**D 103062** **(Pages : 4)** **Name.......................................**

**Reg. No...................................**

## FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
## APRIL 2024

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

**Section A (Short Answer Type Question)**

*All questions can be attended.*
*Each question carries 2 marks.*
*Overall ceiling* 25.

1. Give an example of a system of linear equation with the following properties :
   - (i) Unique solution
   - (ii) Infinite number of solutions
2. Solve the system $x + y = 2, x - y = 0$ by using any method.
3. Give an example to show that the matrix multiplication is need not be commutative.
4. Find the row reduced echelon form of

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}.$$

5. Let $W = \{(x, y, z) : x + y + z = 0\}$. Show that W is subspace of $\mathbb{R}^3$.
6. Show that $\{(1,0),(0,1)\}$ spans $\mathbb{R}^2$.
7. Define Wronskian. Find the Wronskian of $\sin 5x$ and $\cos 5x$.
8. Define linearly independent set. Give an example.
9. Define row space and column space of a matrix.

**Turn over**

10. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

11. Show that the operator $T : \mathbb{R}^2 \to \mathbb{R}^2$, that projects onto the $x$-axis in the $xy$-plane is not one-one.

12. Find the eigen values of $\begin{bmatrix} 3 & 0 \\ 8 & 1 \end{bmatrix}$.

13. Define similar matrices. Show that if A and B are similar the determinant is equal.

14. Let $\boldsymbol{u} = U = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \boldsymbol{v} = V = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$. Evaluate $\langle \boldsymbol{u}, \boldsymbol{v} \rangle$, where, $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \langle U, V \rangle = trace\left(U^T V\right)$.

15. Define orthogonal matrix. Show that $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is orthogonal.

(Ceiling 25 Marks)

**Section B (Paragraph/Problem Type Questions)**

*All questions can be attended.*
*Each question carries* 5 *marks.*
*Overall Ceiling* 35.

16. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, find, det $(A)$, $A^{-1}$, $A^{-2}$, $A^{-3}$ and $A^{-5}$.

17. Using row reduction, evaluate the determinant of :

$$\begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}.$$

18. Determine the set $\left\{6, 3\sin^2 x, 2\cos^2 x\right\}$ is independent or not.

19. Show that the matrices

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Form a basis for the vector space $M_{22}$ of $2 \times 2$ matrices.

20. Find a basis for row space of the matrix

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}.$$

21. Describe the null space of the matrix $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$.

22. Define inner product. Consider $P_2$ with the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)\, q(x)\, dx$. Verify that $x$ and $x^2$ are orthogonal with respective above inner product.

23. Let $f = f(x)$ and $g = g(x)$ be two functions on $C[a, b]$. Show that $\langle f, g \rangle = \int_a^b f(x) f(x)\, dx$ defines an inner product on $C[a, b]$.

(Ceiling 35 Marks)

**Section C (Essay Type Question)**

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

24. (a) For what values of $b_1$, $b_2$ and $b_3$ the following system of equations are consistent ?

$$x_1 + x_2 + 2x_3 = b_1$$
$$x_1 + 0x_2 + x_3 = b_2$$
$$2x_1 + x_2 + 3x_3 = b_3.$$

(b) Let A and B are symmetric matrices of same size. Then show that the followings.

(i) $A^T$ is symmetric

(ii) A + B and A – B are symmetric

(iii) $k$A is symmetric, where $k$ is any scalar.

25. Let $u = \{1, 2, -1\}$, $v = \{6, 4, 2\}$ in $\mathbb{R}^3$.

(a) Show that $w = \{9, 2, 7\}$ is in the linear combination of $u$ and $v$.

(b) Show that $w = \{4, -1, 8\}$ is not in the linear combination of $u$ and $v$.

26. Consider the matrix,

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$$

(a) Verify that $\text{rank}(A) = \text{rank}(A^T)$.

(b) Verify dimension theorem for the matrix A.

27. Find an orthogonal matrix P that diagonalizes :

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}.$$

(2 × 10 = 20 marks)

**C 41231** **(Pages : 4)** **Name.......................................**

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# FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2023

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half — Maximum : 80 Marks

## Section A (Short Answer type Question)

*Each question carries* 2 *marks.*
*All questions can be attended.*
*Overall ceiling* 25.

1. Give an example of a system of linear equation with the following properties :
   - (i) Unique solution ; and
   - (ii) No solution.

