418503

D 50197 **(Pages : 4)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Part A

Answer all the **twelve** *question.*
Each question carries 1 *mark.*

1. State Demorgan's law for three sets.
2. Give an example of a denumerable set.
3. Determine the set A of all real numbers x such that $2x+3 \le 6$.
4. State completness property of $\mathbb{R}$.
5. If $S = \left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$, find sup S.
6. Define supremum of a set.
7. Give an example of a monotone sequence.
8. State Bolzano Weierstrass theorem for sequences.
9. Define contractive sequence.
10. Find $\overline{(2+i)^2}$.
11. State de Moiver's formula.
12. Define connected set.

(12 × 1 = 12 marks)

Turn over

418503

Part B

Answer any **ten** *questions.*
Each question carries 4 marks.

13. Let f and g be two functions on $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 2x$ and $g(x) = 3x^2 - 1$. Find $f \circ g$ and $g \circ f$.

14. Prove that $1 + 2 + \ldots + n = \frac{1}{2} n(n+1)$.

15. State and prove Cantor's theorem.

16. State and prove Bernoulli's inequality.

17. If $a, b \in \mathbb{R}$, prove that $|a + b| \leq |a| + |b|$ and $||a| - |b|| \leq |a - b|$.

18. Let S be a non-empty subset of $\mathbb{R}$ that is bounded above and let a be any number in $\mathbb{R}$. Then show that $\sup(a + S) = a + \sup S$.

19. State and prove Archimedean property.

20. State density theorem. Let x and y be two real numbers with $x < y$. Prove that there exists an irrational number z such that $x < z < y$.

21. Use the definition of the limit of a sequence to establish that $\lim_{n \to \infty} \frac{3n + 2}{n + 1} = 3$.

22. Prove that every convergent sequence is bounded. Is the converse true ? Justify.

23. State squeeze theorem and use this theorem to show that $\lim_{n \to \infty} \frac{\sin n}{n} = 0$.

24. Prove that the multiplication of complex numbers is commutative.

25. Compute $\frac{5i}{(1-i)(2-i)(3-i)}$.

26. Sktech the set of all points determined by $|z + i| \leq 3$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Prove that the following statements are equivalent :

    (i) S is a countable set.

    (ii) There exists a surjection from $\mathbb{N}$ onto S.

    (iii) There exists an injection from S into $\mathbb{N}$.

28. Let a and b be positive real numbers. Prove that $\sqrt{ab} \leq \frac{a+b}{2}$ and the equality occures if and only if $a = b$.

29. Prove that there exists a positive real number x such that $x^2 = 2$.

30. Let S be a subset of $\mathbb{R}$ that contains at least two points. Suppose S has the property that if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$, then prove that S is an interval.

31. Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Then prove that the sequence XY couverges to xy.

32. State Monotone convergence theorem. Let $y_1 = 1, y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Prove that $\lim y_n = \frac{3}{2}$.

33. State and prove characterization of closed subsets of $\mathbb{R}$ in terms of sequences.

34. Define Cantor set. Prove that the length of the Cantor set is zero.

35. Find the roots of $-8i$.

(6 × 7 = 42 marks)

Turn over

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (i) State and prove nested interval property.

    (ii) Using nested interval property, prove that $\mathbb{R}$ is uncountable.

37. (i) State an prove Cauchy convergence criterion for sequences.

    (ii) Let $Y = (y_n)$ be the sequence of real numbers given by

$$y_1 = \frac{1}{1!}, y_2 = \frac{1}{1!} - \frac{1}{2!}, \ldots\ldots, y_n = \frac{1}{1!} - \frac{1}{2!} + \ldots + \frac{(-1)^{n+1}}{n!}, \ldots\ldots$$ Prove that Y converges.

38. (i) Let $f(z) = u(x, y) + iv(x, y)$, $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$. The prove that

$$\lim_{z \to z_0} f(z) = w_0 \text{ if and only if } \lim_{(x, y) \to (x_0, y_0)} u(x, y) = u_0 \text{ and } \lim_{(x, y) \to (x_0, y_0)} v(x, y) = v_0.$$

    (ii) Find $\lim_{z \to \infty} \frac{2z^3 - 1}{z^2 + 1}$.

(2 × 13 = 26 marks)

D 30179 **(Pages : 4)** **Name....................................**

Reg. No................................

FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(2017—2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : Infimum of the set $S = \left\{1/3^m - 1/4^n \,;\, m, n \in N\right\}$ is ....
2. The Set of all real numbers which satisfy the inequality $|x-1| < |x-3|$ is ....
3. The Supremum property of R states that ....
4. State Bernoulli's Inequality.
5. State Cantor's theorem .
6. Give an example of a bounded real sequence which is not monotone.
7. State the Characterization theorem of open sets.
8. Fill in the blanks : $\lim \left(\frac{\sin n}{n}\right) =$ ————.
9. If $b > 0$, then $\lim \left(\frac{1}{1+nb}\right) =$ ————.
10. Define contractive sequence.
11. Fill in the blanks : Arg $(-2\pi) =$ ————.
12. The value of $(1+i)^{24}$ is ————.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Define Supremum and Infimum of set . Find them for the Set $S = \left\{1 - \frac{(-1)^n}{n}; n \in N\right\}$.

