D 110212 (Pages : 3) Name.....

Reg. No.....

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2024

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Not more than 20 *marks can be earned from this unit.*
Each question carries 2 *marks.*

1. State Reflection Property of the Parabola.
2. Determine the equation of the tangent to the ellipse with parametric equations

   $x = 3\cos t, y = \sin t$

   at the point with parameter value $t = \pi/4$.
3. Prove that Euclidean-congruence is a symmetric relation.
4. Give the inverse of the affine transformation $t(\boldsymbol{x}) = \mathbf{A}\boldsymbol{x} + \boldsymbol{b}$.
5. Find the quotient and remainder obtained when $f(x) = 2x^7 - 3x^6 + x^5 - 3x^4 + 5x^3 - 4x^2 + 2x - 1$ is divided by $g(x) = 2x^3 - 3x^2 + x - 1$.
6. Calculate the values of the polynomial $4x^3 - 7x^2 + 5x + 3$ and their derivatives for the value of $x = -2$.
7. State the Fundamental theorem of Algebra.
8. Verify that i is a zero of $f(x) = x^3 + 2x - i$
9. How many real roots has the equation $x^4 - 4ax + b = 0$?

Turn over

10. Verify that the equation $x^3 - 7x + 7 = 0$ has roots in the interval $\left(\frac{3}{2}, 2\right)$.

11. State True/False : Let α, β, γ are the roots of the equation $f(x) = 0$, then $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \cdots$ are the roots of the equation $f\left(\frac{1}{x}\right) = 0$.

12. State True/False : If the equation contains only even powers of x and the co-efficients are all of the same sign, the equation has no real root.

Section B

Not more than 30 *marks can be earned from this unit.*
Each question carries 5 *marks.*

13. Prove that 2×2 matrix **P** represents a rotation of $\mathbb{R}^2$ about the origin if and only if it satisfies the following two conditions :

    (a) **P** is orthogonal ;

    (b) det **P** = 1.

14. Determine the affine transformation which maps the points (2, 3), (1, 6) and (3, – 1) to the points (1, – 2), (2, 1) and (– 3, 5), respectively.

15. Show that the roots of the equation

    $$x^3 + px^2 + qx + r = 0$$

    are in arithmetic progression if $2p^3 - 9pq + 27r = 0$.

16. If α, β, γ are roots of $x^3 + px^2 + qx + r = 0$, find the values of $\sum \frac{1}{\beta\gamma}$ in terms of co-efficients of the equation.

17. Find an upper limit of the positive roots of the equation

    $$x^5 - 7x^4 - 100x^3 - 1000x^2 + 10x - 50 = 0.$$

18. Find the rational roots of the equation $6x^4 - 7x^3 + 8x^2 - 7x + 2 = 0$.

19. Using Descartes' Rule of signs, show that the equation :

$$x^6 - x^3 + 2x^2 - 3x - 1 = 0$$

has four imaginary roots.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*

20. (a) Prove that an affine transformation maps parallel straight lines to parallel straight lines.

(b) If α, β and γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form, the equation whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$.

21. (a) Solve the biquadratic equation $x^4 - 3x^2 + 6x - 2 = 0$.

(b) Solve $x^3 - 6x^2 + 3x - 2 = 0$ by Cardano's method.

(1 × 10 = 10 marks)

D 50669 **(Pages : 2)** **Name.....**

Reg. No.....

FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Ceiling is 20.

1. Find equation of the normal to the parabola $y^2 = x$ at the point (1,1).
2. Find the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.
3. What is the reflection property of the hyperbola ?
4. Find the matrix form of the conic $11x^2 + 4xy + 14y^2 - 4x - 28y - 16 = 0$.
5. Show that $x^3 + x^2 - 5x + 3$ is divisible by $x + 3$.
6. Write a cubic equation with the roots 1,2,3.
7. State the Identity Theorem.
8. Find the multiplicity of the root $x = 1$ of the polynomial $f(x) = x^n - nx + n - 1$.
9. Show that the polynomial $f(x) = x^{11} - 1$ has no roots in the interval $(-1,0)$.
10. Find Δ of the equation $x^3 - 10x - 12 = 0$.
11. Show that $\sqrt{2} - \sqrt{3}$ is a root of the equation $x^4 - 10x + 1 = 0$.
12. Find the cubic resolvent corresponding to the bi quadratic equation $x^4 + 4x - 1 = 0$.

