D 103063 **(Pages : 3)** **Name.................................**

Reg. No...............................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2024

Mathematics

MTS4C04—MATHEMATICS—4

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 20.

1. Solve $dx + e^{3x}\, dy = 0$.

2. Find general solution of $x\dfrac{dy}{dx} - 4y = x^6 e^x$.

3. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y\left(1 - x^2\right)}$, $y(0) = 2$.

4. Determine whether the functions $f_1(x) = x, f_2(x) = x^2, f_3(x) = 4x - 3x^2$ are linearly dependent or linearly independent on the interval $(-\infty, \infty)$.

5. The function $y_1 = \ln x$ is a solution of $xy'' + y' = 0$. Find a second solution $y_2(x)$.

6. Solve the initial value problem $4y'' + 4y' + 17y = 0$, $y(0) = -1$, $y'(0) = 2$.

7. Find a homogeneous linear differential equation with constant co-efficient whose solution is $y = c_1 \cos 8x + c_2 \sin 8x$.

8. Let $f(t) = \begin{cases} t, & 0 \le t \le 1 \\ 1, & t \ge 1 \end{cases}$. Find $\mathcal{L}\left(f(t)\right)$.

Turn over

9. Find $\mathcal{L}^{-1}\left(\frac{4s}{4s^2+1}\right)$.

10. Evaluate $\mathcal{L}(t \sin kt)$.

11. Show that the functions $f_1(x) = x$, $f_2(x) = \cos 2x$ are orthogonal on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - 6\frac{\partial u}{\partial y} = 0$ is parabolic .

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 30.

13. Solve $\left(x^2 + y^2\right) dx + \left(x^2 - xy\right) dy = 0$.

14. Solve the initial value problem $\frac{dy}{dx} = \cos(x + y), y(0) = \frac{\pi}{4}$.

15. Solve $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$.

16. Solve the homogeneous boundary value problem $y'' + \lambda y = 0, y(0), y(\text{L}) = 0$.

17. Solve $x^2y'' - 3xy' + 3y = 2x^4e^x$.

18. Solve $f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau) e^{t-\tau} \, d\tau$ for $f(t)$.

19. Expand $f(x) = x^2, 0 < x < \text{L}$ in a cosine series.

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Section C

Answer any **one** *questions.*

Each question carries 10 *marks.*

20. Solve the initial value problem $y'' + 4y' + 6y = 1 + e^{-t}$, $y(0) = 0$, $y'(0) = 0$ using Laplace transform.

21. Find the Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2, & 0 \le x < \pi \end{cases}$ on the interval $[-\pi, \pi]$.

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C 41232 **(Pages : 3)** **Name..................................**

Reg. No...............................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023

Mathematics

MTS 4C 04—MATHEMATICS—4

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 20.

1. Solve the initial value problem $\frac{dy}{dx} = \frac{-x}{y}$, $y(4) = -3$.

2. Solve $(x^2 - 9)\frac{dy}{dx} + xy = 0$.

3. Find the value of k so that the differential equation $(y^3 + kxy^4 - 2x)\,dx + (3xy^2 + 20x^2y^3)\,dy = 0$ is exact.

4. Verify that the functions e^{-3x}, e^{4x} form a fundamental set of solutions of the differential equation $y'' - y' - 12y = 0$ on $(-\infty, \infty)$.

5. The function $y_1 - e^x$ is a solution of $y'' - y — 0$ on the interval $(-\infty, \infty)$, use reduction of order to find a second solution y_2.

6. Find the general solution of $y''' - 4y'' - 5y' = 0$.

7. Solve the initial value problem $y'' + 4y' + 5y = 35e^{-4x}$, $y(0) = -3$, $y'(0) = 1$.

8. Find $\mathscr{L}(f(t))$, where $f(t) = \sin 2t \cos 2t$.

9. Evaluate $\mathscr{L}^{-1}\left(\frac{s}{(s-2)(s-3)(s-6)}\right)$.

Turn over

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10 Write $f(t)=\begin{cases}2, & 0 \le t < 3\\ -2, & t \ge 3\end{cases}$ in terms of unit step functions and find $\mathscr{L}\,(f(t))$

11. Show that the functions $f_1(x)=e^x, f_2(x)=xe^{-x}-e^{-x}$ are orthogonal on [0, 2],

12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2}+6\frac{\partial^2 u}{\partial x \partial y}+9\frac{\partial^2 u}{\partial y^2}$ is parabolic.

Section B

Answer any number of questions.
Each question carries 5 *marks. Ceiling is* 30.

13. Solve $x\frac{dy}{dx}+y=x^2y^2$.

14. Solve $\frac{dy}{dx}=(x+y+1)^2$ by using an appropriate substitution.

15. $y'''+y''=e^x\cos x$.

