

D 51782

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Name.....

Reg. No.....

**THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023**

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019—2022 Admissions)

Time : Two Hours

Maximum : 60 Marks

*Use of Calculator and Statistical table are permitted.***Section A (Short Answer Type Questions)***All questions can be attended.**Each question carries 2 marks.**Overall Ceiling 20.*

1. Establish the relationship between Geometric distribution and discrete uniform distribution.
2. Obtain the m.g.f of a $N(\mu, \sigma^2)$.
3. Obtain mean and variance of Poisson distribution.
4. Define Pareto distribution.
5. Distinguish between parameter and statistic.
6. What are the advantageous and disadvantageous of Chebycheff's inequality.
7. Use any law of large numbers to prove that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is atleast $19/20$.
8. What is the principal of optimum allocation ?
9. What is non probability sampling ? Give an example.
10. Define Student's t distribution.
11. Give a relationship between t and F distribution.
12. A random sample of 14 independent observations is taken from $N(\mu, \sigma^2)$, what is the mean and variance of Chi-square derived from it ?

(Ceiling 20 marks)

Turn over

Section B (Short Essay/ Paragraph Type Questions)

All questions can be attended.

Each question carries 5 marks.

Overall Ceiling 30.

13. Establish the relationship between Binomial distribution and Poisson distribution.
14. Find the mgf of Normal population $N(\mu, \sigma^2)$.
15. State and prove the weak law of large numbers. Deduce as a corollary Bernoulli theorem and comment on its applications.
16. If X denote the sum of the numbers obtained when two dice are thrown, use Chebychev's inequality to obtain an upper bound for $P[|X - 7| > 4]$. Compare this with the actual probability.
17. Use central limit theorem to establish a relationship between Binomial distribution and Normal distribution.
18. Carry out a comparison between census method and sampling method. Explain systematic random sampling.
19. A sample of size 16 is taken from a normal population with mean 1 and S.D 1.5. Find the probability that the sample mean is negative.

(Ceiling 30 marks)

Section C (Essay Type Questions)

Answer any one question.

The question carries 10 marks.

20. a) If 3 % of electric bulbs are found to be defective, then using Poisson's approximation, find the probability that a sample of 100 bulbs will contain (i) no defective ; and (2) exactly one defective.
b) A random variable X is normally distributed with mean 12 and S.D 2, Find the probability of the event $9.6 \leq x \leq 13.8$.
21. a) How large a sample is to be taken from a normal population $N(10, 3)$ if the sample mean is to lie between 8 and 12 with probability 0.95.
b) Two independent samples of sizes 15 and 20 from a normal population $N(\mu, \sigma^2)$, Find the upper bound to $P\left(\frac{S_1^2}{S_2^2} < 2\right)$.

(1 × 10 = 10 marks)

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NOVEMBER 2022**

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

*Use of calculator and Statistical table are permitted.***Part A (Short Answer Type Questions)***Each question carries 2 marks.**Maximum marks that can be scored from this part is 20.*

1. If X is a random variable following discrete uniform distribution over the numbers $-1, 0$ and 1 ; find the variance of X .
2. For two independent Poisson random variables, $P(X = 0) = e^{-3}$, $P(Y = 0) = e^{-2}$. Find $P(X + Y = 2)$.
3. Write the p.d.f. of an exponential random variable with mean 0.2 .
4. Obtain $P(|X - 5| < 3)$, where X follows $N(5, 3^2)$.
5. Point out one of the strengths and weaknesses of Chebyshev's inequality.
6. Define convergence in distribution.
7. State Weak Law of Large Numbers.
8. Define census and sampling.
9. A box contains 4 black balls and 6 white balls. 2 balls are taken at random one by one. What is the probability that all the balls taken are white : (i) If balls are taken without replacement ; (ii) With replacement.
10. Find the probability that the variance of a sample of size 12 taken from a normal population with mean 10 and variance 9 is greater than 10.28.
11. If X and Y are independent standard normal random variables, identify the probability distributions of (i) X^2 ; and (ii) $[X^2 + Y^2]$.
12. Define t -distribution.