14. If $x > -1$, then show that $(1+x)^n \geq 1 + nx, \forall n \in N$.

15. If $y > 0$, then show that there exist some $n_y \in N$, such that $n_y - 1 \leq y < n_y$.

16. Show that there doesnt exist a real number r such that $r^2 = 5$.

17. If x and y are real numbers satisfying $x < y$, then prove that there exist an irrational number z such that $x < z < y$.

18. State the completeness property of R. Prove or disprove that the sub field Q of rational numbers has this property .

19. Define the limit of a sequence. Using the definition of limit, prove that lim $(1/n) = 0$.

20. State and prove Squeeze theorem for sequence .

21. If $X = (x_n)$ is a converging sequence of non-negative real numbers then prove that lim $(x_n) \geq 0$.

22. Show that every Cauchy sequence of real numbers is bounded.

23. Let $X = (x_n)$ and $Y = (y_n)$ be real sequences that converges to x and y respectively. Prove that $X + Y$ converges to $x + y$.

24. Define contractive sequence. Test whether the sequence $X = (x_n)$ defined by $x_1 = 2$, $x_{n+1} = 2 + 1/x_n$, $n \in N$ is a contractive sequence or not.

25. Find a sequence (x_n) of real numbers such that $\lim \left| x_{n+1} - x_n \right| = 0$, but not a Cauchy sequence.

26. Find the Arg Z, if $Z = \left(\sqrt{3} + i\right)^6$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*

Each question carries 7 *marks.*

27. Show that the unit interval [0, 1] is uncountable.

28. Determine the set $A = \left\{ x \in R : \frac{2x+1}{x+2} < 1 \right\}$.

29. State and prove the characterization theorem of intervals.

30. State and prove the "'Ratio test " for sequence.

31. If $0 < a < b$, then test the convergence of the sequence (x'_n), where $x_n = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$.

32. Test the convergence of (x_n) defined by $x_n = 1 + 1/2! + 1/3! + \ldots\ldots + 1/n!$

33. Prove that a subset of R is closed if and only if it contains its cluster points.

34. Find all values of $(-216\, i)^{\frac{1}{3}}$.

35. Factorise $x^6 - 1$ into linear factors.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*

Each question carries 13 *marks.*

36. (a) State and prove the "Density theorem " of rational numbers.

    (b) Discuss the convergence of the following sequences whose n'th terms are defined by :

    (i) $x_n = \left(1 + \frac{1}{n^2}\right)^{2n^2}$; and (ii) $y_n = \frac{\log n}{n}$.

Turn over

37. (a) State and prove the Cauchy convergence criterion for sequence.

    (b) Discuss the convergence of $X = (x_n)$, defined by $x_n = \ln n$.

38. (a) Define closed sets in R show that the Intersection of an arbitrary collection of closed sets in R is closed.

    (b) Show by an example that the union of infinitely many closed sets in R need not be closed.

(2 × 13 = 26 marks)

15418

D 10232 (Pages : 3) Name.................................

Reg. No...............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : Supremum of the set $S = \{1 - 1/n; n \in N\}$ is ———.
2. Determine the set $A = \{x \in R : |x-1| < |x|\}$.
3. The set of all real numbers which satisfy the inequality $0 \leq b < \epsilon, \forall \epsilon > 0$, then b = ———.
4. Fill in the blanks : The Supremum property of R states that ———.
5. State the Trichotomy Property of R.
6. Give the condition for a subset of R to be an interval of R.
7. State the general Arithmetic Geometric mean inequality of real numbers.
8. Fill in the blanks : The characterization theorem of open sets states that ———.
9. State the Bernoulli's inequality.
10. If $a > 0$, then $\lim(a^{1/n})$ = ———.
11. Fill in the blanks : Arg (-2π) = ———.
12. Fill in the blanks : The Exponential form of $-1-i$ = ———.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Define Supremum an Infimum of a set. Find them for the set $S = \{1/2^m - 1/3^n; m, n \in N\}$.
14. Show that there doesn't exist a rational number r such that $r^2 = 3$.
15. If $a, b \in R$, then prove that $||a| - |b|| \leq |a - b|$.
16. Prove that a real sequence can have at most one limit.

Turn over

17. If $x \in \mathrm{R}$ then prove that there exists $n_x \in \mathrm{N}$ such that $x < n_x$.

18. Discuss the convergence of the following sequences $\mathrm{X} = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$.

19. Show directly that a bounded monotonic sequence is a Cauchy sequence.

20. Define Cauchy sequence. Test whether $(1/n)$ is a Cauchy sequence or not.

21. Show by an example that intersection of infinitely many open sets in R need not be open.

22. Discuss the convergence of $\mathrm{X} = (x_n)$ define by $x_n = n$, if n odd and $x_n = 1/n$, if n even.

23. Show that every bounded sequence of real numbers has a converging subsequence.

24. Test the convergence of the sequence $\left(\frac{\cos n}{n}\right)$.

25. Express the complex number $(\sqrt{3} + i)^7$ in exponential form.

26. Find the principal value of $(-8i)^{\frac{1}{3}}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Prove that the set R of real numbers is uncountable.

