TOPICAL REVIEW

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Topical Review

Radiation of a resonant medium excited by few-cycle optical pulses at superluminal velocity

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Abstract

Recent progress in generation of optical pulses of durations comparable to one optical cycle has presented great opportunities for studies of the fundamental processes in matter as well as time-resolved spectroscopy of ultrafast processes in nonlinear media. It opened up a new area of research in modern ultrafast nonlinear optics and led to appearance of the attosecond science. In parallel, a new research area related to emission from resonant media excited by superluminally propagating ultrashort bursts of electromagnetic radiation has been actively developed over the last few years. In this paper, we review our recent results on theoretical analysis of the Cherenkov-type radiation of a resonant medium excited by few-cycle optical pulses propagating at superluminal velocity. This situation can be realized when an electromagnetic pulse with a plane wavefront incidents on a straight string of resonant atoms or a spot of light rotates at very large angular frequency and excites a distant circular string of resonant dipoles. Theoretical analysis revealed some unusual and remarkable features of the Cherenkov radiation generated in this case. This radiation arises in a transient regime which leads to the occurrence of new frequencies in the radiation spectrum. Analysis of the characteristics of this radiation can be used for the study of the resonant structure properties. In addition, a nonlinear resonant medium excited at superluminal velocity can emit unipolar optical pulses, which can be important in ultrafast control of wave-packet dynamics of matter. Specifics of the few-cycle pulse-driven optical response of a resonant medium composed of linear and nonlinear oscillators is discussed.

Keywords: Cherenkov radiation, few-cycle pulses, attosecond pulses, pulse shaping, unipolar pulses, ultrafast phenomena superluminal motions, resonant periodic structures

(Some figures may appear in colour only in the online journal)

1. Introduction

Since the appearance of the first lasers, studies of the methods of ultrashort pulse production and their interaction with matter developed in parallel as two separate fields of optics. The former area led to the generation of ultrashort optical pulses of durations down to a single optical oscillation. As a result, a modern area of ultrafast optics known as attosecond science appeared, see [1–8]. These extremely short 'bursts' of light can be applied to time-resolved studies of different complex systems with duration of transients of even less than femtoseconds. For example, it is possible to study and control the dynamics of wave packets in matter, thus creating 'snapshots' of complex molecules with femtosecond pulses. Hence, study of the optical response of the medium at such extremely short time scales helps to reveal the mechanisms of their interactions with matter.

Another interesting area of optics related to the study of the optical response of a resonant medium excited by superluminally propagating ultrashort pulses has been developed over the last few years [9–19]. Physical objects moving at a velocity faster than the velocity of light in vacuum c have been the subject of intense research for many years [20–22]. In the pioneer works of Heaviside (1850–1925) and Sommerfeld (1868–1921), the radiation of charged particles moving in vacuum at a velocity greater than c was considered, see [23–25] and [26]. After the appearance of the special theory of relativity in 1905, that does not allow the motion of objects at a velocity greater than c, works on superluminal motion were accepted to be unphysical and forgotten for many years. Later, it was found that only those motions that involve signal (information) transfer at the superluminal velocity are prohibited [20–22, 27]. Hence, if there is no signal transfer, superluminal movement of objects can take place.

A number of objects that can propagate at the superluminal velocity have been found in different areas of physics. Basov *et al* demonstrated experimentally and theoretically the possibility of superluminal propagation of the pulse maximum in an amplifying medium [28–30]. These experiments were performed in the 1960s after the appearance of first lasers. The superluminal propagation of pulse maximum was related to nonlinear amplification of a pulse of light on its leading edge. Rosanov [31] analysed soliton-like light structures propagating superluminally in vacuum and in a nonlinear medium.

Different superluminal objects can be sources of electromagnetic radiation. One of the most pronounced example is Cherenkov or Vavilov–Cherenkov radiation, arising when a charged particle moves in a medium at a velocity greater than the phase velocity of light in this medium [32–34]. This radiation was discovered in 1934 by Cherenkov (1904–1990) and Vavilov (1891–1951) and theoretically explained by Tamm (1895–1971) and Frank (1908–1990) in 1937 [35, 36]. Cherenkov radiation propagates at the angle determined by the ratio of the particle velocity and the phase velocity of light in the medium: $\cos\varphi = c/Vn$. Here n is the refractive index of the medium, V is the velocity of particle. The history of the discovery of this radiation can be found in [37].

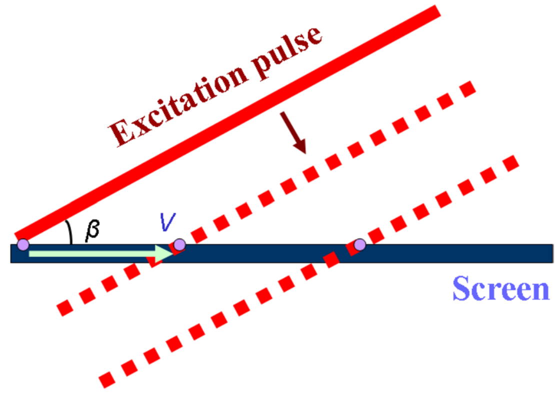

Figure 1. Plane screen is irradiated by an ultrashort pulse with a plane wavefront; the intersection line of the pulse wavefront and the screen moves along the screen at the superluminal velocity $V = c/\sin\beta > c$.

Not only particles, but also spots of light and optical solitons, can propagate faster than the phase velocity of light in particular mediums and in vacuum [38–41]. Askaryan theoretically predicted radiation produced by electromagnetic beams moving at a velocity greater than the phase velocity of light in the medium [42]. The radiation of such sources is also referred to as Cherenkov radiation. Cherenkov radiation of optical solitons was reviewed in [43]. Ginzburg (1916–2009) considered different spots of light propagating at superluminal velocity, particularly the spot of light formed by a rotating projector, searchlight (or pulsar in astrophysics) at a fairly remote screen, see [41, 44]. Next, the intersection point of two interfering laser beams that propagate at the velocity $V = c/\sin\frac{\alpha}{2} > c$ (α is the angle between two interfering waves) also moves at a velocity exceeding that of light in vacuum. The next example of a 'superluminal' source of electromagnetic radiation is an ultrashort plane optical pulse crossing a plane screen, see figure 1 and [41]. In this case, the intersection line of the pulse and screen moves along the screen at the velocity $V = c/\sin\beta > c$ (here β is the angle of wave incidence) [41].

