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Pratiyogita Darpan

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Physical Sciences
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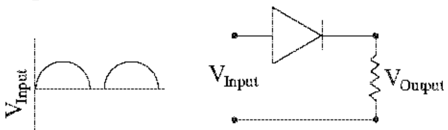
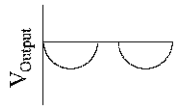
Solved Paper

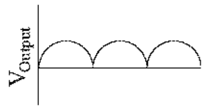
2011
Physical Sciences

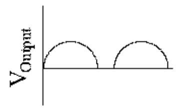
PART A

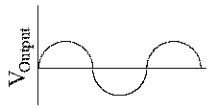
1. If V_{input} is applied the circuit shown, the output would be—



(A) 

(B) 

(C) 

(D) 

2. A physiological disorder X always leads to the disorder Y. However, disorder Y may occur by itself. A population shows 4% incidence of disorder Y. Which of the following inferences is valid ?
   (A) 4% of the population suffers from both X and Y
   (B) Less than 4% of the population suffers from X
   (C) At least 4% of the population suffers from X
   (D) There is no incidence of X in the given population

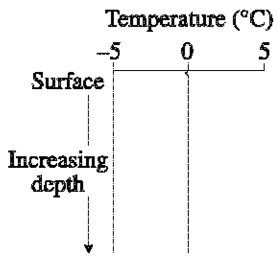
3. The speed of a car increases every minute as shown in the following Table. The speed at the end of the 19th minute would be—

Time (minutes)	**Speed** (m/sec)
1	1·5
2	3·0
3	4·5
.	.
.	.
24	36·0
25	37·5

   (A) 26·5 (B) 28·0
   (C) 27·0 (D) 28·5

4. How many σ bonds are present in the following molecule ?
   HC ≡ CCH = $CHCH_3$
   (A) 4 (B) 6
   (C) 10 (D) 13

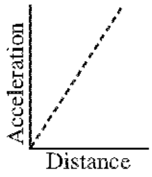
5. The graph represents the depth profile of temperature in the open ocean; in which region this is likely to be prevalent ?

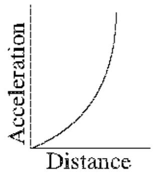


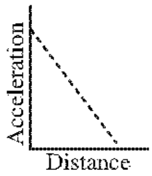
   (A) Tropical region
   (B) Equatorial region
   (C) Polar region
   (D) Sub-tropical region

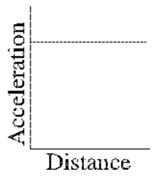
6. A reference material is required to be prepared with 4 ppm calcium. The amount of $CaCO_3$ (molecular weight = 100) required to prepare 1000 g of such a reference material is—
   (A) 10 μg
   (B) 4 μg
   (C) 4 mg
   (D) 10 mg

7. A ball is dropped from a height *h* above the surface of the earth. Ignoring air drag, the curve that best represents its variation of acceleration is—

(A) 

(B) 

(C) 

(D) 

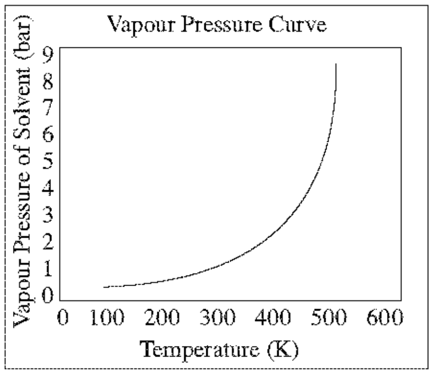
8. If the atmospheric concentration of carbon dioxide is doubled and there are favourable conditions of water, nutrients, light and temperature, what would happen to water requirement of plants ?
   (A) It decreases initially for a short time and then returns to the original value
   (B) It increases
   (C) It decreases
   (D) It increases initially for a short time and then returns to the original value

9. Exposing an organism to a certain chemical can change nucleotide bases in a gene, causing mutation, In one such mutated organism if a protein had only 70% of the primary amino acid sequence, which of the following is likely ?
   (A) Mutation broke the protein
   (B) The organism could not make amino acids
   (C) Mutation created a terminator codon
   (D) The gene was not transcribed

10. The reason for the hardness of diamond is—
   (A) extended covalent bonding
   (B) layered structure
   (C) formation of cage structures
   (D) formation of tubular structures

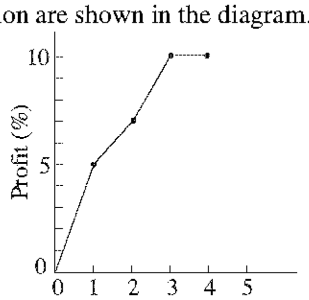
11. The acidity of normal rain water is due to—
   (A) SO_2 (B) CO_2
   (C) NO_2 (D) NO

12. The normal boiling point of a solvent (whose vapour pressure curve is shown in the figure)



on a planet whose normal atmospheric pressure is 3 bar, is about—
   (A) 100 K (B) 273 K
   (C) 400 K (D) 500 K

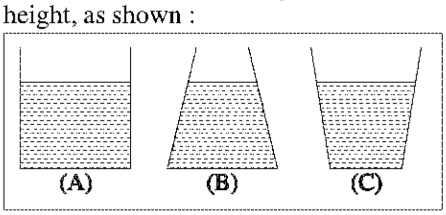
13. The cumulative profits of a company since its inception are shown in the diagram. If the net



worth of the company at the end of 4th year is 99 crores, the principal it had started with was—
   (A) 9·9 crore (B) 91 crore
   (C) 90 crore (D) 9·0 crore

14. Which of the following particles has the largest range in a given medium if their initial energies are the same ?
   (A) Alpha (B) Electron
   (C) Positron (D) Gamma

15. Water is dripping out of a tiny hole at the bottom of three flasks whose base diameter is the same, and are initially filled to the same height, as shown :



Which is the correct comparison of the rate of fall of the volume of water in the three flasks ?

(A) A fastest, B slowest

(B) B fastest, A slowest

(C) B fastest, C slowest

(D) C fastest, B slowest

16. Diabetic patients are advised a low glycaemic index diet. The reason for this is—

(A) they require less carbohydrate than healthy individuals

(B) they cannot assimilate ordinary carbohydrates

(C) they need to have slow, but sustained release of glucose in their blood stream

(D) they can tolerate lower, but not higher than normal blood sugar levels

17. Standing on a polished stone floor one feels colder than on a rough floor of the same stone. This is because

(A) thermal conductivity of the stone depends on the surface smoothness

(B) specific heat of the stone changes by polishing it

(C) the temperature of the polished floor is lower than that of the rough floor

(D) there is greater heat loss from the soles of the feet when in contact with the polished floor than with the rough floor

18. Popular use of which of the following fertilizers increases the acidity of soil ?

(A) Potassium Nitrate

(B) Urea

(C) Ammonium sulphate

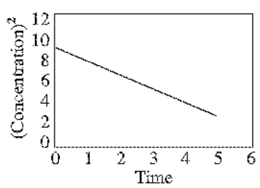
(D) Superphosphate of lime

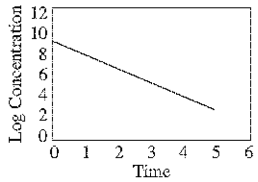
19. Glucose molecules diffuse across a cell of diameter d in time τ. If the cell diameter is tripled the diffusion time would—

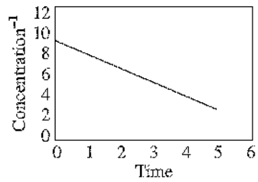
(A) increase to 9τ (B) decrease to $\tau/3$

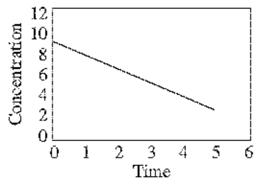
(C) increase to 3τ (D) decrease to $\tau/9$

20. Identify the figure which depicts a first order reaction.

(A)


(B)


(C)


(D)


PART B

21. Let $p_n(x)$ (where $n = 0, 1, 2, ...$) be a polynomial of degree n with real coefficients, defined in the interval $2 \le n \le 4$.

If $\int_2^4 p_n(x)\, p_m(x)\, dx = \delta_{nm}$, then—

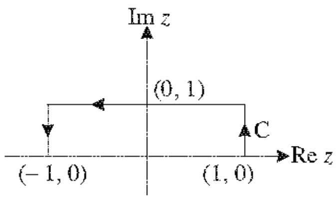
(A) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}\,(-3 - x)$

(B) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}\,(3 + x)$

(C) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}\,(3 - x)$

(D) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}\,(3 - x)$

22. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in the figure below, is—



(A) $\frac{5}{e} + e$ (B) $e - \frac{5}{e}$

(C) $\frac{5}{e} - e$ (D) $-\frac{5}{e} - e$

23. Which of the following matrices is an element of the group SU (2) ?

(A) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$

(C) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (D) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

24. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by—

(A) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{a^2}$ (B) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{b^2}$

(C) $\frac{(\vec{a} \cdot \vec{b})\,\vec{b}}{b^2}$ (D) $\frac{(\vec{b} \cdot \vec{a})\,\vec{a}}{a^2}$

25. A particle of unit mass moves in a potential $V(x) = a x^2 + \frac{b}{x^2}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is—

(A) $\sqrt{8b}$ (B) $\sqrt{8a}$

(C) $\sqrt{\frac{8a}{b}}$ (D) $\sqrt{\frac{8b}{a}}$

26. The acceleration due to gravity (g) on the surface of Earth is approximately 2·6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately—

(A) 1·1 (B) 1·3

(C) 2·3 (D) 5·2

27. The Hamiltonian of a system with n degrees of freedom is given by $H(q_1, \ldots, q_n; p_1, \ldots, p_n; t)$, with an explicit dependence on the time t. Which of the following is correct ?

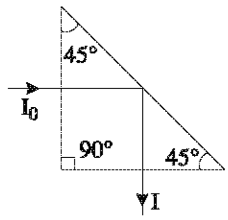
(A) Different phase trajectories cannot intersect each other.

(B) H always represents the total energy of the system and is a constant of the motion.

(C) The equations $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$ are not valid since H has explicit time dependence.

(D) Any initial volume element in phase space remains unchanged in magnitude under time evolution.

28. Circularly polarized light with intensity I_0 is incident normally on a glass prism as shown in the figure. The index of refraction of glass



is 1·5. The intensity I of light emerging from the prism is—

(A) I_0 (B) $0{\cdot}96\, I_0$

(C) $0{\cdot}92\, I_0$ (D) $0{\cdot}88\, I_0$

29. The electrostatic potential V (x, y) is free space in a region where the charge density ρ is zero is given by

$V(x, y) = 4e^{2x} + f(x) - 3y^2$. Given that the x-component of the electric field, E_x, and V are zero at the origin, $f(x)$ is—

(A) $3x^2 - 4e^{2x} + 8x$ (B) $3x^2 - 4e^{2x} + 16x$

(C) $4e^{2x} - 8$ (D) $3x^2 - 4e^{2x}$

30. For constant uniform electric and magnetic fields $\vec{E} = \vec{E}_0$ and $\vec{B} = \vec{B}_0$, it is possible to choose a gauge such that the scalar potential ϕ and vector potential $\vec{A}$ are given by—

(A) $\phi = 0$ and $\vec{A} = \frac{1}{2}(\vec{B}_0 \times \vec{r})$

(B) $\phi = -\vec{E}_0 \cdot \vec{r}$ and $\vec{A} = \frac{1}{2}(\vec{B}_0 \times \vec{r})$

(C) $\phi = -\vec{E}_0 \cdot \vec{r}$ and $\vec{A} = 0$

(D) $\phi = 0$ and $\vec{A} = -\vec{E}_0 t$

31. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by—

$$\vec{E}(x, y, z, t) = E_0(\hat{x} + A\hat{z}) \exp\left[ik_0\left\{-ct + \left(x + \sqrt{3}\,z\right)\right\}\right]$$

where c is the speed of light in vaccum, E_0, A and k_0 are constants and $\hat{x}$ and $\hat{z}$ are unit vectors along the x- and z-axes. The relative dielectric constant of the medium, ε_r and the constant A are—

(A) $\varepsilon_r = 4$ and $A = -\frac{1}{\sqrt{3}}$

(B) $\varepsilon_r = 4$ and $A = +\frac{1}{\sqrt{3}}$

(C) $\varepsilon_r = 4$ and $A = \sqrt{3}$

(D) $\varepsilon_r = 4$ and $A = -\sqrt{3}$

32. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}\cdot\vec{B}$ where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the 2×2 unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to—

(A) $I\cos\frac{\omega t}{2} + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\sin\frac{\omega t}{2}$

(B) $I\cos\omega t + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\sin\omega t$

(C) $I\sin\omega t + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\cos\omega t$

(D) $I\sin 2\omega t + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\cos 2\omega t$

33. If the perturbation $H' = ax$, where a is a constant, is added to the infinite square well potential

$$V(x) = \begin{cases} 0 & \text{for } 0 \le x \le \pi \\ \infty, & \text{otherwise} \end{cases}$$

The correction to the ground state energy, to first order in a, is—

(A) $\frac{a\pi}{2}$ (B) $a\pi$

(C) $\frac{a\pi}{4}$ (D) $\frac{a\pi}{\sqrt{2}}$

34. The energy levels of the non-relativistic electron in a hydrogen atom (*i.e.* in a Coulomb potential $V(r) \propto -1/r$) are given by $E_{nlm} \propto -1/n^2$, where n is the principal quantum number, and the corresponding wave functions are given by Ψ_{nlm}, where l is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement ?

(A) There are exactly $(2l + 1)$ different wave functions Ψ_{nlm}, for each E_{nlm}.

(B) There are $l\,(l + 1)$ different wave functions Ψ_{nlm}, for each E_{nlm}.

(C) E_{nlm} does not depend on l and m for the Coulomb potential.

(D) There is a unique wave function Ψ_{nlm} for each E_{nlm}.

35. The wave function of a particle is given by $\Psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\,\phi_1\right)$, where ϕ_0 and ϕ_1 are the normalized eigenfunctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state Ψ is—

(A) $\frac{E_0}{2} + E_1$ (B) $\frac{E_0}{2} - E_1$

(C) $\frac{E_0 - 2E_1}{3}$ (D) $\frac{E_0 + 2E_1}{3}$

36. Consider the transition of liquid water to steam as water boils at a temperature of 100°C under a pressure of 1 atmosphere. Which one of the following quantities does **not** change discontinuously at the transition ?

(A) The Gibbs free energy

(B) The internal energy

(C) The entropy

(D) The specific volume

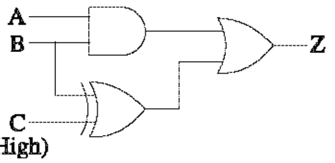
37. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_v = \gamma T^3$, where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is—

(A) $4\gamma T^4$ (B) $2\gamma T^4$
(C) γT^4 (D) $\gamma T^4/4$

38. A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $u(x) = u_0 x$. The mean position of the particle at temperature T is—

(A) $\frac{k_B T}{u_0}$ (B) $(k_B T)^2/u_0$
(C) $\sqrt{\frac{k_B T}{u_0}}$ (D) $u_0 k_B T$

39. Consider the digital circuit shown below in which the input C is always high (1).



The truth table for the circuit can be written as—

A	B	Z
0	0	
0	1	
1	0	
1	1	

The entries in the Z column (vertically) are—
(A) 1010 (B) 0100
(C) 1111 (D) 1011

40. A signal of frequency 10 kHz is being digitized by an A/D converter. A possible sampling time which can be used is—
(A) 100 μs (B) 40 μs
(C) 60 μs (D) 200 μs

PART C
(Compulsory)

41. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$?
(A) $(3 + x - iy)^7$
(B) $(1 + x + iy)^4 (7 - x - iy)^3$
(C) $(1 - 2x - iy)^4 (3 - x - iy)^3$
(D) $(x + iy - 1)^{1/2}$

42. A static, spherically symmetric charge distribution is given by $\rho(r) = \frac{A}{r} e^{-Kr}$ where A and K are positive constants. The electrostatic potential corresponding to this charge distribution varies with r as—
(A) re^{-Kr} (B) $\frac{1}{r} e^{-Kr}$
(C) $\frac{1}{r^2} e^{-Kr}$ (D) $\frac{1}{r}(1 - e^{-Kr})$

43. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic fields is given by $L = \frac{1}{2} m \vec{v}^2 + e\vec{A} \cdot \vec{v} - e\phi$, where $\vec{A}$ and ϕ are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct ?
(A) The canonically conjugate momentum of the particle is given by $\vec{p} = m\vec{v}$
(B) The Hamiltonian of the particle is given by $H = \frac{\vec{p}^2}{2m} + \frac{e}{m} \vec{A} \cdot \vec{p} + e\phi$
(C) L remains unchanged under a gauge transformation of the potentials
(D) Under a gauge transformation of the potentials, L changes by the total time derivative of a function of $\vec{r}$ and t

44. A particle in one dimension moves under the influence of a potential $V(x) = ax^6$, where a is a real constant. For large n the quantized energy level E_n depends on n as—
(A) $E_n \sim n^3$ (B) $E_n \sim n^{4/3}$
(C) $E_n \sim n^{6/5}$ (D) $E_n \sim n^{3/2}$

45. Consider two independently diffusing non-interacting particles in 3-dimensional space, both placed at the origin at time $t = 0$. These particles have different diffusion constants D_1 and D_2. The quantity $\langle [(\vec{R}_1(t) - \vec{R}_2(t))^2] \rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as—
(A) $6t(D_1 + D_2)$ (B) $6t|D_1 - D_2|$
(C) $6t\sqrt{D_1^2 + D_2^2}$ (D) $6t\sqrt{D_1 D_2}$

46. Consider a system of N non-interacting spins, each of which has classical magnetic moment of magnitude μ. The Hamiltonian of this system in an external magnetic field $\vec{H}$ is $\mathcal{H} = -\sum_{i=1}^{N} \vec{\mu}_i \cdot \vec{H}$, where $\vec{\mu}_i$ is the magnetic moment of the i^{th} spin. The magnetization per spin at temperature T is—

(A) $\frac{\mu^2 H}{k_B T}$

(B) $\mu\left[\coth\hbar\left(\frac{\mu H}{k_B T}\right) - \frac{k_B T}{\mu H}\right]$

(C) $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$

(D) $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$

47. Consider the matrix $M = \begin{Bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{Bmatrix}$

A. The eigenvalues of M are—

(A) 0, 1, 2 (B) 0, 0, 3

(C) 1, 1, 1 (D) –1, 1, 3

B. The exponential of M simplifies to (I is the 3 × 3 identity matrix)

(A) $e^M = I + \left(\frac{e^3 - 1}{3}\right) M$

(B) $e^M = I + M + \frac{M^2}{2!}$

(C) $e^M = I + 3^3 M$

(D) $e^M = (e - 1) M$

48. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given by I_1, I_2 and I_3 is rotating freely about a fixed point inside the body. The Euler equations for the components of its angular velocity $(\omega_1, \omega_2, \omega_3)$ are—

$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$, $\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1, \omega_3$,

$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_1\omega_2$

A. The equilibrium points in $(\omega_1, \omega_2, \omega_3)$ space are—

(A) (1, – 1, 0), (– 1, 0, 1) and (0, – 1, 1)

(B) (1, 1, 0), (1, 0, 1) and (0, 1, 1)

(C) (1, 0, 0), (0, 1, 0) and (0, 0, 1)

(D) (1, 1, 1), (– 1, – 1, – 1) and (0, 0, 0)

B. The constants of motion are—

(A) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1\omega_1 + I_2\omega_2 + I_3\omega_3$

(B) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

(C) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $\omega_1 + \omega_2 + \omega_3$

(D) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

49. In a system consisting of two spin $\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)} = \frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)} = \frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators. Here $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ and $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices

A. In the standard basis the matrices for the operators $S_x^{(1)}S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ are respectively,

(A) $\frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(B) $\frac{\hbar^2}{4}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

(C) $\frac{\hbar^2}{4}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$

(D) $\frac{\hbar^2}{4}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

B. These two operators satisfy the relation

(A) $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = S_z^{(1)}S_z^{(2)}$

(B) $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = 0$

(C) $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = i\,S_z^{(1)}S_z^{(2)}$

(D) $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = 0$

50. A resistance is measured by passing a current through it and measuring the resulting voltage drop. If the voltmeter and the ammeter have uncertainties of 3% and 4%, respectively, then

A. The uncertainty in the value of the resistance is—

(A) 7·0% (B) 3·5%
(C) 5·0% (D) 12·0%

B. The uncertainty in the computed value of the power dissipated in the resistance is—

(A) 7% (B) 5%
(C) 11% (D) 9%

Choose any 10 of the remaining questions

51. The character table of C_{3v} the group of symmetries of an equilateral triangle is given below—

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1C_1$	1	1	b
$3C_2$	1	a	c
$2C_3$	1	1	d

In the above C_1, C_2, C_3 denote the three classes of C_{3v} containing 1, 3 and 2 elements respectively, and $\chi^{(0)}$, $\chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducible representations $\Gamma^{(0)}$, $\Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v}.

A. The entries a, b, c and d in this table are, respectively,

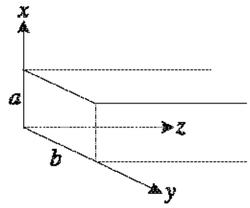
(A) 2, 1, – 1, 0 (B) – 1, 2, 0, – 1
(C) –1, 1, 0, – 1 (D) – 1, 1, 1, – 1

B. The reducible representation Γ of C_{3v} with character = χ = (4, 0, 1) decomposes into its irreducible representations $\Gamma^{(0)}$, $\Gamma^{(1)}$, $\Gamma^{(2)}$ as—

(A) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (B) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$
(C) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (D) $2\Gamma^{(2)}$

52. The magnetic field of the TE_{11} mode of a rectangular waveguide of dimensions $a \times b$ as shown in the figure is given by

$H_z = H_0 \cos(0{\cdot}3\pi x) \cos(0{\cdot}4\pi y)$, where x and y are in cm.



A. The dimensions of the waveguide are—

(A) a = 3·33 cm, b = 2·50 cm
(B) a = 0·40 cm, b = 0·30 cm
(C) a = 0·80 cm, b = 0·60 cm
(D) a = 1·66 cm, b = 1·25 cm

B. The <u>entire</u> range of frequencies f for which the TE_{11} mode will propagate is—

(A) 6·0 GHz < f < 7·5 GHz
(B) 7·5 GHz < f < 9·0 GHz
(C) 7·5 GHz < f < 12·0 GHz
(D) 7·5 GHz < f

53. Light of wavelength 660 nm and power of 1 mW is incident on a semiconductor photo-diode with an absorbing layer of thickness of (ln 4) μm.

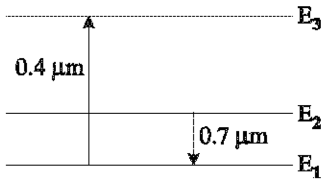
A. If the absorption coefficient at this wavelength is 10^4 cm^{-1} and if 1% power is lost on reflection at the surface, the power absorbed will be—

(A) 750μW (B) 675μW
(C) 250μW (C) 225μW

B. The generated photo-current for a quantum efficiency of unity will be—

(A) 400μA (B) 360μA
(C) 133μA (D) 120μA

54. Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with $1{\cdot}6 \times 10^{19}$ chromium ions per cubic centimeter. All the atoms excited by the 0·4μm radiation decay rapidly to level E_2 which has a lifetime τ = 3 ms.



A. Assuming that there is no radiation of wavelength 0·7μm present in the pumping cycle and that the pumping rate is R atoms per cm^3, the population density in the level N_2 builds up as—

(A) $N_2(t) = R\tau(e^{t/\tau} - 1)$

(B) $N_2(t) = R\tau(1 - e^{-t/\tau})$

(C) $N_2(t) = \frac{Rt^2}{\tau}(1 - e^{-t/\tau})$

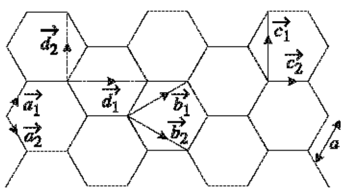
(D) $N_2(t) = Rt$

B. The minimum pump power required (per cubic centimeter) to bring the system to transparency, *i.e.*, zero gain, is—

(A) 1·52 kW (B) 2·64 kW

(C) 0·76 kW (D) 1·32 kW

55. The two dimensional lattice of graphene is an arrangement of carbon atoms forming a honeycomb lattice of lattice spacing a, as shown below. The carbon atoms occupy the vertices.



A. The Wigner-Seitz cell has an area of—

(A) $2a^2$ (B) $\frac{\sqrt{3}}{2}a^2$

(C) $6\sqrt{3}a^2$ (D) $\frac{3\sqrt{3}}{2}a^2$

B. The Bravais lattice for this array is a—

(A) rectangular lattice with basis vectors $\vec{d_1}$ and $\vec{d_2}$

(B) rectangular lattice with basis vectors $\vec{c_1}$ and $\vec{c_2}$

(C) hexagonal lattice with basis vectors $\vec{a_1}$ and $\vec{a_2}$

(D) hexagonal lattice with basis vectors $\vec{b_1}$ and $\vec{b_2}$

56. A flux quantum (fluxoid) is approximately equal to 2×10^{-7} gauss-cm^2. A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is $\frac{2}{\pi} \times 10^5$ gauss.

A. The penetration depth of this superconductor is—

(A) 100 Å (B) 10 Å

(C) 1000 Å (D) 314 Å

B. The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi} \times 10^5$ gauss. The correlation length of the superconductor is—

(A) 20 Å (B) 200 Å

(C) 628 Å (D) 2000 Å

57. A narrow beam of X-rays with wavelength 1·5 Å is reflected from an ionic crystal with an fcc lattice structure with a density of 3·32 g cm^{-3}. The molecular weight is 108 AMU (1 AMU = $1{\cdot}66 \times 10^{-24}$ g).

A. The lattice constant is—

(A) 6·00 Å (B) 4·56 Å

(C) 4·00 Å (D) 2·56 Å

B. The sine of the angle corresponding to (111) reflection is—

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{8}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

58. The radius of a $^{64}_{29}Cu$ nucleus is measured to be $4{\cdot}8 \times 10^{-13}$ cm.

A. The radius of a $^{27}_{12}Mg$ nucleus can be estimated to be—

(A) $2{\cdot}86 \times 10^{-13}$ cm (B) $5{\cdot}2 \times 10^{-13}$ cm

(C) $3{\cdot}6 \times 10^{-13}$ cm (D) $8{\cdot}6 \times 10^{-13}$ cm

B. The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number A in its ground state varies as—

(A) $A^{4/3}$ (B) $A^{1/3}$

(C) $A^{-1/3}$ (D) $A^{-2/3}$

59. A beam of pions (π^+) is incident on a proton target, giving rise to the process $\pi^+ p \to n + \pi^+ + \pi^+$

A. Assuming that the decay proceeds through strong interactions, the total isospin I and its third component I_3 for the decay products, are—

(A) $I = \frac{3}{2}, I_3 = \frac{3}{2}$ (B) $I = \frac{5}{2}, I_3 = \frac{5}{2}$

(C) $I = \frac{5}{2}, I_3 = \frac{3}{2}$ (D) $I = \frac{1}{2}, I_3 = -\frac{1}{2}$

B. Using isospin symmetry, the cross-section for the above process can be related to that of the process

(A) $\pi^- n \to p\, \pi^- \pi^-$

(B) $\pi^- \vec{p} \to \vec{n}\, \pi^- \pi^-$

(C) $\pi^+ n \to p\, \pi^+ \pi^-$

(D) $\pi^+ \vec{p} \to n\, \pi^+ \pi^-$

60. Consider the decay process $\tau^- \to \pi^- + V_\tau$ in the rest frame of the τ^-. The masses of the τ^-, π^- and ν_τ are M_τ, M_π and zero respectively.

A. The energy of π^- is—

(A) $\frac{(M_\tau^2 - M_\pi^2)c^2}{2M_\tau}$ (B) $\frac{(M_\tau^2 + M_\pi^2)c^2}{2M_\pi}$

(C) $(M_\tau - M_\pi)c^2$ (D) $\sqrt{M_\tau M_\pi c^2}$

B. The velocity of π^- is—

(A) $\frac{(M_\tau^2 - M_\pi^2)c}{M_\tau^2 + M_\pi^2}$ (B) $\frac{(M_\tau^2 + M_\pi^2)c}{M_\tau^2 - M_\pi^2}$

(C) $\frac{M_\pi c}{M_\tau}$ (D) $\frac{M_\tau c}{M_\pi}$

61. The Hamiltonian of a particle of unit mass moving in the xy-plane is given to be :

$H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$ in suitable units.

The initial values are given to be—

$(x(0), y(0)) = (1, 1)$ and

$(p_x(0), p_y(0)) = \left(\frac{1}{2}, -\frac{1}{2}\right)$. During the motion, the curves traced out by the particles in the xy-plane and the $p_x p_y$-plane are—

(A) both straight lines

(B) a straight line and a hyperbola respectively

(C) a hyperbola and an ellipse, respectively

(D) both hyperbolas

62. Two gravitating bodies A and B with masses m_A and m_B, respectively, are moving in circular orbit. Assume that $m_B >> m_A$ and let the radius of the orbit of body A be R_A. If the body A is losing mass adiabatically, its orbital radius R_A is proportional to—

(A) $\frac{1}{m_A}$ (B) $\frac{1}{m_A^2}$

(C) m_A (D) m_A^2

63. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately—

(A) 0·68 (B) 0·48

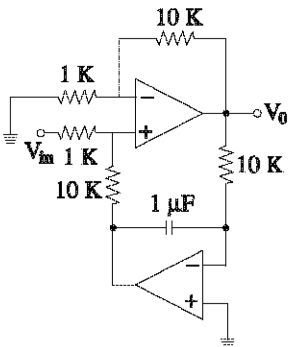
(C) 0·28 (D) 0·91

64. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s > 0$. The range of s for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is—

(A) $1 < s < 3$ (B) $0 < s < 2$

(C) $0 < s < 3$ (D) $0 < s < \infty$

65. A time varying signal V_{in} is fed to an op-amp circuit with output signal V_0 as shown in the figure below.



The circuit implements a—

(A) high pass filter with cutoff frequency 16Hz

(B) high pass filter with cutoff frequency 100 Hz

(C) low pass filter with cutoff frequency 16 Hz

(D) low pass filter with cutoff frequency 100 Hz

Answers with Explanations

Part A

1. (C)	2. (B)	3. (D)	4. (C)	5. (C)
6. (D)	7. (D)	8. (D)	9. (C)	10. (A)
11. (A)	12. (C)	13. (C)	14. (D)	15. (C)
16. (C)	17. (D)	18. (C)	19. (D)	20. (B)

Part B

21. (D) For $p_0(x) = \frac{1}{\sqrt{2}}$

$$p_1(x) = \sqrt{\frac{3}{2}}(3-x)$$

$$\int_0^4 p_0 p_1 dx = \int_2^4 \frac{1}{\sqrt{2}} \times \sqrt{\frac{3}{2}}\,(3-x)\,dx$$

$$= \frac{\sqrt{3}}{2}\left[3x - \frac{x^2}{2}\right]_0^4$$

$$= \frac{\sqrt{3}}{2}\,[12 - 8^4 - 6 + 2]$$

$$= 0$$

$$\int_2^4 p_0 p_0 dx = \frac{1}{2}[x]_0^4 = 1$$

$$\int_2^4 p_1 . p_1 dx = \int_2^4 \sqrt{\frac{3}{2}}\,(3-x)\sqrt{\frac{3}{2}}\,(3-x)\,dx$$

$$= 1$$

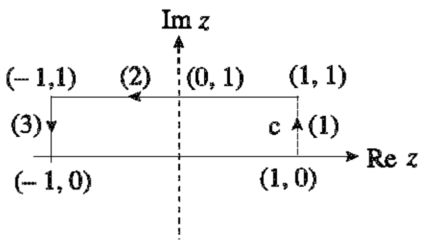
So fulfilling the condition

$$\int_2^4 p_n(x)\, p_m(x)\, dx = \delta_{mn}$$

22. (A) Integral is

$$\int_c dz\, z^2 e^z = \int_c (dx + idy)\,(x+iy)^2\, e^{x+iy}$$

$$= \int_c (dx + idy)\,(x+iy)^2\, e^x \cdot e^{iy}$$



For path 1,

$$x = 1,\, dx = 0$$

y vary from 0 to 1.

So, integral become

$$I_1 = \int_0^1 (idy)\,(1+iy)^2\, e^{1} \cdot e^{iy}$$

For path 2,

$$y = 1,\, dy = 0$$

x vary from 1 to – 1

So, $$I_2 = \int_1^{-1} (dx)\,(x+1)^2 e^x\, e^i$$

For path (3); $x = -1$, y vary from 1 to 0

$$I_3 = \int_1^0 (idy)\,(-1+iy)^2\, e^{-1}\, e^{iy}$$

So, $$I = I_1 + I_2 + I_3 = \frac{5}{e} + e$$

23. (B) Matrix will be unitary in S U (2) group.

24. (A)

25. (C) $$V(x) = ax^2 + \frac{b}{x^2}$$

$$\text{Freq. } \omega = \sqrt{\frac{1}{b}\frac{d^2V}{dx^2}}\Bigg|_{x = x_0}$$

$$\frac{\partial V}{\partial x}\Bigg|_{x = x_0} = 0$$

$$2ax_0 - \frac{2b}{x_0^3} = 0$$

$$2ax_0 = \frac{2b}{x_0^3}$$

$$x_0^4 = \frac{b}{a}$$

Now, $$\frac{d^2V}{dx^2} = 2a + \frac{6b}{x_0^4}$$

$$\frac{d^2V}{dx^2} = 2a + \frac{6b \times a}{b}$$

$$= 8a$$

$$\omega = \sqrt{\frac{8a}{b}}$$

26. (C)

27. (D) This is a statement of Livoulle's theorem in statistical mechanics.

28. (B) As, $I = I_0 \cos^2 \theta$

29. (D) 30. (B) 31. (A) 32. (B)

33. (A) First order correction in energy is given by

$$E_1 = \langle \Psi_0 | ax | \Psi_0 \rangle$$

$$\Psi_0 = \frac{2}{L} \sin \frac{n\pi x}{L}$$

where $L = \pi$

$$= \int \sqrt{\frac{2}{\pi}} \cdot \sin \frac{n\pi x}{\pi} \cdot ax \cdot \sqrt{\frac{2}{\pi}} \sin \frac{n\pi x}{\pi} dx$$

$$= \frac{2a}{\pi} \int_0^\pi x \sin^2 nx \cdot dx$$

$$= \frac{2a}{\pi} \int \left[x \cdot \frac{1 - \cos 2nx}{2} \right] dx$$

$$= \frac{2a}{\pi} \int_0^\pi \frac{x}{2} dx - \frac{2a}{\pi} \times \frac{1}{2} \int_0^\pi \cos 2nx \, dx$$

$$= \frac{2a}{\pi} \times \frac{1}{2} \cdot \frac{x^2}{2} \Big|_0^\pi$$

$$= \frac{a\pi}{2}$$

34. (C)

35. (D) $\Psi = \frac{1}{2} \phi_0 + i\phi_1$

The expectation value for H,

$$\langle H \rangle = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

$$\langle \Psi | \Psi \rangle = \left\langle \frac{1}{\sqrt{2}} \phi_0 - i\phi_1 \middle| \frac{1}{\sqrt{2}} \phi_0 + i\phi_1 \right\rangle$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

$$\langle \Psi | H | \Psi \rangle = \left\langle \frac{1}{\sqrt{2}} \phi_0 - i\phi_1 | H | \frac{1}{\sqrt{2}} \phi_0 + i\phi_1 \right\rangle$$

$$= \frac{1}{2} E_0 - i \times iE_1$$

$$\text{So, } \langle H \rangle = \frac{\frac{1}{2} E_0 + E_1}{3/2}$$

$$= \frac{E_0 + 2E_1}{3}$$

36. (A) 37. (D) 38. (A) 39. (A) 40. (C)

Part C
(Compulsory)

41. (A) For analytic function of complex variable, function should follow Cauchy's Riemann equation in given domain

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

where u and v are imaginary function.

42. (B) 43. (D) 44. (A) 45. (B)

46. (D) Magnetization

$$M(N, T, H) = \left\langle \sum_{i=1}^{H} \mu_i \right\rangle$$

where $\mathcal{H} = -\sum_{i=1}^{N} \bar{\mu}_i \cdot \bar{H}$

$$\langle \mathcal{H} \rangle = -H \cdot M$$

Also, $\langle \mathcal{H} \rangle = E$

So, $M = -\frac{E}{H}$ ...(A)

Now, $E = -\frac{1}{z} \frac{\partial \ln z}{\partial \beta}$

where $z = \sum_{\{Si\}} e^{-\beta \bar{H} \cdot \sum_1 \bar{\mu}_i s_i}$

$$= \sum_{\{Si\}} \prod_{i=1}^{N} e^{-\beta H \mu S_i}$$

$$= \prod_{i=1}^{N} \Sigma e^{-\beta H \mu S_i}$$

$$= (e^{\mu\beta H} + e^{-\mu\beta H})^N$$

$$= (2 \cos \hbar \beta H \mu)^N$$

As, $\beta = \frac{1}{kT}$

So, $E = -\frac{\partial}{\partial \beta} \left(2 \cos \hbar \frac{\mu H}{kT} \right)$

$$= -NH \tan \hbar \frac{\mu H}{kT}$$

So, equation (A) will give

$$M = +N \tan \hbar \frac{\mu H}{kT}$$

47. A. (B)
    B. (A)
48. A. (C)
    B. (B)
49. A. (B)
    B. (B)
50. A. (A)
    B. (A)
51. A. (B)
    B. (B)
52. A. (A) By comparing

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

$$\frac{m}{a} = 0{\cdot}3$$

$$\frac{n}{b} = 0{\cdot}4$$

For TE_{11} mode $m = 1, n = 1$

So, $a = \frac{1}{0{\cdot}3}$

$b = \frac{1}{0{\cdot}4}$

    B. (D)
53. A. (C)
    B. (B)
54. A. (B)
    B. (D)
55. A. (D) The Wigner-Seitz cell is hcp

So, area $\frac{3\sqrt{3}}{2} a^2$

B. (B) The Bravais lattice for any array can be formed by finding the bisectors of given basis vector within same cell $\vec{d}_1$ and $\vec{d}_2$ are from different cells.

Bisectors of $\vec{a}_1$ & $\vec{a}_2$ and bisectors of $\vec{b}_1$ & $\vec{b}_2$ forming neither rectangular nor hexagonal lattice so answer is (B) as bisector of $\vec{c}_1$ and $\vec{c}_2$ forming rectangle.

56. A. (A)
    B. (B)
57. A. (A)
    B. (B)
58. A. (C)
    B. (A)
59. A. (A)
    B. (B)
60. A. (A)
    B. (B)
61. (C)
62. (C) For a spherical shell, the potential energy is

$$dU = -\frac{G}{r}\left(\frac{4}{3}\pi r^2 \rho\right)(4\pi r^2 dr\rho)$$

$$= \frac{-16\pi^2 G{\cdot}\rho^2 M^2}{3 \times 16\pi^2 R^6} \cdot R^4 dr$$

$$M{\cdot}M^2 \propto R^2$$

$$R \propto M$$

$$M = \frac{4}{3}\pi R^3 \rho$$

$$\rho = \frac{3M}{4\pi R^3}$$

63. (A) 64. (A) 65. (C)

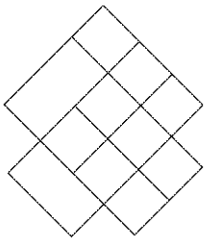
Physical Sciences
CSIR-UGC NET/JRF Exam.
Solved Paper

2012
Physical Sciences

Part–A

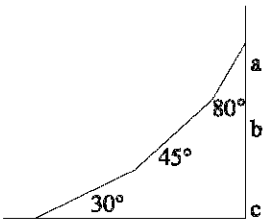
1. In still air, fragrance of a burning incense stick will be smelt by an observer quickest when the experiment is carried out at—
   (A) Low altitude and high air temperature
   (B) High altitude and low air temperature
   (C) Low altitude and low air temperature
   (D) High altitude and high air temperature

2. How many squares are there in this figure ?



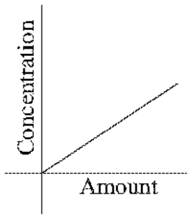
   (A) 9 (B) 14
   (C) 15 (D) 17

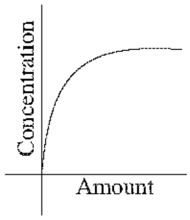
3. A mountain road has 3 sections of different slopes as shown. What is the average slope m of the entire climb ?

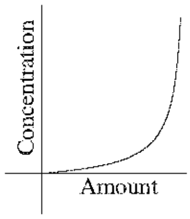


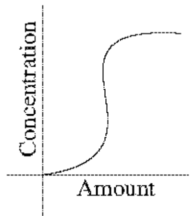
   (A) 1 (B) $(1/3) < m < (1/2)$
   (C) $1 < m < \sqrt{3}$ (D) $(1/\sqrt{3}) < m < 1$

4. Which of the following graphs shows the concentration of a sugar solution as a function of the cumulative amount of sugar added in the process of preparing a saturated solution (the temperature remaining constant) ?

(A)


(B)


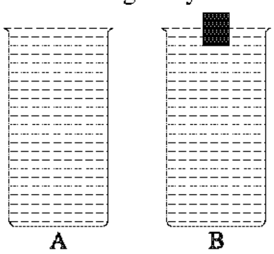
(C)


(D)


5. There are sand-piles which are geometrically similar but of different heights. The ratio of the masses of the sand comprising two randomly chosen piles will be equal to the ratio of the—
   (A) Pile heights
   (B) Squares of the pile heights
   (C) Cubes of the pile heights
   (D) Cube-roots of the pile heights

6. There are two identical vessels of volume V each, one empty, and the other containing a

block of wood of weight w. The vessels are then filled with water up to the brim. The two arrangements are shown as A and B in the figure. If the density of water is ρ and g is the acceleration due to gravity then—



(A) A and B have equal weights

(B) A is heavier than B by an amount w

(C) A is heavier than B by an amount $V\rho g - w$

(D) B is heavier than A by an amount $V\rho g - w$

7. If the father has blood group O and the mother has blood group AB, what are the possible blood groups of their children ?

(A) O, AB, A (B) A, B

(C) A, O (D) B, AB

8. Nuclei of ^{32}P and ^{32}S, accelerated through the same potential difference enter a uniform, transverse magnetic field (Z = 15 for P and Z = 16 for S. As they emerge from the magnetic field—

(A) Both nuclei emerge undeflected

(B) ^{32}P is deflected less than ^{32}S

(C) ^{32}P is deflected more than ^{32}S

(D) Both are equally deflected

9. A person chewing a bubble gum did not experience ear pain in a jet plane while landing whereas another person not chewing a gum had ear pain. The reason could be—

(A) Chewing gum is a pain killer

(B) Chewing equilibrates pressure on both sides of the ear drum

(C) Chewing gum closes the ear drum

(D) Chewing distracts the person

10. The reason why a lunar eclipse does not occur at every full moon is—

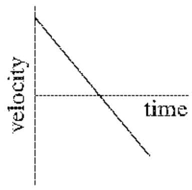
(A) The position of the sun is not favourable at all full moons

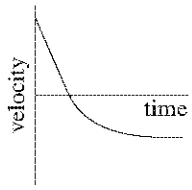
(B) The orbital planes of the moon and that of the earth are inclined to each other by a small angle

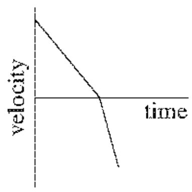
(C) The shape of the earth is not a perfect sphere

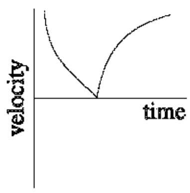
(D) The moon reflects only from one hemisphere

11. A boy throws a stone vertically upwards with a certain initial velocity. Which of the following graphs depicts the velocity as a function of time, if the acceleration due to gravity is assumed to be uniform and constant ?

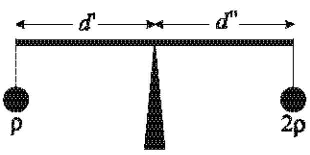
(A) 

(B) 

(C) 

(D) 

12. A rigid uniform bar of a certain mass has two bobs of the same size, but with different densities ρ and 2ρ suspended identically from its ends



When the bar is level on a fulcrum as shown in the figure, d' and d'' are related by—

(A) $2d = d'$ (B) $d > 2d'$

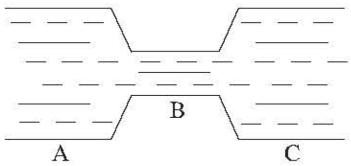
(C) $d = 2d'$ (D) $d < 2d'$

13. There are two points A and A' on the equator at longitudes 0° and 90°E, and two other points B and B' on the same longitudes, respectively, but at latitude 60°S. The distances (along the latitudes) between the points A, A' and B, B' are related by—

(A) AA' = BB' (B) AA' = 2BB'

(C) AA' = $(\sqrt{3})$ BB' (D) AA' = $(\sqrt{2})$ BB'

14.



Water is flowing through a tube as shown. The cross-sectional areas at A and C are equal, and greater than the cross-sectional area at B. If the flow is steady, then the pressure on the walls at B is—

(A) Less than that at A and that at C

(B) More than that at A and that at C

(C) Same as that at A and that at C

(D) More than that at A but less than that at C

15. Match the two lists—

	Raw Material		Product
(a)	Limestone	1.	Porcelain
(b)	Gypsum	2.	Glass
(c)	Silica sand	3.	Plaster of Paris
(d)	Clay	4.	Cement

	(a)	(b)	(c)	(d)
(A)	1	2	3	4
(B)	4	3	2	1
(C)	1	3	4	2
(D)	4	1	3	2

16. The ^{14}C dating method is not usually used for dating organic substances older than ~60,000 years, because—

(A) Such objects rarely contain carbon

(B) Such objects accumulated ^{14}C after their formation

(C) In those times there was no production of ^{14}C

(D) Most of the ^{14}C in the sample would have decayed

17. A seismograph receives a S-wave 60s after it receives the P-wave. If the velocities of P-and S-waves are 7 km/s and 6 km/s respectively, then the distance of the seismic focus from the seismograph is—

(A) 2520 km (B) 42 km

(C) 7070 km (D) 72 km

18. The decay of a radioactive isotope P produces a stable daughter isotope D. The ratio of the number of atoms of D to the number of atoms of P after 2 half lives would be—

(A) 1/4 (B) 3/4

(C) 3 (D) 2

19. The scatter plots represent the values measured by two similar instruments. Point A in the figures represents the true value. Which of the following is a correct description of the quality of these measurements ?

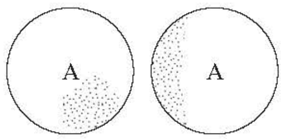


Fig. 1 **Fig. 2**

(A) Fig. 1 : good accuracy, good precision
Fig. 2 : good accuracy, good precision

(B) Fig. 1 : poor accuracy, poor precision
Fig. 2 : good accuracy, poor precision

(C) Fig. 1 : poor accuracy, good precision
Fig. 2 : poor accuracy, poor precision

(D) Fig. 1 : poor accuracy, poor precision
Fig. 2 : poor accuracy, good precision

20. Even though the concentration of CO_2 is the same at sea level and at high altitude, the photosynthetic rate is higher in a plant grown at sea level than in a plant (of the same species) grown at high altitude. The reason for this is—

(A) Light intensity is more at sea level

(B) Temperature is lower at higher altitude

(C) Atmospheric pressure is higher at sea level

(D) Relative humidity is higher at sea level

Part–B

21. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is—

(A) $\hat{i} + \hat{j}$ (B) $\hat{j} + \hat{k}$

(C) $\hat{i} + \hat{j} + \hat{k}$ (D) $2\hat{i} + 3\hat{j} + 5\hat{k}$

22. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ are—

(A) (1, 4, 9) (B) (0, 7, 7)

(C) (0, 1, 13) (D) (0, 0, 14)

23. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \leq |z| \leq 2$, and around $z = 1$ is—

(A) $\frac{1}{2}[1 + z + z^2 + z^3 + \ldots]$

$$\left[1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \ldots\right]$$

(B) $\frac{1}{1-z} + z - (1-z)^2 + (1-z)^3 + \ldots$

(C) $\frac{1}{z^2}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \ldots\right]\left[1 + \frac{2}{z} + \frac{4}{z^2} + \ldots\right]$

(D) $2(z-1) \div 5(z-1)^2 + 7(z-1)^3 + \ldots$

24. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second. What should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ?

(A) $100/\lambda$ (B) $10^4/\lambda^2$

(C) $10^4/\lambda$ (D) $1/\lambda$

25. Let $u(x, y) = x + \frac{1}{2}(x^2 - y^2)$ be the real part of an analytic function $f(z)$ of the complex variable $z = x + iy$. The imaginary part of $f(z)$ is—

(A) $y + xy$ (B) xy

(C) y (D) $y^2 - x^2$

26. Let $y(x)$ be a continuous real function in the range 0 and 2π, satisfying the inhomogeneous differential equation :

$\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of $\frac{dy}{dx}$ at the point $x = \frac{\pi}{2}$—

(A) is continuous

(B) has a discontinuity of 3

(C) has a discontinuity of $\frac{1}{3}$

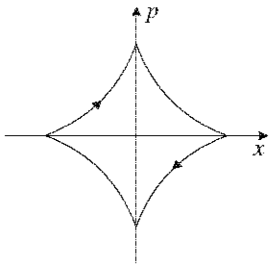
(D) has a discontinuity of 1

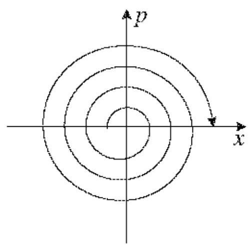
27. A ball is picked at random from one of two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white ?

