Modelling Credit Risk in Indian Bond Markets

by

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Abstract

Government bonds are subject only to interest rate risk. However, corporate bonds are subject to credit risk in addition to interest rate risk. Credit risk subsumes the risk of default as well as the risk of an adverse rating change. Considerable work has been done in the US and other countries on credit rating migrations. However, there is little work done in India in this regard. In this paper therefore, we analyse credit rating migrations in Indian corporate bond market to bring about greater understanding of its credit risk.

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Modelling Credit Risk in Indian Bond Markets

Introduction

A portfolio of government bonds is subject only to interest rate risk. However, a portfolio of corporate bonds is subject to credit risk in addition to interest rate risk. Credit risk subsumes the risk of default as well as the risk of an adverse rating change [Lucas, D. J., and J. G. Lonski, 1992]. In other words, though the bond has not actually defaulted, a rating downgrade indicates an increased likelihood of default in future. As such the market price of the bond falls as the future cash flows from the bond are discounted at a higher yield to maturity (YTM).

Credit-related losses in the United States and many other counties have historically been very low [see Altman et al, 1998, 1998, and 1996, Carty and Fons, 1994]. In these countries the default likelihoods are typically estimated using very long periods of historical data which are not available in India. However, rating downgrades occur at a much higher rate and can be reasonably estimated from much shorter periods of historical data. This implies that an analysis of credit rating migrations is the most useful way to study credit risk in Indian bond markets.

Rating migrations probabilities can be easily converted into probability distributions of bond values using the credit spread for various rating categories. For example if AAA bonds trade at a spread of 150 basis points above the risk free rate, and AA bonds at a spread of 250 basis points, then a AAA bond with a duration of 4 years loses approximately 4% of its value when it is downgraded to AA. Multiplying this loss by the probability of this rating migration yields a probability distribution of losses.

Data

The data for this study consists of ratings of the debentures of manufacturing companies by the Credit Rating and Information Services of India Limited (CRISIL). CRISIL is India's largest and oldest credit rating agency. The ratings were collected from CRISIL's *Rating Scan* for 24 quarters from January 1993 to October 1998.

There were a total of 426 companies in this sample, and though some companies were rated for only part of the period, we had about 4300 data points (company-quarters of rating data). Since we can observe a rating change in any quarter only if the company was rated in the previous quarter as well, the sample size comes down to 3819 company-quarters. Within this sample, there are 255 rating changes implying that about 6.7% of the ratings change during a quarter.

Biases and Limitations of the Data

Downgrade Bias

The data period covers only a part of a complete business cycle and is dominated by a recessionary phase in the Indian economy. Moreover, there is evidence from our own previous studies on Indian credit rating that this period was marked by a secular improvement in rating methodologies and tightening of rating standards. Both these factors would imply a bias towards rating downgrades in the period under study. This is borne out by the fact that about 70% of the rating changes in our sample are rating downgrades.

We therefore studied rating downgrades and rating upgrades separately. A risk manager who wishes to use our results could therefore adopt either of two approaches:

- Regard our estimates of the downgrade probabilities as worst-case bounds on the true probabilities. This would cover the credit losses that could arise if the economy goes through a similar necessary phase of deteriorating corporate creditworthiness.

- Use our estimates of downgrade and upgrade probabilities as the starting point for constructing his/her own estimate of the true probability. At an extreme, a risk manager who foresees a sustained economic boom and steadily improving credit quality could regard our upgrade probabilities (during recession) as the prospective downgrade probabilities (during boom). More realistically, the risk manager could use an appropriate weighted average of the upgrade and downgrade probabilities as an estimate of the true downgrade probability averaged over a complete business cycle.

Small Sample Size: The Granularity Problem

Apart from the problems of bias in the data, we must also contend with the problem of limited sample size. Any attempt to estimate the entire matrix of migration probabilities (probability of migration from rating X to rating Y for every pair of ratings X and Y) directly is doomed to failure as there would be too few instances of migrations between many pairs of rating. If we consider each of the 18 rating notches as a distinct rating, there would be over 300 (18 x 17 = 306) rating pairs to consider if we focus on rating changes. It is obvious that with a sample of a mere 255 quarterly rating changes, the number of observations in many of these 300 odd cells would be zero. In fact, we found that three-fourths of the cells were empty, and about 95% of the cells had five observations or less.