The objective of this review is to present recent results on the study of the optical response of an inhomogeneous resonant medium excited by few-cycle optical pulses at superluminal velocity and to discuss the possible applications of radiation arising in this case [9–14]. This research is motivated by recent progress in the development of fabrication techniques for arrays of microsize particles, plasmonic nanostructures, metallic nanoantennas and semiconductor quantum dots. All of them exhibit multiple beneficial optical properties and can serve as optical waveguides [45, 46], high quality optical resonators [47–49], antennas and detectors [50], or the active elements in laser diodes [51, 52]. Such nanostructures have highly flexible resonance frequencies which are determined by their geometry and size, and can be thus tuned in a wide range, from terahertz up to the visible one. We considered the Cherenkov radiation of linear and circular strings composed of resonant optical oscillators with spatially varying linear density and analysed the features of the frequency spectrum of the emission arising in this case. It was found that this radiation possesses an unusual character of its spectrum in the transient regime and contains both resonance frequency of the medium and the new Doppler-shifted frequency. This latter frequency depends on the velocity of excitation pulse propagation, spatial period of the oscillator density and the angle of observation. If the excitation velocity varies during the excitation process due to the medium geometry or the wavefront curvature of the pump pulse, a frequency continuum of the definite frequency range can be obtained in the spectrum of the emitted field. Analysis of the spectrum of such radiation can be used for the control and study of resonant periodic structures and the optical and material properties of the medium.

We also analysed the emission of strings composed of resonant oscillators with nonlinear coupling to an external electric field and excited by the ultrashort pulses at superluminal velocity [15–19]. In contrast to the linear oscillator case, nonlinear field coupling was shown to provide a novel and effective method of unipolar pulse generation in a wide frequency range by all-optical means using a pair of successive few-cycle light pulses. As opposed to conventional pulses, which are bipolar and thus have an electric field of alternating sign, unipolar pulses exhibit a constant sign of electric field throughout the pulse duration. This distinguishing feature of unipolar pulses makes them ideally suited for controlling the motion of charges in matter and detection of the dynamics of ionic and electronic wave packets [53–56]. We also proposed a new method to control the waveform of an emitted unipolar pulse by multiple-oscillator emission interference when the excitation velocity varies in a prescribed manner along the string of oscillators.

The review is organized as follows. Section 2 discusses the optical response of resonant oscillators to the few-cycle pump pulse. Special attention is paid to the role of the carrier-envelope phase of the excitation pulse and the relative values of the pulse duration and the period of the resonant oscillations of the medium. Section 3 reviews the spectral and temporal characteristics of the Cherenkov radiation arising when a linear or circular string of dipoles is excited at superluminal velocity. We also discuss the possibility of unipolar pulse emission when the resonant oscillators possess nonlinear coupling to the electric field. In section 4 we generalize our findings to the case of excitation velocity varying over the course of time due to the curvature of the pulse wavefront. Finally, in section 5, we discuss both the fundamental and applied aspects of our findings and expound our views of their possible applications.

2. Optical response of a resonant medium to a few-cycle excitation pulse

We treat in the following the resonant medium as being composed of linear optical oscillators corresponding to the elastically bounded charges in atoms or molecules. The oscillator displacement under the influence of an external field is described by the effective medium polarization $P(t)$. We assume the excitation field to be linearly polarized, which gives us the following equation for the temporal evolution of polarization:

$$\ddot{P} + \gamma\dot{P} + \omega_0^2 P = g_0 E(t), \tag{1}$$

where ω_0 is the oscillator resonance frequency, γ is the damping rate and the variable g_0 describes the coupling strength of the medium to the external electric field. Taking into account that the field coupling can be anisotropic in general, equation (1) should be written for each component of the polarization vector, but with the linearly polarized electric field these equations have a form analogous to equation (1). Hence, without loss of generality we can restrict ourselves to the scalar case described by equation (1).

We suppose that the excitation pulse is spectrally broadband and much shorter in duration than the resonant period of medium $T_0 = 2\pi/\omega_0$. This implies that the central frequency of the pulses used for pumping of considered scheme should be very large compared to the medium resonant frequency.

In view of this assumption, the oscillators are driven during the limited amount of time corresponding to the pump pulse duration and execute free oscillations after the excitation pulse has passed through. Integrating equation (1) over the whole duration of the excitation pulse and taking the oscillator damping to be small $\gamma \ll \omega_0$ yield the following expression for this subsequent free-oscillation dynamics:

$$P(t) = [P_0 \sin(\omega_0 t + \phi_0) + \Pi_1 \sin(\omega_0 t) - \Pi_2 \cos(\omega_0 t)]\mathrm{e}^{-\gamma t/2}, \tag{2}$$

where P_0 and ϕ_0 are the integration constants and Π_1, Π_2 are given as:

$$\Pi_1 = \frac{g_0}{\omega_0}\int_{-\infty}^{+\infty} E(t')\cos(\omega_0 t')\mathrm{d}t',$$
$$\Pi_2 = \frac{g_0}{\omega_0}\int_{-\infty}^{+\infty} E(t')\sin(\omega_0 t')\mathrm{d}t'. \tag{3}$$

The integrals on the right-hand side of equation (3) are considered to be taken over the whole pulse duration, as schematically indicated by the infinite integration limits. Since the terms proportional to the field of excitation pulse are equal to zero before the pulse action, variables P_0 and ϕ_0 correspond to the oscillation amplitude and phase at the moment of excitation pulse arrival.