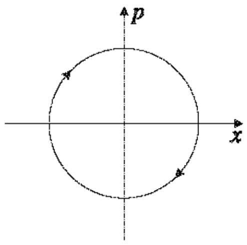
(A) 34/70 (B) 41/70

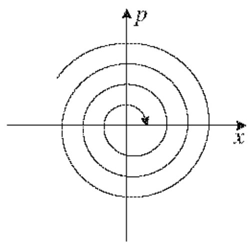
(C) 36/70 (D) 29/70

28. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum ?

(A) 

(B) 

(C) 

(D) 

29. Two events, separated by a (spatial) distance 9×10^9 m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0·8 c (where the speed of light $c = 3 \times 10^8$ m/s) is—
(A) 60 s (B) 40 s
(C) 20 s (D) 0 s

30. If the Lagrangian of a particle moving in one dimensions is given by $L = \frac{\dot{x}^2}{2x} - V(x)$, the Hamiltonian is—
(A) $\frac{1}{2}xp^2 + V(x)$ (B) $\frac{\dot{x}^2}{2x} + V(x)$
(C) $\frac{1}{2}\dot{x}^2 + V(x)$ (D) $\frac{p^2}{2x} + V(x)$

31. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass m horizontally with speed v. The acceleration of the bullet, in the reference frame of the shooter, is—
(A) $2v\Omega$ to his right (B) $2v\Omega$ to his left
(C) $v\Omega$ to his right (D) $v\Omega$ to his left

32. The magnetic field corresponding to the vector potential

$$\vec{A} = \frac{1}{2}\vec{F} \times \vec{r} + \frac{10}{r^3}\vec{r}$$

where $\vec{F}$ is a constant vector, is—
(A) $\vec{F}$ (B) $-\vec{F}$
(C) $\vec{F} + \frac{30}{r^4}\vec{r}$ (D) $\vec{F} - \frac{30}{r^4}\vec{r}$

33. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field, E_0, of the reflected wave into the water is found to remain the same for all angles of incidence. The phase of the magnetic field H—
(A) does not change (B) changes by $3\pi/2$
(C) changes by $\pi/2$ (D) changes by π

34. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to—
(A) IR (B) $\frac{I}{R^2}$
(C) $\frac{I^2}{R^2}$ (D) $\frac{I}{R}$

35. The component along an arbitrary direction $\hat{n}$, with direction cosines (n_x, n_y, n_z) of the spin of a spin $-\frac{1}{2}$ particle is measured. The result is—
(A) 0 (B) $\pm\frac{\hbar}{2}n_z$
(C) $\pm\frac{\hbar}{2}(n_x + n_y + n_z)$ (D) $\pm\frac{\hbar}{2}$

36. A particle of mass m is in a cubic box of size a. The potential inside the box $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinite outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2\hbar^2}{2ma^2}$, its wavefunction is—
(A) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a} \sin\frac{5\pi y}{a} \sin\frac{6\pi z}{a}$
(B) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi x}{a} \sin\frac{4\pi y}{a} \sin\frac{3\pi z}{a}$
(C) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a} \sin\frac{8\pi y}{a} \sin\frac{2\pi z}{a}$
(D) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a} \sin\frac{3\pi z}{a}$

37. Let ψ_{nlm} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential V (r). The wavefunction

$$\psi = \frac{1}{4}\left[\psi_{2lo} + \sqrt{5}\,\psi_{2lt} + \sqrt{10}\,\psi_{2lz}\right]$$

is an eigenfunction only of—
(A) H, L^2 and L_z (B) H and L_z
(C) H and L^2 (D) L^2 and L_z

38. The commutator $[x^2, p^2]$ is—
(A) $2ihxp$ (B) $2ih(xp + px)$
(C) $2ih\,px$ (D) $2ih(xp - px)$

39. Consider a system of non-interacting particles in d-dimensions obeying the dispersion relation $\varepsilon = Ak^s$ ('s') where ε is the energy, k is the wavevector, s is an integer and A a constant. The density of states. N (ε), is proportional to—
(A) $\varepsilon^{\frac{s}{d}-1}$ (B) $\varepsilon^{\frac{d}{s}+1}$
(C) $\varepsilon^{\frac{d}{s}-1}$ (D) $\varepsilon^{\frac{s}{d}+1}$

40. The number of ways in which N identical bosons can be distributed in two energy levels, is—

(A) N + 1 (B) N (N – 1)/2
(C) N (N + 1)/2 (D) N

41. The free energy of a gas of N particles in a volume V and at a temperature T is

$$F = Nk_BT \ln [a_0V (k_BT)^{5/2}/N],$$

where a_0 is a constant and k_B denotes the Boltzmann constant. The internal energy of the gas is—

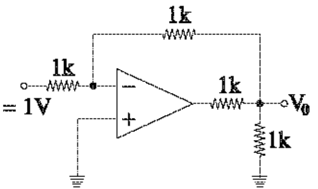
(A) $\frac{3}{2}Nk_BT$

(B) $\frac{5}{2}Nk_BT$

(C) $Nk_BT \ln [a_0V (k_BT)^{5/2}/N] - \frac{3}{2}Nk_BT$

(D) $Nk_BT \ln [a_0V/(k_BT)^{5/2}]$

42. In the op-amp circuit shown in the figure below, the input voltage V_1 is 1V. The value of the output V_0 is—

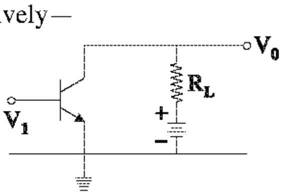


(A) – 0·33V (B) – 0·50V
(C) –1·00V (D) – 0·25V

43. An LED operates at 1·5V and 5 mA in forward bias. Assuming an 80% external efficiency of the LED, how many photons are emitted per second ?

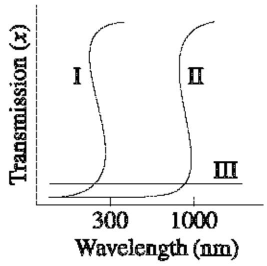
(A) $5{\cdot}0 \times 10^{16}$ (B) $1{\cdot}5 \times 10^{16}$
(C) $0{\cdot}8 \times 10^{16}$ (D) $2{\cdot}5 \times 10^{16}$

44. The transistor in the given circuit has h_{fe} = 35Ω and h_{ie} = 1000Ω. If the load resistance R_L = 1000Ω, the voltage and current gain are, respectively—



(A) –35 and +35 (B) 35 and –35
(C) 35 and –0·97 (D) 0·98 and –35

45. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond, respectively, to—



(A) Insulator, semiconductor and metal
(B) Semiconductor, metal and insulator
(C) Metal, semiconductor and insulator
(D) Insulator, metal and semiconductor

Part–C

46. The eigenvalues of the antisymmetric matrix,

$$A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$$ where n_1, n_2 and n_3 are the components of a unit vector, are—

(A) 0, *i*, –*i* (B) 0, 1, –1
(C) 0, 1 + *i*, –1 – *i* (D) 0, 0, 0

47. Which of the following limits exists ?

(A) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}+\ln N\right)$

(B) $\lim_{N\to\infty}\left(\sum_{\infty=1}^{N}\frac{1}{m}-\ln N\right)$

(C) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{\sqrt{m}}-\ln N\right)$

(D) $\lim_{N\to\infty}\sum_{N\to\infty}^{N}\frac{1}{m}$

48. A bag contains many balls, each with a number painted on it. There are exactly *n* balls which have the number *n* (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to—

(A) $\frac{2N+1}{3}$ (B) $\frac{N}{2}$

(C) $\frac{N+1}{2}$ (D) $\frac{N(N+1)}{2}$

49. The value of the integral

$$\int_{-\infty}^{\infty} \frac{1}{t^2 - R^2} \cos\left(\frac{rt}{2R}\right) dt \text{ is—}$$

(A) $\frac{-2\pi}{R}$ (B) $\frac{-\pi}{R}$

(C) $\frac{\pi}{R}$ (D) $\frac{2\pi}{R}$

50. The Poisson bracket $\{|r|, |p|\}$ has the value—

(A) $|r|\,|p|$ (B) $\bar{r} \cdot \bar{p}$

(C) 3 (D) 1

51. Consider the motion of a classical particle in a one dimensional double-well potential $V(x) = \frac{1}{4}(x^2 - 2)^2$. If the particle is displaced infinitesimally from the minimum on the positive x-axis (and friction is neglected), then—

(A) The particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$

(B) The particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$

(C) The particle will switch between the right and left wells

(D) The particle will approach the bottom of the right well and settle there

52. What is the proper time interval between the occurrence of two events if in one inertial frame the events are separated by $7 \cdot 5 \times 10^8$ m and occur 6·5 s apart ?

(A) 6·50 s (B) 6·00 s

(C) 5·75 s (D) 5·00 s

53. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential

$$V(r) = \begin{cases} V_0 & \text{if } r \le R \\ 0 & \text{if } r > R \end{cases}$$

If V_0 is changed to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the Born approximation—

(A) Increases to four times the original value

(B) Increases to twice the original value

(C) Decreases to half the original value

(D) Decreases to one fourth the original value

54. A variational calculation is done with the normalized trial wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}(a^2 - x^2)$ for the one dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } |x| \le a \\ \infty & \text{if } |x| > a \end{cases}$$

line ground state energy is estimated to be—

(A) $\frac{5h^2}{3ma^2}$ (B) $\frac{3h^2}{2ma^2}$

(C) $\frac{3h^2}{5ma^2}$ (D) $\frac{5h^2}{4ma^2}$

55. A particle in one-dimension is on the potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ -V_m & \text{if } 0 \le x \le t \\ 0 & \text{if } x > t \end{cases}$$

If there at least one bound state, the minimum depth of the potential is—

(A) $\frac{h^2\pi^2}{8mt^2}$ (B) $\frac{h^2\pi^2}{2mt^2}$

(C) $\frac{2h^2\pi^2}{mt^2}$ (D) $\frac{h^2\pi^2}{mt^2}$

56. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r, θ, ϕ) ?

(A) $-\frac{ih}{\sin^2\theta}\frac{\partial}{\partial\theta}$ (B) $-ih\frac{\partial}{\partial\theta}$

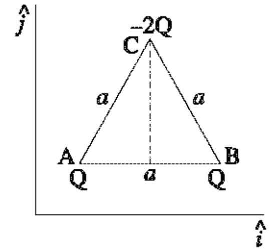
(C) $-\frac{ih}{\sin\theta}\frac{\partial}{\partial\theta}$ (D) $-ih\sin\theta\frac{\partial}{\partial\theta}$

57. Which of the following quantities is Lorentz invariant ?

(A) $|E \times B|^2$ (B) $|E|^2 - |B|^2$

(C) $|E|^2 + |B|^2$ (D) $|E|^2\,|B|^2$

58. Charges Q, Q and – 2Q are placed on the vertices of an equilateral triangle ABC of sides of length a, as shown in the figure



The dipole moment of this configuration of charges, irrespective of the choice of origin, is—

(A) $+2aQ\hat{i}$ (B) $+\sqrt{3}aQ\hat{j}$

(C) $-\sqrt{3}aQ\hat{j}$ (D) 0

59. The vector potential A due to a magnetic moment m at a point r is given by $A = \frac{m \times r}{r^3}$. If m is directed along the positive z-axis, the x-component of the magnetic field, at the point r, is—

(A) $\frac{3myz}{r^3}$ (B) $-\frac{3mxy}{r^3}$

(C) $\frac{3mxz}{r^3}$ (D) $\frac{3m(z^2 - xy)}{r^3}$

60. A system has two normal modes of vibration, with frequencies ω_1 and $\omega_2 = 2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\hbar\omega_1$?

[In the following $x = e^{-\beta\hbar\omega_1}$ and Z is the partition function of the system]

(A) $x^{3/2}(x + 2x^2)/Z$ (B) $x^{3/2}(1 + x + x^2)/Z$

(C) $x^{3/2}(1 + 2x^2)/Z$ (D) $x^{3/2}(1 + x + 2x^2)/Z$

61. The magnetization M of a ferromagnet, as a function of the temperature T and the magnetic field H, is described by the equation $M = \tanh\left(\frac{T_c}{T}M + \frac{H}{T}\right)$. In these units, the zero-field magnetic susceptibility in terms of M (0) – M (H = 0) is given by—

(A) $\frac{1 - M^2(0)}{T - T_c[1 - M^2(0)]}$

(B) $\frac{1 - M^2(0)}{T - T_c}$

(C) $\frac{1 - M^2(0)}{T + T_c}$

(D) $\frac{1 - M^2(0)}{T}$

62. Bose condensation occurs in liquid He^4 kept at ambient pressure at 2·17 K. At which temperature will Bose condensation occur in He^4 in gaseous state, the density of which is 1000 times smaller than that of liquid He^4 ?

(Assume that it is a perfect Bose gas)

(A) 2·17 mK (B) 21·7 mK

(C) 21·7 μK (D) 2·17 μK

63. Consider black body radiation contained in a cavity whose walls are at temperature T. The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to 2T and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of—

(A) 2 (B) 4

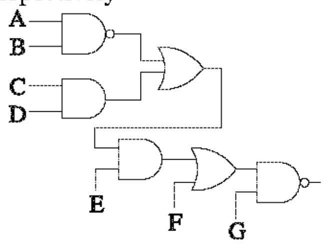
(C) 8 (D) 16

64. The output O, of the given circuit in cases I and II, where

Case I : A, B = 1; C, D = 0; E, F = 1 and G = 0

Case II : A, B = 0; C, D = 0; E, F = 0 and G = 1

are, respectively—



(A) 1, 0 (B) 0, 1

(C) 0, 0 (D) 1, 1

65. A resistance strain gauge is fastened to a steel fixture and subjected to a stress of 1000 kg/m^2. If the gauge factor is 3 and the modulus of elasticity of steel is 2×10^{10} kg/m^2, then the fractional change in resistance of the strain gauge due to the applied stress is—

(**Note :** The gauge factor is defined as the ratio of the fractional change in resistance to the fractional change in length.)

(A) $1{\cdot}5 \times 10^{-7}$ (B) $3{\cdot}0 \times 10^{-7}$

(C) $0{\cdot}16 \times 10^{-10}$ (D) $0{\cdot}5 \times 10^{-7}$

66. Consider a sinusoidal waveform of amplitude 1V and frequency f_c. Starting from an arbitrary initial time, the waveform is sampled at intervals of $1/(2f_0)$. If the corresponding Fourier spectrum peaks at a frequency $\bar{f}$ and an amplitude $\bar{A}$, then—

(A) $\bar{f} = 2f_0$ and $\bar{A} = 1V$

(B) $\bar{f} = f_u$ and $0 \le \bar{A} \le 1V$

(C) $\bar{f} = 0$ and $\bar{A} = 1V$

(D) $\bar{f} = \frac{f_0}{2}$ and $\bar{A} = \frac{1}{\sqrt{2}}V$

67. The first absorption spectrum of $^{12}C^{16}O$ is at $3{\cdot}842\ cm^{-1}$ while that of $^{13}C^{16}O$ is at $3{\cdot}673\ cm^{-1}$. The ratio of their moments of inertia is—
(A) 1·851 (B) 1·286
(C) 1·046 (D) 1·038

68. The spin-orbit interaction in an atom is given by $H = a\mathrm{LS}$, where L and S denote the orbital and spin angular momenta, respectively, of the electron. The splitting between the levels $^2P_{3/2}$ and $^2P_{1/2}$ is—
(A) $\frac{3}{2}ah^2$ (B) $\frac{1}{2}ah^2$
(C) $3ah^2$ (D) $\frac{5}{2}ah^2$

69. The spectral line corresponding to an atomic transition from J = 1 to J = 0 states splits in a magnetic field of 1 kG into three components separated by $1{\cdot}6 \times 10^{-3}$ Å. If the zero field spectral line corresponds to 1849 Å, what is the g-factor corresponding to the J = 1 state ?
(You may use $\frac{hc}{\mu_0} \approx 2 \times 10^4$ cm.)
(A) 2 (B) 3/2
(C) 1 (D) 1/2

70. The energy required to create a lattice vacancy in a crystal is equal to 1eV. The ratio of the number densities of vacancies n (1200K)/n (300K), when the crystal is at equilibrium at 1200 K and 300 K, respectively, is approximately—
(A) exp (–30) (B) exp (–15)
(C) exp (15) (D) exp (30)

71. The dispersion relation of phonons in a solid is given by
$\omega^2 (k) = \omega^2_0 (3 - \cos k_x a - \cos k_y a - \cos k_z a)$.
The velocity of the phonons at large wavelength is—
(A) $\omega_0 a/\sqrt{3}$ (B) $\omega_0 a$
(C) $\sqrt{3}\omega_0 a$ (D) $\omega_0 a/\sqrt{2}$

72. Consider an electron in a box of length L with periodic boundary condition $\psi(x) = \psi(x + L)$. If the electron is in the $\psi_k(x) = \frac{1}{\sqrt{L}} e^{ikx}$ with energy $\varepsilon_\lambda = \frac{h^2k^2}{2m}$, what is the correction to its energy, to second order of perturbation theory, when it is subjected to a weak periodic potential $V(x) = V_0 \cos gx$, where g is an integral multiple of the $2\pi/L$?
(A) $V_0^2 \varepsilon_x / \varepsilon_k^2$
(B) $-\frac{mV^2_0}{2h^2}\left(\frac{1}{g^2 + 2kg} + \frac{1}{g^2 - 2kg}\right)$
(C) $V_0^2 (\varepsilon_k - \varepsilon_g)/\varepsilon_g^2$
(D) $V_0^2/(\varepsilon_k + \varepsilon_x)$

73. The ground state of $^{207}_{82}Pb$ nucleus has spin-parity $J^P = \frac{1^-}{2}$, while the first excited state has $J^P = \frac{5^-}{2}$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to the ground state are—
(A) E2 and E3
(B) M2 and E3
(C) E2 and M3
(D) M2 and M3

74. The dominant interactions underlying the following processes
A. $K^- + p \rightarrow \Sigma^- + \pi^0$
B. $\mu^- + \mu^0 \rightarrow K^- + K^0$
C. $\Sigma^0 \rightarrow p + \pi^0$
are—
(A) A : strong, B : electromagnetic and C : weak
(B) A : strong, B : weak and C : weak
(C) A : weak, B : electromagnetic and C : strong
(D) A : weak, B : electromagnetic and C : weak

75. If a Higgs boson of mass m_H moving with a speed $\beta = \frac{v}{c}$ decays into a pair of photons, then the invariant mass of the photon pair is—
[Note : The invariant mass of a system of two particles, with four momenta p_1 and p_2 is $(p_1 + p_2)^2$]
(A) βm_H (B) m_H
(C) $m_H/\sqrt{1-\beta^2}$ (D) $\beta m_H/\sqrt{1-\beta^2}$

Answers with Explanations

1. (D) At high altitude air is less dense and at high temperature air can travel faster. Let's first consider the altitude, at high altitude there would be lower density of air and *vice-versa*. At higher temperature the fragrance will move quicker.
2. (C) Number of Squares = 9

   Number of Squares with sides twice that of small squares = 5

   Number of Squares with sides three times that of small squares = 1

   Therefore, total number of squares

   $$= 9 + 5 + 1$$
   $$= 15$$
3. (D)
4. (B) As the concentration is directly proportional to the amount of sugar added the graph should be linear with constant inclination.
5. (C) The ratio of the masses of the sand comprising two randomly chosen piles will be equal to the ratio of the cubes of the pile heights

   Ratio of volume = Ratio of the cube of height
6. (A) Weight of jar A and B is same because if we suppose both jars were filled upto brim with water then certainly both would have same weight. Now, if we put a block of weight W in jar B, then the amount of water that will go out of jar B will have same weight as that of block immersed in jar B (Archimedes Principle), so effectively there is no net change in the weight of jar B. Hence, both jar A and B had same weight.
7. (B) ABO blood grouping system—According to the ABO blood group system there are four different kinds of blood groups : A, B, AB or O (null).

   Blood group A—If you belong to the blood group A, you have A antigens on the surface of your red blood cells and B antibodies in your blood plasma.

   Blood group B—If you belong to the blood group B, you have B antigens on the surface of your red blood cells and A antibodies in your blood plasma.

   Blood group AB—If you belong to the blood group AB, you have both A and B antigens on the surface of your red blood cells and no A or B antibodies at all in your blood plasma.

   Blood group O—If you belong to the blood group O (null), you have neither A or B antigens on the surface of your red blood cells but you have both A and B antibodies in your blood plasma.

   As parents have A and B antigens, their children can have A and B blood groups.
8. (B)
9. (B) Chewing gum stretches muscles in the jaw and throat that can cause the Eustachian tube to open. This allows air into the middle ear and equalizes the pressure.
10. (B) A lunar eclipse occurs when the Moon passes behind the Earth so that the Earth blocks the Sun's rays from striking the Moon. This can occur only when the Sun, Earth and Moon are aligned exactly, or very closely so, with the Earth in the middle. Hence, a lunar eclipse can only occur the night of a full moon. The type and length of an eclipse depend upon the Moon's location relative to its orbital nodes. Unlike a solar eclipse, which can only be viewed from a certain relatively small area of the world, a lunar eclipse may be viewed from anywhere on the night side of the Earth.
11. (A)   12. (D)   13. (B)
14. (A) The pressure at B will be less then that at A and C. Let us explain this with the help of Bernoulli's theorem. It states that energy can be neither created nor destroyed, but merely changed from one form to another. To illustrate how this applies, let us consider Figure I which represents a horizontal pipe with air flowing through it. The air in the pipe has two forms of available energy. One is potential energy, which is in the form of air pressure. The other is kinetic energy which the air has by virtue of its motion. Now, notice that the pipe is constricted at (B). Supposing the cross-sectional area at (B) is one half the cross-sectional areas at (A) : the air will have to move about twice as fast past (B), in order to allow the same amount of air by in the same time. This is analogous to a nozzle on a hose. where you obtain a high-

velocity stream of water by passing the water through a small orifice.

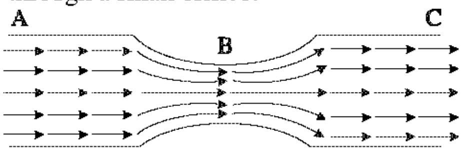


Fig. 1–Air flow through an orifice

Now since the air is going faster past (B), it must have more kinetic energy when passing (B). Recalling the law of conservation of energy, we realize that we must have converted some of the potential energy in order to have more kinetic energy. Since, the only potential energy available in this set-up is in the form of air pressure, there will be a low-pressure area in the construction of the pipe at (B).

15. (B) Gypsum – Plaster of Paris

Porcelain – Clay

Limestone – Cement

Silica sand – Glass

16. (D) The half life of carbon – 14 is 5,700 years; because of this it is only reliable for dating objects up to 60,000 years. Objects older then this will have most ^{14}C decayed.

17. (A)

18. (C) Suppose we take any number of nuclei = 1500

After one half-life no. of parent nuclei left = No. of daughter nuclei formed = 750

After second half-life no. of parent nuclei left

$$= \frac{750}{2} = 375$$

Number of daughter nuclei formed

$$= 375 + 750$$
$$= 1125$$

$$\text{The ratio is } = \frac{1125}{375} = 3$$

19. (C)

20. (C) The pressure at sea level is high, due to which the density of air is also high. This means that the number of CO_2 molecules per unit volume of air is more. Because of this the CO_2 uptake will be more and hence the rate of photosynthesis is also more.

21. (C) 22. (D)

23. (B) Let us consider the Laurent series for

$$f(z) = \frac{1}{(1-z)(2-z)}$$

For series expansion about

$$Z = 1$$

Let
$$Z - 1 = 4$$
$$Z - 2 = 4 - 1$$

$$f(z) = \frac{1}{(z-1)(z-2)}$$

$$= \frac{1}{4(4-1)} \qquad 1 \le z \le 2$$

$$= \frac{1}{-4(1-4)} \qquad 0 \le z - 1 \le 1$$

$$= -\frac{1}{4}(1-4)^{-1} \qquad 0 \le 4 \le 1$$

$$= -\frac{1}{4}[1 + 4 + 4^2 + 4^3]$$

$$= \frac{-1}{4} - 1 - 4 - 4^2 - 4^3$$

$$= \frac{-1}{z-1} - 1 + z + 1 - (z-1)^2 + (z-1)^3$$

$$= \frac{1}{1-z} + z - (1-z)^2 + (1-z)^3$$

24. (C) 25. (A) 26. (D)

27. (B)

Bag 1	Bag 2
2 Black **3 White**	**3 Black** **4 White**

Let E_1 and E_2 be the events of choosing the first and the second bag respectively.

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

Let W be the event of drawing a white ball.

$$\therefore \quad P\left(\frac{W}{E_1}\right) = \frac{3}{2+3} = \frac{3}{5}$$

$$P\left(\frac{W}{E_2}\right) = \frac{4}{4+3} = \frac{4}{7}$$

Now, P (drawing a white ball)

$$= P(W) = P(WE_1 \text{ or } WE_2)$$
$$= P(WE_1) + P(WE_2)$$
$$= P(E_1)\, P\left(\frac{W}{E_1}\right) + P(E_2)\, P\left(\frac{W}{E_2}\right)$$
$$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{4}{7}$$
$$= \frac{3}{10} + \frac{4}{10}$$
$$= \frac{21 + 20}{70}$$
$$= \frac{41}{70}$$

28. (D)

29. (B) $$t_2 - t_1 = \frac{\frac{v}{c^2}(x_2 - x_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{0{\cdot}8\, \frac{c}{c^2} \times 9 \times 10^9}{0{\cdot}6}$$
$$= 40 \text{ sec.}$$

30. (A) $$L = \frac{\dot{x}^2}{2x} - V(x) \text{ given}$$
$$p = \frac{\partial L}{\partial \dot{x}}$$
$$= \frac{\dot{x}}{x} \qquad H = \sum p\dot{q}' - L$$
$$H = \frac{\dot{x}}{x}\dot{x} - \frac{\dot{x}^2}{2x} + V$$
$$= \frac{\dot{x}^2}{2x} + V(x)$$
As $$\dot{x} = xp$$
Or $$H = \frac{p^2x^2}{2x} + V(x)$$
$$= \frac{p^2x}{2} + V(x)$$

31. (A) Person in the centre of the disc rotating with angular momentum $\Omega = 2V\,\Omega$

32. (A) 33. (D)

34. (D) The magnetic field at a point P due to a straight wire carrying a current I is proportional to I

$$M \propto I$$

Magnetic field is inversely proportional to the perpendicular distance R of the point from the wire

$$M \propto \frac{I}{R}$$

So,

$$M \propto \frac{I}{R}$$

35. (D) 36. (D) 37. (C) 38. (B) 39. (C)

40. (A) Let $w(n, g)$ be the number of ways of distributing n particles among g sublevels of an energy level.

There is only one way of distributing n particles with one sublevel, therefore $w(n, 1) = 1$.

It is easy to see that there are $(n + 1)$ ways of distributing n particles in two sublevels which we will write as :

$$w(n, 2) = \frac{(n+1)\,!}{n\,!\,1\,!}$$

41. (B) 42. (C) 43. (D) 44. (A) 45. (A)

46. (A) The eigenvalues of an antisymmetric matrix are all purely imaginary numbers, and occur as conjugate pairs, $+i\omega$ and $-i\omega$. As a corollary it follows that an antisymmetric matrix of odd order necessarily has one eigenvaue equal to zero.

47. (B) 48. (A) 49. (A) 50. (B)

51. (A) $$V(x) = \frac{1}{4}(x^2 - 2)^2$$
$$= \frac{1}{4}(x^4 - 4x^2 + 4)$$

For small oscillation (SHO)

$$V(x) \sim x^2$$
$$\frac{\partial^2 V(x)}{\partial x^2} = 2 \qquad T \sim \frac{1}{2}\dot{x}^2$$
and $$T_1 = \frac{\partial^2 (T)}{0\dot{x}^2} = 1$$
$$V - \omega^2 T = 0$$
Gives $$2 - \omega^2 = 0$$
$$\boxed{\omega = \sqrt{2}}$$

52. (A) 53. (A) 54. (D) 55. (A) 56. (C)

57. (B) The fundamental invariants of the electromagnetic field are :

$$P = \frac{1}{2} F_{ab} F^{ab}$$

$$= \|\vec{B}\| - \|\vec{E}\|$$

$$= -\frac{1}{2} * F_{ab} * F^{ab}$$

$$Q \equiv \frac{1}{4} F_{ab} * F^{ab}$$

$$= \vec{E} \cdot \vec{B}$$

58. (C) Let us place the origin mid-way between the two positive charges Q such that they lie on the X-axis at equal distance from the origin $= \frac{a}{2}$.

So, the charge-2Q is at a distance $\frac{\sqrt{3}a}{2}$.

Dipole moment

$$p = (q \times d)$$

$$= -2Q \times \sqrt{3} \times \frac{a}{2} + Q \times a + Q \times (-a)$$

$$= -\sqrt{3}a\, Q\hat{j}$$

59. (C) 60. (D) 61. (A)

62. (B) Bose condensation occurs in liquid ^{4}He kept at ambient pressure at 2·17K. Superfluid helium-4 is a liquid rather than a gas, which means that the interactions between the atoms are relatively strong. So ^{4}He in gaseous state will condense at 21·7 mk.

63. (C) The entropy in case of black body is proportional to T^3.
As the temperature of the black body is raised from T to 2T.
So, the entropy will be $(2T)^3 = 8$.

64. (D) 65. (A) 66. (B) 67. (C) 68. (A)
69. (C) 70. (D) 71. (D) 72. (B) 73. (C)
74. (A) 75. (B)

Physical Sciences
CSIR-UGC NET/JRF Exam.
Solved Paper

December 2012
Physical Sciences

Directions –

1. This Test Booklet contains seventy five (20 Part 'A' + 25 Part 'B' + 30 Part 'C') Multiple Choice Questions. You are required to answer a maximum of 15, 20 and 20 questions from Part 'A', 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 20 and 20 questions in Parts 'A', 'B' and 'C' respectively, will be taken up for evaluation.
2. Each question in Part 'A' carries 2 marks, Part 'B' 3·5 marks and Part 'C' 5 marks respectively. There will be negative marking @ 25% for each wrong answer.
3. Below each question in Part 'A', 'B' and 'C' four alternatives or responses are given. Only one of these alternatives is the 'correct' option to the question. You have to find, for each question, the correct or the best answer.

Useful Fundamental Constants

m	Mass of electron	9.11×10^{-31} kg
h	Planck's constant	6.63×10^{-34} J-sec
e	Charge of electron	1.6×10^{-19} C
k	Boltzmann constant	1.38×10^{-23} J/k
c	Velocity of Light	3.0×10^{8} m/sec
I_cV	1.6×10^{-19} J	
amu	1.67×10^{-27} kg	
G	6.67×10^{-11} Nm^2 kg^{-2}	
R_y	Rydberg constant	1.097×10^{7} m^{-1}
N_A	Avogadro number	6.023×10^{23} mole^{-1}
ε_0	8.854×10^{-12} Fm^{-1}	
μ_0	$4\pi \times 10^{-7}$ Hm^{-1}	
R	Molar Gas constant	8.314 JK^{-1} mole^{-1}

List of the Atomic Weights of the Elements

Element	Symbol	Atomic Number	Atomic Weight
Actinium	Ac	89	(227)
Aluminium	Al	13	26.98
Americium	Am	95	(243)
Antimony	Sb	51	121.75
Argon	Ar	18	39.948
Arsenic	As	33	74.92
Astatine	At	85	(210)
Barium	Ba	56	137.34
Berkelium	Bk	97	(249)
Beryllium	Be	4	9.012
Bismuth	Bi	83	208.98
Boron	B	5	10.81
Bromine	Br	35	79.909
Cadmium	Cd	48	112.40
Calcium	Ca	20	40.08
Californium	Cf	98	(251)
Carbon	C	6	12.011
Cerium	Ce	58	140.12
Cesium	Cs	55	132.91
Chlorine	Cl	17	35.453
Chromium	Cr	24	52.00
Cobalt	Co	27	58.93
Copper	Cu	29	63.54
Curium	Cm	96	(247)
Dysprosium	Dy	66	162.50
Einsteinium	Es	99	(254)
Erbium	Er	68	167.26
Europium	Eu	63	151.96
Fermium	Fm	100	(253)
Fluorine	F	9	19.00
Francium	Fr	87	(223)
Gadolinium	Gd	64	157.25
Gallium	Ga	31	69.72
Germanium	Ge	32	72.59
Gold	Au	79	196.97
Hafnium	Hf	72	178.49
Helium	He	2	4.003
Holmium	Ho	67	164.93

Hydrogen	H	1	1.0080
Indium	In	49	114.82
Iodine	I	53	126.90
Iridium	Ir	77	192.2
Iron	Fe	26	55.85
Krypton	Kr	36	83.80
Lanthanum	La	57	138.91
Lawrencium	Lr	103	(257)
Lead	Pb	82	207.19
Lithium	Li	3	6.939
Lutetium	Lu	71	174.97
Magnesium	Mg	12	24.312
Manganese	Mn	25	54.94
Mendelevium	Md	101	(256)
Mercury	Hg	80	200.59
Molybdenum	Mo	42	95.94
Neodymium	Nd	60	144.24
Neon	Ne	10	20.183
Neptunium	Np	93	(237)
Nickel	Ni	28	58.71
Niobium	Nb	41	92.91
Nitrogen	N	7	14.007
Nobelium	No	102	(253)
Osmium	Os	76	190.2
Oxygen	O	8	15.9994
Palladium	Pd	46	106.4
Phosphorus	P	15	30.974
Platinum	Pt	78	195.09
Plutonium	Pu	94	(242)
Polonium	Po	84	(210)
Potassium	K	19	39.102
Praseodymium	Pr	59	140.91
Promethium	Pm	61	(147)
Protactinium	Pa	91	(231)
Radium	Ra	88	(226)
Radon	Rn	86	(222)
Rhenium	Re	75	186.23
Rhodium	Rh	45	102.91
Rubidium	Rb	37	85.47
Ruthenium	Ru	44	101.1
Samarium	Sm	62	150.35
Scandium	Sc	21	44.96
Selenium	Se	34	78.96
Silicon	Si	14	28.09
Silver	Ag	47	107.870
Sodium	Na	11	22.9898
Strontium	Sr	38	87.62
Sulfur	S	16	32.064
Tantalum	Ta	73	180.95
Technetium	Tc	43	(99)
Tellurium	Te	52	127.60
Terbium	Tb	65	158.92
Thallium	Tl	81	204.37
Thorium	Th	90	232.04
Thulium	Tm	69	168.93
Tin	Sn	50	118.69
Titanium	Tl	22	47.90
Tungsten	W	74	183.85
Uranium	U	92	238.03
Vanadium	V	23	50.94
Xenon	Xe	54	131.30
Ytterbium	Yb	70	173.04
Yttrium	Y	39	88.91
Zinc	Zn	30	65.37
Zirconium	Zr	40	91.22

* Based on mass of C^{12} at 12.00… . The ratio of these weights of those on the order chemical scale (in which oxygen of natural isotopic composition was assigned a mass of 16.0000…) is 1.000050. (Values in parentheses represent the most stable known isotopes).

PART—A

1. A granite block of 2 m × 5 m × 3 m size is cut into 5 cm thick slabs of 2 m × 5 m size. These slabs are laid over a 2 m wide pavement. What is the length of the pavement that can be covered with these slabs ?
   (A) 100 m (B) 200 m
   (C) 300 m (D) 500 m

2. Which is the least among the following ?
   $0{\cdot}33^{0\cdot33}$, $0{\cdot}44^{0\cdot44}$, $\pi^{-1/\pi}$, $e^{-1/e}$
   (A) $0{\cdot}33^{0\cdot33}$ (B) $0{\cdot}44^{0\cdot44}$
   (C) $\pi^{-1/\pi}$ (D) $e^{-1/e}$

3. What is the next number in this "see and tell" sequence ?
   1 11 21 1211 111221 ....
   (A) 312211 (B) 1112221
   (C) 1112222 (D) 1112131

4. A vertical pole of length a stands at the centre of a horizontal regular hexagonal ground of side a. A rope that is fixed taut in between a vertex on the ground and the tip of the pole has a length—
   (A) a (B) $\sqrt{2a}$
   (C) $\sqrt{3a}$ (D) $\sqrt{6a}$

5. A peacock perched on the top of a 12 m high tree, spots a snake moving towards its hole at the base of the tree from a distance equal to thrice the height of the tree. The peacock flies

towards the snake in a straight line and they both move at the same speed. At what distance from the base of the tree will the peacock catch the snake ?

(A) 16 m (B) 18 m
(C) 14 m (D) 12 m

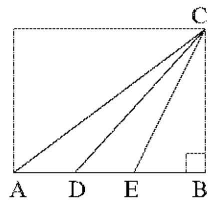
6. The cities of a country are connected by intercity roads. If a city is directly connected to an odd number of other cities, it is called an odd city. If a city is directly connected to an even number of other cities, it is called an even city. Then which of the following is impossible ?

(A) There are an even number of odd cities
(B) There are an odd number of odd cities
(C) There are an even number of even cities
(D) There are an odd number of even cities

7. In the figure $\angle ABC = \pi/2$

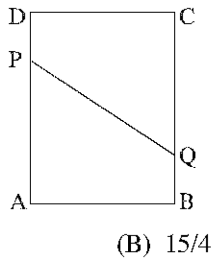
$$AD = DE = EB$$

What is the ratio of the area of triangle ADC to that triangle CDB ?



(A) 1 : 1 (B) 1 : 2
(C) 1 : 3 (D) 1 : 4

8. A rectangular sheet ABCD is folded in such a way that vertex A meets vertex C, thereby forming a line PQ. Assuming AB = 3 and BC = 4, find PQ. Note that AP = PC and AQ = QC.



(A) 13/4 (B) 15/4
(C) 17/4 (D) 19/4

9. A string of diameter 1 mm is kept on a table in the shape of a close flat spiral *i.e.*, a spiral with no gap between the turns. The area of the table occupied by the spiral is 1 m^2. Then the length of the string is—

(A) 10 m (B) 10^2 m
(C) 10^3 m (D) 10^6 m

10. 25% of 25% of a quantity is x% of the quantity where x is—

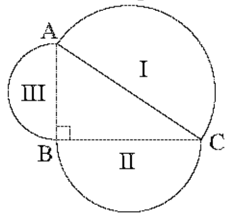
(A) 6·25% (B) 12·5%
(C) 25% (D) 50%

11. In sequence $\{a_n\}$ every term is equal to the sum of all its previous terms.

If $a_0 = 3$, $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$ is—

(A) 3 (B) 2
(C) 1 (D) e

12. In the figure below, angle ABC = $\pi/2$. I, II, III are the areas of semicircles on the sides opposite angles B, A and C, respectively. Which of the following is always true ?



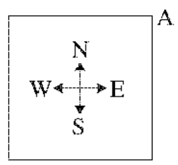
(A) $II^2 + III^2 = I^2$ (B) II + III = I
(C) $II^2 + III^2 > I^2$ (D) II + III < I

13. What is the minimum number of days between one Friday the 13th and the next Friday the 13th ?

(Assume that the year is a leap year)

(A) 28 (B) 56
(C) 91 (D) 84

14. Suppose a person A is at the North-East corner of a square (see the figure below).



From that point he moves along the diagonal and after covering 1/3rd portion of the diagonal, he goes to his left and after sometime he stops, rotates 90° clockwise and moves straight. After a few minutes he stops, rotates 180° anticlockwise. Towards which direction he is facing now ?

(A) North-East (B) North-West
(C) South-East (D) South-West

15. Cucumber contains 99% water. Ramesh buys 100 kg of cucumbers. After 30 days of storing, the cucumbers lose some water. They now contain 98% water. What is the total weight of cucumbers now ?

(A) 99 kg (B) 50 kg
(C) 75 kg (D) 2 kg

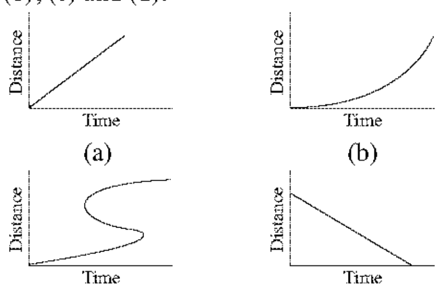
16. In a museum there were old coins with their respective years engraved on them, as follows.

1. 1837 AD 2. 1907 AD
3. 1947 AD 4. 200 BC

Identify the fake coin(s)—

(A) Coin 1 (B) Coin 4
(C) Coins 1 and 2 (D) Coin 3

17. A student observes the movement of four snails and plots the graphs of distance moved as a function of time as given in figures (a), (b), (c) and (d).



(a) (b) (c) (d)

Which of the following is not correct ?

(A) Graph (a) (B) Graph (b)
(C) Graph (c) (D) Graph (d)

18. Find the missing letter—

A	**EGK**	**C**
?		**P**
U		**R**
Q		**V**
B	**OJF**	**D**

(A) H (B) L
(C) Z (D) Y

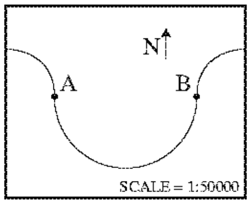
19. Consider the following equation

$$x^2 + 4y^2 + 9z^2 = 14x + 28y + 42z + 147$$

where x, y and z are real numbers. Then the value of $x + 2y + 3z$ is—

(A) 7 (B) 14
(C) 21 (D) Not unique

20. The map given below shows a meandering river following a semi-circular path, along which two villages are located at A and B. The distance between A and B along the east-west direction in the map is 7 cm. What is the length of the river between A and B in the ground ?



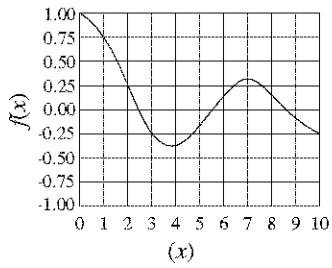
(A) 1·1 km (B) 3·5 km
(C) 5·5 km (D) 11·0 km

PART—B

21. A 2 × 2 matrix A has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of n such that $A^n = I$ is—

(A) 20 (B) 30
(C) 60 (D) 120

22. The graph of the function $f(x)$ shown below is best described by—



(A) The Bessel function $J_0(x)$
(B) $\cos x$
(C) $e^{-x} \cos x$
(D) $\frac{1}{x} \cos x$

23. In a series of five Cricket matches, one of the captains calls 'Heads' every time when the toss is taken. The probability that he will win 3 times and lose 2 times is—

(A) 1/8 (B) 5/8
(C) 3/16 (D) 5/16

24. The unit normal vector at the point

$$\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$$

on the surface of the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, is—

(A) $\frac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (B) $\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$

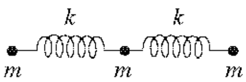
(C) $\frac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (D) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

25. A solid cylinder of height H, radius R and density ρ, floats vertically on the surface of a liquid of density ρ_0. The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is—

(A) $\frac{\rho g}{\rho_0 H}$ (B) $\frac{\rho}{\rho_0}\sqrt{\frac{g}{H}}$

(C) $\sqrt{\frac{\rho g}{\rho_0 H}}$ (D) $\sqrt{\frac{\rho_0 g}{\rho H}}$

26. Three particles of equal mass m are connected by two identical massless springs of stiffness constant k as shown in the figure—



If x_1, x_2 and x_3 denote the horizontal displacements of the masses from their respective equilibrium position, the potential energy of the system is—

(A) $\frac{1}{2} k [x_1^2 + x_2^2 + x_3^2]$

(B) $\frac{1}{2} k [x_1^2 + x_2^2 + x_3^2 - x_2 (x_1 + x_3)]$

(C) $\frac{1}{2} k [x_1^2 + 2x_2^2 + x_3^2 + 2x_2 (x_1 + x_3)]$

(D) $\frac{1}{2} k [x_1^2 + 2x_2^2 + x_3^2 - 2x_2 (x_1 + x_3)]$

27. Let v, p and E denote the speed, the magnitude of the momentum, and the energy of a free particle of rest mass m. Then—

(A) $\frac{dE}{dp}$ = constant

(B) $p = mv$

(C) $v = \frac{cp}{\sqrt{p^2 + m^2c^2}}$

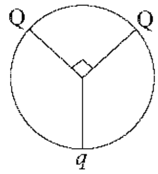
(D) $E = mc^2$

28. A binary star system consists of two stars S_1 and S_2, with masses m and $2m$ respectively separated by a distance r. If both S_1 and S_2 individually follow circular orbits around the centre of mass with instantaneous speeds v_1 and v_2 respectively, the speeds ratio v_1/v_2 is—

(A) $\sqrt{2}$ (B) 1

(C) 1/2 (D) 2

29. Three charges are located on the circumference of a circle of radius R as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge q is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q ?



(A) $q/\sqrt{2}$ (B) $\sqrt{2}q$

(C) $2q$ (D) $4q$

30. Consider a hollow charged shell of inner radius a and outer radius b. The volume charge density is $\rho(r) = \frac{k}{r^2}$ (k is a constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is—

(A) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for all $r > a$

(B) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\varepsilon_0 r^2}$ for $r > b$

(C) $\frac{k(r-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$

(D) $\frac{k(r-a)}{\varepsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 a^2}$ for $r > b$

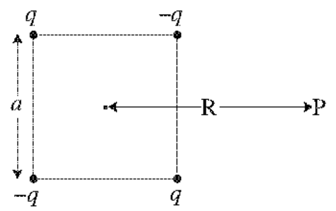
31. Consider the interference of two coherent electromagnetic waves whose electric field

vectors are given by $\vec{E_1} = \hat{i}\, E_0 \cos \omega t$ and $\vec{E_2} = \hat{j}\, E_0 \cos (\omega t + \phi)$ where ϕ is the phase difference. The intensity of the resulting wave is given by $\frac{\varepsilon_0}{2} \langle E^2 \rangle$, where $\langle E^2 \rangle$ is the time average of E^2. The total intensity is—

(A) 0 (B) $\varepsilon_0 E_0^2$

(C) $\varepsilon_0 E_0^2 \sin^2 \phi$ (D) $\varepsilon_0 E_0^2 \cos^2 \phi$

32. Four charges (two + q and two – q) are kept fixed at the four vertices of a square of side a as shown



At the point P which is at a distnace R from the centre (R >> a), the potential is proportional to—

(A) 1/R (B) $1/R^2$

(C) $1/R^3$ (D) $1/R^4$

33. A point charge q of mass m is kept at a distance d below a grounded infinite conducting sheet which lies in the xy-plane. For what value of d will the charge remains stationary ?

(A) $\frac{q}{4\sqrt{mg\pi\varepsilon_0}}$

(B) $\frac{q}{\sqrt{mg\pi\varepsilon_0}}$

(C) There is no finite value of d

(D) $\frac{\sqrt{mg\rho\varepsilon_0}}{q}$

34. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\psi_{21-1}$$

where ψ_{nlm} is the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_Z is the state ψ is—

(A) $\frac{15h}{16}$ (B) $\frac{11h}{16}$

(C) $\frac{3h}{8}$ (D) $\frac{h}{8}$

35. The energy eigen values of a particle in the potential $V(x) = \frac{1}{2} m\omega^2 x^2 - ax$ are—

(A) $E_n = \left(n + \frac{1}{2}\right) h\omega - \frac{a^2}{2m\omega^2}$

(B) $E_n = \left(n + \frac{1}{2}\right) h\omega + \frac{a^2}{2m\omega^2}$

(C) $E_n = \left(n + \frac{1}{2}\right) h\omega - \frac{a^2}{m\omega^2}$

(D) $E_n = \left(n + \frac{1}{2}\right) h\omega$

36. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \frac{\sqrt{15}\,(a^2 - x^2)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty Δp in its momentum is—

(A) $\frac{2h}{5a}$ (B) $\frac{5h}{2a}$

(C) $\frac{\sqrt{10}h}{a}$ (D) $\frac{\sqrt{5}h}{\sqrt{2}a}$

37. Given the usual canonical communtation relations, the commutator [A, B] of $A = i(xp_y - yp_x)$ and $B = (yp_z + zp_y)$ is—

(A) $\hbar (xp_z - p_x z)$ (B) $-\hbar (xp_z - p_x z)$

(C) $\hbar (xp_z + p_x z)$ (D) $-\hbar (xp_z + p_x z)$

38. The entropy of a system, S, is related to the accessible phase space volume Γ by $S = k_B \ln \Gamma (E, N, V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that Γ —

(A) Does not change during evolution to equilibrium

(B) Oscillates during evolution to equilibrium

(C) Is a maximum at equilibrium

(D) Is a minimum at equilibrium

39. Let ΔW be the work done in a quasistatic reversible thermodynamic process. Which of

the following statements about ΔW is correct?

