

D 111973

(Pages : 2)

Name.....

Reg. No.....

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTS 3C 03—MATHEMATICS—3

(2019—2023 Admissions)

Time : Two Hours

Maximum Marks : 60

Part A*All questions can be answered.**Each question carries 2 marks.**(Ceiling 20 marks)*

1. If $r(t) = \langle f(t), g(t), h(t) \rangle$, where f , g and h are differentiable, then prove that $r'(t) = \langle f'(t), g'(t), h'(t) \rangle$.
2. The position of a moving particle is given by $r(t) = t^2i + tj + \frac{5}{2}tk$. Find $v(2)$ and $a(2)$.
3. Describe the level surfaces of the function $f(x, y) = y^2 - x$.
4. If $F = (x^2y^3 - z^4)i + 4x^5y^2zj + y^4z^6k$, find curl F .
5. Evaluate $\int_C xy^2 ds$ on the quarter-circle C defined by $x = 4 \cos t$, $y = 4 \sin t$, $0 \leq t \leq \frac{\pi}{2}$.
6. Find $\int_C y dx + x dy$ on the curve $y = x^3$ between $(0, 0)$ and between $(1, 1)$.
7. Convert $\left(8, \frac{\pi}{3}, 7\right)$ in cylindrical co-ordinates to rectangular co-ordinates.
8. Find the values of $\ln(-2)$.
9. Prove that $\cosh^2 z + \sinh^2 z = 1$.
10. Evaluate $\int_C \bar{z} dz$, where C is given by $x = 3t$, $y = t^2$, $-1 \leq t \leq 4$.
11. Evaluate $\oint_C e^z dz$, where C is the circle $|z| = 2$.
12. Derive Cauchy's inequality.

Turn over

Part B

*All questions can be answered.
Each question carries 5 marks.
(Ceiling 30 marks)*

13. Find the directional derivative $f(x, y) = 2x^2y^3 - 6xy$ at $(1, 1)$ in the direction of a unit vector whose angle with the positive x -axis is $\frac{\pi}{6}$.
14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at $(2, 1, 4)$.
15. Evaluate the double integral $\iint_{\mathbb{R}} e^{x+3y} dA$ over the region bounded by the graphs of $y = 1$, $y = 2$, $y = x$ and $y = -x + 5$.
16. Evaluate $\oint_C (x^5 + 3y) dx + (2x - e^{y^3}) dy$, where C is the circle $(x - 1)^2 + (y - 5)^2 = 4$.
17. Find the volume of the solid in the first Octant bounded by the graphs of $z = 1 - y^2$, $y = 2x$ and $x = 3$.
18. Solve the equation $\cos z = 10$.
19. Find an upper bound for the absolute value of $\oint_C \frac{e^z}{z+1} dz$, where C is the circle $|z| = 4$.

Section C

*Answer any **one** questions.
The question carries 10 marks.*

20. Let S be the part of the cylinder $z = 1 - x^2$ for $0 \leq x \leq 1$, $-2 \leq y \leq 2$. Verify Stoke theorem for the vector field $\mathbf{F} = xy \mathbf{i} + yz \mathbf{j} + xz \mathbf{k}$. Assume S is oriented upward.
21. Find the moment of inertia about the z -axis of the homogeneous solid bounded between the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, $a < b$.

(1 × 10 = 10 marks)

D 51761

(Pages : 3)

Name.....

Reg. No.....

**THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023**

Mathematics

MTS 3C 03—MATHEMATICS—3

(2019—2022 Admissions)

Time : Two Hours

Maximum : 60 Marks

Part A

All questions can be attended.

Each question carries 2 marks.

Overall Ceiling is 20.

1. If $r(t) = \cos 2t i + \sin t j$. Find $r'(0)$.
2. Find the curvature of a circle of radius a .
3. Describe the level surfaces of the function $F(x, y, z) = \frac{(x^2 + y^2)}{z}$.
4. If $F = (x^2y^3 - z^4)i + 4x^5y^2z j + y^4z^6 k$, find $\text{div}(\text{curl } F)$.
5. Evaluate $\int xy^2 dy$ on the quarter-circle C defined by $x = 4 \cos t, y = 4 \sin t, 0 \leq t \leq \frac{\pi}{2}$.
6. Find $\int_C y dx + x dy$ on the curves $y = \sqrt{x}$ between $(0, 0)$ and between $(1, 1)$.
7. Convert $(6, \pi/4, \pi/3)$ in spherical coordinates to rectangular co-ordinates.
8. Find the values of $\ln(-1, -i)$.
9. Prove that $\sinh z = \sinh x \cos y + i \cosh x \sin y$.

Turn over

10. Evaluate $\int (z + 3) dz$, where C is $x = 2t, y = 4t - 1, 1 \leq t \leq 3$.
11. Evaluate $\oint_C z^3 - 1 + 3i dz$, where C is the circle $|z| = 1$.
12. State Cauchy's Integral Formula.

