D 120183 **(Pages : 3)** **Name....................................**

Reg. No..................................

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION MARCH 2025

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half | Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 Marks.

1. Define an analytic function. Give an example.

2. Define L'Hôpital Rule and using it compute $\lim_{z \to i} \frac{z^7 + i}{z^{14} + 1}$.

3. Verify the Cauchy-Riemann equations for $f(z) = \operatorname{Re}(z)$.

4. Define harmonic function and show that $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic.

5. Prove that the two families of curves $u(x, y) = x^2 - y^2 = c_1$ and $v(x, y) = 2xy = c_2$ form an orthogonal system.

6. Find the derivative of $e^{z^2 - (1+i)z + 3}$.

7. Find the value of $\cos i$ and $\sin(2 + i)$.

8. Evaluate $\oint_C x\,dx$ where C is the circle defined by

   $x = \sin t, y = \cos t, 0 \le t \le 2\pi.$

Turn over

9. Using Cauchy's integral formula evaluate $\oint_C \frac{z^2 - 4z + 4}{z + i}$ where C is the circle $|z| = 2$.

10. State Cauchy's integral formula for derivatives.

11. State Ratio test.

12. State Taylor's theorem and write the Maclaurin series expression for $\sin z$.

13. State Laurent's theorem.

14. Prove that $f(z) = \frac{\sin z}{z}$ has a removable singular point.

15. State Cauchy's residue theorem.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 Marks.

16. Show that the function $f(z) = x + 4iy$ is not differentiable at z.

17. Prove that an analytic function $f(z) = u + iv$ is constant if its (a) real part is constant ; and (b) its modulus is constant.

18. Briefly explain the exponential mapping $w = e^z$.

19. Find the values of $(1+i)^i$.

20. Evaluate $\int_C \bar{z} dz$ where C is $x = 3t, y = t^2, -1 \le t \le 4$.

21. State and prove the fundamental theorem for contour integrals.

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22. Find the radius of convergence of :

(a) $\sum_{0}^{\infty} \frac{z^n}{n!}$; and

(b) $\sum_{1}^{\infty} \frac{n!z^n}{n^n}$.

23. Find the Maclaurin series expansion of $f(z) = \frac{1}{(1-z)^2}$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *Marks.*

24. State and prove Cauchy Riemann equations.

25. Expand $f(z) = \frac{1}{(z-1)^2 (z-3)}$ in a Laurent series valid for

(a) $0 < |z-1| < 2$; and

(b) $0 < |z-3| < 2$.

26. State and prove :

(a) Fundamental theorem of algebra ; and

(b) Morera's theorem.

27. Evaluate $\int_{0}^{2\pi} \frac{1}{(2+\cos\theta)^2} d\theta$.

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D 100613 **(Pages : 2)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admissions onwards)

Time : Two Hours and a Half Maximum Marks : 80

Section A

Answer any number of questions.
Each question carries 2 marks. Maximum marks 25.

1. Show that $f(z)=\overline{z}$ is nowhere differentiable.
2. Verify Cauchy-Riemann equations for $f(z) = z^2$.
3. Write the Cauchy-Riemann equations in polar co-ordinates.
4. Define harmonic function and harmonic conjugate function.
5. Solve $e^w = -2$.
6. Express cos $(2 - 4i)$ in the form $a + ib$.
7. Evaluate $\int ydx + xdy$ on the curve $y = x^2$ from (0, 0) to (1, 1).
8. Evaluate $\oint zdz$ over the first quadrant of the circle $|z| = 1$ from $z = i$ *to* $z = 1$.
9. Prove that $\int\limits_C f(z)dz = 0$ for $f(z)=\dfrac{z^2}{z-3}$ where C is the unit circle $|z| = 1$.
10. Evaluate $\oint\limits_C \dfrac{z+1}{z^4+2iz^3}\,dz$ where C is the circle $|z| = 1$.
11. Define (a) Power series ; (b) Circle of convergence.
12. Write the Maclaurin series expansion for $\sin z$ and $\cos z$.
13. Expand $f(z)=e^{\frac{3}{z}}$ in a Laurent series valid for $0<|z|<\infty$.
14. Find zeroes of $f(z)=\dfrac{z-2}{z^2}\sin\dfrac{1}{z-1}$.
15. Find the residue of $f(z)=\dfrac{z}{z^2+1}$ at its poles.

Turn over

472421

Section B

Answer any number of questions.
Each question carries 5 marks. Maximum marks 35.

16. Verify the Cauchy Riemann equations for $f(x+iy) = \dfrac{x-iy}{x^2+y^2}$.

17. If $f(z) = u + iv$ then show that $|f'(z)|^2 = u_x^2 + v_x^2 = u_y^2 + u_y^2$.

18. Find the image of the annulus $2 \le |z| \le 4$ under the mapping $w = \text{Ln}\, z$.

19. Find the principal value of $(-3)^{\frac{i}{\pi}}$.

20. Using Cauchy-Goursat theorem, evaluate $\oint_C \dfrac{5z+7}{z^2+2z-3}\,dz$ where C is the circle $|z-2| = 2$.

21. Find $\oint_C \dfrac{z^2-4z+4}{z+i}\,dz$ where C is the circle $|z| = 2$.

22. Prove that the sequence $\left\{\dfrac{3+ni}{n+2ni}\right\}$ converges to $\dfrac{2}{5}+\dfrac{1}{5}i$.

23. Find the residue of $f(z) = \tan z$ at $z = \dfrac{\pi}{2}$.

Section C

Answer any **two** *questions.*
Each question carries 10 marks. Maximum marks 20.

