INDIAN CONVERTIBLE BONDS WITH UNSPECIFIED TERMS
A VALUATION MODEL

by

Samir K. Barua and Jayanth Rama Varma

The first author is currently with RVB: The Netherlands International Institute for Management, Maastricht, Netherlands, on leave from Indian Institute of Management, Ahmedabad, India. The second author is from the Indian Institute of Management, Ahmedabad, India. This work was partially supported by a research grant from the Indian Institute of Management, Ahmedabad.

Complete address for correspondence:

Prof. Jayanth Rama Varma
Indian Institute of Management
Ahmedabad, INDIA, 380 015
Tel. : (91)-272- 407241
Tlx. : 121 6351 IIMA IN
Fax : (91)-272- 467396

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Abstract

Indian convertible bonds have two peculiar features that make them possibly unique in the world: a) the bonds are compulsorily converted into equity without any option, and b) the conversion terms are not specified at the time of issue but are left to be determined subsequently by the Controller of Capital Issues (CCI) who is the government functionary regulating capital issues in India.

A naive model would say that the market simply forms an estimate of the likely conversion terms and then values the bond as if these terms were prespecified. However, the empirical investigation reported in a companion paper, Barua, Madhavan and Varma (1991) convincingly rejects the naive model and makes a more sophisticated model necessary.

In this paper, we use the general theory of derivative securities (Cox, Ingersoll and Ross, 1985) to obtain a closed form expression for the value of the Indian convertible bond. The testable implications derived from our model are in sharp contrast to those of the naive model and are consistent with the empirical results in Barua, Madhavan and Varma (1991).

Though the valuation formula contains an unobservable state variable, the crucial functional parameters of the pricing relationship can be estimated, allowing the investor to measure and manage his risk. We derive the hedge ratios which an investor needs to control his risk exposure. An investor in Indian convertibles cannot, however, protect himself from both sources of risk (firm value risk and conversion ratio risk) with a single hedge ratio. This is an important difference between the Indian convertible and the ordinary convertible bond.

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Introduction

Convertible bonds are well known securities throughout the world. However, recent Indian convertible bonds have some peculiar features that make them possibly unique. First, the bonds are compulsorily redeemed by conversion into equity. There is no option in this regard either to the bondholder or to the issuing corporation. This is less serious than it might appear because the conversion terms have generally been so favourable as to make the conversion always beneficial to the bondholder. Therefore, even if the bond were to be vested with a genuine option to convert, such a deep-in-the-money option would behave just like the bond with compulsory conversion. The second and more serious problem is that the conversion terms are not specified at the time of issue. In other words, neither the exact time of conversion nor the conversion ratio (i.e. the number of equity shares into which each bond is to be converted) are prespecified. These are left to be determined at a subsequent stage by the Controller of Capital Issues (CCI) who is the government functionary entrusted with the regulation of issues of capital in the primary market.

At first sight, an instrument like this defies rational valuation. Nevertheless, these instruments are actively traded in Indian capital markets, and the market does place a value upon them. In fact, many practitioners and academics seem to think that all that is required is to form an estimate of the likely conversion terms and then value the bond as if these terms were prespecified. Apart from this additional complication of estimating the expected conversion terms, the Indian convertible bond with unspecified terms should, in this view, behave like an ordinary convertible bond.

This view is what we shall call the naive valuation model. In a companion paper to this study, Barua, Madhavan and Varma (1991) analyzed the actual market prices of one of the largest issues of convertible bonds in India, and found that the naive valuation model was convincingly rejected.

The empirical study thus brings out the need for more refined valuation models for Indian convertible bonds. This is what we shall undertake in this paper drawing heavily on an earlier model developed by one of us (Varma 1991) and the general framework of derivative securities as developed by Cox, Ingersoll and Ross (1985).

The paper begins with a description of how the convertible bond could be valued if the terms were prespecified and shows how the naive valuation model tries to extend this idea to the case where the terms are not specified. We then outline the general theory of derivative securities as developed by Cox, Ingersoll and Ross (1985). We then proceed to discuss the context in which the CCI operates while fixing the conversion terms, and proceed to model his behaviour. Building on this we develop our valuation model and obtain a closed form solution. We finally derive the testable implications of our model and show that these are consistent with the empirical results obtained in Barua, Madhavan and Varma (1991).

