D 120184 **(Pages : 4)** **Name.................................**

Reg. No...............................

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION MARCH 2025

Mathematics

MTS6B12—CALCULUS OF MULTI VARIABLE

(Admissions Year—2019 Onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 25.

1. Find the domain of the function f defined by $f(x, y, z) = \sqrt{x + y - z} + xe^{yz}$.

2. Define limit of a function of two variables at a point.

3. Evaluate $\lim_{(x, y) \to (2, 4)} \sqrt[3]{\dfrac{8xy}{2x + y}}$.

4. Show that the function $u(x, y) = e^x \cos y$ is harmonic in the xy plane.

5. Let $f(x, y) = x^2 - 2xy$. Find the gradient of f at the point $(1, -2)$.

6. State second derivative test for a function of two variables.

7. Find the equation of the tangent plane to the graph of the function f defined by $f(x, y) = 4x^2 + y^2 + 2$ at the point $(1, 1)$.

8. Evaluate $\int_1^2 \int_0^1 3x^2 \, y \, dx \, dy$.

9. State Fubini's theorem for rectangular regions.

Turn over

10. Find the volume of the solid lying under the elliptic paraboloid $z = 8 - 2x^2 - y^2$ and above the rectangular region $R = \{(x, y)/0 \leq x \leq 1, 0 \leq y \leq 2\}$.

11. Find the area of the part of the surface with equation $z = 2x + y^2$ that lies directly above the triangular region R in the xy-plane with vertices (0, 0), (1, 1) and (0, 1).

12. A vector field **F** in $\mathbf{R}^2$ is defined by $\mathbf{F}(x, y) = -yi + xj$. Describe **F**, and sketch a few vectors representing the vector field.

13. Find divergence of the vector field $\mathbf{F}(x, y, z) = yz\, i + xz\, j + xy\, k$.

14. Find curl of the vector field $\mathbf{F} = x^2y^3\, i + xz^2\, k$.

15. State Green's theorem.

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 35.

16. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $2x^2z - 3x^2 + yz - 8 = 0$.

17. Use chain rule to find $\frac{dw}{dt}$, where $w = 2x^3y^2z$, $x = t$, $y = \cos t$, $z = t \sin t$.

18. A heat-seeking object is located at the point (2, 3) on a metal plate whose temperature at a point (x, y) is $T(x, y) = 30 - 8x^2 - 2y^2$. Find the path of the object if it moves continuously in the direction of maximum increase in temperature at each point.

19. Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$.

20. Evaluate $\iint_R y\, dA$, where R is the region in the first quadrant that is outside the circle $r = 2$, and inside the cardioid $r = 2(1 + \cos\theta)$.

21. Evaluate $\int_C 2x\, ds$, where C consists of the arc C_1 of the parabola $y = x^2$ from (0, 0) to (1,1) followed by the line segment C_2 from (1, 1) to (0, 0).

22. Evaluate $\int_C kz\, ds$, where k is a constant and C is the circular helix with parametric equations $x = \cos t$, $y = \sin t$ and $z = t$ where $0 \leq t \leq 2\pi$.

23. Let $F(x, y) = 2xy\, i + (1 + x^2 - y^2)\, j$.

    (a) Show that **F** is conservative, and find a potential function f such that $\mathbf{F} = \nabla f$.

    (b) If **F** is a force field, find the work done by **F** in moving a particle along any path from (1, 0) to (2, 3).

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. The production function of a certain country is given by $f(x, y) = 20x^{2/3}y^{1/3}$ billion dollars, when x billion dollars of labor and y billion dollars of capital are spent.

    (a) Compute $f_x(x, y)$ and $f_y(x, y)$.

    (b) Compute $f_x(125, 27)$ and $f_y(125, 27)$ and interpret your result.

    (c) Should the government encourage capital investment rather than investment in labor to increase the country's productivity ?

25. Find the maximum and minimum values of the function $f(x, y) = x^2 - 2y$ subject to $x^2 + y^2 = 9$.

Turn over

26. Find the center of as of the solid T of uniform density bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = z$.

27. State divergence theorem. Let T be a solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 3$, and let S be the surface of T. Calculate the outward flux of the vector field $\mathbf{F}(x, y, z) = xy^2\, i + yz^2\, j + zx^2\, k$ over S.

(2 × 10 = 20 marks)

D 100614 **(Pages : 3)** **Name.................................**

Reg. No...............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions onwards)

Time : Two Hours and a Half Maximum Marks : 80

Section A

Questions 1—15. *Answer any number of questions.*
Each carry 2 *marks. Maximum marks* 25.

