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Name.....

Reg. No.....

**SIXTH SEMESTER UG (CBCSS-UG) DEGREE
EXAMINATION, MARCH 2024**

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

(2019 Admission onwards)

Time : Two Hours

Maximum Marks : 60

Section A (Short Answer Type Questions)

Answer any number of questions.

Each question carries 2 marks. Maximum marks 20.

1. Find the number of edges of $k_{2,3}$.
2. Draw the graph $K_5 - \{e\}$.
3. Define degree of a vertex. Explain with example.
4. Let G be a simple graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ? Justify.
5. Give an example of a self-complementary graph with five vertices.
6. Let G be a simple graph with n vertices and \bar{G} be its complement. Prove that, for each vertex V in G , $d_G(v) + d_{\bar{G}}(v) = n - 1$.
7. A connected graph G has 21 vertices, what is the minimum possible number of edges in G .
8. Define diameter of a graph G . Which simple graphs have diameter 1 ?
9. When can you say that the wheel graph $W_n, n \geq 4$ is Euler ? Justify.
10. Define Jordan curve. Give an example.
11. Define Spanning tree. State Cayleys theorem in spanning trees.
12. Let G be a Hamiltonian graph. Show that G does not have a cut vertex.

Section B (Paragraph/Problem Type Questions)

Answer any number of questions.

Each question carries 5 marks. Maximum marks 30.

13. Prove that k_5 , the complete graph on five vertices, is non-planar.
14. Let G be a planar graph with less than 12 vertices. Prove that G has a vertex V with $d(v) \leq 4$.

Turn over

15. Explain Konigsberg bridge problem.
16. Let G be a graph in which the degree of every vertex is at least two then prove that G contains a cycle.
17. Prove that a vertex V of a tree T is a cut vertex if and only if $d(v) > 1$.
18. Let G be a connected graph, then G is a tree if and only if every edge of G is a bridge.
19. Given any two vertices u and v of a graph G , prove that every u - v walk contains u - v path.

Section C (Essay Type Questions)

*Answer any **one** questions.
The question carries 10 marks.*

20. Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycle.
21. Prove that if T is a tree with n vertices then it has precisely $n-1$ edges.

(1 × 10 = 10 marks)

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Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

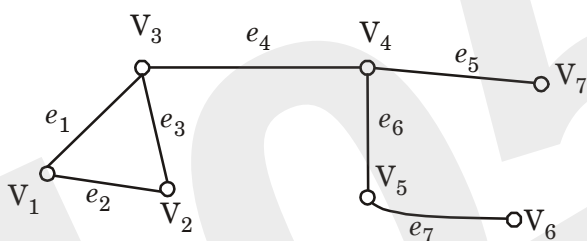
(2019 Admission onwards)

Time : Two Hours

Maximum Marks : 60

Section A (Short Answer Questions)*Each question carries 2 marks.**A maximum of 20 marks can be earned from this section.*

- Find the number of edges of K_7 .
- Draw the graph $K_{3,3} - \{V\}$ where V is a Vertex in $K_{3,3}$.
- Define K -regular graph. Give an example.
- Define union of two graphs G_1 and G_2 .
- Let G be a simple graph with n vertices and \bar{G} be its complement. Prove that, for each vertex V in G , $d_G(V) + d_{\bar{G}}(V) = n - 1$.
- Define the adjacency matrix of a graph G with n vertices.
- A connected graph G has 17 edges, what is the maximum possible number of vertices in G ?
- Let

Find all bridges in G .

Turn over

9. When can you say that the complete graph $K_n, n \geq 3$ is Euler ? Justify.
10. Define critical planar graphs. Which complete graph K_n are critical planar ?
11. State Cayley's theorem on spanning trees.
12. When can you say that a graph G is maximal non-Hamiltonian.

(Ceiling marks = 20 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 marks.

A maximum of 30 marks can be earned from this section.

13. Prove that the complete bipartite graph $K_{3,3}$ is non-planar.
14. Let G be a simple planar graph with less than 12 vertices. Prove that G has a vertex V with $d(v) \leq 4$.
15. Prove that, in any graph G there is an even number of odd vertices.
16. Prove that for any connected graph G , $\text{rad } G \leq \text{diam } G \leq 2 \cdot \text{rad } G$.
17. Let u and v be distinct vertices of a tree T . Then prove that there is precisely one path from u to v .
18. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.
19. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.

(Ceiling marks = 30 marks)

Section C (Essay Type Questions)

*Answer any **one** question.*

The question carries 10 marks.

20. Explain the Konigsberg bridge problem. Give the graph theory model for this problem. Also state the respective theorem to solve this problem.
21. Let G be a graph with n vertices. Then prove that the following statements are equivalent :
 - (i) G is a tree.
 - (ii) G is acyclic graph with $n - 1$ edges.
 - (iii) G is a connected graph with $n - 1$ edges.

(1 × 10 = 10 marks)

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Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

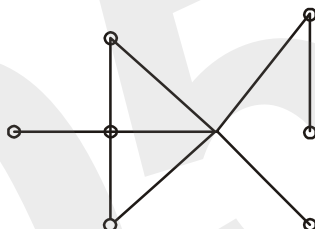
(2019 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A (Short Answer Type Questions)*Answer at least **eight** questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 24.*

1. Find number of edges of $k_{m,n}$.
2. Draw the graph $K_6 - \{v\}$ where v is any vertex in K_6 .
3. Draw a 4-regular graph with ten vertices.
4. Define intersection of two graphs.
5. Let G be a simple graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ? Justify.
6. Draw the graph $K_{2,3,3}$.
7. Define eccentricity and radius.
8. Draw Peterson graph and find a trail of length 5.
9. When can you say that the complete graph $k_n, n \geq 3$ is Euler? Justify.
10. Prove that any subgraph of a planar graph is planar.
11. Find $K(G)$ for the graph.



12. How many different Hamiltonian cycles does K_n have?

(8 × 3 = 24 marks)

Turn over

Section B (Paragraph/Problem Type Questions)

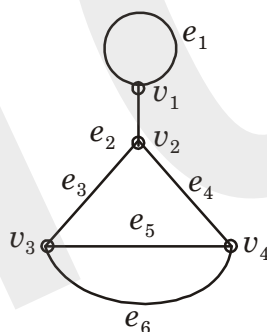
Answer at least **five** questions.

Each question carries 5 marks.

All questions can be attended.

Overall Ceiling 25.

13. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.
14. Prove that the complete bipartite graph $K_{3,3}$ is non-planar.
15. Let G be a graph with n vertices, where $n \geq 2$. Then prove that G has atleast two vertices which are not cut vertices.
16. Explain the Konigsberg bridge problem.
17. Prove that G is connected if and only if it has a spanning tree.
18. Let G be a graph with n vertices. Then prove that G is a tree if and only if G is a connected graph with $n - 1$ edges.
19. Define (i) adjacency matrix of a graph G ; (ii) incidence matrix of a graph G . Find the adjacency and incidence matrix of the following graph G .



(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** question.

The question carries 11 marks.

20. Prove the following :
 - (i) A connected graph G has an Euler trail if and only if it has at most two odd vertices.
 - (ii) A simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.
21. Prove that, a non-empty graph with atleast two vertices is bipartite if and only if it has no odd cycle.

(1 × 11 = 11 marks)