This implies that a direct estimate of the matrix of migration probabilities would be quite unreliable. For example, a direct estimate would set three-fourths of the migration probabilities to zero because these cells are empty. However, in many cases, whether a cell happened to contain one observation or none is a matter of chance, and the probability estimate is quite meaningless. Some authors recommend that in such cases, the probabilities be estimated after adding a small number (usually, one) to all the cell counts [see Good, 1965, for a discussion of this and other approaches]. In our case, this approach is not useful because it would over-estimate some probabilities by a very large margin.

The problem that we face here on account of lack of available data points is known as the granularity problem. With a small sample size, changing just one rating migration can make a significant change to the migration probability. We have a sample size of nearly 4,000 rating-quarters, but when this is distributed over 18 rating notches, we have an average of only around 200 observations for each rating notch. If one of these ratings is downgraded or upgraded, the resulting change in the migration probability is about ½%. For some rating notches where the number of observations is smaller, the probability may change by 1%. In other words, probabilities are measured only with a resolution of about ½% or even 1%. This is somewhat like trying to measure the heights of people with a half-foot ruler that has no inches marked on it at all.

The only real solution to the granularity problem is to wait for a larger sample size to be available. In the meantime, however, the best that we can do is to choose a migration model, which has only dozen or so parameters to be estimated, the available sample size might provide an adequate basis for estimation.

Parsimonious Modelling of the Migration Matrix

We therefore seek to model the entire matrix of migration probabilities using a parsimonious model that requires only a small number of parameters to be estimated from the data. Our parsimonious modelling includes the following ideas:

- There were so few non-default ratings below BB (BB-, B+, B, B-, C+, C, C-) that we decided to merge all these with rating category D (default) which has a relatively large number of ratings. We shall denote the merged category by D* to distinguish it from D. We therefore work with 13 rating categories (AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB-, BB+, BB, D*) and number them from 1 (AAA) to 13 (D*). When we talk of ratings i and j, and calculate $k = |i - j|$, we refer to this numerical scale. The rating migration matrix estimated directly from the actual rating migrations is shown in Table I. As already stated, this direct estimate is quite meaningless because of the large number of cells that are empty or have a small number of observations. Even after merging some of the low frequency ratings, almost three-fourths of the off-diagonal cells in Table I are zero. We present this matrix only to provide a point of comparison for the matrix that we estimate in this paper.

- We estimate a common probability specification for a set of similar rating migrations. For example we can regard a transition from A+ to A- as being similar to a transition from BBB to BB+ in that both represent a downgrade by two notches. We have a much larger sample size available to estimate the probability of a two-notch downgrade, and we can use this common probability specification to estimate the probability of all transitions involving a two-notch downgrade (A+ to A-, A to BBB+, BBB to BB+ and so on).

- We can carry this idea even further. We normally expect a two-notch downgrade to be less likely than a one-notch downgrade and more likely than a three-notch downgrade. Normally, the rating agency reviews all ratings on an ongoing basis and changes ratings as soon as any new information becomes available. This leads to gradual rating changes and makes small downgrades more likely than larger ones.

  This is, in fact, one of three monotonicity conditions that are desirable features of a migration probability matrix:

  1. Higher credit ratings should never have a higher default probability.

  2. The probability of transition to a particular rating should be higher for the ratings closer to that particular rating.

  3. The probability of transition from a particular rating should decrease as the transition distance in terms of rating notches increases

  We decided to impose the third monotonicity condition as part of the estimation process itself, both to reduce the number of parameters, and to ensure this desirable feature in the estimated matrix. We did not impose the first and second monotonicity condition as part of the estimation process, as it is much more difficult to do so. This point is discussed further towards the end of this paper.

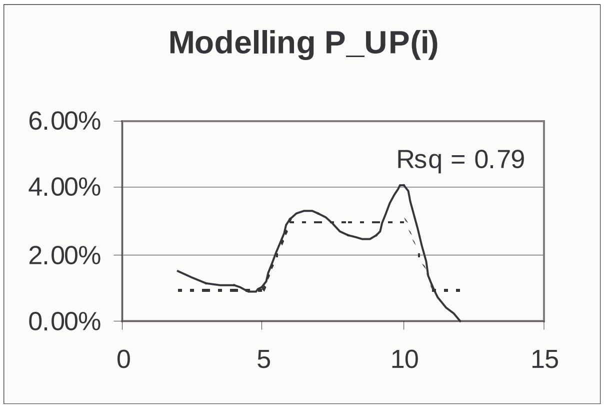
  When we impose the third monotonicity condition, we do not estimate the probabilities of one-notch, two-notch and three-notch downgrades independently, but postulate a monotonically decreasing function *NOTCH_DOWN*(*k*) for the probability that a rating downgrade will be by *k* notches. We endeavour to use a simple functional form (usually linear) to reduce the number of parameters to be estimated. Similarly, we estimate the probability *NOTCH_UP*(*k*) that a rating *upgrade* will be by *k* notches.