(A) ΔW is a perfect differential if the process is isothermal

(B) ΔW is a perfect differential if the process is adiabatic

(C) ΔW is always a perfect differential

(D) ΔW cannot be a perfect differential

40. Consider a system of three spins S_1, S_2 and S_3 each of which can take values + 1 and – 1. The energy of the system is given by $E = -J[S_1 S_2 + S_2 S_3 + S_3 S_1]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively—

(A) J and 1 (B) – 3J and 1

(C) – 3J and 2 (D) – 6J and 2

41. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ and mass m placed in a one dimensional infinite square well potential of width L is—

(A) $\frac{14\pi^2\hbar^2}{mL^2}$ (B) $\frac{91\pi^2\hbar^2}{mL^2}$

(C) $\frac{7\pi^2\hbar^2}{mL^2}$ (D) $\frac{3\pi^2\hbar^2}{mL^2}$

42. A live music broadcast consists of a radio-wave of frequency 7 MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 kHz. The spectrum of modulated output will be zero outside the frequency band—

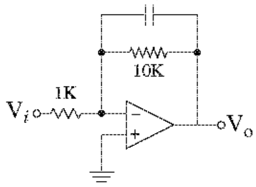
(A) 7·00 MHz to 7·01 MHz

(B) 6·99 MHz to 7·01 MHz

(C) 6·99 MHz to 7·00 MHz

(D) 6·995 MHz to 7·005 MHz

43. In the op-amp circuit shown in the figure, V_i is a sinusoidal input signal of frequency 10 Hz and V_0 is the output signal.

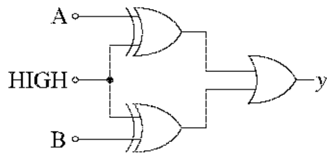


The magnitude of the gain and the phase shift, respectively, close to the values—

(A) $5\sqrt{2}$ and $\pi/2$ (B) $5\sqrt{2}$ and $-\pi/2$

(C) 10 and zero (D) 10 and π

44. The logic circuit shown in the figure below



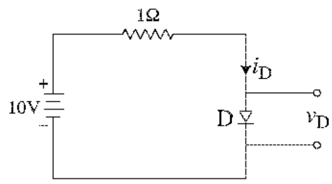
implements the Boolean expression—

(A) $y = \overline{A \cdot B}$ (B) $y = \overline{A} \cdot \overline{B}$

(C) $y = A \cdot B$ (D) $y = A + B$

45. A diode D as shown in the circuit has an $i - v$ relation that can be approximated by—

$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \le 0 \end{cases}$$



The value of v_D in the circuit is—

(A) $\left(-1+\sqrt{11}\right)$ V (B) 8V

(C) 5V (D) 2V

PART—C

46. The Taylor expansion of the function In (cosh x), where x is real, about the point $x = 0$ starts with the following terms—

(A) $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \ldots$

(B) $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \ldots$

(C) $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \ldots$

(D) $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \ldots$

47. Given a 2 × 2 unitary matrix U satisfying $U^tU = UU^t = 1$ with det $U = e^{i\phi}$, one can construct a unitary matrix V ($V^tV = VV^t = 1$) with det V = 1 from it by—

(A) Multiplying U by $e^{-i\phi/2}$

(B) Multiplying any single element of U by $e^{-i\phi}$

(C) Multiplying any row or column of U by $e^{-i\phi/2}$

(D) Multiplying U by $e^{-i\phi}$

48. The value of the integral $\int_C \frac{z^3 dz}{z^2 - 5z + 6}$, where C is a closed contour defined by the equation $2\,|z| - 5 = 0$, traversed in the anti-clockwise direction, is—

(A) $-16\pi i$ (B) $16\pi i$

(C) $8\pi i$ (D) $2\pi i$

49. The function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2} - (3 - 2i)f = 0$ and satisfies the conditions $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is—

(A) $e^{2\pi}$ (B) $e^{-2\pi}$

(C) $-e^{-2\pi}$ (D) $-e^{2\pi i}$

50. A planet of mass m moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axes of the orbit are a and b respectively, the angular momentum of the planet is—

(A) $\sqrt{2GMm^2(a+b)}$

(B) $\sqrt{2GMm^2(a-b)}$

(C) $\sqrt{\frac{2GMm^2 ab}{a-b}}$

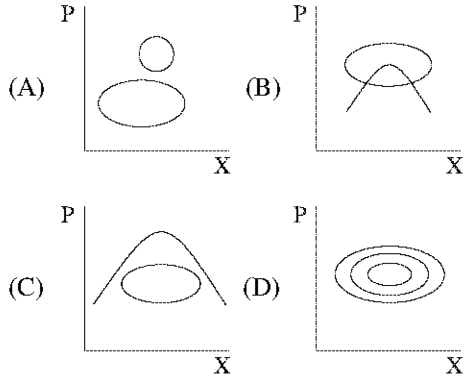
(D) $\sqrt{\frac{2GMm^2 ab}{a+b}}$

51. The Hamiltonian of a simple pendulum consisting of a mass m attached to a massless string of length l is $H = \frac{p_\theta^2}{2ml^2} + mgt(1 - \cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is—

(A) $-\frac{2g}{l} p_\theta \sin\theta$ (B) $-\frac{g}{l} p_\theta \sin 2\theta$

(C) $\frac{g}{l} p_\theta \cos\theta$ (D) $l p_\theta^2 \cos\theta$

52. Which of the following set of phase-space trajectories is not possible for a particle obeying Hamilton's equations of motion ?

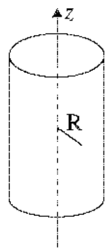


53. Two bodies of equal mass m are connected by a massless rigid rod of length l lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is—

(A) $\frac{ml^2\omega}{4}$ (B) $\frac{ml^2\omega}{2}$

(C) $ml^2\omega$ (D) $2ml^2\omega$

54. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I.



The vector potential $\vec{A}$ at a distance R from the axis—

(A) is constant inside and varies as R outside the solenoid

(B) varies as R inside and is constant outside the solenoid

(C) varies as $\frac{1}{R}$ inside and as R outside the solenoid

(D) varies as R inside and as $\frac{1}{R}$ outside the solenoid

55. Consider an infinite conducting sheet in the xy-plane with a time dependent current density

$Kt\hat{i}$, where K is constant. The vector potential at (x, y, z) is given by $\dot{A} = \frac{\mu_0 K}{4c}(ct - z)^2\,\hat{i}$. The magnetic field $\vec{B}$ is—

(A) $\frac{\mu_0 Kt}{2}\hat{j}$ (B) $-\frac{\mu_0 Kz}{2c}\hat{j}$

(C) $-\frac{\mu_0 K}{2c}(ct - z)\,\hat{i}$ (D) $-\frac{\mu_0 K}{2c}(ct - z)\,\hat{j}$

56. When a charged particle emits electromagnetic radiation, the electric field $\vec{E}$ and the Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ at a large distance r from the emitter vary as $\frac{1}{r^n}$ and $\frac{1}{r^m}$ respectively. Which of the following choices for n and m are correct ?

(A) $n = 1$ and $m = 1$ (B) $n = 2$ and $m = 2$

(C) $n = 1$ and $m = 2$ (D) $n = 2$ and $m = 4$

57. The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential V (x) are – 4 and – 1, respectively, (in units in which $\hbar = 1$). If the corresponding wave functions are related by $\psi_1(x) = \psi_0(x)$ sin h (x), then the ground state eigen function is—

(A) $\psi_0(x) = \sqrt{\text{sech}\,x}$

(B) $\psi_0(x) = \text{sech}\,x$

(C) $\psi_0(x) = \text{sech}^2\,x$

(D) $\psi_0(x) = \text{sech}^3\,x$

58. The perturbation

$$H' = \begin{cases} b(a - x), & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

acts on a particle of mass m confined in an infinite square well potential

$$V(x) = \begin{cases} 0, & -a < x < a \\ \infty, & \text{otherwise} \end{cases}$$

The first order correction to the ground-state energy of the particle is—

(A) $\frac{ba}{2}$ (B) $\frac{ba}{\sqrt{2}}$

(C) $2ba$ (D) ba

59. Let $|0>$ and $|1>$ denote the normalized eigen states corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0> + |1>)$ is—

(A) $\Delta x = \sqrt{\hbar/2m\omega}$ (B) $\Delta x = \sqrt{\hbar/m\omega}$

(C) $\Delta x = \sqrt{2\hbar/m\omega}$ (D) $\Delta x = \sqrt{\hbar/4m\omega}$

60. What would be the ground state energy of the Hamiltonian—

$$H = -\frac{h^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

if variational principle is used to estimate it with the trial wavefunction $\psi(x)\,Ae^{-bx^2}$ with b as the variational parameter ?

Hint :

$$\int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-1/2}\,\Gamma\left(n + \frac{1}{2}\right)$$

(A) $-m\alpha^2/2\hbar^2$ (B) $-2m\alpha^2/\pi\hbar^2$

(C) $-m\alpha^2/\pi\hbar^2$ (D) $m\alpha^2/\pi\hbar^2$

61. The free energy difference between the superconducting and the normal states of a material is given by

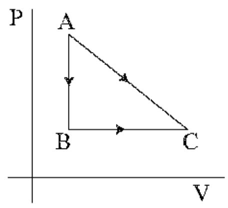
$$\Delta F = F_S - F_N = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4,$$

where ψ is an order parameter and α and β are constants such that $\alpha > 0$ in the normal and $\alpha < 0$ in the superconducting state, while $\beta > 0$ always. The minimum value of ΔF is—

(A) $-\alpha^2/\beta$ (B) $-\alpha^2/2\beta$

(C) $-3\alpha^2/2\beta$ (D) $-5\alpha^2/2\beta$

62. A given quantity of gas is taken from the state A → C reversibly, by two paths, A → C directly and A → B → C as shown in the figure below.



During the process A → C the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process A → B → C the work done by the gas is 30 J, the heat absorbed is—

(A) 20 J (B) 80 J
(C) 220 J (D) 280 J

63. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2J. In a configuration with r spin flips, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^{N}C_r$; r varies from 0 to N. the partition function is—

(A) $\left(\frac{J}{k_B T}\right)^N$ (B) $e^{-NJ/k_B T}$

(C) $\left(\sinh \frac{J}{k_B T}\right)^N$ (D) $\left(\cosh \frac{J}{k_B T}\right)^N$

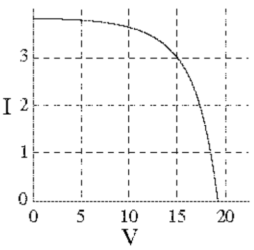
64. A magnetic field sensor based on the Hall effect is to be fabricated by implanting As into a Si film of thickness 1 μm. The specifications require a magnetic field sensitivity of 500 mV/tesla at an excitation current of 1 mA. The implantation dose is to be adjusted such that the average carrier density, after activation, is—

(A) $1{\cdot}25 \times 10^{26}$ m^{-3} (B) $1{\cdot}25 \times 10^{22}$ m^{-3}
(C) $4{\cdot}1 \times 10^{21}$ m^{-3} (D) $4{\cdot}1 \times 10^{20}$ m^{-3}

65. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and high pass filters are ω_0^{LP} and ω_0^{HP}, respectively, the condition required to implement the bandpass and band-reject filters are, respectively—

(A) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0 < \omega_0^{LP}$
(B) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$
(C) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$
(D) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

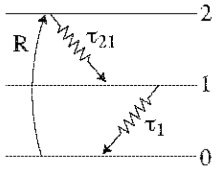
66. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of 5Ω, the power drawn by the load is—

(A) 97 W (B) 73 W
(C0 50 W (D) 45 W

67. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is 10^{20} atoms cm^{-3} s^{-1} and the decay routes are as shown with $\tau_{21} = 20$ ns and $\tau_1 = 1$ μs, the equilibrium populations of states 2 and 1 are, respectively—

(A) 10^{14} cm^{-3} and 2×10^{12} cm^{-3}
(B) 2×10^{12} cm^{-3} and 10^{14} cm^{-3}
(C) 2×10^{12} cm^{-3} and 2×10^{6} cm^{-3}
(D) Zero and 10^{20} cm^{-3}

68. Consider a hydrogen atom undergoing a 2P → 1S transition. The lifetime t_{sp} of the 2P state for spontaneous emission is 1·6 ns and the energy difference between the levels is 10·2 eV. Assuming that the refractive index of the medium $n_0 = 1$, the ratio of Einstein coefficients for stimulated and spontaneous emission $B_{21}(\omega)/A_{21}(\omega)$ is given by—

(A) $0{\cdot}683 \times 10^{12}$ m^{3}J^{-1}s^{-1}
(B) $0{\cdot}146 \times 10^{-12}$ J s m^{-3}
(C) $6{\cdot}83 \times 10^{12}$ m^{3}J^{-1}s^{-1}
(D) $1{\cdot}463 \times 10^{-12}$ J s m^{-3}

69. Consider a He–Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0{\cdot}98$. The mirrors are separated by a distance $d = 20$ cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes δv and the width Δv_p of each mode of the laser cavity are—

(A) $\delta v = 75$ kHz, $\Delta v_p = 24$ kHz
(B) $\delta v = 100$ kHz, $\Delta v_p = 100$ kHz
(C) $\delta v = 750$ MHz, $\Delta v_p = 2{\cdot}4$ MHz
(D) $\delta v = 2{\cdot}4$ MHz, $\Delta v_p = 750$ MHz

70. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to—

(A) $\hbar\omega/2$ (B) $\hbar\omega$

(C) $3\hbar\omega/2$ (D) 0

71. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta\,(\cos k_x a + \cos k_y a + \cos k_z a),$$

where β is a contant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zone is—

(A) $\frac{2\hbar^2}{5\beta a^2}$ (B) $\frac{4\hbar^2}{5\beta a^2}$

(C) $\frac{\hbar^2}{2\beta a^2}$ (D) $\frac{\hbar^2}{3\beta a^2}$

72. The radius of the Fermi sphere of free electrons in a monovalent metal with an *fcc* structure, in which the volume of the unit cell is a^3 is—

(A) $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ (B) $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$

(C) $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ (D) $\frac{1}{a}$

73. The muon has mass 105 MeV/c^2 and mean lifetime 2·2 μs in its rest frame. The mean distance traversed by a muon of energy 315 MeV/c^2 before decaying is approximately—

(A) 3×10^5 km (B) 2·2 cm

(C) 6·6 μm (D) 1·98 km

74. Consider the following particles : the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of **decreasing** lifetime, the correct arrangement is as follows—

(A) π^0, n, p, Δ^+ (B) p, n, Δ^+, π^0

(C) p, n, π^0, Δ^+ (D) Δ^+, n, π^0, p

75. The single particle energy difference between the p-orbitals (*i.e.*, $p_{3/2}$ and $p_{1/2}$) of the nucleus $^{114}_{50}Sn$ is 3 MeV. The energy difference between the states in its $1f$ orbital is—

(A) – 7 MeV (B) 7 MeV

(C) 5 MeV (D) – 5 MeV

Answers with Explanations

1. (C) Total number of slab

$$= \frac{2\text{ m} \times 5\text{ m} \times 3\text{ m}}{2\text{ m} \times 5\text{ m} \times 5\text{ cm}}$$

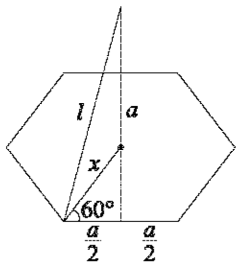
$$= \frac{300\text{ cm}}{5\text{ cm}} = 60$$

Length of pavement that can be covered with these slabs

$$60 \times 5 = 300$$

2. (D) 3. (A)

4. (B)



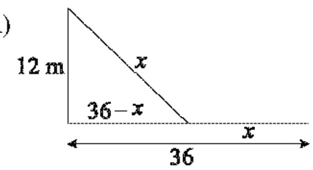
$$\cos 60° = \frac{a/2}{x}$$

$$x = \frac{a/2}{1/2} = a$$

$$l = \sqrt{a^2 + a^2}$$

$$= \sqrt{2}\,a$$

5. (A)



$$(12)^2 + (36 - x)^2 = x^2$$

$$144 + (36)^2 - 72x + x^2 = x^2$$

$$144 + 1296 = 72x$$

$$1440 = 72x$$

$$x = \frac{1440}{72} = 20$$

$$36 - x = 36 - 20$$

$$= 16\text{ m}$$

6. (B) 7. (B) 8. (B) 9. (C)

10. (A) Let quantity is 100.

$$x\% \text{ of } 100 \text{ is } \frac{100 \times x}{100} = x$$

$$25\% \text{ of } 100 = 25$$

$$25\% \text{ of } 25 = \frac{25 \times 25}{100} = 6{\cdot}25$$

$$x = 6{\cdot}25\%$$

11. (B) Sequence

3, 3, 6, 12, 24, 48, ...

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 2$$

12. (B) Let Length of AB = AB

Length of BC = BC

Length of CA = CA

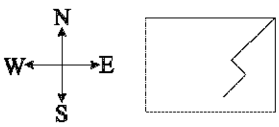
$$(AB)^2 + (BC)^2 = (CA)^2$$

$$\pi(AB)^2 + \pi(BC)^2 = \pi(CA)^2$$

$$III + II = I$$

13. (C)

14. (A)



15. (B) 16. (B)

17. (C) In a graph (C) in one time there are two distance, but it is not possible.

18. (C) 19. (C)

20. (C) Distance between A and B is 2R.

$$2R = 7 \text{ cm}$$

$$R = \frac{7}{2} = 3{\cdot}5 \text{ cm}$$

Length of river between A and B

$$\frac{2\pi R}{2} = \pi R$$

$$= 3{\cdot}14 \times 3{\cdot}5 = 11 \text{ cm}$$

It is given scale = 1 : 50000

Length of river between A and B in the ground

$$= 11 \times 50000 \text{ cm}$$

$$= 5{\cdot}5 \text{ km}$$

21. (C) $e^{i\pi/5}, e^{i\pi/6}$

Eigen value follow characteristic equation

$$\left(e^{i\frac{\pi}{5}}\right)^n = 1 \quad \text{...(i)}$$

$$\left(e^{i\frac{\pi}{6}}\right)^n = 1 \quad \text{...(ii)}$$

$$e^{i\frac{\pi}{5}n} = 1$$

$$e^{i\frac{\pi}{6}n} = 1$$

The minimum value of $n = 60$

$$e^{i\frac{\pi}{5}\times 60} = e^{i\,12\pi}$$

$$= \cos 12\pi + i \sin 12\pi$$

$$= 1$$

$$e^{i\frac{\pi}{6}\times 60} = e^{i\,10\pi}$$

$$= \cos 10\pi + i \sin 10\pi$$

$$= 1$$

22. (A)

23. (D) 3 win, 2 loss

$$\frac{1}{2^5}\,{}^5C_3 = \frac{1}{32} \times \frac{5\times 4}{2}$$

$$= \frac{5}{16}$$

24. (A) Unit normal vector $\nabla\phi/|\nabla\phi|$

$$\phi = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$$

$$\nabla\phi = \frac{2x}{a^2}\hat{i} + \frac{2y}{b^2}\hat{j} + \frac{2z}{c^2}\hat{k}$$

At the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$

$$\nabla\phi = \frac{2}{a^2}\frac{a}{\sqrt{3}}\hat{i} + \frac{2}{b^2}\frac{b}{\sqrt{3}}\hat{j} + \frac{2}{c^2}\frac{c}{\sqrt{3}}\hat{k}$$

$$\nabla\phi = \frac{2}{\sqrt{3}a}\hat{i} + \frac{2}{\sqrt{3}b}\hat{j} + \frac{2}{\sqrt{3}c}\hat{k}$$

Unit normal vector $\frac{\nabla\phi}{|\nabla\phi|}$

$$= \frac{\frac{2}{\sqrt{3}a}\hat{i} + \frac{2}{\sqrt{3}b}\hat{j} + \frac{2}{\sqrt{3}c}\hat{k}}{\sqrt{\left(\frac{2}{\sqrt{3}a}\right)^2 + \left(\frac{2}{\sqrt{3}b}\right)^2 + \left(\frac{2}{\sqrt{3}c}\right)^2}}$$

$$= \frac{\frac{1}{a}\hat{i} + \frac{1}{b}\hat{j} + \frac{1}{c}\hat{k}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

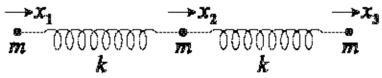
$$= \frac{\frac{1}{a}\hat{i} + \frac{1}{b}\hat{j} + \frac{1}{c}\hat{k}}{\sqrt{\frac{b^2c^2 + a^2c^2 + a^2b^2}{a^2\,b^2\,c^2}}}$$

$$= \frac{\frac{abc}{a}\hat{i} + \frac{abc}{b}\hat{j} + \frac{abc}{c}\hat{k}}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$$

$$= \frac{bc\,\hat{i} + ca\,\hat{j} + ab\,\hat{k}}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$$

No option are **correct**.

25. (D)

26. (D) 

Potential energy

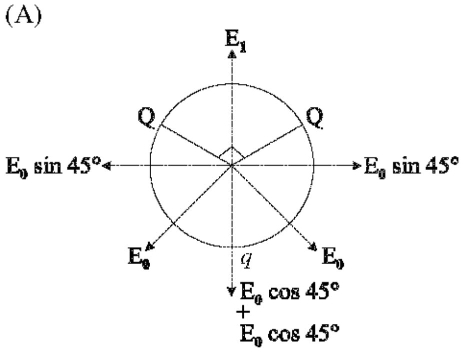
$$= \frac{1}{2}k\,(x_1 - x_2)^2 + \frac{1}{2}k\,(x_3 - x_2)^2$$

$$= \frac{1}{2}k\,[x_1^2 + x_2^2 - 2x_1x_2 + x_3^2 + x_2^2 - 2x_2x_3]$$

$$= \frac{1}{2}k\,[x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_2)]$$

27. (C) 28. (D)

29. (A)



$E_D \sin 45°$ will be cancel to each other.

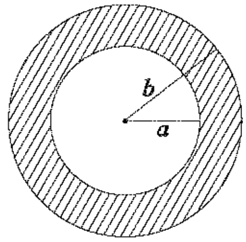
$$E_1 = 2\,E_0 \cos 45°$$

$$\frac{1}{4\pi\epsilon_0}\frac{q}{R^2} = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\cdot\frac{1}{\sqrt{2}}$$

$$q = \sqrt{2}\,Q$$

$$Q = \frac{1}{\sqrt{2}}q$$

30. (C)



The electric field for $a < r < b$

$$\int \vec{E}\cdot\vec{dS} = \frac{q_{en}}{\epsilon_0}$$

$$q_{en} = \int \rho\, dv$$

$$= \int_a^r \frac{k}{r^2} 4\pi r^2\, dr$$

$$q_{en} = 4\pi k\,(r - a)$$

So, $\int \vec{E}\cdot\vec{dS} = \frac{q_{en}}{\epsilon_0}$

$$E\,4\pi r^2 = \frac{4\pi k\,(r-a)}{\epsilon_0}$$

$$E = \frac{4\pi k\,(r-a)}{4\pi\epsilon_0 r^2}$$

$$E = \frac{k\,(r-a)}{\epsilon_0 r^2} \quad \text{For } a < r < b$$

The electric field for $r > b$

$$\int \vec{E}\cdot\vec{dS} = \frac{q_{en}}{\epsilon_0}$$

$$\int \vec{E}\cdot\vec{dS} = \frac{q_{en}}{\epsilon_0}$$

$$q_{en} = \int \rho\, dV = \int_a^b \frac{k}{r^2} 4\pi r^2\, dr$$

$$q_{en} = 4\pi k\,(b - a)$$

So, $\int \vec{E}\cdot\vec{dS} = \frac{q_{en}}{\epsilon_0}$

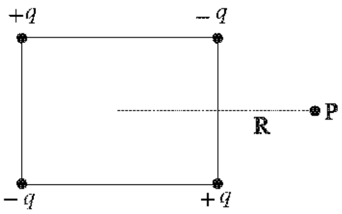
$$E\,4\pi r^2 = \frac{4\pi k\,(b-a)}{\epsilon_0}$$

$$E = \frac{4\pi k\,(b-a)}{4\pi\, r^2\epsilon_0}$$

$$E = \frac{k\,(b-a)}{\epsilon_0 r^2} \quad \text{For } r > b$$

31. (B)

32. (C)



It is quadrupole.

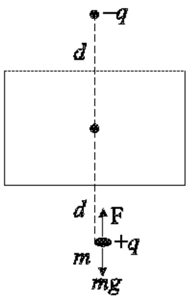
Potential (V) for monopole proportional to $\frac{1}{r}$.

Potential (V) for dipole is proportional to $\frac{1}{r^2}$.

Potential (V) for quadrupole is proportional to $\frac{1}{r^3}$.

Potential (V) for octapole is proportional to $\frac{1}{r^4}$.

33. (A)



$$F_{electrostatic} = F_{gravitational}$$
$$F = mg$$
$$\frac{1}{4\pi\epsilon_0}\frac{q.q}{(2d)^2} = mg$$
$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{4\,mg} = d^2$$
$$d^2 = \frac{q^2}{16\pi\epsilon_0\, mg}$$
$$d = \frac{q}{4\sqrt{mg\,\pi\epsilon_0}}$$

34. (D) $\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}$

For normalization

$$\psi = A[\psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}]$$

$$|A|^2 + (2)^2|A|^2 + (3)^2|A|^2 + (\sqrt{2})^2|A|^2 = 1$$
$$16A^2 = 1$$
$$A^2 = \frac{1}{16}$$
$$A = \frac{1}{4}$$

$$\psi = \frac{1}{4}\psi_{200} + \frac{2}{4}\psi_{211} + \frac{3}{4}\psi_{210} + \frac{\sqrt{2}}{4}\psi_{21-1}$$
$$\langle L_z\rangle = \sum P_i L_{zi}$$
$$\because L_z = m\hbar$$
$$= \left(\frac{1}{4}\right)^2.0\hbar + \left(\frac{2}{4}\right)^2\cdot 1.\hbar + \left(\frac{3}{4}\right)^2.0.\hbar + \left(\frac{\sqrt{2}}{4}\right)^2.(-1)\hbar$$
$$= \frac{1}{4}\hbar - \frac{1}{8}\hbar = \frac{1}{8}\hbar$$

35. (A) $V(x) = \frac{1}{2}m\omega^2 x^2 - ax$

$$V(x) = \frac{1}{2}m\omega^2\left[x^2 - \frac{2}{m\omega^2}ax\right]$$
$$V(x) = \frac{1}{2}m\omega^2\left[x^2 - 2.\frac{a}{m\omega^2}x + \left(\frac{a}{m\omega^2}\right)^2 - \left(\frac{a}{m\omega^2}\right)^2\right]$$
$$V(x) = \frac{1}{2}m\omega^2\left[\left(x - \frac{a}{m\omega^2}\right)^2 - \left(\frac{a}{m\omega^2}\right)^2\right]$$
$$V(x) = \frac{1}{2}m\omega^2\left[\left(x - \frac{a}{m\omega^2}\right)^2\right] - \frac{1}{2}m\omega^2\left(\frac{a^2}{m\omega^2}\right)^2$$
$$V(x) = \frac{1}{2}m\omega^2 X^2 - \frac{1}{2}\frac{a^2}{m\omega^2}$$

where $X = x - \frac{a}{m\omega^2}$

$$V(x) = \frac{1}{2}m\omega^2 X^2 - \frac{1}{2}\frac{a^2}{m\omega^2}\text{ const.}$$

We know that if

$$V(x) = \frac{1}{2}m\omega^2 X^2$$

then total energy is $\left(n + \frac{1}{2}\right)\hbar\omega$

So, here

$$V(x) = \frac{1}{2}m\omega^2 X^2 - \frac{a^2}{2m\omega^2}$$

Total energy

$$= \left(n + \frac{1}{2}\right)\hbar\omega - \frac{a^2}{2m\omega^2}$$

36. (D) $\Delta P = \sqrt{<P^2> - <P>^2}$

$$<P> = \int \Psi^* \left(-i\hbar \frac{d}{dx}\right) \Psi \, dx$$

$$= \int_{-a}^{+a} \frac{\sqrt{15}}{4a^{5/2}} (a^2 - x^2) \left(-i\hbar \frac{d}{dx}\right) \frac{\sqrt{15}}{4a^{5/2}} (a^2 - x^2) \, dx$$

$$<P> = \int_{-a}^{a} \frac{\sqrt{15}}{4a^{5/2}} (a^2 - x^2) \, -i\hbar \, \frac{\sqrt{15}}{4a^{5/2}} (-2x) \, dx$$

It is odd function.

$$<P> = 0$$

Now,

$$<P^2> = \int \Psi^* \left(-\hbar^2 \frac{d^2}{dx^2}\right) \Psi \, dx$$

$$<P^2> = \int_{-a}^{+a} \frac{\sqrt{15}\,(a^2 - x^2)}{4a^{5/2}} (-\hbar^2) \frac{d^2}{dx^2} \frac{\sqrt{15}}{4a^{5/2}} (a^2 - x^2) \, dx$$

$$<P^2> = \int_{-a}^{a} \frac{\sqrt{15}}{4a^{5/2}} (a^2 - x^2)(-\hbar^2) \frac{\sqrt{15}}{4a^{5/2}} (-2) \, dx$$

$$= \frac{15 \times 2\hbar^2}{16a^5} \int_{-a}^{+a} (a^2 - x^2) \, dx$$

$$= \frac{30\hbar^2}{16a^5} \left[a^2 x - \frac{x^3}{3}\right]_{-a}^{+a}$$

$$= \frac{15}{8} \frac{\hbar^2}{a^5} \left[\left(a^3 - \frac{a^3}{3}\right) - \left(-a^3 + \frac{a^3}{3}\right)\right]$$

$$= \frac{15}{8} \frac{\hbar^2}{a^5} \left[\frac{2a^3}{3} + \frac{2a^3}{3}\right]$$

$$<P^2> = \frac{15}{8} \frac{\hbar^2}{a^5} \frac{4a^3}{3} = \frac{5\hbar^2}{2a^2}$$

$$\Delta P = \sqrt{<P^2> - <P>^2}$$

$$\Delta P = \sqrt{\frac{5}{2} \frac{\hbar^2}{a} - 0}$$

$$= \sqrt{\frac{5}{2} \frac{\hbar}{a}}$$

37. (C) Poisson Bracket

$$[x, y]_{q,P} = \sum \left[\frac{\partial x}{\partial q_i} \cdot \frac{\partial y}{\partial p_i} - \frac{\partial x}{\partial p_i} \cdot \frac{\partial y}{\partial q_i}\right]$$

Poisson Bracket [A, B]

where $A = i\,(xP_y - yP_x)$, $B = (yP_z + zP_y)$

$$[A, B] = (iPy.0 - (-y)\, i.0) + (-P_x Z_i - x\, iP_z) + [0.y - 0.P_y]$$

$$= 0 + -i\,[xP_z + P_x z] + 0$$

$$[A, B] = -i\,[xP_z + P_x z]$$

Commutator Bracket = $i\hbar$ (Poisson Bracket)

$$= -i\,(i\hbar)\,[xP_z + P_x z]$$

$$= \hbar\,[xP_z + P_x z]$$

38. (C)

39. (B) $\delta\theta = du + \delta w$

If process is adiabatic

$$\delta S = 0 = \frac{\delta\theta}{T}$$

So, $\delta\theta = 0$

$$0 = du + \delta w$$

$$\delta\omega = -du = dw$$

Δw is perfect differential if process is adiabatic.

40. (C) $E = -J\,[S_1 S_2 + S_2 S_3 + S_3 S_1]$

S_1, S_2 and S_3 each of which can take values + 1 and – 1.

For minimum energy

$$E = -J\,[1{\cdot}1 + 1{\cdot}1 + 1{\cdot}1] \quad \ldots(1)$$

$$E = -3J$$

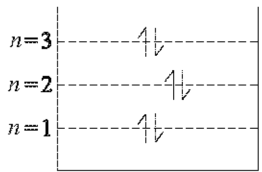
$$E = -J\,[(-1)(-1) + (-1)(-1) + (-1)(-1)] \quad \ldots(2)$$

$$E = -3J$$

So, minimum energy = – 3J

Number of spin configuration = 2.

41. (A)



For one dimensional infinite square well potential.

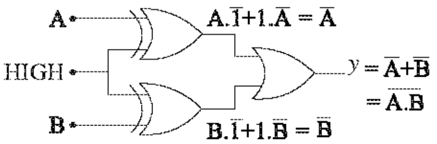
$$\text{Total energy} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$$= 2 \times \frac{(1)^2 \pi^2 \hbar^2}{2mL^2} + 2 \times \frac{(2)^2 \pi^2 \hbar^2}{2mL^2} + 2 \times \frac{(3)^2 \pi^2 \hbar^2}{2mL^2}$$

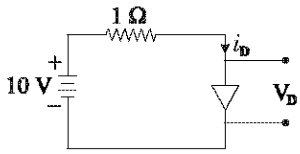
$$= \frac{28 \pi^2 \hbar^2}{2mL^2} = \frac{14 \pi^2 \hbar^2}{mL^2}$$

42. (B) 43. (D)

44. (A)



45. (D)



$$i_D = V_D^2 + 2V_D \quad \text{For } V_D > 0$$
$$= 0 \quad \text{For } V_D \le 0$$

$$\text{Equation} \quad i_D \cdot 1 + V_D = 10$$
$$i_D + V_D = 10$$
$$V_D^2 + 2V_D + V_D = 10$$
$$V_D^2 + 3V_D - 10 = 0$$
$$V_D^2 + 5V_D - 2V_D - 10 = 0$$
$$V_D(V_D + 5) - 2(V_D + 5) = 0$$
$$(V_D - 2)(V_D + 5) = 0$$
$$V_D - 2 = 0$$
$$V_D = 2V$$

46. (B) 47. (A)

48. (A) $\int_c \frac{z^3\, dz}{z^2 - 5z + 6} \quad c : |z| = \frac{5}{2}$

$$\int_c \frac{z^3\, dz}{(z-2)(z-3)}$$

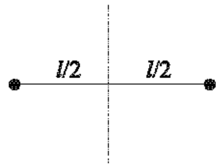
$$\int_c \frac{\left(\frac{z^3}{z-3}\right)}{z-2}\, dz$$

$$2\pi i \frac{\left(\frac{z^3}{z-3}\right)}{z = 2}$$

$$2\pi i \left(\frac{8}{2-3}\right) = -16\pi i$$

49. (C) 50. (D) 51. (A) 52. (C)

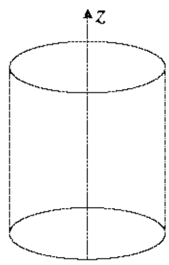
53. (B)



$$\text{Angular momentum} = mvr$$
$$\text{Total number of bodies} = 2$$
$$r = l/2$$
$$L = 2 \times v \times \frac{l}{2} \cdot m$$
$$= 2 \cdot \frac{l}{2} \omega \cdot \frac{l}{2} \cdot m$$
$$= \frac{1}{2} l^2 \omega m$$
$$L = \frac{1}{2} m l^2 \omega$$

54. (C) Inside

$$\int \vec{A} \cdot \vec{dl} = \int (\nabla \times A)\, ds$$
$$A\, 2\pi R = B\, ds$$
$$A\, 2\pi R = \mu_0\, ni\, \pi R^2$$
$$A = \frac{\mu_0\, ni\, \pi R^2}{2\pi R}$$
$$A = \frac{\mu_0\, ni}{2} R$$



$$\therefore \quad \boxed{A \propto R}$$

Outside

$$\int A \cdot dl = \int (\nabla \times A)\, ds$$
$$= B \cdot ds$$
$$A\, 2\pi R = \mu_0\, nI\, \pi r^2$$

where r is radius of solenoid.

$$A = \frac{\mu_0\, nI\, \pi r^2}{2\pi R}$$
$$A = \frac{\mu_0\, nIr^2}{2R}$$
$$A \propto \frac{1}{R}$$

55. (D) $\vec{A} = \frac{\mu_0 K}{4c}(ct - z)^2 \hat{i}$

$$B = \nabla \times A$$

$$B = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\mu_0 K}{4c}(ct-z)^2 & 0 & 0 \end{bmatrix}$$

$$B = -\frac{\mu_0 K}{2c}(ct-z)\hat{j}$$

56. (C) When a charged particle emits electromagnetic radiation

$$E \propto \frac{1}{r}, \quad B \propto \frac{1}{r}$$

$$S = \frac{E \times B}{\mu_0}$$

$$\therefore \quad S \propto \frac{1}{r^2}$$

57. (C)

58. (D) $H^1 = b(a-x) \quad -a < x < a$ otherwise

$= 0$

$$E_n^{(1)} = \int \Psi^* H^1 \Psi \, dx$$

$$= \int_{-a}^{+a} \sqrt{\frac{a}{2a}} \cos\frac{\pi x}{2a}\, b(a-x) \sqrt{\frac{2}{2a}}$$

$$\cos\frac{\pi x}{2a}\ \frac{1}{a} \int_{-a}^{+a} \cos^2\frac{\pi x}{2a}\ (ab-bx)\, dx$$

$$= \frac{1}{a} \int_{-a}^{+a} \cos^2\frac{\pi x}{2a}\ ab\, dx - \frac{b}{a} \int_{-a}^{+a} x \cos^2\frac{\pi x}{2a}\, dx$$

$$= \frac{2ab}{a} \int_0^a \cos^2\frac{\pi x}{2a} - 0$$

$$b \int_0^a \left[1 + \cos\frac{2\pi x}{2a}\right] dx$$

$$b\left[x + \sin\frac{\pi x}{a}\right]_0^a$$

$= ab$

59. (A) $\Delta x = \sqrt{<x^2> - <x>^2}$

$$\Psi = \frac{1}{\sqrt{2}} |0> + |1>$$

$$<x> = \frac{1}{2} <0| + <1| x |0> + |1>$$

$$= \frac{1}{2}\sqrt{\frac{\hbar}{2m\omega}} <0| + <1| a + a^+ |0> + |1>$$

$$<x> = \frac{1}{2}\sqrt{\frac{\hbar}{2m\omega}} \big[<0|a|0> + <0|a|1> + <1|a|0> + <1|a|1> + <0|a^+|0> + <0|a^+|1> + <1|a^+|0> + <1|a^+|1>\big]$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \cdot \frac{1}{2}\ [1+1]$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \cdot \frac{1}{2} \times 2$$

$$= \sqrt{\frac{\hbar}{2m\omega}}$$

$$<x>^2 = \frac{\hbar}{2m\omega}$$

Now,

$$<x^2> = \frac{1}{2}\frac{\hbar}{m\omega} <0| + <1| (a+a^+)^2 |0> + |1>$$

$$= \frac{\hbar}{2m\omega} <0| + <1| a^2 + a^{+2} + aa^+ + a^+ a |0> + |1>$$

$$= \frac{\hbar}{2m\omega} <0| + <1| a^2 + a^{+2} + (2N+1) |0> + |1>$$

$$= \frac{\hbar}{2m\omega} \big[<0| + <1| a^2 + a^{+2} |0> + |1> + <0| + <1| 2N+1 |0> + |1>\big]$$

$$= \frac{\hbar}{2m\omega}[0+2] = \frac{\hbar}{m\omega}$$

$$\Delta x = \sqrt{<x^2> - <x>^2}$$

$$= \sqrt{\frac{\hbar}{m\omega} - \frac{\hbar}{2m\omega}} = \sqrt{\frac{\hbar}{2m\omega}}$$

60. (C) $\Psi(x) = Ae^{-bx^2}$

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx = 1$$

$$|A|^2 \int_{-\infty}^{\infty} e^{-2bx^2} = 1$$

$$|A|^2 \sqrt{\frac{\pi}{2b}} = 1$$

$$A = \left(\frac{2b}{\pi}\right)^{\frac{1}{4}}$$

$$\Psi = \left(\frac{2b}{\pi}\right)^{\frac{1}{4}} e^{-bx^2}$$

$$<H> = <T> + <V>$$

$$<T> = \left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} e^{-bx^2} \left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\right) e^{-bx^2} dx$$

$$= \frac{\hbar^2 b}{2m}$$

$$<V> = \left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} e^{-bx^2} \left(-\alpha\delta(x)\right) e^{-bx^2} dx$$

$$= -\left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \alpha$$

$$<H> = \frac{\hbar^2 b}{2m} - \left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \alpha$$

For ground state energy

$$\frac{dH}{db} = 0$$

$$\frac{\hbar^2}{2m} - \frac{1}{2}\frac{\sqrt{2}}{\sqrt{\pi}}\frac{1}{\sqrt{b}}\alpha = 0$$

$$b = \frac{2m^2\alpha^2}{\pi\hbar^4}$$

So, at $b = \frac{2m^2\alpha^2}{\pi\hbar^4}$

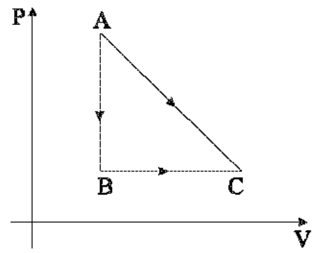
$$\text{Ground state energy} = \frac{\hbar^2}{2m}\frac{2m^2\alpha^2}{\pi\hbar^4} - \left(2\cdot\frac{2m^2\alpha^2}{\pi\hbar^4}\right)^{1/2}\alpha$$

$$<H>_{\text{ground state}} = \frac{-m\alpha^2}{\pi\hbar^2}$$

61. (B)

62. (B) $\delta Q = du + \delta w$

In the process A → C



$$150 = du + 100$$
$$du = 150 - 100$$
$$du = 50$$

Change in enternal energy (du) is state function. It does not depend upon path.

So, for process A → B → C

$$\delta Q = du + \delta w$$
$$\delta Q = 50 + 30$$
$$= 80 \text{ J}$$

63. (D) 64. (B) 65. (B) 66. (D) 67. (B)

68. (A) 69. (C) 70. (C) 71. (D) 72. (A)

73. (D)

74. (C) Proton is stable particle.

Life-time of neutron 16 min.

Life-time of $\pi^0 = 0{\cdot}84 \times 10^{-16}$ sec.

Life-time of $\Delta^+ = 10^{-23}$ sec.

So, in decreasing order life-time

$$p, n, \pi^0, \Delta^+$$

75. (B)

Physical Sciences
CSIR-UGC NET/JRF Exam.
Solved Paper

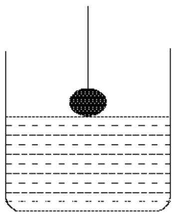
June 2013
Physical Sciences

PART A

1. There is an equilateral triangle in the XY plane with its centre at the origin. The distance of its sides from the origin is 3·5 cm. The area of its circumcircle in cm^2 is—

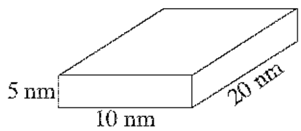
   (A) 38·5 (B) 49
   (C0 63·65 (D) 154

2. A sphere of iron of radius R/2 fixed to one end of a string was lowered into water in a cylindrical container of base radius R to keep exactly half the sphere dipped. The rise in the level of water in the container will be—

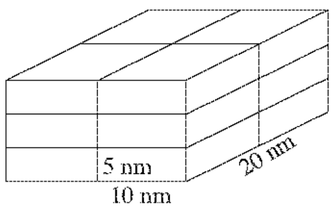


   (A) R/3 (B) R/4
   (C) R/8 (D) R/12

3. A crystal grows by stacking of unit cells of 10 × 20 × 5 nm size as shown in the diagram given below. How many unit cells will make a crystal of 1 cm^3 volume ?



Unit Cell (not to scale)



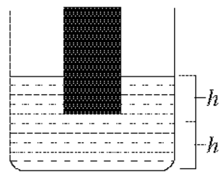
Crystal (not to scale)

   (A) 10^6 (B) 10^9
   (C) 10^{12} (D) 10^{18}

4. What is the value of $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} +$ … to ∞ ?

   (A) 2/3 (B) 1
   (C) 2 (D) ∞

5. A solid cylinder of basal area A was held dipped in water in a cylindrical vessel of basal area 2A vertically such that a length h of the cylinder is immersed. The lower tip of the cylinder is at a height h from the base of the vessel. What will be the height of water in the vessel when the cylinder is taken out ?



   (A) $2h$ (B) $\frac{3}{2}h$
   (C) $\frac{4}{3}h$ (D) $\frac{5}{4}h$

6. Of all the triangles that can be inscribed in a semicircle of radius R with the diameter as one side, the biggest one has the area—

   (A) R^2 (B) $R^2\sqrt{2}$
   (C) $R^2\sqrt{3}$ (D) $2R^2$

7. Choose the largest number—

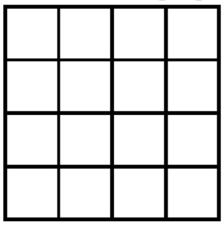
   (A) 2^{500} (B) 3^{400}
   (C) 4^{300} (D) 5^{200}

8. A daily sheet calendar of the year 2013 contains sheets of 10 × 10 cm size. All the sheets of the calendar are spread over the floor of a room of 5m × 7·3 m size. What

percentage of the floor will be covered by these sheets ?

(A) 0·1 (B) 1
(C) 10 (D) 100

9. How many rectangles (which are not squares) are there in the following figure ?



(A) 56 (B) 70
(C) 86 (D) 100

10. Define $a \otimes b$ = lcm (a, b) + gcd (a, b) and $a \oplus b = a^b + b^a$. What is the value of $(1\oplus2) \otimes (3\oplus4)$? Here lcm = least common multiple and gcd = greatest common divisor.

(A) 145 (B) 286
(C) 436 (D) 572

11. A square pyramid is to be made using a wire such that only one strand of wire is used for each edge. What is the minimum number of times that the wire has to be cut in order to make the pyramid ?

(A) 3 (B) 7
(C) 2 (D) 1

12. A crow is flying along a horizontal circle of radius R at a height R above the horizontal ground. Each of a number of men on the ground found that the angular height of the crow was a fixed angle θ (< 45°) when it was closest to him. Then all these men must be on a circle on the ground with a radius—

(A) $R + R \sin\theta$ (B) $R + R \cos\theta$
(C) $R + R \tan\theta$ (D) $R + R \cot\theta$

13. How many pairs of positive integers have gcd 20 and lcm 600 ?

(gcd = greatest common divisor; lcm = least common multiple)

(A) 4 (B) 0
(C) 1 (D) 7

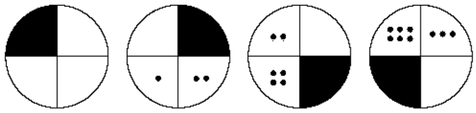
14. During an evening party, when Ms. Black, Ms. Brown and Ms. White met, Ms. Brown remarked, "It is interesting that our dresses are white, black or brown, but for each of us the name does not match the colour of the dress!". Ms. White replied, "But your white dress does not suit you!". Pick the correct answer.

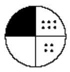
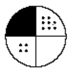
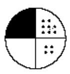
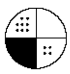
(A) Ms. White's dress was brown.
(B) Ms. Black's dress was white.
(C) Ms. White's dress was black.
(D) Ms. Black's dress was black.

15. Two integers are picked at random from the first 15 positive integers without replacement. What is the probability that the sum of the two numbers is 20 ?

(A) $\frac{3}{4}$ (B) $\frac{1}{21}$
(C) $\frac{1}{105}$ (D) $\frac{1}{20}$

16. Identify the next figure in the sequence—



(A)  (B) 
(C)  (D) 

17. In a customer survey conducted during Monday to Friday, of the customers who asked for child care facilities in super markets, 23% were men and the rest, women. Among them, 19·9% of the women and 8·8% of the men were willing to pay for the facilities.

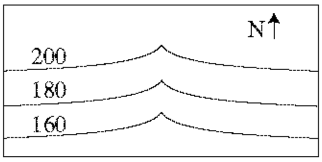
1. What is the ratio of the men to women customers who wanted childcare facilities ?
2. If the survey had been conducted during the weekend instead, how will the result change ?

With the above data,—

(A) Only 1 can be answered
(B) Only 2 can be answered
(C) Both 1 and 2 can be answered
(D) Neither 1 nor 2 can be answered

18. The map given below shows contour lines which connect points of equal ground surface elevation in a region. Inverted 'V' shaped

portions of contour lines represent a valley along which a river flows. What is the downstream direction of the river ?

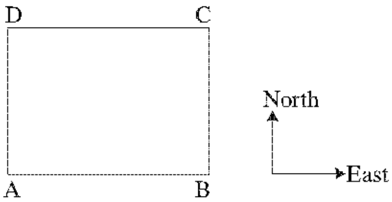


Scale = 1 : 5000

(A) North (B) South
(C) East (D) West

19. During a summer vacation, of 20 friends from a hostel, each wrote a letter to each of all others. The total number of letters written was—
(A) 20 (B) 400
(C) 200 (D) 380

20.