2. For any 2 × 2 matrices, A and B, prove that

   $\text{trace}(A + B) = \text{trace}(A) + \text{trace}(B)$.

3. Define all subspaces of the vector space $\mathbb{R}^3$ over $\mathbb{R}$.

4. Define linear combination of vectors in a vector space. Write $(2, 3)$ as the linear combination of $(1, 0)$ and $(0, 1)$.

5. Define basis of a vector space. Write a basis of $P_n$, where $P_n$ is the polynomials of degree less than or equal to $n$.

6. Consider the basis $B = \{u_1, u_2\}$ and $B' = \{u_1', u_2'\}$ of $\mathbb{R}^2$, where $u_1 = (1, 0)$, $u_2 = (0, 1)$, $u_1' = (1, 1)$ and $u_2' = (2, 1)$. Find the transformation matrix from $B' \to B$.

7. Let $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$. Find the dimension of W.

**Turn over**

8. Give an example of an infinite dimensional vector space.
9. Define rank and nullity of a matrix.
10. Find the image of $x=(1,1)$ under the rotation of $\frac{\pi}{6}$, about the origin.
11. Define eigen values and eigen vectors of a matrix.
12. Find the egien values of $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 2/3 & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$.
13. If $\lambda$ is the eigen values of a matrix A, show that $\lambda^n$ is the eigen values of $A^n$.
14. Show that $(1,1)$ and $(1,-1)$ are orthogonal vectors with respective the Euclidian inner product.
15. Let W be the subspace spanned by the orhonornal vector $v_1=(0,1,0)$. Find the orthogonal projection of $u=(1,1,1)$ on W.

(Ceiling 25 marks)

**Section B (Paragraph/Problem Type Questions)**

*Each question carries 5 marks.*
*All questions can be attended.*
*Overall Ceiling 35.*

16. Solve the following linear system by Gauss-Elimination method,

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10. \end{aligned}$$

17. Prove that, if A and B are invertible matrices of same size, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

18. Show that the set $\{(1,1,2),(1,0,1),(2,1,3)\}$ spans $\mathbb{R}^3$.

19. Show that the operator $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by the equations

$$w_1 = 2x_1 + x_2$$
$$w_2 = 3x_1 + 4x_2$$

is one-one, and find $T^{-1}(w_1, w_2)$.

20. Let T be the operator which is the reflection about the *xz* plane in $\mathbb{R}^3$. Find the matrix of T with respective the standard basis.

21. Find the rank and nullity of the matrix

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

22. Find the bases of the eigen spaces of the matrix

$$\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}.$$

23. Show that a square matrix A is invertible if and only if 0 is not and eigen value of A.

(Ceiling 35 marks)

**Section C (Essay Type Question)**

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

24. (a) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

(b) Define the followings with examples :

(i) Diagonal matrices ;

(ii) Lower triangular matrices ;

(iii) Upper triangular matrices ;

(iv) Symmetric matrices ; and

(v) Singular matrices.

25. Let $v_1 = \{1, 2, 1\}, v_2 = \{2, 9, 0\}$ and $v_3 = \{3, 3, 4\}$.

(a) Show that $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.

(b) Find the co-ordinate vector of $v = (5, -1, 9)$ relative to the basis $\{v_1, v_2, v_3\}$.

26. Consider the following linear system :

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}.$$

(a) Show that the above system is consistent.

(b) Solve the above system of linear equations.

27. (a) Define similar matrices.

(b) Show that the following matrix is not diagonazible :

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}.$$

(2 × 10 = 20 marks)

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**C 21545** **(Pages : 4)** **Name.......................................**

**Reg. No..................................**

## FOURTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

**Section A (Short Answer Type Questions)**

*Answer at least* **ten** *questions.*
*Each question carries* 3 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

1. Show that the linear system of equations $\begin{aligned}4x-2y&=1\\16x-8y&=4\end{aligned}$ has infinitely many solutions.

2. Write any two facts about row echelon forms and reduced row echelon forms.

3. Express the linear system
$$\begin{aligned}4x_1-3x_3+x_4&=1\\5x_1+x_2-8x_4&=3\\2x_1-5x_2+9x_3-x_4&=0\\3x_2-x_3+7x_4&=2\end{aligned}$$
in the form AX = B.

