# Bond Valuation and the Pricing of Interest Rate Options in India

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*Abstract*

In recent years, Indian corporates and institutions have issued a large number of exotic bonds with a variety of embedded interest rate options. Pricing the embedded interest rate options is a pre-requisite for valuing these bonds. The pricing of interest rate options is quite complex and depends crucially on the dynamics of interest rates. International studies show that no model of these dynamics is valid world-wide. Drawing on the author's earlier study of Indian interest rate dynamics (Varma, 1996), this paper expounds a practical methodology for pricing interest rate options in India and valuing bonds with embedded interest rate options. The Black-Derman-Toy model (Black et al., 1994) is shown to be the most attractive tool for valuing interest rate options in India.

A real-life application of the proposed approach shows its practical usability in valuing complex instruments with multiple embedded options. This also serves to show that embedded options in bonds can make a big difference to their valuation.

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## Introduction

For the last few years, India has been preparing for the introduction of full fledged markets for stock and stock index options. While these markets have yet to see the light of day, options on interest rates have become increasingly important in the country's fledgling debt market. Though there is no trading in interest rate options *per se*, there has been a lot of activity in the issue of bonds with various embedded interest rate options. The pricing of the embedded call and put options, is essential to arrive at a rational valuation of these bonds.

Valuation of options on bonds is considerably more complex than the pricing of options on stocks and stock indices mainly because of the vastly greater complexity of the bond price dynamics as compared to the dynamics of stock prices. The probability distribution of stock prices closely resembles a log-normal distribution generated by a random walk. In other words, the distribution of stock market *returns* approximates the familiar bell-shaped normal distribution. The famous Black-Scholes option pricing formula (Black and Scholes, 1973) for valuing options on stocks is based on this distribution and is known to perform quite well in practice.

Bond prices, on the other hand, do not follow a random walk at all. In fact, as the bond approaches maturity, its price approaches the redemption value and all uncertainty rapidly disappears. (A random walk has been compared to the walking of a drunkard; the bond then is a drunkard who becomes increasingly sober as the maturity approaches!). To value options on bonds (or options embedded in them), it is, therefore, usual to regard the interest rate rather than the bond price as the underlying variable. However, the dynamics of interest rates are not straightforward either. Interest rates do not follow a simple random walk, but exhibit the well known phenomenon of mean reversion. This phenomenon refers to the tendency of interest rates to revert to a normal rate over the long run. Whenever the interest rate drifts too far away from the normal rate, it is pulled back towards it. It is also well known that interest rates are more volatile when rates are high than when they are low. (To pursue our previous analogy, the drunkard's swagger increases sharply when he drifts towards the left hand side of the road and is less pronounced when he drifts towards the other end.) This means that the Black-Scholes option pricing formula is of limited applicability in pricing options involving interest rates. Another, though somewhat less serious, problems of the Black-Scholes formula is that it assumes that interest rates are constant - a very odd assumption to make while valuing options on interest rates!

## Interest Rate Dynamics

Researchers around the world have expended a great deal of effort on determining the dynamics of interest rates. Unfortunately, no single model appears to be valid in all countries of the world. A recent study (Tse, 1995) of eight different models in eleven countries found that no model was valid in all countries. Each of the three most popular models found applicability in some countries, but each was rejected in half the countries. This means that we cannot simply pick up one of the models developed elsewhere in the world and apply it to India. It is necessary to study the dynamics of interest rates in India and determine the model which best fits the Indian experience. The author has carried out such a study of Indian interest rates (Varma, 1996) and concluded:

- Indian interest rates exhibit fairly strong mean reversion.
- Interest rates do fluctuate more when they are high rather than when they are low. In fact, the variability of interest rates is proportional to the level of interest rates so that the variability of proportionate changes in interest rates (known as volatility) is level independent. This type of model was initially proposed by Brennan and Schwartz (1979).
- The "normal rate" to which interest rates mean-revert is itself changing over time, and it too undergoes mean-reversion to a "grand" normal rate.

