**D 120181** **(Pages : 3)** **Name.......................................**

**Reg. No...................................**

**SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, MARCH 2025**

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

**Section A**

*Answer any number of questions.*
*Each question carries 2 marks.*
*Ceiling is 25.*

1. Prove that polynomial functions are continuous on $\mathbb{R}$.

2. Give an example of uniformly continuous function which is not a Lipschitz function.

3. State Weierstrass approximation theorem.

4. Prove that every constant function on $[a, b]$ is Reimann integrable on $[a, b]$.

5. Suppose $f$ and $g$ are in $\mathcal{R}[a, b]$. If $f(x) \leq g(x)$ for all $x \in [a, b]$, prove that $\int_a^b f \leq \int_a^b g$.

6. State substitution theorem and use this to evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.

7. Prove that rational numbers in $[0, 1]$ is a null set.

8. If $f \in \mathcal{R}[a, b]$, Prove that $|f| \in \mathcal{R}[a, b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

9. Define uniform norm on a set of bounded functions.

10. Evaluate $\lim \left(e^{-nx}\right)$ for $x \in \mathbb{R}, x \geq 0$.

**Turn over**

11. State and prove Weierstrass M test.

12. Evaluate $\int_{1}^{\infty} \frac{dx}{x^2+1}$.

13. Find P.V. $\int_{-\infty}^{\infty} x\, dx$.

14. Prove that $\int_{2}^{\infty} \frac{1}{\ln(x)}\, dx = \infty$.

15. Prove that $\Gamma(p)$ is strictly convex for $0 < p < \infty$.

## Section B

*Answer any number of questions.*
*Each question carries 5 marks.*
*Ceiling is 35.*

16. Prove that Dirichlet's function is discontinuous on $\mathbb{R}$.

17. State and prove preservation of intervals theorem.

18. If $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$, then prove that $f \in \mathcal{R}[a, b]$.

19. State and prove Taylor's theorem with the remainder.

20. Let $(f_n)$ be sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that $(f_n)$ converges uniformly on A to a function $f : A \to \mathbb{R}$. Prove that $f$ is continuous on A.

21. Prove that for all $q \leq -1$, $\int_{0}^{1} x^q e^{-x}\, dx$ diverges.

22. Prove that $\int_{0}^{\infty} \sqrt{y} e^{-y^2}\, dy = \frac{1}{2}\Gamma(3/4)$.

23. Prove that $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ for all $p > 0$ and $q > 0$.

## Section C

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*
*Maximum* 20 *marks.*

24. State and prove location of roots theorem.

25. State and prove second form of Fundamental theorem of calculus.

26. State and prove Cauchy criterion for uniform convergence.

27. For any $p > 0$, fixed, Prove that $\lim\limits_{0 < q \to \infty} q^p \,\mathrm{B}(p, q) = \Gamma(p)$.

(2 × 10 = 20 marks)

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**D 100611** **(Pages : 3)** **Name...................................**

**Reg. No................................**

# SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions onwards)

Time : Two Hours and a Half Maximum Marks : 80

## Section A

*Questions* 1—15. *Answer any number of questions.*
*Each carry* 2 *marks. Maximum marks* 20.

1. State discontinuity criterion. Hence show that the signum function is not continuous at $x = 0$.
2. State maximum-minimum theroem.
3. Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[a, \infty)$ where $a > 0$.
4. Define Riemann integral of a function $f$ on an integral $[a, b]$.
5. If $f$ and $g$ are in R$[a, b]$ and if $f(x) \leq g(x)$ for all $x$ in $[a, b]$ then show that $\int_a^b f \leq \int_a^b g$.
6. State Lebesgue's integrability criterion.
7. If $f$ and $g$ belong to R$[a, b]$ then the product $fg$ belongs to R $[a, b]$.
8. Show that $\lim \frac{\sin(nx + n)}{n} = 0$ for $x \in \mathbb{R}$.
9. Discuss the uniform convergence of $f_n(x) = \frac{x}{n}$ on A = [0, 1].
10. Evaluate $\lim\left(e^{-nx}\right)$ for $x \in \mathbb{R}, x \geq 0$.
11. Define absolute convergence of series of functions.
12. Evaluate $\int_{-\infty}^{0} e^x \, dx$.
13. Find the principal value of $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$.

**Turn over**

472337

14. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

15. Define Beta fucntion. St B $(p, q)$ = B $(q, p)$.

## Section B

*Questions* 16—23. *Answer any number of questions.*
*Each carry* 5 *marks. Maximum marks* 35.

16. Let $A = \{x \in \mathbb{R} | x > 0\}$. Define $h$ on A by $h(x) = 0$ if $x \in A$ is irrational and $h(x) = \frac{1}{n}$ if $x \in A$ is rational with $x = \frac{m}{n}, m, n \in \mathbb{N}$ have no common factor except 1. Then show that $h$ is continuous at every irrational number in A and discontinuous at every rational number in A.

