519786

D 103769 **(Pages : 3)** **Name.....................................**

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Not more than 25 marks can be earned from this Section.
Each question carries 2 marks.

1. What is the natural domain of the function $f(x) = x^2$. Is the function one-to-one ? Justify your answer.

2. Determine whether the function $f(x) = x \sin x$, even, odd or neither even nor odd.

3. Find $(f \circ g \circ h)(x)$ if $f(x) = \sqrt{x}$, $g(x) = 1/x$, $h(x) = x^3$.

4. Find $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

5. The area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4} D^2$. How fast is the area changing with respect to the diameter is 10 m ?

6. Show that when x is very near 0, and k is any real number, then

   $(1 + x)^k \approx 1 + kx$.

7. Find dy and Δy at $x = 3$ with $dx = \Delta x = 2$ where $y = \sqrt{x}$.

8. State Rolle's Theorem.

Turn over

9. Is $x^5 - x^3 - 2x^2$ increasing or decreasing at -2 ? Justify.

10. For what values of x is the curve $y = 2\sqrt{ax}$ concave to the foot of the ordinate.

11. Find $\int (x+2)\left(x^2-1\right) dx$.

12. Show that $\int_a^b x dx = \frac{b^2 - a^2}{2}$.

13. Show that if f is continuous on $[a, b], a \neq b,$ and if $\int_a^b f(x)\, dx = 0,$ then $f(x) = 0$ at least once in $[a, b]$.

14. State the Fundamental Theorem of Calculus part-1

15. Find the work done in lifting a 1000 lb object 1.25 ft off the ground.

Section B

Not more than 35 marks can be earned from this Section.
Each question carries 5 marks.

16. State The Squeeze Theorem. Use the same to evaluate $\lim_{x \to 0} x^2 \sin \frac{1}{x}$.

17. Find the local linear approximation of $f(x) = \sqrt{x}$ at $x = x_0 = 9$ and use it to approximate $\sqrt{9.02}$, $\sqrt{8.82}$ and $\sqrt{10}$. Also find absolute error

18. Prove that if $f'(x) = 0$ for all x in an interval (a, b) then f is constant on (a, b).

19. Find $\lim_{x \to +\infty} \frac{\sqrt{x^2 + 3}}{5x - 6}$.

20. In a test run of a high-speed train along a straight elevated monorail track, data obtained from reading its speedometer indicated that the velocity (in ft/sec) of the train at time t can be described by the velocity function

$v(t) = 7.8\,t \quad 0 \le t \le 25.$

Find the position function of the train. Assume that the maglev is initially located at the origin of a co-ordinate line.

21. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t\, dt.$

22. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x},\ 1 \le x \le 2,$ about the x-axis.

23. Find the center of mass of a system comprising four particles with masses 6, 2, 3, and 5 slugs, located at the points $(-1, 3), (-2, -1), (2, 6)$ and $(5, 1)$, respectively. (Assume that all distances are measured.

Section C

Answer any **two** *question.*
Each question carries 10 *marks.*

24. (a) State and prove the Lagrange's Mean Value Theorem

(b) Verify that the following functions satisfies the hypothesis of mean value theorem on the given internal and find all value of c $f(x) = x^2, [0, 2].$

25. Sketch a graph of

$f(x) = x^3 - 3x^2 + 1.$

26. A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence ?

27. (a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x.$

(b) For the curve $y = c\cosh\frac{x}{c},$, show that $y^2 = c^2 + s^2,$ where s is the length of the arc measured from its vertex to the point (x, y).