A person has to cross a square field by going from A to C. The person is only allowed to move towards the east or towards the north or use a combination of these movements. The total distance travelled by the person—
(A) Depends on the length of each step
(B) Depends on the total number of steps
(C) Is different for different paths
(D) Is the same for all paths

PART B

21. Two identical bosons of mass m are placed in a one-dimensional potential $V(x) = \frac{3}{2} m\omega^2 x^2$. The bosons interact via a weak potential,

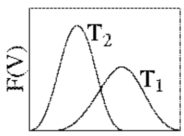
$$V_{12} = V_0 \exp[-m\Omega (x_1 - x_2)^2/4\hbar]$$

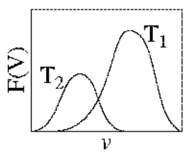
where x_1 and x_2 denote coordinates of the particles. Given that the ground state wavefunction of the harmonic oscillator is $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$. The ground state energy of the two-boson system, to the first order in V_0 is—
(A) $\hbar\omega + 2V_0$
(B) $\hbar\omega + \frac{v_0\Omega}{\omega}$
(C) $\hbar\omega + V_0\left(1 + \frac{\Omega}{2\omega}\right)^{-1/2}$
(D) $\hbar\omega + V_0\left(1 + \frac{\omega}{\Omega}\right)$

22. Ten grams of ice at 0°C is added to a beaker containing 30 grams of water at 25°C. What is the final temperature of the system when it comes to thermal equilibrium ? (The specific heat of water is 1 cal/gm/°C and latent heat of melting of ice is 80 cal/gm).
(A) 0° C (B) 7·5° C
(C) 12·5° C (D) – 1·25° C

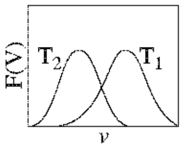
23. A vessel has two compartments of volume V_1 and V_2, containing an ideal gas at pressures p_1 and p_2, and temperatures T_1 and T_2 respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be—
(A) $\frac{p_1 T_1 + p_2 T_2}{p_1 + p_2}$
(B) $\frac{V_1 T_1 + V_2 T_2}{V_1 + V_2}$
(C) $\frac{p_1 T_1 + p_2 V_2}{(p_1 V_1/T_1) + (p_2 V_2/T_2)}$
(D) $(T_1 T_2)^{1/2}$

24. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution $F(v)$ of the speed v of a molecule in three dimensions is correctly represented by the following figure—

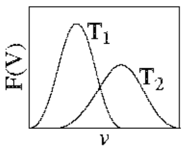
(A)


(B)


(C)



(D)



25. A system of non-interacting spin – 1/2 charged particles are placed in an external magnetic field. At low temperature T, the leading behaviour of the excess energy above the ground state energy, depends on T as—

(c is a constant)

(A) cT

(B) cT^3

(C) $e^{-c/T}$

(D) c (is independent of T)

26. The acceleration due to gravity g is determined by measuring the time period T and the length L, of a simple pendulum. If the uncertainties in the measurements of T and L are ΔT and ΔL respectively, the fractional error $\Delta g/g$ in measuring g is best approximated by—

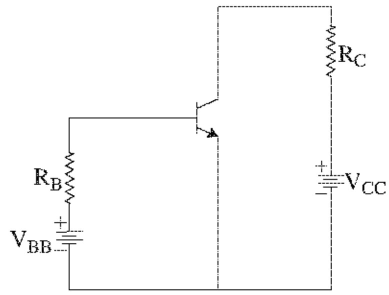
(A) $\frac{|\Delta L|}{L} + \frac{|\Delta T|}{T}$

(B) $\frac{|\Delta L|}{L} + \frac{|2\Delta T|}{T}$

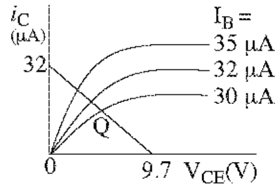
(C) $\sqrt{\frac{|\Delta L|^2}{L} + \frac{|\Delta T|^2}{T}}$

(D) $\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$

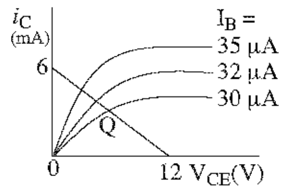
27. A silicon transistor with built-in voltage 0·7 V is used in the circuit shown, with $V_{BB} = 9{\cdot}7$ V, $R_B = 300\ k\Omega$, $V_{CC} = 12$ V and $R_C = 2\ k\Omega$. Which of the following figures correctly represents the load line and the quiescent Q point ?



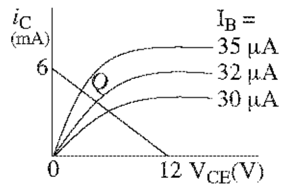
(A)



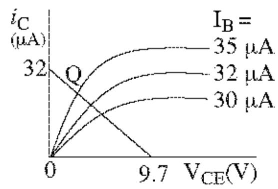
(B)



(C)



(D)



28. If the analog input to an 8-bit successive approximation ADC is increased from 1·0 V to 2·0 V, then the conversion time will—

(A) remain unchanged

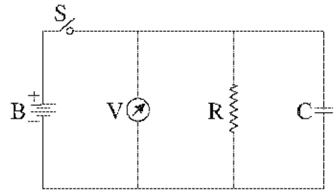
(B) double

(C) decrease to half its original value

(D) increase four times

29. The insulation resistance R of an insulated cable is measured by connecting it in parallel with a capacitor C, a voltmeter, and battery B as shown. The voltage across the cable dropped form 150 V to 15 V, 1000 seconds after the switch S is closed. If the capacitance

of the cable is 5 μF, then its insulation resistance is approximately—

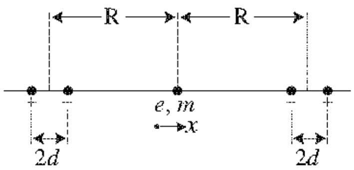


(A) $10^9\ \Omega$ (B) $10^8\ \Omega$
(C) $10^7\ \Omega$ (D) $10^6\ \Omega$

30. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than—
(take $180°/\pi \simeq 57·29°$)
(A) 1·28° (B) 1·81°
(C) 3·28° (D) 4·01°

31. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vacuum)—
(A) $\left(1-\frac{u^2}{c^2}\right)^{1/2}$ (B) $\left(1-\frac{u^2}{c^2}\right)^{-1/2}$
(C) $\left(1-\frac{u^2}{c^2}\right)$ (D) $\left(1-\frac{u^2}{c^2}\right)^{-1}$

32. A particle of charge e and mass m is located at the midpoint of the line joining two fixed collinear dipoles with unit charges as shown in the figure. (The particle is constrained to move only along the line joining the dipoles.) Assuming that the length of the dipoles is much shorter than their separation, the natural frequency of oscillation of the particle is—



(A) $\sqrt{\frac{6e^2R^2}{\pi\varepsilon_0 md^5}}$ (B) $\sqrt{\frac{6e^2R}{\pi\varepsilon_0 md^4}}$
(C) $\sqrt{\frac{6e^2d^2}{\pi\varepsilon_0 mR^5}}$ (D) $\sqrt{\frac{6e^2d}{\pi\varepsilon_0 mR^4}}$

33. A current I is created by a narrow beam of protons moving in vacuum with constant velocity $\vec{u}$. The direction and magnitude, respectively, of the Poynting vector $\vec{S}$ outside the beam at a radial distance r (much larger than the width of the beam) from the axis, are—
(A) $\vec{S} \perp \vec{u}$ and $|\vec{S}| = \frac{I^2}{4\pi^2\varepsilon_0 |\vec{u}| r^2}$
(B) $\vec{S} \parallel (-\vec{u})$ and $|\vec{S}| = \frac{I^2}{4\pi^2\varepsilon_0 |\vec{u}| r^4}$
(C) $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \frac{I^2}{4\pi^2\varepsilon_0 |\vec{u}| r^2}$
(D) $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \frac{I^2}{4\pi^2\varepsilon_0 |\vec{u}| r^4}$

34. If the electric and magnetic fields are unchanged when the vector potential $\vec{A}$ changes (in suitable units) according to $\vec{A} \to \vec{A} + \hat{r}$, where $\hat{r} = r(t)\,\hat{r}$, then the scalar potential Φ must simultaneously change to—
(A) $\Phi - r$ (B) $\Phi + r$
(C) $\Phi - \partial r/\partial t$ (D) $\Phi + \partial r/\partial t$

35. Consider an axially symmetric static charge distribution of the form,
$$\rho = \rho_0 \left(\frac{r_0}{r}\right)^2 e^{-r/r_0}\cos^2\phi$$
The radial component of the dipole moment due to this charge distribution is—
(A) $2\pi\rho_0 r_0^4$ (B) $\pi\rho_0 r_0^4$
(C) $\rho_0 r_0^4$ (D) $\pi\rho_0 r_0^4/2$

36. In a basis in which the z-component S_z of the spin is diagonal, an electron is in a spin state
$$\psi = \begin{pmatrix} (1+i)/\sqrt{6} \\ \sqrt{2/3} \end{pmatrix}$$
The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively,—
(A) 1/2 and 1/2 (B) 2/3 and 1/3
(C) 1/4 and 3/4 (D) 1/3 and 2/3

37. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator—
$$|\psi\rangle = b_1|0\rangle + b_2|1\rangle$$
where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the

displacement x in the state $|\psi\rangle$ will be a minimum when—

(A) $b_2 = 0, (b_1 = 1)$ (B) $b_2 = \frac{1}{\sqrt{2}} b_1$

(C) $b_2 = \frac{1}{2} b_1$ (D) $b_2 = b_1$

38. A muon (μ^-) from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be—

(A) 5·62 Å (B) 6·67 Å

(C) 3·75 Å (D) 13·3 Å

39. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by

$$\psi(\vec{r}) = z f(r)$$

where $f(r)$ is a function of the radial variable r. The eigenvalue of the operator $\vec{L}^2$ (namely the square of the orbital angular momentum) is—

(A) $\hbar^2/4$ (B) $\hbar^2/2$

(C) $\hbar^2$ (D) $2\hbar^2$

40. Given that

$$\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = e^{-t^2 + 2tx}$$

the value of $H_4(0)$ is—

(A) 12 (B) 6

(C) 24 (D) – 6

41. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120° with respect to $\hat{i}$. The angle between the vectors $\vec{u} = a\hat{i} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{i}$ will be 60° if—

(A) $b = \sqrt{3}a/2$ (B) $b = 2a/\sqrt{3}$

(C) $b = a/2$ (D) $b = a$

42. With $z = x + iy$, which of the following functions $f(x, y)$ is not a (complex) analytic function of z ?

(A) $f(x, y) = (x + iy - 8)^3 (4 + x^2 - y^2 + 2ixy)^7$

(B) $f(x, y) = (x + iy)^7 (1 - x - iy)^3$

(C) $f(x, y) = (x^2 - y^2 + 2\,ixy - 3)^5$

(D) $f(x, y) = (1 - x + iy)^4 (2 + x + iy)^6$

43. A planet of mass m and an angular momentum L moves in a circular orbit in a potential, V $(r) = -k/r$, where k is a constant. If it is slightly perturbed radially, the angular frequency of radial oscillations is—

(A) $mk^2/\sqrt{2}L^3$ (B) mk^2/L^3

(C) $\sqrt{2}mk^2/L^3$ (D) $\sqrt{3}mk^2/L^3$

44. The Lagrangian of a particle of mass m moving in one dimension is given by—

$$L = \frac{1}{2} m\dot{x}^2 - bx$$

where b is a positive constant. The coordinate of the particle $x(t)$ at time t is given by : (in the following c_1 and c_2 are constants)—

(A) $-\frac{b}{2m} t^2 + c_1 t + c_2$

(B) $c_1 t + c_2$

(C) $c_1 \cos\left(\frac{bt}{m}\right) + c_2 \sin\left(\frac{bt}{m}\right)$

(D) $c_1 \cosh\left(\frac{bt}{m}\right) + c_2 \sinh\left(\frac{bt}{m}\right)$

45. A uniform cylinder of radius r and length l, and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is—

(A) $V\sqrt{\frac{14rl}{15R^2}}$ (B) $4V\sqrt{\frac{r}{15R}}$

(C) $\frac{4V}{\sqrt{15}}$ (D) $V\sqrt{\frac{14}{15}}$

PART C

46. The components of a vector potential

$$A_\mu \equiv (A_0, A_1, A_2, A_3)$$

are given by

$$A_\mu = k(-xyz, yzt, zxt, xyt)$$

where k is a constant. The three components of the electric field are—

(A) $k(yz, zx, xy)$ (B) $k(x, y, z)$

(C) $(0, 0, 0)$ (D) $k(xt, yt, zt)$

47. In the Born approximation, the scattering amplitude $f(\theta)$ for the Yukawa potential

$$V(r) = \frac{\beta e^{-\mu r}}{r}$$

is given by—

$\left(\text{in the following } b = 2k \sin \frac{\theta}{2}, E = h^2k^2/2m\right)$

(A) $-\frac{2m\beta}{h^2(\mu^2 + b^2)^2}$ (B) $-\frac{2m\beta}{h^2(\mu^2 + b^2)}$

(C) $-\frac{2m\beta}{h^2\sqrt{(\mu^2 + b^2)}}$ (D) $-\frac{2m\beta}{h^2(\mu^2 + b^2)^3}$

48. If ψ_{nlm} denotes the eigenfunction of the Hamiltonian with a potential V = V (r) then the expectation value of the operator $L_x^2 + L_y^2$ in the state

$$\psi = \frac{1}{5}\left[3\psi_{211} + \psi_{210} - \sqrt{15}\,\psi_{21-1}\right]$$

is—

(A) $39h^2/25$ (B) $13h^2/25$

(C) $2\,h^2$ (D) $26h^2/25$

49. An oscillating current $I(t) = I_0 \exp(-i\omega t)$ flows in the direction of the y-axis through a thin metal sheet of area 1·0 cm^2 kept in the xy-plane. The rate of total energy radiated per unit area from the surfaces of the metal sheet at a distance of 100 m is—

(A) $I_0\omega/(12\pi\varepsilon_0c^3)$ (B) $I_0^2\,\omega^2/(12\pi\varepsilon_0c^3)$

(C) $I_0^2\omega/(12\pi\varepsilon_0c^3)$ (D) $I_0\omega^2/(24\pi\varepsilon_0c^3)$

50. Consider a two-dimensional infinite square well

$$V(x, y) = \begin{cases} 0 & 0 < x < a, \quad 0 < y < a \\ \infty & \text{otherwise} \end{cases}$$

Its normalized eigenfunctions are

$$\psi_{n_x, n_y}(x, y) = \frac{2}{a}\sin\left(\frac{n_x\pi x}{a}\right)\sin\left(\frac{n_y\pi y}{a}\right)$$

where $n_x, n_y = 1, 2, 3, \ldots$. If a perturbation—

$$H' = \begin{cases} V_0 & 0 < x < \frac{a}{2}, \quad 0 < y < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state to order V_0 is—

(A) $\frac{V_0}{4}$ (B) $\frac{V_0}{4}\left[1 \pm \frac{64}{9\pi^2}\right]$

(C) $\frac{V_0}{4}\left[1 \pm \frac{16}{9\pi^2}\right]$ (D) $\frac{V_0}{4}\left[1 \pm \frac{32}{9\pi^2}\right]$

51. Consider a system of two Ising spins S_1 and S_2 taking values ± 1 with interaction energy given by $\varepsilon = -J\,S_1S_2$, when it is in thermal equilibrium at temperature T. For large T, the average energy of the system varies as C/k_BT, with C given by—

(A) $-2J^2$ (B) $-J^2$

(C) J^2 (D) 4J

52. Consider three particles A, B and C, each with an attribute S that can take two values ± 1. Let $S_A = 1$, $S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to – S with probability 1/3. The probability that $S_A + S_B + S_C$ remains unchanged is—

(A) 2/3 (B) 1/3

(C) 2/9 (D) 4/9

53. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H = -\frac{h^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

using the variational principle with the trial wavefunction $\psi(x) = A\exp(-bx^2)$ is—

$\left[\text{Note} : \int_0^\infty e^{-t}t^a\,dt = \Gamma(a + 1)\right]$

(A) $-m\alpha^2/4\pi\hbar^2$ (B) $-m\alpha^2/2\pi\hbar^2$

(C) $-m\alpha^2/\pi\hbar^2$ (D) $-m\alpha^2/\sqrt{5}\pi\hbar^2$

54. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies ε_0, $3\varepsilon_0$ and $5\varepsilon_0$, ($\varepsilon_0 > 0$). One system is populated by spin– $\frac{1}{2}$ fermions and the other by bosons. What is the value of $E_F - E_B$. Where E_F and E_B are the ground state energies of the fermionic and bosonic systems respectively ?

(A) $6\varepsilon_0$ (B) $2\varepsilon_0$

(C) $4\varepsilon_0$ (D) ε_0

55. The input to a lock-in amplifier has the form $V_i(t) = V_i \sin(\omega t + \theta_i)$ where V_i, ω, θ_i are the amplitude, frequency and phase of the input signal respectively. This signal is multiplied by a reference signal of the same frequency ω, amplitude V_r and phase θ_r. If the multiplied signal is fed to a low pass filter of

cut-off frequency ω, then the final output signal is—

(A) $\frac{1}{2} V_i V_r \cos(\theta_i - \theta_r)$

(B) $V_i V_r \left[\cos(\theta_i - \theta_r) - \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)\right]$

(C) $V_i V_r \sin(\theta_i - \theta_r)$

(D) $V_i V_r \left[\cos(\theta_i - \theta_r) + \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)\right]$

56. The solution of the partial differential equation

$$\frac{\partial^2}{\partial t^2} u(x, t) - \frac{\partial^2}{\partial x^2} u(x, t) = 0$$

satisfying the boundary conditions $u(0, t) = 0 = u(L, t)$ and initial conditions $u(x, 0) = \sin(\pi x/L)$ and $\frac{\partial}{\partial t} u(x, t)\big|_{t=0} = \sin(2\pi x/L)$ is—

(A) $\sin(\pi x/L)\cos(\pi t/L) + \frac{L}{2\pi}\sin(2\pi x/L)\cos(2\pi t/L)$

(B) $2\sin(\pi x/L)\cos(\pi t/L) - \sin(\pi x/L)\cos(2\pi t/L)$

(C) $\sin(\pi x/L)\cos(2\pi t/L) + \frac{L}{\pi}\sin(2\pi x/L)\sin(\pi t/L)$

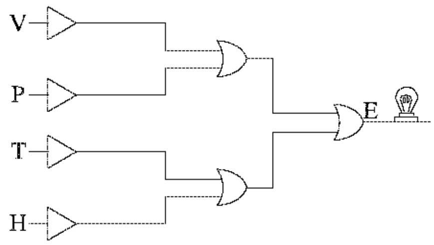
(D) $\sin(\pi x/L)\cos(\pi t/L) + \frac{L}{2\pi}\sin(2\pi x/L)\sin(2\pi t/L)$

57. Consider the hydrogen-deuterium molecule HD. If the mean distance between the two atoms is 0·08 nm and the mass of the hydrogen atom is 938 MeV/c^2, then the energy difference ΔE between the two lowest rotational states is approximately—

(A) 10^{-1} eV (B) 10^{-2} eV

(C) 2×10^{-2} eV (D) 10^{-3} eV

58. Four digital outputs V, P, T and H monitor the speed v, tyre pressure p, temperature t and relative humidity h of a car. These outputs switch from 0 to 1, when the values of the parameters exceed 85 km/hr, 2 bar, 40° C and 50%, respectively. A logic circuit that is used to switch ON a lamp at the output E is shown ahead.



Which of the following conditions will switch the lamp ON ?

(A) $v < 85$ km/hr, $p < 2$ bar, $t > 40°$ C, $h > 50\%$

(B) $v < 85$ km/hr, $p < 2$ bar, $t > 40°$ C, $h < 50\%$

(C) $v > 85$ km/hr, $p < 2$ bar, $t > 40°$ C, $h < 50\%$

(D) $v > 85$ km/hr, $p < 2$ bar, $t < 40°$ C, $h > 50\%$

59. The solution of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition $x(0) = 1$ will blow up as t tends to—

(A) 1 (B) 2

(C) 1/2 (D) ∞

60. Let u be a random variable uniformly distributed in the interval [0, 1] and $V = -c \ln u$, where c is a real constant. If V is to be exponentially distributed in the interval [0, ∞] with unit standard deviation, then the value of c should be—

(A) ln 2 (B) 1/2

(C) 1 (D) – 1

61. The inverse Laplace transform of $\frac{1}{s^2(s+1)}$ is—

(A) $\frac{1}{2}t^2 e^{-t}$ (B) $\frac{1}{2}t^2 + 1 - e^{-t}$

(C) $t - 1 + e^{-t}$ (D) $\frac{1}{2}t^2(1 - e^{-t})$

62. The number of degrees of freedom of a rigid body in d space-dimensions is—

(A) $2d$ (B) 6

(C) $d(d+1)/2$ (D) $d\,!$

63. A particle of mass m is at the stable equilibrium position of its potential energy

$$V(x) = ax - bx^3$$

where a, b are positive constants. The minimum velocity that has to be imparted to the particle to render its motion unstable is—

(A) $(64a^3/9m^2b)^{1/4}$ (B) $(64a^3/27m^2b)^{1/4}$

(C) $(16a^3/27m^2b)^{1/4}$ (D) $(3a^3/64m^2b)^{1/4}$

64. If the operators A and B satisfy the commutation relation [A, B] = I, where I is identity operator, then—

(A) $[e^A, B] = e^A$ (B) $[e^A, B] = [e^B, A]$

(C) $[e^A, B] = [e^{-B}, A]$ (D) $[e^A, B] = I$

65. A system is governed by the Hamiltonian

$$H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$$

where a and b are constants and p_x, p_y are momenta conjugate to x and y respectively.

For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved ?

(A) $a = -3, b = 2$ (B) $a = 3, b = -2$

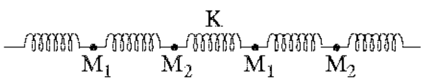
(C) $a = 2, b = -3$ (D) $a = -2, b = 3$

66. Using the frequency–dependent Drude formula, what is the effective kinetic inductance of a metallic wire that is to be used as a transmission line ? [In the following, the electron mass is m, density of electrons is n, and the length and cross-sectional area of the wire are l and A respectively.]

(A) $mA/(ne^2l)$ (B) Zero

(C) $ml/(ne^2A)$ (D) $m\sqrt{A}/(ne^2l^2)$

67. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



is given by

$$\omega^2(q) = K\left(\frac{1}{M_1} + \frac{1}{M_2}\right)\left[1 \pm \sqrt{1 - \frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

where a is the lattice parameter and K is the spring constant. The velocity of sound is—

(A) $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}}\,a$

(B) $\sqrt{\frac{K}{2(M_1+M_2)}}\,a$

(C) $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}}\,a$

(D) $\sqrt{\frac{KM_1M_1}{2(M_1+M_2)^3}}\,a$

68. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A, Z) \approx 16A - 20A^{2/3} - \frac{3}{4}Z^2A^{-1/3} + 30\frac{(N-Z)^2}{A}$$

where N = A – Z is the neutron number. The value of Z of the most stable isobar for a given A is—

(A) $\frac{A}{2}\left(1 - \frac{A^{2/3}}{160}\right)^{-1}$ (B) $\frac{A}{2}$

(C) $\frac{A}{2}\left(1 - \frac{A^{2/3}}{120}\right)^{-1}$ (D) $\frac{A}{2}\left(1 + \frac{A^{4/3}}{64}\right)$

69. Muons are produced through the annihilation of particle a and its antiparticle, namely the process

$$a + \bar{a} \rightarrow \mu^+ + \mu^-$$

A muon has a rest mass of 105 MeV/c^2 and its proper life time is 2 μs. If the centre of mass energy of the collision is 2·1 GeV in the laboratory frame that coincides with the centerofmass frame, then the fraction of muons that will decay before they reach a detector placed 6 km away from the interaction point is—

(A) e^{-1} (B) $1 - e^{-1}$

(C) $1 - e^{-2}$ (D) e^{-10}

70. The conductors in a 0·75 km long two-wire transmission line are separated by a centre-to-centre distance of 0·2 m. If each conductor has a diameter of 4 cm, then the capacitance of the line is—

(A) 8·85 μF (B) 88·5 nF

(C) 8·85 pF (D) 8·85 nF

71. The electron dispersion relation for a one-dimensional metal is given by—

$$\varepsilon_k = 2\varepsilon_0\left[\sin^2\frac{ka}{2} - \frac{1}{6}\sin^2 ka\right]$$

where k is the momentum, a is the lattice constant, ε_0 is a constant having dimensions of energy and $|ka| \le \pi$. If the average number of electrons per atom in the conduction band is 1/3, then the Fermi energy is—

(A) $\varepsilon_0/4$ (B) ε_0

(C) $2\varepsilon_0/3$ (D) $5\varepsilon_0/3$

72. The electronic energy levels in a hydrogen atom are given by $E_n = -13{\cdot}6/n^2$ eV. If a selective excitation to the $n = 100$ level is to be made using a laser, the maximum allowed frequency linewidth of the laser is—

(A) 6·5 MHz (B) 6·5 GHz

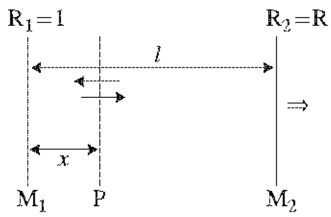
(C) 6·5 Hz (D) 6·5 kHz

73. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states D(E) depends on the energy as—

(A) $1/\sqrt{E}$ (B) $\sqrt{E}$

(C) E (D) Constant

74. Consider the laser resonator cavity shown in the figure.



If I_1 is the intensity of the radiation at mirror M_1 and α is the gain coefficient of the medium between the mirrors, then the energy density of photons in the plane P at a distance x from M_1 is—

(A) $(I_1/c)\, e^{-\alpha x}$

(B) $(I_1/c)\, e^{\alpha x}$

(C) $(I_1/c)\,(e^{\alpha x} + e^{-\alpha x})$

(D) $(I_1/c)\, e^{2\alpha x}$

75. A spin-$\frac{1}{2}$ particle A undergoes the decay

$$A \rightarrow B + C + D$$

where it is known that B and C are also spin-$\frac{1}{2}$ particles. The complete set of allowed values of the spin of the particle D is—

(A) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$

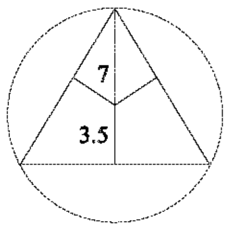
(B) 0, 1

(C) $\frac{1}{2}$ only

(D) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

Answers with Explanations

1. (D)



The area of circumcircle

$$\pi(7)^2 = 154$$

2. (D) Volume of half the sphere

$$\frac{2}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{2}{3}\pi\frac{R^3}{8}$$

$$= \frac{1}{12}\pi R^3$$

Volume of rise water $= \pi R^2 h$

where h is rise in the level of water $\frac{1}{12}\pi R^3$

$$= \pi R^2 h$$

$$h = \frac{R}{12}$$

3. (D)

4. (B) $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots\ldots\ldots\infty$

$$\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \ldots\ldots\ldots$$

$= 1$

5. (B) Volume of rise water due to cylinder

$$2Ah - Ah = Ah$$

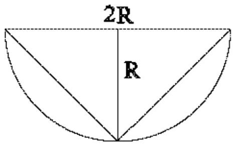
after removing the cylinder, height of water is h_1

$$2Ah_1 = 2Ah + Ah$$

$$Ah_1 = \frac{3}{2}Ah$$

$$h_1 = \frac{3}{2}h$$

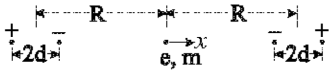
6. (A)



$$\text{Area} = \frac{1}{2} \times 2R \times R$$

$$\text{Area} = R^2$$

7. (B)
8. (C) Area of total sheet in the year 2013
$10 \times 10 \times 365 \text{ cm}^2$
Percentage of floor covered by these sheets
$$\frac{10 \times 10 \times 365}{500 \times 730} \times 100 = 10$$
9. (B)
10. (C) $(1 \oplus 2) \otimes (3 \oplus 4)$
$(1^2 + 2^1) \otimes (3^4 + 4^3)$
$3 \otimes 145$
L.C.M. (3, 145) + G.C.D. (3, 145)
$435 + 1 = 436$
11. (D)
12. (D) $\tan\theta = \frac{R}{x}$
$$x = \frac{R}{\tan\theta} = R\cot\theta$$
So radius $R + x = R + R\cot\theta$
13. (A)
14. (C) Ms. Brown → White
Ms. Black → Brown
Ms. White → Black
15. (B) 16. (C) 17. (A) 18. (B)
19. (D) Each friend wrote 19 letters
Total number of letters $20 \times 19 = 380$
20. (D) Person will be move by path ABC or ADC, both paths have same distance.
21. (C)
22. (A) Heat required to melt 10 gram of ice at 0°C,
$$Q = mL \text{ (latent heat)}$$
$$Q = 10 \times 80 = 800 \text{ Joule}$$
heat loss by 30 gram of water at 25°C to 0°C
$$= mC\,\Delta\theta$$
$$= 30 \times 1 \times (25 - 0)$$
$$= 750 \text{ Joule}$$
heat loss by water at 25°C to cold 0°C
$$= 750 \text{ Joule}$$
but heat required by ice to melt 800 Joule.
So, finally ice will not completely melt.
Final temperature of system when it comes to thermal equilibrium will be 0°C.
23. (C) 24. (A) 25. (C)
26. (D) $T = 2\pi\sqrt{\frac{l}{g}}$
$$T^2 = 4\pi^2\frac{l}{g},$$
So, $g = 4\pi^2\frac{l}{T^2}$
$$g = 4\pi^2\frac{l}{T^2}$$
$$dg = \frac{4\pi^2\,dl}{T^2} + \frac{4\pi^2\,l\,(-2)}{T^3}$$
$$\frac{dg}{g} = \frac{\frac{4\pi^2\,dl}{T^2}}{4\pi^2\frac{l}{T^2}} + \frac{\frac{4\pi^2\,l\,(-2)dT}{T^3}}{4\pi^2\frac{l}{T^2}}$$
$$\frac{dg}{g} = \frac{dl}{l} - \frac{2\,dT}{T}$$
The fractional error R.M.S. $\frac{\Delta g}{g}$ is
$$\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$$
27. (B) 28. (A) 29. (B) 30. (B)
31. (A) In the rest frame 'S' disc looks like disc. and its area is πa^2, where a is radius. But according to moving frame it looks like ellipse and its area πab
where $b = a.\sqrt{1 - \frac{u^2}{c^2}}$
$$\pi a^2\sqrt{1 - \frac{u^2}{c^2}}$$
$$\frac{\text{Area of disc according to Observer 'O'}}{\text{Area of disc according to Observer 'S'}} :$$
$$\frac{\pi a^2\sqrt{1 - \frac{u^2}{c^2}}}{\pi a^2} = \sqrt{1 - \frac{u^2}{c^2}}$$
Area of disc according to Observer 'O'
$$= \sqrt{1 - \frac{u^2}{c^2}}$$

32. (D) 

After the charge displacing x, force acting on charge particle is

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{e^2}{(R-d-x)^2} + \frac{e^2}{(R+d+x)^2} - \frac{e^2}{(R+d-x)^2} - \frac{e^2}{(R+d+x)^2}\right]$$

$$ma = \frac{6e^2d}{\pi\varepsilon_0 R^4}(-x)$$

$$a = \frac{6e^2d}{\pi\varepsilon_0 mR^4}(-x)$$

$$a \propto (-x)$$

So, it performs S.H.M.

$$\omega^2 = \frac{6e^2d}{\pi\varepsilon_0 mR^4}$$

$$\omega = \sqrt{\frac{6e^2d}{\pi\varepsilon_0 mR^4}}$$

33. (C) $E \propto \frac{1}{r}$ $\qquad B \propto \frac{1}{r}$

$S = \frac{E \times B}{\mu_0}$ $\qquad S \propto \frac{1}{r^4}$

Direction of wave is same as Direction of poynting vector. So option (C) is correct.

34. (C)

35. (A) $\rho = rq$ $\qquad$ For radial

$$P = \int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} r\ \rho_0\left(\frac{r_0}{r}\right)^2 e^{-r/r_0} \cos^2\phi\ r^2 \sin\theta\ dr d\theta\ d\phi$$

$$= 2\pi\ \rho_0\ r_0^4$$

36. (D) $$\Psi = \begin{pmatrix} \frac{1+i}{\sqrt{6}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$$

The probabilities that a measurement of S_z will yield the value $\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ are

$$\left|\frac{1+i}{\sqrt{6}}\right|^2 = \frac{1}{3}$$

$$\left|\sqrt{\frac{2}{3}}\right|^2 = \frac{2}{3}$$

37. (D) $|\Psi\rangle = b_1|0\rangle + b_2|1\rangle$

$$|b_1|^2 + |b_2|^2 = 1$$

$$\langle\Psi|x|\Psi\rangle = b_1^2\langle 0|x|0\rangle + b_2^2\langle 1|x|1\rangle + b_1 b_2\langle 0|x|1\rangle + b_1 b_2\langle 1|x|0\rangle$$

$$= b_1^2 + b_2^2 + 2b_1b_2\langle 0|x|1\rangle$$

$$= b_1^2 + b_2^2 + 2b_1b_2\sqrt{\frac{\hbar}{2m\omega}}\langle 0|a + a^{+}|1\rangle$$

$$= b_1^2 + b_2^2 + 2b_1b_2\sqrt{\frac{\hbar}{2m\omega}}$$

It will be minimum when $b_1 = b_2$.

38. (B)

39. (D) $L^2 = l(l+1)\hbar^2$

if $l = 0$ $\qquad L^2 = 0$

if $l = 1$ $\qquad L^2 = 2\hbar^2$

40. (A)

41. (C) $\vec{u}\cdot\vec{v} = (a\hat{i} + b\hat{n}).(a\hat{n} + b\hat{i})$

$$\vec{u}\cdot\vec{v} = a^2(\hat{i}.\hat{n}) + ab + ba + b^2(\hat{n}.\hat{i})$$

$$(\sqrt{a^2 + b^2 + 2ab\cos 120})^2 \cos 60 = a^2\cos 120 + 2ab + b^2\cos 120$$

$$(a^2 + b^2 - ab).\frac{1}{2} = -\frac{a^2}{2} + 2ab - \frac{b^2}{2}$$

$$a^2 + b^2 = 2ab + \frac{1}{2}ab$$

$$a^2 + b^2 = \frac{5ab}{2}$$

$$b = \frac{a}{2}$$

42. (D) For a analytic function C.R. equation should be satisfied but in option (D) C.R. equation is not satisfied.

43. (B) Radial oscillation is

$$\omega = \sqrt{\frac{V''(r)}{m}}$$

$$V_{eef} = \frac{L^2}{2mr^2} - \frac{k}{r}$$

$$V'_{eef} = \frac{-2L^2}{2mr^3} + \frac{k}{r^2} = 0$$

$$\frac{L^2}{mr^3} = \frac{k}{r^2}$$

$$\frac{L^2}{mK} = r$$

$$V''_{eef} = \frac{+3L^2}{mr^4} - \frac{2k}{r^3}$$

$$V''_{eef} = \frac{3L^2}{m\left(\frac{L^2}{mk}\right)^4} - \frac{2k}{\left(\frac{L^2}{mk}\right)^3}$$

$$= \frac{3L^2 \, . \, m^4 \, k^4}{m \, L^8} - \frac{2k \, m^3 \, k^4}{L^6}$$

$$= \frac{3m^3 \, k^4}{L^6} - \frac{2m^3 \, k^3}{L^6}$$

$$= \frac{m^3 \, k^4}{L^6}$$

$$\omega_1 = \sqrt{\frac{V''_{eef}}{m}} = \sqrt{\frac{m^3 \, k^4}{L^6 \, m}}$$

$$= \frac{mk^2}{L^3}$$

44. (A) $$L = \frac{1}{2} m\dot{x}^2 - bx$$

Equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$\frac{\partial L}{\partial x} = -b$$

$$m\ddot{x} + b = 0$$

$$\ddot{x} = -\frac{b}{m}$$

$$\frac{d^2x}{dt^2} = -\frac{b}{m}$$

$$\frac{dx}{dt} = -\frac{b}{m} t + c_1$$

$$dx = -\frac{b}{m} tdt + c_1 dt$$

$$x = -\frac{b}{2m} t^2 + c_1 t + c_2$$

45. (D) 46. (C) 47. (B)

48. (D) $$L_x^2 + L_y^2 + L_z^2 = L^2$$

$$L_x^2 + L_y^2 = L^2 - L_z^2$$

$$< \Psi | L^2 - L_z^2 | \Psi >$$

$$\frac{9}{25}\left[1\,(1+1)\hbar^2 - 1\hbar^2\right] + \frac{1}{25}\left[1\,(1+1)\,\hbar^2 - 0\hbar^2\right] + \frac{15}{25}\left[1\,(1+1)\,\hbar^2 - 1\hbar^2\right]$$

$$\frac{9}{25}\hbar^2 + \frac{2}{25}\hbar^2 + \frac{15}{25}\hbar^2$$

$$= \frac{26}{25}\hbar^2$$

49. (B)

50. (B) $$\Psi_{n_x n_y}(x_1\, y) = \frac{2}{a}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{a}\right)$$

$$E_n^{(1)} = \int \Psi_n^* \, H' \, \Psi_n \, dx\, dy$$

$$= \int_0^{a/2}\int_0^{a/2} \left[\frac{2}{a}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{a}\right)\right]^2 V_0 \, dx\, dy$$

For First excited state $n_x = 1, n_y = 2$

$$= \frac{4V_0}{a^2}\int_0^{a/2}\int_0^{a/2}\left(\frac{1-\cos\frac{2\pi x}{a}}{2}\right)\left(\frac{1-\cos\frac{4\pi y}{a}}{2}\right) dx\, dy$$

$$= \frac{4V_0}{4a^2}\int_0^{a/2}\int_0^{a/2}\left(1-\cos\frac{2\pi x}{a}\right)\left(1-\cos\frac{4\pi y}{a}\right) dx\, dy$$

$$= \frac{V_0}{a^2}\left[x + \frac{\sin\frac{2\pi x}{a}}{\frac{2\pi}{a}}\right]_0^{\frac{a}{2}}\left[x + \frac{\sin\frac{4\pi x}{a}}{\frac{4\pi}{a}}\right]_0^{\frac{a}{2}}$$

$$= \frac{V_0}{4}\left[1 \pm \frac{64}{9\pi^2}\right]$$

51. (B) 52. (D)

53. (C)

$$\Psi(x) = A\, e^{-bx^2}$$

$$\int_{-\infty}^{+\infty} \Psi^* \Psi\, dx = 1$$

$$|A|^2 \int_{-\infty}^{+\infty} e^{-2bx^2} = 1$$

$$|A|^2 \sqrt{\frac{\pi}{2b}} = 1$$

$$A = \left(\frac{2b}{\pi}\right)^{\frac{1}{4}}$$

$$\Psi = \left(\frac{2b}{\pi}\right)^{\frac{1}{4}} e^{-bx^2}$$

$$< H > = < T > + < V >$$

$$< T > = \left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{+\infty} e^{-bx^2} \left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\right) e^{-bx^2}\, dx$$

$$= \frac{\hbar^2 b}{2m}$$

$$< V > = \left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{+\infty} e^{-bx^2} (-\,\alpha\delta(x))\, e^{-bx^2}\, dx$$

$$= -\left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \alpha$$

$$< H > = \frac{-\hbar^2 b}{2m} - \left(\frac{2b}{\pi}\right)^{\frac{1}{2}} \alpha$$

For ground state energy

$$\frac{dH}{db} = 0$$

$$\frac{\hbar^2}{2m} - \frac{1}{2}\frac{\sqrt{2}}{\sqrt{\pi}}\frac{1}{\sqrt{b}}\alpha = 0$$

$$b = \frac{2m^2\alpha^2}{\pi\hbar^4}$$

So, at $b = \frac{2m^2\alpha^2}{\pi\hbar^4}$,

$$< H >_{min} = \frac{\hbar^2 . 2m^2\alpha^2}{2m\pi\hbar^4} - \left(\frac{2}{\pi}\cdot\frac{2m^2\alpha^2}{\pi\hbar^4}\right)^{\frac{1}{2}} \alpha$$

$$< H >_{min} = \frac{-m\alpha^2}{\pi\hbar^2}$$

54. (B) For boson total energy $3\varepsilon_0$

For fermion total energy

$$= 2\varepsilon_0 + 3\varepsilon_0$$

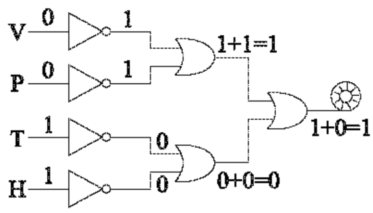
$$= 5\varepsilon_0$$

$$E_F - E_B = 5\varepsilon_0 - 3\varepsilon_0$$

$$= 2\varepsilon_0$$

55. (A) 56. (D) 57. (B)

58. (A)



59. (A)

$$\frac{dx}{dt} = x^2$$

$$\int \frac{dx}{x^2} = \int dt$$

$$-\frac{1}{x} = t + c$$

as given condition

$$x(0) = 1$$

$$-\frac{1}{1} = 0 + c$$

$$c = -1$$

So $$-\frac{1}{x} = t - 1$$

$$t = 1 - \frac{1}{x}$$

If $$x = \infty$$

$$t = 1$$

60. (C)

61. (C) Laplace transform of

$t - 1 + e^{-t}$

$$= \frac{\underline{|1}}{s^2} - \frac{1}{s} + \frac{1}{(s+1)}$$

$$= \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}$$

$$= \frac{s + 1 - s^2 - s + s^2}{s^2\,(s+1)}$$

$$= \frac{1}{s^2\,(s+1)}$$

62. (C) The number of degrees of freedom of rigid body in 3-space-dimension

$$3 \times 3 - 3 = 6$$

Putting $d = 3$

In $\frac{d(d+1)}{2} = \frac{3\,(3+1)}{2}$

$$= 6$$

So number of degrees of freedom of a rigid body in d-space is $\frac{d(d+1)}{2}$

63. (B) 64. (A) 65. (D) 66. (C) 67. (B)

68. (A) $B(A, Z) \approx 16A - 20A^{2/3} - \frac{3}{4} Z^2 A^{-\frac{1}{3}} + 30 \frac{(N-Z)^2}{A}$

$$B(A, Z) = 16A - 20A^{2/3} - \frac{3}{4} Z^2 A^{-\frac{1}{3}} + 30 \frac{(A-2Z)^2}{A}$$

$$\frac{\partial B}{\partial Z} = -\frac{3}{4} \times 2\, A^{-1/3} Z + 30 \times 2 \frac{(A-2Z)}{A} (-2)$$

$$= -\frac{3}{2} A^{-1/3} Z + \frac{-120}{A} (A - 2Z)$$

$$\frac{\partial B}{\partial Z} = -\frac{3}{2} A^{-1/3} Z - 120 + 240 \frac{Z}{A}$$

$$\frac{\partial B}{\partial Z} = 0$$

$$\left(-\frac{3}{2} A^{-1/3} + \frac{240}{A}\right) Z = 120$$

$$Z = \frac{120}{\left(\frac{240}{A} - \frac{3}{2} A^{-1/3}\right)}$$

$$Z = \frac{120}{\frac{240}{A}\left(1 - \frac{3}{2 \times 240} A^{2/3}\right)}$$

$$Z = \frac{A}{2\left(1 - \frac{A^{2/3}}{160}\right)}$$

$$Z = \frac{A}{2}\left(1 - \frac{A^{2/3}}{160}\right)^{-1}$$

69. (B) 70. (D) 71. (A) 72. (B) 73. (C) 74. (C) 75. (D)

Physical Sciences
CSIR-UGC NET/JRF Exam.
Solved Paper

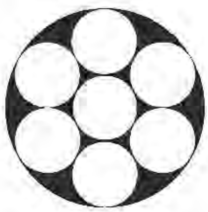
December 2013
Physical Sciences

PART A

1. One of the four –A, B, C and D – committed a crime. A said, "I did it." B said, "I didn't." C said, "B did it." D said, "A did it." Who is lying ?
(A) A (B) B
(C) C (D) D

2. A farmer gives 7 full, 7 half-full and 7 empty bottles of honey to his three sons and asks them to share these among themselves such that each of them gets the same amount of honey and the same number of bottles. In how many ways can this be done ? (bottles cannot be distinguished otherwise, they are sealed and cannot be broken)—
(A) 0 (B) 1
(C) 2 (D) 3

3. A circle circumscribes identical, close-packed circles of unit diameter as shown in the figure below. What is the total area of the shaded portion ?

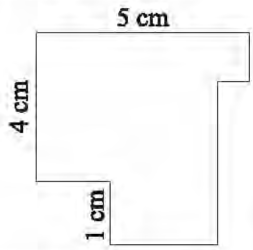


(A) 2 (B) 2π
(C) $\frac{1}{2}$ (D) $\frac{\pi}{2}$

4. What is the next number in the following sequence ?
39, 42, 46, 50, ……
(A) 52 (B) 53
(C) 54 (D) 55

5. A student buys a book from an online shop at 20% discount. His friend buys another copy of the same book in a book fair for ₹ 192 paying 20% less than his friend. What is the full price of the book ?
(A) ₹ 275 (B) ₹ 300
(C) ₹ 320 (D) ₹ 392

6. What is the perimeter of the given figure as below, where adjacent sides are at right angles to each other ?
(A) 20 cm
(B) 18 cm
(C) 21 cm
(D) Cannot be determined



7. Three fishermen caught fishes and went to sleep. One of them woke up, took away one fish and $\frac{1}{3}$rd of the remainder as his share, without others' knowledge. Later, the three of them divided the remainder equally. How many fishes were caught ?
(A) 58 (B) 19
(C) 76 (D) 88

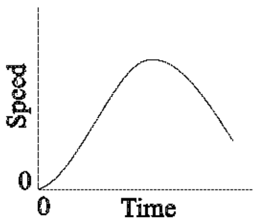
8. Every time a ball falls to ground, it bounces back to half the height it fell from. A ball is dropped from a height of 1024 cm. The maximum height from the ground to which it can rise after the tenth bounces is—
(A) 102·4 cm (B) 1·24 cm
(C) 1 cm (D) 2 cm

9. A circle of radius 7 units lying in the fourth quadrant touches the x-axis at (10, 0). The centre of the circle has coordinates—
(A) (7, 7) (B) (–10, 7)
(C) (10, –7) (D) (7, –7)

10. A cylinder of radius 1 cm and height 1 cm is broken into three pieces. Which of the following must be true ?
(A) At least one piece has volume equal to 1 cm^3
(B) At least two pieces have equal volumes
(C) At least one piece has volume less than 1 cm^3
(D) At least one piece has volume greater than 1 cm^3

11. For real numbers x and y, $x^2 + (y - 4)^2 = 0$. Then the value of $x + y$ is—
(A) 0 (B) 2
(C) $\sqrt{2}$ (D) 4

12. A car is moving along a straight track. Its speed is changing with time as shown below—



Which of the following statements is correct ?
(A) The speed is never zero
(B) The acceleration is zero once on the path
(C) The distance covered initially increase and then decreases
(D) The car comes back to its initial position once

13. If $a + b + c + d + e = 10$ (all positive numbers), then the maximum value of $a \times b \times c \times d \times e$ is—
(A) 12 (B) 32
(C) 48 (D) 72

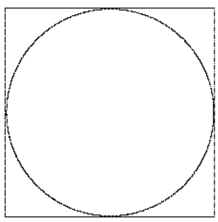
14. How many nine-digit positive integers are three, the sum of squares of whose digits is 2 ?
(A) 8 (B) 9
(C) 10 (D) 11

15. What is the arithmetic mean of $\frac{1}{1 \times 2}, \frac{1}{2 \times 3}, \frac{1}{3 \times 4}, \frac{1}{4 \times 5}, \ldots\ldots\ldots \frac{1}{100 \times 101}$?
(A) 0·01 (B) $\frac{1}{101}$
(C) 0·00111... (D) $\frac{\frac{1}{49 \times 50} + \frac{1}{50 \times 51}}{2}$

16. Consider the sequence of ordered sets of natural numbers :
{1}, {2, 3}, {4, 5, 6}, {7, 8, 9, 10}. What is the last number in the 10th set ?
(A) 10 (B) 19
(C) 55 (D) 67

17. 366 players participate in a knock-out tournament. In each round all competing players pair together and play a match, the winner of each match moving to the next round. If at the end of a round there is an odd number of winners, the unpaired one moves to the next round without playing a match. What is the total number of matches played ?
(A) 366 (B) 282
(C) 365 (D) 418

18. What does the diagram above establish ?



Note : The diagram is a circle inside a square.
(A) $\pi > 3$
(B) $\pi \geq 2\sqrt{2}$
(C) $\pi < 4$
(D) π is closer to 3 than to 4

19. There are 2 hills, A and B, in a region. If hill A is located N 30° E of hill B, what will be the direction of hill B when observed from hill A ? (N 30° E means 30° from north towards east)—
(A) S 30°W (B) S 60°W
(C) S 30°E (D) S 60°E

20. $(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3) =$
(A) 61 (B) 22
(C) $\frac{41}{24}$ (D) 16

PART B

21. If A, B and C are non-zero Hermitian operators, which of the following relations must be false ?
(A) $[A, B] = C$ (B) $AB + BA = C$
(C) $ABA = C$ (D) $A + B = C$

22. The expression
$$\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2}\right) \frac{1}{(x_1^2 + x_2^2 + x_3^2 + x_4^2)}$$
is proportional to—
(A) $\delta(x_1 + x_2 + x_3 + x_4)$
(B) $\delta(x_1)\,\delta(x_2)\,\delta(x_3)\,\delta(x_4)$
(C) $(x_1^2 + x_2^2 + x_3^2 + x_4^2)^{-3/2}$
(D) $(x_1^2 + x_2^2 + x_3^2 + x_4^2)^{-2}$

23. Given that the integral $\int_0^\infty \frac{dx}{y^2 + x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{(y^2 + x^2)^2}$ is—
(A) $\frac{\pi}{y^3}$ (B) $\frac{\pi}{4y^3}$
(C) $\frac{\pi}{8y^3}$ (D) $\frac{\pi}{2y^3}$

24. A loaded dice has the probabilities $\frac{1}{21}$, $\frac{2}{21}$, $\frac{3}{21}$, $\frac{4}{21}$, $\frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6 respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even ?
(A) $\frac{144}{441}$ (B) $\frac{225}{441}$
(C) $\frac{221}{441}$ (D) $\frac{220}{441}$

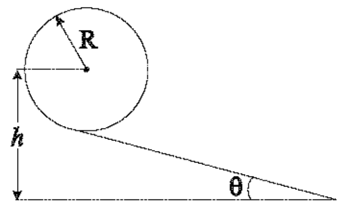
25. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?
(A) x^2y (B) $x^2 - y^2$
(C) $x^3 - 3xy^2$ (D) $3x^2y - y - y^3$

26. A particle moves in a potential $V = x^2 + y^2 + \frac{z^2}{2}$. Which component(s) of the angular momentum is/are constant(s) of motion ?
(A) None (B) L_x, L_y and L_z
(C) Only L_x and L_y (D) Only L_z

27. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If { , } represents the Poisson bracket, the value of $\{A, \{B, C\}\} - \{\{A, B\}, C\}$ is given by—
(A) 0 (B) $\{B, \{C, A\}\}$
(C) $\{A, \{C, B\}\}$ (D) $\{\{C, A\}, B\}$

28. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H = \sqrt{p^2 + m^2} + V(x)$, in units in which the speed of light $c = 1$. The corresponding Lagrangian is—
(A) $L = m\sqrt{1 + \dot{x}^2} - V(x)$
(B) $L = -m\sqrt{1 - \dot{x}^2} - V(x)$
(C) $L = \sqrt{1 + m\dot{x}^2} - V(x)$
(D) $L = \frac{1}{2}m\dot{x}^2 - V(x)$

29. A ring of mass m and radius R rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height h, the angular velocity when the ring reaches the base is—



(A) $\sqrt{g/(h - R)\tan\theta}$ (B) $\sqrt{g/(h - R)}$
(C) $\sqrt{g(h - R)/R^2}$ (D) $\sqrt{2g/(h - R)}$

30. Two monochromatic sources, L_1 and L_2, emit light at 600 and 700 nm, respectively. If their frequency bandwidths are 10^{-1} and 10^{-3} GHz, respectively, then the ratio of linewidth of L_1 to L_2 is approximately—
(A) 100 : 1 (B) 1 : 85
(C) 75 : 1 (D) 1 : 75

31. The force between two long and parallel wires carrying currents I_1 and I_2 and separated by a distance D is proportional to—

(A) I_1I_2/D (B) $(I_1 + I_2)/D$
(C) $(I_1I_2/D)^2$ (D) I_1I_2/D^2

32. Let (V, A) and (V', A') denote two sets of scalar and vector potentials, and ψ a scalar function. Which of the following transformations leave the electric and magnetic fields (and hence Maxwell's equations) unchanged ?