Mathematically, these interest rate dynamics can be described by the following equations:

$$\Delta r_t = \kappa(\mu_t - r_{t-1}) + \sigma r_{t-1} u_t \quad \textbf{(1)}$$

$$\Delta \mu_t = \varphi(\mu - \mu_{t-1}) + \omega \mu_{t-1} v_t \qquad \textbf{(2)}$$

In Eq (1), $r_t$ is the interest rate and $\mu_t$ the normal rate at time $t$, while $\Delta r_t$ is the change in the interest rate during time period t. In these and other models of interest rate dynamics, the time period is usually rather short (a month or less), and the discrete time equations are regarded as approximations to continuous time diffusion processes. The parameter $\kappa$ is the speed of adjustment which tells us how rapidly the interest rate gets pulled back towards its normal level. The parameter $\sigma$ is the volatility of interest rates and $u_t$ is a serially uncorrelated disturbance term which is normally distributed with zero mean and unit variance. Eq **(2)** which models the evolution of the normal rate is quite similar to Eq (1). The normal rate $\mu_t$ mean-reverts to a grand normal rate $\mu$ with a speed of adjustment $\varphi$. The parameter $\omega$ is the volatility of the normal rate and the serially uncorrelated disturbance term $v_t$ is normally distributed with zero mean and unit variance. Varma (1996) provides the following parameter estimates for the above equations in which the unit of time is a week:

$\varphi = 0.0392$, $\mu = 12.12$, $\omega = 0.190$, $\kappa = 0.718$ and $\sigma = 0.2376$.

## Pricing of Interest Rate Options

In recent years, there has been a proliferation of interest rate option pricing models of growing complexity and sophistication in what has been described as an "arms race" (Lochoff, 1993). These models can be classified in several different ways.

- We can distinguish models with analytical (closed-form) solutions like the Black-Scholes model from models which require numerical methods for solution. Most realistic models require numerical solutions and are typically based on the binomial lattice. The binomial lattice is a very flexible tool which can be used for valuing American options and options with many exotic features. The lattice is also computationally quite efficient as compared to alternative numerical methods.

- We can also distinguish between single factor models and multi-factor models. Single factor models are based on the dynamics of just one factor (typically the short term interest rate), while multi-factor models involve several factors (for example, a short term rate and a long term rate). In India very little is known about the dynamics of long term interest rates, and Varma (1996) argues that, apart from the short term rate (call market rate), sufficient reliable data does not exist for other interest rates for estimation of the interest rate dynamics. This suggests the use of single factor models at this stage in India.

- Option pricing models can be classified according to the dynamics assumed for the short term interest rate. Most realistic models allow for mean reversion in some way, but they differ in the assumption made regarding variability of interest rates. For example, variability may be assumed to be independent of the level of interest rates, to be proportionate to the level, proportionate to the square-root of the level, and so forth. It has already been pointed out that Indian interest rate dynamics can be characterized by mean reversion and a level independent volatility (variability proportionate to the level of interest rates).

- Pricing models can also be classified according to whether they endogenously derive the term structure of interest rates or they allow the user to input a term structure. There are strong reasons for preferring models which allow the term structure to be endogenously specified as these models are guaranteed to provide correct pricing of straight bonds (i.e. bonds which do not have any embedded options). This point is forcefully argues in Dattatreya and Fabozzi (1989).

Based on these considerations, the single factor model of Black-Derman-Toy (Black et al., 1994) stands out as a very attractive tool for pricing interest rate options in India. This model is lattice based, incorporates mean reversion, assumes level independent volatility and is calibrated through an exogenously specified yield curve.