28. State and prove the Ratio Test for the convergence of real sequence.

29. Discuss the convergence of the following sequences $\mathrm{X} = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$ and (b) $x_n = \left(\frac{1-2}{n}\right)^n$.

30. $\mathrm{X} = x_n$ and $\mathrm{Y} = y_n$ be sequences of real numbers converges to x and y respectively, then prove that X . Y converges to xy.

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

    (b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim\left(\frac{x_{n+1}}{x_n}\right) = 1$.

32. If $X = (x_n)$ is a real sequence and $X_m = (x_{m+n} : n \in N)$ is the m-tail of X ; $m \in N$, then show that X_m converges to x if and only if X converges to x.

33. Let $X = (x_n)$ be a bounded sequence of real numbers and $x \in R$ has the property that "every converging subsequence of $X = (x_n)$ converges to x". Prove that $X = (x_n)$ converges to x.

34. Prove or disprove the following statement : $\|z_1\| - |z_2| \le |(z_1)| - |(z_2)|, \forall z_1, z_2 \in C$.

35. Test the convergence of (x_n) defined by $x_n = 1 + 1/2 + 1/3 + \ldots + 1/n$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Show that there exists a positive real number x such that $x^2 = 2$.

37. (a) If $I_n = [a_n, b_n], n \in N$ is a nested sequence of closed and bounded intervals, then prove that there exist a common point in every I_n.

(b) Test the convergence of (x_n) defined by $x_n = 1 + 1/2! + 1/3! + \ldots + 1/n!$.

38. (a) Define a closed set and "cluster point" of a set. Give examples for each of them.

(b) Prove that a subset of R is closed in R if and only if it contains all of its cluster points.

(2 × 13 = 26 marks)

D 90232 (Pages : 4) Name.............................

Reg. No.............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Find $f \circ g$ for the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x, g(x) = 3x^2 - 1$.
2. Is there exists a bijection between $\mathbb{N}$ and a proper subset of itself ? Justify.
3. State the principle of Strong Induction.
4. If $a \in \mathbb{R}$ satisfies $a \cdot a = a$, prove that either $a = 0$ or $a = 1$.
5. Define absolute value of a real number.
6. Find $\sup\left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$.
7. Prove that $\lim \frac{1}{n} = 0$.
8. Give an example of two divergent sequences X and Y such that their sum X + Y converges.
9. Give an example of an unbounded sequence that has a convergent subsequence.
10. Define an open subset of $\mathbb{R}$.
11. Find Re z and Im z for $z = \frac{2+i}{(1+i)(1-2i)}$.
12. Show that $\operatorname{Re}(iz) = -\operatorname{Im} z$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Let $f : A \to B$ be a function and let $G, H \subseteq B$. Prove that $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.

14. Prove that if $c \geq 0$, then $|a| \leq c$ if and only if $-c \leq a \leq c$; $a, b, c \in \mathbb{R}$.

15. Prove that set $\mathbb{N} \times \mathbb{N}$ is denumerable.

16. Prove that a sequence of real numbers can have at most one limit.

17. Prove that every convergent sequence of real numbers is bounded.

18. Prove that the sequence (n) is divergent.

19. Prove that $\lim \left(\frac{1}{n^2+1}\right) = 0$.

20. Prove that every convergent sequence of real numbers is a Cauchy sequence.

21. Prove or disprove : The arbitrary intersection of open sets in $\mathbb{R}$ is open.

22. Show that if G is an open set and F is a closed set, then $G|F$ is an open set and $F|G$ is a closed set.

23. Show that $\left|e^{i\theta}\right| = 1$.

24. Prove that z is real if and only if $\bar{z} = z$.

25. Sketch the given set and determine whether it is a domain : $|2z+3| > 4$.

26. Define accumulation point of a set. Determine the accumulation points, if any, for the set $z_n = i^n$; $n = 1, 2, 3, \ldots$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 marks.
All questions can be attended.
Overall Ceiling 45.

27. If $I_n = [a_n, b_n]; n \in \mathbb{N}$ is a nested sequence of closed bounded intervals, prove that there exists a real number ξ such that $\xi \in I_n$ for all $n \in \mathbb{N}$.

28. Prove that the set $\mathbb{R}$ of real numbers is not countable.

29. State and prove Monotone Convergence Theorem.

30. State and prove Bolzano-Weierstrass Theorem.

31. Let (x_n) and (y_n) be sequences of real numbers such that $x_n \leq y_n \; \forall n \in \mathbb{N}$. Prove that :

    (a) If $\lim (x_n) = +\infty$, then $\lim (y_n) = +\infty$.

    (b) If $\lim (y_n) = -\infty$, then $\lim (x_n) = -\infty$.

32. Let $F \subseteq \mathbb{R}$. Prove that the following are equivalent :

    (a) F is a closed subset of $\mathbb{R}$.