We suppose that the excitation pulse possesses a symmetric envelope with respect to the middle of the pulse (e.g. of Gaussian shape) and an arbitrary phase shift of the carrier:

$$E(t) = E_0 \mathrm{e}^{-t^2/\tau_p^2}\sin(\Omega t + \vartheta_{\mathrm{CE}}), \tag{4}$$

where Ω is the central frequency and ϑ_{CE} stands for the carrier-envelope phase (CEP).

According to equations (3) and (4), we obtain the following dependence of the response amplitudes of medium oscillators Π_1, Π_2 on the carrier-envelope phase ϑ_{CE}:

$$\Pi_1 = \sqrt{\pi}E_0\frac{g_0}{\omega_0}\tau_p \mathrm{e}^{-(\Omega^2+\omega_0^2)\tau_p^2/4}\cosh(\Omega\,\omega_0\tau_p^2/2)\sin\vartheta_{\mathrm{CE}},$$
$$\Pi_2 = \sqrt{\pi}E_0\frac{g_0}{\omega_0}\tau_p \mathrm{e}^{-(\Omega^2+\omega_0^2)\tau_p^2/4}\sinh(\Omega\,\omega_0\tau_p^2/2)\cos\vartheta_{\mathrm{CE}}, \tag{5}$$

that are illustrated in figure 2.

According to equations (5), the response amplitudes Π_1, Π_2 are the harmonic functions of the carrier-envelope phase ϑ_{CE} and vary reversely to each other. Thus control of CEP value allows us to obtain both exactly sine and exactly cosine

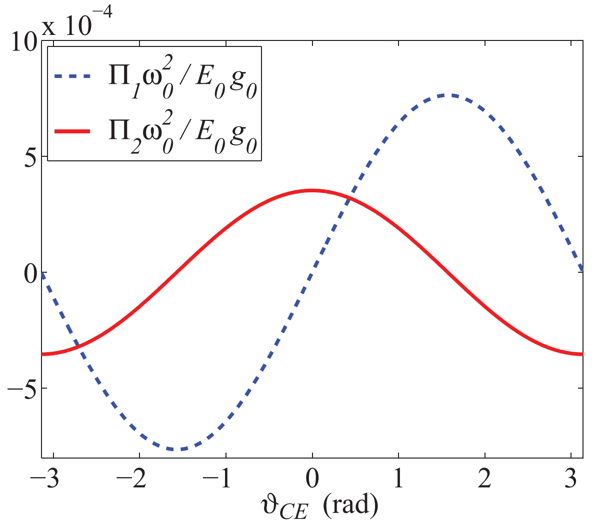

Figure 2. Dependence of the amplitudes of the oscillator response Π_1, Π_2 on the carrier-envelope phase $\vartheta_{\rm CE}$; $\omega_0\tau_p = 0.2$, $\Omega/\omega_0 = 25$.

responses in equation (2). It also directly follows from equation (5) that:

$$\max_{\vartheta_{\rm CE}} \Pi_2 / \max_{\vartheta_{\rm CE}} \Pi_1 = \tanh(\Omega\,\omega_0\tau_p^2/2). \qquad (6)$$

It is seen from equation (6) that the relative values of both terms in equation (2) are determined by the value of the product $\Omega\,\omega_0\tau_p^2$. If the excitation pulse contains just a few optical periods $\Omega\,\tau_p \sim 1$ and is much shorter in duration than the resonant period of medium $\omega_0\tau_p \ll 1$, it is seen from equation (5) that cosine term amplitude Π_2 equals zero for $\vartheta_{\rm CE} = \pm\frac{\pi}{2}$ only, but except for the vicinity of $\vartheta_{\rm CE} = 0$ and $\vartheta_{\rm CE} = \pi$, the sine term amplitude Π_1 is much greater. When considering $\omega_0\tau_p \sim 1$, the response amplitudes Π_1, Π_2 according to equation (6) turn out to be of the same order of magnitude (see figure 2).

Another interesting point is the response of an oscillator with nonlinear coupling to an external electric field [15–19]. In contrast to the constant coupling strength g_0 in equation (1), it means that the field coupling itself depends on the electric field $g(E)$. Let us consider the simplest case when this dependence is linear: $g(E) = g_1 E$. Such a form of field coupling nonlinearity naturally holds for Raman-active media [57], which are usually modelled as nonlinearly bonded electronic and nuclear oscillators. Since they have the widely different resonance frequencies and effective masses, the motion of high-frequency oscillator in oscillating electrical field can be adiabatically excluded what results in the equation for the low-frequency oscillator similar to the equation (1), but with nonlinear coupling function $g(E)$. Similar field coupling can be also realized in different hybrid optical materials containing nonlinearly bonded resonances, such as coupled localized plasmonic resonances, quantum dots or microcavities. It should be noted that such nonlinear field coupling essentially differs from the usual instantaneous second-order nonlinear response when the medium oscillators are anharmonic and the

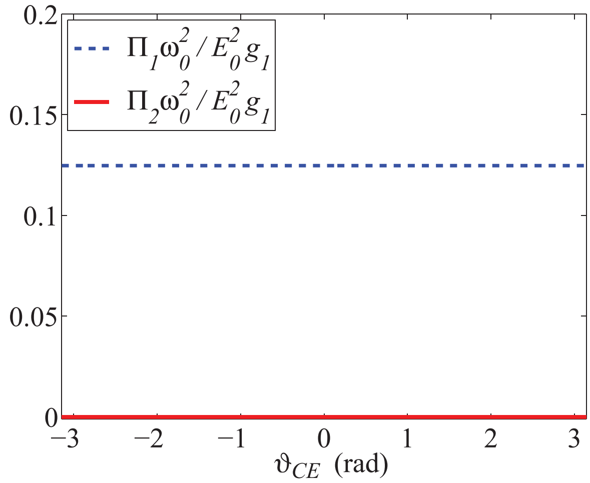

Figure 3. Dependence of the amplitudes of the oscillator response on the carrier-envelope phase for the field-coupling function $g[E(t)] = g_1 E(t)$ (7); $\omega_0\tau_p = 0.2$, $\Omega/\omega_0 = 25$.

second-order response term arises as a small additive to the linear response term.

