469562

D 53678 **(Pages : 3)** **Name..................................**

Reg. No..............................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2023

Mathematics

MTS 1C 01—Mathematics—I

(2019—2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum Marks 20.

1. Find the tangent line to the parabola $y = x^2 - 3x + 1$ when $x_0 = 2$. Sketch.

2. Find limit if exists, $\lim_{x \to 3} \sqrt{|x-3|}$.

3. Calculate an approximate value for $\sqrt{10}$ using a linear approximation around $x_0 = 9$.

4. Calculate the second derivative of $\frac{1+x}{\sqrt{x}}$.

5. Find the critical points of the function $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$.

6. Find the intervals on which the function $f(x) = \frac{x}{x-1}$ is concave upward and those which they are concave downward,

7. A shoe repair shop can produce $2x - x^2 - 3$ dollars of revenue every hour when x workers are employed. Find the marginal productivity when 5 workers are employed.

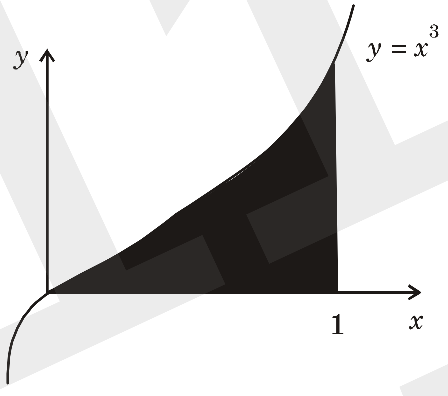
8. Find $\lim_{x \to 0^+} x \log x$.

9. Find the rate of increase of are of circle with radius r.

Turn over

469562

10. Compute the area of the region shown in Fig.



11. Using the fundamental theorem of calculus, compute $\int_a^b x^2 \, dx$.

12. Verify the formula $\frac{d}{dx}\int_a^x f(s)\,ds = f(x)$ f or $f(x) = x$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum Marks 30.

13. (a) Find $\lim_{x \to 2} -\frac{3x}{x^2 - 4x + 4}$; (b) Find $\lim_{x \to 0} \frac{3x + 2}{x}$.

14. Calculate the linear approximation to the area of a square whose side is 2.01. Draw a geometric figure, obtained from a square of side 2, whose area is exactly that given by the linear approximation.

15. A race car travels mile in 6 seconds, its distance from the start in feet after t seconds being $f(t) = \frac{44}{3}t^2 + 132\,t$. (a) Find its velocity and acceleration as it crosses the finish line ; and (b) How fast was it going halfway down the track ?

16. If $y = f(x)$ and $x^2 + y^2 = 1$ express $\frac{dy}{dx}$ interms of x and y.

17. State Mean Value Theorem .Verify Mean Value Theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in [1, 3].

18. Find the volume of a ball' of radius r.

19. (a) Find the average value of $f(x) = x^2$ on [0, 2].

    (b) Find the volume of the solid obtained by revolving the region under the graph of $3x + 1$ on [0, 2] about the x axis.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum Marks 10.

20. (a) Prove the power rule $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$.

    (b) The velocity of a particle moving along a line is $2t + 3$, at a time t. At time 2 the particle is at position 6, where is it at time 15 ?

21. (a) Show that a good approximation to $\frac{1}{1+x}$ when x is small is $1 - x$.

    (b) Find the equation of the tangent line to the curve to $x^6 + y^4 = 9xy$ at the point (1, 2).

(1 × 10 = 10 marks)

D 32374 **(Pages : 3)** **Name..................................**

Reg. No..............................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. Calculate the slope of the tangent line to the graph of $y = x^2$ at $x = 1$.