    (b) If $X = (x_n)$ is any convergent sequence of elements in F, then $\lim X = x$ belongs to F.

33. Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all of its cluster points.

34. Let z be a complex number. Prove that $|1+z| = 1 + |z|$ if and only if z is real.

35. Find all roots of $8^{1/6}$ in rectangular co-ordinates.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. Prove that there exists a positive real number x such that $x^2 = 2$.

37. Let $X = (x_n)$ be a sequence of real numbers and let $x \in \mathbb{R}$. Prove that the following are equivalent :

    (a) X converges to x.

    (b) For every $\varepsilon > 0$, there exists a natural number K such that for all $n \geq K$ the terms x_n satisfy $|x_n - x| < \varepsilon$.

    (c) For every $\varepsilon > 0$, there exists a natural number K such that for all $n \geq K$ the terms x_n satisfy $x - \varepsilon < x_n < x + \varepsilon$.

    (d) For every ε-neighborhood $V_\varepsilon(x)$ of x, there exists a natural number K such that for all $n \geq K$ the terms x_n belong to $V_\varepsilon(x)$.

38. Prove that a subset of $\mathbb{R}$ is open if and only if it is the union of countably many disjoint open intervals in $\mathbb{R}$.

(1 × 15 = 15 marks)

D 70322 (Pages : 3) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : Infimum of the $S = \{1/m - 1/n; m, n \in N\}$ is ————.

2. The Set of all real numbers which satify the inequality $|x^2 - 1| \leq 3$ is ————.

3. Fill in the blanks : The $\in$ neighborhood of $a \in R$ is ————.

4. State the Infimum Property of R.

5. State Bernoulli's Inequality.

6. Fill in the blanks : If X is a converging sequence of non-negative real numbers, then lim X = ————.

7. Fill in the blanks : The intersection of infinitely many open sets in R is ————.

8. State the Density Theorem.

9. State the Monotone Sub-sequence Theorem.

10. Give an example of a bounded real sequence which is not a Cauchy sequence.

11. Fill in the blanks : The Polar form of $1 + i\sqrt{3} =$ ————.

12. Find the ArgZ, if $Z = \dfrac{-2}{1 + i\sqrt{3}}$.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Define Supremum and Infimum of set. Find them for the Set $S = \left\{1 - \dfrac{(-1)^n}{n}; n \in N\right\}$.

14. Prove that the set N of positive integers is not bounded above.

Turn over

15. If a is a real number such that $0 \leq a \leq \varepsilon$ for every $\varepsilon > 0$, then prove that $a = 0$.

16. Show that there does not exist a rational number r such that $r^2 = 5$.

17. State and prove Squeeze theorem on sequence.

18. If $S = \{1/n; n \in N\}$, then prove that Infimum of $S = 0$.

19. If X and Y are convergent sequences of real numbers satisfying $(x_n) \leq (y_n), \forall n \in N$, then prove that $\lim(x_n) \leq \lim(y_n), \forall n \in N$.

20. Prove that every bounded sequence of real numbers has a converging sub-sequence.

21. Define Cauchy sequence. Show that $(1/n)$ is a Cauchy sequence.

22. Prove that every converging sequence is a Cauchy sequence.

23. Prove or disprove that "the union of infinitely many closed sets in R is closed ".

24. Let S and T be bounded non-empty subsets of real numbers such that $S \subset T$. Prove that $\text{Inf } T \leq \text{Inf } S \leq \text{Sup } S \leq \text{Sup } S$.

25. Test the convergence of the sequence $\left(\frac{\log n}{n}\right)$.

26. Find all values of $(-27i)^{\frac{1}{3}}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. State and prove Characterization theorem of Intervals.
28. Prove that [0, 1] is uncountable.
29. State and prove the "Betweeness Property" of Irrational numbers.
30. Determine the set $A = \left\{x \in R: \frac{2x+1}{x+2} < 1\right\}$.
31. Let $X = (x_n)$ be a non-negative sequence of real numbers with $\lim(x_n) = x$. Prove that $\lim\left(\sqrt{x_n}\right) = x$.

32. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

(b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

(c) Justify the property of the sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

33. Test whether the (x_n) defined by $x_n = 1 + 1/2 + 1/3 + ... + 1/n$ is Cauchy sequence or not.

34. Prove that every contractive sequence is a Cauchy sequence.

35. Give an algebraic proof for the triangle inequality of complex numbers,

$$|(z_1 + z_2)| \le |(z_1)| + |(z_2)|, \forall z_1, z_2 \in C.$$

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. If $I_n = [a_n, b_n], n \in N$ is a nested sequence of closed and bounded intervals, then prove that there exist a common point in every I_n.

37. State and prove the monotone convergence theorem of sequence.

38. (a) State and prove the Ratio Test for the convergence of real sequence.

(b) Define "cluster point" of a set. Prove that a subset of R is closed in R if and only if it contains all of it's cluster points.

(2 × 13 = 26 marks)

D 50601 **(Pages : 3)** Name.................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

*Answer **all** the twelve questions.*
Each question carries 1 mark.