4. Let V = $R^2$ and define addition and scalar multiplication as follows. For $\bar{u}=(u_1,u_2),\bar{v}=(v_1,v_2)$, $\bar{u}+\bar{v}=(u_1+v_1,u_2+v_2)$ and for a real number $k, k\bar{u}=(ku_1,0)$. For $\bar{u}=(1,1)$ and $\bar{v}=(-3,5)$ find $\bar{u}+\bar{v}$ and for $k$ = 5, find $k\bar{u}$ . Also show that one axiom for vector space is not satisfied.

5. Define basis for a vector space.

6. How will you relate the dimension of a finite dimensional vector space to the dimension of its subspace. Give two facts.

7. Give a solution to the change of basis problem.

8. When you can say that a system of linear equation A$x$ = $b$ is consistent. What is meant by a particular solution of the consistent system A$x$ = $b$.

9. Find the rank of a 5 × 7 matrix A for which A$x$ = 0 has a two-dimensional solution space.

**Turn over**

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10. If $T_A : R^n \to R^m$ is a matrix transformation. Then define its kernel ker $(T_A)$ and Range of $(T_A)$. What is ker $(T_A)$ in terms of null-space of A.

11. Discuss the geometric effect on the unit square of multiplication by a diagonal matrix $A = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$.

12. Confirm by multiplication that $x$ is an eigen vector of A and find the corresponding eigen value, if $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

13. Let $R^2$ have the weighted Euclidean inner product $< u,v > = 2u_1v_1 + 3u_2v_2$. For $u = (1,1), v = (3,2)$, compute $d(u,v)$.

14. If $u$ and $v$ are orthogonal vectors in a real inner product space, then show that $\| u+v \|^2 = \| u \|^2 + \| v \|^2$.

15. State four properties of orthogonal matrices.

(10 × 3 = 30 marks)

## Section B (Paragraph/ Problem Type Questions)

*Answer at least* **five** *questions.*
*Each question carries* 6 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

16. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$ solve the system.

17. If A is an invertible matrix, then show that $A^T$ is also invertible and $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

18. Let V be a vector space and $\overline{u}$, a vector in V and $k$ a scalar. Then show that (i) $O\overline{u} = 0$ ; (ii) $(-1)\overline{u} = -\overline{u}$.

19. If $S = [v_1, v_2, \ldots, v_n]$ is a basis for a vector space V, then show that every vector $v$ in V can be expressed in form $v = c_1v_1 + c_2v_2 + \ldots + c_nv_n$ in exactly one way. What are the co-ordinates of V relative to the basis S ?

20. Consider the basis $B = [u_1, u_2]$ and $B' = [u_1', u_2']$ for $R^2$, where $u_1 = (2,2)$ $u_2 = (4,-1)$ $u_1' = (1,3)$ $u_2' = (-1,-1)$.

    (a) Find the transition matrix from $B'$ to B.

    (b) Find the transition matrix from B to $B'$.

21. If A is a matrix with $n$ columns, then define rank A, nullity of A and establish a relationship between them.

22. Define eigen space corresponding to an eigen value $\lambda$ of a square matrix A. Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

23. Use the Gram–Schmidt process for an orthonormal basis corresponding to the basis vectors $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$.

(5 × 6 = 30 marks)

**Section C (Essay Type Questions)**

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

24. Show that the following statements are equivalent for an $n \times n$ matrix A :

    (a) A is invertible.

    (b) $Ax = 0$ has only the trivial solution.

    (c) The reduced row echelon form of A is $I_n$.

    (d) A is expressible as a product of elementary matrices.

25. (a) Define Wronskian of the functions $f_1 = f_1(x), f_2 = f_2(x) \ldots f_n = f_n(x)$ which are $n - 1$ times differentiable in $(-\infty, \infty)$. Use this to show that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $c^\infty(-\infty, \infty)$.

    (b) Show that the vectors $v_1 = (1,2,1), v_2 = (2,9,0)$ and $v_3(3,3,4)$ form a basis for $R^3$.

26. (a) If A is the matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, then find a basis for the row space consisting on entirely row vectors from A.

(b) Find the standard matrix for the operator $T : R^3 \to R^3$ that first rotates a vector counter clockwise about $z$-axis through an angle $\theta$, reflects the resulting vector about $yz$ plane and then projects that vector orthogonally onto the $xy$ plane.

27. (a) On $P_2$, polynomial in $[-1,1]$, define innerproduct as $< p,q > = \int_{-1}^{1} p(x)q(x)dx$. Find $\| p \|, \| q \|$ and $< p,q >$ for $p = x$ and $q = x^2$.