- The foregoing analysis tells us how large a rating upgrade or downgrade will be given that a rating change has occurred. It still leaves us with the task of estimating the probability that a rating upgrade or downgrade will occur at all. There is no reason to believe that this probability is the same for all ratings. On the contrary, factors like regression towards the mean would suggest that the probability would not be the same for all ratings. We therefore postulate separate functional forms *P_DOWN*(*i*) for the probability that a bond rated *i* will be downgraded and *P_UP*(*i*) for the probability that it will be upgraded.

Empirical Results

The empirical results can be summarised in terms of the answers to the following questions:

Given the current rating of a bond, what is the probability that it would be upgraded during the next quarter? (What is P_UP(i)?).



The probability of a rating upgrade from AAA and D are zero by definition. The probability of an upgrade for ratings A to BBB- (*i* ranging from 6 to 10) is about 3%, while an upgrade from any other rating is about 1%. In other words, P_UP(i) is 0 if i is 1, it is 3% if i is 6, 7, 8, 9 or 10, and it is 1% for all other i. This step function is depicted in the accompanying figure and the R-Square of the fit is 0.79. This inverted U shaped curve is perhaps a little counter-intuitive, but one can think of a plausible reason. Upgrades from very high ratings are unlikely because these ratings are so high that they cannot rise much higher. On the other hand, upgrades of very low rated bonds are also unlikely because these companies are financially quite weak and are less likely to see a significant improvement in credit quality. Therefore, it is the middle ratings that have the highest potential for being upgraded. It is of course possible that the inverted U shaped curve is also a reflection in part of inadequacies in the rating process.

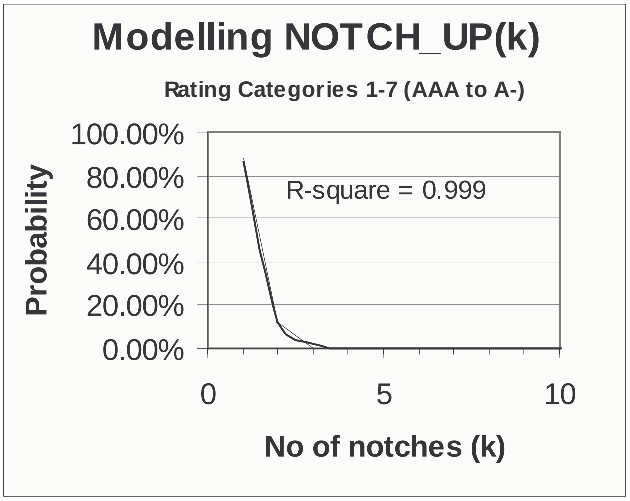
Given that a bond has been upgraded during a particular quarter what is the probability that the upgrade will be by k notches? (What is NOTCH_UP(k)?).

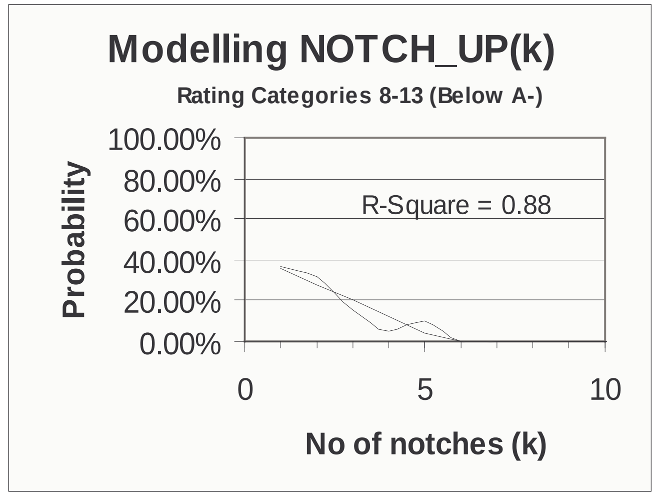
Examination of the data showed that ratings below A- behave very differently from ratings of A- and above. Bonds rated A- or above are almost never upgraded by more than two notches, while lower rated bonds are often subject to larger upgrades. We were thus forced to modify our initial specification and let *NOTCH_UP*(*k*) depend on the current rating *i* to a limited extent.

