when the opposite takes place. The construction of a mill-tally log rule requires the measurement of the yield actually recovered from logs of different diameters and lengths. The resultant rule, which is derived by regression analysis, is valid for a specific mill or group of mills.

Diagram log rules assume that logs have a cylindrical shape. A circle is drawn with a diameter equal to that at the thin end of the log. The recoverable boards are drawn within this circle, but the width of the sawkerf and the expected amount of shrinkage are taken into account. The board foot content is determined for each thin-end log diameter class and the estimates are multiplied by the ratio of log lengths to obtain the board foot content for other log lengths.

The widely used *Scribner rule* belongs to the group of diagram log rules. It also assumes a cylindrical shape of the logs, is standardized for 1 in. thick timber, with a 1/4 in. allowance for sawkerf and shrinkage. It produces an approximately 30% overrun for logs under 14 in. (Husch et al. 1982). The following equation, developed by Bruce et al. (1950) produces smoothed estimates of the board-foot volume with sawlog diameter and sawlog length as predictors:

$$v = \left(0.79d^2 - 2d - 4\right) \cdot \frac{L}{16}$$

The Doyle rule

$$v = \left[\frac{1}{4}(d - 4)^2\right] \cdot L$$

is easy to apply but produces an underrun for large logs. The *International log rule* starts with a straightforward calculation of the board-foot content of a 4 ft long cylindrical log. For each 1 in. thick board, an allowance of 1/8 in. for sawkerf and 1/16 in. for shrinkage is made. The board-feet content of this log is then equal to $0.226d$. A further allowance equal to $0.71d$ for slabs and edgings is also subtracted. Hence

$$v = 0.226 \cdot d^2 - 0.71 \cdot d$$

Allowing a taper of 1/2 in. per 4 ft log section, this formula was expanded to obtain estimates for the board-foot content of longer logs.

2.2.2   Log grades

General rules and guidelines for a qualitative grading of logs exist in many countries. The US Forest Products Laboratory Hardwood Log Grading System distinguishes between *factory class, construction class, local-use class* and *veneer class*. The factory class is subdivided into the categories F1, F2 and F3, according to the diameter and length of the logs, and provides a further subdivision of the categories F1 and F2. The softwood log grading systems distinguish between veneer and sawlog class. Additional grading criteria are applied, which are determined by defects, log diameter and length, sweep and cull (Husch et al. 1982).

2.2.3   Log classes

In Germany, there are two main systems which provide a size-related grading of long-length logs ("Langholz"), with only one system being used in a given region. The first classification is based on the mid-diameter of the logs and produces the classes L0–L6, with a further subdivision within the classes L1, L2 and L3 (Table 4-1). The *Heilbronner classification* is based on minimum length and diameter of the logs. In addition, the long-length timber with an under bark diameter below 14 cm at 1 m above the thick end, is subdivided into 11 size-related subclasses. Stacked wood is classified into seven size-related subclasses. The timber is independently classified according to the four qualitative classes A, B, C, and D, which indicate the occurrence of wood defects.

In South Africa, the timber products are classified as sawtimber, poles and pulpwood. For sawlogs, the diameter and length specification based on the class midpoints is given in Table 4-2. The matrix of dimensions in the table defines the sawlog classes. For veneer and construction timber, there is no formalized specification for quality-based grading of logs, but the number of knots and ring width patterns are of decisive importance for qualifying as veneer timber.

Table 4-1. Grading long-length logs and *Heilbronner grading* in Germany

Long-length logs		Heilbronner classes		
Class	$d_{m,u.b.}$ (cm)	Class	Minimum length (m)	Minimum top diameter (cm)
L0	<10	H1	8	10
L1a	10–14	H2	10	12
L1b	15–19	H3	12	14
L2a	20–24	H4	14	17
L2b	25–29	H5	16	22
L3a	30–34	H6	18	30
L3b	35–39			
L4	40–49			
L5	50–59			
L6	>59			

Table 4-2. South African sawlog classification

Length	Diameter at thin end, *under bark(cm)*												
	13.5	15	17	19	21	23	25	27	29	31	33	35	37
2.7													
3.0	Class *a*			Class b_1				Class c_1				Class d_1	
3.3													
3.6													
3.9													
4.2				Class b_2				Class c_2				Class d_2	
4.5													
≥4.8													

2.3   Weight

It is easier to determine the weight of a quantity of small roundwood than to measure its volume. Pulp yield is a function of timber weight, rather than of timber volume. The volume over weight ratio, however, is partly dependent on the period of time between harvesting and weighing, as well as upon the weather conditions during this period. The length of this period, however, can be controlled by mutual agreement between buyer and seller. Bearing in mind that it is cheaper to determine the weight, the recorded weight can also be used to convert weight into volume. This is occasionally done for small-sized sawlogs, which are sold on a volume basis.

The green density of the timber is the sum of basic density and moisture content. The basic density expresses the ratio weight of oven-dried timber (in grams) over green volume (in cubic centimeters), the green density is obtained as the ratio green weight over green volume. The moisture content is expressed as the difference between green and oven-dry weight, expressed as a percent of oven-dry weight.

The green density varies with regions, with species, stem diameter and season and is different for different trees within a given stand. In the USA, equations were derived, which estimate the green weight from log volume in board feet, for different log rules (Row et al. 1966). Markstrom et al. (1982) derived empirical equations to estimate the cubic foot wood volume, the oven dry weight of wood and that of bark from the total green weight of wood and bark. In Germany, conversion factors are used to convert the weight of oven dry and air-dry timber into cubic volume.

In South Africa, the ICFR conducted a study to assess the advantage and feasibility of using volume instead of weight as the unit of measurement of small dimension debarked roundwood logs for the mining industry (Coetzee 1984). The density of the roundwood sold is, usually expressed as mass in kilogram per unit volume in cubic meters. Delivery of wood on the basis of weight is inaccurate because of the varying water content of the logs. The mining timber industry requires that the logs with a thin end under bark diameter between 8 and 20 cm are air-dried for a period of 6 weeks. The corresponding standard conversion factor is $1.47 \, m^3/ton$, *which is the equivalent of a density of* $680 \, kg/m^3$. *A survey, however, showed that this ratio varied between 1.35 and 1.53* m^3/ton. *A 10% variation in density was recorded in* Eucalyptus grandis *and a 16% variation in* Acacia mearnsii, *whereas a 17.5% variation of the basic density of* Pinus radiata *was found in New Zealand (Cown et al. 1982). Studies in South Africa (Coetzee 1984) as well as those conducted in other countries, indicated differences between species, although the variability within species tended to be greater. A density gradient within the individual tree has been found in South African studies, with the oven-dry density of* Eucalyptus nitens *increasing from an average of* $665.3 \, kg/m^3$ *at a height of 2.4 m, to* $706.9 \, kg/m^3$ *at a height of 19.5 m. These conclusions contra-indicated the studies of southern pines in the USA (Zobel 1982). Studies by Frederick et al. (1982) furthermore revealed an increasing density with increasing age.*

The moisture content is defined as the difference between mass before drying and oven dry mass, expressed as a proportion of mass before drying. South African studies of Eucalyptus grandis *showed a decrease in moisture content from 83.7 immediately after felling to 23.1% after 28 weeks air-drying (Schönau 1975). The reduction in moisture content of debarked logs of Pine*

species is also considerably faster than in undebarked logs (Stöhr 1983), and there is a significant effect of age as well as drying method on the drying rate.

Conventional methods based on the measurement of diameter and length of logs have been compared with others based on measurements on the stack face. Measurements on $1\,m^2$ grids using a photographic method with polaroid-type cameras showed no significant differences.

Chapter 5

MEASUREMENT OF STANDS

1 INTRODUCTION

The forest stand is defined as a group of trees occupying a specific area, which is sufficiently uniform in species composition, age arrangement, and condition as to be distinguishable from the forest on adjoining areas. It represents the unit for which one and the same silvicultural treatment is prescribed. Quantitative information about stands or compartments, therefore, relates directly to silvicultural and management decisions. Stand measurements provide information about:
- Age
- Diameter distribution, mean, and other parameters of the diameter distribution
- Height distribution, relationship between diameter and height, mean and top heights
- Stand density
- Diversity and spatial structure
- Volume and biomass
- Site index, site class, or yield class
- Present and future growth
- Stand quality and vitality
- Yield
- Damages

and possibly other relevant stand characteristics and site parameters.

2 AGE

The age of an even-aged stand is usually defined as age from germination, although this rule is not universally adopted. In Great Britain, for example, stand age, as recorded in yield tables reflects the length of time since planting.

In consequence, the reported age is lower than the true age of the trees. In the
northern hemisphere, the resultant difference is of the order of magnitude of
1–2 years, depending upon the age of the plants at the time of stand establish-
ment. In uneven-aged stands various definitions for age have been proposed.
If the stand consists of an upper storey of even-aged trees, and a similarly
even-aged but younger lower storey, the two age classes might be weighted,
for example, by assigning a weight to each age class proportional to volume
(Anuchin 1970). If the stand consists of several identifiable clusters of even-
aged trees, weights might be assigned proportional to the area occupied by
these clusters (Kramer et al. 1982).

$$A = \frac{f_1 \cdot a_1 + f_2 \cdot a_2 + \cdots + f_k \cdot a_k}{\sum f}$$

where

> A = mean age
>
> f_i = area occupied by the ith age class
>
> a_i = age class i
>
> k = number of age classes

Information about stand age is usually obtained from management plans, but
a direct determination either by counting the number of annual rings on incre-
ment cores, extracted from standing trees, or from ring counts on stem cross
sections, obtained from felled trees, are required when no management plans
or other sources of information are available. Because of off-center boring and
the possibility of false or missing rings, which can be identified on stem cross
sections, but less easily on increment cores, stem analysis produces ring counts
which are more reliable than those obtained from increment cores. Both meth-
ods fail when no distinct annual rings are discernible. In young stands of conifer
the stand the age can also be determined by counting annual internodes on
selected sampling trees.

3   MEAN DIAMETER

3.1   Arithmetic mean diameter

The arithmetic mean diameter of a stand is

$$\mu_d = \frac{\sum\limits_{1}^{N} d_i}{N}$$

and is usually estimated by sampling

$$\bar{d} = \frac{\sum\limits_{i=1}^{n} d_i}{n}.$$

It gives an unbiased estimate of the population mean, if the assumption of random sampling is satisfied. Normally the population is identified by the forest stand from which the sample was taken and in experiments by the sample plot on which the treatment was applied. The arithmetic mean is appropriate for certain types of experimental studies, for example, single-tree short-term fertilizer experiments, pruning trials, and species as well as progeny trials, primarily to measure the responses of the trees to the experimental treatments during the first years after stand establishment. It is less useful for management inventories because it does not represent the tree with the mean basal area or mean volume and is affected by the method and degree of thinning.

3.2 Quadratic mean diameter

The quadratic mean diameter of the stand represents the tree with the mean basal area. The estimator is

$$d_q = \sqrt{\frac{\sum\limits_{i=1}^{n} d_i^2}{n}}$$

In the case of grouped data, with n_i trees in the ith class is calculated from

$$d_q = \sqrt{\frac{\sum\limits_{i=1}^{k} n_i d_i^2}{\sum\limits_{i=1}^{k} n_i}}$$

It represents a slightly negatively biased estimate of the diameter of the tree with the mean volume. The mean stand diameter recorded in yield tables and used as a target variable in growth modeling, is always calculated as quadratic mean diameter. The relationship between $\bar{d}$ and d_q is

$$d_q = \sqrt{\bar{d}^2 + s_d^2}$$

Weise (1880) introduced a rule of thumb for estimating the quadratic mean diameter and proposed the 60th percentile of the ordered set of diameters.

The quadratic mean is usually assumed to represent the tree with the mean volume, although a small negative bias is involved (Essed 1957).

Example 5.1   The arithmetic and quadratic mean diameter of the observed diameters in Appendix B are as follows

$$\bar{d} = \frac{18.0 + 26.9 + \cdots + 23.8}{253} = 21.8\,\text{cm} \quad d_q = \sqrt{\frac{18.0^2 + 26.9^2 + \cdots + 23.8^2}{253}} = 22.3\,\text{cm}$$

or

$$d_q^2 = \bar{d}^2 + s_d^2, \quad d_q^2 = 21.8^2 + 4.97^2 = 499.74, \quad d_q = 22.3\,\text{cm},$$

3.3   Basal area central diameter

The calculation of the basal area central diameter (d_{mg}) requires an estimate of the median of the ordered set of basal areas, which partitions the dataset into two subsets with equal total basal areas. It is defined as the median of the distribution. Compared to the quadratic mean diameter, it is less severely affected by suppressed trees and by the thinning method, but its calculation is more time-consuming and requires a computer program. Normally, the ordered set of basal areas is derived from the tally sheet of diameters, which is converted into a table with the cumulative basal areas for the upper class limits of the diameter distribution. The basal area central diameter for a given diameter distribution is obtained from

$$d_{mg} = d_u + w \cdot \left[\frac{G/2 - \sum n_i \cdot g_i}{g_k} \right]$$

Where

$\quad W = $ class width

$\quad D_u = $ lower limit of the kth diameter class, containing the basal area
$\qquad\quad$ central tree

$\quad \sum n_i g_i = $ accumulated basal area below the lower limit of the kth
$\qquad\qquad$ diameter

$\quad g_k = $ basal area of the kth diameter class

$\quad G = $ total basal area

Alternatively, the cumulative squared diameters could be entered into this table. The basal area central diameter is approximately represented by the 70th percentile of the ordered set of tree diameters. In some regions within Germany,

the height of the tree with the basal area central diameter is used to derive the site class of a stand from existing yield tables and to calculate the stand volume with the aid of the form height method.

Example 5.2 The diameters (measured in millimeters) of all 253 trees in *Pinus radiata* in a sample plot with a size of 0.3750 ha were grouped in 1 cm diameter classes. The original data are given in Appendix B and those for the grouped diameters in Table 5-1.

The stand characteristics for ungrouped data are:

$$n = 253 \quad \bar{d} = 21.8 \text{ cm}, d_q = 22.4 \text{ cm}, s_d = 4.96 \text{ cm}, d_M = 22.0 \text{ cm}$$

For the basal area central tree we have

$$\sum_{1}^{28} (n_i \cdot d_i^2) = 126992 \quad \frac{1}{2} \sum_{1}^{28} (n_i \cdot d_i^2) = 63496$$

The cumulative sum of squared diameters for the lower and upper limit of the 24 cm diameter class are 57167 and 70415, respectively. The required mean diameter is located within this class.

$$d_{mg} = 23.5 + 1 \cdot \frac{63496 - 57167}{70415 - 57167} = 23.98 \text{ cm} \cong 24.0 \text{ cm}$$

Weise's rule of thumb for the 60th percentile gives $d_W = 23.0$ cm. The quadratic mean diameter of the 100 thickest trees per hectare is $d_{100} = 27.9$ cm.

4 DIAMETER DISTRIBUTIONS

The diameter distribution of a stand is required to construct stand tables, to estimate the total or merchantable stand volume, and to estimate the volume of the wide range of products, which are recovered from a stand of a given mean diameter and mean height.

The unimodal diameter distribution, which is frequently observed in young even-aged stands before the first thinning, can be approximated as a normal distribution, but deviations from normality occur occasionally (Gates et al. 1983). With increasing stand age, the distribution tends to become increasingly skewed, partly because of mortality in the lower tree strata, partly because of thinnings from below, which remove the dominated trees or crown thinning (thinning from above), which remove dominating and codominating trees.

Table 5-1. Grouped diameters

i	Dbh (cm)	No of Trees	i	Dbh (cm)	No of Trees	i	Dbh (cm)	No of Trees	i	Dbh (cm)	No of Trees	i	Dbh (cm)	No of Trees
1	**8**	2	7	**14**	6	13	**20**	24	19	**26**	15	25	**32**	2
2	**9**	4	8	**15**	6	14	**21**	21	20	**27**	14	26	**33**	–
3	**10**	1	9	**16**	13	15	**22**	20	21	**28**	8	27	**34**	1
4	**11**	–	10	**17**	9	16	**23**	17	22	**29**	13	28	**35**	2
5	**12**	3	11	**18**	12	17	**24**	23	23	**30**	3	–	–	–
6	**13**	1	12	**19**	17	18	**25**	14	24	**31**	2	–	–	–

4.1 Weibull distribution

The Weibull distribution was initially developed to describe the lifetime distribution of systems under stress, consisting of a large number of components, each with its own lifetime distribution. Although this is the basic philosophy which underlies the Weibull distribution, the function has been successfully used for modeling diameter distributions in even-aged stands, and for decreasing diameter distributions in all-aged forests, although no systems under stress are necessarily involved.

4.1.1 Three-parameter distribution

The density function of the three-parameter Weibull function is

$$f(x; \alpha, \beta, \gamma) = \frac{\gamma}{\beta} \left\{ \left(\frac{x-\alpha}{\beta} \right)^{\gamma-1} e^{-\left(\frac{x-\alpha}{\beta}\right)^{\gamma}} \right\}$$

Where

$\alpha =$ location parameter, expressing the lower bound

$\beta =$ scale parameter ($\beta < 0$)

$\gamma =$ shape parameter ($\gamma > 0$)

Distribution curves for different values of the shape parameter γ are shown in Figure 5-1.
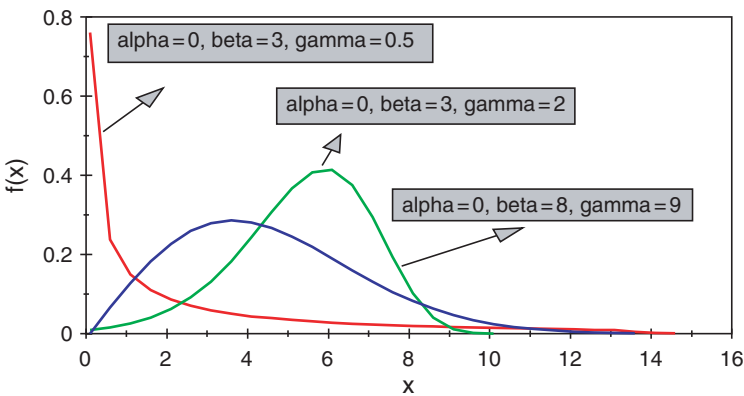
Figure 5-1. Weibull distributions for specified parameters.

Integrating the density function generates the cumulative distribution function

$$f(x) = 1 - e^{-\left(\frac{x-\alpha}{\beta}\right)^{\gamma}}$$

The log likelihood function of the three-parameter Weibull distribution

$$\log(n) = (\gamma - 1) \sum_{n}^{n} \log(x - \alpha) - n(\gamma - 1) \log \frac{\gamma}{\beta} - \sum^{n} \left(\frac{x-\alpha}{\beta}\right)^{\gamma}$$

(Rennols et al. 1985) is differentiated with respect to α, β, and γ, respectively. The resultant three equations are solved iteratively to obtain estimates for these unknown parameters.

4.1.2   Two-parameter Weibull distribution

The density function of the two-parameter version with no location parameter is

$$f(x, \beta, \gamma) = \frac{\gamma}{\beta} \left[\frac{x}{\beta}\right]^{\gamma-1} e^{-(x/\beta)\gamma}$$

To fit the distribution, the variable x is expressed as a deviation from x_{min}. The maximum likelihood function for the two-parameter function is obtained by solving the following equations by trial and error to estimate γ

$$\frac{\sum f_i x_i^c \ln x_i}{\sum f_i x_i^c} - \frac{1}{c} = \frac{\sum f_i \ln x_i}{\sum n_i}$$

The iterations are discontinued as soon as the defined convergence criterion has been met. The parameter β is estimated from

$$b = \left(\frac{\sum f_i x_i^c}{\sum f_i}\right)^{1/c}.$$

The function is less flexible than the three-parameter version, but is preferred when a more parsimonious model is required.

Example 5.3   Both versions of the Weibull distribution were fitted to the dataset of Appendix B, after grouping the diameters in 1 cm classes. The parameter estimates and χ^2 were:

Distribution	A	B	c	χ^2	$\chi^2\alpha = 0 \cdot 05$
Two-parameter	–	23.72	3.92	30.4	21.03
Three-parameter	0	23.77	4.66	18.9	21.03

The observed and expected frequencies in the first and last two classes were pooled to ensure that no expected class frequency was below (Figure 5-2 and Table 5-2). The lower value of χ^2 for the three-parameter version confirms the superior performance of this function.

4.1.3 Percentile estimators

Estimates of the parameters a and b can be obtained from selected points of the distribution, for example, the 17th and the 97th percentile (p_{17}, p_{97}),
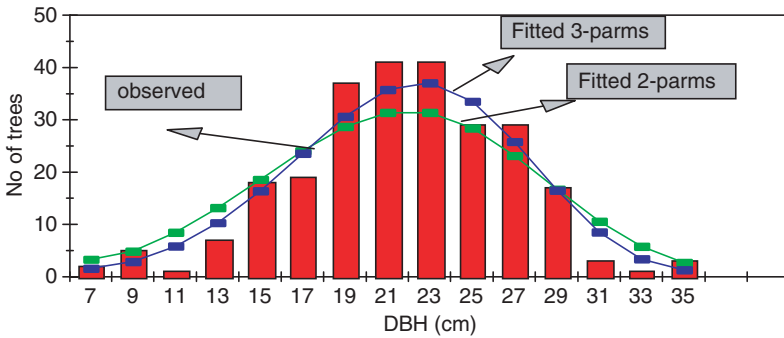
Figure 5-2. Fitted two- and three-parameter Weibull distributions.

Table 5-2. Fitted two- and three-parameter Weibull distribution

Dbh (cm)		Two-parameter		Three-parameter	
	f_{obs}	f_{fit}	χ^2	f_{fit}	χ^2
7	2	3.31	0.52	1.58	0.11
9	5	4.87	0.00	2.86	1.60
11	1	8.49	6.61	5.82	3.99
13	7	13.17	2.89	10.32	1.07
15	18	18.59	0.02	16.40	0.16
17	19	24.08	1.07	23.57	0.89
19	37	28.69	2.41	30.63	1.32
21	41	31.39	2.94	35.77	0.76
23	41	31.38	2.95	37.07	0.42
25	29	28.43	0.01	33.50	0.60
27	29	23.13	1.49	25.81	0.39
29	17	16.70	0.01	16.51	0.01
31	3	10.57	5.42	8.50	3.56
33	1	5.78	3.95	3.40	1.69
35	3	2.69	0.15	1.27	2.36

although such estimates are less efficient than maximum likelihood estimators and require a larger sample (Dubey 1967). The sample estimate of the parameter γ is obtained from

$$c = \frac{\ln(-\ln(1 - p_{17})) - \ln(-\ln(1 - p_{97}))}{\ln x_{p_{17}} - \ln x_{p_{97}}}$$

where x_{p17} and x_{p97} are the diameters which correspond with p_{17} and p_{97}. The parameter b is estimated from

$$b = e^{(w \cdot \ln x_1 + (1-w) \cdot \ln x_2)}$$

where

$$w = 1 - \frac{\ln(-\ln(1 - p_{17}))}{\ln[-\ln(1 - p_{17})] - \ln[-\ln(1 - p_{97})]}$$

Numerous authors applied the two- and three-parameter Weibull distribution. Zarnoch et al. (1985) compared percentile estimators of the three-parameter Weibull distribution, proposed by Zanakis (1985) with maximum likelihood estimators, which were obtained with the routine FITTER, written by Bailey (1974). Sample bias, sample variance and mean squared error, the latter combining bias and variance, were evaluated for computer-simulated Weibull distributions. The studies indicated the superiority of maximum likelihood over percentile estimators. Shiver (1988) who conducted simulation studies to determine the sample size which is required to estimate the three parameters of the Weibull distribution, based on the percentile, the maximum likelihood, and the modified moments method, respectively, reached a similar conclusion. Furthermore the largest reduction in variance, bias, and mean square error was recorded if the sample size increased from 30 to 50, which implies that a sample size of not less than 50 is required to obtain satisfactorily accurate estimates of the Weibull parameters. Comparisons with the beta distribution (Burkhart et al. 1974), in loblolly pine, indicated that the Weibull function was superior and produced a closer fit. Dippel (1988) fitted the two- and three-parameter Weibull function to distributions recorded in mixed beech-larch stands, for each species separately. Saborowski (1994) investigated the minimum sample size, which is required to estimate the three parameters of the Weibull distribution. The assumption was that the trees are measured in clusters with a maximum cluster size not greater than 12. Based on five case studies for which the real distribution was known, simulation studies indicated that a sample with $n = 80$ could generally be expected to produce satisfactory results.

Zanakis (1979) proposed the following percentile estimator for the location parameter a of the three-parameter Weibull distribution:

$$a = \frac{(x_1 x_2 - x_2^2)}{(x_1 + x_n - 2x_2)}$$

The scale parameter ß was estimated from

$$b = -a + x_{0.63n}$$

and the shape parameter γ from

$$c = \frac{\ln(\ln(1 - p_k)/\ln(1 - p_i))}{\ln((x_{npk} - 1)/(x_{npi} - 1))}$$

with $p_i = 0.167$ and $p_k = 0.974$.

Cao et al. (1984) applied a segmented distribution approach for modeling diameter distributions. The cumulative Weibull distribution F(x) was fitted to segments, in such a manner that the jth cumulative distribution $F(x_j)$ was monotone nondecreasing, continuous. In addition, $F(x_j)$ as well as its derivative $f(x_j)$, were required to be continuous at the joins. A modified form of the cumulative Weibull function was necessary, with two additional parameters, which are described as scale parameters of the probability distribution. They were adjusted when no convergence could be reached. The 0th, 25th, 50th, 75th, and 100th percentile points were selected as joint points. The goodness of fit was compared with the conventional three-parameter Weibull distribution. In unthinned stands, the segmented approach did not perform better than the three-parameter function fitted with conventional algorithms, but in thinned plantations it produced a closer fit. Because of its greater flexibility, the segmented approach was thought to be advantageous in stands with an irregular diameter distribution, generated by thinning, although the computations are more tedious and the location of the joint points remains uncertain.

Ek et al. (1975) developed a method to obtain the parameter estimates a and b of the Weibull distribution for a specified quadratic mean diameter, which was based of the assumption that the parameter γ is known. The expected value of the quadratic mean is

$$E(d^2) = b^2 \sqrt{b^2 \left(\Gamma_1^2 - \Gamma_2 \right) + E\left(d^2\right)}$$

The estimates for a and b were obtained from

$$a = -b\frac{\Gamma_1}{\Gamma_2}\sqrt{b^2 \left(\Gamma_1^2 - \Gamma_2 \right) + E\left(d^2\right)}$$

$$b = -a\frac{\Gamma_1}{\Gamma_2} + \sqrt{\frac{a^2}{\Gamma_2^2}\left(\Gamma_1^2 - \Gamma_2 \right) + \frac{E\left(d^2\right)}{\Gamma_2}}$$

The equations are solved iteratively, with initial estimates being obtained from the general shape of the density curve. There is no explicit solution for the parameter γ. The authors proposed a procedure for estimating c, when a and b are known. McTague et al. (1987) presented a method to estimate the three parameters of the Weibull distribution from its 10th and 63rd percentile, compatible with a basal area projection model. The 10th and 63rd percentile were predicted from equations with age, stand density, and site index as independent variables, with unthinned versus thinned stands being introduced as a dummy variable. The Weibull parameter a was estimated from an equation with d_{10}, stand density and site index as predictor variables; b was estimated from the equation $b = d_{63} - a$; the 90th percentile was estimated from an equation with d_{63}, d_{10}; site index and age as predictors and the Weibull parameter c was obtained from an equation with the estimated d_{90}, and the parameter estimates for a and b as independent variables.

4.2   Beta distribution

Fitting a beta distribution requires a transformation of the variable x

$$y = \frac{x - a}{b - a}$$

where a, b = lower and upper bound of the distribution. It converts the distribution into the standard beta distribution. The density function of y is

$$f(y; \alpha, \beta) = \frac{y^{\alpha - 1} \cdot (1 - y)^{\beta - 1}}{B \cdot (\alpha, \beta)}$$

with α and β representing the two parameters of the function and

$$B(\alpha, \beta) = \frac{\Gamma(\alpha) \cdot \Gamma(\beta)}{\Gamma(\alpha + \beta)}$$

being obtained from the gamma functions $\Gamma(\alpha)$ and $\Gamma(\beta)$. Density curves for different values of α and β are shown in Figure 5-3.

   Estimates for α and β are obtained from the sample mean and sample variance of y, with β being a function of the mean and variance of y

$$\widehat{\beta} = \frac{\dfrac{k}{s_y^2 \cdot (1 + k)^2}}{1 + k}$$

where $k = \dfrac{\bar{x}}{1 - \bar{x}}$ and α is estimated as follows: $\hat{\alpha} = \hat{\beta} \cdot k$
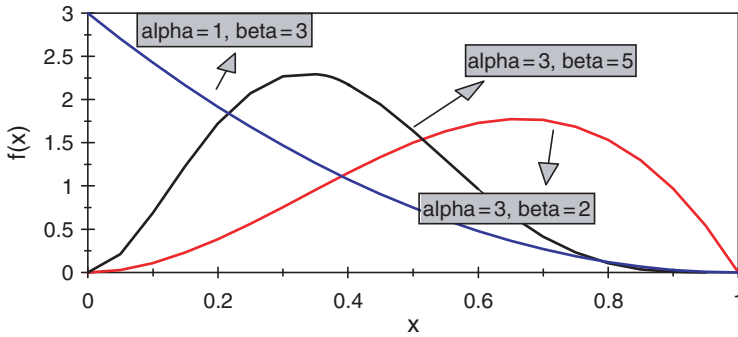
Figure 5-3. Beta distributions for specified parameters.

Table 5-3. Observed and fitted diameter distribution

dbh	n_{obs}	n_{fit}	dbh	n_{obs}	n_{fit}
8.5	6	0.18	**22.5**	37	36.82
10.5	1	2.69	**24.5**	37	34.06
12.5	4	8.33	**26.5**	29	28.01
14.5	12	16.09	**28.5**	21	19.79
16.5	22	24.33	**30.5**	5	11.11
18.5	29	31.38	**32.5**	2	4.01
20.5	45	35.84	**34.5**	3	0.34

$\chi^2 = 1.58$, 7 degrees of freedom

Example 5.4   The beta distribution is fitted to the previous dataset (Table 5-3 and Figure 5-4)

In forestry, the beta distribution has been applied to model diameter distributions in *Picea abies* (Zöhrer 1969), *Fagus silvatica* (Kennel 1972), and *P. taeda* (Burkhart et al. 1974), in tropical forests (Zöhrer 1969), and in old-field slash pine plantations (Clutter et al. 1965). In order to construct stand tables, the distribution is fitted and the parameter estimates are regressed on stand characteristics.

4.3   Gamma distribution

The density function of the gamma distribution is as follows:

$$f(x) = \frac{\left[\frac{\Gamma(\beta+1)}{\bar{x}}\right]^{\beta+1}}{\Gamma(\beta+1)} x^{\beta} e^{-\left[\frac{\beta+1}{\bar{x}}\right]}$$
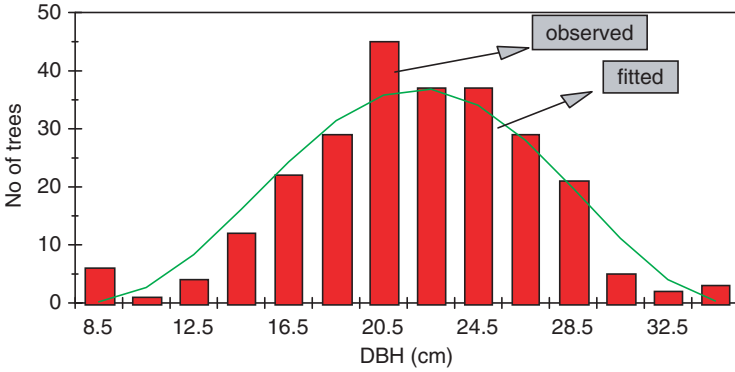
Figure 5-4. Fitted beta distribution.

To obtain the densities for different values of x, we calculate the gamma function $\Gamma(x)$

$$\Gamma(x) = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot \Gamma(y)$$

The gamma function $\Gamma(y)$ for $1 < y < 2$ is calculated from the approximation

$$\Gamma(y) = 1 - 0.57710(y-1) + 0.9858(y-1)^2 - 0.8764(y-1)^3$$
$$+ 0.8328(y-1)^4 - 0.5685(y-1)^5 + 0.2548(y-1)^6$$
$$- 0.05150(y-1)^7$$

The parameter β is estimated from mean and variance

$$\hat{\beta} = \frac{\bar{x}^2}{s_x^2} - 1$$

The calculated densities are multiplied by the ratio total frequency/sum of densities.

Example 5.5   The gamma distribution is fitted to the data of Appendix B. The parameter estimates are

$$\hat{\alpha} = 16.97 \quad \hat{\beta} = 0.7788$$

The fitted frequencies are shown in Table 5-4 and Figure 5-5.

Table 5-4. Fitted gamma distribution

dbh (cm)	n_{obs}	n_{fit}	dbh (cm)	n_{obs}	n_{fit}
<9.6	6	0.5	22.4–24.0	32	27.5
9.6–11.2	1	1.7	24.0–25.6	25	23.0
11.2–12.8	4	4.6	25.6–27.2	26	18.0
12.8–14.4	6	9.7	27.2–28.8	14	13.2
14.4–16.0	16	16.4	28.8–30.4	10	9.3
16.0–17.6	14	23.2	30.4–32.0	3	6.2
17.6–19.2	23	28.5	32.0–33.6	1	4.2
19.2–20.8	38	31.0	33.6–35.2	3	2.4
20.8–22.4	31	30.5	>35.2	–	3.3
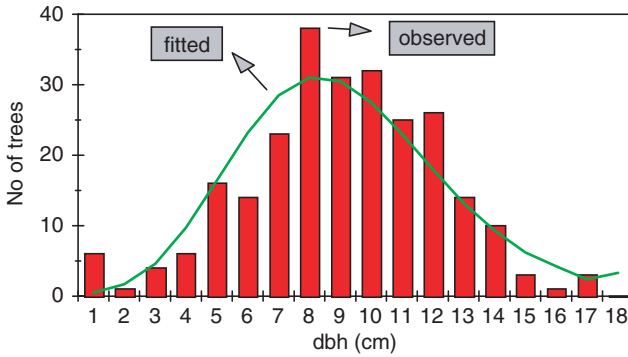
Figure 5-5. Fitted gamma distribution.

4.4   Johnson's S_B distribution

Johnson (1949) introduced the four parameter S_B distribution with the following density function:

$$f(x) = \left(\frac{\delta}{\sqrt{2\pi}}\right)\left(\frac{\lambda}{(x-\varepsilon)(\varepsilon+\lambda-x)}\right) e^{\left[-\frac{1}{2}\left(\gamma+\delta\ln\left(\frac{x-\varepsilon}{\varepsilon+\lambda-x}\right)\right)^2\right]}$$

The parameters γ represents the skewness with δ expressing the degree of peakedness of the distribution. They are calculated as follows

$$\hat{\gamma} = -\frac{\overline{f}}{s_f} \quad \hat{\delta} = \frac{1}{s_f} \quad \text{where} \quad \overline{f} = \frac{\sum\limits_{i=1}^{k} f_i}{k} \quad \text{and} \quad s_f = \sqrt{\frac{\sum\limits_{i=1}^{k} f_i}{k\sum\limits_{i=1}^{k}\left(f_i - \overline{f}\right)^2}}$$

with k = number of classes and

$$f_i = \sum_{i=1}^{k} f_i \log \frac{x_i - \xi}{\xi + \lambda - x_i} \quad \text{where} \quad \xi = \text{lower limit and } \lambda = \text{range.}$$

The function is flexible and provides a satisfactory fit if the distribution of the subject variable indicates skewness and kurtosis. It has no closed form and requires a translation of the distribution of the unit normal variate. When a variable x has an S_B distribution then

$$z = \gamma + \delta \ln \left(\left(\frac{(x - \varepsilon)}{\varepsilon + \lambda - x} \right) \right)$$

follows the distribution of the unit normal variate (0, 1).

Example 5.6   Johnson's S_B distribution is fitted to the data of Appendix B (Table 5-5 and Figure 5-6). The parameter estimates are:

$$\hat{\xi} = 6.0 \quad \hat{\lambda} = 30.0 \quad \hat{\gamma} = 3.101 \cdot 10^{-9} \quad \hat{\delta} = 4.573 \cdot 10^{-4}$$

Table 5-5. Observed and fitted frequencies, Johnson's S_B distribution

dbh (cm)	n_{obs}	n_{fi}	dbh (cm)	n_{obs}	n_{fi}
7	2	3.2	23	41	24.8
9	5	9.1	25	29	23.4
11	1	14.0	27	29	21.2
13	7	18.1	29	17	18.1
15	18	21.2	31	3	14.0
17	19	23.4	33	1	9.1
19	37	24.8	35	3	3.2
21	41	25.2	–	–	
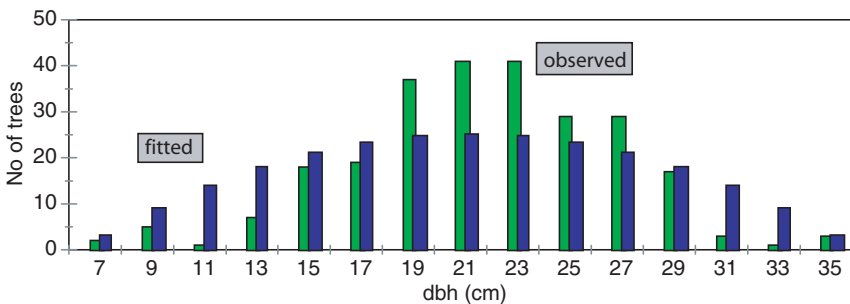
Figure 5-6. Observed values and fitted Johnson S_B distribution.

4.5 Decreasing distributions

Decreasing distributions occur in all-aged natural forests. The properties of inverse J-shaped diameter distributions in natural forests of coniferous tree species in France was studied by de Liocourt (1898), who hypothesized that the ratio of the number of trees in successive diameter classes was constant within a given forest and typifies this forest, but varied for different forests. The underlying model is

$$N_i = e^{b_0 + b_1 \cdot d_i}$$

where N_i = number of tree per hectare in the ith diameter class with class midpoint d_i. Alternatively this equation is written as

$$N_i = k \cdot e^{-a \cdot d_i}$$

which is known as the negative exponential function, with $k = \exp(b_0)$ and $a = -b_1$. For $d = d_i$ and $d = d_{i+1}$ we obtain

$$N_i = k \cdot e^{-a \cdot d_i}$$
$$N_{i+1} = k \cdot e^{-a \cdot (d_{i+1})}$$

Hence

$$q = \frac{N_i}{N_{i+1}} = e^a$$

can be determined by regression analysis.

In earlier studies, Meyer et al. (1943, 1951) expanded the Liourt law and introduced the concept of balanced diameter distribution, defined as that distribution which generates a sustainable yield. A given distribution is balanced if the relationship between ln(N) and dbh is linear, whereas a nonlinear trend indicates that the distribution is unbalanced. In many all-aged natural forests, however, the relationship between ln(N) and dbh reveals nonlinear trends (ZÖhrer et al. 1973). LEAK (1964) investigated unbalanced distributions in natural hardwood stands in USA and characterized these distributions through a linearization of the relationship between q and dbh by using 4 in. instead of 2 in. diameter classes. Moser (1976) presented an alternative method to specify an inverse J-shaped diameter distribution, which was based on Schumacher's tree area equation

$$Tree\ area = b_0 + b_1 d + b_2 d^2$$

and on Brender's formula (Brender 1973) to derive m from a specified basal area G. It is

$$m = \frac{G}{0.005454 \sum\limits_{i=1}^{n} \left(d_i^2 q^{i-1}\right)}$$

Expressing density (e.g., in terms of basal area) as a function of dbh and N/ha produces an equation to estimate k from density and to reconstruct the diameter distribution for a given value of q

$$k = \frac{\text{density}}{\sum\limits_{i=1}^{n} \left(b_1 + b_2 \cdot d + b_3 \cdot d^2\right) \, e^{-d_i (\ln q)/w}}.$$

$$k = \frac{\text{density}}{\sum\limits_{i=1}^{n} \left(b_1 + b_2 \cdot d + b_3 \cdot d^2\right) \cdot e^{-d_i \frac{\ln q}{w}}}$$

Chevrou (1990) introduced the truncated Liocourt law to be used in all-aged forests. It is described by the negative exponential function with

$$k = N \cdot (q - 1) \cdot q^{d_0 - 1}$$

where w = class width and d_0 = midpoint of the lowest recorded diameter class. The relationship between the coefficients k and a is

$$k = N . w . a,$$

where N = number of trees per hectare and w = class width. The arithmetic mean diameter is equal to the reciprocal value of a (Zeide 1984). The relationship between k and a is, therefore, dependent upon the relationship between trees per hectare and mean diameter. Assuming that the latter can be expressed by Reineke's function and assuming unit class width we obtain

$$k = a \cdot e^{b_0 + b_1 \cdot \ln(d)}$$

This contradicts Meyer's assumption of a linear relationship between k and a (Meyer et al. 1943).

Example 5.7   The equation $N = \exp(b_0 + b_1 d)(\rightarrow \ln(N) = b_0 + b_1 d)$ was fitted to sample plot data in an all-aged *P. abies* stand in the forest district Wolfach (data by courtesy of Prof. M. Prodan). The parameter estimates were $b_0 = 5.291, b_1 = -0.06096$. In this particular case, the model failed to produce a satisfactory fit, with negative residuals in the lower and upper diameter range and positive residuals in the center.

The ratio of expected successive class frequencies N_j/N_{j+1} plotted over dbh (Figures 5-7 and 5-8) seems to confirm that the distribution is balanced.
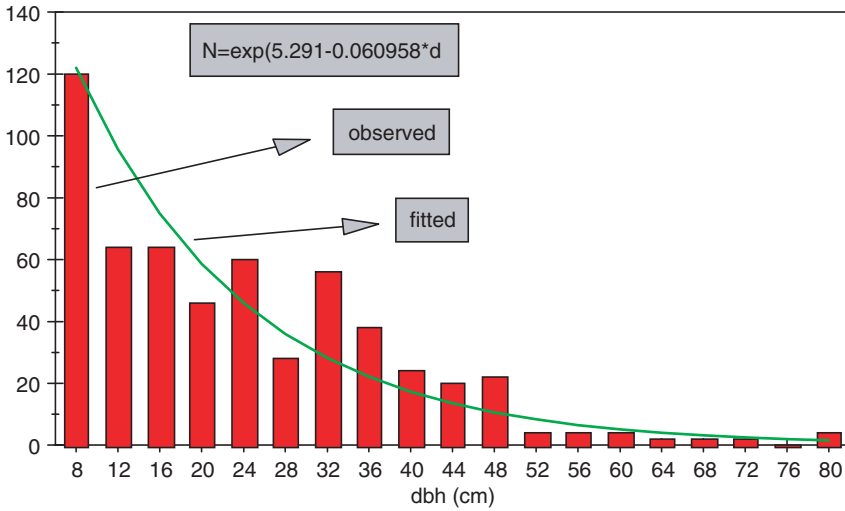
Figure 5-7. Fitted negative exponential distribution and observed values.
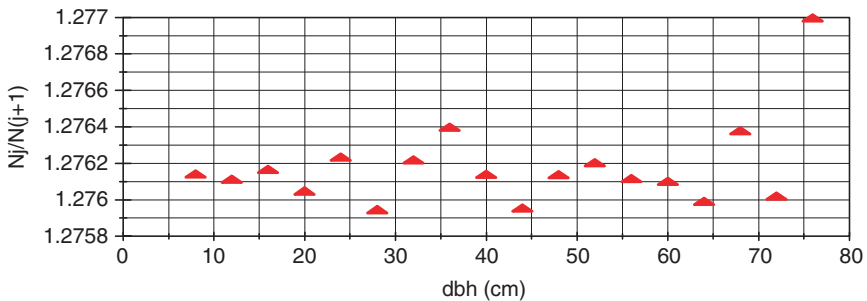
Figure 5-8. Relationship between the ratio of successive class frequencies and dbh.

4.6   Other approaches

Bliss et al. (1964) fitted the lognormal distribution to diameter distributions in even-aged Pinus elliottii *stands. The function, which implies that the logtrans-formed variable follows the normal distribution, was suitable to account for the observed skewness of these distributions. In general, however, other distri-bution functions are preferred for even-aged stands.*

Borders et al. (1987) fitted diameter distributions based on the percentiles of the diameter distribution, obtained from stand tables. The method was based on the assumption of a uniform diameter distribution within the individual diame-ter classes. A system of 12 equations was developed to estimate the percentiles.

The "driver" percentile d_{65} was a function of the quadratic mean diameter and stand age. Stand treatment, i.e., thinned versus unthinned stands, was introduced as a dummy variable. Bailey et al. (1981) developed diameter distribution models for slash pine plantations, based on the three-parameter Weibull distribution. Because of the relationship between the parameters and the percentiles of the distributions, equations were fitted to predict the 24th, 63rd, and 93rd percentile for unthinned and thinned stands. The model predicting the 24th percentile of unthinned stands used age, ln(N) and ln(SI) as predictors, the 63rd percentile was estimated from ln(A), 1/SI and 1/N, whereas ln(H), 1/N and 1/SI were used to predict the 93rd percentile. Similar models were developed for thinned stands. The Weibull parameter c was estimated from an equation with all three percentiles as predictors, b was estimated from an equation with $(d_{63} - d_{24})$ and c as independent variables, whereas a was obtained from an equation with d_{24}, b, and c as independent variables.

5   STAND TABLES

5.1   Introduction

The *stand table* gives the expected number of stems per unit area in each diameter class within a given stand. It may also reflect the average distribution for all age classes together, independently of site index, in which case the expected stem numbers are summarized per height class. A *stock table* gives similar information but expressed in terms of volume.

A stand table, based on the sampled diameter distribution, may be constructed for an individual stand. When applied to the metric measurement system, 2 cm wide diameter classes are usually adequate, but the stand tables for conifers of the British Forestry Commission are based on 5 cm wide classes, a class width of 4 cm is customary for working plans in German forestry. It is desirable that the stand table contains at least 10 diameter classes. In young and medium age stands, information about the diameter distribution is sacrificed when selecting a class width of more than 3 cm.

5.2   Parameter prediction and parameter recovery

In order to be useful in the practice of forest management, stand tables should reflect the effect of all influential variables on the parameters of the diameter distribution.

- The *parameter prediction method* fits distributions (Weibull, beta, gamma) and relates the parameter estimates to stand characteristics, possibly with the addition of influential variables, for example, to account for the effect of thinnings.
- The *parameter recovery method* recovers the parameters of the distribution from the moments of a given distribution from actual or estimated stand attributes.

A considerable amount of research has been done to develop algorithms for a parameter recovery method for the three-parameter Weibull distribution. Burk et al. (1984) presented the following procedure:

- Estimate the first, second, and third noncentral moment of the Weibull distribution, for example, from equations with stand attributes (age, site index, stand density) as predictor variables
- The estimation requires a series of iterations since

$$\Gamma_k = \Gamma\left(1 + \frac{k}{c}\right)$$

- Use this relationship to solve the following equation for c:

$$\mu_3' = b^3\left(\Gamma_3 - 3\Gamma_1\Gamma_2 + 2\Gamma_1^3\right) + 3\mu_1'\mu_2' - 2\left(\mu_1'\right)^3$$

- Calculate the parameter b of the Weibull distribution from

$$b = \sqrt{\frac{\mu_2' - \left(\mu_1'\right)^2}{\Gamma_2 - \Gamma_1^2}}$$

- The parameter a is calculated as follows:

$$a = \mu_1' - b\Gamma_1$$

Pienaar et al. (1988) developed a stand table projection method, based on the relationship between age and the relative size of the ith surviving tree. The latter was defined as the basal area of this tree divided by the basal area of the mean tree. The initial hypothesis of this ratio being constant over time was rejected and required the estimation of the parameter b of the function

$$\frac{g_{2i}}{g_{1i}} = \left\{\frac{g_{1i}}{\bar{g}_1}\right\}^{(A_2/A_1)^b}$$

Estimates for b were obtained from permanent sample plots. The projected basal area of a tree in the ith diameter class ($i = 1, \ldots, k$) was then obtained

from

$$g_{2i} = G_2 \cdot \left(\frac{g_{1i}}{\bar{g}_1} \right)^a \cdot \frac{n_i}{\displaystyle\sum_{i=1}^{k} \left[\left(\frac{g_{1i}}{\bar{g}_1} \right)^a \cdot n_i \right]}$$

with

$$a = \left(\frac{A_2}{A_1} \right)^b$$

In order to project the stand table, the Clutter and Jones (1980) survival function, as well as a projection equation for basal area

$$\ln g_2 = \ln g_1 + b_1 \cdot \left(\frac{1}{A_2} - \frac{1}{A_1} \right) + b_2 \cdot (\ln h_2 - \ln h_1) + b_3 \cdot (\ln N_2 - \ln N_1)$$
$$+ b_4 \cdot \left(\ln \frac{N_2}{A_2} - \ln \frac{N_1}{A_1} \right) + b_5 \cdot \left(\ln \frac{h_2}{A_2} - \ln \frac{h_1}{A_1} \right)$$

and height

$$\ln h_2 = \ln h_1 - b_1 \cdot \left(\frac{1}{A_2} - \frac{1}{A_1} \right) \quad \text{were fitted.}$$

6   STAND HEIGHT

The stand height is required to determine the site index of a stand, to calculate the stand volume, to predict the future growth from stand characteristics, and to represent a target variable in provenance, progeny, and species trials and silvicultural experiments.

6.1   Mean height

The mean stand height is a useful target variable for the early analysis and evaluation of silvicultural trials and tree breeding experiments. Provenance and species trials usually require that a large number of treatments are tested in an experiment. The mean height of a stand can be calculated as the arithmetic mean of a sample of tree heights, but alternatively by regressing tree height on dbh.

The arithmetic mean height of a stand is calculated as follows

$$\mu_h = \frac{\displaystyle\sum_{1}^{N} h_i}{N}.$$

and is usually estimated by sampling

$$\bar{h} = \frac{\sum\limits_{1}^{n} h_i}{n}.$$

Regression estimators however, are preferred, for example, the regression height of the tree with the arithmetic ($h_{\bar{d}}$) or quadratic mean diameter (h_{d_q}) or basal area central diameter (h_{d_M}) as predictor.

In forest inventories the mean height of the stand is required to estimate the volume of the tree with the quadratic mean diameter. In case of sampling the mean height is estimated for each plot separately. For practical reasons and because of cost considerations, it is usually assumed that the height curves of the individual sample plots coincide, i.e., the height curves are thought to have a common intercept and shape. In consequence, the observed heights and the corresponding diameters are pooled and a single height curve is fitted to the data of the individual compartment or for the single forest stand. The sampling error of the volume estimates will be underestimated if the dbh–height relationship within a given stand or stratum is affected by site differences. The calculated confidence intervals for the total volume also will tend to be underestimated by pooling data because between-plots variability of the parameters of the dbh–height regression is ignored. In silvicultural and tree breeding trials it is commonly accepted that diameter and height growth are response variables in their own right and respond differently to experimental treatments. For practical reasons again the dbh–height data of the replicates of a given treatment are pooled, a single height curve is fitted and the mean height of each plot is obtained as the regression height of the tree with the quadratic mean diameter of a given replicate.

Lorey (1878) introduced a weighted mean height, with the individual trees being weighted proportional to their basal area. The computational procedure is as follows:

- The diameters are grouped in classes
- The total basal area (g_i) is calculated for each diameter class
- The dbh–height regression equation is used to obtain the estimated tree height for the midpoint of each diameter class. The mean height is calculated as a weighted mean, with a weight of g_i ($i = 1, \ldots, k$) being assigned to each estimated height

$$h_L = \frac{g_1 h_1 + g_2 h_2 + \cdots + g_k h_k}{g_1 + g_2 + \cdots + g_k h_k}$$

Where

n_i = number of trees in the ith diameter class,

g_i = corresponding basal area for the class midpoint, and

h_i = regression height for the class midpoint.

Lorey's mean height necessitates the grouping of the observed diameters in classes of equal number of trees or of equal basal area. The mean height is calculated from

$$h_L = \frac{h_1 + h_2 + h_3 + h_4 + h_5}{5}$$

with $h_1, \ldots, h_5$ = the regression height of the tree with the mean basal area in each of the five classes.

The German yield tables, constructed by Schwappach and Wiedemann were based on Lorey's mean height (Schober 1987), but those constructed more recently (Assmann and Franz 1965; Bergel 1985) are based on the regression height of the quadratic mean diameter.

6.2   Top height

The mean height is needed to estimate the stand volume but, being calculated as the regression height of the tree with the mean dbh, it is sensitive to thinnings from below or from above (section 4). Thinnings from below, for example, emphasize the removal of dominated and suppressed trees, with a mean diameter substantially below that of the stand before thinning. In consequence, there is an *arithmetic shift* in the mean diameter of the remaining stand, due to thinnings from below or above. Modern yield tables give the estimated mean height for different site classes and ages, but in addition, the estimated top height of the stand, which is less sensitive to thinnings and more suitable for predicting the site index. The following measures for top height have been proposed for growth modeling and for constructing yield tables:

- Hart (1928) defined the top height of the stand as the arithmetic mean of the 100 tallest trees per hectare. In order to remove bias due to a fertility gradient within a sample plot, the population was subdivided into blocks with a size of 10×10 m, and the tallest tree in each block was measured. In a 40×40 m research plot, this would require a subdivision in 16 blocks, and the measurement of the 16 tallest trees. The top height is defined as the regression height of the mean diameter of the 100 thickest trees per hectare, i.e., as the regression height of the quadratic mean diameter of the 16 thickest trees within a 40×40 m sample plot. In Great Britain, Hummel (1953) introduced

the regression height of the mean diameter of the 100 thickest trees per acre as a measure for top height. Both definitions assume that the tallest trees are uniformly distributed within a given area. Because of site variability, this is not necessarily true and the highest trees may occur in clusters within a given sample plot. Furthermore, the mean height of the 100 thickest trees per hectare is always less than the mean of the 100 tallest trees. In Germany the top height is defined as the regression height of quadratic mean diameters of the 100 or 200 thickest trees per hectare.

- Dietrich (1923) introduced the mean height of the dominating trees, defined as those belonging to Kraft's classes 1 and 2, Etter (1949) those belonging to classes 1, 2, and 3 as a quantitative expression for "biological top height."
- Swedish mensurationists proposed to define top height or maximum height as the regression height of the tree with a dbh equal to the arithmetic mean of the diameter distribution plus 3 times the standard deviation (Nåslund 1935).
- In Germany, Weise (1880) introduced the regression height of the quadratic mean diameter of the 20% thickest trees as top height.
- When using aerial photographs to estimate the stand volume per hectare from height and crown cover, top height is sometimes defined as the mean of the k tallest trees per photo-plot, for example; with $k = 3$. On aerial photographs; the tallest trees can be identified more easily than trees representing the mean height.

Top height is less sensitive to an arithmetic shift of the mean diameter; due to thinning than mean height. The tallest trees are found in the dominant tree stratum; which is biologically of greater importance than other social classes, more particularly to measure site productivity. In general, the 20% thickest trees at a higher age originate from the category of the 20% thickest trees at a younger age, although some trees move to lower crown strata and others move in upward direction. There is a consensus of opinion that top height is useful for growth predictions. Trees with a height equal to the top height of the stand are usually selected for stem analysis to reconstruct the growth of forest stands.

In order to estimate the top height of a stand, when defined as the regression height of the tree with a quadratic mean diameter of the 100 thickest trees (d_t) per hectare, a prediction equation is required to estimate d_t from d_q with trees per hectare being introduced as a second predictor variable. Top height (h_t) could be obtained from the stand height curve or from a fitted equation.

Example 5.8 Permanent sample plots in *P. radiata* were used to examine the relationship between d_t and d_q (Figure 5-9). It was found that stem number has a statistically significant effect on the regression equation

$$d_t = 4.21 + 1.087 \cdot d_q - 0.00129 \cdot N$$
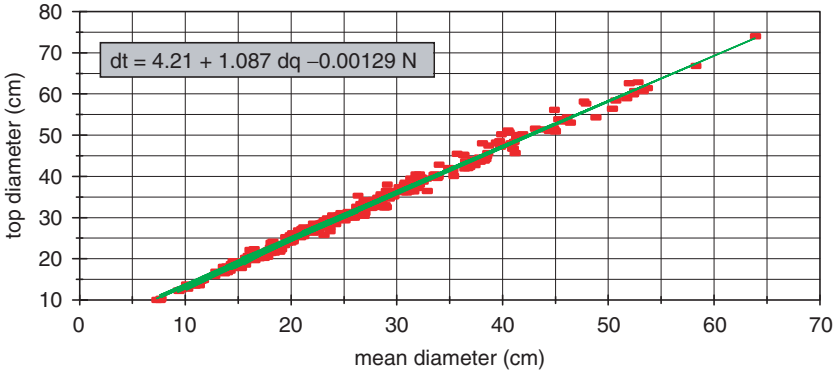
Figure 5-9. Relationship between mean diameter of 100 thickest trees and quadratic mean stand diameter.

In order to remove the effect of N/ha, a regression equation was fitted with N/ha as the dependent and d_q as the independent variable. The resultant regression estimates for N/ha were inserted into the above equation (see Figure 5-1). The mean diameter of the 100 thickest trees increases by 1.09 cm per cm increase of d_q, but decreases by 1.29 cm for an increase of stand density of 1000 trees per hectare.

Example 5.9   Sample plot data in *P. radiata* were used to examine the relationship between mean height (h_m) and top height (h_t). A second-degree equation was fitted with top height as dependent, h_m, h_m^2, and N/ha as predictor variables. The quadratic term was statistically significant in presence of the linear term but, contrary to German studies (Kramer 1962), trees per hectare were not significant, primarily because of the uniform thinning regime in South African *P. radiata* plantations, which are managed to produce sawtimber. The improvement of R^2, due to h_m^2 was almost negligible, with R^2 increasing from 0.989 to 0.990, but Mallows' CP was 7.2 for the linear model and 3.0 for the three-parameter model. The regression equation is

$$h_t = 0.270 + 1.0158 \cdot h_m - 0.001881 \cdot h_m^2$$

Example 5.10   The diameter distribution of Appendix B is used to calculate the regression height of the tree with the arithmetic and quadratic mean diameter, the median, the diameter of the tree with the mean volume, the central area basal area tree, the mean derived from the Weise rule, for the quadratic mean of the 100 thickest trees per hectare (d_{100}), and those of the 10th and 90th percentile of the diameter distribution. The calculations are based on the prediction

equations, which performed satisfactorily in this specific stand. Equation (a) has an upper asymptote with the estimated height converging to $\exp(b_0)$ as $d \to \infty$. Equation (b) is monotone increasing for all values of dbh.

(a) $\ln(h) = b_0 + \dfrac{b_1}{d}$

(b) $h = b_0 + b_1 \ln(d)$

Mean diameter	dbh (cm)	Estimated mean height	Eq. (a) (m)	Eq. (b) (m)
$\overline{d}$	21.8	$h_{\overline{d}}$	16.9	16.9
d_q	22.4	h_{dq}	17.1	17.1
d_M	22.0	h_M	17.0	17.0
d_v	22.5	h_v	17.2	17.1
d_{mg}	24.0	h_{mg}	17.7	17.5
d_{100}	27.9	h_{d100}	19.0	18.5
d_{Weise}	23.0	h_{Weise}	17.4	17.3
$d_{10\%}$	15.5	$h_{10\%}$	14.1	14.3
$d_{90\%}$	28.0	$h_{90\%}$	19.0	18.5*

The differences between the height estimates based on Eq. (a) and (b) are almost negligible for central values, but substantial for top height.

6.3 Fitting height curves

The relationship between diameter and height within an even-aged stand is curvilinear but nonlinearity is not always detectable, for example, because the sample was too small to detect lack of fit of the linear model or due to excessive random variability of tree heights within a given diameter class, which is sometimes caused by inaccurate height measurements. Many nonlinear regression models, the majority being linearizable by a transformation of variables, have been proposed to fit a stand height curve. The fitted curve should satisfy certain requirements. The function should be monotone increasing with increasing dbh. In practice, it does not eliminate the use of a second-degree equation ($h = b_0 + b_1 d + b_2 d^2$), although the resultant height curve has a maximum value. Its location is found by equating the first derivative of the equation to zero and solving the resultant equation for d

<div align="center">*Table 5-6.* Functions for fitting height curves in practice</div>

$$(1) \quad h = 1.3 + \frac{d^2}{b_0 + b_1 \cdot d + b_2 \cdot d^2} \qquad (6) \quad \ln(h) = b_0 + b_1 \cdot \ln(d)$$

$$(2) \quad h = b_0 + b_1 \cdot d + b_2 \cdot d^2 \qquad\qquad (7) \quad \ln(h) = b_0 + b_1 \cdot \frac{1}{d}$$

$$(3) \quad h - 1.3 = b_1 \cdot d + b_2 \cdot d^2 \qquad\quad (8) \quad h - 1.37 = b_1 \cdot \left(1 - e^{-b_2^d}\right)$$

$$(4) \quad h = b_0 + b_1 \ln(d) \qquad\qquad\qquad (9) \quad h = b_1 \cdot \left(1 - e^{-b_2 \cdot d}\right)$$

$$(5) \quad \frac{1}{\sqrt{(h-1.3)}} = b_0 + b_1 \cdot \frac{1}{d} \qquad (10) \quad h = b_1 \cdot \left[\frac{d^{b_2}}{b_3 + d^{b_2}}\right]^{b_3}$$

$$h_{\max} = -2\frac{b_1}{b_2}$$

The maximum, however, must be located outside the observed range of diameters.

A variety of functions has been proposed to fit height curves. The equations in Table 5-6 have been proposed and applied in practice.

Equation (1), which was introduced by Prodan (1944), produces a satisfactory fit for the all-aged selection forests in Germany and has an inflection point. Equation (2) has a maximum for $d = -b_1/2b_2$. Equation (3) is a second-degree equation without the intercept parameter b_0. The model forces the height curve through the point $d = 0$ for $h = 1.3$. Equation (5) has similar properties. The equivalent equation, which is used to obtain the estimated heights is as follows:

$$h = 1.3 + \left(\frac{1}{b_0 + b_1 \cdot \ln d}\right)^2$$

Equation (5) can be generalized by replacing $\sqrt{h - 1.3}$ with the transformed height $\sqrt[k]{h - 1.3}$, is characterized by an inflection point and has asymptotic properties. Equations (6) and (7) are monotone increasing with increasing dbh. Equations (8) and (9) are identical to Mitscherlich's growth function (Chapter 9). They have asymptotic properties but no inflection point. The following functions might be more flexible:

$$h = b_1 \cdot \left(1 - b_2 e^{-b_3 \cdot d}\right)$$
$$h = b_1 \cdot \left(1 - e^{-b_3 \cdot (d - c)}\right)$$

with c being a constant. Equation (5) was introduced by Petterson (1955) and is favored by German forest mensurationists. It has asymptotic properties and an inflection point. Several authors compared the performance of different functions (Michailoff 1943; Schmidt 1967; Brewer et al. 1985; van Laar 1986).

Example 5.11   The dbh and height of 53 trees in a *P. radiata* stand (Appendix C) were used to fit a height curve. The fitted equations are:

(1) $h = \dfrac{d^2}{1.624 + 0.59 \cdot d + 0.033 \cdot d^2} + 1.3$

(2) $h = 2.44 + 0.93 \cdot d - 0.012 \cdot d^2$

(3) $h = 1.04 \cdot d - 0.015 \cdot d^2 + 1.3$

(4) $h = -8.4 + 8.215 \cdot \ln(d)$

(5) $h = \left(\dfrac{1}{0.187 + 1.44 \cdot \dfrac{1}{d}}\right)^2 + 1.3$

(6) $h = e^{1.0993 + 0.5584 \cdot \ln(d)}$

(7) $h = e^{3.258 - 9.2486 \cdot (1/d)}$

(8) $h = 23.505 \cdot \left(1 - e^{-0.0588d}\right) + 1.3$

(9) $h = 23.505 \cdot \left(1 - e^{0.0588 \cdot d}\right)$

(10) $h = 437 \cdot \left[\dfrac{d^{0.02}}{-1.021 + d^{0.02}}\right]^{-1.021}$

Figure 5-10 shows the fitted height curves for Equations (2), (4), and (7).

Bias, expressed by the mean deviations between the observed and estimated heights and the sum of squared deviations between the observed and estimated heights were as follows:

Equation	$\dfrac{\sum\left(h - \hat{h}\right)}{n}$	$\dfrac{\sum\left(h - \hat{h}\right)^2}{n}$	Equation	$h - \hat{h}$	$\dfrac{\sum\left(h - \hat{h}\right)^2}{n}$
(1)	0.0001	1.189	(6)	0.0070	1.280
(2)	−0.0000	1.211	(7)	−0.0061	1.250
(3)	−0.0179	1.232	(8)	0.0044	1.197
(4)	−0.0000	1.188	(9)	−0.0017	1.194
(5)	−0.0014	1.190	(10)	−0.0016	1.211
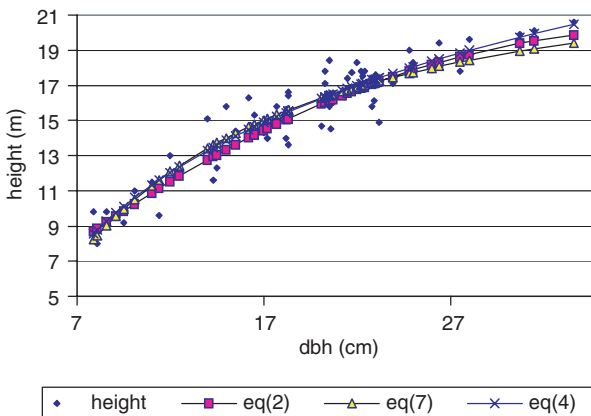
Figure 5-10. Fitted height curves.