2. Find $\lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5}$.

3. If f has a derivative at $x = c$, then prove that f is continuous at $x = c$.

4. Find the derivative of $y = \frac{2x + 5}{3x - 2}$.

5. Find the linearization of $f(x) = x^4$ when $x = 1$.

6. Find $\frac{d}{dx}\left[\tan\left(x^2 + 1\right)\right]$.

7. Find $\lim_{x \to 0} \frac{(1 + x)^n - 1}{x}$.

8. Find points of inflection on the curve $y = 3x^4 - 4x^3 + 1$.

9. Find the intervals on which the function $g(t) = -t^2 - 3t + 3$ is increasing and decreasing.

Turn over

10. Evaluate $\sum_{k=1}^{7} -2k$.

11. Using limits of Riemann sums, establish the equation $\int_a^b c\,dx = c\,(b-a)$, where c is a constant.

12. Find $\int_1^2 \frac{x^2+2x+2}{x^4}\,dx$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 30 marks.

13. If $\lim_{x\to 4} \frac{f(x)-5}{x-2} = 1$, find $\lim_{x\to 4} f(x)$.

14. Show that the line $y = mx + b$ is its own tangent at any point $(x, mx + b)$ on the line.

15. An oil slick has area $y = 30x^3 + 100x$ square meters x minutes after a tanker explosion. Find the average rate of change in area with respect to time during the period from $x = 2$ to $x = 3$ and from $x = 2$ to $x = 2.1$. What is the instantaneous rate of change of area with respect to time at $x = 2$?

16. State and prove power rule for positive integers.

17. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval [– 10, 10].

18. Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$.

19. Find the area of the region in the first quadrant bounded by the line $y = x$.the line $x = 2$,the curve $y = 1/x^2$, and the axis.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum 10 *marks.*

20. (a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

(b) Evaluate $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t \, dt$.

21. (a) Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.

(b) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

(c) State and prove the product rule of differentiation.

D 13605 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019—2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. Find the derivative of $f(x) = x^2 - x$ at $x = 2$.

2. Find $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

3. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

4. Find the derivative of $y = \left(x^2 + 1\right)\left(x^3 + 3\right)$.

5. Give the parameterization of the circle $x^2 + y^2 = 1$.

6. Find $\lim_{y \to 1} \sec\left(y \sec^2 y - \tan^2 y - 1\right)$.

7. Suppose that $F'(x) = x$ for all x and that $F(3) = 2$. What is $F(x)$?

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

9. Prove that for the curve $y = c \sin \frac{x}{a}$, every point at which it meets the x-axis is a point of inflection.

Turn over

10. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval $[-10, 10]$.

11. Find $\sum_{k=1}^{7} \left(3 - k^2\right)$.

12. Find $\int_0^1 \frac{\left(3x^2 + x^4\right)}{\left(1 + x^2\right)^2} dx$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 30 marks.

13. If $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \to 0} f(x)$.

14. Find the linearization of $f(x) = \sqrt{x + 1} + \sin x$ at $x = 0$. How is it related to the individual linearizations for $\sqrt{x + 1}$ and $\sin x$?

15. An oil slick has area $y = 30x^3 + 100\,x$ square meters x minutes after a tanker explosion. Find the average rate of change in area with respect to time during the period from $x = 2$ to $x = 3$ from $x = 2$ to $x = 2.1$. What is the instantaneous rate of change of area with respect to time at $x = 2$?

16. Use implicit differentiation to find dy/dx if $6y^2 + \cos y = x^2$.

17. Prove that the curve $y = \frac{x}{1 + x^2}$ has three points of inflection and they are collinear.

18. Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}$, where n is natural number.

19. Find the area of the region enclosed by the curves $x + y^2 = 3$ and $4x + y^2 = 0$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum 10 *marks.*

20. (a) State and prove the quotient rule of differentiation for positive integers.

(b) Prove that $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1)$.

(c) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder.The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge.

21. (a) On what interval is $f(x) = x^3 - 2x + 6$ increasing or decreasing ?

(b) Find the asymptotes of the graph of $f(x) = -\frac{8}{x^2 - 4}$.

(c) Find the equation of the line tangent to the parametric curve given by the equations $x = (1 + t^3)^4 + t^2, y = t^5 + t^2 + 2$ at $t = 1$.

(1 × 10 = 10 marks)

D 12646 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Calculate the slope of the tangent line to the graph of $f(x) = x^2 + 1$ when $x = -1$.

