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Correlation, coherence and context

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Abstract

The modern theory of coherence is based on correlation functions. A generic example could be written $\langle V^*(t_1)V(t_2)\rangle$, denoting an average of products of the values of a signal $V(t)$ at two specified times. Here we infer that t is a degree of freedom that the signal depends on. Typically, physical variables depend on more than one degree of freedom, and recognition of this has prompted attention to some interesting questions for the correlation functions and the several coherences that can be attributed to the same optical field. We examine some of the questions arising from the standpoint of experimental contexts. Degree of polarizability and degree of entanglement (classical non-separability) can serve as starting points for quantitative assignments.

Keywords: context, coherence, decoherence, correlation, polarization, entanglement

(Some figures may appear in colour only in the online journal)

Background

In the 1850s Sir George Stokes first touched the topic of coherence in a quantitative way by showing how to determine the degree of optical polarization. The concept of correlation function itself has a remarkably short history, barely a century old [1][1]. Optical physics is perhaps the field in which the study of coherence has become most sophisticated, and correlation functions were an esential element of the pioneering studies of Emil Wolf in the 1950s [2–4], which showed how polarization is determined by the amount of correlation existing in the optical field.

The foundation of our analysis will be the optical field itself. It has several independent degrees of freedom available to it [5][2]. These are space, time, and spin (intrinsic polarization). Idealized optical beams can be given a well-defined direction of propagation, and this allows a slight simplification, which we take for granted by ignoring the propagation degree of freedom and writing each field's complex amplitude in terms of the orthonormal bases for each of its other degrees of freedom. These are the two-dimensional transverse coordinate $r_\perp$, time t and spin (polarization) s, and we denote the bases as follows. The spin unit vectors satisfy $\hat{s}_1 \cdot \hat{s}_2 = 0$. The temporal basis functions $F_k(t)$ are orthonormal eigenvectors of the integral equation that has the field's temporal correlation function as kernel (see [6][3] and section 4.7.1 in [7]), and the transverse beam basis functions $G_m(r_\perp)$ are taken as orthonormal in integration across the beam.

We will use Dirac notation with the abbreviations $\langle i|s_i\rangle \equiv \hat{s}_i$; $\langle t|t_k\rangle \equiv F_k(t)$; $\langle r|r_m\rangle \equiv G_m(r_\perp)$, allowing replacement of the field by its Dirac-abbreviated form:

$$\vec{E}(r_\perp, t) = E_0 \sum_{i=1,2}\sum_{k,m} D_{ikm}\hat{s}_i F_k(t) G_m(r_\perp), \tag{1}$$

$$\rightarrow \quad |\mathbf{E}\rangle = E_0 \sum_{i=1,2}\sum_{k}\sum_{m} D_{ikm}|s_i\rangle|t_k\rangle|r_m\rangle, \tag{2}$$

where D_{ikm} are complex coefficients. The Dirac notation has not made the field quantum mechanical, but makes it easier to follow the vector spaces that are engaged. We will normalize to unit total intensity and go to lower-case $|\mathbf{e}\rangle$ for that:

$$|\mathbf{e}\rangle = \sum_{i=1,2}\sum_{k}\sum_{m} D_{ikm}|s_i\rangle|t_k\rangle|r_m\rangle, \tag{3}$$

[1] See Taylor's investigations of fluid flow for a very early introduction of correlation functions into theoretical physics: [1].

[2] Attention is directed to different roles for different degrees of freedom and vector spaces for the optical field in [5].

[3] A complete orthonormal set of time functions directly determined by the random process itself can be obtained as eigenfunctions of the integral equation in which the kernel is the random signal's autocorrelation function. See [6].

where the normalization $\langle \mathbf{e}|\mathbf{e}\rangle = 1$ implies

$$\sum_{i=1,2}\sum_{k}\sum_{m}\langle |D_{ikm}|^2\rangle = 1. \tag{4}$$

The unit-trace tensor outer product, the equivalent of a quantum density matrix, is then written

$$|\mathbf{e}\rangle\langle\mathbf{e}| = \sum_{ikm}\sum_{jln}(D_{ikm})(D_{jln})^* \times |s_i\rangle\langle s_j| \otimes |t_k\rangle\langle t_l| \otimes |r_m\rangle\langle r_n|. \tag{5}$$

Each of the degrees of freedom clearly defines (occupies) one of the independent vector spaces of the field, and for convenience we will continue to label them s for spin, t for time, and r for transverse spatial location.