6.4   Precision of height estimates

The regression analysis estimates the parameters of the regression equation and calculates the mean square error of the regression equation. The latter expresses that component of the total sum of squares, not explained by the regression of height on dbh. It can be used to determine the precision of the estimated mean height and to calculate confidence limits for the true mean height. This assumes that the target variable has not been transformed, which occurs, for example, if $\log(h)$ instead of h has been regressed on dbh or on a function of dbh or if any other height transformation has been applied. The positive and negative deviations of the measured heights from the regression curve are associated with biological factors, for example, leaning trees, broken tops, diseases, but also competition amongst trees and in addition are due to random errors of height measurements. In many instances, it is necessary to report on the precision of the regression estimate, either for the mean height of the stand or for each of the specified diameter classes. In Germany the standard deviation expressed as a percent of the mean height varies between 5% for Norway spruce, pine, and fir to 8% for beech and Douglas fir (Assmann 1957; van Tuyll et al. 1981).

Example 5.12   The previous dataset is used to calculate conditional confidence limits for the population mean, i.e., confidence limits for given diameters. The mean, the lower 0.95 confidence limit (lcl) and upper limit (ucl) for the mean height, corresponding with the 10th, 50th, and 90th percentile of the diameter distribution of Appendix C, were calculated for the regression equations:

$$h = b_0 + b_1 \cdot \ln(d) \tag{4}$$

$$\ln(h) = b_0 + b_1 \cdot \left(\frac{1}{d}\right) \tag{7}$$

The transformation of the dependent variable in Eq. (7) requires the calculation of confidence limits on the logarithmic scale and their retransformation.

	Breast height diameter					
	15.5 cm		22.0 cm		28.0 cm	
	Eq. (4)	Eq. (7)	Eq. (4)	Eq. (7)	Eq. (4)	Eq. (7)
	Height estimates, lower (lcl) and upper (ucl) confidence limits					
Mean	14.1	14.3	17.0	17.0	19.0	18.5
lcl*	13.7	14.0	16.6	16.5	18.5	18.0
ucl*	14.5	14.7	17.4	17.4	19.6	19.1

*lcl = lower confidence limit, ucl = upper confidence limit
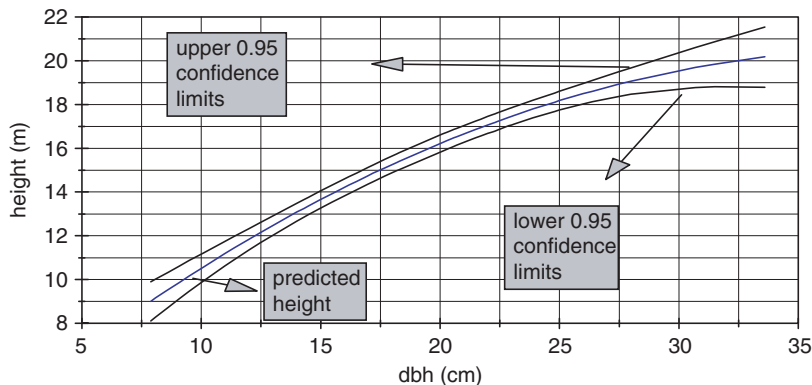
Figure 5-11. Lower and upper 0.95 confidence limits, second-degree equation.

The 0.95 confidence limits for the population mean, based on the fitted second-degree equation are shown in Figure 5-11.

6.5   The standardized height curve

The standard procedure in stand inventories, fitting a height curve for each stand separately, produces unbiased estimates of the mean height of the stand. It is generally accepted that at least 20–25 heights should be measured in each stand to obtain sufficiently accurate estimates. The basic idea of standardized height curves was introduced by German forest mensurationists, its purpose being to rationalize the field work required for forest inventories (Wiedemann 1936; Lang 1938). Tables were constructed, which indicated how much should be added to or subtracted from the observed mean height to estimate the mean height, if the diameter deviates a specific number of units of 1 (or 2) cm from the mean. In order to use these tables, it was necessary to estimate the mean diameter by conventional methods and to measure a limited number of tree heights, around the quadratic mean diameter. This method, however, was improved by more efficient approaches. Many studies indicated that the location and shape of the height curve changes with increasing age, but is also related to site and differs for different tree species. In other words, if it were possible to develop models, which incorporate such stand variables into the regression model, more efficient estimates of the mean height could be obtained. For growth monitoring in permanent sample plots and for growth modeling, for example, it might be advantageous, to pool the height measurements of successive remeasurements and relate the parameters of the function being used to age. When the nature of this relationship is known, age is introduced as an additional predictor variable

and a single equation is fitted to the sequence of remeasured plots (Sadiq et al. 1983; Pollanschütz 1974; van Laar 1986).

Hui et al. (1993) applied the function

$$h = 1.3 + b_0 d^{b_1}$$

to describe the relationship between dbh and height and used a log–log transformation to estimate the two parameters from top height. EK et al. (1984) introduced basal area per hectare and site index as additional predictor variables to model the dbh–height relationship. Zakrewski et al. (1988) fitted the function in its unlinearized form

$$\ln(h) = b_0 + b_1 \frac{1}{d}$$
$$h = e^{b_0 + b_1/d}$$

and regressed b_0 and b_1 on the quadratic mean diameter and stand mean height, respectively.

Gaffrey (1988) developed a model based on Michailoff's function (Michailoff, 1943):

$$h = 1.3 + (h_m - 1.3) e^{[a_1(1 - d_q/d) + a_2(1/d_q - 1/d)]}$$

Pienaar (1991) introduced the equation:

$$h = b_1 h_t \left(1 - b_2 e^{-b_3(d/d_q)}\right)^{b_4}$$

Nagel (1991) compared standardized height curves, derived from the Petterson function with those based on Sloboda's function (Sloboda et al. (1993)):

$$h = 1.3 + (h_m - 1.3) e^{(b_0(1 - d/d_q))} e^{b_1(d/d_q - 1/d)}$$

The Petterson function implies the existence of an inflection point, which is a function of b_0 and b_1

$$d_{\text{infl.}} = \frac{b_1}{b_0}$$

It can be shown that

$$b_0 = \frac{1 + \dfrac{d_{\text{infl}}}{d}}{\sqrt[3]{(h - 1.3)}}$$

Thus

$$b_1 = b_0 d_{\text{infl.}}$$

and

$$h_i = (b_0 + b_1/d_i)^3 + 1.3$$

Based on these considerations, the construction of a system of standardized height curves should be preceded by a sampling study to estimate b_1 from age.

7 STAND VOLUME

Stand volume is the most important stand characteristic in stand inventories. It is a function of the number of trees, basal area, mean height, and the individual or the average form of the trees and may be expressed in terms of:

- Either stem or total tree volume
- Either total or merchantable volume
- Either over or under bark volume

The stand volume is usually estimated from the diameter and height of the mean tree, but sometimes for each diameter class separately.

7.1 Standard tree volume tables and functions

Two-entry tree volume tables give the estimated volume for specified diameter and height strata, whereas the corresponding volume functions use dbh and height as predictor variables. The use of a two-entry volume table assumes that the form factor of trees of a certain species, for a given dbh and height, is not affected by external factors, for example, provenance, site, and stand treatment. The commonly applied *mean tree method* in stand inventories requires the estimation of the quadratic mean diameter and its regression height. This mean diameter is usually obtained by sampling, unless the diameter distribution of the stand is determined by a complete enumeration. In both cases, the mean height is calculated from the dbh–height regression equation or read off from the height curve. The resultant volume estimate does not provide information about the volume distribution in terms of size classes. When this additional information is required, either standardized height curves, which reflect the average shape and location of the height curve, can be used to estimate mean height for each diameter class separately, or the parameters of the dbh–height equation are estimated for each compartment separately. The tree volume equation is subsequently used to estimate the average tree volume in each diameter class.

The majority of the existing volume tables in the metric system of measurement give the estimated volume for 1 cm diameter classes and 1 m height classes, whereas 1 in. classes for diameter and 1 ft classes for height are customary in the English system. A bold underlining within the table serves to demarcate which diameter–height strata were represented in the sample used for the construction of the table. Theoretically, the table should not be applied outside these boundaries, but in practice this may be unavoidable. One major disadvantage of using the volume table is the necessity to interpolate in order to avoid rounding-off errors.

Example 5.13   The following equation was fitted to a sample of 302 *Eucalyptus grandis* trees:

$$v\,(\mathrm{m}^3) = \exp(-11.021 + 1.764\ln(d) + 1.433\ln(h))$$

where v = volume in cubic meters, d = dbh in centimeters and h = height in meters. The regression estimate for the volume of a tree with a dbh of 22.4 cm and a height of 21.4 m is 0.318 m^3. Rounding off to $d = 22$ cm and $h = 21.0$ m gives an estimated volume of 0.300 m^3, i.e., it produces a rounding-off error of -5.7%. Linear interpolation between 22.0 and 23.0 cm and between 21 and 22 m gives an estimated volume of 0.318 m^3.

7.2   Volume estimation with form height and volume series

Based on form factors, derived from German tree volume tables (Grundner and Schwappach 1942) and heights, obtained from Wiedemann's (1936) standardized height curves. Laer (1936) developed the concept of *form height series* to estimate stand volume. The form height of a tree was defined as the product of tree height and form factor, the form height of a stand as the average form factor multiplied by the mean height of the stand. Tables, which disclosed the estimated stand form height as a function of the mean height of the stand, were constructed for different tree species. For a mean height of 14 m, the form height (which is usually expressed in meters) varies between 5.6 m for beech and 7.3 m for fir, for a mean height of 20 m it varies between 8.9 m for ash and 10.5 m for fir, and at 26 m between 13.8 m (oak) and 10.6 m (birch). The form height of a given diameter class is multiplied by its basal area to obtain the estimated volume in this diameter class. The method, although now obsolete, has a moderate advantage over the conventional two-entry tree volume table, since height measurements can be restricted to the subpopulation of trees around the basal area central tree. A rough estimate of the diameter of the basal area central tree can be obtained from the 70th percentile of the ordered set of diameters with its height being estimated by sampling.

The *volume series* method, introduced by Spiecker (1948), is closely related to the form height series method. It prescribes the calculation of the diameter of the basal area central tree, approximated by the 70th percentile of the ordered set of diameters. Its height is estimated by sampling. Similar to von Laer's form height series, Spiecker's volume series estimate the average stem volume per diameter class. The mean diameter and mean height of a given stand are required as entries to a table, indicating which volume series is to be used for that particular stand. These volume series were constructed in the form of subtables, with a number being assigned to each of them, which reflected the

relative volume. The volume estimates for volume series 50, for example, were exactly 50% of those for volume series 100. In 1951, the form height method was combined with the volume series approach. Since then, the diameter of the basal area central tree and its regression height are calculated to read off a common number, which is used to either enter the volume or the form height tables.

Example 5.14   The diameter of the basal area central tree in a *P. abies* stand is 24 cm and its regression height is 22 m. The auxiliary table indicates that series 67 of the Laer–Spiecker table is to be used to estimate the volume per diameter class. The relevant information derived from this series is summarized below:

dbh (cm)	Stem volume (m^3)	Form height	dbh (cm)	Stem volume (m^3)	Form height (m)
12	0.02	3.6	28	0.55	12.3
16	0.10	8.9	32	0.77	12.5
20	0.22	10.9	36	1.03	12.7
24	0.37	11.8	–	–	–

This information is sufficient to calculate the stem volume in 4 cm classes.

7.3   Stand volume tables and functions

The accuracy of stand volume tables, giving the estimated stand volume per hectare as a function of stand basal area and mean height, was investigated by Stoate (1945) and Spurr (1952). The "Australian" formula

$$V = b_0 + b_1 G + b_2 h_q + b_3 G \cdot h_q$$

where V = volume per hectare, G = basal area per hectare, h_m = mean height was originally proposed by Stoate and applied in *P. radiata* plantations in Australia (Cromer et al. 1956). The combined variable equation

$$\ln(V) = b_0 + b_1 \ln(G \cdot h_q)$$

was recommended by Spurr (1952), but some revealed evidence of nonlinearity in the lower domain of the predictor variable. Rondeux (1985) developed an equation to estimate the volume per hectare for Norway spruce from a regression equation with basal area per hectare, dominant height, and their linear interaction as predictor variables. The more parsimonious version of this model with basal area multiplied by height as the independent variable, required the

estimation of the basal area per hectare by point sampling. It was suggested that the resultant equation should be used primarily to obtain quick estimates of the stand volume.

Example 5.15   The following equation was fitted to sample plots in plantations of *P. radiata.* The resultant equation is

$$V = -17.62 + 2.376 \cdot G + 1.1068 \cdot h_m + 0.256 \cdot G \cdot h_m$$

with $R^2 = 0.993$. The equations

$$V = -16.260 + 3.608 \cdot G - 31.7546 \cdot \ln(G) + 32.648 \cdot \ln(h_m) + 0.2456 \cdot h_m \cdot G$$

and

$$V = -24.290 + 2.935 \cdot G - 23.053 \cdot \ln(G) + 27.4435 \cdot \ln(h_t) + 0.0.2482 \cdot h_t \cdot G$$

performed equally well, with $R^2 = 0.993$ for both models and $CP = 7.8$ and 6.1, respectively. In terms of Mallows' *CP*, the model with top height being used to represent one of the predictor variables, is superior in predicting stand volume.

7.4   Estimation with yield tables

Yield tables of a given tree species provide an estimate of volume and growth per hectare as a function of age, site class, and a "normal" stocking density. In general, such tables represent the average growth pattern within large regions. Sometimes they were constructed for different degrees and methods of thinning and for different yield levels (Assmann and Franz 1963; Bergel 1985). The following information must be available to estimate the stand volume from stand volume yield tables:
- Stand age
- Mean or top height
- Stocking density in terms of basal area per hectare
- The area occupied by the individual species (if the table is used for estimates in mixed stands)

In order to obtain the estimated stand volume, the site class or site index is estimated from age and mean or top height. The tabulated volume is multiplied by stocking density, the latter being calculated as the ratio of the observed over the "normal" basal area, reflected by the yield table. To a large extent, the accuracy of the stand volume estimation with yield tables depends upon the accuracy of the estimation of age, mean height, stocking density and – in the case of mixed stands – the ground area occupied by the single tree species. The method is

primarily used for low-intensity surveys, for example, with the basal area per hectare being measured with the relascope method and the stand height estimated by measuring sample trees with a dbh around the estimated mean stand diameter. The estimates may be subject to operator-bias.

Example 5.16   We assume that a mixed stand consists of 86-year-old beech and 86-year-old maple trees. The estimated mean height is 25 and 27 m and the corresponding site classes are II and I, respectively. The basal area per hectare estimates, obtained by field measurements, are 25.1 and 4.3 m^2/ha, respectively, the estimated mixture (in terms of basal area per hectare) is 85% beech and 15% maple. The volume estimates, obtained from the appropriate yield tables, are 295 and 61 m^3/ha, respectively.

7.5   Volume estimation from felled sample trees

The diameter distribution and mean height of the stand is obtained by sampling, but the stem form factor is either obtained from tables or the form factor is estimated from regression equations, with diameter and height as independent variables. The determination of the stem form factor by measuring felled sample trees is justified if the true stem form is expected to differ substantially from the tabulated values (e.g., because of genetic differences), or if such tables are not available. The latter may the case in developing countries with a young forestry history. In European forestry, the estimation of stand volume based on sample tree measurements has a long tradition. Draudt (1860), Kopezky (1899), Gehrhardt (1909), and others proposed sampling methods and developed methods and algorithms to estimate the volume of stands, based on felled sample trees. In Central Europe, these methods have lost their popularity, in spite of more accurate volume estimates, in part because of the prohibitively high measuring cost. Recently developed instruments for measuring upper-stem diameters, however, which can be carried out conveniently and accurately, is one of the reasons for a renewed interest in the measurement of one or more than one upper diameter on standing trees. This can be combined with modern sampling methods to estimate the stand volume on small tracts (Pelz 1980). The following methods may be used:

- Selection of sample trees at random, which ensures an equal chance for each tree to be included in the sample. A strict application of this rule may be impractical because it necessitates drawing up a list of sampling units and identifying each sampling unit.
- Quasi-random selection of sample trees, together with the constraint that they are spatially uniformly distributed within the stand

- Selection of sample trees with a probability proportional to prediction, with either basal area or volume used as a size variable. The method, which is known as *3-P sampling*, has been found to be more efficient than the previously discussed method.

The stand volume may be obtained as follows:

1. The selected n sample trees are felled, their dbh and volume are determined by sectionwise measurements. Alternatively, the volume of the sample trees is determined on the standing tree. Tree volume is regressed on basal area or squared diameter. An equation is usually fitted with unit weight being assigned to each tree, but since the variance about the regression line increases with tree size, it is appropriate to assign weights, for example

   $$w_i = 1/(d_i) \text{ or } w_i = 1/(di^2)$$

2. Alternatively, the regression equation $\log(v) = b_0 + b_1 \log(d)$ is fitted to the data. The logarithmic transformation tends to remove heteroscedasticity and no weighting procedures are required. The volume estimates are slightly biased, but this bias may be removed by applying Baskerville's adjustment factor (see Chapter 8). The model provides an adequate fit, but contrary to the previous model, it has the disadvantage that confidence intervals are obtained on a logarithmic scale, which implies that confidence limits have to be retransformed.

3. Sample trees are selected with a mean diameter approximately equal to the estimated quadratic mean stand diameter. Their mean volume is subsequently multiplied by the ratio R^*

$$R^* = \bar{d}_1^2 / \bar{d}_2^2$$

where $\bar{d}_2$ = quadratic mean diameter of the sample trees

$\bar{d}_1$ = quadratic mean diameter of the forest stand.

In consequence, a two-phase sampling procedure is required:

- When sample trees across the range of diameters are selected and measured. Fixed-radius plots are established to determine the diameter distribution of the stand.
- When sample trees of the mean stand diameter are measured, the fixed radius plots serve to estimate the quadratic mean stand diameter and the number of trees per hectare.

Different methods for estimating the stand volume can be combined. For example, the basal area may be measured by a complete enumeration or estimated by sampling in fixed sample plots or through angle count sampling. The resultant

Table 5-7. DBH and volume of sample trees

dbh (cm)	Volume (m^3)	dbh (cm)	Volume (m^3)	dbh (cm)	Volume (m^3)
8.1	0.02014	24.8	0.35083	20.6	0.19331
10.1	0.03788	33.7	0.69915	20.5	0.23411
14.0	0.09449	25.0	0.34556	22.0	0.26746
15.5	0.10862	9.5	0.02928	28.0	0.46018
18.3	0.16887	12.5	0.06130	30.7	0.56222
21.2	0.22831	15.0	0.11218	23.0	0.28261
22.2	0.26578	17.2	0.13001	–	–

information may be used in combination with volume and form height tables, volume tariffs, and yield tables.

Example 5.17   Suppose that no volume table or volume equation was available to estimate the stand volume of the sample plot data of Appendix B. Twenty sample trees, which were uniformly distributed over diameter classes, were felled and their volume determined by sectionwise measurement. The resultant volumes are given in Table 5-7. The observed volumes are regressed on squared dbh

$$v_{est} = -0.0389 + 0.00063095 \cdot d^2.$$

The quadratic mean diameter of the 20 sample trees is 22.95 cm, that of the sample plot is 22.38 cm. The regression estimate of the tree volume for $d_q = 22.38$ is 0.2771 m^3 and represents an almost unbiased estimate of the mean volume. In fact, the estimated volume is slightly higher because the quadratic mean diameter underestimates the diameter of the tree with the mean volume.

7.6   Critical height sampling

The basic concept was introduced by Kitamura (1962). The critical height is defined as the distance between the base of the tree and a second point, where the stem diameter subtends a certain angle to the sampling point. The latter is equal to the horizontal sighting angle, which corresponds to a given basal area factor, with the vertical being erected at the sampling point. The stand volume per hectare is expressed as the product of the basal area factor (*BAF*) and the sum of the critical heights:

$$V = BAF \cdot \sum h_c$$

In practice, the critical height is measured for those trees which fall within the variable-radius sample plot and are counted "in," since they are located inside the imaginary plot (Chapter 10). For most species, the "critical point" on the bole, which corresponds to the critical height, is located within the live crown and not clearly visible. It has been suggested to obtain an indirect estimate of the critical height, for example, with the aid of taper functions, although the use of such functions may give biased estimates when applied to specific stands.

8   SPATIAL DISTRIBUTION OF TREES

8.1   Tests of randomness

A random spatial distribution of trees may occur in natural forests, in which case the number of individuals within sample plots of a fixed size has a Poisson distribution. In many instances, and more particularly in mixed forests which originated from natural regeneration, certain species occur in clumps, for example, because of the proximity of parent trees, because of site differences within the stand or due to competing undergrowth. For several reasons it may be necessary to model the spatial distribution, to test the hypothesis of a random distribution, and to calculate an index which expresses the degree of nonrandomness. If species occur in clumps, which in turn may have a Poisson distribution, the distribution is called *contagious*. The contagion is positive if the occurrence of a tree of a certain species within a given quadrate, induces an increased probability of the occurrence of a second tree within the same quadrate. In plant communities, a negative contagion occurs less frequently. It would mean that the occurrence of a tree of a certain species reduces the probability of a second occurrence. The spatial distribution of individuals within populations with a positive contagion can frequently be described either by the *negative binomial* or by the *Neymann distribution*.

The hypothesis of a random distribution is to be tested either with the χ^2 or with the likelihood-ratio test for goodness of fit. The likelihood-ratio test is usually preferred and is compulsory when fitting loglinear models. In both cases, the test statistic has a χ^2 *distribution* with $(k-1-p)$ degrees of freedom, where k = number of classes and p = number of parameters of the distribution. In the case of a Poisson distribution, the observed χ^2 value is therefore associated with $(k-2)$ degrees of freedom, since this is a one-parameter distribution. However, since no expected frequencies are allowed to be smaller than one, it may be necessary to combine adjacent classes. This grouping of classes has an adverse effect on the power of the test. If, for example, the number of classes is 7 and

the frequencies in the last 3 classes are pooled, the observed χ^2 is associated with three degrees of freedom. With so few degrees of freedom, it is unlikely to detect a nonrandom underlying distribution. Alternatively, a Poisson, a negative binomial and a Neyman distribution are fitted to the observed data and χ^2 is calculated for each distribution fitted. The associated probability P expresses the probability of obtaining a larger χ^2, if H_o is true. If the Neyman distribution produces a P-value in excess of that which is calculated for the other two distributions, this distribution produces a superior fit, although it does not disprove that the hypothesis of a random distribution was wrong. Alternatively, the following statistic, which quantifies the degree of nonrandomness, is calculated

$$R = \frac{s^2}{\overline{x}}$$

which is equal to 1, if the hypothesis of a Poisson distribution holds true. The ratio variance over mean is a useful numerical expression to express the degree of aggregation, although it cannot be used to test the hypothesis of a random distribution. Other tests have been developed by ecologists and botanists. Pielou (1959) proposed the index

$$\alpha = N_{ha}\overline{W}$$

where N_{ha} = estimated plant density and W = distance between a random point and the nearest plant. Large values of α indicate aggregation, small values occur in a uniform distribution. In a spatially random distribution, the expected value of α is

$$E\,(\alpha) = \frac{n-1}{n}$$

where n = number of quadrates. The quantity $2n\alpha$ is a χ^2 variate with $2n$ degrees of freedom and serves to test the hypothesis of randomness.

Morisita (1957) introduced the following formula to calculate plant density in populations with a nonrandom spatial distribution of individuals:

$$N_{ha} = \frac{n}{\pi \cdot \sum_{i=1}^{n} r^2}$$

where n = number of sampling points and r = distance between individuals and sampling point. Zeide (1985) showed that the expected mean distance between a point and the sample trees, which falls inside an angle count sample plot, is a function of mean diameter and basal area factor:

$$E\,(\overline{r}) = \frac{\overline{d}}{3\sqrt{BAF}}$$

When selecting a dominant tree as plot center, the mean of the observed distances will tend to exceed its expected value, because the dominant tree suppresses surrounding trees belonging to a lower social stratum. The opposite holds true for suppressed trees selected as plot center. The difference between the observed and expected mean distance, $\bar{r}$ therefore, measures the degree of spatial nonrandomness.

Clark et al. (1954) basic their statistical test of the distribution of the distance r between an individual tree and its nearest neighbor. Their mean, $\bar{r}_{observed} = \sum r/N$, in a spatially random distribution of trees has an expected value of $\bar{r}_{expected} = 1/2\sqrt{\rho}$ with $\rho = $ number of plants per unit area. The ratio $\bar{r}_{observed}/\bar{r}_{expected}$ varies between 0 (maximum aggregation) and 1 (random distribution). The ratio

$$z = \frac{\bar{r}_{observed} - \bar{r}_{expected}}{\sigma_{\bar{r}_{expected}}}$$

with

$$\sigma_{\bar{r}_{expected}} = \frac{0.26136}{\sqrt{samplesize \cdot \rho}}$$

follows the distribution of the unit normal variate. The hypothesis of a random distribution is rejected if z exceeds the specified level of significance.

Holgate (1965) introduced the ratio

$$R = \frac{y_s}{y_t}$$

where $y_s, y_t = $ distance between a random point and the sth and the tth individual respectively. In randomly distributed populations the expected value of R is

$$E(R) = \sqrt{\frac{s(t-s)}{t^2(t+1)}}$$

The variate R is approximately normally distributed (0, 1), unless s/t is near to 0 or 1.

Hopkins (1954) introduced the coefficient of aggregation

$$A = \frac{\sum\limits^n P^2/n}{\sum\limits^n I^2/n}$$

where P = distance between a random point and the nearest individual and I = distance between a random individual and its nearest neighbor. In random distributions, the two distance measures are equal.

Payandeh (1970) compared five methods for measuring the degree of nonrandomness based on data from five forest types, and redistributed populations, which simulated semi-random, random, semi-uniform, and uniform distributions. The quadrate method, which necessitates the layout of a random number of sample plots to calculate the ratio variance: mean, performed best, but plot size affected the outcome. Using the distance between a random point and the nearest neighbor was the next-best method and was not affected by plot size.

8.2   Spatial structure

It is now generally accepted that the extent of horizontal and vertical heterogeneity of stand structures has a impact on their ecological stability. This explains the increasing emphasis on the quantification of spatial structures of forest stands, primarily to determine habitats and species diversity. The spatial structure can be quantified in terms of spatial distribution, species diversity, and variability of tree dimensions. The indices are either distance-independent or distance-dependent. The latter quantify either species mingling and differentiation, or they describe forest structure at stand level or represent functions which incorporate distance measures with the aid of pair correlation functions. Pommerening (2003) discussed the merits of the following indices

- The Clark and Evans aggregation index as described in the previous section.
- The single-tree contagion index, proposed by Gadow et al. (1998) quantifies the degree of regularity of the spatial distribution of the trees and requires a specification of the number of neighbors involved in the calculation of the index. For a group of n neighbors and a perfectly regular spatial distribution, the standard angle is $360/n$ degrees. For example, for $n = 4$, the standard angle is $90°$ and the sum of the two angles α, $\beta (\alpha \leq \beta)$ which are shared by two neighbors is $360°$. The contagion index is defined as the proportion of angles smaller than the standard angle and is calculated as follows:
- $W_i = \sum_{j=1}^{n} w_{ij}$, with $w_{ij} = 1$ if the angle α is smaller than the standard angle
- It is either obtained by comparing the observed with the expected angle by visual inspection or by making use of a stand map with the known coordinates of each tree.

- The distance-independent *Shannon index* (Shannon et al. 1949) describes species mingling as the extent of mixture when a stand consists of two species and is distance-dependent.

Reference tree	Species of nearest neighbor		
	Species A	Species B	Total
		No of trees	
Species A	*A*	*B*	*M*
Species B	*C*	*D*	*N*
Total	*R*	*S*	*N*

- The segregation index is defined

$$S = 1 - \frac{N \cdot (b + c)}{m \cdot s. + b \cdot r}$$

- The single-tree mingling index *M* quantifies the proportion of the 3 nearest neighbors which do not belong to the species of the reference tree.

- Gadow and Füldner (1992, 1995) introduced a simple diameter differentiation index for pairwise comparison

$$TDn = 1 - \frac{DBH_i}{DBH_j}$$

where

N = ordered position of the neighbor tree

DBH_i = diameter of the thinner tree

DBH_j = diameter of the thicker tree.

- The following diameter differentiation index was introduced by Pommerening (2003)

$$T = 1 - \frac{\min\left(DBH_i, DBH_j\right)}{\max\left(DBH_i, DBH_j\right)}$$

- The pair correlation function which incorporates all possible intertree distances.

Three indices were compared to test diversity, in terms of crown cover and leaf biomass with watershed being used as an additional discrete variable. The general form of the index was

$$\Delta = \sum_{i=1}^{s} \pi_i R\left(\pi_i\right)$$

with s = total number of species, π_i = proportion abundance of the ith species and $R\left(\pi_i\right)$ = measure of rarity of the ith species. The index Δ was calculated for species count, the Shannon index and the Simpson index

$$\text{Species count} \quad \Delta_{\text{count}} = \sum \pi_i \left(\pi_i / 1 - \pi_i \right)$$

$$\text{Shannon index} \quad \Delta_{\text{Shannon}} = - \sum \pi_i \ln \pi_i$$

$$\text{Simpson index} \quad \Delta_{\text{Simpson}} = \sum \pi_i \left(1 - \pi_i \right)$$

Five plant life-forms were defined as independent variables. A principal components transformation, together with an orthogonal rotation of the axes was applied to obtain meaningful uncorrelated predictor variables. The first and third principal component being subsequently used to estimate the three diversity indices. Together they explained more than 98% of the variation. For all three diversity indices an equation with the first and third principal component as continuous and a two-level watershed variable as a discrete predictor were fitted.

8.3 Structural diversity

The structural diversity of a stand is defined as the diversity of trees within stands and can be indicative of overall biodiversity (Staudhammer et al. 2001). Forest stands with a large number of tree species and highly variable tree sizes are characterized by high biodiversity and this in turn has a positive impact on the stability of forest ecosystems. Biodiversity indices have been developed to quantify the degree of diversity, which are based on the distribution of individuals by species. The Shannon–Weaver index (Shannon et al. 1949) is based on the probability that a randomly selected individual belongs to a specific species. The index is defined as

$$H' = - \sum_{i=1}^{S} p_i \ln p_i$$

with p_i = proportion of the ith species, S = number of species involved. Because of the effect of a varying tree size on the index, modified indices were introduced. Specific variables were correlated with the proportion of individual trees of a given species such as number of individuals, basal area, stems per hectare, and foliar cover. If all proportions are equal, Shannon's index is equal to the number of species. Habitat heterogeneity was introduced to account for varying tree sizes (Orloci 1970; Freemark et al. 1986)

$$HH = - \sum_{i=1}^{r} \sum_{j=1}^{c} x_{ij} \ln \left(x_{ij} / \bar{x}_i \right)$$

where c = number of plots, r = number of classes, and x_{ij} = proportion of individuals in the ith class within the jth plot. The index was calculated for different tree and stand level variables or was derived from stand variables alone (Lähde 1999). In all instances, continuous variables were grouped into classes to calculate proportions.

Staudthammer et al. (1999) extended Shannon's index and emphasized the necessity to quantify horizontal as well as vertical diversity. The post hoc method was based on proportion basal area, height, and species by dbh class. The mean of the three values was calculated to define the modified Shannon index. The combination method was based on the proportional basal area in each dbh/height/species class. The observed proportions were averaged to produce a single index value. In addition a structural index based on R^2 index was defined by the departure of the observed dbh, height, and species distribution from a theoretical maximum, i.e., from a uniform distribution. Furthermore a structure index was derived from the difference between the empirical basal area distribution and the univariate uniform distribution. The empirical variance was calculated as follows

$$S^2 = \frac{\sum\limits_{i=1}^{n} \left[w_i \, (x_i - \bar{x})^2 \right]}{\sum\limits_{i=1}^{n} w_i}$$

with x = diameter, height, w_i = weight of the ith observation proportional to basal area per hectare and the variance of the hypothesized uniform distribution is

$$S_u^2 = \frac{(b - a)^2}{12}$$

The extensions of the Shannon index performed satisfactorily, with the post hoc method giving more information about the overall diversity. Diameter and height, however, had to be grouped in classes for calculating proportions.

9   STAND DENSITY

9.1   Area-related indices

The degree of stocking of a stand expresses the current stocking, usually in terms of basal area, but expressed as a percentage of the basal area which is considered "normal" for a stand of a given species, age, site index, thinning regime, and possibly for a given yield level. It does not adequately measure

the biological stand density. The latter should reflect the degree of competition between trees within the biogroup and is a useful predictor variable in growth and yield modeling.

The stand density index introduced by Reineke (1933), although frequently identified as an index for density, falls within the category of stocking indices. The method is as follows:

- The number of trees per hectare and mean diameter is determined in temporary plots, which are established at randomly selected locations in stands representing a wide range of sites
- The following regression equation is fitted:

$$\ln (N_{\text{ha}}) = b_0 + b_1 \ln (d_q)$$

- An arbitrarily selected mean diameter (d_I), for example, $d_I = 40\,\text{cm}$ serves as the reference diameter. The regression equation is used to estimate the expected number of trees in a stand for the index diameter.
- The stand density index (*SDI*) of a new stand, is found by sampling to estimate the mean diameter (d_i) and the number of trees per hectare (N_i), for example, in each of n sample plots
- The stand density index is calculated for each sample plot by

$$SDI_i = \frac{\exp \lfloor \ln N_i + b_1 \cdot (\ln(d_I - \ln d_i)) \rfloor}{\exp (b_0 + b_1 \cdot \ln d_I)}$$

- The resultant index values are averaged. Alternatively, the data of n plots are pooled and a single index value determined for the pooled observations, although this produces a slightly biased estimate of the stand density index.
- Reineke's stand density concept is based on the assumption that the line which expresses the relationship between the logtransformed number of trees per hectare and the logtransformed mean diameter may be located above or below the reference line. However, since the expected number of trees at some point in time either in the past or in the future is to be calculated, the method assumes a constant slope of the regression line.
- The parameters of the basic equation depend on management objectives, more specifically, whether the forest is either managed for the production of pulpwood or for sawtimber. It is therefore a stocking guide rather than an index of competition. However, when applied to unthinned natural forests, it represents a biological stand parameter.

Conventional fixed-radius sample plots, as well as variable-radius sample plots, provide unbiased estimates of the number of trees and basal area per hectare. Both variables are used to numerically express stand density, but they fail to measure the degree of competition for growing space amongst trees within the biogroup, unless age and site index are specified. In growth modeling, the number of trees per hectare as a predictor variable in growth models is less suitable than basal area for representing stand density, since it ignores tree size.
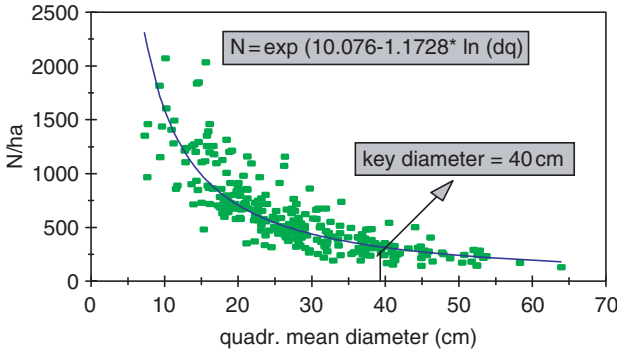
Figure 5-12. Reineke's reference curve for *Pinus radiata*.

Example 5.18   Sample plot data in *P. radiata* were used to fit the following equation with $\ln(d_q)$ and $\ln(N)$ as independent and dependent variables, respectively (Figure 5-12):

$$\ln(N) = 10.076 - 1.1728. \ln(d_q)$$

The equation generates the reference curve, with the key diameter being arbitrarily fixed at $d_I = 40$ cm. The corresponding stem number is

$$N_{key} = \exp[10.076 - 1.1728 \cdot \ln(40)] = 314$$

The model could be improved by the addition of a quadratic term. The latter was significant in the presence of $\ln(d_q)$ with R^2 increasing from 0.727 to 0.738. The equation is

$$\ln(N) = 7.344 + 0.591 \cdot \ln(d_m) - 0.2794 \cdot (\ln(d_m))^2$$

The expected stem number for $d_q = 40$ cm is 305 instead of 314. Suppose that the SDI of a specific stand with $d_q = 28$, $N = 644$ has to be determined. Then

$$b_0 = \ln 644 - 0.591. \ln 28 - 0.2794. (\ln 28)^2$$

The expected stem number for $d_q = 40$ then is

$$N = \exp[7.601 + 0.591 \cdot \ln 40] - 0.2794 \cdot [\ln 40]^2$$

Thus $SDI = 395/305 = 1.30$

Hart (1928) introduced the $S\%$ to express stand density

$$S\% = \frac{a}{h_t} \cdot 100$$

where a = mean distance between trees and h_t = top height. The calculation of the mean distance is based on the assumption of a square lattice, i.e., an unthinned plantation forest, a square spacing, and zero mortality, in which case

$$a\,(m) = \frac{100}{\sqrt{N_{ha}}}$$

Hart's index is an artifact, which ignores the spatial distribution of the trees and the diameter distribution of the stand. Nevertheless, it remains a simple and useful index, which incorporates plant density as well as height.

Example 5.19 The stand density of even-aged forests in Central Europe is notably higher than that observed in the plantation forests of fast-growing species in the southern hemisphere; grown to produce sawtimber. The management of these plantations is characterized by a severe first thinning and a more moderate regime at a later age. In consequence, the relationship between age and $S\%$ index in these plantations can be expected to differ substantially from those in Central Europe. The following equation was fitted to the *P. radiata* dataset:

$$S\% = 19.59 - 62.367 \cdot \frac{1}{\text{age}} + 914.24 \cdot \left(\frac{1}{\text{age}}\right)^2$$

with $R^2 = 0.974$ (see Figure 5-13). The relationship between age and $S\%$ index for site class 40 of the Assmann–Franz yield table for Norway spruce is shown in Figure 5-14. The two diagrams illustrate the different thinning regimes in South Africa and Germany, respectively. *P. radiata* plantations in South Africa are based on a planting espacement of 2.7×2.7 m and an early first thinning,
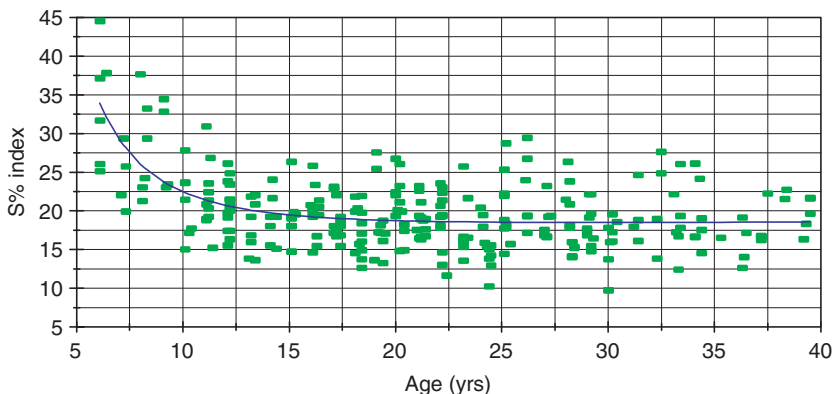
Figure 5-13. Relationship between S% and age, *Pinus radiata*, South Africa.
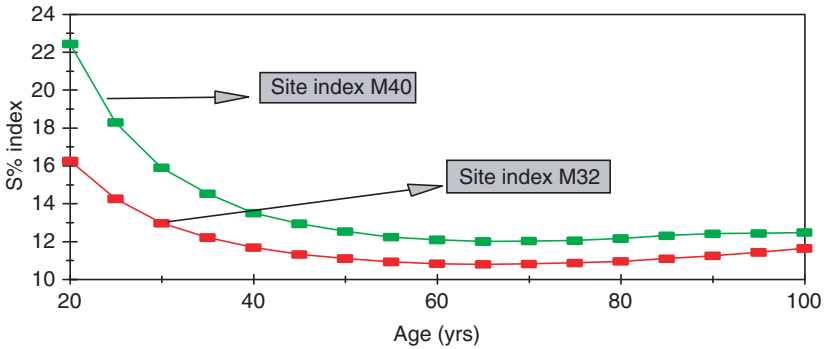
Figure 5-14. Relationship between S% and age, *Pinus abies*, Assmann and Franz (1963).

which reduces the number of trees per hectare from 1200 to 650, whereas the Assmann–Franz yield table is based on an initial density of 4500 trees per hectare and a first moderate thinning between 20 and 25 years.

Krajicek et al. (1961) introduced the concepts maximum crown area (*MCA*) and crown competition factor (*CCF*). The maximum crown area, defined as the maximum size of the projected crown area, was determined for free-growing trees and calculated from their crown width. Since crown width is a linear function of dbh, the relationship between *MCA* and dbh can be expressed by a second-degree equation

$$MCA = b_0 + b_1 \cdot d + b_2 \cdot d^2$$

The crown competition factor was defined as the sum of the *MCA* values per hectare (in square meters) divided by 10000

$$CCF = \frac{\sum_{i=1}^{N} \left(b_0 + b_1 \cdot d + b_2 \cdot d^2\right)}{10000}$$

Vezina (1962) examined the usefulness of the *CCF* index for expressing stand density, but found the index to be poorly correlated with other measures, such as basal area per hectare and Reineke's stand density index.

Chisman et al. (1940) introduced the tree-area-ratio model, which assumes that the relationship between the area occupied by the individual trees and its dbh can be expressed by a second-degree equation

$$\text{Tree area} = b_0 + b_1 d + b_2 d^2$$

The data which are required to estimate the parameters were obtained from measurements in temporary sample plots with $\sum d$ and $\sum d^2$ being converted

to their equivalents per hectare. They were introduced as predictor variables into a multiple regression equation with the target variable being equal to 1 for all n sample plots. The tree area equation assumes that the ground area is fully utilized and ignores the possible existence of unutilized or underutilized gaps. The assumption is, furthermore, that the growing space of a single tree is uniformly utilized by the root system.

The previously described stand density indices refer to the stand in its entirety, but do not take cognizance of density variations within the stand. Point density estimators have been proposed, which quantify the density around a given subject tree or around a given point within the stand. Spurr (1962) introduced an estimator, based on the angle subtended by the tree sighted from the sampling point. For a one-tree plot, the largest angle corresponds to the largest ratio dbh to distance (D). The basal area per hectare, based on the one-tree plot is

$$G_{ha} = \frac{1}{4} \cdot \left[\frac{d_1}{D_1} \right]^2$$

where

G = basal area (m^2/ha)

d_1 = diameter (cm)

D_1 = distance (m) between the sampling point and the tree with the largest angle.

This basal area is to be multiplied by an adjustment factor 0.5, since one-half the basal area is thought to fall outside the plot boundaries. Hence

$$G_{ha} = \frac{1}{4} \cdot 0.5 \cdot \left[\frac{d_1}{D_1} \right]^2$$

The basal area per hectare for a plot containing two trees, which subtend the largest and next-largest angle, was obtained by adding the basal area per hectare of the first to half that of the second tree. The n-tree density estimator, expressed in basal area per hectare is then given by the formula

$$G_{ha} = 0.25 \cdot \frac{0.5 \cdot \left(\frac{d_1}{D_1} \right)^2 + 1.5 \cdot \left(\frac{d_2}{D_2} \right)^2 + \cdots (n - 0.5) \cdot \left(\frac{d_n}{D_n} \right)^2}{n}$$

In order to quantify competition between individual trees within stands of Pinus radiata, *van Laar (1973) proposed an extended version of Spurr's index, which assigned different weights to the n potential competitors. The trees surrounding the subject tree were arranged in a descending order of the angle subtended*

with the subject tree. The basal area per hectare was estimated from the measurements for the first tree, the mean of the first and second tree, the mean of the first, second, and third tree, etc. The index values were calculated for $n = 1$ to $n = 9$ and generated nine competition indices, which were subsequently regressed on the diameter growth of the subject tree, with tree height being used as covariate. In unthinned sample plots, the coefficient of determination increased from $n = 1$ to $n = 7$, but such a trend was not apparent in moderately and heavily thinned plots.

Brown (1965) linked the point density concept with the area potentially available for the individual tree. After preparing a stand map, lines were drawn which connected the subject tree with its neighbors. They were bisected with the line segments being proportional to stem diameter. Lines, drawn at an angle of 90° to those bisected, formed the boundaries of the area which was available to the subject tree.

Seymour et al. (1987) proposed a stocking index based on the crown competition factor concept. The starting point was the allometric relationship between stem volume (v_s) and the crown volume (v_c)

$$v_s = k_0 v_c^{b_1}$$

where $b_0 = coefficient$, which is related to foliar efficiency. The (assumed circular) vertical crown projection area (CPA) is a function of the crown radius so that

$$v_c = k_2 \cdot CPA \cdot h$$

where $k_2 = $ shape coefficient. Based on Schumacher's logarithmic equation for predicting the stem volume from dbh and height, it was shown that dbh could be estimated from CPA and height

$$d = b_0 \cdot CPA^{b_1} \cdot h^{b_2}$$

This equation was fitted to eastern White pine and inverted in order to express CPA as a function of dbh and height. Assuming that the ground area is fully covered by the live crowns, it was possible to estimate that value of CPA, which is required to grow trees of a certain dbh. However, since the geometry of the tree crown inhibits an unrestricted increase of the crown radius, there were upper limits for the CPA needed to obtain a given diameter.

9.2   Distance-related indices

Estimators based on distance measurements were introduced by plant ecologists. Although not necessarily producing unbiased estimates of plant density

in plant communities with a nonrandom spatial distribution, such estimators are useful for ecological sampling studies. The quadrant method, as used in plant ecology, prescribes that the distance between a randomly selected point and the nearest neighbor is determined in each of the four quadrants with the sampling point as the center (Greig-Smith 1964). Alternatively, the distance to the nearest tree is replaced by that to the ith nearest tree. Diggle (1978) proposed an estimator, which combines the distance between a random point and the nearest individual with that of a randomly selected individual to the nearest neighbor. The estimator was recommended when there was statistical evidence of a nonrandom spatial pattern. Batcheler (1971) introduced an index, which incorporates the distance between a random point and its nearest neighbor as well as that between this individual and its nearest neighbor An estimator for density was also developed for truncated samples (Batcheler 1973)

$$\text{density} = \frac{n}{\pi \cdot \left(r_1^2 +, \cdots, r_n^2 + (N - n) \cdot R^2\right)}.$$

where

N = number of sample points

$R_1, \ldots, r_n$ = distance to the 1$^{\text{st}}$, ..., nth individual

R = truncation distance

n = number of sample points with a distance smaller than R.

Chapter 6

TAPER TABLES AND FUNCTIONS

1 TAPER TABLES

Forest managers require information about the diameter of the bole at fixed distances from the base of the tree, for example, to predict the recovery of sawlogs of different diameter and length or the yield of poles of varying dimensions, for trees of different dbh and height. Taper tables are usually based on dbh and height as table entries. Those which give the estimated upper diameter, expressed as a percent of dbh, are sometimes described as "false taper" tables, since their information about the real shape of the stem is obscured by giving estimates for the diameter at fixed instead of relative distances from the base of the tree. The tables are; nevertheless, useful for reconstructing the stem profile of trees of different sizes. Such tables were already constructed by Behre (1923), those for different species were developed by Mitscherlich (1939), for *Picea abies* by Zimmerle (1949), and recently by Bergel (1981).

"True taper" tables give the predicted diameters at fixed relative heights above the base of the tree, expressed as a fraction of the stem diameter at 10% of the height also use dbh and height as table entries and. They are useful in examining and comparing the stem profile of trees of different species and sizes. In order to reconstruct the stem profile of a tree of given dimensions, the relative diameters at different relative heights are to be multiplied by dbh and total height, respectively.

The construction of true taper tables requires the following procedure:

- Felling of a representative sample of trees, selected from a wide range of tree diameters, heights, ages, and site classes
- Tree measurements, including dbh$_{\text{o.b.}}$, tree height, stem diameter (either over- or under-bark), at predefined positions along the bole, for example at 5%, 10%, 15%, ..., 95% of the tree height
- Fitting of regression equations with $d_{i\%}$ (under or over bark) as the target variable and $d_{10\%,\text{u.b.}}$ or $d_{10\%,\text{o.b.}}$ as predictors

- A regression analysis with the parameter estimates of these equations as the target variable and the position of the point of measurement on the bole as the independent variable. Its purpose is to harmonize the regression curves obtained in previous steps, i.e., to smoothen the parameter estimates.
- Böckmann (1990) used the stem diameter at intervals of 1/10 of the tree height, with the lowest point of measurement being located at 1/20 of the height. A sixth-degree polynomial with stem diameter at the ith position as the dependent and dbh as predictor variable was fitted with the resultant parameter estimates being regressed on relative height above ground
- Fitting an equation to estimate $d_{10\%,\text{u.b.}}$ or $d_{10\%,\text{o.b.}}$ from $dbh_{\text{o.b}}$ with tree height as additional predictor variable
- Construction of a taper table, based on parameter estimates obtained in previous steps

Example 6.1   Measurements on 25 *Pinus patula* sample trees are used to demonstrate the construction of a taper table. Linear equations were fitted with stem diameters at 20%, 30%, ..., 90% as the dependent and that at 10% of the tree height as the independent variable. The regression statistics were as follows:

Target variable	b_0	b_1	R^2
$d_{0.2}$	-0.480	0.938	0.988
$d_{0.3}$	-1.565	0.925	0.984
$d_{0.4}$	-1.611	0.854	0.971
$d_{0.5}$	-1.793	0.786	0.960
$d_{0.6}$	-2.174	0.721	0.900
$d_{0.7}$	-1.914	0.598	0.847
$d_{0.8}$	-0.925	0.407	0.698
$d_{0.9}$	0.236	0.172	0.667

The accuracy of predicting upper-stem diameters from the diameter at 10% of the height decreases significantly with increasing height above ground of the target variable. The relationship between the coefficients b_0 and b_1 and relative height above the base of the tree is shown in Figure 6-1 and can be expressed by the following equations:

$$b_0 = 2.169 - 16.3386 \cdot h_{\text{rel}} + 15.602 \cdot h_{\text{rel}}^2$$
$$b_1 = 0.835 + 0.7803 \cdot h_{\text{rel}} - 1.6601 \cdot h_{\text{rel}}^2$$
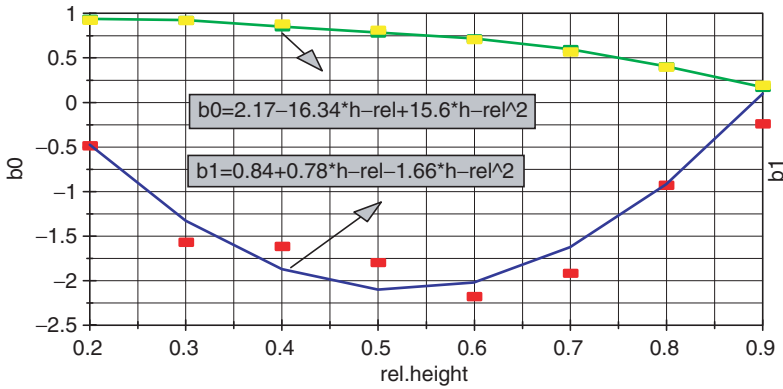
Figure 6-1. Relationship between parameter estimates and relative height.

In order to construct a taper table with dbh and height as table entries, a regression equation is required to predict $d_{0.1}$ from dbh and height. A stepwise screening of variables produced the following equation:

$$d_{0.1} = -0.912 + 1.140 \cdot d - 0.00747 d \cdot h$$

It was used to estimate $d_{0.1}$ for any combination of dbh and height, although the latter should not fall outside the range of dbh and height, represented by the sample. For $d = 25\,\text{cm}$ and $h = 20\,\text{m}$, the estimated $d_{0.1}$ is $23.85\,\text{cm}$. The previous set of equations is subsequently used to estimate the diameter at different heights above ground.

The regression equations can be applied to calculate true form quotients, possibly after harmonizing the regression lines, by assuming a nonlinear relationship between b_1 and upper stem position h_u, for example

$$b_1 = c_0 + c_1 \cdot h_u + c_2 \cdot h_u^2$$

and a linear relationship between b_0 and h_u.

2   STEM PROFILE MODELS

2.1   Introduction

In many countries, more particularly in the USA, Canada, New Zealand, and Australia, forest mensurational research has developed in a different direction and moved away from the classical approach of predicting form quotients for

the construction of taper tables. There has been an increasing emphasis on the development of models and equations, which describe the variable rate of decrease of the stem diameter between the base and the top of the tree. In order to obtain a model which is valid for trees of different size, equations which express this rate of decrease might be fitted to each sample tree separately. Thereafter, their parameters are regressed on tree characteristics, for example on dbh and height, with the resultant equations being used to reconstruct the stem form of a given tree, but also to estimate its merchantable height and upper diameter. Similar to the previous taper tables, stem measurements at different positions along the stem are obtained by destructive sampling, based on a representative sample of trees across the entire range of tree ages, tree sizes, and site indices. In exceptional cases, taper functions were developed from measurements on standing trees (James et al. 1984).

In general, however, the sample tree measurements are pooled with the upper diameter and upper height expressed as a fraction of dbh and tree height respectively. In consequence, the relative instead of actual upper diameters and heights are entered into the database. Pooling nevertheless implies that the total sample consists of a number of subsamples, one for each sample tree. The assumption of uncorrelated residuals is thereby violated. A similar situation occurs in growth modeling with multiple measurements in permanent sample plots.

In recent years, researchers emphasized the benefits of developing taper equations which are compatible with stem volume equations. Integrating the taper function produces an equation with total or merchantable volume as the dependent and dbh, as well as height as independent variable. The parameters of the volume equations which are compatible with the taper equations are not estimated independently by least-squares procedures. To ensure compatibility, they are derived from those of the taper function, for which least-squares estimates were obtained. A volume function, however, which is compatible with a taper function, does not usually give the best estimates of stem volume. Conversely, stem taper estimates will tend to be biased by deriving taper functions which are compatible with volume functions derived by least-squares analysis.

2.2   Taper functions

Taper functions were developed by Newberry (1986), Kozak et al. (1969), Demaerschalk et al. (1977), and others.

Newberry et al. (1986)

The coefficient b_0, multiplied by diameter, expresses stem taper, whereas b_1 explained the shape of the stem. The estimation proceeded in two stages. In stage 1, the coefficients b_0 and b_1 were estimated from single-tree data,

ordinary least squares was applied in stage 2 to estimate the parameters of two equations with dbh, height, height to live crown, crown ratio, and age as predictors and b_0 as well as b_1 as target variables.

Kozak et al. (1988)

Variable-exponent taper function, with d_I = diameter at the inflection point of the stem curve, h_I = corresponding height above ground. The prediction model to estimate the exponent C from the ratios h_i/h and d/h was

$$C = b_0 + b_1 \cdot Z + b_2 \cdot Z^2 + b_3/Z + b_4 \cdot \ln(Z + 0.001)$$
$$+ b_5 \cdot \sqrt{Z} + b_6 \cdot e^z + b_7 \cdot (d/h)$$

where $Z = h_i/h$. Breast height diameter under bark was estimated from $d_{o.b.}$

$$d_{u.b.} = a_o \cdot d_{o.b}^{a_1} \cdot a_2^{d_{o.b}}$$

and back substituted into the previous prediction equation. The equation is linearizable by a logarithmic transformation of the variables and its parameters are found by ordinary least-squares procedures. The equation has three important properties: $d_i = 0$ for $h_i/h = 1$, $d_i = d_{u.b.}$ when $h_i/h = $ p and a change of direction where $h_i/h = p$ *(Table 6-1)*.

Perez et al. (1990)

A more parsimonious form of Kozak's variable-form taper function

$$d_i = b_0 \cdot d^{b_1} \cdot b^{d_2} \cdot x^c$$

with $x = h_i/h$. Kozak's model was superior in terms of total squared error, whereas estimates obtained from Perez's model were less biased.

Biging (1984)

Taper equation was derived from a model resembling the integral form of the Chapman–Richards growth function. Constraining the function to obtain the estimate $d_i = 0$ for $h_i = h$ gives

$$d_i = d \cdot \left[b_1 + b_2 \cdot \ln\left(1 - \frac{h_i}{h}\right) \right]^{\frac{1}{m}} \cdot \left[1 - e^{\frac{-b_1}{b_2}} \right]$$

The estimated diameter for $h_i = 0$ (i.e., for the base of the tree) is equal to $b_1 d$, which implies that the coefficient b_1 can be interpreted as a ratio

$$b_1 = \frac{d_0}{d}$$

where d_0 = diameter at the base of the tree. Fitting this equation to sample tree data in Douglas fir, produced the estimate $1/m = 0.334$. In order to obtain an integrable function, m was to be an integer and $1/m$ was therefore fixed at 1/3. The volume function, obtained by integration, was compatible with the

Table 6-1. Selected taper equations

Author	Equation
Newberry et al. (1986)	$d_i = b_0 d \left(\dfrac{h - hi}{h - 1.3} \right)^{b_1}$
Kozak et al. (1969)	$\dfrac{d_i^2}{d^2} = b_1 \cdot \left(\dfrac{h_i}{h} - 1 \right) + b_2 \cdot \left(\dfrac{h_i^2}{h^2} - 1 \right)$
Kozak et al. (1988)	$y = x^c$
	$x = \dfrac{1 - \sqrt{\dfrac{h_I}{h}}}{1 - \sqrt{p}}, \; y = \dfrac{d_I}{d} \; and \; p = 100 \cdot \dfrac{h_I}{h}$
Perez et al. (1990)	$d_i = b_0 \cdot d^{b_1} \cdot b_2{}^d \cdot x^c$
Biging (1984)	$d_i = d \cdot \left[b_1 + b_2 \cdot \ln \left(1 - \dfrac{h_i}{h} \right) \right]^{\frac{1}{m}} \cdot \left[1 - e^{\frac{-b_1}{b_2}} \right]$
Demaerschalk (1973)	$d_i = b_1 \cdot d^{b_2} \cdot \dfrac{(h - h_i)^{b_3}}{h^{b_4}}$
Riemer et al. (1995)	$r_{h_i} = b_0 + (r_{1.3} - b_0)$ $\dfrac{\exp (b_1 \cdot (1.3 - h_i)) - \exp \left(b_{1(1.3-h)} \right)}{1 - \exp (b_1 \cdot (1.3 - h))}$ $-b_0 \dfrac{\exp (b_2 \cdot (1.3 - h_i)) - \exp (b_2 \cdot (1.3 - h))}{1 - \exp (b_2 \cdot (1.3 - h))}$
Reed (1984)	$d_i = b_1 \cdot d^{b_2} \cdot \dfrac{(h - h_i)^{b_3}}{h^{b_4}}$ (Eq.(1)) $v = a_1 \cdot d^{a_2} \cdot h^{a_3}$ (Eq.(2)) $a_1 = \dfrac{cb_1^2}{2b_3 + 1}, \; a_2 = 2 \cdot b_2 \; and \; a_3 = 2 \cdot b_3 + 1 - 2 \cdot b_4$

taper equation and contained dbh and height as predictor variables, with its coefficients being a function of the parameters of the taper equation.
Brink et al. (1986)
The version of the Weibull function, proposed by Yang et al. (1978), Eq. (1), with $r = $ stem radius at height h was used for modeling. A 90° clockwise rotation of the coordinate axes was implemented by expressing the stem radius at height h as the deviation between the observed stem radius at this position and either the diameter at the stem base or dbh as reference diameter. Selecting the diameter at the base of the stem as reference diameter, forces the fitted taper curve through the observed stem radius at this position. Equation (2) with b_0

and b_1 represent a scale and shape parameter respectively and b_2 representing the upper asymptote for height. The model however failed to produce a zero estimate for the stem diameter at the top of the tree.

Riemer et al. (1995)

Improved Brink et al. model through conditioning their function, such that the estimated bole diameter is zero at the tip of the tree and is equal to dbh at a height of 1.3 m. The parameter b_0 represents common asymptote of the decay function, b_1 = parameter expressing stem curvature in the lower part of the bole and b_2 = parameter expressing stem curvature in the upper part of the bole.

Reed et al. (1984)

Compatible stem taper and volume ratio equations for the constant form factor function, the combined variable equation as well as Schumacher's and Honer's equations. The authors presented a taper function, a tree volume equation, a volume ratio function (characterized by the upper stem height h_u) and the volume ratio function (defined by the upper diameter d_u), compatible with the volume model. Demaerschalk's model (Eq. (1)) is compatible with Schumacher's function (Eq. (2))

Example 6.2 The Newberry–Ormerod model was modified by including a quadratic term and fitted to the taper data of *P. patula* (Appendix B) (Figure 6-2). The regression equation is

$$y = \exp\left(-0.0482 + 0.72126 \cdot x_1 - 0.02417 \cdot x_2\right)$$

where $x_1 = \ln\left[(h - h_i) / (h - 1.3)\right], \quad x_2 = \left(\ln\left[(h - h_i) / (h - 1.3)\right]\right)^2$

and $y = d_i / d$.

Example 6.3 The Riemer–Gadow model was applied to a single tree from the previous dataset. The deviations between the observed and estimated upper diameters are shown in Figure 6-3.
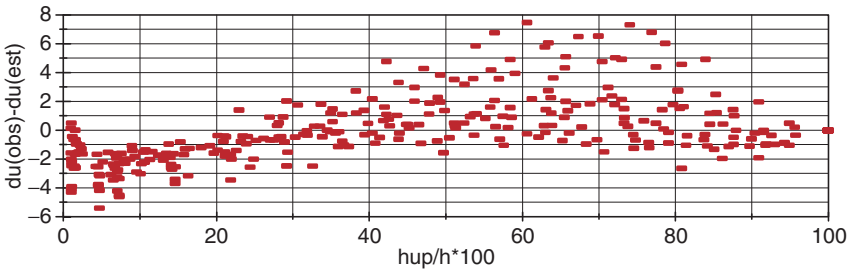
Figure 6-2. Residuals for modified Ormerod model.
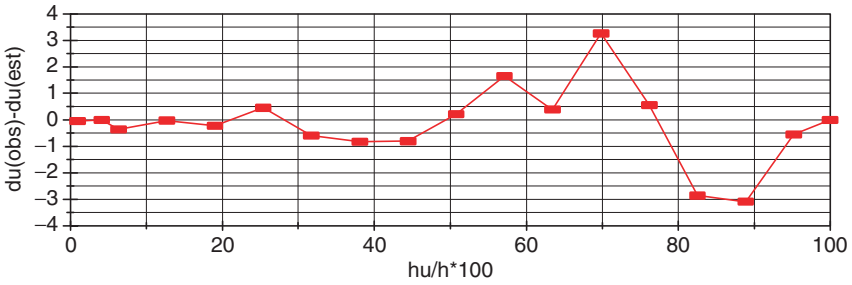
Figure 6-3. Fitted Riemer–Gadow model.
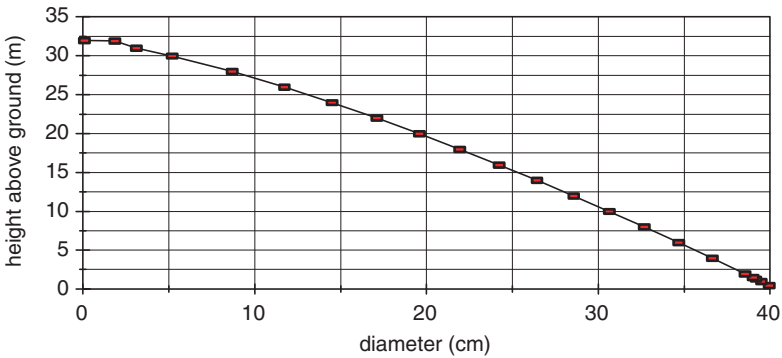
Figure 6-4. Stem profile based on Demaerschalk's 1973 model.

Example 6.4   The Demaerschalk (1973) model was fitted to the data of Appendix B. The regression coefficients were

$$b_1 = -0.262 \ b_2 = 0.8628 \ b_3 = 0.7389 \ b_4 = -0.4809$$

with $R_2 = 0.946$. The stem profile for dbh $= 35$ and $h = 30$ is shown in Figure 6-4.

2.3   Polynomials and segmented polynomials

Because of the general shape of the stem, many researchers investigated the usefulness of polynomials. Madsen (1983) described the stem profile of *P. abies* with a fourth-degree polynomial as

$$x = \ln\left(\frac{h_u}{h} + 0.1\right) \text{ and } y = \frac{d_u}{d}$$

The estimates of the five parameters of the model were subsequently regressed on mean stand diameter and height. The intercept for example was a linear function of h^2, d^2, $1/h$ and $1/d$.

Goulding et al. (1976) developed compatible taper equations for *Pinus radiata*, based on a polynomial of the fifth degree as starting point

$$d_i{}^2 = \frac{v}{k \cdot h} \left(b_1 \cdot \left(\frac{l}{h}\right) + b_2 \cdot \left(\frac{l}{h}\right)^2 + \cdots + b_5 \cdot \left(\frac{l}{h}\right)^5 \right)$$

with $l = \dfrac{h - h_u}{h}$ and $k = \dfrac{\pi}{40000}$ (metric system)

To ensure compatibility, the following constraint is imposed

$$\sum_{i=1}^{5} \frac{b_i}{i+1} = 1$$

and the equation to be fitted by least squares was

$$\frac{d_i{}^2 kh}{v} - \frac{2l}{h} = b_2{}^* \left[3\left(\frac{l}{h}\right)^2 - \frac{2l}{h} \right] + b_3{}^* \left[4\left(\frac{l}{h}\right)^3 - \frac{2l}{h} \right]$$
$$+ b_4{}^* \left[5\left(\frac{l}{h}\right)^4 - \frac{2l}{h} \right] + b_5{}^* \left[6\left(\frac{l}{h}\right)^5 - \frac{2l}{h} \right]$$

The coefficients of the two equations are related:

$$b_1 = 2\left(1 - (b_2{}^* + b_3{}^* + b_4{}^* + b_5{}^*)\right), \; b_2 = 3b_2{}^*, \; b_3 = 4b_3{}^*, \; b_4 = 5b_4{}^*,$$
$$b_5 = 6b_5{}^*$$

Example 6.5 The Goulding model was fitted to the previously used dataset. The deviations between observed and estimated upper-stem diameters as a function of h_u/h are shown in Figure 6-5.