Valuing the Indian Convertible with Prespecified Terms

Valuing the Indian convertible bond with prespecified terms is quite straightforward. Since there is no option to convert, we do not even need the option pricing model (Black and Scholes, 1973). Suppose that the convertible bond is convertible into K shares at time T. If we compare the cashflows received by holding one bond with those received by holding K shares, we see that the flows are identical after the conversion date (T). Prior to T, these flows are different as the bondholder gets interest while the shareholder receives dividends. We subtract the present value of the interest stream from the value of the bond and call the balance the conversion value of the bond. Similarly, we subtract the present value of the expected dividends from the market price of the share and call the balance the adjusted share price. The foregoing argument then implies that the conversion value of the bond must be exactly K times the adjusted share price.

The Naive Valuation Model for Indian Convertible Bonds

When conversion terms are not prespecified, the naive valuation model would use the same approach as above, but replace the unknown conversion ratio K by its expected value. The market is assumed to form an expectation about the conversion ratio from whatever information is available to it. The naive valuation model then asserts that the conversion value of the bond is simply this expected conversion ratio times the adjusted share price.

The naive valuation model is intuitively a quite plausible one, and is in fact popular among practitioners and academics in India. The model has the following implications:

1. The conversion value of the bond is proportional to the adjusted share price. By regressing the former on the latter, we could try to empirically determine the conversion ratio that the market is expecting.
2. A 1% change in the value of the share should cause a 1% change in the conversion value of the bond.
3. If we obtain the CAPM betas of the shares and the conversion value of the bond by regressing the respective returns on the market return, these betas must be the same.

As already stated, the empirical tests reported in Barua, Madhavan and Varma (1991) strongly rejected the naive model and established the need for more sophisticated valuation models. We now turn to the general theory of derivative securities to develop our model.

The Cox Ingersoll Ross Model

The basic model for valuing derivative securities is that of Cox, Ingersoll and Ross (1985) henceforth referred to as CIR. We present below the salient features of the CIR model with some simplifications and in a somewhat altered notation. There are n state variables Y_i which evolve according to a diffusion process:

$$\frac{dY_i}{Y_i} = \mu_i \, dt + \sum_j \sigma_{ij} \, dw_j \qquad (1)$$

where the w_j are a set of standardized Wiener processes. Some of the Y_i may be traded (non derivative) securities, but some may be state variables which are not traded but affect the prices of other (derivative) securities. A derivative security or contingent claim F pays an amount $\theta(Y(T))$ at time T. In other words, the payoff at time T depends on the values of the state variables Y_i at that time. CIR show that associated with the Y_j are a set of factor risk premiums and the return on F is equal to the riskfree interest rate r plus the risk premiums for each of the factors Y_j:

$$R = r + \sum_j \Phi_j \frac{\partial F/F}{\partial Y_j/Y_j} \qquad (2)$$

Here R is the expected return on the security F, Φ_j is the risk premium per unit of the j'th factor risk (if a security's payoff is equal to Y_j, its expected return equals $r + \Phi_j$), and the coefficient $(\partial F/F)/(\partial Y_j/Y_j)$ is the measure of the security's exposure to the j'th factor risk (the extent to which returns on Y_j influence returns on F).

The principle of risk neutral valuation asserts that the value of the security F can be computed by the following procedure:

1. Change the system dynamics by reducing the drift of the state variables by the risk adjustment Φ_j:

   $$\frac{dY_i}{Y_i} = (\mu_i - \Phi_i)dt + \sum_j \sigma_{ij} \, dw_j \qquad (3)$$

   If Y_j is a traded security, Eqn 0 applies to it, $\mu_i = \Phi_i + r$, and the risk adjustment amounts to choosing the dynamics that the security would have in a risk neutral world.

2. Compute the expected payoff of the derivative security F under the above altered dynamics and discount this at the riskfree rate r to get the value of F:

   $$F(t) = \exp(-r(T-t)) \, E'_t(\theta(Y(T))) \qquad (4)$$

where E_t denotes the expectation at time t with respect to the altered dynamics.