Let *NOTCH_UP*(*i*, *k*) denote the probability that a rating upgrade of a bond rated *i* will be by *k* notches. Letting *NOTCH_UP*(*i*, *k*) depend in any arbitrary way on *i* and *k* would at one stroke remove all the benefits of parsimonious modelling with which we started. We therefore fitted two separate linear functions to the two categories of ratings as shown in the accompanying figures. Within each category, *NOTCH_UP*(*i*, *k*) depends only on *k* and not on *i*; the only role that *i* plays is in deciding which of the two fitted lines to use:

Let *NOTCH_UP*(*i*, *k*) denote the probability that a rating upgrade of a bond rated *i* will be by *k* notches. Then we have:

$$NOTCH\_UP(i, k) = \begin{cases} \mathrm{MAX}(0, 1.64 - 0.76\, i), & i \leq 7 \text{ (A- and above)} \\ \mathrm{MAX}(0, 0.44 - 0.08\, i), & i > 7 \text{ (below A-)} \end{cases}$$

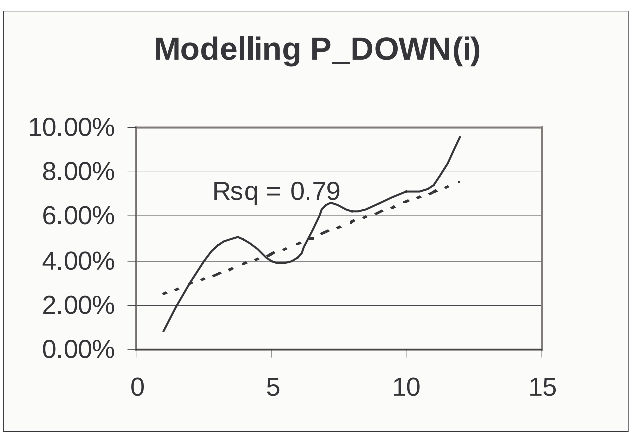




It may be noted that 1.64-0.76i and 0.44-0.08I represent ordinates of the fitted straight lines for $i \leq 7$ and $i > 7$. For ratings of A- and above (i from 1 to 7), our fitted straight line allows upgrades of only one or two notches: we have $NOTCH\_UP(i, 1) = 0.88$ and $NOTCH\_UP(i, 2) = 0.12$ and $NOTCH\_UP(i, k) = 0$ for $k > 2$. Since upgrades of more than two notches are extremely rare for these ratings, this specification provides a near-perfect fit (R-square = 0.999) to the actual rating upgrades for these ratings.

For ratings below A-, that is i >7 the, fit is less perfect, but still quite good (R-square = 0.88) and allows upgrades up to five notches (k = 5): we have NOTCH_UP(i,k) = 36% (k=1), 28% (k=2), 20% (k=3), 12% (k=4), 4% (k=5), and zero otherwise.

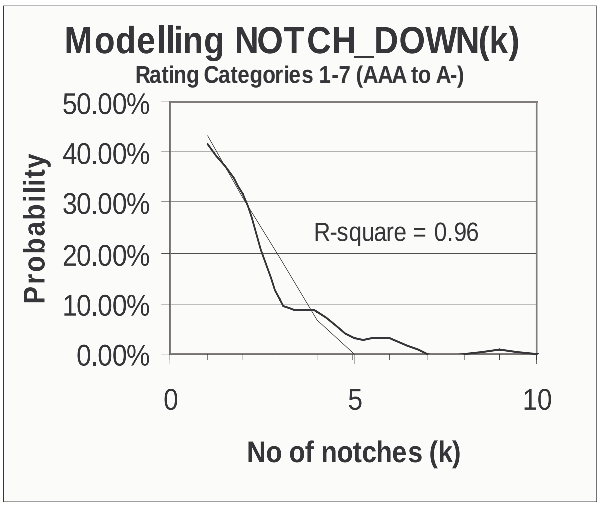
Given the current rating of a bond what is the probability that it would be downgraded during the next quarter? (What is P_DOWN(i)?).