Gordon (1983) modified Goulding's stem profile equations and found that a superior fit was obtained by the addition of a higher order term to the model. The predictor variables z^{2}, z^5, and z^{16} where $z = (h - h_i)/h$, performed better than other combinations of predictor variables and produced negligible bias in the estimation of upper-stem diameters, although the amount of bias tended to increase slightly with increasing height above the base of the tree. Allen (1991) fitted a polynomial taper equation for Pinus caribaea, *which represented an improved and more flexible version of the model presented by Real et al. (1988). The latter was as follows:*

$$y = b_1 \left(z^3 - z^2\right) + b_2 \left(z^8 - z^2\right) + b_3 \left(z^{40} - z^2\right)$$
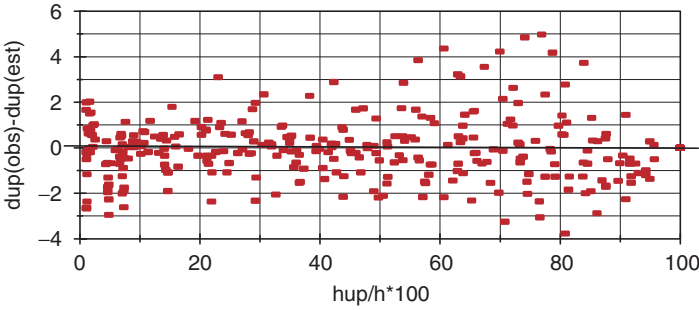
Figure 6-5. Residuals for fitted Goulding's stem profile equation.

where

$$z = \frac{h - h_u}{h - 1.3}, \; y = \frac{d_u}{d}$$

The modified model, introduced by Allen was as follows:

$$y = b_1 \cdot \left(z^{f(dbh,h)} - z^2 \right) + b_2 \cdot \left(z^5 - z^2 \right) + b_3 \cdot \left(z^8 - z^2 \right)$$
$$+ b_4 \cdot \left(z^{40} - z^2 \right)$$

and provided a superior fit in the upper section of the bole.

In general, a single taper equation fails to describe the stem profile in the lower- and upper-stem section accurately. The basic idea to fit polynomials to different segments of the bole was introduced by Max et al. (1976). Four models to describe the stem taper were compared. Model 1 was identical to Kozak's 1969 quadratic taper model, model 2 was based on the method of segmented polynomials with two quadratic functions and a single join, model 3 consisted of a quadratic model for the lower- as well as upper-stem section and a linear model for the middle section, model 4 used quadratic functions for all 3 sections.

$$y = b_1 \cdot z + b_2 \cdot z^2 - b_3 \cdot (z - a_1)^2_+ + b_4 \cdot (z - a_2)_+^2$$

with

$$z = 1 - \frac{h_i}{h}, \; y = \frac{d_i^2}{d^2}$$

and

$$(z - a_1)_+ = 1 \text{ if } z \geq a_1 \; and \; (z - a_1)_+ = 0, \text{ if not}$$
$$(z - a_2)_+ = 1 \text{ if } z \geq a_2 \; and \; (z - a_2)_+ = 0, \text{ if not}$$
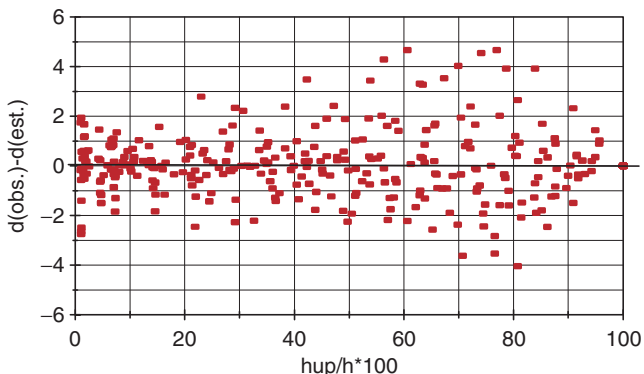
Figure 6-6. Residuals for fitted Max–Burkhart's model.

A subsample of the research data, not used for fitting, served to validate the model and thereafter, added to the dataset for refitting. The position of the joins may be fixed, in which case the equation has four paremeters, or they may be estimated by fitting the above equation as a six-parameter model. A practical example of fitting the four-parameter Max–Burkhart model was given in Chapter 2. The residuals for this model for the presently used dataset are shown in Figure 6-6.

Valenti (1986) applied the Max–Burkhart three-segment polynomial, rewrote the equation in the following modified form:

$$d_u{}^2 = d^2 \left(b_1 z + b_2 z^2 + b_3 (z - a_1)^2{}_+ + b_4 (z - a_2)^2{}_+ \right)$$

and introduced crown ratio (CR) as an additional variable. The resultant final equation with a single join was:

$$d^2{}_u = d^2 \left(\left(c_0 + \frac{c_1}{CR} \right) z + \left(c_2 + \frac{c_3}{CR} \right)^2 z^2 + c_4 (z - a_1)^2{}_+ \right)$$

with $(z - a_1) = 1$ *if* $(z - a_1) \geq 0$ *and* $(z - a_1) = 0$, *if not. The addition of crown ratio to the two-segment polynomial, however, produced biased estimates in the middle- and upper-stem section. Burkhart et al. (1985) incorporated crown ratio as an additional predictor into the three-segment taper equation but did not find evidence of the joins being related to crown ratio.*

Gordon (1983) fitted a taper function, which produced the equation

$$d_i{}^2 = \frac{v}{kh \left(b_1 z + b_2 z^2 + b_3 z^3 + b_4 z^p \right)}$$

with the restriction $5 < p < 41$. *The function was similar to the taper function developed by Goulding et al. (1976).*

Parresol et al. (1987) examined a segmented polynomial consisting of two joint cubic–cubic polynomials

$$\frac{d_i^2}{d} = z^2\,(b_1 + b_2 z) + (z - a)^2_+\,[b_3 + (z + 2a)]$$

where

$D = diameter\ at\ 10\%\ of\ the\ height$

$Z = ((h{-}h_u)/h)^2$

$(z{-}a)_+ = 1\ if\ z \geq a\ and$

$(z{-}a)_+ = 0\ if\ not$

Zakrewski et al. (1988) conditioned the parameter b_3 of the Max–Burkhart six-parameter segmented profile model, to ensure that the predicted and observed diameter at breast height is equal. The conditioned parameter is substituted back into the original equation and produced a six-parameter model with a transformed set of predictor variables, which simultaneously estimated the parameters of two submodels. A further refinement is to simultaneously condition the parameter b_1 of the Max–Burkhart model for dbh and its parameter b_2 for an upper-stem diameter measurement.

Saborowski et al. (1981) applied cubic splines with subsequent smoothing and joins at 1.3, 7 m, and the top of the tree. Tietze et al. (1979) applied locally fitted spline functions in the construction of stem profile functions. Czaplewski (1989) suggested scatter plots with the empirical rate of change of taper on the ordinate axis and the relative position of the point of measurement on the abscissa. This could serve, to locate the joins, which are required for segmented polynomials. Clark et al. (1991) introduced a segmented stem-profile model with dbh, diameter at 17.3 ft. and tree height as predictor variables. The stem was subdivided into 4 sections, the butt log extending to breast height, the lower stem located between breast height and 17.3 ft (Girard's form class), the middle stem between 17.3 ft and 40–70% of the height from the top and the upper stem located above this point. Different equations were fitted to these sections. The Max–Burkhart segmented approach was used for the sections of the middle and upper stem, with a linear segment for the upper section and a quadratic equation for the lower section. Conditioning ensured continuity at the joins.

Several authors explored the merits of principal components transformations. Newcomer et al. (1984) performed a principal components analysis on the following logtransformed-tree variables: dbh, $d_{5,24m}$, the largest diameter and a second diameter, at an angle of $90°$ to the previous one, height, height to live crown, and crown depth. The first principal component represented a compound size variable, whereas the second one was highly loaded by crown depth.

The minor principal component represented the logarithmic contrast between $d_{5,24m}$ and breast height diameter. Liu et al. (1978) applied principal components transformation to the diameter measurements at 14 positions along the stems of Pinus elliottii. *The first principal component explained more than 99% of the total variance. The elements of the eigenvector associated with the dominant eigenvalue, plotted over the corresponding position of the diameter variable, produced a curve which resembled the shape of the tree. A regression equation with various powers of the upper height was fitted and applied to predict taper. Real et al. (1989) performed principal components analysis on the stem profiles of felled Douglas fir trees and simulated stem profiles, representing a cone, a paraboloid, and a neiloid, respectively. The stem profile was described by the stem diameter at 3%, 5%, 10%, 20%, ..., 90% of the tree height. The resultant 11 variables were subjected to a principal components transformation, which showed that the first principal component represented 98% of the total variance within the group of simulated trees and 95.4% within the group of felled trees. It was interpreted as being a variable associated with the average tree form.*

Czaplewski et al. (1990) examined the amount of bias in profile models, associated with retransformations. An unbiased profile model, with the ratio d_i/dbh as the target variable, produced positively biased estimates of log volumes. For the same reason, the use of $(d_i/dbh)^2$ produced unbiased estimates for basal area, but biased estimates for taper. Stem diameters were underestimated by 0.2–2.1%. The amount of bias, obtained from the second term of a Taylor series expansion remained high, unless heteroscedasticity was properly accounted for. Bias, associated with retransformation of the target variable, was greatest near the position of the merchantable top diameter.

Chapter 7

TREE VOLUME TABLES AND EQUATIONS

1 INTRODUCTION

Tree volume tables, which give the average volume of single trees of given dimensions, have been used ever since the early 19th century. They provide estimates for:

- Either the stem volume or the tree volume, including branches
- Either the stem or total tree volume or the merchantable volume
- Either the over or the under bark volume

The early German tree volume tables estimated the total tree volume, i.e., the volume of bole and branches, above the traditional 7 cm diameter limit of merchantability. Most volume tables, however, exclude branch volume unless such tables are constructed to estimate the volume of trees utilized for the production of energy. The modern trend is to construct volume tables for the estimation of the total stem volume and to develop functions, which give the merchantable stem volume, usually for variable upper-diameter limits.

Volume tables and equations can be classified according to the number of entries to the table and predictor variables of the volume function:

- The single-entry volume table has its origin in the *méthode du contrôle*, which was developed towards the end of 19th century for the all-aged forests in France and adopted for the management of the mixed uneven-aged forests of Switzerland. The method preceded the modern concept of the *Continuous Forest Inventory*, and prescribed a complete stand enumeration. All trees above a fixed minimum diameter were marked at breast height and remeasured on successive occasions. The volume estimates were based on local or regional tables, which were denoted as *tariffs*, with dbh being used as the entry to the table. They were sometimes constructed for each main species separately, sometimes for species groups. The tariffs produced volume estimates in *sylve* units. Theoretically, the *sylve* is

equal to $1\,m^3$ roundwood, but when applied to specific stands, the stand volume expressed in *sylve* units may differ from the actual volume. The deviations were primarily due to site factors, which influenced the h/d ratio. To obtain more accurate estimates, these *tariffs* were gradually replaced by multiple single-entry tables, each of them characterized by a tariff number

- The standard volume table uses both dbh and height as table entries. The multispecies Bavarian tables were based on an excessively large number of field measurements of the volume of felled trees, which formed the basis of the tree volume tables constructed by Grundner and Schwappach (1942). Although being useful for forest inventories within large regions, specific silvicultural systems or sites differences within subregions prescribed the construction of similar tables for these subregions, for example, those produced by Zimmerle (1949) for Norway spruce and other species in the Black Forest. The two-entry volume table, although easy to apply in practice, assumes that the variability of the tree form factor is sufficiently explained by dbh and height

- Several studies, however, indicated that the addition of a third predictor variable, for example, height above ground of the base of the live crown (Nåsslund 1947) or stem diameter at 30% of the tree height (Pollanschütz 1965) or at a height of $7\,m$ (Schmidt et al. 1971) reduces the amount of unexplained variation and makes it possible to estimate the tree volume more accurately. Stem form studies in Finland showed that a single two-entry volume table produced biased volume estimates for Scots pine in North and South Finland, respectively. The incorporation of an upper-stem diameter and the resultant three-entry volume table was necessary to obtain unbiased volume estimates for each of these regions. Alternatively, two or more standard volume tables might have been constructed for Scots pine, for example, one for the northern and a second for the southern part of the country. Mensurational studies in Germany (Akça 1996), however, indicated that the additional measurement of an upper-stem diameter did not significantly improve the accuracy of volume estimates in hardwoods.

Tree volume tables are useful when no computer facilities are available, but volume functions are preferred because the estimated parameters can be stored in the memory of a computer and retrieved whenever necessary.

2 VOLUME EQUATIONS WITH ONE PREDICTOR VARIABLE

2.1 Simple tariff functions

The earliest equations for estimating the tree volume from dbh were developed in France, at the onset of this century. Algan's (1901) *tarif rapide* was based on a hypothesized relationship between tree volume, its dbh, and its assumed volume of a 45-year-old tree. Similarly *tarif lent* was proposed by Schaeffer (1949). The early tariffs, developed by Huffel (1919), were constructed with graphic methods. The more recent single-entry volume tables are invariably based on regression analysis with either log (dbh) as the independent and log (volume) as dependent variable or with volume as dependent and squared dbh as predictor variable. Regression analysis is preferred for several reasons. It eliminates the necessity to read off the estimated volume from a graph or to interpolate in a table, for a given model and fitting procedure, the results are unique and confidence limits can be calculated for the conditional population mean, i.e., for the mean volume, for given dbh. More importantly, the parameters of the equation can be stored in the memory of a computer and retrieved for volume calculations.
Meyer (1953) introduced an equation based on the model

$$\ln(v_i) = b_0 + b_1 \ln(d_i) + e_i$$

Ordinary unweighted least squares assumes that the residuals ε_i are independently and normally distributed. The model is also based on the condition of homoscedasticity, i.e., it assumes that $s_{ln(v)}^2$ is independent of the expected value of ln (v). It is generally accepted that s_v^2 increases with increasing tree size, but to a large extent, the logarithmic transformation eliminates heteroscedasticity. The logarithmic transformation, however, produces negatively biased estimates of the tree volume. *Baskerville's correction factor* (Chapter 8) can be applied to correct for bias.

The function used by Hummel (1955) and others, is based on the *Kopezky–Gehrhardt volume line* (Kopezky 1899), which assumes a linear relationship between tree cross-sectional area at breast height and stem volume. This is equivalent with the model

$$v = b_0 + b_1 d^2$$

with the same assumptions about the distribution of residuals. The function produces unbiased estimates of the stem volume, but the assumption

of homoscedasticity does not usually hold true and requires weighting, e.g., $w_i = 1/d_i$ or $w_i = 1/(d_i)^2$. Hoffmann (1982) used the following function:

$$v = e^{b_0 + b_1 \ln(d) + b_2 (\ln(d))_2}$$

to construct a *volume tariff table* for tree species in Switzerland. A non-linear algorithm was used to estimate the parameters, which minimized the sum of weighted squared deviations

$$\sum_{i=1}^{n} w_i \left(v_i - v_{i(\text{est})} \right)^2$$

Example 7.1   Tree volume data of *Eucalyptus grandis*, obtained by courtesy of the Institute for Commercial Forestry Research (ICFR), were used to test regression models for predicting tree volume of the over 5 cm diameter. Due to rounding-off errors, which occurred in some cases, dbh was expressed in units of 10 cm, height in units of 10 m. The following equations were tested to construct a tariff function with dbh as the predictor variable:

1. $\ln(v) = b_0 + b_1 \ln(d)$
2. $v = b_0 + b_1 d + b_2 d^2$
3. $v = b_0 + b_1 d + b_2 d^2$, $w_i = 1/(d_i^2)$

The resultant R^2-values were 0.943, 0.927 and 0.943, respectively. The means of the squared deviations between the observed and estimated volumes were 310.3, 310.4 and 301.4 for equations 1, 2 and 3, respectively. In consequence, there is no evidence of one model performing any better than the two others. The fitted regression curves for models (1) and (3) are shown in Figure 7-1.
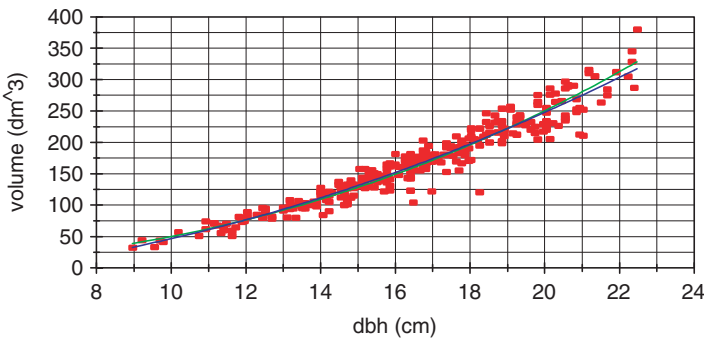
Figure 7-1. Fitted volume equation with a single predictor variable.

2.2   The incorporation of height into the tariff function

The single-entry volume table assumes that the parameters of the regression equation are not related to age, site, and stand treatment or genetic factors. It is generally accepted, however, that this simplification tends to produce biased estimates of the stand volume, whenever the tables or functions are applied to individual stands. Three methods, which retained the basic idea of tariffs were introduced to obtain more accurate volume estimates.

2.2.1   Hummel's method

Hummel (1955) constructed a set of volume lines, representing the relationship between basal area and stem volume for trees over 4 in. diameter:

$$v = b_0 + b_1 g$$

The common intercept on the abscissa was $0.087\,\text{ft}^2$. On this assumption, the volume-basal area equation was rewritten as follows:

$$v = b_1(g - 0.087)$$

The tariff number (*TN*) was defined as the volume (in cubicfeet) of a tree with a basal area of $1\,\text{ft}^2$.

$$TN = b_1(1 - 0,087)$$

Hence:

$$b_1 = \frac{TN}{0.913}$$

For a known tariff number, the tree volume is then given by:

$$v = \frac{TN}{0.913}(g - 0.087)$$

A field-sampling procedure was applied to decide which tariff to use in specific cases. Sample trees of known dbh were felled and their volume determined with one of the conventional formulae for sectionwise diameter measurements. After plotting the tree volumes over basal area, in a preprinted diagram, which showed the relationship between basal area and volume for the standard set of tariffs, the appropriate tariff number *TN* was determined for each sample tree. They were subsequently averaged to obtain the tariff to be used for a given population of trees. The procedure, although producing unbiased estimates, is obviously time-consuming and costly.

2.2.2 Stoffels' method

Stoffels (1953), who was influenced by earlier research in Germany, proposed a different approach, based on the allometric relationship between diameter and volume:

$$\ln(v) = b_0 + b_1 \ln(d)$$

It was assumed that b_1 is not influenced by stand characteristics; whereas b_0 is related to the quadratic mean diameter and the mean height of the stand:

$$\ln(b_0) = c_0 + c_1 \ln(v) + c_2 \ln\left(\overline{h}\right)$$

The parameter estimates were obtained by sampling. Random samples of n_1 trees were drawn from each of n_2 stands. A representative sample of n_1 trees in the each of the n_2 subsamples was felled and their volume determined with one of the conventional formulae, for example with the aid of Hohenadl's five-sections method, or otherwise. The logtransformed stem volume was regressed on log (dbh), for each subsample separately. A regression equation was fitted with the resultant n_2 parameter estimates of b_0 as dependent and mean diameter as well as mean height as independent variables. The common regression coefficient b_1 was estimated as the weighted mean of the n_2 estimates of b_1. The resultant equation was:

$$\ln(v_i) = c_0 + c_1 \ln\left(\overline{d}\right) + c_2 \ln\left(\overline{h}\right) + b_1 \ln(d_i)$$

Alternatively, an equation $\ln v = b_0 + b_1 \ln(d)$ with the constraint $b_1 = 2.21$ may be fitted to the data of each stand, with the resultant b_0-values being regressed on mean dbh and height. In order to apply this method, it is necessary to estimate the mean diameter and mean height of the stand. This method of stand volume estimation might be slightly more cost-efficient than the conventional method of measuring the heights of n randomly selected trees within a given stand and using a two-entry volume table. The advantage of the modified tariff method is to be found in the smaller sample, which is needed to estimate the mean height of the stand, since height measurements can be restricted to trees between the 40th and 60th percentile of the diameter distribution.

2.2.3 Brister's method

Brister et al. (1985) proposed a tariff system for loblolly pine, along lines similar to those followed by Stoffels. It used tree diameter to predict stem volume with the mean diameter and mean height of the dominants being introduced as covariates. The tariffs were linked with Hummel's volume-basal area line. Sample trees in 36 study stands of loblolly pine were felled and measured to

establish the volume-basal area line. The subsequent analysis did not indi-
cate the existence of a common x-intercept. Two volume models were examined.
Model 1 was:

$$v = ad^b \overline{h}^{-c}$$

where h_m expressed stand height. A non-linear least-squares algorithm, with
weights assigned inversely proportional to d^2, was applied to estimate the para-
meters. The parameters of the second model:

$$\ln(v) = b_0 + b_1 \ln(d) + b_2 \ln\left(\overline{h}\right)$$

were estimated by ordinary least squares. The next step was to fit the following
model to the data of each stand separately:

$$\ln(h) = c_0 + c_1 \ln(d) + c_2 \ln\left(\overline{h}\right) + c_3 \ln\left(\overline{d}\right)$$

It was substituted back into the previous equation and produced the tariff equa-
tion:

$$\ln(v) = d_0 + d_1 \ln(d) + d_2 \ln\left(\overline{h}\right) + d_3 \ln\left(\overline{d}\right)$$

For a basal area of $1 \, ft^2$, the volume is equal to the TN and with $h_t = $ corre-
sponding height

$$\ln(v) = c_0 + c_1 \ln(v) + c_2 \ln(TN).$$

The TN was obtained from the equation

$$TN = b_1 \overline{h}^{-b_2} \overline{d}^{b_3}$$

In order to construct volume tariffs for Picea mariana, *Ung (1990) carried out
stem analysis on 4 or 5 trees each in 26 localities. After first fitting an equation
based on the classical allometric relationship between dbh and volume, the
following more suitable prediction equation was introduced*

$$v_i = \left(b_0 + b_1 \frac{\overline{h}}{\overline{d}}\right) d_i^{b_2}$$

where $\overline{d}$ and $\overline{h}$ represent the mean diameter and mean height of the stand,
respectively, and $d_i =$ breast height diameter of the ith sample tree.

When several species, for example, a group of Eucalyptus species are
involved for constructing one or more volume functions, the usual procedure is
to apply ordinary least squares to each of them, although it might be possible
to combine some of them with the aid of dummy variables. In order to develop
a volume function for 3 species, the additive species effect could be accounted

for by two dummy variables: $Z_1 = Z_2 = 0$ *if species A, $Z_1 = 1$, $Z_2 = 0$ if species B and $Z_1 = 0$, $Z_2 = 1$ if species C.*

Alternatively, a specific model is used for all species involved and a Stein estimator is obtained for the parameters, which reduces the total sum of squared error and "shrinks" the parameter vector. Green et al. (1984) applied this method in the construction of volume functions based on the constant form factor function $v = b(d^2 h)$ for 18 hardwood species. Using available data material, the Stein rule, based on unweighted and weighted least squares was compared with ordinary and weighted least squares. The results were evaluated in terms of bias and precision of the estimated volumes and indicated that the weighted Stein rule was superior in terms of total squared error.

3   EQUATIONS WITH TWO PREDICTOR VARIABLES

3.1   Graphic methods

During the period which preceded the widespread application of computers, alignment charts were frequently used as a graphic tool to estimate the tree volume from dbh and height. These graphic substitutes for tables were proposed by Bruce (1919) and have the advantage of avoiding interpolation when two-entry volume tables are used. Because of the widespread availability of personal computers, alignment charts are obsolete.

The conventional standard tree volume tables are based on dbh and tree height as table entries. Graves (1914) proposed graphic techniques, similar to those applied by Grundner and Schwappach (1942) to construct two-entry tree volume tables:

- The sample trees were subdivided into height strata. A scatter diagram was prepared within each stratum, with stem volume being plotted over dbh or basal area.
- A free-hand curve for the non-linear relationship between volume and dbh or a free-hand straight line for the linear relationship between volume and basal area was drawn, with the constraint that the average deviation from the free-hand curve, within each of preselected domains of the predictor variable was zero.
- The resultant set of curves or lines was harmonized to remove inconsistencies and irregularities among the free-hand curves. This harmonization procedure remained largely subjective and the resultant set of curves was not uniquely defined.

These time-consuming graphic methods of construction were gradually abandoned in favor of regression methods, with parameter estimates being obtained from a multiple-regression equation to be fitted by ordinary or weighted least squares.

3.2   Regression equations

Schumacher et al. (1933) introduced the equation

$$v = b_0 d^{b_1} h^{b_2}$$

which was linearized by a logarithmic transformation of the dependent and predictor variables. The resultant equation

$$\ln(v) = a_0 + a_1 \ln(d) + a_2 \ln(h)$$

assumes that the residuals are normally distributed with constant variance. Similar to the equations with a single variable, the volume estimates are slightly biased because of the logarithmic transformation of the target variable. For this reason, some authors prefer to use a non-linear algorithm to estimate the parameters, together with some weighting procedure.

Other non-linear equations, some of them being linearizable, were proposed:
- *Deadman et al. (1979) modified the Schumacher–Hall volume equation, with ln(h) being replaced by $\ln(h^2/(h-1.4))$*
- *Lockow (1977) proposed equations with ln(v), as well as ln(form factor) as dependent and functions of ln(d) and ln(h) as predictor variables*
- *Scott (1981) introduced the 6-parameter equation*

$$v = b_0 + b_1 d^{b_2} + b_3 d^{b_4} h^{b_5}.$$

- *Ernst et al. (1984) compared functions to estimate stem volume. A model which used d^2, h, d^2 and dh^3 as predictor variables provided the best fit in terms of R^2 and mean-square residuals. Recovery, defined as the volume lumber produced, was estimated from the fitted non-linear equation*

$$\text{recovery} = b_1 h^{b_2} + b_3 d^{b_4} h^{b_5} + b_6 I \cdot d^{b_7} + b_8 I \cdot d^{b_9} h^{b_{10}}$$

with I = dummy variable for presence versus absence of defects
- *Böckmann et al. (1990) proposed a five-parameter non-linear model for predicting the total stem volume for Tilia species*

$$v = b_1 \left(b_2 d^2 h + b_3 dh + b_4 h \right)^{b_5}$$

A large number of models, which are linear in their parameters, have been proposed with d, d^2, h, h^2 and interaction terms as predictor variables, volume as the dependent variable.

Author	Predictor variables	Author	Predictor variables
Spurr (1952)	$d^2 d^2 h$	Nåslund (1940)	$d^2, d^2 h, dh^2, h^2$
Stoate (1945)	$d^2, h, d^2 h$	Dwight	$d \cdot h, h, d^2, d^2 h$
Quebec	$d, d^2, h, dh, d^2 h$	Berry	$d^2 h, (d^2 h)^2$
Bruce (1974)	$h, d/h, d, dh, 1/h$	Myburgh	$d^2 h, h, d, dh, d^2, d/h$
Tremblay (1976)	$d, h, dh, d^2 h$	Ernst (1984)	$d^2 h, d^2, d^3$
Barnard (1973)	$d^2 h$	Myers (1972)	$d, d^2, dh, d^2 h, d^2 h^2$
Murphy (1968)	$d, d^2, dh, d^2 h,$	Eriksson (1973)	(1) $d^2, dh, d^2 h, d^2 h^2, d^2 h, dh^2$
	$h^2, dh^2, d^2 h^2$		(2) $d^2, d^2 h, d^2 h^2, dh, dh^2$
			(*Pinus contorta*)
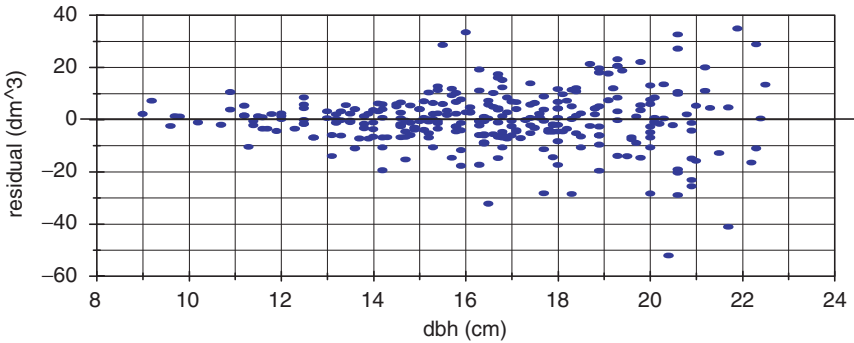
Figure 7-2. Relationship between residuals and dbh.

Example 7.2   A regression equation with d, d^2, h and $d^2 h$ as predictor variables and the volume of trees over 5 cm diameter as target variables is fitted to a sample of 303 tree volumes of *Eucalyptus grandis*. Figure 7-2 shows the deviations between the observed and estimated volume, plotted over dbh.

There is no evidence of a trend within the observed range of diameters, which indicates that the function being used produces unbiased estimates of tree volume. Figure 7-3, however, confirms heteroscedasticity and this in turn justifies weighting inversely proportional to the observed variance.
The histogram of studentized residuals (Figure 7-4), based on unit weight being assigned, reveals a symmetric but leptocurtic distribution, i.e., the degree of peakedness exceeds that of the normal distribution.

Example 7.3   The data in Example 7.1 are used to test the performance of models with dbh, height and other variables generated by dbh and height, such as quadratic terms and interactions. The forward, backward, stepwise and R^2 procedures were used to select the best subset of variables from a model with
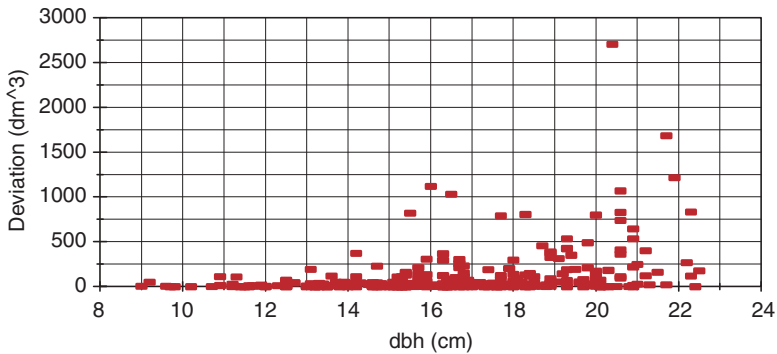
Figure 7-3. Relationship between squared residuals and dbh.
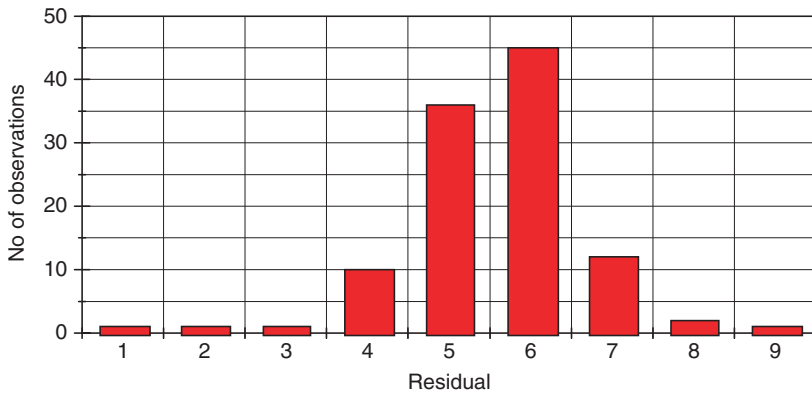
Figure 7-4. Distribution of residuals.

the following potential predictor variables: $d, h, d^2, h^2, d^2h, dh^2, d^2h^2, d/h$. The following variables were retained at the 0.05 level of significance:

1. Forward d, d^2, d^2h^2
2. Backward $d, d^2, d^2h, d/h$
3. Stepwise d, d^2, d^2h^2

All three R^2 values were 0.972. The mean-squared deviation between observed and estimated volumes was 115.4, 114.7, and 115.4, respectively. The RSQUARE screening procedure produced as "best" four-variable model an equation with d, d^2, h and d^2h as independent variables, with $R^2 = 0.973$. The partial regression coefficients were affected by weighting inversely proportional to d^2, but weighting did not improve the overall fit. The model with $\ln(d)$ and $\ln(h)$ as the independent and $\ln(v)$ as the dependent variable produced $R^2 = 0.979$, whereas the mean of the squared deviations was 119.9.

These evaluations of Example 7.3 show that variability is an inherent characteristic of the population, for which a volume equation is to be constructed. Using dbh as a predictor variable produced $R^2 = 0.943$. It increased to 0.973 by including height as an additional predictor. The different selection procedures produced equations, which performed almost equally well. Even the model which used all potential predictor variables, with $R^2 = 0.973$, did not perform better than models with fewer predictor variables. In this specific case, it is doubtful whether it was justified to use more than two independent variables. The "best" model with two predictors used h^2 and dh^2 to predict stem volume, with $R^2 = 0.972$. Models with few independent variables are usually preferred above others and the additional advantage of using a model with few predictor variables is the suppression of variance inflation. The regression model *with* d, d^2, h and d^2, for example, produced a correlation matrix with the highest and lowest eigenvalue being 4.896 and 0.0000421, respectively. The ratio of these eigenvalues is 116294, which indicates severe multicollinearity and variance inflation. This in turn produces large confidence intervals for the partial regression coefficients, although this does not necessarily imply that poor estimates are obtained with such equations.

The results of the evaluation are different when other criteria, for example, Mallows' CP index is used as selection criterion. In terms of CP, the model with four predictor variables (h, d^2h, dh^2, and d^2h^2) was the best performer with CP = 5.0.

In some cases (Spurr 1952; Barnard 1973), the combined variable d^2h has been used to estimate tree volume. Fitting the regression line through the origin implies a constant form factor. The equation which includes the intercept parameter provides a greater flexibility and makes provision for a tree-size-dependent form factor. In order to illustrate the implications, we fit the equation $v = b_0 + b_1(d^2h)$, with unit weight, as well as $w_i = 1/(d_i^2)$, respectively. The results are as follows:

Unit weight: $v = 5.884 + 29.326\ d^2h$

Weighted: $v = 3.913 + 29.712\ d^2h$.

Many of these volume functions use tree diameter, tree height and transformation variables as predictor variables. An alternative method, although rarely applied in practice, is the equation proposed by Sadiq et al. (1983) to estimate the tree volume from dbh and age. The model used the interaction variable $age \cdot d^3$ and $age \cdot d^4$ as predictor variables.

In those cases where tree volume tables are constructed for a large number of tree species, the question arises whether and how several species can be combined and the data pooled to obtain a single equation for a given group of species. When few species are involved, dummy variables might be introduced

to represent these species. Leech et al. (1991) introduced a novel approach and applied principal coordinate analysis as an aid for aggregating tree species in the construction of volume tables in Myanmar. Although 110 tree species were involved, the core group consisted of 27 species. For each species, a weighted multiple regression analysis was carried out with dbh and higher powers of dbh as independent variables. This produced 11 prediction equations. In order to assign the other 83 species to one of these groups, a distance measure was defined and used in principal coordinate analysis.

Morton et al. (1990) compared the following functions for the construction of two-entry volume tables: Honer's 1965 weighted volume function with $w = 1/(d^2h)^2$, Schumacher's 1933 unweighted logarithmic equation, Spurr's unweighted 1952 equation with d^2h as predictor variable, Spurr's weighted 1952 equation with $w = 1/(d^2h)^2$, the Quebec unweighted equation with d, d^2, h, dh and d^2h as predictor variables, and its weighted version with $w = 1/(d^2h)^2$. It was found that Schumacher's model and the Quebec equations performed marginally better than the others. Roebbelen et al. (1981) compared Dwight's modified equation with d, h, d^2 and d^2h as independent variables to Berry's equation based on d^2h and $(d^2h)^2$ as predictors and the product-form equation with P^2 and $d.P$ as independent variables ($P = decrease$ in diameter per unit increase in height above ground). Freese's test criterion, which was used to evaluate the models, indicated that the product-form equation outperformed the other models. Bruce et al. (1974) used a model with $d, 1/h^2, d/h^2$ and d/h to predict the form factor of second-growth Douglas fir, if $h > 3.5\,m$.

Stoate (1945) proposed the function

$$v = b_0 + b_1 g + b_2 h + b_3 gh$$

where $g = tree$ cross-sectional area at breast height. The equation is usually referred to as the Australian equation and is more flexible than Barnard's model. Equivalently, the following equation could be used:

$$v = b_0 + b_1 d^2 + b_2 h + b_3 d^2 h$$

4   EQUATIONS WITH MORE THAN TWO PREDICTOR VARIABLES

Volume equations, with dbh and height as predictor variables, assume that the form of the tree is sufficiently controlled by diameter and height. In order to obtain more accurate tree volume estimates, various authors proposed the inclusion of a third variable, for example, an upper-stem diameter.

- *Smalley (1973) proposed an equation with $d, d^2, d^3, h, dh, d^2h, h/d,$ $ln(d^2h), h_c$ and $(h - h_c)/h$ as predictors, where $h_c = height$ above ground of the base of the live crown.*
- *Eriksson (1973) developed a volume equation for alder, with $d^2, d^2h, d^2h_c,$ dh, h and d^3 as independent variables.*
- *Pollanschütz (1965) introduced a model for estimating the form factor of Norway spruce:*

$$F = b_0 + b_1 \frac{d_{0.3h}}{d} + b_2 \frac{H}{d^2}$$

The model simultaneously removed heteroscedasticity and eliminated bias associated with the use of two-entry tables on specific sites or within specific regions. Multiplying both sides of the equation by d^2h produces the prediction equation for stem volume.
- *The Swiss National Continuous Forest Inventory, based on the regular remeasurement of permanent sample plots, uses volume functions with dbh, height and d_7 as independent variables.*
- *Rustagi et al. (1990) introduced the model*

$$v = b_1 + b_2 d^2h + b_3 d^2 h_{0.67 \cdot d}$$

which incorporates the height at which the diameter is two-thirds of the breast height diameter as an additional and useful predictor variable. To some extent, it takes cognizance of the variability of the stem form of trees with a given dbh and height. In a follow-up study, Rustagi et al. (1991) compared the performance of tree volume equations with dbh and an upper height, defined as that relative height where the diameter is equal to a fixed fraction of breast height diameter as the predictor variable. A comparison between equations based on the fractions 0.5, 0.67 and 0.75 indicated that those models which used $h_{0.5\,d.}, h_{0.67d}$ and $h_{0.75\,d}$ performed better than other models.
- *Wagner (1982) proposed a volume function with logtransformed volume as dependent, logtransformed dbh, height and diameter at 30% of the tree height as predictor variables.*
- *Nåslund (1947) developed the tree volume equation*

$$v = b_1 d^2 + b_2 d^2h + b_3 dh^2 + b_4 d^2h_c + b_5 dh\,BT$$

where $h_c = height$ above ground at the base of the live crown and $BT = bark$ thickness.

- *Hann (1987) examined the suitability of the following equations in predicting stem volume from dbh, height and crown ratio:*

$$\frac{v}{d^2h} = \frac{b_0}{d^2h} + b_1 e^{b_2 \frac{CR}{h}}$$

$$\ln(v) = b_1 + b_2 \ln(H) + b_3 \ln(D) + b_4 \left(\frac{CR}{h}\right)$$

Example 7.4 Tree volume data were used to fit volume equations with the upper-stem diameter at one-third of the tree height as an additional predictor variable. The following models were tested with a stepwise screening of variables:

$$\ln(v) = f(\ln(d), \ln(h), \ln(d_{33\%}))$$
$$v = f(d, h, d_{33\%}, d^2, h^2, d_{33\%}^2, dh, d^2h, dh^2, d.d_{33\%}, h.d_{33\%}).$$

The resultant logarithmic equation was:

$$\ln(v) = -10.249 + 1.2475 \ln(d) + 0.64353 \ln(h) + 0.64353 \ln(d_{33\%}).$$

The linear model produced the following equation:

$$v = 0.476 - 0.0210\, h - 0.057038\, d_{33\%} + 0.0028137\, h.d_{33\%}$$

It is of interest to note that dbh was non-significant in the presence of the more influential upper-stem diameter at one-third of the tree height. The logarithmic volume equation based on dbh and height as predictors was:

$$\ln(v) = -10.387 + 2.0089 \ln(d) + 1.04156 \ln(h)$$

5 MERCHANTABLE VOLUME

For forest managers, information about the merchantable stand volume is of greater importance than total volume. The early German tree volume tables were based on a fixed minimum upper diameter of 7 cm over bark. The modern trend, however, is to construct tables and functions for a variable upper diameter.

- In a volume study on Douglas fir, Hann et al. (1987) distinguished between the stem volume above and below breast height. The volume of the stem section below breast height was obtained with a conventional formula, whereas

the following prediction equation performed satisfactorily for the above breast height stem section:

$$\frac{v}{d^2h} = b_1 e^{b_2(CR/h)} \left(\frac{h}{d}\right)^{b_3}$$

The prediction equation is a weighted regression model, with $w_i = 1/(d_i^2 h)$.
- Burkhart (1977) and other authors used the equation

$$\frac{v - v_m}{v} = b_0 d_m^{b_1} d^{b_2}$$

to estimate the stem volume up to a specific upper diameter d_m. The equation was linearized to estimate its parameters by ordinary least squares.

Example 7.5   The Burkhart model was applied in estimating the merchantable volume ratio as a function of dbh and upper diameter with the 10 cm limit being used as the cut-off point (see Figure 7-5).

- Newberry et al. (1989) developed equations to estimate the merchantable volume over the ratio to total volume, based on geometric solids. Parameter-free, as well as parameterized equations with differently defined basal diameters, were proposed. The parameterized equation based on dbh as basal diameter was:

$$R = 1 - b_0 \left(\frac{d_i}{d}\right)^{b_2} \left(\frac{h - h_i}{h - h_{st}}\right)^{b_2}$$

where h_{st} = height of stump diameter, d_i = diameter limit for merchantability, h_i = corresponding height above ground and $(h - h_{st})$ = total height minus stump height.
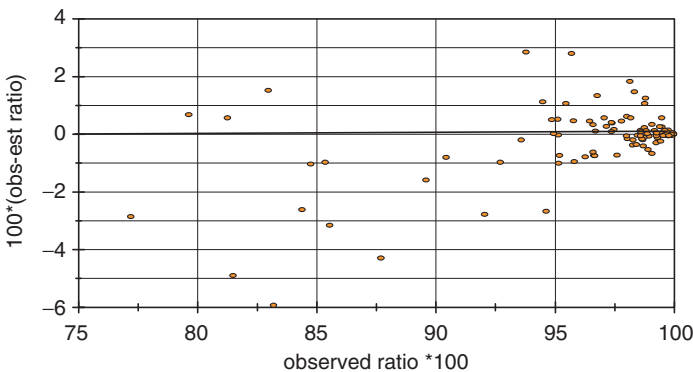
Figure 7-5. Residuals for Burkhart equation fitted to *Pinus patula*.

- Cao et al. (1980) evaluated two approaches to estimate the merchantable volume for a given upper diameter or height, one being obtained from volume ratio models, the second by integrating taper equations to obtain the volume of stem segments. In addition to existing volume ratio models and taper functions, new models were introduced, tested and compared in terms of their performance in estimating merchantable volumes and taper. Bias, mean absolute difference and standard deviation of the differences were used as criteria. The Max–Burkhart model was superior for estimating taper, but the model is not compatible. A segmented compatible polynomial model

$$d^2 k\, h/v - 2z = b_1(3z^2 - 2z) + b_2(z - a_1)^2 I_1 + b_3(z - a_2)^2 I_2$$

with $z = (h - h_u)/h$, $I = 1$, if $z < a_i$, $I = 0$, if not and $k = 0.00007854$ (English system) performed satisfactorily in estimating the merchantable volume for a specified upper-stem diameter. CAO also tested the following modified Burkhart model:

$$\frac{v_m}{v} = 1 + b_1 \frac{(h - h_u)^{b_2}}{h^{b_3}}$$

The model performed well in estimating the merchantable volume for specified merchantable heights.

- Amateis et al. (1986) developed a ratio approach to predict merchantable volume per unit area total yield (V_t), mean diameter (d_q), number of surviving trees (N), merchantable top diameter (d_u) and threshold dbh limit ($d_{thr.}$):

$$V_m = V_t e^{b_1(d_u/d_q)^{b_2} + b_3 N^{b_4}\left(\frac{d_{thr.}}{d_q}\right)^{b_5}}$$

In order to provide an estimate of total yield, a model was used with log yield as target variable and those generated by age, top height, trees per unit area and basal area as predictors. The prediction model was improved by the incorporation of a sub-model to estimate the number of trees per hectare above a given merchantability limit from the total number of trees, basal area and (d_u/d):

$$N_{mer.} = N_{tot.} e^{-b_1 G^{b_2}\left(\frac{d_{mer.}}{d}\right)^{b_3}}$$

- Alemdag (1988) fitted a number of equations, using either a relative merchantable diameter (group 1) or a relative merchantable height (group 2), as the predictor variable to estimate the merchantable volume ratio K. In group 1 the equation

$$K = 1 + b_1 \left(\frac{d_u}{d}\right)^{b_2}$$

performed slightly better than other models tested, with R^2 being equal to 0.907. The equation

$$K = e^{\left(b_1\left(1-\frac{h_u}{h}\right)^{b_2}\right)}$$

represented the best model in group 2, with R^2 being equal to 0.976. The corresponding compatible taper equation, which performed best was:

$$d_u = d\left(\frac{e^{c_1\left(1-\frac{h_u}{h}\right)^{c_2}} - 1}{b_1}\right)^{1/b_2}$$

The coefficients of these equations were derived from fitted equations, with K as the dependent and either d_u/d or h_u/h as the predictor variable.

- Turnbull et al. (1965) proposed an equation with a single predictor variable to estimate the ratio of the total to the merchantable volume:

$$\frac{v_t}{v_m} = b_1 + b_2 e^{-b_3 d}$$

Example 7.6   The Turnbull equation was applied to the previously used data set. The resultant equation was

$$\frac{v_t}{v_m} = b_1 + b_2 e^{-b_3 d} \text{ (see Figure 7-6).}$$

- Strub et al. (1986) developed a model to predict the merchantability of individual loblolly pine trees of given dbh with stand age, mean height of the dominants and trees per hectare as predictor variables. The proportion of merchantable trees within the selected plots, being zero for dbh < 7.55 in.,
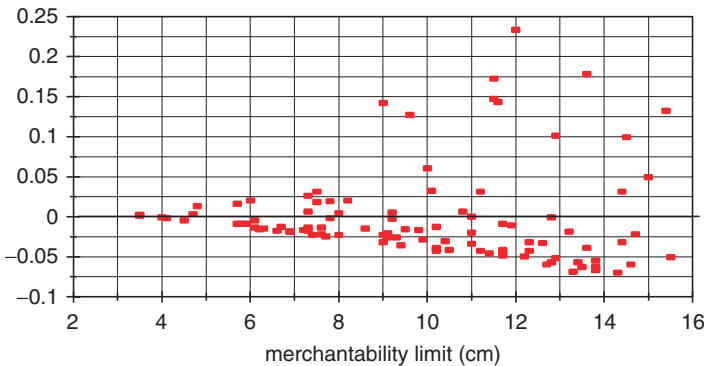
Figure 7-6. Residuals for fitted Turnbull equation.

increases exponentially with dbh and has an upper asymptote of 1. The resultant prediction equation is:

$$y = 1 - e^{a(7.55-d)} \quad for \quad d \ \rangle \ 7.55 \text{ in.}$$

$$y = 0 \quad\quad\quad\quad for \quad d \ s \ 7.55 \text{ in.}$$

where y = probability that a tree with a diameter d_i is merchantable and b = parameter of the equation. Further investigations produced the three-parameter logistic model

$$y = \frac{\left(1 - e^{\beta_1(7.55-d)}\right)}{1 + e^{\beta_2 + \beta_3 h}}$$

for $d > 7.55$ in. $= 0$ and for $d < 7.55$ in.

- Honer (1967) proposed a second-degree equation to estimate the merchantable volume ratio v_m/v

$$\frac{v_m}{v} = b_0 + b_1 x + b_2 x^2$$

where

$$x = \frac{d_u}{d_{st}} \left(1 - \frac{h_{st}}{h}\right)$$

with d_u = upper diameter, d_{st} = stump diameter, h_{st} = stump height

- Matney et al. (1982) developed equations to estimate the merchantable volume ratios and merchantable heights, assuming that the total volume for a given dbh and height is known. Various powers of the ratio of merchantable top diameter to dbh, merchantable ratio of merchantable to total volume as a function of tree height, total volume and dbh were used as predictor variables.

- Mctague et al. (1987) developed and tested equations to simultaneously estimate the total and merchantable volume of loblolly pine as a function of dbh, height and minimum diameter for merchantability:

$$v_m = a_0 d^{a_1} h^{a_2} + a_3 \left(\frac{2d_m^4}{d^2} - \frac{d_m^5}{d^3}\right) \cdot (h - 1, 3) + a_4 \left(\frac{2d_m^4}{d^2} + \frac{d_m^5}{d^3} - \frac{d_m^3}{d}\right)$$
$$\cdot (h - 1, 3)$$

The function predicts total stem volume for $d_{mer} = 0$. The coefficients of the equation were estimated with the restriction

$$a_4 = - \left(3 \cdot 0,0000785 + \frac{7a_3}{8}\right)$$

to ensure that the taper function, which is compatible with the volume function predicts a merchantable height of 1.3 m when the upper-stem diameter d_m is equal to the breast height diameter.

Chapter 8

TREE AND STAND BIOMASS

1   INTRODUCTION

Until recently, forest mensuration has emphasized the estimation of the total and utilizable volume rather than weight, partly because timber is usually sold on a volume basis and partly because the volume of standing trees can be estimated more easily than their weight. In many countries and regions there is an increasing need to express the productivity of forests in terms of weight, more particularly in those plantation forests which are managed for the production of pulpwood and mining timber or when by-products, for example bark for the production of tannins, are involved. A similar situation arises when trees are planted or natural forests are managed to produce wood for energy, since mass rather than volume is a yardstick to quantify the production of wood for energy. Other reasons for the increased interest in forest biomass, initiated in the early 1960s, was the necessity to measure biological productivity in terms of dry weight of the organic matter, and the oil crisis, which induced a greater emphasis on the utilization of wood as a renewable natural resource.

Because of these developments, the need arose to develop sampling methods and to construct functions and tables which give the estimated oven-dry biomass of trees as a function either of dbh, or dbh as well as height. Whenever the tree is converted into cellulose products, the *oven-dry weight* of its merchantable part expresses its value more adequately then its green weight. For this reason *biomass tables* were constructed, which give the average oven-dry instead of *green weight* as a function of one or more than one tree characteristic.

The first extensive sampling studies to estimate the weight of the above-ground tree components were carried out much earlier, primarily because of the necessity to measure the biological productivity of tree species. These studies, undertaken in *Pinus strobus* in Switzerland (Burger 1929), were followed by similar studies in *Larix decidua* (Burger 1945) and *Picea abies* (Burger 1953). Concurrently, ecologists and physiologists became increasingly interested in

this research direction and made a contribution towards the development of more efficient sampling methods to estimate the quantity of foliage, either expressed in terms of their oven-dry mass or in terms of the *leaf surface area* per hectare. The latter was converted into the leaf-area index, defined as the ratio of leaf-surface area over ground area. Studies conducted by Kittredge (1944) and Ovington (1957) emphasized their importance within the framework of ecological research. Studies in Danish beech forests, conducted by Møller (1946), which were preceded by those carried out by Boysen–Jensen (1932), were primarily undertaken to establish the relationship between degree, as well as kind of thinning and yield in terms of mass instead of volume. They had a considerable impact on thinning research in Europe and the USA. The studies were also conducive for the initiation of sampling studies for the construction of biomass tables and functions.

An evaluation of the extensive world literature about this subject shows that the majority of the sampling studies estimate the oven-dry weight of each biomass component separately. In many of these studies, *root biomass* is ignored, probably because of the prohibitively high cost of estimating root biomass sufficiently accurately. The estimation of biomass components in relation to tree size requires that they are oven-dried separately. The drying temperature is normally around 70° C, but in *Pinus radiata*, Forrest (1968) recorded a 2% weight loss by increasing the temperature from 70° C to 105° C. In *Picea mariana* a weight loss of 3% was observed by increasing the drying temperature from 65° C to 103° C (Barney et al. 1978).

2   BIOMASS COMPONENTS

The total aboveground green weight of young trees is conveniently measured by felling sample trees and weighing the entire tree. This method, which was applied by Young et al. (1976), ensures that no sampling errors are involved to determine the green weight of the entire tree. This is feasible for young trees but prohibitively expensive for mature trees. Methods were therefore developed to estimate biomass by sampling. The following notes serve as a general guideline for sampling. In practical situations it is usually necessary to modify the proposed procedure.

2.1   Branches

Two-stage sampling is an efficient method to estimate the branchwood weight of the single tree. The diameter at the base of the branch (preferably at a

fixed distance of 4 or 5 cm from the main stem) is measured on all branches. A subsample of branches is drawn at random to estimate either the green or the oven-dry branchwood weight of the single branch. The observed weights are subsequently regressed on branch diameter or branch basal area.

In biomass studies in Pinus radiata, *branch length was a statistically significant additional predictor variable but the moderate increase in R^2 did not justify its inclusion into the prediction equation (van Laar 1973). Other researchers included branch position to improve the prediction model.*

The estimated weights per tree are subsequently regressed on dbh or on dbh as well as height to obtain a regression estimator for this biomass component for the entire stand. A direct measurement of the oven-dry weight eliminates the necessity to convert the green weight into the oven-dry weight. For practical reasons it is sometimes preferred to measure the branchwood weight as green weight. In that case, a subsample of branches of known green weight is drawn and oven-dried to estimate the ratio oven-dry over green weight. In *P. radiata* this ratio was about 0.45 for live and 0.80 for dead branches (Satoo 1982). In consequence, the conversion of green into oven-dry weight should be carried out for live and dead branches separately.

2.2 Foliage

In young trees all leaves or needles are removed from the tree, dried and weighed, but sampling is required when mature trees are involved. The needles or leaves, for example, from a 25% random sample of branches, are removed and their green or oven-dry weight is determined and recorded for each branch separately. For reasons of cost-efficiency the same branches will be used for measuring the branch-wood and foliage weight. The foliage weight may be measured as green weight, in which case subsampling is required to convert green weight into oven-dry weight. In regions with hot dry summers and wet winters, the ratio green weight to oven-dry weight shows a seasonal trend. Ratio estimates obtained during summer cannot be used when sampling continues during winter. The regression model used to estimate the branch-wood weight from branch diameter can also be used to estimate leaf weight.

2.3 Stemwood weight

Sampling usually proceeds in two stages. In stage 1 the volume of the felled sample tree is determined by measuring the diameter at the midpoint of 1 m sections, with Smalian's formula being used to obtain the volume of each stem section. This produces an estimate of the stem volume, although it may be

negatively biased if too few stem sections are used. In stage 2 wood discs of a predetermined thickness are extracted, preferably at the midpoint or else at the thin end of each stem section. Since the disc volume is proportional to the volume of the stem section, this method implies self-weighting and their volume, as well as green weight is determined. A subsample is drawn for drying and to convert green weight into oven-dry weight. The use of a single-conversion factor erroneously assumes that the dry weight over green weight ratio is unrelated to the position within the stem. This simplification could produce biased estimates, since the ratio tends to decrease with increasing height above the base of the tree. In many regions this ratio might also reveal a seasonal trend.

2.4 Bark weight

The previously described sampling procedure is repeated to estimate the oven-dry bark weight. It can be combined with the measurement of stemwood weight by measuring the bark thickness of each stem disc from four directions under an angle of 90°, or by removing the bark from each disc to measure its weight. Alternatively bark-thickness functions, such as those constructed by Deetlefs (1957), can be used to estimate the bark volume which is subsequently multiplied by a single oven-dry/green-weight ratio. This method will be applied if prediction errors associated with bark thickness equations are negligible.

2.5 Root weight

The estimation of the weight of roots requires a complete excavation of the root system, which in practice is virtually impossible. Some kind of subsampling may be necessary to estimate the weight of the root sections which remain in the soil. The green weight of all roots together is determined and a subsample is drawn to convert green weight into dry weight.

The above sampling methods serve as general guidelines but are modified in specific situations. In a biomass study in Pinus radiata, *carried out by Forrest (1969), the breast height diameter and height of all trees within a given sample plot were measured. After grouping the trees in five-diameter classes, two trees were selected at random from each class. The oven-dry weight of each bole was determined in its entirety. Discs were cut and their bark removed to estimate the ratio bark to total weight. Foliage and branch weight were obtained by drying and weighing all branches and needles. This was done for each age stratum separately. Spank (1982) conducted sampling studies to estimate the crown and needle biomass of* Pinus sylvestris. *Samples were taken to estimate the needle biomass from the biomass of the branchlets. The crown and branchlet*

biomass per tree were regressed on tree diameter and tree height, with a logarithmic transformation of the dependent and predictor variables. Ranasinghe et al. (1991) conducted biomass studies on two sites in Eucalyptus camaldulensis *plantations of different ages. The mean tree within each sample plot was felled to determine the green weight of the leaves, large branches, bole, bark, and roots. Subsamples were dried at* 70° *during 48 h to estimate the conversion factor from green to oven-dry weight.*

2.6 Pooling of data

The construction of general biomass tables and equations requires a large sample representing the entire range of tree sizes, ages, sites, and silvicultural treatments. It is feasible to rationalize the previously described sampling procedures by pooling some observations obtained from single trees. In order to estimate the ratio oven-dry/green weight of branchwood, foliage, stem wood, and bark, for example, it can be expected that the between-trees and within-trees variability of these ratios, within a given stand, might be of the same order of magnitude. Pooling the ratios observed for single trees seems justified but might produce slightly biased estimates of the biomass of trees in different stands, for example because of the effect of site. When regressing tree biomass estimates on dbh and other tree characteristics, the assumption of uncorrelated residuals will then be violated, although not necessarily seriously.

2.7 Randomized branch and importance sampling

In order to eliminate the necessity to determine the weight of the entire tree, Valentine et al. (1984) compared *randomized branch sampling* (RBS) with the closely related *importance sampling* (IS), which is based on an earlier concept, introduced by Jessen (1955), to obtain estimates for the aboveground biomass of individual trees. RBS selects a path, which is defined as a series of connected branch segments or internodes. Within the context of RBS a branch is defined as the entire stem system which develops either from a terminal or from a lateral bud. A segment is defined as a part of a branch between two successive internodes. In RBS no distinction is made between main stem and branch. The first segment extends between the base of the tree and the first node. At this point the diameter and length of the two branches (the diameter of the branch and that of the main stem) are measured. A selection probability which is proportional to d^2l is calculated. with l being the length of the segment between the node and the tip of the branch and main stem respectively. If the two selection probabilities are 0.30 and 0.70 respectively and a random number is drawn which

is greater than 0.70, the second branch is selected. Its associated probability of selection is equal to 0.70. When three branches (segments) occur at the first node, with selection probabilities of 0.20, 0.35, and 0.45 and a random number equal of 0.60 is drawn, the second branch will be selected with a selection probability of 0.35, since 0.60 > (0.20 + 0.35). After having selected the third, and possibly last branch, for example with a conditional probability of 0.45, the latter is weighed and the unconditional probability of selection is obtained by multiplying conditional probabilities 1*0.70*0.35*0.45 = 0.1103. The weight is divided by 1/0.1103 to obtain the estimated weight of the entire tree.

Importance sampling is a related method which eliminates the necessity to determine the weight of heavy segments of a path. Weighing is limited to a single stem disc within each path. The procedure starts with the measurement of the diameter at various points along the bole. These are used to estimate the timber volume of the path. A random number is drawn from a uniform distribution which extends between 0 and 1. In order to determine the position of sampling it is multiplied by the estimated volume. The weight is determined and converted into weight per unit thickness of the disc. This value is multiplied by an expansion factor to obtain the estimated total weight.

3   TREE-LEVEL REGRESSION MODELS

Several models have been proposed and used to estimate the aboveground biomass components and the total tree biomass from tree characteristics. Many models are based on the assumed allometric relation between biomass (y) and the tree or branch characteristic (x)

$$\ln y = b_0 + b_1 \ln x$$

Allometry deals with the relationship between the relationship between the growth rates of two organs of an individual. The assumption that the relative growth rate of x is a fixed proportion of that of y can be stated as follows:

$$\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}t} = k \cdot \frac{1}{x} \frac{\mathrm{d}x}{\mathrm{d}t}$$

where k represents a proportionality coefficient. The terms of the equation can be rearranged as follows:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y}{x} \cdot k \cdot \frac{\mathrm{d}x}{\mathrm{d}t}$$

which represents a differential equation. It has been shown (Batscheler 1975) that the solution of this equation is:

$$y = c.x^k$$

or

$$\ln y = \ln c + k \ln x$$

Although the allometric model produces a satisfactory fit, the resultant estimates are biased. Several formulae have been proposed to correct for bias. Baskerville (1972) approximated the regression estimate corrected for bias as follows:

$$y_{\text{adj.}} = e^{\ln b_o + b_1 \ln x + MS_{\text{error}}/2}$$

where MS_{error} = mean square for error, obtained from the regression analysis with the transformed variables. Using Baskerville's correction for bias, Wiant et al. (1979) estimated bias as follows:

$$\text{Bias} = \frac{e^{MS_{\text{error}}/2 - 1}}{e^{MS_{\text{error}}/2}}$$

A more complex correction formula was presented by Finney (1941). Yandle (1981) compared Finney's estimator with Baskerville's function. For small samples and a large-error mean square, Baskerville's function performed better than Finney's estimator. This was confirmed in similar studies carried out by Lee (1982). Snowdon (1991) proposed to multiply the estimate obtained from the log–log regression of biomass on dbh by a correction factor C, obtained from the observed and estimated biomass of the n sampling units. In a later simulation study Snowdon (1992) compared this method with the ratio of mean methods, based on basal area as auxiliary variable. This was done for simple random sampling, for sampling with selection probability proportional to size (PPS) and for sampling with selection PPS for the first sampling unit being drawn and at random for the others. The estimates obtained by combining PPS sampling with the simple ratio method of adjustment produced highly biased results.

One disadvantage of the log–log model is that the sum of the estimated biomass components differs from the regression estimate obtained from the equation with total biomass as dependent variable. A second problem is the estimation of confidence intervals. Meyer (1944) suggested to adjust the variance formula

$$s_y^2 = e^{y^2 + MS_{\text{error}}/2}$$

and to use the corrected variance for obtaining confidence intervals for the estimated biomass.

Campbell et al. (1985) examined the allometric relationship between tree biomass and breast height diameter (model 1) and that between tree biomass and the combined variable d^2h (model 2). An analysis of covariance

for model 1, with region as additional variable and dbh as covariate, did not indicate significant differences amongst intercepts but those between slopes were significant, whereas the analysis based on model 2 disclosed significant differences amongst slopes. It was suggested that the d/h ratio might be a useful additional variable for predicting the biomass of trees growing in different regions, but the study did not disclose which of the two models performed better.

Ruark et al. (1987) introduced the concept VAR (variable allometric relationship) for estimating the tree biomass from a tree-size characteristic x. It was hypothesized that the allometric ratio b_1 of the equation

$$y = b_0 x^{b_1}$$

is a function of x. The proposed equation

$$y = b_0 x^{b_1} e^{b_2 x}$$

is linearizable and produces the equation

$$\ln y = c_0 + c_1 \ln x + c_2 x$$

The model performed satisfactorily if the relationship between x and y differs for different ages of the organism. For some biomass components it produced a significantly better fit than the constant allometric model. Geron et al. (1988) tested the usefulness of Ruark's variable allometric ratios for predicting the foliar biomass of different tree species. In Populus tremuloides, *it compensated the negative bias associated with the constant allometric ratio model, which overpredicted foliage biomass in the lower and upper ends of the diameter distribution. In a study with a similar objective, Crow (1980) compared the performance of the allometric model $\ln y = b_0 + b_1 \ln x$ with others, for example:*

$$w = b_0 d^{b_1}$$
$$w = b_0 d^{b_1} h^{b_2}$$

combined with different weighting functions. The weighted nonlinear equations and the allometric model performed equally well. Clark (1990) predicted biomass components of planted southern pines from models which used either d^2 or $d^2 h$, or $d^2 h_4$ ($h_4 = $ height to 4 in. top) or sawlog merchantable height as predictor and log(biomass) as target variable. Schlaegel (1982) estimated biomass components of Acer negundo *from fitted regression equations with $d^2 h$ as predictor variable. In order to ensure homoscedasticity, the data were weighted inversely proportional to $d^2 h$. Payandeh (1981) discussed regression models for biomass prediction equations*

$$W = b_0 + b_1 d \tag{1}$$

$$W = b_0 d^{b_1} \tag{2}$$

$$W = e^{b_0 + b_1 \ln d} \tag{3}$$

$$W = b_0 + b_1 d^2 + b_2 h + b_3 d^2 h \tag{4}$$

$$W = b_0 d^{b_1} h^{b_2} \tag{5}$$

The performance of the models was based on an index of fit, which is identical with R^2 in models with no constraints on the parameter b_0. The models 1 and 2 performed equally well and were superior to the others. In biomass studies, based on a sample of size 38, which were carried out in the beech forests of Romania, Decei (1981) used a second-degree equation to estimate various aboveground biomass components as well as root biomass from breast height diameter.

