D 103770 (Pages : 3) Name.....................................

Reg. No.................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2024

Mathematics

MAT 2C 02—MATHEMATICS—2

(2019 Admissions Only)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum marks : 20.

1. Describe the set of points P whose polar coordinates (r, θ) satisfy $0 \le r \le 2$ and $0 \le \theta < r$.

2. Differentiate : (a) $(\tan 3x)/1 + \sin^2 x)$; (b) $1 - \csc^2 5x$.

3. Show that $e^x = \cosh x + \sinh x$.

4. Find $\int_1^b \frac{1}{x^4}\,dx$. What happens as b goes to infinity ?

5. State the comparison test for integrals.

6. Briefly explain Taylor's and Maclaurin's series.

7. Write down the criteria for checking whether a given subset W is a subspace of a vector space V.

8. Define linear independence.

9. State a condition for the consistency of the matrix equation AX = B.

10. Find the inverse of $A = \begin{pmatrix} 1 & 4 \\ 2 & 10 \end{pmatrix}$.

11. Define the adjoint of an $n \times n$ matrix.

12. State Cayley Hamilton theorem.

Turn over

Section B

Answer any number of questions.

Each question carries 5 marks.

Maximum marks : 30.

13. Convert from cartesian to polar co-ordinates : $(2, -4)$; and from polar to cartesian coordinates : $(6, -\pi/8)$.

14. Calculate : (a) $\frac{d}{dx}\sinh^{-1}(3x)$; and (b) $\frac{d}{dx}[\sinh^{-1}(3\tanh 3x)]$.

15. (a) For which values of the exponent r is $\int_1^{\infty} x^r\, dx$ convergent ?

    (b) Find $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

16. Let $f(x) = \cos x$. Evaluate $\int_0^{\pi/2} \cos x\, dx$. by the method of Riemann sums, taking 10 equally spaced points : $x_0 = 0,\ x_1 = \pi/20,\ x_2 = 2\pi/20,\ \ldots,\ x_{10} = 10\pi/20 = \pi/2$ and $c_i = x_i$. Compare the answer with the actual value.

17. Define a vector space.

18. Reduce to echelon form the augmented matrix :

$$\left(\begin{array}{ccc|c} 2 & 6 & 1 & 7 \\ 1 & 2 & -1 & -1 \\ 5 & 7 & -4 & 9 \end{array}\right).$$

19. Find the inverse of $A = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}$.

Section C

Answer any **one** *question.*

The question carries 10 *marks.*

Maximum 10 *marks.*

20. Describe Newton's Method for solving $f(x) = 0$.

    Use Newton's method to find the first few approximations to a solution of the equation $x^2 = 4$, taking $x_0 = 1$.

21. (a) Describe Gram Schmidt Process in R^2 and R^3.

    (b) Orthonormalize $B = \{u_1, u_2\}$, where $u_1 = \langle 3, 1 \rangle$, $u_2 = \langle 1, 1 \rangle$.

    (c) Orthonormalize $u_1 = \langle 1, 1, 1 \rangle$, $u_2 = \langle 1, 2, 2 \rangle$, $u_3 = \langle 1, 1, 0 \rangle$.

D 103771 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024

Mathematics

MAT 2C 02—MATHEMATICS—2

(2020—2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 20.

1. Find the cartesian co-ordinates of $(r, \theta) = (6, -\pi/8)$.

2. Let $y = x^3 + 2$. Find $\frac{dx}{dy}$ when $y = 3$.

3. Compute $\int \coth x \, dx$.

4. Find $\lim_{n \to \infty} \left(\frac{n^2 + 1}{3n^2 + n} \right)$.

5. Sum the series $\sum_{i=0}^{\infty} \frac{3^i - 2^i}{6^i}$.

6. Show that $\sum_{i=1}^{\infty} \frac{2}{4 + i}$ diverges.

Turn over

7. Verify that the basis $B = \left\{ \left\langle \frac{12}{13}, \frac{5}{13} \right\rangle, \left\langle \frac{5}{13}, \frac{-12}{13} \right\rangle \right\}$ is an orthonormal basis for $\mathbb{R}^2$.

8. Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 1 \end{bmatrix}$.