The Black-Derman-Toy (BDT) model of option pricing is based on a binomial lattice of interest rates. The lattice approach breaks time into discrete periods (years, months, weeks, days or whatever). The larger the number of periods (the shorter, the time interval), the more accurate the valuation. The term “binomial” means that given the interest rate in any period, the interest rate in the next period can take only two values (usually called the up-state and the down-state). When we use short time intervals, this ceases to be restrictive because though there are only two possible values one period hence, a large number of values may be possible after a few periods. In a general lattice, there are two possible values one period hence and each of them can have two values two periods hence and so on. The number of values (states) explodes exponentially (2, 4, 8, 16, 32 and so on) and the lattice soon becomes unmanageable even with a powerful computer. Most practical lattice models (including the BDT lattice), therefore, “recombine”. Recombination means that if the interest rate moves up in one period and moves down the next, the resulting value is the same as would result if it moved down first and then moved up. With recombination, there are only 3 possible values of the interest rate after 2 periods, 4 values after 3 periods and so on. Recombination allows lattices with a large number of time periods to be analysed on a computer. In any lattice, it is also possible to specify the probabilities of an up-move and a down-move at each node. In the Black-Derman-Toy model, both these probabilities are set equal to half throughout the lattice and we shall do likewise.

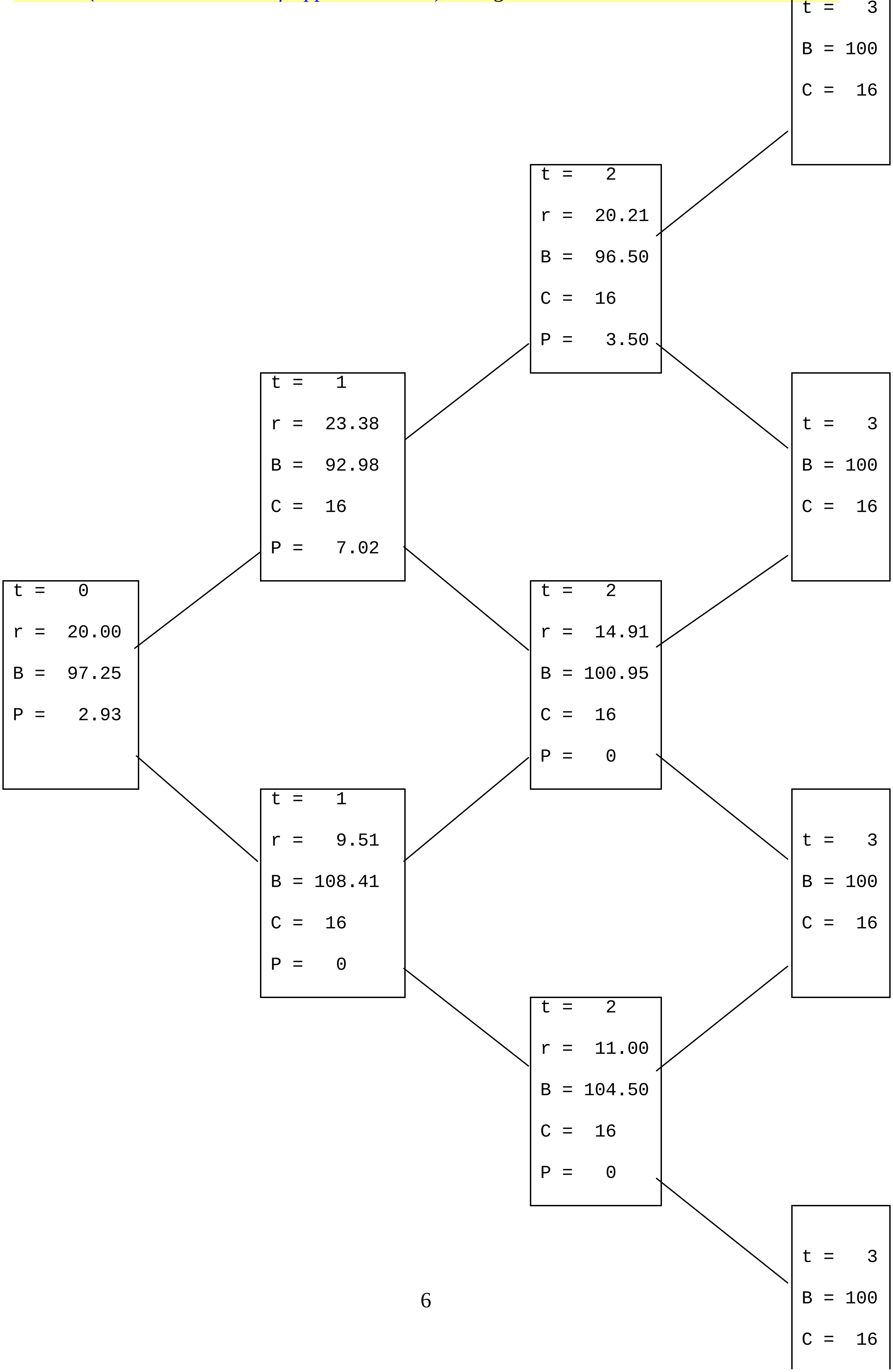
t = 0
r = 20.00
B = 97.25
P = 2.93
t = 1
r = 23.38
B = 92.98
C = 16
P = 7.02
t = 1
r = 9.51
B = 108.41
C = 16
P = 0
t = 2
r = 20.21
B = 96.50
C = 16
P = 3.50
t = 2
r = 14.91
B = 100.95
C = 16
P = 0
t = 2
r = 11.00
B = 104.50
C = 16
P = 0
t = 3
B = 100
C = 16
t = 3
B = 100
C = 16
t = 3
B = 100
C = 16
t = 3
B = 100
C = 16