Landis et al. (1975) regressed bole, stem bark, branchwood, and foliage biomass, as well as the total aboveground biomass on breast height diameter. The equation $W = b_0 + b_1 d^2$ was preferred before those with logtransformed-dependent and predictor variables, because of nonnormality of the distribution of residuals, generated by this transformation. Mitchell et al. (1981) developed regression equations with stem, branch, and foliage dry weight of conifers as target variables and dbh as predictor, after weighting the sampling observations proportional to the inverse of dbh. Apparently there was no evidence of a nonlinear relationship between dbh and weight. In South African Eucalyptus plantations, Schönau et al. (1981) regressed the biomass components on the squared dbh.

Alemdag et al. (1981) tested the performance of four equations for estimating stemwood, stem bark, live branches, twigs, leaves, and the total biomass of some hardwood species. One model used d^2, h and d^2h as regressors, and contained an intercept, a second equation used the same predictor variables but with zero-intercept, a third function was based on d^2h as predictor (with intercept), and the last one represented the concurrent zero-intercept model. The first and last model performed equally well in terms of R^2.

Madgwick (1984) investigated the amount of bias when estimating the aboveground biomass per unit area with the basal area ratio method, the unweighted regression of biomass on squared diameter approach and the log–log regression of biomass on dbh. Five different adjustment factors to correct estimates, based on the allometric model tended to produce biased estimates. The best results were obtained with the basal area ratio method, which is a

ratio of means estimator, requiring the multiplication of the observed biomass of the n sample trees by the ratio of plot basal area over sample trees basal area.

Snowdon (1986) conducted simulation studies to devise a sampling strategy for estimating the biomass of crown components of individual trees of Pinus radiata. *The efficiency of simple random sampling was compared with various versions of stratified random sampling and ratio estimators. Not surprisingly, stratified random sampling produced better results than simple random sampling. In a subsequent regression analysis branch position and branch diameter were used as predictor variables and branch-wood biomass as target variable. Both independent variables were entered as a linear, quadratic, and cubic term, together with the linear interaction term.*

4   ADDITIVITY OF BIOMASS COMPONENTS

Kozak (1970) proposed to fit identical models for estimating the total tree biomass and biomass components, in order to ensure additivity of biomass components, which implies that the sum of these estimates is equal to that obtained from the total-biomass equation. However, the question arises whether different subsets of independent variables should be used to estimate different biomass components. Krumlik (1974) compared several models for predicting biomass components. Linear models with d^2h as predictor variable were suitable to estimate stem biomass, $d^2h*crown\ width$ to estimate the biomass of big branches, basal area was more suitable to predict bark biomass, $d*crown\ length$ for estimating the twig and foliage biomass. Models based on the same predictor-variables, but with a logtransformed-dependent variable, were also tested and performed satisfactorily.

Example 8.1   Regression equations were fitted to data from a biomass study in *P. radiata* (Forrest 1969). The equations to predict total biomass are shown in Table 8-1.

The adjusted R^2-values based on stepwise elimination of variables and the corresponding values of $\Sigma(w - w_{est})^2$ and the associated ranks are given in Table 8-2.

The two criteria being used produce different ranks. Model (2) is superior in terms of R^2 but is of rank 3 when expressed in terms of total squared error. Models (3) and (4) with d and d^2 as predictors perform well in terms of sum of squared deviations but correspond with ranks 3 and 4 when expressed in terms of R^2. Models (5) and (6) are the poorest performers independently of

Table 8-1. Regression models for predicting biomass in *P. radiata*

Eq.	Dependent variable	Predictor variable	Weight
(1)	ln(mass)	ln(d)	unit weight
(2)	Predictor	ln(d), ln(h)	unit weight
(3)	mass	d, d^2	unit weight
(4)	mass	d, d^2	$w_i = 1/d^2$
(5)	mass	d^2h	unit weight
(6)	mass	d^2h	$w_i = 1/d^2$

Table 8-2. Regression statistics and ranks for predicting total biomass

Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank	Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank
(1)	0.978	2	33.18	4	(4)	0.977	3	26.59	1
(2)	0.979	1	30.64	3	(5)	0.909	6	59.66	5
(3)	0.960	4	26.62	2	(6)	0.949	5	60.78	6

Table 8-3. Regression statistics for predicting stem biomass

Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank	Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank
(1)	0.954	2	14.84	6	(4)	0.950	3	9.01	3
(2)	0.968	1	10.52	5	(5)	0.899	6	8.65	2
(3)	0.905	5	8.19	1	(6)	0.949	4	9.97	4

the criterium being used. Weighting produces the best results when using R^2 as criterium, but not necessarily when the performance of the model is based on total squared error.

4.1   Stem biomass

The same data set was also used to compare and test the same set of regressor variables. The results are given in Table 8-3.

The criterium being used has a considerable impact on the ranking results. Eq. (1) was the best performer when based on R^2 but performed poorly when total squared error was used as quality criterium. The weighted models (4) and (6) were superior to unweighted equations, when based on R^2 but the opposite was true when total squared error was used as criterium for goodness of fit.

4.2   Branch and needle biomass

The previous models were applied to estimate the sum of branch and leaf biomass. The results are shown in Table 8-4.

Models (5) and (6) were the poorest performers, independently of the criterium being used. Eqs. (1) and (2) were superior in terms of R^2 but Eq. (1) did not perform well in terms of total squared error. The weighted models (4) and (6) were superior to unweighted models (3) and (5), when based on R^2 but not when based on total squared error was used as criterium.

4.3   Root biomass

The regression statistics for predicting root biomass are given in Table 8-5.

The evaluation of the root biomass models reveals the same inconsistency as that observed for other biomass components and shows similar trends. Eqs. (4) and (6) are superior to Eqs. (3) and (5) when assessed in terms of R^2 but weighting had no influence on the total squared error. Model (6) was the poorest performer in terms of R^2 but the best model when based on total squared error.

The criteria R^2 and total-squared error present an overall evaluation of the goodness of fit. Additional information about the performance of the models is obtained by calculating the observed and estimated biomass within the lower, middle, and upper range of diameters. In the present case the calculations were carried out for the trees below the 10th and above the 90th percentile of the diameter distribution and for 10 trees around the median. In the present case

Table 8-4. Regression statistics for predicting the sum of branch and needle biomass

Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank	Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank
(1)	0.944	2	13.61	4	(4)	0.941	3	13.55	3
(2)	0.946	1	13.08	1	(5)	0.807	6	30.35	5
(3)	0.916	4	13.40	2	(6)	0.879	5	30.64	6

Table 8-5. Regression statistics for predicting root biomass

Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank	Eq.	R^2	Rank	$\Sigma(w - w_{est})^2$	Rank
(1)	0.919	3	2.91	4	(4)	0.922	2	2.79	2
(2)	0.923	1	2.80	3	(5)	0.829	6	3.47	5
(3)	0.866	5	2.75	1	(6)	0.904	4	3.48	6

these calculations were carried out for each of the four biomass components, with estimates being obtained from the equations

$$\text{total biomass} = \exp(a_0 + a_1 \ln(d) + a_2 \ln(h)) \tag{1}$$

$$\text{total biomass} = b_0 + b_1 d + b_2 d^2 (\text{weights proportional to } 1/d^2) \tag{2}$$

$$\text{stem biomass} = \exp(a_0 + a_1 \ln(d) + a_2 \ln(h)) \tag{3}$$

$$\text{stem biomass} = b_0 + b_1 d + b_2 d^2 (\text{weights proportional to } 1/d^2) \tag{4}$$

$$\text{branches} + \text{foliage} = \exp(a_0 + a_1 \ln(d) + a_2 \ln(h)) \tag{5}$$

$$\text{branches} + \text{foliage} = b_0 + b_1 d + b_2 d^2 (\text{weights proportional to } 1/d^2) \tag{6}$$

$$\text{root biomass} = \exp(a_0 + a_1 \ln(d) + a_2 \ln(h)) \tag{7}$$

$$\text{root biomass} = b_0 + b_1 d + b_2 d^2 (\text{weighting proportional to } 1/d^2) \tag{8}$$

The results are given in Table 8-6.

In addition to the above 6 models (Example 8.1), an equation was tested with ln(*biomass*) as well as ln(*stem biomass*), and ln(*leaves* + *branches*) as dependent, ln(*d*), ln(*h*), and ln(*crown volume*) as predictors. To predict total biomass, crown volume was significant in presence of ln(*d*), but ln(*h*) was non-significant. R^2 however increased only marginally from 0.979 to 0.980 when ln(*h*) was replaced by ln(*crown volume*). The addition of crown volume did not improve the accuracy of the prediction of stem biomass and that of branch + leaf biomass and in both cases dropped out as predictor variable. The estimation of ln(*root biomass*) from ln(*d*), ln(*h*) and ln(*CL*) produced an R^2 of 0.928. In this case crown dimension improved the precision of root-biomass estimates.

Additivity

In order to illustrate nonadditivity of biomass estimates for the logarithmic model equations were fitted with ln(*biomass*) as dependent and ln(*d*) as well as

Table 8-6. Estimates in the lower, middle, and upper ranges

		Biomass component			
		Total	Stem	Fol. + branches	Root
Middle range	Eq. (1)	96.7	106.5	106.9	94.5
	Eq. (2)	98.3	103.0	105.9	97.2
Upper range	Eq. (1)	117.6	107.6	109.7	116.6
	Eq. (2)	94.3	96.7	98.6	99.7
Lower range	Eq. (1)	101.6	94.2	97.1	99.6
	Eq. (2)	102.6	104.7	44.1	43.4

Table 8-7. Regression coefficients for equations to predict the biomass components stem, leaves, and branches

Component	Logarithmic model (1)			Linear model (2)	
	b_0	b_1	b_2	c_0	c_1
Stems	−3.598	2.156	0.5147	3.758	0.01206
Leaves	−4.968	2.542	0.2265	−0.056	0.00575
Branches	−4.686	3.148	−0.4425	−0.812	0.00975

$\ln(h)$ predictor variables and biomass as dependent and d^2h as predictor variable. The calculations were carried out for stem, foliage, and for live branches as dependent variables. The parameter estimates are given in Table 8-7.

5   DUMMY-VARIABLES FOR TREE SPECIES

Jacobs et al. (1980) developed prediction equations for ten tree species. In order to harmonize tree biomass tables dummy-variables were introduced which represented biomass components, based on stem-biomass categories demarcated by specified top diameters. The regression analysis did not give evidence of nonparalellism, but the relationship between the intercept parameter and top diameter could be expressed by a second-degree equation. Crow et al. (1980) applied a two-variable linear model, with a logtransformed-dependent variable and a logtransformed (d^2h) as predictor variable, to estimate the biomass of tropical forest trees and found that habitat as dummy-variable was not influential in presence of dbh as predictor variable. In sampling studies conducted by Brown et al. (1989) however, equations for estimating the total aboveground biomass of tropical trees were developed, with a model using dbh and d^2 as predictor variables. This model was used for trees from the dry life zone whereas an equation with $\ln(d^2h^*SG)$ with SG = specific gravity for trees was applied to those growing in the dry life zone.

The objective of many sampling studies has been to estimate biomass components such as branch weight and needle weight. Hepp et al. (1982) developed branch, tree, and stand level equations to estimate the branch and needle weight of loblolly pine. The branch model was

$$\ln W_{br} = b_0 + b_1 \ln d_i + b_2 \ln h \cdot h_i + b_3 \ln A$$

where h_i = branch height is aboveground, d_i = branch basal diameter, h = tree height and A = age. The tree model was

$$\ln W_{cr} = b_0 + b_1 d + b_2 \ln CR$$

where W_{cr} = crown weight and CR = crown ratio. The stand level model was

$$\ln W = b_0 + b_1 A + b_2 \ln G$$

where W = weight per hectare and G = basal area per hectare

Madgwick et al. (1974) applied the following equation for predicting branch weight

$$\ln W = b_0 + b_1 \ln d^2 l + b_2 RH + b_3 RH^2$$

with l = branch length, RH = relative height above ground of the sample branch, W = branch weight. Branch weight estimates in different crown sections, however, were not unbiased. Ek (1979) proposed a model for estimating branch and foliage weight as a function of branch diameter at 25 mm from the stem. Starting point for the model development was the allometric relationship between branch diameter and the target variable. Several models which took cognizance of the effect of height aboveground level of the ith branch and of spacing, the latter either expressed in terms of distance between the trees or by the height over dbh ratio, were tested for their predictive ability. The relative height aboveground was less influential than spacing in predicting branch biomass, but the variable which expressed branch position was significant in presence of the spacing variable. The following model was more suitable to estimate foliage mass

$$y = b_1 d_i^{b_2} (h - h_i)^{b_3}$$

but the addition of a spacing variable served no useful purpose. The analysis suggested the possibility of pooling data from stands of different ages and spacings. Piene (1983) established a close linear relationship between needle length and weight per 1000 needles, as well as between-needle density (expressed as the number of needles per centimeter shoot length) as dependent and the reciprocal of needle length as predictor variable. The relevant regression equations were combined to estimate the needle biomass for a branch with estimation of needle biomass required the measurement of shoot lengths, needle length, and needle density. The author determined sampling positions within the shoot and within the crown, which should produce unbiased estimates.

Several studies were undertaken to compare the efficiency of different sampling methods. Attiwill et al. (1968) compared four methods to estimate the biomass per unit area

- From the average of four weight measurements of trees with a dbh equal to the mean stand diameter.
- From the average weight of four trees in each of five dbh classes, to be multiplied by the stem number in these classes.
- From weight estimates obtained from regression equations based on a sample of size 20.
- From the expected weight of the tree with the mean dbh, to be estimated from the regression equation and to be multiplied by the number of trees.

Since the third method was assumed to produce the most accurate estimates, those based on the others were compared with this method. The negative bias was -8.5%, -3.1%, and -11.6% for the first, second, and fourth method respectively. Williams et al. (1991) investigated the relationship between site index and biomass production within four drainage classes in even-aged spruce–fir stands in Maine. Stand age and site index were used as predictor variables for biomass production per hectare. The regression model was

$$y = b_0 + b_1 A + b_2 SI \cdot A^2 + b_3 SI \cdot A^3$$

It was found however that site index was a poor predictor for biomass yield.

6   RATIO ESTIMATORS AND CLUSTER SAMPLING

The purpose of the previously described sampling procedures is to obtain the most accurate estimate of the biomass components at the lowest possible cost. The ultimate goal of sampling for biomass, however, is to obtain estimates per unit area. If a random sample of n trees is selected from a population containing N trees, the sample mean, i.e., the mean of the n biomass estimates, multiplied by the ratio N/n, represents un unbiased estimate of the population total. This assumes that no measurement bias is involved in the determination of the biomass components and that sampling for biomass, within the selected sample trees, produces unbiased biomass estimates per sample tree. However, for given fixed cost, a more efficient estimate can be obtained by making use of ratio estimators (Chapter 10) than would be possible by simple random sampling. It requires the measurement of an auxiliary variable x, which can be measured quickly and cheaply. The assumption is that the relationship between the target variable y and this variable x can be expressed by the equation $y = b.x$, which represents a zero intercept regression equation and implies that $y = 0$ for $x = 0$. In order to be useful the population total for the ancillary variable must be known. For practical reasons the latter will be measured in sample plots with biomass being calculated for each plot. The biomass of the stand is obtained by multiplying the mean plot biomass by an expansion factor. Cunia

(1981) suggested the ancillary variable $x = d^2 h$. This means that all diameters within a given population should be measured and a height curve fitted to obtain the estimated heights for given dbh. Alternatively basal area, which is equivalent with squared dbh could be used as auxiliary variable. In both instances the assumption of a zero intercept of the regression line which reflects the relationship between basal area and biomass does not hold true, since tree biomass tends to a value which is equal to the expected biomass of a tree which is 1.3 m high as breast height basal area tends to zero.

In cluster sampling groups of trees, usually all those within a given sample plot, are selected at random, their biomass is determined either by a complete measurement or possibly by subsampling and the tree biomass (or biomass component) for the cluster total represents the subject variable. This method ensures that the residuals of the model are distributed independently. In biomass studies based on linear regression methods, Briggs et al. (1982) investigated the implications of cluster sampling. Because of heteroscedasticity ordinary least squares estimates were replaced by weighted least squares estimates and the dependent and predictor variables expressed the sample totals for each cluster. The efficiency of cluster sampling was investigated by Cunia et al. (1980, 1981). The ratio of means produced the best, the mean of ratios estimator the poorest estimates. Kotimaki et al. (1981) compared various auxiliary variables within the framework of cluster sampling to estimate biomass components. Snowdon (1992) conducted simulation studies in which three sampling strategies were involved

- Simple random sampling
- Sampling with probability of a given sample proportional to the sum of the sizes
- Sampling with probability of selecting each tree proportional to its size

These strategies were combined with two ratio estimators. RATIO1 was a ratio of means estimator, with cross-sectional area as auxiliary variable, RATIO2 defined the auxiliary variable as d^{b_1}, where b_1 represented the regression coefficient of the log–log regression of biomass on dbh. The results were evaluated in terms of precision as well as accuracy. RATIO1 tended to perform better when estimating total stem biomass, RATIO2 was superior to estimate other biomass components, but invariably PPS sampling outperformed other strategies in terms of precision as well as accuracy, usually in combination with RATIO2. The average bias was highest by combining the simple ratio estimator with PPS sampling.

Chapter 9

GROWTH AND YIELD

1 DEFINITIONS

Growth is a biological process, which applies to the organism in its entirety, including all tree components (stem, branches, roots, foliage). *Increment* expresses the observed growth of the organism during a given period of time and normally applies to the tree and stand variables diameter, basal area, height, volume, and biomass. Yield is defined as the harvested or harvestable accumulated increment per unit area.

The *current annual increment* is defined as that between year k and $k+1$, the *periodic annual increment* as the increment between year k and $k + a(a > 1)$, divided by a and the *total increment* at year k is defined as the sum of the annual increments. When applied to stands these estimates include the trees removed by intermediate fellings and those resulting from mortality.

The *mean annual volume increment* (MAI) of a stand is defined as total increment divided by age. We distinguish between

- $MAI_t = MAI$ at t years
- $MAI_{max} = $ maximum *MAI*,
- $MAI_{ra} = MAI$ at rotation age.

MAI_{100} and MAI_{max} are used in Germany and some other countries and are interpreted as the absolute site index.

In continuous inventories of commercial forests all trees within a given sampling unit are measured from a certain minimum diameter of a cm. In Germany this minimum diameter is 7 cm, in other cases it may be lower, for example 5 cm in plantations managed for pulpwood production. A minimum of 12–14 cm was applied in the early continuous forest inventory (*méthode du contrôle*) in Switzerland. In tropical nature forest the minimum diameter a is usually 10 cm. In that case, some trees thinner than a cm at occasion A will exceed this diameter at occasion B. The volume or biomass of these trees is called *ingrowth*. The stand growth estimate therefore is positively biased by ingrowth. The ingrowth

trees can be identified if the coordinates of all trees, measured at occasion A, are known. This is feasible in experimental plots, in continuous inventories with permanent plots and in regularly spaced plantation forests but in other cases this will be prohibitively expensive. The total increment should include fellings and mortality during the growth period. If not, the estimate is negatively biased.

2   THE GROWTH OF SINGLE TREES

2.1   Growth parameters

2.1.1   Diameter and basal area

The diameter growth of the single tree is usually calculated as the difference between the diameter measured at the beginning and the end of a given period:

$$i_d = d_2 - d_1$$

Both measurements are affected by measurement errors. When expressing the standard deviation as a percent of the actual increment, the rate of growth can therefore be expected to be estimated more accurately by extending the length of the period between successive measurements. The standard deviation of a diameter measurement is partially controlled by the instrument being used. It can be reduced by using a tape instead of calliper, by measuring the diameter in millimeter instead of centimeter, by measuring the diameter from two directions, under an angle of 90° to one another and by marking the point of measurement in such a way that it is identifiable when remeasuring the tree.

2.1.2   Height

The estimation of the height increment of a single tree, by repeated measurements, is inaccurate because of the relatively large measurement error. In young trees of a uninodal species, the measurement error can be reduced substantially by measuring the height increment during the past k years with the aid of telescopic poles. When trees are felled for complete stem analysis, it may be possible to measure the length of the annual shoot of uninodal conifers and that of poplars. When direct measurements of growth are not feasible, growth estimates should be obtained by modelling height growth or by sampling methods, which are combined with sampling procedures to estimate the volume per hectare.

Height growth may also be estimated by stem analysis, although the estimates tend to be biased. Suppose, for example, that stem discs are extracted at 1 m distances from the base of a 27-year-old tree with 15 and 14 annual rings

being counted at a height of 9 and 10 m, respectively. The actual tree height at 12 years in that case is between 9 and 10 m. Dyer (1987) compared the accuracy of different methods to estimate the true height at different ages. The following formula, proposed by Carmean (1972), produced the best estimates

$$h_{ij} = h_i + \frac{h_{i+1} - h_i}{2\,(r_i - r_{i+1})} + (j-1) \left[\frac{h_{i+1} - h_i}{r_i - r_{i+1}} \right]$$

where

h_i = height at the ith cross section

t_{ij} = age of the tree associated with jth inner ring at the ith cross section

$\quad = n - r_i + j$

h_{ij} = estimated height at age t_{ij}.

The method assumes that the annual height growth within a given stem section is constant and the crosscut – on the average – occurs in the middle of a given year's height growth. A different method to avoid or minimize this error in estimating height growth is to measure the length of internodes on felled trees (section 2.2).

2.1.3 Stem volume

Several methods may be used to estimate the volume growth of single trees. Stem analysis is an efficient, but time-consuming method to reconstruct the volume growth of the single tree since planting or germination and more generally during the past k years. An alternative method is the extraction of increment cores with an increment borer, to determine the diameter increment in centimeter or expressed as a percentage. The latter is doubled to obtain the approximate the basal area increment percentage. The unknown height increment percent is usually obtained from growth models or from yield tables and added to the basal area increment percent. In Switzerland, the volume of permanently marked sample trees is estimated from an equation with dbh, an upper stem diameter, and tree height as predictor variables.

2.2 Stem analysis

Stem analysis has been applied as a forest mensurational tool since the second half of 19th century. Its objective is to reconstruct the growth of individual trees by determining the annual diameter, basal area, height, and volume growth. Stem analysis involves the following measurements.

- The over-bark dbh and total tree height of selected and subsequently felled sample trees.
- The length of annual internodes on felled sample trees.
- Ring measurements on stem discs extracted at the base of the tree, at breast height, and at regularly spaced distances from the base of the tree. The number of rings on each stem disc is counted and the annual diameter increment measured accurately, usually from at least four directions. These observations are subsequently used to estimate the height of the tree at successive ages, the latter being obtained from ring counts on the extracted stem discs at different heights above the ground

The ring width measurements serve to estimate the annual diameter and basal area growth at breast height and at other stem positions, whereas ring counts are required to reconstruct the height development of the sample trees. The above information is subsequently summarized to obtain the estimated volume growth, the true form factor, and the *h/d* ratio.

Stem analyses is applied to estimate growth in temporary sample plots, which are measured once only. The height development of a given stand, for example, may be estimated by pooling the growth data of sample trees from the dominant crown stratum. Information about the estimated diameter and volume growth of the sample trees is used advantageously to test hypotheses about the effect of thinning, fertilizer, pruning, meteorological variables, and environmental stresses on tree growth. The observed changes in the *h/d* ratio may be useful to assess changes in the competitive status of the sample trees within the stand. In addition, stem analysis is used in growth studies of uneven-aged mixed stands (Kramer 1994).

Example 9.1   Species: *Picea abies*. Age: 48 years. After felling the sample tree, the length of internodes starting from the top of the tree was measured and ring counts were carried out at a height of 0.3 m, at breast height and at 2 m distances from breast height. Stem discs, with a thickness varying between 3 and 10 cm, were cut at the midpoint of each section and at the base of the tree. The tree number, compass direction of the points of measurement, aspects of the site and height above ground of each disc were recorded. The discs were planed to obtain more accurate estimates of the diameter growth. They were transported to the laboratory and ring widths were measured with a digital measuring instrument for annual ring widths, in this case with the *Digitalpositiometer*, designed by Johann (1977). The tree age was obtained by adding the estimated number of years which the tree requires to reach a height of 0.3 m, in the present example 4 years. Ring widths were measured at four positions at an angle of 90° to one another, with the first measuring point being positioned at an angle of 22.5° to the largest diameter (Siostrzonek 1958). The number of

rings at 0.3 m was 44, the estimated tree age 48 years. The rings were marked at age 45, and thereafter at 5-year intervals. In consequence, the resultant diameter measurements reflect the under-bark diameter at 48, 45, 40, 35, ... years, at 0.3, 1.3, 3.3, 5.3, ... m above the base of the tree (see Figure 9-1).
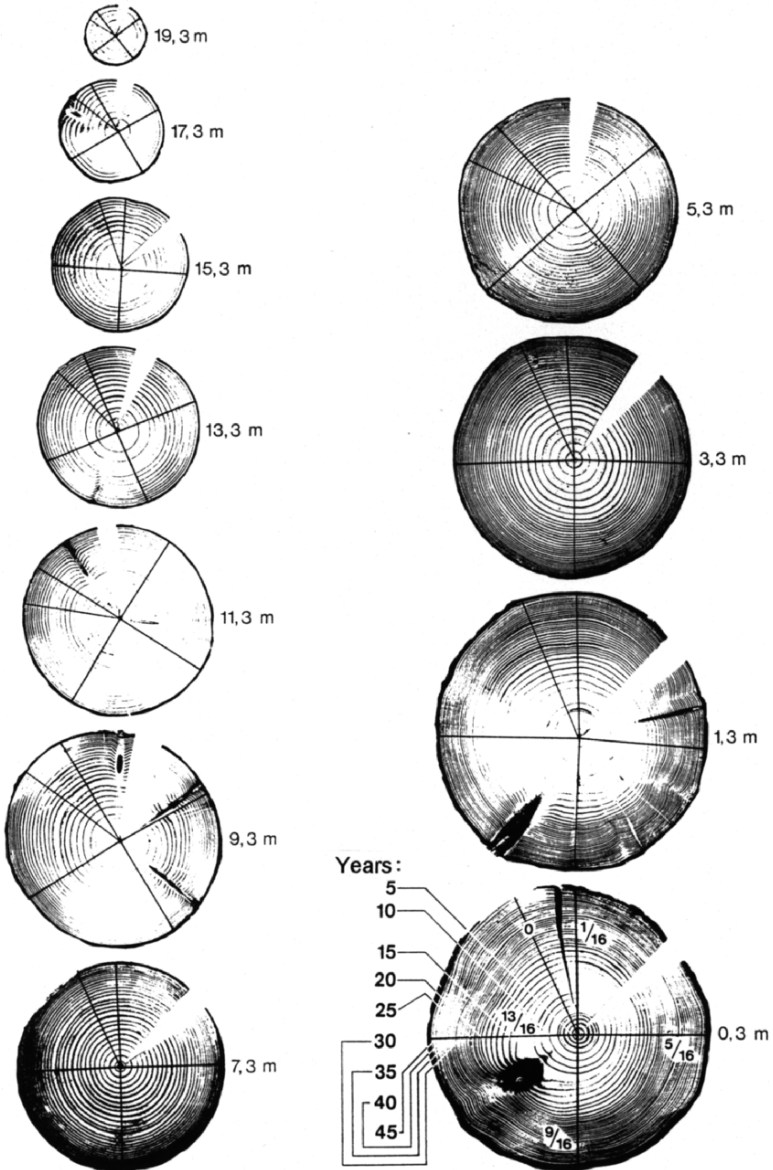
Figure 9-1. Stem discs of a 48-year old Norway spruce tree.

Table 9-1. Summary of diameter and height records

Disc No.	No. of rings	Disc height (m)	Under-bark diameter(cm) at age									
			5	10	15	20	25	30	35	40	45	48
1	44	0.3	0.8	4.10	8.81	12.44	14.05	15.56	17.10	18.45	19.64	19.94
2	40	1.3		1.30	7.44	11.54	13.41	14.95	16.30	17.44	18.37	18.57
3	38	3.3			4.76	9.20	11.35	13.33	14.83	15.93	16.75	16.90
4	35	5.3			1.47	6.55	9.26	11.75	13.56	14.83	15.70	15.80
5	31	7.3				2.55	6.19	9.49	12.40	13.72	14.75	14.85
6	28	9.3					3.50	7.47	10.57	12.60	13.85	14.35
7	24	11.3					0.59	4.84	8.48	11.10	12.72	13.20
8	20	13.3						1.23	4.81	8.32	10.61	11.30
9	15	15.3							1.12	5.36	8.47	9.55
10	11	17.3								1.93	5.77	7.30
11	7	19.3									2.31	4.35

Table 9-1 gives the mean diameter at different ages for discs extracted at 0.3, 1.3, … 19.3 m above the base of the tree and the number of rings counted at each position.

All relevant information about the diameter increment at different heights above the ground and the height developments of the sample tree were subsequently obtained. The under-bark volume at different ages is obtained from Table 9-1 and summarized in Figure 9-2.

$$v = 2\,(g_1 + g_2 + \cdots + g_{n-1}) + \frac{1}{3}g_n l_{\text{top}}$$

where $l = $ length of top section. The calculation of the volume of the top section is based on the assumption that the latter is a cone. The cross-sectional area of the cone base is located 1 m above the midpoint of the last 2 m section. The breast height form factor is calculated from the estimated stem volume, tree height, and basal area at breast height. The resultant estimates are found in Table 9-2. Much useful information can be obtained from such tables. The h/d ratio (with height expressed in meters, dbh in centimeters), for example, increased from 81 at 15 years, to 112 at 45 years.

The absolute and relative 5-year diameter increment (the latter expressed as a percent of the increment at breast height) is given in Table 9-3.

In many instances, missing rings occur within the ring series, either due to senescence or because of the effect of stresses (environmental pollution, pathogens) on radial growth. In order to eliminate this error source,
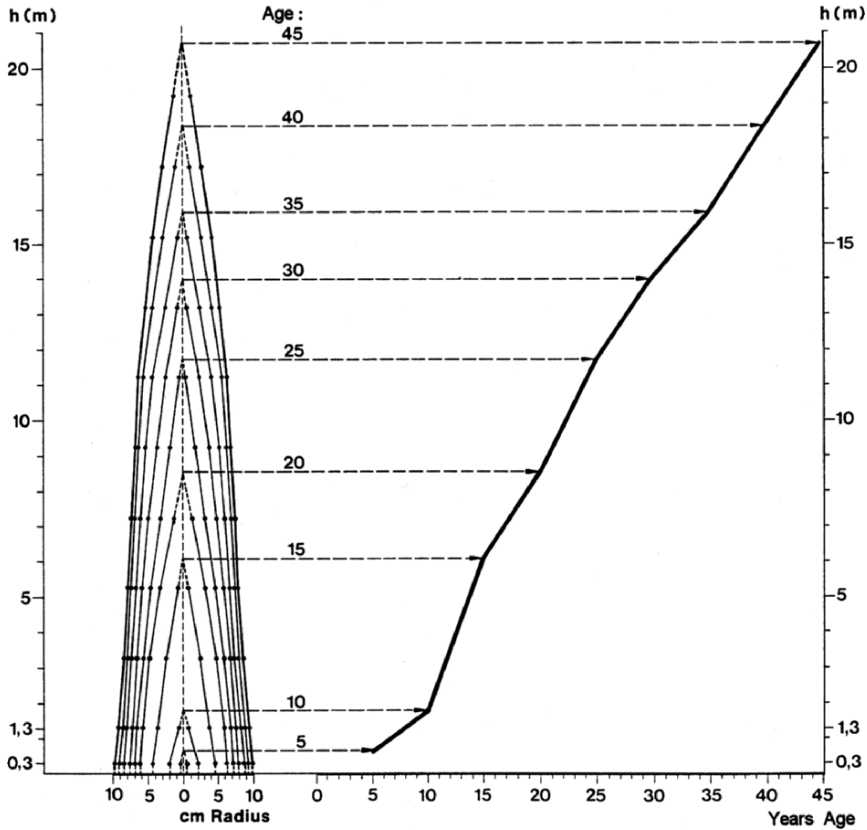
Figure 9-2. Stem profile and height at different ages.

it is necessary to synchronize the ring series; similar to cross-dating in dendrochronological and dendroclimatological studies. Either the observed ring widths of a given tree are compared optically with a standard ring width series (Figure 9-3), which is based on a study of the impact of year-to-year variations of climatic parameters on ring width, for example, the rainfall during the growing season, or the individual series within a given stand are compared by visual examination.

The standard series represents an error-free image of the average ring width pattern, observed for a number of sample trees within a given region, recorded during a sequence of years. Because of macroclimatic differences, the standard series must be constructed for each climatic region separately. It is useful to additionally compare ring width patterns in the lower part of the bole with those in the upper part, since missing rings are unlikely to occur in the upper bole

Table 9-2. Summary of growth estimates

Age	dbh*	i_d	g	i_g	h	i_h	V	i_v	f	Δf	fh	Δfh	h/dbh
	(cm)	(cm)	(m²)	(m²)	(m)	(m)	(m³)	(m³)	–	–	m	m	
10	1.30		0.0001		1.8		0.0009		5.000		9.00		–
		6.14		0.0042		4.2		0.0166		–4.322		–4.93	–
15	7.44		0.0043		6.0		0.0175		0.678		4.07		81
		4.10		0.0062		2.6		0.0274		–0.181		0.20	
20	11.44		0.0105		8.6		0.0449		0.497		4.27		75
		1.87		0.0036		3.1		0.0460		0.054		2.18	
25	13.41		0.0141		11.7		0.0909		0.551		6.45		87
		1.54		0.0035		2.3		0.0109		–0.138		–0.67	
30	14.95		0.0176		14.0		0.1018		0.413		5.78		94
		1.35		0.0033		1.9		0.0645		0.087		2.17	
35	16.30		0.0209		15.9		0.1663		0.500		7.95		98
		1.14		0.0030		2.5		0.0525		–0.002		1.21	
40	17.44		0.0239		18.4		0.2188		0.498		9.16		106
		0.93		0.0026		2.2		0.0488		–0.008		0.93	
45	18.37		0.0265		20.6		0.2676		0.490		10.09		112

*dbh = under-bark diameter

Table 9-3. Absolute and relative diameter growth at different heights

H (m)	15–20 (cm)	(%)	20–25 (cm)	(%)	25–30 (cm)	(%)	30–35 (cm)	(%)	35–40 (cm)	(%)	40–45 (cm)	(%)
					Diameter growth for 5-year period							
0.3	3.63	86	1.61	86	1.51	98	1.54	114	1.35	118	1.19	128
1.3	4.10	100	1.87	100	1.54	100	1.35	100	1.14	100	0.93	100
3.3	4.44	108	2.15	115	1.98	129	1.50	111	1.10	96	0.82	88
5.3	5.08	124	2.71	145	2.49	162	1.81	134	1.27	111	0.87	94
7.3			3.64	1.95	3.30	214	2.55	189	1.68	147	1.03	111
9.3					3.97	258	3.10	230	2.03	178	1.25	134
11.3					4.25	276	3.64	270	2.62	230	1.62	174
13.3							3.58	265	3.51	308	2.29	246
15.3									4.24	372	3.11	334
17.3											3.84	413
19.3												

section (Athari 1980). Modern measuring devices make provision for digital storage of the observed ring widths on magnetic tape or diskettes. The data are subsequently analyzed with a computer program and may be reproduced graphically (Figure 9-4).

Alternatively, estimates for the rate of diameter growth are obtained from increment cores. In both cases, however, instruments for mass-processing of wood samples, such as the Austrian "Digitalpositiometer" should be available.

2.3   Single-tree models

Equations and models to estimate the growth of single trees within even-aged stands are usually subdivided into distance-dependent and distance-independent models.

2.3.1   Distance-dependent models

Distance to and size of the trees, which surround the subject tree, are important building stones for growth models in this category. The size of the subject tree is assumed to be proportional to its growing space, whereas the distance to and size of surrounding trees determines the amount of overlap amongst the individual two-dimensional growing spaces of the subject tree and its competitors. Opie (1968) introduced the concept "zone of overlap," which quantifies the basal area density of the trees which fall within the zone of influence of

Annual ring diagrams from smoke-damaged spruce;
extracted at different heights

9 = 29,3m
8 = 25,3m
7 = 21,3m
6 = 17,3m
5 = 13,3m
4 = 9,3m
3 = 5,3m
2 = 1,3m
1 = **Standard curve**
• = **missing annualring**
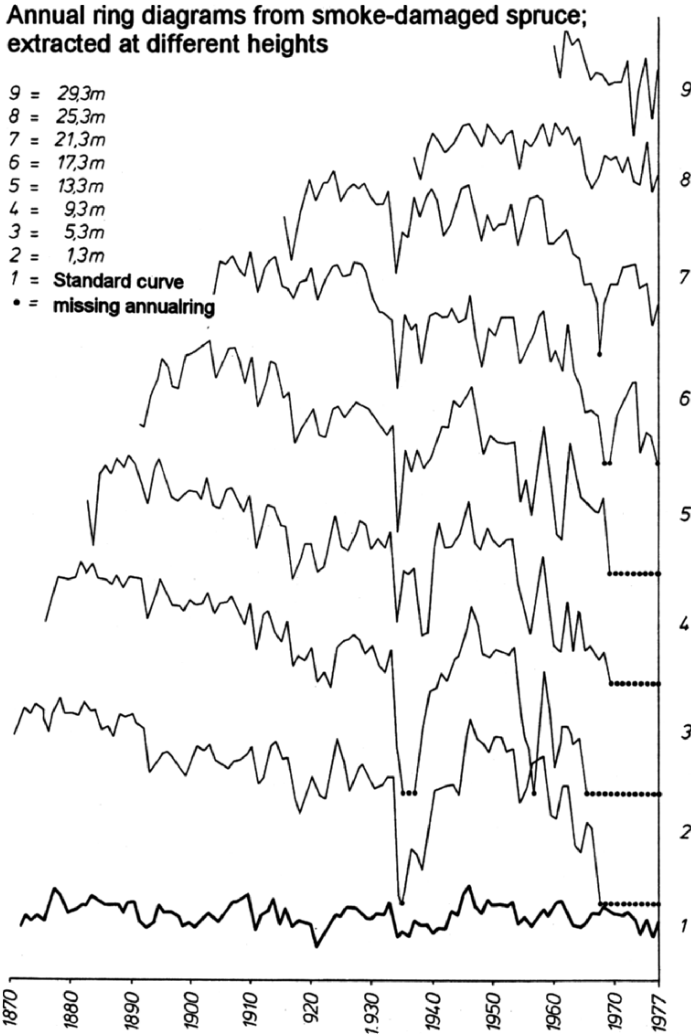
Figure 9-3. Synchronization of the ring series.

the subject tree. Gillespie et al. (1986) constructed a growth model for individual trees of *Pinus strobus* to quantify the effect of competition and crown class on postthinning growth, with prethinning initial diameter as covariate. Crown class as well as prethinning diameter were introduced as dummy variables and Opies's "zone of influence" index was used as competition index. Bella (1971) proposed a competition model, based on influence zones. Hegyi (1974) introduced an index of competition, based on the hypothesis that trees
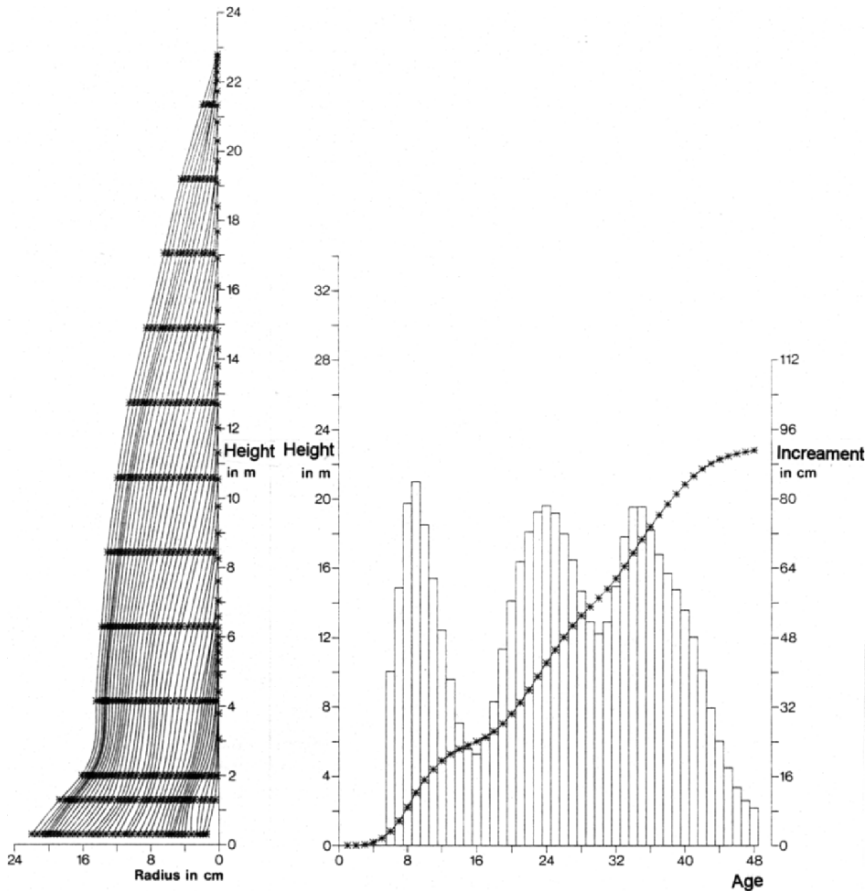
Figure 9-4. Stem profile and heights. (From a computer program by Nagel and Athari 1982.)

outside the influence zone (which does not assumed to extend beyond the edge of the crown), may well induce competitive stresses upon the subject tree. This assumption is reasonable, because of the possible effect of trees further away from the subject tree with no crown overlap. The following *competition index* (*CI*) was introduced in managed *P. banksiana* stands

$$CI = \sum_{i=1}^{n} \left[\frac{d_i/d_j}{D_{ij}} \right]$$

where d_j = dbh of the subject tree, d_i = dbh of a competitor, and D_{ij} = distance between the subject tree and the competitor.

Pretsch (1992) developed a growth model for single trees in mixed stands in Germany and introduced a three-dimensional concept to describe and quantify the crown development of individual trees. Competition amongst trees was described in terms of lateral restrictions and shading. The model required information concerning the crown diameter of the single tree under ideal conditions, with no growth restrictions associated with competition. Only those competitors, which belonged to the same social stratum as the subject tree, were used to determine the lateral restriction. One coefficient (ε) measured lateral growth restriction based on a three-dimensional crown model. For a given subject tree the restriction was assumed to occur within a circle with a radius equal to the maximum radius under conditions of free growth. The live crown of the competitor was assumed to either have a conical or a more spherical shape. To determine the amount of shading, which represented the second structural parameter, a light cone was established for each subject tree, with its apex being located at 70% of the tree height and with an opening angle of 60°. A neighboring tree was identified as competitor if it intruded into the light cone. A dot count sample served to determine the number of times the crown space of the subject tree was intruded by a competitor. The model allowed for more than one competitor intruding at a given point within the live crown and expressed this number as a fraction of the total number of dots. The resultant parameter (Φ) determined the magnitude of shading. The structural parameters were used to predict height growth, lateral crown growth, the age-related change in the position of the base of the live crown, basal area increment, and mortality. The updated tree dimensions were subsequently used for the second simulation run. The model was subsequently incorporated into the growth model SILVA 1, which simulates the expected growth of stands

Daniels (1976) modified Hegyi's index, by redefining N such that competitors were selected on the basis of their size and distance from the subject tree. In addition three new indices were introduced. Physiological considerations played a role in the competition index proposed by Ek et al. (1974); Adlard (1974) used the growing space available to the individual tree which was identical with Brown's growing space concept (Brown 1965) as index of competition in growth studies in East Africa. Faber (1991) developed a distance-dependent growth model, which consisted of a number of submodels. Eriksson (1976) compared competition models in a study to assess the effect of cleaning operations on the rate of diameter growth, for example the sum of the basal areas of overlapping competitors, the unweighted and weighted area of overlapping competition zones and others. Brand (1986) examined and compared competition indices to quantify brush competition in young Douglas fir stands. Daniels et al. (1986) compared the usefulness of various competition indices for predicting

the diameter and basal area growth of individual loblolly pine trees. Tome et al. (1989) examined distance-dependent competition indices to predict the growth of individual trees and introduced modified versions of the existing distance-weighted size ratio and the area overlap indices.

2.3.2 Distance-independent models

These models predict the growth of individual trees based on information obtained from different sources, with the exclusion of distance variables. The predictor variables may represent stand-level variables, variables describing the growth potential of the individual tree or a combination of both variables.

In studies to predict plantation mortality, Glover et al. (1979), defined a competition index, which was calculated as the basal area of the subject tree, divided by the total basal area of the trees within the sample plot. Lorimer (1983) introduced a competition index, defined as the sum of the diameters of trees which surround the subject trees, divided by the diameter of the latter. Hamilton (1970) regressed the volume increment of individual trees of Picea sitchensis *on dbh, crown projection, crown surface area, and basal area per hectare. Alder (1979) introduced a distance-independent tree model for pines in East Africa. Hix et al. (1990) introduced competition measures to evaluate the effect of competition on the height growth of hardwoods on contrasting sites in Wisconsin.*

3   SITE CLASS AND SITE INDEX

3.1   Introduction

The concept site index is crucially important to construct yield tables and growth models. The site index of a stand is a mensurational statistic which is used as an easily accessible variable expressing the combined effect of those edaphic and climatic characteristics which have an impact on growth and yield of a given tree species. These site–growth correlation studies relate site variables to site index based on a statistical model which explains the relationship between site and site index. Because of the interactions amongst site variables, the impact of a single variable should not be assessed in isolation but only in presence of the other potentially influential variables. The analysis shows which site variables are statistically significant and should be retained and which subset of variables is most useful to explain growth patterns. Nevertheless, even under almost ideal research conditions, it remains highly uncertain whether the resultant model includes all influential variables. The model may furthermore

include site variables, which are significantly correlated with growth, but are not readily available. Forest managers prefer parsimonious models, which are useful to take decisions about the selection of species, suitable for specific sites, and about management alternatives. In order to be useful for management and silviculture, the model should be based on accessible predictor variables. The mean temperature during the growing season, for instance, may be an influential predictor variable, is frequently available for the individual plantation or forest district but seldom for the individual sample plot. In tree plantations of short rotation *Eucalyptus* species, tree breeding has a profound effect on the yield and growth of single-clone plantations. In addition the site index of a given stand is affected by the introduction of new silvicultural regimes, more particularly by better soil preparation, improved nursery material, site-related fertilization, and more adequate planting techniques. This implies that the effect of site variables on growth is confounded with and obscured by silvicultural methods.

Research and practical experience in the northern countries of Europe, more particularly in Finland, has indicated that the site may be adequately described in terms of dominant features of the lesser vegetation. This more qualitative approach produced forest types, which were shown to be closely related to site productivity. In Central Europe this approach was less successful, primarily because of the greater complexity of the forest environment.

The early German yield tables produced estimates about stand characteristics at different ages for different site classes. The latter were defined by a set of curves reflecting the relationship between age and mean height of the stand and were denoted as site class I, II, . . . , etc. The site classes in Wiedemann's yield tables (Wiedemann 1936), are of unequal class width in terms of mean height as well as mean annual volume increment. The more recently constructed tables, for example, the Assmann–Franz yield table for *P. abies* in Bavaria (Assmann and Franz 1963) express site class in terms of the top height at the age of 100 years, but expressed in MAI at this reference age. The yield tables of the British Forestry Commission (Hamiltion and Chistie 1971) are based on the age–top height relationship but the site classes are identified by the mean annual volume increment at the age of its culmination.

In the USA and many other countries the site index of a stand is defined as the mean height of the dominants and codominants (Husch et al. 1982; Avery et al. 1983), for a predefined reference age. In principle this requires the identification of the social status of the single trees within the stand and introduces an element of uncertainty and subjectivity. In Germany, site index is defined as the regression height of the quadratic mean diameter of the 100 thickest trees per hectare at a reference age which is usually 100 years. The British yield

tables are based on the regression height of the 100 thickest trees per acre at a reference age of 50. A much lower reference age is used for fast-growing pine plantations in South Africa, for example, 20 years for Pines, 10 years for black wattle, and 8 years for *Eucalypts.*

In young stands the estimation of the expected height of the dominants at reference age is inaccurate since silvicultural regimes and weather conditions during the first years after planting are more influential than the potential productivity of the site. Many studies of the effect of site on growth revealed that the *growth intercept*, defined as the total length of a fixed number of internodes, usually measured from the first internode above breast height, is more closely correlated with environmental variables than site index (Wakeley et al. 1958; Ferree et al. 1958; Day et al. 1960; Oliver 1972). In studies of the relationship between site and site index, Brown et al. (1981) modified the conventional intercept method by correlating site factors with the 5-year height growth, from the age of 2 years with age at breast height as reference point. The response equation for height was

$$\ln h = b_0 + b_1 \left(\frac{1}{A} \right) + b_2 (\text{int}) + b_3 Z$$

where $Z =$ site variable and int $=$ growth intercept.

3.2   Site index curves

Prior to the advent of electronic data processing, graphical methods were generally applied to construct site index curves. They are still used when no adequate computer facilities are available. The procedure is as follows

- Temporary sample plots are established in each of the N stands, selected for sampling. The stands should be representative for the entire range of sites and age classes. In consequence it requires a prestratification of the population based on site productivity and age. To eliminate bias, the age class distribution of the sample plots should be approximately the same on all sites. The quadratic mean diameter of the 100 thickest trees per unit area is estimated from the diameter distribution, the top height is obtained from the fitted height curve. Alternatively, age–height data are obtained from periodically remeasured permanent sample plots. Such datasets, however, violate the basic assumption of statistical independence, since each sample plot is represented by a sequence of plot measurements.

- A freehand curve is fitted to the distribution of top heights, plotted over age. It is denoted as the index, reference, or guide curves and used to generate a family of site index curves. If necessary the freehand curve is adjusted to

satisfy the condition of a zero mean deviation from the fitted curve in each of k age classes.

- A set of anamorphic site index curves is generated on the basis of an assumed proportionality. For example, if an index age of 20 years is selected for fast-growing plantations, and the height–age curve gives an expected height of 21 m at 20 years, the site index curve for site index $SI = SI(i)$ is obtained by multiplying the heights at age $A(i)$ by $SI(i)/20$. The resultant family of site index curves is of identical shape and assumes that the height of a given stand, with increasing age, remains $p\%$ above or below the guide curve. It is an artifact which seldom reflects the real-life situation in forests. When fitting a regression curve through the data points, for example with $\ln(height)$ as dependent and reciprocal age as predictor variable, the slope of the regression line is not affected by multiplying each observed height by a factor q. Only the intercept would decrease or increase by the amount $\ln(q)$. By virtue of the definition of the shape of a curve all members of a family of anamorphic site index curves have the same shape. The introduction of an age-dependent value for q would generate a curve with a different value for the intercept b_0 and the slope b_1. In consequence, this modification generates a polymorphic family of site index curves.

- To obtain a set of polymorphic site index curves, Bruce and Scumacher (1950) suggested to group the data in age classes, to determine the coefficient of variation (CV) in each age class and to plot CV over age. If a linear or nonlinear trend is apparent, the constant proportionality coefficient, which forms the basis for the construction of the anamorphic family of site index curves, is multiplied by

$$K_l = \frac{s_l(\%)}{s_k(\%)}$$

where $s_i(\%)$ = coefficient of variation at i years and $s_k(\%)$ = coefficient of variation at age k. This modification was the first step towards the construction of polymorphic site index curves, which hypothesizes that each site has its own family of site index curves. A further improvement of the Bruce–Schumacher method would be to fit a weighted regression equation with CV and age as target- and predictor variable, respectively, and to use the resultant regression estimates to determine K_l.

The current trend is to apply stem analysis for the construction of polymorphic site index curves. Curtis (1964) discussed the merits of the stem analysis approach which leads to a realistic assessment of the potential site productivity, because so much more information is recovered about the past growth of individual trees. Stem analysis also allows for the construction of polymorphic site index curves if the height at index age has been recorded for each sample tree.

The selection of a suitable reference age is problematic. A reference age slightly below the normal rotation age is a good choice. However, when a rotation age of 5 years is imposed in forests managed to produce timber for energy, 10 years for the production of pulpwood and 25 years for the production of saw-timber, no single index age is simultaneously optimal for all three categories. It might then be necessary to use a reference age of 5 years for the first category and a common age of 15 years for the second and third group.

3.3   Site index equations

The construction of a set of site index equations requires one or more than one equation which estimates stand height from age. A simple equation was presented by Schumacher (1939)

$$\ln h = b_0 + b_1 \left(\frac{1}{A}\right)$$

but has the restriction that the underlying model assumes that height increases monotonely with increasing age and excludes the existence of an inflection point. In consequence, it can be used to model the age–height relationship beyond the age at which the inflection point occurs. The yield table for Norway spruce in North Germany (Wiedemann 1936) indicates an inflection point which occurs at approximately 30 years, the Bavarian tables (Assmann and Franz 1963) show an inflection point between 20 and 25 years. In fast-growing plantations of *Eucalyptus grandis* the highest current height growth is recorded during the first year after planting (van Laar 1961), in *Acacia mearnsii* it occurs before the third year after planting (Schönau 1969).

The Schumacher equation estimates stand height from age. Substituting the index age A_I gives

$$\ln SI = b_0 + b_1 \left(\frac{1}{A_I}\right)$$

The resultant site index equation expresses the site index of a given stand as a function of its height h_j and age A_j

$$SI = \exp\left(\ln h_j + b_1 \left(\frac{1}{A_I} - \frac{1}{A_j}\right)\right)$$

The prediction equation is inverted to construct a set of site index curves

$$h_j = \exp\left(\ln SI + b_1 \left(\frac{1}{A_j} - \frac{1}{A_I}\right)\right)$$
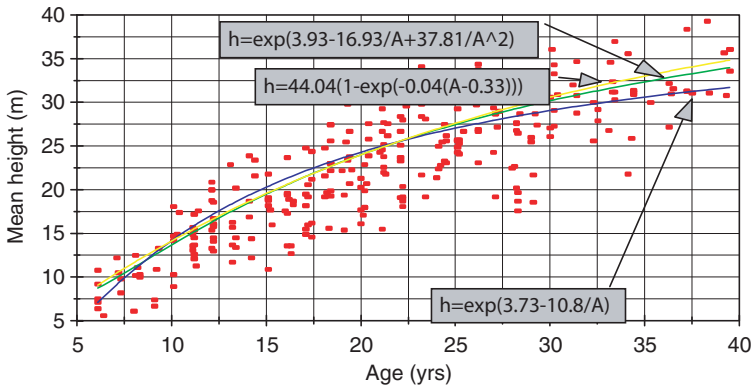
Figure 9-5. Fitted age–height curves for *Pinus radiata*.

In *E. grandis*, Schönau (1976) added the quadratic term $1/(A^2)$ to remove bias in the younger age classes, in which case the site index equation is as follows

$$SI = \exp\left[\left(\ln h + b_1\left(\frac{1}{A_I} - \frac{1}{A}\right) + b_2\left(\frac{1}{A_I^2} - \frac{1}{A^2}\right)\right)\right]$$

Example 9.2   The unmodified and a modified Schumacher age–height equation and a nonlinear model were fitted to sample plot data in *P. radiata* (Figure 9-5). Mean height instead of top height was used as target variable, in order to be consistent with the conventional mean height of dominants being used in South African sawtimber plantations.

The nonlinear model produces a better fit in the lower age classes but a poorer fit in the upper range of stand ages. The addition of a quadratic term to the basic Schumacher model failed to produce a better fit. The application of the nonlinear model would imply to define site classes in terms of asymptotic height (Figure 9-6).

Many linear and nonlinear models have been tested for their performance (Almeida et al. 1989; Brewer et al. 1985; Carmean 1972; Farrar 1973). The study carried out by Carmean indicated that intrinsically nonlinear models were more flexible and performed better than models which were linear in their parameters. Several authors applied Chapman–Richards equation to fit age–height curves. Brickell (1968) introduced the constraint $b_4 = 1/(1 - m) = 1$ into the four-parameter equation

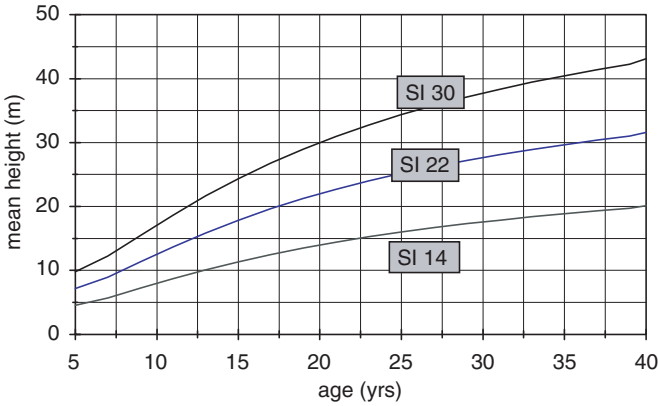$$h = b_1\left(1 - b_2 e^{-b_3 A}\right)^{b_4}$$

Figure 9-6. Unmodified Schumacher model, fitted to *Pinus radiata*.

Ek (1971a,b) developed a function for site index modelling, which produces a sigmoidal growth curve with double asymptotic property and produces a set of polymorphic site index curves

$$h = b_1 SI^{b_2} \left(1 - e^{b_3 A}\right)^{b_4 SI^{b_5}}$$

The model is an extension of the Chapman–Richards equation and implies that the coefficient b_1 which expresses the upper asymptote for stand height and b_4 in Brickell's equation, being related to the location of the inflection point, are a function of site index. The inverse of the function, with site index as target variable was used to predict the site index of a given stand from its age and mean or top height.

  Lundgren et al. (1970) introduced the model

$$h = b_1 SI \left(1 - e^{b_2 A}\right)^{b_3}$$

which represents an extension of the Chapman–Richards model, with its parameter b_1 being a function of site index. Hägglund (1973, 1974) constructed site index curves for Picea abies *and* Pinus silvestris *in Sweden, based on remeasured permanent sample plots, with height above breast height replacing total height.*

$$h_{ij} - 1.3 = b_{1i} \left(1 - e^{-b_{2i} t_{ij}}\right)^{\frac{1}{1-m}}$$

where

$$h_{ij} = height\ of\ the\ ith\ plot\ at\ age\ j$$

$$t_{ij} = corresponding\ age$$

To obtain polymorphic site index curves, the coefficients b_2 and m were regressed on age

$$m = d_0 + d_1 A_i^{d_2}$$

Graney et al. (1973) used the three-parameter Chapman–Richards equation

$$h = b_1 \left(1 - e^{-b_2 A}\right)^{b_3}$$

with b_1 and b_2 being a linear function of site index. Burkhart et al. (1977) applied this model to predict the stand height of Pinus radiata *from age. The equation was furthermore conditioned to ensure that the predicted height at reference age was equal to site index. The resultant two-parameter equation was*

$$h_i = SI \left(1 - e^{b_1 SI \cdot A_{ref}}\right)^{b_2} \left(1 - e^{b_1 SI^* A_i}\right)^{b_2}$$

Payandeh (1974) applied the model

$$h = b_1 SI \left(1 - e^{b_2 A}\right)^{b_3}$$

and assumed that the upper asymptote was a function of site index, Ek's (1971a,b) extension

$$h = b_1 SI^{b_2} \left(1 - e^{b_3 A}\right)^{b_4 SI^{b_5}}$$

which additionally assumes that the parameter m of the Chapman–Richards model is a function of site index.

Lyle et al. (1975) used the following five-parameter function to predict height from site index (SI) and age and to construct a set of polymorphic site index curves, derived from the three-parameter Chapman–Richards equation

$$h = b_1 \left(1 - e^{-b_2 A}\right)^{b_3}$$

by assuming that b_1 as well as b_2 are a linear function of site index. The resultant equation

$$h = (b_0 + b_1 SI) \left(1 - e^{-(b_2 + b_3 SI) A}\right)^{b_4 + b_5 SI}$$

which is a modified Graney–Burkhart equation. Alder (1975) fitted the following six-parameter function to estimate height from site index and age for tree species in East Africa

$$h = b_0 \left(1 - e^{b_1 SI}\right) \left(1 - e^{b_2 A}\right)^{b_3^{b_4 - b_5/SI}}$$

Alder (1975) fitted the following equation for tree species in East Africa

$$h = b_0 \left(1 - e^{b_1 SI}\right) \left(1 - e^{b_2 A}\right)^{b_3^{b_4 - b_5/SI}}$$

Example 9.3 In many cases it is useful to convert site index curves derived from existing yield tables into a single regression equation. Wiedemann's yield table for *P. abies* in Germany is based on site index curves which were fitted by graphical methods. Fitting the Schumacher equation

$$\ln (h_t) = b_0 + b_1 \ln \left(\frac{1}{A}\right)$$

to the recorded heights for site classes 1, 2, 3, and 4, with a mean height of 21, 25, 29.3, and 33.3 m at age 100 produced the following parameter estimates:

Site index	b_0	b_1
21.0		-68.36
25.0		-58.15
29.3		-51.64
33.3		-44.51

Third-degree polynomials with site index as target variable and b_0 as well as b_1 as predictors were fitted. The resultant set of site index curves is given in Figure 9-7.
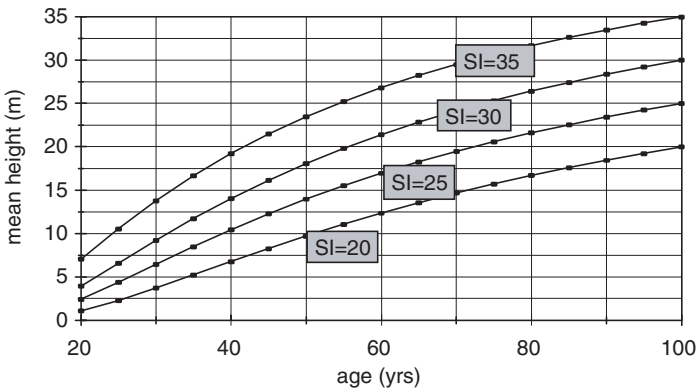

Figure 9-7. Site index curves for *Picea abies*.

4   THE GROWTH OF STANDS

Growth is a biological process, yield quantifies the volume (or weight) of the whole stand or that of a single tree, which is potentially available at the time of harvesting. This differentiation between growth and yield is in line with the forest mensurational terminology used in North America (Husch et al. 1982; Avery et al. 1983). The merchantable yield quantifies the volume (or weight) of the merchantable part of the trees, which varies with the product being recovered from the tree. Accurate estimates of the growth stands are of crucial importance for decision-making in forest management, but the ultimate goal of growth modelling is the estimation of the expected final yield at the time of harvesting and of the intermediate total or merchantable yields during the rotation.

In order to obtain a biologically meaningful estimate for growth, the volume should reflect the total, rather than merchantable stand volume, although the latter is required for management inventories. The construction of volume ratio models, which convert total volume into the merchantable volume, was discussed in Chapter 7. German yield tables reflect the average current and cumulative growth as well as mean annual increment as growing stock volume. They usually refer to the merchantable yield of an upper diameter limit of 7 cm over bark. However, when used to draw up working plans, these estimates are multiplied by a reduction factor, which adjusts for the average losses due to timber defects and gaps within the stand, caused by wind damage and other factors. Practical experience has shown that such losses are of the magnitude of 10–15% of the recorded growth and yield.

4.1   Estimating stand growth based on actual measurements

4.1.1   Complete enumerations in continuous forest inventories

The basic principle of continuous forest inventories was introduced during the last decade of the 19th century, for the management of all-aged selection forests in France (Algan 1901; Biolley 1922). All trees within the forest, above a certain lower diameter limit, were permanently marked and remeasured at 10-year intervals, with single-entry local volume tables (Chapter 7) used in estimating the tree and stand volume. Because of the high cost involved, complete enumerations have been abandoned in favor of modern CFI systems, which are based on sampling methods (Chapter 10).

4.1.2   Permanent sample plots in continuous forest inventories

Periodically remeasured sample plots, established within the framework of continuous forest inventories, produce unbiased estimates of the growth for the

forest in its entirety during the period of measurement, if the condition of a random layout has been satisfied. When this method is applied to estimate the rate of growth of the single stand, the estimate is usually unreliable because of the low sampling fraction within a given stand.

4.1.3 Increment cores

The estimation of the growth of stands, based on the measurement of increment cores, is time-consuming and requires a large sample, because radial growth is measured on one stem position only. For Central and Northern European species, growth can be measured accurately by using instruments which provide optical magnification. In plantations of fast-growing pines in South Africa and other countries in the southern hemisphere, the boundary between earlywood and latewood is less distinct because growth is not completely discontinued during winter. Under these circumstances, the method cannot be recommended when a high accuracy is specified. The measurements on increment cores, when extracted from trees representing the entire diameter distribution, are used to obtain regression estimators either for the diameter growth of trees within each diameter class or to estimate the diameter growth of the mean tree. The growth measurements are to be adjusted for a different bark thickness at the beginning and end of the period. In order to estimate volume growth, either the location of the height curve k years ago must be estimated from standardized height curves or from appropriate site index curves. The usual assumption is that the number of trees per hectare during the growth period of k years did not change. The diameter distribution as well as number of trees are to be estimated by sampling.

4.1.4 Yield tables

Yield tables reflect the "normal" development of stands in terms of number of trees per hectare, as well as basal area and volume per hectare as a function of age, for different site indices or site classes. Their construction is based on standard silvicultural practices, more particularly in terms of stocking. It may be necessary to adjust the yield table estimates for volume growth, in order to account for an observed stocking, which is far below the tabulated density, for example, by multiplying the estimates by an adjustment factor, which is calculated as the estimated stand basal area per hectare (obtained by sampling), divided by the tabulated basal area. Such adjustment factors, however, tend to underestimate the rate of volume growth. The first yield tables were constructed in Germany around the end of the 18th century (Kramer 1988). They were empirical tables, partly based on temporary sample plots, partly on forest inventory data and they estimated the expected yield at the end of the rotation. Several regional tables were constructed during the 19th century, but real

progress was made after the establishment of the German Union of Forestry Research Institutes, in 1872, and the prescribed of methods of construction.