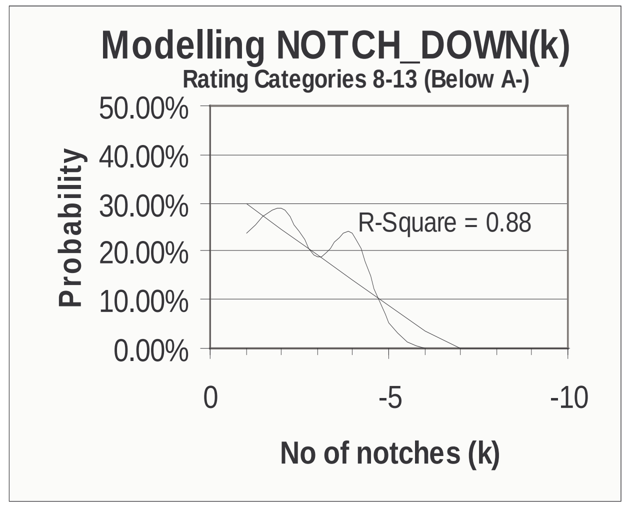


The probability of a rating downgrade from AAA ($i = 1$) is about 2½ %, and it increases by about ½ % for each rating notch, reaching over 7½ % for BB. To be precise, P_DOWN(i) is 2.5%, 2.9%, 3.4%, 3.9%, 4.3%, 4.8%, 5.3%, 5.7%, 6.2%, 6.7%, 7.1%, and 7.6% for i = 1, 2 ... 12 and zero for i = 13. This is a simple linear relationship as depicted in the accompanying figure. The R-square of the fitted straight line is 0.79, indicating an excellent fit. It is evident that the rating agency had a very high propensity to downgrade low rated bonds. This is partly due to the sample bias that we have already mentioned, but it might also reflect inadequacies in the rating process.

Given that a bond has been downgraded in a particular quarter, what is the probability that 'the downgrade will be by k notches? (What is NOTCH_DOWN(k)?)

Once again, bonds rated below A- behave differently from bonds rated A- or above as is evident from the figures below for these two categories. For bonds rated A- or better, the rating downgrades tend to be small and there is a large probability that the downgrade is by only one or two notches. We therefore fitted two different functional forms for bonds rated A- or above and those rated below A- as shown below. For bonds rated below A-, the fitted line is much flatter indicating a significant probability of large downgrades.





The linear fit for both categories is fairly good (R-square of 0.96 and 0.88 respectively). But on close examination of the data, we find that the fit for ratings below A- is misleading and arises solely due to aggregation. Looking at the data in a disaggregated manner shows that the line for this category ought to be flatter than it is. The reason why the fitted line[1] is not so flat is simple: for bonds with low ratings, large downgrades are not possible since the rating is already near the bottom.

At the disaggregated level, ratings below A- behave quite badly in terms of possible downgrades. We argued at the outset that *NOTCH_DOWN*(*i*,*k*) should be a monotonically declining function of *k*. In other words, small downgrades should be more likely than large downgrades. Unfortunately, ratings below A- violate the monotonicity condition quite badly. For many speculative grade ratings, two-notch and even three-notch downgrades are more likely than one-notch downgrades. This is almost certainly a reflection of deficiencies in the rating process during the period under study.

Therefore, we decided to accommodate the observed tendency for large downgrades of rating below A- as much as possible without actually violating monotonicity. We do this by

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[1] The NOTCH_DOWN(i,k) for ratings below A- (i>7) for this discarded line was 30% (k=1), 25%(k=2), 19%(k=3), 14%(k=4), 9%(k=5), 3%(k=6), and zero otherwise.

postulating that for a bond rated below A, all possible downgrades are equally likely. In other words, *NOTCH_DOWN*(i, k) = 1/(13-i) and does not depend on k at all. This leads to the following specification.

NOTCH_DOWN(i, k) =	MAX(0, 0.48 – 0.22 k),	$i \le 7$ (A- and above)
	1/(13-i),	$i > 7$ (below A-)

It is clear that while notationally, *NOTCH_DOWN*(i, k) appears to depend on i and k, in the actual specification, we have a function that depends only on one variable (on k for $i \le 7$ and on i for $i > 7$). Thus the modelling remains parsimonious.

For i ≤ 7, we have NOTCH_DOWN(i,k) = 43% (k=1), 31% (k=2), 19% (k=3), 7% (k=4), and zero otherwise. For i > 7, we have NOTCH_DOWN(i,k) = (1/13-i) = 20% (i=8), 25%(i=9), 33%(i=10), 50%(i=11), and 100%(i=12) regardless of k.

Putting it all together – given the current rating of the bond, what is the rating of the bond likely to be next quarter?

If the bond's current rating is i, then it will be upgraded with probability *P_UP*(i), and downgraded with probability *P_DOWN*(i) during the next quarter. It will therefore retain its current rating with probability 1 – *P_UP*(i) – *P_DOWN*(i). It will be upgraded to rating i-k with probability *P_UP*(i)**NOTCH_UP*(i, k) and downgraded to rating i+k with probability *P_DOWN*(i)**NOTCH_DOWN*(i, k). The implied matrix *P_ QUARTER*(i, j) of quarterly rating migration probabilities is shown in Table II.