9. Evaluate determinant of $A = \begin{bmatrix} 2 & 4 & 7 \\ 6 & 0 & 3 \\ 1 & 5 & 3 \end{bmatrix}$.

10. Find the value of x such that the matrix $A = \begin{bmatrix} 4 & -3 \\ x & -4 \end{bmatrix}$ is its own inverse.

11. Find the eigenvalues of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$.

12. Verify that the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ satisfies its characteristic equation.

Section B

Answer any number of questions.

Each question carries 5 *marks.*

Ceiling is 30.

13. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on $[1, 2]$.

14. Find the area of the surface obtained by revolving the graph of x^3 on $[0, 1]$ about the x-axis.

15. Show that the improper integral $\int_0^{\infty} \frac{e^{-x}}{\sqrt{x}}\, dx$ is convergent.

16. Let $f(x) = \cos x$. Evaluate $\int_0^{\pi/2} \cos x\, dx$ by the Simpson's rule, taking 10 equally spaced points.

17. Let $u_1 = \langle 1, -1, 1, -1 \rangle, u_2 = \langle 1, 3, 0, -1 \rangle$ be the vectors span a subspace W of $\mathbb{R}^4$. Use the Gram-Schmidt orthogonalization process to construct a orthonormal basis for the subspace W.

18. Find nontrivial solution for the homogeneous system of equations

$$\begin{aligned} 2x_1 - 4x_2 + 3x_3 &= 0 \\ x_1 + x_2 - 2x_3 &= 0. \end{aligned}$$

19. Find the inverse of $A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$.

Section C

Answer any **one** *questions.*
The question carries 10 *marks.*

20. Use Gaussian elimination or Gauss-Jordan elimination to solve

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 3 \\ 3x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 3 \\ 4x_1 + 5x_2 - 2x_3 + x_4 &= 16. \end{aligned}$$

21. Determine whether the matrix $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$ is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^T AP$.

(1 × 10 = 10 marks)

D 103770 **(Pages : 3)** **Name..................................**

Reg. No...............................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024

Mathematics

MAT 2C 02—MATHEMATICS—2

(2019 Admissions Only)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum marks : 20.

1. Describe the set of points P whose polar coordinates (r, θ) satisfy $0 \leq r \leq 2$ and $0 \leq \theta < r$.

2. Differentiate : (a) $(\tan 3x)/1 + \sin^2 x)$; (b) $1 - \csc^2 5x$.

3. Show that $e^x = \cosh x + \sinh x$.

4. Find $\int_1^b \frac{1}{x^4} dx$. What happens as b goes to infinity ?

5. State the comparison test for integrals.

6. Briefly explain Taylor's and Maclaurin's series.

7. Write down the criteria for checking whether a given subset W is a subspace of a vector space V.

8. Define linear independence.

9. State a condition for the consistency of the matrix equation AX = B.

10. Find the inverse of $A = \begin{pmatrix} 1 & 4 \\ 2 & 10 \end{pmatrix}$.

11. Define the adjoint of an $n \times n$ matrix.

12. State Cayley Hamilton theorem.

Turn over

Section B

Answer any number of questions.

Each question carries 5 marks.

Maximum marks : 30.

13. Convert from cartesian to polar co-ordinates : $(2, -4)$; and from polar to cartesian coordinates : $(6, -\pi/8)$.

14. Calculate : (a) $\dfrac{d}{dx}\sinh^{-1}(3x)$; and (b) $\dfrac{d}{dx}\left[\sinh^{-1}(3\tanh 3x)\right]$.

15. (a) For which values of the exponent r is $\int_1^{\infty} x^r\, dx$ convergent ?

    (b) Find $\int_{-\infty}^{\infty} \dfrac{dx}{1+x^2}$.

16. Let $f(x) = \cos x$. Evaluate $\int_0^{\pi/2} \cos x\, dx$. by the method of Riemann sums, taking 10 equally spaced points : $x_0 = 0,\ x_1 = \pi/20,\ x_2 = 2\pi/20,\ \ldots,\ x_{10} = 10\pi/20 = \pi/2$ and $c_i = x_i$. Compare the answer with the actual value.