At the beginning of the 20th century, Eichhorn (1904) introduced the function

$$\text{yield} = f(h_m)$$

where h_m = mean height.

In Germany it is widely known as the "general yield level." (Kramer 1988). Eichhorn's law states that yield – for a given stand treatment – is a function of stand height only. The modern yield tables is invariably based on two basic relations, defined by Assmann (1970)

$$h = f(A) \tag{1}$$
$$\text{yield} = f(A) \tag{2}$$

where h = stand height and $A = stand\ age$. Initially, stand height was defined as *Lorey's mean height* (section 5.6) or as the regression height of the tree with the quadratic mean diameter, but because of the effect of stand density, it was replaced by top height, which is less sensitive to stand treatment. Eichhorn's law has been incorporated into the construction of yield tables for Norway spruce in Southern Germany, for the main tree species in the Netherlands and for the yield tables of the Forestry Commission in Great Britain.

Extensive growth studies in Norway spruce in southern Germany, however, indicated the necessity of a further differentiation between a lower, a medium, and an upper yield level within each site quality class (Figure 9-8).
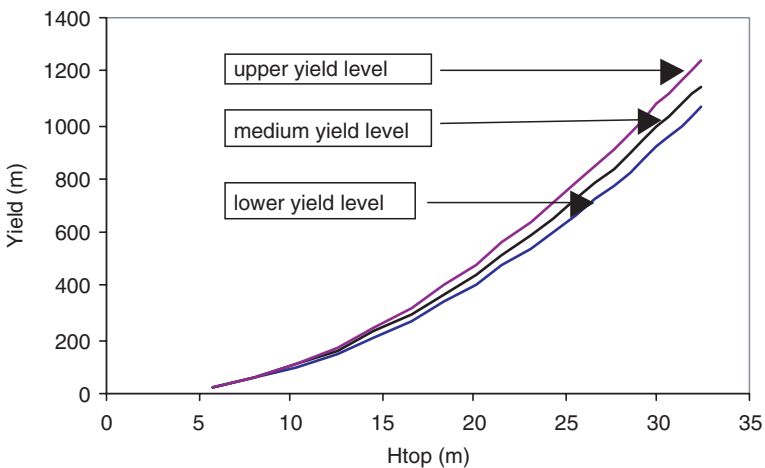

Figure 9-8. Yield levels for Norway spruce in Southern Germany.

This generated the concept "special yield level." In consequence the parameters of the function $yield = f(stand\ height)$ are identical for all site quality classes but for a given site quality class they differ for different yield levels. A similar difference has been noted for *E. grandis* in the Eastern Transvaal and Zululand regions of South Africa.

The *British yield tables* of the Forestry Commission distinguish between the concepts General Yield Class (*GYC*) and Local Yield Class (*LYC*). The *GYC* is based on the age–top height relationship but contrary to the German tables the classes are identified by the mean annual volume increment at the age of maximum mean annual volume increment.

A review and evaluation of the methods being used in the construction of yield tables falls outside the scope of this book. The majority of these tables is based on data, obtained from remeasured sample plots, for example, those constructed by Schwappach (1902), Wiedemann (1936, 1942), Wiedemann and Schober (1957), Assmann and Franz (1963, 1972), Hamilton and Christie (1971), Jansen, Sevenster, and Faber (1996) and several others. Schwappach, however, used inventory plots as well as remeasured sample plots to construct the tables and some tables, for example, those constructed by Schmidt (1971) for *P. abies* in the Upper Pfalz were based on inventory data. For the construction of these tables it is generally required that the research plots are fully stocked and the generally accepted thinning regime has been applied throughout the research period. In case of unthinned stands of short-rotation crops, for example, in stands which are managed for the production of pulpwood, wood for energy and industrial wood, the yield tables are normally based on the currently applied silvicultural practice to produce growth and yield estimates at rotation age. The first step is invariably to construct a set of site index curves, which reflect the expected development of stand height as a function of site index. The relationship between yield and height is subsequently established for each site quality class separately and other relevant relationships are established for each site quality class separately.

The modern yield tables and functions are invariably based on statistical growth and yield functions, with their parameters being estimated with the aid of modern electronic data processing. The *empirical model* is a regression model which predicts the volume or yield per unit area from relevant stand characteristics, for example age, site index, and stand density. Transformations may be required to obtain a model which is linear in its parameters and interaction variables may be added to obtain a better fit. The addition of variables which are generated from the three basic variables age, site index, and stand density will tend to induce multicollinearity, inflated standard errors of the regression coefficients and possibly overfitting. The contribution of each predictor variable to the regression sum of squares is therefore tested for its significance,

for example, by stepwise fitting, forward and backward procedures. It has been found that the empirical model will produce a vector of parameter estimates which fits the current data satisfactorily but may not be suitable for a second independent dataset (Bredenkamp 1988). The *analytical model* on the contrary is based on a certain knowledge of the growth process in its entirety. The modeller formulates a hypothesis about the behavior of model components and fits an equation based on the hypothesis.

It is necessary to distinguish between *prediction* and *projection models*. The previously mentioned empirical model predicts yield as a function of statistically significant independent variables. The model, however, can be converted into a *projection model*, which estimates the expected growth per annum between age A_1 and A_2. Clutter (1963) introduced the basic idea of compatibility of growth and yield models and fitted the yield equation

$$\ln(V) = b_0 + b_1 SI + b_2 \ln BA + b_3 A^{-1}$$

with SI = site index, BA = basal area, A = age, and V = volume per unit area. Differentiation with respect to age produced a growth equation with reciprocal basal area, basal area growth, and squared reciprocal age as predictors. An additional equation was required to estimate basal area growth. A follow-up study, presented a prediction equation for volume yield (Eqs. (1) and (2)) and a projection equation for basal area (Eq. (3))

$$\ln V_1 = b_0 + b_1 SI + b_2 A_1^{-1} + b_3 \ln BA_1 \tag{1}$$
$$\ln V_2 = b_0 + b_1 SI + b_2 A_2^{-1} + b_3 \ln BA_2 \tag{2}$$
$$\ln BA_2 = [A_1/A_2] \ln BA_1 + a_1 [1 - A_1/A_2] + a_2 [1 - A_1/A_2] SI \tag{3}$$

Equation (1) was fitted to the data. The parameter estimates were used to estimate the stand volume at age 1 and 2. Equation 3 estimated the basal area at age 2 and was used to replace the basal area at age 2 in Eq. (2) by a regression estimate obtained from Eq. (3). Basal area and/or volume projection equations were also developed by Pienaar (1979), Chang (1987), and Amateis et al. (1986). Other projection equations were subsequently developed for stands of *E. grandis* (Harrison 1991, 1993; Pienaar et al. 1986).

4.2   Stand table projection

Several studies have been undertaken to project the current *stand table* at age *k* as starting point. Pienaar et al. (1988) proposed a projection equation for total basal area which required an equation to predict survival and a second one to project total basal area. The relative size of the single tree was defined as its

basal area expressed a proportion of the basal area of the mean tree. The procedure was based on the expected change in time of the relative tree size. Initially it was hypothesized that the change of relative tree size is constant over time. It was tested in South African long-term spacing trials in *P. elliotti* stands. The model was acceptable for short-term projections but when applied to longer growth periods, it could be shown that the relative size of the trees with a diameter below the mean diameter decreased, whereas that of the dominant trees increased over time. This revised hypothesis was incorporated into the stand table projection model. Nepal et al. (1992) developed an algorithm for projecting stand tables, which was based on the parameter recovery method to estimate the parameters of the Weibull distribution, which was developed in an earlier study (Cao et al 1982). The parameters were recovered from the mean as well as quadratic mean diameter at breast height and from the minimum diameter at breast height. In a later study, Cao et al. (1999) developed a new method which required an estimate of survival rates and the location of mortality, to estimate diameter growth for each diameter class. The projected diameters are adjusted to ensure consistency between future mean diameter and stand basal area. Trincado et al. (2003) evaluated these methods for projecting the stand table of *E. nitens* in Chile. Both methods were suitable for projecting the stand table, but the method proposed by Nepal, produced marginally better results than that proposed by Cao.

4.3   Recent developments

Pretzsch (1992) developed a stand-level model which consists of a number of submodels. The height growth submodel estimates the potential height growth which is subsequently adjusted to account for the effect of competition on height growth. Similarly the age-related change in crown diameter is estimated from the potential growth of the tree crown, adjusted with the aid of competition-related variables. Other submodels deal with the age-dependent shift of the position of the height above ground of the live crown, the effect of crown surface area, tree basal area at the beginning of the growth period, local light conditions and management-related changes in light conditions. They are combined to estimate the diameter increment. Models were also developed to quantify the effect of mortality. In Germany the models SILVA 1 and more recently SILVA 2 are now widely used to simulate the growth of pure and mixed stands, which are based on the single-tree models. If no information is available about the coordinates and crown dimensions of all single trees within the sample plot, the program STRUGEN provides estimates of the structure parameters as initial values for a simulation run.

Chapter 10

SAMPLING FOR FOREST INVENTORIES

1   INTRODUCTION

The objective of forest inventories is to obtain qualitative and quantitative information about forest resources and their physical environment, at a specified point in time and at reasonable cost. Its main goal is to report on the status quo of the forest: areas, volume, and volume distribution in terms of size classes, but also on the expected changes (growth, vitality, mortality). Because of the increasing multiple use of forest resources during recent years, its scope has been enlarged and may include information about the potential of the forest for wildlife, recreation, and other uses. Forest inventories may therefore be classified as follows:

- National and regional forest inventories, which form the basis for forest policy decisions and long-term planning of the forest industry in its entirety
- Management inventories which are required for management decisions and more particularly for the construction of working plans
- Inventories for the appraisal of stumpage value
- Surveys required for planning logging operations
- Multiresource inventories, for example, those required for land-use management, wildlife and recreation
- Inventories to assess and estimate the impact of pollution on vitality and growth of trees and forests

Each of these categories requires specific information and prescribes a specified accuracy. A stand inventory to assess the timber value requires accurate estimates of the growing stock and, to a lesser extent, this applies also to a stand inventory for planning logging or thinning operations. For a survey undertaken to estimate the forestry potential of a region, however, it may be more important to estimate site productivity than stand volume per unit area. National forest surveys are carried out to provide a basis for forest policy decisions.

The reasons for sampling in forest inventories are:

1. A total enumeration of large forest tracts is not feasible and prohibitively expensive
2. Complete tree measurement implies destructive sampling which, for example, is implemented in stem analysis, but cannot be carried out on a large scale
3. Sampling is advantageous for updating forest inventories at reasonable cost
4. Sampling makes it possible to obtain more accurate information for selected sampling units

The sample consists of n sampling units on which tree, stand, or site characteristics are counted, estimated, or measured. It is drawn from the population and represents the aggregate of sampling units for which information is required. In a regeneration survey, the population may be defined as the aggregate of trees higher than 10 cm, in a management inventory as the aggregate of trees with a diameter exceeding a defined threshold value, which is usually related to merchantability.

During the early days of forest inventory, sample plots were frequently laid out subjectively. The enumeration team decided how and where to establish those plots considered to represent "average" conditions. In practice, this procedure tended to overestimate the volume per hectare. Subjective sampling was gradually replaced by probability sampling. This requires a subdivision of the population into N non-overlapping sampling units, covering the entire forest or the part of the forest being sampled. The condition of "*non-overlapping sampling units*," however, is not satisfied in *angle count sampling* and the condition of a 100% cover of the sampling units is violated when circular instead of square sample plots are established. A probability of selection is assigned to each of the sampling units within the population. A single probability of $1/N$ implies that each of the N sampling units has the same chance of being included into the sample. However, it is frequently advantageous to apply a different strategy, for example, *selection probability proportional to size* (PPS).

An important objective of sampling is to obtain information of a given accuracy, i.e., a specified precision together with negligible bias at the lowest cost or the highest accuracy at given cost. In many instances, the required precision and the risk of exceeding the acceptable maximum error are specified. It is then necessary to estimate the total cost of the field survey, prior to sampling, in order to preempt cost overruns. Most commonly, either volume or growth per unit area represents the target variable. In that case, the cost of the field survey – for a given accuracy – is controlled primarily by stand parameters (species mixture, even-aged or uneven-aged stands, mortality due to stresses

and calamities, etc.). In mixed forests, the variability of the volume per hectare among plots of a given size tends to be larger than in single-species forests and in natural forests it exceeds that in even-aged plantations. This has a profound impact on the variability of the target variable.

In those instances where the total cost are specified prior to sampling, a decision must be made as to which sampling method is likely to produce the most accurate estimate of the target variable. It should be realized, however, that the cost of a poor estimate, due to such rigid cost constraints, should be added to the actual cost of the survey. Inventory reports, which state the achieved accuracy, invariably assume that areas are determined error-free. This assumption is seldom satisfied. Forest maps may require updating to obtain nearly error-free stand areas. Such information is frequently obtained from existing, recent aerial photographs, in which case the quality of this information depends upon the quality and scale of the aerial photograph. An inventory produces quantitative information about the growing stock and its rate of change per unit area. Reports on such inventories give information about volume and expected growth, expressed per unit area and for the stand as well as forest in its entirety. In many countries, it is feasible to obtain reliable area information from existing aerial photographs but, particularly in developing countries, the information may be outdated. In such instances, reports on the accuracy of timber estimates should be scrutinized carefully.

The inventory cost per unit area varies considerably and is influenced by:
- The type of information required (volume, density and diversity, size distribution, growth, mortality, etc.)
- The specified maximum error of the estimates
- The probability that this error will be exceeded
- The variability of the subject variable
- The size of the population or stratum
- The degree of stratification
- The shape of the forest tract being sampled
- Accessibility of the forest, topographic, and terrain features
- The extent of information available prior to the survey
- The availability of trained field staff
- The available time

The majority of forest inventories specifies the accuracy required either for the entire population or for predefined strata. The size of the stratum for which information is to be obtained, with a specified maximum error, is an important factor, which influences the sampling fraction and thereby the inventory cost per unit area. In management inventories, for example, the compartment serves as a unit of forest management. In order to make management decisions on final

and intermediate fellings, as well as pruning and other tending operations and for harvest scheduling, the forest manager requires reliable information for each compartment, more particularly about volume, growth, and other stand characteristics. The associated inventory cost, however, may be excessively high, unless the specified accuracy requirements are adjusted drastically to reduce the cost of sampling. Alternatively, single compartments should be combined with others to generate subpopulations sufficiently large to allow the application of scientific sampling methods. Prior to World War II, working plans for the all-aged selection forests in Switzerland and France were usually based on a complete enumeration of all trees above a fixed threshold diameter, for example, 14 cm. This practice has been abandoned due to the prohibitively high cost. Because of these cost and time constraints, scientific sampling methods were developed to obtain information with the optimum cost–benefit ratio. Many of these sampling techniques were developed by researchers in other disciplines, for example in the social sciences, and adapted to meet the requirements specified by decision-makers in forestry.

1.1   Sampling units

The sampling unit is defined as the smallest unit on which the target variable is obtained. The individual tree is suitable as a sampling unit for estimating the population mean of tree characteristics such as diameter, height, and volume or in studies of the effect of environmental pollution on vitality. In case of selection *without replacement*, it implies that the total number of possible sampling units is equal to the size of the population, although the latter is usually unknown. The majority of forest inventories, however, deal with the estimation of volume and growth per unit area and for the population in its entirety. It is difficult to determine the expansion factor for converting volume per tree into volume per hectare, since the measurement of the growing space of the individual tree is inaccurate and may well be biased. The trees are therefore usually measured in clusters, more particularly in stand inventories for estimating the volume and growth per unit area.

2   PLOT SAMPLING

2.1   Plot shape

In practice, the sample plots are most often of circular, square, or rectangular shape. The sample strip is a specially shaped rectangular, which is particularly useful in inaccessible forests.

A sample tree is considered to fall inside a plot of given boundaries, if the center of the bole, at the base of the tree, falls inside the plot. In consequence, each sample plot contains edge trees, with a growing space which is partly located outside the plot boundaries. A circular plot shape has two advantages over other plot shapes:

- It represents the geometric shape with the smallest perimeter for a given plot size. It tends to produce less borderline trees than plots of the same size, but different shape.
- In stands without undergrowth, the plot boundaries can be conveniently located with the aid of optical devices, for example; with the optical Blume-Leiss or Suunto and an ultrasonic or laser (Vertex or Ledha) range-finder

In plantation forests, it is convenient to establish rectangular plots, with the longest side coinciding with the direction of the planting rows. The two longest sides are positioned halfway between two adjoining rows. It remains imperative to determine the shortest side of the rectangle (in addition to the longest one), instead of relying on the recorded planting espacement, because the actual distance between the planting rows may vary within a given stand. In cases of excessive differences between the row distances, the layout of square or rectangular plots should be abandoned in favor of circular plots. In many tropical forests, however, it is more convenient to lay out square or rectangular plots or sample strips, the extreme version of the rectangular plot. They can be cleared prior to measuring the dbh and height of the trees within the plot boundaries and are therefore more efficient than other shapes. The much greater number of edge trees, due to the unfavorable plot shape, may induce systematic errors. Sampling studies in Indonesia have indicated that either square or rectangular plots, established within systematically or randomly distributed sampling lines within the population, are more cost-efficient than a total enumeration of sample strips. They reduced sampling cost without sacrificing accuracy (Akça 1996).

2.2   Plot size

The variability of the study variable, e.g., volume (converted into volume per hectare or acre), tends to decrease with increasing plot size and this in turn reduces the sample size required to obtain a predefined precision of the estimated population mean or total, but the coefficient of variation tends to decrease curvilinearly with increasing plot size. At the same time, however, measurement as well as travel time between plots increases and this again reduces the greater efficiency of large plots. In consequence, it is necessary to determine the optimal plot size and to devise a sampling strategy which either

produces the highest accuracy for a given cost or the lowest cost for a given accuracy.

In relatively homogeneous plantation forests, however, the coefficient of variation is less severely affected by plot size than in natural forests. Sampling studies in South African *Pinus radiata* plantations (van Laar 1981) showed a decrease of measuring time per unit area with decreasing plot size, but in this particular study, only borderline trees were checked whether they were located inside or outside the plot boundaries. The opposite was found in sampling studies in Germany (Akça et al. 1986), where the measurement of distances was not limited to borderline trees. In the case of random sampling, the location of the plots is determined prior to sampling, for example on maps or aerial photographs. In consequence, time is needed to locate the sampling points. In addition, travel time for moving from one plot to the next, although not strictly proportional to the number of plots, tends to increase with increasing number of plots. It is therefore necessary to estimate the optimum plot size, which produces the best results at the lowest cost.

For a given sampling intensity, the required number of plots decreases proportionally to plot size. In order to estimate the volume of a stand with an area of 15 hectares, a sampling intensity of 5%, and sample plots with a radius of 7, 10, and 13 m, respectively, the required number of sample plots is

$$R = 7\,\text{m} : n = 7500/154.0 \approx 49$$
$$R = 10\,\text{m} : n = 7500/314.3 \approx 24$$
$$R = 13\,\text{m} : n = 7500/531.2 \approx 14$$

The optimum plot size is also related to forest type, stand structure, site, and genetic parameters. Inventories of multispecies tropical forests with a low density of commercially utilizable trees, require large plots, whereas small plots are adequate in clonal plantations. To some extent, the selection of an appropriate plot size is also determined by terrain and vegetation features, which affect the traveling time component. Moving from one plot to the next one in inaccessible forests with heavy undergrowth is time-consuming. Larger plots may therefore be more efficient than small plots, particularly when the undergrowth has to be removed prior to measuring. Plot size also affects the sampling cost per unit area, which increases with decreasing plot area. In consequence, it is necessary to determine:

- Either the optimum plot size which will produce the highest precision for a given cost (if there is a cost constraint), or
- A plot size which gives the lowest cost for a given precision (when the precision specification prevails), or

- A plot size which is associated with the lowest value of variance multiplied by cost

Smith (1938) and O'Regan et al. (1966) proposed the formula

$$s_y^2 = k \cdot x^{-c}$$

where x = plot size (in hectares), s_y^2 = variance of the target variable (basal area, volume, number of trees, etc.), expressed per hectare, and k, $c(k, c > 0)$ representing the coefficients of the equation. Sampling studies are required to express c as a function of age, site index, and forest composition. In Germany, the coefficient c varies between 0.3 and 0.7 and, in many cases, is close to 0.5. However, it can be expected that k and c are inversely related.

Freese (1961) used Smith's formula to derive the relationship between the ratio of the squared coefficients of variation and the square root of the inverse ratio of the plot sizes (*PS*):

$$s_2^2(\%) = s_1^2(\%) \cdot \sqrt{\frac{PS_1}{PS_2}}$$

O'Regan et al. (1966) simulated forests, based on randomly distributed circular plots of a fixed size. A quadratic equation with log (variance) as dependent, plot size, and squared plot size as independent variables was fitted. The following budget function was proposed

$$c_{\text{tot.}} = c_1 . \sqrt{n} \cdot \text{tract area} + c_2 n + c_2 N_{\text{tree}}$$

where c_1 = walking cost per unit distance, c_2 = cost of establishing a plot, c_3 = cost of measurement per tree, $\sqrt{n}$.tract area = approximated minimum travel distance between randomly distributed points, n = sample size and N_{tr} = average number of trees measured at a sampling point. The introduction of the component $\sqrt{n}$.tract area was due to sampling studies by Jessen (1942), reported by Sukathme (1954), which indicated that the minimum travel distance between random sampling positions is proportional to $\sqrt{n}$.

Zeide (1980) presented a method to determine the optimum plot size which minimizes the total time (= T) involved in the field work of a forest inventory, and necessary to meet the specified accuracy. For a given sample size n, the total time requirement is

$$T = n (t_{tr.} + t_m)$$

where t_m = measuring time and t_{tr} = travel time. The required sample size is obtained from the conventional formula

$$n = s^2 (\%) \frac{t^2}{E^2}$$

*where E = maximum allowable error (in %), $t = t_{1/2\alpha,(n-1)}$ and $s_y(\%) =$
coefficient of variation. The latter is related to and can be estimated from
plot size (PS)*

$$s_y(\%) = b_0 PS^{b_1}$$

*The formula for estimating the travel time was based on systematic sampling,
with the sample centers arranged in a square lattice. In that case, the total
travel time for a given tract is*

$$t_{tr.} = \frac{\sqrt{TA}}{TS\sqrt{n}}$$

*where TA = total area and TS = travel speed. After some manipulation, it is
shown that the relationship between total time T and plot size is as follows:*

$$T = \frac{k}{\sqrt{PS}}\left(a \cdot PS^{1/4} + b \cdot PS^{3/4}\right)$$

where

$$k = \frac{s_y^2(\%)\sqrt{PS} \cdot t_{1/2\alpha}^2}{E^2}$$

and

$$a = \sqrt{\frac{TA}{k \cdot TS}}$$

*The coefficient b is described as a proportionality constant, but in fact is deter-
mined by sampling and regression analysis, with measuring time t_m as a depen-
dent and plot size as an independent variable*

$$t_m = b \cdot PS^c$$

*with b and c estimated after linearizing the relationship between m and PS.
It was then shown that:*

$$PS_{opt} = \left(\frac{a}{b}\right)^2$$

*Lang et al. (1971) compared the plot sizes $10 \times 20\,m$, $10 \times 40\,m$, and $20 \times 40\,m$
for estimating the density of the rich tropical forests in the Panama Canal
Zone. The $10 \times 20\,m$ size required the lowest sampling intensity. Sampling
intensity required to meet specifications increased with the increasing degree
of species aggregation. A 60–70% sampling fraction was needed for $10 \times 20\,m$
plots. In Central Europe, and more particularly in Germany, plot sizes for for-
est inventories vary between 0.01 and 0.10 ha with plot areas of 0.01, 0.02,
and 0.03 ha being used in young, dense stands and plots of 0.05 ha and more*
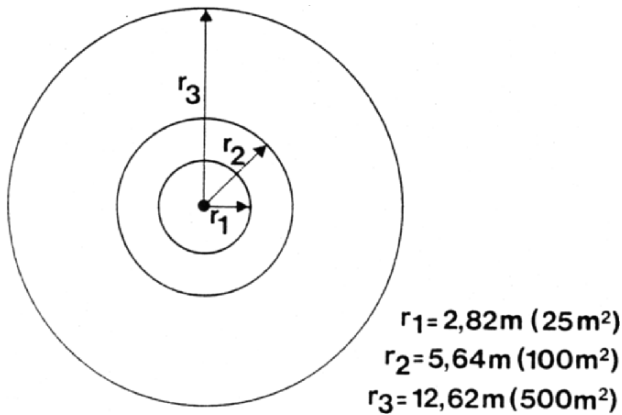
Figure 10-1. Concentric circular sample plots.

in mature, thinned stands. The rule of thumb, that the individual sample plot should contain 15–20 trees, is widely adopted.

2.2.1 Concentric sample plots

Concentric circular plots have been successfully used in the inventory of all-aged forests. Three circles with different radii and the same center are superimposed (Figure 10-1). All trees with a dbh of more than 7 cm are measured within the inner plot, those with a dbh of more than 20 cm within the second-smallest circular plot and the trees with a dbh of more than 40 cm within the outer circular plot. Volumes and confidence intervals have to be calculated for each of the tree size categories separately.

The basic principle of plots of different sizes can obviously be applied if square or rectangular plots are laid out.

Prodan (1968) introduced the six-tree sample plot for estimating the basal area, volume, and number of trees per unit area. The distance between randomly or systematically positioned sampling points and the sixth nearest tree is measured, together with their diameters. The basal area of the six-tree sample plot is estimated as follows:

$$G\left(m^2/ha\right) = \frac{2500}{r_6^2}\left(\sum_{i=1}^{5} d_i^2 + \frac{1}{2}d_6^2\right)$$

where d_i $(i = 1, \ldots, 5) =$ diameter (cm) of the ith nearest tree and $r_6 =$ distance (m) to the *sixth* nearest tree. The six-tree sampling method is a *mean ratio estimator* but the estimate is not unbiased (Figure 10-2).
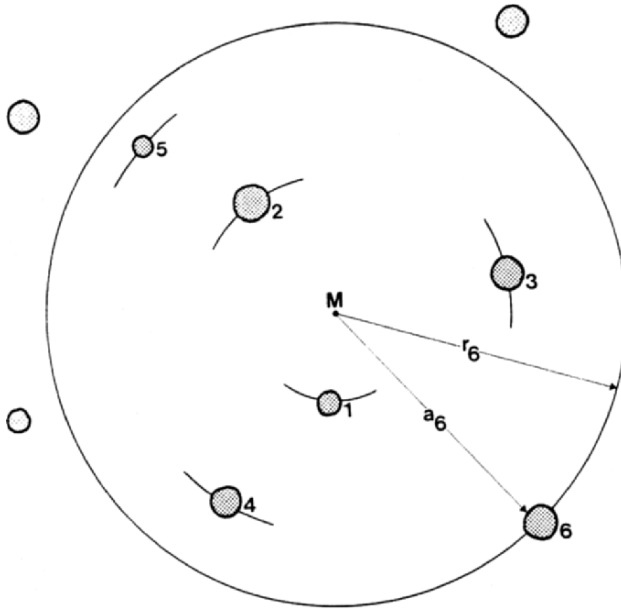
Figure 10-2. Prodan's six-trees sampling method.

2.3   Plots on stand boundaries

Trees along the stand and forest boundary usually grow faster than those grow-ing in the inner stand, primarily because edge trees are less severely exposed to inter-tree competition. On the other hand, trees near the wind-exposed edge of a stand develop a different stem and crown form, which is not necessarily con-ducive to growth and may reduce the rate of height growth. Unbiased volume estimates for a given stand are obtained only if the trees on the stand boundary are correctly represented.

2.3.1   Relocating of plots

When sample plots are established at random, the sample plot is sometimes par-tially located outside the stand. The sample plot center may then be relocated by moving the plot center away from the edge. This method produces a substantial systematic error, especially in small stands with irregular boundaries, particu-larly if the stand structure along the boundary differs from the rest. The basic principle of random (and systematic) sampling requires that a sample plot will be established at the selected position, if the randomly selected location of the plot center falls inside the stand. The area of the plot section falling within the

stand must be determined and recorded, together with its tree diameters. The determination of the true plot size can be time-consuming and the calculations required to estimate the mean diameter or volume and their standard errors are more complex when dealing with sample units of varying sizes. In such situations, the population mean must be estimated as a regression or ratio estimator. Due to the increased field work and the complexity of the calculations involved, this method is not usually preferred.

2.3.2 Mirage method

The second method of dealing with plots falling on the stand boundary is known as the mirage method and produces unbiased estimates of the population parameters (Schmid 1969). The field application of this method is simple and the calculations are identical to those for sample plots of equal sizes. Figure 10-3 illustrates the method for circular sample plots. After locating the plot center, all trees falling within the sample plot are measured, together with the distance between the plot center and the edge of the stand. A "mirror" plot center is located outside the stand, at the same distance from the edge. The trees within the "mirror" plot are measured, those falling in the shaded section of the circular plot are measured twice. When it is not technically possible to establish a "miraged" plot center directly outside the stand, the method can be implemented by relocating the mirror plot (Figure 10-3).

The mirage method produces unbiased estimates, because the probability of a sample point within the boundary zone, falling inside the stand with a center located outside the stand, is equal to the probability of falling outside the stand with the plot the center being located inside the stand. The unbiasedness of the mirage method was shown by Gregoire (1982).
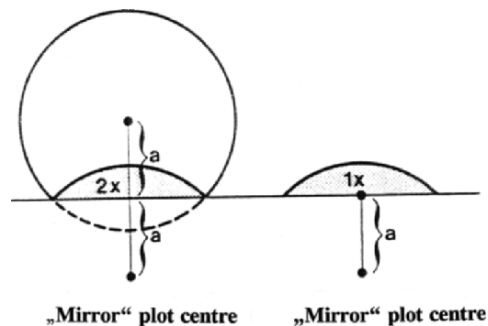

Figure 10-3. Mirage method.

2.4   Slope Correction

In forest inventories, stand and plot areas are expressed in terms of their area, projected on the horizontal plane. The number of trees, basal area, volume, and growth estimates are adjusted accordingly. It is therefore necessary to convert the plot size into its equivalent on the horizontal plane. The circular plot, established on a slope at an angle of α degrees, when projected on a horizontal plane (Figure 10-4), generates an ellipse with its longest and shortest axis being r_s and $r_h = r_s \cdot \cos \alpha$, respectively. The area of this ellipse is

$$a_h = \pi \cdot r_s^2 \cdot \cos \alpha = a_s \cdot \cos \alpha$$

a_s = ground area on the slope

$\quad\quad a_h$ = projected area on the horizontal plane

$\quad\quad r_s$ = radius of circular plot measured on the slope and longest axis
$\quad\quad\quad$ of the ellipse respectively.

Slope is frequently measured as a percentage and then requires conversion to degrees. For example, when a circular plot with an area of 0.05 ha is established on a slope with an angle of 45%, the corresponding angle α is equal to $\tan^{-1}(0.45) = 24.23°$ and the correction factor is 0.912.
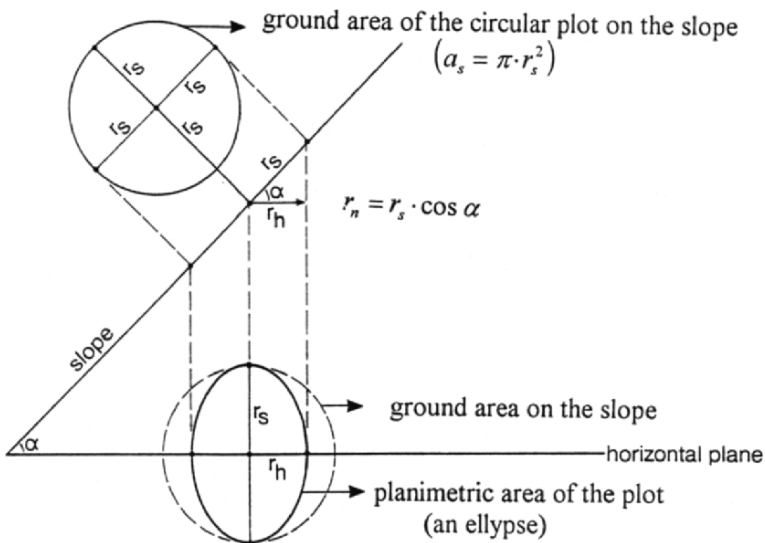

Figure 10-4. Slope correction.

The application of a single correction factor, based on the average slope for a given compartment, produces slightly biased estimates of the stand volume if volume per unit hectare and angle of slope are correlated. A more accurate and efficient adjustment for slope can be implemented by laying out a circular plot with a radius which has been corrected for slope. In that case, the horizontal plot area is identical to the corrected plot size.

$$r_e = \frac{r}{\sqrt{\cos \alpha}}$$

where

r_e = Enlarged radius of the circular plot

r = Plot radius projected onto horizontal plane

α = angle of slope

When established with the aid of the *Blume-Leiss measuring cylinders*, the correction for slope of circular plots can be carried out optically by increasing the distance between the two cylinders. In the case of square or rectangular sample plots, the four sides of the square or rectangle have to be corrected due to their angles β_1 and β_2 with the horizontal plane, and their lengths multiplied by $1/\cos \beta_1$ and $1/\cos \beta_2$, respectively. The square or rectangular plots, however, should preferably be established in such a way that two sides are parallel to the direction of the slope, in which case only one side is corrected for slope.

3   POINT SAMPLING

3.1   Basic principles

The theory and technique of point sampling, which is also known as *angle count sampling*, *plotless sampling*, *Bitterlich's method*, and relascope sampling, and is widely recognized as a breakthrough in forest mensuration. It was developed by the Austrian forester Bitterlich, in 1948. In its original version, it was designed to estimate the basal area per hectare. Grosenbaugh (1963) extended the basic principle of point sampling and redefined the method as PPS sampling. Point sampling estimates stand parameters from sample plots with imaginary plot boundaries. A critical angle is introduced, which defines the ratio between the diameter of a tree and its distance from the sampling point. For each of the trees, surrounding the sampling point, a decision is made whether
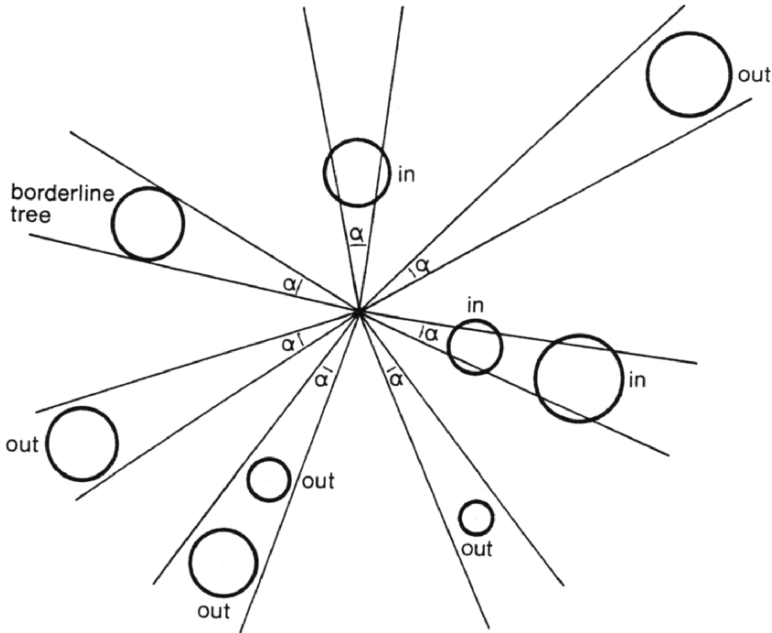
Figure 10-5. Tallying of trees in point sampling.

it is "in" or "out," i.e., whether it falls inside or outside the plot with its imaginary boundaries. The number of trees counted "in" is used to predict the stand parameter. A tree is "in" if it exceeds the critical angle being used. Plot centers are selected by any conventional sampling method. A device with a fixed angle of view is used to sight the surrounding trees at breast height, in a 360° sweep. All trees with an apparent dbh, which exceeds the angle of viewing, are tallied "in" (Figure 10-5). The basal area per unit area is obtained by multiplying this number of trees by a basal area factor (BAF), which is a function of the critical angle.

In order to illustrate the basic principle underlying point sampling, we consider a rod with a length of c units, with a cross-arm or blade with a width of 1 unit, attached at a right angle to the rod. If unit width is used for the cross-arm, the critical angle α, is obtained from the ratio of 1:c. For each tree with a dbh of d_i cm only one distance r_i exists, for which the ratio $d : R$ is equal to 1:c. At this distance, the two lines of sight are tangent to the stem at breast height (Figure 10-6).

We have $\dfrac{d_i}{r_i} = \dfrac{1}{e}$ and thus $r_i = c \cdot d_i$
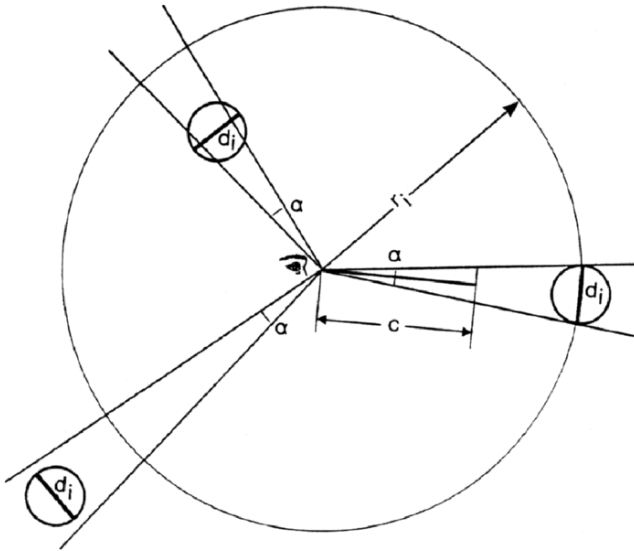
Figure 10-6. Basic principle of point sampling (r_i = the limiting plot radius for the tree with $dbh = d_i$ and α = viewing angle for c *and diameter d_i*). Tree 1 is a borderline tree, tree 2 is "in," and tree 3 is "out."

and the corresponding plot area is

$$\text{Area} = \pi \cdot c^2 \cdot d_i^2$$

The total basal area of all n_i trees with $dbh = d_i$ is

$$G_i = n_i \cdot \frac{\pi}{4} \cdot d_i^2$$

and is converted into basal area per unit area

$$G_i\,(\text{m}^2/\text{ha}) = n_i \cdot \frac{2500}{c^2}$$

where G_i = basal area per hectare of all trees with a dbh of d_i cm. This relationship holds true for any diameter and therefore, the total basal area per unit hectare is

$$G = BAF.N$$

where

$$BAF = \text{Basal Area Factor} = 2500/c^2$$
$$N = \text{total number of trees counted} \ (= \Sigma n_i)$$

The plot radius factor c is calculated as follows:

$$c = \frac{50}{\sqrt{BAF}}$$

The resultant c values for different basal area factors are given below:

BAF (m^2/ha)	1	2	3	4
Value of c	50.0	35.4	28.9	25.0

The c values are not entirely correct, since a given tree is not a flat object with a width of d_i units, but it gives a sufficiently accurate approximation and is adequate for forest inventories. The derivation of the critical angle α is shown in Figure 10-7.

Calculation of the critical angle α:

We have $\sin \dfrac{\alpha}{2} = \dfrac{d_i/2}{r_i}$ and $r_i = \dfrac{d_i}{2 \cdot \sin \dfrac{\alpha}{2}}$

The corresponding ground area is $A = \pi \cdot r^2 = \pi \dfrac{d_i^2}{4 \left(\sin \dfrac{\alpha}{2} \right)^2}$

and $G_i \left(\text{m}^2/\text{ha} \right) = 10^4 \cdot \dfrac{n_i \cdot \dfrac{\pi}{4} \cdot d_i^2}{\pi \cdot \dfrac{d_i^2}{4 \cdot \left(\sin \dfrac{\alpha}{2} \right)^2}} = 10^4 \cdot n_i \cdot \left(\sin \dfrac{\alpha}{2} \right)^2$

and the total basal area per hectare is given by

$$G_{\text{ha}} = 10^4 \cdot \left(\sin \dfrac{\alpha}{2} \right)^2 \cdot \sum_{i=1}^{k} n_i$$
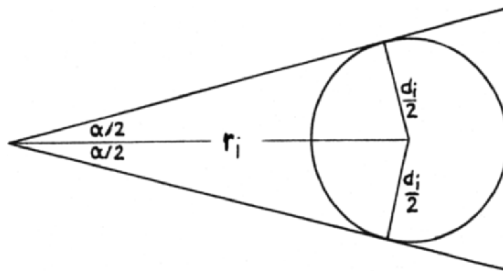

Figure 10-7. Critical angle α.

Table 10-1. Basal Area Factors, corresponding plot radius factors and angle sizes, for the metric system

BAF (m²/ha)	1	2	3	4
Critical angle (°)	1.146	1.621	1.985	2.292
Plot radius factor	49.997	35.351	28.863	24.995

and

$$G_{\text{ha}} = BAF \cdot \sum_{i=1}^{k} n_i \quad \text{where } BAF = 10^4 \cdot \left(\sin \frac{\alpha}{2} \right).$$

Hence

$$\sin \frac{\alpha}{2} = \frac{\sqrt{BAF}}{100}$$

The results are given in Table 10-1.

For $BAF = 1\,\text{m}^2/\text{ha}$, a tree with a diameter of exactly $30\,\text{cm}$, located at a distance of exactly $50 \cdot 0.30 = 15.00\,\text{m}$ is evaluated as a borderline tree. Unless the distance between the sampling point and the tree, as well as its diameter are measured, it remains uncertain whether it falls inside or outside the plot. One way to deal with this problem is to assign a numerical value of 1/2 instead of 1 to this borderline tree.

The general formula for estimating the basal area per hectare at the selected location of the center of the imaginary plot may be generalized to estimate other stand characteristics per unit area, for example, tree volume per hectare. The general formula is

$$y = BAF \cdot \sum^{n} \frac{y_i}{g_i}$$

where y_i = stand characteristic measured on the ith tree, e.g., basal area, volume or number of trees, g_i = basal area of the tree counted "in," $y = G/\text{ha}$, V/ha, or N/ha. The resultant formulae are

Basal area per hectare : $G_{\text{ha}} = BAF \sum^{n} \dfrac{g_i}{g_i} = BAF \cdot n$

Volume per hectare : $V_{\text{ha}} = BAF \sum^{n} \dfrac{v_i}{g_i}$

Number of trees per hectare : $N_{\text{ha}} = BAF \sum^{n} \dfrac{1}{g_i}$

Sukwong et al. (1971) compared the precision of fixed-radius and variable-radius plots for basal area estimates using spatial distribution models to generate artificial forests. The logtransformed coefficient of variation was a linear

function of the logtransformed number of trees per hectare. The following relationship was established:

$$\frac{s_y^2\,(\%)_{\text{fixed radius}}}{s_y^2\,(\%)_{\text{variable radius}}} = b_0 N^{b_1}$$

The coefficient of variation for fixed radius plots was consistently greater than the variable-radius coefficient of variation, if the number of trees sampled was the same for both methods.

3.2   Choice of basal area factor

A decrease in the basal area factor produces a proportional increase in the average number of sample trees counted "in." In consequence, a smaller sample is required to obtain a predefined precision. However, the number of borderline trees increases, whereas its checking, because of the greater distance between the sampling point and the subject tree, is more time-consuming. The optimum basal area factor depends on the stand structure, but the ultimate choice is usually based on practical experience and general guidelines. In Europe, *BAF* values of $1\,\text{m}^2$/ha in low density stands and 2 or $4\,\text{m}^2$/ha in high-density stands are normally used, whereas basal area factors of 5 and $10\,\text{ft}^2$/acre are common in North America. Husch et al. (1982) recommended relating the *BAF* to be used to the number of trees counted "in"

$$BAF = \frac{0,4046(\text{estimated basal area per hectare})}{n}$$

where $n = $ predefined average number of trees to be counted at a given sample point. In European forestry, satisfactory results have been obtained with n varying between 6 and 16, whereas $n = 10$ has given good results in North America.

3.3   Nonsampling errors

Theoretically, point sampling produces unbiased estimates of the basal area per hectare. However, several other error sources, which have an impact on precision and bias, play a role.

• *Instruments*

  The instruments used must be regularly checked and calibrated if necessary, prior to the beginning of the field work. This includes checking and correcting the viewing angle and checking the ratio $1/c$ for a given basal area factor.

- *Sampling position*

  The sampling position must be located objectively, i.e., concurring with a rigidly controlled sampling plan. Although observers tend to select sampling points away from large trees, underbrush, or hanging branches, because they obstruct viewing, this method might induce bias and should be avoided. The same holds true for the practice of temporarily moving away from the sampling point to evaluate hidden trees.

- *Instrument position*

  The instrument used should be positioned vertically above the sampling point, which implies that the position of the vertex of the generated angle must be correct. Most point sampling devices make provision that the vertex point is located at eye level. In the case of a wedge prism, the vertex of the viewing angle is located on the surface of the prism, which is therefore, positioned vertically above the sampling point (Figure 10-8).

  An important source of bias also occurs when the instrument is not positioned perpendicularly to the line of sight. The rotation in the vertical plane, at an angle of 90° to the line of sight, for example, reduces the critical angle, and induces an overestimate of tree count.

- *Incorrect decisions in checking borderline trees*

  Strictly speaking, borderline trees cannot occur, since a given tree either exceeds the critical angle or not. A tree is classified as a borderline tree, if the distance between the sampling point and the stem center is about equal to the tree diameter, multiplied by the plot radius factor. Field checking to eliminate an incorrect decision error due to borderline trees, is time-consuming. In practice, borderline trees are not checked but counted as half the unit value. It is generally assumed that the errors involved are of a random nature and their expected value zero. Experience and careful execution of the field
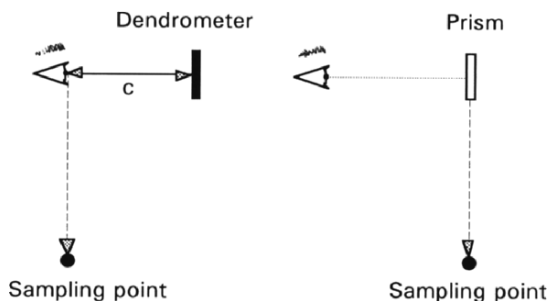

Figure 10-8. Correct methods of positioning the dendrometer and wedge prism.

work are necessary to avoid or minimize systematic errors. In order to implement the checking procedure, the dbh of the borderline tree and its distance from the sampling point must be measured. Suppose they are 43 cm and 22.5 m, respectively, and a mirror relascope with $BAF = 1\,\mathrm{m^2/ha}$, i.e., $c = 50$ is used. The limiting distance of $0.43 \cdot 50 = 21.5\,\mathrm{m}$ is exceeded and the subject tree is "out."

- *Hidden trees and double counting*
  Hidden trees can represent the most important source of bias in high-density stands or in stands with heavy undergrowth. Although hidden, they belong to the population and have to be evaluated. If the line of sight is obstructed by other trees or underbrush, the observer moves temporarily away from the sampling point, but the viewing distance must not be affected. In some cases, precautionary measures are necessary to avoid double-counting. Initially evaluated subject trees are sometimes recounted when the 360° sweep has been completed. For this reason, the first tree counted at a given sampling point should be marked.

- *Neglecting slope corrections*
  A negative bias in basal area estimates occurs when instruments, such as the originally introduced dendrometer without a device for automatic correction for slope, are used. In this case, a slope correction must be made at each sample point. The best way to obtain an unbiased estimate is the arithmetical correction after sampling:

$$G_i = \frac{BAF \cdot n_i}{\cos \alpha_i}$$

were

   G = corrected basal area (m^2/ha) at the ith sampling point

   n_i = number of trees tallied at the sampling point

   α_i = angle of slope at the ith sampling point

On slopes of less than 10%, the slope correction can be ignored because the correction of the basal area is then less than 1% (Table 10-2).

   Suppose that $BAF = 4\,\mathrm{m^2/ha}$ and 7.5 trees are counted. The slope of the sampling unit, measured with a *Suunto clinometer* is 60%. The corrected basal area per hectare is $47.5 \cdot \sec(60°) = 35.1\,\mathrm{m^2}$.

- *Neglecting boundary-overlap correction*
  Boundary overlap in point sampling occurs when a section of the imaginary plot extends beyond the boundary of the forest stand. Ignoring overlap may cause considerable bias in small stands with a large perimeter:area ratio. The mirage method for boundary-overlap correction may then be applied to obtain an unbiased estimate. A second sampling point is selected outside

Table 10-2. Correction factors for slope

Slope (α)	Correction factor ($1/\cos\alpha$)	Slope (%)	Correction factor ($1/\cos\alpha$)
5	1.00	10	1.00
10	1.02	20	1.02
15	1.04	30	1.04
20	1.06	40	1.08
25	1.10	50	1.12
30	1.15	60	1.17
35	1.22	70	1.22
40	1.31	80	1.28
45	1.41	90	1.35
50	1.56	100	1.41

the forest stand and a partial sweep within the sampled stand, is carried out to count trees, which were evaluated in the first sweep. The mirage method should be applied in all cases where the distance between the sampling point and the stand boundary is less than the limiting radius for the largest tree diameters within the boundary zone.

3.4 Efficiency of point sampling

Husch (1955) compared the efficiency of the *BAF* values 2 1/2, 10, and 40 and found a *BAF* of 40 to be more efficient than the others. Clutter (1957) found that *BAF* values of 20 and 40, in terms of degree of unbiasedness, performed better than *BAF* values of 5 and 10. Wiant et al. (1984) concluded that *BAF*-values of 5 and 10 produced underestimates of the basal area per hectare. In order to ensure a sufficient number of trees, counted "in," different *BAF* values may be used for point sampling within a given population. Schreuder et al. (1981) evaluated bias resulting from varying the basal area factor within a given population. It was of the magnitude of 10% and more. The authors proposed applying cluster sampling, with the same *BAF* applied within a given cluster, but different basal area factors for different clusters.

Oderwald (1981) compared plot sampling with point sampling in random, clumped and systematic forests represented by squared lattices. The Poisson function applies when the pattern is random, the negative binomial distribution describes clumped spatial distributions. Point sampling produced the most precise basal area estimates in random and clumped populations, plot sampling performed better in systematic forests.

Gambill et al. (1985) applied the basic idea proposed by Zeide (1980) to optimize plot size and developed a method for determining the optimum basal area factor. The time needed for a single measurement is given by the sum of travel and measurement time. The required sample size was estimated with the aid of the standard formula.

$$n = \left(s_y(\%) \cdot \frac{t}{E} \right)^2$$

The relationship between log $(s_y(\%))$ and log (BAF) is linear, but the parameters of the regression equation

$$s_y(\%) = b_0 BAF^{b_1}$$

vary according to inventory conditions. The authors proposed using the parameter estimate b_1 to estimate the coefficient of variation $s_y(\%)$, corresponding with Q, but the coefficient of variation $s_{y_j}(\%)$ for BAF_j must be known or is estimated:

$$s_{y_i}(\%) = s_{y_j}(\%) \frac{BAF_i^{b_1}}{BAF_j^{b_1}}$$

This approach makes it possible to estimate the sample size required to meet the specified precision. The estimated mean travel time between successive sampling positions within the population, which is a function of sampling intensity and travel rate, must still be added. The optimum fixed-radius plot sizes vary between 200 and $800 \, m^2$, the optimum *BAF* between 6 and 72 in the English system.

Iles et al. (1988) investigated the amount of bias, which results from changing the basal area factor during sampling within a given population, for example, when too few trees are counted. In general, a change of BAF changes the basal area proportionally, but not when the number of trees counted drops below the acceptable minimum. Since doubling the basal area factor in such cases means that the stand is resampled in a partially denser sector, this method will produce positively biased estimates. The same author investigated the necessity of checking the 5–15 borderline trees, which needed to be evaluated (Iles 1988). It was found that about 75% of the borderline trees were correctly classified as "in" or "out," whereas the errors classifying the remaining 25% were two-directional.

Ulbricht (1986) evaluated the cost-efficiency of a stand inventory for variable radius plots and different BAF values. Based on data from two Picea abies *stands, Akça et al. (1986) investigated the efficiency of 0.01 and 0.1 ha fixed-radius sample plots and variable radius plots with BAF values varying between 2 and 10, to determine the optimum plot size. Palley et al. (1961) showed that*

point sampling in random as well as systematic sampling with a random start produces an unbiased estimate of the basal area per hectare. The introduction of the ratio of volume to basal area for trees, which are counted "in" produces an unbiased estimate of the volume per hectare. The method, proposed by Bell et al. (1957) for determining the ratio volume to basal area within a subsample of the angle count trees, produced an almost negligible bias in volume estimates. Barrett et al. (1966) compared point with line sampling in terms of bias and variance of the estimated basal area. Edge bias was approximately 5% for point sampling and 3% for line sampling. Depending upon the critical angle, the coefficient of variation for point sampling, as well as line sampling, varied between 40% for a critical angle of 104.18 min and 60% for an angle of 180.46 min.

4 SIMPLE RANDOM SAMPLING

Suppose that a 10 ha forest stand is subdivided into 40 sampling units of 50 × 50 m. Three sampling units are selected and measured to estimate the volume and other stand characteristics. This procedure generates $40!/(3!37!) = 9880$ possibilities for selecting a sample of size 3 (section 2.6.1). Simple random sampling requires that each of these combinations has the same chance of being selected. The method guarantees that no selection bias is involved. In practice, selection bias occurs quite frequently. In order to prepare a height curve for a given stand, the field team, for example, will possibly tend to exclude excessively leaning trees and trees with broken tops and in other instances might avoid inaccessible locations.

In the above case, field sampling is preceded by a subdivision of the target population into N sampling units. It requires a list of these units, with each of them identified by a number between 1 and 40. A table of random numbers, or a random numbers generator, is subsequently used to draw the three sampling units. In such inventories each sampling unit is not allowed to occur more than once in a given sample. The method is described as *sampling without replacement*. This procedure does not violate the basic concept of random sampling, but the method requires an adjustment of the variance formulae. The sample size cannot exceed N and the ith sampling unit in the population cannot occur more than once in a given sample. The probability of the ith sampling unit being drawn first is 1/40. A second sampling unit is drawn at random. The probability of the jth unit being included is 1/39, since this element is drawn from a population which contains $N - 1$ sampling units. For the third sampling unit drawn, the probability of the kth unit being selected is 1/38. The sampling units are

drawn consecutively, but independently. The probability of the joint occurrence of the ith, jth, and kth sampling unit is $1/40 \; 1/39 \; 1/38 = 0.00001687$, but in sampling *with replacement*, this probability would be $(1/40)^3 = 0.00001525$. For large populations and a small sampling fraction, the difference between sampling with and without replacement is negligible.

In order to obtain an unbiased estimate of the variance of the population of sample means, based on sampling without replacement, the variance of the sample mean is multiplied by the *correction factor for the finite population* $f = (1 - n/N)$. The ratio n/N is denoted as the *sampling fraction*, the ratio N/n as the *expansion factor*. The target population may contain the aggregate of tree heights, breast height diameters, or single-tree volumes within a given stand, with the individual tree serving as the sampling unit. In such cases, the size of the population is probably unknown, since its determination would require a stem count. On the other hand, when ten 20×20 m sample plots are established, for example, to estimate the total volume of the trees within a 50 ha compartment, the population contains (approximately) 1250 sampling units, the sampling fraction is 0.008 and the expansion factor is 125. A sample with the same number of sampling units, but based on plots with an area of $200 \, \text{m}^2$, generates a population containing 2500 sampling units and the sampling fraction and expansion factor are 0.004 and 250, respectively.

When sample plots are established at random and measured to determine the plot volume, the mean of these volumes, multiplied by the expansion factor stand area: plot area, estimates the total stand volume. The estimate is unbiased if the sample was drawn at random and no other sources of bias were involved. It does not necessarily imply that the sample mean and sample sum produce reliable information about the true population mean or total. For practical purposes, the information derived from sampling may be useless, either because the sample was too small or due to excessive variability. A random selection of sampling units, however, makes it possible to draw conclusions about the quality of the information obtained by sampling. The application of the statistical theory to study the sampling distribution of the inventory data, assumes that the sample means are independently and normally distributed. Unless plot size is excessively small, this assumption is usually approximately satisfied, although confidence intervals are inevitably distorted, if the assumption of a normal distribution is not satisfied.

The sample of size n also provides an unbiased estimate of the population variance. It should be emphasized that this variance is defined as the expected value of the squared deviation of an observed value from the population mean. Establishing plots of different sizes generates populations with different variances. In consequence, it makes no sense to compare variances which were

obtained from sampling the same stand with different plot sizes, unless these variances are multiplied by an expansion factor to estimate the variance of the population aggregate or when they are converted into their equivalents per unit area.

Usually, sampling based on individual trees as sampling units and sampling based on either fixed-or variable-radius sample plots, serve a different purpose. The individual tree is a useful sampling unit for estimating the mean wood density or the mean vitality of trees within a stand, for example. Sampling for forest inventories, however, is usually undertaken to estimate physical entities such as basal area, number of trees, and volume on a per hectare basis. When a random sample of n single trees is used to estimate the stand volume, the sum of the volumes, multiplied by the expansion factor N/n provides an unbiased estimate of the total volume, but the precision of this estimate is low unless a large sample has been drawn. A more accurate estimate will be obtained, if the volume of a single tree is multiplied by the expansion factor stand area:tree area to estimate the stand volume. The resultant variance of the population aggregate would certainly be substantially lower, but it is difficult, if not impossible, to determine the tree area of the individual tree. The same problem arises in small sample plots. The difficulty of determining the area occupied by all 25 trees in a sample plot with a size of 1/20 ha is usually overlooked and this total tree area is invariably equated with the size of the sample plot. The determination of the areas occupied by edge trees in a sample plot is technically impossible without destructive sampling, since the extent of the root system is to be assessed. It is even more difficult in tree breeding experiments, where the single sampling unit may consist of a row of six trees, with one or two of them dying during the first 5 years. In that case, the expansion factor to be used in estimating the volume per hectare from the remaining 4 or 5 trees is inaccurate and may not reflect the true area occupied by the remaining trees.

In management inventories the assumption of a random selection of sampling units is sometimes violated

- The need of drawing up a list of all sampling units, of necessity identified by their location within the population may have been overlooked.
- The estimation of stand volume requires independent estimates of the mean height of each sample plot. The usual procedure, however, is to draw a random sample of trees within a given compartment, to measure their heights and to fit a single height curve. Theoretically, the assumption of independent volume estimates is violated, since the relationship between diameter and height may well be affected by site quality differences within a given compartment and, in that case, should not be estimated from pooled data.

The standard procedures for calculating stand characteristics and their variances furthermore assume that diameter and height, as well as other stand characteristics, are measured without measurement-bias being involved. The error calculation furthermore assumes that errors associated with the volume function being used, are negligible and can be ignored.

The sample mean ($\overline{y}$), derived from simple random sampling

$$\overline{y} = \frac{\sum y_i}{n}$$

is a consistent estimate of the population mean, i.e., it converges to the population mean with increasing n and is equal to this mean if $n = N$ and the sample units are selected without replacement. The sample mean represents a point estimate of the population mean μ, the sample total $\hat{Y}$ is a point estimate of the population total Y. The standard error of the sample mean, based on sampling without replacement is

$$s_{\overline{y}} = \frac{s_y}{\sqrt{n}} \cdot \sqrt{1 - \frac{n}{N}} = \sqrt{\frac{SS_{yy}}{n \cdot (n-1)} \cdot \left(1 - \frac{n}{N}\right)}$$

where $SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$

The estimated population total is

$$\hat{Y} = N\overline{y}$$

Its standard error is

$$s_{\hat{Y}} = N s_{\overline{y}}.$$

When preparing working plans, it is often useful to calculate the mean volume per hectare and a $(1 - \alpha)$ confidence interval for this estimate. This requires multiplying the sample mean and its standard error by the expansion factor q:

$$q = \frac{10000}{\text{plot size in m}^2}$$

The calculation of a confidence interval assumes a normal distribution of the subject variable. In the case of skewed distributions, the observed relative frequency of the exceeded confidence limits is always greater than the stated probability. Independent of the degree of skewness and kurtosis of a distribution, the sample means, obtained from nonnormal distributions, converge to a normal distribution with increasing sample size. This is known as the *central limit theorem* and it has a considerable impact on inferences in satisfying the assumption

of normally distributed sample means. The $(1 - \alpha)$ confidence interval for the population mean and total are found as follows:

Population mean : $\bar{y} \pm t_{\alpha,(n-1)} \cdot s_{\bar{y}}$

Population total : $\hat{Y} \pm t_{\alpha,(n-1)} \cdot s_{\hat{Y}}$

The quality of information derived from forest inventories is partly controlled by the sampling fraction. Forest managers require information on the accuracy of these estimates. The information is imperative, for example, where the amount of timber to be delivered to the saw or pulp mill is to be estimated. In other instances, accuracy assessments are needed for forest management plans and to prepare logging operations.

Although the forest manager is primarily interested in estimates of the total volume and volume per hectare, it may be necessary to calculate a confidence interval for other stand parameters, for example, the mean stand diameter. In the case of fixed-or variable-radius sample plots, the mean diameter is obtained from clusters of trees instead of randomly selected individual sample trees. In consequence, the distribution of the mean diameters of the *n* sample plots, instead of the diameters of single trees, should be used to calculate a confidence interval for the mean stand diameter. A weighting procedure may be necessary, due to unequal tree numbers in these sample plots. The number of degrees of freedom is equal to the number of clusters (= plots) minus 1. In this situation, the *central limit theorem* also ensures that these mean diameters are approximately normally distributed. The mean height and its confidence interval are based on regression analysis. The standard deviation of the *n* observed tree heights is, therefore, replaced by the standard deviation of the regression.

Example 10.1 A compartment with an area of 6 ha is surveyed by simple random sampling. The size of the sampling units is 0.01 ha and sample size is 25, hence $N = 600$. The plot volumes in cubic meters are:

1.6	3.8	1.6	1.5	3.5
5.7	1.2	1.5	3.3	4.8
3.6	6.0	1.1	4.7	6.2
1.2	7.5	1.6	6.6	1.7
6.8	3.5	4.1	3.4	1.5

The mean and total volume and their standard errors are:

$$n = 25; \ \overline{y} = 3.52 \, \text{m}^3/0.01 \, \text{ha}; \ \hat{Y} = 600 \cdot 3.52 = 2112 \, \text{m}^3;$$

$$\Sigma y = 88.0; \ \Sigma y^2 = 410.68$$

$$SS_{yy} = 410.68 - \frac{88.0^2}{25} = 100.92; \ s_y^2 = \frac{100.92}{24} = 4.205$$

$$s_{\overline{y}} = \sqrt{\frac{4.205}{25} \cdot \left(1 - \frac{25}{600}\right)} = \pm 0.4015; \ s_{\hat{Y}} = 600 \cdot 0.4015 = \pm 240.9 \, \text{m}^3$$

and the corresponding confident intervals are:

95% C.I. for μ_y (per plot) : $3.52 \pm 2.064 \cdot 0.4015 \Rightarrow$ 2.69 4.35 m^3

95% C.I. for μ_y (per hectare) : $3.52 \pm 2.064 \cdot 0.4015 \Rightarrow$ 269 435 m^3

95% C.I. for aggregate : $2112 \pm 2.064 \cdot 240.9 \Rightarrow$ 1615 2609 m^3

4.1   Sample size

Sampling based on scientific methods requires a precalculation of the sample size, before conducting the field survey. The estimate should ensure that a given maximum error will not be exceeded with a specified probability equal to or less than $(1 - \alpha)$. The selection of a 90% probability implies a risk of 1 in 10 that the specification will not be achieved, a 99% probability is associated with a risk of 1 in 100. Suppose that the mean volume per hectare is to be estimated with an error not in excess of E units. The maximum error, for an associated probability of 0.95, and based on sampling without replacement is as follows:

Population mean: $E = t_\alpha \cdot \dfrac{s_y}{\sqrt{n}} \cdot \sqrt{1 - \dfrac{n}{N}}$

Population total: $E = t_\alpha \cdot N \cdot \dfrac{s_y}{\sqrt{n}} \cdot \sqrt{1 - \dfrac{n}{N}}$

In practice, it may be more useful to express the maximum error in terms of a percentage of the true, but unknown volume, per hectare. This implies that the standard deviation s_y in the equations for sampling mean should be replaced by the coefficient of variation $s_y(\%)$:

$E\% = t_\alpha \cdot \dfrac{s_y(\%)}{\sqrt{n}} \cdot \sqrt{1 - \dfrac{n}{N}}$

Furthermore, if the variance of the population is known, the formulae reads as follows

Population mean: $E = z_{\alpha/2} \cdot \dfrac{\sigma_y}{\sqrt{n}} \cdot \sqrt{1 - \dfrac{n}{N}}$

Population total: $E = z_{\alpha/2} \cdot N \cdot \dfrac{\sigma_y}{\sqrt{n}} \cdot \sqrt{1 - \dfrac{n}{N}}$

where $z = $ unit normal variate. Since the true variance is usually unknown, the sample variance replaces the population variance and the t-statistic replaces the unit normal variate. The above equation must be solved by trial and error. An estimate for σ^2 may be available from previous surveys or can be obtained by sampling. In that case, the sampling procedure is carried out in two stages. The stage 1 sample serves primarily to estimate σ^2, which is necessary to determine the required sample size. If the stage 1 sample, because of cost constraints might contain few observations, the population variance σ^2 is estimated inaccurately. Confidence intervals for the population variance based on sampling are notoriously wide. To illustrate the dilemma, we consider a dataset consisting of the basal areas, measured on four 12×12 m sample plots in a *P. radiata* plantation. The observed basal areas were $0.771\,\mathrm{m}^2$, $0.488\,\mathrm{m}^2$, $0.911\,\mathrm{m}^2$, and $1.183\,\mathrm{m}^2$, respectively. The sample variance is 0.2514 and limits of the 0.90 confidence interval for the population variance is obtained from the equation

$$\frac{s_y^2}{\chi_{0.05,4}^2} < \sigma^2 < \frac{s_y^2}{\chi_{0.95,4}^2}$$

with $\chi_{0.05,4\,\mathrm{df}}^2 = 9.49$ and $\chi_{0.95,4\,\mathrm{df}}^2 = 0.71$. The resultant limits are

$$0.026 < \sigma^2 < 0.354.$$

The confidence limits are obviously too wide to be useful for a determination of the required sample size. Alternatively, it may be possible to recover the coefficient of variation from past inventories and to fit a regression equation with age, site index, species, and management regime as independent variables and the coefficient of variation as the target variable.