In this case we get the following equations for the response amplitudes Π_1, Π_2:

$$\Pi_1 = \frac{1}{2}\sqrt{\frac{\pi}{2}}\,E_0^2\frac{g_1}{\omega_0}\tau_p {\rm e}^{-\omega_0^2\tau_p^2/8} \left[1 - {\rm e}^{-\Omega^2\tau_p^2/2}\cosh(\Omega\,\omega_0\tau_p^2/2)\cos 2\vartheta_{\rm CE}\right],$$
$$\Pi_2 = \frac{1}{2}\sqrt{\frac{\pi}{2}}\,E_0^2\frac{g_1}{\omega_0}\tau_p {\rm e}^{-(4\Omega^2+\omega_0^2)\tau_p^2/8}\sinh(\Omega\,\omega_0\tau_p^2/2)\sin 2\vartheta_{\rm CE}, \qquad (7)$$

that are illustrated in figure 3.

When the condition ${\rm e}^{-\Omega^2\tau_p^2/2} \ll 1$ is satisfied, equation (7) exhibits a weak dependence of oscillator response on the CEP. As long as this condition is fulfilled, the value of the cosine term amplitude Π_2 in equation (7) is negligibly small, as compared with the sine term amplitude Π_1, although it is exactly equal to zero just when $\vartheta_{\rm CE}$ is multiple of $\frac{\pi}{2}$. Thereby, the response of an oscillator with nonlinear field coupling is extremely close to sine regardless of the CEP value, thus making CEP control insignificant in this case.

3. Transient Cherenkov radiation of linear and circular strings of resonant particles excited at constant superluminal velocity

In this section, we consider the features of Cherenkov radiation from an inhomogeneous string excited by few-cycle pulses superluminally propagating over the medium. The details of this research can be found in [9, 10]. The medium of length L consists of linear harmonic oscillators with resonance frequency ω_0 and decay rate γ. The spatial density of oscillators $N(z)$ varies harmonically along the z-axis at the spatial period Λ_z. This medium is excited by the ultrashort pulse obliquely incident on the string, as shown in figure 4. We assume that the excitation pulse duration is smaller than (or comparable to) the resonant period of oscillators $T_0 = 2\pi/\omega_0$. According

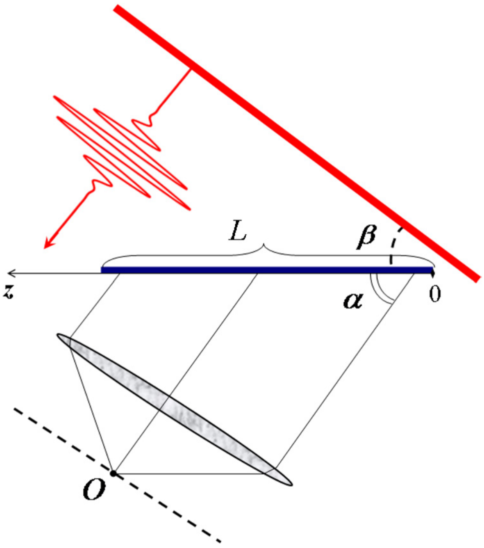

Figure 4. Linear string of oscillators with a periodic density, excited at a superluminal velocity by a few-cycle pulse with a plane wavefront. Emission from the string is measured at a far-distant point with the observation angle α.

to the results of the previous section, the oscillator response function mainly depends on the carrier-envelope phase of the excitation pulse. To be definite, we shall consider in the following the most simple case when $\vartheta_{CE} = 0$. The medium response in equation (2) is then given by:

$$P(t) = -\Pi_2 \cos(\omega_0 t)\Theta(t)\mathrm{e}^{-\gamma t/2}, \tag{8}$$

where Θ is the Heaviside step function.

The electric field is measured at some angle α to the z-axis and in view of equation (8) is given by the integral:

$$E(t) = E_0 \int_0^L N(z')\cos[\omega_0(t - f_{z'})]\Theta[t - f_{z'}]\mathrm{e}^{-\gamma(t-f_{z'})/2}\mathrm{d}z'. \tag{9}$$

Here $f_{z'} = \frac{z'}{V} + \frac{L-z'}{c}\cos\alpha + \frac{r}{c}$ describes the emission delay from the oscillator placed at a point with the coordinate z, E_0 is the scaling constant and r is the distance between the observation point and the string end $z = L$. From analytical calculation of equation (9) one can see that the medium response contains the radiation at the resonance frequency ω_0 as well as the new component given by:

$$\Omega_1 = 2\pi \frac{V/\Lambda_z}{\left|\frac{V}{c}\cos\alpha - 1\right|}. \tag{10}$$

The numerator of equation (10) is the frequency of excitation of a linear string of oscillators. The denominator contains typical Doppler term V/c. Equation (10) is valid in the case of subluminal excitation as well as in the case when the excitation velocity $V = c$. The physical reason for the occurrence of the new frequency is as follows. When a medium is excited at a velocity greater than c, $V > c$, the medium emission succeeds the excitation remaining behind it. In this case, the radiation from different parts of the medium will come to the observation point in turn. Since $V > c$, the radiation from points close to point $z = L$ arrives at the observation point first.

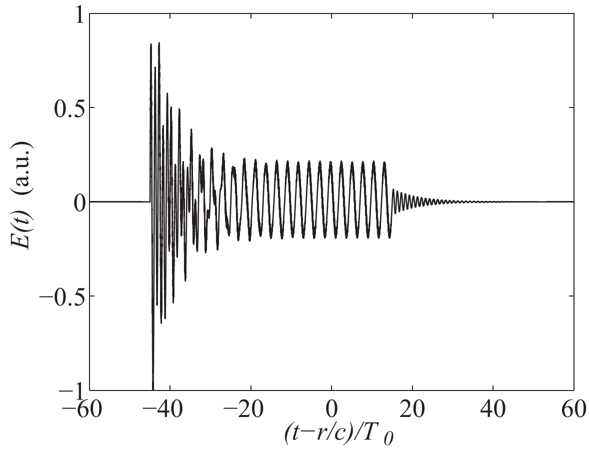

Figure 5. Time dependence of the electric field obtained from a string of linear oscillators at the observation point for the parameter values $V/c = 3$, $L = 45\lambda_0$, $\Lambda_z = 2\lambda_0$, $\gamma/\omega_0 = 0.05$, $\alpha = \pi$.

