D 111994 **(Pages : 2)** **Name..................................**

Reg. No...............................

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019—2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

All *questions can be attended.*
Each questions carries 2 *marks.*
Overall ceiling 20.

1. State the conditions that must be fulfilled for using a binomial distribution.
2. Obtain the m.g.f. of a Poisson distribution with parameter m.
3. Define Negative Binomial distribution.
4. Define Rectangular distribution
5. State Tchebycheff's inequality.
6. State Weak law of large numbers.
7. What do you meant by convergence in probability ?
8. What are the merits of sample survey ?
9. Explain the lottery method of selecting a simple random sample.
10. Define Chi square distribution and state its applications.
11. What is the square of a random variable following t distribution with n degrees of freedom
12. If X has a F distribution with n_1 and n_2 degrees of freedom, what is the distribution of 1/X ?

(Ceiling : 20 marks)

Section B (Short Essay/Paragraph Type Questions)

All *questions can be attended.*
Each questions carries 5 *marks.*
Overall ceiling 30.

13. State and prove recurrence relation for central moments for a Binomial distribution.
14. Find the mean and S.D. of normal population $N(\mu, \sigma^2)$, if 10% of the items are under 40 and 95% of the items are under 75.

Turn over

15. State and prove Bernoulli's law of large numbers.
16. Using Central Limit Theorem prove that one parameter Gamma distributin tends to normal distribution.
17. Explain stratified random sampling.
18. Give the procedure of selecting a linear systematic sample of size 5 from a population of size 30.
19. Prove that square of a t random variable is F random variable.

(Ceiling : 30 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*

20. (a) If X follow $N(\mu, \sigma^2)$, find k if $P(X \le k) = 4P(X \ge K)$

(b) In a single throw a two dice, find the chance of throwing (1) eight and (2) eleven.

21. (a) The following data on the measurements of the fat content of two kinds of ice creams brand A and brand B yielded the following results :

Brand A	...	13.5	14.0	13.6	12.9	13.0
Brand B	...	12..9	13.0	12.4	13.5	12.7

Find $P\left[\sigma_1^2 > 5.75\sigma_2^2\right]$ where σ_1^2 and σ_2^2 are the population variances.

(1 × 10 = 10 marks)

D 51782 **(Pages : 2)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

All questions can be attended.
Each question carries 2 *marks.*
Overall Ceiling 20.

1. Establish the relationship between Geometric distribution and discrete uniform distribution.
2. Obtain the m.g.f of a $N\left(\mu, \sigma^2\right)$.
3. Obtain mean and variance of Poisson distribution.
4. Define Pareto distribution.
5. Distinguish between parameter and statistic.
6. What are the advantageous and disadvantageous of Chebycheff's inequality.
7. Use any law of large numbers to prove that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is atleast 19/20.
8. What is the principal of optimum allocation ?
9. What is non probability sampling ? Give an example.
10. Define Student's *t* distribution.
11. Give a relationship between t and F distribution.
12. A random sample of 14 independent observations is taken from $N\left(\mu, \sigma^2\right)$, what is the mean and variance of Chi-square derived from it ?

(Ceiling 20 marks)

Turn over

Section B (Short Essay/ Paragraph Type Questions)

All questions can be attended.
Each question carries 5 marks.
Overall Ceiling 30.

13. Establish the relationship between Binomial distribution and Poisson distribution.

14. Find the mgf of Normal population $N(\mu, \sigma^2)$.

15. State and prove the weak law of large numbers. Deduce as a corollary Bernoulli theorem and comment on its applications.

16. If X denote the sum of the numbers obtained when two dice are thrown, use Chebychev's inequality to obtain an upper bound for $P\left[\,|X-7|>4\right]$. Compare this with the actual probability.

17. Use central limit theorem to establish a relationship between Binomial distribution and Normal distribution.

18. Carry out a comparison between census method and sampling method. Explain systematic random sampling.

19. A sample of size 16 is taken from a normal population with mean 1 and S.D 1.5. Find the probability that the sample mean is negative.

(Ceiling 30 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 marks.

20. a) If 3 % of electric bulbs are found to be defective, then using Poisson's approximation, find the probability that a sample of 100 bulbs will contain (i) no defective ; and (2) exactly one defective.

    b) A random variable X is normally distributed with mean 12 and S.D 2, Find the probability of the event $9.6 \le x \le 13.8$.

21. a) How large a sample is to be taken from a normal population $N(10, 3)$ if the sample mean is to lie between 8 and 12 with probability 0.95.

    b) Two independent samples of sizes 15 and 20 from a normal population $N(\mu, \sigma^2)$, Find the upper bound to $P\left(\frac{S_1^2}{S_2^2} < 2\right)$.