(2 × 10 = 20 marks)

C 43189 **(Pages : 4)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2023

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2022 Admissions)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. Sketch the graph of the absolute value function $f(x) = |x|$.
2. By translating the graph of $y = x^2$, sketch the graphs of $y = x^2 + 2$.
3. Show that $\lim_{x \to 2} [x]$ does not exist.
4. State The Squeeze Theorem.
5. Show that zero is a critical number of each of the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ but that neither function has a relative extremum at 0.
6. State Rolle's theorem and Mean Value Theorem.
7. Show that the function $f(x) = x^3 + x + 1$ has exactly one zero in the interval $[-2, 0]$.
8. Determine the intervals where the graph of $f(x) = x^{\frac{2}{3}}$ is concave upward and where it is concave downward.
9. Find (i) $\int 2x^3 \, dx$; and (ii) $\int (2x + 3 \sin x) \, dx$.

Turn over

1105 1105 1105

10. Verify the mean value theorem and find c for $f(x) = x(x-1)(x-2)$ for $a = 0, b = \frac{1}{2}$.

11. Evaluate $\frac{d}{dx}\int_0^x \frac{1}{2+t^4}\,dt$.

12. Using Riemann sum show that $\int_0^b x\,dx = \frac{b^2}{2}$.

13. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

14. Use differentials to obtain an approximation of the arc length of the graph of $y = 2x^2 + x$ from $P(1, 3)$ to $Q(1.1, 3.52)$.

15. Find the area of the surface obtained by revolving the graph of $x = y^3$ on the interval $[0, 1]$ about the y-axis.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. Show that the function $f(x) = |x|$ is differentiable everywhere except at 0.

17. Suppose that the total cost in dollars incurred per week by the Polaraire Corporation in manufacturing x refrigerators is given by the total cost function

$$C(x) = -0.2x^2 + 200x + 9000 \qquad 0 \le x \le 400.$$

   (i) What is the cost incurred in manufacturing the 201^{st} refrigerator ?

   (ii) Find the rate of change of C with respect to x when $x = 200$.

18. Find the points of inflection of $f(x) = (x-1)^{1/3}$.

19. Find $\lim_{x \to -\infty} \frac{x^2+1}{x-2}$.

20. If $y = \int_0^{x^3} \cos t^2 \, dt$, what is $\frac{dy}{dx}$?

21. Evaluate $\int_{-2}^{2} \frac{\sin x}{\sqrt{1+x^2}} dx$.

22. Find the volume of the solid obtained by revolving the region bounded by the graphs of $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

23. Find the center of mass of a system comprising three particles with masses 2, 3. and 5 slugs, located at the points $(-2, 2), (4, 6)$ and $(2, -3)$, respectively. (Assume that all distances are measured in feet.)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 marks.

24. (a) The total cost incurred in operating an oil tanker on an 800-mi run, traveling at an average speed of v mph. is estimated to be

$$C(v) = \frac{10,00,000}{v} + 200v^2$$

dollars. Find the approximate change in the total operating cost if the average speed is increased from 10 mph to 10.5 mph.

(b) Prove that $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$.

25. (a) Sketch the graph of the function $f(x) = \frac{1}{1+\sin x}$.

(b) Find the vertical asymptotes of the graph of $f(x) = \tan x$.

26. (a) Using Riemann sum evaluate $\int_{-1}^{3} \left(4 - x^2\right) dx$.

(b) Using the property of definite integral estimate the integral $\int_1^3 \sqrt{3 + x^2}\, dx$.

27. Find the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$.

(a) With respect to x ; and

(b) With respect to y.

($2 \times 10 = 20$ marks)

C 23878 **(Pages : 4)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. By reflecting the graph of $y = \sqrt{x}$ sketch the graphs of $y = \sqrt{-x}$ and $y = -\sqrt{x}$.