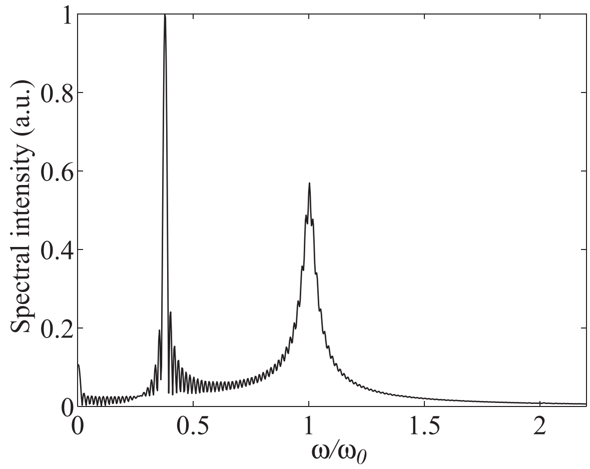

Figure 6. Spectral intensity of the field measured from a string of linear oscillators for the parameter values $V/c = 3$, $L = 45\lambda_0$, $\Lambda_z = 2\lambda_0$, $\gamma/\omega_0 = 0.05$, $\alpha = \pi$.

The radiation from the points of the medium close to $z = 0$ arrives much later. Because of the interference from incoming waves, a complex transient process occurs and the new frequency (10) appears in the spectrum of the medium response. This fact is illustrated in figures 5 and 6 where an example of electric field time dependence and the spectral intensity are presented for the case when the medium is excited by a single ultrashort pulse. The presence of the transient process as well as resonance frequency ω_0 and the new one Ω_1 can be easily seen.

Figure 7 shows the dependence of the field spectrum on the angle of observation α. It is seen that the spectrum contains two branches corresponding to the resonance frequency ω_0 and the new frequency equation (10), respectively. When the angle of observation is equal to:

$$\cos\alpha_0 = \frac{c}{V}, \tag{11}$$

the additional frequency (10) becomes infinite, implying that a single-peaked spectrum occurs. The observation angle (11)

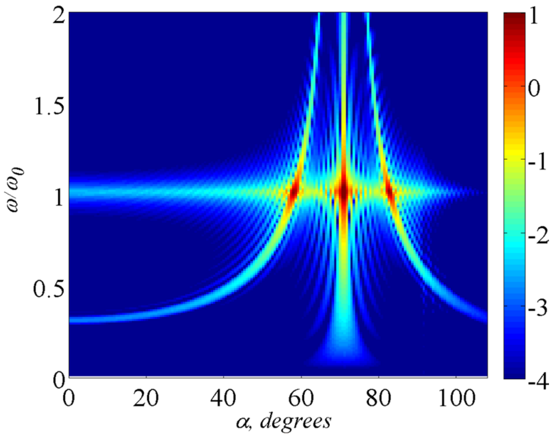

Figure 7. Diagram illustrating the dependence of the output spectrum of the string of linear oscillators by the observation of observation α; $V/c = 3$, $L = 47.75\lambda_0$, $\Lambda_z = 5\lambda$, $\gamma/\omega_0 = 0.05$.

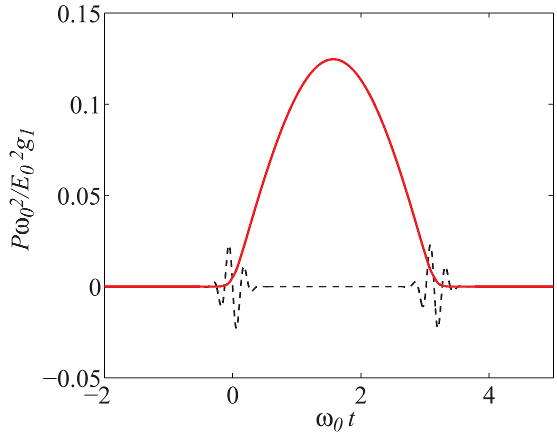

Figure 8. Unipolar half-cycle pulse emitted by an oscillator with the field coupling $g[E(t)] = g_1 E(t)$ (red solid line) together with the electric field of the excitation few-cycle pulse (black dashed lines); $\omega_0\tau_p = 0.2$, $\Omega/\omega_0 = 25$, $\vartheta_{CE} = 0$.

coincides with the angle of Cherenkov radiation. This fact can be naturally expected to occur since the Cherenkov angle corresponds to the direction of the in-phase summation of the secondary waves emitted by different parts of the medium. Hence, no transient processes will be registered when measuring at this angle.

The transient Cherenkov frequency equation (10) closely resembles the well-known relation for Doppler-shifted frequency. Indeed, if the relation $VT_0 = \Lambda_z$ is fulfilled, i.e. the excitation spot during one period of resonant oscillations of the medium covers one period of the modulation of medium density, equation (10) is precisely the Doppler frequency in the nonrelativistic limit. It is also interesting to note that the frequency (10) has the same form as the frequency of the Smith–Purcell effect [58] arising when the charged particle moves along the surface of the metallic periodic grating.

A periodic train of ultrashort pulses can be used for effective control of the parameters of Cherenkov radiation described above. This fact was pointed out in [12, 13]. The idea of this control is very simple. When the frequency of the pulse train is equal to or a multiple of the resonance frequency of the medium, the radiation efficiency at the resonance frequency can be increased. At the same time, the radiation intensity at the frequency Ω_1 becomes suppressed. In addition, if the pulse repetition period is a multiple of the period $T_1 = 2\pi/\Omega_1$ the radiation intensity at the frequency Ω_1 will be increased, and resonance radiation will be suppressed. This fact is very similar to the control of molecular vibrations by a train of femtosecond pulses [59, 60]. Thus we see that using a train of ultrashort pulses allows one to control and study the properties of resonant periodic structures and the radiation emitted by them.

