C 41697 (Pages : 11) Name..................................

Reg. No..............................

PG/INTEGRATED PG ENTRANCE EXAMINATION, APRIL 2023

MATHEMATICS

Time : Two Hours Maximum : 200 Marks

All questions carries 4 marks.

1 *mark will be deducted for each wrong answer.*

1. Let G be a non-abelian group. Let $\alpha \in G$ have order 4 and let $\beta \in G$ have order 3. Then the order of the element $\alpha\beta$ in G :

   (a) Is 6.

   (b) Is 12.

   (c) Is of the form $12k$ for $k \geq 2$.

   (d) Need not be finite.

2. Which one of the following series is divergent ?

   (a) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$.

   (b) $\sum_{n=1}^{\infty} \frac{1}{n} \log.$

   (c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$.

   (d) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$.

3. The matrix equation

   $$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$$

   represents

   (a) A circle of radius $\sqrt{15}$.

   (b) An ellipse of semi major axis $\sqrt{5}$.

   (c) An ellipse of semi major axis 5.

   (d) A hyperbola.

Turn over

4. In a H.P., p^{th} term is q and the q^{th} term is p. Then pq^{th} term is :

(a) 0. (b) 1.

(c) pq. (d) $pq(p+q)$.

5. Newton Raphson scheme for solving $x^2 - 153 = 0$, is :

(a) $x_{n+1} = 0.5\left[x_n + \frac{153}{x_n}\right]$. (b) $x_{n+1} = 0.5\left[x_n - \frac{153}{x_n}\right]$.

(c) $x_{n+1} = \left[x_n + \frac{153}{x_n}\right]$. (d) $x_{n+1} = \left[x_n - \frac{153}{x_n}\right]$.

6. The Jacobian $\frac{d(x, y)}{d(u, v)}$ of the transformation

$x = \frac{-u+v}{6}$

$y = \frac{u+2v}{3}$ is

(a) $\frac{1}{6}$. (b) $-\frac{1}{6}$.

(c) 6. (d) -6.

7. An urn contains 3 balls numbered 1, 2 and 3 . The co-efficient of equation $px^2 + qx + c = 0$ is determined by drawing the numbered balls with replacement. What is the probability that the equation will have imaginary roots ?

(a) $\frac{4}{27}$. (b) $\frac{23}{27}$.

(c) $\frac{16}{27}$. (d) None of the above.

8. If $f(x)=x^{a}\log x$ and $f(0)=0$, then the value of a for which Rolle's theorem can be applied to $[0,1]$ is :

(a) -2.
(b) -1.
(c) 0.
(d) 1/2.

9. Which among the following is not possible ?

(a) $\exists A$ s.t $P(A)=\{\ \}$.
(b) $\exists A$ s.t $P(A)=\{\phi\}$.
(c) $\exists A$ s.t $P(A)$ is countable.
(d) $\exists A$ s.t $P(A)$ is uncountable.

10. The number of group homomorphisms from the A_5 to the symmetric group S_4 is :

(a) 1.
(b) 12.
(c) 20.
(d) 6.

11. Which of the following is not a definition of Gamma function ?

(a) $\Gamma(n)=n!$.
(b) $\Gamma(n)=\int_0^\infty x^{n-1}e^{-x}\,dx$.
(c) $\Gamma(n+1)=n\Gamma(n)$.
(d) $\Gamma(n)=\int_0^1\left(\log\frac{1}{y}\right)^{n-1}$.

12. If a 3 × 3 matrix A has eigen values 1, – 1 and 0 then which among the following is not an eigen value $A^{100}+I$:

(a) 2.
(b) 1.
(c) -1.
(d) All of above.

13. Let $T:R^7\to R^7$ be a linear transformation with Nullity $(T)=2$. Then, the minimum possible value of Rank (T^2) is :

(a) 2.
(b) 1.
(c) 3.
(d) 4.

14. Which among the following iteration technique is having non linear rate of convergence :

(a) Bisection. (b) Regula-falsi.

(c) Newton-Raphson method. (d) All of the above.

15. Which among the following functions is not uniformly continuous on the given domain :

(a) $f(x) = x^2$ on R. (b) $f(x) = \sin \frac{1}{x}$ on $(0, 1)$.

(c) $f(x) = \ln(x)$ on $(0, \infty)$. (d) All of the above.

16. Second degree Maclaurin polynomial of $f(x) = \frac{1}{\sqrt{x+1}}$ is :

(a) $1 - \frac{1}{2}x + \frac{3}{8}x^2$. (b) $1 - \frac{1}{2}x + \frac{3}{4}x^2$.