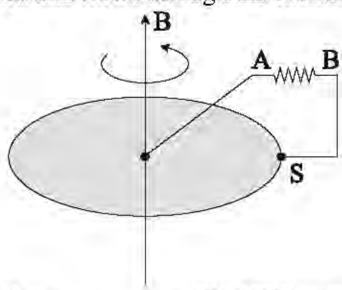
(A) $A' = A + \nabla\psi$ and $V' = V - \frac{\partial\psi}{\partial t}$

(B) $A' = A - \nabla\psi$ and $V' = V + 2\frac{\partial\psi}{\partial t}$

(C) $A' = A + \nabla\psi$ and $V' = V + \frac{\partial\psi}{\partial t}$

(D) $A' = A - 2\nabla\psi$ and $V' = V - \frac{\partial\psi}{\partial t}$

33. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field pointing up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contact S. The current that flows through the resistor is—



(A) Zero (B) DC from A to B
(C) DC from B to A (D) AC

34. A spin $\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{11}}\begin{pmatrix}1+i\\3\end{pmatrix}$ in the eigenbasis of S^2 and S_z. If we measure S_z, the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are—

(A) $\frac{1}{2}$ and $\frac{1}{2}$ (B) $\frac{2}{11}$ and $\frac{9}{11}$

(C) 0 and 1 (D) $\frac{1}{11}$ and $\frac{3}{11}$

35. The motion of a particle of mass m in one dimension is described by the Hamiltonian H $= \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 + \lambda x$. What is the difference between the (quantized) energies of the first two levels ? (In the following, $\langle x\rangle$ is the expectation value of x in the ground state)—

(A) $\hbar\omega - \lambda\langle x\rangle$ (B) $\hbar\omega + \lambda\langle x\rangle$

(C) $\hbar\omega + \frac{\lambda^2}{2m\omega^2}$ (D) $\hbar\omega$

36. Let ψ_{nlm} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential V(r). The expectation value of L_z in the state

$$\psi = \frac{1}{6}\left[\psi_{200} + \sqrt{5}\psi_{210} + \sqrt{10}\psi_{21-1} + \sqrt{20}\psi_{211}\right]$$

is—

(A) $-\frac{5}{18}\hbar$ (B) $\frac{5}{6}\hbar$

(C) $\hbar$ (D) $\frac{5}{18}\hbar$

37. Three identical spin $\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is—

(A) 8 (B) 4
(C) 3 (D) 2

38. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition ?

(A) Internal energy
(B) Helmholtz free energy
(C) Gibbs free energy
(D) Entropy

39. Two different thermodynamic systems are described by the following equations of state—

$$\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}}$$

and $$\frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$$

where $T^{(1,2)}$, $N^{(1,2)}$ and $U^{(1,2)}$ are respectively the temperatures, the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{tot} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{tot}}$ is—

(A) $\frac{5N^{(2)}}{3N^{(1)} + 5N^{(2)}}$ (B) $\frac{3N^{(1)}}{3N^{(1)} + 5N^{(2)}}$

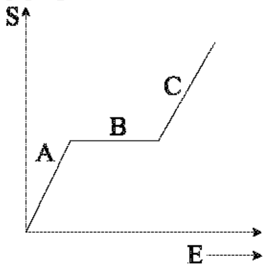
(C) $\frac{N^{(1)}}{N^{(1)} + N^{(2)}}$ (D) $\frac{N^{(2)}}{N^{(1)} + N^{(2)}}$

40. The speed v of the molecules of mass m of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x, v_y, v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constant, is—

(A) $(\alpha^2 - \beta^2)\, k_B T/m$ (B) $(\alpha + \beta^2)\, k_B T/m$

(C) $(\alpha + \beta)^2\, k_B T/m$ (D) $(\alpha^2 - \beta)^2\, k_B T/m$

41. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperature of the phases A, B and C, denoted by T_A, T_B and T_C, respectively, satisfy the following inequalities—

(A) $T_C > T_B > T_A$ (B) $T_A > T_C > T_B$

(C) $T_B > T_C > T_A$ (D) $T_B > T_A > T_C$

42. The physical phenomenon that cannot be used for memory storage applications is—

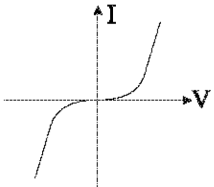
(A) Large variation in magneto resistance as a function of applied magnetic field

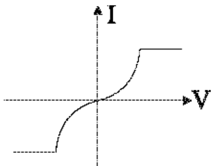
(B) Variation in magnetization of a ferromagnet as a function of applied magnetic field

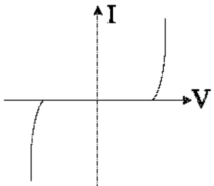
(C) Variation in polarization of a ferroelectric as a function of applied electric field

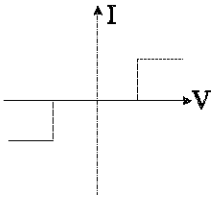
(D) Variation in resistance of a metal as a function of applied electric field

43. Two identical Zener diodes are placed back to back in series and are connected to a variable DC power supply. The best representation of the I–V characteristics of the circuit is—

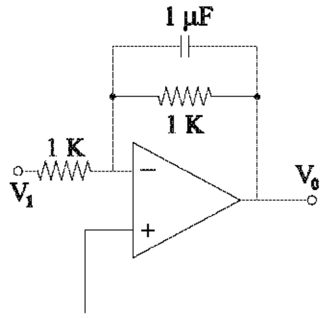
(A) 

(B) 

(C) 

(D) 

44. Consider the op-amp circuit shown in the figure.



If the input is a sinusoidal wave $V_i = 5\sin(1000t)$, then the amplitude of the output V_0 is—

(A) $\frac{5}{2}$ (B) 5

(C) $\frac{5\sqrt{2}}{2}$ (D) $5\sqrt{2}$

45. If one of the inputs of a J-K flip flop is high and the other is low, then the outputs Q and $\bar{Q}$—
(A) Oscillate between low and high in race-around condition
(B) Toggle and the circuit acts like a T flip flop
(C) Are opposite to the inputs
(D) Follow the inputs and the circuit acts like an R-S flip flop

PART C

46. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij} = A_iB_j$—
(A) Reduces to a direct sum of three 3-dimensional representations
(B) Is an irreducible 9-dimensional representation
(C) Reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations
(D) Reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representations

47. The Fourier transform of the derivative of the Dirac δ-function, namely $\delta'(x)$, is proportional to—
(A) 0 (B) 1
(C) $\sin k$ (D) ik

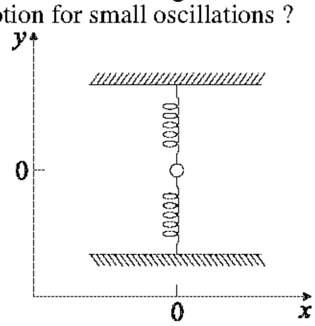
48. If $A = \hat{i}yz + \hat{j}xz + \hat{k}xy$, then the integral $\oint \mathbf{A} \cdot d\mathbf{l}$ (where C is along the perimeter of a rectangular area bounded by $x = 0$, $x = a$ and $y = 0$, $y = b$) is—
(A) $\frac{1}{2}(a^3 + b^3)$ (B) $\pi(ab^2 + a^2b)$
(C) $\pi(a^3 + b^3)$ (D) 0

49. Consider an $n \times n$ ($n > 1$) matrix A, in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$). The matrix A—
(A) Has one degenerate eigenvalue with degeneracy $(n - 1)$
(B) Has two degenerate eigenvalues with degeneracies 2 and $(n - 2)$
(C) Has one degenerate eigenvalue with degeneracy n
(D) Does not have any degenerate eigenvalue

50. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is—
(A) $2\pi\sqrt{\frac{l + R}{g}}$
(B) $\frac{2\pi}{\sqrt{g}}(l^2 + R^2)^{1/4}$
(C) $2\pi\sqrt{\frac{2R^2 + 2Rl + l^2}{g(R + l)}}$
(D) $\frac{2\pi}{\sqrt{g}}(2R^2 + 2Rl + l^2)^{1/4}$

51. Spherical particles of a given material of density ρ are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, *i.e.*, $F_d = 6\pi\eta R\upsilon$, where η is the viscosity of the medium, R the radius of a particle and v its instantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass m to reach half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is—
(A) 8 (B) $\frac{1}{8}$
(C) 4 (D) $\frac{1}{4}$

52. Consider a particle of mass m attached to two identical springs each of length l and spring constant k (see the figure below). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations ?



(A) $m\ddot{x} + \frac{kx^3}{l^2} = 0$ (B) $m\ddot{x} + kx = 0$

(C) $m\ddot{x} + 2kx = 0$ (D) $m\ddot{x} + \frac{kx^3}{l} = 0$

53. If $\psi(x) = A \exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue $E = 0$, the potential $V(x)$ (in units where $h = 2m = 1$) is—

(A) $12x^2$ (B) $16x^6$

(C) $16x^6 + 12x^2$ (D) $16x^6 - 12x^2$

54. A particle is in the ground state of an infinite square well potential given by,

$$V(x) = \begin{cases} 0 & \text{for } -a \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is—

(A) $\frac{1}{2}$ (B) $\frac{1}{2} + \frac{1}{\pi}$

(C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{\pi}$

55. The expectation value of the x-component of the orbital angular momentum L_x in the state

$$\psi = \frac{1}{5}[3\psi_{2,1,-1} + \sqrt{5}\,\psi_{2,1,0} - \sqrt{11}\,\psi_{2,1,+1}]$$

(where ψ_{nlm} are the eigenfunctions in usual notation), is—

(A) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11} - 3)$

(B) 0

(C) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11} + 3)$

(D) $\hbar\sqrt{2}$

56. A particle is prepared in a simultaneous eigenstate of L^2 and L_z. If $l(l+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of L^2 and L_z, then the expectation value $\langle L_x^2 \rangle$ of the particle in this state satisfies—

(A) $\langle L_x^2 \rangle = 0$

(B) $0 \le \langle L_x^2 \rangle = l^2\hbar$

(C) $0 \le \langle L_x^2 \rangle \le \frac{l(l+1)\hbar^2}{3}$

(D) $\frac{l\hbar^2}{2} \le \langle L_x^2 \rangle \le \frac{l(l+1)\hbar^2}{2}$

57. If the electrostatic potential $V(r, \theta, \phi)$ in a charge free region has the form $V(r, \theta, \phi) = f(r)\cos\theta$, then the functional form of $f(r)$ (in the following a and b are constants) is—

(A) $ar^2 + \frac{b}{r}$ (B) $ar + \frac{b}{r^2}$

(C) $ar + \frac{b}{r}$ (D) $a \ln\left(\frac{r}{b}\right)$

58. The electric field of an electromagnetic wave is given by

$$\bar{E} = E_0 \cos[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\,\hat{k}.$$

The associated magnetic field $\vec{B}$ is—

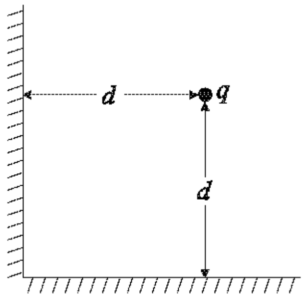
(A) $10^{-3} E_0 \cos[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\,\hat{k}$

(B) $10^{-4} E_0 \cos[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\,(4\hat{i} - 3\hat{j})$

(C) $E_0 \cos[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\,(0{\cdot}3\hat{i} + 0{\cdot}4\hat{j})$

(D) $10^2 E_0 \cos[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\,(3\hat{i} + 4\hat{j})$

59. A point charge q is placed symmetrically at a distance d from two perpendicularly placed grounded conducting infinite plates as shown in the figure. The net force on the charge (in units of $1/4\pi\varepsilon_0$) is—



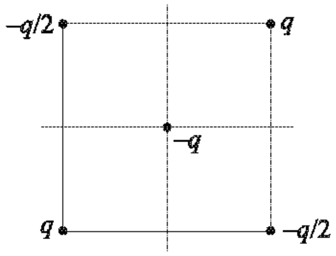
(A) $\frac{q^2}{8d^2}(2\sqrt{2} - 1)$ away from the corner

(B) $\frac{q^2}{8d^2}(2\sqrt{2} - 1)$ towards the corner

(C) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner

(D) $\frac{3q^2}{8d^2}$ away from the corner

60. Let four point charges q, $-q/2$, q and $-q/2$ be placed at the vertices of a square of side a. Let another point charge $-q$ be placed at the centre of the square (see the figure)—



Let V (r) be the electrostatic potential at a point P at a distance $r >> a$ from the centre of the square. Then V ($2r$)/V (r) is—

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

61. A system of N classical non-interacting particles, each of mass m, is at a temperature T and is confined by the external potential $V(r) = \frac{1}{2} A r^2$ (where A is a constant) in three dimensions. The internal energy of the system is—

(A) $3Nk_BT$ (B) $\frac{3}{2}Nk_BT$

(C) $N(2mA)^{3/2} k_BT$ (D) $N\sqrt{\frac{A}{m}} \ln\left(\frac{k_BT}{m}\right)$

62. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0·225, 0·255, 0·245, and 0·245, respectively. After a large number of steps, N, the expected position of the child with respect to the starting point is at a distance—

(A) $\sqrt{2} \times 10^{-2}$ Na in the north-east direction

(B) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction

(C) $2\sqrt{2} \times 10^{-2}$ Na in the south-west direction

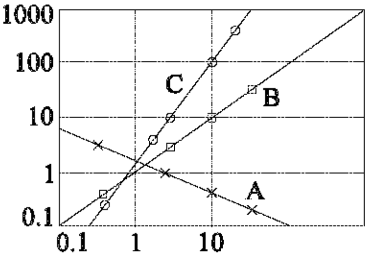
(D) 0

63. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are T_1 and T_2, respectively, and $T_1 > T_2$ then their common final temperature is—

(A) T_1^2/T_2 (B) T_2^2/T_1

(C) $\sqrt{T_1T_2}$ (D) $\frac{1}{2}(T_1 + T_2)$

64. Three sets of data A, B and C from an experiment, represented by ×, □ and O, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure.



The functional dependence $y(x)$ for the sets A, B and C are, respectively—

(A) $\sqrt{x}$, x and x^2 (B) $-\frac{x}{2}$, x and $2x$

(C) $\frac{1}{x^2}$, x and x^2 (D) $\frac{1}{\sqrt{x}}$, x and x^2

65. A sample of Si has electron and hole mobilities of 0·13 and 0·05 m^2V-s respectively at 300K. It is doped with P and Al with doping densities of $1{\cdot}5 \times 10^{21}/m^3$ and $2{\cdot}5 \times 10^{21}/m^3$ respectively. The conductivity of the doped Si sample at 300 K is—

(A) $8\Omega^{-1} m^{-1}$ (B) $32\Omega^{-1} m^{-1}$

(C) $20{\cdot}8\ \Omega^{-1} m^{-1}$ (D) $83{\cdot}2\Omega^{-1} m^{-1}$

66. A 4-variable switching function is given by $f = \sum (5, 7, 8, 10, 13, 15) + d\,(0, 1, 2)$, where d is the do-not-care-condition. The minimized form of f in sum of products (SOP) form is—

(A) $A\bar{C} + \bar{B}\bar{D}$ (B) $A\bar{B} + C\bar{D}$

(C) $AD + BC$ (D) $\bar{B}\bar{D} + BD$

67. A perturbation $V_{pert} = aL^2$ is added to the Hydrogen atom potential. The shift in the energy level of the 2P state, when the effects of spin are neglected upto second order in a, is—

(A) 0 (B) $2a\hbar^2 + a^2\hbar^4$

(C) $2a\hbar^2$ (D) $a\hbar^2 + \frac{3}{2}a^2\hbar^4$

68. A gas laser cavity has been designed to operate at $\lambda = 0{\cdot}5\ \mu$m with a cavity length of 1m. With this set-up, the frequency is found to be larger than the desired frequency by 100 Hz. The change in the effective length of the cavity required to retune the laser is—
(A) $-0{\cdot}334 \times 10^{-12}$ m
(B) $0{\cdot}334 \times 10^{-12}$ m
(C) $0{\cdot}167 \times 10^{-12}$ m
(D) $-0{\cdot}167 \times 10^{-12}$ m

69. The spectroscopic symbol for the ground state of $_{13}$Al is $^2P_{1/2}$. Under the action of a strong magnetic field (when L–S coupling can be neglected) the ground state energy level will split into—
(A) 3 levels (B) 4 levels
(C) 5 levels (D) 6 levels

70. A uniform linear monoatomic chain is modeled by a spring-mass system of masses m separated by nearest neighbor distance a and spring constant $m\omega_0^2$. The dispersion relation for this system is—
(A) $\omega(k) = 2\omega_0 \left(1 - \cos\left(\frac{ka}{2}\right)\right)$
(B) $\omega(k) = 2\omega_0 \sin^{-2}\left(\frac{ka}{2}\right)$
(C) $\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$
(D) $\omega(k) = 2\omega_0 \tan\left(\frac{ka}{2}\right)$

71. The energy of an electron in a band as a function of its wave vector k is given by $E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$, where E_0B and a are constants. The effective mass of the electron near the bottom of the band is—
(A) $\frac{2\hbar^2}{3Ba^2}$ (B) $\frac{\hbar^2}{3Ba^2}$
(C) $\frac{\hbar^2}{2Ba^2}$ (D) $\frac{\hbar^2}{Ba^2}$

72. A DC voltage V is applied across a Josephson junction between two superconductors with a phase difference ϕ_0. If I_0 and k are constants that depend on the properties of the junction, the current flowing through it has the form—
(A) $I_0 \sin\left(\frac{2eV_t}{\hbar} + \phi_0\right)$
(B) $kV \sin\left(\frac{2eV_t}{\hbar} + \phi_0\right)$
(C) $kV \sin \phi_0$
(D) $I_0 \sin \phi_0 + kV$

73. Consider the following ratios of the partial decay widest
$R_1 = \frac{\Gamma(\rho^+ \to \pi^+ + \pi^0)}{\Gamma(\rho^- \to \pi^- + \pi^0)}$ and
$R_2 = \frac{\Gamma(\Delta^{++} \to \pi^+ + P)}{\Gamma(\Delta^- \to \pi^- + n)}$
If the effects of electromagnetic and weak interactions are neglected, then R_1 and R_2 are, respectively—
(A) 1 and $\sqrt{2}$ (B) 1 and 2
(C) 2 and 1 (D) 1 and 1

74. The intrinsic electric dipole moment of a nucleus $^A_Z X$—
(A) Increases with Z, but independent of A
(B) Decreases with Z, but independent of A
(C) Is always zero
(D) Increases with Z and A

75. According to the shell model, the total angular momentum (in units of $\hbar$) and the parity of the ground state of the $^7_3$Li nucleus is—
(A) $\frac{3}{2}$ with negative parity
(B) $\frac{3}{2}$ with positive parity
(C) $\frac{1}{2}$ with positive parity
(D) $\frac{7}{2}$ with negative parity

Answers with Explanations

1. (C) A said I did it
B said I didn't it
C said B did it
D said A did it
So, from four sentence C is lying.
2. (C)

3. (D) Area of one small circle

$$= \pi\left(\frac{1}{2}\right)^2$$

$$\text{Area of 7 circle} = 7\pi\left(\frac{1}{2}\right)^2$$

$$= \frac{7\pi}{4}$$

$$\text{Area of big circle} = \pi\left(\frac{3}{2}\right)^2$$

$$= \pi\frac{9}{4}$$

Area of shaded portion

$$\frac{9\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{2}$$

4. (D)
5. (B) Let full price of book is x from the question

$$\frac{x \times 80}{100} \times \frac{80}{100} = 192$$

$$x = \frac{192 \times 100 \times 100}{80 \times 80}$$

$$= 300$$

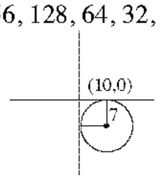
6. (A)
7. (B) From the option

$$19 - 1 = 18$$

$$\frac{18}{3} = 6$$

Rest fish 18 – 6 = 12
they equally divided so each get 4
so total fish was 19.

8. (C) 512, 256, 128, 64, 32, 16, 8, 4, 2, 1
9. (C)



So centre of circle has co-ordinate (10, –7)

10. (D) 11. (D) 12. (B) 13. (B)
14. (A) Nine digit positive integers, the sum of square of whose digit is 2 is

110000000
101000000
100100000
100010000
100001000
100000100
100000010
110000001

15. (B)

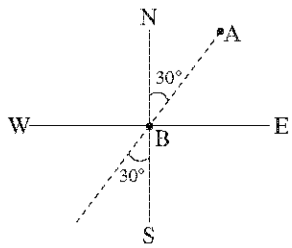
$$\frac{\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5} + \ldots\ldots\ldots \frac{1}{100\times 101}}{100}$$

$$\frac{1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5} \ldots\ldots \frac{1}{100}-\frac{1}{101}}{100}$$

$$\frac{1-\frac{1}{101}}{100} = \frac{1}{101}$$

16. (B) {1}, {2, 3}, {4, 5, 6}, {5, 6, 7, 8}
{6, 7, 8, 9, 10}, {7, 8, 9, 10, 11, 12}
{8, 9, 10, 11, 12, 13, 14}, {9, 10, 11, 12, 13, 14, 15, 16}
{10, 11, 12, 13, 14, 15, 16, 17, 18}
{11, 12, 13, 14, 15, 16, 17, 18, 19}
So last number in the 10th set is 19

17. (C) 18. (C)
19. (A)



B will be located 30° from south towards west

20. (A) $(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3)$

$$\left(\frac{25}{5} + 3 - 8\right) + 64 - 3 = 61$$

21. (A) $[A, B] = C$

L.H.S. $[A, B]^+ = (AB - BA)^+$

$$= B^+A^+ - A^+B^+$$

Since A and B and C hermitian operator

So, $A^+ = A$, $B^+ = B$, $C^+ = C$

$$[A, B]^+ = (BA - AB)$$

$$= -(AB - BA)$$

R.H.S. $C^+ = C$

So, [A, B] = C is not hermitian operator.

22. (B)

23. (B) $$\int_0^\infty \frac{dx}{x^2+y^2} = \frac{\pi}{2y}$$

$$\int_0^\infty \frac{dx}{(x^2+y^2)^2}$$

Now $$\int_{-\infty}^\infty \frac{dz}{(z^2+y^2)^2} = \int_{-\infty}^\infty \frac{dz}{(z^2-i^2y^2)^2}$$

$$\int_{-\infty}^\infty \frac{dz}{(z+iy)^2(z-iy)^2}$$

Residue at $(z = iy)$

$$\frac{1}{\lfloor 1}\frac{d}{dz}\frac{(z-iy)^2}{(z+iy)^2(z-iy)^2}\Bigg|_{z=iy}$$

$$= \frac{d}{dz}(z+iy)^{-2}\Big|_{z=iy}$$

$$= \frac{1}{4iy^3}$$

$$\int_{-\infty}^\infty \frac{dz}{(z^2+y^2)^2} = 2\pi i\frac{1}{4iy^3}$$

$$\int_{-\infty}^\infty \frac{dx}{(x^2+y^2)^2} = \frac{\pi}{2y^3}$$

$$\int_0^\infty \frac{dx}{(y^2+x^2)^2} = \frac{\pi}{4y^3}$$

24. (B)

25. (A) $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$$

If $$u = x^2y$$

$$\frac{\partial^2 x^2y}{\partial x^2}+\frac{\partial^2 x^2y}{\partial y^2}$$

$$2y+0 \neq 0$$

So x^2y cannot real past.

26. (D)

27. (D) $$\{A,\{B,C\}\}+\{B,\{C,A\}\}+\{C,\{A,B\}\} = 0$$

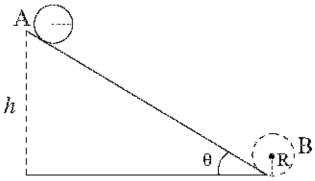
$$\{A,\{B,C\}\}+\{B,\{C,A\}\}-\{\{A,B\},C\} = 0$$

$$\{A,\{B,C\}\}-\{\{A,B\},C\} = -\{B,\{C,A\}\}$$

$$\{A,\{B,C\}\}-\{\{A,B\},C\} = \{\{C,A\},B\}$$

28. (B) $$L = -m\sqrt{1-\dot{x}^2} - V(x)$$

29. (C)



Total energy at A = total energy at B

K.E. + P.E. at A = K.E. + P.E. at B

$$0+mgh = \frac{1}{2}mv^2+\frac{1}{2}I\omega^2+mgR$$

$$mgh = \frac{1}{2}mR^2\omega^2 + \frac{1}{2}mR^2\omega^2 + mgR$$

$$mgh = mR^2\omega^2 + mgR$$

$$gh = R^2\omega^2 + gR$$

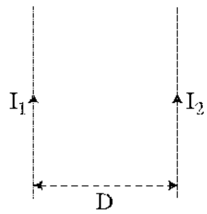
$$g(h-R) = R^2\omega^2$$

$$\omega^2 = \frac{g(h-R)}{R^2}$$

$$\omega = \sqrt{\frac{g(h-R)}{R^2}}$$

30. (C)

31. (A)



Force between two parallel wire at separation D is

$$\frac{dF}{dl} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{D}$$

If current is flowing in same direction, force will be attractive and if current is flowing in opposite direction force will be repulsive.

32. (A) $$A' = A+\nabla\psi$$

$$V' = V-\frac{\partial\psi}{\partial t}$$

33. (C) If disc is rotated as shown in the fig., positive charge will accumulate at boundary of the disc and negative charge will accumulate at centre of disc. So current will from B to A.

34. (B)

$$\chi = \frac{1}{\sqrt{11}}\begin{pmatrix}1+i\\3\end{pmatrix}$$

$$= \frac{1+i}{\sqrt{11}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{3}{\sqrt{11}}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\text{Prob of } \frac{\hbar}{2} = \left|\frac{1+i}{\sqrt{11}}\right|^2 = \frac{2}{11}$$

$$\text{Prob of } \frac{-\hbar}{2} = \left|\frac{3}{\sqrt{11}}\right|^2 = \frac{9}{11}$$

35. (D)

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2x^2 + \lambda x$$

$$\frac{P^2}{2m} + \frac{1}{2}m\omega^2\left[x^2 + \frac{2}{m\omega^2}\lambda x + \left(\frac{\lambda}{m\omega^2}\right)^2 - \left(\frac{\lambda}{m\omega^2}\right)^2\right]$$

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2\left[x + \frac{\lambda}{m\omega^2}\right]^2 - \frac{\lambda^2}{2m\omega^2}$$

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2(X')^2 - \frac{\lambda^2}{2m\omega^2}$$

where $X' = x + \frac{\lambda}{m\omega^2}$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{\lambda^2}{2m\omega^2}$$

$$E_0 = \frac{1}{2}\hbar\omega - \frac{\lambda^2}{2m\omega^2}$$

$$E_1 = \frac{3}{2}\hbar\omega - \frac{\lambda^2}{2m\omega^2}$$

$$\Delta E = \hbar\omega$$

36. (D) $\psi = \frac{1}{6}[\psi_{200} + \sqrt{5}\psi_{210} + \sqrt{10}\psi_{21-1} + \sqrt{20}\psi_{211}]$

$$\langle L_z\rangle = \sum_{i=1} P_iL_{zi}$$

$$= \frac{1}{36}[0\times\hbar + 5\times 0\times\hbar + 10\times(-1)\times\hbar + 20\times 1\times\hbar]$$

$$= \frac{5}{18}\hbar$$

37. (B)

38. (C) Melting of ice into water at constant pressure is first, order phase transition Gibbs free energy does not show discontinuous change across the phase transition.

39. (B)

$$\frac{1}{T^1} = \frac{3RN'}{2U'}$$

$$\frac{1}{T^2} = \frac{5RN^2}{2U^2}$$

$$U_{total} = U_1 + U_2$$

at equilibrium

$$\frac{1}{T^1} = \frac{1}{T^2}$$

$$\frac{3RN^1}{2U^1} = \frac{5RN^2}{2U^2}$$

$$\frac{3N^1}{U^1} = \frac{5N^2}{U^2}$$

$$\frac{U^2}{U^1} = \frac{5}{3}\frac{N^2}{N^1}$$

$$\frac{U_1}{U_{total}} = \frac{U^1}{U^1 + U^2}$$

$$\frac{U_1}{U_{total}} = \frac{U^1/U^2}{1 + \frac{U^1}{U^2}}$$

$$= \frac{\frac{3N^1}{5N^2}}{1 + \frac{3N^1}{5N^2}} = \frac{3N^1}{2N^1 + 5N^2}$$

40. (B) $\langle(\alpha v_x - \beta v_y)^2\rangle$

$\langle\alpha^2 v_x^2 + \beta^2 v_y^2 - 2\alpha\beta v_x v_y\rangle$

$\alpha^2\langle v_x^2\rangle + \beta^2\langle v_y^2\rangle - 2\alpha\beta\langle v_x v_y\rangle$

$$\alpha^2\left(\sqrt{\frac{kT}{m}}\right)^2 + \beta^2\left(\sqrt{\frac{kT}{m}}\right)^2 - 0$$

$$(\alpha^2 + \beta^2)\frac{kT}{m}$$

41. (C)

$$S \propto \frac{E}{T}$$

$$T \propto \frac{E}{S}$$

$$T_B \propto \frac{E_B}{S}$$

$$T_A \propto \frac{E_A}{S_A}$$

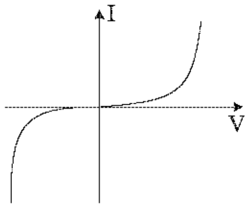
So $T_B > T_C > T_4$

Since in A, S is increase rapidly V/S E but in C, S is increase slowly V/S E so

$$T_B > T_C > T_4$$

42. (D)

43. (C)



44. (C) $V_0 = \left(\dfrac{\dfrac{1\ K\Omega}{1 + S \times 1 \times 10^{-6} \times 10^{3}}}{1\ K\Omega}\right) V_{in}$

$$V_0 = \left(\frac{1}{1 + S \times 10^{-3}}\right) V_{in}$$

$$V_0 = \left(\frac{1}{1 + J\omega \times 10^{-3}}\right) V_{in}$$

$$\omega = 1000$$

$$V_0 = \left(\frac{1}{1 + j}\right) V_{in}$$

$$V_0 = \left(\frac{1}{1 + j}\right) 5 \sin 1000t$$

$$\text{amplitude} = \left|\frac{5}{1 + j}\right| = \left|\frac{5(1 - j)}{(1 + j)(1 - j)}\right|$$

$$= \left|\frac{5(1 - j)}{2}\right| = \frac{5}{2}\sqrt{2}$$

45. (D) Follow the input and the circuits acts like an R – S flip–flop.

46. (C)

47. (D) Fourier transform $\delta'(x)$

$$= \int_{-\infty}^{\infty} e^{-ikx}\,\delta'(x)\,dx - (e^{-ikx})'|_{x=0}$$

$$= ike^{-ikx}|_{x=0}$$

$$= ik$$

48. (D) $A = iyz + jxz + \hat{k}xy$

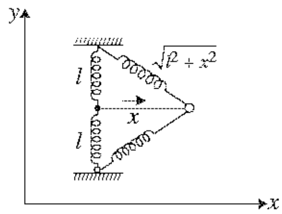
$$\oint A \cdot dl = \int (\nabla \times A)\, \vec{ds}$$

$$(\nabla \times A) = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = 0$$

$$\oint \vec{A} \cdot \vec{dl} = 0$$

49. (A) 50. (C) 51. (C)

52. (A)



$$T = \frac{1}{2} m\dot{x}^2$$

$$V = 2 \times \frac{1}{2} k$$

[Change in length of spring]^2

$k\sqrt{x^2 + l^2} - l)^2$

$$L = \frac{1}{2} m\dot{x}^2 - k(\sqrt{x^2 + l^2} - l)^2$$

$$\frac{\partial L}{\partial \dot{x}} = m2\dot{x} = m\dot{x}$$

$$\frac{\partial L}{\partial x} = -2k[\sqrt{x^2 + l^2} - l]\frac{1}{2}\frac{2x}{\sqrt{x^2 + l^2}}$$

$$= -2kx\left[1 - \frac{l}{\sqrt{x^2 + l^2}}\right]$$

$$= -2kx\left[1 - \frac{l}{l\sqrt{1 + \dfrac{x^2}{l^2}}}\right]$$

$$\frac{\partial L}{\partial x} = -2kx\left[1 - \left(1 + \frac{x^2}{l^2}\right)^{1/2}\right]$$

$$= -2kx\left[1 - \left(1 - \frac{1}{2}\frac{x^2}{l^2}\right)\right]$$

$$= -2kx \cdot \frac{1}{2} \cdot \frac{x^2}{l^2}$$

$$= \frac{-kx^3}{l^2}$$

$$\frac{d}{dt}(m\dot{x}) + \frac{kx^3}{l^2} = 0$$

$$m\ddot{x} + \frac{kx^3}{l^2} = 0$$

53. (D) Schrodinger eqn.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\,\psi = 0$$

$$\psi(x) = A \exp.(-x^4)$$

$$\frac{d^2}{dx^2}[A \exp.(-x^4)] + \frac{2m}{\hbar^2}[E - V(x)]$$

$$A \exp.(-x^4) = 0$$

$$\hbar = 2m = 1 \text{ given}$$
$$E = 0 \text{ given}$$
$$[(16x^6 - 12x^2) - V(x)]\, A \exp.(-x^4) = 0$$
$$V(x) = 16x^6 - 12x^2$$

54. (B) $V(x) = \begin{cases} 0 & -a \le x < a \\ \infty & \text{otherwise} \end{cases}$

$$P = \int_{-a/2}^{a/2} \sqrt{\frac{1}{a}} \cos\frac{\pi x}{2a} \sqrt{\frac{1}{a}} \cos\frac{\pi x}{2a}\, dx$$
$$= \frac{2}{a}\int_0^{a/2} \cos^2\frac{\pi x}{2a}\, dx$$
$$= \frac{2}{a}\int_0^{a/2} \left(\frac{1+\cos\frac{\pi x}{a}}{2}\right) dx$$
$$\frac{2}{a}\cdot\frac{1}{2}\left[\sin\frac{\pi x}{a}\cdot\frac{a}{\pi} + x\right]_0^{a/2}$$
$$= \left[\frac{1}{2} + \frac{1}{\pi}\right]$$

55. (A) $\psi = \frac{1}{5}[3\psi_{2,1,-1} + \sqrt{5}\psi_{2,1,0} - \sqrt{11}\psi_{2,1,+1}]$

$$\langle L_x \rangle = \langle \psi | L_x | \psi \rangle$$
$$\langle \psi \left| \frac{L_+ + L_-}{2} \right| \psi \rangle$$
$$\frac{1}{50}[(\langle 3\psi_{2,1,-1}| + \sqrt{5}\,\langle \psi_{2,1,0}| - \sqrt{11}\,\langle \psi_{2,+1,+1}|)$$
$$(L_+ + L_-)\,(|3\psi_{2,1,-1}\rangle + \sqrt{5}\,|\psi_{2,1,0}\rangle$$
$$- \sqrt{11}\,|\psi_{2,1,1}\rangle)] - \frac{\hbar\sqrt{10}}{25}(\sqrt{11} - 3)$$

56. (D) Expectation value of $\langle L_x^2 \rangle$

$$\langle L_x^2 \rangle = \frac{1}{2}[l(l+1) - m^2]\,\hbar^2$$

For l, m have take value $-l$ to $+l$

i.e. $-l \le m < l$

If $m = 0$, then

$$\langle L_x^2 \rangle = \frac{1}{2}[l(l+1)]\,\hbar^2$$

If $m = \pm l$, then

$$\langle L_x^2 \rangle = \frac{1}{2}[l(l+1) - l^2]\,\hbar^2$$
$$\langle L_x^2 \rangle = \frac{1}{2}\, l\,\hbar^2$$
$$\frac{1}{2}\, l\hbar^2 \le \langle L_x^2 \rangle < \frac{l}{2}(l+1)\,\hbar^2$$

57. (B) $V(r, \theta, \phi) = F(r)\cos\theta$ ...(1)

We know that

$$V(r,\theta) = \sum\left(a_l r^l + \frac{b_l}{r^{l+1}}\right) p_l \cos\theta$$

Since $V(r, \theta)$ contains $\cos\theta$, so $l = 1$

$$V(r,\theta) = \sum\left(a_1 r^n + \frac{b_1}{r^{1+1}}\right) P_1(\cos\theta)$$
$$V(r,\theta) = \left(ar + \frac{b}{r^2}\right)\cos\theta \quad ...(2)$$

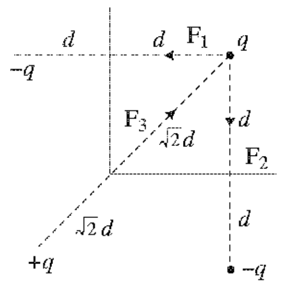
From equation (1) and (2)

$$F(r) = ar + \frac{b}{r^2}$$

58. (B) $E = E_0 \cos[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\hat{k}$

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega} = \frac{|K|\,\hat{k}\,\vec{E}}{1000\,\pi}$$
$$|k| = \pi\sqrt{\frac{9+16}{100}} = \frac{\pi\cdot 5}{10} = \frac{\pi}{2}$$
$$\vec{B} = \frac{\pi}{2}\left(\frac{3\hat{x} + 4\hat{y}}{10}\right) \times \frac{\cos\pi}{1000\,\pi}$$
$$[(0{\cdot}3\hat{x} + 0{\cdot}4\hat{y} - 1000t)\,\hat{k}$$
$$= \frac{10^{-4}}{2}(4\hat{i} - 3\hat{j})\,E_0\cos$$
$$[\pi(0{\cdot}3x + 0{\cdot}4y - 1000t)]\,(4\hat{i} - 3\hat{j})$$

59. (B)



$$F_1 = F_2 = \frac{Kq^2}{4d^2}$$
$$F_3 = K\frac{q^2}{(2\sqrt{2}\,d)^2} = \frac{Kq^2}{8d^2}$$
$$F_{net} = \sqrt{F_1^2 + F_2^2} - F_3$$

towards the corner

$$F_{net} = \frac{Kq^2}{2\sqrt{2}} - \frac{Kq^2}{8d^2}$$
$$= \frac{Kq^2}{8d^2}[2\sqrt{2} - 1]$$

F_{net}, in units of $\frac{1}{4\pi\varepsilon_0}$

$$F_{net} = \frac{q^2}{8d^2}[2\sqrt{2} - 1]$$

towards the corner.

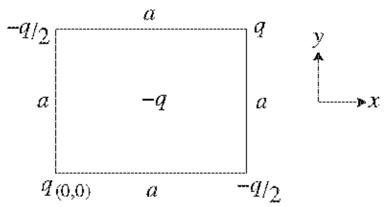
60. (D) Monopole moment = Sum of the charge

$$= -\frac{q}{2} + q - \frac{q}{2} + q - q = 0$$

Dipole moment = $\sum q_i r_i$

Here monopole moment is zero

So we can take origin at any point. It is arbitary choice



Dipole moment = $q \times (0\hat{i} + 0\hat{j}) - \frac{q}{2} a\hat{i} + qa(\hat{i} + \hat{j}) - \frac{q}{2} a(\hat{j}) - q\frac{a}{2}(\hat{i} + \hat{j}) = 0$

It is quadrupole

Potential due to quadrupole is proportional to $\frac{1}{r^3}$

$$\frac{V(2r)}{V(r)} = \frac{\left(\frac{1}{2r}\right)^3}{\frac{1}{r^3}} = \frac{1}{8}$$

61. (A) $$E = \frac{P^2}{2m} + \frac{1}{2}Ar^2$$

$$\frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{P_z^2}{2m} + \frac{1}{2}A(x^2 + y^2 + z^2)$$

$$\downarrow \quad \downarrow \quad \downarrow \qquad \downarrow \quad \downarrow \quad \downarrow$$

$$\frac{1}{2}kT \quad \frac{1}{2}kT \quad \frac{1}{2}kT \qquad \frac{1}{2}kT \quad \frac{1}{2}kT \quad \frac{1}{2}kT$$

$$E = 6 \times \frac{1}{2}NkT$$

$$= 3NkT$$

62. (A)

63. (C) $\sqrt{T_1 T_2}$

64. (D) 65. (A)

66. (D)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	X (0)	X (1)	(3)	X (2)
$\bar{A}B$	(4)	1 (5)	1 (7)	(6)
AB	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$	1 (8)	(9)	(11)	1 (10)

$$F = BD + A\bar{B}\bar{D} + \bar{A}\bar{B}\bar{D}$$

$$= BD + \bar{B}\bar{D}(A + \bar{A})$$

$$F = BD + \bar{B}\bar{D}$$

67. (C) $V_{pert} = aL^2$

$J = L + S \qquad (S = 0)$

$J = L$

So $V_{pert} = aJ^2 = al(l+1)\hbar^2$

For 2P state $l = 1$

$V_{pert} = a \cdot 1(l+1)\hbar^2$

$= 2a\hbar^2$

68. (D) 69. (C) 70. (C)

71. (D) $E(K) = E_0 - B(\cos K_x a + \cos K_y a + \cos K_z a)$

$$= E_0 - B\left(1 - \frac{K_x^2 a^2}{2} + 1 - \frac{K_y^2 a^2}{2} + 1 - \frac{K_z^2 a^2}{2}\right)$$

$$= E_0 - B\left[3 - \left(\frac{K_x^2 + K_y^2 + K_z^2}{2}\right)a^2\right]$$

$$E(K) = E_0 - B\left[3 - \frac{K^2 a^2}{2}\right]$$

$$mx = \frac{\hbar^2}{2\frac{Ba^2}{2}} = \frac{\hbar^2}{Ba^2}$$

72. (A) 73. (D) 74. (C)

75. (A) ${}^{7}_{3}Li$

$P = 3 \qquad n = 4$

$(1S_{1/2})^2 \quad (P_{3/2})^1$

$J = 3/2, \quad P = (-1)^1 = -1$

$$\boxed{J^P = \frac{3^-}{2}}$$

Physical Sciences
CSIR-UGC NET/JRF Exam.
Solved Paper

June 2014
Physical Sciences

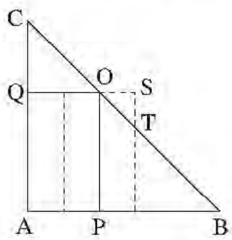
PART A

1. Find the missing letter

A	B	C	D
F	I	L	O
K	P	U	Z
P	W	D	?

(A) P (B) K
(C) J (D) L

2. Consider a right-angled triangle ABC where AB = AC = 3. A rectangle APOQ is drawn inside it, as shown, such that the height of the rectangle is twice its width. The rectangle is moved horizontally by a distance 0·2 as shown schematically in the diagram (not to scale).



What is the value of the ratio $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta OST}$?

(A) 625 (B) 400
(C) 225 (D) 125

3. 80 gsm paper is cut into sheets of 200 mm × 300 mm size and assembled in packets of 500 sheets. What will be the weight of a packet ? (gsm = g/m^2)

(A) 1·2 kg (B) 2·4 kg
(C) 3·6 kg (D) 4·8 kg

4. Three indentical flat equilateral-triangular plates of side 5 cm each are placed together such that they form a trapezium. The length of the longer of the two parallel sides of this trapezium is—

(A) $5\sqrt{\frac{3}{4}}$ cm (B) $5\sqrt{2}$ cm
(C) 10 cm (D) $10\sqrt{3}$ cm

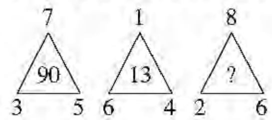
5. An archer climbs to the top of a 10 m high building and aims at a bird atop a tree 17 m away. The line of sight from the archer to the bird makes an angle of 45° to the horizontal. What is the height of the tree ?

(A) 17 m (B) 27 m
(C) 37 m (D) 47 m

6. Consider the set of numbers $\{17^1, 17^2, \ldots, 17^{300}\}$. How many of these numbers end with the digit 3 ?

(A) 60 (B) 75
(C) 100 (D) 150

7. Find the missing number in the triangle.



(A) 16 (B) 96
(C) 50 (D) 80

8. The time gap between the two instants, one before and one after 12·00 noon, when the angle between the hour hand and the minute hand is 66°, is—

(A) 12 minute (B) 16 minute
(C) 18 minute (D) 24 minute

9. A merchant buys equal number of shirts and trousers and pays ₹ 38000. If the cost of 3 shirts is ₹ 800 and that of a trouser is ₹ 1000, then how many shirts were bought ?

(A) 60 (B) 30
(C) 15 (D) 10

10. In the growing years of a child, the height increases as the square root of the age while the weight increases in direct proportion to the age. The ratio of the weight to the square of the height in this phase of growth—
(A) is constant
(B) reduces with age
(C) increases with age
(D) is constant only if the weight and height at birth are both zero

11. In 450 g of pure coffee powder 50 g of chicory is added. A person buys 100 g of this mixture and adds 5 g of chicory to that. What would be the rounded-off percentage of chicory in this final mixture ?
(A) 10 (B) 5
(C) 14 (D) 15

12. Suppose in a box there are 20 red, 30 black, 40 blue and 50 white balls. What is the minimum number of balls to be drawn, without replacement, so that you are certain about getting 4 red, 5 black, 6 blue and 7 white balls ?
(A) 140 (B) 97
(C) 104 (D) 124

13. Suppose

$$x \Delta y = (x-y)^2$$
$$x \text{ o } y = (x+y)^2$$
$$x * y = (x \times y)^{-1}$$
$$x \cdot y = x \times y$$

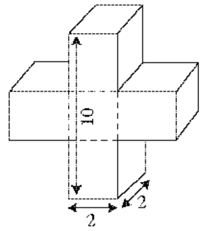
+, – and × have their usual meanings. What is the value of
$\{(197 \text{ o } 315) - (197 \Delta 315)\} \cdot (197 * 315)$?
(A) 118 (B) 512
(C) 2 (D) 4

14. Students in group A obtained the following marks : 40, 80, 70, 50, 60, 90, 30. Students in group B obtained 40, 80, 35, 70, 85, 45, 50, 75, 60 marks. Define
dispersion (D) = (maximum marks – minimum marks), and

$$\text{relative dispersion (RD)} = \frac{\text{dispersion}}{\text{mean}}$$

Then,
(A) RD of group A = RD of group B
(B) RD of group A > RD of group B
(C) RD of group A < RD of group B
(D) D of group A < D of group B

15. The following diagram shows 2 perpendicularly inter-grown prismatic crystals (twins) of identical shape and size. What is the volume of the object shown (units are arbitrary) ?