The above procedure may be summarized as follows :

> "The equilibrium price of a claim is given by its expected discounted value with discounting done at the riskfree rate, where the expectation is taken with respect to a risk adjusted process for wealth and the state variables. The risk adjustment is accomplished by reducing the drift of each underlying variable by the corresponding factor risk premium." (CIR, page 380).

How Does the CCI Fix the Conversion Ratio?

Before we develop a formal model in terms of the CIR framework, it is worthwhile to describe briefly the considerations that the CCI takes into account while fixing the conversion ratio.

Until mid 1991, the interest rates on corporate bonds were subject to a ceiling fixed by the government. The ceiling rate on convertible bonds was lower than that on ordinary bonds. To compensate for this, the conversion was usually at favourable terms; on conversion, the bondholder received equity shares with a market value exceeding the face value of the bond. Typically, conversion at favourable terms acts like a redemption premium that boosts the effective yield on the convertible bond above even the ceiling rate on ordinary bonds. The investor in Indian convertible bonds has thus come to expect a substantial part of his return to come from the conversion.

In fact, part of the rationale for leaving the conversion terms to be determined subsequently by the CCI rather than specifying them at the time of issue was to ensure that the conversion terms are favourable not only ex-ante but also ex-post. This is the context in which the CCI operates.

The CCI can, therefore, be expected to look closely at the value of the shares transferred to the bondholders on the conversion date. The CCI would want to ensure that this terminal value of the bond (the conversion ratio times the market price of the share on the conversion date) exceeds the face value of the bond by an amount adequate to compensate the bondholder for the low interest rate that he has received till conversion. Strict adherence to just this consideration would force the CCI to set the conversion ratio equal to a target terminal value divided by the market price of the share at the time of conversion:

$D = D_0$. $\qquad\qquad$ $K = D_0/S$.

where K is the conversion ratio, D_0 is the target terminal value, S is the share price on the conversion date, and D = KS is the actual terminal value.

At the other extreme, would be a hypothetical policy of setting the conversion ratio equal to a value which would have been considered fair at the time of issue of the bond. This policy, if implemented, would make the terminal value directly proportional to the market price at the time of conversion:

$K = K_0$. $D = K_0S$.

Yet another possibility arises because the CCI can regard his job as one of fixing the conversion price that the bond holder pays for the equity share. The conversion ratio would then be the face value of the bond divided by the conversion price. This would suggest that the CCI could use the same procedure that he has evolved for valuation of shares for various other purposes. This valuation procedure of the CCI relies on historical accounting data and keeps market prices in the background (Varma and Venkiteswaran, 1990). The result would probably be something midway between the two extreme policies discussed above.

This discussion about the range of possible policies leads us to specify the conversion ratio as a flexible function of the stock price on the conversion date. This functional relationship is not, however, an unchanging one; it is strongly influenced by the CCI's own norms, policies and discretion. We assume, for modelling purposes, that these norms, policies and discretion can be captured by a single scalar variable denoted by C. The conversion ratio (K) would then be a function of the stock price (S) and the parameter (C):

$K = K(S,C)$. $D = K(S,C)\ S$.

The functional form of this relationship will be specified in the next section.

The Valuation Model

To fit the Indian convertible bond into the CIR framework would require us to specify the state variables and the payoff function θ. The discussion in the previous section would suggest that the most natural state variables would be the stock price S and the parameter C representing the CCI's norms and discretion. The payoff to the bond at the conversion date T is what we have been calling the terminal value $K(S(T),C(T))*S(T)$. For the purpose of this model we shall assume that T is known ignoring the uncertainty that exists about the exact conversion date itself.

On closer reflection, we find that it is inappropriate to treat the stock price S as a state variable. This is because, if the outstanding amount of convertible bonds is a large fraction of the equity capital, then the conversion would lead to a substantial dilution of the existing share capital and a corresponding reduction in the share price. The share price is thus as much a derivative security as the convertible bond. The uncertainty about conversion ratio affects both securities though in opposite directions. To deal with this situation, we treat the total value of the firm (market value of all the equity shares and all the convertible bonds taken together) as the fundamental state variable instead of the stock price. We then view the conversion ratio as deciding how this value is split up between shareholders and bondholders.