1. Fill in the blanks : Supremum of the set $S = \{1/m - 1/n \,;\, m, n \in N\}$ is ———.

2. Determine the set $A = \{x \in R : |2x + 3| < 7\}$.

3. The Set of all real numbers which satify the inequality $0 \leq a < \epsilon, \forall \epsilon > 0$, then a = ———.

4. Fill in the blanks : The Supremum property of R states that ———.

5. State the Trichotomy Property of R.

6. Give the condition for a subset of R to be an Interval of R.

7. State the general Arithmetic-Geometric mean Inequality of real numbers .

8. Fill in the blanks : The Characterization Theorem of Open sets states that ———.

9. State the Archimedian Property of positive Integers.

10. If $c > 0$, then $\lim (c^{1/n})$ = ———.

11. State the Bolzano-Weierstrass Theorem on Sequence .

12. Fill in the blanks : The Exponential form of $-1 - i$ = ———.

(12 × 1 = 12 marks)

Part B

*Answer any **ten** questions.*
Each question carries 4 marks.

13. State and prove Bernoulli's Inequality of Real numbers.

14. Show that there doesnot exist a rational number r such that $r^2 = 2$.

15. If $a, b \in R$, then prove that $\| a | - | b \| \leq | a - b \|$.

Turn over

16. Prove that a real sequence can have atmost one limit.

17. If $x \in$ R then prove that there exists $n_x \in$ N such that $x < n_x$.

18. State and prove the "Betweeness Property" of Irrational numbers.

19. If $X = (x_n)$ is a convergent sequences of real numbers satisfying $\lim (x_n) = x$, then show that $\lim(|x_n|) = |x|$.

20. Prove that the set of irrational numbers is uncountable.

21. Let A and B be bounded non-empty subsets of real numbers such that $a \leq b, \forall a \in A, b \in B$. Prove that Sup A $\leq$ Inf B.

22. Discuss the convergence of $X = (x_n)$ defined by $x_n = n$, if n odd and $x_n = 1/n$, if n even.

23. Show that every bounded sequence of real numbers has a converging sub-sequence.

24. Test the convergence of the sequence $\left(\frac{\sin n}{n}\right)$.

25. Define Cauchy sequence test whether $(1/n)$ is a Cauchy sequence or not.

26. Find all values of $(-8i)^{1/3}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. State and prove the Nested Interval Property.

28. Define denumerable set. Show that the set Q of rational numbers is denumerable.

29. If the set A_m is countable for each m $\in$ N, then prove that $A = \bigcup_{m=1}^{m=\infty} A_m$ is countable.

30. $X = x_n$ and $Y = y_n$ be sequences of real numbers converges to x and y respectively, then prove that X.Y converges to xy.

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(b) Give an example of a dververgent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

32. If $X = (x_n)$ is a real sequence and $X_m = (x_{m+n} : n \in N)$ is the m-tail of X ; $m \in N$, then show that X_m converges to x if and only if X converges to x.

33. Find a sequence (x_n) of real numbers such that $\lim |x_{n+1} - x_n| = 0$, but not a Cauchy sequence.

34. (a) Find the Arg Z, if $Z = \frac{i}{-2-2i}$.

(b) Express the complex number $\left(\sqrt{3}+i\right)^7$ in Rectangular form.

35. Discuss the convergence of the following sequences whose n'th terms are defined by $(a)\ x_n = \left(1+\frac{1}{n^2}\right)^{2n^2}$ and $(b)\ y_n = \frac{\log n}{n}$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. State and prove the Cauchy convergence criterion for sequence.

37. (a) Show that the unit interval [0,1] is uncountable.

(b) State and prove the Ratio Test for the convergence of real sequence.

38. (a) Define closed sets in R. Show that the Intersection of an arbitrary collection of closed sets in R is closed.

(b) Show by an example that the union of infinitely many closed sets in R need not be closed.

(2 × 13 = 26 marks)

C 30307 **(Pages : 3)**

Name.............................

Reg. No.............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define a denumerable set.
2. State trichotomy law of real numbers ?
3. State triangle inequality for real numbers.
4. Find all x satisfying $|x-3|=|x-5|$.
5. State the Archimedian property of the set of natural numbers.
6. Write the condition for a nested sequence of real numbers to have a unique common point.
7. State the ration test for the convergence of a real sequence.
8. Explicitly state monotone subsequence theorem.
9. Write the statement of Bolzano-Weierstrass theorem for a sequence.
10. Define Cauchy sequence.
11. Find Arg (z) if $z=\dfrac{i}{-2-2i}$.
12. Express $\left(\sqrt{3}-i\right)^7$ in the exponential form.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of* **fourteen** *questions.*
Each question carries 4 marks.

13. Prove that the collection of all finite subsets of $\mathbb{N}$ is countable.

14. If $x > 1$, prove that $(1+x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$.

15. If $a \geq 0$ and $b \geq 0$, prove that $a < b$ if and only if $\sqrt{a} < \sqrt{b}$.

16. Sate and prove density theorem.

17. If $y > 0$ prove that there is an n_y in $\mathbb{N}$ such that $n_y - 1 \leq y < n_y$.

18. Define the supremum and the infimum of a set S. Find them for the set $S = \left\{1 - \frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$.