(A) 60 (B) 65
(C) 72 (D) 80

16. If A × B = 24, B × C = 32, C × D = 48 then A × D—
(A) cannot be found (B) is a perfect square
(C) is a perfect cube (D) is odd

17. Suppose n is a positive integer. Then $(n^2 + n)(2n + 1)$—
(A) may not be divisible by 2
(B) is always divisible by 2 but may not be divisible by 3
(C) is always divisible by 3 but may not be divisible by 6
(D) is always divisible by 6

18. There is a train of length 500 m, in which a man is standing at the rear end. At the instant the rear end crosses a stationary observer on a platform, the man starts walking from the rear to the front and the front to the rear of the train at a constant speed of 3 km/hr. The speed of the train is 80 km/hr. The distance of the man from the observer at the end of 30 minutes is—
(A) 41·5 km (B) 40·5 km
(C) 40·0 km (D) 41·0 km

19. A rectangular area of sides 9 and 6 units is to be covered by square tiles of sides 1, 2 and 5 units. The minimum number of tiles needed for this is—
(A) 3 (B) 11
(C) 12 (D) 15

20. If all horses are donkeys, some donkeys are monkeys, and some monkeys are men, then which statement must be true ?
    (A) All donkeys are men
    (B) Some donkeys may be men
    (C) Some horses are men
    (D) All horses are also monkeys

PART B

21. One gram of salt is dissolved in water that is filled to a height of 5 cm in a beaker of diameter 10 cm. The accuracy of length measurement is 0·01 cm while that of mass measurement is 0·01 mg. When measuring the concentration C, the fractional error $\Delta C/C$ is—
    (A) 0·8% (B) 0·14%
    (C) 0·5% (D) 0·28%

22. A system can have three energy levels : E = 0, $\pm\varepsilon$. The level E = 0 is doubly degenerate, while the others are non-degenerate. The average energy at inverse temperature β is—
    (A) $-\varepsilon \tan h(\beta\varepsilon)$ (B) $\dfrac{\varepsilon(e^{\beta\varepsilon} - e^{-\beta\varepsilon})}{(1 + e^{\beta\varepsilon} + e^{-\beta\varepsilon})}$
    (C) zero (D) $-\varepsilon \tan h\left(\dfrac{\beta\varepsilon}{2}\right)$

23. For a particular thermodynamic system the entropy S is related to the internal energy U and volume V by

$$S = c\, U^{3/4} V^{1/4}$$

where c is a constant. The Gibbs potential G = U – TS + pV for this system is—
    (A) $\dfrac{3pU}{4T}$ (B) $\dfrac{cU}{3}$
    (C) zero (D) $\dfrac{US}{4V}$

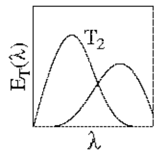
24. An op amp-based voltage follower—
    (A) is useful for converting a low impedance source into a high impedance source
    (B) is useful for converting a high impedance source into a low impedance source
    (C) has infinitely high closed loop output impedance
    (D) has infinitely high closed loop gain

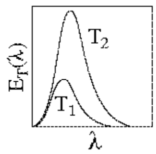
25. A particle of mass m in three dimensions is in the potential

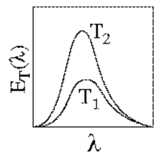
$$V(r) = \begin{cases} 0 & r < a \\ \infty & r \geq a \end{cases}$$

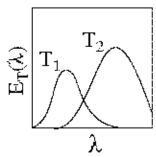
Its ground state energy is—
    (A) $\dfrac{\pi^2\hbar^2}{2ma^2}$ (B) $\dfrac{\pi^2\hbar^2}{ma^2}$
    (C) $\dfrac{3\pi^2\hbar^2}{2ma^2}$ (D) $\dfrac{9\pi^2\hbar^2}{2ma^2}$

26. Which of the graphs below gives the correct qualitative behaviour of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and T_2 ($T_1 < T_2$) ?

(A)


(B)


(C)


(D)


27. Given that $\hat{p}_r = i\hbar\left(\dfrac{\partial}{\partial r} + \dfrac{1}{r}\right)$, the uncertainty Δp_r in the ground state

$$\psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

of the hydrogen atom is—
    (A) $\dfrac{\hbar}{a_0}$ (B) $\dfrac{\sqrt{2}\hbar}{a_0}$
    (C) $\dfrac{\hbar}{2a_0}$ (D) $\dfrac{2\hbar}{a_0}$

28. An RC network produces a phase-shift of 30°. How many such RC networks should be cascaded together and connected to a Common Emitter amplifier so that the final circuit hebaves as an oscillator ?
    (A) 6 (B) 12
    (C) 9 (D) 3

29. The free energy F of a system depends on a thermodynamic variable Ψ as

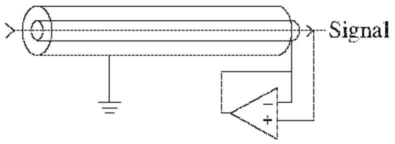
$$F = -a\Psi^2 + b\Psi^6$$

with $a, b > 0$. The value of Ψ, when the system is in thermodynamic equilibrium, is—

(A) zero (B) $\pm\left(\frac{a}{6b}\right)^{1/4}$

(C) $\pm\left(\frac{a}{3b}\right)^{1/4}$ (D) $\pm\left(\frac{a}{b}\right)^{1/4}$

30. The inner shield of a triaxial conductor is driven by an (ideal) op-amp follower circuit as shown. The effective capacitance between the signal-carrying conductor and ground is—



(A) unaffected (B) doubled

(C) halved (D) made zero

31. Consider a system of two non-interacting identical fermions, each of mass m in an infinite square well potential of width a. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy $E = \frac{5\pi^2\hbar^2}{2ma^2}$ is—

(A) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(B) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(C) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right) - \sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(D) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right) - \sin\left(\frac{\pi x_2}{a}\right)\cos\left(\frac{\pi x_1}{a}\right)\right]$

32. A particle of mass m in the potential $V(x, y) = \frac{1}{2} m\omega^2 (4x^2 + y^2)$, is in an eigenstate of energy $E = \frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is—

(A) $y \exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]$

(B) $x \exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]$

(C) $y \exp\left[-\frac{m\omega}{2\hbar}(x^2 + y^2)\right]$

(D) $xy \exp\left[-\frac{m\omega}{2\hbar}(x^2 + y^2)\right]$

33. A particle of mass m and coordinate q has the Lagrangian

$$L = \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$$

where λ is a constant. The Hamiltonian for the system is given by—

(A) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2m^2}$

(B) $\frac{p^2}{2(m - \lambda q)}$

(C) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2(m - \lambda q)^2}$

(D) $\frac{p\dot{q}}{2}$

34. If $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by $z = 1$, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}\cdot\vec{dl}$ is—

(A) $\pi/2$ (B) π

(C) $\pi/4$ (D) 0

35. Given

$\sum_{n=0}^{\infty} P_n(x)t^n = (1 - 2xt + t^2)^{-1/2}$

for $|t| < 1$, the value of $P_5(-1)$ is—

(A) 0·26 (B) 1

(C) 0·5 (D) -1

36. A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance $d/2$ from the plane. At what distance from the plane will the particle reach the speed $2u$?

(A) $d/6$ (B) $d/3$

(C) $d/4$ (D) $d/5$

37. Consider the matrix—

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are—

(A) $-5, -2, 7$ (B) $-7, 0, 7$

(C) $-4i, 2i, 2i$ (D) $2, 3, 6$

38. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

with the initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$. The solution $x(t)$ attains its maximum value when t is—

(A) 1/2 (B) 1
(C) 2 (D) ∞

39. A light source is switched on and off at a constant frequency f. An observer moving with a velocity u with respect to the light source will observe the frequency of the switching to be—

(A) $f\left(1-\frac{u^2}{c^2}\right)^{-1}$ (B) $f\left(1-\frac{u^2}{c^2}\right)^{-1/2}$
(C) $f\left(1-\frac{u^2}{c^2}\right)$ (D) $f\left(1-\frac{u^2}{c^2}\right)^{1/2}$

40. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral

$$\oint_c \frac{dz}{\sin^2 z}$$

is—

(A) ∞ (B) $2\pi i$
(C) 0 (D) πi

41. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be—

(A) 0° to the vertical and $\sqrt{3}$T
(B) 30° to the vertical and T/2
(C) 60° to the vertical and T/$\sqrt{2}$
(D) 0° to the vertical and T/$\sqrt{3}$

42. Consider an electromagnetic wave at the interface between two homogeneous dielectric media of dielectric constants ε_1 and ε_2. Assuming $\varepsilon_2 > \varepsilon_1$ and no charges on the surface, the electric field vector $\vec{E}$ and the displacement vector $\vec{D}$ in the two media satisfy the following inequalities—

(A) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$
(B) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$
(C) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$
(D) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

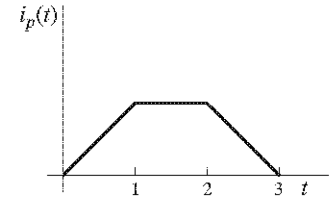
43. If the electrostatic potential in spherical polar co-ordinates is

$$\varphi(r) = \varphi_0 e^{-r/r_0}$$

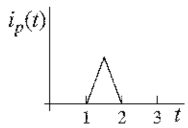
where φ_0 and r_0 are constants, then the charge density at a distance $r = r_0$ will be

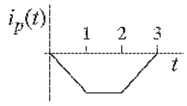
(A) $\frac{\varepsilon_0 \varphi_0}{e r_0^2}$ (B) $\frac{e\varepsilon_0 \varphi_0}{2 r_0^2}$
(C) $-\frac{\varepsilon_0 \varphi_0}{e r_0^2}$ (D) $-\frac{2e\varepsilon_0 \varphi_0}{r_0^2}$

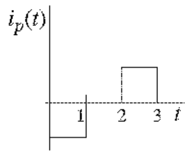
44. A current i_p flows through the primary coil of a transformer. The graph of $i_p(t)$ as a function of time t is shown in the figure below.

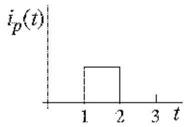


Which of the following graphs represents the current i_S in the secondary coil ?

(A)


(B)


(C)


(D)


45. A time-dependent current $I(t) = Kt\hat{z}$ (where K is a constant) is switched on at $t = 0$ in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance a from the wire is given (for time $t > a/c$) by

(A) $\hat{z}\,\frac{\mu_0 K}{4\pi c}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{ct - \sqrt{a^2+z^2}}{(a^2+z^2)^{1/2}}$

(B) $\hat{z}\,\frac{\mu_0 K}{4\pi}\int_{-ct}^{ct} dz \frac{t}{(a^2+z^2)^{1/2}}$

(C) $\hat{z}\,\frac{\mu_0 K}{4\pi c}\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{(a^2+z^2)^{1/2}}$

(D) $\hat{z}\,\frac{\mu_0 K}{4\pi}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{t}{(a^2+z^2)^{1/2}}$

PART C

46. The pressure of a nonrelativistic free Fermi gas in three-dimensions depends, at T = 0, on the density of fermions n as—

(A) $n^{5/3}$ (B) $n^{1/3}$

(C) $n^{2/3}$ (D) $n^{4/3}$

47. A double slit interference experiment uses a laser emitting light of two adjacent frequencies ν_1 and ν_2 ($\nu_1 < \nu_2$). The minimum path difference between the interfering beams for which the interference pattern disappears is—

(A) $\frac{c}{\nu_2+\nu_1}$ (B) $\frac{c}{\nu_2-\nu_1}$

(C) $\frac{c}{2(\nu_2-\nu_1)}$ (D) $\frac{c}{2(\nu_2+\nu_1)}$

48. The recently-discovered Higgs boson at the LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are 125 GeV/c^2 and 90 GeV/c^2 respectively, and the decaying Higgs particle is at rest, the energy of the photon will approximately be—

(A) $35\sqrt{3}$ GeV (B) 35 GeV

(C) 30 GeV (D) 15 GeV

49. A permanently deformed even-even nucleus with $J^p = 2^+$ has rotational energy 93 keV. The energy of the next excited state is—

(A) 372 keV (B) 310 keV

(C) 273 keV (D) 186 keV

50. How much does the total angular momentum quantum number J change in the transition of Cr ($3d^6$) atom as it ionizes to Cr^{2+} ($3d^4$) ?

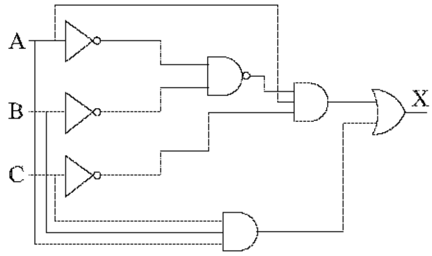
(A) Increases by 2

(B) Decreases by 2

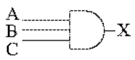
(C) Decreases by 4

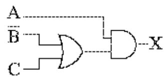
(D) Does not change

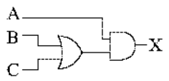
51. For the logic circuit shown in the figure below :

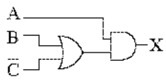


a simplified equivalent circuit is—

(A) 

(B) 

(C) 

(D) 

52. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is—

(A) 10 (B) 3

(C) 6 (D) 9

53. A particle in the infinite square well

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x) = \begin{cases} A \sin^3\left(\frac{\pi x}{a}\right) & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

The expectation value of the energy of the particle is—

(A) $\frac{5\hbar^2\pi^2}{2ma^2}$ (B) $\frac{9\hbar^2\pi^2}{2ma^2}$

(C) $\frac{9\hbar^2\pi^2}{10ma^2}$ (D) $\frac{\hbar^2\pi^2}{2ma^2}$

54. The average local internal magnetic field acting on an Ising spin is $H_{int} = \alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_C, the magnetic susceptibility at zero external field is proportional to (k_B is the Boltzmann constant)—

(A) $k_B T - \alpha$ (B) $(k_B T + \alpha)^{-1}$

(C) $(k_B T - \alpha)^{-1}$ (D) $\tanh(k_B T + \alpha)$

55. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps ?

(A) $\frac{3}{8}$ (B) $\frac{5}{16}$

(C) $\frac{1}{4}$ (D) $\frac{1}{16}$

56. Consider an electron in a b.c.c. liattice with lattice constant a. A single particle wave-function that satisfies the Bloch theorem will have the form $f(\vec{r})\exp(i\vec{k}.\vec{r})$, with $f(\vec{r})$ being—

(A) $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$

(B) $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

(C) $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$

(D) $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

57. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation, by

$$\varepsilon(k) = -4\varepsilon_0\left[\cos\frac{k_xa}{2}\cos\frac{k_ya}{2}+\cos\frac{k_ya}{2}\cos\frac{k_za}{2}+\cos\frac{k_za}{2}\cos\frac{k_xa}{2}\right]$$

where a is the lattice constant and ε_0 is a constant with the dimension of energy.

The x-component of the velocity of the electrons at $\left(\frac{\pi}{a}, 0, 0\right)$ is—

(A) $-2\varepsilon_0 a/\hbar$ (B) $2\varepsilon_0 a/\hbar$

(C) $-4\varepsilon_0 a/\hbar$ (D) $4\varepsilon_0 a/\hbar$

58. The following data is obtained in an experiment that measures the viscosity η as a function of molecular weight M for a set of polymers.

M (Da)	η (kPa-s)
990	0·28 ± 0·03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is—

(A) $\eta \sim M^{4/9}$ (B) $\eta \sim M^{3/2}$

(C) $\eta \sim M^2$ (D) $\eta \sim M^3$

59. The integral $\int_0^1 \sqrt{x}\,dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is—

(A) 0·683 (B) 0·667

(C) 0·657 (D) 0·638

60. In a classical model, a scalar (spin-0) meson consists of a quark and an antiquark bound by a potential

$$V(r) = ar + \frac{b}{r}$$

where $a = 200$ MeV fm^{-1} and $b = 100$ MeV fm. If the masses of the quark and antiquark are negligible, the mass of the meson can be estimated as approximately

(A) 141 MeV/c^2 (B) 283 MeV/c^2

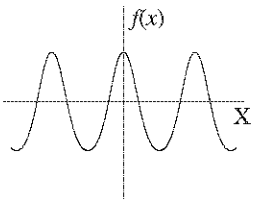
(C) 353 MeV/c^2 (D) 425 MeV/c^2

61. Let $y = \frac{1}{2}(x_1 + x_2) - \mu$, where x_1 and x_2 are independent and identically distributed. Gaussian random variables of mean μ and standard deviation σ. Then $\langle y^4\rangle/\sigma^4$ is—

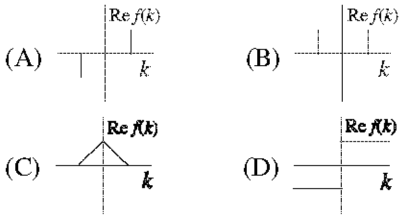
(A) 1 (B) 3/4

(C) 1/2 (D) 1/4

62. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below



Which of the following graphs represents the real part of its Fourier transform ?



63. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relations—

(A) $[A, B] = B + C, [B, C] = 0, [C, A] = B + C$

(B) $[A, B] = C, [B, C] = A, [C, A] = B$

(C) $[A, B] = B, [B, C] = 0, [C, A] = A$

(D) $[A, B] = C, [B, C] = 0, [C, A] = B$

64. The function $\Phi(x, y, z, t) = \cos(z - v\,t) + \text{Re}(\sin(x + iy)$ satisfies the equation—

(A) $\frac{1}{v^2}\frac{\partial^2 \Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$

(B) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi$

(C) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

(D) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

65. The coordinates and momenta x_i, p_i $(i = 1, 2, 3)$ of a particle satisfy the canonical Poisson bracket relations $\{x_i, p_j\} = \delta_{ij}$. If $C_1 = x_2p_3 + x_3p_2$ and $C_2 = x_1p_2 - x_2p_1$ are constants of motion, and if

$C_3 = \{C_1, C_2\} = x_1p_3 + x_3p_1$, then

(A) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = C_2$

(B) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = -C_2$

(C) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = C_2$

(D) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = -C_2$

66. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q = q^2$ and $P = p/2q$. The corresponding time-independent generating function is—

(A) P/q^2 (B) q^2P

(C) q^2/P (D) qP^2

67. The time evolution of a one-dimensional dynamical system is described by

$$\frac{dx}{dt} = -(x + 1)(x^2 - b^2).$$

If this has one stable and two unstable fixed points, then the parameter b satisfies—

(A) $0 < b < 1$ (B) $b > 1$

(C) $b < -1$ (D) $b = 2$

68. A charge $(-e)$ is placed in vacuum at the point $(d, 0, 0)$, where $d > 0$. The region $x \le 0$ is filled uniformly with a metal. The electric field at the point $\left(\frac{d}{2}, 0, 0\right)$ is—

(A) $-\frac{10e}{9\pi\varepsilon_0 d^2}(1, 0, 0)$

(B) $\frac{10e}{9\pi\varepsilon_0 d^2}(1, 0, 0)$

(C) $\frac{e}{\pi\varepsilon_0 d^2}(1, 0, 0)$

(D) $-\frac{e}{\pi\varepsilon_0 d^2}(1, 0, 0)$

69. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to—

(A) 0·60 (B) 0·90

(C) 0·16 (D) 0·32

70. A beam of light of frequency ω is reflected from a dielectric-metal interface at normal incidence. The refractive index of the dielectric medium is n and that of the metal is $n_2 = n(1 + i\rho)$. If the beam is polarised parallel to the interface, then the phase change experienced by the light upon reflection is—

(A) $\tan(2/\rho)$ (B) $\tan^{-1}(1/\rho)$

(C) $\tan^{-1}(2/\rho)$ (D) $\tan^{-1}(2\rho)$

71. The scattering amplitude $f(\theta)$ for the potential $V(r) = \beta e^{-\mu r}$, where β and μ are positive constants, is given, in the Born approximation, by

(in the following $b = 2k \sin\frac{\theta}{2}$ and $E = \frac{\hbar k^2}{2m}$)

(A) $-\frac{4m\beta\mu}{\hbar^2(b^2+\mu^2)^2}$ (B) $-\frac{4m\beta\mu}{\hbar^2 b^2(b^2+\mu^2)}$

(C) $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$ (D) $-\frac{4m\beta\mu}{\hbar^2(b^2+\mu^2)^3}$

72. The ground state eigenfunction for the potential $V(x) = -\delta(x)$, where $\delta(x)$ is the delta function, is given by $\psi(x) = Ae^{-\alpha|x|}$, where A and $\alpha > 0$ are constants. If a perturbation $H' = bx^2$ is applied, the first order correction to the energy of the ground state will be—

(A) $\frac{b}{\sqrt{2}\alpha^2}$ (B) $\frac{b}{\alpha^2}$

(C) $\frac{2b}{\alpha^2}$ (D) $\frac{b}{2\alpha^2}$

73. A thin, infinitely long solenoid placed along the z-axis contains a magnetic flux ϕ. Which of the following vector potentials corresponds to the magnetic field at an arbitrary point (x, y, z) ?

(A) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2}, 0\right)$

(B) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2+z^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2+z^2}, 0\right)$

(C) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, 0\right)$

(D) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x}{x^2+y^2}, \frac{\phi}{2\pi}\frac{y}{x^2+y^2}, 0\right)$

74. The van der Walls' equation of state for a gas is given by

$$\left(P + \frac{a}{v^2}\right)(V - b) = RT$$

where P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by—

(A) $\frac{a}{9b}$ (B) $\frac{a}{27b^2}$

(C) $\frac{8a}{27bR}$ (D) $3b$

75. An electromagnetically-shielded room is designed so that at a frequency $\omega = 10^7$ rad/s the intensity of the external radiation that penetrates the room is 1% of the incident radiation. If $\sigma = \frac{1}{2\pi} \times 10^6\ (\Omega\,m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that ln 10 = 2·3)

(A) 4·60 mm (B) 2·30 mm

(C) 0·23 mm (D) 0·46 mm

Answers with Explanations

1. (B) In first line

A B C D difference 1

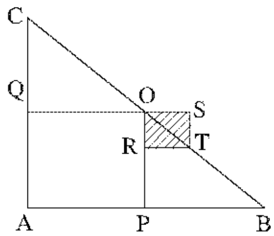
F I L O difference 2

K P U Z difference 4

P W D ? difference 6

So Ans. K

2. (C)



ΔOST is similar for actually congruent to ΔORT.

ΔORT similar to ΔCAB

Area of ORT $= \frac{1}{2} \times 0{\cdot}2 \times 0{\cdot}2$

Area of ABC $= \frac{1}{2} \times 3 \times 3$

$$\text{Ratio} = \frac{\frac{1}{2} \times 3 \times 3}{\frac{1}{2} \times 0{\cdot}2 \times 0{\cdot}2} = 225$$

3. (B) Area of one sheet

$= 200 \times 10^{-3} \times 300 \times 10^{-3} m^2$

$= 6 \times 10^{-2} m^2$

Area of 500 sheet

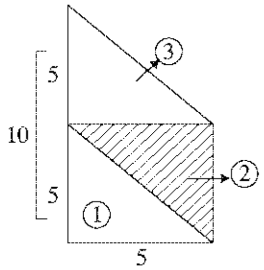
$= 6 \times 10^{-2} m^2 \times 5 \times 10^2$

$= 30\ m^2$

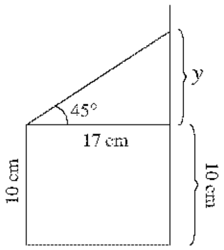
$1 m^2 = 80$ gm

$$30\text{m}^2 = 80 \times 30 = 2400 \text{ gm} = 2{\cdot}4 \text{ kg}$$

4. (C)



5. (B)



$$\frac{y}{17} = \tan 45° = 1$$

$$y = 17 \text{ cm}$$

$$\text{Height of tree} = y + 10 = 17 + 10 = 27 \text{ m}$$

6. (B) $17^3 = 491(3) \leftarrow$ Last digit is 3

$17^7 = 41033867(3) \leftarrow$ Last digit is 3

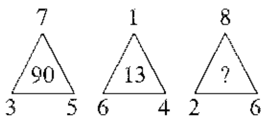
different in exponent

$= 7 - 3 = 4$

Numbers which end with digit

$$3 = \frac{300}{4} = 75$$

7. (D)



$$(3 \times 5 \times 7) - (3 + 5 + 7) = 105 - 15 = 90$$

$$(6 \times 4 \times 1) - (6 + 4 + 1) = 24 - 11 = 13$$

$$(2 \times 6 \times 8) - (2 + 6 + 8) = 96 - 16 = \boxed{80}$$

8. (D) $\frac{360}{66} = 10{\cdot}90 \sim 12$ min.

before 12 time : 11·48 min.

after 12 time : 12·12 min.

difference : 12 : 12 $\rightarrow$ 11·48

= 24 min.

9. (B) $nC_s + nC_t = 38000$

$$3C_s = 800$$

$$C_t = 1000$$

$$n\left[\frac{800}{3} + 1000\right] = 38000$$

$$n\left[\frac{3800}{3}\right] = 38000$$

$$\Rightarrow \boxed{n = 30}$$

10. (D) height $\propto x^{1/2}$

height $= k_1 x^{1/2}$

weight $\propto x$

weight $= k_2 x$

$$\frac{\text{weight}}{(\text{height})^2} = \frac{k_2 x}{k_1^2 x^{1/2}} = \frac{k_2}{k_1^2}$$

~ constant if at birth weight and height are both zero.

11. (C) 450 gm coffee + 50 gm chicory = 500 gm of mixture.

100 gm of mixture = 90 gm coffee + 10 gm chicory.

Add 5 gm chicory $\Rightarrow$ 90 gm coffee + 15 gm chicory

$$\% \text{ chicory} = \frac{15}{(15 + 90)} \times 100 = \frac{15}{105} \times 100 = 14{\cdot}28\%$$

12. (D)

13. (D) $\{(197 + 315)^2 - (197 - 315)^2\} \cdot \frac{1}{197 \times 315}$

$$\frac{(197^2 + 315^2 + 2 \times 197 \times 315) - (197^2 + 315^2 - 2 \times 197 \times 315)}{(197 \times 315)}$$

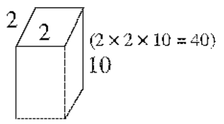
$$= \frac{4 \times 197 \times 315}{197 \times 315} = \boxed{4}$$

14. (B) **Group A** **Group B**

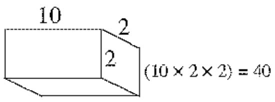
$$D = 90 - 30 \qquad D = 85 - 35$$
$$= 60 \qquad = 50$$
$$\text{Mean} = \frac{420}{7} \qquad \text{Mean} = \frac{540}{9}$$
$$= 60 \qquad = 60$$
$$\text{RD} = \frac{60}{60} = 1 \qquad \text{RD} = \frac{50}{60} = \frac{5}{6}$$

RD of Group A(1) > RD of group B $\left(\frac{5}{6}\right)$.

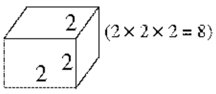
15. (C)



(1)



(2)



(3)

$$\text{Volume} = (1) + (2) - (3)$$
$$= 40 + 40 - 8$$
$$= \boxed{72}$$

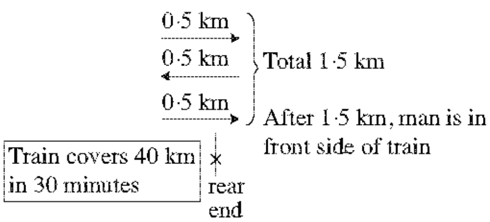
16. (B)
$$AB^2C^2D = 24 \times 32 \times 48$$
$$AD(BC)^2 = (192)^2$$
$$AD = \left(\frac{192}{BC}\right)^2 \text{ perfect square}$$

17. (D) $(n^2 + n)(2n + 1) = 2n^3 + 3n^2 + 1n$

L.C.M. of 1, 2, 3 = 6

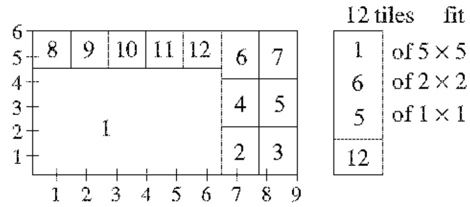
always divisible by 6.

18. (B) In 30 minutes man covers 1·5 km walking from rear to final end

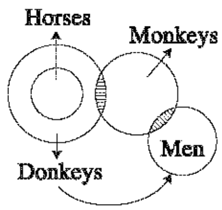


$$\text{To distance} = 40{\cdot}0 + 0{\cdot}5$$
$$= \boxed{40{\cdot}5 \text{ km}}$$

19. (C)



20. (B)



Venn diagram see the overlapping region

Some Donkeys are monkeys (Some monkeys are ⟶ men)

So, some donkeys may be men.

21. (D) If
$$z = xy$$
or
$$z = \frac{x}{y}$$

$$\text{fractional error} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$
$$\frac{\Delta c}{c} = \left[\left(\frac{0{\cdot}01}{1000}\right)^2 + \left(\frac{0{\cdot}01}{5}\right)^2 + \left(\frac{0{\cdot}01}{5}\right)^2\right]^{1/2} \times 100$$
$$= 0{\cdot}28\,\%$$

22. (D)
$$g_s = 1 \text{ ——— } +\varepsilon$$
$$g_2 = 2 \text{ ——— } 0$$
$$g_1 = 1 \text{ ——— } -\varepsilon$$

Partition function
$$z = \sum_i g_i e^{-\beta\varepsilon_i}$$
$$z = e^{\beta\varepsilon} + 2 + e^{-\beta\varepsilon}$$
$$\langle E \rangle = -\frac{\partial}{\partial\beta}\ln z$$
$$= -\frac{\partial}{\partial\beta}\ln\left[2 + e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right]$$
$$= -\left(\frac{\varepsilon e^{\beta\varepsilon} - \varepsilon e^{-\beta\varepsilon}}{2 + e^{\beta\varepsilon} + e^{-\beta\varepsilon}}\right)$$

$$= -\varepsilon\left[\frac{e^{\beta\varepsilon}-e^{-\beta\varepsilon}}{2+\frac{e^{\beta\varepsilon}+e^{-\beta\varepsilon}}{2}\times 2}\right]$$

$$= -\varepsilon\frac{2\sin h\beta\varepsilon}{2[1+\cos h\beta\varepsilon]}$$

$$= -\varepsilon\frac{\sin h\,\beta\varepsilon}{1+\cos h\,\beta\varepsilon}$$

$$= -\varepsilon\frac{2\sin h\frac{\beta\varepsilon}{2}\cos h\frac{\beta\varepsilon}{2}}{1+2\cos h^2\frac{\beta\varepsilon}{2}-1}$$

$$= \boxed{-\varepsilon\tan h\left(\frac{\beta\varepsilon}{2}\right)}$$

23. (C)

$$S = cu^{3/4}V^{1/4}$$

$$G = U - TS + pV$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V$$

$$= C\frac{3}{4}U^{-1/4}V^{1/4}$$

$$= \frac{3}{4}\frac{CU^{3/4}V^{1/4}}{U}$$

$$= \frac{3}{4}\frac{S}{U}$$

$$\boxed{TS = \frac{4}{3}U}$$

$$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_U$$

$$= CU^{3/4}\frac{1}{4}V^{-3/4}$$

$$= \frac{1}{4}\frac{CU^{3/4}V^{1/4}}{V}$$

$$= \frac{5}{4V}$$

$$pV = \frac{1}{4}TS$$

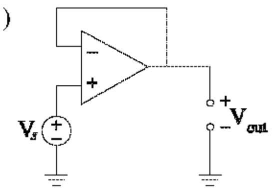
$$= \frac{1}{4}\times\frac{4}{3}U$$

$$= \boxed{\frac{U}{3}}$$

$$\therefore \quad G = U - \frac{4}{3}U + \frac{U}{3} = 0,$$

$$\boxed{G = 0}$$

24. (B)



(Diagram of a voltage follower)

$$\text{Voltage gain} = \frac{V_{out}}{V_s}$$

$$= \frac{V_-}{V_+}$$

$$V_+ = V_-$$

$$= \frac{V_+}{V_+} = 1$$

Voltage follower is a unity gain buffer. The benefit of using a voltage follower is that the high impedance source operated via op-amp allows all of the voltage to be dropped across a low impedance source, due to the fact that voltage follower is a unity gain buffer.

25. (A) $V(r) = 0, \quad r < a$

$\infty, \quad r \geq a$

Potential is sphericaly symmetric

$$\psi(r, \theta, \phi) = R(r)\,Y_{lm}(\theta, \phi)$$

Radial function $R(r)$ satisfies

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\frac{2m}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2}\right]R(r)$$

$$= 0$$

$$\rho = Kr$$

$$\frac{d^2R}{dr^2}+\frac{2}{\rho}\frac{dR}{d\rho}+\left[1-\frac{l\,(l+1)}{\rho^2}\right]R(\rho) = 0$$

$$\rho = Kr, \qquad K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$R_\rho(\rho) = Aj_\rho(\rho) + B\eta_\rho(\rho)$$

$j_\rho(\rho)$ and $\eta_\rho(\rho)$ are spherical Bessel function and spherical Neumann function respectively.

B = 0, otherwise wave function will diverge at $\rho = 0$

$$R_\rho(\rho) = 0 \text{ for } r = a$$

$$\therefore \quad j_\rho(Ka) = 0$$

Roots of this *eqn*. gives energy eigen value.

$$\rho = 0, \qquad j(\rho) = \frac{\sin\rho}{\rho}$$

$$\sin(K_n a) = 0$$

$$K_n a = 0$$

$$K_n = \frac{n\pi}{a}$$

$$E_n \text{ (for } \rho = 0) = \frac{\hbar^2 K_n^2}{2m}$$

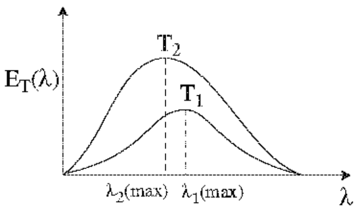
$$= \frac{\hbar^2 n^2\pi^2}{2ma^2}$$

$$E_n (\rho = 0) = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

ground state $n = 1$,

$$\boxed{E_1 = \frac{\pi^2\hbar^2}{2ma^2}}$$

26. (C)



Plank's black body energy density

$$E_T(\lambda) \propto T^4$$

Wien's displacement law :

$$\lambda_{(max)} T = \text{Constant}$$

Since $T_2 > T_1$, $E_{T_2}(\lambda) > E_{T_1}(\lambda)$

The maximum $\lambda_1 >$ maximum λ_2.

27. (A) $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$,

$$\psi_0(r) = \frac{1}{\sqrt{\pi}} e^{-r} \, (a = 1, \hbar = 1)$$

$$p_r\psi_0 = \frac{-i}{\sqrt{\pi}} \cdot e^{-r}\left(1 + \frac{1}{r}\right)$$

$$\langle pr \rangle = \int_0^\infty \psi_0 \, pr \, \psi_0 \, 4\pi r^2 \, dr$$

$$= -4i\int_0^\infty e^{-2r}\left(1 + \frac{1}{r}\right) r^2 dr$$

$$= -4i\int_0^\infty [e^{-2r}(-r^2 + r)dr]$$

$$= -4i\left[-\frac{2!}{(2)^3} + \frac{1!}{(2)^2}\right] = 0$$

$$\langle p_r \rangle = 0$$

$$\hat{p}_r \psi = -\hbar^2 \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)$$

[you can proceed directly from here but involve lot of algebra].

It can be shown that for any nl state of hydrogen atom

$$\langle p_r \rangle_{\eta\rho} = 0$$

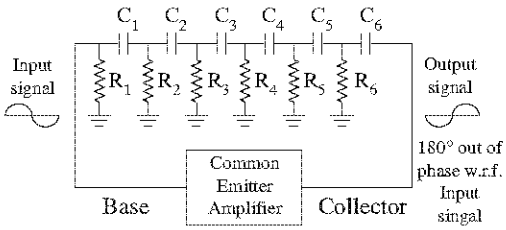
$$(\Delta p_r)_{nl} = \frac{Z}{n}\frac{\hbar}{a_0}\sqrt{1 - \frac{2l(l+1)}{\eta(2l+1)}}$$

[Memorize this formula for future exam.]

Z = Nuclear charge for an one-electron system.

$$(\Delta p_r) = \frac{\hbar}{a_0}$$

28. (A)



For an RC oscillator, we need negative feedback output signal is 180° out of phase with input signal.

No. of RC circuit

$$= \frac{\text{Total phase shift required}}{\text{phase shift produced by each RC circuit}}$$

$$= \frac{180°}{30°} = \boxed{6}$$

29. (C) $F = -a\psi^2 + b\psi^6$

$$\frac{\partial F}{\partial \psi} = -2a\psi + 6b\psi^5 = 0$$

$\psi = 0$, or $\psi[-2a + 66\psi^4] = 0$

$$-2a + 6b\psi^4 = 0$$

$$\psi^4 = \frac{2a}{6b} = \frac{a}{3b}$$

$$\psi = \pm\left(\frac{a}{3b}\right)^{1/4}$$

$$\frac{\partial^2 F}{\partial \psi^2} = -2a + 30b\psi^4$$

$$= \underbrace{(-2a + 6b\psi^4)}_{\text{zero}} + 24b\psi^4$$

$$= 0 + 24b\psi^4$$

$$\frac{\partial^2 F}{\partial \psi^2} = 24b\psi^4$$

If $\psi = 0$, $\frac{\partial^2 F}{\partial \psi^2} = -2a > 0$ (maximum)

$$\psi = \pm\left(\frac{a}{3b}\right)^{1/4},$$

$$\frac{\partial^2 F}{\partial \psi^2} = 24b\left(\frac{a}{3b}\right) = 8a > 0 \text{ (min.)}$$

$\therefore$ for thermodynamic equilibrium

$$\boxed{\psi = \pm\left(\frac{a}{3b}\right)^{1/4}}$$

30. (D)

31. (A) $$E_{n_1} = \frac{n_1^2 \pi^2 \hbar^2}{2ma^2},$$

$$E_{n_2} = \frac{n_2^2 \pi^2 \hbar^2}{2ma^2}$$

$$\psi_{n_1} = \sqrt{\frac{2}{a}} \sin \frac{n_1 \pi x_1}{a},$$

$$\psi_{n_2} = \sqrt{\frac{2}{a}} \sin \frac{n_2 \pi x_2}{a}$$

$$E_{n_1 n_2} = \frac{(n_1^2 + n_2^2)\pi^2 \hbar^2}{2ma^2}$$

$n_1 = 1, n_2 = 2,$

$$E_{12} = \frac{(1^2 + 2^2)\pi^2 \hbar^2}{2ma^2} = \frac{5\pi^2 \hbar^2}{2ma^2}$$

One fermion is in ground state

Second fermion is in first excited state

Ignoring spin, the wavefunction of two fermion must be antisymmetric in variable x_1 and x_2 to satisfy pauli exclusion principle.

$$\boxed{\psi_{n_1 n_2} = \frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]}$$

$$\psi_{n_1 n_2}(x_1, x_2) = -\psi_{n_1 n_2}(x_2, x_1)$$

32. (A) $$V(x, y) = \frac{1}{2} m\omega^2 (4x^2 + y^2),$$

$$E = \frac{5}{2}\hbar\omega.$$

$$H = \left(\frac{p_x^2}{2m} + \frac{1}{2} m (2\omega)^2 x^2\right) + \left(\frac{p_y^2}{2m} + \frac{1}{2} m\omega^2 y^2\right)$$

$$E_{nx} = \left(n_x + \frac{1}{2}\right)(2\hbar\omega),$$

$$E_{ny} = \left(n_y + \frac{1}{2}\right)\hbar\omega$$

$n_x = 0, n_y = 1,$

$$E = \hbar\omega + \frac{3}{2}\hbar\omega = \frac{5}{2}\hbar\omega$$

$n_x = 0$ (ground state),

$n_y = 1$ (first excited state)

$$\psi_0(x) \propto e^{-\frac{m(2m)}{2\hbar}x^2}$$

$$\psi_1(y) \propto e^{-\frac{m\omega}{2\hbar}y^2}$$

$$\boxed{\psi \sim \psi_0(x)\,\psi_1(y) \propto y \exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]}$$

33. (B) $$L = \frac{1}{2} m\dot{q}^2 - \frac{\lambda}{2} q\dot{q}^2$$

$$H = p\dot{q} - L,$$

$$p = \frac{\partial L}{\partial \dot{q}} = m\dot{q} - \lambda q\dot{q}$$

$$p = \dot{q}(m - \lambda q)$$

$$\dot{q} = \frac{p}{m - \lambda q}$$

$$H = p\left(\frac{p}{m - \lambda q}\right) - \frac{1}{2}(m - \lambda q)\left(\frac{p}{m - \lambda q}\right)^2$$

$$= \frac{p^2}{(m - \lambda q)} - \frac{p^2}{2(m - \lambda q)}$$

$$\boxed{H = \frac{p^2}{2(m - \lambda q)}}$$

34. (D) $$\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$$

$$\text{Curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{i} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$= \hat{i}(x - x) - \hat{j}(y - y) + \hat{k}(z - z) = 0$$

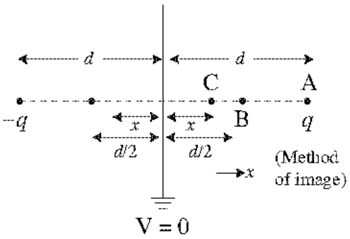
Since curl $\vec{A} = 0$, $\vec{A}$ is a conservative vector field and its line integral around a closed loop is zero. This means work done $\oint_C \vec{A}.\vec{dl}$ around a closed loop is zero.

$$\boxed{\oint_C \vec{A}\cdot\vec{dl} = 0}$$

35. (D) Legendre polynomial are symmetric or antisymmetric

$$P_n(-x) = (-1)^n P_n(x)$$
$$P_n(1) = 1$$
$$P_5(-1) = (-1)^5 P_5(1) = -1$$

36. (D)



U_A = potential energy at

$$A = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d}$$

(Half of usual coulomb potential on only $x > 0$ region contain a non-zero field).

T_A = Kinetic energy at A = 0

U_B = Potential energy at B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2d}$$

T_B = Kinetic energy at B

$$= \frac{1}{2}mu^2$$

U_C = Potential energy at C

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4x}$$

T_C = Kinetic energy at C

$$= \frac{1}{2}m(2u)^2$$

$$\boxed{U_A + T_A = U_B + T_B = U_C + T_C}$$

$$-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d} + 0 = \frac{1}{2}mu^2 - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2d}$$

$(U_A + T_A = U_B + T_B)$

$$\frac{1}{2}mu^2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d} \quad \ldots(1)$$

$$-\frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{2d}\right) + \frac{1}{2}mu^2$$
$$= -\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{4x}\right)^2 + \frac{1}{2}m(2u)^2 \ldots(2)$$

Eliminate u^2 from (1) and (2)

$$-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2d} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d}$$
$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4x} + \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{4d}\right)u$$
$$-\frac{1}{2d} + \frac{1}{4d} = -\frac{1}{4x} + \frac{1}{d}$$
$$\frac{1}{4x} = \frac{1}{2d} - \frac{1}{4d} + \frac{1}{d}$$
$$= \frac{5}{4d}$$
$$\boxed{x = \frac{d}{5}}$$

[Note : If you have taken $U_A = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2d}$, you will get answer as $d/4$, which wrong. The potential energy between a charge q and a conductor plane is half of energy between two charges q and $-q$.]

37. (B) $M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$,

$$\det(M) = -2i(18i^2) + 3i(12i^2) = 0$$

Product of eigen value = det (M) = 0

Trace = Sum of diagonal element = 0.

Sum of eigen value = Trace = 0

only option (B), satisfies these two properties

Eigen value : – 7, 0, + 7

$$\text{Prouct} = -7 \times 0 \times 7 = 0$$
$$\text{Sum} = -7 + 0 + 7 = 0$$

38. (B) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$

Auxillary $eqn.$ $\lambda^2 + 2\lambda + 1 = 0$

$$\lambda = \frac{-2 \pm \sqrt{4-4}}{2} = -1$$

General solution :

$$x(t) = At\,e^{-t} + Be^{-t}$$

$x(0) = 0 \Rightarrow \boxed{B = 0}$

$$x(t) = A t e^{-t}$$

$$\dot{x}(t) = \frac{dx(t)}{dt}$$

$$= A\cdot[e^{-t} - te^{-t}]$$

$$\dot{x}(0) = 1, \Rightarrow 1 = A[e^{-0} - 0e^{-0}], \boxed{A = 1}$$

$$\therefore \quad x(t) = te^{-t}$$

for maximum value, set $\dot{x}(t) = 0$,

$$e^{-t}(1-t) = 0 \quad \boxed{t = 1}$$

Check if it is maximum,

$$\ddot{x}(t) = -e^{-t} - (1-t)e^{-t}$$

$$\ddot{x}(1) = -\frac{1}{e} < 0 \text{ [Implies max.]}$$

39. (D) Dilated time

$$T' = \gamma T$$

$$\frac{1}{f'} = \gamma \frac{1}{f}$$

$$f' = \frac{1}{\gamma} f, \quad \gamma = \text{Lorentz factor}$$

$$\gamma = \frac{1}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}},$$

$$\frac{1}{\gamma} = \left(1 - \frac{u^2}{c^2}\right)^{1/2}$$

$$\boxed{f' = f\left(1 - \frac{u^2}{c^2}\right)^{1/2}}$$

40. (C) $I = \oint_C \frac{dz}{\sin^2 z}$

$$\cos^2 z - \sin^2 z = \cos 2z$$

$$1 - 2\sin^2 z = \cos 2z$$

$$\sin^2 z = \frac{1 - \cos 2z}{2}$$

$$I = \oint_{|z| = \frac{1}{2}} \frac{2dz}{1 - \cos 2z},$$

Poles are at I – cos 2z = 0

$$\cos 2z = 1$$

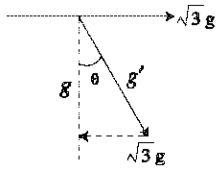
Poles $\quad z = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$z = 0$, lies with in $|z| = \frac{1}{2}$

There is no residue left at the origin.

$$\therefore \quad \boxed{I = 0}$$

41. (C)



Resultant acceleration :

$$g' = \sqrt{g^2 + (\sqrt{3}g)^2}$$

$$= \sqrt{g^2 + 3g^2}$$

$$= 2g$$

$$T' = 2\pi\sqrt{\frac{l}{g'}}$$

$$= 2\pi\sqrt{\frac{l}{2g}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} 2\pi\sqrt{\frac{l}{g}} = \frac{1}{\sqrt{2}} T$$

$$\boxed{T' = \frac{T}{\sqrt{2}}}$$

$$\tan\theta = \frac{\sqrt{3}g}{g} = \sqrt{3}$$

$$= \tan 60°$$

$$\boxed{\theta = 60°}$$

42. (C) $\vec{D} = \varepsilon \vec{E}$

$$\frac{\vec{D}_2}{\vec{D}_1} = \frac{\varepsilon_2}{\varepsilon_1}\frac{\vec{E}_2}{\vec{E}_1}$$

Since $\varepsilon_2 > \varepsilon_1$

$$|\vec{D}_2| > |\vec{D}_1|$$

for equality to hold $|\vec{E}_2| < |\vec{E}_1|$

43. (A) $\nabla^2 V(r) = -\frac{\rho(r)}{\varepsilon_0}$ (Poisson equation)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial\phi}{\partial r}\right) = -\frac{\rho(r)}{\varepsilon_0}$$

$$\phi(r) = \phi_0 e^{-r/r_0},$$

$$\frac{\partial\phi}{\partial r} = -\frac{\phi_0}{r_0} e^{-r/r_0}$$

$$r^2\frac{\partial\phi}{\partial r} = -\frac{\phi_0}{r_0} r^2 e^{-r/r_0}$$

$$\frac{\partial}{\partial r}\left[-\frac{\phi_0}{r_0} r^2 e^{-r/r_0}\right]$$

$$= -\frac{\phi_0}{r_0}\left[e^{-r/r_0}(2r) - \frac{r^2}{r_0} e^{-r/r_0}\right]$$

$\nabla^2 V(r = r_0)$

$$= \left[-\frac{\phi_0}{r_0 r^2}\left[e^{-r/r_0}(2r) - \frac{r^2}{r_0} e^{-r/r_0}\right]\right]$$

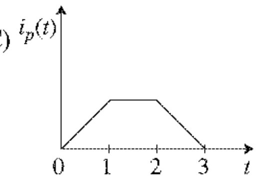
$$= -\frac{\phi_0}{r_0 r_0^2}[2r_0 e^{-1} - r_0 e^{-1}]$$

$$= -\frac{\phi_0}{r_0 r_0^2} r_0 e^{-1} = -\frac{\phi_0}{r_0^2 e}$$

$$-\frac{\phi_0}{r_0^2 e} = -\frac{\rho}{\varepsilon_0}$$

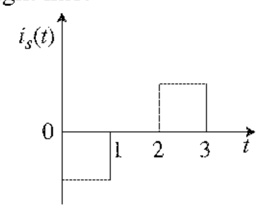
$$\Rightarrow \quad \boxed{\rho = \frac{\phi_0 \varepsilon_0}{r_0^2 e}}$$

44. (C)



$$i_s(t) \propto -\frac{d}{dt} i_p(t)$$

i_s is 180° out of phase with i_p from 0 to 1, $i_p(t)$ is a straight line.