Our intention is simply to emphasize via several examples how intimately context, correlation and coherence are interrelated. The importance of context is often not recognized at all. This is especially true in discussion of so-called 'hidden' coherences. A number of such examples of hidden optical coherence have been pointed out recently [5, 8–13][4]. As is always the case in physics, experimental setups determine context.

Coherence matrices

We can begin by bringing to view the so-called polarization coherence matrix [14, 15]. This is written, for our unit-normalized field, as

$$\mathcal{W} = \begin{bmatrix} \langle e_x^* e_x\rangle & \langle e_y^* e_x\rangle \\ \langle e_x^* e_y\rangle & \langle e_y^* e_y\rangle \end{bmatrix}, \tag{6}$$

where the degree of correlation captured in the matrix is traditionally written:

$$\gamma_{xy} = \frac{|\langle e_x^* e_y\rangle|}{\sqrt{\langle |e_x|^2\rangle\langle |e_y|^2\rangle}}, \tag{7}$$

where the Schwarz inequality guarantees that $0 \leqslant \gamma \leqslant 1$.

This is familiar, but we note that the arguments of the field components are not specified. The components have arguments, but in this format there is nothing specific about them in the sense that the argument can justifiably be claimed to refer to a time instant (t), a space location ($r_\perp$), both of those jointly ($r_\perp, t$), or no argument at all. This odd situation points to interesting questions regarding detection. In addition, a rationale for arranging the four correlation averages in (6) in the form of a 2×2 matrix is rarely, almost never, given. What vector space identifies these correlations as a matrix?

Contextual issues

There is only one vector space available, but it is a tensor product of several smaller spaces, the three spaces (s, r, t) used to define the field in (1) or (3), and again in (5) to form the dyadic tensor $|\mathbf{e}\rangle\langle\mathbf{e}|$. This is where context matters, and where detection determines context. While retaining spin (conventional polarization) identity there are several recognizable approaches to detection.

(a) Projected space function:
Spatial dependence is chosen by projecting on $|\mathbf{e}\rangle$ a chosen spatial configuration (e.g. an arbitrary transmission mask in the optical beam, denoted $|R\rangle$) to obtain

$$\begin{aligned}|\mathbf{e}(R)\rangle &\equiv \langle R|\mathbf{e}\rangle = \sum_{ik}|s_i\rangle|t_k\rangle\sum_m D_{ikm}\langle R|r_m\rangle \\ &\equiv \sum_{ik}C_{ik}(R)|s_i\rangle|t_k\rangle.\end{aligned} \tag{8}$$

In this r-projection example, and in the following, the C is a reduced coefficient obtained by obvious summation on the preceding D coefficient.

(b) Projected time function:
Temporal dependence is chosen by projecting on $|\mathbf{e}\rangle$ a distinct temporal configuration (e.g. arbitrarily switching out a pulse from the optical beam, denoted by $|T\rangle$) to obtain

$$\begin{aligned}|\mathbf{e}(T)\rangle &\equiv \langle T|\mathbf{e}\rangle = \sum_{im}|s_i\rangle|r_m\rangle\sum_k D_{ikm}\langle T|t_k\rangle \\ &\equiv \sum_{im}C_{im}(T)|s_i\rangle|r_m\rangle.\end{aligned} \tag{9}$$

The result of a t projection can be seen as sr coherence, the existence of which was highlighted in 2006 by Gori *et al* [16], and then identified as a hidden coherence by Abouraddy *et al* [8].

(c) Projection jointly in space-time :
Joint spatial and temporal dependence is chosen by projecting on $|\mathbf{e}\rangle$ a joint spatio-temporal functional arrangement, an extension of the two previous projections:

$$\begin{aligned}|\mathbf{e}(R,T)\rangle &\equiv \langle TR|\mathbf{e}\rangle = \sum_{i}|s_i\rangle\sum_{km} D_{ikm}\langle TR|t_k, r_m\rangle \\ &\equiv \sum_{i}C_{i}(R,T)|s_i\rangle.\end{aligned} \tag{10}$$

Another coherence matrix, previously unknown, abandons spin sensitivity. This is an arbitrary projection that fixes the spin degree of freedom, easily done experimentally. It fits the projection process perfectly and has only recently been identified and also explored experimentally [13].

