D 120187 **(Pages : 2)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, MARCH 2025

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Part A

Answer any number of questions.
Each question carries 2 *marks.*
Ceiling is 20.

1. Define complete graph. Draw K_6.
2. Draw all non isomorphic simple graphs of order 4.
3. How many edges are there for a k-regular graph of order n ?
4. Differentiate proper subgraphs and spanning subgraphs.
5. State Cayley's theorem on spanning trees of a complete graph.
6. Define vertex connectivity of a graph. Find the vertex connectivity of the cycle C_8.
7. State Whitney's theorem for 2-connected graphs.
8. Distinguish between bridge and cut vertex of a graph with example.
9. Find the Hamiltonian closure of the cycle C_4.
10. Give a characterization of Eulerian graphs. Show that K_5 is Euler.
11. Define a maximal non-Hamiltonian graph.
12. If e is an edge of the complete graph K_5, then illustrate that $K_5 - e$ is planar.

Turn over

Part B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 30.

13. Prove that, in any graph G, the number of vertices of odd degree is even.

14. Define union and intersection of two graphs. Explain with example.

15. Prove that any tree T with at least twoverticeshas more than one vertex of degree 1.

16. Prove that a connected graph G with n vertices has at least $n - 1$ edges.

17. Let v be a vertex of the connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G, both different from v, such that v is on every $u - w$ path in G.

18. Prove that a simple graph G is Hamiltonian if and only if its closure c (G) is Hamiltonian

19. Prove that $K_{3,3}$ is nonplanar.

Part C

Answer any **one** *question.*
The question carries 10 marks.

20. (i) Explain adjacency matrix and incidence matrix of a graph G with an example.

    (ii) Given any *two* vertices u and v of a graph G, prove that every $u - v$ walk contains a $u - v$ path.

21. Define spanning tree. Prove that a graph G is connected if and only if it has a spanning tree.

(1 × 10 = 10 marks)

D 100617 **(Pages : 2)** **Name.................................**

Reg. No..............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

(2019 Admission onwards)

Time : Two Hours Maximum Marks : 60

Section A (Short Answer Type Questions)

Answer any number of questions.
Each question carries 2 *marks. Maximum marks* 20.

1. Find the number of edges of $k_{2,3}$.
2. Draw the graph $K_5 - \{e\}$.
3. Define degree of a vertex. Explain with example.
4. Let G be a simple graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ? Justify.
5. Give an example of a self-complementary graph with five vertices.
6. Let G be a simple graph with n vertices and $\overline{G}$ be its complement. Prove that, for each vertex V in G, $d_G(v) + d_{\overline{G}}(v) = n - 1$.
7. A connected graph G has 21 vertices, what is the minimum possible number of edges in G.
8. Define diameter of a graph G. Which simple graphs have diameter 1 ?
9. When can you say that the wheel graph $W_n, n \geq 4$ is Euler ? Justify.
10. Define Jordan curve. Give an example.
11. Define Spanning tree. State Cayleys theorem in spanning trees.
12. Let G be a Hamiltonian graph. Show that G does not have a cut vertex.

Section B (Paragraph/Problem Type Questions)

Answer any number of questions.
Each question carries 5 *marks. Maximum marks* 30.

13. Prove that k_5, the complete graph on five vertices, is non-planar.
14. Let G be a planar graph with less than 12 vertices. Prove that G has a vertex V with $d(v) \leq 4$.

Turn over

15. Explain Konigsberg bridge problem.
16. Let G be a graph in which the degree of every vertex is at least two then prove that G contains a cycle.
17. Prove that a vertex V of a tree T is a cut vertex if and only if $d(v) > 1$.
18. Let G be a connected graph, then G is a tree if and only if every edge of G is a bridge.
19. Given any two vertices u and v of a graph G, prove that every u-v walk contains u-v path.

Section C (Essay Type Questions)

Answer any **one** *questions.*
The question carries 10 *marks.*

20. Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycle.
21. Prove that if T is a tree with n vertices then it has precisely n-1 edges.

(1 × 10 = 10 marks)

333840

C 40608 **(Pages : 2)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS–UG)

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

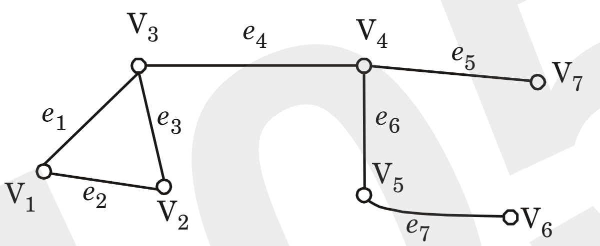
(2019 Admission onwards)

Time : Two Hours Maximum Marks : 60

Section A (Short Answer Questions)

Each question carries 2 marks.
A maximum of 20 marks can be earned from this section.