19. Prove that there is no rational number whose square is two.

20. Prove that there is at most one limit for the sequence of real numbers.

21. For $0 < b < 1$ show that $\lim b^n = 0$.

22. Prove that every Cauchy sequence of real numbers is bounded.

23. Discuss the convergence of the sequence $x_n = \left(1 + \frac{1}{n}\right)^n$, n in $\mathbb{N}$.

24. Prove the triangle inequality for the complex numbers algebraically.

25. Find all values of $(-8i)^{\frac{1}{3}}$.

26. Find the rectangular form of $\left(\sqrt{3} - i\right)^6$ and the principal value of the amplitude.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 *marks.*

27. Show that the set of real numbers is uncountable.

28. Prove that there is no rational number x whose square is 3.

29. Sate and prove Cantor's theorem.

30. State and prove the characterization theorem for intervals.

31. Define contractive sequence and show that every contractive sequence is a Cauchy sequence.

32. Establish that every monotone sequence is convergent if and only if it is bounded.

33. Discuss the convergence of the following (x_n) where :

    (i) $x_n = \left(1 + \frac{1}{n^2}\right)^{2n^2}$, (ii) $x_n = \frac{\sin n}{n}$.

34. Show that a real sequence is convergent if and only if it is Cauchy.

35. Find the square roots of $-\sqrt{3i} + 1$ and express them in rectangular form.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* **three** *questions.*
Each question carries 13 *marks.*

36. (a) Show the existence of a positive real number in detail whose square is 2.

    (b) Show that between any two real numbers there is an irrational number.

37. (a) Prove that every bounded sequence of real numbers has a converging subsequence.

    (b) Show that a monotone sequence of real numbers is properly divergence if and only if it is unbounded.

38. (a) If $\lim (x_n) = x$ and $\lim (y_n) = n$ be sequences of non-zero reals that converge to x and $y \neq 0$ respectively. Prove that $\lim \left(\frac{x_n}{y_n}\right) = \frac{x}{y}$.

    (b) Discuss the convergence of (i) $x_n = \frac{(-1)^n n}{n^2 + 1}$ and (ii) $y_n = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ where $0 < a < b$.

(2 × 13 = 26 marks)

D 11547 (Pages : 3) Name.................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2016

(UG–CCSS)

Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer *all twelve* questions :

1 Let $f(x) = \dfrac{2x}{x-1}$, for all $x \in A = \{x \in R : x \neq 1\}$, then range of f is ————.

2 Using algebraic properties of R, prove that $a + b = 0 \Rightarrow b = -a$.

3 Find the supremum of $S = \left\{1 - \dfrac{(-1)^n}{n} : n \in N\right\}$.

4 State nested intervals property.

5 Give an example of a convergent sequence (x_n) of positive numbers with $\lim x_n^{1/n} = 1$.

6 Show that $\{2^n\}$ cannot converge.

7 Define a Cauchy sequence.

8 State True *or* False :

"If $\{x_n\}$ converges to l, then every subsequence of $\{x_n\}$ also converges to l".

9 Give an example of an open set in R.

10 State True *or* False :

"Arbitrary union of closed sets in R is closed".

11 Prove that $\text{Im}\,(iz) = \text{Re}\,z$, where z is a complex number.

12 Prove that 1 + 2's is closer to the origin then 3 + 4 *i*.

(12 × ¼ = 3 weightage)

Turn over

II. Very short answer questions. Answer *all nine* questions :

13 Suppose S and T are sets such that $T \subseteq S$ then prove that If S is finite then T is finite.

14 Given $f = x^2 + 1$ and $g = \sqrt{x-1}$ find $f \circ g$ and $g \circ f$.

15 If x and y are reals with $x < y$, then prove that if an irrational number of such that $x < z < y$.

16 Give examples to show that supremum and infimum may not belong to the set.

17 Find the supremum and Infimum of the set S where $S = \{x : 3x^2 - 10x + 3 < 0\}$.

18 Using the definition of limit, show that $\lim\left(\frac{3x+2}{x+1}\right) = 3$.

19 If $X = (x_n)$ is a convergent sequence of reals and if $x_n \geq 0$ for all n, then prove that :

$x = \lim(x_n) \geq 0$.

20 Establish the proper divergence of $(\sqrt{n})$.

21 Find the least positive integer (non-zero) in such that $\left(\frac{1+i}{1-i}\right)^n = 1$.

(9 × 1 = 9 weightage)

III. Short answer questions. Answer any *five* questions :

22 Prove that there does not exist a rational number r such that $r^2 = 2$.

23 If $a, b \in R$, prove that :

$||a| - |b|| \leq |a - b|$.

24 Let S be a non-empty bounded set in R and $a > 0$ and $aS = \{as ; s \in S\}$, prove that :

$\sup(aS) = a \cdot \sup S$.

25 Prove that a sequence of real numbers is convergent iff it is a Cauchy sequence.

26 Prove that the intersection of arbitrary collection of closed sets is closed.

27 Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.

