D 51761 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2023

Mathematics

MTS 3C 03—MATHEMATICS—3

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Part A

All questions can be attended.
Each question carries 2 marks.
Overall Ceiling is 20.

1. If $r(t) = \cos 2t\, i + \sin t\, j$. Find $r'(0)$.

2. Find the curvature of a circle of radius a.

3. Describe the level surfaces of the function $F(x, y, z) = \frac{(xr + y^2)}{z}$.

4. If $F = (x^2y^3 - z^4)\, i + 4x^5y^2z\, j + y^4\, z^6\, k$, find div $(curl\ F)$.

5. Evaluate $\int xy^2\, dy$ on the quarter-circle C defined by $x = 4\cos t,\ y = 4\sin t,\ 0 \le t \le \frac{\pi}{2}$.

6. Find $\int_C ydx + xdy$ on the curves $y = \sqrt{x}$ between $(0, 0)$ and between $(1, 1)$.

7. Convert $(6, \pi/4, \pi/3)$ in spherical coordinates to rectangular co-ordinates.

8. Find the values of $\ln(-1, -i)$.

9. Prove that $\sinh z = \sinh x \cos y + i \cosh x \sin y$.

Turn over

10. Evaluate $\int (z+3)\,dz$, where C is $x = 2t,\ y = 4t - 1,\ 1 \le t \le 3$.

11. Evaluate $\oint_C z^3 - 1 + 3i\,dz$, where C is the circle $|z| = 1$.

12. State Cauchys Integral Formula.

Part B

All questions can be attended.
Each question carries 5 *marks.*
Overall Ceiling is 30.

13. Find an equation of the tangent plane to the graph of $\frac{1}{2}x^2 + \frac{1}{2}y^2 - z = = 4$ at $(1, -1, 5)$.

14. Find the maximum value of the directional derivative of $F(x, y, z) = xy^2 - 4x^2 y + z^2$ at $(1, -1, 2)$ in the direction of $6i + 2j + 3k$.

15. Find the moment of inertia about the y-axis of the thin homogeneous disk $x^2 + y^2 = r^2$ of mass m. Given $\rho(x, y) = \frac{m}{\pi r^2}$.

16. Find the volume of the solid that is under the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and above the region bounded by the graph of the circle $x^2 + y^2 - y = 0$. $V = \iint_R \sqrt{1 - x^2 - y^2}\,dA$.

17. (a) Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane.

    (b) Find the harmonic conjugate function of u.

18. Solve the equation $\cos z = 10$.

19. Evaluate $\oint_C \frac{dz}{z^2 + 1}$ where C is the circle $|z| = 3$.

Part C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Verify Stokes theorem. Assume that the surface S is oriented upward. Given $F = z\,i + x\,j + y\,k$; S that portion of the plane $2x + y + 2z = 6$ in the first octant.

21. Let D be the region bounded by the hemisphere $x^2 + y^2 + (z-1)^2 = 9, 1 \le z \le 4,$ and the plane $z = 1.$ Verify the divergence theorem if $F = xi + yj + (z-1)\,k.$

(1 × 10 = 10 marks)

D 31819 **(Pages : 3)** **Name.......................................**

Reg. No..................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019 Admission Onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Maximum 20 *marks.*

1. Find the derivative of the vector function $\vec{r}(t) = \sin t\,\hat{i} - e^{-t}\hat{j} + \left(3t^3 - 4\right)\hat{k}$.

2. If $z = 4x^3y^2 - 6x^2 + y^2 + 5$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

3. If $f(x,y) = e^{xy}$, find $\nabla f(x,y)$.

4. Find the level curve of $f(x,y) = y^2 - x^2$ passing through the point (–1, 2).

5. Find div $\vec{F}$ for $\vec{F} = \left(x^2y^3 - z^4\right)\hat{i} + 4x^5y^2z\hat{j} - y^4z^6\hat{k}$.

6. Evaluate $\int_1^3 \int_{-1}^1 (2x-4)\,dx$.

7. State Stoke's theorem.

8. Find the Jacobian of $x = r\cos\theta,\ y = r\sin\theta$.

9. Express $1 + i$ in polar form.

10. Evaluate $\lim_{z \to i} \frac{z^4 - 1}{z - i}$.

Turn over

11. Evaluate $\oint_C \frac{e^z}{z-3}\, dz$ where C is $|z|=1$.

12. Evaluate $\oint_C \bar{z}\, dz$ where C is $x = t, y = t^2, 0 \le t \le 1$.

Section B

Answer any number of questions.
Maximum 30 *marks.*

13. Use chain rule to find $\frac{\partial z}{\partial u}$ at $(\pi, 1)$ for $z = x^2 - y^2 \tan x$, where $x = \frac{u}{v}, y = uv$.

14. Find an equation of the tangent plane to the graph of $z = \frac{x^2}{2} + \frac{y^2}{2} + 4$ at (1, –1, 5).

15. Show that $\int_C \left(y^2 - 6xy + 6\right) dx + \left(2xy - 3x^2\right) dy$ is independent of any path C between (–1, 0) and (3, 4). Hence evaluate $\int_{(-1,0)}^{(3,4)} \left(y^2 - 6xy + 6\right) dx + \left(2xy - 3x^2\right) dy$.