1. Find the domain of the function $f(x, y) = \frac{\ln(x+y+1)}{y-x}$.

2. Evaluate $\underset{(x,y)\to(1,2)}{\text{Lt}} \frac{2x^2 - 3y^3 + 4}{3 - xy}$.

3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $\ln\left(x^2 + y^2\right) + yz^3 + 2x^2 = 10$.

4. Find the gradient of $f(x, y) = x^2 + y^2 + 1$ at the point (1, 2). Use the resutl to find the directional derivative of f at (1, 2) in the direction from (1, 2) to (2, 3).

5. Find the equation of the tangent plane to the hyperboloid $z^2 - 2x^2 - 2y^2 = 12$ at the point (1, – 1, 4).

6. Find the critical points of $f(x, y) = -x^3 + 4xy - 2y^2 + 1$.

7. Evaluate $\int_0^2 \int_{y^2}^4 dx\, dy$.

8. Set up a triple integral for the volume of the solid region in the first octant bounded above by the sphere $x^2 + y^2 + z^2 = 6$ and below by the parabolid $z = x^2 + y^2$.

9. Evaluate $\iint_R 1 - 2xy^2 dA$ where R is the regoin $\{(x, y) | 0 \le x \le 2, -1 \le y \le 1\}$.

Turn over

10. Find the volume of the solid S below the hemisphere $z=\sqrt{9-x^2-y^2}$ above the xy plane and inside the cylinder $x^2+y^2=1$.

11. Find the gradient vector field $\vec{F}$ of the function $f(x,y,z)=\frac{-K}{\sqrt{x^2+y^2+z^2}}$ and hence deduce that the inverse square field $\vec{F}$ is conservative.

12. Find a parametric representation for the cone $x^2+y^2=z^2$.

13. State Stoke's theorem.

14. Using Divergence theorem evaluate $\iint_S \vec{F}.n\, dS$ where $\vec{F}=x\hat{i}+y^2\hat{j}+z\hat{k}$ and S is the surface bounded by the co-ordinate planes and the plane $2x+2y+z=6$.

15. Find an equation of the tangent plane to the paraboloid $\vec{r}(u,v)=u\hat{i}+v\hat{j}+(u^2+v^2)\hat{k}$ at the point (1, 2, 5).

Section B

Questions 16—23. *Answer any number of questions.*
Each carry 5 *marks. Maximum marks* 35.

16. Let $w=2x^2y$ where $x=u^2+v^2$ and $y=u^2-v^2$. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

17. The dimensions of a closed rectangular box are measured as 30 in 40 in and 60 in with a maximum error of 0.2 inches in each measurement. Using differentials find the maximum error in calculating the volume of the box.

18. Show that the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ at (x_0,y_0,z_0) is $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}+\frac{zz_0}{c^2}=1$.

19. Sketch the level curve corresponding to $c=0$ for the function $f(x,y)=y-\sin x$ and find a normal vector at the point $\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$.

20. Using polar co-ordinates find the volume of the solid region bounded above by the hemisphere $z=\sqrt{16-x^2-y^2}$ and below by the circular region $x^2+y^2\le 4$.

21. Find the surface area S of the portion of the hemisphere $f(x, y) = \sqrt{25 - x^2 - y^2}$ that lies above the region R bounded by the circle $x^2 + y^2 \leq 9$.

22. Evaluate $\oint_C \left(y^2 + \tan x\right) dx + \left(x^3 + 2xy + \sqrt{y}\right) dy$ where C is the circle $x^2 + y^2 = 4$ and is oriented in the positive direction.

23. Find the surface area of the unit sphere $\vec{r}(u, v) = \sin u \cos v \hat{i} + \sin u \sin v \hat{j} + \cos u \hat{k}$ where the domain D is $0 \leq u \leq \pi$ and $0 \leq v \leq 2\pi$.

Section C

Questions 24—27. *Answer any* **two** *questions.*
Each carry 10 *marks.*

24. (a) Show that $\lim_{(x,y)\to(0,0)} \dfrac{2x^2 y}{x^2 + y^2} = 0$.

(b) The production function of a certain company is $f(x, y) = 20x^{2/3} y^{1/3}$ Billion dollars, when x billion dollars of labour and y billion dollars of capital are spent :

(i) Compute $f_x(x, y), f_y(x, y)$.

(ii) Compute f_x (125, 27) and f_y (125, 27) and interpret your result.

25. Let $T(x, y, z) = 20 + 2x + 2y + z^2$ represent the temperature at each point on the sphere $x^2 + y^2 + z^2 = 11$. Find the extreme temperatures on the curve formed by the intersection of the plane $x + y + z = 3$ and the sphere.

26. Find the volume of the solid that lies below the paraboloid $z = 4 - x^2 - y^2$ above the xy plane and inside the cylinder $(x - 1)^2 + y^2 = 1$.