We let D(t) be the conversion value of the bond at time t and S(t) the adjusted stock price at time t. Let V(t) be the value of the firm defined as N_D*D(t) + N_S*S(t) where N_D and N_S are the number of convertible bonds and the number of shares respectively. Recall that the conversion value of the bond is the value of the bond excluding the present value of the interest payments upto the conversion date T and the adjusted share price similarly excludes the present value of the dividend payments.

The definition of V implies that

$$S(t) = [V(t) - N_D{*}D(t)]/N_S$$
$$= pV(t) - qD(t) \qquad (5)$$

where $p = 1/N_S$ and $q = N_D/N_S$

Hence if we take V as a state variable and develop a valuation formula for D(t), we obtain the value of S(t) automatically from the above relation.

At time T, the conversion value D(T) must obviously equal the terminal value of the bond, i.e., S(T)*K(S(T),C(T)). We can use Eqn. 0 to eliminate S from this expression allowing us to write D(T) as a function of the state variables:

$$D(T) = F(V(T),C(T)) \qquad (6)$$

We assume the following logarithmic form for the function F:

$$\ln F(V,C) = \ln D_0 + f_v{*}\ln V + f_c{*}\ln C + f_{vc}{*}\ln V{*}\ln C \qquad (7)$$

where D_0, f_v, f_c and f_{vc} are functional parameters which do not depend on any of the state variables.

Letting lower case letters represent the logarithms of the corresponding upper case quantities, we can rewrite this as:

$$f(v,c) = d_0 + f_v v + f_c c + f_{vc} vc \qquad 0 \le f_v, f_c \le 1. \qquad (8)$$

This functional form is quite rich in that it includes most of the special cases discussed in the previous section:

1. $f_v = f_c = f_{vc} = 0, F(V,C) = D_0, \quad K = D_0/S$
   Terminal value is a constant (does not depend on the state variables). Conversion ratio inversely proportional to stock price.

2. $f_v = 0,\ f_c = 1,\ f_{vc} = 0, F(V,C) = C{*}D_0, \quad K = C{*}D_0/S$

Terminal value independent of stock price but depends on CCI's discretion. Conversion ratio inversely proportional to S but also depends on CCI's discretion.

3. $f_v = 1$, $f_c = 0$, $f_{vc} = 0$, $F(V,C) = D_0*V = K*S$, $K = D_0*N_S/(1-D_0*N_D)$.
   Conversion ratio is a constant (does not depend on the state variables). Terminal value proportional to S.

4. $f_v = 1$, $f_c = 1$, $f_{vc} = 0$, $F(V,C) = C*D_0*V = K*S$, $K = C*D_0*N_S/(1-C*D_0*N_D)$.
   Conversion ratio depends on CCI's discretion but independent of stock price. Terminal value proportional to S but also depends on CCI's discretion.

These special cases arise when f_v is set to the extreme values of zero or unity. The more complex and realistic cases arise when f_v takes an intermediate value; in this case the conversion value increases with S, but less than proportionately. Though the term $f_{vc}vc$ has been included in Eqn. 0 for greater generality, it may, in practice, be worthwhile to simplify the expression by setting f_{vc} equal to zero.

We now specify the dynamics of the two state variables:

$$\frac{dV}{V} = \mu_V\, dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R \qquad (9)$$

where dw_M represents the unanticipated component of the market return (i.e., the return on the market portfolio), and dw_R represents the unanticipated component of the firm specific return on V. It is assumed that w_M and w_R are uncorrelated. This decomposition of the random shocks to V is designed to allow the Capital Asset Pricing Model (CAPM) to be applied.

We let C(t) denote not the CCI's policies at time t but the market's expectation at time t of what the policies will be at time T. We assume a strong form of rational expectations and a steady flow (leakage ?) of information from the CCI's office about his likely policies. This allows us to conclude that the market's expectation converges to the true value C(T) as t approaches T, and justifies our use of the same symbol C to denote both the true value C(T) and the expectation thereof C(t):

$$\lim_{t \to T} C(t) = C(T)$$

We assume the dynamics of C to be:

$$\frac{dC}{C} = \mu_C\, dt + \sigma_C\, dw_C \qquad (10)$$

Under the earlier assumption of rational expectations, it would make sense to assume μ_C to be equal to zero. We, however, retain μ_C for greater generality. Similarly, though it is natural to assume that w_C is uncorrelated with w_M and w_R, we shall allow a correlation of ρ_{MC} between w_M and w_C as well as a correlation of ρ_{RC} between w_R and w_C. We define the covariance terms $\sigma_{MC} = \rho_{MC}\sigma_M\sigma_C$ and $\sigma_{MR} = \rho_{RC}\sigma_R\sigma_C$.