We remark that a linear string is not the only string that allows generation of new frequencies like (10). One can imagine a circular string composed of classical harmonic oscillators. This system can be superluminally excited by a spot of light propagating along the circle of radius R. This situation was studied in [10–12]. The quick rotation of a laser pulse along the circle can be realized using angular laser beam deflectors. Let us consider that the resonant particles are periodically distributed along the circle with the angular period Λ_φ and the radiation is measured at some point on the axis passing through the centre perpendicular to the circle. In this case, the expression for the new frequency component is given by:

$$\Omega_2 = 2\pi \frac{V/\Lambda_\varphi}{R}. \tag{12}$$

The frequency (12) is proportional to the velocity of excitation and depends on the angular period of the oscillator density Λ_φ and the radius of the circle R. This frequency also arises in the transient regime due to the reasons described in the case of a linear string. One can also imagine the less realistic situation when the radiation is measured at an observation point placed directly on the circle. In this speculative case, described in [11], the expression for the new frequency slightly differs from equation (10) and is given by:

$$\Omega_3 = 2\pi \frac{V/\Lambda_\varphi}{\left|\frac{V}{c} - 1\right|} \tag{13}$$

and thus depends once again on the ratio V/c.

Use of a circular string of oscillators seems promising because circular arrays of resonant nanoparticles are actively studied nowadays and their resonant interaction with an electromagnetic field can find various applications in optics (see [48–50] and references therein).

The described phenomena remain valid for oscillators with nonlinear field coupling as well. However, the specifics of the optical response in the case of nonlinear field coupling pave the way for other interesting effects. Namely, let us assume that the oscillators are excited by the sequence of two few-cycle pulses with the delay equal to the half-period of resonant oscillations $T_0/2$. As it was shown in the previous section (see figure 3), the response on the first excitation pulse will have the sine form. Then the second pulse will stop the oscillator just after the half-period, so that the emitted field will be the half-sine unipolar impulse (except the short splashes at the start and stop of the oscillation) (see figure 8) [15–19].

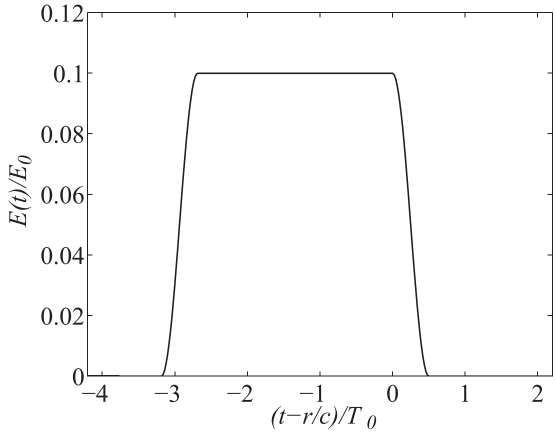

Figure 9. Rectangular unipolar pulse obtained from equation (14) generated by a linear homogeneous string of oscillators with the field coupling $g[E(t)] = g_1E(t)$; $V/c = 3, \omega_0L/c = 30, \alpha = 0$.

Unipolar pulses seems to be unphysical because emitted electric field is proportional to the acceleration of bounded system of oscillating charges (dipoles) and this acceleration is bipolar. However, unipolar pulses can exist in some cases as it was shown by several authors, see [61–67] and review [68]. Unipolar pulses are usually produced when an initially bipolar pulse propagates in a nonlinear medium [61–64] or in high-intensity light–matter interactions typical in extreme nonlinear optics [65–67]. In some cases unipolar halfcycle pulses appear in the form of solitonic solutions of nonlinear optics equations, see review [68]. The main advantage of the proposed method is the possibility of efficient control over the profile of the obtained unipolar pulse by varying the spatial arrangement of excited oscillators. Specifically, if we consider the oscillators placed along a linear string with the spatial density $N(z)$, the resulting field obtained by the integration over the whole string will be given as:

$$E(t) = E_0 \sum_{k=0}^{1} \int_0^L \sin[\omega_0(t - f_{z'})]\Theta[t - f_{z'}]N(z')\mathrm{d}z', \qquad (14)$$

where E_0 is scaling constant and the time delay $f_{z'} = \frac{z'}{V} + \frac{L-z'}{c}\cos\alpha + \frac{r}{c} + (k-1)\frac{T_0}{2}$. If the string is homogeneous $N(z) = const$ the rectangular-shaped unipolar pulse is emitted, which is illustrated by an example in figure 9.

For the inhomogeneous string, the shape of the resulting pulse can vary greatly based on the particular form of the oscillator density. It is important to state that the generated pulse will be unipolar in all cases. Compared to the Cherenkov radiation caused by the single excitation pulse and having the well-pronounced peak in the angular spectrum, the unipolar pulse equation (14) has the same form regardless of the value of the observation angle α. Varying the observation angle allows for only stretching the pulse in the time domain, thus inversely tuning the pulse duration and amplitude. Different methods of unipolar pulse generation and fundamental limitations on their existence are described in review [68].

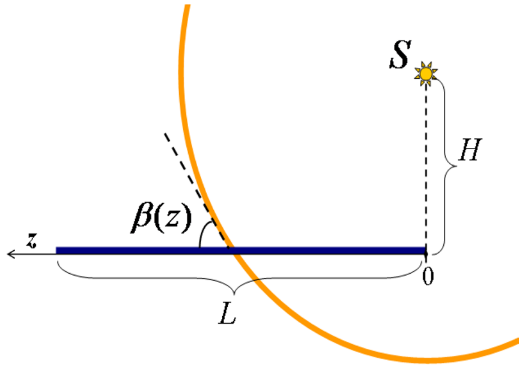

Figure 10. Linear string with periodic density of oscillators excited with a superluminal velocity by a few-cycle pulse with a cylindrical wavefront. Emission from the string is measured at a far-distant point with the angle α to observer.

