D 113925 **(Pages : 3)** **Name...**

Reg. No....................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2024

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 25 *marks.*
Ceiling is 25.

1. Does the statement "$x^2 + 2 = 1$" is a proposition ? Justify your answer.

2. Define the logical operator "Conjunction" and illustrate with an example.

3. Evaluate the Boolean expression $\sim[a > b] \wedge (b < c)$ for $a = 3, b = 2$ and $c = 4$

4. Write the converse and inverse of the statement : If Δ ABC is equilateral, then it is isosceles.

5. Using Mathematical Induction, prove that $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$.

6. Show that $n^4 + n$ is divisible by 2.

7. State the "Euclidean Algorithm"

8. Determine whether 631 is a prime or not.

9. Prove or disprove : $(2n)! + 1$ is a prime for every positive integer n.

10. Let $(a, b) = d$. Prove that $(a, a - b) = d$.

11. Express $(12, 32)$ as a linear combination of 12 and 32.

Turn over

12. Prove the following : if $d = (a, b)$ and d' be any common divisor of a and b, then d' divides d.

13. Find the l cm of 105 and 65.

14. Is $199 \equiv 1 \pmod 3$? Justify your answer.

15. Show that $2^{2^5} + 1$ is divisible by 641.

Section B

Answer any number of questions.
Each question carries 5 marks.
Overall Ceiling is 35.

16. Evaluate $(18, 30, 60, 75, 132)$.

17. Find the number of trailing zeros in $234!$.

18. Find the positive integer a if $[a, a+1] = 132$.

19. Find the general solution of the LDE $6x + 8y + 12z = 10$.

20. Prove that there are infinitely many primes of the form $4n + 3$.

21. State and Prove the Pigeonhole Principle.

22. Construct a truth table for $(p \to q) \wedge (q \to p)$.

23. Prove that an implication is logically equivalent to its contrapositive.

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Prove indirectly : If the square of an integer is odd, then the integer is odd.

    (b) Prove directly that the product of any two odd integers is an odd integer.

    (c) Prove by contradiction, $\sqrt{5}$ is irrational.

25. State and Prove the Division Algorithm.

26. State and Prove The Fundamental theorem of Arithmetic.

27. (a) Find the positive integer n for which $\sum_{k=1}^{n} k!$ is a square.

(b) Find the reminder when 3^{247} is divided by 17.

(2 × 10 = 20 marks)

D 53677 **(Pages : 2)** **Name.....................................**

Reg. No.................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Does the statement "$x + 2 = 5$" is a proposition ? Justify your answer.

2. Define the logical operator "disjunction" and illustrate with an example.

3. Evaluate the Boolean expression $\sim[a > b] \wedge (b < c)$ for $a = 3$, $b = 5$ and $c = 6$.

4. Prove that there is no positive integer between 0 and 1.

5. Prove that $1 + 2$ +------+ $n = \frac{n(n+1)}{2}$.

6. Show that $n^3 - 3n^2 + 2n$ is divisible by 2.

7. Prove that a product of 3 consecutive natural numbers is divisible by 6.

8. Determine whether 1661 is a prime or not.

9. Prove or disprove : $n! + 1$ is a prime for every positive integer n.

10. Let $(a, b) = d$. Prove that $(a/d, b/d) = 1$

11. Express (12, 28) as a linear combination of 12 and 28..

12. Prove the following : if p is a prime and p/ab. Then p/a or p/b.

13. Find the 1 cm. of 1050 and 574 .

Turn over

461115

14. Is 99 $\equiv$ 7 $(m0d3)$?. Justify your answer.

15. Show that $2^{32} + 1$ is divisible by 641.

Section B

Answer any number of questions.
Each question carries 5 marks.
Overall Ceiling is 35.

16. Find the reminder when $(n^2 + n + 41)^2$ is divided by 12.

17. Solve $12x \equiv 48(\text{mod}18)$.

18. Let a, b are positive integers. Prove that $[a, b]\ (a, b) = ab$.

19. Find the general solution of the LDE $6x + 8y + 12z = 10$.

20. Prove that there are infinitely many primes.

21. State and Prove the Pigeonhole Principle.

22. Construct a truth table for $(p \to q) \leftrightarrow (\sim p \vee q)$.

23. State and prove the associate laws for Conjunction and Dis junction.

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Prove that there is a positive integer that can be expressed in two different ways as the sum of two cubes.

    (b) Prove that there is a prime number > 3.

    (c) Prove by contradiction, $\sqrt{2}$ is irrational.