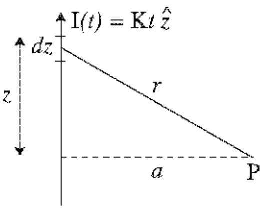


$i_p(t) \propto t$, $\therefore$ $i_s(t)$ = constant

from 1 to 2, $i_p(t)$ is constant in time $i_s(t) = 0$

from 2 to 3, i_p is $\propto -t$ is a constant in upper place.

45. (A)



$$\vec{A} = \frac{\mu_0}{4\pi}\hat{z}\int_{-\infty}^{\infty}\frac{I_r\,dz}{r},$$

I_r = retarded time current.

$$r = \sqrt{a^2 + z^2}$$

$$|z| = \sqrt{c^2 t^2 - a^2}$$

$c^2 t^2 > a^2$ or $t > \frac{a}{c}$ [time like event]

$$ct_r = ct - \sqrt{a^2 + z^2}$$

$$t_r = \frac{ct - \sqrt{a^2 + z^2}}{c}$$

$$I_r = Kt_r$$

$$\vec{A} = \hat{z}\frac{\mu_0 K}{4\pi c}\int_{-\sqrt{c^2t^2 - a^2}}^{\sqrt{c^2t^2 - a^2}}\frac{dz\,[ct - \sqrt{a^2 + z^2}]}{(z^2 + a^2)^{1/2}}$$

46. (A) $U_0(N, V) = \frac{3}{5} NE_f$

$$= \frac{3}{5} N\left[\frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}\right]$$

$$P = -\left(\frac{\partial U_0}{\partial V}\right)_N$$

$$= \frac{3}{5} N \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} N^{2/3}\frac{2}{3} V^{-5/3}$$

$$= \frac{2}{5}\frac{N}{Y} E_f$$

$$= \frac{2}{5} n E_f$$

$n = \frac{N}{V}$, $\boxed{E_f \propto n^{2/3}}$

$\boxed{\rho \propto n \cdot n^{2/3} \propto n^{5/3}}$

47. (C) Calculate laser band width Δv :

$$v = \frac{c}{\lambda}$$

$$\Delta v = \frac{c\,|\Delta\lambda|}{\lambda^2}$$

$$\frac{c}{\Delta v} = \frac{\lambda^2}{\Delta\lambda}$$

$$= \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} (\lambda_1 = \lambda_2 = \lambda)$$

$$\frac{c}{\Delta v} = \frac{1}{\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)},$$

$$\frac{c}{\Delta v} = \frac{c}{\left(\frac{c}{\lambda_2} - \frac{c}{\lambda_1}\right)}$$

$$= \frac{c}{(v_2 - v_1)}$$

Minimum path difference

$$= \frac{1}{2}\frac{c}{\Delta v}$$

$$= \boxed{\frac{c}{2(v_2 - v_1)}}$$

48. (C) $\underset{\underset{125\text{ GeV}}{\downarrow}}{H} = \underset{\underset{90\text{ Gev}}{\downarrow}}{Z} + V,$

$$E_H = E_V + E_Z$$

$$p_\mu^2 = E^2 - c^2p^2$$

$$p_H^2 = p_r^2 - p_z^2$$

$$(E_H^2 - c^2p_H^2) = (E_v^2 - c^2p_v^2) + (E_Z^2 - c^2p_Z^2)$$

Standard four-momenta analyis yield

$$m_\mu^2 c^4 = E_V^2$$

$$E_\gamma = \frac{E_H^2 - E_Z^2}{2E_H}$$

$$= \frac{(125)^2 - (90)^2}{2 \times 125}$$

$$= \boxed{30{\cdot}1 \text{ Gev}}$$

49. (B) even Z, even N nucleide ⇒ first excited state to be 2^+.

8^+ ———— 1116 keV
6^+ ———— 651 keV
4^+ ———— 310 keV
2^+ ———— 93 keV
0^+ ———— 0·0 keV

$$\Delta E = BI(I+1), I = 2$$

$$93 = 6B$$

$$B = \frac{93}{6}$$

$$E(4^+) = B4(4+1) = 20B$$

$$= 20 \times \frac{93}{6}$$

$$= \frac{930}{3} = 310 \text{ keV}$$

50. (C) By Hund's Rule

Cr ($3d^6$)
Cr^{2+} ($3d^4$)

m_L	2	1	0	–1	–2
m_S	⇅	↑	↑	↑	↑

$$L = \Sigma m_L = 4 + 1 + 0 - 1 - 2 = 2$$

$$S = \Sigma m_S = 0 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

This is the case of more than half filled

So $J = L + S = 4$

For Cr^{2+} ($3d^4$)

2	1	0	–1	–2
↑	↑	↑	↑	

$$L = \Sigma m_L = 2 + 1 + 0 - 1 = 2$$

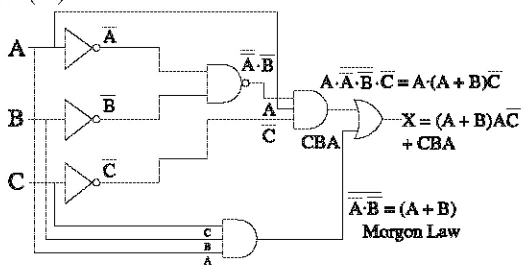
$$S = \Sigma m_S = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

This is the case of less than half filled so

$$J = 2 - 2 = 0$$

So decreases by 4.

51. (D)



$(A + B)\,A\overline{C} + CBA$

$AA\overline{C} + BA\overline{C} + CBA$

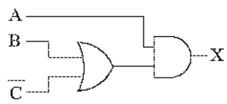
$A\overline{C} + BA\overline{C} + CBA$

$A\overline{C} + BA\overline{C} + BAC$

$A\overline{C} + BA(\overline{C} + C)$ $\qquad \overline{C} + C = 1$

$A\overline{C} + BA$

$A(B + \overline{C})$ is equivalent to



52. (A) **p state** : $l = 1, s = \frac{1}{2}, j = \frac{1}{2}, \frac{3}{2}$

for $j = \frac{1}{2}$, $\quad m_j = -\frac{1}{2}, +\frac{1}{2}$ (2 level)

$j = \frac{3}{2}, \qquad m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ (4 level)

s state : $l = 0,\ s = \frac{1}{2}, j = \frac{1}{2},$

$m_j = -\frac{1}{2}, +\frac{1}{2}$ (2 level)

There are 4 + 2 = 6 level for p state

There are 2 level for s state

Total no. of combinations

$= 6 \times 2 = 12$

Use $\Delta m_j = 0, \pm 1$ rule for dipole allow transition

$\Delta m_j = \pm 2$ not allowed

$\Delta m_j = +2$ arises from

$m_j = \frac{3}{2}$ to $m_j = -\frac{1}{2}$

$= -2$ arises from

$m_j = -\frac{3}{2}$ to $m_j = \frac{1}{2}$ — Not allowed

↑ p-level, S-level

No. of spectral line = 12 – 2 = $\boxed{10}$

53. (C) $\psi(x) = A \sin^3\left(\frac{\pi x}{a}\right)$

$$\langle H \rangle = \frac{\int_{-\infty}^{+\infty} \psi^* \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi\, dx}{\int_{-\infty}^{\infty} \psi^* \psi\, dx}$$

$$\frac{d}{dx}\left(\sin^3\frac{\pi x}{a}\right) = \frac{3\pi}{a}\sin^2\frac{\pi x}{a}\cos\frac{\pi x}{a}$$

$$\frac{d^2}{dx^2}\left(\sin^3\frac{\pi x}{a}\right) = \frac{3\pi}{a}\left[2\frac{\pi}{a}\sin\frac{\pi x}{a}\cos^2\frac{\pi x}{a} - \frac{\pi}{a}\sin^3\frac{\pi x}{a}\right]$$

$$= \frac{3\pi^2}{a^2}\left[2\sin\frac{\pi x}{a} - 3\sin^3\frac{\pi x}{a}\right]$$

$\langle H \rangle =$

$$\frac{-\frac{\hbar^2}{2m}\int_{-\infty}^{\infty}\frac{3\pi^2}{a^2}\left[2\sin\frac{\pi x}{a} - 3\sin^3\frac{\pi x}{a}\right]\sin^3\left(\frac{\pi x}{a}\right)}{\int_{-\infty}^{\infty}\sin^6\frac{\pi x}{a}}$$

$$= -\frac{\hbar^2}{2m}\frac{3\pi^2}{a^2}\frac{\left[2\int_{-\infty}^{\infty}\sin^4\frac{\pi x}{a} - 3\int_{-\infty}^{\infty}\sin^6\frac{\pi x}{a}\right]dx}{\int_{-\infty}^{\infty}\sin^6\frac{\pi x}{a}\,dx}$$

$$= -\frac{\hbar^2}{2m}\left[\frac{2I_1}{I_2} - 3\right]$$

$$I_1 = \int_0^{\infty}\sin^4\frac{\pi x}{a}\,dx = \frac{3a}{8}$$

$$I_2 = \int_0^{a}\sin^6\frac{\pi x}{a}\,dx = \frac{5a}{16}$$

$$\langle H \rangle = -\frac{\hbar^2}{2m}\left[2\left(\frac{3a}{8}\right)\left(\frac{16}{5a}\right) - 3\right]\frac{3\pi^2}{a^2}$$

$$= \boxed{\frac{9}{10}\frac{\pi^2\hbar^2}{ma^2}}$$

54. (C) $M = \chi_P(B_0 + B_E)$,

$BE = \alpha M$

B_0 : applied field causes a fraction magnetization.

χ_p : Paramagnetic region magnetic susceptibility

$$M = \chi_p(B_0 + \alpha M)$$

$$M = \chi_p B_0 + \chi_p \alpha M$$

$$M(1 - \chi_p{}^{\alpha}) = \chi_p B_0$$

$$M = \frac{\chi_p B_0}{1 - \chi_p{}^{\alpha}}$$

Cure law : $\chi_p = \frac{C}{T}$

$$\chi_F = \frac{M}{B_0} = \frac{\chi_p}{1 - \chi_p{}^{\alpha}}$$

χ_F : ferromagnetic region susceptibility

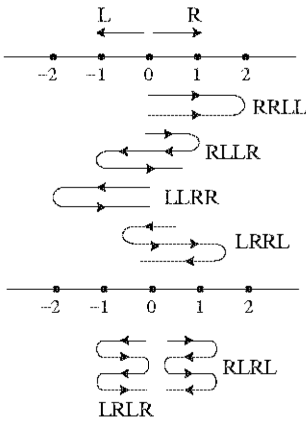
$$= \frac{C}{T\left[1 - \frac{C\alpha}{T}\right]} = \frac{C}{T - C\alpha}$$

$$= \frac{C}{C\left(\frac{T}{C} - \alpha\right)}, \frac{T}{C} \sim K_B T.$$

$$\chi_F \sim \frac{1}{K_B T - \alpha} \sim \boxed{(K_B T - \alpha)^{-1}}$$

(Curie Weiss law)

55. (A) Visual method :



There are only six accepted events as shown to reach O point after 4 steps,

RRLL
RLLR
LLRR
LRRL
RLRL
LRLR

No. of accepted events

$$= 4 + 2 = 6,$$

$$\text{No. of state} = 2^4 = 16$$

$$\text{Sample space} = \text{(No. of options)}$$

$$\text{Probability} = \frac{6}{16} = \boxed{\frac{3}{8}}$$

Short Method

For symmetric random walks probability of reaching origin after n steps

$$= \frac{\binom{n}{n/2}}{2^n}$$

for $n = 4$, Probability

$$= \frac{\binom{4}{2}}{2^4} = \frac{4!}{2!\,16} = \boxed{\frac{3}{8}}$$

56. (B) For b.c.c. lattice, the primitive vertices are

$$\vec{a}_1 = \frac{a}{2}[-\hat{i} + \hat{j} + \hat{k}]$$

$$\vec{a}_2 = \frac{a}{2}[\hat{i} - \hat{j} + \hat{k}]$$

lattice constant = a

$$\vec{a}_3 = \frac{a}{2}[\hat{i} + \hat{j} - \hat{k}]$$

Bloch theorem

$$\psi(\vec{k}, \vec{r}) = u(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$$

$$= f(\vec{r})e^{i\vec{k}\cdot\vec{r}}$$

then $f(\vec{r}) = f(\vec{r} + \vec{a})$,

then $\vec{a}$ is a lattice vector *i.e.*, $f(\vec{r})$ is periodic w.r.t. translational vector $\vec{a}$.

$$f = 1 + \cos\left[\frac{2\pi}{a}(x+y)\right] + \cos\left[\frac{2\pi}{a}(y+z)\right] + \cos\left[\frac{2\pi}{a}(z+x)\right]$$

$$f(\vec{r} + \vec{a}_1) = 1 + \cos\left[\frac{2\pi}{a}(x+y) - \frac{a}{2} + \frac{a}{2}\right] + \cos\left[\frac{2\pi}{a}\left(y + z + \frac{a}{2} + \frac{a}{2}\right)\right] + \cos\left[\frac{2\pi}{a}\left(x + z - \frac{a}{2} + \frac{a}{2}\right)\right]$$

$$= 1 + \cos\left[\frac{2\pi}{a}(x+y) + \cos\left[\frac{2\pi}{a}(y+z+a)\right] + \cos\left[\frac{2\pi}{a}(z+x)\right]\right.$$

$$= 1 + \cos\left[\frac{2\pi}{a}(x+y) + \cos\left[\frac{2\pi}{a}(y+z) + 2\pi\right] + \cos\left[\frac{2\pi}{a}(z+x)\right]\right.$$

$$= 1 + \cos\left[\frac{2\pi}{a}(x+y)\right] + \cos\frac{2\pi}{a}(y+z) + \cos\frac{2\pi}{a}(z+x) = f(\vec{r})$$

$$f(\vec{r} + \vec{a}_2) = 1 + \cos\left[\frac{2\pi}{a}(x+y) + \frac{a}{2} - \frac{a}{2}\right] + \cos\left[\frac{2\pi}{a}(y+z) - \frac{a}{2} + \frac{a}{2}\right] + \cos\left[\frac{2\pi}{a}\left(z + x + \frac{a}{2} + \frac{a}{2}\right)\right]$$

$$f(\vec{r} + \vec{a}_2) = f(\vec{r})$$

$$f(\vec{r} + \vec{a_3}) = 1 + \cos\left[\frac{2\pi}{a}\left(x + y + \frac{a}{2} + \frac{a}{2}\right)\right] + \cos\left[\frac{2\pi}{a}(y+z) + \frac{a}{2} - \frac{a}{2}\right] + \cos\left[\frac{2\pi}{a}\left(z + x + \frac{a}{2} - \frac{a}{2}\right)\right]$$

$$= f(\vec{r})$$

Other functions are not periodic *i.e.*,

$$f(\vec{r} + \vec{a}) \neq f(\vec{r}).$$

57. (D) $\varepsilon(k) = -4\varepsilon_0 \left[\cos\frac{k_x a}{2}\cos\frac{k_y a}{2} + \cos\frac{k_y a}{2}\cos\frac{k_z a}{2} + \cos\frac{k_z a}{2}\cos\frac{k_x a}{2}\right]$

$$\varepsilon(k) = \frac{p^2}{2m} \cdot \frac{\hbar^2}{2m}(k_x^2 + k_y^2 + k_z^2)$$

$$\frac{\partial \varepsilon}{\partial k_x} = \frac{\hbar^2}{2m} 2k_x = \frac{\hbar(\hbar k_x)}{m}$$

$$= \frac{\hbar p_x}{m} = \hbar V_x$$

$$V = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k_x}$$

$$\frac{\partial \varepsilon}{\partial k_x} = -4\varepsilon_0 \left[\left(-\sin\frac{k_x a}{2}\right)\left(\frac{a}{2}\right)\cos\frac{k_y a}{2} + \cos\frac{k_z a}{2}\left(-\sin\frac{k_x a}{2}\right)\left(\frac{a}{2}\right)\right]$$

$$= \frac{4\varepsilon_0 a}{2}\sin\frac{k_x a}{2}\left(\cos\frac{k_y a}{2} + \cos\frac{k_z a}{2}\right)$$

$$\left[\frac{\partial \varepsilon}{\partial k_x}\right]_{\left(\frac{\pi}{a}, 0, 0\right)} = 2\varepsilon_0 a \sin\frac{\pi a}{2a}(\cos 0 + \cos 0)$$

$$= 4\varepsilon_0 a$$

$$\boxed{[V_x]_{(\pi/a, 0, 0)} = \frac{4\varepsilon_0 a}{\hbar}}$$

58. (D) In numerical data, avoid the end point, so use the two middle points

M	η
5032	30 ± 2
10191	250 ± 10

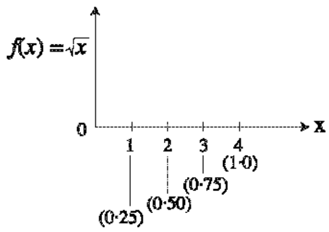
Check for $\frac{M^3}{\eta}$ (one other if you like)

$$\left(\frac{10191}{5032}\right)^3 \times 30 = 249{\cdot}2 \text{ is very close to } 250.$$

$$M^3 \propto \eta$$

$$\boxed{\eta \propto M^3}$$

59. (C)



$$\text{Integral} = \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

$$h = \frac{1{\cdot}0 - 0{\cdot}0}{4} = 0{\cdot}25$$

$$f(x_0) = 0$$

$$f(x_1) = \sqrt{0{\cdot}25},\ 4f(x_1) = 2{\cdot}000$$

$$f(x_2) = \sqrt{0{\cdot}50},\ 2f(x_2) = 1{\cdot}4142$$

$$f(x_3) = \sqrt{0{\cdot}75},\ 4f(x_3) = 3{\cdot}4641$$

$$f(x_4) = \sqrt{1{\cdot}00},\ f(x_4) = \frac{1{\cdot}000}{7{\cdot}8783}$$

$$I = \int_0^1 \sqrt{x}\, dx$$

$$= \frac{0{\cdot}25}{3} \times 7{\cdot}8783$$

$$= \boxed{0{\cdot}657}$$

60. (B) Greatest distance a quark can be estimated from uncertainty principle.

$$r_m = ct,\ \Delta E\, \Delta t \sim \hbar$$

$$r_m = c\frac{\hbar}{\Delta E} = \frac{200 \text{ MeV } fm}{\Delta E \text{ (MeV)}}$$

$$V(r) = ar + \frac{b}{r}$$

$$V'(r) = a - \frac{b}{r^2} = 0,$$

$$r_0\sqrt{\frac{b}{a}} = \sqrt{\frac{100 \text{ MeV } fm}{200 \text{ MeV } fm^{-1}}}$$

$$= \frac{1}{\sqrt{2}} fm.$$

$$\frac{1}{\sqrt{2}} fm = \frac{200 \text{ MeV } fm}{\text{E (MeV)}}$$

$$\text{E (MeV)} = 200\sqrt{2}$$

$$= \boxed{283 \text{ MeV}}$$

61. (D)

$$y = \frac{1}{2}(x_1 + x_2) - \mu$$

$$y^2 = \frac{1}{4}(x_1^2 + x_2^2 + 2x_1x_2) + \mu^2 - \mu(x_1 + x_2)$$

$$\langle y^2\rangle = \frac{1}{4}[\langle x_1^2\rangle + \langle x_2^2\rangle + 2\langle x_1\rangle\langle x_2\rangle] + \mu^2 - \mu[\langle x_1\rangle + \langle x_2\rangle]$$

$$\langle x_1^2\rangle = \langle x_2^2\rangle = \mu^2 + \sigma^2$$

$$\langle x_1\rangle = \langle x_2\rangle = \mu$$

$$\langle y^2\rangle = \frac{1}{4}[2\mu^2 + 2\sigma^2 + 2\mu^2] + \mu^2 - \mu(\mu + \mu)$$

$$= \frac{1}{4}[4\mu^2 + 2\sigma^2] + \mu^2 - 2\mu^2$$

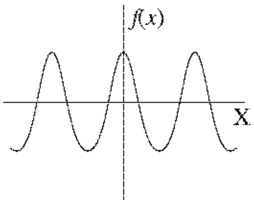
$$= \mu^2 + \frac{1}{2}\sigma^2 - \mu^2$$

$$\langle y^2\rangle = \frac{1}{2}\sigma^2$$

$$\Rightarrow \langle y^4\rangle = \langle y^2\rangle\langle y^2\rangle = \frac{1}{4}\sigma^4$$

$$\Rightarrow \frac{\langle y^4\rangle}{\sigma^4} = \frac{1}{4}$$

62. (B)

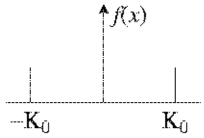


It is an even function $\cos(2\pi k_x x)$

$$f[\cos(2\pi k_0 x)] = \int_{-\infty}^{\infty} e^{-2\pi ikx}\left[\frac{e^{2\pi ik_x x} + e^{-2\pi ik_x x}}{2}\right] dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} [e^{-2\pi i(k_0 - k)x} + e^{-2\pi i(k_0 + k)x}]dx$$

$$\text{Re}\, f(x) = \frac{1}{2}[\delta(k - k_0) + \delta(k + k_0)]$$



Infinite function centered at $-k_0$ and k_0.

63. (D)

$$[A, B] = AB - BA$$

$$[A, B] = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = C$$

$$\boxed{[A, B] = C}$$

$$[B, C] = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= 0$$

$$\boxed{[B, C] = 0}$$

$$[C, A] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = B$$

$$[C, A] = B$$

64. (A)

$$\phi(x, y, z, t) = \cos(z - vt) + \text{Re}(\sin(x + iy))$$

$$\sin(x + iy) = \sin x \cos(iy) + \cos x \sin(iy)$$

$$\sin(x + iy) = \sin x \cos hy + \cos x \sin hy$$

$$\text{Re}[\sin(x + iy)] = \sin x \cos hy.$$

$$\frac{\partial\phi}{\partial x} = \cos x \cos hy$$

$$\frac{\partial^2\phi}{\partial x^2} = -\sin x \cos hy$$

$$\frac{\partial\phi}{\partial y} = \sin x \sin hy$$

$$\frac{\partial^2\phi}{\partial y^2} = \sin x \cos hy$$

$$\frac{\partial\phi}{\partial z} = -\sin(z - vt)$$

$$\frac{\partial^2\phi}{\partial z^2} = -\cos(z - vt)$$

$$\frac{\partial\phi}{\partial t} = v \sin(z - vt)$$

$$\frac{\partial^2\phi}{\partial t^2} = -v^2 \cos(z - vt),$$

$$\frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2} = \boxed{-\cos(z - vt)}$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -\sin x \cos hy + \sin x \cos hy - \cos(z - vt)$$

$$\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -\cos(z - vt)}$$

$$\boxed{\frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi}$$

65. (D)

$$\{c_2, c_3\} = \{x_1p_2 - x_2p_1, x_1p_3 + x_3p_1\}$$

$$= \frac{\partial}{\partial x_1}(x_1p_2 - x_2p_1)\frac{\partial}{\partial p_1}(x_1p_3 + x_3p_1) - \frac{\partial}{\partial p_1}(x_1p_2 - x_2p_1)\frac{\partial}{\partial x_1}(x_1p_3 + x_3p_1)$$

$$+ \frac{\partial}{\partial x_2}(x_1p_2 - x_2p_1)\frac{\partial}{\partial p_2}(x_1p_3 + x_3p_1) - \frac{\partial}{\partial p_2}(x_1p_2 - x_2p_1)\frac{\partial}{\partial x_2}(x_1p_3 + x_3p_1)$$

$$+ \frac{\partial}{\partial x_3}(x_1p_2 - x_2p_1)\frac{\partial}{\partial p_3}(x_1p_3 + x_3p_1) - \frac{\partial}{\partial p_3}(x_1p_2 - x_2p_1)\frac{\partial}{\partial x_3}(x_1p_3 + x_3p_1)$$

$$= p_2x_3 + x_2p_3 = c_1$$

$$\{c_3, c_1\} = \{x_1p_3 + x_3p_1, x_2p_3 + x_3p_2\}$$

$$= \frac{\partial}{\partial x_3}(x_1p_3 + x_3p_1)\frac{\partial}{\partial p_3}(x_2p_3 + x_3p_2) - \frac{\partial}{\partial p_3}(x_1p_3 + x_3p_1)\frac{\partial}{\partial x_3}(x_2p_3 + x_3p_2)$$

+ other terms which vanish

$$= p_1x_2 - x_1p_2 = -c_2$$

$$\boxed{\{c_3, c_1\} = -c_2}$$

66. (B)

$$q^2 = Q$$

$$P = \frac{p}{2q}$$

Old coordinate q, p (one from old, one from new : qp)

New coordinate Q, p (one from new, and from old : Q, p)

Generating function $F_2(q, P)$

Choose $\boxed{F_2 = q^2p}$

$$Q = \frac{\partial}{\partial p}F_2 = \frac{\partial}{\partial p}(q^2p) = q^2$$

$Q = q^2$ (given)

$$p = \frac{\partial}{\partial q}F_2 = \frac{\partial}{\partial q}q^2P = 2qp$$

} check for given eqn.

$$\frac{p}{2q} = P \text{ (given)}$$

67. (C)

$$f(x) = \frac{dx}{dt} = -(x + 1)(x^2 - b^2)$$

$$= -[x^3 + x^2 - xb^2 - b^2]$$

$$= b^2 + b^2x - x^2 - x^3$$

$$= b^2(1 + x) - x^2(1 + x) = 0$$

$$= (b^2 - x^2)(1 + x) = 0$$

Fixed point are

$$x = -1, x_1 = -1$$

$$x = \pm b, x_2 = \pm b, x_3 = -b$$

for stable point $f'(x_i) < 0$, for unstable point $f'(x_i) > 0$

$$f'(-1) = b^2 - 2(-1) - 3(-1)^2$$

$$= b^2 + 2 - 3$$

$$= (b^2 - 1) < 0$$

only one first stable point : $\boxed{(b^2 - 1) < 0}$

(options B and D are eliminated)

$$f'(b) = b^2 - 2(b) - 3b^2$$
$$= -2b^2 - 2b$$
$$= \boxed{-2b\,(b+1) > 0}$$

(unstable) first unstable point)

$$f'(-b) = b^2 - 2(-b) - 3b^2$$
$$= -2b^2 + 2b$$
$$= \boxed{-2b\,(b-1) > 0}$$

(2nd unstable point)

for a stable point $\boxed{b < \pm 1}$

So we point for choice $0 < b < 1$ or $\boxed{b < -1}$

$\left\{\begin{matrix} -2b\,(1+b) > 0 \\ -2b\,(1-b) > 0 \end{matrix}\right\}$ for two unstable fixed points but keep $b < 1$

It is not possible to choose option 1 or 3 to satisfy all the conditions.

Choose option A : Take any value of b (say $0{\cdot}3) - 2 \times \frac{1}{2}\left(1 + \frac{1}{2}\right) \not> 0$.

Choose option C : $b < -1$ (satisfies stable point condition)

Take $b = -1, 1$ so that $b < -1$

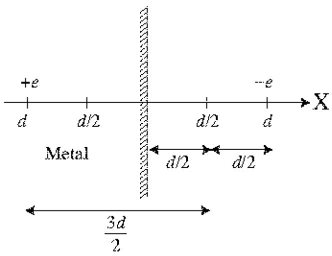
$-2(-1, 1)\,[1 - 1{\cdot}1] \not> 0$

($i.e.$, < 0 so it is a stable point)

$-2(-1,1)\,[1 + 1{\cdot}1] > 0$

Choice is 3 ($b < -1$) provided there is one unstable and two stable points. Somebody must point this to the conserved authority.

68. (B)



$$E(r) = \frac{1}{4\pi\varepsilon_0}\Sigma\frac{q_i}{x_1^2} = E_1 + E_2$$

E field in × dirk^n

$$= \frac{e}{4\pi\varepsilon_0}\left[\frac{1}{\left(\frac{d}{2}\right)^2} + \frac{1}{\left(\frac{3d}{2}\right)^2}\right]$$

$$\boxed{E\,(1, 0, 0) = \frac{10e}{9\pi\varepsilon_0 d^2}}$$

69. (D)

$$I = \int_0^a |\psi|^2 r^2 dr \underbrace{\int_0^\pi \sin\theta\, d\theta}_{2} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$$= 4\pi\int_0^{a_0} |\psi|^2\, r^2 dr$$

$$= \frac{4}{a_0^3}\int_0^{a_0} a^{-2r/a}\, r^2 dr$$

$$x = \frac{r}{a_0},\ dx = \frac{dr}{a_0}$$

$$I' = \int_0^1 x^2 e^{-2x}\, dx$$

$$= -\left|\left(\frac{x^2}{2} + \frac{x}{2} + \frac{1}{4}\right) e^{-2x}\right|_0^1$$

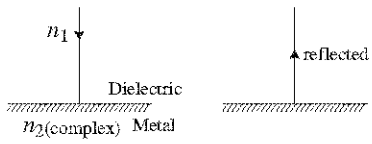
$$= -\left[\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\right)e^{-2}\right] + \frac{1}{4}$$

$$= \frac{1}{4} - \frac{5}{4}e^{-2}$$

$$I = 4I' = 1 - 5e^{-2}$$

$$= \boxed{0{\cdot}323}$$

70. (C)



When an electromagnetic wave travels in a metal, the Maxwell equation have to be solved with Ohm's law $\vec{J} = \sigma\vec{E}$. This gives rise to attraction of the wave and the electric and magnetic component of the wave are no longer in phase. The phase difference for normal incidence is given by fresnel equation

$$X = \frac{E_r}{E_i} = \frac{n_2 - n_1}{n_2 + n_1}$$

E_r – reflected component of $\vec{E}$ field.

E_i – incidence component of $\vec{E}$ field.

$$= \frac{n\,(1 + i\rho) - n}{n\,(1 + i\rho) + n}$$

$$= \frac{ni\rho}{n[2+i\rho]} = \frac{i\rho}{2+i\rho}$$

$$X = \frac{i\rho\,(2-i\rho)}{(2+i\rho)\,(2-i\rho)}$$

$$= \frac{\rho^2 + 2i\rho}{4+\rho^2}$$

$$\tan\theta = \frac{\text{Imaginary component of } x}{\text{Real component of } x}$$

$$= \frac{2i\rho}{\rho^2} = \frac{2}{\rho}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{2}{\rho}\right)}$$

71. (A) $V(r) = \beta e^{-\mu r}$

$$f(\theta) = -\frac{2m}{\hbar^2}\int_0^\infty dr\, r^2\, V(r)\,\frac{\sin br}{br}$$

$$b = 2k\sin\frac{\theta}{2}$$

$$= -\frac{2m\beta}{b\hbar^2}\int_0^\infty dr\,(re^{-\mu r}\sin br)$$

$$\int_0^\infty r\sin br\, e^{-\mu r}\, dr$$

$$= \frac{2\mu b}{(b^2+\mu^2)^2}\,[\text{standard integral}]$$

$$\boxed{f(\theta) = -\frac{4m\beta\mu}{\hbar^2\,(b^2+\mu^2)^2}}$$

72. (D) $\psi(x) = Ae^{-\alpha|x|}$

$H' = bx^2$

ΔE (first order correction)

$$= \frac{\int \psi^*(x)\, H'\, \psi(x)\, dx}{\int \psi^*(x)\,\psi(x)}$$

$$\Delta E = \frac{2A^2\int_0^\infty e^{-2\alpha|x|}\, bx^2\, dx}{2A^2\int_0^\infty e^{-2\alpha|x|}\, dx}$$

$$= \frac{b\int_0^\infty x^2\, e^{-2\alpha|x|}\, dx}{\int_0^\infty e^{-2\alpha|x|}\, dx}$$

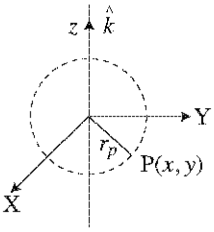
$$= b\,\frac{\frac{2!}{(2\alpha)^3}}{\left(\frac{1}{2\alpha}\right)} = \frac{2b}{8\alpha^3}\times 2\alpha$$

$$= \boxed{\frac{b}{2\alpha^2}}$$

73. (A) $\oint \vec{B}\cdot\vec{ds} = \phi$

$$B\pi r_p^2 = \phi,\ B = \frac{\phi}{\pi r_p^2}$$

$$\vec{B} = \frac{\phi}{\pi(x^2+y^2)}\hat{k}$$



This $\vec{B}$ field is uniform,

$r_p \rightarrow$ distance from the wire in XY plane.

$$\vec{A} = \frac{1}{2}(\vec{B}\times\vec{r})$$

$$= \frac{1}{2}\left[\frac{\phi}{\pi(x^2+y^2)}\hat{k}\times(x\hat{i}+y\hat{j}+z\hat{k})\right]$$

$$= \left[\frac{\phi x}{2\pi(x^2+y^2)}, \frac{-\phi y\hat{i}}{2\pi(x^2+y^2)}, 0\right]$$

$$(A_x, A_y, A_z) = \left[-\frac{1}{2}\frac{\phi y}{\pi(x^2+y^2)}, \frac{1}{2}\frac{\phi x}{\pi(x^2+y^2)}, 0\right]$$

74. (D) $\left(p+\frac{a}{V^2}\right)(V-b) = RT$

Hard way—Solve the cubic eqn. for V, minimized it to find the volume. It is well known that the critical volume.

$$V_c = 3b$$

Easy way—Use dimensional analysis.

(1) $\frac{a}{b} \sim \frac{PV^2}{V} \sim PV$ (Energy)

(2) $\frac{a}{b^2} \sim \frac{PV^2}{V^2} \sim P$ (Pressure)

(3) $\frac{a}{bR} \sim \frac{PV^2}{V\frac{PV}{T}} \sim T$ (Temperature)

(4) $b \sim V \sim$ (Volume)

$$\boxed{V_c = 3b}$$

75. (B) $I = I_0\, e^{-\alpha x}$

α = attenuation constant,

$$\alpha = \frac{2}{\text{Skin depth}}$$

$$\alpha = \sqrt{2\mu\omega\sigma}$$

$$\text{Skin depth} = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$$= \sqrt{2 \times 4\pi \times 10^{-7} \times 10^{7} \times \frac{10^6}{2\pi}}$$

$$\alpha = 2 \times 10^3 \text{ m}^{-1}$$

$$\ln\left(\frac{I}{I_0}\right) = -\alpha x$$

$$\ln(10^{-2}) = -\alpha x$$

$$-2\ln I_0 = -\alpha x$$

$$x = \frac{2 \ln 10}{\alpha} = \frac{2 \times 2{\cdot}3 \text{ m}}{2 \times 10^3}$$

$$= 2{\cdot}3 \times 10^{-3} \text{ m}$$

$$\boxed{x = 2{\cdot}3 \text{ mm}}$$

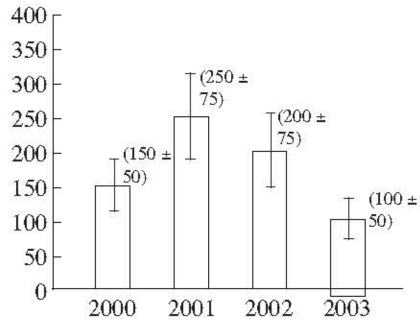
[Note : If attenuation of only E or B field is asked then $\alpha = \frac{1}{\text{skin depth}}$, and your option will be **4·6 mm** but since intensity α square of amplitude of σ, one must be $\alpha = \frac{2}{\text{skin depth}}$].

Physical Sciences
CSIR-UGC NET/JRF Exam.
(December 2014)
Solved Paper

December 2014
Physical Sciences

PART A

1. Average yield of a product in different years is shown in the histogram. If the vertical bars indicates variability during the year, then during which year was the per cent variability over the average of that year the least ?



(A) 2000 (B) 2001
(C) 2002 (D) 2003

2. A rectangle of length d and breadth $d/2$ is revolved once completely around its length and once around its breadth. The ratio of volumes swept in the two cases is—
(A) 1 : 1 (B) 1 : 2
(C) 1 : 3 (D) 1 : 4

3. A long ribbon is wound around a spool up to a radius R. Holding the tip of the ribbon, a boy runs away from the spool with a constant speed maintaining the unwound portion of the ribbon horizontal. In 4 minutes, the radius of the wound portion becomes $\frac{R}{\sqrt{2}}$. In what further time, it will become R/2?

(A) $\sqrt{2}$ min (B) 2 min
(C) $2\sqrt{2}$ min (D) 4 min

4. If n is a positive integer, then
$n(n + 1) (n + 2) (n + 3) (n + 4) (n + 5) (n + 6)$
is divisible by—
(A) 3 but not 7 (B) 3 and 7
(C) 7 but not 3 (D) neither 3 nor 7

5. The area (in m^2) of a triangular park of dimensions 50 m, 120 m and 130 m is—
(A) 3000 (B) 3250
(C) 5550 (D) 7800

6. Lunch-dinner pattern of a person for m days is given below. He has a choice of a VEG or a NON-VEG meal for his lunch/dinner :
   1. If he takes a NON-VEG lunch, he will have only VEG for dinner.
   2. He takes NON-VEG dinner for exactly 9 days.
   3. He takes VEG lunch for exactly 15 days.
   4. He takes a total of 14 NON-VEG meals.

   What is m ?
(A) 18 (B) 24
(C) 20 (D) 38

7. A bank offers a scheme wherein deposits made for 1600 days are doubled in value, the interest being compounded daily. The interest accrued on a deposit of Rs. 1000/- over the first 400 days would be Rs.
(A) 250 (B) 183
(C) 148 (D) 190

8. What is the next number of the following sequence ?
2, 3, 4, 7, 6, 11, 8, 15, 10 …
(A) 12 (B) 13
(C) 17 (D) 19

9. Two locomotives are running towards each other with speeds of 60 and 40 km/h. An object keeps on flying to and fro from the front tip of one locomotive to the front tip of the other with a speed of 70 km/h. After 30 minutes, the two locomotives collide and the object is crushed. What distance did the object cover before being crushed ?

(A) 50 km (B) 45 km
(C) 35 km (D) 10 km

10. Weights (in kg) of 13 persons are given below :

70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94

Two new persons having weights 100 kg and 79 kg join the group. The average weight of the group increases by—

(A) 0 kg (B) 1 kg
(C) 1·6 kg (D) 1·8 kg

11. A code consists of at most two identical letters followed by at most four identical digits. The code must have at least one letter and one digit. How many distinct codes can be generated using letters A to Z and digits 1 to 9 ?

(A) 936 (B) 1148
(C) 1872 (D) 2574

12. Two solid iron spheres are heated to 100°C and then allowed to cool. One has the size of a football; the other has the size of a pea. Which sphere will attain the room temperature (constant) first ?

(A) The bigger sphere
(B) The smaller sphere
(C) Both spheres will take the same time
(D) It will depend on the room temperature

13. Find the missing letter :

A	?	Q	E
C	M	S	C
E	K	U	A
G	I	W	Y

(A) L (B) Q
(C) N (D) O

14. The least significant bit of an 8-bit binary number is zero. A binary number whose value is 8 times the previous number has—

(A) 12 bits ending with three zeroes
(B) 11 bits ending with four zeroes
(C) 11 bits ending with three zeroes
(D) 12 bits ending with four zeroes

15. A person sells two objects at Rs. 1035/- each. On the first object he suffers a loss of 10% while on the second he gains 15%. What is his net loss/gain percentage ?

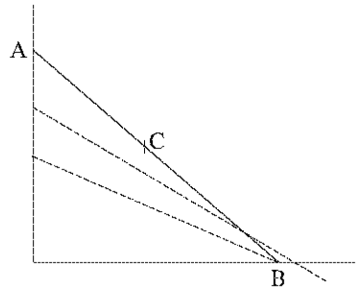
(A) 5% gain (B) < 1% gain
(C) < 1% loss (D) no loss, no gain

16. Continue the sequence

2, 5, 10, 17, 28, 41, —, —, —

(A) 58, 77, 100 (B) 64, 81, 100
(C) 43, 47, 53 (D) 55, 89, 113

17. A ladder rests against a wall as shown. The top and the bottom ends of the ladder are marked A and B. The base B slips. The central point C of the ladder falls along—



(A) A parabola
(B) The arc of a circle
(C) A straight line
(D) A hyperbola

18. 20% of students of a particular course get jobs within one year of passing. 20% of the remaining students get jobs by the end of second year of passing. If 16 students are still jobless, how many students had passed the course ?

(A) 32 (B) 64
(C) 25 (D) 100

19. Binomial theorem in algebra gives $(1 + x)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, where $a_0, a_1, \ldots\ldots, a_n$ are constants depending on n. What is the sum $a_0 + a_1 + a_2 + \ldots + a_n$?

(A) 2^n (B) n
(C) n^2 (D) $n^2 + n$

20. A sphere is made up of very thin concentric shells of increasing radii (leaving no gaps). The mass of an arbitrarily chosen shell is—
(A) Equal to the mass of the preceding shell
(B) Proportional to its volume
(C) Proportional to its radius
(D) Proportional to its surface area

PART B

21. A particle of mass m is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ where a, b are positive constants. The frequency of small oscillations about a point of stable equilibrium is—
(A) $\sqrt{a/m}$ (B) $\sqrt{2a/m}$
(C) $\sqrt{3a/m}$ (D) $\sqrt{6a/m}$

22. The radius of Earth is approximately 6400 km. The height h at which the acceleration due to Earth's gravity differs from g at the Earth's surface by approximately 1% is—
(A) 64 km (B) 48 km
(C) 32 km (D) 16 km

23. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is—
(A) $v = c\sqrt{1 - \frac{mc^2}{E}}$
(B) $v = \sqrt{\frac{2E}{m}}\left(1 + \frac{mc^2}{E}\right)$
(C) $v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$
(D) $v = c\left(1 + \frac{mc^2}{E}\right)$

24. Let $\vec{r}$ denote the position vector of any point in three-dimensional space, and $r = |\vec{r}|$. Then—
(A) $\vec{\nabla} \cdot \vec{r} = 0$ and $\vec{\nabla} \times \vec{r} = \vec{r}/r$
(B) $\vec{\nabla} \cdot \vec{r} = 0$ and $\nabla^2 r = 0$
(C) $\vec{\nabla} \cdot \vec{r} = 3$ and $\nabla^2 \vec{r} = \vec{r}/r^2$
(D) $\vec{\nabla} \cdot \vec{r} = 3$ and $\vec{\nabla} \times \vec{r} = 0$

25. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of A = $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and B = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if—
(A) $b = 0$ or $a = 0$
(B) $b = a$ or $b = -2a$
(C) $b = 2a$ or $b = -a$
(D) $b = a/2$ or $b = -a/2$

26. The principal value of the integral $\int_{\infty}^{-\infty} \frac{\sin(2x)}{x^3}\, dx$ is—
(A) -2π (B) $-\pi$
(C) π (D) 2π

27. Two independent random variables m and n, which can take the integer values 0, 1, 2, ..., ∞, follow the Poisson distribution, with distinct mean values μ and ν respectively. Then—
(A) The probability distribution of the random variable $l = m + n$ is a binomial distribution
(B) The probability distribution of the random variable $r = m - n$ is also a Poisson distribution
(C) The variance of the random variable $l = m + n$ is equal to $\mu + \nu$
(D) The mean value of the random variable $r = m - n$ is equal to 0

28. The Laurent series expansion of the function $f(z) = e^z + e^{1/z}$ about $z = 0$ is given by—
(A) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ for all $|z| < \infty$
(B) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}$ only if $0 < |z| < 1$
(C) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}$ for all $0 < |z| < \infty$
(D) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$, only if $|z| < 1$

29. The equation of motion of a system described by the time-dependent Lagrangian

$$L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$$

is—

(A) $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$

(B) $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$

(C) $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$

(D) $m\ddot{x} + \frac{dV}{dx} = 0$

30. A solid sphere of radius R has a charge density, given by

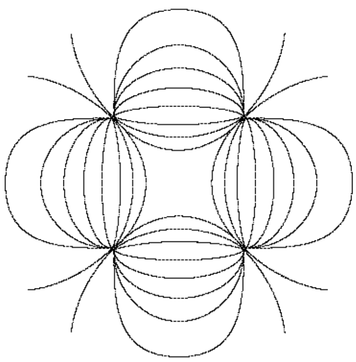
$$\rho(r) = \rho_0\left(1 - \frac{ar}{R}\right),$$

where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r = R/2$ is 1·25 times that at $r = R$, then the value of a is—

(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

31. The electrostatic lines of force due to a system of four point charges is sketched below.

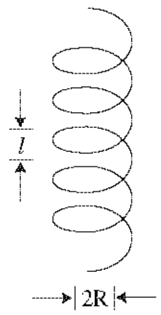


At a large distance r, the leading asymptotic behaviour of the electrostatic potential is proportional to—

(A) r (B) r^{-1}

(C) r^{-2} (D) r^{-3}

32. A charged particle moves in a helical path under the influence of a constant magnetic field. The initial velocity is such that the component along the magnetic field is twice the component in the plane normal to the magnetic field.



The ratio l/R of the pitch l to the radius R of the helical path is—

(A) $\pi/2$ (B) 4π

(C) 2π (D) π

33. A parallel beam of light of wavelength λ is incident normally on a thin polymer film with air on both sides. If the film has a refractive index $n > 1$, then second order bright fringes can be observed in reflection when the thickness of the film is—

(A) $\lambda/4n$ (B) $\lambda/2n$

(C) $3\lambda/4n$ (D) λ/n

34. Consider the normalized wavefunction

$$\phi = a_1\,\psi_{11} + a_2\psi_{10} + a_3\,\psi_{1-1}$$

where ψ_{lm} is a simultaneous normalized eigenfunction of the angular momentum operators L^2 and L_z, with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. If ϕ is an eigenfunction of the operator L_x with eigenvalue $\hbar$, then—

(A) $a_1 = -a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(B) $a_1 = a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(C) $a_1 = a_3 = \frac{1}{2}, a_2 = -\frac{1}{\sqrt{2}}$

(D) $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

35. Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x, p] = i$ in natural units ($\hbar = 1$). Then the commutator $[x, pe^{-p}]$ is—

(A) $i(1-p)e^{-p}$ (B) $i(1-p^2)\,e^{-p}$
(C) $i(1-e^{-p})$ (D) ipe^{-p}

36. Suppose Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is—

(A) $-i\hbar\omega\left(x\frac{\partial}{\partial x}-\frac{1}{2}\right)$ (B) $-i\hbar\omega\left(x\frac{\partial}{\partial x}+\frac{1}{2}\right)$

(C) $-i\hbar\omega x\frac{\partial}{\partial x}$ (D) $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

37. Let ψ_1 and ψ_2 denote the normalized eigenstates of a particle with energy eigenvalues E_1 and E_2 respectively, with $E_2 > E_1$. At time t = 0 the particle is prepared in a state

$$\psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2).$$

The shortest time T at which $\psi(t = T)$ will be orthogonal to $\psi(t = 0)$ is—

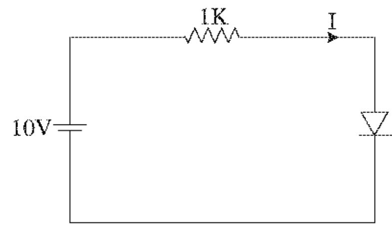
(A) $\frac{2\hbar\pi}{(E_2-E_1)}$ (B) $\frac{\hbar\pi}{(E_2-E_1)}$

(C) $\frac{\hbar\pi}{2(E_2-E_1)}$ (D) $\frac{\hbar\pi}{4(E_2-E_1)}$

38. The I–V characteristics of the diode in the circuit below is given by

$$I = \begin{cases} (V-0\cdot7)/500 & \text{for } V \geq 0\cdot7 \\ 0 & \text{for } V < 0\cdot7 \end{cases}$$

where V is measured in volts and I is measured in amperes.