2. Find $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

3. Find the derivative of $y = \sqrt{x}$ for $x > 0$.

4. Find $\frac{d}{dx}\left[\cos\left(\sqrt{1 + \cos x}\right)\right]$.

5. Find the linearization of $f(x) = \cos x$ at $x = \pi/2$.

6. Show that there is a number c such that $c^3 - c^2 = 10$.

7. Find $\lim_{t \to 0} \cos\left(\frac{x}{\sqrt{19 - 3\sec 2t}}\right)$.

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

Turn over

9. Find the critical points of $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$.

10. Find the inflection points of $f(x) = x^2 + (1/x)$.

11. Using limits of Riemann sums, establish the equation $\int_a^b c\,dx = c(b-a)$, where c is a constant.

12. Find $\int_0^2 \left(\frac{t^2}{4} - 7t + 5\right) dt$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*

Each question carries 5 *marks.*

All questions can be attended.

Overall Ceiling 25.

13. Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.

14. Show that the line $y = mx + b$ is its own tangent at any point $(x, mx+b)$ on the line.

15. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 1 ft/s. How fast is the area of the spill increasing when the radius of the spill is 20 ft ?

16. Use implicit differentiation to find d^2y/dx^2 if $5x^3 - 7y^2 = 10$.

17. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval $[-10, 10]$.

18. Use l'Hôpital's Rule to find $\lim_{x \to 0} \frac{\sin x - x}{x^3}$.

19. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

(b) Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

21. (a) Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

(b) Find the volume of the solid generated by the revolution about the x-axis of the loop of the curve $y^2 = x^2 \frac{3a - x}{a + x}$.

(c) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

(1 × 11 = 11 marks)

D 93937 (Pages : 3) Name.........................

Reg. No.........................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION

NOVEMBER 2020

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*

Each question carries 3 marks.

All questions can be attended.

Overall Ceiling 24.

1. A train has position $x = 3t^2 + 2 - \sqrt{t}$ at time t. Find the velocity of the train at $t = 2$.

2. Find $\lim\limits_{x \to 2} \dfrac{-3x}{x^2 - 4x + 4}$.

3. Find the slope of the line tangent to the graph of $f(x) = x^8 + 2x^2 + 1$ at $(1, 4)$.

4. Suppose that $f(t) = \dfrac{1}{4}t^2 - t + 2$ denotes the position of a bus at time t. Find and plot the speed as a function of time.

5. Find $\dfrac{d^2}{dr^2}\left(8r^2 + 2r + 10\right)$.

6. If $x^2 + y^2 = 3$, compute $\dfrac{dy}{dx}$ when $x = 0$ and $y = \sqrt{3}$.

7. On what interval is $f(x) = x^3 - 2x + 6$ increasing or decreasing ?

Turn over

8. Use the second derivative test to analyze the critical points of the function $f(x) = x^3 - 6x^2 + 10$.

9. Discuss the concavity of $f(x) = 4x^3$ at the points $x = -1$ and $x = 1$.

10. Find $\int_2^6 \left(x^2 + 1\right) dx$.

11. Find the area between the graph of $y = x^2$ and $y = x^3$ for x between 0 and 1.

12. Find the average value of $f(x) = x^2$ on $[0, 2]$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*

Each question carries 5 *marks.*

All questions can be attended.

Overall Ceiling 25.

13. (a) Find $\frac{d}{dx}\left(\frac{\sqrt{x}}{1 + 3x^2}\right)$.

(b) Calculate approximate value for $\sqrt{9.02}$ using linear approximation around $x_0 = 9$.

14. Find the equation of the tangent line to the curve $2x^6 + y^4 = 9xy$ at the point $(1, 2)$.

15. Find the slope of the parametric curve given by $x = \left(1 + t^3\right)^4 + t^2, y = t^5 + t^2 + 2$ at $t = 1$.

16. State mean value theorem. Verify mean value theorem for the function $f(x) = x^2 - x + 1$ on $[-1, 2]$.

17. Find $\lim_{x \to 0}\left(\frac{1}{x \sin x} - \frac{1}{x^2}\right)$.

18. An object on the x-axis has velocity $v = 2t - t^2$ at time t. If it starts out at $x = -1$ at time $t = 0$, where is at time $t = 3$? How far has it traveled ?

19. Find average value of $f(x) = x^2 \sin x^3$ on $[0, \pi]$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Using product rule, differentiate $(x^2 + 2x - 1)(x^3 - 4x^2)$. Check your answer by multiplying out first.