How good is the fit between our parsimonious model for P_QUARTER and the actual rating migrations?

To measure the goodness of fit, we regressed the actual number of migrations from rating i to rating j on the predicted number of migrations between the same ratings. The R-Square of this relationship was 0.80 indicating a fairly good fit.

Extensions and Refinements

Correcting for sample period bias

The probabilities computed in the matrix *P_ QUARTER* are based on the historical experience of the period covered by this study. As already stated at the outset, the period of study is biased towards downgrades and therefore leads to highly conservative estimates of credit loss. A risk manager, who wishes to correct for this bias, could use a weighted average of *P_UP* and *P_DOWN* to obtain a bias corrected *P_DOWN′* and *P_UP′*. Similarly the probabilities *NOTCH_UP* and *NOTCH_DOWN* could be suitably weighted to yield a bias corrected *NOTCH_DOWN′* and *NOTCH_UP′*. All these could be combined to yield a bias corrected *P_QUARTER′* matrix. If we use equal (*i*.e., 50:50) weights, the estimates *P_DOWN′* and

P_UP′ would be equal, as would *NOTCH_DOWN′* and *NOTCH_UP′*. In this case, the *P_QUARTER′* matrix would be symmetric.

Obtaining annual transition matrices

The risk manager who wishes to derive risk estimates over a one-year horizon can easily do so by computing the matrix *P_ANNUAL* as the fourth power of the one-quarter transition matrix *P_ QUARTER*. The estimated *P_ANNUAL* is shown in Table III.

Deriving the Probability Distribution of Loss

We have already pointed out that conversion of rating migrations probabilities into probability distributions of bond values require another key piece of data – the credit spread for various rating categories. There is a paucity of reliable published data on credit spreads in Indians bond markets. We therefore refrain from computing value at risk estimates for corporate bond portfolios in this paper. However, all market participants make in-house estimates of credit spreads for valuation purposes. Therefore these participants can readily convert rating migration probabilities into a probability distribution of bond values and compute the desired value at risk statistics.

Reasonableness of estimated matrix

The migration matrix that we have estimated can be evaluated against *a priori* criteria like the monotonicity conditions that we have stated above. It can also be compared with migration matrices estimated for other countries like the United States using a larger and more comprehensive sample. That is what we attempt to do now.

Evaluation against a priori criteria

The first monotonicity condition states that "higher credit ratings should never have a higher default probability". Our estimation process used the rating category D* which is slightly different from the default category D. As such, it is not possible to verify this condition in the strict sense. However, the estimated matrix does satisfy this condition if we interpret it in terms of "near-default", D*, rather than D. The quarterly probability of "near-default" rises monotonically from about 1% for BBB+ rated bonds to about 8% for BB rated bonds (Table II). On the other hand, the raw matrix (Table I) violated this quite badly: the quarterly probability of "near-default" of a BBB bond was nearly 3% while the corresponding probability for a BBB- bond was only about 2%.

The second monotonicity condition states that "the probability of transition to a particular rating should be higher for the ratings closer to that particular rating". This requires that in each column of the estimated matrix, the numbers should decline monotonically on either side of the diagonal element (this can be regarded as a generalisation of the first condition which postulates this property only for the D (or D*) column.). The estimated matrix conforms to this reasonably well with one major exception at the boundary between A- and BBB+. We found in our estimation process that bonds rated A- and above behave very differently from those rated below A-, and we estimated two different models for these two categories. In most cases, we see that the monotonicity is violated at the boundary between the two categories. For example, if we look at the A+ column, the probabilities decline as we go down from the diagonal element (94.68%) to 2.64% (A) and then to 0.36%(A-). As we cross the boundary, the probability jumps up to 0.60% (BBB+) and then declines monotonically once again to 0.36%, 0.13% and 0. Similar behaviour may be seen in many other columns. This monotonicity violation could perhaps have been eliminated at the cost of greater complexity by smoothly splicing together the different models that we used for the two categories (A- and above versus below A-). However, since the modelling was based on an observed dichotomy in the data, we chose not to introduce this additional level of complexity in the estimation process.

The third monotonicity condition requires that "the probability of transition from a particular rating should decrease as the transition distance in terms of rating notches increases". This was imposed as part of the estimation process, and is therefore automatically satisfied by the estimated matrix (Tables 2 and 3). On the other hand, the raw matrix (Table I) estimated directly from the data violated this requirement very badly: for example, the probability of transition from BBB- to the next rating of BBB is only 1%, while the probability of transition to the more distant rating of BBB+ is 2%.