4. Transient Cherenkov radiation of strings of resonant particles excited at varying superluminal velocity

In previous studies [9–13], it was assumed that the excitation pulse propagates along the medium at a constant velocity. If the excitation velocity varies we can expect the emitted field to have a more complex form of frequency spectrum [14].

Suppose first that the oscillators are arranged along the linear string but the excitation pulse has a curved wavefront. As provided by relation $V = c/\sin\beta$, this means that the excitation velocity varies during the pulse propagation over the string. For definiteness, we suggest a point light source to be placed above one of the string ends at height H (see figure 10).

From the system geometry it follows that the angle of incidence β monotonically increases from its maximum value

$$\beta_1 = 0 \qquad (15)$$

at the nearest end of the string to the

$$\beta_2 = \arcsin\frac{1}{\sqrt{1+\left(\frac{H}{L}\right)^2}} \qquad (16)$$

at the opposite end. According to equation (10), the corresponding instantaneous values of the additional frequency Ω_1 are given as:

$$\Omega_{\beta_1} = \frac{\omega_0}{\frac{\Lambda_z}{\lambda_0}|\cos\alpha|}; \qquad (17)$$

$$\Omega_{\beta_2} = \frac{\omega_0\sqrt{1+\left(\frac{H}{L}\right)^2}}{\frac{\Lambda_z}{\lambda_0}\left|\sqrt{1+\left(\frac{H}{L}\right)^2}\cos\alpha - 1\right|}. \qquad (18)$$

In this case, the electric field measured at the observation point is expressed as follows:

$$E(t) = E_0 \int_0^L N(z')\cos[\omega_0(t - f_{z'})]\Theta[t - f_{z'}]\mathrm{e}^{-\gamma(t-f_{z'})/2}\mathrm{d}z', \qquad (19)$$

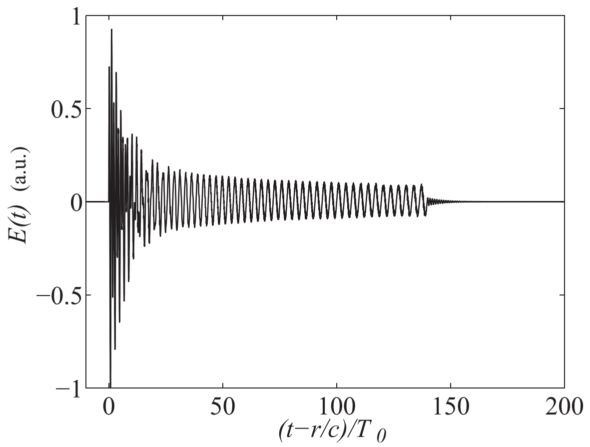

Figure 11. Time dependence of the electric field obtained at the observation point for parameter values $L = 99\lambda_0$, $\Lambda_z = 2\lambda_0$, $H = 100\lambda_0$, $\gamma/\omega_0 = 0.05$, $\alpha = \pi$.

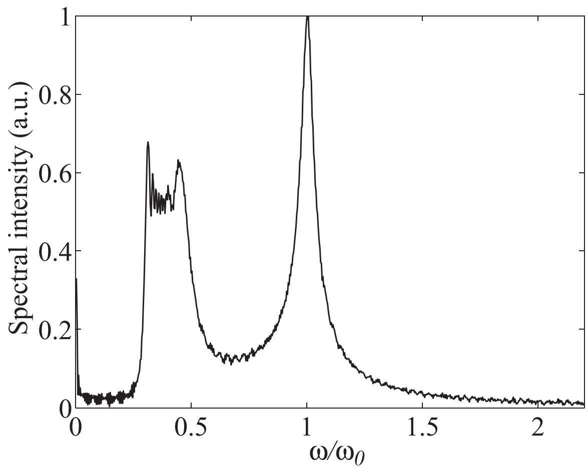

Figure 12. Spectral intensity of the measured field for parameter values $L = 99\lambda_0$, $\Lambda_z = 2\lambda_0$, $H = 100\lambda_0$, $\gamma/\omega_0 = 0.05$, $\alpha = \pi$.

where $f_{z'} = \frac{1}{c}\left(\sqrt{H^2 + z'^2} - H\right) + \frac{L-z'}{c}\cos\alpha + \frac{r}{c}$ describes the emission delay from the oscillator placed at the point with the coordinate z and r is the distance between the observation point and the string end $z = L$. Results of the numerical integration of equation (19) are shown in figure 11.

It is seen from equation (10) that the additional frequency Ω_1 becomes infinite at the excitation velocity:

$$V_\infty = \frac{c}{\cos\alpha}. \tag{20}$$

Equation (20) physically describes the situation where the observation point is placed in the direction of the excitation pulse propagation, so that the medium geometry has no effect on the temporal dynamics of the measured field.

In the case when the velocity (20) satisfies the condition:

$$V_\infty \notin \left[\frac{c}{\sin\beta_2}; \frac{c}{\sin\beta_1}\right] = \left[c\sqrt{1+\left(\frac{H}{L}\right)^2}; +\infty\right) \tag{21}$$

the spectrum of emitted field will contain frequencies filling the finite continuous range:

$$\Omega \in [\min(\Omega_{\beta_1}, \Omega_{\beta_2}); \max(\Omega_{\beta_1}, \Omega_{\beta_2})]. \tag{22}$$

In the opposite case, when:

$$V_\infty \in \left[\frac{c}{\sin\beta_2}; \frac{c}{\sin\beta_1}\right] = \left[c\sqrt{1+\left(\frac{H}{L}\right)^2}; +\infty\right) \tag{23}$$

the measured spectrum extends to infinity:

$$\Omega \in [\min(\Omega_{\beta_1}, \Omega_{\beta_2}); +\infty). \tag{24}$$

In this latter case (24), the new frequency component Ω_1 (10) during the finite time interval of the medium excitation takes on values within the infinite range that allow us to expect that the registered emission will have no pronounced features in its spectrum. Thus the former case (22) is of particular interest when the frequencies of the transient radiation (10) fill in the finite frequency range, forming a continuous spectrum with definite boundaries. As it follows from equations (17)–(18), these boundary frequencies can be tuned within wide limits depending on the spatial period of the modulation of oscillator density, the geometrical parameters of the system and the observation angle.