(c) $1 - \frac{1}{2}x - \frac{3}{8}x^2$. (d) $1 + \frac{1}{2}x - \frac{3}{8}x^2$.

17. The product AB of any two real, symmetric matrices A and B is :

a) Symmetric for all A and B.

(b) Never symmetric.

(c) Symmetric, if AB = BA.

(d) Anti-symmetric for all A and B.

18. Order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 2 & 1 & 8 & 7 & 6 & 4 & 5 \end{pmatrix}$ is :

(a) 4. (b) 2.

(c) 6. (d) 5.

19. Which among the following is not a generator of Z_9 :

(a) 1. (b) 2.

(c) 3. (d) 4.

20. In the Fourier series of the periodic function

$$f(x) = |\sin x| = \sum_{n=0}^{\infty} [\alpha_n \cos nx + \beta_n \sin nx]$$

Which of the following co-efficients are non-zero ?

(a) α_n for odd n.

(b) α_n for even n.

(c) β_n for odd n.

(d) β_n for even n.

21. In a restaurant, 17 men and 7 women are seated on 24 chairs at a round table. Find the total number of possible ways such that 17 men are always sitting next to each other.

(a) 23 !.

(b) 7 ! × 17 !.

(c) 6 ! × 16 !.

(d) 8 ! × 17 !.

22. The number of real solutions of $2\sin(e^x) = 5^x + 5^{-x}$ in $[0,1]$ is / are :

(a) 0.

(b) 1.

(c) 2.

(d) Infinite.

23. Tangent line at $(1,1)$ to the curve $x^2 + xy + 5 = 0$ is:

(a) $y = 3x - 2$.

(b) $y = -3x + 4$.

(c) $x = 3y - 2$.

(d) $x = -3y + 4$.

24. The remainder when $2^{20} + 3^{30} + 4^{40} + 5^{50} + 11^7$ is divided by 7 is :

(a) 3.

(b) 0.

(c) 6.

(d) 5.

25. The Sieve of Eratosthenes is used for finding :

(a) All primes below a given integer.

(b) All even numbers below a given integer.

(c) All odd numbers below a given integer.

(d) All composite numbers below a given integer.

26. Which of the following can only be used in disproving the statements ?

(a) Direct proof. (b) Contrapositive proofs.

(c) Counter Example. (d) Mathematical Induction.

27. The argument of the Complex number $z=\left(1+i\sqrt{3}\right)(1+i)(\cos\theta+i\sin\theta)$ is :

(a) $\frac{\pi}{3}+\theta$. (b) $\frac{\pi}{4}+\theta$.

(c) $\frac{7\pi}{12}+\theta$. (d) None

28. Polar form of Complex number $\frac{1+7i}{(2-i)^2}$ is :

(a) $\sqrt{2e}^{(3i\pi)/4}$. (b) $\sqrt{2e}^{-3i\pi/4}$.

(c) $\sqrt{2e}^{3i\pi/2}$. (d) $\sqrt{2}$.

29. The value of $\left|\int_0^{3+i}(\bar{z})^2\,dz\right|^2$, along the line $3y=x$, where $z=x+iy$ is :

(a) $\frac{1000}{9}$. (b) $\frac{1000}{27}$.

(c) $\frac{100}{9}$. (d) $\frac{100}{3}$.

30. The value of $\int_C \frac{z^2+5z+6}{(z-2)}\, dz$, where C is $|z|=1$ is :

(a) $2\pi i$. (b) 1.

(c) 0. (d) $40\pi i$.

31. For $f(x, y) = x^2 + xy - 2y - 2x + 1, (2, -2)$ is a :

(a) Maximum. (b) Saddle point.

(c) Minimum. (d) None of these.

32. If u and v are harmonic functions then $f(z) = u + iv$ is :

(a) Analytic function.

(b) Need not be analytic function.

(c) Analytic function only at $z = 0$.

(d) None of the above.

33. Fixed points for the transformation $w = \frac{z-1}{z+1}$ is :

(a) $Z = \pm i$. (b) $Z = \pm 2i$.

(c) $Z = \pm 1$. (d) $Z = \pm 2$.

34. Image of the circle $|z| = 2$ under the transformation $w = 3z$ is :

(a) $|w| = 2$. (b) $|w| = 3$.

(c) $|w| = 6$. (d) $|w| = 4$.

