**D 110210** **(Pages : 3)** **Name.......................................**

**Reg. No...................................**

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION**
**NOVEMBER 2024**

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

**Section A**

*Answer any number of questions.*
*Each question carries 2 marks.*
*Ceiling is 20.*

1. Find the polynomial of degree one passing through the points $(1, 2)$ and $(2, -1)$.

2. What you mean by Interpolation.

3. Write The Newton's Forward difference formula.

4. State Fixed Point Theorem.

5. Show that the equation $f(x) = x^5 - x - 5$ has a root between 1 and 2.

6. State Weierstrass approximation theorem.

7. Find the zeroth divided difference of the function $f(x) = x^2 - 1$ at $x_1 = 2$.

8. Write Three Point Mid Point Formula

9. Write the Trapezoidal rule for $\int_0^2 (x^2 + 1)\, dx$.

10. Write Newton's iteration formula for computing $\sqrt[3]{7}$

11. Does the set $\{(t, y), -1 < t < 1, -1 < y < 1\}$ is a convex set ? Justify your answer.

12. State Lipschitz condition.

**Turn over**

## Section B

*Answer any number of questions.*
*Each question carries 5 marks.*
*Ceiling is 30.*

13. Use Lagrange interpolating polynomial of degree three to approximate $f(10)$ if $f(5)=12, f(6)=13, f(9)=14, f(11)=16$.

14. The following table lists the values of $f$ at various points.

| $x$ | $f(x)$ |
|---|---|
| 20 | 0.3420 |
| 23 | 0.3907 |
| 26 | 0.4384 |
| 29 | 0.4848 |

Use the Newton forward difference formula to construct interpolating polynomial for this data. Also find $f(21)$.

15. Find the real positive root of $f(x)=x-\cos x-1=0$ by Newton's method.

16. Consider the following table of data :

| $x$ | $f(x)$ |
|---|---|
| 50 | 3.6840 |
| 51 | 3.7084 |
| 52 | 3.7325 |
| 53 | 3.7563 |
| 54 | 3.7798 |
| 55 | 3.8030 |
| 56 | 3.8259 |

Use forward difference formula to approximate the value of $f'(50)$.

17. Evaluate $\int_{-3}^{3} x^4 \, dx$ by using (i) Trapezoidal rule ; and (ii) Simpson's rule.

18. Apply Taylor's method of order two to approximate the solution for the initial value problem $y' = e^{t-y}, 0 \leq t \leq 1, y(0) = 1, h = 0.5$.

19. Use Euler's method to approximate the solution for $y' = y + e^t, y(0) = 0, h = 0.2$.

**Section C**

*Answer any* **one** *question.*
*The question carries* 10 *marks.*

20. Find the positive root of $x^4 - x^3 - 2x^2 - 6x - 4 = 0$ by Bisection method within $10^{-4}$ accuracy.

21. Use the Runge - Kutta method of order four with $h = 0.2, \mathrm{N} = 10, t_i = 0.2i$ to obtain approximations to the solutions of the initial value problem $y' = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$.

(1 × 10 = 10 marks)

**D 50667** **(Pages : 3)** **Name.........................................**

**Reg. No....................................**

## FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
## NOVEMBER 2023

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

### Section A

*Answer any number of questions.*
*Each question carries 2 marks.*
*Ceiling is 20.*

1. State the formula for Newton's method.

2. Find all fixed points of the function $f(x) = \dfrac{x^3 - 1}{x^2 + 1}$.

3. What is an algebraic polynomial ? Give an example.

4. State Fixed Point Theorem.

5. State one advantage of Secant method over Newton's method.

6. Write Newton's backward difference formula.

7. Write the Simpson's rule for $\int_0^2 x^2 \, dx$.

8. Write the formula for the method of false position

9. Does the set $\{(t, y), -1 < t < 2, 0 < y < 1\}$ is a convex set ? Justify your answer.

10. What is Lipschitz constant ?

11. What is a well posed problem ?

12. What is the 'Degree of Accuracy' of a quadrature formula ?

**Turn over**

## Section B

*Answer any number of questions.*
*Each question carries* 5 *marks.*
*Ceiling is* 30.