28 Locate the points in the complex plane for which $|z-1|^2 + |z+1|^2 = 4$.

(5 × 2 = 10 weightage)

I. Essay questions. Answer any *two* questions :

29. (a) Show that the set of all real numbers between 0 and 1 is uncountable.

(b) If $x_n = \sqrt{n+1} - \sqrt{n}$, show that $[x_n]$ converges.

30. Show that a monotonic sequence of real numbers is convergent iff it is bounded.

31. (a) Prove that $\arg(z_1 z_2) = \arg z_1 + \arg z_2$.

(b) Find the value of $\sqrt{2i}$.

(2 × 4 = 8 weightage)

D 11163

Reg

(Pages : 3)

Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer **all** *the* **twelve** *questions.*
Each question carries 1 *mark.*

1. Define a countable set.
2. What do you mean by trichotomy law of real numbers ?
3. State Bernoulli's inequality.
4. Find all x satisfying $|x-1|<|x|$.
5. State the completeness property of the set of real numbers.
6. What are the conditions for a subset of real numbers to be an interval ?
7. If $a>0$ find $\lim\left(\dfrac{1}{1+na}\right)$.
8. State Squeeze theorem for limit of sequences.
9. Give the divergence criteria for a sequence of real numbers.
10. Find $Arg(z)$ if $z=-1-i$.
11. Define contractive sequence.
12. Find the exponential form of $\left(\sqrt{3}-i\right)^{6}$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of* **fourteen** *questions.*
Each question carries 4 *marks.*

13. Verify that the set of all integers $\mathbb{Z}$ is denumerable.
14. If $a \geq 0$ and $b \geq 0$, prove that $a < 6$ if and only if $a^2 < b^2$.
15. State and prove arithmetic-geometric mean inequality.

Turn over

16. Define infimum of a set. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, prove that inf (S) = 0.

17. If $t > 0$ prove that there is an n_t in $\mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.

18. State and prove the betweenness property of irrational numbers.

19. Determine the set A of all x satisfying $|2x+3| < 7$.

20. Test the convergence of the sequence (x_n) if $x_n = \frac{\sin n}{n}$.

21. Define Cauchy sequence. Find a sequence (x_n) which is not Cauchy such that $\lim |x_n - x_{n+1}| = 0$.

22. Prove that every convergent sequence of real numbers is a Cauchy sequence.

23. Show that subsequence of a converging real sequence always converge to the same limit.

24. State and prove Bolzano-Weierstrass theorem.

25. Find all values of $(-27\,i)^{\frac{1}{3}}$.

26. Prove that $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$ for all $z_1, z_2 \in \mathbb{C}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 marks.

27. Show that the unit interval $[0, 1]$ is uncountable.

28. Prove that there is a real x whose square is 2.

29. If A is any set, prove that there is no surjection of A on to the set $\mathcal{P}(A)$ of all subsets of A. Deduce that power set of natural numbers is uncountable.

30. If $I_n = [a_n, b_n], n \in \mathbb{N}$ is a nested sequence of closed and bounded intervals, prove that there is a real number which lies in I_n for all n.

31. Sate and prove monotone convergence theorem for a sequence.

32. Show that every contractive sequence is convergent.

33. Discuss the convergence of the following (x_n) where (i) $x_n = \left(1+\frac{1}{2n}\right)^n$; (ii) $x_n = \sum_{m=1}^{n} \frac{1}{m!}$.

34. State Cauchy's convergence criterion. Use it to test the convergence of $x_n = \sum_{m=1}^{n} \frac{1}{m}$.

35. Find the square roots of $\sqrt{3} + i$ and express them in rectangular form.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* **three** *questions.*
Each question carries 13 *marks.*

36. (a) State and prove the characterization theorem for intervals.

(b) Show that between any two real numbers there is a rational number.

37. (a) State and prove the ratio test for the convergence of real sequences.

(b) If $a > 0$ construct a sequence of real numbers which will converge to the square root of a.

38. (a) Let $X = (x_n)$ and $Y = (y_n)$ be real sequences that converge to x and y respectively. Prove the following :

(i) $\lim(x_n + y_n) = x + y$.

(ii) $\lim(x_n - y_n) = x - y$.

(iii) $\lim(x_n y_n) = xy$.

(iv) $\lim(cx_n) = cx, c \in \mathbb{R}$.

(b) Discuss the convergence of $\frac{n!}{n^n}$.

(2 × 13 = 26 marks)

D 90907

(Pages : 3)

Name.....................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

I. Objective type questions : Answer all *twelve* questions :

1 Let $f(x) = \frac{3x}{x+1}$ for $x \in A = \{x \in R ; x \neq -1\}$. Then range of f is ———.

2 Using algebraic properties of R, prove $a \cdot b = 6 \Rightarrow a = 1$.

3 State completeness property of R.

4 Write the supremum of $S = \left\{\frac{1}{n} ; n \in N\right\}$.

5 Give an example of a convergent sequence (x_n) of positive numbers with $\lim \frac{x_{n+1}}{x_n} = 1$.

6 Give an example of a Cauchy sequence.

7 State True *or* False. "Every bounded sequence is convergent".

8 If (x_n) and (y_n) are two sequences, such that $x_n < y_n$ and $\lim x_n = x$; $\lim y_n = y$. What is the relation between x and y ?