35. The bilinear transformation that maps the points $z = 0, 1, \infty$ into the points $w = -1, -2, -i, i$ respectively is :

(a) $w = \dfrac{(iz-2)}{z+2}$.

(b) $w = \dfrac{(iz-2)}{z-2}$.

(c) $w = \dfrac{(iz+2)}{z+2}$.

(d) $w = \dfrac{(-iz-2)}{z+2}$.

36. The radius of convergence of the power series $\sum_{n=1}^{\infty} \left(\dfrac{n+2}{n}\right)^{n^2} x^n$ is :

(a) e^2.

(b) $\dfrac{1}{\sqrt{e}}$.

(c) $\dfrac{1}{e}$.

(d) $\dfrac{1}{e^2}$.

37. Let $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$. A point at winch the gradient of the function f is equal to zero is :

(a) $(-1, 1, -1)$.

(b) $(-1, -1, -1)$.

(c) $(-1, 1, 1)$.

(d) $(1, -1, 1)$.

38. Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$:

(a) Diverges.

(b) Converges to $3S - a_1 - a_2$.

(c) Converges to $3S - a_1 - 2a_2$.

(d) Converges to $3S - 2a_1 - a_2$.

39. Let $M=\begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of $\det\left((8I-M)^3\right)$ is :

(a) -216.
(b) 116.
(c) 256.
(d) 216.

40. If vector field $\bar{v}=(x+3y)\bar{i}+(y-2z)\bar{j}+(x+az)\bar{k}$ is solenoidal then value of a is :

(a) 0.
(b) 3.
(c) 2.
(d) -2.

41. The n^{th} derivative of $y=\sin px+\cos px$ is :

(a) $\sin(px+n\pi)+\cos(px+n\pi)$.

(b) $\sin\left(px+\frac{n\pi}{2}\right)+\cos\left(px+\frac{n\pi}{2}\right)$.

(c) $p^n\sin(px+n\pi)+p^n\cos\left(px+\frac{n\pi}{2}\right)$.

(d) $p^n\sin\left(px+\frac{n\pi}{2}\right)+p^n\cos\left(px+\frac{n\pi}{2}\right)$.

42. Residue of

$\dfrac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z=1$ is :

(a) $\frac{100}{6}$.
(b) $\frac{101}{6}$.
(c) $\frac{101}{16}$.
(d) $\frac{1}{116}$.

43. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a/an :

(a) Circle.
(b) Parabola.
(c) Hyperbola.
(d) Ellipse.

44. The values of $P_n(1)$ and $P_n(-1)$, where $P_n(x)$ is Legendra polynomial of degree n respectively are :

(a) $(-1)^n, 1$.
(b) $1, -1$.
(c) $-1, 1$.
(d) $1, (-1)^n$.

45. Solution of the LP problem

Maximize $z = 2x + 6y$ subject to $-x + y \le 1$, $2x + y \le 2$ and $x \ge 0, y \ge 0$ is :

(a) $\frac{4}{3}$.
(b) $\frac{1}{3}$.
(c) $\frac{26}{3}$.
(d) No feasible region.

46. The orthogonal trajectories of the family of curve $y = ax^2$ is (a and c being constants) :

(a) $\frac{x^2}{1} + \frac{y^2}{2} = c$.
(b) $\frac{x^2}{2} + \frac{y^2}{3} = c$.
(c) $\frac{x^2}{2} + \frac{y^2}{1} = c$.
(d) $\frac{x^2}{3} + \frac{y^2}{2} = c$.

47. If the order of every element of a group is 2 , then this group :

(a) Is Abelian.
(b) Is cyclic.
(c) Is of infinite order.
(d) Is definitely non-abelian.

48. The sum of interior angles of a geodesic triangle on the surface of a sphere radius R is :

(a) Less than π.

(b) π.

(c) Greater than π.

(d) Not constant.

49. Let (X, d) be a metric space where X is an infinite set and d is the discrete metric. Then :

(a) Heine-Borel theorem holds for (X, d).

(b) Heine-Borel theorem does not hold for (X, d).

(c) X is not bounded.

(d) X is compact.

50. Which of the following sets is in one-to-one correspondence with N

(I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots\right\}$.

(II) $\{\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$.

(III) $\left\{\frac{p}{q} : p, q \in Z, q \neq 0\right\}$.

(IV) $\left\{\frac{p}{q} : p, q \in N\right\}$.

(a) (I) and (II).

(b) (I), (II) and (III)

(c) (I) and (IV).

(d) All of the above.