The risk adjusted dynamics of V and C are obtained by subtracting the risk premium from the drift terms as follows (we preface the equation with the symbol (RA) to emphasize that the equations represent not the actual dynamics but the risk adjusted ones):

$$\text{(RA)} \quad \frac{dV}{V} = r\ dt + \sigma_{VM}\ dw_M + \sigma_{VR}\ dw_R \tag{11}$$

$$\text{(RA)} \quad \frac{dC}{C} = (\mu_C - \Phi_C)\ dt + \sigma_C\ dw_C \tag{12}$$

We define D'(t) as follows:

$$D'(t) = F(V(t),C(t)) \tag{13}$$

This estimates the terminal value using the current values of the state variables (V(t) and C(t)) rather than the values at the conversion date (V(T) and C(T)). Obviously,

$$D'(T) = D(T) \tag{14}$$

Using Ito's lemma, we can write the risk adjusted dynamics of D'(t) as follows:

$$\begin{aligned}\text{(RA)}\ dD' = {} & [F_V rV + F_C(\mu_C - \Phi_C)C + (1/2)F_{VC}(\sigma_{MC} + \sigma_{RC})VC]\ dt + \\ & F_V V(\sigma_{VM}\ dw_M + \sigma_{VR}\ dw_R) + F_C C\sigma_C\ dw_C\end{aligned} \tag{15}$$

where the subscripts on F indicate partial derivatives. Observe that Eqns. 0 implies that F_{VV} and F_{CC} are zero.

Using Eqns. 0 to express the derivatives of F in terms of the derivatives of f, we get

$$\text{(RA)} \quad \frac{dD'}{D'} = g\ dt + f_v(\sigma_{VM}\ dw_M + \sigma_{VR}\ dw_R) + f_c\sigma_C\ dw_C \tag{16}$$

where $g = f_v r + f_c(\mu_C - \Phi_C) + (1/2)(f_{vc} + f_v f_c)(\sigma_{MC} + \sigma_{RC})$

Risk neutral valuation now gives:

$$D(t) = \exp(-r(T-t))\ E_t^{,}\ (D(T)) = \exp(-r(T-t))\ E_t^{,}\ (D'(T)) \tag{17}$$

where the second equality comes from Eqn. 0.

Using Ito's lemma to compute the expectation of D'(T) and substituting the result in Eqn. 0, we get:

$$
\begin{aligned}
D(t) &= \exp[-r(T-t)] \exp [g(T-t)] D'(t) \\
&= \exp[(g-r)(T-t)] D'(t) \\
&= \exp[(g-r)(T-t)] F(V(t),C(t))
\end{aligned} \tag{18}
$$

This is the valuation formula for the Indian convertible bond.

<u>Testable Implications and Empirical Estimation</u>

The valuation formula in Eqn. 0 involves the unobservable state variable C representing the CCI's policy. In addition, it involves the factor risk premium Φ_C and the functional parameters f_v, f_c and f_{vc}. We can nevertheless derive several testable implications from our model. We shall also show how some of the functional parameters can be estimated.

First of all, by Ito's lemma, we derive the dynamics of D:

$$
\frac{dD}{D} = [r + f_v(\mu_v - r) + f_c\Phi_C]\, dt + f_v(\sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R) + f_c C\sigma_C\, dw_C \tag{19}
$$

The expected return on the bond, $[r + f_v(\mu_v - r) + f_c\Phi_C]$ equals the risk free rate plus compensation for two sources of risk:

1. firm value risk: f_v times the risk premium for the firm, and
2. conversion risk: f_c times the risk premium Φ_C for conversion policy parameter C.