(Ceiling 20)

Turn over

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Ceiling is 30.

13. Show that $t(x)=\begin{pmatrix}1 & 3\\ 1 & 2\end{pmatrix}x+\begin{pmatrix}4\\ -2\end{pmatrix}$ is an affine transformation and find the inverse.

14. (i) State the Fundamental theorem of Affine Geometry.

    (ii) Determine the affine transformation which maps the points (0, 0), (1, 0) and (0, 1) to the points (3, 2), (5, 8) and (7, 3), respectively.

15. Find the rational roots of the equation $6x^4-7x^3+8x^2-7x+2=0$.

16. Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

17. Factorize in to real linear and quadratic factors of the polynomial $f(x)=x^4+1$.

18. Show that the necessary and sufficient condition for an equation $x^3+px+1=0$ to have three real and distinct roots is $p^3<-27/4$.

19. How many real roots of the equation $f(x)=x^4-32x+1=0$.

(Ceiling 30)

Section C

Answer any **one** *question.*
The question carries 10 marks.

20. Prove that the conic with the equation $3x^2-10xy+3y^2+14x-2y+3=0$ is a hyperbola. Determine its centre, and its major and minor axis.

21. Solve the cubic equation $x^3+x^2-2=0$ by using Carden's formula.

(1 × 10 = 10 marks)

D 30572 **(Pages : 2)** **Name.....**

Reg. No.....

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks. Ceiling is* 20.

1. Does the equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ represents a hyperbola ? Justify your answer.

2. Find the vertex and directrix of the parabola $y^2 = 4x$.

3. Show that the parametric equation $x = 3\cos t, y = 2\sin t; -\pi < t \leq \pi$ represents the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

4. State the reflection property of the parabola.

5. Find the reminder when $f(x) = x^7 - 7x^3 + 1$ is divisible by $x + 1$.

6. Find the sum of the squares of the roots of the equation $x^4 - 2x + 1 = 0$.

7. State the Fundamental Theorem of Algebra.

8. Solve the equation $(a-b)x^2 - (b-c)x + (c-a) = 0$.

9. Find Δ of the equation $x^3 + 10x - 7 = 0$.

10. Show that $\sqrt[3]{\sqrt{5}+2} - \sqrt[3]{\sqrt{5}-2} = 1$.

Turn over

11. Show that the equation $x^6 - x^4 + 4x - 11 = 0$ has a root lies between 1 and 2.

12. Find the real root of the cubic equation $x^3 + 9x - 2 = 0$.

Section B

Answer any number of questions.
Each question carries 5 *marks. Ceiling is* 30.

13. (i) State the Fundamental theorem of Affine Geometry.

    (ii) Determine the affine transformation which maps the points $(0, 0), (1, 0)$ and $(0, 1)$ to the points $(3, 2), (5, 8)$ and $(7, 3)$ respectively.

14. Determine the image of the line $y = -x$ under the affine transformation $t(x) = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} \times + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

15. Solve $x^3 - 9x^2 + 26x - 24 = 0$ if the roots form an arithmetic progression $\alpha - \beta, \alpha, \alpha + \beta$.

16. Factorize into real linear and quadratic factors of the polynomial $f(x) = x^4 + 9$.

17. Find the rational roots of the equation $4x^3 - 4x^2 - x + 1 = 0$.

18. Separate the roots of the equation $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$.

19. Show that the necessary and sufficient condition for an equation $x^3 + px + q = 0$ to have three real and distinct roots is $4p^3 + 27q^2 < 0$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. (i) Prove that a perpendicular from a focus of a parabola to a tangent meets the tangent on the directrix of the parabola.

    (ii) Determine the equation of the tangent to the ellipse with parametric equations $x = 3\cos t, y = \sin t$ at the point with parameter $t = \pi/4$. Deduce the co-ordinates of the point of intersection of this tangent with the x-axis.

21. Solve the cubic equation $x^3 - 3x^2 + 12x + 16 = 0$ by using Carden's formula.

(1 × 10 = 10 marks)