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D 100615 **(Pages : 3)** Name.................................

Reg. No...............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half | Maximum Marks : 80

Section A (Short Answer Type Questions)

Answer any number of questions.
Each carry 2 *marks. Maximum marks* 25.

1. State Existence and Uniqueness Theorem for First Order Linear Differential Equations.
2. Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.
3. Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.
4. Show that the given differential equation is exact :

$$\left(x^3 + 3xy^2\right)dx + \left(3x^2y + y^3\right)dy = 0.$$

5. Find the Wronskian of the functions $e^{\lambda_1 x}, e^{\lambda_2 x}$.
6. Solve the differential equation $y'' - 2y' - 3y = 3e^{2t}$.
7. Let $y = \phi(x)$ be a solution of the initial value problem

$$\left(1 + x^2\right)y'' + 2xy' + 4x^2y = 0, \; y(0) = 0, \; y'(0) = 1.$$

Determine $\phi'''(0)$.

8. Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.
9. Find the Laplace transform of $2t + 6$.
10. Find the inverse Laplace transform of $\frac{s-4}{s^2+4}$.

Turn over

472587

11. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \geq 0,$ and if c is a constant. Show that

$$\mathcal{L}\left(e^{ct} f(t)\right) = F(s - c), s > a + c.$$

12. If $\mathcal{L}(f)$ denote the Laplace transform of the function $f(x)$. Show that

$$\mathcal{L}(f_1 + f_2) = \mathcal{L}(f_1) + \mathcal{L}(f_2), \ \mathcal{L}(cf) = c\mathcal{L}(f).$$

13. Solve the boundary value problem :

$$y'' + y = 0, y(0) = 1, y(\pi) = a.$$

14. Define an even function and show that if $f(x)$ is an even function then

$$\int_{-L}^{L} f(x)\, dx = 2\int_{0}^{L} f(x)\, dx.$$

15. Verify that the method of separation of variables may be used to solve the equation $xu_{xx} + u_t = 0.$

Section B (Paragraph/Problem)

Answer any number of questions.
Each carry 5 *marks. Maximum marks* 35.

16. Show that the equation $\dfrac{dy}{dx} = \dfrac{x^2}{1 - y^2}$ is separable, and then find an equation for its integral curves.

17. Find the value of b for which the following equation is exact, and then solve it using that value of b.

$$\left(xy^2 + bx^2 y\right) + (x + y)x^2 y' = 0.$$

18. Solve the initial value problem $y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2.$

19. Find the general solution of the differential equation $y'' + y = \tan t$ on $0 < t < \pi / 2.$

20. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0, y(0) = 3, y'(0) = -1.$$

21. Find the inverse Laplace transform of the following function using the convolution theorem :

$$F(s) = \frac{1}{(s+1)^2 \left(s^2 + 4\right)}.$$

22. Determine the coefficients in the Fourier series of the function

$$f(x)=\begin{cases}-x, & -2\le x\le 0,\\ x, & 0\le x<2\end{cases}$$

with $f(x+4)=f(x)$.

23. Find the solution of the following heat conduction problem :

$$\begin{aligned} 100u_{xx} &= u_t, && 0<x<1,\ t>0;\\ u(0,t)=0,\ u(1,t)&=0, && t>0;\\ u(x,0)=\sin(2\pi x)&-\sin(5\pi x), && 0\le x\le 1.\end{aligned}$$

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each carry 10 *marks.*

24. Find the general solution of the following differential equaton using the method of integrating factors

$$\frac{dy}{dt}+\frac{1}{2}y=\frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0, 1).

25. Find a series solution of the differential equation :

$$y''+y=0,\ -\infty<x<\infty.$$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau\, d\tau$.

27. Find the Fourier series of the following periodic function $f(x)$ of period $p = 2L$ defined by

$$f(x)=3x^2 -1<x<1.$$

(2 × 10 = 20 marks)

C 40606 **(Pages : 4)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Short Answer Type Questions.
Ceiling 25 *Marks.*

1. Find the solution of the differential equation $\frac{dp}{dt} = 0.5p - 450.$

2. Solve the differential equation $\left(4+t^2\right)\frac{dy}{dt} + 2ty = 4t.$

3. State Existence and Uniqueness Theorem for First-Order Linear Differential Equations.

4. Solve the initial value problem $y' = y^2, \quad y(0) = 1.$

5. Find the general solution of $y'' + 5y' + 6y = 0.$

6. If y_1 and y_2 are two solutions of the differential equation, $y'' + p(x)\,y' + q(x)\,y = 0.$

   Show that $c_1y_1 + c_2y_2$ is also a solution for any values of the constants c_1 and c_2.

7. Let $y_1 = e^t \sin t, y_2 = e^t \cos t.$ Find the Wronskian $W[y_1, y_2]$.

8. Solve the differential equation $y'' - 2y' - 3y = 3e^{2t}.$

9. Find the Laplace transform of e^{at}.

Turn over

10. Find $\mathcal{L}^{-1}\left(\frac{s}{s^2-a^2}\right)$ for $s > |a|$.

11. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \geq 0$, and if c is a constant, Show that

    $\mathcal{L}\left(e^{ct} f(t)\right) = F(s-c), \quad s > a + c.$

12. Solve the boundary value problem $y'' + y = 0,\ y(0) = 1,\ y(\pi) = a,$

    where a is a given number.

13. Find the fundamental period of the function $\sin(5x)$.

14. Define an odd function. Prove that if $f(x)$ is an odd function then

    $\int_{-L}^{L} f(x)\,dx = 0.$

15. Verify that the method of separation of variables may be used to solve the equation $xu_{xx} + u_t = 0$.

(2 Marks each)

Section B

Paragraph / Problem Type Questions.
Ceiling 35 *Marks.*

16. Show that the equation

    $$\frac{dy}{dx} = \frac{x^2}{1-y^2}.$$

    is separable, and then find an equation for its integral curves.