17. Define a vector space.

18. Reduce to echelon form the augmented matrix :

$$\left(\begin{array}{ccc|c} 2 & 6 & 1 & 7 \\ 1 & 2 & -1 & -1 \\ 5 & 7 & -4 & 9 \end{array}\right).$$

19. Find the inverse of $A = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}$.

Section C

Answer any **one** *question.*

The question carries 10 *marks.*

Maximum 10 *marks.*

20. Describe Newton's Method for solving $f(x) = 0$.

    Use Newton's method to find the first few approximations to a solution of the equation $x^2 = 4$, taking $x_0 = 1$.

21. (a) Describe Gram Schmidt Process in R^2 and R^3.

    (b) Orthonormalize $B = \{u_1, u_2\}$, where $u_1 = \langle 3, 1\rangle$, $u_2 = \langle 1, 1\rangle$.

    (c) Orthonormalize $u_1 = \langle 1, 1, 1\rangle$, $u_2 = \langle 1, 2, 2\rangle$, $u_3 = \langle 1, 1, 0\rangle$.

C 43191 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2023

Mathematics

MTS 2C 02—MATHEMATICS—2

(2020—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum marks that can be earned from this Section is 20.

1. Sketch the set of points whose polar co-ordinates (r, θ) satisfy the conditions $0 < r < 4$ and $-\pi/2 < \theta < \pi/2$.

2. Let $f(x) = x^2 + 2x + 3$. Restrict f to a suitable interval so that it has an inverse. Find the inverse $f^{-1}(x)$ of the given function.

3. Find the slope of the line tangent to the graph of $r = \cos(3\theta)$ at $\theta = \pi/3$.

4. Prove $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$.

5. Show that determinant of a square matrix A is the product of its eigenvalues

6. Find $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ using the notion of improper integrals.

7. Evaluate $\lim_{n \to \infty} \frac{n^2+1}{3n^2+n}$.

8. Define rank of a matrix.

Turn over

9. Find A^2 using Cayley-Hamilton theorem, if $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

10. When will you say that a series $\sum_{n=1}^{\infty} a_n$ an converges to the sum s.

11. Use Doolittle's method to find an LU-factorization of $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$.

12. Selecting a proper test of convergence, decide whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - \ln n}$ converges or diverges.

Section B

Answer any number questions from this section.

Each question carries 5 marks.

Maximum that can be earned from this Section is 30.

13. Find the maxima and minima of $f(\theta) = 1 + 2\cos(\theta)$. Sketch the graph of $r = 1 + 2\cos\theta$ in the xy–plane.

14. Find :

    (a) $\frac{d}{dx}\left(\sinh^{-1}(3x)\right)$; and

    (b) $\int \sinh^{-1}(3x)\, dx$.

15. Prove that the length of the parabola $y = x^2$ from $x = 0$ to $x = 1$ is $\frac{1}{2}\left[\sqrt{5} + \frac{1}{2}\ln\left(2 + \sqrt{5}\right)\right]$.

16. A bouncing ball loses half of its energy on each bounce. The height reached on each bounce is proportional to the energy. Suppose that the ball is dropped vertically from a height of one meter. How far does it travel ?

17. Find the Taylor's series expansion for $\ln x$ around $x = 1$.

18. Find the rank of the matrix $A + 3I$ where $A = \begin{bmatrix} 0 & 1 & 1 & 7 \\ 1 & 0 & 1 & -3 \\ 4 & 1 & 0 & 3 \\ 4 & 1 & 0 & 3 \end{bmatrix}$.

19. The set $B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = \langle 1, 1, 1 \rangle, \mathbf{u}_2 = \langle 1, 2, 2 \rangle, \mathbf{u}_3 = \langle 1, 1, 0 \rangle$ is a basis for $\mathbb{R}^3$. Transform B into an orthonormal basis.

Section C

Answer **one** *question from this section.*
The question carries 10 *marks.*
Maximum that can be earned from this Section is 10.

20. (a) Find the surface area of a sphere of radius r using the method of integration.

(b) Use Simpson's rule of integration to evaluate $\int_0^1 \frac{dx}{x^2 + 1}$ and hence find an approximate value for π.