25. State and Prove the Division Algorithm.

26. State and Prove The Fundamental theorem of Arithmetic.

27. (a) Find the reminder when 1! + 2! + ... + 100! is divided by 15.

    (b) Show that $11.14^n + 1$ is a composite number.

(2 × 10 = 20 marks)

D 32373 **(Pages : 2)** **Name....................................**

Reg. No...............................

FIRST SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS–UG)

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020—2022 Admissions)

Time : Two Hours and a Half Maximum Marks : 80

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum marks that can be earned from this section is 25.

1. Draw the truth table of the disjunction of the proposition p and q.
2. Write down the negation of the statement with UD = the set of integers : $(\forall x)(\exists |x| = x)$.
3. State the prime number theorem.
4. Express (28, 12) as a linear combination of 28 and 12.
5. State the fundamental theorem of arithmetic.
6. Define Euler's phi function.
7. Show that $a \equiv b \pmod{m}$ if and only if $a = b$ + km for some integer k.
8. Briefly describe Euclidean algorithm.
9. Define the least common multiple of two positive integers.
10. What is a tautology ?
11. Prove that there is no positive integer between 0 and 1.
12. What do you mean by the well-ordering principle ?
13. Write down the recursive definition of the factorial function $f(n) = n!$
14. If $a | b$ and $b \}a$, show that $a = b$.
15. Show that $11 \times 14^n + 1$ is a composite number.

(Maximum ceiling 25 marks)

Turn over

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum marks that can be earned from this section is 35.

16. Distinguish between Converse, Inverse and Contrapositive of a proposition by giving example for each.
17. Prove that there is no polynomial $f(n)$ with integral co-efficients that will produce primes for all integers n.
18. Find the remainder when 1! + 2! + … + 100! is divided by 15.
19. Show that $2^{2^5}+1$ is divisible by 641.
20. If p is a prime, then show that $(p-1)! \equiv -1 (\text{mod } p)$.
21. Solve the recurrence relation $h(n) = h(n-1) + (n-1)$ if $h(1) = 0$.
22. If a cock is worth five coins, a hen three coins, and three chicks together one coin, how many cocks, hens and chicks, totalling 100, can be bought for 100 coins ?
23. Show that the LDE $ax + by = c$ is solvable if and only if $d \mid c$, where $d = (a, b)$.

(Maximum ceiling 35 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 marks.
Maximum marks that can be earned from this section is 20.

24. (a) Conjecture a formula for the sum of the first n odd positive integers and then use induction to establish the conjecture.

    (b) Prove that no integer of the form $8n + 7$ can be expressed as a sum of three squares.
25. (a) State and prove Wilson's Theorem.

    (b) State and prove Euler's Theorem.
26. (a) Show that $a \equiv b (\text{mod } m)$ if and only if a and b leave the same remainder when divided by m.

    (b) Prove the divisibility criterion to test the divisibility by 11 and use it to check whether n = 243, 506, 076 is divisible by 11 or not.
27. (a) Define contrapositive of a statement and show that an implication is logically equivalent to its contrapositive by constructing the truth table.

    (b) Prove by contradiction that there is no largest prime.

(2 × 10 = 20 marks)

D 13604 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

(Answer any number of questions.
Each carries 2 marks.
Maximum marks that can be earned from this section is 25)

1. What is a logical statement or a proposition ?
2. Define a vacuously true statement and give an example.
3. If $a \equiv b \pmod n$ and $c \equiv d \pmod n$, show that $ac \equiv bd \pmod n$.
4. Prove or disprove : Every composite number n has a prime factor $\leq \lfloor \sqrt{n} \rfloor$.
5. If $(a, b) = d$, show that a/d and b/d are relatively prime.
6. State Dirichlet's Theorem.
7. If n is a positive integer such that $(n-1)! \equiv -1 \pmod n$, show that n is a prime.
8. State Euler's theorem.
9. State the Division algorithm for integers.
10. What are linear diophantine equations in two variables x and y.
11. State the necessary and sufficient condition(s) for a linear congruence $ax \equiv b \pmod m$ to have a unique solution.
12. Use the divisibility criterion to check whether 10000234 divisible by eight or not.
13. Express the g.c.d (100,13) as a linear combination of 100 and 13.