(d) Projected spin function:

$$\begin{aligned}|\mathbf{e}(S)\rangle &\equiv \langle S|\mathbf{e}\rangle = \sum_{km}|t_k\rangle|r_m\rangle\sum_i D_{ikm}\langle S|s_i\rangle \\ &\equiv \sum_{km}C_{km}(S)\ |t_k\rangle|r_m\rangle,\end{aligned} \tag{11}$$

where $|S\rangle$ is any combination of $|s_1\rangle$ and $|s_2\rangle$.

Projection is not the only experimental tool that can provide context. What is sometimes called a 'bucket detector' is employed in many situations. This is a detector that captures everything indiscriminately. An ordinary bucket left in the garden during a rainstorm will catch all raindrops falling into it, without registering arrival times or locations drop by drop. Many analogs are familiar in physics. An unfiltered photon

[4] A particularly wide-ranging examination is provided by De Zela [9].

counter is of this type, since it counts photons without wavelength registration. In reverse, a spectrometer can respond to wavelength intensity without counting photons. The idea is clear.

The mathematical-theoretical equivalent of bucket detection is the trace operation. By definition, a trace incorporates everything within a given vector space. The complete trace of any density matrix equals 1, signalling that with probability 1 every possible event has been included. It is easy to imagine optical experiments in which the projections listed above are replaced by traces. We can extend (a) above as a sufficient illustration of this.

(e) Traced space location: Spatial bucket detection is designated by the indiscriminate sum of projections onto all possible r modes. In that case the field's multi-space dyad is reduced appropriately:

$$|\mathbf{e}\rangle\langle\mathbf{e}| \to \sum_q \langle r_q|\mathbf{e}\rangle\langle\mathbf{e}|r_q\rangle$$
$$= \sum_{ik}\sum_{jl} |s_i\rangle\langle s_j| \otimes |t_k\rangle\langle t_l| \sum_q d_{ikq}d^*_{jlq}$$
$$= \sum_{ik}\sum_{jl} C_{ikjl}|s_i\rangle\langle s_j| \otimes |t_k\rangle\langle t_l|. \tag{12}$$

We point out that this reduction generally does not reduce the field dyad to a pure-state reduced outer product $|\mathbf{e}'\rangle\langle\mathbf{e}'|$. Such a special reduction is possible only if it happens that C_{ikjl} can be factored in the form $b_{ik}b^*_{jl}$. In general the reduction by tracing produces a mixed field state (for illustration, see [12]).

It is clear that the same bucket tracing applied to the t and s spaces will have exactly similar effects, and it is equally clear that all of these tracings lead to different fields to be observed, although all start from the same original field $|\mathbf{e}\rangle$, and all tracings differ from all projections obtained from the same $|\mathbf{e}\rangle$. Different experimental detection arrangements, amounting to different contexts, obviously have to be devised to accomplish either projected fields or bucket fields. Further analysis to determine residual coherence will naturally depend on the contexts, producing different, even greatly different, results.

Interpretation of a coherence matrix

We have now presented several reductions of the original field, but the question about the vector-space nature of the 2×2 matrix (6) remains open. Let us begin to address it by fixing the field's spatial character, as in (8), obtaining $|\mathbf{e}(R)\rangle$ with s and t degrees of freedom remaining active. Two different coherence matrices can be formed by tracing over either s or r. The result for a temporal-space trace is:

$$\sum_q \langle t_q|\mathbf{e}(R)\rangle\langle\mathbf{e}(R)|t_q\rangle$$
$$= \sum_{ik}\sum_{jl} |s_i\rangle\langle s_j| C_{ik}(R)C^*_{jl}(R) \sum_q \langle t_q|t_k\rangle\langle t_l|t_q\rangle$$
$$= \sum_{ij} \langle\mathbf{e}_i(R)|\mathbf{e}^*_j(R)\rangle\ |s_i\rangle\langle s_j|. \tag{13}$$

The corresponding result for a spin-space trace is:

$$\sum_f \langle s_f|\mathbf{e}(R)\rangle\langle\mathbf{e}(R)|s_f\rangle$$
$$= \sum_{ik}\sum_{jl} |t_k\rangle\langle t_l| C_{ik}(R)C^*_{jl}(R) \sum_f \langle s_f|s_i\rangle\langle s_j|s_f\rangle$$
$$= \sum_{kl} \langle\mathbf{e}_k(R)|\mathbf{e}^*_l(R)\rangle\ |t_k\rangle\langle t_l|. \tag{14}$$