Part B

*All questions can be attended.
Each question carries 5 marks.
Overall Ceiling is 30.*

13. Find an equation of the tangent plane to the graph of $\frac{1}{2}x^2 + \frac{1}{2}y^2 - z = 4$ at $(1, -1, 5)$.
14. Find the maximum value of the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6i + 2j + 3k$.
15. Find the moment of inertia about the y -axis of the thin homogeneous disk $x^2 + y^2 = r^2$ of mass m .
Given $\rho(x, y) = \frac{m}{\pi r^2}$.
16. Find the volume of the solid that is under the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and above the region bounded by the graph of the circle $x^2 + y^2 - y = 0$. $V = \iint_R \sqrt{1 - x^2 - y^2} dA$.
17. (a) Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane.
(b) Find the harmonic conjugate function of u .
18. Solve the equation $\cos z = 10$.
19. Evaluate $\oint_C \frac{dz}{z^2 + 1}$ where C is the circle $|z| = 3$.

Part C

*Answer any **one** question.
The question carries 10 marks.*

20. Verify Stokes theorem. Assume that the surface S is oriented upward. Given $F = z i + x j + y k$; S that portion of the plane $2x + y + 2z = 6$ in the first octant.
21. Let D be the region bounded by the hemisphere $x^2 + y^2 + (z - 1)^2 = 9, 1 \leq z \leq 4$, and the plane $z = 1$. Verify the divergence theorem if $F = xi + yj + (z - 1)k$.

(1 × 10 = 10 marks)

D 31819

(Pages : 3)

Name.....

Reg. No.....

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019 Admission Onwards)

Time : Two Hours

Maximum : 60 Marks

Section A

Answer any number of questions.
Maximum 20 marks.

- Find the derivative of the vector function $\vec{r}(t) = \sin t \hat{i} - e^{-t} \hat{j} + (3t^3 - 4) \hat{k}$.
- If $z = 4x^3y^2 - 6x^2 + y^2 + 5$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
- If $f(x, y) = e^{xy}$, find $\nabla f(x, y)$.
- Find the level curve of $f(x, y) = y^2 - x^2$ passing through the point $(-1, 2)$.
- Find $\text{div } \vec{F}$ for $\vec{F} = (x^2y^3 - z^4) \hat{i} + 4x^5y^2z \hat{j} - y^4z^6 \hat{k}$.
- Evaluate $\int_{-1}^3 \int_{-1}^1 (2x - 4) dx$.
- State Stoke's theorem.
- Find the Jacobian of $x = r \cos \theta$, $y = r \sin \theta$.
- Express $1 + i$ in polar form.
- Evaluate $\lim_{z \rightarrow i} \frac{z^4 - 1}{z - i}$.

Turn over

11. Evaluate $\oint_C \frac{e^z}{z-3} dz$ where C is $|z|=1$.
12. Evaluate $\oint_C \bar{z} dz$ where C is $x=t, y=t^2, 0 \leq t \leq 1$.

Section B

*Answer any number of questions.
Maximum 30 marks.*

13. Use chain rule to find $\frac{\partial z}{\partial u}$ at $(\pi, 1)$ for $z = x^2 - y^2 \tan x$, where $x = \frac{u}{v}, y = uv$.
14. Find an equation of the tangent plane to the graph of $z = \frac{x^2}{2} + \frac{y^2}{2} + 4$ at $(1, -1, 5)$.
15. Show that $\int_C (y^2 - 6xy + 6) dx + (2xy - 3x^2) dy$ is independent of any path C between $(-1, 0)$ and $(3, 4)$. Hence evaluate $\int_{(-1,0)}^{(3,4)} (y^2 - 6xy + 6) dx + (2xy - 3x^2) dy$.
16. Change the order of integration and hence evaluate $\int_0^4 \int_y^4 \frac{x}{x^2 + y^2} dx dy$.
17. Show that $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic. Find the harmonic conjugate of u .
18. Evaluate $\int_C z^2 dz$ where C is the line $x = 2y$ from $z = 0$ to $z = 2 + i$.
19. Evaluate $\oint_C \frac{z+1}{z^4 + 4z^3} dz$ where C is $|z| = 1$.

Section C

Answer any **one** question.

Maximum 10 marks.

20. Use Green's theorem to evaluate $\oint_C (x^5 + 3y) dx + (2x - e^{y^3}) dy$ where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

21. Find the volume bounded by the cylinder $x^2 + y^2 = 4$, the plane $y + z = 3$ and $z = 0$.

(1 × 10 = 10 marks)

D 12034

(Pages : 3)

Name.....

Reg. No.....