Turn over

14. Compute the value of $\varphi(100)$.

15. Explain how canonical decomposition is useful in finding the least common multiple of two positive integers.

Section B

(Answer any number questions from this section.
Each question carries 5 marks.
Maximum that can be earned from this section is 35)

16. Evaluate the boolean expression $\sim\left[(x \leq y) \wedge (y > z)\right]$ when $x = 3, y = 4$ and $z = 5$.

17. Construct the truth table of $(p \to q) \wedge (q \to p)$.

18. Prove that there is no positive integer between 0 and 1.

19. There are n guests at a party. Each person shakes hands with everybody else exactly once. Define recursively the number of handshakes $h(n)$ made.

20. Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.

21. Compute the remainder when 3^{247} is divided by 25.

22. Solve the linear congruence $35x \equiv 47 \pmod{24}$.

23. Let b be an integer $\geq 2.$ Suppose $b + 1$ integers are randomly selected. Prove that the difference of two of them is divisible by b.

Section C

(Answer any **two** *questions from this section.*
Each question carries 10 *marks.*
Maximum that can be earned from this section is 20*)*

24. (a) Construct truth table for $p \to q \leftrightarrow \sim p \wedge q$

    (b) Prove by the method of contradiction : The square of an odd integer is odd. Moreover, rewrite the proposition symbolically with UD = set of all integers.

25. (a) State and prove the weak version of principle of mathematical induction.

(b) Define the least common multiple $[a, b]$ of two positive integers and show that $[a, b] = \frac{ab}{(a, b)}$.

26. (a) Find the remainder when 24^{1947} is divided by 17.

(b) State and prove the Fermat's little theorem.

27. State and prove the Fundamental theorem of arithmetic.

D 12645 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2021 Admissions)

Time : Two Hour and a Half Maximum : 80 Marks

Section A

Answer atleast **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall ceiling 30.

1. Verify that $p \vee p \equiv p$ and $p \wedge p \equiv p$.

2. Let P(x)denote the statement "$x > 3$." What is the truth value of the quantification $\exists x\ \mathrm{P}(x)$, where the universe of discourse is the set of real numbers ?

3. State the barber paradox presented by Bertrand Russell in 1918.

4. Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

5. Prove the following formula for the sum of the terms in a "geometric progression" :

$$1 + r + r^2 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

6. Let a and b positive integers such that $a \mid b$ and $b \mid a$. Then prove that $a = b$.

7. Briefly explain Mahavira's puzzle.

8. Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.

9. Prove that every composite number n has a prime factor $\leq \lfloor \sqrt{n} \rfloor$.

10. Show that any two consecutive Fibonacci numbers are relatively prime.

Turn over

11. Let a and b be integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers α and β such that $1 = \alpha a + \beta b$.

12. Prove that if $a\,|$ and $b\,|\,c$, and $(a, b) = 1$, then $ab\,|\,c$.

13. Prove that every integer $n \geq 2$ has a prime factor.

14. Let f_n denote the n^{th} Fermat number. Then prove that $f_n = f_{n-1}^2 - 2f_{n-1} + 2$, where $n \geq 1$.

15. Express gcd (28, 12) as a linear combination of 28 and 12.

(10 × 3 = 30 marks)

Section B

Answer atleast **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall ceiling 30.

16. Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

17. Show that theassertion "All primes are odd" is false.

18. Let b be an integer ≥ 2. Suppose $b + 1$ integers are randomly selected. Prove that the difference of two of them is divisible by b.

19. If p is a prime and $p\,|\,a_1 a_2 \ldots a_n$, then prove that $p\,|\,a_i$ at for some i, where $1 \leq i \leq n$.

20. Show that $11 \times 14n + 1$ is a composite number.

21. There are infinitely many primes of the form $4n + 3$.

22. Show that $2^{11213} - 1$ is not divisible by 11.

23. Prove that if $n \geq 1$ and gcd $(a, n) = 1$,then $a^{\phi(n)} \equiv 1 \pmod{n}$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) State six standard methods for proving theorems and briefly explain any two of them with the help of examples.

    (b) Using the laws of logic simplify the Boolean Expression $(p \wedge \neg q) \vee q \vee (\neg p \wedge q)$.

25. (a) Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

    (b) State the prime number theorem and find six consecutive integers that are composites.

26. (a) State and prove Fundamental Theorem of Arithmetic.

    (b) Find the largest power of 3 that divides 207!

27. (a) Let p be a prime and a any integer such that $p | a$. Then show that the least residues of the integers a, $2a$,$3a$,. .. ,$(p - 1)\, a$ modulo p are a permutation of the integers

    1, 2, 3,...,$(p - 1)$.

    (b) Find the remainder when 24^{1947} is divided by 17.

(2 × 10 = 20 marks)

D 93936-B (Pages : 2)

Name..............................