13. Find the positive root of $x = \cos x$ using Newton's method.

14. Use Newton's forward difference formula to find a polynomial of degree four which takes the values :

| $x$ | $f(x)$ |
|---|---|
| 2 | 0 |
| 4 | 0 |
| 6 | 1 |
| 8 | 0 |
| 10 | 0 |

15. Using Lagrange's formula of interpolation find $f(9.5)$ given :

| $x$ | $f(x)$ |
|---|---|
| 7 | 3 |
| 8 | 1 |
| 9 | 1 |
| 10 | 9 |

16. Approximate the integral $\int_0^6 \frac{1}{1+x^2}$ using Simpson's rule.

17. Consider the following table of data :

| $x$ | $f(x)$ |
|---|---|
| 50 | 3.6840 |
| 51 | 3.7084 |
| 52 | 3.7325 |
| 53 | 3.7563 |
| 54 | 3.7798 |
| 55 | 3.8030 |
| 56 | 3.8259 |

Use backward difference formula to approximate the value of $f'(56)$.

18. Use Euler's method to approximate the solution for $y' = t + y, \ y(0) = 1, \ h = 0.2$.

19. Apply Taylor's method of order two to approximate the solution for the initial value problem $y' = y - t^2 + 1, \ 0 \le t \le 2, \ y(0) = 0.5$.

## Section C

*Answer any* **one** *question.*
*The question carries* 10 *marks.*

20. Show that the Mid point method and Modified Euler method give the same approximations to the initial value problem $y' = -y + t + 1, \ 0 \le t \le 1, \ y(0) = 1$ for any choice of $h$. Why is this true ?

21. Find the positive root of $x^3 - 9x + 1 = 0$ by Bisection method within $10^{-4}$ accuracy.

(1 × 10 = 10 marks)

**D 30570** **(Pages : 2)** **Name.......................................**

**Reg. No..................................**

# FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

## Section A

*Answer any number of questions.*
*Each question carries 2 marks. Ceiling is* 20.

1. Compute the absolute error and relative error in the approximation of $p$ by $p^*$, where $p = \pi$, $p^* = \frac{22}{7}$.
2. Write Newton-Raphson Formula and state Intermediate Value theorem.
3. Let $f(x) = e^x - x - 1$. Show that $f$ has a zero of multiplicity 2 at $x = 0$.
4. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$.
5. Express $\Delta^2 f_0$ and $\Delta^3 f_0$ in terms of the values of the function.
6. State Neville's Method.
7. Write Newton's Divided Difference Formula.
8. What is the degree of accuracy of a quadrature formula ?
9. Write the Legendre Polynomials $P_0(x), P_1(x), P_2(x), P_3(x)$ and $P_4(x)$.
10. Define a convex set in $\mathbb{R}^2$.
11. Give the difference equation form of Runge-Kutta method of order four.
12. Define an implicit three-step method known as the fourth-order Adams-Moulton Technique.

**Turn over**

247254

## Section B

*Answer any number of questions.*
*Each question carries 5 marks. Ceiling is 30.*

13. Show that $g(x) = \dfrac{x^2 - 1}{3}$ has a unique fixed point on the interval $[-1, 1]$.

14. The function $f \in C'[a, b]$ has a simple zero at $p$ in $(a, b)$ if and only if $f(p) = 0$ and $f'(p) \neq 0$.

15. Suppose $x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, x_4 = 6$ and $f(x) = e^x$. Determine the interpolating polynomial $P_{(1,2,4)}$ and use this to approximate $f(5)$.