(b) Find the dimensions of a rectangular box of minimum cost if the manufacturing costs are 10 cents per square meter on the bottom, 5 cents per square metre on the sides, and 7 cents per square metre on the top. The volume is to be 2 cubic meters and height is to be 1 metre.

21. (a) The curves $y = x^2$ and $x = 1 + \frac{1}{2}y^2$ divide the xy plane into five regions, only one of which is bounded. Sketch and find the area of this bounded region.

(b) The region between the graph of x^2 on $[0, 1]$ is revolved about the x-axis. Sketch the resulting solid and find its volume.

(1 × 11 = 11 marks)

D 73288 (Pages : 3) Name..............................

Reg. No..............................

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CBCSS—UG)

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum Marks 20.

1. Find the derivative of $f(x) = 3x^2 + 8x$ at $x_0 = -2$ and $x_0 = \frac{1}{2}$.

2. A rock thrown down from a bridge has fallen $4t + 4.9t^2$ meter after t seconds. Find its velocity at $t = 3$.

3. Find $\lim_{x \to \infty} \frac{5x^2 - 3x + 2}{x^2 + 1}$.

4. Suppose that $f(t) = \frac{1}{4}t^2 - t + 2$ denotes the position of a bus at time t. Find the acceleration.

5. A bagel factory produces $30x - 2x^2 - 2$ dollars worth of bagels for each x worker hours of labour. Find the marginal productivity when 5 worker hours are employed.

6. The velocity of a particle moving along a line is $3t + 5$ at time t. At time 1, the particle is at position 4. Where is at time 10 ?

7. Use the second derivative test to analyze the critical points of the function $f(x) = x^3 - 6x^2 + 10$.

Turn over

8. Find inflection point of the function $f(x) = x^2 + \frac{1}{x}$.

9. Find $\lim_{x \to 0^+} x \ln x$.

10. Draw the graph of the step function g on $[0,1]$ defined by $g(x) = \begin{cases} -2, \text{ if } 0 \le x < \frac{1}{3} \\ 3, \text{ if } \frac{1}{3} \le x \le \frac{3}{4} \\ 1, \text{ if } \frac{3}{4} < x \le 1 \end{cases}$. Compute the signed area of the region between its graph and the x-axis.

11. Find the sum of the first n integers.

12. Find $\int_0^4 \left(t^2 + 3t^{\frac{7}{2}}\right) dt$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum Marks 30.

13. (a) Differentiate $\frac{1}{(x^3+3)(x^2+4)}$.

(b) Calculate approximate value for $\sqrt{8}$ using the linear approximation around $x_0 = 9$.

14. Find the equation of the tangent line to the curve $2x^6 + y^4 = 9xy$ at the point (1, 2).

15. Water is flowing into a tub at $3t + \frac{1}{(t+1)^2}$ gallons per minute after t minutes. How much water is in the tub after 2 minutes if it started out empty.

16. State mean value theorem. Let $f(x) = \sqrt{x^3 - 8}$. Show that somewhere between 2 and 3 the tangent line to graph of f has slope $\sqrt{19}$.

17. Find the dimensions of a box of minimum cost if the manufacturing costs are 10 cents per square meter on the bottom, 5 cents per square meter on the sides, and 7 cents per square meter on the top. The volume is to be 2 cubic metres and height is to be 1 metre.

18. The region between the graph of x^2 on $[0, 1]$ is revolved about the x-axis. Sketch the resulting solid and find its volume.

19. Find the area between the graphs of $y = x^3$ and $y = 3x^2 - 2x$ between $x = 0$ and $x = 2$.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum Marks 10.

20. (a) Differentiate $\dfrac{x^{\frac{1}{2}} + x^{\frac{3}{2}}}{x^{\frac{3}{2}} + 1}$.

(b) Find inflection point of the function $f(x) = x^2 + \dfrac{1}{x}$.

21. (a) Find $\lim\limits_{x \to 0} \left(\dfrac{1}{x \sin x} - \dfrac{1}{x^2} \right)$.

(b) Find average value of $f(x) = x^2 \sin x^3$ on $[0, \pi]$.