Reg. No..............................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020 Admissions)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. Find the truth table for the *Disjunction* of two propositions.

2. Define a proposition. Give an example.

3. Define Valid and Invalid Arguments.

4. Show that the proposition P(0) is true where P(n) is the propositional function "If $n > 1$ then $n^2 > n$".

5. Prove that every non-empty set of non-negative integers has a least element.

6. Find the quotient q and the remainder r when :

   (i) 207 is divided by 15.

   (ii) –23 is divided by 5.

7. Prove that 2 and 3 are the only two consecutive integers that are primes.

8. State and prove Handshake Problem.

9. Define the Fibonacci sequence and write the first four Fibonacci Numbers and Lucas numbers.

10. Using the Euclidean algorithm, express (4076, 1024) as a linear combination of 4076 and 1024.

11. Find the number of trailing zeros in 234 !.

12. Find the largest power of 2 that divides 109 ! .

13. Find all solutions of the congruence $9x \equiv 21 \pmod{30}$.

14. If 2p +1 is a prime number, prove that $(p)^2 + (-1)^r$ is divisible by $2p + 1$.

15. Find the remainder obtained when 5^{38} is divided by 11.

($10 \times 3 = 30$ marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 30.

16. Give a proof by contradiction of the theorem "if n^2 is even, then n is even."
17. Write any 5 inference rules.
18. Prove that there is no positive integer between 1 and 2.
19. Obtain an explicit formula corresponding to the recursive relation :

    $h(n) = h(n-1) + (n-1), n \geq 2$.
20. Let $(a,b) = d$. Then prove that $(a, a-b) = d$.
21. Explain Jigsaw Puzzle.
22. Find the remainder obtained upon dividing the sum

    $1! + 2! + 3! + 4! + ... + 99! + 100!$ by 12.
23. Let p be a prime and a any integer such that p does not divide a. Then prove that the solution of the linear congruence $ax = b \pmod{p}$ is given by $x = a^{p-2}b \pmod{p}$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

    (b) Prove the implication "If n is an integer not divisible by 3, then $n^2 \equiv 1 \pmod{3}$."
25. (a) Prove that a palindrome with an even number of digits is divisible by 11.

    (b) Let a and b be any positive integers, and r the remainder, when a is divided by b. Then prove that gcd (a, b) = gcd (b, r).
26. (a) Prove that the gcd of the positive integers a and b is a linear combination of a and b.

    (b) State Duncan's identity. Using recursion, evaluate (18, 30, 60, 75, 132).
27. (a) Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.

    (b) Prove that the linear congruence $ax \equiv b \pmod{m}$ is solvable if and only if $d|b$, where $d = (a, m)$ and if $d|b$, then it has d incongruent solutions.

(2 × 10 = 20 marks)

D 93936 (Pages : 3) Name.........................

Reg. No.........................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. Define what is meant by *disjunction* of two statements. Construct the disjunction of the following statements :

   Statement 1) Vinod watches cinemas during holidays .

   Statement 2) Vinod enjoys poetry during holidays.

2. What is meant by the *converse* of an implication ? Give an example.

3. What is the difference between *tautology* and a *contradiction* ?

4. Show that if $c|a$ and $c|b$, then $c|\alpha a + \beta b$ for any integers α, β.

5. Express $(1092)_{10}$ in base 8.

6. Define the GCD of integers a, b. When do we say that they are relatively prime ?

7. Find the canonical decomposition of 1980.

8. Find $(252, 350)$ and hence find $[252, 360]$.

9. If $a \equiv 5 \pmod{25}$ give four possibilities of a.

10. Which of the following is a complete set of residues modulo 8? $\{1, 2, 4, 5, 7, 8, 11, 14\}$ or $\{1, 2, 4, 5, 7, 8, 19, 24\}$ or both ? Why ?

11. Find the inverse of 7 modulo 50.

Turn over

12. Evaluate $\phi(35), \phi(48)$ directly without using formula.

13. State Fermat's Little Theorem. Prove it.

14. Define the function τ. Evaluate $\tau(19)$ and $\tau(23)$.

15. Compute $\phi(666)$ and $\phi(1976)$ using canonical decomposition.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. Test the validity of the following argument :

    A_1 : There are more residents in New Delhi than there are hairs in the head of any resident.

    A_2 : No resident is totally bald.

    Hence At least two residents must have the same number of hairs on their heads.

17. State the Inclusion-Exclusion Principle. Use it to find the number of positive integers ≤ 2076 that are divisible by neither 4 nor 5.