We can comment on these results separately. If we interpret, as we may do, $\hat{s}_1, \hat{s}_2$ as $\hat{x}, \hat{y}$, then the first result in (13) recovers exactly the matrix in (6), and additionally supplies for the fields a spatial character R that was not evident in (6):

$$\mathcal{W} = \begin{bmatrix} \langle e^*_x(R)e_x(R)\rangle & \langle e^*_y(R)e_x(R)\rangle \\ \langle e^*_x(R)e_y(R)\rangle & \langle e^*_y(R)e_y(R)\rangle \end{bmatrix}. \tag{15}$$

It was the original projection that prescribed the spatial character of the coherence. There is no doubt that the matrix entries are spin-component correlations of the field and are matrix elements in the vector space of the spins, spanned by $|x\rangle\langle x|,\ |x\rangle\langle y|,\ |y\rangle\langle x|,\ |y\rangle\langle y|$.

Moreover, if we reverse the projection and trace operations that led to (13), and project first temporally and then trace spatially, we obtain exactly the same matrix of spin component correlations, in exactly the same vector space, except that the fields are evaluated with a specific temporal character T, not space character R. The fact that the matrices look the same has nothing to do with the various numerical values of the elemental correlation functions such as $\langle\mathbf{e}_x(T)|\mathbf{e}^*_y(T)\rangle$ in contrast to $\langle\mathbf{e}_x(R)|\mathbf{e}^*_y(R)\rangle$. Those values are fully context-dependent and different.

All of this makes it obvious that the second result above, shown in (14), leads to an analogous matrix in a different vector space. But that space is importantly different—it is infinite dimensional, implying an infinite-dimensional coherence matrix. The infinite dimensionality was noticeable earlier in (8) where the k sum includes infinitely many modes to accommodate the temporal continuum. However, the Schmidt Theorem of analytic function theory [17][5] provides an exact decomposition of the double summation in (8) and it has only two terms. In effect, there are only two distinct combinations of temporal modes that can couple to a similarly specific pair of spin modes, simply because the number of spin modes is fixed at two. Thus the apparently infinite-dimensional coherence matrix implied in (14) is only 2×2.

Contexts for non-lossy decoherences

The contextual character of coherence leads to practical consequences related to decoherence. It can, for example, provide a clear distinction between intrinsic and induced decoherences. We emphasize that none of the decoherences described below are related to absorption or other lossy mechanisms. These decoherences are observable in tests of a field for which total intensity remains fixed, i.e. $\langle\mathbf{e}|\mathbf{e}\rangle = 1$.

[5] The original paper is: [17]. For background, see Fedorov and Miklin (2014).

We proceed by making use of a simplified example presented recently [12]. The simplification is the assumption that experimental control is available to restrict participation to the usual two spin modes, a small number of orthonormal spatial modes, in this case $G_1(r_\perp)$ and $G_2(r_\perp)$, and two new temporal modes $f_1(t)$ and $f_2(t)$ that are unit-normalized but not orthogonal: $\langle f_1^*(t)f_1(t)\rangle = \langle f_2^*(t)f_2(t)\rangle = 1$ and $\langle f_1^*(t)f_2(t)\rangle \neq 0$. Thus the z-propagating beam field, here denoted $|\mathbf{a}\rangle$, will be assembled from those modes as a sum of x and y spin components:

$$|\mathbf{a}_x\rangle = \alpha|f_1\rangle|G_1\rangle + \beta|f_2\rangle|G_2\rangle \tag{16}$$

$$|\mathbf{a}_y\rangle = \alpha|f_1\rangle|G_2\rangle + \beta|f_2\rangle|G_1\rangle, \tag{17}$$

where the real coefficients α and β allow an arbitrary shifting of amplitudes within the components without affecting the unit-normalized beam intensities.