16. Use the forward difference formula to approximate the derivative of $f(x) = ln\, x$ at $x_0 = 1.8$ with $h = 0.1, 0.05$ and $h = 0.01$ and determine bounds for the approximation errors.

17. Determine value of $h$ that will ensure an approximation error of less than 0.00002 when approximating $\int_0^\pi \sin x\, dx$ using composite Simpson's rule.

18. Approximate $\int_{-1}^{1} e^x \cos x\, dx$ using Gaussian quadrature with $n = 3$.

19. Show that the initial value problem

    $\dfrac{dy}{dt} = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$ is well posed on $D = \{(t, y) : 0 \leq t \leq 2, -\infty < y < \infty\}$.

## Section C

*Answer any* **one** *question.*
*The question carries 10 marks.*

20. By fixed point iteration method determine a solution accurate to within $10^{-2}$ for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

21. Apply Taylor's method of order 2 with $N = 10$ to the initial value problem $y' = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$.

(1 × 10 = 10 marks)

247254

247192

**D 30561** **(Pages : 2)** **Name.........................................**

**Reg. No....................................**

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2022**

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2019 Admissions only)

Time : Two Hours Maximum : 60 Marks

**Section A**

*Answer any number of questions.*
*Each question carries 2 marks.*
*Ceiling is 20 marks.*

1. State Intermediate value Theorem.
2. Determine the fixed points of the function $f(x) = x^2 - 2$.
3. Set up Newton's iteration formula for computing $\sqrt[3]{24}$.
4. State the formula for method of false Position.
5. Write the Lagrange Interpolating polynomial through (2, 4) and (5, 1).
6. Write Newton's Forward difference formula.
7. Write second derivative Mid Point formula.
8. Write Simpson's Three- Eighths Rule formula.
9. Define Numerical quadrature.
10. Show that $f(t, y) = t|y|$ satisfies a Lipschitz condition on the interval

    D = $\{(t, y) : 1 \le t \le 2, \text{ and } -3 \le y \le 4\}$.
11. What does local truncation error at a specified step of an approximation method measure ?
12. What is the local truncation error, if Taylor's method of order $n$ is used to approximate the solution to $y'(t) = f(t, y(t)), a \le t \le b, y(a) = \alpha$ and with step size $h$ and if $y \in C^{n+1}[a, b]$ ?

**Turn over**

## Section B

*Answer any number of questions.*
*Each question carries 5 marks.*
*Ceiling is 30 marks.*

13. Approximate the root of the function $f(x) = \cos x - x = 0$ using Newton's method with $p_0 = \pi/4$.

14. Given $f(2) = 5$, $f(2.5) = 6$. Evaluate $f(2.2)$ using Lagrange's Method.

15. Using Newton's divided difference interpolation formula evaluate $f(3)$ from the following table :

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | : | 1 | 2 | 4 | 5 | 6 |
| $y$ | : | 14 | 15 | 5 | 6 | 19 |

16. Use Newton's forward-difference formula to approximate the derivative of $f(x) = In\ x$ at $x_0 = 1.8$ using $h = 0.01$, $h = 0.05$ and $h = 0.01$ and determine bounds for the approximation errors.

17. The values for $f(x) = xe^x$ are given. Use Three-point end point formula to approximate $f'(2.0)$ with $h = 0.1, -0.1$ :

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | : | 1.8 | 1.9 | 2 | 2.1 | 2.2 |
| $xe^x$ | : | 10.889365 | 12.703199 | 14.778112 | 17.148957 | 19.855030 |

18. Approximate the integral $\int_{0.5}^{1} x^4 dx$ using Trapezoidal Rule.

19. Use Euler Method to approximate the solution of the initial value problem

$y' = 1 + (t - y)^2, 2 \le t \le 3, y(2) = 1$ with $h = 0.5$.