27. Verify Stoke's theorem for the vector field $\vec{F}(x, y, z) = 2z\hat{i} + 3x\hat{j} + 5y\hat{k}$ and S is the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \leq 0$ with upward orientation and C is the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of S in the xy-plane.

(2 × 10 = 20 marks)

C 40605 **(Pages : 4)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTI VARIABLE

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Questions 1–15, *Answer any number of questions.*
Each question carries 2 *marks.*
Maximum marks 25.

1. Find the domain and range of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$.

2. Show that $\lim_{(x, y) \to (0, 0)} \frac{xy^2}{x^2 + y^4}$ does not exist.

3. Find f_x and f_y if $f(x, y) = y^x$.

4. Find the Directional Derivative of $f(x, y) = 4 - x^2 - \frac{y^2}{4}$ at $(1, 2)$ in the direction of $\vec{u} = \cos\frac{\pi}{3}\hat{i} + \sin\frac{\pi}{3}\hat{j}$.

5. Find the gradient of $f(x, y) = y \ln x + xy^2$ at the point $(1, 2)$.

6. Find the relative extrema of $f(x, y) = 1 - \left(x^2 + y^2\right)^{1/3}$.

Turn over

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7. Evaluate $\iint_R 2x - y \, dA$ where R is the region bounded by the parabola $x = y^2$ and the straight line $x - y = 2$.

8. Find the surface area of the portion of the plane $z = 2 - x - y$ that lies above the circle $x^2 + y^2 \le 1$ in the first quadrant.

9. Find a transformation T from a region S to the region R bounded by the lines $x - 2y = 0, x - 2y = -4, x + y = 4$ and $x + y = 1$ such that S is a rectangular region.

10. Sketch the region of integration and reverse the order of integration in $\int_1^e \int_0^{\ln x} f(x, y) \, dy \, dx$.

11. Check whether the vector field $\vec{F}(x, y, z) = 2xy \, \hat{i} + \left(x^2 + z^2\right) \hat{j} + 2yz \, \hat{k}$ is irrotational.

12. Find div $\left(\text{curl } \vec{F}\right)$ if $\vec{F}(x, y, z) = xyz \, \hat{i} + y\hat{j} + z\hat{k}$.

13. Use Green's theorem to evaluate $\oint_C \left(x^2 y + x^3\right) dx + 2xy \, dy$ where C if the boundary of the region bounded by $y = x$ and $y = x^2$.

14. State Gauss Divergence theorem.

15. Evaluate the surface integral $\iint_S x + z \, dS$ where S is the first octant portion of the cylinder $y^2 + z^2 = 9$ between $x = 0$ and $x = 4$.

Section B

Questions 16–23, *Answer any number of questions.*
Each question carries 5 *marks.*
Maximum marks 35.

16. Show that the function $z = \sqrt{x^2 + y^2} \tan^{-1} \frac{y}{x}$ satisfies $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$.

17. Let $w = x^2 y - xy^3$ where $x = \cos t$ and $y = e^t$. Find $\frac{dw}{dt}$ at $t = 0$.

18. Find the points on the sphere $x^2 + y^2 + z^2 = 14$ at which the tangent plane is parallel to the plane $x + 2y + 3z = 12$.

19. Find the relative extrema of $f(x, y) = 2x^2 + y^2 + 8x - 6y + 20$.

20. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, y = 0, z = 0$ and $2x + y = 2$.

21. Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x} \, dx \, dy$ by changing the order of integration.

22. Find an equation of the tangent plane to the paraboloid

$\vec{r}(u, v) = u\hat{i} + v\hat{j} + (u^2 + v^2)\hat{k}$ at the point $(1, 2, 5)$.

23. Find the surface integral $\iint_S y^2 + 2yz \, dS$ where S is the first octant portion of the plane $2x + y + 2z = 6$.

Turn over

Section C

Questions 24–27, Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Find the second order partial derivatives of $w = \cos(2u - v) + \sin(2u + v)$.

    (b) Let $z = f(x, y) = 2x^2 - xy$. Find Δz and use the result to find the change in z if (x, y) changes from $(1, 1)$ to $(0.98, 1.03)$.

25. Find the absolute maximum and absolute minimum values of the function $f(x, y) = 2x^2 + y^2 - 4x - 2y + 3$ on the rectangle $D = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.

26. Evaluate $\iiint_T \sqrt{x^2 + z^2}\, dv$ where T is the region bounded by the cylinder $x^2 + z^2 = 1$ and the planes $y + z = 2$ and $y = 0$.

27. verify Divergence Theorem for $\vec{F}(x, y, z) = 2z\hat{i} + x\hat{j} + y^2\hat{k}$ and T is the solid region bounded between the paraboloid $z = 4 - x^2 - y^2$ and the xy plane.

