D 110209 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2024

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. Prove that there does not exist a rational number r such that $r^2 = 2$.
2. Determine the set A of $x \in \mathbb{R}$ such that $|2x+3| \leq 7$.
3. If $a, b \in \mathbb{R}$, prove that $|a+b| \leq |a| + |b|$.
4. State the supremum property of $\mathbb{R}$.
5. If $S = \left\{\frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N}\right\}$, find inf S and sup S.
6. State and prove Archimedean property.
7. Let x and y be real numbers with $x < y$, prove there exists an irrational number z such that $x < z < y$.
8. State and prove squeeze theorem.
9. If a sequence (x_n) of real numbers converges to a real number x, prove that any subsequence (x_{nk}) of (x_n) also converges to x.

Turn over

10. Prove that every Cauchy sequence of real numbers is bounded.

11. Let (x_n) and (y_n) be two sequence of real numbers and suppose that $x_n \leq y_n$ for all $n \in \mathbb{N}$. If $\lim x_n = +\infty$, prove that $\lim y_n = +\infty$.

12. Prove that the intersection of any finite collection of open sets in $\mathbb{R}$ is open.

13. Compute $\left(1+\sqrt{3}i\right)^9$.

14. Find the real and imaginary parts of $f(z) = z^2 - (2+i)z$ as a function of x and y.

15. Show that the complex function $f(z) = z + 3i$ is a one to one on the entire complex plane and find a formula for its inverse function.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 marks.

16. State and prove Cantor's theorem.

17. Let a and b be positive real numbers, prove that $\sqrt{ab} \leq \dfrac{a+b}{2}$ and the equality occurs if and only if $a = b$.

18. State and prove density theorem.

19. Prove that unit interval $[0,1]$ is not countable.

20. State and prove monotone convergence theorem.

21. Let $F \subseteq \mathbb{R}$; prove that the following are equivalent :

    (a) F is a closed subset of $\mathbb{R}$;

    (b) If $X = (x_n)$ is any convergent sequence of element in F, then lim X belongs to F.

22. Find an upper bound for $\left|\dfrac{-1}{z^4 - 5z + 1}\right|$ if $|z| = 2$.

23. For any *two* complex numbers, prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|\right)^2$.

Section C

Answer any **two** *questions.*

Each question carries 10 *marks.*

24. a) If A_m is a countable set for each $m \in \mathbb{N}$, prove that $A = \bigcup_{m=1}^{\infty} A_m$ is countable.

b) State and prove Bernoulli's inequality.

25. a) State and prove monotone convergence theorem.

b) Let $s_1 = 1$ and $s_{n+1} = \frac{1}{2}\left(s_n + \frac{a}{s_n}\right)$ for $n \in \mathbb{N}$. Prove that (s_n) converges to $\sqrt{a}$.

26. a) Prove that every contractive sequence is a Cauchy sequence.

b) Let $f_1 = 1, f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Define $x_n = \frac{f_n}{f_{n+1}}$. Prove that $\lim x_n = \frac{-1+\sqrt{5}}{2}$.

27. Find a complex linear function that maps the equilateral triangle with vertices $1+i, 2+i$ and $\frac{3}{2} + \left(1 + \frac{1}{2}\sqrt{3}\right)i$ onto the equilateral triangle with the vertices $i, \sqrt{3} + 2i$ and $3i$.

(2 × 10 = 20 marks)

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D 50666 **(Pages : 3)** **Name.......................................**

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Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. State Cantor's theorem.

2. Prove that there does not exist a rational number r such that $r^2 = 3$.

3. If $a, b \in \mathbb{R}$, prove that $\left\| a \right| - \left| b \right\| \leq \left| a - b \right|$.

4. Prove that an upper bound u of a nonempty set S in $\mathbb{R}$ is the supremum of S if and only if for every $\in > 0$ there exist an $s_{\in} \in \mathrm{S}$ such that $u - \in < s_{\in}$.

5. State and prove Archimedean property.

6. Prove that a sequence in $\mathbb{R}$ can have at most one limit point.

7. Prove that $\lim \left(\frac{\sin n}{n} \right) = 0$.

8. Let $e_n = \left(1 + \frac{1}{n} \right)^n$ for $n \in \mathbb{N}$. Prove that $2 < e_n < 3$ for all $n \in \mathbb{N}$.

9. Give an example of an unbounded sequence that has a convergent subsequence.

10. If (x_n) and (y_n) are Cauchy sequences, prove that $(x_n + y_n)$ is a Cauchy sequence.

Turn over

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11. Prove that the sequence $\left(\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{n}\right)$ diverges.