## Section C

*Answer any* **one** *question.*
*The question carries 10 marks.*
*Maximum marks 10.*

20. Find a positive root of the equation $f(x) = xe^x - 1$ correct to 3 decimal places using Bisection Method.

21. Use the Midpoint method with $N = 10$, $h = 0.2$, $t_i = 0.2i$, and $w_0 = 0.5$ to approximate the solution to $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$.

14777

**D 10668** **(Pages : 3)** **Name..................................**

**Reg. No..............................**

# FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2019 Admissions)

Time : Two Hours | Maximum : 60 Marks

## Section A

*Answer at least* **eight** *questions.*
*Each question carries 3 marks.*
*All questions can be attended.*
*Overall Ceiling 24.*

1. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$.

2. Determine fixed points of the function $g(x) = x^2 - 2$.

3. Write the equation of Lagrange's interpolating polynomial through $(x_0, y_0)$ and $(x_1, y_1)$.

4. State three point end point formula of differentiation.

5. Using Trapezoidal rule find $\int_0^2 x^2 \, dx$.

6. Show that $f(t, y) = t|y|$ satisfies a Lipschitz condition on the interval $D = \{(t, y)/1 \le t \le 2$ and $-3 \le y \le 4\}$.

7. Define a convex set.

8. For all $x \ge -1$ and any positive $m$ show that $0 \le (1 + x)^m \le e^{mx}$.

9. When is the initial value problem $\frac{dy}{dt} = f(t, y), \; a \le t \le b, \; y(a) = \alpha$ well posed.

10. What is the degree of accuracy or precision of a quadrature formula ?

**Turn over**

11. Write Newton's Forward difference formula.

12. Set up Newton-Raphson formula for computing $\sqrt{N}$.

(8 × 3 = 24 marks)

**Section B**

*Answer at least* **five** *questions.*
*Each question carries* 5 *marks.*
*All questions can be attended.*
*Overall Ceiling* 25.

13. Find a root of $f(x) = x^3 - 3x - 5 = 0$ correct to 3 decimal places using Newton-Raphson method. Start with $x_0 = 3$.

14. Using Lagrange's interpolation formula find $y$ (10) if :

| | | | | | |
|---|---|---|---|---|---|
| $x$ | : | 5 | 6 | 9 | 11 |
| $y$ | : | 12 | 13 | 14 | 16 |

15. Using Newton's forward interpolation formula find the cubic polynomial for the data :

| | | | | | |
|---|---|---|---|---|---|
| $x$ | : | 0 | 1 | 2 | 3 |
| $y$ | : | 1 | 2 | 1 | 10 |

16. Approximate $\int_1^2 \frac{1}{x}\, dx$ using Simpson's $\frac{3}{8}^{\text{th}}$ rule with step value $h = 0.25$

17. Using Second derivative midpoint formula approximate $f^{11}(1.3)$ if $f(x) = 3xe^x - \cos x$ with $h = 0.1$. Given :

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | : | 1.2 | 1.29 | 1.30 | 1.31 | 1.40 |
| $y$ | : | 11.59006 | 13.78176 | 14.04276 | 14.30741 | 16.86187 |

18. Use Euler's method to find approximate solution for the initial value problem $y^1 = 1 + \frac{y}{t}$, $1 \leq t \leq 2$, $y(1) = 2$ with $h = 0.25$.

19. Use Newton's Backward difference formula to construct interpolating polynomial of degree 1 if $f(-0.75) = -.07181250$, $f(-0.5) = -.02475000$, $f(-.25) = .33493750$, $f(0) = 1.10100000$.

(5 × 5 = 25 marks)

**Section C**

*Answer any* **one** *question.*
*The question carries* 11 *marks.*

20. Find by the method of Regula Falsi a root of the equation $x^3 + x^2 - 3x - 3 = 0$ lying between 1 and 2.

21. Use the Modified Euler method to approximate the solutions to the IVP $y^1 = \frac{1+t}{1+y}, 1 \leq t \leq 2,$ $y(1) = 2$ with $h = 0.5$.

(1 × 11 = 11 marks)