16. Change the order of integration and hence evaluate $\int_0^4 \int_y^4 \frac{x}{x^2 + y^2}\, dx\, dy$.

17. Show that $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic. Find the harmonic conjugate of u.

18. Evaluate $\int_C z^2 dz$ where C is the line $x = 2y$ from $z = 0$ to $z = 2 + i$.

19. Evaluate $\oint_C \frac{z+1}{z^4 + 4z^3}\, dz$ where C is $|z| = 1$.

Section C

Answer any **one** *question.*

Maximum 10 *marks.*

20. Use Green's theorem to evaluate $\oint_C \left(x^5 + 3y\right) dx + \left(2x - e^{y^3}\right) dy$ where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

21. Find the volume bounded by the cylinder $x^2 + y^2 = 4$, the plane $y + z = 3$ and $z = 0$.

(1 × 10 = 10 marks)

D 12034 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Evaluate $\int_0^1 \left(t\hat{i} + 3t^2\hat{j} + 4t^3\hat{k}\right)dt$.

2. The position of a moving particle is $\bar{r}(t) = t^2\hat{i} + t\hat{j} + t^3\hat{k}$. Find velocity and acceleration of the particle at $t = 2$.

3. If $z = e^{-y}\cos x$ find $\frac{\partial^2 z}{\partial x\,\partial y}$.

4. Find the level surface of $F(x, y, z) = x^2 + y^2 + z^2$ passing through (1, 1, 1).

5. Evaluate $\oint_C x\,dx$, where C is the circle $x = \cos t, y = \sin t, 0 \le t \le 2\pi$.

6. Show that curl $\vec{r} = \vec{0}$.

7. State Green's theorem in the plane.

8. Evaluate $\int_0^3\int_0^2\int_0^1 xyz\,dx\,dy\,dz$.

9. Write the equation of the circle with centre (1, 2) and radius 4 in the complex plane.

Turn over

10. Find the value of i^{2i}.

11. Evaluate $\oint_C \frac{Ze^z}{(z-3)}\,dz,$ where C is $|z|=2$.

12. Evaluate $\oint_C \frac{dz}{z}$, where C is $|z|=1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Use chain rule to find $\frac{dw}{dx}$ at (0,1, 2) for $w = xy + yz$; $x = \cos x$, $y \sin x$, $z = e^x$.

14. Find the directional derivative of $f(x,y) = \sqrt{x^2y + 2y^2z}$ at $(-2, 2, 1)$ in the direction of the negative z-axis.

15. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$ using double integrals.

16. Use polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2}\,dy\,dx$.

17. Show that $f(z) = \left(2x^2 + y\right) + i\left(y^2 - x\right)$ is not analytic at any point.

18. Evaluate $\oint_C \frac{5z+7}{z^2+2z-3}\,dz,$ where C is the circle $|z-2|=2$.

19. Evaluate $\int \text{Re}\, z\, dz$ along a line segment from $z=0$ to $z=1+2i$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Let $\vec{F}(x,y,z) = z\hat{j} + z\hat{k}$ represents the flow of a liquid. Find the flux of $\vec{F}$ through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward.

21. Use triple integrals to find the volume of the solid with in the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

(1 × 11 = 11 marks)

D 92960 (Pages : 3) Name.....................................

Reg. No.....................................

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3C 03—MATHEMATICS – 3

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. If $\bar{r}(t)=2\cos t\,\hat{i}+6\sin t\,\hat{j}$, find $\frac{d\bar{r}}{dt}$ at $t=\frac{\pi}{2}$.

2. Find the curvature of a circle whose radius is 2.

3. If $z=e^{x}\sin(xy)$, find $\frac{\partial^2 z}{\partial y^2}$.

4. Find the gradient of $f(x,y,z)=xy^2+3x^2-z^3$ at (1, 1, 1).

5. Show that $\operatorname{div}\bar{r}=3$.

6. Evaluate $\int_2^4\int_1^3(40-2xy)\,dx\,dy$.

7. Use double integrals to find the area of the plane region enclose by the curves $y \sin x$ and $y=\cos x$ for $0\le x\le \pi/4$.

8. Find the Jacobian of $u=\frac{y}{x^2}, v=xy$.

9. Sketch the graph of the region $|z-2i|=2$.

10. Write the real and imaginary part of $f(z)=\sin z$.

Turn over

11. Evaluate $\oint_C \frac{z^2}{z-1}\, dz$, where C is $|z| = 2$.

12. Evaluate $\int_C z\, dz$ where C is given by $x = t^2, y = t$ from $0 \le t \le 1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at $(1,-1,2)$ in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$.

14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at (2, 1, 4).

15. Use Green's theorem to evaluate $\oint_C \left(x^2 - y^2\right) dx + (2y - x)\, dy$, where C consists of the boundary of the region in the first quadrant that is bounded by $y = x^2$ and $y = x^3$.

16. Change the order of integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$.

17. Use divergence theorem to evaluate $\iint_S \left(\vec{F} \cdot \hat{n}\right) dS$ where $\vec{F} = xy\,\hat{i} + y^2 z\hat{j} + z^3\hat{k}$ and S is the unit cube defined by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$.

18. Evaluate $\oint_C \left(z + \frac{1}{z}\right) dz$, where C is the unit circle $|z| = 1$.

19. Evaluate $\oint_C \frac{z^4 - 3z^2 + 6}{(z+i)^3}\, dz$, where C is $|z| = 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Use Stoke's theorem to evaluate $\oint_C z\,dx + x\,dy + y\,dz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$ counter clockwise as viewed from above.

21. Find the volume of the solid in the first octant bounded by the graphs of $z = 1 - y^2$, $y = 2x$ and $x = 3$.

(1 × 11 = 11 marks)