21. (a) If consistent solve the system using Gauss-Jordan elimination :

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 3 \\ 3x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 3 \\ 4x_1 + 5x_2 - 2x_3 + x_4 &= 16. \end{aligned}$$

(b) Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(1 × 10 = 10 marks)

C 23880 **(Pages : 3)** **Name.......................................**

Reg. No..................................

**SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION
APRIL 2022**

Mathematics

MAT 2C 02—MATHEMATICS—II

(2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. Find the inverse of the function $f(x) = \sqrt{3x-2}$.

2. Find the Cartesian form of the polar equation $r = \dfrac{2}{\sin\theta - 2\cos\theta}$.

3. Find the slope of the line tangent to the graph of $r = \cos 3\theta$ at $(r, \theta) = (-1, \pi/3)$.

4. Show that $\lim\limits_{n\to\infty} \dfrac{2n+1}{n} = 2$.

5. Find $\dfrac{dy}{dx}$, where $y = x \sinh x - \cosh x$.

6. Find the norm of the vector $\langle 3, 4, 0, 1, -1\rangle$. Also normalize the vector.

7. Compute $\|\cos x\|$ in $C[0, 2\pi]$.

8. Using Maclaurin's series find the expansion of $\sin x$.

9. Find the determinant of the matrix $C = \begin{bmatrix} 3 & 4 & 8 \\ 2 & -4 & -18 \\ -4 & 7 & 27 \end{bmatrix}$.

Turn over

10. Let I be an identity matrix of order $n \times n$. Show that I is an orthogonal matrix.

11. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ find A^3 using Cayley Hamilton theorem.

12. Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 30 *marks.*

13. Find the length of the curve $y = (x/2)^{2/3}$ from $y = 0$ to $y = 2$.

14. Find $\left(f^{-1}\right)'(2)$, if $f(x) = x^5 + x$.

15. Find the length of the curve $r = a \sin^2\left(\frac{\theta}{2}\right), 0 \le \theta \le \pi, a > 0$.

16. Evaluate $\int_0^1 e^{-x^2}\, dx$ by means of Trapezoidal rule with $n = 10$.

17. Using Maclaurin's series expand $\tan^{-1} x$. Hence deduce the Gregory series :

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

18. $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = \langle 2, -1, 1\rangle$, $\mathbf{u}_2 = \langle 1, 5, 1\rangle$, $\mathbf{u}_3 = \langle 0, 1, 2\rangle$, is a basis for $\mathbb{R}^2$. Transform it into an orthonormal basis $B_2 = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$.

19. Find the inverse of $\begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$ if it exists.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum 10 *marks.*

20. (a) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

(b) For which values of r is $\int_0^1 x^r \, dx$ convergent ? Justify your answer.

21. (a) Solve the system of equations :

$$\begin{aligned} x_1 - 2x_2 + x_5 - x_6 + x_7 &= 0 \\ x_3 - x_4 + x_5 - 2x_6 + 3x_7 &= 0 \\ x_1 - x_5 + 2x_6 &= 0 \\ 2x_1 - 3x_4 + x_5 &= 0. \end{aligned}$$

(b) Diagonalize the matrix $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$.

C 23879 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2022

Mathematics

MAT 2C 02—MATHEMATICS–2

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum Marks 20.

1. State inverse function test. Verify that $f(x) = x^2 + x$ has an inverse if f is defined on $\left[\frac{-1}{2}, \infty\right)$.

2. Find the slope of the line tangent to the graph of $r = 3\cos^2 2\theta$ at $\theta = \frac{\pi}{6}$.

3. Compute $\int \frac{\sinh x dx}{1 + \cosh^2 x}$.

4. Prove that $\tanh^{-1} x = \frac{1}{2} \ln\left[\frac{1+x}{1-x}\right], -1 < x < 1$.

5. For which values of the exponent r is $\int_1^{\infty} x^r \, dx$ convergent ?

6. State Simpson's rule.

7. Sum the series $\sum_{i=0}^{\infty} \frac{3^i - 2^i}{6i}$.

Turn over

8. State alternating series test and test the convergence for the series $\sum_{i=1}^{\infty} \frac{(-i)^i}{(1+i)^2}$.

9. Prove that the vectors $w_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), w_2 = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $w_3 = \left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are orthonormal vectors.