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019–2020 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A*Answer at least **eight** questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 24.*

1. Evaluate $\int_0^1 (t\hat{i} + 3t^2\hat{j} + 4t^3\hat{k}) dt$.
2. The position of a moving particle is $\vec{r}(t) = t^2\hat{i} + t\hat{j} + t^3\hat{k}$. Find velocity and acceleration of the particle at $t = 2$.
3. If $z = e^{-y} \cos x$ find $\frac{\partial^2 z}{\partial x \partial y}$.
4. Find the level surface of $F(x, y, z) = x^2 + y^2 + z^2$ passing through $(1, 1, 1)$.
5. Evaluate $\oint_C x dx$, where C is the circle $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$.
6. Show that $\text{curl } \vec{r} = \vec{0}$.
7. State Green's theorem in the plane.
8. Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz \, dx \, dy \, dz$.
9. Write the equation of the circle with centre $(1, 2)$ and radius 4 in the complex plane.

Turn over

10. Find the value of i^{2i} .
11. Evaluate $\oint_C \frac{ze^z}{(z-3)^2} dz$, where C is $|z|=2$.
12. Evaluate $\oint_C \frac{dz}{z}$, where C is $|z|=1$.

(8 × 3 = 24 marks)

Section B

*Answer at least five questions.
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.*

13. Use chain rule to find $\frac{dw}{dx}$ at (0,1, 2) for $w = xy + yz$; $x = \cos x$, $y = \sin x$, $z = e^x$.
14. Find the directional derivative of $f(x, y) = \sqrt{x^2y + 2y^2z}$ at (-2, 2, 1) in the direction of the negative z-axis.
15. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$ using double integrals.
16. Use polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2} dy dx$.
17. Show that $f(z) = (2x^2 + y) + i(y^2 - x)$ is not analytic at any point.
18. Evaluate $\oint_C \frac{5z+7}{z^2+2z-3} dz$, where C is the circle $|z-2|=2$.
19. Evaluate $\int \operatorname{Re} z dz$ along a line segment from $z=0$ to $z=1+2i$.

(5 × 5 = 25 marks)

Section C

*Answer any one question.
The question carries 11 marks.*

20. Let $\vec{F}(x, y, z) = z\hat{j} + z\hat{k}$ represents the flow of a liquid. Find the flux of \vec{F} through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward.
21. Use triple integrals to find the volume of the solid with in the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

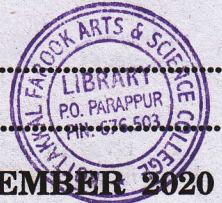
(1 × 11 = 11 marks)

D 92960

(Pages : 3)

Name.....

Reg. No.....



THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3C 03—MATHEMATICS – 3

Time : Two Hours

Maximum : 60 Marks

Section A

Answer at least eight questions.

Each question carries 3 marks.

All questions can be attended.

Overall Ceiling 24.

1. If $\vec{r}(t) = 2\cos t \hat{i} + 6 \sin t \hat{j}$, find $\frac{d\vec{r}}{dt}$ at $t = \frac{\pi}{2}$.
2. Find the curvature of a circle whose radius is 2.
3. If $z = e^x \sin(xy)$, find $\frac{\partial^2 z}{\partial y^2}$.
4. Find the gradient of $f(x, y, z) = xy^2 + 3x^2 - z^3$ at $(1, 1, 1)$.
5. Show that $\text{div } \vec{r} = 3$.
6. Evaluate $\int_2^4 \int_1^3 (40 - 2xy) dx dy$.
7. Use double integrals to find the area of the plane region enclosed by the curves $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{4}$.
8. Find the Jacobian of $u = \frac{y}{x^2}$, $v = xy$.
9. Sketch the graph of the region $|z - 2i| = 2$.
10. Write the real and imaginary part of $f(z) = \sin z$.

Turn over

11. Evaluate $\oint_C \frac{z^2}{z-1} dz$, where C is $|z|=2$.

12. Evaluate $\int_C z dz$ where C is given by $x=t^2, y=t$ from $0 \leq t \leq 1$.

(8 × 3 = 24 marks)

Section B

Answer at least five questions.

Each question carries 5 marks.

All questions can be attended.

Overall Ceiling 25.

13. Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$.

14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at $(2, 1, 4)$.

15. Use Green's theorem to evaluate $\oint_C (x^2 - y^2) dx + (2y - x) dy$, where C consists of the boundary of the region in the first quadrant that is bounded by $y = x^2$ and $y = x^3$.

16. Change the order of integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$.

17. Use divergence theorem to evaluate $\iiint_S (\vec{F} \cdot \hat{n}) dS$ where $\vec{F} = xy\hat{i} + y^2z\hat{j} + z^3\hat{k}$ and S is the unit cube defined by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.

18. Evaluate $\oint_C \left(z + \frac{1}{z}\right) dz$, where C is the unit circle $|z|=1$.

19. Evaluate $\oint_C \frac{z^4 - 3z^2 + 6}{(z+i)^3} dz$, where C is $|z|=2$.

(5 × 5 = 25 marks)

Section C

*Answer any one question.
The question carries 11 marks.*

20. Use Stoke's theorem to evaluate $\oint_C z dx + x dy + y dz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$ counter clockwise as viewed from above.
21. Find the volume of the solid in the first octant bounded by the graphs of $z = 1 - y^2$, $y = 2x$ and $x = 3$.

(1 × 11 = 11 marks)