16. Solve $x^2\,y''-3xy'+3y=2x^4\,e^x$.

17. Solve $y'+6y=e^{4t}, y(0)=2$ using the Laplace transform.

18. Evaluate $\mathscr{L}^{-1}\left(\frac{1}{\left(s^2+k^2\right)^2}\right)$.

19. Expand $f(x)=x, -2<x<2$ in a Fourier series.

Section C

Answer any **one** *question.*

The question carries 10 *marks.*

20. Solve the initial value problem $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 17$ using Laplace transform.

21. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$ in a Fourier series.

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C 21546 **(Pages : 2)** **Name.......................................**

Reg. No...................................

FOURTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4C 04—MATHEMATICS – 4

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Write the order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+4y=\sin x$.

2. Verify that $y = xe^x$ is a solution of $y''-2y'+y=0$.

3. Show that $\left(25x^2-5y\right)dx+\left(3y^2-5x\right)dy=0$ is an exact differential equation.

4. Find the integrating factor corresponding to the differential equation $\frac{dy}{dx}+y\tan x=\cos x$.

5. Reduce $\frac{dy}{dx}=\left(y-2x^2\right)-7$ to an equation with separable variables.

6. Find the general solution of $y''-y'-2y=0$.

7. Find the particular integral of $y''+5y'+6y=e^{2x}$.

8. Find the Laplace transform of $\sin 3t\cos 2t$.

9. Find the Laplace transform of $e^{-3t}t^3$.

10. Write the inverse Laplace transform of $\frac{s}{s^2+16}$.

11. Show that the functions $f_1(x)=x^3$ and $f_2(x)=x^2+1$ are orthogonal on $[-1,1]$.

12. Show that the partial differential equation $3\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}$ is parabolic.

(8 × 3= 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Solve $(1+x)y\,dx+(1-y)x\,dy=0$.

14. Solve $\left(x^2+y^2\right)\frac{dy}{dx}=xy$.

15. Solve $y''+y=\tan x$ using the method of variation of parameter.

16. Find the Laplace transform of $\frac{1-\cos t}{t^2}$.

17. Find the inverse Laplace transform of $\frac{s^2+2s+5}{s^3}$.

18. Apply convolution theorem to evaluate the inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$.

19. Solve $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}-u=0$ using method of separation of variables.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Solve $x^3y'''-x^2y''+2xy'-2y=\cos(2\log x)$.

21. Expand $f(x)=x\sin x$ as a Fourier series in $0<x<2\pi$..

(1 × 11 = 11 marks)

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(Pages : 2)

Name...

Reg. No.....................................

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FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MTS 4C 04—MATHEMATICS—4

Time : Two Hours

Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. If $x = C_1 \cos 4t + C_2 \sin 4t$ is a solution of $x'' + 16x = 0$, $x(\pi/2) = -2$, $\pi'(\pi/2) = 1$, then find C_1 and C_2.

2. Write any two solutions of $y'' = y'$ by inspection..

3. Solve $\dfrac{dy}{dx} = \sin 5x$.

4. Check whether $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y(1-x^2)}$ is an exact differential equation.

5. Write the integrating factor corresponding to the differential equation $\dfrac{xdy}{dx} - 4y = x^6 e^x$.

6. Show that $y = 3e^{2x} + e^{-2x} - 3x$ is a unique solution of $y'' - 4y = 12x, y(0) = 4, y'(0) = 1$.

7. Check whether $y_1 = e^{3x}$ and $y_2 = e^{-3x}$ are linearly independent solutions of $y'' - 9y = 0$.

8. If $y_1 = x^2$ is a solution of $x^2 y'' - 3xy' + 4y = 0$, find a second solution.

9. Find the Laplace transform of e^{-3t}.

10. Find the inverse Laplace transform of $\dfrac{6-2s}{s^2+4}$.

11. Check whether $f_1(x) = x, f_2(x) = \cos 2x$ are orthogonal in $[-\Pi/2, \Pi/2]$.

12. Show that the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ is elliptic.

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Solve $\left(x^2+y^2\right)dx+\left(x^2-xy\right)dy=0$.

14. Solve the initial value problem $\frac{dy}{dx}=(-2x+y)^2-7, y(0)=0$.

15. Find the particular integral of $(D-3)^3 y=xe^{-2x}$.

16. Using the method of variation of parameters solve $y''+3y'+2y=3e^{-2x}+x$.

17. Convert the equation $xy''-3y'+x^{-1}y=x^2$ as a linear equation with constant co-efficients.

18. Find the Laplace transform of $\frac{\cos at-\cos bt}{t}$.

19. Find the half range sine series of $f(x)=\cos x$ in $0<x<\pi$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
Each question carries 11 marks.

20. Solve the differential equation $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4, y(0)=2, y'(0)=3$, using Laplace Transform.

21. Obtain the Fourier series of $f(x)=x^2-2$ in the interval $(-2,2)$.

(1 × 11 = 11 marks)