We assume all of the spin contributions to the field are detected indiscriminately, which means summing the two contributions that are in principle observable, namely $|\langle x|\mathbf{a}\rangle|^2 + |\langle y|\mathbf{a}\rangle|^2 = \langle x|\mathbf{a}\rangle\langle\mathbf{a}|x\rangle + \langle y|\mathbf{a}\rangle\langle\mathbf{a}|y\rangle$. As in the spatial sector tracing in (12), this is the trace of the dyadic $|\mathbf{a}\rangle\langle\mathbf{a}|$ over an entire sector, in this case the spin sector. The result is a field state with contributions from both t and r sectors:

$$\begin{aligned}|\mathbf{a}\rangle\langle\mathbf{a}| &\rightarrow \sum_f \langle s_f|\mathbf{a}\rangle\langle\mathbf{a}|s_f\rangle \\ &= \sum_{km}\sum_{ln}|t_k\rangle\langle t_l| \otimes |r_m\rangle\langle r_n| \sum_f d_{fkm}d^*_{fln} \\ &= \sum_{km}\sum_{ln} C_{kmln}|t_k\rangle\langle t_l| \otimes |r_m\rangle\langle r_n|,\end{aligned} \tag{18}$$

where (16) supplies the elements C_{kmln}. The result is a mixed field state except in the special factorization case: $C_{kmln} = c_{km}c^*_{ln}$.

A small technical point needs to be addressed before proceeding. Tracing over $|t_k\rangle$s will not produce a useful result because $|f_1\rangle$ and $|f_2\rangle$ don't provide an orthogonal basis. But there is always a unit-normalized orthogonal partner to $|f_1\rangle$ that we denote $|\bar{f}_1\rangle$, satisfying $\langle f_1^*|\bar{f}_1\rangle = 0$. This allows $|f_2\rangle$ to be written in an orthogonal basis:

$$|f_2\rangle = \alpha|\bar{f}_1\rangle + \gamma|f_1\rangle, \tag{19}$$

and from $\langle f_2|f_2\rangle = 1$ we easily find

$$\langle f_1|f_2\rangle \equiv \gamma \quad \text{and} \quad |\alpha|^2 + |\gamma|^2 = 1. \tag{20}$$

After this assignment of an orthonormal two-state basis in t space (relying on the Schmidt decomposition without actually using it) we can proceed with a trace over $|f_1\rangle$ and $|\bar{f}_1\rangle$.

We introduce contrasting results by addressing the field in two ways. The first will be a projection, treating each component as a separate field, which is easily accomplished experimentally in a variety of ways. For example, a polarizing beam splitter will provide a separately accessible beam for each spin component. After tracing over the t sector in each case, we obtain two coherence matrices whose elements are correlations in the r sector—(i) for the x spin component considered as a unit-normalized field itself, and (ii) for the y spin component considered as a unit-normalized field itself. We easily find:

$$\text{(i)} \quad \mathcal{W}_r^x = \begin{bmatrix} \alpha^2 & \gamma^*\alpha\beta \\ \gamma\alpha\beta & \beta^2 \end{bmatrix} \tag{21}$$

and

$$\text{(ii)} \quad \mathcal{W}_r^y = \begin{bmatrix} \beta^2 & \gamma\alpha\beta \\ \gamma^*\alpha\beta & \alpha^2 \end{bmatrix}. \tag{22}$$

Alternatively, a different coherence matrix (iii) is found when the field can be treated as a whole. Its total t-space trace is obtained by adding matrices (i) and (ii): $\mathcal{W}_r^x + \mathcal{W}_r^y \equiv \mathcal{W}_r$ (after dividing by 2 to ensure unit trace), to obtain:

$$\text{(iii)} \quad \mathcal{W}_r = \frac{1}{2}\begin{bmatrix} 1 & (\gamma+\gamma^*)\alpha\beta \\ (\gamma+\gamma^*)\alpha\beta & 1 \end{bmatrix}. \tag{23}$$

Several interesting points show up in comparisons of (i)–(iii). We see that γ controls incoherence at the level where only one spin component is active, i.e. in each of (21) and (22) separately. This has been labeled [12] as 'intrinsic' incoherence in the sense that γ is a feature of the field itself, arising from $|\langle f_1|f_2\rangle| = |\gamma| \leqslant 1$. Thus we see that if we simply remove its effect, by taking $|\gamma| = 1$, we find both (21) and (22) to have zero determinant, i.e. to represent fully coherent pure states.

Notice however, that even after we remove such 'intrinsic' incoherence by taking $|\gamma| = 1$, the final total coherence matrix (23) remains mixed. Where did such incoherence come from? Clearly, it was introduced by the bucket detection, the failure to make precise enough observation, i.e. by failing to segregate the two spin components for individual attention. It might be called observational or 'extrinsic' incoherence.