10. Determine whether the set of all functions *f* with $f(1) = 0$ is a subspace of $C(-\infty, \infty)$.

11. Find the inverse of $A = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}$.

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} -1 & 2 \\ -7 & 8 \end{pmatrix}$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum Marks 30.

13. Find the length of the graph of $f(x) = (x-1)^{\frac{3}{2}} + 2$ on $[0, 2]$.

14. Find the area of the surface obtained by revolving the graph of x^3 on $[0, 1]$.

15. State ratio test for the power series. For which x does $\sum_{n=0}^{\infty} \frac{i}{i+1} x^i$ converge.

16. Find the Maclaurin series for $f(x) = \sin x$.

17. Use Grami-Schmidt orthonormalization process to transform the basis $\{u_1, u_2, u_3\}$ for $\mathbb{R}^3$ into an orthonormal basis $B' = \{w_1, w_2, w_3\}$, where $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 2)$ and $u_3 = (1, 1, 0)$.

18. State Cayley's theorem and using this theorem find the inverse of $A = \begin{pmatrix} -2 & 4 \\ -1 & 3 \end{pmatrix}$.

19. Diagonalize $A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum Marks 10.

20. (a) Find the area enclosed by the cardiod $r = 1 + \cos\theta$.

    (b) Using LU factorization to solve the system linear equations $AX = B$ where $A = \begin{pmatrix} 2 & -2 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$.

21. (a) Calculate $\sin\left(\frac{\pi}{4} + 0.06\right)$ to whithin 0.0001 by using Taylor series about $x_0 = \frac{\pi}{4}$.

    (b) Determine whether the vectors $u_1 = (1, -1, 3, -1)$, $u_2 = (1, -1, 4, 2)$ and $u_3 = (1, -1, 5, 7)$ are linearly dependent or independent.

(1 × 10 = 10 marks)

C 22093 **(Pages : 3)** **Name.....................................**

Reg. No.................................

SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 2C 01—MATHEMATICS—2

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find the inverse of $f(x)=\dfrac{2x-3}{5x-7}$, where the domain of f excludes $x=\dfrac{7}{5}$.

2. Find the Cartesian form of the polar equation $r = \sin 2\theta$.

3. Express the number $\coth^{-1}(5/4)$ in terms of natural logarithms.

4. Prove that $\tanh^2 x + \text{sech}^2 x = 1$.

5. Show that the series $1+\dfrac{1}{2}+\dfrac{1}{2^2}+\cdots$ converges and also find its sum.

6. Find the norm of the vector (3, 4, 0, 1, – 1). Also normalize the vector.

7. Determine the radius of convergence of $\sum_{k=0}^{\infty}\dfrac{k^5}{(k+1)!}x^k$.

8. Find a basis and then give the dimension of solution space of.

9. Find the inner product of the vectors $\mathbf{a}=\langle 1,2,3\rangle$ and $\mathbf{b}=\langle 0,-2,1\rangle$ in R^3. Are the vectors orthogonal ?

10. Show that $A=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$ is an orthogonal matrix.

Turn over

11. If $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$ find A^3 using Cayley Hamilton theorem.

12. Find the inverse of the 2 × 2 matrix $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on [1, 2].

14. Diagonalize the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

15. Find the length of the perimeter of the cardioid $r = a(1+\cos\theta)$.

16. Find an approximation value of $\int_0^1 x^2 dx$ by Simpson's rule with $n = 10$.

17. Expand $\log x$ in ascending powers of $x - 1$ as for the term containing $(x - 1)^4$.

18. $B_1 = \{u_1, u_2, u_3\}$, where $u_1 = \langle 2,-1,1 \rangle, u_2 = \langle 1,5,1 \rangle, u_3 = \langle 0,1,2 \rangle$, is a basis for $\mathbb{R}^2$. Transform it into an orthonormal basis $B_2 = \{w_1, w_2, w_3\}$.

19. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing it to the echelon form.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20\. (a) Find the area of the region shared by the circles $r = 1$ and $r = 2 \sin \theta$.

(b) Show that the set $B = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ is a basis for R^3.

21\. (a) Using Gauss-Jordan elimination method, solve the system of equations :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4. \end{aligned}$$

(b) Find the eigen values of $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$.