2. Let $f(x) = x - (\pi/2)$, $g(x) = 1 + \cos^2 x$, and $h(x) = \sqrt{x}$. Find $h \circ g \circ f$.

3. Find $\lim_{x \to 0} \frac{\tan x}{x}$.

4. Find the intervals where the function $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ is continuous.

5. Find the critical numbers of $f(x) = x - 3x^{\frac{1}{3}}$.

6. Verify Rolle's theorem for $f(x) = x^3 - x$ in $[-1, 1]$.

7. Find the relative extrema of $f(x) = 15x^{\frac{2}{3}} - 3x^{\frac{5}{3}}$.

8. Define inflection point and state the second derivative test.

Turn over

1105 1105 1105

9. Find the definite integral $\int_0^b x\,dx$ considering it as an area under the graph of a nonnegative function.

10. Find the value of c guaranteed by the Mean Value Theorem for Integrals for $f(x) = 4 - x^2$ on the interval $[0, 3]$.

11. Evaluate $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\,dt$.

12. Evaluate $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\,dx$.

13. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on $[0, 2]$ about the x-axis.

14. Find the work done by the force $F = 3x^2 + x$ (measured in pounds) in moving a particle along the x-axis from $x = 2$ to $x = 4$ (measured in feet).

15. Define the Center of Mass of a System of Masses on a Line and The Center of Mass of a System of Particles in a Plane.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. Prove that if f is differentiable at a, then f is continuous at a.

17. Suppose the weekly revenue realized through the sale of x Pulsar cell phones is

$$R(x) = -0.000078x^3 - 0.0016x^2 + 80x \qquad 0 \le x \le 800 \qquad \text{dollars.}$$

   (i) Find the marginal revenue function.

   (ii) If the company currently sells 200 phones per week, by how much will the revenue increase if sales increase by one phone per week ?

18. During a test dive of a prototype of a twin-piloted submarine, the depth in feet of the submarine at time t in minutes is given by $h(t) = t^3 (t-7)^4$ where $0 \le t \le 7$. Find the inflection points of h.

19. Prove that $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

20. State The Fundamental Theorem of Calculus, Part 2 and evaluate

$$\int_{-2}^{2} f(x)\, dx, \text{ where } f(x) = \begin{cases} -x^2 + 1 & \text{if } x < 0 \\ x^3 + 1 & \text{if } x \ge 0 \end{cases}.$$

21. A car moves along a straight road with velocity function $v(t) = t^2 + t - 6 \quad 0 \le t \le 10$ where $v(t)$ is measured in feet per second.

    (i) Find the displacement of the car between $t = 1$ and $t = 4$.

    (ii) Find the distance covered by the car during this period of time.

22. Find the length of the graph of $x = \frac{1}{3}y^3 + \frac{1}{4y}$ from $P\left(\frac{7}{12}, 1\right)$ to $Q\left(\frac{67}{24}, 2\right)$.

23. Find the area of the surface obtained by revolving thegraph of $y = \sqrt{x}$ on the interval $[0, 2]$ about the x-axis.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. (a) Show that the function f defined by $f(x) = \sqrt{4 - x^2}$ is continuous on the closed interval $[-2, 2]$.

    (b) Assume that the moon is a perfect sphere, and suppose that we have measured its radius and found it to be 1080 mi with a possible error of 0.05 mi. Estimate the maximum error in the computed surface area of the moon.

11105

25. (a) Let $f(x) = \dfrac{2x^2 - x + 1}{3x^2 + 2x - 1}$. Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$, and find all horizontal asymptotes of the graph of *f*.

(b) A man has 100 ft of fencing to enclose a rectangular garden in his backyard. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.

26. (a) By computing Riemann sum, evaluate $\int_{-1}^{3} \left(4 - x^2\right) dx$.

(b) Find the average value of $4 - x^2$ over the interval $[-1, 3]$.

27. (a) Prove that the length *s* of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $s = \frac{3}{2} \sqrt[3]{ax^2}$. Also find the entire length

(b) Find the area of the surface generated by revolving the arc of the catenary $y = c \cosh \frac{x}{c}$ from $x = 0$ to $x = c$ about the *x*-axis.

C 22091 **(Pages : 3)** **Name.....................................**

Reg. No.................................

SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—I

(2021 Admissions)

Time : Two Hours and a Half — Maximum Marks : 80

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Let $f(x)=\sin x$ and $g(x)=1-2x$. Find the functions *gof* and *fog*. What are their domains ?

2. Find $\lim\limits_{x\to -3}\dfrac{x^2+2x-3}{x^2+4x+3}$.

3. Let $H(t)=\begin{cases}0 & \text{if } t<0\\ 1 & \text{if } t\geq 0.\end{cases}$ Determine whether H is continuous from the right at 0 and/or from the left at 0.

4. Find $\lim\limits_{x\to 0}\dfrac{\sin 2x}{3x}$.

5. Find the instantaneous rate of change of $f(x)=\dfrac{2}{x}+x$ at $x=1$.

6. Find the derivative of $f(x)=3\sqrt{x}+2e^x$.

7. Find the critical points of $f(x)=x-3x^{1/3}$.

8. State Mean value theorem.

9. Find $\lim\limits_{x\to -\infty}\dfrac{x^2+1}{x-2}$.

10. Find the horizontal and vertical asymptotes of $f(x)=\dfrac{1}{x+2}$.

11. Find $\int\dfrac{2x^2-1}{x^2}\,dx$.

Turn over

199926

11105

12. Find $\int \frac{e^{2/x}}{x^2}\,dx$.

13. Evaluate $\int_{-1}^{2} |x|\,dx$.

14. Find the area of the region between the graphs of $y = e^x$ and $y = x$ and the vertical lines $x = 0$ and $x = 1$.

15. Find the work done by the force $F(x) = 3x^2 + x$ in moving a particle along the x-axis from $x = 2$ to $x = 4$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Find $\lim_{x \to 0} x^2 \sin\frac{1}{x}$.

17. Let $f(x) = 2x^3 + x$ (a) Find $f'(x)$. (b) What is the slope of the tangent line to the graph of f at (2, 18) ; (c) How fast is f changing when $x = 2$.

18. Find the relative extrema of $f(x) = x^3 - 3x^2 - 24x + 32$ using second derivative test.

19. Let $f(x) = x^3 - x$ for x in $[-1, 1]$:

    (a) Show that f satisfies the hypothesis of Rolle's theorem on $[-1, 1]$.

    (b) Find the numbers c in $(-1, 1)$ such that $f'(c) = 0$ by Roll's theorem.

20. (a) In a test run of a maglev along a straight elevated monorail track, data obtained from reading its speedometer indicated that the velocity of the maglev at time t can be described by the velocity function $v(t) = 8t, 0 \le t \le 30$. Find the position of the maglev. Assume that the maglev is initially located at the origin of a co-ordinate line.

    (b) Find $\int \frac{dx}{1 - \sin x}$.

21. (a) State fundamental theorem of Calculus.

    (b) Find $\frac{d}{dx}\left[\int_{1}^{x} t^3\,dt\right]$ by using the above theorem and by performing the integration and differentiation.

22. Let R be the region bounded by the graphs of $x = -y^2 + 6y$ and $x = 0$. Find the volume of the solid obtained by revolving R about the x-axis.

23. Find the area of the surface obtained by revolving the graph of $x = y^3$ on the interval [0, 1] about y-axis.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) By translating the graph of $y = x^2$, sketch the graphs of $y = x^2 + 2$ and $y = (x - 2)^2$.

(b) Let $f(x) = \begin{cases} -x^5 + x^3 + x + 1 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ x^2 + \sqrt{x+1} & \text{if } x > 0 \end{cases}$

Find $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$. Does $\lim_{x \to 0} f(x)$ exist. Justify your answer.

25. Sketch the graph of the function $f(x) = 2x^3 - 3x^2 - 12x + 12$.

26. Using the definition of the definite integral evaluate $\int_{-1}^{3} (4 - x^2)\, dx$.

27. (a) Find the area of the region enclosed by the graphs of $y = \frac{x^2}{4}$ and $y = \frac{8}{x^2 + 4}$.