(b) If A is an $n \times n$ matrix with real entries, show that A is orthogonally diagonalizable if and only if A has an orthonormal set of $n$ eigenvectors.

(2 × 10 = 20 marks)

**C 3555** (Pages : 4) Name..............................

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# FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MTS 4B 04—LINEAR ALGEBRA

Time : Two Hours and a Half Maximum : 80 Marks

## Section A (Short Answer Type Questions)

*Answer at least* **ten** *questions.*
*Each question carries 3 marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

1. Describe different possibilities for solution $(x, y)$ of a system linear equations in the $xy$ plane. What are consistent system ?

2. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ solve the system.

3. Define trace of a square matric. Find the trace of the matrix $A = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$.

4. Show that the standard unit vectors

   $e_1 = (1, 0, \ldots.0), e_2 = (0, 1, 0\ldots.0), e_3 = (0, 0, 1, 0\ldots.0) \ldots\ldots e_n = (0, 0\ldots\ldots1)$ span $R^n$.

5. Find the co-ordinate vector of $w = (1, 0)$ relative to the basis $s = [\bar{u}_1, \bar{u}_2]$ of $R^2$, where $\bar{u}_1 = (1, -1)$ and $\bar{u}_2 = (1, 1)$.

6. Write two important facts about the vectors in a finite dimensional vector space V.

**Turn over**

7. Consider the bases $B = [\bar{u}_1, \bar{u}_2]$ and $B' = [\bar{u}_1', \bar{u}_2']$ where

$\bar{u}_1 = (1, 0), \bar{u}_2 = (0, 1), \bar{u}_1' = (1, 1), \bar{u}_1' = (2, 1)$. Find the transition matrix $P_{B' \to B}$ from B′ to B.

8. Define row spaces and null spaces an $m \times n$ matrix.

9. If $R = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is the row reduced echelon form of a $3 \times 3$ matrix A, then verify the rank-nullity formula.

10. Show that the operator $T : R^2 \to R^2$ that rotates vectors through an angle $\theta$ is one-one.

11. Find the image of the line $y = 4x$ under multiplication by the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

12. Confirm by multiplication that $x$ is an eigen vector of A and find the corresponding eigen value if

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

13. Let A be an $n \times n$ matrix. Define inner product on $R^n$ generated by A. Also write the generating matrix of the weighted Euclidear inner product $< u, v > = w_1 u_1 v_1 + w_2 u_2 v_2 + .... + w_n u_n v_n$.

14. If $u, v$ are vectors in a real inner product space V, then show that $\| u + v \| \leq \| u \| + \| v \|$.

15. If A is an $n \times n$ orthogonal matrix, then show that $\| Ax \| = \| x \|$ for all $x$ in $R^n$.

(10 × 3 = 30 marks)

## Section B (Paragraph/Problem Type Questions)

*Answer at least* **five** *questions.*
*Each question carries* 6 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

16. Describe Column Row Expansion method for finding the product AB for two matrices A and B. Use this to find the product $AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 4 \\ -3 & 5 & 1 \end{bmatrix}$.

17. If A is an invertible matrix, then show that $A^T$ is also invertible and $(AT)^{-1} = (A^{-1})^T$.

18. Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in $R^3$. Show that $w = (9, 2, 7)$ is a linear combination of $u$ and $v$ and that $w' = (4, -1, 8)$ is not a linear combination of $u$ and $v$.

19. If $s = \{v_1, v_2 \ldots\ldots\ldots v_n\}$ is a basis for a vector space V, then show that every vector $v$ in V can be expressed in form $v = c_1v_1 + c_2v_2 + \ldots. + c_nv_n$ in exactly one way. What are the co-ordinates of $v$ relative to the basis $s$.

20. If A is a matrix with $n$ columns, then define rank of A and show that $\text{rank}(A) + \text{nullity}(A) = n$.

21. Find the standard matrix for the operator $T : R^3 \to R^3$ that first rotates a vector counter clockwise about $z$-axis through an angle $\theta$, then reflects the resulting vector about $yz$ plane and then projects that vector orthogonally onto the $xy$ plane.

22. Define eigen space corresponding to an eigen value $\lambda$ of a square matrix A. Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

23. If $w$ is a sub-space of real inner product space $v$, then show that :

    (a) $w^{\perp}$ is subspace of $v$.

    (b) $w \cap w^{\perp} = \{0\}$.

(5 × 6 = 30 marks)