Similarly, using Eqn. 0 and Ito's lemma, we derive the dynamics of S as follows:

$$
\frac{dS}{S} = [r + (1 + \tau)(\mu_v - r) - \delta\, f_c\Phi_C]\, dt + (1+\tau)(\sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R) - \delta\, f_c\sigma_C\, dw_C \tag{20}
$$

where $\tau = \delta(1 - f_v)$ and $\delta = N_D D/N_S S$. Both τ and δ can be regarded as leverage ratios; τ takes into account the fact that the convertible bond has the characteristics partly of debt and partly of equity, and treats part of the value of the convertible bond as equity while computing the leverage. The expected return on the share equals the risk free rate plus compensation for two sources of risk:

1. firm value risk: $(1 + \tau)$ times the risk premium for the firm, and
2. conversion risk: $-\delta f_c$ times the risk premium Φ_C for conversion policy parameter C.

It is clear that the expected return on the bond can exceed that of the share if f_v is close to unity and $f_c\Phi_c$ is positive. It is <u>a</u>

priori not clear whether the risk premium Φ_C would be positive or negative. The reason is that the conversion risk affects shareholders and bondholders in opposite directions.

We know proceed to examine the random components of the returns on the share and the bond to see how they are related. We assume now that the correlations ρ_{MC} and ρ_{RC} are zero.

$$Cov(dD/D, dS/S) = f_v\ (1+\tau)\sigma_V^2 - \delta f_c^2 \sigma_C^2$$

$$Var(dS/S) = (1+\tau)^2\ \sigma_V^2 + \delta f_c^2 \sigma_C^2$$

where $\sigma_V^2 = \sigma_{VM}^2 + \sigma_{VR}^2$.

The quantities, dD/D and dS/S are instantaneous returns, but in empirical work, we can replace these by their discrete counterparts, viz., the logarithmic returns $\ln(D_t/D_{t-1})$ and $\ln(S_t/S_{t-1})$.

This implies that if the bond returns are regressed on the stock returns, the regression slope will be:

$$\frac{Cov(dD/D, dS/S)}{Var(dS/S)} = \frac{f_v}{(1+\tau)} \frac{1 - \alpha/f_v}{1 + \alpha/(1+\tau)} \leq \frac{f_v}{1+\tau} \leq 1 \qquad (21)$$

where $\alpha = [\delta f_c^2 \sigma_C^2] / [(1+\tau)\sigma_v^2]$.

Not only is the regression slope always less than unity, it can even be negative if α exceeds f_v.

If the bond returns and stock returns are regressed on the market return dw_M, the regression slopes (CAPM betas) will be:

$$\beta_D = f_v \sigma_{VM} = f_v \beta_V \qquad (22)$$

$$\beta_S = (1+\tau)\ \sigma_{VM} = (1+\tau)\ \beta_V \qquad (23)$$

where β_D, β_S and β_V are the betas respectively of the bond, the stock and the firm as a whole. The ratio of the betas of the bond and the stock is also less than unity:

$$\frac{\beta_D}{\beta_S} = \frac{f_v}{1+\tau} \leq 1 \qquad (24)$$

Now consider the residuals when the bond and stock returns are respectively regressed on the market return. We see that when the bond residual is regressed on the stock residual, the regression slope is also less than unity:

$$\frac{Cov(res_D, res_S)}{Var(res_S)} = \frac{f_v}{(1+\tau)} \frac{1 - \alpha'/f_v}{1 + \alpha'/(1+\tau)} \leq \frac{f_v}{1+\tau} \leq 1 \qquad (25)$$

where $\alpha' = [\delta f_c^2 \sigma_C^2] / [(1+\tau)\sigma_{VM}^2]$ and res_D and res_S are the residuals when the bond and stock returns are respectively regressed on the market return. The regression slope can be negative if α' exceeds f_v.

Since $\sigma_V^2 \geq \sigma_{VM}^2$, it follows that $\alpha' \geq \alpha$, and we have the following inequalities:

$$\frac{\mathrm{Cov}(res_D, res_S)}{\mathrm{Var}(res_S)} \leq \frac{\mathrm{Cov}(dD/D, dS/S)}{\mathrm{Var}(dS/S)} \leq \frac{\beta_D}{\beta_S} \leq 1 \tag{26}$$

By contrast, in the naive valuation model described earlier, all these quantities should be equal to unity. It may also be mentioned that in our companion paper (Barua, Madhavan and Varma 1991), we find the above inequalities are strongly supported by the data. In this sense, we can regard our model as being validated by the data.