(1 × 11 = 11 marks)

C 4388-B **(Pages : 3)** **Name..**

Reg. No....................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS—2

(2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Prove that $\cosh^2 x - \sinh^2 x = 1$.

2. Find the Cartesian form of the polar equation $r = \dfrac{8}{1-2\cos\theta}$.

3. Find the slope of the line tangent to the graph of $r = 3\cos^2 2\theta$ at $\theta = \pi/6$.

4. Evaluate $\int \sinh^2 x dx$.

5. Show that $\lim\limits_{n\to\infty} \dfrac{2n}{n^2+1} = 0$.

6. Test the convergence of the series $1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{4}} - \dfrac{1}{\sqrt{8}} + \dfrac{1}{\sqrt{16}} \ldots$

7. Compute $\|\cos x\|$ in $C[0, 2\pi]$.

8. Examine whether the set of vectors $u_1 = \langle 1,2,3\rangle, u_2 = \langle 2,4,3\rangle$, and $u_3 = \langle 3,2,1\rangle$ is linearly independent or not.

9. Find the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$.

10. Find the determinant of the matrix $C = \begin{bmatrix} -1 & 2 & 9 \\ 2 & -4 & -18 \\ 5 & 7 & 27 \end{bmatrix}$.

Turn over

11. Show that $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is an orthogonal matrix.

12. Find the eigen values of the matrix $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*

Each question carries 5 *marks.*

All questions can be attended.

Overall Ceiling 25.

13. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1$.

14. Find the equation of the tangent line when $t = 1$ for the curve $x = t^4 + 2\sqrt{t}, y = \sin(t\pi)$.

15. Find the length of the perimeter of the cardioid $r = a(1 - \cos\theta)$.

16. Use the Trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. Compare the estimate with the exact value of the integral.

17. Using Maclaurin's series expand $\tan^{-1} x$. Hence deduce the Gregory series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

18. Show that the set B = {(1, 2, 1), (2, 1, 0), (1,–1, 2)} is a basis for R^3.

19. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Evaluate $\int_{1}^{\infty} \frac{ln x}{x^2} dx$, if it exists.

(b) Find the area of the region shared by the cardioids $r = 2(1 + \cos\theta)$ and $r = 2(1 - \cos\theta)$.

21. (a) Solve :

$$x_1 + x_2 + x_3 + x_4 = 0$$
$$x_1 + 3x_2 + 2x_3 + 4x_4 = 0$$
$$2x_1 + x_3 - x_4 = 0.$$

(b) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

(1 × 11 = 11 marks)

C 4388 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS—2

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Compute the derivative of $\sqrt{x}$ using inverse function rule. Evaluate the derivative at $x = 2$.

2. Convert the relation $r = 1 + 2\cos\theta$ to Cartesian co-ordinates.

3. Compute $\int \cosh^2 x\, dx$.

4. Find $\frac{d}{dx}\cosh^{-1}\sqrt{x^2+1}, x \neq 0$.

5. Find $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

6. Show that $\int_0^{\infty} \frac{\sin x}{\left(1+x^2\right)}\, dx$ converges.

7. A bouncing ball loses half of its energy on each bounce. The height reached on each bounce is proportional to the energy. Suppose that the ball is dropped vertically from a height of one meter. How far does it travel ?

Turn over

8. State Ratio comparison test and show that $\sum_{i=1}^{\infty} \frac{2}{4+i}$ diverges.

9. Prove that the vectors $w_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), w_2 = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $w_3 = \left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are orthonormal vectors.

10. Define basis of a vector space. Give a basis for vector space P_n of all polynomial of degree less than or equal to n.

11. Find the inverse of $A = \begin{pmatrix} 1 & 8 \\ 2 & 10 \end{pmatrix}$.

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Let $f(x) = x^2 + 2x + 3$. Restrict f to a suitable interval so that it has an inverse. Find the inverse function and sketch its graph.

14. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on $[0, 2]$.

15. State root test and test the convergence for the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$.

16. For which x does the series $\sum_{n=0}^{\infty} \frac{4^n}{\sqrt{2n+5}} (x+5)^n$ converge.

17. Let $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 2)$ and $u_3 = (1, 1, 0)$ be basis of $\mathbb{R}^3$. Using Gram Schimdt process find an orthonormal basis of $\mathbb{R}^3$.