The formula for the maximum error E as a function of sample variance, confidence coefficient, and sample size

$$E = t_{\alpha,n-1}\sqrt{\frac{s^2}{n}}$$

is solved for n

$$n = \left[t_{\alpha,n-1}\frac{s}{E} \right]^2$$

Adjusting the formula for the finite population gives

$$n = \cfrac{1}{\cfrac{E^2}{t_{\alpha,n-1}^2} + \cfrac{1}{N}}$$

The standard deviation is replaced by the coefficient of variation when the maximum allowable error is defined in terms of a percentage.

Stauffer (1982) improved the commonly used iterative procedure for estimating sample size in unrestricted random sampling, since the latter does not always converge to the population mean. The formula for the required sample size

$$n = t_\alpha^2 \left(\frac{s_y\%}{E\%} \right)^2$$

where E, expressed as a percentage of the mean, is rewritten as follows:

$$\frac{n}{t^2} = \left(\frac{s_y\%}{E\%} \right)^2$$

The expression on the left-hand side is evaluated for a series of ascending values of n, until convergence has been reached and $n/t^2 = (s_y\%/E\%)^2$.

Example 10.2 Sample plots were established in a compartment of 15 ha. The size of the sample plots was 0.04 ha, hence $N = 375$. From previous, inventories it was known that the coefficient of variation of the plot volumes was about 32%. It was required that there were to be a less than 1:20 chance of the error exceeding 10% (probability of risk). As a first approximation, we assume $n = 25$. The associated number of degrees of freedom is 24 and $t_{0.05}$(two tailed) $= 2.064$. Hence

$$n = \frac{2,064^2 32^2}{10^2} = 44$$

Since $t_{0.05(43df)} = 2.021$, the estimated sample size is adjusted accordingly

$$n = \frac{2.021^2 \cdot 32^2}{10^2} = 42$$

Two or three iterations are usually sufficient to reach convergence. The resultant estimate is inevitably conservative because of sampling without replacement. Adjusted for the finite population, the required sample size is approximately 38.

The formula for calculating sample size may be simplified by inserting a t value of 2 into the above equation. This approximation is acceptable,

if a 5% probability to exceed the maximum error is specified and sample size is greater than 30.

Random sampling, although being a prerequisite to obtain unbiased estimates of the population mean and total and their variance, does not guarantee that the estimate is indeed unbiased. Bias may be associated with the following sources

- Instrument errors (see Chapter 3)
- Operator bias, for example, when there is a tendency to either include or exclude trees growing on plot boundaries
- Bias associated with the method used to estimate parameters, for example, by using the arithmetic mean diameter to estimate the diameter of the tree with the mean volume
- Applying ridge regression instead of ordinary least squares to estimate the parameter vector, or calculating an unweighted mean in those cases where weighting is more appropriate

Possible sources of bias should be identified, but at the same time the implications of reducing or completely eliminating bias should be considered. Some sampling methods, for example, produce slightly biased estimates, but are preferred to others because of a higher precision or lower cost.

Yandle et al. (1981) noted the fundamental difference between the basic principle of simple random sampling and fixed-radius circular plot sampling. Simple random sampling requires a subdivision of the forest into N mutually exclusive, equally large plots and assumes that any part of the forest is located within one of the sampling units. However, when the forest is subdivided into mutually exclusive circular plots, 21% of the forest area is excluded, i.e., is not located within this population of circular plots. The authors concluded that "the definition of the population as non-overlapping circular plots is not satisfactory, either when sampling is to be with randomly located plots or when plot centers are placed on a systematic grid with a random start." It was suggested that plots must be allowed to overlap, as in point sampling.

5   ERROR PROPAGATION

In many sampling studies, the subject estimate ($\bar{z}$) is defined as the sum, difference, product, or ratio of two or more estimates, $\bar{x}$ and $\bar{y}$, or it is defined as a linear combination of $\bar{x}$ and $\bar{y} : \cdot \bar{z} = c_1\bar{x} + c_2\bar{y} : \bar{z} = c_1\bar{x} + c_2\bar{y}$. In such cases, the variance of $\bar{z}$ is a function of the variances of $\bar{x}$ and $\bar{y}$.

Equation   **Variance**

1. $\bar{z} = \bar{x} + \bar{y}$ $s_{\bar{z}}^2 = s_{\bar{x}}^2 + s_{\bar{y}}^2 + 2s_{\overline{xy}}$

2. $\bar{z} = c_1\bar{x} + c_2\bar{y}$ $s_{\bar{z}}^2 = c_1^2 s_{\bar{x}}^2 + c_2^2 s_{\bar{y}}^2 + 2c_1c_2 s_{\overline{xy}}$

3. $\bar{z} = \bar{x} - \bar{y}$ $s_{\bar{z}}^2 = s_{\bar{x}}^2 + s_{\bar{y}}^2 - 2s_{\overline{xy}}$

4. $\bar{z} = \bar{x} \cdot \bar{y}$ $s_{\bar{z}}^2 = \bar{y}^2 s_{\bar{x}}^2 + \bar{x}^2 s_{\bar{y}}^2 + 2 \cdot \bar{x} \cdot \bar{y} \cdot s_{\overline{xy}}$

5. $\bar{z} = \dfrac{\bar{x}}{\bar{y}}$ $s_{\bar{z}}^2 = \bar{z}^2 \left[\dfrac{s_{\bar{x}}^2}{\bar{x}^2} + \dfrac{s_{\bar{y}}^2}{\bar{y}^2} - \dfrac{2s_{\overline{xy}}}{\bar{x} \cdot \bar{y}} \right]$

6. $\bar{z} = c \cdot \bar{x} \cdot \bar{y}$ $s_{\bar{z}}^2 = c^2(\bar{y}^2 s_{\bar{x}}^2 + \bar{x}^2 s_{\bar{y}}^2 + 2 \cdot \bar{x} \cdot \bar{y} \cdot s_{\overline{xy}})$

where

$c, c_1, c_2 = $ Constants

$s_{\bar{x}}^2, s_{\bar{y}}^2 = $ Variances of $\bar{x}$ and $\bar{y}$

$$ s_{\bar{x}}^2 = \frac{\sum x^2 - \dfrac{(\sum y)^2}{n}}{n(n-1)} \quad \text{and} \quad s_{\bar{y}}^2 = \frac{\sum y^2 - \dfrac{(\sum y)^2}{n}}{n(n-1)} $$

$s_{\overline{xy}} = $ Covariance between $\bar{x}$ and $\bar{y}$

$$ s_{\overline{xy}} = \frac{\sum xy - \dfrac{\sum x \sum y}{n}}{n(n-1)} \quad \text{or } s_{\overline{xy}} = r \cdot s_{\bar{x}} s_{\bar{y}} $$

when $\bar{x}$ and $\bar{y}$ are

when $\bar{x}$ and $\bar{y}$ are independent $\Rightarrow s_{\overline{xy}} = 0$.

Case (1) and (2) arise in a sampling study, for example, to estimate the tree biomass defined as the sum of the biomass of crown and bole. The estimates are usually obtained by matched sampling, with both biomass components determined on a sample of n trees from the population. Case (3) occurs in a similar sampling study for estimating the difference between the above- and below-ground tree biomass and in continuous inventories to estimate changes. Case (4) arises when calculating the variance of the estimated volume of logs obtained by measuring the cross-sectional area at the log midpoint and its length. Case (5) is a ratio of means estimator and occurs, e.g., when the mean volume is estimated from the estimated total volume and estimated total area. In cases (4) and (5) the variance of $\bar{z}$ could also be obtained as follows:

$$ (s_{\bar{z}}\%)^2 = (s_{\bar{x}}\%)^2 + \left(s_{\bar{y}}\%\right)^2, $$

but only if $\bar{x}$ and $\bar{y}$ are independent.

The above formulae apply when a given variable represents a linear combination of other variables or is defined as a ratio or product. Other, more complex situations occur in forest inventories.

Example 10.3 Sample plot measurements in a mixed forest, consisting of species A and B, produced the following plot volumes:

Plots	1	2	3	4	5	6	7	8	9	10
Species A	35	27	41	43	32	38	39	31	38	36
Species B	36	40	31	28	45	41	38	44	41	47
Species A + B	71	67	72	71	77	79	77	75	79	83

The mean and total plot volume and their standard errors are:
   Based on error propagation formula

$$\bar{v}_A = 36 \, \mathrm{m}^3/\mathrm{plot},$$

$$\bar{v}_B = 39.1 \, \mathrm{m}^3/\mathrm{plot}, \, \bar{v}_{\mathrm{total}} = 75.1 \, \mathrm{m}^3/\mathrm{plot}; \, \cdot s_{\bar{v}_A}^2 = 2.38 \cdot (\mathrm{m}^3/\mathrm{plot})^2,$$

$$\cdot s_{\bar{v}_B}^2 = 3.65 (\mathrm{m}3/\mathrm{plot})^2,$$

$$s_{\bar{A}\bar{B}} = -1.856, \, s_{\bar{v}_{\mathrm{total}}}^2 = 2.38 + 3.65 + 2 \cdot (-1.856) = 2.32 (\mathrm{m}^3/\mathrm{plot})^2$$
$$s_{\bar{v}_{\mathrm{total}}} = \pm 1.51 \, \mathrm{m}^3/\mathrm{plot}$$

$$s_A^2 = 23.78; \, s_B^2 = 36.54; \, s_{AB} = -18.56;$$
$$s_{v(\mathrm{tot})}^2 = 23.78 + 36.54 - 2 \cdot 18.56 = 23.2; \, s_{v(\mathrm{tot})} = 4.81$$

Based on total volumes

$$\bar{v}_{\mathrm{total}} = 75.1 \, \mathrm{m}^3/\mathrm{plot}, \, s_{\bar{v}_{\mathrm{total}}}^2 = 2.32 (\mathrm{m}^3/\mathrm{plot})^2, \, s_{\bar{v}_{\mathrm{total}}} = \pm 1.51 \, \mathrm{m}^3/\mathrm{plot}$$

Example 10.4 Sample plot measurements in a beech stand produced a volume estimate of $320 \, \mathrm{m}^3/\mathrm{ha}$. The standard error of the estimated volumes was $15 \, \mathrm{m}^3/\mathrm{ha}$. The estimated compartment area was 10.0 ha, with a standard error of 0.02% (0.2 ha). The estimated total volume is $3200 \, \mathrm{m}^3$. Defining volume and area as variables x and y, with $z = x \cdot y$, we obtain an estimated total volume of $3200 \, \mathrm{m}^3$. The standard error in % is

$$s_{\bar{v}}(\%) = \sqrt{\frac{0.2^2}{10^2} + \frac{15^2}{320^2}} = 5$$

Cunia (1965) summarized the error sources in timber volume inventories and their consequences:

- *Sampling error*
- *Error associated with the volume function*
- *Measurement errors*

*Gertner (1990) analyzed each of these error sources in greater detail and in many publications, emphasized a shortcoming dealing with the precision obtained in a forest inventory. The latter is usually evaluated in terms of the standard deviation of the mean volume, which expresses sampling error but ignores errors due to sources previously mentioned by Cunia (1965). To remedy this situation, the author presented a method for estimating the total error and its three components. The variance associated with the regression estimate for a model which is linear in its parameters was estimated with conventional formulae for linear regression. When using a function which is nonlinear in its parameters, the error is approximated with the aid of a Taylor series expansion about the parameter vector **b**, which is truncated after the first term. In order to evaluate the effect of measurement errors, it is assumed that the observed attribute is the sum of its true value and a component, associated with the error of measurement, which is a normally distributed variate. The variance of the mean volume per hectare is calculated as the sum of the variances associated with sampling error, the error of the regression function, and measurement errors of the variables in question. In addition, it contains a component expressing the squared bias. Assuming unbiased measurements of the variables involved (dbh, height), the mean square for sampling error as well as the total error decreased curvilinearly with the number of plots being measured. The assumption of a 2% bias in diameter measurements, however, induced a dramatic increase in the mean square for total error.*

6   STRATIFIED RANDOM SAMPLING

6.1   Basic principles

The purpose of stratification is to group sampling units on the basis of homogeneity of the target variable. In order to generate homogeneous strata, the variables used to stratify the population (site quality, age, stand composition, stand treatment) should be closely related to the quantity being measured. In many instances, particularly in the inventories of large forest areas aerial photographs are used as a basis for stratification, while it is sometimes feasible to stratify on the basis of soil differences, identifiable on soil maps. In other situations,

a random sample of a given size, possibly proportional to the size of the compartment, is drawn from all compartments within a plantation or forest district. In that case, the population might be poststratified on the basis of tree species (or forest type) as well as age, or on the basis of species and height classes.

Assuming that the strata and their sizes are known prior to sampling, a random sample is drawn within each of the L strata. They represent subpopulations, assumed to be more homogeneous than the parent population. The sample mean of the jth stratum is

$$\bar{y}_j = \frac{\sum\limits_{i=1}^{n_j} y_{ji}}{n_j}$$

where

y_{ji} = measurement on the ith sampling unit in the jth stratum

n_j = sample size in stratum j

and the estimated population mean and total are as follows:

$$\bar{y}_{\text{str}} = \frac{\sum\limits_{j=1}^{L} N_j \cdot \bar{y}_j}{N_1 + \div \cdots \div N_L} = \frac{\sum\limits_{j=1}^{L} N_j \cdot \bar{y}_j}{N}$$

$$\hat{Y} = N \cdot \bar{y}_{\text{str}}$$

where

L = number of strata

N_j = size of the jth stratum

N = population size

The mean of the stratified population is therefore calculated as a weighted mean of the stratum means with weights assigned in proportion to stratum sizes. The estimated variance of the jth stratum is

$$s_j^2 = \frac{\sum\limits_{i=1}^{n_j} \left(y_{ji} - \bar{y}_j\right)^2}{n_j - 1}$$

and the variance of the j-stratum mean, corrected for the finite population, is

$$s_{\bar{y}_j}^2 = \frac{s_j^2}{n_j} \cdot \left(1 - \frac{n_j}{N_j}\right)$$

If required, the standard error of a stratum mean and total might be used to calculate a confidence interval either for the mean and aggregate of the jth stratum. An unbiased estimate of the variance of the population aggregate is obtained by adding the variances of the estimated stratum aggregates. The covariances are zero, since the L strata are sampled independently. It can be shown that the variance of the stratified population mean and total can be estimated from

$$s^2_{\bar{y}_{\text{str}}} = \frac{1}{N^2} \cdot \sum_{j=1}^{L} \left[\frac{N_j^2}{n_j} \cdot s_j^2 \cdot \left(1 - \frac{n_j}{N_j} \right) \right] \text{ or } s^2_{\bar{y}_{\text{str}}} = \frac{1}{N^2} \sum_{j=1}^{L} \left[\cdot \frac{N_j}{n_j} s_j^2 (N_j - n_j) \right]$$

$$s^2_{\hat{Y}} = \sum_{j=1}^{L} \left[\frac{N_j^2}{n_j} \cdot s_j^2 \cdot \left(1 - \frac{n_j}{N_j} \right) \right] \text{ or } s^2_{\text{str}} = \frac{1}{N^2} \sum_{j=1}^{L} \left[\cdot \frac{N_j}{n_j} s_j^2 (N_j - n_j) \right]$$

The formulae can be simplified, if the strata were sampled proportional to stratum size.

Example 10.5   Fixed-radius 0.1 ha sample plots were laid out in a compartment with an area of 60 ha, subdivided into three age strata. The stratum areas were 18.4, 17.6, and 24.0 ha, respectively. The observed plot volumes in cubic meters were

Stratum	n	y_j									
1	8	44	37	31	38	28	40	33	40		
2	7	13	16	17	20	15	15	20			
3	10	59	55	57	65	56	65	58	44	44	50

Stratum	N_j	n_j	$\bar{y}_j$	SS_y	s_j^2	s_j	$s_{\bar{y}_j}$
1	184	8	36.38	197.9	28.27	5.32	1.88
2	176	7	16.57	41.7	6.95	2.64	1.00
3	240	10	55.30	496.1	55.12	7.42	2.35

$$\bar{y}_{\text{str}} = \frac{184 \cdot 36.38 + 176 \cdot 16.57 + 240 \cdot 55.30}{600} = 38.14 \, \text{m}^3/0.1\,\text{ha}$$

$$\hat{Y} = 600 \cdot 38.14 = 22884 \, \text{m}^3$$

The variance of the mean is

$$s_{\bar{y}_{str}}^2 = \frac{1}{600^2} \cdot \left[\frac{184}{8} \cdot 28.27 \cdot (184 - 8) + \frac{176}{7} \cdot 6.95 \cdot (176 - 7) \right.$$
$$\left. + \frac{240}{10} \cdot 55.12 \cdot (240 - 10) \right] = 1.245$$

$$s_{\bar{y}_{str}} = 1.115; \quad s_{\hat{Y}}^2 = 448200; \quad s_{\hat{Y}} = 669.5 \, \text{m}^3$$

The $(1-\alpha)$ confidence interval for the population mean and total are as follows:

$$\bar{y}_{str} \pm t_{\alpha, \sum (n_j - 1)} \cdot s_{\bar{y}_{str}} \quad 38.14 \pm 2.074 \cdot 1.115 : 35.86 \quad 40.41$$

$$\hat{Y} \pm t_{\alpha, \sum (n_j - 1)} \cdots \cdot s_{\hat{Y}} \quad 22884 \pm 2.074 \cdot 669.477 : 21496 \quad 24272$$

The efficiency of stratified random sampling can be assessed by estimating the variance from the pooled data obtained from the L strata. It gives a reliable estimate of the variance of an unrestricted random sample in the case of proportional allocation. Cochran (1977) introduced the more correct estimate

$$s_{\bar{y}_{srs}}^2 = \frac{N - n}{n (N - 1)} \cdot \left[\frac{1}{N} \sum_{j=1}^{L} \frac{N_j}{n_j} \sum_{i=1}^{n_j} y_{ji}^2 - \bar{y}_{str}^2 + s_{\bar{y}_{str}}^2 \right]$$

This formula applies in case of a nonproportional allocation.

Example 10.5 (continued)
In the present example the variance estimate obtained by pooling is

$$s_y^2 = \frac{\sum_{j=1}^{L} \sum_{i=1}^{n_j} (y_{ij} - \bar{y})^2}{24} = 290$$

and

$$s_{\bar{y}_{srs}} \approx \sqrt{\frac{290}{25} \cdot \left(1 - \frac{25}{600}\right)} = 3.334$$

Applying the correct formula gives

$$s_{\bar{y}_{srs}} = \sqrt{\left(\frac{\frac{184}{8} \cdot 10783 + \frac{176}{7} \cdot 1964 + \frac{240}{10} \cdot 31077}{600} - 38.14^2 + 1.245 \right) \cdot \left(\frac{600 - 25}{599 \cdot 25} \right)}$$

$$= 3.31 \, \text{m}^3 / 0.1 \, \text{ha}$$

The standard error of the mean of the stratified population is 1.115. The ratio of the squared standard errors is 9 to 1. In consequence, a ninefold increase of the sample size is required to match the precision of stratified random sampling.

6.2   Allocation methods

6.2.1   Proportional allocation

The simplest rule is to allocate the n sampling units proportional to the size of the strata

$$n_h = n\frac{N_h}{N}$$

which is rounded off to the nearest integer.
The required total sample size is

$$n = \frac{\sum_{j=1}^{L} \frac{N_j}{N} \cdot s_j^2}{s_{\bar{y}_{str}}^2} \quad \text{(sampling with replacement) or}$$

$$n = \frac{\sum_{j=1}^{L} \frac{N_j}{N} \cdot s_j^2}{s_{\bar{y}_{str}}^2 + \sum_{j=1}^{L} \frac{N_j \cdot s_j^2}{N^2}} \quad \text{(sampling without replacement)}$$

Proportional allocation is appropriate if the stratum variances are equal and is furthermore applied when no prior knowledge is available on the variances within the L strata.

6.2.2   Optimum allocation

There are frequently significant differences between the variances of the plot volumes observed in the L strata. In that case, the optimum allocation rule performs better than proportional allocation. It represents an optimum strategy, i.e., it minimizes the variance of the estimated population mean and total for the total sample size to be fixed. The allocation rule is

$$n_h = \frac{N_h s_h}{\sum\limits^{L} N_h s_h}$$

Total sample size is

$$n = \frac{\left(\sum_{j=1}^{L} \frac{N_j}{N} \cdot s_j\right)^2}{s_{\bar{y}_{str}}^2} \quad \text{(sampling with replacement)}$$

or

$$n = \frac{\left(\sum_{j=1}^{L} N_j \cdot s_j \right)^2}{N^2 \cdot s_{\bar{y}_{str}}^2 + \sum_{j=1}^{L} \left(N_j \cdot s_j^2 \right)} \quad \text{(sampling without replacement)}$$

6.2.3 Optimum allocation with variable sampling cost

This modified allocation rule is an improvement over the previous one, and should be applied if the sampling cost per sampling unit varies across strata. Certain parts of the forest, for example, may be more easily accessible than others, in which case the enumeration cost per sampling unit are lower. The following rule produces the highest precision for fixed total sampling cost:

$$n_j = \frac{\left(N_j \cdot s_j \right) / \sqrt{c_j}}{\sum_{j=1}^{L} \left(\left(N_j \cdot s_j \right) / \sqrt{c_j} \right)} \cdot n$$

The required total sample size is

$$n = \frac{\sum_{j=1}^{L} \left(\frac{N_j}{N} \cdot s_j \cdot \sqrt{c_j} \right) \cdot \sum_{j=1}^{L} \frac{\frac{N_j}{N} \cdot s_j}{\sqrt{c_j}}}{s_{\bar{y}_{str}}^2} \quad \text{(sampling with replacement)}$$

or

$$n = \frac{\sum_{j=1}^{L} \left(\frac{N_j}{N} \cdot s_j \cdot \sqrt{c_j} \right) \cdot \sum_{j=1}^{L} \frac{\frac{N_j}{N} \cdot s_j}{\sqrt{c_j}}}{s_{\bar{y}_{str}}^2 + \left(\sum_{j=1}^{L} \frac{N_j}{N} \cdot s_j^2 \right) / N} \quad \text{(sampling without replacement)}$$

where c_j = the cost per sampling unit in the jth stratum.

Example 10.6 The total sample size for the data in Example 5 is 25. *Proportional allocation* gives the following estimates:

$$n_1 = \frac{184}{600} \cdot 25 \cong 8, \quad n_2 = \frac{176}{600} \cdot 25 \cong 7, \quad n_3 = \frac{240}{600} \cdot 25 \cong 10$$

The optimum allocation rule gives

$$n_1 = \frac{184 \cdot 5.32}{184 \cdot 5.32 + 176 \cdot 2.64 + 240 \cdot 7.42} \cdot 25 \cong 7$$

$$n_2 = \frac{176 \cdot 2.64}{184 \cdot 5.32 + 176 \cdot 2.64 + 240 \cdot 7.42} \cdot 25 \cong 4$$

$$n_3 = \frac{240 \cdot 9.63}{184 \cdot 5.32 + 176 \cdot 2.64 + 240 \cdot 7.42} \cdot 25 \cong 14$$

Assuming $c_1 = 8, c_2 = 10$, and $c_3 = 6$, *optimum allocation with variable sample* cost gives

$$n_1 = \frac{184 \cdot 5.32/\sqrt{8}}{\left(184 \cdot \dfrac{5.32}{\sqrt{8}} + 176 \cdot \dfrac{2.64}{\sqrt{10}} + 240 \cdot \dfrac{7.42}{\sqrt{6}}\right)} \cdot 25 \cong 7$$

$$n_2 = \frac{176 \cdot 2.64/\sqrt{10}}{\left(184 \cdot \dfrac{5.32}{\sqrt{8}} + 176 \cdot \dfrac{2.64}{\sqrt{10}} + 240 \cdot \dfrac{7.42}{\sqrt{6}}\right)} \cdot 25 \cong 3$$

$$n_3 = \frac{240 \cdot 7.42/\sqrt{6}}{\left(184.\dfrac{5.32}{\sqrt{8}} + 176.\dfrac{2.64}{\sqrt{10}} + 240.\dfrac{7.42}{\sqrt{6}}\right)} .25 \cong 15$$

The optimum allocation with variable cost requires a smaller sample in strata 1 and 2 and a larger sample in stratum 3.

6.3   Poststratification

The conventional formulae for estimating the mean and its variance of a stratified population assume that the subdivision of this population into homogeneous strata is carried out prior to the selection of sampling units. In certain situations, however, this may be either impossible or prohibitively time-consuming. When stratifying a population on the basis of forest types, for example, it is frequently possible to define the strata and to obtain their area from aerial photographs. In that case, the population ratio N_j/N is known for each of the L strata, but it may not be feasible to draw the sample before conducting the field survey. In such cases, it is more efficient to poststratify the population, i.e., to allocate the sampling units on the basis of quantitative or qualitative information obtained from the sample. It implies that each sampling unit is allocated to one of the strata, but only after completing the field work. The sample mean of the jth stratum is obtained from the formula, which

is identical to that applied in calculating the estimated population mean of a *prestratified* population with proportional allocation:

$$\bar{y}_{\text{p.str}} = \sum_{j=1}^{L} W_j \cdot \bar{y}_j$$

We note that the true value of $W_j = N_j/N$ is assumed to be known. The variance of the mean is obtained from the following formula:

$$s^2_{\bar{y}_{\text{p.str}}} = \frac{N-n}{n \cdot N} \sum_{J=1}^{L} \left(\frac{N_j}{N} \cdot \sigma_j^2 \right) + \frac{1}{n^2} \sum_{j=1}^{L} \left(\frac{N-N_j}{N} \cdot \sigma_j^2 \right)$$

The first term expresses the variance of the estimated mean based on proportional allocation. The second term, which has the effect of increasing the calculated variance, is due to the nonproportional distribution of n_j. For computational purposes, the stratum variances are estimated from the sample.

Example 10.7 The data in Example 10.5 are used to illustrate the effect of poststratification on the variance of the mean. In order to use this dataset, the above formula, which applies to proportional allocation, was modified to account for nonproportional allocation in this example.

The resultant estimates of the variance components were

$$\frac{N-n}{n \cdot N} \cdot \sum_{j=1}^{L} \left(\frac{N_j}{N} \cdot s_j^2 \right) = 1.25565$$

$$\frac{N-n}{n^2 \cdot (N-1)} \cdot \sum_{j=1}^{L} \left(\frac{N-N_j}{N} \cdot s_j^2 \right) = 0.08844.$$

The variance and the standard error of the mean were

$$s^2_{\bar{y}_{\text{pstr}}} = 1.34409 \; 1.34778 \text{ and } s_{\bar{y}_{\text{pstr}}} = \pm 1.16 \, \text{m}^3/0.1 \, \text{ha}$$

For comparison: $s_{\bar{y}_{\text{srs}}} = \pm 3.31 \, \text{m}^3/0.1 \, \text{ha}$; and $s_{\bar{y}_{\text{str}}} = \pm 1.12 \, \text{m}^3/0.1 \, \text{ha}$

6.4   Block Sampling

Block sampling is sometimes defined as a modified version of stratified random sampling. Instead of pre- or post-stratifying the population on the basis of uniformity and homogeneity of the forest within the strata, block sampling subdivides the population into equally large blocks, which are statistically equivalent with strata. It can be expected that the block means differ significantly, since

sampling units which are located far apart, will tend to be less alike than those located nearer to one another, for example, because of a one or two-directional trend. In that case, the subdivision into blocks, which are sampled at random and independently, is slightly more efficient than unrestricted random sampling, although less efficient than stratified random sampling, because of the rigid layout of blocks. In the case of approximately equal large blocks, and equal sample sizes within blocks, the sampling fractions are approximately the same for all blocks. The estimated population mean is found as the mean of the block means and the variance calculations are identical to those for stratified sampling. However, assuming that there are no significant differences between the stratum variances, the variance formula can be somewhat simplified by pooling the data from the L blocks.

The variance of the mean can be estimated by

$$s_{\bar{y}_{\text{bl}}}^2 = \frac{\displaystyle\sum_{j=1}^{L}\sum_{i=1}^{n_j} x_{ji}^2 - \frac{\displaystyle\sum_{J=1}^{L}\left(\sum_{i=1}^{n_j} x_{ji}\right)^2}{n_j}}{n\cdot(n-L)}\cdot\left(1-\frac{n}{N}\right)$$

Example 10.8   The previously described *P. radiata* stand is used to illustrate block sampling. After rigidly subdividing the stand into 6 blocks, a random sample of size 5, representing a sampling fraction of 0.10, is drawn within each block. The sampling units consist of 4×4 rows of trees. The observed basal areas are given in Table 10-3.

$$\bar{y}_{\text{bl}} = \frac{10.059}{30} = 0.3353\,\text{m}^2$$

$$s_{\bar{y}_{\text{bl}}}^2 = \frac{3.792879 - \dfrac{16.860419}{5}}{30\cdot 24} = 0.0005844 \quad\text{and}\quad s_{\bar{y}_{\text{bl}}} = 0.024\,\text{m}^2$$

Table 10-3. Observed basal areas of 4×4 row plots

Block	Basal Area (m^2)					Mean
1	0.291	0.408	0.253	0.274	0.185	0.282
2	0.291	0.365	0.238	0.463	0.483	0.368
3	0.155	0.351	0.300	0.096	0.281	0.237
4	0.577	0.137	0.252	0.225	0.431	0.324
5	0.274	0.359	0.403	0.347	0.363	0.349
6	0.406	0.370	0.450	0.429	0.602	0.451

The estimated population mean in example 8 is $0.3353\,\text{m}^2$ and its standard error is $0.024\,\text{m}^2$. In order to evaluate the efficiency of the arbitrary subdivision into blocks, an analysis of variance was calculated to test the difference between block means. The resultant mean square within blocks was 0.0118, the calculated F value for testing differences between block means was 1.52, with 5 and 24 degrees of freedom and indicated that the differences between block means were nonsignificant. Ignoring the subdivision into blocks and pooling the data produces a variance estimate of 0.0144 and a standard error of 0.022. It confirmed that the error variance could not be reduced by blocking. Due to the loss of degrees of freedom, block sampling does not perform better than unrestricted random sampling, unless there are significant differences between block means. This occurs if there is a linear or nonlinear trend across the population to be sampled, for example, when sampling a stand located on a slope, with shallow soil in the upper and a much deeper soil in the lower section of the stand. In the present example, block sizes were equal, but block sampling can also be applied to unequal stratum sizes, which are likely to occur in the practice of forest inventories.

7 REGRESSION AND RATIO ESTIMATORS

7.1 Regression estimators

Suppose that the volume of a stand is to be estimated by felling and sectionwise measurement of sample trees. The mean dbh of the stand is known from a recent complete stand enumeration. The tree basal area, which is closely and linearly correlated with tree volume, is introduced as an auxiliary variable to obtain a higher precision for the estimated total stand volume. The estimator is

$$y_{\text{reg}} = \bar{y} + b\,(\mu_x - \bar{x})$$

which is equivalent with the equation

$$y_{reg} = a + bx$$

Hence

$$\bar{y}_{\text{reg}} = \bar{y} + b\,(\mu_x - \bar{x}) \quad \text{or} \quad \bar{y}_{reg} = a + b\mu_x$$

where μ_x and $\bar{x}$ = population and sample mean of x, respectively. The sample is drawn at random, but without replacement, in which case the standard error of this estimate is approximately

$$s_{\bar{y}_{\text{reg}}} = \frac{N-n}{N} MS_{\text{error}} \left(\frac{1}{n} + \frac{(\bar{x} - \mu_x)^2}{SS_{xx}} \right)$$

$$SS_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i \right)^2}{n}$$

In large samples, the quantity $(x - \mu_x)^2 / SS_{xx}$ can be ignored. Cochran (1977) introduced an approximation based on sampling without replacement in finite populations

$$s_{\bar{y}_{\text{reg}}}^2 = \frac{1 - \dfrac{n}{N}}{n} s_{\bar{y}}^2 \left(1 - r^2 \right)$$

where r = correlation coefficient between x and y.

Example 10.9   A complete enumeration of a 2.7 ha beech stand was carried out to determine the quadratic mean stand diameter. The number of trees and quadratic mean diameter were 1523 and 23.2 cm respectively. A random sample of 15 trees was felled and their volume determined by sectionwise diameter measurements (Table 10-4).

$$y = -0.103 + 0.00118 \cdot x, \quad \text{where} \quad x = d^2 \quad \text{and} \quad y = \text{volume}$$

$$r = 0.983; \quad s_{y.x}^2 = \frac{0.01672}{13} = 0.001286; \quad SS_{xx} = 344975.6$$

$$s_{\bar{y}_{\text{reg}}}^2 = 0.001292 \cdot \left(\frac{1}{15} + \frac{(565.83 - 538.24)^2}{344975.6} \right) \cdot \frac{1523 - 15}{1523} = 0.0000881$$

$$s_{\bar{y}_{\text{reg}}} = \pm 0.009 \, \text{m}^3 / \text{tree}$$

Table 10-4. DBH and tree volume (m^3) of sample trees

dbh (cm)	Volume (m^3)	dbh (cm)	Volume (m^3)	dbh (cm)	Volume (m^3)
17.2	0.235	27.7	0.770	20.5	0.445
25.2	0.690	28.5	0.880	22.0	0.444
25.3	0.635	20.4	0.331	22.4	0.524
26.4	0.692	24.2	0.553	23.6	0.530
27.6	0.820	18.5	0.324	23.9	0.608

Based on Cochrans's formulae (Cochran 1977) we obtain

$$s_y^2 = 0.03561 \quad \text{and} \quad s_{\bar{y}}^2 = 0.002374$$

$$s_{\bar{y}_{\text{reg}}}^2 = \frac{1523 - 15}{1523 \cdot 15} 0.03561 \cdot (1 - 0.983^2) = 0.0000792 \rightarrow s_{\bar{y}_{\text{reg}}}$$

$$= \pm 0.009 \, \text{m}^3/\text{tree}$$

$$Q = \frac{s_{\bar{y}_{\text{reg}}}^2}{s_{\bar{y}}^2} = 0.037 \quad \text{or} \quad \frac{1}{Q} = 26.9 \quad \bar{y}_{\text{rs}} = \bar{v} = \frac{8.482}{15} = 0.565$$

$$\bar{x} = (d_q)^2 = \frac{\sum (d^2)}{n} = \frac{8487.46}{15} = 565.83 \, (\text{sample}); \dots \mu_x = 23.2^2$$

$$= 538.24 \, (\text{population})$$

$$\bar{y}_{\text{reg}} = 0.565 + 0.00118 \cdot (538.24 - 565.83) = 0.532 \, \text{m}^3/\text{tree or}$$

$$\bar{y}_{\text{reg}} = -0.103 + 0.00118 \cdot 538.24 = 0.532 \, \text{m}^3/\text{tree}$$

$$\bar{v}_{\text{reg}} = 0.532 \pm 0.09 \, \text{m}^3/\text{tree}$$

The ratio Q of the variance of regression estimate over that of the SRS estimate is always smaller than 1. The regression estimator therefore, reduces the standard error of the estimated population mean. The gain is substantial if x and y are closely and linearly correlated. In example 10.9, the ratio $(1/Q)$ is 26.9. In simple, i.e., in unrestricted random sampling (SRS), a sample size of 404 ($=15 \cdot 26.9$) would have been necessary to match the precision of the regression estimator. The regression estimate, however, is biased if the assumption of a linear relationship between x and y is not satisfied.

7.2   Ratio estimators

The previously discussed regression model assumes that x and y are linearly related, with the regression line, which does not necessarily pass through the origin. Assuming that $y = 0$ for $x = 0$, it is appropriate to fit the zero-intercept equation $y = bx$. Schumacher et al. (1954), for example, conducted a sampling study in a nursery to estimate the number of plantable seedlings from the total number of seedlings in seedbeds. Assuming that the variance of y is independent of x, the least squares estimate of the parameter β is

$$b = \frac{\sum\limits^{n} xy}{\sum\limits^{n} x^2}.$$

The variance of the regression coefficient b is

$$s_b^2 = \frac{MS_{\text{error}}}{\sum x^2} \quad \text{with} \quad MS_{\text{error}} = \frac{\sum(y - bx)^2}{n - 1}$$

The $(1 - \alpha)$ confidence interval for β is

$$b \pm t_{1/2\alpha, n-1} s_b$$

Assigning a weight of $1/x_i$ to the ith observation generates the *ratio of means estimator*

$$\widehat{R}_{rm} = \frac{\sum^n y}{\sum^n x} = \frac{\bar{y}}{\bar{x}}$$

with variance

$$s_{\widehat{R}_{rm}}^2 = \frac{N - n}{nN\bar{x}^2} \cdot \left[s_y^2 + \widehat{R}^2 \cdot s_x^2 - 2\widehat{R} \cdot s_{yx} \right] = \frac{1 - f}{n\bar{x}^2} \left[s_y^2 + \widehat{R}^2 \cdot s_x^2 - 2\widehat{R} \cdot s_{yx} \right]$$

where $f = n/N$ (Cochran 1977). The estimated population total is

$$\widehat{Y}_{mr} = X\widehat{R}_{rm}$$

and because of the relationship $\widehat{Y}_{mr} = N\overline{X}\widehat{R}_{rm}$, the estimated variance for the population total is

$$s_{\widehat{Y}_{rm}}^2 = \frac{N^2}{n} \cdot (1 - f) \cdot \left[s_y^2 + \widehat{R}^2 \cdot s_x^2 - 2\widehat{R} \cdot s_{yx} \right]$$

The ratio of means estimator is slightly biased, but the magnitude of bias decreases with increasing n (Hansen et al. 1953; Cochran 1977; de Vries 1986).

Assuming that the regression line $y = bx$ passes through the origin, but the variance of y is proportional to x^2, a weight proportional to $1/x_i^2$ is assigned to the ith observation. Minimizing the sum of weighted residuals gives the *mean of ratios estimator*

$$R_{mr} = \frac{\sum \frac{y_i}{x_i}}{n} = \frac{\sum r_i}{n}$$

The resultant variance of $\widehat{Y}$ is

$$s_{\widehat{Y}_{mr}}^2 = X^2 \frac{s_r^2}{n} \left(1 - \frac{n}{N} \right) \quad \text{where} \quad X = \sum_{i=1}^{N} x_i$$

and

$$s_r^2 = \frac{\sum_{i=1}^{n} (r_i - \bar{r})^2}{n - 1}$$

Table 10-5. Squared dbh and crown volume of sample trees

dbh^2	v_{cr}	dbh^2	v_{cr}	dbh^2	v_{cr}	dbh^2	v_{cr}
343.8	6.1	534.2	36.3	1124.1	57.7	1024.3	87.2
382.5	9.4	570.1	28.1	619.6	34.8	1073.6	88.6
392.5	31.0	684.4	59.8	853.2	61.7	1356.4	90.9
412.9	19.9	838.4	61.9	976.1	81.1	1394.1	139.9
522.6	31.4	883.2	56.7	976.1	61.2	1413.2	46.0
477.2	43.4	1057.0	58.8	992.0	54.8	–	–

However, the estimator is biased. Several authors reported on its erratic behavior (Cunia 1981; Van Hensbergen 1994).

Example 10.10 The diameter, crown width, crown diameter at the base of the light crown (*cw*) and the length of the light crown (*cl*) of 23 trees in a 4.7 ha *P. radiata* stand, with a total of 2524 trees, were measured to estimate the volume of the light crown (v_{cr}). The latter was calculated by assuming a conical shape of the light crown:

$$v_{cr.} = \frac{\pi}{12} cw^2 \cdot cl$$

The population mean of the auxiliary variable d^2 was 28.1^2 (=789.61).

The calculation of the ratio of means estimator is as follows:

$$\sum x = 18901.4; \quad \bar{x} = 821.8; \quad \sum y = 1246.7; \quad \bar{y} = 54.21$$

$$\sum xy = 1203786; \quad \hat{R}_{rm} = \frac{54.21}{821.8} = 0.066;$$

$$\hat{X} = 2524 \cdot 28.1^2 = 1992976$$

$$s_x^2 = 112856; \quad s_y^2 = 910.6; \quad s_{yx} = \frac{SP_{xy}}{n-1} = 8144.6$$

$$\hat{Y} = 1992976 \cdot 0.066 = 131536 \, m^3 \left(= 27986 \, m^3/ha\right)$$

$$s_{\hat{R}_{rm}}^2 = \frac{1}{821.8^2}\left(\frac{1-23/2524}{23}\right) \cdot \left(910.6 + 0.066^2 \cdot 112858\right.$$

$$\left. - 2 \cdot 0.066 \cdot 8144.6\right) = 0.0000210$$

$$s_{\hat{R}}^2 = 2524^2 \cdot \frac{2524-23}{2524.23} \cdot \left[910.6 + 0.066^2 \cdot 112858 - 2 \cdot 0.066 \cdot 8144.6\right]$$

$$= 90582160$$

$$s_{\hat{y}} = 9517.47$$

For the data in the previous example, the mean of ratio estimators and its standard error are

$$\hat{R}_{mr} = 0.0638; \quad s_R^2 = 0.00044; \quad s_R = 0.021; \quad X = 1992976 \left(= 2524 \cdot 28.1^2\right)$$

$$s_{\hat{Y}_{mrr}}^2 = 1992976 \cdot \frac{0.00044}{23} \cdot \left(\frac{2501}{2524}\right) = 37.779; \quad s_{\widehat{Y}_{mr}} = 6.15$$

In an attempt to overcome bias associated with ratio of means and means of ratio estimators, Hartley et al. (1954) suggested the following estimator for the population total

$$\hat{y} = X \ R + \frac{n(N-1)}{n-1}(\bar{y} - R\bar{x})$$

where $X = population\ total\ for\ x$ and $R_{rmb} = ratio\ of\ means$. Mickey (1959) proposed dividing the sample into g groups of size m, with $n = mg$. The estimator for the population total is

$$\hat{y} = x\hat{r}_g + (N - n + m) \cdot g \cdot \left(\bar{y} - r_g\bar{x}\right)$$

with

$$\bar{r}_g = \frac{\sum\limits_{}^{g} r_j}{g}$$

and

$$r_j = \frac{n\bar{y} - m\bar{y}_j}{n\bar{x} - m\bar{x}_j}$$

8   DOUBLE SAMPLING (TWO-PHASE SAMPLING)

8.1   Double sampling for regression estimators

The regression and ratio estimators assume that the population mean and total of the auxiliary variable are known. This situation arises if the mean height of a research plot is to be estimated, with the auxiliary variable dbh measured on each tree within a given plot. In many cases however, the population mean of x is unknown, for example, when estimating the mean height of the entire stand. However, in this case also, dbh is a useful auxiliary variable, which can be measured cheaply and quickly in a large sample of n_1 trees drawn at random and height measurements restricted to a subsample of size n_2. The equation

$$y = b_0 + b_1 x$$

is fitted to the subsample and produces an unbiased estimate for tree height for given dbh.

An unbiased estimate for the population mean of y is obtained from

$$\bar{y}_{ds} = \bar{y} + b\,(\bar{x}_1 - \bar{x}_2)\ \text{or}$$
$$\bar{y}_{ds} = b_0 + b_1\bar{x}_1$$

where $\bar{x}_1 = $ sample mean for main sample, $\bar{x}_2 = $ sample mean for subsample. The variance of the regression estimator of the population mean is

$$s_{\bar{y}_{reg}} = s_y^2 \frac{1 - r_{xy}^2}{n_2} + r^2 \frac{s_y^2}{n_1} - \frac{s_y^2}{N} = s_y^2 \cdot \left(\frac{1 - r^2}{n_2} + \frac{r^2}{n_1} - \frac{1}{N} \right)$$

and assumes that the population size is approximately known. For $r = 0$, we get

$$s_{\bar{y}_{ds}}^2 = s_y^2 \cdot \left(\frac{1}{n_2} - \frac{1}{N} \right) = \frac{s_y^2}{n_2} \cdot \left(1 - \frac{n_2}{N} \right)$$

which is identical to the formula for unrestricted random sampling with replacement.

Example 10.11   The data in Example 10.9 are used to illustrate double sampling with regression estimates. We assume the mean of the squared diameters, obtained from a random sample of size 299, was $542.89\,\text{cm}^2$. The total number of trees was 1523, the estimated quadratic mean diameter and the mean of the squared diameters, based on the subsample, were $23.8\,\text{cm}$ and $565.83\,\text{cm}^2$, the variance of the observed volumes was 0.03561 and $r = 0.983$.

Hence $\bar{y}_{ds} = -0.103 + 0.00118 \cdot 542.89 \cong 0.538\,\text{m}^3/\text{tree}$

$$s_{\bar{y}_{ds}}^2 = 0.03561 \cdot \left[\frac{1 - 0.983^2}{15} + \frac{0.983^2}{299} - \frac{1}{1523} \right] = 0.0001717\ \left(\text{m}^3/\text{tree}\right)^2$$

$$s_{\bar{y}_{ds}} = \pm 0.013\,\text{m}^3/\text{tree}$$

If the present data had been generated by an unrestricted random sample, the variance of the mean would have been $0.03561/15 = 0.002374$. The regression estimator produced a variance of 0.0000881 and the variance estimate based on double sampling is 0.0001717. The latter represents a considerable improvement over simple random sampling, but exceeds that obtained with the regression estimator because of the unknown population mean of the quadratic mean diameter.

The data in the previous example, with $N = 1523$ and $n_2 = 50$ may be used to illustrate the effect of the correlation coefficient between the auxiliary variable y and the subject variable x as well as the effect of the size of the
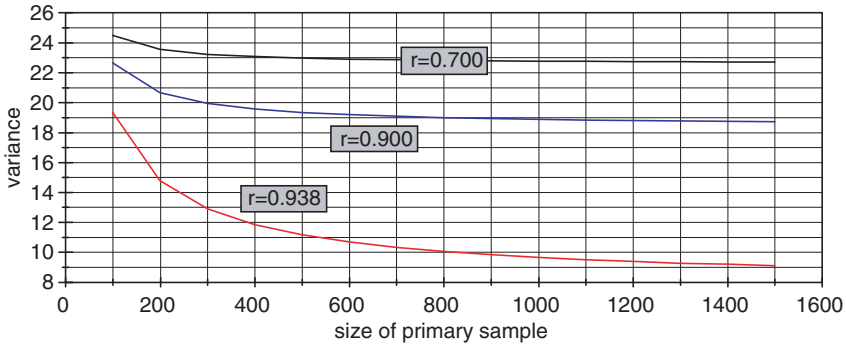
Figure 10-9. Effect of *r* and size of the primary sample on the variance of the mean.

initial sample on the variance of the double-sampling estimator for the mean. The calculations are carried out for the observed $r = 0.938$ and for $r = 0.5$ and $r = 0.7$, respectively. The results are shown in Figure 10-9.

The previous formulae apply for large n_1 and n_2. Cochran (1977) derived the more accurate formula representing a hybrid between conditional and average variance

$$s^2_{\bar{y}_{ds}} = s^2_{y.x}\left[\frac{1}{n} + \frac{(\bar{x}_2 - \bar{x}_1)^2}{\sum(x_2 - \bar{x}_2)^2}\right] + \frac{s^2_y - s^2_{y.x}}{n_2} - \frac{s^2_y}{N}$$

Double sampling can be extended to more than one auxiliary variable to be measured in phase 1, especially when using aerial photographs in phase 1. The variance formula is modified accordingly

$$s^2_{\bar{y}_{ds}} = \frac{s^2_y \cdot (1 - R^2)}{n_2} \cdot \left[1 + \frac{n_1 - n_2}{n_2} \cdot \frac{k}{n_2 - k - 2}\right] + \frac{R^2 \cdot s^2_y}{n_1} - \frac{s^2_y}{N}$$

where R = multiple correlation coefficient, k = number of auxiliary variables (khan et al. 1967). However, this approach is not effective when then number of auxiliary variables is less then 3.

Double sampling finds useful applications in forest inventories since aerial photographs are available to obtain quick and inexpensive estimates of the auxiliary variable. A large number of photoplots is established either at random or in a square lattice. The auxiliary variable or variables are measured on all photoplots. A random subsample of size n_2 is drawn from the main sample and measured on the ground. In general, two-phase sampling is efficient only if the auxiliary variables and the subject variable *y* are closely correlated. In this particular example, there may be practical problems, for example, in relocating the photoplots on the ground. If there is no complete overlap of photo-and ground

plots, the two estimates may be poorly correlated and two-phase sampling will not be more cost-efficient than simple random sampling.

8.1.1   Optimum allocation

The measuring costs have to be taken into account, upon deciding how many sampling units will be measured in phase 1 and how many in phase 2. The total cost of sampling is

$$C = n_1 c_1 + n_2 c_2$$

where $c_1 =$ cost per unit of phase 1 sample and $c_2 =$ cost per unit of the phase 2 sample of the subject variable y. For the fixed total cost C, the optimum values of n_1 is

$$n_1 = \frac{C}{c_1 + c_2 \sqrt{\frac{1 - r^2}{r^2} \cdot \frac{c_1}{c_2}}}$$

For a specified maximum error of the mean, the size of the phase 1 sample is obtained from

$$n_1 = \frac{s_y^2}{s_{\bar{y}_{ds}}^2} \cdot \left(\left(\frac{c_2}{c_1} \cdot r^2 \cdot \left(1 - r^2 \right) \right)^{0.5} + r^2 \right)$$

Let

$$Q = \left(\left(\frac{c_2}{c_1} \cdot r^2 \cdot \left(1 - r^2 \right) \right)^{0.5} + r^2 \right)$$

The effect of cost ratio and r on Q which expresses the ratio of the variance of the mean obtained by regression analysis to that obtained from random sampling is shown in Figure 10-10. In both cases n_2 is obtained from

$$n_2 = n_1 \cdot \sqrt{\frac{1 - r^2}{r^2} \cdot \frac{c_1}{c_2}}$$

Example 10.12   If the total sampling cost in the previous example is fixed at 1000 units and c_1 and c_2 are 5 and 25 units, respectively, the optimum values of n_1 and n_2 are

$$n_1 = \frac{1000}{5 + 25 \cdot \sqrt{\frac{1 - 0.983^2}{0.983^2} \cdot \frac{5}{25}}} \cong 141; \quad n_2 = 141 \cdot \sqrt{\frac{1 - 0.983^2}{0.983^2} \cdot \frac{5}{25}} \cong 12$$
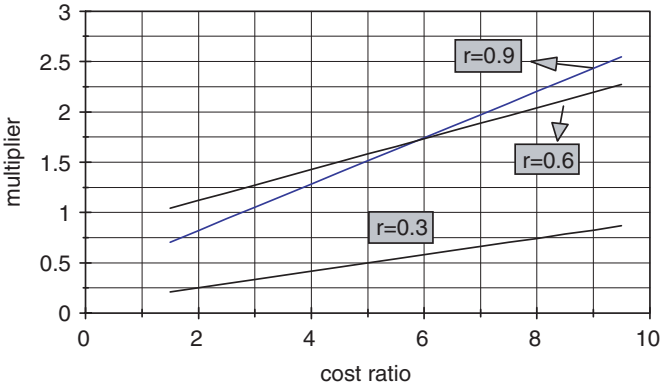
Figure 10-10. Influence of cost ratio and correlation coefficient on Q.

Specifying a variance of the mean of 0.0002 units, the optimum size of the initial sample is

$$n_1 = \frac{0.0356}{0.0002} \cdot \left(\frac{25}{5} \cdot 0.983^2 \cdot \left(1 - 0.983^2 \right) + 0.983^2 \right) \cong 201;$$

$$n_2 = 201 \cdot \sqrt{\frac{1 - 0.983^2}{0.983^2} \cdot \frac{5}{25}} \cong 17$$

8.2 Double sampling for stratification

Prestratification assumes that the relative sizes of the strata are known. In many situations however, this assumption is not satisfied. In such cases, an initial random sample of size n^* is drawn to stratify the population. The subject variable is measured on a random subsample of size n^*. Hence, $w_j = n^*_j / n$. This *double sampling for stratification* strategy requires a large initial sample, which can be measured cheaply, for example with the aid of aerial photographs and the layout and measurement of photoplots. The mean of the stratified population is estimated from

$$\bar{y}_{\text{str,ds}} = \sum_{j=1}^{j=L} w_j \bar{y}_j$$

and its variance from

$$s_{\bar{y}ds.}^2 = \frac{N-1}{N} \cdot \sum_{j=1}^{L} \left[\left(\frac{n_j^* - 1}{n^*} \right) - \left(\frac{n_j - 1}{N-1} \right) \right] \cdot \frac{w_j s_j^2}{n_j}$$

$$+ \frac{N-n^*}{N \cdot (n^* - 1)} \cdot \sum_{j=1}^{L} \left[w_j \left(\bar{y}_j^- \bar{y}_d \right)^2 \right]$$

Example 10.13 The data in Example 10.5 are used to illustrate the calcula-
tion of mean and variance. The initial random sample contained 100 sampling
units. The number of units belonging to stratum 1, 2, and 3 was 32, 28 and 40,
respectively and the resultant estimated population proportions were 0.32, 0.28,
and 0.40, respectively. Contrary to prestratification, the real proportions were
unknown. As previously, the subsample was of size 25, of which 8, 7 and 10
sampling units were drawn from stratum 1, 2 and 3, respectively. The statistics
required for the calculation of the mean and variance were:

n_j^*	n_j	w_j	s_j^2	$\bar{y}_j$	$\bar{y}_{ds}$
32	8	0.32	28.27	36.38	38.14
28	7	0.28	6.95	16.57	38.14
40	10	0.40	55.12	55.30	38.14

A weight of n_j^*/n is assigned to each of the stratum means obtained from the
subsample. The resultant estimated population mean remains the same and is
38.14, but the revised variance of the mean is 5.554, whereas this variance,
obtained in Example 10.5 was 1.245. The much greater variance is due to the
unknown population proportions of the three strata.

8.3   Double sampling for ratio estimators

The ratio estimator assumes that the population total X for the auxiliary vari-
able x is known. In double sampling for ratio estimation, the population mean
X is estimated from the initial sample of size n_1. A subsample n_2 is drawn to
calculate the ratio estimator R_{rm}. The ratio estimate for the population mean of
y is

$$\bar{y}_{ds_{rm}} = \widehat{R}_{rm} \cdot \bar{x}_1 = \frac{\bar{y}}{\bar{x}_2} \cdot \bar{x}_1$$

with variance

$$s_{\bar{y}ds_{rm}}^2 = \frac{s_y^2 - 2 \cdot R_{rm} \cdot s_{xy} + R_{rm}^2 \cdot s_x^2}{n_2} + \frac{2 \cdot R_{rm} \cdot s_{xy} + R_{rm}^2 \cdot s_x^2}{n_1} - \frac{s_y^2}{N}$$
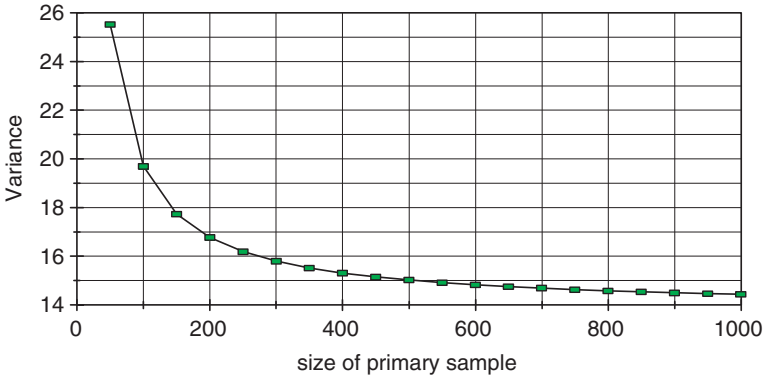
Figure 10-11. Effect of size of primary sample on variance.

Example 10.14   The data in Example 10.10 are used to illustrate double sampling for ratio estimation. We have

$$R_{rm} = 0.066; s_y^2 = 910.6; s_x^2 = 112865; s_{xy} = 8144.6; n_1 = 100; n_2 = 23$$

$$s_{\bar{y}_{ds_{rm}}}^2 = \frac{910.6 - 2 \cdot 0.066 \cdot 8144.6 - 0.066^2 \cdot 112865}{23}$$

$$+ \frac{2 \cdot 0.066 \cdot 8144.6 - 0.066^2 \cdot 1128656}{100} - \frac{910.6}{2524}$$

$$s_{\bar{y}_{ds_{rm}}}^2 = 19.70$$

The variance of the mean based, on the assumption of a simple random sample of the same size is

$$s_{\bar{y}_{srs}}^2 = \frac{910.6}{23} = 39.59$$

Double sampling reduced the variance by 50%. The effect of the size of the initial sample on the variance is shown in Figure 10-11.

9   CLUSTER SAMPLING

9.1   Definitions

Because of the relatively low sampling fractions, which typify forest inventories, the amount of time needed to travel from one sampling unit to the next is considerable, when compared with measuring time. In the case of unrestricted

or stratified random sampling, it is necessary to mark the randomly selected locations of the plot centers on a stand map or aerial photograph. Thereafter, they are relocated in the field and either fixed-radius or angle-count estimates of the subject variable are made. In tropical forests with difficult access, even more time is needed to relocate the plot centers. For this reason, the measurement of sampling units in clusters may be more efficient. The following designs, which are known as satellite designs, illustrate cluster sampling in forest inventories:

- The US Forest Service (USDA For. Serv., 1968) introduced the ten-point cluster in combination with angle count sampling and a basal area factor of approximately 8.6. The sampling points were located at the corner points of 10 equilateral triangles, with 21 ft long sides.
- Loetsch (1957) introduced the camp unit system for a forest inventory in Thailand. The recording units consisted of 0.05 ha plots located at the four corners of a $400 \times 400\,\text{m}^2$ tract. The tracts were combined in units, positioned along the perimeter of a circle and 7 units were positioned with the camp as survey center. The design was described as a triple satellite system (Figure 10-12).
- The Swedish and German national forest inventories are also based on the tract system, with a systematic spatial distribution of the tracts on survey
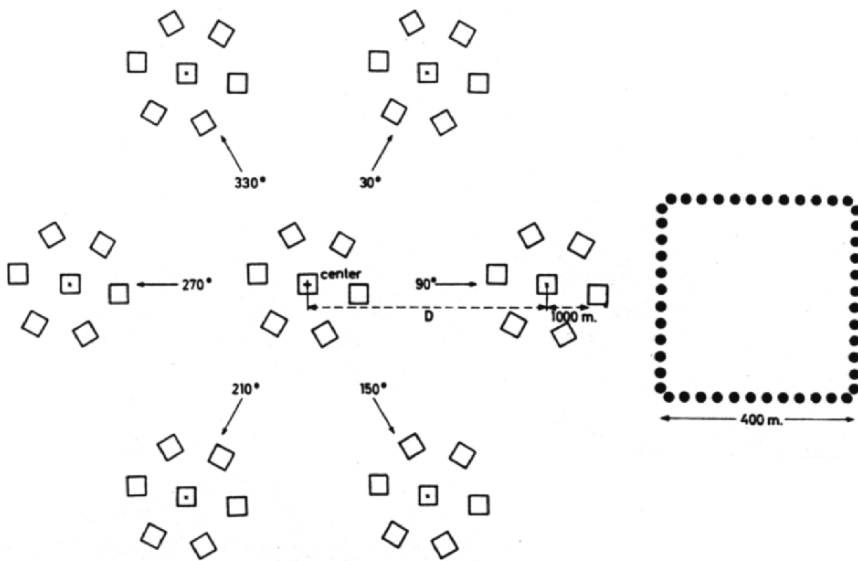
Figure 10-12. Camp unit system of the forest inventory in Thailand.
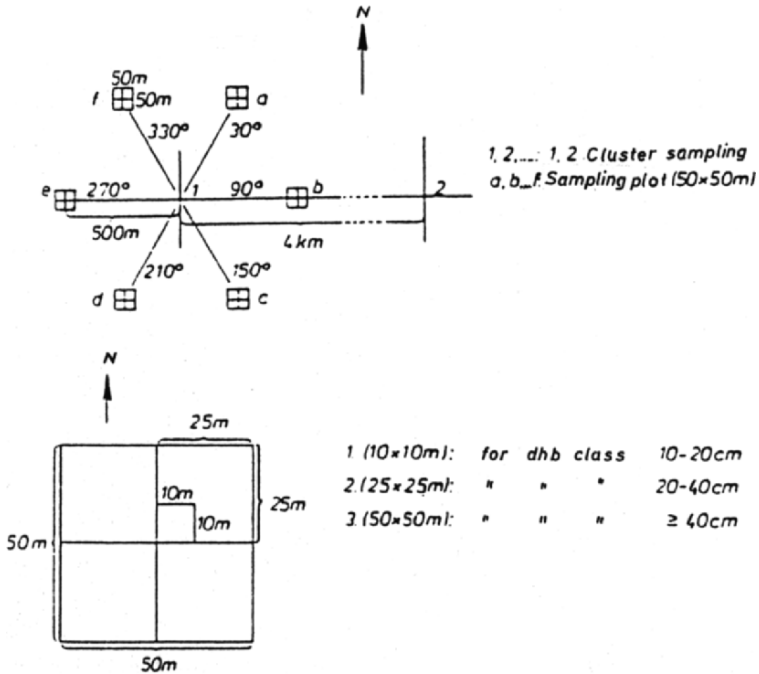
Figure 10-13. Satellite system for the forest inventory in East Kalimantan.

lines running east-west. Similar designs are used for the forest inventory in Finland and Austria.
- The *six-tree sample*, introduced by Prodan (1968) and the *n-tree sample*, introduced by Trisl (1998) for estimating peeling damage in Germany can also be interpreted as small-cluster samples.
- The design, developed for a forest inventory in East Kalimantan (Soeyitno 1989) is shown in Figure 10-13.

9.2   Estimators

We assume that the population consists of N clusters of which n clusters are selected at random. Each selected cluster contains M subunits, on which the subject variable is measured. The following notation is introduced:

$Y_L, \bar{y}_L$ = total and mean of the M observations in the ith cluster

$Y, \bar{y}$ = estimated population total and mean

σ_b^2 = variance between clusters

σ_w^2 = variance within clusters

The unbiased estimator for the population mean μ_y is given by

$$\bar{\bar{y}} = \frac{\sum\limits^n \sum\limits^M y_{ij}}{nM}.$$

The sample estimator

$$s_w^2 = \frac{\sum\limits^n \left(y_{ij} - \bar{y}_i\right)^2}{n \cdot (M - 1)}$$

represents an unbiased estimator of the variance within clusters, whereas

$$s_b^2 = \frac{\sum\limits^n \left(\bar{y}_i - \bar{\bar{y}}\right)^2}{n - 1}$$

estimates the variance between clusters. The sample estimate of the variance among subunits, ignoring clusters, is

$$s^2 = \frac{\sum\limits^n \sum\limits^M \left(y_{ij} - \bar{\bar{y}}\right)^2}{n(M - 1)}$$

The variance of the overall mean is

$$\sigma_{\bar{\bar{y}}}^2 = \frac{\sigma_b^2}{n}$$

and is estimated from

$$s_{\bar{\bar{y}}}^2 = \frac{s_b^2}{n}$$

In order to quantify the efficiency of cluster sampling, the between-clusters variance may be written as follows

$$\sigma_b^2 = \frac{s^2 \left[1 + (M - 1)r_I\right]}{M}$$

where r_I = intracluster correlation coefficient, expressing the correlation between the elements of pairs of sampling units within clusters, averaged over the N clusters. The variance s^2 ignores clusters and, to some extent, represents an estimate resulting from simple random sampling. Since the variance of the mean of the n clusters is estimated as s_b^2/n and the variance of the mean of the nM observations (ignoring clusters) is estimated from s^2/nM, we get

$$s_{\bar{y}_{cl}}^2 = s_{\bar{y}_{rs}}^2 \left[1 + (M - 1)r_I\right]$$
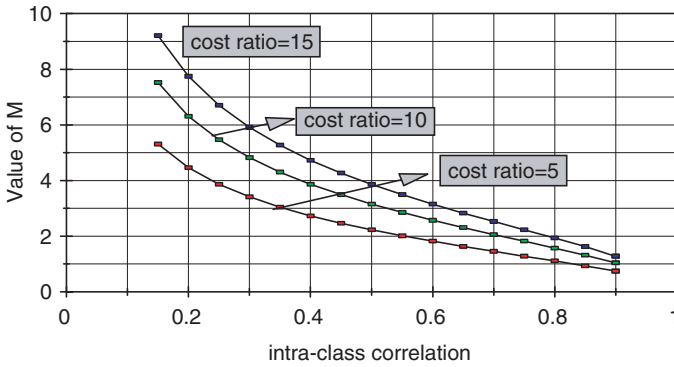
Figure 10-14. Influence of cost ratio c_1/c_2 and r_I.

It can be seen that

$$\left(s_{\bar{y}_{cl}}^2 / s_{\bar{y}_{rs}}^2\right) > 1$$

if r_I is positive. In that case, cluster sampling is less efficient than simple random sampling, although it may be more cost-efficient since less travel time is involved. A simple cost function (Som 1976) with a total fixed cost is

$$M = \sqrt{\frac{c_1}{c_2}\left(\frac{1 - r_I}{r_I}\right)}$$

where c_1 = cost per cluster, mainly for traveling, c_2 = cost of measuring a subunit. For example, for $c_1/c_2 = 10$ and $r_I = 0.15$, we get $M = 7.5$. The value of M increases proportionally to the square root of the cost ratio c_1/c_2 (see Figure 10-14).