1. Find the number of edges of K_7.
2. Draw the graph $K_{3,3} - \{V\}$ where V is a Vertex in $K_{3,3}$.
3. Define K-regular graph. Give an example.
4. Define union of two graphs G_1 and G_2.
5. Let G be a simple graph with n vertices and $\overline{G}$ be its complement. Prove that, for each vertex V in G, $d_G(V) + d_{\overline{G}}(V) = n - 1$.
6. Define the adjacency matrix of a graph G with n vertices.
7. A connected graph G has 17 edges, what is the maximum possible number of vertices in G ?
8. Let



Find all bridges in G.

Turn over

333840

9. When can you say that the complete graph $K_n, n \geq 3$ is Euler ? Justify.
10. Define critical planar graphs. Which complete graph K_n are critical planar ?
11. State Cayley's theorem on spanning trees.
12. When can you say that a graph G is maximal non-Hamiltonian.

(Ceiling marks = 20 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 marks.
A maximum of 30 marks can be earned from this section.

13. Prove that the complete bipartite graph $K_{3,3}$ is non-planar.
14. Let G be a simple planar graph with less than 12 vertices. Prove that G has a vertex V with $d(v) \leq 4$.
15. Prove that, in any graph G there is an even number of odd vertices.
16. Prove that for any connected graph G, rad G $\leq$ diam G $\leq$ 2. rad G.
17. Let u and v be distinct vertices of a tree 7. then prove that theme is precisely one path from u to v.
18. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.
19. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.

(Ceiling marks = 30 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 marks.

20. Explain the Konigsberg bridge problem. Give the graph theory model for this problem. Also state the respective theorem to solve this problem.
21. Let G be a graph with n vertices. Then prove that the following statements are equivalent :
    (i) G is a tree.
    (ii) G is acyclic graph with $n - 1$ edges.
    (iii) G is a connected graph with $n - 1$ edges.

(1 × 10 = 10 marks)

112989

C 20650 **(Pages : 2)** **Name..**

Reg. No......................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

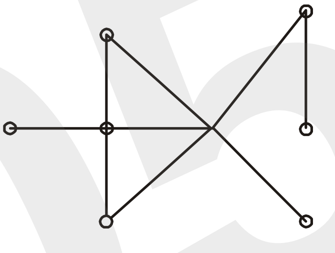
(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Find number of edges of $k_{m, n}$.
2. Draw the graph $K_6 - \{v\}$ where v is any vertex in K_6.
3. Draw a 4-regular graph with ten vertices.
4. Define intersection of two graphs.
5. Let G be a simple graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ? Justify.
6. Draw the graph $K_{2,3,3}$.
7. Define eccentricity and radius.
8. Draw Peterson graph and find a trial of length 5.
9. When can you say that the complete graph $kn, n \geq 3$ is Euler ? Justify.
10. Prove that any subgraph of a planar graph is planar.
11. Find K (G) for the graph.



12. How many different Hamiltonian cycles does K_n have ?

(8 × 3 = 24 marks)

Turn over

112989

Section B (Paragraph/Problem Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.

14. Prove that the complete bipartite graph $k_{3,3}$ is non-planar.

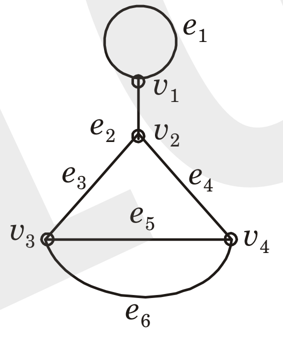
15. Let G be a graph with n vertices, where $n \geq 2$. Then prove that G has atleast two vertices which are not cut vertices.

16. Explain the Konigsberg bridge problem.

17. Prove that G is connected if and only if it has a spanning tree.

18. Let G be a graph with n vertices. Then prove that G is a tree if and only if G is a connected graph with $n - 1$ edges.

19. Define (i) adjacency matrix of a graph G ; (ii) incidence matrix of a graph G. Find the adjacency and incidence matrix of the following graph G.



(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. Prove the following :

   (i) A connected graph G has an Euler tail if and only if it is atmost two odd vertices.

   (ii) A simple graph G is Hamiltonian if and only if it closure c (G) is Hamiltonian.

21. Prove that, a non-empty graph with atleast two vertices is bipartite if and only if it has no odd cycle.

(1 × 11 = 11 marks)