18. Compute A^m for $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

19. Identify the conic whose equation is $2x^2 + 4xy - y^2 = 1$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Find the area of the surface obtained by revolving the graph $y = x^2$ about the y-axis for $1 \le x \le 2$.

    (b) Determine whether the set of vectors $u_1 = (1, 2, 3)$, $u_2 = (1, 0, 1)$ and $u_3 = (1, -1, 5)$ is linearly dependent or linearly independent.

21. (a) Find the terms through cubic order in the Taylor series for $\frac{1}{1+x^2}$ at $x_0 = 1$.

    (b) Find an LU factorization of $A = \begin{pmatrix} -1 & 2 & -4 \\ 2 & -5 & 10 \\ 3 & 1 & 6 \end{pmatrix}$.

(1 × 11 = 11 marks)

C 82446 (Pages : 2) Name.....................................

Reg. No.................................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Mathematics

MAT 2C 02—MATHEMATICS—II

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. If $f(x) = x^3 + 2x + 1$, show that f has an inverse on [0, 2], Find the derivative of the inverse function at $y = 4$.
2. Calculate the slope of the line tangent to $r = f(\theta)$ at (r, θ) if f has a local maximum there.
3. Prove that $\tanh^2 x + \operatorname{sech}^2 x = 1$.
4. Find $\int \frac{dx}{\sqrt{4+x^2}}$.
5. Show that $\int_0^\infty \frac{dx}{\sqrt{1+x^8}}$ is convergent, by comparison with $\frac{1}{x^4}$.
6. Find $\lim_{n\to\infty} \left(\frac{n^2+1}{3n^2+n}\right)$.
7. Sum the series $\sum_{i=1}^{\infty} \left(\frac{7}{8}\right)^i$.
8. State integral test and show that $\sum_{m=2}^{\infty} \frac{1}{m(\ln m)^2}$ converges.
9. Define dimension of a vector space. Find the dimension of the vector space P_n of all polynomial of degree less than or equal to n.
10. Determine whether the set of all functions f with $f(1) = 0$ is a subspace of the vector space $C(-\infty, \infty)$.

Turn over

11. Use inverse of coefficient matrix to solve the system :

$$\begin{aligned} 2x_1 - 9x_2 &= 15 \\ 3x_1 + 6x_2 &= 16. \end{aligned}$$

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 30 *marks.*

13. Polygonal line joining the points (2, 0), (4, 4), (7, 5) and (8, 3) is revolved about the x-axis. Find the area of the resulting surface of revolution.

14. Find the length of the cardiod $r = 1 + \cos\theta, 0 \leq \pi \leq 2\pi$.

15. Find the power series of the form $\sum_{i=0}^{\infty} a_i x^i$ fo. $\frac{23-7x}{(3-x)(4-x)}$. Also find the radius of convergence.

16. Evaluate $\lim_{x \to \infty} \frac{\sin x - x}{x^3}$ using a Macluarin's series.

17. Use Gram Schmidt orthonormalization process to transform the basis $\{u_1, u_2, u_3\}$ for $\mathbb{R}^3$ into an orthonormal basis $B' = \{w_1, w_2, w_3\}$, where $u_1 = (1, 1, 0)$, $u_2 = (1, 2, 2)$ and $u_3 = (2, 2, 1)$.

18. Compute A^m for $A = \begin{pmatrix} 8 & 5 \\ 4 & 0 \end{pmatrix}$.

19. Find LU factorization of $A = \begin{pmatrix} 2 & -8 \\ 3 & 0 \end{pmatrix}$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum 10 *Marks.*

20. (a) Find the area enclosed by the cardiod $r = 1 + \cos\theta$.

(b) Calculate $\sin\left(\frac{\pi}{4} + 0.06\right)$ to within 0.0001 by using Taylor's series about $x_0 = \frac{\pi}{4}$.

21. (a) Use an LU factorization to evaluate the determinant of $A = \begin{pmatrix} -1 & 2 & -4 \\ 2 & -5 & 10 \\ 3 & 1 & 6 \end{pmatrix}$.

(b) Find the rank of $A = \begin{pmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 8 \end{pmatrix}$.

(1 × 10 = 10 marks)