(1 × 10 = 10 marks)

D 31839 **(Pages : 2)** **Name.................................**

Reg. No..............................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Use of calculator and Statistical table are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 *marks.*
Maximum marks that can be scored from this part is 20.

1. If X is a random variable following discrete uniform distribution over the numbers -1,0 and 1; find the variance of X.
2. For two independent Poisson random variables, P (X = 0) = e^{-3}, P (Y = 0) = e^{-2}. Find P (X + Y = 2).
3. Write the p.d.f. of an exponential random variable with mean 0.2.
4. Obtain $P(|X - 5| < 3)$, where X follows N $(5, 3^2)$.
5. Point out one of the strengths and weaknesses of Chebyshev's inequality.
6. Define convergence in distribution.
7. State Weak Law of Large Numbers.
8. Define census and sampling.
9. A box contains 4 black balls and 6 white balls. 2 balls are taken at random one by one. What is the probability that all the balls taken are white : (i) If balls are taken without replacement ; (ii) With replacement.
10. Find the probability that the variance of a sample of size 12 taken from a normal population with mean 10 and variance 9 is greater than 10.28.
11. If X and Y are independent standard normal random variables, identify the probability distributions of (i) X^2 ; and (ii) $[X^2 + Y^2]$.
12. Define *t*-distribution.

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Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 *marks.*
Maximum marks that can be scored from this part is 30.

13. Obtain the mode of X following B (n, p)

14. Show that the sum of independent exponential random variables with common parameter λ-follows gamma distribution.

15. State Chebyshev's inequality. Let X be a random variable following rectangular distribution over [5, 15]. Use Chebyshev's inequality to find an upper bound for P $(|X - 10| > 4.33)$.

16. State and prove Bernoulli's Law of Large numbers.

17. Using central limit theorem, obtain the probability distribution of the mean of a large sample of size n taken from rectangular distribution over [0, 5].

18. Explain systematic random sampling.

19. If F follow F (n_1, n_2), show that 1/F follow F(n_2, n_1)

Part C (Essay type Questions)

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. (i) Define normal distribution. State any *four* of the properties of normal distribution.

    (ii) If the random variable X following N $\{\mu, \sigma^2)$, obtain the quartile deviation of X.

21. (i) Derive the (a) m.g.f. ; (b) mean ; and (c) variance of a random variable X following chi square distribution with n degrees of freedom.

    (ii) State and prove the additive property of chi-square distribution.

(1 × 10 = 10 marks)

D 12050 **(Pages : 2)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find the mean of a uniform random variable with possible values 1,2,3,4 and 5.
2. Define negative binomial distribution.
3. Obtain the m.g.f. of exponential distribution.
4. If X follow normal distribution with mean 10 and variance 9, find (i) $P(X > 13)$; (ii) $P(7 < X < 13)$.
5. Define parameter and statistic.
6. Define convergence in distribution.
7. State Central Limit theorem.
8. Define the terms (i) population ; (ii) sampling frame.
9. Define simple random sampling.
10. Find the probability that the sample mean of a random sample of size 16 taken from a normal population with mean 2 and variance 4 is less than 1.
11. If X and Y are independent standard normal random variables, identify the probability distribution of $\left[\frac{X-Y}{X+Y}\right]^2$.
12. Define t-distribution.

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If the $(r-1)^{th}$, r^{th}, and $(r+1)^{th}$ central moments of X following binomial distribution with parameters n and p are, μ_{r-1}, μ_{r-1} and μ_r, show that $\mu_{r+1} = pq\left[nr\,\mu_{r-1} + \frac{d}{dp}\mu_r\right]$.

14. State and prove lack of memory property of exponential distribution.

15. State and prove Chebyshev's inequality.

16. Examine whether Weak Law of Large Numbers hold for the sequence of random variable $\{X_i\}$, where $P\left(X_i = \pm\sqrt{2i-1}\right) = \frac{1}{2}$.

17. State and prove Bernoulli's law of large numbers.

18. Explain cluster sampling.

19. A random sample of size 10 is taken from a normal population with mean 10 and unknown variance. If the sample variance are is 18.23, find the probabilities of the sample mean (i) less than 9 ; (ii) greater than 11.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (i) Show that odd order central moments of X following $N\left(\mu, \sigma^2\right)$ are zeroes.

    (ii) Prove that the mean deviation about the mean of X following $N\left(\mu, \sigma^2\right)$ is $\sqrt{\frac{2}{\pi}}\,\sigma$.

21. (i) Define Chi-square distribution. Obtain the m.g.f., of X following $\chi^2_{(n)}$.

    (ii) State and prove the additive property of Chi-square distribution.

(1 × 11 = 11 marks)