17. Solve the differential equation

    $$\left(y\cos x + 2xe^y\right) + \left(\sin x + x^2e^y - 1\right)y' = 0.$$

18. Given that $y_1(t) = t^{-1}$ is a solution of

$$2t^2 y'' + 3ty' - y = 0, t > 0,$$

find a fundamental set of solutions.

19. Find the general solution of the differential equation $y'' + y = \tan t$ on $0 < t < \pi/2$.

20. Find the Laplace transform of the following function $f(t) = \int_0^t (t-\tau)^2 \cos(2\tau)\, d\tau$.

21. Find the inverse Laplace transform of the following function using the convolution theorem

$$F(s) = \frac{1}{(s+1)^2 (s^2+4)}.$$

22. Determine the co-efficients in the Fourier series of the function

$$f(x) = \begin{cases} -x, & -2 \le x \le 0, \\ x, & 0 \le x \le 2 \end{cases}$$

with $f(x+4) = f(x)$.

23. Find the displacement $u(x, t)$ of the vibrating string of length $L = 30$ that satisfies the wave equation

$$4u_{xx} = u_{tt}, \quad 0 < x < 30, t > 0.$$

Assume that the ends of the string are fixed and that the string is set in motion with no initial velocity from the initial position

$$u(x, 0) = f(x) = \begin{cases} x/10, & 0 \le x \le 10, \\ (30-x)/20, & 10 < x \le 30 \end{cases}.$$

(5 marks each)

Section C (Essay Type Questions)

two *out of four.*

24. (a) Find the general solution of the differential equation $\frac{dy}{dt} - 2y = 4 - t$ by the method of integrating factors.

(b) Find the value of b for which the following equation is exact, and then solve it using that value of b

$$\left(ye^{2xy} + x\right) + bxe^{2xy}y' = 0.$$

25. Find a series solution in powers of x of Airy's equation

$$y'' - xy = 0, \quad -\infty < x < \infty.$$

26. Use the Laplace transform and solve the following initial value problem

$$y'' + 3y' + 2y = 0\,; \quad y\left(0\right) = 1,\ y'\left(0\right) = 0.$$

27. Find the Fourier series of the following periodic function $f\left(x\right)$ of period $p = 2\mathrm{L}$ defined by

$$f\left(x\right) = 3x^2 \quad -1 < x < 1.$$

(10 marks each)

C 20648 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the general solution of the differential equation $\frac{dy}{dt} = -ay + b$ where a,b are positive real numbers.

2. Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.

3. Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.

4. Find the solution of the differential equation :

   $$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, y(0) = -1.$$

5. Find the Wronskian of the functions $\cos^2\theta, 1 + \cos(2\theta)$.

6. Find the general solution of the differential equation $y'' + 2y' + 2y = 0$.

7. Let $y = \phi(x)$ be a solution of the initial value problem :

   $$\left(1 + x^2\right)y'' + 2xy' + 4x^2y = 0,\ y(0) = 0, y'(0) = 1.$$

   Determine $\phi'''(0)$.

8. Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.

Turn over

9. Find the Laplace transform of' the function sin (at).

10. Find the inverse Laplace transform of $\frac{n!}{(s-a)^{n+1}}$ where $s > a$.

11. Let $u_c(t)$ be unit step function and $L(f(t)) = F(s)$. Show that :

    $L(u_c(t)f(t-c)) = e^{cs}F(s)$.

12. Find the inverse Laplace transform of the following function by using the convolution theorem $\frac{1}{s^4(s^2+1)}$.

13. Solve the boundary value problem :

    $y'' + y = 0, y(0) = 0, y(\pi) = 0$.

14. Define an even function and show that if $f(x)$ is an even function then :

    $\int_{-L}^{L} f(x)dx = 2\int_{0}^{L} f(x)\, dx$.

15. Define the following partial differential equations :
    (a) heat conduction equation.
    (b) one-dimensional wave equation.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Let $y_1(t)$ be a solution of $y' + p(t)y = 0$ and let $y_2(t)$ be a solution of $y' + p(t)y = g(t)$.

    Show that $y(t) = y_1(t) + y_2(t)$ is also a solution of equation $y' + p(t)y = g(t)$.

17. Find the value of b for which the following equation is exact, and then solve it using that value of b.

    $(xy^2 + bx^2y) + (x+y)x^2y' = 0$.

18. Solve the initial value problem

    $y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2$.

19. Use method of variation of parameters find the general solution of :

$y'' + 4y = 8\tan t, -\pi/2 < t < \pi/2$.

20. Find the solution of the initial value problem :

$2y'' + y' + 2y = \delta(t-5), y(0) = 0, y'(0) = 0.$

here $\delta(t)$ denote the unit impulse function.

21. Using Laplace transform solve the initial value problem :

$y'' + 4y = 0,\ y(0) = 3,\ y'(0) = -1.$

22. Find the co-efficients in the Fourier series for f :

$$f(x) = \begin{cases} 0, & -3 < x < -1 \\ 1, & -1 < x < 1 \\ 0, & 1 < x < 3 \end{cases}$$

Also suppose that $f(x + 6) = f(x)$.

23. Find the solution of the following heat conduction problem :

$100u_{xx} = u_t, 0 < x < 1, t > 0$

$u(0,t) = 0, u(1,t) = 0, t > 0$

$u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Find the general solution of the following differential equation using the method of integrating factors :

$$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0,1).

25. Find a series solution of the differential equation :

$y'' + y = 0, -\infty < x < \infty.$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau \ d\tau$

27. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

(2 × 10 = 20 marks)