Example 10.15   To illustrate single-stage cluster sampling, the dbh of all five trees within each of 10 sample plots which were systematically distributed within a given *P. radiata* stand, was measured. The data were as follows:

				Plots					
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
				dbh (cm)					
20.3	24.1	19.5	23.6	26.5	19.5	19.8	22.4	23.2	21.1
18.0	18.7	22.5	24.0	23.5	23.7	20.1	26.0	22.5	14.1
29.0	22.0	22.1	18.0	28.1	21.5	21.0	26.5	20.0	19.8
19.4	23.6	24.3	21.8	25.7	33.1	22.0	21.8	24.0	20.5
17.6	23.0	18.3	19.7	26.0	22.9	14.0	25.2	27.1	24.5

The analysis of variance is as follows:

Source	Sum of squares	df	Mean square
Between clusters	199.0808	9	22.1201
Within clusters	417.5120	40	10.4378

The between-clusters mean square estimates $\sigma_w^2 + 5\sigma_b^2$. In consequence, the estimated variance between clusters is 2.336 and is considerably less that the within-clusters variance of 10.438. The estimated variance, ignoring clusters, is $616.5928/49 = 12.584$. The estimated intraclass correlation coefficient is

$$r_I = \frac{\dfrac{5 \cdot 2.336}{12.584} - 1}{5 - 1} = -0.018$$

10   MULTISTAGE SAMPLING

In multistage sampling, the sampling units are selected in a hierarchical order. The population is partitioned into *primary units*, which in turn are subdivided into *secondary units*, the latter into *tertiary units*, etc. An important property of multistage sampling is the random selection of sampling units in each stage. We consider the following sampling problem. Suppose that the mean site index of *P. radiata* grown in different countries and regions is to be estimated. Three regions are selected at random from a total of ten, with four plantations (*primary sampling units*) selected at random in each region and ten stands (*secondary sampling units*) within each plantation.

10.1   Two-stage sampling

In two-stage sampling, *n* primary units are drawn from a population of *N* units. In the case of equal cluster sizes, each of the *N* primary units is subdivided into *M* secondary units with *m* units drawn at random within each primary unit. The sampling fractions are n/N for the primary and m/M for the secondary units. The total size of the sample is *mn* and the overall sampling fraction is *mn/MN*. Unbiased estimates of the population mean and total are

$$\overline{\overline{y}} = \frac{\sum\limits^{m}\sum\limits^{n} y_{ij}}{mn} \quad Y = MN\overline{\overline{y}}$$

An analysis of variance, which partitions the total sum of squared deviations from the overall mean into components associated with variability amongst the

primary units and variability between secondary units within primary units, is required to estimate the variances involved.

Source	Sum of squares	df	Mean square
Total	$\displaystyle\sum^{m}\sum^{n}\left(y_{ij}-\overline{\overline{y}}\right)^2$	$mn-1$	
Primary units	$\displaystyle m\sum^{n}\left(\bar{y}_j-\overline{\overline{y}}\right)^2$	$n-1$	$\dfrac{SS_{\text{pu.}}}{(n_1-1)}$
Secondary units	$\displaystyle\sum^{n}\left(\sum^{m}\left(y_{ij}-\bar{y}_j\right)^2\right)$	$N(m-1)$	$\dfrac{SS_{su.}}{n(m-1)}$

In sampling from an infinite population, the expected values of the mean squares between primary units and between secondary units within primary units are

between primary units $\sigma_w^2 + m\sigma_b^2$

between secondary units σ_w^2

where

$\sigma_b^2 =$ variance amongst primary units

$\sigma_W^2 =$ variance amongst secondary units

The variance of the overall mean

$$\sigma_{\bar{\bar{y}}}^2 = \frac{m\sigma_b^2+\sigma_w^2}{mn} = \frac{\sigma_b^2}{n}+\frac{\sigma_w^2}{mn}$$

is to be adjusted for the finite population

$$s_{\bar{\bar{y}}}^2 = f_1\cdot\frac{s_b^2}{n}+f_1\cdot(1-f_2)\cdot\frac{s_w^2}{nm}\text{(Cochran 1977)}$$

Example 10.16   The mean of a population consisting of 2000 primary units and 100 secondary units per primary unit is estimated by two-stage sampling with $n=10$ and $m=2$

Primary unit	Data	Sum	Primary unit	Data	Sum
1	49, 34	83	6	5, 6	11
2	12, 21	33	7	37, 50	87
3	6, 16	22	8	28, 40	68
4	27, 27	54	9	23, 30	53
5	8, 7	15	10	42, 51	93

$$\sum_{}^{n}\sum_{}^{m} y_{ij} = 519; \qquad \frac{\left(\sum_{}^{n}\sum_{}^{m} y_{ij}\right)^2}{nm} = 13\,468.05; \qquad ss_{\text{total}} = 4\,644.95$$

$$SS_b = \frac{83^2 + \cdots 93^2}{2} - 13\,468.05 = 4219.54; \quad MS_b = \frac{4219.54}{9} = 468.8$$

$$SS_W = 4\,644.95 - 4\,219.45 = 415.5; \quad MS_W = \frac{425.5}{10} = 42.55$$

The quantity MS_w estimates the variance amongst secondary units within primary units, MS_b estimates $\sigma_w^2 + 2\sigma_b^2$. Hence

$$s_b^2 = \frac{468.8 - 42.55}{2} = 213.12$$

The observed mean is $519/20 = 25.95$ and its variance is

$$s_{\bar{\bar{y}}}^2 = \frac{990/1\,000}{10} \cdot 468.8 + 0.001 \frac{90}{100} \cdot 42.55 = 46.44, \quad s_{\bar{y}} = 6.81$$

The 0.95 confidence interval for $\bar{y}$ is $25.95 \pm 2.23 \cdot 6.81 :$ ′10.76 41.14. The standard error of the estimated population total is

$$s_{\hat{Y}} = NMs_{\bar{y}} = 200\,000 \cdot 6.01 = 1\,362\,000$$

and the 0.95 confidence interval for $\hat{Y}$ is

$$5190\,000 \pm 2.23 \cdot 1\,362\,000 \quad 2152740 \quad 8227260$$

10.1.1 Optimum allocation

The allocation of sampling units requires prior knowledge of the variance amongst primary and secondary units, respectively. In exceptional cases, such estimates can be derived from external sources, for example, from sampling studies in similar populations. In theory, estimates could be obtained by first sampling the population to obtain useful estimates of these variances in order to be able to apply the allocation rule and to decide on the required sample size, but this is seldom feasible or considered worthwhile. Two cost components are involved

- Sampling cost per primary unit (c_1)
- Sampling cost per secondary unit (c_2)

The cost element c_1 represents the cost of traveling from one primary unit to the second, whereas c_2 gives the cost of measuring the secondary unit. Other cost elements, such as traveling to reach the location of the population to be sampled during a given working day, as well as social expenses should be added to the

cost for the inventory in its entirety. The optimum allocation rule prescribes the number of primary sampling units to be measured:

$$m = \sqrt{\frac{\sigma_w^2}{\sigma_b^2} \cdot \frac{c_1}{c_2}}$$

The number of primary units is given by

$$n = \frac{\sigma_b^2 + \dfrac{\sigma_w^2}{m}}{\left[s_{\bar{\bar{y}}}^2 + \dfrac{1}{N} \cdot \left(\sigma_b^2 + \dfrac{\sigma_w^2}{M}\right)\right]} \quad \text{with } s_{\bar{\bar{y}}}^2 \text{ being specified.}$$

Example 10.17   For the data in Example 10.16, we assume: $c_1 = 220$, $c_2 = 15$. Hence

$$m = \sqrt{\frac{213.12}{468.8} + \frac{220}{15}} \cong 3$$

The estimated sample mean is 51.9. It is required that the maximum error should not exceed 5% of the mean, with a stated probability of 0.95. The t value to be multiplied by the standard error depends upon the degrees of freedom and is approximately 2. Hence, since

$$t \cdot s_{\bar{\bar{y}}} = 2.60$$

the standard error of the mean should not be greater than 1.30. The required number of primary units is

$$n = \frac{468.8 + \dfrac{213.12}{3}}{2.60 + \dfrac{1}{2000} \cdot \left(468.8 + \dfrac{213.12}{100}\right)} \cong 190$$

10.2   Three-stage sampling

The basic principle of two-stage sampling may be extended to multistage sampling. When three stages are involved, n primary units are drawn at random from a population of N units, m secondary units from a population of M subunits per primary unit and k tertiary units from a population of K units per secondary unit. The total sample size is nmk and represents a sampling fraction of nmk/NMK. The following examples illustrate potential applications of three-stage sampling:

- In a national forest inventory, a given area is covered by N satellite images, from which a sample of size n is drawn at random. Suppose that medium-scale aerial photography of a 1:12000 scale provides aerial coverage of each

of the n selected primary units. The next step is to select at random m photographs within each of these satellites images and k sample plots within each of the m selected photographs per satellite image. They are measured by conventional ground methods.

In a study to estimate the mean nitrogen content of the foliage of trees within a given stand, a primary sample of n trees is drawn at random from a population of N units, a subsample of m branches is selected from a population of M branches in each tree and a subsubsample of k leaves within each of these branches, from a population of K leaves in each branch. We assume that N, M, and K are known, and each tree has the same number of branches and each branch the same number of leaves per branch. The resultant sample fractions are

Trees $f_1 = n/N$

Branches $f_2 = m/M$

Leaves $f_3 = k/K$

Obviously, the assumption of an equal population size of branches within each tree and of leaves within branches and that of known N, M, and K is an oversimplification.

The analysis of variance partitions the total variance into three components. One component expresses variability amongst trees, a second quantifies the variability amongst branches within trees, and a third is associated with the variability amongst leaves within branches. The mean squares estimate the following quantities:

	$E(MS)$
Between trees	$\sigma_l^2 + k \cdot \sigma_{br}^2 + m \cdot k \cdot \sigma_{tr}^2$
Between branches (within trees)	$\sigma_l^2 + k \cdot \sigma_{br}^2$
Between leaves (within branches)	σ_l^2

Where σ_{tr}^2 = variance among trees, σ_{br}^2 = variance among branches (within trees) and σ_l^2 = variance among leaves (within branches). The variance of the population mean, corrected for the finite population is

$$s_{\bar{\bar{y}}}^2 = \left(\frac{1}{nmk}\right)^2 \left[(1 - f_1) \cdot s_{tr}^2 + f_1 \cdot (1 - f_2) \cdot s_{br}^2 + (1 - f_3) \cdot f_1 \cdot f_2 \cdot s_l^2 \right].$$

The decision of how to distribute the sample amongst stands, trees and branches, however, is also affected by cost considerations.

Example 10.18   A sampling study was carried out in *P. radiata* to estimate the mean needle dry weight of branches of a given basal diameter of 2.5 cm. Two stands were selected within a given forest district, with three trees

Table 10-6. Observed needle dry weights (g)

	Stand A				Stand B			
Branch	Tree 1	Tree 2	Tree 3	Tree 4	Tree 5	Tree 6	Tree 7	Tree 8
1	91.6	162.5	88.7	248.0	110.9	96.0	98.2	104.6
2	127.7	240.3	162.8	281.0	124.1	117.5	100.3	123.7
3	120.0	163.0	217.6	303.3	117.7	76.8	167.3	77.6
4	176.0	229.7	227.0	278.0	154.2	195.2	93.8	202.4
5	171.4	212.3	222.4	310.7	135.6	156.1	88.2	321.0
6	223.0	171.2	252.7	420.2	166.8	169.3	91.8	258.1
7	181.0	300.2	227.6	321.3	252.3	217.8	172.7	236.1

selected within each stand and seven branches within each tree. The sampling observations are given in Table 10-6.

The analysis of variance produces the following mean squares:

Between stands $MS = 64810.8$

Between trees (within stands) $MS = 17726.8$

Between branches (within trees) $MS = 3154.9$

The expected values of the relevant mean squares are:

Branches σ_b^2; Trees $\sigma_b^2 + 7\sigma_{tr}^2$; Stands $\sigma_b^2 + 7\sigma_{tr}^2 + 28\sigma_{st}^2$

The estimated variances are

$$\hat{\sigma}_{br}^2 = 3154,9 \quad \hat{\sigma}_{tr}^2 = 2081.692 \quad \hat{\sigma}_{st}^2 = 1681.6$$

The overall mean is 184.95. The variance and standard error of the estimated mean are:

$$s_{\bar{y}}^2 = \frac{3154.94}{56} + \frac{2081.69}{8} + \frac{1681.6}{2} = 1157.3$$
$$s_{\bar{y}} = 34.02$$

The 0.95 confidence interval for the population mean is:

$$\bar{y} \pm t_{0.025,48\,\text{df}} \cdot s_{\bar{y}}; \quad 184.95 \pm 2.011 \cdot 34.02; \quad 116.5 \quad 253.4$$

11   STRIP SAMPLING

In tropical forests, strip sampling is a simple and therefore a favored inventory design. It does not necessarily represent the best method because of the disproportion between the sampled area and accuracy. Randomly or systematically distributed sample strips with a width of 5, 10, or 20 m are established and all trees either on one or both sides of a line. The layout of these strips is more

efficient and less time-consuming that the establishment of square or circular sample plots. Usually strip lengths differ, the area of strips varies accordingly and ratio estimators are appropriate for calculating the mean and variance.

The mean of the target variable z and its variance are

$$\bar{z} = \frac{\bar{y}}{\bar{x}} = \frac{\sum\limits_{1}^{n} y_i}{\sum\limits_{1}^{n} x_i}$$

$$s_{\bar{z}}^2 = \bar{z}^2 \cdot \frac{1-f}{n \cdot (n-1)} \left[\frac{\sum\limits_{1}^{n} y_i^2}{\bar{y}^2} + \frac{\sum\limits_{1}^{n} x_i^2}{\bar{x}^2} - 2 \cdot \frac{\sum\limits_{1}^{n} (x_i \cdot y_i)}{\bar{x} \cdot \bar{y}} \right]$$

With

y_i = observed or measured target variable on the ith strip

x_i = strip size in m^2

f = Sampling fraction (f = n/N)

n = Sample size (number of strips).

Example 10.19   Four rows were selected at random from a population consisting of 31 strips.. The results are given below

Strip no.	Row no	Strip volume y_i (m^3)	Area x_i (ha)
1	21	531.2	2.3
2	25	448.8	2.0
3	14	568.0	2.5
3	16	566.0	2.4
	Sum	2114.0	9.2

The estimated mean volume is:

$$\bar{z} = \frac{528.5}{2.3} = \frac{2114.0}{9.2} = 229.8 \, \text{m}^3/\text{ha}$$

and its variance is

$$s_{\bar{z}}^2 = 229.8^2 \cdot \frac{1 - \dfrac{9.2}{60.0}}{4 \cdot (4-1)} \cdot \left[\frac{1126574.88}{528.5^2} + \frac{21.3}{2.3^2} - 2 \cdot \frac{4897.76}{528.5 \cdot 2.3} \right]$$

$$= 5.01 \, (\text{m}^3/\text{ha})^2$$

$$\bar{z} = 229.8 \, \text{m}^3/\text{ha} \quad s_{\bar{z}} = 2.24 \, \text{m}^3/\text{ha}$$

An alternative estimate of the mean and variance is given below

$$\bar{x} = \frac{\sum\limits_{i=1}^{n}(w_i \cdot x_i)}{\sum\limits_{i=1}^{n} w_i}$$

$$s_{\bar{x}}^2 = \frac{s_x^2}{\sum\limits_{i=1}^{n} w_i} \cdot (1 - f)$$

with

w_i = weight of the ith strip (area in ha)

x_i = strip volume per hectare

F = sampling percent.

$$f = \frac{\sum\limits_{i=1}^{n} w_i}{W} \text{ with } W = \text{total size of the population in hectares}$$

s_x^2 = Variance amongst strips

$$s_x^2 = \frac{\sum\limits_{i=1}^{n}(w_i(x_i - \bar{x})^2)}{n - 1}$$

Example 10.20   (Data from Example 10.19)

Strip No.	Row No	Strip Volume $w_i^2 \cdot x_i$ (m^3)	Strip area w_i (ha)	Volume x_i (m^3/ha)	$x_i - \bar{x}$	$w_i(x_i - \bar{x})^2$
1	21	**531.2**	2.3	231.0	1.4	4.508
2	25	448.8	2.0	224.4	−5.2	54.080
3	14	**568.0**	2.5	227.2	−2.4	14.400
4	16	**566.0**	2.4	235.8	6.2	92.256
Sum		**2114.0**	**9.2**	–	–	**165.244**

The estimated mean volume is:

$$\bar{x} = \frac{2114.0}{9.2} = 229.8 \, \text{m}^3/\text{ha}$$

with variance:

$$s_x^2 = \frac{165.244}{4-1} = 55.0813 \,(\text{m}^3/\text{ha})^2, \quad s_{\bar{x}}^2 = \frac{55.0813}{9.2} = 5.9871 \,(\text{m}^3/\text{ha})^2,$$
$$s_{\bar{x}} = 2.45 \,\text{m}^3/\text{ha}$$

In both cases all trees on the selected strips were measured. The random layout of strips, however, could be combined with a systematical sample of plots within each strip. Each strip mean represents a single observation.

Example 10.21 Five systematically distributed 0.1 ha sampling units were selected in each of the four strips from the previous example The observed volumes were:

>Strip 1: 27.2 26.0 27.2 12.8 22.4 **23.10**

>Strip 2: 30.4 26.8 30.4 12.8 22.4 **24.56**

>Strip 3: 32.0 22.4 23.2 12.8 11.2 **20.32**

>Strip 4: 26.8 30.4 26.6 11.2 12.8 **21.56**

The overall mean is $22.39 \,\text{m}^3/\text{plot}$, with a standard error of $0.85 \,\text{m}^3/\text{plot}$. The corresponding values per hectare are 224 and $8.5 \,\text{m}^3$, respectively.

12 SAMPLING WITH UNEQUAL SELECTION PROBABILITIES

In simple random sampling with replacement, each unit within the population has the same chance of being selected. This would be appropriate in a sampling study where n stands are selected from a population of N stands, with approximately equal stand volumes. In forest inventories with stands of unequal size and a varying volume per hectare serving as primary units, this assumption is seldom satisfied. In such cases, sampling with PPS or *probability proportional to prediction* (PPP) produces a more efficient estimate of the population mean and population total.

12.1 List sampling (PPS sampling)

Single-stage list or *PPS sampling* requires a list of the N sampling units in the population, is also referred to as *priori list sampling* (Loetsch 1971). If the forest consists of N stands, and the total sample size is fixed at n, the following procedure is applied.

Table 10-7. List of sampling units and volume estimates

No	Area (ha)	V (m³/ha)	Cum. Area (ha)	Cum. V (m³)	$V_{1(ha)}$ (Area)	$V_{2(ha)}$ (Volume)	Ratio 1	Ratio 2
1	2	3	4	5	6	7	8	9
1	7.3	110	7.3	803	–	–	–	–
2	16.2	80	23.5	2099	–	85	–	1.063
3	11.5	270	35.0	5204	–	–		
4	17.3	130	52.3	7453	135	–	1.038	
5	6.4	240	58.7	8989	–	–		
6	16.3	170	75.0	11760	185	185	1.088	1.088
7	22.1	60	97.1	13086	–	–		
8	12.3	130	109.4	14685	–	–		
9	18.1	80	127.5	16133	–	–		
10	13.5	210	141.0	16968	–	–		
11	7.1	270	148.1	20885	–	–		
12	15.8	130	163.9	22939	–	–		
13	12.4	250	176.3	27279	234	–	0.936	
14	6.8	80	183.1	27823	–	87		1.088
15	14.3	150	197.4	29968	–	–		
16	12.0	120	209.4	31408	–	–		
17	11.7	280	221.1	34684	–	–		
18	17.3	160	238.4	37452	–	–		
19	8.9	250	247.3	39677	293	–	1.172	
20	11.1	310	258.4	43118	–	336		1.084
21	5.8	70	264.2	43524	–	–		
22	3.7	140	267.9	44042	126	–	0.900	
23	12.5	190	280.4	46417	–	–		
24	16.4	60	296.8	47401	–	–		
25	13.5	220	310.3	50371	–	255		

- A list of either stand areas or quick ocular estimates of the stand volumes $x_1, x_2, \ldots, x_N$ is drawn up (column 3 in Table 10-7). Compiling of a list of stand areas (column 2, in Table 10-7) is simpler and less expensive than a list containing ocular volume estimates. However, the efficiency of list sampling is greater when stand volumes, instead of areas, are estimated, since the former is more closely correlated with the real volume.
- A number n_i, which is proportional to the size of the auxiliary variable x, is assigned to each sampling unit (column 3). The number n_i, divided by $\sum n_i$, is equivalent to the probability of including the ith element of the population into the sample.
- The list is augmented with a column containing the accumulated total $\sum n_i$

- (column 4).
- A random number between 1 and $\sum n_i$ is drawn. The jth sampling unit (stand) is included into the sample, if the selected random number falls within the group of numbers assigned to the jth stand. For example, $\sum n_i = 50371 \, \text{m}^3$ and the numbers 27280–27823 are allocated to stand no. 14. The probability of its selection is $544/50371 = 0.0108$. It should be noted that the sample is selected with replacement. In consequence, a given stand may be selected more than once.
- The selected stands are either completely enumerated or their volume is estimated by subsampling. They are considered to represent the (approximately) true volumes.
- A column containing the true volumes y_1, y_n is added to the table (column 5)
- The ratio of true volume over ocular estimate is calculated for each of the n stands of the sample

$$R = \frac{\sum\limits_{L=1}^{n} \frac{y_L}{x_L}}{n}$$

- In this equation, x_L represents either the stand area or a rough estimate of the stand volume. The estimator for the population total is:

$$\hat{y} = \sum_{}^{N} x_L R$$

- This is equivalent to Cochran's estimator

$$\hat{y} = \frac{\sum\limits_{L=1}^{n} \frac{y_L}{p_L}}{n} \text{ since } p_L = \frac{x_L}{\sum^{N} x}$$

- The variance estimator for $\hat{Y}$ is:

$$s_Y^2 = \frac{\sum\limits^{n} \left(R - \overline{R}\right)^2}{n \cdot (n-1)}$$

Example 10.22 In a hypothetical situation, the list consists of 25 stands with known areas for which rough estimates of the volumes per hectare are available (column 3 in Table 10-7). Columns 6 and 7 give the accurate estimates for stands selected on the basis of stand area and volume, respectively.

The resultant sample statistics are

$$\overline{R}_1 = 1.027 \qquad \overline{R}_2 = 1.096$$
$$s_{R_1} = 0.111 \qquad s_{R_2} = 0.037$$
$$s_{\overline{R}_1} = 0.050 \qquad s_{\overline{R}_2} = 0.016$$

The estimated population volumes are 51731 and 55206 m³, respectively.

12.2   3P sampling

Sampling with PPP is sometimes referred to as *posteriori list sampling* (Loetsch 1971) because the list is not drawn up prior to, but during sampling. Grosenbaugh (1963) developed the theory and introduced the method for conducting timber sales surveys. The 3P sampling method requires that each sampling unit in the population, for example, each single tree, is visited twice. The first visit is necessary to obtain a rough estimate of the size variable (in terms of volume or value) and to assign a number to each sampling unit. The probability of selecting a given sampling unit is proportional to the estimated size, i.e., the decision whether or not to include a given sampling unit into the sample depends upon its size. The second visit is required to measure the selected elements accurately, for example, by using a dendrometer. In consequence, the estimate obtained prior to sampling is used to assign a value and acts as an auxiliary variable, which is needed to decide whether or not to include a given tree. The sampling procedure is as follows:

- Select an integer L, which controls the sampling intensity. The value of L is estimated from a rough estimate of the population total $X^* = \sum(y_i)$ and from the required sample size n

  $L = X^*/n$

- After visiting the ith sampling unit and estimating its size ($= x_i$), a random number n^R between 1 and L is drawn from a random integer dispenser
- The ith sampling unit is included into the sample and measured accurately, if $n^R < x_i$. When not, only x_i will be recorded
- After examining each sampling unit in the population, the population size N, as well as x_i, for $L = 1, \ldots, N$ are known and n sampling units are measured accurately
- The estimated population total is

$$\hat{Y} = \overline{q} \sum_{i=1}^{N} x_i$$

  where

$$\overline{q} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{x_i} \right)$$

- The variance of $\overline{q}$ is

$$s_{\overline{q}}^2 = \frac{\sum\limits_{i=1}^{n} (q - \overline{q})^2}{n \cdot (n-1)}$$

and the sample variance of the estimated population total $\hat{Y}$ is

$$s_{\hat{Y}}^2 = \left(\sum_{i=1}^{N} x_i \right)^2 \cdot s_{\bar{q}}^2 \cdot \left(1 - \frac{n}{N} \right)$$

Example 10.23 The following data represent a hypothetical example, with an auxiliary variable x measured on each sampling unit in a population consisting of 15 trees. The estimated sample total is 85 and the required sample size is $n = 5$. Hence, $L = 85/5 = 17$. Random numbers (N^R) between 1 and 17 were generated and those trees for which $N^R < x_i$ were included.

Tree	x_i	N^R	y_i	q_i
1	5	13	–	–
2	3	5	–	–
3	4	2	4.5	1.125
4	6	13	–	–
5	7	7	7.2	1.029
6	8	9	–	–
7	5	14	–	–
8	4	1	3.8	0.950
9	6	2	6.3	1.050
10	9	13	–	–
11	2	4	–	–
12	5	5	5.6	1.120
13	4	15	–	–
14	6	8	–	–
15	6	9	–	–
Total	80	–	–	5.274

$$\bar{q} = 1.0548; \quad \hat{Y} = 1.0548 \cdot 80 = 84.4; \quad \sum_{i=1}^{15} q_i = 5.274; \quad \sum_{i=1}^{15} q_i^2 = 5.5839$$

$$s_{\bar{q}}^2 = 0.001044; \quad s_{\hat{Y}}^2 = 80^2 \cdot 0.001044 \cdot \left(1 - \frac{5}{15} \right) = 4.4544; \quad s_{\hat{Y}} = 2.1$$

The advantages of 3P sampling are

- The list of sampling elements is compiled during sampling. There is no need to draw up this list prior to sampling. The elements of the population are visited once only.
- The auxiliary variable x is used as a decision aid and not to predict the target variable.

13   SYSTEMATIC SAMPLING

In *systematic sampling*, the sampling units are not selected at random, but, according to a rigid scheme. For small populations, the systematic sample tends to be more representative than random sampling, since it implies the subdivision of the population into blocks, with each block represented by a single sampling unit. The main advantage of systematic sampling in forest inventories is the simplicity of its implementation in the field. The main disadvantages are

- Strictly speaking the systematic sample is a cluster of nonrandomly selected sampling units and therefore represents a sample of size 1. In consequence it is not possible to estimate of the population variance. For practical reasons, however, the data obtained by systematic sampling are analyzed with those formulae which apply to random sampling. This is justified if the individual units are uncorrelated or no spatial trend is apparent.
- Sampling units on the stand edge tend to be avoided, in which case estimates of the population mean are negatively biased.
- For a single systematic sample, available formulas for estimating the variance of a mean, require knowledge of distributional patterns of the population.

In order to draw a systematic sample of size n from a given population, numbers between 1 and N are allocated to the N sampling units of the population, for example, by superimposing a grid upon the stand map and numbering the units, consistent with a systematic pattern. A random number n_r between 1 and k is drawn to locate the first sampling unit. The second unit corresponds to the number $(n_r + k)$, the third to $(n_r + 2k)$, etc. The constant k represents the sampling interval. For example, if the 5th unit in the first row of sampling units is selected at random and the sampling interval is 10, the second unit to be included into the sample is the 15th unit within the first row. The total number of sampling units within a row, however, is seldom a multiple of k. In consequence, the location of sampling units is continued on the rth row after the first one. Because of irregular stand boundaries also, the number of sampling units within rows varies.

A systematic sample based on a square lattice implies $k = r$, but it may be more convenient to choose a shorter interval within and a longer interval between rows. Systematic sampling can also be applied when sampling units are separated in time, for example, to measure labor productivity of logging within a working day. The first measurement might then be made at a random point in time during the first hour, the second measurement one hour later, etc. In this instance, systematic sampling may produce biased estimates, because of a time-related periodicity in labor productivity. A similar situation could arise

in a stand inventory, with a nonrandom spatial distribution of the volume per unit area, although this is less likely to occur. The following notes are relevant for the field application of systematic sampling:

- Systematic sampling can be interpreted as stratified random sampling with n strata and k sampling units per stratum (Cochran 1977).
- Several studies indicate an increase of the relative precision of systematic sampling with increasing sampling intensity.
- Systematic sampling with multiple random starts resembles cluster sampling with sample means and sample variances determined for each cluster separately. The estimated within-cluster variances, however, are biased upward. Shiue (1960) showed that systematic sampling with a single random start produces unbiased estimates of the population mean, if the population is a random forest. It also produces unbiased estimates of the population variance.
- A systematic sample is more effective than random sampling in removing a linear trend of the recorded volumes per unit area. If such a trend occurs, it produces a more precise estimate of the target variable than that obtained by other designs (Madow et al. 1944). In the case of a periodic trend, the systematic sample either over- or underestimates the mean, if the location of the sample plots coincides with the turning point of the trend curve.
- More serious is the positively biased estimates of the variance, which occurs in the case of spatial correlation (Saborowski 1991). The spatial distribution of plot volume per unit area, within a given population, is essentially of a nonrandom nature. However, the extent of the bias varies and there is no simple method of correcting for bias. Because of the positive systematic errors, the results of systematic sampling, measured in terms of precision, are always superior to those predicted from variance estimates.

When systematic sampling is redefined as stratified random sampling with n strata and k sampling units per stratum, it might, in certain situations, be feasible to draw more than one sampling unit from each of the strata. The variance of the population, ignoring strata, is σ^2, those between and within strata are σ_b^2 and σ_w^2, respectively. Thus

$$\sigma^2 = \sigma_h^2 + \sigma_w^2$$

with σ_b^2 being the variance of the sample means. When two sampling units are selected within each stratum, the between-strata variance can be expressed in terms of the intraclass correlation coefficient r_I

$$\sigma_b^2 = \frac{\sigma^2}{n\left(1 + (n-1)\,r_I\right)}$$

with $\sigma_b^2 = \sigma^2/n$, if the intraclass correlation coefficient, expressing the degree of association amongst the two sampling units within the strata is zero. The variance of the mean of a systematic sample is as follows:

$$s_{\bar{y}}^2 = s_w^2 \cdot \left(\frac{N-n}{N}\right) \cdot \left[1 + \left(\frac{(n-1)/r_I}{n}\right)\right]$$

In forest inventories, a perfect subdivision of the population into blocks is seldom feasible because of the irregular stand boundaries. In practice, lines are drawn onto the stand map and sampling units within these strips are positioned at equal distances. Sometimes, the variance is calculated from successive sampling units within strips. When, for example, the sampling observations on the first line are $x_{11}, x_{12}, x_{13},$ and x_{14}, the variance estimate, derived from x_{11} and x_{12} is

$$s_{(1)}^2 = \left[x_{11} - \frac{1}{2}(x_{11}+x_{12})\right]^2 + \left[x_{12} - \frac{1}{2}(x_{11}+x_{12})\right]^2 = \frac{1}{2}(x_{11}-x_{12})^2$$

A second estimate is obtained from x_{12} and x_{13} and a third from x_{13} and x_{14}, etc. These three estimates are averaged. The complete variance formula of successive differences is

$$s_{y_{sy}}^2 = \frac{\displaystyle\sum_{j=1}^{k}\sum_{i=1}^{n_j}(x_{ij}-x_{(i+1)j})^2}{2n \cdot \displaystyle\sum_{j=1}^{k}(n_j-1)} \cdot \left(1 - \frac{n}{N}\right)$$

where

$\quad k = $ number of rows

$\quad n_j = $ number of sample units in row j

Example 10.24   Sample plots consisting of 3×3 rows were established in a square lattice in a *P. radiata*. The plot basal areas are given in Table 10-8.

Table 10-8. Plot basal areas in square centimeters

Row 1	589.2	442.9	487.2	589.2	442.9	487.2
Row 2	393.7	460.3	366.8	515.2	535.2	608.6
Row 3	320.0	816.1	537.6	478.8	849.6	699.0
Row 4	532.0	908.5	542.0	592.6	521.0	587.3
Row 5	385.9	714.9	466.4	786.4	669.3	412.1
Row 6	459.9	659.6	479.3	221.8	541.0	516.8

The variance, based on the erroneous assumption of random sampling, results in $s^2 = 21770$. The variance estimated from pairs of adjoining plots is:

$$s^2 = [(589.2 - 442.9)^2/2 + (442.9 - 487.2)^2/2 + \ldots, (487.2 - 608.6)^2/2 \\ + (659.6 - 459.9)^2/2]/35 = 21885$$

Calculating the variance from blocks of four observations produces the following estimates for the first block of four observations:

$$s^2 = [(589.2^2 + 442.9^2 + 393.7^2 + 460.3^2) - 1886.1^2/4]/3 = 5604.9$$

The average of the 9 variance estimates is 22789.

The variance of the estimated mean according to the formula of successive differences is:

$$s_{\bar{y}_{sy}}^2 = \Big[(589.2 - 442.9)^2 + (442.9 - 487.2)^2 + \cdots + (442.9 - 498.7)^2 \\ + (393.7 - 460.3)^2 + \cdots + (541.0 - 516.8)^2\Big]/(2 \cdot 36 \cdot 30) = 679.17$$

The standard error of the estimated mean is 26.1 cm^2.

14 SAMPLING PROPORTIONS

14.1 Simple random sampling

Many sampling studies deal with *qualitative* characteristics. They are *discrete variables*, which are not measured on a metric scale. In some situations, the trees are classified instead of measured, for example, as dead or alive, sick or healthy, species A or species B. In other instances, the variables are measured on an ordinal scale, for example, when trees are classified on the basis of the estimated needle loss, or on the basis of their social class within the stand as dominating, dominated, and suppressed, etc. When two classes are involved, and the population is sampled with replacement of the sampling units, the subject variable follows the binomial distribution, with a mean of p and a variance of $p(1 - p)/n$, where p = true proportion and n = sample size. When expressed in terms of the sample sum, the mean and variance are np and $np(1 - p)$, respectively.

In the case of a binomial distribution of the subject variable, the number of sample elements (y) belonging to class 1, is counted in a random sample of size n and expressed as an observed proportion

$$\hat{p} = y/n.$$

The $(1 - \alpha)$ confidence limits for the population proportion are

$$\hat{p} \pm z_{1/2\alpha} \sqrt{\frac{p(1-p)}{n}}$$

Sampling with replacement of the sampling units generates a binomial distribution, if the assumption of statistically independent outcomes of sampling holds true. Sampling without replacement generates a hypergeometric distribution with

$$\mu = p$$
$$\sigma_p^2 = \frac{p(1-p)}{n} \cdot \frac{N-n}{N-1}$$

The large-sample approximation of the hypergeometric distribution as a normal distribution produces the following estimate for the confidence interval for p:

$$p \pm z_{1/2\alpha} \sqrt{\sigma_p^2}$$

When p is near to 0 or 1, the normal approximation is valid only for large n and N.

Example 10.25   In a sampling trial to estimate the proportion of trees, affected by environmental pollution, within a given stand, large-scale photographs were used to evaluate the vitality of 100 trees, which were selected randomly, but without replacement, from a population containing an estimated 4000 trees. Twenty-one trees were classified as sick. The sample estimate $\bar{p} = 0.21$ represents an unbiased estimate of the population proportion p and

$$s_{\bar{p}}^2 = \frac{0.21 \cdot 0.79}{100} \cdot \frac{3900}{3999} = 0.00162 \quad s_{\bar{p}} = 0.040$$

estimate the variance and standard error, respectively. The large-sample approximation of the 0.95 confidence limits for the true proportion gives the following results:

$$0.210 \pm 1.96 \cdot 0.040 \Rightarrow 0.1316 \quad 0.2884$$

The approximation as a binomial distribution produces a variance estimate of 0.00166 instead of 0.00162.

14.1.1   Sample size for proportions

The $(1 - \alpha)$ confidence interval for a population proportion, ignoring the correction for the finite population, is given by

$$p \pm z_{1/2\alpha} \sqrt{\frac{p(1-p)}{n}}$$

Hence

$$E = z_{1/2\alpha}\sqrt{\frac{p\,(1-p)}{n}}$$

is to be solved for n

$$n = p(1-p)\frac{\left(z_{1/2\alpha}\right)^2}{E^2}$$

In order to apply this formula, the population proportion p is replaced by a prior estimate of the true proportion. If the latter is completely unknown, sampling should be carried out in two stages. The stage 1 sample serves to obtain a preliminary estimate of p, which will be used in calculating the required sample size.

Example 10.26 The afforested area in a region is estimated by dot sampling on a satellite image. Existing records indicate that this area is approximately 15% of the total land area. It is required that the maximum error shall not exceed 10% of the estimated proportion, with a probability of 0.99. Hence

$$n = 0.15 \cdot 0.85 \cdot \frac{2.576^2}{0.015^2} = 3760$$

An incorrect prior estimate of p has a profound effect on the required sample size. The product $p(1-p)$ increases from 0 for $p = 0$ to 0.25 for $p = 0.5$. In consequence, if sampling produces an estimate of p substantially greater than the assumed 0.15, the calculation of the required sample size underestimates n and vice versa.

14.2   Cluster sampling

In estimating proportion, the cost of locating sample units is usually very high in relation to the cost of observing certain attributes in sample units. Therefore in practice, simple random sampling is seldom used to estimate proportions. *Cluster sampling* with *equal* as well as *unequal sizes* is a useful alternative to simple random sampling.

Due to a possible correlation between the observations on individual sampling units within the cluster, this sample design is generally less efficient than simple random sampling. In cluster sampling, however, the time required to select and locate the units is reduced substantially by grouping sampling units. Expressed per unit cost the method may therefore produce more accurate estimates of the target variable than simple random sampling.

14.2.1 Equal cluster sizes

In case of large clusters with at least 100 units of equal size within each cluster, which is the case when k-tree sampling is applied, the sample mean and variance can be obtained from the standard formulae for continuous variables.

The mean proportion is calculated as a mean of ratio estimator

$$\overline{p} = \frac{\sum p_i}{n}$$

Where

$$p_i = \frac{y_i}{k}$$

y_i = number individuals having the characteristics within cluster or plot

k = total number of the individuals within in clusters or plots

n = number of clusters

and the variance of estimated mean proportion is

$$s_{\overline{p}} = \frac{\sum p_i^2 - \frac{(\sum p_i)^2}{n}}{n(n-1)}$$

Example 10.27 (equal cluster sizes) The proportion of healthy plants in a forest nursery is to be estimated. Ten clusters, each containing 100 plants ($k = 100$), were selected at random. The data are given in Table 10-9.

$$\overline{p} = \frac{\sum p_i}{n} = \frac{901/100}{10} = 0.901 \quad \sum p_i = 9.01 \quad \sum p_i^2 = 8.1293$$
$$s_{\overline{p}}^2 = 0.000012544 \quad s_{\overline{p}} = 0.011$$

The $(1 - \alpha)$ confidence interval for the mean proportion, for $\alpha = 0.05$ is

$$\overline{p} \pm t_{1/2\alpha, n-1} s_{\overline{p}} \quad : \quad 0.901 \pm 2.262 \cdot 0.011 \quad : \quad 0.876 \quad 0.926$$

Table 10-9. Number of healthy plants

Cluster No.	Number of healthy plants	Cluster No.	Number of healthy plants
1	92	6	89
2	97	7	88
3	87	8	91
4	85	9	94
5	90	10	88
		Total	901

If proportions occur either greater than 0.80 or smaller than 0.20, it is advisable to apply a square root transformation to the data and to retransform the calculated means and confidence limits.

14.2.2 Clusters of unequal size

When applying fixed-radius sample plots for sampling, the total number of individuals can be expected to vary. In consequence, a weighting procedure is required to calculate the mean. In such cases, the ratio-of-means should be calculated.

$$\overline{p} = \frac{\overline{y}}{\overline{x}} = \frac{\sum y_i}{\sum x_i}$$

where

x_i = total number of individuals in the ith cluster

y_i = number of individuals in the ith cluster possessing the attribute.

The estimated variance is

$$s_{\overline{p}}^2 = \frac{1}{\overline{x}^2} \cdot \frac{s_y^2 + \overline{p}^2 \cdot s_x^2 - 2\overline{p}s_{yx}}{n} \left(1 - \frac{n}{N}\right)$$

Where

s_x^2, s_y^2 = variance of x and y, respectively

s_{yx} = covariance between x and y

n = sample size.

Example 10.28 (unequal cluster sizes) The proportion of ash trees in a mixed stand consisting of the common beech and ash, is to be estimated. The variables x and y refer to the total number of trees and the number of ash trees on fixed area plots (Table 10-10).

Table 10-10. Number of total and ash trees on fixed area plots

Plot No.	Total number of trees (x_i)	Number of ash trees (y_i)	Plot No.	Total number of trees (x_i)	Number of ash trees (y_i)
1	16	1	7	20	2
2	15	3	8	15	1
3	22	3	9	19	1
4	18	2	10	21	2
5	15	–	Total	186	19
6	25	4	Mean	18.6	1.9

$$\overline{p} = \frac{1.9}{18.6} = 0.1022; s_x^2 = \frac{3566 - \dfrac{186^2}{10}}{9} = 11.8222; s_y^2 = \frac{49 - \dfrac{18^2}{10}}{9} = 1.4333$$

$$s_{yx} = \frac{\sum xy - \dfrac{\sum x \sum y}{n}}{n-1} = \frac{379 - \dfrac{186 \cdot 18}{10}}{9} = 2.8444$$

$$s_{\overline{p}}^2 = \frac{1}{18.6^2} \frac{1.4333 + 0.1022^2 \cdot 11.8222 - 2 \cdot 0.1022 \cdot 2.8444}{9} = 0.000313260$$

$$s_{\overline{p}} = \pm \cdot 0.0177$$

The 0.95 confidence interval is

$$0.102 \pm \cdot 2.262 \cdot 0.0177 \rightarrow 0.102 \pm 0.040$$

15 ESTIMATING CHANGES

Four methods are available for estimating and monitoring changes, more particularly to estimate the rate of growth:

- Estimating growth from temporary sample plots, which are measured once only, with *independent random samples* drawn on successive occasions.
- Estimating growth from *permanent sample* plots, which are remeasured at regular time intervals.
- Estimating growth based on subsamples representing measurements of permanent sample plots on successive occasions.
- *Sampling with partial replacement* (SPR) of sample plots. A number of sample plots is established on the first occasion and are partially remeasured on the second, whereas the abandoned plots are replaced by new units.

15.1 Independent and matched sampling

The measurements of temporary sample plots to estimate changes is recommended when permanent plots on successive occasions are poorly correlated, for example, because of thinnings being carried out during the growth period, or because of excessive mortality or due to high ingrowths. In case of temporary sample plots the covariance between successive measurements is zero and the variance of the estimated growth is

$$s_{(y_2 - y_1)}^2 = s_{y_1}^2 + s_{y_2}^2$$

The $(1 - \alpha)$ confidence interval for the true growth is

$$\left(\bar{y}_2 - \bar{y}_1\right) \pm t_{1/2\alpha, n_1 + n_2 - 2} \cdot \sqrt{\frac{s_{y_1}^2}{n_1} + \frac{s_{y_2}^2}{n_2}}$$

Matched sampling, which occurs when monitoring growth in permanent sample plots, reduces the variance of the estimated growth if y_1 and y_2 are closely and positively correlated. The variance of the estimated growth obtained by matched sampling is

$$s_{(\bar{y}_2 - \bar{y}_1)}^2 = s_{\bar{y}_1}^2 + s_{\bar{y}_2}^2 - 2\text{cov}\left(\bar{y}_1, \hat{y}_2\right)$$

and the confidence limits for the true growth, based on a sample of size n is

$$\left(\bar{y}_2 - \bar{y}_1\right) \pm t_{1/2\alpha, n-1} \sqrt{\frac{s_{\bar{y}_2 - \bar{y}_1}^2}{n}}$$

The number of degrees of freedom is reduced from $2(n - 1)$ to $(n - 1)$. Due to the higher associated t values, it produces a wider confidence interval, i.e., a less precise estimate of the population parameter, if successive measurements were uncorrelated. The difference between $t_{1/2\alpha}, 2(n - 1)$ and $t_{1/2\alpha}, (n - 1)$ is substantial for small, but negligible for large samples. The cost per sampling unit, however, is higher than in temporary plots, because of the necessity to relocate the plots.

15.2 Sampling with partial replacement (SPR)

This method was introduced by Cochran (1953), the underlying theory was further developed by Ware and Cunia (1962) and the application in forest inventories was highlighted by Bickford et al. (1963). Cochran (1953) considered a sampling problem, whereby the population is sampled on occasions 1 and 2. A random sample of n sampling units is measured on occasion 1, with m plots selected for remeasurement on occasion 2. The remaining $u = n - m$ units are discarded and new sampling units selected and measured. The mean of the u unmatched sampling units on occasion 2 estimates the mean of this part of the sample, whereas a regression estimator is used to estimate the mean of the matched part obtained from measurements on occasion 2. The m observations of y on occasion 2 are regressed on those of occasion 1 and the regression coefficient b_1 is used to adjust the sample mean on occasion 2

$$y_{2m(\text{adj.})} = y_{2m} + b_1 \left(y_1 - y_{1m}\right)$$

with

$\bar{y}_1$ = occasion 1 sample mean of the complete sample

$\bar{y}_{1m}$ = occasion 2 mean of the subsample.

The variances of the unmatched and matched portions on occasion 2 are identical to those for simple random and double sampling respectively. Their reciprocal variances are used to assign weights w_1 and w_2 to the two estimates:

$$w_u = \frac{1}{us_2^2}$$

$$w_m = \frac{1}{s_2^2 \left(1 - r^2\right)/m + r^2 s_2^2/n}$$

$$\bar{y}_2 = w_u \bar{y}_{2u} + w_m \bar{y}_{2m}$$

with

$$s_{\bar{y}_2}^2 = s_2^2 \left[\frac{n - ur^2}{n_2 - u^2 p^2} \right]$$

(Cochran 1953; de Vries 1986)

Cochran showed that 30–40% of the first set of sampling units should be used for remeasurements, if the cost per sampling units are the same for occasions 1 and 2 and the total number of sampling units is fixed prior to sampling. Fewer plots should be selected for remeasurement when the costs per sampling unit on occasion 2 are higher than on occasion 1. At least 40 plots are required to estimate the correlation coefficient. Ware et al. (1962) derived the following formula for estimating the number of remeasured plots, which ignores double sampling:

$$m = n\sqrt{\frac{1 - r^2}{r^2}} \cdot \left[\sqrt{\frac{c_1}{c_2}} - \sqrt{1 - r^2} \right]$$

The advantage of continuous forest inventory with partial replacement is the greater precision of current estimates of the target variable and the flexibility in sampling design. Sampling with partial replacement is always at least as efficient as sampling based on fixed remeasured sample plots, and usually more efficient (Cunia 1964).

The method "sampling with partial replacement" (SPR) differentiates between three groups of sample plots at occasions A and B respectively:

1. permanent sample plots, which are measured at both occasions
2. temporary plots which are measured only at the beginning of the period
3. temporary plots which are selected and measured at the end of the period

The third group replaces the sample plots of group 2. In case of a third occasion inventory, however, they could be partly incorporated as permanent sample plots. The permanent sample plots are used to regress the volume at the end of the period on that of the beginning. The relevant regression equation is subsequently applied to estimate the volume of those plots which were not

remeasured. They are combined with the new set of plots and their volumes. This makes it possible to estimate the volume of the initial plots at the beginning and end of the period, some of them being measured directly others obtained by regression. The volumes, volume changes and the increment can therefore be derived from the pooled data of permanent and temporary plots.

	Temporary plots	Permanent plots	Temporary plots	
Occasion A	n_{1t}	n_p		$\rightarrow n_1 = n_{1t} + n_p$
Occasion B		n_p	n_{2t}	$\rightarrow n_2 = n_{2t} + n_p$

The mean at occasion B is estimated from

$$\bar{x}_2 = a\left(\bar{x}_{1t} - \bar{x}_{1p}\right) + b\bar{x}_{2p} + (1 - b)\,\bar{x}_{2t}$$

with

$$a = \frac{r \cdot s_{x_{2p}}}{s_{x_{1p}}} \cdot \frac{n_p \cdot n_{1t}}{n_1 \cdot n_2 - n_{1t} \cdot n_{2t} \cdot r^2} \quad \text{and } b = \frac{n_p \cdot n_{1t}}{n_1 \cdot n_2 - n_{1t} \cdot n_{2t} \cdot r^2}$$

Where

$\bar{x}_{1t}$ = mean volume of temporary plots at occasion A

$\bar{x}_{1p}$ = mean volume of permanent plots at occasion A

$\bar{x}_{2p}$ = mean volume of permanent plots at occasion B

$\bar{x}_{2t}$ = mean volume of temporary plots at occasion B

r = correlation coefficient between plot volumes at occasions A and B, respectively (permanent plots)

$^s x_{1p}$ = standard deviation of permanent plots at occasions A and B
$^s x_{2p}$

The variance of the estimated means is as follows

$$s_{\bar{x}_2}^2 = a^2 \cdot s_{x_{1p}}^2 \cdot \left(\frac{1}{n_{1t}} + \frac{1}{n_p}\right) + s_{x_{2p}}^2 \cdot \left(\frac{b^2}{n_p} + \frac{1 - b}{n_{2t}}\right)^2 - \frac{2a \cdot b \cdot r \cdot s_{x_{1p}} \cdot s_{x_{2p}}}{n_p}$$

The volume change is obtained from

$$\Delta\bar{x}_{12} = A \cdot \bar{x}_{2p} - B \cdot \bar{x}_{1p} + (1 - A)\,\bar{x}_{2t} - (1 - B) \cdot \bar{x}_{1t}$$

with

$$A = \frac{n_p \cdot (b_{12} \cdot n_{2t} + n_1)}{n_1 \cdot n_2 - n_{1t} \cdot n_{2t} \cdot r^2} \quad \text{and } B = \frac{n_p \cdot (b_{21} \cdot n_{1t} + n_2)}{n_1 \cdot n_2 - n_{1t} \cdot n_{2t} \cdot r^2}$$

$$b_{12} = \frac{r \cdot s_{x_{1p}}}{s_{x_{2p}}} \quad \text{and } b_{21} = \frac{r \cdot s_{x_{2p}}}{s_{x_{1p}}}$$

Example 10.29   The volume change of a forest stand is to be estimated for a period of 5 years. Five 0.1 ha sample plots were measured in 1985 and remeasured in 1990. In 1990 a subset of five temporary plots, measured in 1985, were replaced by five new temporary plots. Plot volumes were converted to volume per hectare. They are summarized below.

Plot no.	$V_{1985} (\mathrm{m^3/ha})$	$V_{1990} (\mathrm{m^3/ha})$	$\Delta V (\mathrm{m^3/ha})$
1	318	–	–
2	216	–	–
3	300	–	–
4	275	–	–
5	254	–	–
6	246	282	36
7	199	231	32
8	295	324	29
9	205	230	25
10	274	301	27
11	–	352	–
12	–	247	–
13	–	340	–
14	–	301	–
15	–	290	–

$n_1 = 10, n_2 = 10, n_p = 5, n_{1t} = 5, n_{2t} = 5 r = 0.995$

$$\bar{x}_{1p} = 243.8 \,\mathrm{m^3/ha} \qquad s^2_{x_{1p}} = 1762.7 (\mathrm{m^3/ha})^2 \qquad s_{x_{1p}} = 41.9845 \,\mathrm{m^3/ha}$$
$$\bar{x}_{1t} = 272.6 \,\mathrm{m^3/ha} \qquad s^2_{x_{1t}} = 1591.8 (\mathrm{m^3/ha})^2 \qquad s_{x_{1t}} = 39.8974 \,\mathrm{m^3/ha}$$
$$\bar{x}_{2p} = 273.6 \,\mathrm{m^3/ha} \qquad s^2_{x_{2p}} = 1769.3 (\mathrm{m^3/0.1ha})^2 \qquad s_{x_{2p}} = 42.0630 \,\mathrm{m^3/ha}$$
$$\bar{x}_{2t} = 306.0 \,\mathrm{m^3/ha} \qquad s^2_{x_{2t}} = 17.585 \left(\mathrm{m^3/0.1ha}\right)^2 \qquad s_{x_{2t}} = 41.9345 \,\mathrm{m^3/ha}$$

$$a = \frac{0.995 \cdot 42.0630}{41.9845} \cdot \frac{5 \cdot 5}{10 \cdot 10 - 5 \cdot 5 \cdot 0.995^2} = 0.3312$$

$$b = \frac{5 \cdot 5}{10 \cdot 10 - 5 \cdot 5 \cdot 0.995^2} = 0.3322$$

The "best"estimate for the mean volume ($\bar{x}_2$) at the end of the period (1990) is:

$$\bar{x}_2 = 0.3312 \cdot (272.6 - 243.8) + 0.3322 \cdot 273.6 + (1 - 0.3322) \cdot 306.0$$
$$= 304.8 \, \text{m}^3/\text{ha}$$

$$s_{\bar{x}_2}^2 = 0.3312^2 \cdot 1762.7 \cdot \left(\frac{1}{5} + \frac{1}{5}\right) + 1769.3 \cdot \left(\frac{0.3322}{5} + \frac{1 - 0.3322}{5}\right)^2$$
$$- \frac{2 \cdot 0.3312 \cdot 0.3322 \cdot 0.995 \cdot 41.9845 \cdot 42.0630}{5} = 70.7514 (\text{m}^3/\text{ha})^2$$

$$s_{\bar{x}_2} = 8.4 \, \text{m}^3/\text{ha}$$
$$\bar{x}_2 = 304.8 \, \text{m}^3/\text{ha and } s_{\bar{x}} = 8.4 \, \text{m}^3/\text{ha}$$

$$b_{12} = \frac{0.995 \cdot 41.9845}{42.0630} = 0.9931, \quad b_{21} = \frac{0.995 \cdot 42.0630}{41.9845} = 0.9969$$

$$A = \frac{5 \cdot (0.9931 \cdot 5 + 10)}{10 \cdot 10 - 5 \cdot 5 \cdot 0.995^2} = 0.9944, \quad B = \frac{5 \cdot (0.9969 \cdot 5 + 10)}{10 \cdot 10 - 5 \cdot 5 \cdot 0.995^2} = 0.9957,$$

The "best" estimate for the mean volume change is

$$\Delta\bar{x}_{12} = 0.9944 \cdot 273.6 - 0.9957 \cdot 243.8 + (1 - 0.9957) \cdot 272.6$$
$$\Delta\bar{x}_{12} = 29.9 \, \text{m}^3/\text{ha}$$

The variance of the estimated mean volume change is obtained from

$$s_{\Delta\bar{X}_{1''}}^2 = \frac{0.9944^2 \cdot 1769.3 + 0.9957^2 \cdot 1762.7}{5}$$
$$- \frac{2 \cdot 0.9944 \cdot 0.9957 \cdot 0.995 \cdot 1762.7^{0.5} \cdot 1769.3^{0.5}}{5}$$
$$+ \frac{(1 - 0.9957)^2 \cdot 1762.7}{5} + \frac{(1 - 0.9944)^2 \cdot 1769.3}{5}$$
$$= 3.5148 (\text{m}^3/\text{ha})^2$$

$$s_{\Delta\bar{x}_{12}} = 1.87 \, \text{m}^3/\text{ha}$$
$$\Delta\bar{x}_{12} = 9.9 \, \text{m}^3/\text{ha with standard with standard error} = 1.9 \, \text{m}^3/\text{ha} \quad (\cong 6.3\%)$$

The SPR method combines the advantages of permanent and temporary sample plots. The flexibility of the design increases by removing some of the occasion A sample plots and the addition of new plots, thereby making it possible to modify the sampling design in line with a changed forestry situation in terms of age classes and tree species. The danger of the number of permanent sample plots becoming too small to adequately represent the population, can be avoided by adding sample plots at successive occasions. By replacing existing plots by new ones at the next occasion, the set of sample plots will remain representative

for a long period of time. The varying accuracy which is obtained for certain subpopulations was a disadvantage of the permanent sample plots design.

In case of using the subset of new sample plots, which was added at occasion B, as permanent sample plots at occasion C, the trees within these plots should be permanently identifiable and remeasured. This, however, has an adverse effect on the cost of the inventory. For a given set of sample plots, which are measured at the beginning and end of the period between successive inventories, the sampling method which is based on permanent sample plots, without the addition of temporary plots, produces more accurate information about volume changes. In addition the final sample, which is generated by a combination of permanent and temporary plots may no longer be representative for the population in its entirety.

The third method of subsampling is closely related to SPR sampling. The second-occasion data are used to continue subsampling. A subsample is selected from the first-occasion permanent sample plots and only these plots are measured. To estimate changes this subsample is considered to represent permanent sample plots, the remaining first-occasion plots as temporary plots. Volume, volume changes, and growth on the second occasion could be obtained as regression estimates. The method is not suitable to estimate long-term changes in a series of successive inventories, because toe few data will available to produce accurate estimates of changes. It may be suitable in intermediate inventories, for example to estimate the quantity and distribution of the wood mass resulting from wind damage or other natural disasters.

16   LINE INTERSECT SAMPLING

Line intersect sampling, based on *Buffon's needle problem* (Buffon 1777), is a sampling technique in its own right and has been applied in the following situations:
- Estimation of logging waste (Warren et al. 1964)
- Sampling for fuel volume (Brown 1971; van Wagner 1968)
- Sampling for biomass in arid regions
- Estimation of the area of wooded strips in Kansas (Hansen 1985)
- Estimation of the length of roads (Matern 1964)
- Estimation of the length of hedges in the French national forest inventory (Chevrou 1973)
- Estimating the length of boundaries between ecosystems (Hildebrandt 1975)

The theory underlying line intersect sampling for those sampling elements which are either linear or circular was developed by de Vries (1973, 1974,

1979), although the basic problem was recognized and formulated by Buffon. A thin needle of length l_i is placed on a flat area with a width of W and a length of L units, located within an area of irregular shape and size A. Parallel lines with a length of L units are drawn, one of them passing through the center. It is assumed and required that the needle length l_i is less than the distance between two adjacent lines. The probability that the jth needle intersects the center line is a function of the needle length, the length of the sampling line, and the orientation of the needle. The latter is assumed to be a random variable, with a uniform distribution, within a domain extending between 0 and 180°. The probability of intersection (de Vries 1986) is

$$p_i = \frac{2L \cdot l_i}{\pi A}$$

In the case of a fixed orientation at an angle of α degrees to the sampling line, this probability is to be multiplied by $\sin(\alpha)$. Hansen (1985) applied line intersect sampling to estimate the area of wooded strips in Kansas. A systematic sample of 57 counties was laid out, with one township selected at random from each county. A cross-hatched grid was placed over the photo mosaic of the townships, whereafter the intersections with wooded strips were counted to obtain the value of the auxiliary variable x and the total length of all wooded strips ($= x$). There followed an exact measurement of the total length of all wooded strips ($= y$). The ratio of means estimator was used to estimate the total length of wooded strips

$$\hat{L} = \frac{y}{x} \cdot X$$

where X = total number of intersections in the entire state, obtained by counting.

Chapter 11

REMOTE SENSING IN FOREST MENSURATION

1   INTRODUCTION

The first attempts to introduce aerial photographs as a remote-sensing tool in forestry were made in 1887. An airborne balloon was used as a photographic platform to produce photographs of forests in the vicinity of Berlin. The objective was to examine the possibility of preparing forest maps from aerial photographs and, in addition, to classify and describe the forest on the basis of a visual examination of the photographs. Aerial photography from aircraft was introduced during World War II, primarily for military purposes. It stimulated rapid technical developments in aerial photography and photogrammetry, which in turn induced applications in other fields, for example, in the exploration of natural resources. Since then aerial photographs have been increasingly used to rationalize mapping operations, but in addition, they are widely used to facilitate orientation in the forest and to stratify the forest.

The subsequent technical advances in photography widened the scope of aerial photography. The quality of the photographs improved gradually, partly because more advanced aerial cameras and high-quality lenses were developed, partly because of the increasing availability of aerial films of higher resolution. Recently, the development of nonphotographic sensors, the application of digital photogrammetry and the widespread use of geographic information systems have widened the scope and usefulness of remote-sensing technology for mapping and for the classification of forests. In recent years, satellite imagery has been integrated successfully with the inventory of large forest tracts, sometimes in combination with double sampling.

Technical progress with regard to the integration of aerial photo interpretation with forest mensurational and forest inventory sampling techniques has been less spectacular. Aerial photographs, however, are routinely used in the inventory of large forest tracts in North America, Scandinavia, and tropical forests, primarily in combination with two-phase sampling. The phase 1 sample

served measure quantitative characteristics of the forest on aerial photographs and phase 2 involved the selection of a subsample. The latter was remeasured by conventional ground surveying. In many other countries, the usefulness of aerial photographs for forest inventories remains a controversial issue. It has been found and is generally accepted that the extraction of forest mensurational data from aerial photographs in the closed forests of Central Europe has certain limitations. Recent German studies, however, produced evidence that photo measurements with the aid of modern digital technology provide dendrometric information of nearly the same accuracy as that obtained by conventional ground surveying. At the same time, this information is obtained at lower cost since less traveling is involved and many more sampling units can be measured per unit time. In forests of average management intensity, the aerial photograph, in combination with ground surveying, is therefore useful in estimating the site index, the stand density, and the stand volume of individual forest stands. In addition, a forest specialist can extract that information from aerial photographs, which cannot be easily obtained by ground surveying, or it can be obtained only at greater cost, such as the estimation of the extent of stresses in forests.

Satellite imagery is increasingly used for mapping and classification of forests, and has been found to be indispensable for such operations in large forest tracts in remote and inaccessible regions. Concurrently, computers and software were developed to manage and analyze the vast amount of digital information obtained from satellite images of ever-increasing resolution. In consequence, the forest manager is now able to obtain a considerable amount of forestry-relevant information of higher quality at substantially lower cost than previously obtained by conventional ground surveying.

The applications of aerial photographs in forestry, prior to the early 1960s, have been reviewed by Loetsch et al. (1964), Huss et al. (1984), and Hildebrandt (1996). The authors summarized advances in modern remote-sensing technology and analogue methods of photo interpretation. The present chapter discusses the applications of photogrammetry in forest mensuration: acquisition of dendrometric tree, stand information, and the use of remote sensors in forest inventories.

2   FUNDAMENTALS OF AERIAL PHOTOGRAPHY

Aerial photographs taken from an aircraft represent a perspective view of the area beneath the aircraft. All light rays are projected through the perspective center, i.e., through the center of the camera lens, to form an image of the photographic film. A photograph is called vertical if the camera axis has been

pointed vertically down and oblique when tilted to a predetermined angle to the vertical. Prior to World War II, oblique photographs were extensively used for mapping large tracts, since they cover a larger area. However, all vertical aerial photographs are slightly tilted, due to airplane movements during exposure. This tilt, however, should not exceed 3–4° in either direction.

The scale of the photograph is a function of the focal length of the camera lens and flying height:

$$\text{scale} = \frac{\text{focal length (m)}}{\text{flying height (m) above ground}}$$

Alternatively it is defined as:

$$\text{scale} = \frac{\text{distance between objects on aerial photo}}{\text{ground distance}}$$

For example, for a focal length of 15 cm and a flying height of 1500 m the scale of the photograph is 1:10000. A distance of 100 m on the ground is recorded as a distance of 1 cm on the aerial photo. In consequence, a single 23×23 cm photograph covers an area of 529 ha although, because of overlap, the effective area is much smaller. Scale variations within a given aerial photograph are due to elevation differences in hilly country and camera tilt, those between adjoining photographs within a flight line are, in addition, caused by variations in flying height (see Figure 11-1). The *principal point* of an aerial
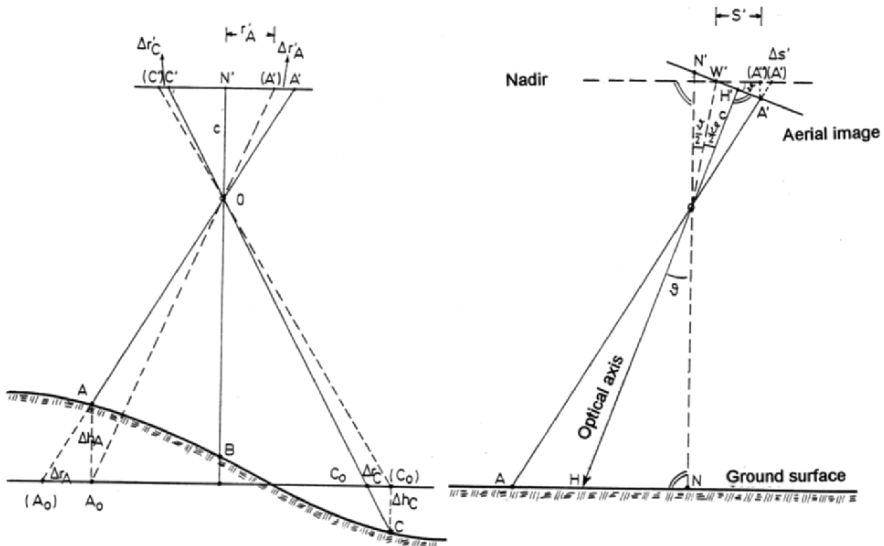
Figure 11-1. Geometry of the aerial photograph.

photograph represents the geometric center of the photograph. It is identified by connecting the opposite *fiducial marks* on the edge of the photo by (penciled) lines and determining the point of intersection. The photographic position of the principal point is required for stereoscopic viewing. The *nadir* is defined as the point where a vertical line from the ground up, which passes through the center of the camera lens intersects the plane of the photograph. In the case of perfectly vertical photographs, the principal point coincides with the nadir. *Radial displacement* of an image results primarily from changes in ground elevation and is described as topographic displacement. It occurs radially in an outward direction from the nadir. Displacement due to camera tilt takes place radially outward from the *vanishing point*.