9 Give an example of a monotonic sequence.

10 State True *or* False. "Every open interval in an open set".

11 Prove that $z\bar{z} = |z|^2$.

12 Write the multiplicative inverse of the non-zero complex number $z = x + iy$.

(12 × ¼ = 3 weightage)

Turn over

II. Very short answer questions. Answer all *nine* questions :

13 Let $f : A \to B$; $g : B \to C$ be functions. Show that if gof is injective then f is injective.

14 Use mathematical induction to prove that $n^3 + 5n$ is divisible by 6.

15 Define $\in$-neighbourhood of $a \in R$.

16 Find the supremum and infimum of the set $S = \left\{1 - \frac{(-1)^n}{n} ; n \in N\right\}$.

17 If $S \subseteq T \subseteq R$, where $S \neq \phi$, then show that if T is bounded above then $\text{Sup } S \leq \text{Sup } T$.

18 "A sequence in R can have atmost one limit"—Prove.

19 Using definition of limit, prove that $\lim \left(\frac{1}{n}\right) = 0$.

20 Prove that a Cauchy sequence in bounded.

21 Find arg of z where $z = \frac{i}{-2 - 2i}$.

(9 × 1 = 9 weightage)

III. Short answer questions. Answer any *five* questions :

22 State and prove Bernoulli's inequality.

23 If $a, b \in R$, prove that $|a + b| \leq |a| + |b|$.

24 Let A and B be bounded non-empty subsets of R and $A + B = (a + b ; a \in A, b \in B)$. Prove that $\text{Sup }(A + B) = \text{Sup } A + \text{Sup } B$.

25 State and prove Squeeze theorem.

26 If a sequence $X = (x_n)$ of real numbers converges to a real number x, then prove that any subsequence $X' = (x_{nk})$ also converges to x.

27 Show that z is either real or purely imaginary iff $(\bar{z})^2 = z^2$.

28 Locate the points in the camplex plane for which $|z-1| = |z+i|$.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29 (a) Prove that the set Q of all rational numbers is denumerable.

(b) Suppose S and T are sets such that $T \subseteq S$. Prove that z + T is infinite, then S is infinite.

30 (a) Prove that the union of arbitrary collection of open subsets is R is open.

(b) Give an example to show that the arbitrary intersection of open set is not open.

31 (a) If z_1 and z_2 are two non-zero complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then prove that $z_1 = -z_2$.

(b) Evaluate $\sqrt{1-\sqrt{3}\,i}$.

(2 × 4 = 8 weightage)

C 30801 (Pages : 2) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2017

(UG—CCSS)

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. Define bijection.
2. Give an example of a denumerable set.
3. Give an example of a bounded below set which is not bounded above.
4. State nested interval property.
5. Is the sequence (n) convergent ?
6. Give an example of an unbounded sequence that has a convergent subsequence.
7. If (x_n) is an unbounded increasing sequence find $\lim x_n$.
8. Given an example of an open set which is not an interval.
9. Define Cantor set.
10. State Cauchy convergence criterion.
11. If z is real show that $z = \bar{z}$.
12. State de Moivres formula.

(12 × ¼ = 3 weightage)

Part B

Answer **all** *questions.*

13. By Mathematical Induction, prove that $1+2+3+\ldots\ldots+n = \frac{n(n+1)}{2}$.
14. Determine the set $A = \{x \in R / |2x+3| < 7\}$.
15. Show that $\lim (1/n) = 0$.

Turn over

16. Prove that a convergent sequence of real numbers is bounded.

17. Give an example of a bounded sequence that is not a Cauchy sequence.

18. Show that the set of Natural numbers is a closed set.

19. Show that $\overline{iz} = -i\bar{z}$.

20. Find $(\text{Arg}\, z_1 z_2)$.

21. If $z_1 = 2i$, $z_2 = \frac{2}{3} - i$, find $z_1 + z_2$.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions.*

22. Determine the set of all real numbers x such that $2x + 3 < 6$.

23. Find the infimum and supremum of $\left\{\frac{1}{n} - \frac{1}{m}; n, m \in \mathbb{N}\right\}$.

24. Find $\lim n^{1/n}$.

25. Is a Cauchy sequence of real numbers bounded ?

26. Show that a convergent sequence of real numbers is Cauchy.

27. Sketch the set of points determined by $|z + i| \leq 3$.

28. Prove that $\cos 3\theta = \cos^3\theta - 3\cos\theta \sin^2\theta$.

(5 × 2 = 10 weightage)

Part D

Answer any **two** *questions.*

29. Prove that there exists a positive real number x such that $x^2 = 2$.

30. State and prove Bolzano Weierstrass Theorem.

31. Find the exponential form of the complex number $-1 - i$, $\frac{-1 + \sqrt{3}i}{2}$.

(2 × 4 = 8 weightage)