**Figure 1: Black-Derman-Toy lattice used to value an American put option**

A recombining lattice is depicted in Figure 1 in which the symbol t denotes time (measured in years) and the symbol r denotes the rate of interest in percent (the other symbols are explained subsequently). At time 0, the interest rate is 20%, and from this level, it can either go up to 23.38% or go down to 9.51% at time 1. It may be noted that the up-move from 20% to 23.38 is much smaller than the down-move from 20% to 9.51%. This kind of asymmetry is the mechanism through which a lattice can reflect mean reversion; in this case, the 20% rate prevailing at time 0 is significantly above the normal rate of interest, and mean reversion is pulling the rate of interest down. Now consider the two possible rates of interest (23.38% and 9.51% ) at time 1. From 23.38%, the rate could go to either 20.21% or 14.91%; in this case, mean reversion is so strong that even the up-move is actually to a lower rate of interest. Similarly from 9.51%, the rate can go to either 14.91% or 11%; in this case mean reversion is pulling the rate up so that even the down-move is to a higher rate of interest. The lattice recombines because the down-move from 23.38% and the up-move from 9.51% are both equal to 14.91%.

The lattice can be used to value any security whose value depends only on interest rates. The simplest example is that of an ordinary bond. In this case, we need a lattice that goes up to the maturity of the bond or beyond. At the maturity date, the value of the bond is clearly equal to the redemption value of the bond regardless of the prevalent structure of interest rates. To find the value of the bond before its maturity, the lattice is folded back by calculating a discounted expected value as in any decision tree. To find the value of a security at a node, we average the two possible values one period hence (at the up and down states) and discount the average (expected value) at the interest rate prevailing at the node to get the value of the security at the node. (If, unlike the BDT lattice, the probabilities of the up and down moves are not equal, we must use a weighted average with the probabilities as weights to compute the expected value). Typically, a bond also pays interest at regular intervals; while calculating the value of the bond at any node, the coupon or any other cash flows to be received during the ensuing period must also be discounted and added to the value at the node as computed above.