17. Let I be an interval and $f : I \to \mathbb{R}$ be a continuous function on I then show that $f$(I) is an interval.

18. If $f \in R\,[a, b]$ then show that $f$ is bounded on $[a, b]$.

19. Show that if $\phi : [a, b] \to R$ is a step function then $\phi \in R[a, b]$.

20. Evaluate $\lim \frac{x^2 + nx}{n}, x \in R$. Is the convergence uniform on $\mathbb{R}$ ?

21. Let $(f_n)$ be a sequence of bounded functions on $A \subseteq \mathbb{R}$. Then show that $(f_n)$ converges uniformly on A to a bounded function $f$ iff for each $\varepsilon > 0$ there is a number H($\varepsilon$) in $\mathbb{N}$ such that for all $m, n \geq H(\varepsilon)$ then $\|f_m - f_n\|_A \leq \varepsilon$.

22. Discuss the convergence of $\int_0^\infty \frac{\sin^2 x}{x^2} dx$.

23. Define Beta function and show that $\forall p > 0, q > 0, B(p, q) = 2\int_0^{\pi/2} \sin^{2p-1}\theta \cos^{2p-1}\theta \, d\theta$.

## Section C

*Questions* 24—27. *Answer any* **two** *questions.*
*Each carry* 10 *marks.*

24. (a) Show that if $f$ and $g$ are uniformly continuous on $A \subseteq \mathbb{R}$ and they are bounded on A then their product $fg$ is also uniformly continuous.

    (b) Show that $f(x) = \sqrt{x}$ is uniformly continuous on $[a, \infty)$ where $a > 0$.

25. Suppose $f$ and $g$ are in R $[a, b]$. Then

(a) if $k \in \mathrm{R}$, show that $kf \in \mathcal{R}[a,b]$ and $\int_a^b kf = k\int_a^b f$.

(b) $f + g \in \mathcal{R}[a,b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

26. Discuss the pointwise and uniform convergence of :

(a) $f_n(x) = \dfrac{\sin(nx+n)}{n}$ for $x \in \mathbb{R}$.

(b) $g_n(x) = \dfrac{x^2 + nx}{n}$ for $x \in \mathbb{R}$.

27. Show that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

(2 × 10 = 20 marks)

**C 40602** **(Pages : 3)** **Name.......................................**

**Reg. No...................................**

## SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

### Section A

*Answer any number of questions.*
*Each question carries* 2 *marks.*
*Maximum marks* 25.

1. State sequential criterian for continuity.
2. Show that the sine function is continuous on $\mathbb{R}$.
3. Define Lipchitz function. If $f : A \to \mathbb{R}$ is a Lipschitz function then show that $f$ is uniformly continuous on A.
4. Define tagged partition.
5. Show that every constant function on $[a,b]$ is in $\mathbb{R}[a,b]$.
6. State Cauchy's criterion for Riemann integrability.
7. Let F, G be differentiable on $[a,b]$ and let $f = F'$ and $g = G'$ belong to $\mathbb{R}[a,b]$, then show that $\int_a^b f\,G = [FG]_a^b - \int_a^b F\,g$.
8. Show that $\lim\left(\frac{x}{n}\right) = 0$ for $x \in \mathbb{R}$.
9. Define uniform convergence of a sequence of functions.
10. State bounded convergence theorem.
11. State Weirstrass M-test for the uniform convergence of series of functions.
12. Evaluate $\int_1^\infty \frac{dx}{x^2+1}$.

**Turn over**

13. Find the principal value of $\int_{-2}^{3} \frac{dx}{(x-1)^3}$.

14. Discuss the absolute convergence of $\int_{0}^{\infty} \frac{\sin x}{n+1}\, dx$ for $n\pi \le x \le (n+1)\,\pi$, $n = 0, 1, 2, \ldots$

15. If $\int_{0}^{b} \frac{dx}{1+ax} = \frac{1}{a} \ln(1+ab)$. Evaluate $\int_{0}^{b} \frac{x dx}{(1+ax)^2}$.

## Section B

*Questions* 16–23, *answer any number of questions.*
*Each question carries* 5 *marks.*
*Maximum marks* 35.

16. State and prove Boundedness theorem for continuous function.

17. Show that $f(x) = \frac{1}{1+x^2}$, $x \in \mathbb{R}$ is uniformly continuous in $\mathbb{R}$.

18. State and prove Squeeze theorem for Riemann integrable functions.

19. If $f \in \mathbb{R}[a,b]$ and $f$ is continuous at a point $c \in [a,b]$. Then show that the indefinite integral $F(z) = \int_{a}^{z} f$ for $z \in [a,b]$ is differentiable at $c$ and $F'(c) = f(c)$.

20. Show that a sequence $(f_n)$ of bounded functions on $A \subset \mathbb{R}$ converges uniformly on A to $f$ iff $\| f_n - f \| n \to 0$.