On the whole, therefore, the estimated matrices (Tables 2 and 3) are in reasonable conformity with the a priori requirements of monotonicity.

Comparison with S&P Matrix

The US rating agency S&P publishes data about migrations of their ratings [Standard & Poor's, 1996). The raw migration matrix provided by S&P is shown in Table IV. As part of their modelling of credit risk JP Morgan [J.P. Morgan & Company, 1997] has published an imputed migration matrix (Table V) that imposes the third monotonicity condition. Even the imputed matrix violates the second monotonicity condition just as in the case of our estimated matrix. These matrices show the migration behaviour at the one-year horizon, and should, therefore, be compared with our estimated matrix of annual migrations (Table III). Moreover, the published S&P tables are also without rating notches; appropriate adjustments must therefore be made while comparing these with our tables.

It is apparent that, on the whole, our estimated matrices are similar to the S&P tables in terms of rough order of magnitudes. The major difference is in terms of the bias towards downgrades mentioned at the outset. In our matrix, downgrades outnumber upgrades by about 3:1. S&P's raw matrix (Table IV) reveals a much smaller bias (about 1.75:1) towards downgrades, while the imputed matrix for S&P shows much less bias at about 1.2:1. It may also be seen that in case of S&P, high ratings (A and above) are more likely to be downgraded than upgraded while the reverse is true for low ratings (below A). This phenomenon is known as regression towards the mean – ratings tend to be pulled towards the middle as high ratings are pulled down and low ratings are pulled up. The Indian ratings behave very differently – downgrades outnumber upgrades throughout the rating spectrum. As already stated, this reflects the bias in the sample period, and is not due to the estimation method that has been employed.

Conclusion

We believe that despite the acute limitations of the data, we have been able to produce usable estimates of the rating migration probabilities for modelling credit risk in Indian bond markets.

However, these estimates reflect the biases of the sample period, which was characterised by declining corporate creditworthiness and rising rating standards. Users may need to correct for this and modify the estimates in line with their beliefs in this regard. The migration probabilities may be made more symmetric by averaging the downgrade and upgrade probabilities with suitable weights.

At several points in this paper, we have pointed out anomalies in the rating migration data that are suggestive of inadequacies in the rating process itself. It is possible to argue that over a period of time, these deficiencies would be removed. If a risk manager believes that rating migration behaviour in India would move closer to what is observed in the United States, the estimated matrix may be modified to a suitable extent in that direction by making use of the published migration matrices of the US rating agencies.

Whichever route is adopted to produce a customised migration matrix, risk managers also need to estimate the rating spreads in the Indian market to convert the migration matrix into a loan loss distribution. Though published data on this is limited, in-house estimates of these spreads that are employed for valuation purposes can be used for this purpose.

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Table I: Raw Empirical Quarterly Transition Matrix
(Observed Probability (%) from row ratings to column ratings over the period 1993-98)

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	D*
AAA	99.15	0.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA+	1.48	95.56	2.59	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	1.17	94.15	2.92	0.88	0.58	0.29	0.00	0.00	0.00	0.00	0.00	0.00
AA-	0.00	0.28	0.83	93.91	2.22	2.22	0.00	0.00	0.28	0.00	0.00	0.00	0.28
A+	0.00	0.15	0.00	0.88	95.01	1.76	1.17	0.59	0.44	0.00	0.00	0.00	0.00
A	0.00	0.00	0.00	0.44	2.64	92.82	1.32	1.32	1.03	0.15	0.15	0.15	0.00
A-	0.00	0.00	0.00	0.00	0.59	2.66	90.24	1.18	2.37	0.00	1.48	0.59	0.89
BBB+	0.00	0.00	0.00	0.00	0.37	1.10	1.10	91.21	0.73	1.10	1.10	2.20	1.10
BBB	0.00	0.00	0.00	0.36	0.00	0.73	0.36	1.09	90.88	0.73	0.73	2.19	2.92
BBB-	0.00	0.00	0.00	0.00	0.00	1.02	0.00	2.04	1.02	88.78	0.00	5.10	2.04
BB+	0.00	0.00	0.00	0.00	0.00	1.05	0.00	0.00	0.00	0.00	91.58	0.00	7.37
BB	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90.48	9.52
D*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table II: Estimated Transition Matrix *P_QUARTER*
(Probability (%) of migrating from row rating to column rating in one quarter)