12. Define Cantor set.

13. Describe the set of points z in the complex plane that satisfy the equation $|z-2|=\text{Re}(z)$.

14. Find the image of the line segment from 1 to i under the complex mapping $\omega=\overline{iz}$.

15. Find the image of the rectangle with vertices $-1+i,\ 1+i,\ 1+2i$ and $-1+2i$ under the linear mapping $f(z)=4iz+2+3i$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. Prove that the following statements are equivalent :

    a) S is a countable set ; and

    b) There exists a surjection of $\mathbb{N}$ onto S.

    c) There exists an injection of S onto $\mathbb{N}$.

17. Determine the set $B=\{x\in\mathbb{R}:|x-1|<|x|\}$.

18. Prove that $\mathbb{R}$ of real numbers is not countable.

19. Let $X=(x_n)$ and $Y=(y_n)$ be sequences of real numbers that converge to x and y respectively. Prove that $X\cdot Y$ converges to xy.

20. State and prove Cauchy convergence criterion for sequences.

21. Prove that

    a) the union of an arbitrary collection of open subsets in $\mathbb{R}$ is open.

    b) the intersection of any finite collection of open sets in $\mathbb{R}$ is open.

22. Determine whether the points $z_1 = -2 - 8i, z_2 = 3i, z_3 = -6 - 5i$ are the vertices of a right triangle.

23. Let $S = \{z \in \mathbb{C} : 1 \leq |z - 1 - i| < 2\}$. Determine whether the set S is :

   a) Open ;

   b) Closed ;

   c) Domain ;

   d) Bounded ; and

   e) Connected.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Let a and b be positive real numbers, prove that $\sqrt{ab} \leq \frac{1}{2}(a+b)$ and the equality holds if and only if $a = b$.

    (b) State and prove Bernoulli's inequality.

25. Prove that there exits a positive real number x such that $x^2 = 2$.

26. (a) Prove that every contractive sequence is a convergent sequence.

    (b) The polygonal equation $x^3 - 7x + 2 = 0$ has a root r with $0 < r < 1$. Use an appropriate contractive sequence to calculate r within 10^{-5}.

27. (a) Solve the simultaneous equations $|z| = 2$ and $|z - 2| = 2$.

    (b) Find the image of the triangle with vertices $0, 1+i$ and $1-i$ under the mapping $\omega = z^2$.

(2 × 10 = 20 marks)

D 30569 **(Pages : 3)** **Name.......................................**

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FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 Marks.
Maximum 25 Marks.

1. Define denumerable set. Give an example.
2. If $a \in \mathbb{R}$, then prove that $a \cdot 0 = 0$.
3. Let a, b, c be elements of $\mathbb{R}$ and if $a > b$ and $b > c$, then prove that $a > c$.
4. Prove that $|-a| = |a|$ for all $a \in \mathbb{R}$.
5. Describe Fibonacci sequence.
6. State Monotone Convergence Theorem.
7. Define Cauchy sequence. Give an example.
8. Define properly divergent sequence.
9. Show that $\mathbb{R} = (-\infty, \infty)$ is open.
10. Describe any two properties of Cantor Set.
11. Whether the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \ldots\right)$ is convergent ? Justify your answer.
12. Find the principal cube root at the point $z = i$.
13. Define bounded subset of the complex plane.

Turn over

14. Find the reciprocal of $z = 2 - 3i$.

15. Express $-\sqrt{3} - i$ in polar form.

Section B

Answer any number of questions.
Each question carries 5 Marks.
Maximum 35 marks.

16. State and prove Cantor's Theorem.

17. Prove that there does not exist a rational number r such that $r^2 = 2$.

18. (a) Define supremum of a set of real numbers.

    (b) Prove that there can be only one supremum of a given subset S of $\mathbb{R}$, if it exists.

19. Prove that $\lim\left(\frac{1}{n^2}\right) = 0$.

20. If $0 < b < 1$, then prove that $\lim (b^n) = 0$.

21. Prove that the intersection of an arbitrary collection of closed sets in $\mathbb{R}$ is closed.

22. Show that the complex function $f(z) = z + 3i$ is one-to-one on the entire complex plane and find a formula for its inverse function.

23. If $f(z) = \frac{z}{\bar{z}}$ then show by two path test that $\lim_{z \to 0} f(z)$ does not exist.

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. State and prove Monotone Subsequence Theorem.

25. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded. If $X = (x_n)$ is a bounded increasing sequence, then prove that :

    $\lim(x_n) = \sup\{x_n : n \in N\}$.

26. Let X = (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then prove that the sequence $\left(\sqrt{x_n}\right)$ converges and $\lim\left(\sqrt{x_n}\right) = \sqrt{x}$.

27. Show that the function f defined by :

$$f(z) = \sqrt{r}e^{i\theta/2}, -\pi < \theta < \pi$$

is a branch of the multiple-valued function $F(z) = z^{1/2}$.