Coherence and entanglement

Coherence cannot escape entanglement, and this is becoming widely appreciated. Again, context matters. To discuss coherence quantitatively one commonly resorts to degree of polarization $\mathcal{P}$, easily obtained from the eigenvalues $\lambda_1 \geqslant \lambda_2$ of any of the 2×2 coherence matrices. When a field is normalized to unit intensity, as in our examples, one has $\lambda_1 + \lambda_2 = 1$, and the known result [14, 15] is

$$\mathcal{P} = \lambda_1 - \lambda_2, \tag{24}$$

guaranteeing $0 \leqslant \mathcal{P} \leqslant 1$. At the same time, the degree of entanglement may be found from the same eigenvalues. We will measure entanglement via concurrence [18][6] and denote it by $\mathcal{C}$. One finds

$$\mathcal{C} = 2\sqrt{\lambda_1\lambda_2}, \tag{25}$$

where $0 \leqslant \mathcal{C} \leqslant 1$. We have noted [13] that a significant quadratic constraint emerges to quantify what may be called coherence sharing which unites degree of polarization and degree of concurrence (non-separability, entanglement). For st polarization one finds this constraint:

$$\mathcal{C}_{st}^2 + \mathcal{P}_{st}^2 = 1. \tag{26}$$

[6] Concurrence is basis independent and is conveniently bounded to address entanglement (inseparability): $0 \leqslant \mathcal{C} \leqslant 1$.

The *st* constraint has an exact partner constraint in the independent (and contextually distinct) *sr* coherence:

$$\mathcal{C}_{sr}^2 + \mathcal{P}_{sr}^2 = 1. \tag{27}$$

We have likened the partnership of entanglement and polarizability to a form of complementarity [13].

Entanglement is becoming widely understood to be present in classical as well as in quantum physics, with notable examples already appearing in optical studies, both classical and quantum. One can easily understand entanglement as Schrödinger described it [19]: if a function $\Psi(x, y)$ of two variables cannot be written as $f(x)g(y)$, i.e. with the independent variables x and y in factored form, then $\Psi(x, y)$ is said to be entangled. In his own words 'What constitutes the entanglement is that Ψ is not a product of a function of x and a function of y'. Many treatments have been made of classical optical fields entangled across degrees of freedom. Essentially all of our examples have been of this type.

The possibility that classical entanglement is open for experimental observation via optical wave functions was apparently suggested first by Spreeuw in 1998 [20]. This was supported strongly by Ghose and Samal [21] in 2001. An early experimental report by Lee and Thomas [22] came in 2002 and other experimental and theoretical treatments appeared in the following decade [8, 9, 23–30]. Interest in the issues raised by classical entanglement continues to grow. Various different directions of investigation and applications have been reported in the past two years [31–36].

Summary

Our discussion has been based on an optical field. Because it has several independent degrees of freedom available to it, namely space, time, and spin (intrinsic polarization), the field admits application in a wide variety of contexts. We have derived coherences arising from different forms of projective-detection and from different forms of tracing, identified with bucket detection. All have been directed to the lowest-order coherence matrices available to a single field, which all take the form

$$\mathcal{W} = \begin{bmatrix} \langle e_a^* e_a \rangle & \langle e_b^* e_a \rangle \\ \langle e_a^* e_b \rangle & \langle e_b^* e_b \rangle \end{bmatrix}, \tag{28}$$

where the labels a and b identify any of the vector spaces open to the field under study. Notational similarities of this kind conceal wide differences, and the differences are beginning to attract attention, most notably in the wide notice given lately to so-called 'hidden coherences'. Our discussion here illuminates the origin of hidden character. Perhaps not all of the consequences have been revealed, but it is already clear that 'hiding' is mainly a matter of the context in which detection is undertaken. It is best not associated with issues still to be examined in detail, such as the presence or absence of mixedness of the state of an optical field, but rather more simply with the availability of the degree of freedom required. For example, there is no way to examine the *st* degree of polarization if the *s* degree of freedom is not actually operationally free, say because of a projection. These and other questions of recent interest are defined by methods of detection. Our last trivial calculation showed that contexts associated with slight differences in detection can lead even to the presence or absence of decoherence, without engaging any loss process. By design, the coherences examined here have features common to both classical and quantum optics and are open to investigations in both domains.

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