An example of the field intensity spectrum is shown in figure 12. It is seen that besides the maximum on the resonance frequency ω_0, another pronounced frequency continuum arises, which corresponds to the transient Cherenkov radiation. The boundaries of the frequency range in figure 12 exactly agree with the equations (17)–(18).

The amplitude of the obtained continuum can be varied by changing the damping rate of the oscillator. As the damping rate increases, the peak at the medium resonance frequency becomes less intensive, while the amplitudes of the frequency components related to the Cherenkov radiation increase.

Varying excitation velocity also allows us to control the shape of unipolar pulses in the case of oscillators with nonlinear field coupling. For the scheme presented in figure 10, when assuming a homogeneous string and $\gamma \ll \omega_0$, the mathematical expression for the generated pulse shape is written as:

$$E(t) = E_0 \sum_{k=0}^{1} \int_0^L \sin[\omega_0(t - f_{z'})]\Theta[t - f_{z'}]\mathrm{d}z', \tag{25}$$

where the emission delay from the oscillator located at the point with coordinate z $f_{z'} = \frac{1}{c}\left(\sqrt{H^2 + z'^2} - H\right) + \frac{L-z'}{c}\cos\alpha + \frac{r}{c} + (k-1)T_p$ has the same form as in equation (19). The result of the numerical calculation of the integral equation (25) for some parameter values is shown in figure 13.

The generated unipolar pulse has a nonuniform profile monotonically decreasing from its highest level at the leading edge to the lowest at the trailing edge. This shape originates from the fact that the intersection point of the excitation pulses and the string of oscillators moves along the string at varying superluminal velocity due to wavefront curvature. Depending on the geometry of oscillator arrangement and the wavefront curvature of the excitation pulses, the resulting pulse shape can be tuned in wide limits according to the desired waveform.

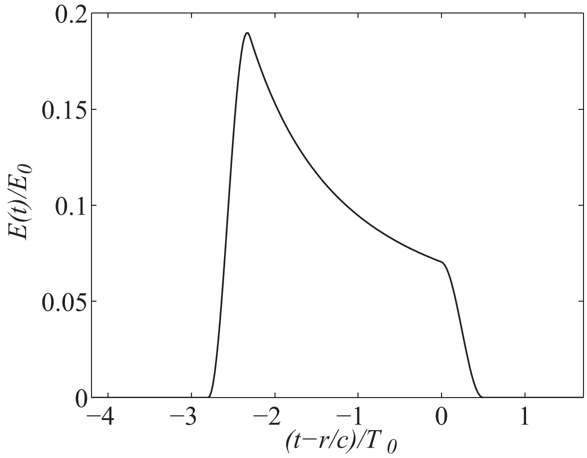

Figure 13. Unipolar pulse generated by a linear homogeneous string of oscillators with nonlinear field coupling excited at a superluminal velocity by two successive few-cycle pulses with cylindrical wavefronts; $H/L = 1$, $\omega_0 L/c = 30$, $\alpha = 0$.

5. Possible applications of the phenomena discussed above

The features of Cherenkov radiation discussed above represent only theoretical predictions. Despite this, one can think about its possible practical applications. From equations (10)–(13) one can see that generated frequencies depend on the excitation velocity V, and parameters of the excited medium (spatial period Λ_z and radius R). Therefore, analysing the spectrum of the emitted Cherenkov radiation enables us to extract information on the medium properties or to detect some objects moving at the velocity V. Furthermore, analysis of the spectrum near the new frequency in the case when the medium is excited at the variable velocity makes it possible to recover information about the character of the particle motion.

Recently, the transient character of Cherenkov radiation described above led to the theoretical prediction of a novel way of unipolar pulse generation in a nonlinear medium [15–19]. Unipolar pulses are pulses with a constant sign of electric field. Conventional electromagnetic waves are bipolar and the integral of the electric field with respect to the time is equal to zero. Unipolar pulses, due to their unidirectionality, can efficiently transfer impulsive momentum to an electron and thus can be effectively used for the control of wave-packet dynamics [53–56, 68]. A detailed description of this phenomena is beyond the scope of this review. However, we remark the following. Unipolar pulse generation described in [15–19] also takes place when the linear or circular string of nonlinear oscillators (for example, Raman-active medium) is excited by a train of few-cycle pulses at superluminal velocity, as plotted in figure 4. In addition, the formation of the unipolar pulse at the observation point takes place in the transient regime as discussed in section 3.

6. Conclusions

Discovery of the Cherenkov radiation in 1934 has led to detailed analysis of its properties in different systems and paved the way to several important applications in various fields of physics. However, in most situations, Cherenkov radiation is rather unstructured and has no clear frequency resonance. In the present review, we have discussed recent advances in Cherenkov radiation, which has other remarkable properties in contrast to common ones. It is excited by an ultrashort pulse, which propagates at a velocity greater than the velocity of light in vacuum. This Cherenkov radiation demonstrates resonant properties as well as containing a new Doppler-shifted frequency. This frequency arises when dipole density varies periodically along the string and depends on the velocity of excitation and medium parameters. The unique feature of this radiation is that it has a transient character when radiation from certain points of the string are summed up at the observation point upon arriving with some delays. As soon as the transient process is finished, the dynamics of the system represents the decaying oscillations at the resonance frequency.

We tried to illustrate to the reader that the field of superluminal motions and light sources is full of examples in various fields of physics and has a long history. The study of superluminal physical objects is interesting from both fundamental and practical points of view.

We pointed out that the features described above can serve as a formidable tool to explore the properties of resonant periodic structures, as well as to detect the localized structures which can move at the arbitrary velocity. Furthermore, we briefly mentioned that superluminally excited nonlinear chains of oscillators under certain conditions can be an efficient tool for unipolar pulse generation. Effective generation of unipolar pulses and their remarkable feature of unidirectionality will open new opportunities in the ultrafast control of wave-packet dynamics and in the construction and engineering of compact particle accelerators.

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