24. Verify that the function $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane and find the harmonic conjugate function.

25. State and prove (a) Liouville's theorem ; (b) Morerea's thereom.

26. State Cauchy's residue theorem, and using this show that $\int_C \dfrac{dz}{z \sin z} = 0$, where C is the unit circle about the origin described in the positive sense.

27. Show that $\int_0^{2\pi} \dfrac{d\theta}{2+\cos\theta} \dfrac{2\pi}{\sqrt{3}}$.

(2 × 10 = 20 marks)

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C 40604 **(Pages : 3)** **Name....................................**

Reg. No................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 Marks.

1. Define entire function. Give an example.
2. State a necessary condition for analyticity.
3. Prove that $u(x, y) = e^{-x} \sin y$ is harmonic.
4. Prove that $\overline{e^{z}} = e^{\overline{z}}$.
5. Find all values of z satisfying the equation $e^{z-1} = -ie^{3}$.
6. Find the real and imaginary parts of $\sin(\overline{z})$.
7. Evaluate $\oint_{C} xydx + x^{2}\,dy$ where C is the curve $y = x^{3}, -1 \leq x \leq 2.$.
8. Define simply and multiply connected domains. Give examples for each.
9. State Cauchy's - Goursat theorem and find $\oint_{C} e^{z}\,dz$ on a simple closed contour C.
10. Evaluate the integral $\int_{\frac{i}{2}}^{i} e^{\pi z}\,dz$ and write it in the form $a + ib$.

Turn over

338732

11. By using Cauchy's integral formula evaluate $\int_C \frac{z}{z^2+9}\,dz$ where C is the circle $|z-2i|=4$.

12. State root test.

13. Find the Taylor expansion of $f(z)=\frac{1}{1-z}$.

14. Find the Laurent's series expansion of $f(z)=\frac{\cos z}{z}$ in $0<|z|$.

15. Find the pole of $\frac{\sin z}{z^2}$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 Marks.

16. Prove that if f is differentiable at a point z_0 in a domain D then f is continuous at z_0

17. Find the real constants a, b, c and d so that $f(z)=(3x-y+5)+i(ax+by-3)$ is analytic.

18. Compute the principal value of the complex logarithm Ln z for $z=i$ and $z=1+i$.

19. Find the derivative of the principal value of z^i at the point $z=1+i$.

20. Find the upper bound of the absolute value of $\oint_C \frac{e^z}{z+1}\,dz$ where C is the circle $|z|=4$.

21. Evaluate $\oint_C \frac{1}{\sqrt{z}}\,dz$ where C is the line segment between $z_0=i$ and $z_1=9$.

22. Examine the covergence of the following series on their circle of convergence (a) $\sum_{0}^{\infty} z^n$; and

(b) $\sum_{0}^{\infty} \frac{z^n}{n^2}$.

23. Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $|z| > 1$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *Marks*

24. (a) State and prove Cauchy's integral formula.

(b) Evaluate $\int_C \frac{z}{z^2+9} dz$ where C is the circle $|z - 2i| = 4$.

25. Evaluate $\int_C \frac{dz}{z^2+1}$.

26. (a) State and prove Cauchy's inequality.

(b) State Maximum modulus theorem and find the maximum modulus of $f(z) = 2z + 5i$ on the closed circular region $|z| \le 2$.

27. State and prove Cauchy's residue theorem and using it evaluate $\int_C \frac{dz}{z^3(z-1)}$ where C is $|z| = 2$.

C 20646 **(Pages : 2)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Define holomorphic function in a domain D. And give an example for an entire function.
2. Prove or disprove : if f is differentiable a point z_0, then f is continuous at that point.
3. Define harmonic function with example.
4. Prove that $\sin^2 z + \cos^2 z = 1$.
5. State ML inequality.
6. Define the path independence for a contour integral.
7. State maximum modulus theorem.
8. Prove that $\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$.
9. Prove or disprove if $\lim_{n \to \infty} z_n = 0$, then $\sum_{k=1}^{\infty} z_k$ converges.
10. Find the radius of convergence of $\sum_{k=1}^{\infty} \frac{z^k}{k}$.
11. Define pole of order n. Give an example of a function with simple pole at $z = 1$.
12. Find the principal part in the Laurent series expansion about the origin of the function $f(z) = \frac{\sin z}{z^4}$.

Turn over

13. State Rouche's theorem.

14. Find the residue of $\dfrac{\sin z}{z}$ at $z = 0$.

15. How many zeroes of are in the disc $|z| = 1$ for the function $f(z) = z^9 - 8z^2 + 5$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Check whether the function U is harmonic or not if so find its harmonic conjugate $U(x, y) = x^3 - 3xy^2 - 5y$.

17. Find all the solutions of the equation $\sin z = 5$.

18. State and prove Fundamental theorem of algebra.

19. State and prove Morera's theorem.

20. Find the Taylor's series expansion with centre $z_0 = 2i$ of $f(z) = \dfrac{1}{1-z}$.

21. Identify the singular points and classify them $f(z) = \dfrac{\sin z e^{\left(\frac{1}{z-1}\right)}}{z(1+z)}$.

22. Find residue of $e^{e^{\left(\frac{1}{z}\right)}}$ at $z = 0$.

23. Find $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^4+1}\, dx$.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Cauchy Riemann Equation. Also state the sufficient condition for differentiability.

25. State and prove Cauchy's integral formula for derivatives.

26. Expand $f(z) = \dfrac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$.

27. State and prove Cauchy's residue theorem.

(2 × 10 = 20 marks)