The lattice of Figure 1 discussed earlier also shows the valuation of a three year bond with an annual coupon of 16%. The ex-interest prices of the bond are denoted by the symbol B while the cash flows (coupon payments) are denoted by the symbol C. Valuation begins from the maturity of the bond at time 3 at which point, the ex-interest value of the bond is clearly 100 (par) and there is a cash flow of 16. We now start folding the lattice back to time 2; consider the topmost node where the interest rate is 20.21%. From this node, both the up and down moves lead to a bond price of 100 and a cash flow of 16 adding up to 116 in all. Discounting this amount at the rate of 20.21% gives a bond value of 96.50 at this node. The same procedure can be applied to get the bond values at the remaining nodes at time 2. We can then fold back to time 1 starting with the topmost node where the interest rate is 23.38%. From here the up-move leads to a bond price of 96.50 and the down-move to 100.95 yielding an average of 98.73. To this the coupon of 16 is added and the resulting sum of 114.73 is discounted at 23.38% to get the bond value of 92.98 at this node. The procedure goes on in this manner to yield a bond value of 97.25 at time 0.

The general principle for valuing any interest rate dependent security is the same as the procedure outlined above for the bond: the lattice is folded backward from some terminal point where the security has a well defined value. In the case of an option, the value is well defined at the expiry of the option provided we know the value of the underlying asset at that point of time. For example, the value at expiry of a call option on a bond is the maximum of zero and the excess of the bond price over the exercise price. Before we can value an option, therefore, we must first value the bond using the method described in the preceding paragraph using a lattice which terminates at the maturity of the bond or beyond. We must then use the same lattice to value the option starting from the expiry date of the option. At this point, the value of the option is known, and from this point, we can fold the lattice backward. At each node we calculate the discounted expected values from the up-state and down-states. If we are valuing a European option which is exercisable only at expiry, this is all that there is to do and we can continue folding the lattice backward using this value. In the case of an American option, however, at each node the holder of the option has two choices - either to exercise the option immediately or to keep it alive for being exercised at a later date. The discounted expected value described above is actually the “live” value of the option (the value if the options is not exercised, but kept alive). This must now be compared with the “dead” value of the option (the value if the option is exercised immediately) which can be readily computed from the value of the bond at that node. The value of the option is clearly the higher of its "live" value and its "dead" value since the holder would exercise the option immediately if and only if that would give him a higher value than he could obtain by keeping it alive.

The lattice of Figure 1 which was used earlier to value a three year 16% bond also shows the valuation of a put option on the same bond. The American put option has an exercise price of 100 and expires at time 2. In the lattice the symbol P has been used to denote the value of this put option. Valuation begins at time 2 when the put value is simply the maximum of 0 and 100-B where B is the bond price. In the top node where the interest rate is 20.21%, the bond price is 96.50 and the put value is 100 - 96.50 = 3.50. In the remaining two nodes at time t, the put is worth 0 because the bond price exceeds the exercise price of 100. We then fold back to time 1 and consider the top node where r is 23.38% and the bond price is 92.98. From here, the up-move leads to a put value of 3.50 and the down-move leads to a put value of 0 yielding an average of 1.75. Discounting this at the rate of 23.38% gives the "live" value of the option of 1.42 which would be the value of a European put at this node. However, since the put we are valuing is American, we must compare this with the value resulting from immediate exercise which is 100 - 92.98 = 7.02. Immediate exercise is profitable and the value of the American put is 7.02. In the same manner, the lattice is folded back right up to time 0 to obtain the current value of the American put as 2.93.

Much of the above is applicable to most lattice based models for pricing interest rate options. The defining features of the BDT model are three:

1. In each time period, the volatility of the interest rate is level independent, but the volatility may vary over time. Black et al. (1994) have shown that the volatility at any node is simply half the logarithm of the ratio between the up-state and the down-state from that node. In the lattice of Figure 1, for example, from the upper node at time 1 (r = 23.38%), the up and down states are 20.21% and 14.91% giving a ratio of 1.36 and a volatility of 15.4% . From the lower node at time 1 (r = 9.51%), the up and down states are 14.91% and 11.00% once again giving a ratio of 1.36 and a volatility of 15.4%. Clearly, therefore, in any time period ,the ratio between successive nodes (for example, 20.21%, 14.91% and 11.00%) must be the same to keep the volatility constant in the preceding time period.