Turn over

Part B (Short Essay/Paragraph Type Questions)

*Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.*

13. Obtain the mode of X following $B(n, p)$
14. Show that the sum of independent exponential random variables with common parameter λ follows gamma distribution.
15. State Chebyshev's inequality. Let X be a random variable following rectangular distribution over $[5, 15]$. Use Chebyshev's inequality to find an upper bound for $P(|X - 10| > 4.33)$.
16. State and prove Bernoulli's Law of Large numbers.
17. Using central limit theorem, obtain the probability distribution of the mean of a large sample of size n taken from rectangular distribution over $[0, 5]$.
18. Explain systematic random sampling.
19. If F follow $F(n_1, n_2)$, show that $1/F$ follow $F(n_2, n_1)$

Part C (Essay type Questions)

*Answer any **one** question.
Each question carries 10 marks.
Maximum marks that can be scored from this part is 10.*

20. (i) Define normal distribution. State any *four* of the properties of normal distribution.
(ii) If the random variable X following $N(\mu, \sigma^2)$, obtain the quartile deviation of X .
21. (i) Derive the (a) m.g.f. ; (b) mean ; and (c) variance of a random variable X following chi square distribution with n degrees of freedom.
(ii) State and prove the additive property of chi-square distribution.

(1 × 10 = 10 marks)

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THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019–2020 Admissions)

Time : Two Hours

Maximum : 60 Marks

*Use of Calculator and Statistical table are permitted.***Section A (Short Answer Questions)***Answer at least **eight** questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 24.*

1. Find the mean of a uniform random variable with possible values 1,2,3,4 and 5.
2. Define negative binomial distribution.
3. Obtain the m.g.f. of exponential distribution.
4. If X follow normal distribution with mean 10 and variance 9, find (i) $P(X > 13)$; (ii) $P(7 < X < 13)$.
5. Define parameter and statistic.
6. Define convergence in distribution.
7. State Central Limit theorem.
8. Define the terms (i) population ; (ii) sampling frame.
9. Define simple random sampling.
10. Find the probability that the sample mean of a random sample of size 16 taken from a normal population with mean 2 and variance 4 is less than 1.
11. If X and Y are independent standard normal random variables, identify the probability distribution of $\left[\frac{X - Y}{X + Y} \right]^2$.
12. Define t -distribution.

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least five questions.

Each question carries 5 marks.

All questions can be attended.

Overall Ceiling 25.

13. If the $(r - 1)^{\text{th}}$, r^{th} , and $(r + 1)^{\text{th}}$ central moments of X following binomial distribution with parameters n and p are, μ_{r-1} , μ_{r-1} and μ_r , show that $\mu_{r+1} = pq \left[nr \mu_{r-1} + \frac{d}{dp} \mu_r \right]$.
14. State and prove lack of memory property of exponential distribution.
15. State and prove Chebyshev's inequality.
16. Examine whether Weak Law of Large Numbers hold for the sequence of random variable $\{X_i\}$, where $P(X_i = \pm \sqrt{2i-1}) = \frac{1}{2}$.
17. State and prove Bernoulli's law of large numbers.
18. Explain cluster sampling.
19. A random sample of size 10 is taken from a normal population with mean 10 and unknown variance. If the sample variance are is 18.23, find the probabilities of the sample mean (i) less than 9 ; (ii) greater than 11.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any one question.

The question carries 11 marks.

20. (i) Show that odd order central moments of X following $N(\mu, \sigma^2)$ are zeroes.
- (ii) Prove that the mean deviation about the mean of X following $N(\mu, \sigma^2)$ is $\sqrt{\frac{2}{\pi}} \sigma$.
21. (i) Define Chi-square distribution. Obtain the m.g.f., of X following $\chi^2_{(n)}$.
- (ii) State and prove the additive property of Chi-square distribution.

(1 × 11 = 11 marks)