(2 × 10 = 20 marks)

C 20647 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the domain and rang of the function $f(x,y) = x + 3y - 1$.

2. Evaluate $\lim\limits_{(x,y,z)\to\left(\frac{\pi}{2},0,1\right)} \dfrac{e^{2y}(\sin x + \cos y)}{1 + y^2 + z^2}$.

3. Find f_x and f_y if $f(x,y) = x\cos xy^2$.

4. Find the directional derivative of $f(x,y) = x^2 \sin 2y$ at $\left(1, \frac{\pi}{2}\right)$ in the direction of $\vec{v} = 3\hat{i} - 4\hat{j}$.

5. Find $\nabla f(x,y,z)$ if $f(x,y,z) = x^2 + y^2 - 4z$ and find the direction of maximum increase of f at the point (2, –1, 1).

6. Find the relative extrema of the function $f(x,y) = x^2 + y^2 - 2x + 4y$.

7. Find the volume of the solid lying under the elliptic paraboloid $z = 8 - 2x^2 - y^2$ above the rectangular region given by $0 \le x \le 1, 0 \le y \le 2$.

8. Find the mass of the triangular lamina with vertices $(0,0),(0,3)$ and $(2,3)$ given that the density at (x, y) is $\rho(x,y) = 2x + y$.

Turn over

9. Find the Jacobian for the change of variables defined by $x = r\cos\theta, y = r\sin\theta$.

10. Evaluate $\iint_R 2x - y\, dA$ where R is the region bounded by the parabola $x = y^2$ and the line $x - y = 2$.

11. Find whether the vector field $\vec{F} = x^2 y\,\hat{i} + xy\,\hat{j}$ is conservative.

12. State Green's theorem.

13. Find a parametric representation for the cone $x = \sqrt{y^2 + z^2}$.

14. Find the surface area of the torus given by $\vec{r}(u,v) = (2+\cos u)\cos v\,\hat{i} + (2+\cos u)\sin v\,\hat{j} + \sin u\,\hat{k}$ where the Domain D is given by $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

15. Compute $\iint_S \vec{F}\cdot\hat{n}\, dS$ given $\vec{F}(x,y,z) = (x+\sin z)\,\hat{i} + (2y+\cos x)\,\hat{j} + (3z+\tan y)\hat{k}$ and S is the unit sphere $x^2 + y^2 + z^2 = 1$.

(10 × 3 = 30 marks)

Section B (Paragraph Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Find f_{xyx} and $f_y x_y$ if $f(x,y) = x\cos y + y\sin x$.

17. Find the differential of $w = x^2 + xy + z^2$. Compute the value of dw if (x, y, z) changes from (1, 2, 1) to $(0.98, 2.03, 1.01)$ and compare the value with that of Δw.

18. Find the equation of the tangent plane and normal line to the surface $x^2 - 2y^2 - 4z^2 = 4$ at (4, –2, –1).

19. Find the relative extrema of $f(x,y) = -x^3 + 4xy - 2y^2 + 1$.

20. Find the volume of the solid region bounded by the paraboloid $z = 4 - x^2 - 2y^2$ and the xy-plane.

21. Evaluate $\int_0^2 \int_0^x \int_0^{x+y} e^x (y+2z)\, dz\, dy\, dx.$

22. Determine whether the vector field $\vec{F} = e^x \left(\cos y\, \hat{i} - \sin y\, \hat{j}\right)$ is conservative. If so, find a potential function for the vector field.

23. Evaluate $\oint_C \left(e^x + y^2\right) dx + \left(x^2 + 3xy\right) dy$, where C is the positively oriented closed curve lying on the boundary of the semi annular region R bounded by the upper semicircles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ and the x-axis.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Sketch the graph of $f(x,y) = 9 - x^2 - y^2$.

(b) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

25. (a) Find the relative extrema of $f(x,y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

(b) Find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$. subject to the constraint $2x - 3y - 4z = 49$.

26. Let R be the region bounded by the square with vertices $(0,1), (1,2), (2,1)$ and $(1,0)$. Evaluate $\iint_R (x+y)^2 \sin^2(x-y)\, dA$.

27. Let $\vec{F} = (x,y,z) = 2xyz^2\, \hat{i} + x^2 z^2\, \hat{j} + 2x^2 yz\, \hat{k}$.

(a) Show that $\vec{F}$ is conservative and find a scalar function f such that $\vec{F} = \nabla f$.

(b) If $\vec{F}$ is a force field, find the work done by $\vec{F}$ in moving a particle along any path from (0, 1, 0) to (1, 2, –1).

(2 × 10 = 20 marks)