2. The probabilities of up-moves and down-moves are both equal to half as already stated.

3. The lattice must reproduce the specified yield curve and the specified "term structure of volatility". The "term structure of volatility" or volatility curve is the mechanism through which mean reversion is incorporated into the Black-Derman-Toy model. Just as the yield curve gives the interest rate for various maturities of bonds, the volatility curve gives the volatilities of these yields for various maturities. Mean reversion manifests itself in the fact that long term interest rates fluctuate much less than short term rates. The term structure of volatility is today regarded as an extremely important concept particularly in the light of the Heath-Jarrow-Morton (1992) results on its crucial importance in the pricing of interest rate options.

It can be shown that the above requirements determine the lattice completely. The lattice can be constructed step by step starting from the first period. In each time period, the interest rates at various nodes at that period are determined by two parameters - the upper yield envelope (the highest interest rate among all nodes at that time period) and the ratio between the interest rates at successive nodes at that time period. These two parameters must be chosen so as to satisfy two requirements in each time period - the lattice must match the yield curve and the volatility curve for that maturity. In general, these two requirements determine both the parameters uniquely.

The lattice of Figure 1 which we have already discussed at length was constructed using the following yield and volatility term structure

| **Maturity** | **Yield** | **Volatility** |
|---|---|---|
| 1 year | 20% | irrelevant |
| 2 years | 18% | 45% |
| 3 years | 17% | 30% |

The yields and volatilities are for zero-coupon bonds of the given tenor, and, for simplicity, the yields are on an annually compounded basis instead of the semi-annual compounding which is customary in quoting bond yields. In practice, yield curves are generally constructed for coupon bearing bonds which are currently trading close to par, but one can readily convert back and forth between such a par-bond yield curve and its equivalent zero coupon yield curve.

Using the specified term structure of yield and volatility, the lattice is constructed as follows beginning at time 0. The rate at time 0 is simply the yield on a one year bond which is given to be 20%. At time 1, the two available parameters - the upper yield envelope of 23.38% and the ratio between the two rates at this time (23.38/9.51) = 2.46 have been chosen to satisfy the two requisite conditions - the two year zero must have an yield of 18% and a volatility of 45%. The parameters are obtained by trial and error, but the end result may be readily confirmed by valuing a two year bond using the lattice. It may be verified that a two year zero would have values of 81.05 and 91.32 at the two nodes at time 1 and a value of 71.82 at time 0. The value of 71.82 at time 0 verifies that the yield is 18% (71.82 x 1.18 x 1.18 = 100). When we move to time 1, however, the yield of the bond for the remaining period of one year is no longer 18% - it can either go up to 23.38% or go down to 9.51%. The Black-Derman-Toy formula gives the volatility as 0.5 ln(23.38/9.51) = 45% as required.

At time 2, the upper yield envelope of 20.21% and the common ratio (20.21/14.91 = 14.91/11 = 1.36) have been chosen to match the three year zero's yield of 17% and volatility of 30%. To confirm this, the lattice may be used to value a three year zero. It may be verified that the value of this zero at time 0 is 62.44 corresponding to an yield of 17%. The two possible values at time 1 would be 68.98 and 80.87 corresponding to yields (for the remaining two years) of 20.40% and 11.20% respectively. The Black-Derman-Toy formula gives the volatility as 0.5 ln(20.40/11.20) = 30% as required.