(b) Find the volume of a right pyramid with a square base of side b and height h.

(2 × 10 = 20 marks)

C 4387 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE – I

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find two functions f and g such that $F = g \circ f$ if $F(x) = (x+2)^4$.

2. Let $f(x) = \begin{cases} -x+3 & \text{if } x < 2 \\ \sqrt{x-2+1} & \text{if } x \geq 2 \end{cases}$.

   Find $\lim_{x \to 2} f(x)$ if it exists.

3. Find the values of x for which the function $f(x) = x^8 - 3x^4 + x + 4 + \frac{x+1}{(x+1)(x-2)}$ is continuous.

4. Find $\lim_{x \to o} \frac{\tan x}{x}$.

5. Find the instantaneous rate of change of $f(x) = 2x^2 + 1$ at $x = 1$.

6. If $f(x) = 2x^3 - 4x$. Find $f'(-2)$ and $f'(0)$.

7. Find the extreme values $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.

8. Determine where the graph of $f(x) = x^3 - 6x$ is concave upward and where it is concave downward.

9. Find $\lim_{x \to -1} \frac{1}{x+1}$.

10. Find the horizontal asymptote of the graph of $f(x) = \frac{1}{x-1}$.

Turn over

70450

11. Find $\int \frac{\sin t}{\cos^2 t}\, dt$.

12. Find $\int \frac{1}{x \log x}\, dx$.

13. Given that $\int_{-2}^{2} f(x)\, dx = 3$ and $\int_{0}^{2} f(x)\, dx = 2$, evaluate $\int_{2}^{0} f(x)\, dx$.

14. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

15. Find the volume of the solid obtained by revolving the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Show that the function $f(x) = |x|$ is differentiable everywhere except at 0.

17. Show that if the function f is differentiable at a, then f is continuous at a.

18. Verify that the function $f(x) = x^2 + 1$ satisfies the hypothesis of the mean value theorem on [0, 2] and find all values of c that satisfy the conclusion of the theorem.

19. Find the relation extrema if any of the function $h(t) = \frac{1}{3}t^3 - 2t^2 - 5t - 10$.

20. The velocity function of a car moving along a straight road is given by $v(t) = t - 20$, for $0 \le t \le 40$, where $r(t)$ is measured in feet per second and t in seconds. Show that at $t = 40$, the car will be in the same position as it was initially.

21. (a) State mean value theorem for integrals.

    (b) Verify mean value theorem for $f(x) = x^2$ on [1, 4].

70450

22. (a) Use differentials to obtain an approximation of the arc length of the graph of $y = 2x^2 + x$ from P (1,3) to Q (1.1, 3.52).

    (b) Find the work done in lifting a 50 – lb sack of potatoes to a weight of 4 ft above the ground.

23. Find the length of the graph of $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ on the interval [1, 3].

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find the slope and an equation of the tangent line to the graph $f(x) = x^2$ at the point (1, 1).

    (b) Suppose that the total cost in dollars incurred per weak by a company in manufacturing x refrigerators is given by the total cost function $c(x) = -0.2x^2 + 200x + 9000$, $0 \le x \le 400$.

    (i) What is the cost incurred in manufacturing the 201 st refrigerator ?

    (ii) Find the rate of change of c with respect to x when $x = 200$.

25. A man has 100 ft of fencing to enclose a rectangular garden. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.

26. (a) Estimate $\int_0^t e^{-\sqrt{x}}\,dx$ using the property of definite integral.

    (b) Use the geometric interpretation of the integral to evaluate $\int_{-1}^{2} |x-1|\,dx$ by making a sketch of f.

27. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

(2 × 10 = 20 marks)

C 82445 **(Pages : 3)** **Name..................................**

Reg. No...............................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—I

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. Let f and g be functions defined by $f(x) = x + 1$ and $g(x) = \sqrt{x}$. Find the functions gof and fog. What is the domain of gof ?