Intuitively, one can see why these inequalities should hold if we recall that changes in S and D are caused by changes either in V or in C. When V changes, C remaining the same, S and D should change in the same direction, but not in the same proportion. When V rises by 1%, D rises by f_v% and S rises by $(1+\tau)$%. On the other hand, a change in C, V remaining the same, causes S and D to move in opposite directions. If C rises by 1%, D rises by f_c and S falls by δf_c. When we look at the changes in S and D in general, these changes are partly caused by changes in C where we expect the changes to be in opposite directions, and partly by changes in V where the changes are in the same direction. Consequently, the overall correlation between these changes would be small if not negative. The situation is different when we look at changes in association with the market return. Under the assumption that w_M is uncorrelated with w_C, changes in the market return (w_M) change V leaving C unchanged. Now D and S must move in the same direction. The only reason for β_D to be below β_S is that f_v is below unity; in fact, if f_v equals unity, the two betas would be equal. On the other hand when we look at residuals from the market model regression, one source of changes in V has been eliminated, and the remaining changes in S and D are more likely to have been caused by changes in C. As a result the correlations are expected to be lowest here.

We now show how some of the functional parameters can be estimated. The easiest parameter to estimate is f_v; if we estimate the CAPM betas of D and S, then Eqn. 0 can be used to estimate f_v quite readily since $\tau = \delta(1-f_v)$ where δ is an observable quantity. The next step is to regress the bond returns (dD/D) on the stock returns (dS/S) and use Eqn. 0 to estimate α. In the expression for α, the quantity σ_v^2 is easily estimated; it is nothing but the variance of (dV/V). We thus have an estimate of $f_c\sigma_C$. Looking at our equations, we find that we do not really need an estimate of f_c and σ_C separately. This is because, when the unobservable state variable C is multiplied by a constant, σ_C gets multiplied by the same constant, and f_c gets divided by the same constant leaving the product unchanged. In fact, one can normalize C so that one of these two quantities is identically equal to unity.

The parameters f_v and $f_c\sigma_C$, which we have just shown to be estimable are the crucial parameters of the pricing relationship which allow an investor to measure and manage his risk exposure. As one application, we show how we can develop the hedge ratios which an investor requires to protect himself from the different sources of risk. We know from the option pricing model (Black and Scholes, 1973) that an ordinary convertible bond can be combined with a short position in the underlying stock in a suitable proportion to construct a portfolio which is instantaneously riskfree. This is not possible with the bond whose terms are unspecified even if the unobservable state variable C were known. It is, however, possible to construct a portfolio which is insulated from firm value risk. The hedge ratio for this is seen from Eqns. 0 and 0 to be $-f_v/(1+\tau)$; in other words, the investor must shortsell shares worth $f_v/(1+\tau)$ times his investment in bonds. Similarly, it is also possible to construct a portfolio which is insulated from conversion ratio risk; the hedge ratio for this is simply the ratio of market values δ. This proposition simply restates the obvious fact that if an investor hold say 1% of the firm's bonds and 1% of its stock, he is unaffected by how the CCI chooses to carve up the firm value between bondholders and stockholders.

Conclusion

We have developed a closed form expression (Eqn. 0) for the value of the Indian convertible bond. Though this formula contains an unobservable state variable, we have been able to derive several testable implications from our model. These testable implications are in sharp contrast to those of the naive model. As stated earlier, the empirical results in Barua, Madhavan and Varma (1991), reject the naive model and are consistent with the predictions of our model.

From an investor's point of view, the usefulness of the model comes from the fact that the functional parameters of the pricing relationship can be estimated. This allows him to measure and manage his risk. We have, for example, derived the hedge ratios which an investor can use to control his risk exposure. In the process, we have also shown that an investor in Indian convertibles cannot protect himself from both sources of risk (firm value risk and conversion ratio risk) with a single hedge ratio. This is another important dimension in which the Indian convertible with unspecified terms differs from the ordinary convertible bond.

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