21. Discuss the convergence of $f_n(x) = \frac{x^n}{n + x^n}$, $x \ge 0$. Is the convergence uniform on $[0, \infty]$.

22. Evaluate $\int_{-1}^{1} \frac{dx}{x^2 - 1}$.

23. Show that $\cancel{r}\ q \in \mathbb{R}$, $\int_{1}^{\infty} x^q e^{-x}\, dx$ converges.

## Section C

*Questions* 24–27, *answer any* **two** *questions.*
*Each question carries* 10 *marks.*

24. State and prove Maximum Minimum Theorem.

25. State and prove Cauchy's criterion of Riemann integrability.

26. Let $(f_n)$ be a sequence of functions in $\mathbb{R}[a,b]$ and suppose that $(f_n)$ converges uniformly on $[a,b]$ to $f$. Then show that $f \in \mathbb{R}[a,b]$.

27. Show that $\int_0^{\infty} \frac{\sin x}{x} dx$ exists and converges to a finite real value and that this integral does not converge absolutely.

(2 × 10 = 20 marks)

**C 20645** **(Pages : 3)** **Name.......................................**

**Reg. No...................................**

## SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

### Section A

*Answer at least* **ten** *questions.*
*Each question carries* 3 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

1. Define continuity of a function. Show that the constant function $f(x)=b$ is continuous on $\mathbb{R}$.

2. State Boundedness theorem. Is boundedness of the interval, a necessary condition in the theorem ? Justify with an example.

3. If $f : \mathrm{A} \to \mathrm{IR}$ is uniformly continuous on $\mathrm{A} \subseteq \mathbb{R}$ and $(x_n)$ is a Cauchy sequence in A. Then show that $f(x_n)$ is a Caychy sequence in $\mathbb{R}$.

4. Define Riemann sum of a function $f:[a,b] \to \mathbb{R}$.

5. Suppose $f$ and $g$ are in $\mathbb{R}[a,b]$ then show that $f+g$ is in $\mathbb{R}[a,b]$.

6. State squeeze theorem for Riemann integrable functions.

7. If $f$ belong to $\mathbb{R}[a,b]$, then show that its absolute value $|f|$ is in $\mathbb{R}[a,b]$.

8. Define pointwise convergence of a sequence $(f_n)$ of functions.

9. Discuss the uniform convergence of $f_n(x)=x^n$ on $(-1,1]$.

10. If $h_n(x)=2nxe^{-nx^2}$ for $x \in [0,1], n \in \mathbb{N}$ and $h(x)=0$ for all $x \in [0,1]$, then show that :

$$\lim \int_0^1 h_n(x)\,dx \neq \int_0^1 h(x)\,dx.$$

11. State Cauchy criteria for uniform convergence series of functions.

**Turn over**

12. Evaluate $\int_{-1}^{0} \frac{dx}{\sqrt[3]{x}}$.

13. What is Cauchy principle value. Find the principal value of $\int_{-1}^{1} \frac{dx}{x}$.

14. State Leibniz rule for differentiation of Ramann integrals.

15. State that $\Gamma(p+1) = p\Gamma p$ for $p > 0$.

(10 × 3 = 30 marks)

## Section B

*Answer at least* **five** *questions.*
*Each question carries* 6 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

16. Show that the Dirichlet's function :

$f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$ is not continuous at any point of $\mathbb{R}$.

17. State and prove Bolzano intermediate value theorem.

18. Show that the following functions are not uniformly continuous on the given sets :

    (a) $f(x) = x^2$ on $A = [0, \infty]$.

    (b) $g(x) = \sin \frac{1}{x}$ on $B = (0, \infty)$.

19. If $f : [a,b] \to \mathbb{R}$ is continuous on $[a,b]$, then show that $f \in \mathbb{R}[a,b]$.

20. Let $(f_n)$ be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that $(f_n)$ converges uniformly on A to a function $f : A \to \mathbb{R}$. Then show that $f$ is continuous on A.

21. Let $f_n : [0,1] \to \mathrm{I\!R}$ be defined for $n \geq 2$ by :

$$f_n(x) = \begin{cases} n^2 x & , 0 \leq x \leq \frac{1}{n} \\ -n^2(x - 2/n), & \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & , \frac{2}{n} \leq x \leq 1. \end{cases}$$

Show that the limit function is Riemann integrable. Verify whether $\lim \int_0^1 f_n(x) = \int_0^1 f(x)\,dx$.

22. Given $\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dx\, dy = \pi$, find the value of $\int_0^\infty e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$.

23. Show that $\forall p > 0, q > 0$ $B(p,q) = \frac{\Gamma p\, \Gamma q}{\Gamma(p+q)}$.

(5 × 6 = 30 marks)

## Section C

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

24. State and prove Location of roots theorem.

25. State and prove Additivity theorem.

26. Evaluate (a) $\lim \frac{x^n}{1+x^n}$ for $x \in \mathbb{R}, x \geq 0$. (b) $\lim \frac{\sin nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.

    Discuss about their uniform convergence.

27. (a) Show that $\forall\, q > -1, \int_0^1 x^q e^{-x} dx$ converges.

    (b) Show that $\forall\, q \leq -1, \int_0^1 x^q e^{-x} dx$ diverges.

(2 × 10 = 20 marks)