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	D*
AAA	97.52	1.07	0.77	0.47	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA+	0.86	96.07	1.27	0.91	0.56	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.12	0.86	95.61	1.47	1.06	0.65	0.23	0.00	0.00	0.00	0.00	0.00	0.00
AA-	0.00	0.12	0.86	95.14	1.67	1.20	0.73	0.27	0.00	0.00	0.00	0.00	0.00
A+	0.00	0.00	0.12	0.86	94.68	1.87	1.35	0.82	0.30	0.00	0.00	0.00	0.00
A	0.00	0.00	0.00	0.36	2.64	92.19	2.07	1.49	0.91	0.33	0.00	0.00	0.00
A-	0.00	0.00	0.00	0.00	0.36	2.64	91.72	2.27	1.64	1.00	0.36	0.00	0.00
BBB+	0.00	0.00	0.13	0.36	0.60	0.84	1.07	91.26	1.15	1.15	1.15	1.15	1.15
BBB	0.00	0.00	0.00	0.13	0.36	0.60	0.84	1.07	90.79	1.55	1.55	1.55	1.55
BBB-	0.00	0.00	0.00	0.00	0.13	0.36	0.60	0.84	1.07	90.32	2.22	2.22	2.22
BB+	0.00	0.00	0.00	0.00	0.00	0.04	0.12	0.20	0.27	0.35	91.88	3.57	3.57
BB	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.12	0.20	0.27	0.35	91.41	7.60
D*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.12	0.20	0.27	0.35	99.02

Table III: Estimated Transition Matrix *P_ANNUAL*
(Probability (%) of migrating from row rating to column rating in one year)

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	D*
AAA	90.51	3.92	2.87	1.81	0.73	0.09	0.04	0.02	0.00	0.00	0.00	0.00	0.00
AA+	3.14	85.32	4.56	3.35	2.14	0.84	0.12	0.06	0.02	0.01	0.00	0.00	0.00
AA	0.47	3.06	83.70	5.22	3.91	2.39	0.98	0.15	0.07	0.03	0.01	0.00	0.00
AA-	0.01	0.45	3.02	82.13	5.97	4.28	2.68	1.13	0.17	0.08	0.04	0.02	0.02
A+	0.00	0.01	0.45	3.02	80.76	6.41	4.67	2.99	1.21	0.18	0.11	0.08	0.09
A	0.00	0.00	0.04	1.34	8.78	72.90	6.80	5.04	3.12	1.28	0.25	0.21	0.23
A-	0.00	0.00	0.02	0.12	1.65	8.44	71.32	7.33	5.31	3.34	1.51	0.46	0.51
BBB+	0.00	0.01	0.43	1.24	2.14	2.88	3.54	69.71	3.71	3.65	3.81	3.96	4.91
BBB	0.00	0.00	0.02	0.46	1.32	2.08	2.77	3.50	68.25	4.80	5.04	5.25	6.52
BBB-	0.00	0.00	0.01	0.04	0.51	1.28	1.97	2.71	3.38	66.83	6.95	7.26	9.06
BB+	0.00	0.00	0.00	0.01	0.02	0.17	0.42	0.68	0.94	1.19	71.46	11.14	13.97
BB	0.00	0.00	0.00	0.00	0.01	0.02	0.15	0.41	0.67	0.93	1.25	70.10	26.44
D*	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.16	0.43	0.69	1.00	1.30	96.38

Table IV: S&P one-year transition matrix
(Observed Probability (%) of migrating from row ratings to column ratings)

Initial	Rating at Year-end							
Rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0.00	0.00	0.00
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0.00
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.3	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0.00	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0.00	0.22	1.3	2.38	11.24	64.86	19.79

Source: Standard & Poor's Credit Week (15 April 1996)

Table V: S&P One-year transition matrix
(Imputed Probability (%) of migrating from row ratings to column ratings)

Initial	Rating at Year-end							
Rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	43.78	53.42	1.65	0.71	0.29	0.11	0.02	0.01
AA	0.60	90.60	6.20	1.45	.93	.16	.04	.01
A	0.22	2.84	92.97	3.12	0.56	0.14	0.04	0.07
BBB	2.67	3.29	12.77	75.30	5.07	0.60	0.14	0.17
BB	0.19	3.58	8.28	9.97	55.20	17.17	4.53	1.08
B	0.12	0.50	20.69	1.05	0.25	55.40	17.05	4.95
CCC	0.04	0.11	6.28	0.30	0.12	41.53	32.46	19.15

Source: J.P. Morgan & Company, 1997 -- Technical Document