## Estimating the Yield Curve and the Volatility Curve

The two main inputs which drive the valuation of options in the Black-Derman-Toy model are the prevailing yield curve and volatility curve. The most natural way to estimate the yield curve is to use market quotations. Essentially, one obtains the yields from market quotations for various maturities and then draws a smooth curve through these points in some way. One could use splines and other sophisticated tools to draw this smooth curve, but the procedure is otherwise quite straightforward. In India, too, the yield curve can be estimated from market prices or indicative quotations, but, given the highly illiquid nature of our debt markets, the estimation of the yield curve does involve some degree of subjectivity even in the case of sovereign debt. In case of corporate debt, the market is so illiquid that the only practical method is add an estimated spread to the sovereign yield curve. Estimation of the yield curve for long maturities does pose some problems as the markets are very thin at these maturities. However, none of these problems are peculiar to pricing interest rate options; all of them have to be faced in valuing even plain bonds with no embedded options.

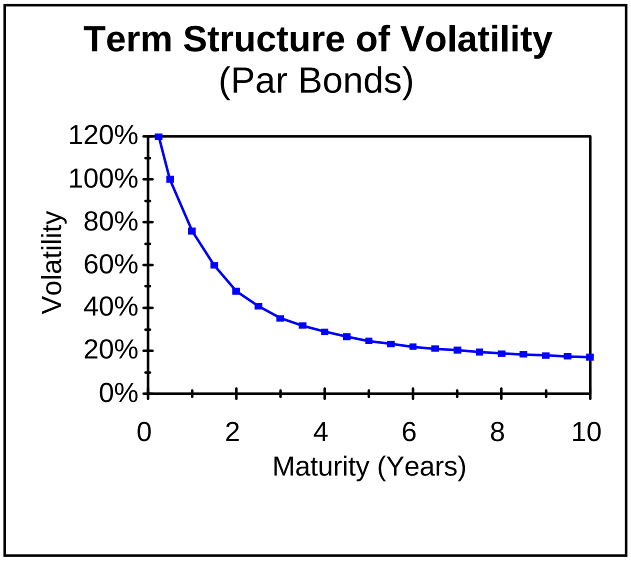


**Figure 2**

As far as the term structure of volatility is concerned, the international practice is to estimate this from historical data on yields. In India, this is, quite clearly, not a feasible proposition as there is no reliable series of long term yields going back far enough for any volatility estimates to be derived. Varma(1996) proposed an alternative method which was to estimate the term structure of volatility directly from the estimated interest rate dynamics (Eqs (1) and (2) above). These dynamics are too complex to admit of any simple closed form solution for the term structure of volatility, but the methodology of Monte Carlo simulation can be used. The volatility curve resulting from the application of this methodology is plotted in Figure 2 which is taken from Varma (1996).

The following international comparison (drawing on Hsieh, 1993) is presented to show that the estimated term structure of volatility is not unreasonable. Though the volatilities in India are the highest in the sample, they are not very different from those in Japan.

| Country | Term Structure of Volatility | |
|---|---|---|
| | 90 days | 10 years |
| India | 120% | 18% |

| | | |
|---|---|---|
| United Kingdom | 36% | 12% |
| Germany | 34% | 10% |
| Japan | 113% | 17% |
| United States | 30% | 13% |

## Application of the Option Pricing Model

This section presents an application of the proposed methodology for the valuation of interest rate options. The example considered is the deep discount bond issued by the Industrial Development Bank of India (IDBI) in February-March 1996. This bond carried as many as eight embedded options making it an interesting example to show how the proposed methodology can value an instrument with multiple embedded options. This example will also serve to show how misleading an issue advertisement can be if it fails to disclose all the embedded options in a bond.

The IDBI deep discount bond was a zero coupon bond with a redemption value at the end of 25 years of Rs 200,000 and an issue price of Rs 5,300. The investors had four European style put options allowing them to redeem the bond at Rs 10,000, Rs 25,000, Rs 50,000 and Rs 100,000 respectively at the end of 4 years and 4 months, 10 years and 8 months, 15 years and 5 months, and 20 years and 2 months respectively. The issuer (IDBI) also had four European style call options with the same exercise dates and exercise prices as the put options.