The current I in the circuit is—

(A) 10·0 mA (B) 9·3 mA
(C) 6·2 mA (D) 6·7 mA

39. A junction is made between a metal of work function W_M, and a doped semiconductor of work function W_S with $W_M > W_S$. If the electric field at the interface has to be increased by a factor of 3, then the dopant concentration in the semiconductor would have to be—

(A) Increased by a factor of 9
(B) Decreased by a factor of 3
(C) Increased by a factor of 3
(D) Decreased by a factor of $\sqrt{3}$

40. In a measurement of the viscous drag force experienced by spherical particles in a liquid, the force is found to be proportional to $V^{1/3}$ where V is the measured volume of each particle. If V is measured to be 30 mm^3, with an uncertainty of 2·7 mm^3, the resulting relative percentage uncertainty in the measured force is—

(A) 2·08 (B) 0·09
(C) 6 (D) 3

41. The pressure P of a fluid is related to its number density ρ by the equation of state $P = a\rho + b\rho^2$ where a and b are constants. If the initial volume of the fluid is V_0, the work done on the system when it is compressed so as to increase the number density from an initial value of ρ_0 to $2\rho_0$ is—

(A) $a\rho_0V_0$
(B) $(a + b\rho_0)\rho_0V_0$
(C) $\left(\frac{3a}{2}+\frac{7\rho_0 b}{3}\right)\rho_0V_0$
(D) $(a \ln 2 + b\rho_0)\rho_0V_0$

42. The Hamiltonian of a classical particle moving in one dimension is

$$H = \frac{p^2}{2m} + \alpha q^4$$

where α is a positive constant and p and q are its momentum and position respectively. Given that its total energy $E \leq E_0$ the available volume of phase space depends on E_0 as—

(A) $E_0^{3/4}$
(B) E_0
(C) $\sqrt{E_0}$
(D) is independent of E_0

43. An ideal Bose gas is confined inside a container that is connected to a particle reservoir. Each particle can occupy a discrete set of single-particle quantum states. If the

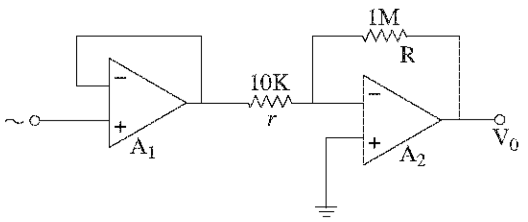
probability that a particular quantum state is unoccupied is 0·1, then the average number of bosons in that state is—

(A) 8 (B) 9
(C) 10 (D) 11

44. In low density oxygen gas at low temperature, only the translational and rotational modes of the molecules are excited. The specific heat per molecule of the gas is—

(A) $\frac{1}{2}k_B$ (B) k_B
(C) $\frac{3}{2}k_B$ (D) $\frac{5}{2}k_B$

45. Consider the amplifier circuit comprising of the two op-amps A_1 and A_2 as shown in the figure.



If the input ac signal source has an impedance of 50 $k\Omega$, which of the following statements is true ?

(A) A_1 is required in the circuit because the source impedance is much greater than r
(B) A_1 is required in the circuit because the source impedance is much less than R
(C) A_1 can be eliminated from the circuit without affecting the overall gain
(D) A_1 is required in the circuit if the output has to follow the phase of the input signal

PART C

46. A plane electromagnetic wave incident normally on the surface of a material is partially reflected. Measurements on the standing wave in the region in front of the interface show that the ratio of the electric field amplitude at the maxima and the minima is 5. The ratio of the reflected intensity to the incident intensity is—

(A) 4/9 (B) 2/3
(C) 2/5 (D) 1/5

47. The scalar and vector potentials $\varphi(\vec{x}, t)$ and $\vec{A}(\vec{x}, t)$ are determined up to a gauge transformation $\varphi \to \varphi' = \varphi - \frac{\partial \xi}{\partial t}$ and $\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\xi$ where ξ is an arbitrary continuous and differentiable function of $\vec{x}$ and t. If we further impose the Lorenz gauge condition

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0$$

then a possible choice for the gauge function $\xi(\vec{x}, t)$ is (where ω, $\vec{k}$ are non zero constants with $\omega = c\,|\vec{k}|$)—

(A) $\cos \omega t \cosh \vec{k}\cdot\vec{x}$
(B) $\sinh \omega t \cos \vec{k}\cdot\vec{x}$
(C) $\cosh \omega t \cos \vec{k}\cdot\vec{x}$
(D) $\cosh \omega t \cosh \vec{k}\cdot\vec{x}$

48. A non-relativistic particle of mass m and charge e, moving with a velocity $\vec{v}$ and acceleration $\vec{a}$, emits radiation of intensity I. What is the intensity of the radiation emitted by a particle of mass $m/2$, charge $2e$, velocity $\vec{v}/2$ and acceleration $2\vec{a}$?

(A) 16 I (B) 8 I
(C) 4 I (D) 2 I

49. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication ?

(A) $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}$, where $\alpha\beta \neq 1$

(C) $\begin{pmatrix} \alpha & \alpha^* \\ \beta & \beta^* \end{pmatrix}$, where $\alpha\beta^*$ is real

(D) $\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

50. The expression

$$\sum_{i,j,k=1}^{3} \in_{ijk} \{x_i, (p_j, L_k)\}$$

(where $\in_{ijk}$ is the Levi-Civita symbol, $\vec{x}$, $\vec{p}$, $\vec{L}$ are the position, momentum and angular momentum respectively, and {A, B} represents the Poisson Bracket of A and B) simplifies to—

(A) 0 (B) 6

(C) $\vec{x} \cdot (\vec{p} \times \vec{L})$ (D) $\vec{x} \times \vec{p}$

51. A mechanical system is described by the Hamiltonian $H(q, p) = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$. As a result of the canonical transformation generated by $F(q, Q) = -\frac{Q}{q}$, the Hamiltonian in the new coordinate Q and momentum P becomes—

(A) $\frac{1}{2m} Q^2P^2 + \frac{m\omega^2}{2} Q^2$

(B) $\frac{1}{2m} Q^2P^2 + \frac{m\omega^2}{2} P^2$

(C) $\frac{1}{2m} P^2 + \frac{m\omega^2}{2} Q^2$

(D) $\frac{1}{2m} Q^2P^4 + \frac{m\omega^2}{2} P^{-2}$

52. Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{a}$ and $\vec{b}$ are two arbitrary constant vectors in three dimensions, the commutator $[\vec{a} \cdot \vec{\sigma}, \vec{b} \cdot \vec{\sigma}]$ is equal to (in the following I is the identity matrix)—

(A) $(\vec{a} \cdot \vec{b})(\sigma_1 + \sigma_2 + \sigma_3)$

(B) $2i\,(\vec{a} \times \vec{b}) \cdot \vec{\sigma}$

(C) $(\vec{a} \cdot \vec{b})\, I$

(D) $|\vec{a}|\,|\vec{b}|\, I$

53. Consider the function $f(z) = \frac{1}{z} \ln(1 - z)$ of a complex variable $z = re^{i\theta}$ $(r \geq 0, -\infty < \theta < \infty)$. The singularities of $f(z)$ are as follows :

(A) Branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ only for $0 \leq \theta < 2\pi$

(B) Branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all θ other than $0 \leq \theta < 2\pi$

(C) Branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all θ

(D) Branch points at $z = 0$, $z = 1$ and $z = \infty$

54. The function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!} \left(\frac{x}{2}\right)^{2n+1}$ satisfies the differential equation—

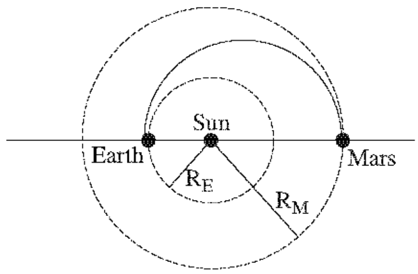
(A) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2 + 1)f = 0$

(B) $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2 - 1)f = 0$

(C) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2 - 1)f = 0$

(D) $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2 - 1)f = 0$

55. The probe *Mangalyaan* was sent recently to explore the planet Mars. The interplanetary part of the trajectory is approximately a half-ellipse with the Earth (at the time of launch), Sun and Mars (at the time the probe reaches the destination) forming the major axis. Assuming that the orbits of Earth and Mars are approximately circular with radii R_E and R_M, respectively, the velocity (with respect to the Sun) of the probe during its voyage when it is at a distance $r(R_E << r << R_M)$ from the Sun, neglecting the effect of Earth and Mars, is :



(A) $\sqrt{2GM\frac{(R_E + R_M)}{r(R_E + R_M - r)}}$

(B) $\sqrt{2GM\frac{(R_E + R_M - r)}{r(R_E + R_M)}}$

(C) $\sqrt{2GM\frac{R_E}{rR_M}}$

(D) $\sqrt{\frac{2GM}{r}}$

56. A large MOS transistor consists of N individual transistors connected in parallel. If the only form of noise in each transistor is $1/f$ noise, then the equivalent voltage noise spectral density for the MOS transistor is—
(A) 1/N times that of a single transistor
(B) $1/N^2$ times that of a single transistor
(C) N times that of a single transistor
(D) N^2 times that of a single transistor

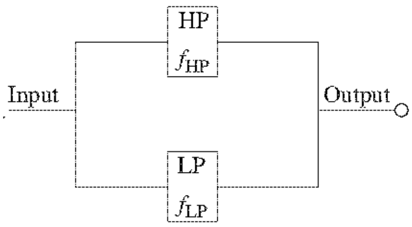
57. Consider a particle of mass m in the potential $V(x) = a\,|x|, a > 0$. The energy eigen-values E_n ($n = 0, 1, 2, \ldots$), in the WKB approximation, are—
(A) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{1/3}$
(B) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$
(C) $\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)$
(D) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{4/3}$

58. When laser light of wavelength λ falls on a metal scale with 1 mm engravings at a grazing angle of incidence, it is diffracted to form a vertical chain of differential spots on a screen kept perpendicular to the scale. If the wavelength of the laser is increased by 200 nm, the angle of the first-order diffraction spot changes from 5° to—
(A) 6·60° (B) 5·14°
(C) 5·018° (D) 5·21°

59. Let $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ (where c_0 and c_1 are constants with $c_0^2 + c_1^2 = 1$) be a linear combination of the wavefunctions of the ground and first excited states of the one-dimensional harmonic oscillator. For what value of c_0 is the expectation value $\langle x\rangle$ a maximum ?
(A) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}, c_0 = \frac{1}{\sqrt{2}}$
(B) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}, c_0 = \frac{1}{2}$
(C) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}, c_0 = \frac{1}{\sqrt{2}}$
(D) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}, c_0 = \frac{1}{2}$

60. Consider a Low Pass (LP) and a High Pass (HP) filter with cut-off frequencies f_{LP} and f_{HP}, respectively, connected in series or in parallel configurations as shown in the Figures A and B below.

A.


B.


Which of the following statements is correct ?
(A) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B acts as a Band Reject filter
(B) For $f_{HP} > f_{LP}$, A stops the signal from passing through and B passes the signal without filtering
(C) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B passes the signal without filtering
(D) For $f_{HP} > f_{LP}$, A passes the signal without filtering and B acts as a Band Reject filter

61. A collection N of non-interacting spins S_i, $i = 1, 2, \ldots, N$, ($S_i = \pm 1$) is kept in an external magnetic field B at a temperature T. The Hamiltonian of the system is $H = -\mu B\ \Sigma_i S_i$. What should be the minimum value of $\frac{\mu B}{k_B T}$ for which the mean value $\langle S_i\rangle \geq \frac{1}{3}$?
(A) $\frac{1}{2}$N ln 2 (B) 2 ln 2
(C) $\frac{1}{2}$ln 2 (D) N ln 2

62. When a gas expands adiabatically from volume V_1 to V_2 by a quasi-static reversible process, it cools from temperature T_1 to T_2. If

now the same process is carried out adiabatically and irreversibly, and T_2' is the temperature of the gas when it has equilibrated, then—

(A) $T_2' = T_2$

(B) $T_2' > T_2$

(C) $T_2' = T_2\left(\frac{V_2 - V_1}{V_2}\right)$

(D) $T_2' = \frac{T_2 V_1}{V_2}$

63. A random walker takes a step of unit length in the positive direction with probability 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is—

(A) $n/3$ (B) $n/8$
(C) $2n/3$ (D) 0

64. The Hamiltonian H_0 for a three-state quantum system is given by the matrix

$$H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$ When perturbed by

$$H' = \in \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ where $\in << 1$, the resulting shift in the energy eigenvalue $E_0 = 2$ is—

(A) $\in, -2\in$ (B) $-\in, 2\in$
(C) $\pm\in$ (D) $\pm 2\in$

65. The ground state energy of the attractive delta function potential

$$V(x) = -b\,\delta(x),$$

where $b > 0$, is calculated with the variational trial function

$$\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for } -a < x < a \\ 0\ , & \text{otherwise} \end{cases}$$

is—

(A) $-\frac{mb^2}{\pi^2\hbar^2}$ (B) $-\frac{2mb^2}{\pi^2\hbar^2}$

(C) $-\frac{mb^2}{2\pi^2\hbar^2}$ (D) $-\frac{mb^2}{4\pi^2\hbar^2}$

66. If the binding energy B of a nucleus (mass number A and charge Z) is given by

$$B = a_V A - a_s A^{2/3} - a_{sym}\frac{(2Z - A)^2}{A} - \frac{a_c Z^2}{A^{1/3}}$$

where $a_V = 16$ MeV, $a_s = 16$ MeV, $a_{sym} = 24$ MeV and $a_c = 0{\cdot}75$ MeV, then the Z for the most stable isobar for a nucleus with A = 216 is—

(A) 68 (B) 72
(C) 84 (D) 92

67. Consider the crystal structure of sodium chloride which is modeled as a set of touching spheres. Each sodium atom has a radius r_1 and each chlorine atom has a radius r_2. The centres of the spheres form a simple cubic lattice. The packing fraction of this system is—

(A) $\pi\left[\left(\frac{r_1}{r_1 + r_2}\right)^3 + \left(\frac{r_2}{r_1 + r_2}\right)^3\right]$

(B) $\frac{2\pi}{3}\frac{r_1^3 + r_2^3}{(r_1 + r_2)^3}$

(C) $\frac{r_1^3 + r_2^3}{(r_1 + r_2)^3}$

(D) $\pi\frac{r_1^3 + r_2^3}{2(r_1 + r_2)^3}$

68. Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the c-direction is half the effective spring constant between atoms in the a and b directions. At low temperatures, the behaviour of the lattice contribution to the specific heat will depend as a function of temperature T as—

(A) T^2 for the tetragonal solid, but as T^3 for the simple cubic solid
(B) T for the tetragonal solid and as T^3 for the simple cubic solid
(C) T for both solids
(D) T^3 for both solids

69. An atomic transition $^1P \to {}^1S$ in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton $\mu_B = 9{\cdot}27 \times 10^{-24}$ J/T, and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately—

(A) 0·01 nm (B) 0·1 nm
(C) 1·0 nm (D) 10 nm

70. If the leading anharmonic correction to the energy of the n-th vibrational level of a diatomic molecule is $-x_e\left(n+\frac{1}{2}\right)^2 \hbar\omega$ with $x_e = 0{\cdot}001$, the total number of energy levels possible is approximately—
(A) 500 (B) 1000
(C) 250 (D) 750

71. A superconducting ring carries a steady current in the presence of a magnetic field $\vec{B}$ normal to the plane of the ring. Identify the **incorrect** statement—
(A) The flux passing through the superconductor is quantized in units of hc/e
(B) The current and the magnetic field in the superconductor are time independent
(C) The current density $\vec{J}$ and $\vec{B}$ are related by the equation $\vec{\nabla} \times \vec{J} + \Lambda^2 \vec{B} = 0$, where Λ is a constant
(D) The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

72. The effective spin-spin interaction between the electron spin $\vec{S}_e$ and the proton spin $\vec{S}_p$ in the ground state of the Hydrogen atom is given by $H' = a\,\vec{S}_e \cdot \vec{S}_p$. As a result of this interaction, the energy levels split by an amount—
(A) $\frac{1}{2}a\hbar^2$ (B) $2a\hbar^2$
(C) $a\hbar^2$ (D) $\frac{3}{2}a\hbar^2$

73. In deep inelastic scattering electrons are scattered off protons to determine if a proton has any internal structure. The energy of the electron for this must be at least—
(A) $1{\cdot}25 \times 10^9$ eV (B) $1{\cdot}25 \times 10^{12}$ eV
(C) $1{\cdot}25 \times 10^6$ eV (D) $1{\cdot}25 \times 10^8$ eV

74. The power density of sunlight incident on a solar cell is 100 mW/cm^2. Its short circuit current density is 30 mA/cm^2 and the open circuit voltage is 0·7 V. If the fill factor of the solar cell decreases from 0·8 to 0·5 then the percentage efficiency will decrease from—
(A) 42·0 to 26·2
(B) 24·0 to 16·8
(C) 21·0 to 10·5
(D) 16·8 to 10·5

75. Consider the four processes :
1. $p^+ \rightarrow n + e^+ + \nu_e$
2. $\Lambda^0 \rightarrow p^+ + e^+ + \nu_e$
3. $\pi^+ \rightarrow e^+ + \nu_e$
4. $\pi^0 \rightarrow \gamma + \gamma$

Which of the above is/are forbidden for free particles ?
(A) Only 2 (B) 2 and 4
(C) 1 and 4 (D) 1 and 2

Answers with Hints

1. (B) Least variation in percentage

For year 2000 $= \frac{50}{150} \times 100 = 33{\cdot}33\%$

For year 2001 $= \frac{75}{250} \times 100 = 30\%$

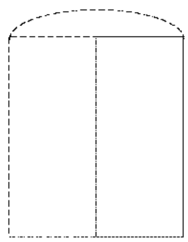
For year 2002 $= \frac{75}{200} \times 100 = 37{\cdot}5\%$

For year 2003 $= \frac{50}{100} \times 100 = 50\%$

So per cent variability over the average of 2001 year was the least.

2. (B) If revolution about $d \rightarrow$ height $\rightarrow d$

radius $= \frac{d}{2}$

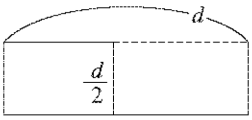


Volume $\alpha\ \pi r^2 h$

$$\pi \times \left(\frac{d}{2}\right)^2 \times d = \frac{\pi d^3}{4}$$

I revolution about $\frac{d}{2}$

$$\pi(d)^2 \times \frac{d}{2}$$



around length : around breadth

$$\frac{\pi d^3}{4} : \frac{\pi d^3}{2}$$

$$1 : 2$$

3. (B) In 4 min, the radius of the wound portion becomes $\frac{R}{\sqrt{2}}$ × area swept will be $\frac{\pi R^2}{2}$.

In x min radius becomes $\frac{R}{2}$ with area $\frac{\pi R}{4}$.

So ratio becomes

Time	Area
4	$\frac{\pi R^2}{2}$
x	$\frac{\pi R^2}{4}$

$$4 : x = \frac{\pi R^2}{2} : \frac{\pi R^2}{4}$$

$$4 \times \frac{\pi R^2}{4} = x \times \frac{\pi R^2}{2}$$

$$= 0$$

$$k = 2 \text{ min}$$

4. (B) Let $n = 1$

Then, the given number will become

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$

divisible by 3 and 7

similar check for $n = 2, n = 3$.

5. (A) Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$

$$= \frac{50+120+130}{2}$$

$$= \frac{50+250}{2}$$

$$= \frac{300}{2}$$

$$= 150$$

$$\text{Area} = \sqrt{150(150-50)(150-120)(150-130)}$$

$$= \sqrt{150 \times 100 \times 30 \times 20}$$

$$= \sqrt{3 \times 50 \times 2 \times 50 \times 3 \times 200}$$

$$= 50 \times 3 \times 20$$

$$= 3000$$

6. (B) 7. (D)

8. (D)

2, 3, 4, 7, 6, 11, 8, 15, 10, 19

Second series varies cos $2\left(\frac{2n+3}{2}\right)$

where $n = 0, 1, 2, \ldots\ldots$

If we put $n = 0 \rightarrow 2\left(\frac{0+3}{2}\right) = 3$

$$n = 1 \rightarrow 2\left(\frac{2+3}{2}\right) = 7$$

$$n = 2 \rightarrow 2\left(\frac{4+3}{2}\right) = 11$$

$$n = 3 \rightarrow 2\left(\frac{6+3}{2}\right) = 15$$

$$n = 4 \rightarrow 2\left(\frac{8+3}{2}\right) = 19$$

9. (C)

10. (B) Old average

$$= \frac{70+72+74+76+78+80+82+84+86+88+90+92+94}{13}$$

$$= 82 \text{ kg}$$

Now new average

$$= \frac{1066+100+79}{15}$$

$$= \frac{1245}{15}$$

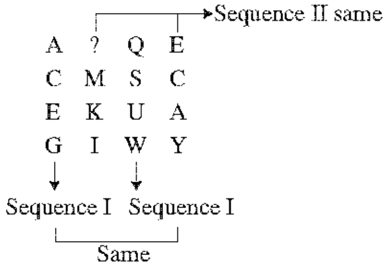
$$= 83 \text{ kg}$$

So, difference $= 83 - 82$

$$= 1 \text{ kg}$$

11. (C) 12. (B)

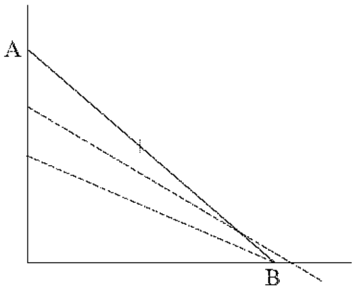
13. (D)



So letter should be 0.

14. (B) 15. (B) 16. (A)

17. (B)



The arc of a circle.

18. (C)

19. (A) Total 2^n terms are there.

20. (D) The mass of arbitrarily chosen shell is proportional to its surface area.

21. (B)

$$V(x) = \frac{-1}{2}ax^2 + \frac{1}{4}bx^4$$

$$\frac{dV(x)}{dx} = \frac{-2ax}{2} + \frac{4}{4}bx^3$$

$$= -xa + bx^3 = 0$$

$$\Rightarrow \quad x(-a + bx^2) = 0$$

$$\Rightarrow \quad x = 0$$

and $\quad x = \pm\sqrt{\frac{a}{b}}$

For stable equilibrium

$$\frac{d^2V}{dx} = -a + 3bx^2 > 0$$

Only at $\quad x = \sqrt{\frac{a}{b}}$ is stable point

$$= -a + 3b \times \frac{a}{b}$$

$$= 2a$$

$$\text{Frequency} = \sqrt{\frac{1}{m}\frac{d^2V}{dx^2}\Big|_{x=\pm\sqrt{a/b}}}$$

$$= \sqrt{\frac{1}{m} \times 2a}$$

$$= \sqrt{\frac{2a}{m}}$$

22. (C) The change in g outside the earth is—

$$g\left(\frac{1-2h}{R_e}\right)$$

Here the acceleration due to Earth's gravity differs from g at Earth's surface approximately by 1%.

So $\quad \frac{2h}{R_e} = \frac{1}{100}$

Or $\quad h = \frac{R_e}{2 \times 100}$

$$= \frac{6400}{2 \times 100}\text{km}$$

$$= 32 \text{ km}$$

23. (C)

24. (D)

$$\overline{\nabla} \cdot \overline{r} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(\hat{i}x + \hat{j}y + \hat{k}z\right)$$

$$= 1 + 1 + 1 = 3$$

whereas $\quad \overline{\nabla} \times \overline{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$

$$= 0$$

25. (B) 26. (A)

27. (C) Two independent random variables m and n, which can take the integer value 0, 1, 2, …… ∞, follow the Poisson distribution, with distinct mean values μ and ν respectively. Then the variance of the random variable $l = m + n$ is equal to $\mu + \nu$.

28. (C) $\quad f(z) = e^z + e^{1/z}$

about $\quad z = 0$

Expansion of $\quad e^z = 1 + z + \frac{z^2}{2!} + \ldots \frac{z^n}{n!} + \ldots \quad \ldots(1)$

whereas for $e^{1/z}$ we obtain

$$= 1 + \sum_{n=1}^{\infty} \frac{1}{n!} z^{-n} \quad \ldots(2)$$

Combining equations (1) and (2) we get

$$f(z) = \sum_{n=0}^{\infty}\left(z^n + \frac{1}{z^n}\right)\frac{1}{n\,!}$$

for all $0 < |z| < \infty$.

29. (A) Given $\quad L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$

The equation of motion is given by

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \qquad \ldots(1)$$

$$\frac{\partial L}{\partial \dot{q}} = m\dot{x}\,e^{\gamma t}$$

whereas $\quad \frac{\partial L}{\partial q} = -e^{+\gamma t}\frac{\partial V(x)}{\partial x}$

Substitute these values in equation (1) we get

$$\frac{\partial}{\partial t}(m\dot{x}\,e^{\gamma t}) + \frac{e^{\gamma t}\partial V(x)}{\partial x} = 0$$

$$m\ddot{x}\;e^{\gamma t} + m\dot{x}\,\gamma e^{\gamma t} + e^{\gamma t}\frac{\partial V(x)}{\partial x} = 0$$

devide the whole equation by $e^{\gamma t}$, we get

$$m\ddot{x} + m\dot{x}\gamma + \frac{\partial V(x)}{\partial x} = 0$$

30. (B) Using Gauss law

$$\int E\cdot ds = \frac{q_{en}\,C}{\epsilon_0}$$

$$= \frac{\int \rho dV}{\epsilon_0}$$

For $\quad r = \frac{R}{2}$

Let $\quad E = E_1$

$$E_1 \cdot 4\pi\frac{R^2}{4} = \frac{1}{60}$$

$$\int_0^{R/2}\int_0^{\pi}\int_0^{2\pi}\rho_0\left(1 - \frac{ar}{R}\right)r^2dr\sin\theta d\theta d\phi$$

Or, $\quad E_1 = \frac{4R}{\epsilon_0}\left[\frac{1}{3\times 8} - \frac{a}{4\times 16}\right]$

$$= \frac{R}{2\epsilon_0}\left(\frac{1}{3} - \frac{a}{8}\right)$$

Now, $\quad E_2 \cdot 4\pi R^2 = \frac{1}{\epsilon_0}$

$$\int_0^{R}\int_0^{\pi}\int_0^{2\pi}\rho_0\left(1 - \frac{ar}{R}\right)r^2dr\sin\theta d\theta d\phi$$

$$E_2 \cdot 4\pi R^2 = \frac{4\pi}{\epsilon_0}\left(\frac{R^3}{3} - \frac{aR^4}{4R}\right)$$

$$E_2 = \frac{R}{\epsilon_0}\left(\frac{1}{3} - \frac{a}{4}\right)$$

Now according to question

$$E_1 = 1{\cdot}25 \times E_2$$

$$\frac{R}{2\epsilon_0}\left(\frac{1}{3} - \frac{a}{8}\right) = \frac{1{\cdot}25\,R}{\epsilon_0}\left(\frac{1}{3} - \frac{a}{4}\right)$$

$$\left(\frac{1}{3} - \frac{a}{8}\right) = 2{\cdot}50\left(\frac{1}{3} - \frac{a}{4}\right)$$

$$\frac{1}{3} - \frac{2{\cdot}5}{3} = \frac{a}{8} - \frac{2{\cdot}50a}{4}$$

$$\frac{-1{\cdot}5}{3} = \frac{a - 5a}{8}$$

$$\frac{-1{\cdot}5}{3} = \frac{4a}{8}$$

Or, $\quad \frac{1}{2} = \frac{1}{2}a$

Or, $\quad \boxed{a = 1}$

31. (D) The electrostatic lines of force due to a system of four point charges as shown in question, at large distance, r, it will behave as quadruple, so the leading behaviour of the electrostatic potential is proportional to $\frac{1}{r^3}$.

32. (B) For a uniform magnetic field

$$d = v_{11}\left(\frac{2\pi r}{u_\perp}\right)$$

$$d = \frac{2\pi m v_{11}}{qB}$$

Now velocity is double

$$d_1 \propto \frac{2\pi m(2v_{11})}{qB}$$

$$\propto \frac{4\pi m v_{11}}{qB}$$

33. (C) For thin film of refractive index $n > 1$, for bright fringes we have

$$2nt = (2m - 1)\lambda/2$$

where $\quad m = 1, 2, 3, \ldots$

For $\quad m = 2$

$$2nt = (4 - 1)\frac{\lambda}{2}$$

$$t = \frac{3\lambda}{4n}$$

34. (B) $L_x \psi_{lm} = \frac{L_+ + L_- \psi_{lm}}{2}$

$$= \frac{\hbar}{2}\left[\sqrt{l(l+1) - m(m+1)}\ \psi_{lm+1} + \sqrt{l(l+1) - m(m-1)}\ \psi_{lm-1}\right]$$

$$\phi = a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1\bar{1}}$$

Given that ϕ

$$Lx\ \phi = \hbar\phi$$

$$= \hbar[a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1\bar{1}}] \quad \ldots(1)$$

Here $L_x[a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1\bar{1}}]$

$$= a_1 \cdot \left[\frac{\hbar}{2}\left\{\sqrt{2-2}\,\psi_{12} + \sqrt{2}\,\psi_{10}\right\}\right] + a_2\left[\frac{\hbar}{2}\left\{\sqrt{2}\,\psi_{11} + \sqrt{2}\,\psi_{1\bar{1}}\right\}\right] + a_3\left[\frac{\hbar}{2}\left\{\sqrt{2}\,\psi_{10} + 0\right\}\right]$$

$$= \frac{\hbar}{2}\left[a_1\sqrt{2}\,\psi_{10} + \left(\sqrt{2}\,\psi_{11} + \sqrt{2}\,\psi_{1\bar{1}}\right)(a_2 + a_3\sqrt{2}\,\psi_{10}])\right]$$

$$= \hbar\left[\frac{\sqrt{2}}{2}a_2\,\psi_{11} + \left(\frac{\sqrt{2}\,a_1 + \sqrt{2}\,a_3}{2}\right)\psi_{10} + \frac{\sqrt{2}}{2}a_2\psi_{1\bar{1}}\right] \quad \ldots(2)$$

Compare (1) and (2) we get

$$a_1 = \frac{\sqrt{2}}{2}a_2$$

$$a_2 = \frac{\sqrt{2}a_1 + \sqrt{2}a_3}{2}$$

$$a_3 = \frac{\sqrt{2}}{2}a_2$$

Suggest $a_1 = a_3$

and $a_2 = \frac{2\sqrt{2}\,a_1}{2}$

$$= \sqrt{2}\,a_1$$

$$a_1 = a_3 = \frac{1}{2}$$

and $a_2 = \frac{\sqrt{2}\times 1}{2}$

$$= \frac{1}{\sqrt{2}}$$

35. (A) $[x, p] = i$

in natural unit ($\hbar = 1$) using the property

$$[x, f(p)] = i\frac{\partial}{\partial p}f(p)$$

$$= i\frac{\partial}{\partial p}[pe^{-p}]$$

$$= i[-pe^{-b} + e^{-b}]$$

$$= i(1-p)\,e^{-b}$$

36. (B)

37. (B) At $t = 0$

$$\psi(t) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

At any time $\psi(t) = \frac{1}{\sqrt{2}}\psi_1 e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\psi_2 e^{-iE_2 t/\hbar}$

For orthogonal condition—

$$<\psi(t) \mid (\psi)(t)> = <\frac{1}{\sqrt{2}}\psi_1 e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\psi_2 e^{-iE_2 t/\hbar}\left|\frac{1}{\sqrt{2}}\psi_1 e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\psi_2 e^{-iE_2 t/\hbar}\right.>$$

$$= \frac{1}{2}[<\psi_1 \mid \psi_1> + <\psi_2 \mid \psi_2> + <\psi_1 \mid \psi_2>\ e^{iE_1 t/\hbar} e^{-iE_2 t/\hbar} + <\psi_2 \mid \psi_1>\ e^{iE_2 t/\hbar}\ e^{-iE_1 t/\hbar}]$$

$$= \frac{1}{2}[1 + 1 + <\psi_1 \mid \psi_2> [e^{i(E_2 - E_1)t/\hbar} + e^{-i(E_2 - E_1)t/\hbar}]]$$

$$= 0$$

$$= \frac{1}{2}[2 + 2 <\psi_1 \mid \psi_2> \cos(E_2 - E_1)t/\hbar] = 0$$

Or, $<\psi_1 \mid \psi_2> = 0$

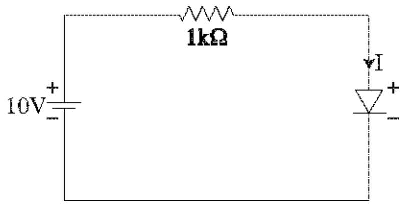
$$\Rightarrow \quad \cos(E_2 - E_1)\frac{t}{\hbar} = -1$$

Or $\frac{(E_2 - E_1)t}{\hbar} = \pi$

At $t = T$

Or $T = \frac{\pi\hbar}{(E_2 - E_1)}$

38. (C) $I = \begin{cases} (V - 0{\cdot}7)/500\text{A}, & V \geq 0{\cdot}7\text{V} \\ 0 & , V < 0{\cdot}7\text{V} \end{cases}$



Diode will be forward biased (ON state)

$$V \geq 0{\cdot}7$$

Given $I = \dfrac{V - 0{\cdot}7}{500}$

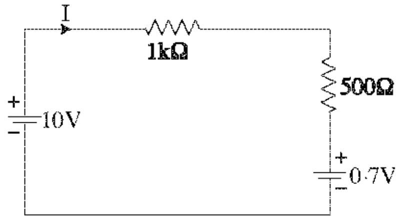
Differentiating with respect to V

$$\frac{dI}{dV} = \frac{1}{500}$$

$$R_\phi = \frac{dV}{dI}$$

$$= 500\ \Omega$$

Now equivalent circuit is



Now, apply KVL for linear circuit,

$$-10 + 1\ k\Omega \times I + 500\Omega \times I + 0{\cdot}7 = 0$$

Or, $$-9{\cdot}3 + 1500\ I = 0$$

$$I = \frac{9{\cdot}3}{1500}$$

$$= 6{\cdot}2\ \text{mA}$$

39. (A)

40. (D) $$F \propto V^{1/3}$$

$$\Delta F = \sqrt{\left(\frac{\partial F}{\partial V} \cdot \Delta V\right)^2}$$

$$= \sqrt{\left(\frac{1}{3} V^{-2/3}\, \Delta V\right)^2}$$

Or, $$\frac{\Delta F}{F} = \sqrt{\left(\frac{1}{3}\frac{\Delta V}{V^{2/3}}\right)^2} \cdot \frac{1}{V^{1/3}}$$

$$= \frac{\Delta V}{3r^{2/3}} \cdot \frac{1}{V^{1/3}}$$

$$= \frac{\Delta V}{3 \times V}$$

$$\Delta V = 2{\cdot}7\ \text{mm}^3$$

and $$V = 3\ \text{mm}^3$$

given percentage uncertainty

$$\frac{\Delta F}{F} \times 100\% = \frac{\Delta V}{3 \times V} \times 100$$

$$= \frac{2{\cdot}7}{3 \times 3} \times 100$$

$$= 3\%$$

41. (D) Work done

$$W = \int_{V_1}^{V_2} \rho\, dV)$$

$$= \int_{V_1}^{V_2} (a\rho + b\rho^2) dV)$$

$$\rho = \frac{n}{V} \to \text{limit } \rho_0 \text{ to } 2\rho_0$$

$$V = \frac{n}{\rho}$$

$$= 0$$

$$dV = -\frac{n}{\rho^2} d\rho$$

$$= \int_{2\rho_0}^{\rho_0} (a\rho + b\rho^2) \left(\frac{-d\rho}{\rho^2}\right) \cdot n$$

$$= n \left[-\int_{2\rho_0}^{\rho_0} \left(\frac{a}{\rho} + b\right)\right] d\rho$$

$$= -n \left[a \ln \rho \int_{\rho_0}^{\rho_0} + b\rho \int_{2\rho_0}^{\rho_0}\right]$$

$$= n \left[a \ln \rho \int_{\rho_0}^{2\rho_0} + b\rho \int_{\rho_0}^{2\rho_0}\right]$$

$$= n\,[a \ln 2 + b\rho_0]$$

$$n = \rho_0 V_0$$

So work done

$$= \rho_0 V_0\, [a \ln 2 + b\rho_0]$$

42. (A) 43. (B)

44. (D) Oxygen is diatomic molecule with $C_V = \frac{5}{2} k_B$.

45. (A) In given circuit, First Op-Amp is like a buffer and help to keep the positive half cycle of input signal and hence it is required because source independence is much greater than r.

46. (A)

47. (D) $\phi \to \phi' = \phi - \frac{\partial \xi}{\partial t}$

$$\left[\text{it should be } \phi - \frac{1}{c}\frac{\partial \xi}{\partial t}\right]$$

$$\vec{A} \to \vec{A'} = A + \nabla\xi,$$

So, $\nabla \cdot A' + \frac{1}{c}\frac{\partial \phi'}{\partial t} = 0$

$$\nabla \cdot (A + \nabla\xi) + \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\phi - 1}{c}\frac{\partial \xi}{\partial t}\right) = 0$$

$$\nabla A + \frac{1}{c}\frac{\partial \phi}{\partial t} + \nabla \cdot (\nabla\xi) - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

For Lorentz condition we must ξ such that

$$\nabla \cdot (\nabla\xi) - \frac{1}{c}\frac{\partial^2 \xi}{\partial t^2} = 0$$

$$\hat{i}\,\frac{\partial}{\partial x} \cdot (k \cosh \omega t \cdot \sinh \bar{k} \cdot \bar{x}) - \frac{1}{c^2}\frac{\partial}{\partial t}$$

$$[\omega \sinh \omega t \cosh \bar{k} \cdot x)]$$

$$\Rightarrow k^2 \cosh \omega t \cosh \bar{k} \cdot x - \frac{1}{c^2} \cdot \omega^2 \cosh \omega t \cosh (k \cdot x)$$

$$\left(k^2 - \frac{\omega^2}{c}\right) \cosh \omega t \cosh k \cdot x = 0$$

$$k^2 - \frac{\omega^2}{c^2} = 0$$

Hence, proved as $\omega = c\,|\vec{k}|$ given.

48. (A) The intensity of the radiation emitted by a particle of mass m, charge e, velocity v and acceleration a is given by—

$$I = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{e^2a^2}{c^2}$$

Independent of mass m and velocity v.

So for $e \to 2e$ and $a \to 2\vec{a}$

$$\text{Intensity} \to \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{(2e)^2\,(2a)^2}{c^2}$$

$$\alpha\ 4 \times 4\frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{e^2a^2}{c^2}$$

$$\alpha\ 16\,I.$$

49. (D) 50. (B)

51. (D) $P = -\frac{\partial F}{\partial Q}(q, Q)$

$$= \frac{-1}{q}$$

and $p_i = \frac{\partial F_1}{\partial q_i}$

$$= -\frac{Q}{q^2}$$

$$Q = -pq^2$$

$\Rightarrow$ $p = -\frac{Q}{q^2}$

$$= -Qp^2$$

$$p = -\frac{1}{q}$$

$\Rightarrow$ $q = -\frac{1}{p}$

Now $H(q, p) = \frac{b^2}{2m} + \frac{1}{2}m\omega^2q^2$

$$= \frac{(-Qp^2)^2}{2m} + \frac{1}{2}m\omega^2\left(-\frac{1}{p}\right)^2$$

$$= \frac{Q^2p^4}{2m} + \frac{1}{2}m\omega^2p^{-2}$$

52. (B) $[\vec{a} \cdot \vec{\sigma}, \vec{b} \cdot \vec{\sigma}] = [\vec{a_i} \cdot \vec{\sigma_i}, \vec{b_j} \cdot \vec{\sigma_j}]$

$$= a_ib_j\,[\sigma_i, \sigma_i]$$

$$= a_{ibj} \in_{ijk} (2_i\ \sigma_k)$$

$$= 2i\,(\vec{a} \times \vec{b}) \cdot \vec{\sigma}$$

53. (B) 54. (C) 55. (B)

56. (A) Noise form of noise in each transistor is $\frac{1}{f}$ then equivalent voltage noise spectral density for MOS transistor $\propto \frac{1}{N^2}$

So, there are N transistors

$$\frac{N}{N^2} \Rightarrow 0\frac{1}{N}$$

57. (B) For stationary bound states we want

$$\int_{x_{min}}^{x_{max}} \hbar k\,(x)\,dx = \left(n + \frac{1}{2}\right)\pi\hbar$$

Here $k^2(x) = \frac{2m}{\hbar^2}(E - a\,|\,x\,|)$

We need

$$\sqrt{2m}\int_{x_{min}}^{x_{max}} \sqrt{(E - a|x\,|)}dx = 2\sqrt{2m}\int_0^{x_{max}} \sqrt{(E - a|x\,|)}\,dx$$

$$= \left(n + \frac{1}{2}\right)\pi\hbar$$

$$a\,x_{max} = E$$

$$\Rightarrow \quad x_{max} = \frac{E}{a}$$

$$\Rightarrow 2\sqrt{2m}\,\frac{2}{3a}\sqrt{(E - a|x|)^3}\,\Big|_0^{x_{max}}$$

$$= 2\sqrt{2m}\,\frac{(-2)}{3a}\sqrt{E^3}$$

$$= \left(n + \frac{1}{2}\right)\pi\hbar$$

$$E = \frac{(a\hbar)^{2/3}}{m^{1/3}}\left[\frac{3\pi\left(n+\frac{1}{2}\right)}{4\sqrt{2}}\right]^{2/3}$$

58. (B)

59. (C) $|\psi> = c_0|0> + c_1|1>$

$$c_0^2 + c_1^2 = 1$$

Let $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$

$$<x> = <\psi|x|\psi>$$

$$= c_0<0| + c_1<1|$$

$$\left|\left(\sqrt{\frac{\hbar}{2m\omega}}\,a_+ + a_-\right)\right|\; c_0|0> + c_1|1>$$

$$\sqrt{\frac{\hbar}{2m\omega}}\,[\{c_0<0| + c_1<1|(a_+|c_0|0> + c_1|1>)\}$$

$$+ c_0<0\,1 + c_1<1|a_-|c_0|0> + c_1|1>\}$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\,[c_0^2<0|a_+|0> + c_1c_0<_1|a_+|0> + c_0c_1<0|a_+|1> + c_1^2<1|a_+|1>$$

$$+ c_0^2<0\,1\,a_-\,1\,0> + c_0c_1<0|a_-|1> + c_0c_1<1|a_-|0> + c_1^2<1|a_-|1>]$$

$$a_+|n> = \sqrt{n+1}\,|n+1>$$

$$a_-|n> = \sqrt{n}\,|n-1>$$

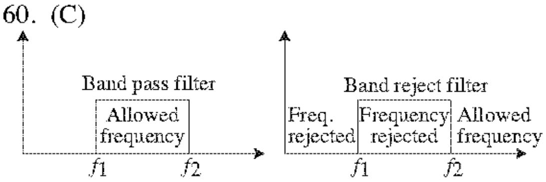
$$= \sqrt{\frac{\hbar}{2m\omega}}\,[0 + c_1c_0 + 0 + 0 + 0 + c_0c_1 + 0 + 0]$$

$$= 2\sqrt{\frac{\hbar}{2m\omega}}\,c_1c_0$$

For c_0 max $\quad c_0 = \frac{1}{\sqrt{2}}, c_1 = \frac{1}{\sqrt{2}}$

$$c_0 = \frac{1}{2}, c_1 = \frac{\sqrt{3}}{2}.$$

60. (C)



If the cut off frequencies of the low pass and high pass filters are f_{LP} and f_{HP}, respectively, the condition required to implement the band pass and band-reject filters are $f^{HP} < f^{LP}$ and $f^{HP} > f^{LP}$ respectively.

61. (C) We use the expression

$$S = \ln W$$

where W is the probability of the arrangement of spins.

For N spins we assume that N_u are up and N_d are down, where $N = N_u + N_d$

The difference arrangements are—

$$W = \frac{N!}{N_u!\,N_d!}$$

$$N_u = \frac{N(1+f)}{2}$$

where her $f = \Sigma\frac{S_i}{N}$

$$N_d = \frac{N(1-f)}{2}$$

using stirling approximation for the factorial to obtain $S = \ln W$

$$= [N\ln N - N_u\ln N_u - N_d\ln N_d]$$

$$= \frac{-N}{a}\left\{\frac{1+f}{2}\ln\frac{1+f}{2} + \frac{1-f}{2}\ln\frac{1-f}{2}\right\}$$

$$= -N\left\{\frac{1+\Sigma S_i/N}{2}\right\}\ln\left\{\frac{1+\Sigma S_i/N}{2}\right\}$$

$$f = \frac{+1-1}{N} = 0$$

$$= -N\left\{\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2}\ln\frac{1}{2}\right\}$$

$$= -N\left\{\ln\frac{1}{2}\right\} N = \frac{1}{2}$$

$$= -1\,N\ln 2$$

62. (B) When a gas expands adiabatically from volume V_1 to V_2 by a quasi-static reversible process, it cools from temp T_1 to T_2. If now the same process is carried out adiabatically and irreversibly, when it has equalibrated then $T_2^1 > T_2$, because internal energy increased and hence temperature increased.

63. (A) $n\frac{2}{3} - n\frac{1}{3} = \frac{n}{3}$

64. (C) 65. (B)

66. (C)

$$B = a_V A - a_S A^{2/3} - a_{sym}\frac{(2z - A)^2}{A} - \frac{ac\, z^2}{A^{1/3}}$$

For stable nucleus

$$\frac{dB}{dz} = 0$$

$$\Rightarrow -4\, a_{sym}\frac{(2z - A)}{A} - \frac{a_c \times 2z}{A^{1/3}} = 0$$

$$\frac{4a_{sym}}{A}(2z - A) = -\frac{a_c \times 2z}{A^{1/3}}$$

$$\frac{4a_{sym}}{A}(A - 2z) = \frac{a_c \times 2z}{A^{1/3}}$$

$$4a_{sym} - \frac{8\, a_{sym}\, z}{A} = \frac{a_c \times 2z}{A^{1/3}}$$

$$4a_{sym} = \frac{a_c \times 2z}{A^{1/3}} + \frac{8\, a_{sym}\, z}{A}$$

$$z = \frac{4a_{sym}}{\frac{a_c}{A^{1/3}} + \frac{8a_{sym}}{A}}$$

Now $A = 216$,

$A^{1/3} = 6$

$a_{sym} = 16$ MeV

$ac = 0{\cdot}75$ MeV

Substitute $z = \dfrac{4 \times 16\text{ MeV}}{\frac{0{\cdot}75}{6} + \frac{8 \times 16}{216}}$

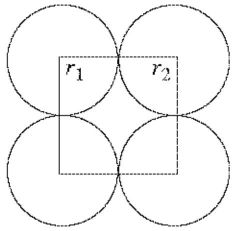
$= 84$

67. (B) Packing fraction $= \dfrac{\text{Volume of Atom}}{\text{Volume of Cubic Cell}}$

$= a$

$a = r_1 + r_2$

$$\frac{1}{2}\left(\frac{4\pi}{3}r_1^3 + \frac{4\pi}{3}r_2^3\right) = \text{Volume of atom occupied}$$



$$\frac{2\pi}{3}r_1^3 + \frac{2\pi r_2^3}{3} = \frac{2\pi}{3}\left(r_1^3 + r_2^3\right)$$

and $a = r_1 + r_2$

$$\text{Packing fraction} = \frac{\frac{2\pi}{3}\left(r_1^3 + r_2^3\right)}{(r_1 + r_2)^3}$$

68. (D) For all solids, at low temperature the behaviour of the lettice contribution to the specific heat will $\alpha\ T^3$.

69. (A) The term $^1p_1$ split into 3 lines with

$m_e = \pm 1, 0$

$\Delta E = -g\, \mu_B\, BM_e$

$$g = 1 + \frac{1 \times 2 + 0 - 1 \times 2}{2 \times 1 \times 2}$$

$= 1$

$^1s_0$ does not split in week magnetic field

$$E = \frac{hc}{\lambda}$$

$$\Delta E = -hc\frac{\Delta\lambda}{\lambda^2}$$

$$g\, \mu_B B = -hc\frac{\Delta\lambda}{\lambda^2}$$

$$\Delta\lambda = \left|\frac{g\, \mu_B\, B\, \lambda^2}{hc}\right|$$

$$= \frac{1 \times 9{\cdot}27 \times 10^{-24}\ \frac{J}{T} \times 1\ T \times (250\text{ nm})^2}{6{\cdot}634 \times 10^{-34} \times 3 \times 10^8 \cdot m}$$

$$= \frac{9{\cdot}27 \times 10^{-24} \times 250\text{ nm} \times 250 \times 10^{-9}\text{ m}}{6{\cdot}634 \times 10^{-34} \times 3 \times 10^{+8}\text{ m}}$$

$$= \frac{9{\cdot}27 \times 2{\cdot}5 \times 2{\cdot}5}{6{\cdot}634 \times 3} \times 10^{-3}\text{ nm}$$

$= 0{\cdot}01$ nm

70. (A) The expression for the vibrational quantum number leading to dissociation of the molecule is—

$$v_{max} = \frac{1}{2x_e} - \frac{1}{2}$$

$$= \frac{1}{2}\left[\frac{1}{x_e} - 1\right]$$

$$x_e = 0{\cdot}001 \text{ (given)}$$

$$= \frac{1}{2}\left[\frac{1}{0{\cdot}001} - 1\right]$$

$$= \frac{1}{2}[1000 - 1]$$

$$= \frac{1}{2} \times 999$$

$$= 499{\cdot}5$$

$$\approx 500$$

71. (A) A flux passing through the superconductor is quantized in units of $\frac{hc}{2e}$ because this flux is due to cooper pair with charge $2e$ not e.

72. (C) The Hamiltonian is given by

$$H' = a\, S_e \cdot S_p$$

$$S = S_e + S_p$$

Squaring both side $S^2 = (S_e + S_p)^2$

$$= S_e^2 + S_p^2 + 2S_e \cdot S_p$$

So, $$S_e \cdot S_p = \frac{S^2 - S_e^2 - S_p^2}{2}$$

$$= \left[\frac{S(S+1) - S_e(S_e+1) - S_p(S_p+1)}{2}\right]\hbar^2$$

Now the splitting in triplet state and singlet state $\Delta E = E_{triplet} - E_{singlet}$

$$E = \frac{a\hbar^2}{2}$$

$$\left[\frac{1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)}{2}\right]$$

$$-\frac{a\hbar^2}{2}\left[\frac{0(0+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)}{2}\right]$$

$$= a\hbar^2$$

73. (A) 74. (D)

75. (D) As in (1) is forbidden because spin is not conserved.

(2) as charge is not conserved.

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