Radial displacement should be explained in terms of the perspective view of a camera lens, which points down vertically. A tree exactly beneath the camera is viewed in its correct position. In absence of image displacement, the characteristic shape of the crown cannot be identified. However, the image of the same tree, if it were located near the edge of the photograph, will be displaced. The amount of displacement of the top of the tree differs from that of its base. This phenomenon is of essential importance for object recognition and to measure heights. The real reason for this displacement is scale variation due to elevation differences (in this case, that between the top and base of the tree, respectively).

Vertical photographs are taken along a series of parallel flight lines so that each object within the study area is photographed from two adjacent positions of the aircraft along the flight line. The overlapping part of a photograph is called *endlap* and the area covered by each of two adjoining photographs forming a stereopair is referred to as the *stereoscopic overlap area*, which can be viewed stereoscopically. The amount of overlap varies between 55% and 65%. In order to ensure that all objects between adjoining flight lines are properly exposed to the aerial camera, a lateral *sidelap* is prescribed. It is dependent on topographic conditions and navigation supports and varies between 15% and 25% (Figure 11-2).

Example 11.1

> Flight planning
>
> Total area $(A_t) = 7500$ ha
>
> Scale of aerial photographs $(1/m_{ph}) = 1 : 10000$, aerial photo scale figure $(m_{ph}) = 10000$
>
> Size of aerial photographs $(s') = 23$ cm $\cdot$ 23 cm (9 inch $\cdot$ 9 inch)
>
> Endlap $= 60\%$ $(p = 0.60)$, sidelap $= 15\%$ $(q = 0.15)$
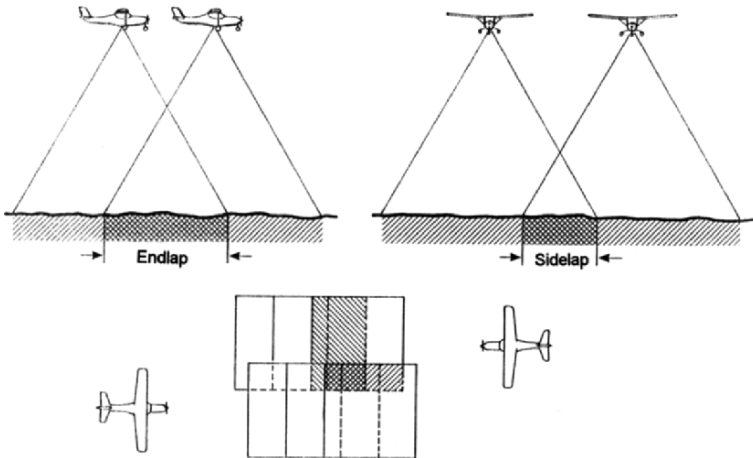
Figure 11-2. Endlap and sidelap in vertical aerial photography.

Size of aerial photographs on the ground $(s = s' \cdot m_b) = 2300\,\text{m}$

Area covered by a single aerial photograph $(A_{ph} = s^2 = (s' \cdot m_{ph})^2 = 5290000\,\text{m}^2 = 529\,\text{ha}$

Stereoscopic model area of a stereopair $(A_{sm} = s(1 - p)) = 3174000\,\text{m}^2 = 317.4\,\text{ha}$

New stereoscopic area $(A_n = s^2 \cdot (1 - p)(1 - q)) = 1798600\,\text{m}^2 = 179.86\,\text{ha}$

Required number of aerial photographs $(n = A_t/A_n) \cong 42$

Parallax is defined as the apparent displacement of the position of the image of an object and is associated with the basic principle of central projection in photography. The amount of displacement of an object differs on the two adjacent photographs of a stereopair, because of a shift in the position of the point of observation. Within each of the parallel flight strips required to cover the study area, the aerial photographs are so spaced that the image of each object on the ground appears on each of the two adjoining photographs within the flight strip. The absolute stereoscopic parallax, or x-parallax, is defined as the distances between the images of a given point measured on the two oriented photographs of the stereopair. Projected on the line of flight, these distances correspond to the sum of distances between images of the point and the principal points on both photographs. The apparent displacement of the top and

base of an object, for example a tree, are determined and the parallax of the top differs from that of the base. *The parallax difference* (Δp or Δx) *between the top and base of the tree can be used to calculate the height as a function of the parallax difference, flying height related to the base of the object and the photographic base.* The latter is obtained by averaging the distance between the principal points and their conjugates on the two adjoining photographs. Parallax measurements require stereoscopic viewing of those adjacent photographs on which the object can be located. The line of flight is reconstructed on each photograph of the stereopair by connecting the principal point on each photograph with its conjugate principal point, i.e., with the image of the other photograph of the stereopair (Figure 11-3).

The two resultant straight lines are properly oriented to form a single common line, which reflects the flight direction during exposure. For stereoscopic viewing, the two photographs are separated along the common line to such an extent that a fused image of selected points on the photos is obtained. The *air base* is defined as the distance between the principal point and its conjugate, measured on one of the photos of the stereomodel.

(1) Determination of the principal points M_1 and M_2
(2) Determination of the conjugate principal points $M_{1'}$ and $M_{2'}$
(3) Relative orientation of the stereopair under a pocket stereoscope
(4) Orientation of the stereopair under a mirror stereoscope
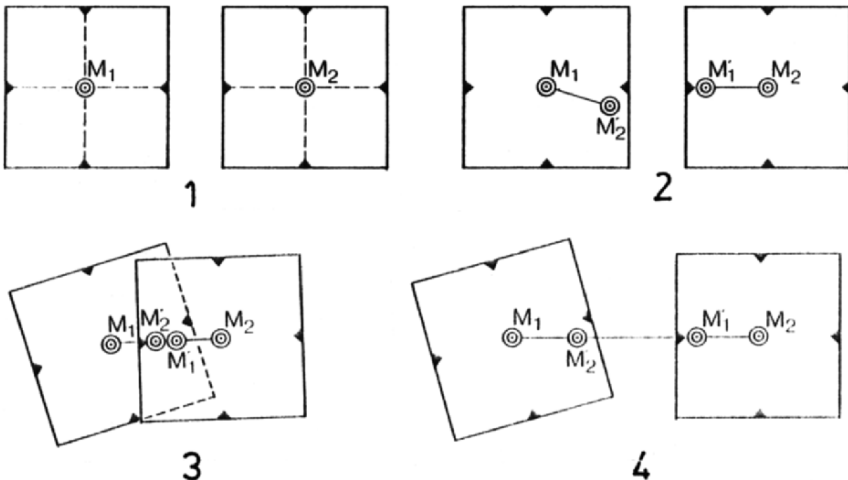
Figure 11-3. Relative orientation of the stereo pair.

3   DENDROMETRIC DATA

On aerial photographs the majority of dendrometric characteristics cannot be measured directly. They are derived from relationships between photo and dendrometric variables. The accuracy depends upon:

- The reliability of the assessment of photogrammetric characteristics and quantitative information from photo interpretation
- The correlation between the aerial dendrometric variables and the "true" values obtained by ground surveys

3.1   Tree height

One of the most important dendrometric variables, which can be directly measured from aerial photographs, is the tree height. Tree heights may be derived from:

- Radial displacement of images on single photographs (Figure 11-4)
- Shadow length of individual trees measured on single photographs (Figure 11-5)
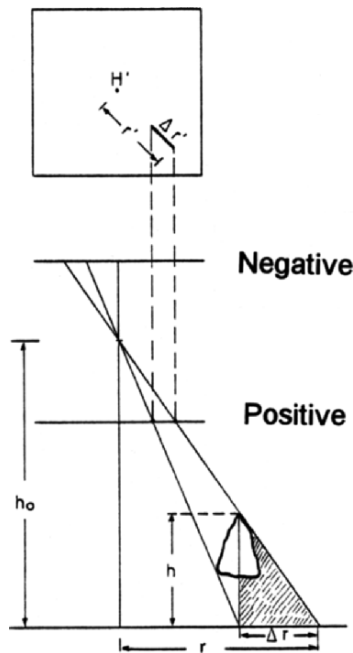
Figure 11-4. Measuring radial displacement.
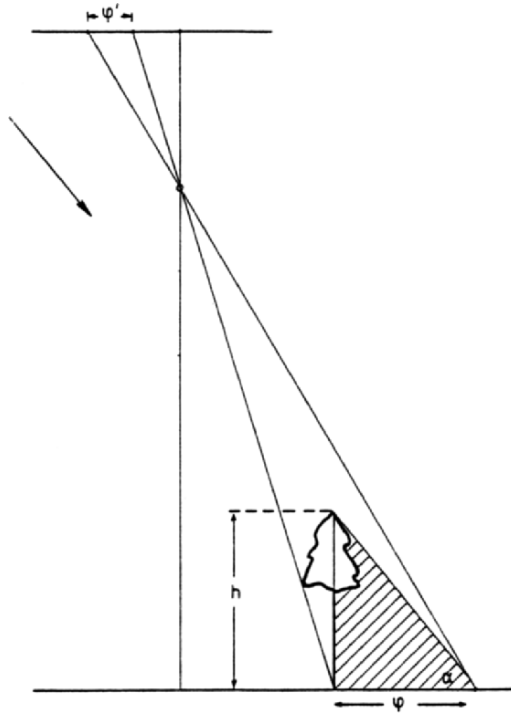
Figure 11-5. Measuring shadow length.

- Parallax differences measured on stereopairs of photographs, with a stereo-scope equipped with a micrometer wedge or stereomicrometer
- Stereoscopic measurements on aerial photographs with either analogue or digital plotters

The determination of tree heights by measuring the *radial displacement* and *shadow length* of the object has severe limitations. Displacement is usually too small to obtain accurate estimates and can be measured only near the edges of a stand on photographs taken from a low altitude. The shadow-length method also has its limitations and is feasible only in stands of low-stand density.

The shadow-length method requires a calculation of the sun's elevation, which is a function of its angle of declination on the day of photography, the latitude of the location of the object and the angle between the true north and the direction of the shadow. The limitations of the shadow-length method for estimating tree height are:

(1) The possible occurrence of a hot spot within the photograph, which is apparent if topographic displacement of the tree matches its shadow.

In tropical regions, it occurs throughout the year, in the temperate zone during summer, around midday.

(2) Shadow-length measurements of trees in the inner part of the stand are inaccurate, if not impossible. In practice, these measurements are obtained at the edge of the stand. In certain cases, the sample of tree heights obtained along the edge of the stand is not representative for the stand in its entirety.

(3) The shadow of a given tree growing uphill is shorter, that of a downhill tree is longer than the shadow of the same tree on horizontal terrain. Depending upon the season of photography, the tree height estimates are either positively or negatively biased.

(4) Trees leaning away from the sun cast longer shadows than those leaning towards the sun.

(5) Shadow lengths are shortened by undergrowth and, in northern regions, by snow.

Nash (1949) reported standard errors of the estimated heights of approximately 0.65 m within the 10 m height category, Nyyssonen (1955) noted standard errors of the same size as those obtained by parallax measurements.

In conclusion, a stereoscopic measurement with analogue or digital plotters produces the best results in terms of cost as well as accuracy. It requires overlapping stereopairs of photographs (60%).

3.1.1   Measuring parallax differences

A stereomicrometer, consisting of two glass plates with two small marks etched onto its surface is placed over the stereopair (Figure 11-6). One of the two marks is stationary, the other is allowed to move towards or away from the first one.

• The two marks are viewed under a mirror stereoscope, with the mobile mark being shifted until it is fused into a single stereoscopic mark.
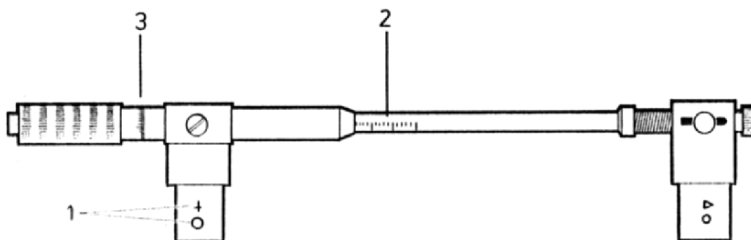
Figure 11-6. Stereomicrometer.
(1: Measuring marks 2: Millimeter scale 3: Micrometer scale)

- By moving the two marks closer together, the parallax increases and the fused image appears to float at a higher elevation. If they are moved in opposite directions, thereby increasing the distance between the two marks, the parallax is reduced. In consequence, a changing parallax influences the apparent elevation of the fused image, which is also called the floating mark or stereoscopic mark.
- The floating mark is positioned at the top and base of the tree.
- The stereoscopic parallax is measured on the millimeter and micrometer scale of the stereomicrometer (Figure 11-6). The parallax difference between the top and base of the tree is obtained by subtracting the two recorded values.
- The tree height is obtained from the following equation:

$$h = \frac{h_g \cdot \Delta p}{b + \Delta p}$$

where

Δh = tree height

h_g = flying height above the base of the tree

Δp = parallax difference between the top and base ($= p_1 - p_2$)

b = photographic base at the position of the base of the tree (Figure 11-7)

Example 11.2   The height of a tree is to be estimated by measuring the simple parallax with a mirror stereoscope. Flying height (h_g) and air base are 950 m and 83.4 mm, respectively. The parallax for the top and base of the tree are 11.53 and 7.98 mm, respectively. The resultant parallax difference is 2.55 mm. Hence:

$$h = \frac{95 \cdot 2.55}{83.4 + 2.55} = 28.2 \, \text{m}$$

with flying height expressed in meters, air base and parallax difference in millimeters.

Flying height and photo base refer to the position of the base of the tree. This is acceptable for measurements on stereopairs on flat terrain. In areas with significant topographic relief, it is necessary to determine the flying height with reference to the principal point of the left photograph and the photo base on the right photograph (Figure 11-8).
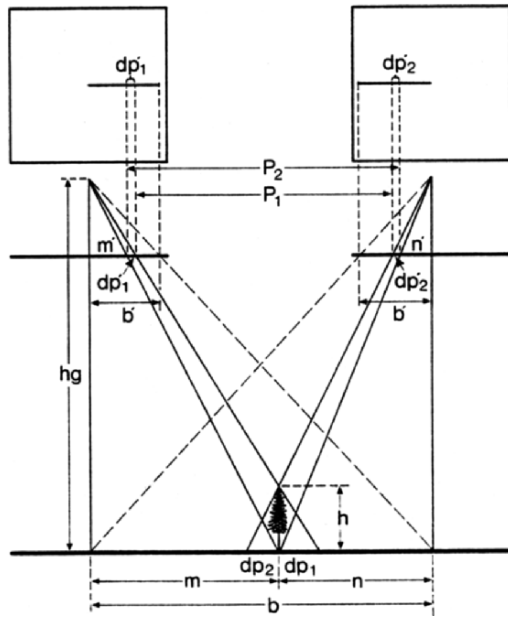
Figure 11-7. Geometric relationships to determine tree height from stereoscopic parallax.
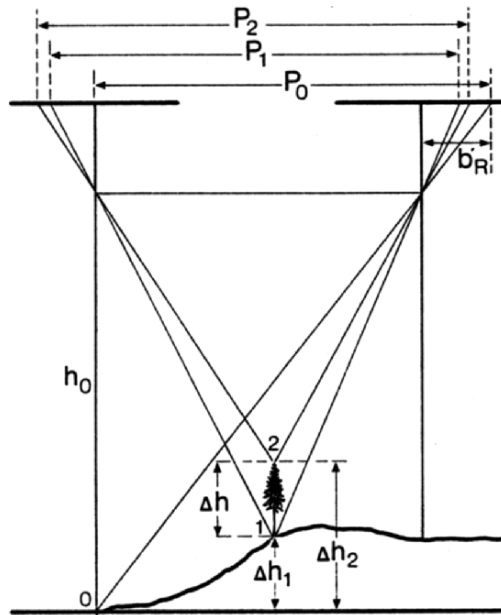
Figure 11-8. Geometric relationships between the stereopair and tree height in areas of significant topographic relief.

Tree height is obtained from the following formula:

$$h = \frac{h_0\,(\Delta p_1 - \Delta p_2)}{b + \Delta p_1 + \Delta p_2 + \dfrac{\Delta p_1 \cdot \Delta p_2}{b}} \simeq \frac{h_0\,(\Delta p_1 - \Delta p_2)}{b + \Delta p_1 + \Delta p_2}$$

where h_0 = flying height above the plane of reference or above the principal point of the left photo of the stereopair, Δp_1 = parallax difference between the principal point of the left photo and the top of the tree, Δp_2 = parallax difference between the principal point of the left photo and the base of the tree, b = photo base measured on the right photo of the stereopair

Example 11.3   The observed parallax differences between the top and base of the tree and the principal point on the left photo are 4.78 mm and 2.23 mm, respectively. The photo base measured with a ruler on the right photo is 81.5 mm, the flying height measured on the left photo is 940 m. Hence:

$$h \simeq \frac{940 \cdot (4.78 - 2.23)}{81.5 + 4.78 + 2.23} = 27.1\,\text{m}$$

Parallax differences may also be measured on stereomodels using simple wedge instruments, which have no mobile parts. The parallax wedge consists of two diverging rows of dots printed on a transparent overlay with the distance between matching dots increasing by a constant amount. In order to measure the height of a tree, the overlay is superimposed over the relevant position on the photo and viewed with a stereoscope. A mirror stereoscope might be used, but a pocket stereoscope is usually adequate to obtain sufficiently accurate readings. When viewing the object under the stereoscope, some of the rows are fused stereoscopically and appear as a single row of dots rising within the stereoscopic model. The tree height may be determined by obtaining readings of the apparent elevation of the top and base of the tree. The fused dots occupying the same position as the top and base of the tree are recorded. The two readings are subsequently multiplied by a constant to obtain the estimated tree height. In North America, wedge instruments are frequently used in field work, in combination with pocket stereoscopes and small-size aerial photos. Viewing aerial photographs of normal size under laboratory conditions, with the aid of more powerful stereoscopes, however, is more convenient and produces more accurate results.

Several studies were carried out to determine the accuracy of tree height measurements by simple parallax measurements. Schultz (1970) and Akça et al. (1971) reviewed the nature and extent of the error sources involved. When using a mirror stereoscope together with a stereomicrometer, the mean error on medium-scale photographs, varying between 1:10000 and 1:15000,

is 1 – 1.5 m. Stellingwerf (1962) reported a standard error of 1.2 m for tree height estimates on aerial photographs of a 1:10000 scale.

3.1.2 Analogue and digital photogrammetric methods

Tree heights are preferably measured with either *analogue*, *digital* (*analytical* or *softcopy*) *plotting devices*. *Analogue instruments* establish the relationship between the aerial photograph and terrestrial coordinate systems by reconstructing the perspective bundles with the aid of optical, mechanical or optical-mechanical devices, either equipped with space rods or by optical projection or both. *Analytical plotters* provide the link between the image and ground coordinates. The basic configuration of the analytical plotter consists of three main functional components: the optical-mechanical stereo viewer, main and ancillary computers, and peripheral equipment such as printers, plotters. The stereo viewer usually consists of a binocular viewing system, two stages for supporting the stereopair of photographs, control devices for moving the stages and/or the optical systems and the illumination systems for the aerial photo. The binocular system is of high quality and usually fitted with a zoom lens, which allows comfortable plotting. The interior, relative, and absolute orientation will be carried out accurately and with considerable speed. The photogrammetric use of the system requires the application of programs, which create the stereomodel and use the latter to produce digital information. Modern *softcopy digital plotting devices* are based on digital pictures in pixel format. The softcopy plotters consist of several software modules and a powerful computer without any optical or optical-mechanical projection devices. The interior, relative, and absolute orientation parameters of aerial photos for creating the stereomodel is calculated with the aid of special modules and the stereomodel of digital pictures is viewed, for example, with the aid, of electro-optical shutters.

Focal length, size of the photo, and magnitude of lens distortions are factors which restrict the use of conventional analogue photogrammetric plotters. In addition, the required relative and absolute interior orientation is time consuming. The use of analytical or softcopy plotters are not restricted by the above factors. Its computer calculates corrections in real time, which are also implemented in real time. The time required to run the three orientation phases of a given stereopair is less than that required with analogue instruments. The analytical and softcopy plotters allow the operator to read, record, and store coordinates in any coordinate system. In addition, the analytical and softcopy plotter have the capability of recalling stored data with the aid of the computer. The orientation data for the stereomodel, i.e., the coordinates of single trees or sample plots, can be stored. The correct stereomodel can be retrieved at a later point in time and reproduced exactly. It is therefore possible to relocate

the previously used trees or plots in the stereomodel. This feature is useful in monitoring changes in tree and stand characteristics such as tree height, crown dimension, and stand density. (Figure 11-9, Akça et al. 1995).

Furthermore the softcopy plotters could be used for classification and plotting of satellite images and in addition they serve as geographic information systems. They could be considered as *total station of remote sensing* (Akça and Radberger, 2000).

Analogue photogrammetric instruments are still used in topographic mapping and in the practice of forestry, but they are gradually being replaced by the digital analytical and softcopy plotter because of the previously mentioned advantage of this category of devices.

The measurement of tree heights with analogue plotting instruments may be difficult, if the base of the tree has to be identified in dense forest stands. It may then be impossible to find a ground point with the same elevation as that of the base point. This represents the most important error source. The analytical and softcopy assessment may be useful to solve this technical problem. In the stereomodel the coordinates x, y, and the elevation z of a visible ground point, as near as possible to the subject tree, have to be measured digitally. A regression model $z = f(x, y)$ based on information from the immediate surroundings of the sample tree is fitted. The fitted equation is used to estimate the elevation of the tree base (Z_b) from the coordinates (x_t, y_t) of the top of the tree (Figure 11-10). The concurrent computer program calculates the estimated height. In softcopy plotting, also, the elevation of the base of trees could be derived from a digital terrain model.

In comparison with the parallax method, tree heights are estimated more accurately when analogue or digital plotters are used, because of the more
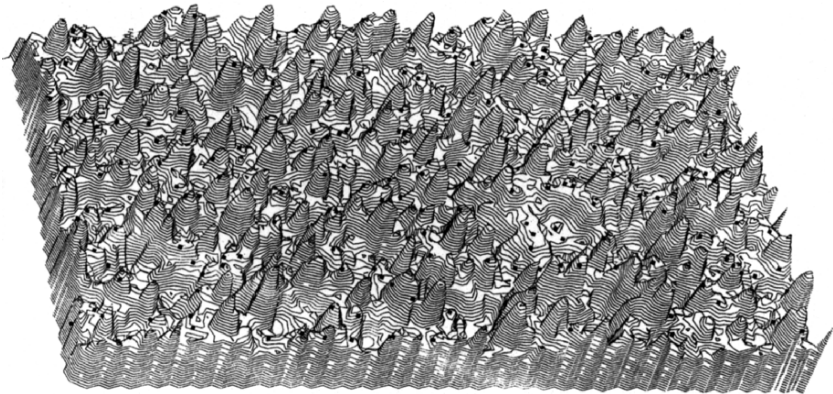
Figure 11-9. Mapping of tree crowns in the fertilizer trial Hilchenbach/Germany. (Hoffmann 2001.)
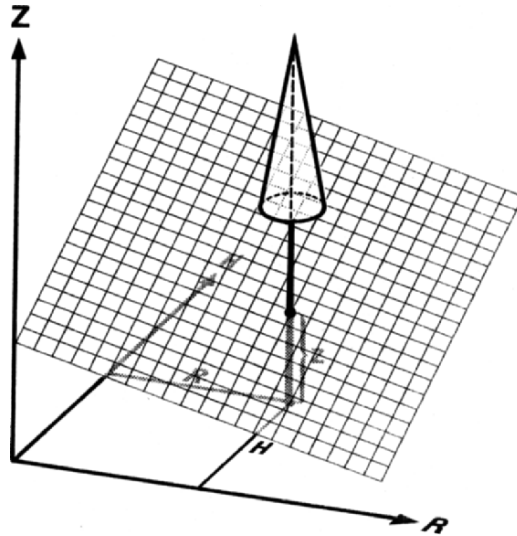
Figure 11-10. Regression surface and elevation of the base of the tree. (Akça 1989.)

accurate orientation of the stereo pair and the superior optical viewing of the stereomodel or viewing with the electro-optical shutters. The mean error of the photogrammetric height measurement, on photos of a medium-scale between 1:5000 and 1:15000 varies between 0.5 and 1.0 m. Akça (1973) and Hildebrandt (1996) reported a mean error of approximately 1 m on aerial photos with a scale varying between 1:5000 and 1:15000, if a mirror stereoscope equipped with stereometer is used and a mean error of 0.3 m when using a more powerful device. When measuring heights with a simple stereometer, the measurement errors, in 80–85% of the cases, are less than 1.5 m.

For many purposes, e.g., for estimating stand volume and for growth modeling and yield forecasting, stand height is a more important characteristic than the height of individual trees. The following error sources are involved:

- Inaccurate identification of the base of the trees
- Because of the limited resolution of the photographic film and paper and also dependent upon the photo scale, the real apex of the tree may not be depicted on the aerial photographs. This in turn produces a negatively biased estimate of the height of the selected sample trees
- A random sample of trees selected to estimate the mean height of the stand produces an unbiased estimate of the arithmetic mean height of the stand and, therefore, underestimates the height of the tree with the quadratic mean diameter

- In practice, suppressed trees are unlikely to be selected for measuring and this in turn tends to produce a positively biased estimate of the mean height

The compound effect of these error sources is likely to be a negatively biased estimate of tree height. Selecting sample trees from the 20–30 tallest trees within a stand, or measuring the k tallest trees within a sample plot, however, tends to reduce the magnitude of bias and is usually thought to produce a fairly reliable estimate of the top height of the stand. A subsample of trees, measured by conventional ground surveying and by photogrammetric methods, could eventually be used to fit a regression equation with the latter as the predictor and the former as the target variable. A standard error of the photogrammetric measured mean or top height varying between 0.7 and 0.8 m by using medium-scale aerial photographs has been reported (Akça 1983).

In conifers up to medium age, a stereophotogrammetric measurement of tree height of necessity tends to produce negatively biased estimates, since the tip of the tree cannot be identified because of the limited resolution of aerial photographs. In these cases the point of measurement coincides with the first whorl below the tip of the tree. It has been found that different operators consistently select the same point of measurement. In old conifer stands with no recognizable terminal shoot as well as in stands of broad-leafed tree species, it may be impossible to clearly identify the tip of the tree. Different operators, therefore, tend to select different points of measurement. However, a more important stumbling block is the identification of the base of the tree. To overcome this problem, the measuring mark is usually fixed at a visible point as near as possible to the tree to be measured, if possible on the contour line of the subject tree. By using an analytical or softcopy plotter, the elevation of the tree base can be obtained by establishing a regression surface (see Figure 11-10) or it can be directly gathered from a digital terrain model by using softcopy plotter.

3.2   Number of tree crowns

Due to overlapping, overshadowing, and clumped crowns, the number of crowns per unit area, counted on aerial photographs inevitably underestimates the actual true number of trees per unit area, with the exception of widely spaced plantation forests (Figure 11-11).

The dominating and codominating trees within a stand, which constitute *Kraft's crown classes* 1 and 2, are usually visible and therefore, countable without systematic errors. Those belonging to *Kraft's class* 3 are partly visible, whereas the majority of trees belonging to classes 4 and 5 are not resolved and can not be counted (Table 11-1). The social classes 4 and 5, however, represent
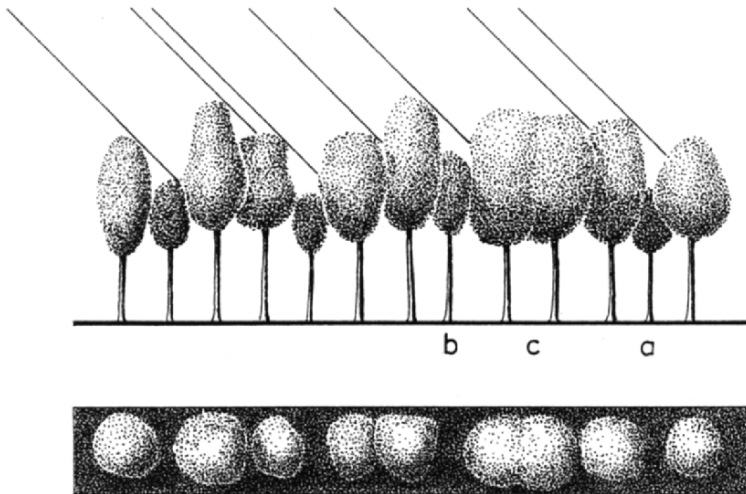
Figure 11-11. Crown canopy (a), overshadowing (b), and clumping of tree crowns (c).

Table 11-1. Observed and recovered number of trees (Akça 1979)

Treatment	Tree class (Kraft)	No. of trees	On aerial photgraphs	
			Recovered	Not recovered
	1	56	56	–
	2	76	76	–
Unthinned	3	68	60	8
	4 + 5	86	25	61
	Total	**286**	**217**	**69**
	1	44	44	–
	2	42	42	–
Row thinning	3	32	30	2
	4 + 5	78	33	45
	Total	**196**	**149**	**47**

a small portion of the total stand volume. For this reason, the recoverable number of crowns and stand volume are more closely correlated than the actual tree number and volume, both expressed per unit area (Tandon 1974).

The difference between the observed number or crowns and the real number of trees per unit area, as well as the variance of these differences, are related to photo scale, resolution of film and paper, stand age, stand structure, and stand density. In hardwoods, the differences and their variance tend to be greater than

in conifers (Spurr 1952; Tandon 1974). Regression analysis, in combination with subsampling, may then be applied to correct for bias (Spellmann 1984). Aerial photographs of medium-scale, varying between 1:10000 and 1:15000 are adequate to obtain estimates of acceptable accuracy.

3.3   Crown closure

The crown closure of a forest stand is determined by expressing the area of the crown, projected on a horizontal plane, as a fraction of the total ground area of these trees. On aerial photographs, it is determined as the relative area covered by tree crowns. Crown closure and stocking density represent different concepts, since the former represents an absolute measure, and the latter is dependent upon management practices and objectives. There is, however, a statistical relationship between crown closure and stand density, the latter expressing the degree to which the site is utilized by the trees. The relationship is influenced by site and silvicultural parameters such as stand treatment, and differs for different regions. Crown closure represents the most frequently used aerial stand parameter being used for stand volume estimations. In addition, it may serve as stratification criteria in the inventories of large forest tracts and is a useful characteristic when evaluating the necessity of intermediate thinnings.

In comparison with terrestrial methods, the estimation of crown closure on aerial photos is easier, less subjective and more accurate. Some training and experience is required to estimate crown closure ocularly on stereopairs of aerial photos viewed under a stereoscope. Alternatively, single photographs may be used to estimate crown closure.

More accurate results are obtained by making use of a *crown-density scale*, which is obtained either by cutting out small sections of existing photographs of known crown closure or by preparing a set of drawings of increasing crown closure (Figure 11-12). The recorded average error varies between 10% and 20% of the true crown closure.

The ocular estimation of crown closure may be improved by making use of a transparent dot grid device (Figure 11-13), to be placed on the aerial photo, either on single photographs or on stereopairs. In this case, the number of dots falling on tree crowns is counted and expressed as a proportion of the total number of dots.

Crown closure may be estimated more accurately by photogrammetric plotting of tree crowns with the aid of an analogue or analytical instrument.

Due to the nature and pattern of shadows and because of the perspective view associated with aerial photographs, the lower part of the crowns of
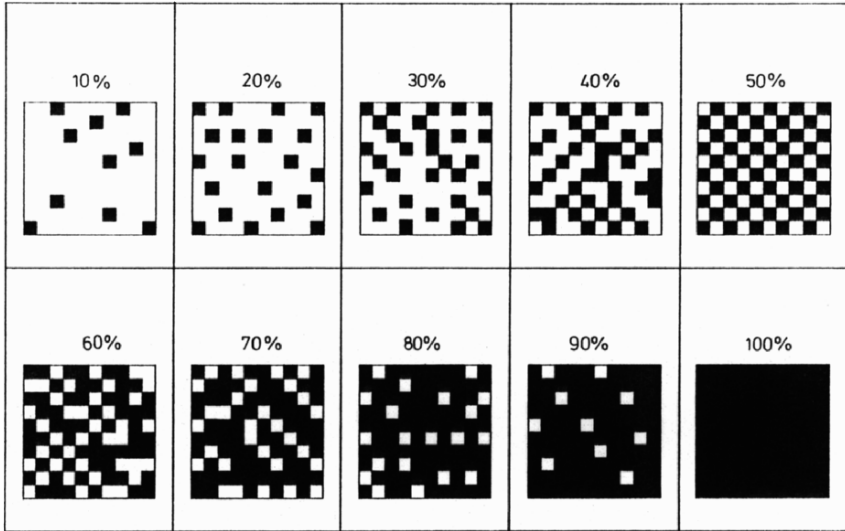
Figure 11-12. Crown closure diagram.
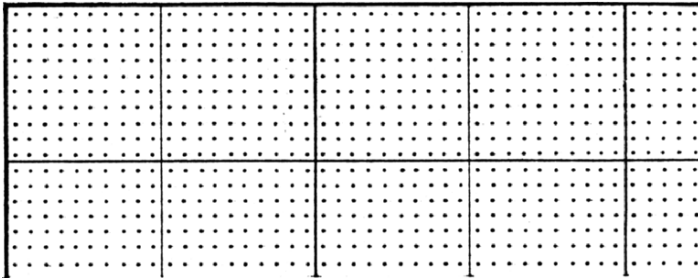
Figure 11-13. Dot grid to estimate crown closure.

dominants and a large section of those of the dominated trees, can neither be detected nor plotted in a stereomodel. This produces negatively biased estimates (Figure 11-14).

Klier (1969) proposed a method based on the angle count procedure, with BAF = 100, for estimating crown closure on single photographs, which is similar to the terrestrial version of the angle count method (Figure 11-15). Each tree falling inside the sample plot with imaginary boundaries is counted and represents a 1% crown closure. In order to facilitate counting, a stereoscopic angle-count-measuring device has been developed (Denstorf 1981).

Strip and row thinning   Unthinned   Strip thinning   Strip and selective thinning

20% + 15% = 35% removed   33% removed   20% + 13% = 33% removed
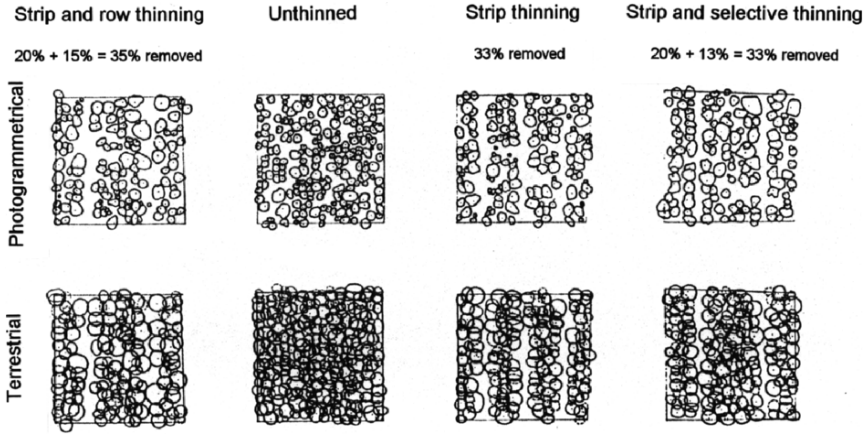
Figure 11-14. Crown maps prepared from photo measurements and terrestrial surveys.
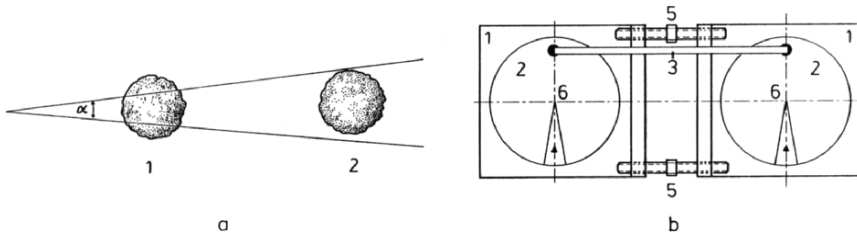
Figure 11-15. Angle count wedge, count factor 100.

  (a) Single photograph
  (b) Stereopair of photographs

3.4   Crown dimensions

It is technically possible to measure crown width, crown-projection area, and crown length on aerial photographs. The lateral crown-surface area and crown volume are based on certain geometric models, which reflect the shape and form of the tree crown. Usually, these measurements express the dimensions of the visible light crown. (see Figure 4-6). The resolution and visibility of small branches and irregular crown perimeters are dependent upon the scale of the photo. Due to inadequate resolution, the estimates are usually negatively biased (Figure 11-16).

A close correlation exists between crown diameter and stem characteristics such as dbh (Figure 11-17). Breast height diameter and photogrammetrically measured crown diameter are closely correlated. In *Pinus silvestris*, Perlewitz (1970) found a correlation coefficient of 0.9 and a standard deviation of the
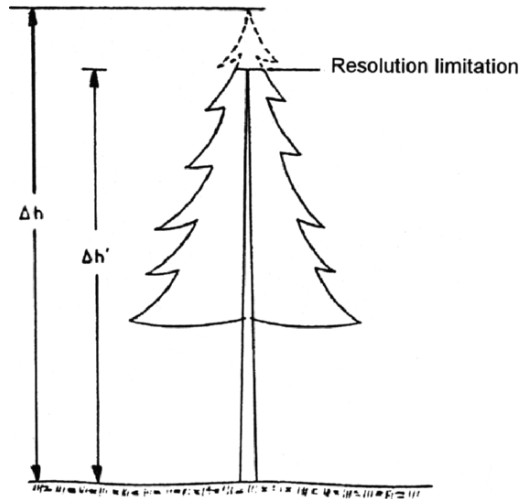
Figure 11-16. Photographic resolution and image-limitation of the crown of conifers.
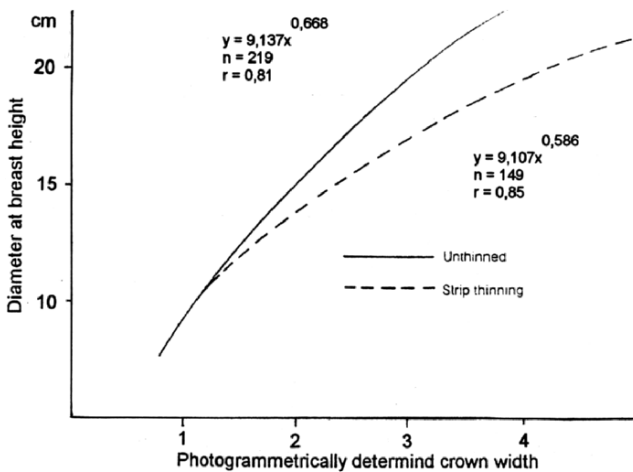
Figure 11-17. Relationship between crown diameter and dbh in a 34-year-old Norway spruce stand, from 1:2000 scale aerial photograph.

regression of 2.5 cm. Klier (1970) emphasized the influence of scale, image quality, species, and species mixture. The influence of method and degree of thinning was analyzed by Akça (1979). The close relationship between these variables motivated Moessner (1959); Sayn-Wittgenstein et al. 1967 and other researchers to construct tree aerial volume tables with crown diameter as the table entry. Regression equations may be fitted to estimate dbh from crown

diameter. In consequence, it is feasible to construct an aerial tree volume function or table with crown width as the predictor variable.

Crown widths can be easily measured from single aerial photographs or stereomodels by using crown wedges or crown-diameter scales and magnifying glasses or stereoscopes. On single photographs, the crown width of trees near the edge of the stand should be measured at a right angle to the radial line, which connects the principal point of the photo to the center of the image of the subject tree. Due to radial displacement, these tree crowns are imaged obliquely on aerial photographs. Measurements taken along the radial lines produce positively biased measures of the crown width. In general, the measurement error does not exceed 10% of the true value.

Nash (1949) and Nyyssonen (1955) found standard errors of 0.6 m, Worley et al. (1955) obtained standard errors between 0.9 and 1.2 m on photographs in scale of 1:12000. Ilvessalo (1950) and other authors noted that the photogrammetrically measured crown diameter tends to underestimate the true crown diameter. Hildebrandt (1996) recovered the dbh distribution of beech stands from the observed distribution of crown widths (Figure 11-18).

The crown projection area can be estimated with the aid of a transparent dot grid. After superimposing the transparency onto the photograph, the number of dots falling within the crown is counted and multiplied by the area represented by each dot. Alternatively, an analogue or analytical plotter is used to prepare a crown map, thereafter the area covered by tree crowns is measured or calculated. The measurement error is fixed at 5%.

Crown lengths can be measured on the stereoscopic model by analogue or digital plotting. Crown surface area and crown volume are a function of crown width and length and are useful variables in forest mensuration and growth studies. The relationship between crown surface area and basal area increment of single trees is shown in Figure 11-19.
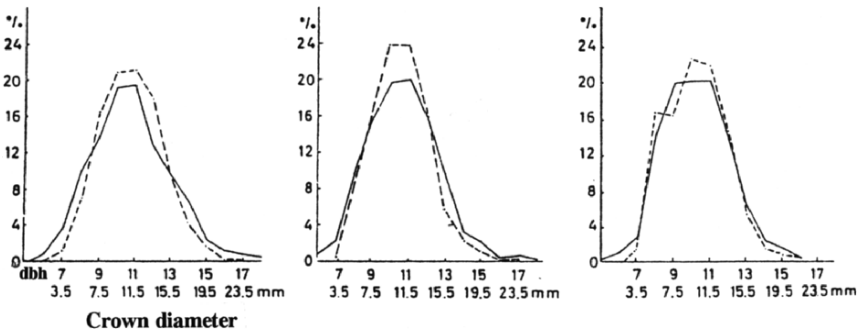
Figure 11-18. Crown diameter and dbh distribution in beech stands. (Hildebrandt 1969.)
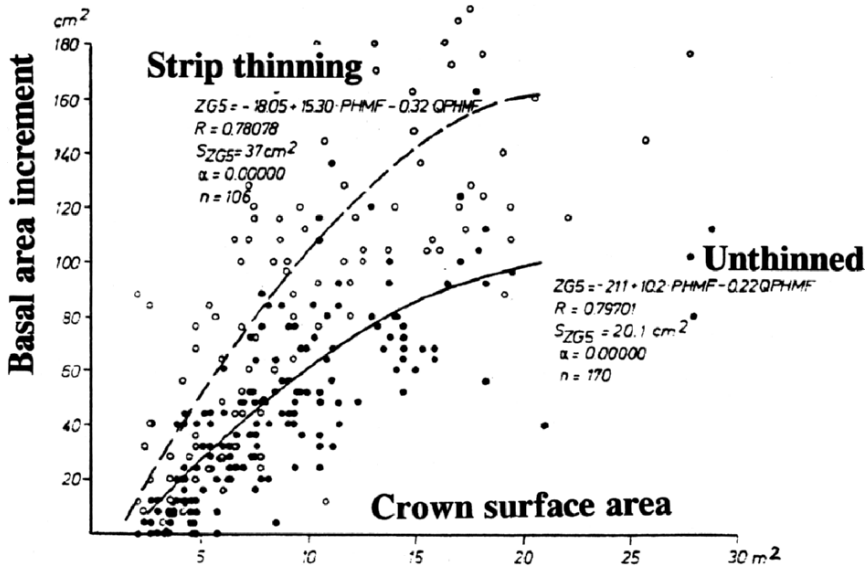
Figure 11-19. Relationship between photogrammetrically determined crown surface area and basal area increment.

3.5   Age

Stand age classes can be estimated from a regression equation with photogrammetrically determined stand height and crown size as predictor variables. Because of the inherent uncertainties, a given stand is usually assigned to one of 20-year-age classes or natural development stages. Studies in Germany (Akça 1996) indicated that the age class of a stand can be estimated from photo measurements of its stage of development.

3.6   Profile of the stand's growing space

The vertical profile of the growing space of a forest stand is useful in estimating the growing stock of a stand. Its measurement was introduced by Hugershoff (1933) and Neumann (1933), but the method was abandoned because of the difficulty to determine these profiles with analogue instruments. The development of modern digital and analytical plotters, however, facilitated the construction of profiles (see Figure 11-20). There is an increasing interest in the reconstruction of growing space profiles, particularly because of unsatisfactory resolution, which is evident in young stands. In such cases, growing space profiles are used to estimate mean and top height.
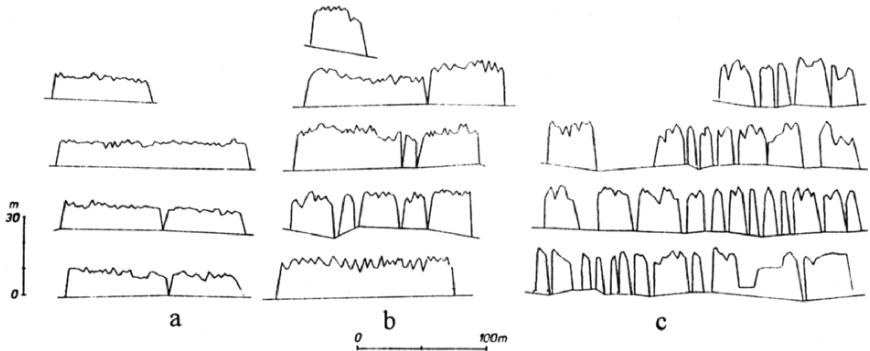
Figure 11-20. Vertical profiles in pine stands, obtained with digital plotters, on 1:6000 aerial photographs (a) young, fully closed stand; (b) closed mature stand; (c) open mature stand.
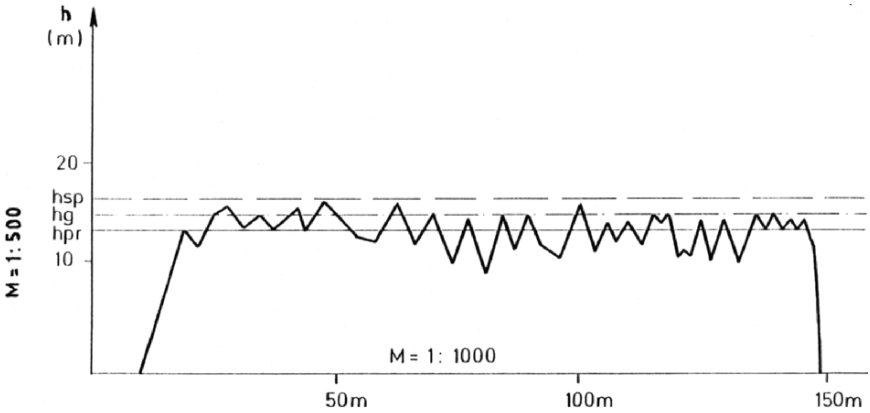
Figure 11-21. Vertical stand profile: height estimates

h_{pr} = photogrammetrically measured mean height of the profile

h_g = estimated height of the tree with the mean basal area

h_{sp} = mean height of dominants

The mean height of the vertical stand profile does not produce a reliable and unbiased estimate of the mean height of the stand, since the mean height of the profile measures the height of the dominants (Figure 11-21). An unbiased estimate of the mean height of the stand may be obtained from a fitted regression equation with profile height as the predictor and true stand mean height as the target variable.

4 ESTIMATION OF STAND VOLUME

The estimation of stand volume, based on photographic interpretation and photogrammetric measurements, requires either the interpretation or measurement of dendrometric variables either on single photographs or on stereopairs and a regression equation with these variables as predictors and stand volume as the target variable. The accuracy of the resultant stand volume estimates depends largely upon the accuracy of the measurement of these predictor variables and the performance of the model (expressed by the coefficient of determination). Aerial stand volume inventory methods may be classified on the basis of the variables being used (Figure 11-22).

The *stereogram method* is based on a visual inspection and comparison between the photographic image of the stand of interest and that of a stand with known volume per hectare (measured by ground surveying). A series of such stereograms representing different stand volume classes, obtained from one and the same photo source, is merged. Photogrammetric and photographic information about the image of the stand, such as grey tone, texture, pattern, crown closure, and stand height is evaluated to allocate the stand to one of the categories of the stereogram assembly. The method is useful for reconnaissance surveys, but the volume estimates have to be adjusted with the aid of terrestrial sampling methods.

Aerial tree and stand volume tables are common in North America. Table 11-2 exemplifies such a stand volume table for even-aged Douglas
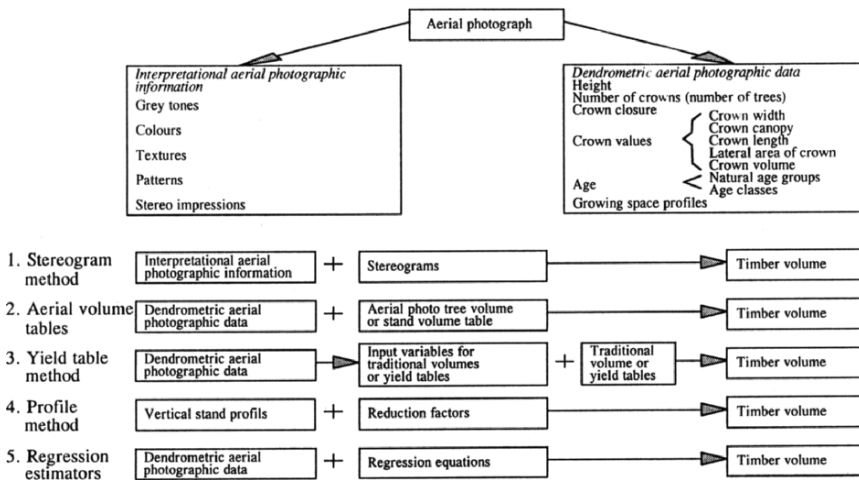
Figure 11-22. Schematic summary of methods of stand volume estimation.

Table 11-2. Aerial stand volume table (Pope 1962)

Stand height	Crown closure (%)								
(feet)	15	25	35	45	55	65	75	85	95
50	1	3	4	5	5	4	3	1	–
60	2	5	7	8	8	8	6	4	–
70	4	8	10	12	13	12	11	8	4
80	6	11	14	17	18	18	16	14	10
90	8	14	19	22	23	24	23	20	16
100	10	18	23	28	30	31	30	28	24
110	13	22	29	34	37	39	39	37	34
120	16	26	34	41	46	48	49	48	44
130	19	31	41	49	54	58	60	59	56
140	22	36	48	57	64	69	72	72	70
150	25	41	55	66	75	81	85	86	85
160	29	47	63	76	86	94	99	101	101
170	33	53	71	86	98	107	114	118	118
180	37	60	80	97	111	122	130	135	137
190	41	67	89	108	125	138	148	154	158
200	46	74	99	121	139	154	166	174	179
210	50	82	109	134	154	172	185	196	202
220	55	90	120	147	170	190	206	218	226
230	60	98	132	161	187	209	228	242	252
240	66	106	143	176	205	230	250	267	279
250	71	116	156	192	223	251	274	293	308
260	77	125	168	208	242	273	299	320	337

fir in the Pacific Northwest. (Pope 1950, 1962) recommended the construction of aerial stand volume tables based on stand height, crown closure, and crown diameter as independent variables. This idea was implemented by Gingrich et al. (1955) and Moessner (1960). Meyer (1961) constructed aerial stand volume tables based on mean height and crown closure as independent variables. Stellingwerf et al. (1977) constructed aerial stand volume tables based upon crown closure as independent variable and table entry. In an earlier study (Stellingwerf 1973) it was suggested the use of terrestrially measured control plots to reduce bias associated with aerial volume tables. The *aerial tree volume table* gives the estimated tree volume as a function of tree variables measured by photogrammetric methods, primarily tree height and crown width, the latter as a substitute for stem diameter. The method gives satisfactory results in open stands, since the dimensions of the single tree must be identified. The stand volume is obtained after the additional estimation of the number of trees per hectare. In stands of high density, such as those prevailing in the European

and tropical forests, however, the *aerial stand volume table* produces more accurate estimates of the stand volume, since mean height and crown closure are highly significant influential variables in estimating the stand volume per hectare. The *single-entry stand volume table*, with total or mean stand height as the table entry, gives satisfactory results in fully stocked stands on uniform sites. *Two-entry stand volume tables* perform better in forests of highly variable stand density, with crown closure being a useful additional predictor variable in stands of normal stocking, number of crowns per unit area in forests of low stocking. In order to obtain more accurate stand volume estimates, separate tables may be constructed for predefined age and site index categories.

Alternatively, *conventional yield tables* constructed for specific species and based on a common stand treatment regime, may be used to estimate the volume of stands of known age, with either photogrammetrically measured mean height or top height being used to determine the site index (Haselhuhn 1983). A different method, which requires the determination of the stand profile, was proposed by Neumann (1933) and Hugershoff (1983). It is obtained as follows:

$$R = a \cdot D$$

where R = growing space of the stand, a = area of the profile, and D = distance between profiles. The actual growing space of a specific stand is obtained by multiplying the calculated growing space by a *stocking index*, obtained by ground surveys or from conventional yield tables. The method produces satisfactory results in even-aged dense forest stands.

Although the above methods were developed and may be applied to estimate the stand volume, there is an increasing emphasis on two-phase sampling with *regression estimators* (see sections 10.61 and 10.71). A large number of plots are established and measured on the aerial photographs to obtain quick and rough estimates of the volume per unit area, based on variables such as crown closure, stand height and number of trees per hectare. A subsample of plots is selected at random from the primary sample of photo plots and measured by conventional terrestrial methods. Several authors reported satisfactory results obtained by regression sampling (Schade 1980; Zindel 1983; Spellmann 1984; Akça et al. 1993). The following models have been used in mensurational studies:

Target variable	*Predictor variables*	*Predictor variables*
Volume per hectare	h_d, A	h_d, h_d^2, cc
	h_m, A	$cc \cdot h_d^2$
	NN^2	$cc, cc \cdot h_d, cc \cdot h_d^2$
	h_d, cc	

where h_d = mean height of dominants, cc = crown closure, N = number of crowns, A = estimated age class

5   ESTIMATION OF VOLUME INCREMENT

German studies (Akça 1984; Akça et al. 1991) confirmed a close relationship between crown projection area, as well as crown surface area and the basal area increment of single trees. Based upon measurements in permanent sample plots, a regression analysis with basal area increment as target variable, and crown surface area as predictor revealed that R^2 values were between 0.64 and 0.72 (Akça 1979, see Figure 11-19).

Two-phase sampling may also be applied to estimate the volume increment, and in the case of intermediate thinnings, to estimate volume changes. Two methods have been developed to estimate changes.

- Method 1 requires an independent estimation of the stand volume per hectare at the beginning and the end of the period. In both instances, the aerial photograph is used to measure ancillary variables, for example, the mean height of the dominants, crown closure, crown mixture (phase 1 sample). A subsample of the photo plots (phase 2 sample) is used to determine the volume per hectare by ground surveying. The mean volume is obtained as a regression estimator. The difference between the estimates at the beginning and end of the period estimates the volume change.
- Similar to the approach above, method 2 requires photo plots to measure the ancillary variables. In phase 2, a subsample of photo plots is drawn and managed as permanent sample plots, on which the volume at the beginning and end of the period is determined. Similar to method 1, a regression estimator is subsequently calculated to estimate volume changes.
- Permanent sample plots should be used in both methods to determine changes and the photogrammetric stand parameters should be determined with the aid of digital or analytical devices. Akça et al. (1991) regressed volume increment on top height and crown closure, both obtained photogrammetrically. The equation used stand height, crown closure and stand height $\times \cdot$ crown closure as predictor variables. The results are summarized in Table 11-3.

In both cases, the standard error is $0.98\,\mathrm{m}^3$/ha. The difference between the recorded means is due to sampling errors. The two-phase sampling procedure used 40 ground plots and 150 photo plots, whereas the method based on permanent plots required 185 ground plots. In this study, the cost ratio photoplots: permanent ground plots was 1:5. In that case, the total cost of two-phase

Table 11-3. Estimation of stand volume in Norway spruce by two-phase sampling with regression estimators and by sampling with permanent sample plots

	Two-phase sampling	Permanent sample plots
Current volume increment	12.62 m^3/ha	13.96 m^3/ha
Standard error	0.98 m^3/ha	0.98 m^3/ha
Sample size	–	–
Terrestrial	40	185
Aerial photographs	150	–

sampling is the equivalent of measuring 70 ground plots. In consequence the two-phase design reduces the total cost by 60%.

Other sampling studies for estimating stand volume increment were carried out by de Gier et al. (1988), de Gier (1989), and Stellingwerf (1973).

APPENDIX

Appendix A

Symbols

N, n = population, sample size
σ^2, s^2 = population, sample variance
$S_{\bar{x}}, S_{\bar{y}}$ = conditional or standard error of the sample mean of x, y
σ_X^2 = variance of the population total of X
s_X^2 = estimated variance of the population total of X
s_X = standard error of the estimated population total of X
$s_{y(x=x_i)}^2$ = conditional sample variance of y for $x = xi$
r^2, R^2 = simple, multiple coefficient of determination
R^2_{adj} = adjusted coefficient of determination
r, R = simple, multiple correlation coefficient
r_I = intraclass correlation coefficent
$s_{y.x}^2$ = sample variance adjusted for regression
t = Student's t-statistic
F = Snedecor's F-statistic
z = unit normal variate
g_1 = skewness
g_2 = kurtosis
E = allowable error
CP = Mallows' CP index
LA = leaf surface area
LAI = leaf area index
BT = bark thickness

MAI	= mean annual increment
CW	= crown width
CL	= crown length
CR	= crown ratio
f	= false form factor
λ	= true form factor
g	= tree basal area
G_{ha}	= basal area per hectare
$q_{0.5h}$	= false form quotient
$\eta_{0.5h}$	= true form quotient
q_M, q_5	= Mitscherlich's form quotient
q_H	= Hohenadl's form quotient
q_G	= Girard's form quotient
d, *dbh*	= diameter at 1.3 m
N_{ha}	= number of trees per hectare
$\bar{d}$	= arithmetic mean diameter
d_q	= quadratic mean diameter
d_{mg}	= diameter of the central basal area tree
d_v	= diameter of the tree with the mean volume
h_m	= mean height
h_L	= Lorey's mean height
h_t	= top height
h_c	= Kitamura's critical height
SDI	= stand density index
S%	= *S%* index of the stand
d_w	= Weise's mean diameter
CCF	= crown competition factor
MCA	= maximum crown area
CPA	= crown projection area
SI	= site index

Greek letters

A α	Alpha	H η	Eta	N ν	Nu	T τ	Tau
B β	Beta	Θ θ	Theta	Ξ ξ	Xi	Y υ	Upsilon
Γ γ	Gamma	I ι	Iota	O o	Omicron	Φ ζ	Phi
Δ δ	Delta	K κ	Kappa	Π π	Pi	X χ	Chi
E ε	Epsilon	Λ λ	Lambda	P ρ	Rho	Ψ ψ	Psi
Z ζ	Zeta	M μ	Mu	Σ σ	Sigma	Ω ω	Omega

Appendix B

Diameter data of sampling trees

dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)	dbh (cm)
18.0	22.5	23.7	22.0	20.1	25.5	17.2	18.9	22.2	19.0	22.8
26.9	22.5	21.1	25.7	22.3	21.5	20.0	22.1	12.3	26.3	20.5
17.0	24.0	20.1	16.0	23.0	21.1	22.0	22.4	8.6	16.0	15.5
19.0	17.1	20.6	14.7	23.5	22.8	23.5	16.0	14.3	22.3	23.1
27.5	19.4	22.6	23.5	22.5	18.1	25.8	20.4	20.5	24.2	29.3
21.8	28.0	25.2	13.6	18.5	21.4	30.0	27.0	28.1	16.5	20.7
29.1	24.3	16.6	20.1	18.2	22.3	15.9	28.0	12.5	26.2	12.1
20.3	23.5	19.7	26.5	25.3	20.0	7.9	29.2	15.7	18.6	26.0
29.0	20.2	21.9	26.6	26.5	23.0	9.7	18.9	9.1	24.4	19.7
17.6	29.4	24.0	25.0	24.1	13.6	18.1	20.6	22.5	28.5	23.8
25.0	18.7	23.0	25.0	24.5	27.2	26.1	23.7	17.1	18.8	20.3
16.3	19.6	23.5	18.0	28.5	18.0	17.1	29.2	24.0	25.1	22.0
26.5	16.4	19.3	24.8	20.5	26.4	8.0	28.0	20.7	34.8	34.3
15.4	25.5	23.0	20.0	21.0	25.7	27.2	27.0	21.2	19.7	28.0
19.3	22.8	21.5	21.4	18.0	15.1	24.3	26.1	8.5	27.2	26.3
24.3	22.5	22.4	28.6	16.4	27.5	17.0	19.1	24.3	19.7	17.2
27.0	14.7	27.8	24.0	20.3	20.6	19.0	28.8	27.2	27.2	14.0
25.3	19.3	29.5	26.6	14.6	25.2	20.8	22.1	24.7	19.7	20.0
19.7	22.1	20.0	21.4	20.0	13.7	30.0	16.0	18.3	21.1	22.0
14.2	15.5	24.7	19.2	24.0	29.0	30.8	21.0	11.6	26.1	17.6
15.4	23.0	18.7	20.6	25.2	31.2	19.1	26.1	22.5	28.7	24.3
21.4	21.5	23.1	17.8	23.5	21.6	18.2	9.4	20.2	32.0	16.0
21.2	25.5	24.9	21.7	15.9	19.6	28.8	20.0	32.2	34.7	23.8

Appendix C

Sample tree data for fitting height curves

No.	dbh	h	No	dbh	h	No	dbh	h	No	dbh	h
–	(cm)	(m)	–	(cm)	(m)	–	(cm)	(m)	–	(cm)	(m)
1	11.4	9.6	15	17.7	15.8	29	22.2	17.8	43	28.0	19.6
2	11.0	11.5	16	18.2	14.0	30	22.3	17.5	44	20.5	15.8
3	12.5	12.1	17	18.3	13.6	31	22.4	17.8	45	20.5	18.4
4	14.0	15.1	18	18.3	16.4	32	22.8	15.8	46	7.9	9.8
5	14.3	11.6	19	18.3	16.6	33	22.9	16.1	47	8.6	9.8
6	14.5	12.3	20	20.1	14.7	34	23.0	17.6	48	9.1	9.5
7	15.0	15.8	21	20.3	17.1	35	23.1	17.1	49	10.1	11.0
8	15.5	14.4	22	20.3	17.8	36	23.2	14.9	50	31.5	20.1
9	16.2	16.3	23	20.6	14.5	37	23.9	17.1	51	30.7	19.9
10	16.5	14.3	24	20.7	16.2	38	24.8	19.0	52	33.6	20.6
11	16.5	15.3	25	21.2	16.6	39	25.0	18.3	53	8.1	8.0
12	17.0	14.3	26	21.5	17.4	40	26.0	17.9	64	9.5	9.2
13	17.2	14.0	27	21.7	17.7	41	26.4	19.4	55	12.0	13.0
14	17.2	15.0	28	22.0	18.3	42	27.5	17.8	–	–	–

Appendix D

Conversion factors for linear, square, cubic, and weight measures

English measures			
Linear measures		*Square measures*	
1 inch (in.)	2.54 cm	1 sq in.	6.4516 cm^2
1 foot (ft)	0.3048 m	1 sq ft	0.0929 m^2
1 yard (yd)	0.9144 m	1 sq yd	0.8361 m^2
1 mile (mi)	1609 m	1 sq mi	258.888 hectares (ha)
1 chain	20.1168 m	1 sq ft quarter girth (q.g.)	0.11893 m^2
1 inch q.g.	3.234 cm	1 acre	0.4047 ha
1 inch true girth	0.8086 cm	1 sq ft (q.g.) per acre	0.2923 m^2/ha
		1 sq ft (true) per acre	0.2296 m^2/ha

English measures

Cubic measures		Weight measures	
1 cu in.	$16.387\,\text{m}^3$	1 ounce	$28.35\,\text{g}$
1 cu ft	$0.0283\,\text{m}^3$	1 pound	$453.6\,\text{g}$
1 cu yd	$0.7645\,\text{m}^3$	1 long ton	$1016\,\text{kg}$
1 hoppus ft	$0.0361\,\text{m}^3$	1 short ton	$907.2\,\text{kg}$
1 cord	$3.6246\,\text{m}^3$		
1 hoppus ft per acre	$0.0891\,\text{m}^3/\text{ha}$		
1 cu ft per acre	$0.06997\,\text{m}^3/\text{ha}$		

Russian measures

Linear measures		Square measures		Cubic measures	
1 linja	$0.254\,\text{cm}$	1 q-djujim	$6.4516\,\text{cm}^2$	1 cu.djujm	$16.387\,\text{cm}^3$
1 djujm	$2.54\,\text{cm}$	1 q-werschok	$19.758\,\text{cm}^2$	1 cu ft	$28.317\,\text{dm}^3$
1 werschok	$4.445\,\text{cm}$	1 q-ft	$929.030\,\text{cm}^2$	1 cu aschin	$0.3597\,\text{m}^3$
1 Russ. Ft	$30.48\,\text{cm}$	1 q-arschin	$0.5058\,\text{m}^2$	1 cu sashen	$9.713\,\text{m}^3$
1 arschin	$71.12\,\text{cm}$	1 q-sashen	$4.552\,\text{m}^2$	1 Pt. standard	$4.672\,\text{m}^3$
1 sashen	$213.34\,\text{cm}$	1 q-desjatine	$1.0925\,\text{ha}$		
1 werst	$1066.8\,\text{m}$				

Japanese measures

Linear measures		Square measures		Cubic measures	
1 sun	$3.030\,\text{cm}$	1 tsubo	$3.3058\,\text{m}^2$	1 sai	$0.01815\,\text{m}^3$
1 sasi	$30.303\,\text{cm}$	1 tan	$991.7\,\text{m}^2$	1 ken	$6.0105\,\text{m}^3$
1yozjo	$303.03\,\text{cm}$	1 cho	$0.9917\,\text{ha}$		
1 ri	$3927.3\,\text{m}$				

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