18. Define Fermat numbers. Derive a recurrence formula for the n^{th} Fermat number f_n.

19. Let a and b be any positive integers, and r the remainder, when a is divided by b. Prove that $(a, b) = (b, r)$.

20. Find the general solution to the LDE $12x + 20y = 28$.

21. Prove that no integer of the form $8n + 7$ can be expressed as a sum of three squares.

22. Let p be a prime and a any positive integer. Prove that $a^p \equiv a \pmod{p}$. Does this result hold for some non-prime integer p ? Justify.

23. Prove that if n is an odd pseudoprime, then $N = 2^n - 1$ is also an odd pseudoprime.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. (a) Prove directly that the product of any even integer and any odd integer is even.

(b) Prove by cases that for any integer $n, n^2 + n$ is an even integer.

(c) Prove by contradiction that $\sqrt{5}$ is an irrational number.

25. (a) Let p be a prime and $p \mid a_1 a_2 \ldots\ldots a_n$, where $a_1, a_2, \ldots, a_n$ are positive integers. Prove that $p \mid a_i$ for some i, where $1 \leq i \leq n$.

(b) State and prove the Fundamental Theorem of Arithmetic.

26. Prove that the linear congruence $ax \equiv b \pmod{m}$ is solvable if and only if $d \mid b$, where $d = (a, m)$. Also, prove that if $d \mid b$, then it has d incongruent solutions.

27. Prove that the function ϕ is multiplicative. Use it to evaluate $\phi(221)$ and $\phi(6125)$.

D 73287 (Pages : 2) Name..............................

Reg. No..............................

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CBCSS—UG)

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. What is a *compound proposition* ? Give two simple propositions and combine it to make compound.
2. If $a = 3, b = 5, c = 6$, explain and evaluate the boolean expression $[\sim (a > b)] \wedge (b < c)$.
3. What is meant by a *biconditional statement* ? Give an example.
4. Suppose that in a party, there are n guests. Each person shakes hands with everybody else exactly once. Define recursively the number of handshakes $h(n)$ made.
5. Find the sum of $(100)_2$ and $(110)_2$.
6. Define Fibonacci numbers and Lucas numbers.
7. Evaluate (18, 30, 60, 75, 132).
8. Prove that two positive integers a and b are relatively prime if and only if $[a,b] = ab$.
9. If $a \equiv b$ and $b \equiv c$ modulo n, can we say that $a \equiv c$ modulo n ? Justify.
10. Is {2, 4, 6, 8, 10} a complete set of residues modulo 5 ? Why ?
11. When will we say that a is invertible modulo m ? Give an example for an a invertible modulo 8.
12. What is a pseudoprime ? Give an example and verify it.
13. Let m be a positive integer and a any integer with $(a,m) = 1$. Prove that $a^{\phi(m)} \equiv 1 (\bmod m)$.
14. What is the value of $\phi(p^e)$ where p is a prime ? Use it to compute $\phi(81)$ and $\phi(625)$.
15. State the fundamental theorem for multiplicative functions and the Gauss theorem for natural number n on sum of $\phi(d)$ where $d|n$.

Turn over

11:50 – 12:30 – NHA
3 – 3:45 – Mn

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 marks.

16. Give a nonconstructive proof to show that there is a prime number > 3.

17. Prove that any postage of Rupees $n \geq 2$ can be made with two and three rupees stamps.

18. Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

19. State and prove the Duncan's identity.

20. Find the general solution to the LDE $63x - 23y = -7$.

21. Prove that $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder when divided by m.

22. Let p be a prime and a any integer such that $p|a$. Prove that the least residues of the integers a, $2a, 3a, \ldots, (p-1)a$ modulo p are a permutation of the integers $1, 2, 3, \ldots, (p-1)$.

23. Prove that the tau, sigma functions are multiplicative.

Section C

Answer any **two** *questions.*
Each question carries 10 marks.
Maximum 20 marks.

24. (a) Verify that $p \to q \equiv \sim q \to \sim p$ constructing the truth table.

    (b) Simplify the boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ using the laws of logic.

25. (a) If $(a,b) = d$, prove that $(a/d, b/d) = 1$.

    (b) Prove that the gcd of the positive integers a and b is a linear combination of a and b.

26. What is the necessary and sufficient condition for the LDE $ax + by = c$ to be solvable ? State it and prove it.

27. (a) If p is a prime, prove that $(p-1)! \equiv -1 \pmod{p}$.

    (b) Let p be a prime and n any positive integer. Prove that $\frac{(np)!}{n!\,p^n} \equiv (-1)^n \pmod{p}$.