2. Find $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{x}$.

3. Let $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2. \end{cases}$ Show that f has a removable discontinuity at 2. Redefine f at 2 so that it is continuous everywhere.

4. Find $\lim\limits_{x \to \pi/4} \dfrac{\sin x}{x}$.

5. Show that $f(x) = |x|$ is continuous everywhere.

6. Find the derivative of $\sqrt[3]{x} + \dfrac{1}{\sqrt{x}}$.

7. Find the critical points of $f(x) = x^3 - 6x + 2$.

8. Find $\lim\limits_{x \to \infty} \left(2x^3 - x^2 + 1\right)$ and $\lim\limits_{x \to -\infty} 2x^3 - x^2 + 1$.

9. Find the interval on which $f(x) = x^2 - 2x$ is increasing or decreasing.

10. Find the vertical asymptote of the graph of $f(x) = \dfrac{1}{x-1}$.

Turn over

11. Find $\int \frac{\cos x}{1-\cos^2 x} dx$.

12. Find $\int x e^{-x^2} dx$.

13. Suppose $\int_1^6 f(x)dx = 8$ and $\int_4^6 f(x)dx = 5$, what is $\int_1^4 f(x)dx$.

14. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on [0, 2] about the x-axis.

15. Find the work done in lifting a 2.4 kg. package 0.8 m. off the ground (given $g = 9.8$ m./sec.^{2}).

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 marks.

16. Find the slope and an equation of the tangent line to the graph of the equation $y = -x^2 + 4x$ at the point p (2, 4).

17. Suppose that $g(x) = (x^2+1) f(x)$ and it is known that $f(2) = 3$ and $f'(2) = -1$. Evaluate $g'(2)$.

18. (a) Show that $f(x) = x^3$ satisfies the hypothesis of the mean value theorem on [– 1, 1].

    (b) Find the numbers c in (– 1, 1) that satisfies the equation as guaranteed by the mean value theorem.

19. Find the slant asymptotes of the graph of $f(x) = \frac{2x^2-3}{x-2}$.

20. A car moves along a straight road with velocity function $v(t) = t^2 + t - 6, 0 \le t \le 10$, where $v(t)$ is measured in feet per second.

    (a) Find the displacement of the car between t = 1 and t = 4.

    (b) Find the distance covered by the car during this period.

21. (a) Evaluate $\int_{-3}^{0} (x^2 - 4x + 7) dx$ by Fundàmental theorem of Calculus.

    (b) Use the definition of definite integral to show that if $f(x) = c$, a constant function, then $\int_a^b f(x)\, dx = c(b-a)$.

22. Find the center of mass of a system comprising three particles with masses 2, 3 and 5 slugs, located at the points (– 2, 2), (4, 6) and (2, – 3) respectively.

23. Find the lengh of the graph of $x = \frac{1}{3}y^3 + \frac{1}{4y}$ from $P\left(\frac{7}{12}, 1\right)$ to $G\left(\frac{67}{24}, 2\right)$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}$.

(b) Use intermediate value theorem to find the value of c such that $f(c) = 7$, where $f(x) = x^2 - x + 1$ on $[-1, 4]$.

(c) In a fire works display, a shell is launched vertically upwards from the ground, reaching a height $S = -16t^2 + 256t$ feets after t seconds. The shell burst when it reaches its maximum height :

(i) A what time after launch will the shell burst.

(ii) What will be the altitude of the shell when it explodes ?

25. Find the dimensions of the rectangle of greatest area that has its base on the x-axis and is inscribed in the parabola $y = 9 - x^2$.

26. Using the definiton of definite integral evaluate $\int_a^b x\,dx$.

27. Find the aera of the surface obtained by revolving the graph of $f(x) = \sqrt{x}$ on the interval $[0, 2]$ about the x-axis.

(2 × 10 = 20 marks)