Actually, this bond with all its eight embedded options can be valued quite simply without using any option pricing model at all. An important relationship known as the put-call parity implies that this bond is actually nothing but a simple discount bond with a maturity value of Rs 10,000 at the end of 4 years and 4 months. This is because the bond will be extinguished on that date either by the investors exercising their put option or by the issuer exercising its call option. Whatever the yield curve may look like at the end of 4 years and 4 months, one of these two options will be worth exercising.

What makes the matter interesting is that in all its advertisements, the issuer highlighted the 25 year tenure of the bond and also mentioned the put options to the investor while being silent about the call options that the issuer itself enjoyed. We thus need to consider three different bonds -

- the straight 25 year discount bond (referred to below as the *straight bond*),
- the bond sweetened with put options but no call option (referred to below as the *advertised bond*), and
- the bond with both put and call options (referred to below as the *actual bond)*.

Investors could have been drawn to the straight bond because it offered the ability to lock-in for 25 years at a high interest rate. (The bonds were issued in February-March 1996 when interest rates were close to their peak.) The advertised bond was even more attractive because in addition to locking-in at high rates, it offered down-side protection. If, for some unexpected reason, interest rates actually rose, investors could exercise the put option and limit their losses.

Unwary investors who did not read the offer document carefully might have bought the actual bond under the misconception that they were buying the straight bond or the advertised bond. How serious would such a misconception be?

The Black-Derman-Toy model can be used to value the advertised bond if we specify the term structure of interest rates and their volatility. Actually, in India there were hardly any long maturity zero coupon instruments and therefore even the estimation of the yield curve is a little problematic beyond a maturity of 4-5 years. The most reasonable assumption is that the yield curve was very gently downward sloping at long maturities. If we assume that the straight bond was correctly priced, then the zero coupon yield curve touches 15.06% (semi-annually compounded) at 25 years. Let us assume therefore that the yield curve slopes down gently to this level. The term structure of volatility up to 10 years is available from the simulation study in Varma (1996) discussed earlier and we can assume that this also slopes down gently from that point.

Under these assumptions, the valuation of the three bonds is as follows:

| | | |
|---|---|---|
| Straight bond | Rs 5,300 | (equals issue price) |
| Advertised bond | Rs 6,300 | (19% above issue price) |
| Actual bond | Rs 5,050 | (20% below advertised bond, and 5% below issue price) |

This is a very shocking result; not only is the actual bond worth 20% less than the advertised bond, but it actually worth less than the issue price.

In the above calculations, it is the straight bond which is fairly priced, i.e., issue price equals fair value of the straight bond. What would the actual bond look like to an investor who thought that it was the advertised bond which was fairly priced? To answer this, we can use the option pricing model to determine an option adjusted spread (OAS) which when added to the yield curve values the advertised bond at par. This OAS turns out to be a little over 1%. Using this OAS, the three bonds are priced as follows:

| | |
|---|---|
| Straight bond | Rs 4,100 (22% below issue price) |
| Advertised bond | Rs 5,300 (equals issue price) |
| Actual bond | Rs 4,800 (9% below issue price) |

This result is equally striking with the actual bond being worth about 9% less than the issue price.

In either case, the conclusion is inescapable that the issue advertisements were grossly misleading in this case.

## Conclusion

This paper has argued that the most practical and sensible way of valuing interest rate options in India is to use a single factor lattice based model. Single factor models (which are based on the short term interest rate) are to be preferred not only for their greater simplicity, but also because multi-factor models would require knowledge of the dynamics of long term rates which is not available in India at present. Within the class of single factor lattice based models, the Black-Derman-Toy model emerged as the most serious contender because its assumptions are consistent with the behaviour of interest rates in India as documented in Varma (1996). Specifically, the Black-Derman-Toy model allows for mean reversion and assumes that the volatility of interest rates is independent of their level. A real-life application of the proposed approach demonstrated its practical usability in valuing complex instruments with multiple embedded options. It also served to show that embedded options in bonds can make a big difference to their valuation and that issue advertisements that do not fully disclose some of these options can be grossly misleading.

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